Co-ordinate Geometry

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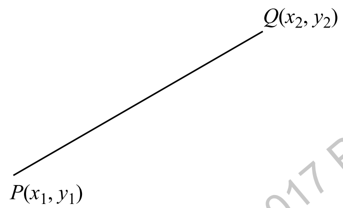
INTRODUCTION

Geometry begins with a point and straight line. Uptil now, we have studied geometry without any use of algebra. In 1637, Descartes used algebra in the study of geometrical relationships. Thus, a new type of geometry was introduced which was given the name analytical geometry or co-ordinate geometry. Thus, co-ordinate geometry is that branch of mathematics in which geometry is studied algebraically, i.e., geometrical figures are studied with the help of equations.

SOME BASIC FORMULAE

1. Distance Formula Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by



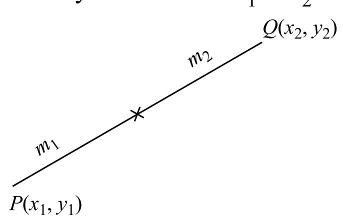
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Illustration 1: Find the distance between the pair of points $A(2, 5)$ and $B(-3, 7)$.

Solution: $AB = \sqrt{(-3-2)^2 + (7-5)^2} = \sqrt{25+4} = \sqrt{29}$.

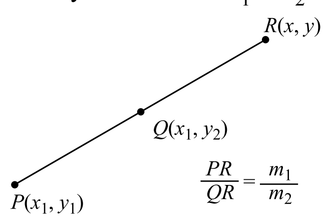
2. Section Formulae

(a) *Formula for internal division* The coordinates of the point $R(x, y)$ which divides the join of two given points $P(x_1, y_1)$ and Q (x_2, y_2) internally in the ratio $m_1 : m_2$ are given by



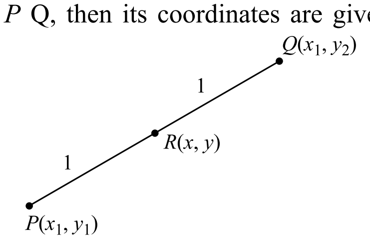
$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

(b) *Formula for external division* The coordinates of the point $R(x, y)$ which divides the join of two given points $P(x_1, y_1)$ and Q (x_2, y_2) externally in the ratio $m_1 : m_2$ are given by



$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right).$$

(c) *Mid-point formula* If R is the mid point of P Q, then its coordinates are given by



$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Illustration 2: Find the coordinates of the point which divides:

(i) the join of (2, 3) and (5, –3) internally in the ratio 1 : 2

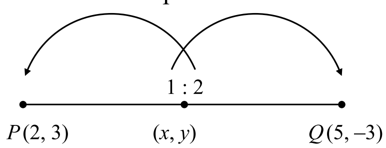
(ii) the join of (2, 1) and (3, 5) externally in the ratio 2 : 3

Solution: (i) Let, (x, y) be the coordinates of the point of division. Then,

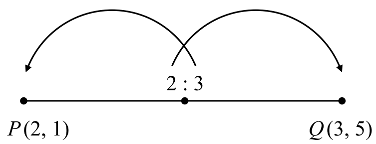
$$x = \frac{1(5)+2(2)}{1+2} = \frac{5+4}{3} = 3$$

$$y = \frac{1(-3)+2(3)}{1+2} = \frac{-3+6}{3} = 1.$$

$\therefore$ Coordinates of the point of division are (3, 1).



(ii) Let, (x, y) be the coordinates of the point of division.



Then, $$x = \frac{2(3)-3(2)}{2-3} = \frac{6-6}{-1} = 0$$

$$y = \frac{2(5)-3(1)}{2-3} = \frac{10-3}{-1} = -7$$

$\therefore$ Coordinates of the point of division are (10, –7).

Illustration 3: Find the coordinates of the mid point of the join of points $P(2, -1)$ and Q(–3, 4).

Solution: The coordinates of the mid-point are

$$x = \frac{2-3}{2} = -\frac{1}{2}$$

$$y = \frac{-1+4}{2} = \frac{3}{2}.$$

$\therefore$ Coordinates of the mid point are $\left(-\frac{1}{2}, \frac{3}{2}\right)$.

Note:

If the point R is given and we are required to find the ratio in which R divides the line segment PQ, it is convenient to take the ratio k:1.

Then, the coordinates of R are

$$\left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{x+1}\right).$$

Illustration 4: In what ratio does the point (6, – 6) divide the join of (1, 4) and (9, –12)?

Solution: Let, the point R (6, – 6) divides the join of P(1, 4) and Q (9, –12) in the ration k:1.

By section formula, the coordinates of R are

$$\left(\frac{k(9)+1(1)}{k+1}, \frac{k(-12)+1(4)}{k+1}\right), \text{ i.e., } \left(\frac{9k+1}{k+1}, \frac{-12k+4}{k+1}\right).$$

But the coordinates of R are given to be (6, –6).

$$\therefore \quad \frac{9k+1}{k+1} = 6 \text{ and } \frac{-12k+4}{k+1} = -6$$

$\Rightarrow$ $9k + 1 = 6k + 6$ and $-12k + 4 = -6k - 6$

$\Rightarrow$ $3k = 5$ and $-6k = -10$

In either case, $k = \frac{5}{3}$(+ve)

$\therefore$ R divides PQ internally in the ratio $\frac{5}{3}:1$

i.e., 5:3.

3. Centroid of a Triangle

The point of concurrence of the medians of a triangle is called the centroid of triangle. It divides the median in the ratio 2:1.

The coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

Illustration 5: Find the centroid of the triangle whose angular points are (3, –5), (–7, 4) and (10, –2), respectively.

Solution: The coordinates of centroid are

$$\left(\frac{3-7+10}{3}, \frac{-5+4-2}{3}\right) = (2, -1).$$

4. Incentre of a Triangle

Incentre of a triangle is the point of concurrence of the internal bisectors of the angles of a triangle.

The coordinates of the incentre of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right).$$

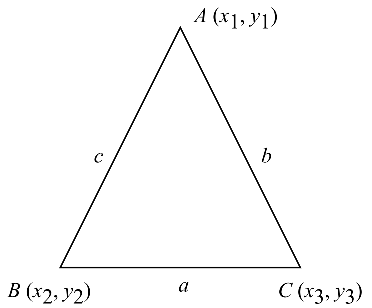


Illustration 6: Find the coordinates of incentre of a triangle having vertices $A(0, 0)$, $B(20, 15)$ and $C(-36, 15)$.

Solution: We have,

$$a = BC = \sqrt{(20+36)\ (15+15)^2} = 56$$

$$b = AC = \sqrt{(36)^2 + (15)^2} = 39$$

$$c = AB = \sqrt{(20) + (15)^2} = 25.$$

$\therefore$ Coordinates of incentre are

$$x = \frac{ax_1 + bx_2 + cx_3}{a+b+c} = \frac{56 \cdot 0 + 39 \cdot 20 + 25 \cdot (-36)}{56+39+25} = -1.$$

$$y = \frac{ay_1 + by_2 + cy_3}{a+b+c} = \frac{56 \cdot 0 + 39 \cdot 15 + 25 \cdot 15}{56+39+25} = 8$$

Thus, $I \equiv (-1, 8)$.

5. **Area of a Triangle** The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

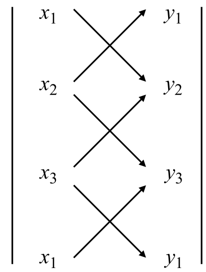
$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Condition of Collinearity of Three Points

The points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ will be collinear (i.e., will lie on a straight line) if the area of the triangle, assumed to be formed by joining them is zero.

Shortcut Method for Finding the Area

1. Write the coordinates of the vertices taken in order in two columns. At the end, repeat the coordinates of the first vertex.



2. Mark the arrow-heads as indicated. Each arrow-head shows the product.
3. The sign of the product remains the same for downward arrows while it changes for an upward arrow.
4. Divide the result by 2.
5. Thus, $\Delta = \frac{1}{2}[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3)]$.

Illustration 7: Find the area of a triangle whose vertices are (4, 4) (3, –2) and (–3, 16).

Solution: Required area

$$= \frac{1}{2}|-8 - 12 + 48 - 6 - 12 - 64|$$

$$= \frac{1}{2}|-54| = \frac{1}{2}(54) = 27 \text{ sq units.}$$

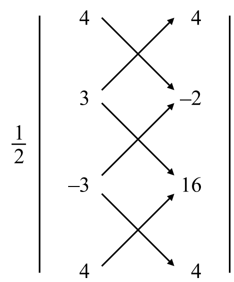
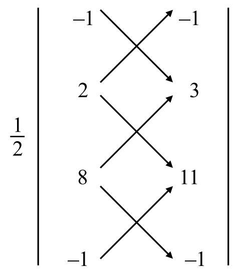


Illustration 8: Show that the three points (–1, –1), (2, 3) and (8, 11) lie on a line.

Solution: The area of the triangle whose vertices are (–1, – 1), (2, 3) and (8, 11) is

$$\Delta = \frac{1}{2}|-3 + 2 + 22 - 24 - 8 + 11| = \frac{1}{2}|0| = 0$$

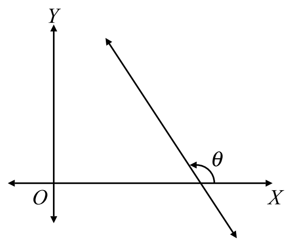
Since the area of the triangle is zero, the given points are collinear.



Slope or Gradient of a Line

The tangent of the angle which a line makes with the positive direction of x-axis is called the slope or the gradient of the line. It is generally denoted by m. If a line makes an angle θ with x-axis, then its slope

$= \tan \theta$, i.e., $m = \tan \theta$.



Note:

1. If a line is parallel to x-axis, $m = \tan 0 = 0$.
2. If a line is parallel to y-axis, $m = \tan 90° = \infty$.

Illustration 9: Find the slope of a line whose inclination with x-axis is 30°.

Solution: Slope, $m = \tan 30° = \frac{1}{\sqrt{3}}$.

Slope of a Line Joining Two Given Points

The slope of the line joining two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Difference of ordinates}}{\text{Difference of abscissae}}$$

Illustration 10: Find the slope of the line passing through the points (2, 3) and (4, 9).

Solution: Slope of the line $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{9-3}{4-2} = 3.$$

Parallel and Perpendicular Lines

(a) Two lines are parallel if and only if their slopes m_1, m_2 are equal, i.e., if $m_1 = m_2$.

(b) Two lines are perpendicular if and only if their slopes m_1, m_2 satisfy the condition $m_1\, m_2 = -1$.

Illustration 11: Show that the line joining (2, –3) and (–5, 1) is:

(*a*) parallel to the line joining (7, –1) and (0, 3),

(*b*) perpendicular to the line joining (4, 5) and (0, –2).

Solution: Let, l_1 be the line joining the points (2, –3) and (–5, 1).

$\therefore$ Slope of $l_1 = \frac{1-(-3)}{-5-2} = -\frac{4}{7}$.

(*a*) Let l_2 be the line joining the points (7, –1) and (0, 3).

$\therefore$ Slope of $l_2 = \frac{3-(-1)}{0-7} = -\frac{4}{7}$.

$\because$ Slope of l_1 = slope of l_2 (each = –4/7).

$\therefore$ Lines l_1 and l_2 are parallel.

(*b*) Let l_3 be the line joining the points (4, 5) and (0, –2).

$\therefore$ Slope of $l_3 = \frac{-2-5}{0-4} = \frac{-7}{-4} = \frac{7}{4}$.

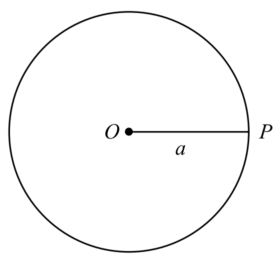
$\because$ Slope of $l_1 \times$ slope of $l_3 = -\frac{4}{7} \times \frac{7}{4} = -1$,

$\therefore$ the lines l_1 and l_3 are perpendicular.

Locus

When a point moves so that it always satisfies a given condition or conditions, the path traced out by it is called its locus under these conditions.

Illustration 12: Let, O be a given point in the plane of the paper and let a point P move on the paper so that its distance from O is constant and is equal to a. All the positions of the moving point must lie on a circle whose centre is O and radius is a. This circle is, therefore, the locus of P when it moves under the condition that its distance from O is equal to a constant a.



Shortcut Method to Find the Locus

1. Take a point on the locus and suppose that its coordinates are (x, y).
2. Apply the given conditions(s) to (x, y) and simplify the algebraic equation so formed.
3. The simplified equation is the required equation of the locus.

Illustration 13: A point moves so that its distance from (3, 0) is twice its distance from (–3, 0). Find the equation of its locus.

Solution: Let $P(x, y)$ be any point on the locus. And, A (3, 0) and B (–3, 0) be the given points.

By the given condition, $PA = 2\ PB$.

$$\Rightarrow \sqrt{(x-3)^2+(y-0)^2} = 2\sqrt{(x+3)^2+(y-0)^2}$$

Squaring both sides,

$x^2 + y^2 - 6x + 9 = 4\ (x^2 + y^2 + 6x + 9)$

or $\quad 3x^2 + 3y^2 + 30x + 27 = 0$

or $\quad x^2 + y^2 + 10x + 9 = 0$,

which is the required equation of the locus.

Exercise-I

1. The distance between the points (7, 9) and (3, −7) is:

(a) $4\sqrt{15}$ (b) $4\sqrt{17}$
(c) $17\sqrt{4}$ (d) $17\sqrt{5}$

2. The distance between $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$ is:

(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) 1 (d) 0

3. The distance between the points (4, p) and (1, 0) is 5, then p =

(a) ± 4 (b) 4
(c) −4 (d) 0

4. The distance between the points $(a\sin 60°, 0)$ and $(0, a\sin 30°)$ is:

(a) $a/\sqrt{2}$ (b) $a\sqrt{2}$
(c) $a/\sqrt{3}$ (d) None of these

5. The distance between the two points is 5. One of them is (3, 2) and the ordinate of the second is −1, then its x-coordinate is:

(a) 7, −1 (b) −7, 1
(c) −7, −1 (d) 7, 1

6. A line is of length 10 and one end is (2, −3). If the abscissa of the other end is 10 then its ordinate is

(a) 3 or 9 (b) −3 or −9
(c) 3 or −9 (d) −3 or 9

7. The nearest point from origin is:

(a) (2, −3) (b) (5, 0)
(c) (2, −1) (d) (1, 3)

8. The vertices of a triangle are A(2, 2), B(−4, −4), C (5, −8). Then, the length of the median through C is

(a) $\sqrt{65}$ (b) $\sqrt{117}$
(c) $\sqrt{85}$ (d) $\sqrt{113}$

9. P(3, 4), Q(7, 7) are collinear with the point R where PR = 10. Then, R =

(a) (5, 2) (b) (−5, 2)
(c) (−5, −2) (d) (5, −2)

10. The third vertex of an equilateral triangle whose two vertices are (2, 4), (2, 6) is:

(a) $(\sqrt{3}, 5)$ (b) $(2\sqrt{3}, 5)$
(c) $(2+\sqrt{3}, 5)$ (d) (2, 5)

11. Three points (0, 0), $(3, \sqrt{3})$, $(3, \lambda)$ form an equilateral triangle. Then, λ =

(a) 2 (b) −3
(c) −4 (d) None of these

12. The perimeter of a triangle formed by (0, 0), (1, 0), (0, 1) is:

(a) $1 \pm \sqrt{2}$ (b) $\sqrt{2}+1$
(c) 3 (d) $2+\sqrt{2}$

13. ABC is an isosceles triangle with B ≡ (1, 3) and C ≡ (−2, 7) then A =

(a) (5/6, 6) (b) (6, 5/6)
(c) (7, 1/8) (d) None of these

14. The area of the triangle formed by (a, a), $(a + 1, a + 1)$, $(a + 2, a)$ is:

(a) a^3 (b) $2a$
(c) 1 (d) $\sqrt{2}$

15. The ratio in which (−3, 4) divides the line joining (1, 2) and (7, −1) is:

(a) 2 : −5 (b) 5 : 2
(c) 1 : −5 (d) 1 : 5

16. Mid-points of the sides AB and AC of ΔABC are (3, 5) and (−3, −3) respectively, then the length of BC =

(a) 10 (b) 15
(c) 20 (d) 30

17. The point $(t, 2t)$, $(-2, 6)$ and $(3, 1)$ are collinear then $t =$

(a) 3/4 (b) 4/3
(c) 3 (d) 4

18. The base vertices of a right angled isosceles triangle are (2, 4) and (4, 2) then its third vertex is

(a) (1, 1) or (2, 2)
(b) (2, 2) or (4, 4)
(c) (1, 10) or (3, 3)
(d) (2, 2) or (3, 3)

19. The points $(1, -1)$, $(2, -1)$, $(4, -3)$ are the mid points of the sides of a triangle. Then its centroid is

(a) $(7, -5)$ (b) $(1/3, -1)$
(c) $(-7, 5)$ (d) $(7/3, -5/3)$

20. The points $(k, 2 - 2k)$, $(1 - k, 2k)$ and $(-4 - k, 6 - 2k)$ are collinear then $k =$

(a) -1 or $\frac{1}{2}$
(b) $-\frac{1}{2}$ or 1
(c) -1 or 1
(d) None of these

21. The point $(k, 3)$ is the centroid of the triangle formed by $(2, 4)$, $(3, k)$ and $(4, 2)$ then $k =$

(a) 2 (b) 3
(c) 4 (d) 5

22. The centroid of a triangle formed by $(7, p)$, $(q, -6)$, $(9, 10)$ is $(6, 3)$, then $(p, q) =$

(a) (4, 5) (b) (5, 4)
(c) $(-5, -2)$ (d) (5, 2)

23. The points (2, 1), (8, 5) and $(x, 7)$ lie on a straight line. Then, the value of x is:

(a) 10 (b) 11
(c) $11\frac{2}{3}$ (d) 12

24. The points (2, 3) and (4, 1) are collinear with the point:

(a) (7, 2) (b) $(7, -2)$
(c) $(-7, 2)$ (d) $(-7, -2)$

25. The point (22, 23) divider the join of P(7, 5) and Q externally in the ratio 3:5, then Q =

(a) (3, 7) (b) $(-3, 7)$
(c) $(3, -7)$ (d) $(-3, -7)$

26. The incentre of the triangle formed by (0, 0), (5, 0) and (0, 12) is:

(a) (3, 3) (b) (2, 2)
(c) (7, 7) (d) (9, 9)

27. The locus of the point equidistant from $(-1, 2)$ and (3, 0) is:

(a) $2x - y - 1 = 0$
(b) $2x + y + 1 = 0$
(c) $x + y + 1 = 0$
(d) $x + y - 2 = 0$

28. A point moves so that its distance from y-axis is half of its distance from the origin. The locus of point is

(a) $2x^2 - y^2 = 0$
(b) $x^2 - 3y^2 = 0$
(c) $3x^2 - y^2 = 0$
(d) $x^2 - 2y^2 = 0$

29. The locus of the point, the sum of whose distances from the coordinate axes is 9 is:

(a) $x^2 - y^2 = 9$ (b) $x^2 - y^2 = -9$
(c) $y^2 - x^2 = 9$ (d) None of these

30. If A(4, 0) and B = $(-4, 0)$, then the locus of P such that PA − PB = 4 is:

(a) $3x^2 + y^2 = 12$ (b) $3x^2 - y^2 = 12$
(c) $3x^2 - y^2 = 8$ (d) None of these

31. The points $(k, 2k)$, $(3k, 3k)$ and $(3, 1)$ are collinear then $k =$

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

32. The centroid of the triangle formed by (1, 2), $(-2, 2)$ and (1, 5) is:

(a) (1, 2) (b) (0, 3)
(c) $(-2, 2)$ (d) (5, 1)

33. The ratio in which (4, 5) divides the join of (2, 3), (7, 8) is:

(a) -2:3 (b) -3:2
(c) 3:2 (d) 2:3

34. The line segment joining $(-3, -4)$ and $(1, -2)$ is divided by y-axis in the ratio.

(a) 1:3 (b) 2:3
(c) 3:1 (d) 3:2

Exercise-2
(Based on Memory)

1. The curve described parametrically by
$x = t^2 + t + 1$ and $y = t^2 - t + 1$ represents
(a) A pair of straight lines
(b) An ellipse
(c) A parabola
(d) A hyperbola

2. Area of the triangle formed by the graph of the straight lines $x - y = 0, x + y = 2$ and the x-axis is:
(a) 1 square units (b) 2 square units
(c) 4 square units (d) None of these
[SSC, 2014]

3. The area (in sq. units) of the triangle formed in the first quadrant by the line $3x + 4y = 12$ is:
(a) 8 (b) 12
(c) 6 (d) 4
[SSC Assistant Grade III, 2013]

4. The area of the triangle, formed by the graph of $ax + by = c$ (where a, b are two positive real numbers) and the coordinate axes, is:
(a) $\frac{c^2}{ab}$ square unit (b) $\frac{a^2}{2bc}$ square unit
(c) $\frac{c^2}{2ab}$ square unit (d) $\frac{a^2}{bc}$ square unit
[SSC Assistant Grade III, 2012]

5. The graph of the linear equation $3x + 4y = 24$ is a straight line intersecting x-axis and y-axis at the points A and B respectively. P(2, 0) and Q$\left(0, \frac{3}{2}\right)$ are two points on the sides OA and OB respectively of ΔOAB, where O is the origin of the co-ordinate system. Given that AB = 10 cm, then PQ = ?
(a) 20 cm
(b) 2.5 cm
(c) 40 cm
(d) 5 cm
[SSC, 2012]

6. The area of the triangle formed by the straight line $3x + 2y = 6$ and the co-ordinate axes is:
(a) 3 square units
(b) 6 square units
(c) 4 square units
(d) 8 square units
[SSC, 2012]

7. The length of the intercept of the graph of the equation $9x - 12y = 108$ between the two axes is:
(a) 15 units
(b) 9 units
(c) 12 units
(d) 18 units
[SSC, 2012]

Answer Keys

Exercise-1

1. (b)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (c)	**11.** (d)	**12.** (d)	**13.** (a)
14. (c)	**15.** (a)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (d)	**20.** (a)	**21.** (b)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (d)	**26.** (b)
27. (a)	**28.** (c)	**29.** (d)	**30.** (b)	**31.** (b)	**32.** (b)	**33.** (d)	**34.** (c)					

Exercise-2

1. (a)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (a)

EXPLANATORY ANSWERS

EXERCISE-I

1. (b) Distance between (7, 9) and (3, −7) is

$$\sqrt{(7-3)^2+(9+7)^2} = \sqrt{16+256} = 4\sqrt{17}.$$

2. (b) $\sqrt{(\cos\theta-\sin\theta)^2+(\sin\theta+\cos\theta)^2}$

$= \sqrt{1+1} = \sqrt{2}.$

3. (a) $9 + p^2 = 25 \Rightarrow p^2 = 16 \Rightarrow p = \pm 4.$

4. (d) Distance between (a sin 60°, 0) and (O, a sin 30°) is $\sqrt{(a\sin 60°)^2+(a\sin 30°)^2}$

$$= \sqrt{a^2\left(\frac{3}{4}+\frac{1}{4}\right)} = a.$$

5. (a) $\sqrt{(3-x)^2+(2+1)^2} = 5$

$\Rightarrow (3-x)^2 + 9 = 25 \Rightarrow 9 + x^2 - 6x + 9 - 25 = 0$

$\Rightarrow x^2 - 6x - 7 = 0 \Rightarrow x = 7, -1.$

6. (c) $\sqrt{(10-2)^2+(y+3)^2} = 10$

$\Rightarrow 64 + y^2 + 9 + 6y - 100 = 0$

$\Rightarrow y^2 + 6y - 27 = 0 \Rightarrow y = -9, 3.$

7. (c) $\sqrt{(2-0)^2+(-1-0)^2} = \sqrt{5}$

$\Rightarrow$ (2, −1) is the nearest point.

8. (c) Mid-point of $AB = \left(\frac{2-4}{2}, \frac{2-4}{2}\right)$

$= (-1, -1) = D$

Now, $CD = \sqrt{(5+1)^2+(-8+1)^2}$

$= \sqrt{36+49} = \sqrt{85}$

9. (c)

10. (c) Third vertex

$$= \left\{\frac{(2+2)\pm\sqrt{3}(6-4)}{2}, \frac{6+4\pm\sqrt{3}(2-2)}{2}\right\}$$

$= (2\pm\sqrt{3}, 5).$

11. (d) Third vertex

$$= \left\{\frac{(3+0)\pm\sqrt{3}(\sqrt{3}-0)}{2}, \frac{(\sqrt{3}+0)\mp\sqrt{3}(3-0)}{2}\right\}$$

$$= \left\{\frac{3\pm 3}{2}, \frac{\sqrt{3}\mp 3\sqrt{3}}{2}\right\} = (3, -\sqrt{3}) \text{ or, } (0, 2\sqrt{3}).$$

12. (d) Perimeter $= 1 + 1 + \sqrt{2} = 2 + \sqrt{2}.$

13. (a) $AB^2 = \left(1-\frac{5}{6}\right)^2 + (3-6)^2 = \frac{1}{36} + 9 = \frac{325}{36}.$

14. (c) Area of $\Delta = \frac{1}{2}\begin{vmatrix} a & a & 1 \\ a+1 & a+1 & 1 \\ a+2 & a & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} a & a & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$= \frac{1}{2}|(-2)| = 1.$$

15. (a) Let, the ratio be $l{:}m$, then,

$\frac{7l+m}{l+m} = -3 \Rightarrow \frac{l}{m} = -\frac{2}{5}.$

16. (c) Let, $D = (3, 5)$, $E = (-3, -3)$, then

$DE = \sqrt{(3+3)^2+(5+3)^2} = \sqrt{36+64}$

$= \sqrt{100} = 10 \Rightarrow BC = 2\,DE = 2\times 10 = 20.$

17. (b) $\frac{2t-6}{t+2} = \frac{6-1}{-2-3} \Rightarrow 15t = 20 \Rightarrow t = \frac{4}{3}.$

18. (b) $A = (2, 4)$, $B = (4, 2)$, then C

$$= \left\{\frac{(2+4)\pm(4-2)}{2}, \frac{(4+2)\mp(2-4)}{2}\right\}$$

$$= \left(\frac{6\pm 2}{2}, \frac{6\pm 2}{2}\right) = (2, 2) \text{ or, } (4, 4).$$

19. (d) Centroid $= \left(\frac{1+2+4}{3}, \frac{-1-1-3}{3}\right) = \left(\frac{7}{3}, \frac{-5}{3}\right).$

20. (a) $\begin{vmatrix} k & 2-2k & 1 \\ 1-k & 2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} k & 2-2k & 1 \\ 1-2k & 4k-2 & 0 \\ -4-2k & 4 & 0 \end{vmatrix} = 0$$

$\Rightarrow (4-8k) - (4k-2)(-4-2k) = 0$

$\Rightarrow (2k-1)(k+1) = 0 \Rightarrow k = 1/2, -1.$

23. (b) Equating the slopes, $\frac{5-1}{8-2} = \frac{7-5}{x-8}$

$\Rightarrow x - 8 = 3 \Rightarrow x = 11.$

25. (d) $\left(\frac{35-3x}{5-3}, \frac{25-3y}{5-3}\right) = (22, 23) \Rightarrow Q = (-3, -7).$

27. (a) $(x + 1)^2 + (y - 2)^2 = (x - 3)^2 + y^2$

$\Rightarrow\ x^2 + 1 + 2x + y^2 + 4 - 4y = x^2 + 9 - 6x + y^2$

$\Rightarrow\ 8x - 4y - 4 = 0$ or, $2x - y = 1$.

28. (c) $x = \frac{1}{2}\sqrt{x^2 + y^2} \Rightarrow 2x = \sqrt{x^2 + y^2}$

$\Rightarrow 4x^2 = x^2 + y^2$

$\Rightarrow 3x^2 - y^2 = 0.$

29. (d) Sum of the distances from the axis $= |x| + |y| = 9$.

30. (b) $PA = PB + 4 \Rightarrow PA^2 = (PB + 4)^2$

$\Rightarrow PA^2 - PB^2 = 8PB + 16$

$\Rightarrow [(x - 4)^2 + y^2] - [(x + 4)^2 + y^2] = 8PB + 16$

$\Rightarrow -16x = 8PB + 16 \Rightarrow PB^2 = 4(x + 4)^2$

$\Rightarrow [(x + 4)^2 + y^2] = 4x^2 + 8x + 4$

$\Rightarrow 3x^2 - y^2 = 12.$

Exercise-2
(Based on Memory)

1. (a) Eliminating t from the given equations, we get $x^2 + y^2 - 2xy - 2x - 2y + 4 = 0$, which is an equation of the pair of straight lines.

2. (a) On putting $x = 0$ in

$x + y = 2$,

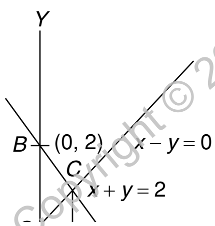
$0 + y = 2 \Rightarrow y = 2$

$\therefore$ Point of intersection on y-axis = (0, 2)

Again, putting $y = 0$ in $x + y = 2$, $x = 2$

$\therefore$ Point of intersection on x-axis = (2, 0)

$x - y = 0$ will pass through origin and be equally inclined to axes.



On putting $x = y$ in $x + y = 2$, we have,

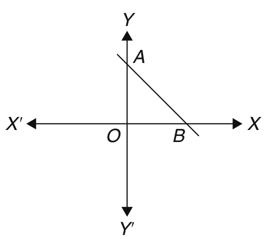
$2y = 2 \Rightarrow y = 1$

$\therefore CD = 1$ and $OA = 2$

$$\text{Area of } \Delta OAC = \frac{1}{2} \times OA \times CD = \frac{1}{2} \times 2 \times 1$$

$$= 1 \text{ square unit}$$

3. (c)



Putting $y = 0$ in the equation $3x + 4y = 12$, we have

$3x + 0 = 12 \Rightarrow x = 4$

$\therefore$ Coordinates of point $B = (4, 0)$

Again, putting $x = 0$ in the equation $3x + 4y = 12$, we have,

$0 + 4y = 12 \Rightarrow y = 3$

$\therefore$ Coordinates of point $A = (0, 3)$

$\therefore OB = 4$ and $OA = 3$

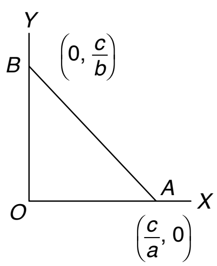
$$\therefore \text{ Area } (\Delta OAB) = \frac{I}{2} \times OB \times OA = \frac{1}{2} \times 4 \times 3$$

$$= 6 \text{ sq units}$$

4. (c) $ax + by = c$ (given)

When $x = 0$, $y = \frac{c}{b}$

When $y = 0$, $x = \frac{c}{a}$



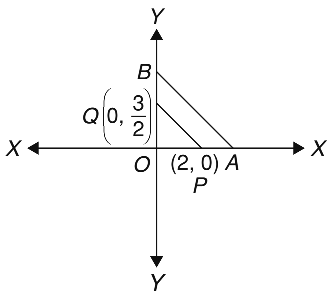
$\therefore OA = \frac{c}{b}$

$OB = \frac{c}{b}$

$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times \frac{c}{a} \times \frac{c}{b} = \frac{c^2}{2ab}$ sq units

5. **(b)** $OP = 2$

$OQ = \frac{3}{2}$



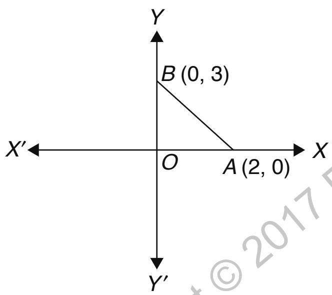
$\therefore \quad PQ = \sqrt{OP^2 + OQ^2}$

$= \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{16+9}{4}} = \sqrt{\frac{25}{4}}$

$= \frac{5}{2} = 2.5 \text{ cm}$

6. **(a)** Putting $y = 0$ in the equation $3x + 2y = 6$,

$3x + 0 = 6 \quad \Rightarrow \quad x = 2$



$\therefore$ Point of intersection on x-axis = (2, 0) putting $x = 0$, in the equation $3x + 2y = 6$, $0 + 2y = 6 \Rightarrow \quad y = 3$

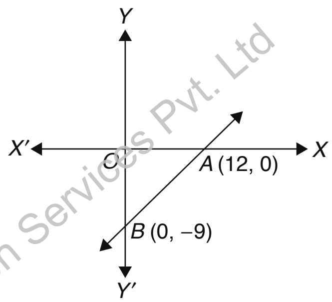
$\therefore$ Point of intersection on y-axis = (0, 3)

$\therefore \quad OA = 2,\ OB = 3$

$\Rightarrow \quad \Delta OAB = \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 2 \times 3 = 3$ sq units

7. **(a)**



Putting $x = 0$ in $9x - 12y = 108$,

$0 - 12y = 108 \quad \Rightarrow \quad y = -9$

Putting $y = 0$ in $9x - 12y = 108$

$9x - 0 = 108 \quad \Rightarrow \quad x = 12$

$\therefore \quad OA = 12,\ OB = 9$

$\therefore \quad AB = \sqrt{OA^2 + OB^2}$

$= \sqrt{12^2 + 9^2} = \sqrt{144 + 81} = \sqrt{225} = 15$ units