# H.C.F. and L.C.M. of Polynomials

## INTRODUCTION

We have already learnt in Chapter 2 how to find the greatest common divisor (G.C.D.) or highest common factor (H.C.F.) and least common multiple (L.C.M.) of two integers. In this chapter, we will study how to find the G.C.D. and L.C.M. of polynomials which have integral coefficients.

#### **Divisor**

A polynomial d(x) is said to be a divisor of polynomial p(x) if d(x) is a factor of p(x), i.e., p(x) can be written as p(x) = d(x) q(x), where q(x) is a polynomial.

For example, (x - 2) is a divisor of the polynomial  $(x - 2)^3(x + 3)$ .

#### **Common Divisor**

A polynomial d(x) is said to be a common divisor of the polynomials p(x) and q(x), if d(x) is a factor of each of p(x) and q(x).

For example, (x + 4) is a common divisor of the polynomials  $(x + 4)^3$  (x - 2) (x + 3) and (x + 4)  $(x - 2)^3$  (x + 5).

## G.C.D. (H.C.F) of Two Polynomials

The G.C.D. of two polynomials p(x) and q(x) is the common divisor which has highest degree among all common divisors and which has the highest degree term coefficient as positive.

**Illustration 1:** Find the G.C.D. of (3x - 2)(4x + 3);  $(3x - 2)^2(2x + 5)$ .

**Solution:** Here we find that (3x - 2) is a polynomial which is a common divisor and has highest degree among all common divisors. Further, the coefficient of the highest degree term (3x) is 3 which is positive. Hence, (3x - 2) is the G.C.D. of the given polynomial.

# G.C.D. by Factorization Method

- **Step 1** Resolve the given polynomials p(x) and q(x) in the complete factored form.
- **Step 2** Find the G.C.D of the numerical factors of p(x) and q(x).
- **Step 3** Find the factors of highest degree common to the two polynomials p(x) and q(x).
- **Step 4** The product of all such common factors and the G.C.D. of the numerical factors is the G.C.D. of the two given polynomials p(x) and q(x).

**Illustration 2:** Find the G.C.D. of  $4 + 9x - 9x^2$  and  $9x^2 - 24x + 16$ .

Solution: We have the factorization

$$p(x) = 4 + 9x - 9x^{2} = -(9x^{2} - 9x - 4)$$

$$= -(9x^{2} - 12x + 3x - 4)$$

$$= -(3x (3x - 4) + 1 (3x - 4))$$

$$= -(3x + 1) (3x - 4)$$

$$q(x) = 9x^{2} - 24x + 16 = (3x - 4)^{2}.$$

 $\therefore$  G.C.D. of numerical factors = 1

and the highest degree common divisor = (3x - 4),

: Required G.C.D. = (3x - 4).

**Illustration 3:** Find the G.C.D. of  $8(x^4 + x^3 + x^2)$  and  $20(x^3 - 1)$ .

**Solution:** Here, 
$$p(x) = 8(x^4 + x^3 + x^2)$$
  
=  $2^3 \cdot x^2 \cdot (x^2 + x + 1)$ .

$$q(x) = 20 (x^3 - 1) = 2^2 \cdot 5 \cdot (x - 1) \cdot (x^2 + x + 1).$$

 $\therefore$  G.C.D. of numerical factors =  $2^2$ 

and the highest degree common divisor =  $x^2 + x + 1$ ,

:. Required G.C.D. = 
$$2^2 (x^2 + x + 1)$$
  
=  $4 (x^2 + x + 1)$ .

### 26.2 Chapter 26

# L.C.M. of Two Polynomials

We know that if a and b are two natural numbers, the product of a and b is equal to the product of their G.C.D. and L.C.M., i.e.,

 $a \times b = (G.C.D. \text{ or H.C.F. of } a \text{ and } b) \cdot (L.C.M. \text{ of } a)$ a and b)

or, L.C.M. of a and 
$$b = \frac{a \times b}{G.C.D. \text{ of } a \text{ and } b}$$

Similarly, if p(x) and q(x) are two polynomials, then

L.C.M. of 
$$p(x)$$
 and  $q(x) = \frac{p(x) \times q(x)}{\text{G.C.D. of } p(x) \text{ and } q(x)}$ 

Thus, L.C.M. of two polynomials Product of two polynomials  $= \overline{\text{G.C.D. of the two polynomials}}$ 

#### *Note:*

L.C.M. of two or more given polynomials is a polynomial of smallest degree which is divided by each one of the given polynomials.

## L.C.M. by Factorization Method

- **Step 1** Resolve the given polynomials p(x) and q(x) in the complete factored form.
- The required L.C.M. is the product of each factor of p(x) and q(x) and if a factor is common, we take that factor which has the highest degree in p(x) or q(x).

**Illustration 4:** Find the L.C.M. of the polynomials

$$(x + 2)^2 (x - 1) (x + 4)^2$$

 $(x + 4)^3 (x + 2) (x + 7)$ and.

**Solution:** We have,  $p(x) = (x + 2)^2 (x - 1) (x + 4)^2$  $q(x) = (x + 4)^3 (x + 2) (x + 7)$ 

Take the highest powers of factors common to both p(x) and q(x) and remaining terms for L.C.M.

$$\therefore$$
 L.C.M. =  $(x + 4)^3 (x + 2)^2 (x - 1) (x + 7)$ 

Illustration 5: Find the L.C.M. of the polynomials  $(2x^2 - 3x - 2)$  and  $(x^3 - 4x^2 + 4x)$ .

**Solution:** We have,  $p(x) = 2x^2 - 3x - 2$ 

$$= (x-2)(2x+1)$$

$$q(x) = x^3 - 4x^2 + 4x = x(x^2 - 4x + 4) = x(x - 2)^2.$$

Hence, L.C.M. =  $\frac{p(x) \cdot q(x)}{\text{H.C.F.}} = \frac{x(x-2)^3 \cdot (2x+1)}{(x-2)}$ 

$$= x(x-2)^2 (2x + 1).$$

or, Taking the highest powers of factors common to both p(x) and q(x) and remaining terms for L.C.M., we have

L.C.M. = 
$$x(x-2)^2 (2x + 1)$$
.

## **EXERCISE-I**

- 1. Find the G.C.D. of  $3 + 13x 30x^2$ ;  $25x^2 30x + 9$ .

- 2. Find the L.C.M. of the polynomials.

$$(x + 3)^2 (x - 2) (x + 1)^2$$
;  $(x + 1)^3 (x + 3) (x + 4)$ .

- (a)  $(x + 3) (x + 1)^2 (x + 4)$
- (b)  $(x + 3)^2 (x + 1) (x 2)$
- (c)  $(x + 3)^2 (x + 1)^3 (x 2) (x + 4)$
- (d) None of these.
- **3.** Find the L.C.M. of the polynomials:  $2x^2 - 3x - 2$ ;  $x^3 - 4x^2 + 4x$ .
  - (a)  $x(x-2)^2(2x+1)$
  - (b)  $x(x-2)(2x+1)^2$
  - (c) x(x-2)(2x+1)
  - (d) None of these.

- **4.** Find the G.C.D. of  $8(x^3 x^2 + x)$ ; 28  $(x^3 + 1)$ .
  - (a)  $6(x^2 + x 1)$  (b)  $4(x^2 x + 1)$
  - (c)  $8(x^2 + 2x 1)$  (d) None of these
- **5.** Find the G.C.D. of  $4x^4 + y^4$ ,  $2x^3 xy^2 y^3$  and  $2x^2$  $+ 2xv + v^2$ 
  - (a)  $2x^2 + 2xy + y^2$  (b)  $2x^3 + 4xy + y^2$
  - (c)  $3x^2 + 2xy + y^2$  (d) None of these
- **6.** Find the G.C.D. of  $(x + 4)^2 (x 3)^2$  and (x 1) $(x + 4) (x - 3)^2$ .
  - (a)  $(x + 3) (x + 9)^2$  (b)  $(x + 4) (x 3)^3$
  - (c)  $(x + 4) (x 3)^2$  (d) None of these
- 7. Find the L.C.M. of the polynomials.  $16 - 4x^2$ ;  $x^2 + x - 6$ .
  - (a)  $-4(x^2-4)(x+3)$  (b)  $6(x^2-4)(x+4)$
  - (c)  $8(x^2 6)(x + 3)$  (d) None of these

- **8.** Find the G.C.D. of  $x^2 4$  and  $x^3 5x + 6$ .
  - (a) x 3
- (b) x 2
- (c) x + 4
- (d) None of these
- 9. The H.C.F. (Highest Common Factor) of two Polynomials is (y-7) and their L.C.M. is  $y^3 - 10y^2$ + 11v + 70. If one of the polynomials is  $v^2 - 5v -$ 14, find the other.
  - (a)  $y^2 12y + 35$
- (b)  $y^2 8y + 35$
- (c)  $v^2 14v + 45$
- (d) None of these
- **10.** If (x 4) is the G.C.D. of  $x^2 x 12$  and  $x^2 mx$ -8, find the value of m.
  - (a) 4
- (b) 6
- (c) 2
- (d) None of these
- 11. Find the G.C.D. of the polynomials

$$(x-2)^2$$
  $(x+3)$   $(x-4)$ ;  $(x-2)$   $(x+2)$   $(x-5)$ .

- (a) (x 4)
- (b) (x 6)
- (c) (x-2)
- (d) None of these
- 12. For what value of a, the G.C.D. of  $x^2 2x 24$ and  $x^2 - ax - 6$  is (x - 6)?
  - (a) 7
- (c) 9
- (d) None of these
- 13. The L.C.M. and H.C.F. of two polynomials p(x)and q(x) are  $36x^3(x + a)(x^3 - a^3)$  and  $x^2(x - a)$ , respectively. If  $p(x) = 4x^2(x^2 - a^2)$ , find q(x).
  - (a)  $12x^3(x^3 a^3)$  (b)  $6x^3(x^3 a^3)$
  - (c)  $9x^3(x^3 a^3)$
- (d) None of these
- **14.** If (x a) is the G.C.D. of  $x^2 x 6$  and  $x^2 + 3x 18$ , find the value of a.
  - (a) 3
- (c) 9
- (d) None of these
- 15. The G.C.D and D.C.M. of two polynomials p(x) and q(x + a) and  $12x^2(x + a)(x^2 - a^2)$ , respectively. If  $p(x) = 4x (x + a)^2$ , find q(x).
  - (a)  $3x^2(x^2 a^2)$
- (b)  $5x^2(x^3 a^3)$
- (c)  $4x^2(x^2-a^2)$
- (d) None of these
- **16.** Find the G.C.D. of  $8(x^4 16)$  and  $12(x^3 8)$ .
  - (a) 6(x-2)
- (b) 4(x-2)
- (c) 8(x-2)
- (d) None of these
- 17. Find the L.C.M. of the polynomials  $(x + 3) (-6x^2)$ +5x + 4;  $(2x^2 + 7x + 3)(x + 3)$ .

- (a)  $-(x + 3)^2(3x 4)(2x + 1)$
- (b)  $(x + 3)^2(3x 4)(2x + 1)$
- (c)  $(x + 3)^2(3x + 4)(2x + 1)$
- (d) None of these
- **18.** Find the G.C.D. of the polynomials  $36x^2 49$  and  $6x^2 - 25x + 21$ .
  - (a) 8x 9
- (b) 9x 5
- (c) 6x 7
- (d) None of these
- **19.** Find the L.C.M. of the polynomials:

$$30x^2 + 13x - 3$$
;  $25x^2 - 30x + 9$ .

- (a)  $-(5x-3)^2(5x+3)(6x-1)$
- (b)  $(5x-3)^2(5x+3)(6x-1)$
- (c)  $(5x + 3)^2 (6x 1)$
- (d) None of these
- **20.** Find the G.C.D. of the polynomials  $6x^2 + 11x$  and  $2x^2 + x - 3$ .
  - (a) 4x + 5
- (b) 2x 3
- (c) 2x + 3
- (d) None of these
- **21.** The H.C.F of two expressions p and q is 1. Their D.C.M. is:
  - (a) (p + q)
- (b) (p q)
- (c) p q
- **22.** The H.C.F. of  $(2x^2 4x)$ ,  $(3x^4 12x^2)$  and  $(2x^5 - 2x^4 - 4x^3)$  is:
  - (a) 2x(x + 2)
- (b) 2x(2-x)
- (c) 2x(x-2)
- (d) x(x-2)
- 23. The product of two non-zero expressions is (x + y)(z) + (z) (z) = (z) (z)
  - (a) (x + y)p
- (b) (y + 2)p
- (c) (z + x)p
- (d) (x + y + z)p
- **24.** If (x-1) is the H.C.F. of  $(x^2-1)$  and  $(px^2-q)$ (x + 1) then:
  - (a) p = 2q
- (b) q = 2p
- (c) 3p = 2q
- (d) 2p = 3p
- **25.** The L.C.M. of  $(x^2 y^2)$ ,  $(x^3 y^3)$ ,  $(x^3 x^2y xy^2)$  $+ v^{3}$ ) is:
  - (a)  $(x + y) (x y) (x^2 + y^2 + xy)$
  - (b)  $(x + y) (x y)^2 (x^2 + y^2 + xy)$
  - (c)  $(x + y) (x y)^2 (x^2 + y^2 xy)$
  - (d)  $(x + v)^2 (x v)^2$

#### 26.4 Chapter 26

#### ANSWER KEYS

#### **EXERCISE-I**

- **1.** (b) **2.** (c) **3.** (a) **4.** (b) **5.** (a) **6.** (c) **7.** (a) **8.** (b) **9.** (a) **10.** (c) **11.** (c) **12.** (b) **13.** (c)
- 14. (a) 15. (a) 16. (b) 17. (a) 18. (c) 19. (b) 20. (c) 21. (c) 22. (d) 23. (d) 24. (a) 25. (b)

## **EXPLANATORY ANSWERS**

#### Exercise-I

- 1. (b) Here,
  - $p(x) = 3 + 13x 30x^2 = 3 + 18x 5x 30x^2$
  - = 3(1 + 6x) 5x(1 + 6x)
  - = (3 5x) (1 + 6x)
  - = -(5x 3)(1 + 6x)
  - $q(x) = 25x^2 30x + 9 = (5x 3)^2$
  - :. G.C.D. of numerical factors = 1 and highest degree of common division.
  - = (5x 3) = G.C.D.
- **2.** (c)  $p(x) = (x+3)^2 (x-2) (x+1)^2$ 
  - $q(x) = (x + 1)^3 (x + 3) (x + 4)$
  - $\therefore \text{ L.C.M.} = (x+3)^2 (x+1)^3 (x-2) (x+4).$
- **3.** (a) We have,
  - $p(x) = 2x^2 3x 2 = 2x^2 4x + x 2$
  - = 2x(x-2) + 1(x-2) = (2x+1)(x-2)
  - $q(x) = x^3 4x^2 + 4x = x(x^2 4x + 4) = x(x 2)^2$
  - $\therefore$  L.C.M. =  $x(x-2)^2 (2x+1)$ .
- 4. (b) We have the factorization
  - $p(x) = 8(x^3 x^2 + x) = 2^3 \cdot x \cdot (x^2 x + 1)$
  - $q(x) = 28(x^3 + 1) = 2^2 .7. (x + 1) (x^2 x + 1)$
  - $\therefore$  G.C.D. of numerical factors =  $2^2$  and, the highest degree common divisor =  $x^2 - x + 1$ .

  - Therefore, required G.C.D. =  $2^2(x^2 x + 1)$
  - $=4(x^2-x+1).$
- **5.** (a) 1st Expression =  $(2x^2 + y^2)^2 (2xy)^2$ 
  - $= (2x^2 + v^2 + 2xv) (2x^2 + v^2 2xv)$
  - 2nd Expression =  $(2x^3 2y^3) y^2(x y)$
  - $= 2 (x y) (x^2 + xy + y^2) y^2) (x y)$
  - $= (x y) (2x^2 + 2xy + 2y^2 y^2)$
  - $= (x y) (2x^2 + 2xy + y^2)$
  - Hence, G.C.D. =  $2x^2 + 2xy + y^2$ .
- **6.** (c) Let,  $p(x) = (x + 4)^2 (x 3)^2$  and q(x) = (x 1) $(x + 4) (x - 3)^2$ 
  - The highest degree common divisor is  $(x + 4) (x 3)^2$
  - $\therefore$  The G.C.D. of given polynomial is  $(x + 4) (x 3)^2$ .

- 7. (a) We have,
  - $p(x) = 16 4x^2 = 4(4 x^2)$
  - = 4(2-x) (2+x) = -4(x-2) (x+2)
  - $q(x) = x^2 + x 6 = x^2 3x 2x 6$
  - = x(x + 3) = 2(x + 3) = (x + 3)(x 2).
  - $\therefore$  L.M. = -4(x-2)(x+2)(x+3) $= -4 (x^2 - 4) (x + 3).$
- **8** (3) Let,  $p(x) = x^2 4 = x^2 2^2$ 
  - =(x+2)(x-2)
  - And,  $q(x) = x^2 5x + 6 = x^2 2x 3x + 6$
  - = x(x-2) 3(x-2) = (x-2)(x-3).

The highest degree common divisor is x - 2.

- $\therefore$  The G.C.D. of p(x) and q(x) is x-2.
- **9.** (a) H.C.F. = (y 7)
  - L.C.M. =  $v^3 10v^2 + 11v + 70$
  - $p(x) = y^2 5y 14$
  - q(x) = ?
  - L.C.M. of two polynomials
  - $= \frac{\text{Ist Polynomial} \times \text{IInd Polynomial}}{\text{IInd Polynomial}}$ 
    - H.C.F. of two polynomials

$$\therefore$$
 L.C.M. =  $\frac{p(x) \ q(x)}{\text{H.C.F.}}$ 

- $y^3 10y^2 + 11y + 70 = \frac{(y^2 5y 14) \times q(x)}{(y 7)}$
- $\therefore q(x) = \frac{(y-7)(y^3-10y^2+11y+70)}{(y^2-5y-14)}$
- = (y-7)(y-5)
- $= y^2 12y + 35$
- **10.** (c) H.C.F. = (x 4)
  - $p(x) = x^2 x 12 = (x 4)(x + 3)$
  - $q(x) = x^2 mx 8$
  - As (x 4) is common in p(x) and q(x). Hence, x 4should be a factor of  $x^2 - mx - 8$ .

Thus, putting (x - 4) = 0 in q(x), we get (Remainder theorem)

$$q(x) = x^{2} - mx - 8$$

$$q(4) = 4^{2} - m \times 4 - 8 = 0 \Rightarrow 16 - 4m - 8 = 0$$

$$\Rightarrow m = 2.$$

11. (c) Let, 
$$p(x) = (x-2)^2 (x+3) (x-4)$$
  
and,  $q(x) = (x-2) (x+2) (x-5)$   
the highest degree common divisor of the given  
polynomials is  $x-2$ .

$$\therefore$$
 The G.C.D. is  $x - 2$ .

12. **(b)** Here, 
$$p(x) = x^2 - 2x - 24$$
  
and,  $q(x) = x^2 - ax - 6$   
Since  $(x - 6)$  is the G.C.D. of  $p(x)$  and  $q(x)$ ,  $(x - 6)$  is a factor of  $p(x)$  and  $q(x)$  both  $\Rightarrow p(6) = q(6)$   
 $\Rightarrow 36 - 2 \times 6 - 24 = 36 - a \times 6 - 6$   
 $\Rightarrow a = 5$ .

13. (c) We know that,  

$$p(x) \times q(x) = \text{L.C.M.} \times \text{H.C.F.}$$

$$4x^{2}(x^{2} - a^{2}) \times q(x) = 36 x^{3}(x + a) (x^{3} + a^{3}) x^{2}(x - a)$$

$$\Rightarrow q(x) = \frac{36x^{5}(x^{2} - a^{2})(x^{3} - a^{3})}{4x^{2}(x^{2} - a^{2})} = 9x^{3}(x^{3} - a^{3}).$$

**14.** (a) Let, 
$$p(x) = x^2 - x - 6$$
  
and,  $q(x) = x^2 + 3x - 18$   
Since  $(x - a)$  is the G.C.D. of  $p(x)$  and  $q(x)$ ,  $(x - a)$  is a divisor of  $p(x)$  and  $q(x)$  or  $(x - a)$  is a Factor of  $p(x)$  and  $q(x)$  both.  
 $\Rightarrow p(a) = 0$  and  $q(a) = 0 \Rightarrow p(a) = q(a)$   
 $\Rightarrow a^2 - a - 6 = a^2 + 3a - 18$   
 $\Rightarrow 4a = 12 \Rightarrow a = 3$ .

15. (a) 
$$q(x) = \frac{\text{L.C.M.} \times \text{G.C.F.}}{p(x)}$$
  
=  $\frac{12x^2(x+a)(x^2-a^2)x(x+a)}{4x(x+a)^2}$ 

 $=3x^2(x^2-a^2).$ 

**16. (b)** 
$$p(x) = 8(x^4 - 16)$$
  
=  $4 \times 2(x^2 + 4) (x + 2) (x - 2)$   
 $q(x) = 12(x^3 - 8)$   
=  $4 \times 3(x - 2) (x^2 + 2x + 4)$   
Hence, G.C.D. =  $4(x - 2)$ .

17. (a) 
$$p(x) = (x + 3) (-6x^2 + 5x + 4)$$
  
=  $(x + 3) (-6x^2 + 8x - 3x + 4)$   
=  $-(x + 3) (3x - 4) (2x + 1)$ 

$$q(x) = (2x^2 + 7x + 3) (x + 3)$$

$$= (2x + 1) (x + 3) (x + 3)$$

$$\therefore \text{ L.C.M.} = -(x + 3)^2 (3x - 4) (2x + 1).$$

18. (c) 
$$p(x) = 36x^2 - 49$$
  
=  $(6x)^2 - (7)^2 = (6x + 7)(6x - 7)$   
 $q(x) = 6x^2 - 25x + 21$   
=  $6x^2 - 18x - 7x + 21$   
=  $6x(x - 3) - 7(x - 3)$   
=  $(6x - 7)(x - 3)$ ;  
∴ G.C.D. =  $(6x - 7)$ .

19. **(b)** 
$$30x^2 + 13x - 3 = 30x^2 + 18x - 5x - 3$$
  
  $= 6x(5x + 3) - 1(5x + 3)$   
  $= (5x + 3)(6x - 1)$   
  $q(x) = 25x^2 - 30x + 9$   
  $= 25x^2 - 15x - 15x - 9$   
  $= 5x(5x - 3) - 3(5x - 3) = (5x - 3)^2$   
 L.C.M.  $= (5x - 3)^2(5x + 3)(6x - 1)$ .

20. (c) 
$$p(x) = 6x^2 + 11x + 3$$
  
 $= 6x^2 + 9x + 2x + 3$   
 $= 3x(2x + 3) + 1(2x + 3)$   
 $= (2x + 3)(3x + 1)$   
 $q(x) = 2x^2 + x - 3 = 2x^2 + 3x - 2x - 3$   
 $= (2x + 3)(x - 1)$   
 $\therefore$  G.C.D.  $= (2x + 3)$ .

21. (c) L.C.M=
$$\frac{\text{Product of expressions}}{\text{H.C.F.}}$$

$$=\frac{pq}{1}=pq.$$

22. (d) 
$$2x^2 - 4x = 2x(x - 2)$$
  
 $3x^4 - 12x^2 = 3x^2(x^2 - 4) = 3x^2(x - 2) (x + 2)$   
 $2x^5 - 2x^4 - 4x^3 = 2x^3(x^2 - x - 2)$   
 $= 2x^3(x - 2) (x + 1)$   
 $\therefore$  H.C.F.  $= x(x - 2)$ .

23. (d) L.C.M. = 
$$\frac{\text{Product}}{\text{H.C.F.}} = \frac{(x+y+z)p^3}{p^2}$$
  
=  $(x+y+z)p$ .

24. (a) Since 
$$(x - 1)$$
 is the H.C.F., it will divide each one of the given expressions. So,  $x = -1$  will make each one zero

$$p \times 1^{2} - q(1+1) = 0 \text{ or } p = 2q.$$
**25. (b)**  $x^{2} - y^{2} = (x-y)(x+y),$ 
 $x^{3} - y^{3} = (x-y)(x^{2} + xy + y^{2}),$ 
 $x^{3} - x^{2}y - xy^{2} + y^{3} = x^{2}(x-y) - y^{2}(x-y)$ 
 $= (x-y)(x^{2} - y^{2})$ 
 $= (x-y)^{2}(x+y)$ 
 $\therefore \text{ L.C.M.} = (x-y)^{2}(x+y)(x^{2} + y^{2} + xy).$ 

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