Binary Number System

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INTRODUCTION

A number system is nothing more than a code. For each distinct quantity there is an assigned symbol. The most familiar number system is the decimal system which uses 10 digits, that is, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The main advantage of this system is its simplicity and long use. Most of the ancient societies used this system. Even in our everyday life we use this system and is sometimes being taken as the natural way to count. Since this system uses 10 digits it is called a system to base 10.

A *binary number system* is a code that uses only two basic symbols, that is, 0 and 1. This system is very useful in computers. Since, in this system, only two symbols are there, it can be used in electronic industry using 'on' and 'off' positions of a switch denoted by the two digits 0 and 1.

Decimal Number System

Decimal number system used 10 digits, 0 through 9, that is, the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Binary Number System

Binary means two. The binary number system uses only two digits, i.e., 0 and 1.

Base or Radix

The *base* or *radix* of a number system is equal to the number of digits or symbols used in that number system. For example, decimal system uses 10 digits, so that base of decimal system (that is, decimal numbers) is 10. Binary numbers have base 2.

A subscript attached to a number indicates the base of the number. For example, 100_2 means binary 100. 100_{10} stands for decimal 100.

Weights

In any number to a given base, each digit, depending on its position in the number has a weight in powers of the base.

Illustration 1: In the number $(5342)_x$.

The weight of 2 is x^0

The weight of 4 is x^1

The weight of 3 is x^2

The weight of 5 is x^3.

The sum of all the digits multiplied by their respective weights is equal to the decimal equivalent of that number and gives the total amount represented by that number.

$$(5342)_x = (5x^3 + 3x^2 + 4x + 2x^0)_{10}$$

Illustration 2:

5	7	0	3	4	Number to the base 10,
10^4	10^3	10^2	10^1	10^0	that is, decimal number weights

$\therefore$ $5 \times 10^4 + 7 \times 10^3 + 0 \times 10^2 + 3 \times 10 + 4 \times 10^0$

= Value represented or decimal equivalent

Illustration 3:

1	1	0	0	1	Number to the base 2
2^4	2^3	2^2	2^1	2^0	that is, binary number weights

$\therefore$ $1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 16 + 8 + 1 = 25$

= Decimal equivalent or value represented by 11001_2.

Decimal to Binary Conversion

Step 1 Divide the number by 2.

Step 2 Divide Quotient of Step 1 by 2

Continue the process till we get quotient = 0 and remainder as 1.

Then, the remainders from down upwards written from left to right give the binary number.

Illustration 4: Convert decimal 23 to binary.

Solution:

		Remainders
2	23	
2	11	1
2	5	1
2	2	1
	1	0
	0	1

Reading the remainders upwards and writing from left to right we get the binary equivalent of decimal 23 as 10111.

That is, Binary 10111 is equivalent to decimal 23 or we can write $10111_2 = 23_{10}$.

Binary to Decimal Conversion

Following steps are involved to convert a binary number to its decimal equivalent

Step 1 Write the binary number.

Step 2 Write the weights $2^0, 2^1, 2^2, 2^3, \ldots$ under the binary digits starting from extreme right.

Step 3 Cross out any weight under a zero, that is, weights under zeros in the binary number should be deleted.

Step 4 Add the remaining weights.

Illustration 5: Convert binary 1101 to its decimal equivalent.

Solution: 1 1 0 1 Binary number

2^3 2^2 2^1 2^0 weights

The weight 2^1 is under 0 so it can be deleted. Sum of the remaining weights

$= 2^3 + 2^2 + 2^0 = 8 + 4 = 1 = 13.$

$\therefore$ Decimal equivalent of binary 1101 = 13, that is, $1101_2 = 13_{10}$.

Binary Addition

In binary number system there are only 2 digits, that is, 0 and 1. In decimal system we carry 1 for every 10 whereas in binary system we carry 1 for every 2. Hence, rules of addition are as under:

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 10$

Illustration 6: Add 1010 to 10100

Solution:

$$\begin{array}{r} 10100 \\ +1010 \\ \hline 11110 \end{array}$$

Binary Subtraction

1. 0 – 0 = 0
2. 1 – 0 = 1
3. 1 – 1 = 0
4. 10 – 1 = 1
5. 0 – 1 = –1

[Complement of a binary number is the exact reverse of the given number]

Complement of 0 = 1

Complement of 1 = 0

For subtraction of binary number the following method known as one's complement method is used.

Subtraction of a Lower Number from a Higher Number

To determine which binary number is lower and which is higher, it is advisable to find their decimal equivalents.

Step 1 Make the number of digits equal in both the numbers.

Step 2 Take the complement of the second number, that is, take the complement of the number to be subtracted.

Step 3 Add the complement obtained in Step II to the first number. The carry over obtained from this addition indicates that the answer shall be positive.

Step 4 This carry over is taken out and added to the first digit on the right, that is, extreme right digit.

Step 5: The digits so obtained is the final answer.

Illustration 7: Subtract 11 from 101.

Solution: Now, $101_2 = 4 + 1 = 5_{10}$, $11_2 = 2 + 1 = 3_{10}$. Clearly, 11 is smaller than 101. Making the number of digits equal, we write 11 as 011.

Complement of 011 = 100.

Adding 100 to 101, we get

$$\begin{array}{rr} & 101 \\ & 100 \\ \hline (1) & 001 \end{array} \qquad \text{[Carry over is 1]}$$

Taking out the carry over and adding to extreme right digit, we get

$$\begin{array}{r} 001 \\ 1 \\ \hline 010 \end{array}$$

$\therefore$ The answer is 010 or 10.

Subtraction of a Higher Number from a Lower Number.

Step 1 Take the complement of the second number.

Step 2 Add the complement obtained in Step I to the first number. In this case there is no carry over indicating that the answer is negative.

Step 3 Recomplement the digits obtained after adding the complement of the second number to the first number.

Step 4 Put a negative sign before the result obtained in Step 4.

Illustration 8: Subtract 1110 from 1001.

Solution: Now, $1110_2 = 8 + 4 + 2 = 14_{10}$;

$1001 = 8 + 2 = 10_{10}$.

Clearly, $1110_2 > 1001_2$.

Complement of 1110 = 0001.

Adding 0001 to 1001, we get

$$\begin{array}{r} 1001 \\ \underline{0001} \\ 1010 \end{array}$$

[There is no carry over]

Complement of 1010 = 0101.

∴ The answer is –0101 or – 101.

Binary Multiplication

Rules: $1 \times 1 = 1$, $1 \times 0 = 0$.

Illustration 9: Multiply 1111 by 11.

Solution:

$$\begin{array}{r} 1111 \\ \underline{11} \\ 1111 \\ \underline{1111} \\ \underline{101101} \end{array}$$

EXERCISE-I

1. Find the binary equivalent of decimal 117.

(a) 1010101 (b) 1110101
(c) 1111101 (d) None of these

2. Find the binary equivalent of decimal 52.

(a) 110100 (b) 111100
(c) Remainder (d) None of these

3. Find the decimal equivalent of binary 1110101.

(a) 110_{10} (b) 111_{10}
(c) 117_{10} (d) None of these

4. Find the binary equivalent of decimal 235.

(a) 1010111_2 (b) 1010111_2
(c) 11101011_2 (d) None of these

5. Find the binary equivalent of decimal 701.

(a) 1010111101_2 (b) 1011101101_2
(c) 1110111101_2 (d) None of these

6. Find the decimal equivalent of binary 101001.

(a) 31 (b) 41
(c) 51 (d) None of these

7. Find the decimal equivalent of binary 10000010011.

(a) 1043 (b) 1023
(c) 1033 (d)

8. Find the decimal equivalent of binary 111011.

(a) 69 (b) 49
(c) 59 (d) None of these

9. Add 1001 to 0101.

(a) 1111 (b) 1110
(c) 1010 (d) None of these

10. Add 11010 to 11100.

(a) 110110 (b) 111110
(c) 110111 (d) None of these

11. $11111_2 + 10001_2 + 1011_2 =$

(a) 110111 (b) 111001
(c) 111011 (d) None of these

12. $11001_2 + 11011_2 + 11111_2 =$

(a) 1010011 (b) 111011
(c) 1110011 (d) None of these

13. $11_2 + 111_2 + 1111_2 + 11111_2 =$

(a) 101010 (b) 111000
(c) 101100 (d) None of these

14. $111_2 + 101_2 =$

(a) 1111 (b) 10111
(c) 1100 (d) None of these

15. $1000_2 + 1101_2 + 1111_2 =$

(a) 100100 (b) 111100
(c) 101010 (d) None of these

16. $111_2 + 101_2 + 011_2 =$

(a) 1011 (b) 1111
(c) 1101 (d) None of these

17. $111000_2 - 11001_2 =$

(a) 11111 (b) 10111
(c) 11011 (d) None of these

18. $10001_2 - 1111_2 =$

(a) 101 (b) 11
(c) 10 (d) None of these

19. $111101_2 - 10111_2 =$

(a) 111110 (b) 100110
(c) 101110 (d) None of these

20. $11111_2 - 10001_2 =$

(a) 1010 (b) 1111
(c) 1110 (d) None of these

21. $100001_2 - 11110_2 =$

(a) 11 (b) 111
(c) 10 (d) None of these

22. Multiply 1111 by 11:

(a) 110101 (b) 101101
(c) 110100 (d) None of these

23. Multiply 101 by 11:

(a) 1111 (b) 1011
(c) 1110 (d) None of these

24. Multiply 101101 by 1101:

(a) 1111001001
(b) 1001101001
(c) 1001001001
(d) None of these

25. Multiply 11001 by 101:

(a) 1111101 (b) 1110101
(c) 1011101 (d) None of these

ANSWER KEYS

Exercise-I

1. (b)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (a)	**11.** (c)	**12.** (a)	**13.** (b)
14. (c)	**15.** (a)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (c)	**21.** (a)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (a)	

EXPLANATORY ANSWERS

Exercise-I

1. (b)

2	117	Remainder
2	58	1
2	29	0
2	14	1
2	7	0
2	3	1
	1	1
	0	1

∴ The binary equivalent of decimal 117 is 1110101.

2. (a)

2	52	Remainder
2	26	0
2	13	0
2	6	1
2	3	0
	1	1
	0	1

∴ The binary equivalent of decimal 52 is 110100.

3. (c)

1	1	1	0	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Delete the weights 23 and 21.

Adding the remaining weights, we get

$2^6 + 2^5 + 2^4 + 2^2 + 2^0 = 64 + 32 + 16 + 4 + 1 = 117$

i.e., $1110101_2 = 117_{10}$.

4. (c)

2	235	Remainder
2	117	1
2	58	1
2	29	0
2	14	1
2	7	0
2	3	1
	1	1
	0	1

∴ $235_{10} = 11101011_2$

5. **(a)**

		Remainder
2	701	
2	350	1
2	175	0
2	87	1
2	43	1
2	21	1
2	10	1
2	5	0
2	2	1
	1	0
	0	1

$\therefore\ (701)_{10} = 1010111101_2$.

6. **(b)**
$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

Decimal equivalent

$= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 + 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 2^5 + 2^3 + 1 \times 2^0 = 32 = 32 + 8 + 1 = 41.$

7. **(a)**
$$\begin{array}{ccccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 2^{10} & 2^9 & 2^8 & 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

Decimal equivalent

$= 1 \times 2^{10} + 0 \times 2^9 + 0 \times 2^8 + 0 \times 2^7$

$\quad + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3$

$\quad + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 2^{10} + 2^4 + 2^1 + 2^0 = 1043.$

8. **(c)**
$$\begin{array}{cccccc} 1 & 1 & 1 & 0 & 1 & 1 \\ 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

Decimal equivalent

$= 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 2^5 + 2^4 + 2^3 + 2^1 + 2^0 = 59.$

9. **(b)**
$$\begin{array}{r} 0101 \\ +1001 \\ \hline 1110 \end{array}$$

10. **(a)**
$$\begin{array}{r} 11100 \\ +\ 11010 \\ \hline 110110 \end{array}$$

11. **(c)**
$$\begin{array}{r} 11111 \\ 10001 \\ 1011 \\ \hline 111011 \end{array}$$

12. **(a)**
$$\begin{array}{r} 0011 \\ +\ 1 \\ \hline 0100 \end{array}$$

Column 1: $1 + 1 + 1 = 3;\ \dfrac{3}{2}$ = Quotient 1, Remainder 1

Column 2: $0 + 1 + 1 + 1$ (carry from first column)

$= 3;\ \dfrac{3}{2}$ = Quotient 1 and Remainder 1

Column 3: $0 + 0 + 1 + 1$ (carry from second column)

$= 2;\ \dfrac{2}{2}$ = Quotient 1 and Remainder 0

Column 4: $1 + 1 + 1$ (carry from column 3)

$= 4;\ \dfrac{4}{2}$ = Quotient 2 and Remainder 0

Column 5: $1 + 1 + 1 + 2$ (carry from column 4)

$= 5,\ 5_{10} = 101_2$.

Note:

Quotient in any column is carry for next column.

13. **(b)**
$$\begin{array}{r} 11 \\ 111 \\ 1111 \\ 11111 \\ \hline 111000 \end{array}$$

14. **(c)**
$$\begin{array}{r} 101 \\ 101 \\ \hline 1100 \end{array}$$

15. **(a)**
$$\begin{array}{r} 1000 \\ 1101 \\ 1111 \\ \hline 100100 \end{array}$$

16. **(b)**
$$\begin{array}{r} 111 \\ 101 \\ 011 \\ \hline 1111 \end{array}$$

17. **(a)** $111000_2 = 32 + 16 + 8 = 56$

$11001_2 = 16 + 8 + 1 = 25$

Since $11001_2 < 111000_2$, so we are to subtract a lower number from a higher number.

Making the digits equal in the number to be subtracted, we get 011001.

Complement of 011001 = 100110.

Adding 100110 to 111 000, we get
$$\begin{array}{r} 111000 \\ 100110 \\ \hline [1]011110 \end{array}$$

[1 in the [ ] is the 1 carried over]

Adding 1 to the extreme right digit in 011 110, we get
$$\begin{array}{r} 011110 \\ 1 \\ \hline 11111 \end{array}$$

$\therefore\ 111000_2 - 11001_2 = 11111.$

18. **(c)** $10001_2 = 2_4 + 1 = 17$,

$1111_2 = 2^3 + 2^2 + 2^1 + 1 = 15$.

Since $1111_2 < 10001_2$, we are to subtract a lower number from a higher number.

Making the digits equal in the number to be subtracted, we get

01111.

Complement of 01111 is 10000.

Adding 10000 to 10001, we get

$$\begin{array}{r} 10001 \\ 10000 \\ \hline [1]00001 \end{array}$$

Adding 1 to 1 in 00001, we get

$$\begin{array}{r} 00001 \\ 1 \\ \hline 00010 \\ \hline \end{array}$$

$\therefore\ 10001_2 - 1111_2 = 10.$

19. **(b)** Complement of $010111_2 = 101000$

Now,

$$\begin{array}{r} 111101 \\ +101000 \\ \hline [1]100101 \end{array}$$

Adding 1 to the extreme right digit in 100101, we get

$$\begin{array}{r} 100101 \\ + \quad\ 1 \\ \hline 100110 \end{array}$$

$\therefore\ 111101_2 - 10111_2 = 100110.$

20. **(c)** Complement of $10001_2 = 01110$.

Now,

$$\begin{array}{r} 11111 \\ +01110 \\ \hline [1]01101 \end{array} \qquad \begin{array}{r} 01101 \\ + \quad 1 \\ \hline 01110 \end{array}$$

$\therefore\ 11\ 111_2 - 10001_2 = 1110.$

21. **(a)** Complement of 011110 = 100001

Now,

$$\begin{array}{r} 100001 \\ +100001 \\ \hline [1]000010 \end{array} \qquad \begin{array}{r} 000010 \\ + \quad\ 1 \\ \hline 000011 \end{array}$$

$\therefore\ 100001_2 - 11110_2 = 11.$

22. **(b)**

$$\begin{array}{r} 1111 \\ 11 \\ \hline 11\ 11 \\ 1\ 11\ 1 \\ \hline 10\ 1101 \end{array}$$

23. **(a)**

$$\begin{array}{r} 101 \\ 11 \\ \hline 101 \\ 101 \\ \hline 1111 \end{array}$$

24. **(c)**

$$\begin{array}{r} 101101 \\ 1101 \\ \hline 101101 \\ 000000 \\ 101101 \\ \hline 1001001001 \end{array}$$

25. **(a)**

$$\begin{array}{r} 11001 \\ 101 \\ \hline 11001 \\ 00000 \\ 11001 \\ \hline 1111101 \end{array}$$