Quadratic Equations 28

INTRODUCTION

An equation of degree two is called a *quadratic equation*. The general form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$ and x is a real variable. Some examples of quadratic equations are $x^2 + 4x + 3 = 0$, $3x^2 - 4x + 5 = 0$ and $3x^2 + 2x - 3 = 0$.

Roots of a Quadratic Equation

A *root* of the equation $f(x) = 0$ is that value of x which makes $f(x) = 0$. In other words, $x = a$ is said to be a root of $f(x) = 0$, where $f(a)$ is the value of the polynomial $f(x)$ at $x = a$ and is obtained by replacing x by a in $f(x)$.

For example, -1 is a root of the quadratic equation $x^2 + 6x + 5 = 0$ because $(-1)^2 + 6(-1) + 5 = 0$.

Solution of a Quadratic Equation

If there is a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the roots of this equation are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

Illustration 1: Solve the following quadratic equations:

(i) $6x^2 + x - 2 = 0$

(ii) $2x^2 + x - 1 = 0$

Solution: (i) Using formula:

$$\text{The roots are } x = \frac{-1 \pm \sqrt{(1)^2 - 4(6)(-2)}}{2\times 6}$$

$$= \frac{-1 \pm \sqrt{49}}{12} = \frac{6}{12}, \frac{-8}{12}.$$

i.e., $\frac{1}{2}, \frac{-2}{3}$.

Using factorization:

$$6x^2 + x - 2 = 0 \Leftrightarrow 6x^2 + 4x - 3x - 2 = 0$$

$$\Leftrightarrow 2x(3x + 2) - 1(3x + 2) = 0$$

$$\Leftrightarrow (2x + 1)(3x + 2) = 0$$

$$\Leftrightarrow x = \frac{1}{2} \text{ or } x = -\frac{2}{3}.$$

(ii) Using formula:

$$\text{The roots are } x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-1)}}{2\times 2}$$

$$= \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$$

$$= \frac{2}{4}, \frac{-4}{4} \text{ i.e., } \frac{1}{2}, -1.$$

Using factorization:

$$2x^2 + x - 1 = 0 \Leftrightarrow 2x^2 + 2x - x - 1 = 0$$

$$\Leftrightarrow 2x(x + 1) - 1(x + 1) = 0$$

$$\Leftrightarrow (2x - 1)(x - 1) = 0.$$

$$\Leftrightarrow x = \frac{1}{2} \text{ or } x = -1.$$

Nature of Roots

A quadratic equation has exactly two roots may be real or imaginary or coincident.

If $ax^2 + bx + c$, $a \neq 0$, then $D = b^2 - 4ac$ is called *discriminant*.

1. If $D > 0$, then there are two distinct and real roots given by

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}, \ \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}.$$

2. If $D = 0$, then there is a repeated real root given by $\alpha = -\frac{b}{2a}$ i.e., roots are real and equal.

3. If $D < 0$, then there are no real roots.

Note:

The roots are rational if $D > 0$ and D is a perfect square whereas the roots are irrational if $D > 0$ but D is not a perfect square.

Illustration 2: Find the nature of the roots of the equations:

(i) $2x^2 + x - 1 = 0$

(ii) $x^2 + x + 1 = 0$

(iii) $x^2 + 5x + 5 = 0$

(iv) $\frac{4}{3}x^2 - 2x + \frac{3}{4} = 0$

Solution: (i) $D = (1)^2 - 4 \times 2 \times (-1) = 9 > 0$.

Also, D is a perfect square.

So, the roots are real, distinct and rational.

(ii) $D = (1)^2 - 4 \times 1 \times 1 = -3 < 0$

So, the roots are imaginary.

(iii) $D = (5)^2 - 4 \times 1 \times 5 = 5 > 0$.

Also, D is not a perfect square.

So, the roots are real, distinct and irrational.

(iv) $D = (-2)^2 - 4 \times \frac{4}{3} \times \frac{3}{4} = 0$.

So, the roots are real and equal.

Illustration 3: For what value of k will the quadratic equation $kx^2 - 2\sqrt{5}\,x + 4 = 0$ have real and equal roots.

Solution: $D = (-2\sqrt{5})^2 - 4 \times k \times 4 = 20 - 16k$.

The given equation will have real and equal roots if $D = 0$.

i.e., $20 - 16k = 0$ or $k = \frac{20}{16} = \frac{5}{4}$.

Note:

1. If $p + \sqrt{q}$ is a root of a quadratic equation, then its other root is $p - \sqrt{q}$.

Illustration 4: If $2 + \sqrt{3}$ is one root of a quadratic equation, find the other root.

Solution: The other root is $2 - \sqrt{3}$.

2. $ax^2 + bx + c$ can be expressed as a product of two linear factors only when $D \geq 0$.

Illustration 5: For what value of k, the quadratic polynomial $kx^2 + 4x + 1$ can be factorized into two real linear factors.

Solution: $D = (4)^2 - 4 \times k \times 1 = 16 - 4k$.

The given quadratic polynomial can be factorized into real linear factors if $D \geq 0$.

i.e., $16 - 4k \geq 0$ or $-4k \geq -6$ or $k \leq 4$.

Relation Between Roots and Coefficients

Let, α, β be the roots of the equation,

$$ax^2 + bx + c = 0$$

Then, sum of the roots

$$= \alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of the roots

$$= \alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

Illustration 6: Find the sum and the product of the roots of the quadratic equation $2x^2 + 5\sqrt{3}\,x + 6 = 0$.

Solution: Here $a = 2$, $b = 5\sqrt{3}$, $c = 6$.

$\therefore$ Sum of the roots $= -\frac{b}{a} = -\frac{5\sqrt{3}}{2}$.

Product of the roots $= \frac{c}{a} = \frac{6}{2} = 3$.

Formation of a Quadratic Equation with Given Roots

If α, β are the roots of a quadratic equation the equation can he written as

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e., $x^2 -$ (sum of roots)x + product of roots = 0.

Illustration 7: Find the quadratic equation whose roots are 5 and -6.

Solution: Sum of roots $= 5 + (-6) = -1$,

Product of roots $= 5 \times (-6) = -30$.

$\therefore$ The required quadratic equation is

$x^2 - (-1)x + (-30) = 0$ i.e., $x^2 + x - 30 = 0$.

EXERCISE–I

1. In the following determine the set of value of P for which the given quadratic equation has real roots.

$$Px^2 + 4x + 1 = 0$$

(a) $P \neq 4$ (b) $P > 4$
(c) $P \leq 4$ (d) $P \geq 4$

2. If one root of the quadratic equation $2x^2 + Px + 4 = 0$ is 2, find the second root and value of P.

(a) 1, −6 (b) 1, 6
(c) −1, 6 (d) −1, −6

3. One root of the quadratic equation $x^2 - 5x + 6 = 0$ is 3. Find the other root.

(a) 2 (b) −2
(c) 1 (d) −1

4. The roots of the equation $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$ are

(a) $-\sqrt{7}, \dfrac{-13\sqrt{7}}{7}$ (b) $\sqrt{7}, \dfrac{-13\sqrt{7}}{7}$
(c) $-\sqrt{7}, \dfrac{13\sqrt{7}}{7}$ (d) None of these

5. The roots of the equation $3a^2x^2 - abx - 2b^2 = 0$ are

(a) $\dfrac{b}{a}, \dfrac{-2b}{3a}$ (b) $\dfrac{b}{a}, \dfrac{2b}{3a}$
(c) $\dfrac{-b}{a}, \dfrac{-2b}{3a}$ (d) None of these

6. The roots of the equation $a^2x^2 - 3abx - 2b^2 = 0$ are

(a) $\dfrac{2b}{a}, \dfrac{-b}{a}$ (b) $\dfrac{2b}{a}, \dfrac{b}{a}$
(c) $\dfrac{-2b}{a}, \dfrac{b}{a}$ (d) None of these

7. Construct a quadratic equation whose roots are $\sqrt{2}$ and $2\sqrt{2}$

(a) $x^2 - 3\sqrt{2}x - 4 = 0$ (b) $x^2 - 3\sqrt{2}x + 4 = 0$
(c) $x^2 + 3\sqrt{2}x - 4 = 0$ (d) $x^2 + 3\sqrt{2}x + 4 = 0$

8. The roots of the equation $ax^2 + (4a^2 - 3b)x - 12ab = 0$ are

(a) $4a, \dfrac{3b}{a}$ (b) $-4a, \dfrac{3b}{a}$
(c) $-4a, \dfrac{3b}{a}$ (d) $-4a, \dfrac{-3b}{a}$

9. Construct a quadratic equation whose roots have the sum = 6 and product = −16.

(a) $x^2 - 6x - 16 = 0$ (b) $x^2 + 6x - 16 = 0$
(c) $x^2 - \sqrt{3}x - 6 = 0$ (d) None of these

10. In the following, find the value (s) of P so that the given equation has equal roots

$3x^2 - 5x + P = 0$

(a) −25/12 (b) 25/6
(c) 25/12 (d) − 25/6

11. If α and β are the roots of the equation $ax^2 + bx + c = 0$, find the value of $\alpha^2 + \beta^2$

(a) $\dfrac{b^2 - 2ac}{2a^2}$ (b) $\dfrac{b^2 + 2ac}{a^2}$
(c) $\dfrac{b^2 + 2ac}{2a^2}$ (d) $\dfrac{b^2 - 2ac}{a^2}$

12. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, the value of $\alpha^3 + \beta^3$ is

(a) $\dfrac{b(b^2 - 3ac)}{a^3}$ (b) $\dfrac{b(3ac - b^2)}{a^3}$
(c) $\dfrac{b(3ac + b^2)}{a^3}$ (d) None of these

13. If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then the value of $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ is

(a) $\dfrac{b^2 - 2ac}{ac}$ (b) $\dfrac{b^2 - 2ac}{2ac}$
(c) $\dfrac{b^2 - ac}{2ac}$ (d) $\dfrac{b^2 + 2ac}{ac}$

14. The quadratic equation with rational coefficients, whose one root is $\sqrt{5}$, is:

(a) $x^2 + 5 = 0$ (b) $x^2 - 10 = 0$
(c) $x^2 - 5 = 0$ (d) None of these

15. The equation $x^2 - px + q = 0$, $p, q \in R$ has on real root if:

(a) $p^2 \le 4q$ (b) $p^2 < 4q$
(c) $p^2 > 4q$ (d) None of these

16. Determine p so that the equation $x^2 + 5px + 16 = 0$ has on real root.

(a) $\frac{-4}{5} < p < \frac{4}{5}$ (b) $\frac{-8}{5} < p < \frac{8}{5}$
(c) $p < -\frac{4}{5}$ or $p > \frac{4}{5}$ (d) None of these

17. For what value of k the quadratic polynomial $3z^2 + 5z + k$ can be factored into product of real linear factors?

(a) $k \le \frac{25}{6}$ (b) $k \le \frac{25}{12}$
(c) $k \ge \frac{25}{12}$ (d) $k \ge \frac{25}{6}$

18. $x = 3$ i a solution of the equation $3x^2 + (k - 1)x + 9 = 0$ if k has value

(a) 13 (b) −13
(c) 11 (d) −11

19. One root of the equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{36}$. Find the other root.

(a) 3 (b) 1/3
(c) −3 (d) None of these

20. The expression $x^4 + 7x^2 + 16$ can be factored as:

(a) $(x^2 + x + 1)(x^2 + x + 16)$
(b) $(x^2 + x + 1)(x^2 - x + 16)$
(c) $(x^2 + x + 4)(x^2 - x + 4)$
(d) $(x^2 + x - 4)(x^2 - x - 4)$

21. The common root of the equations $x^2 - 7x + 10 = 0$ and $x^2 - 10x + 16 = 0$ is:

(a) −2 (b) 3
(c) 5 (d) 2

22. The roots of the equation $x^2 + px + q = 0$ are equal if:

(a) $p^2 = 2q$ (b) $p^2 = 4q$
(c) $p^2 = -4q$ (d) $p^2 = -2q$

23. An equation equivalent to the quadratic equation $x^2 - 6x + 5 = 0$ is:

(a) $6x^2 - 5x + 1 = 0$ (b) $x^2 - 5x + 6 = 0$
(c) $5x^2 - 6x + 1 = 0$ (d) $|x - 3| = 2$

24. Divide 16 into 2 parts such the twice the square of the larger part exceeds the square of the smaller part by 164

(a) 10, 6 (b) 8, 8
(c) 12, 4 (d) None of these

25. With respect to the roots of $x^2 - x - 2 - 0$, we can say that:

(a) both of them are natural numbers
(b) both of them are integers
(c) the latter of the two is negative
(d) None of these

26. The solution of $2 - x = \frac{x-2}{x}$ would include:

(a) −2, −1 (b) 2, −1
(c) −4, 2 (d) 4, −2

27. If $\log_{10}(x^2 - 6x + 45) = 2$, then the values of x are

(a) 6, 9 (b) 9, −5
(c) 10, 5 (d) 11, −5

28. If α, β are the roots of the equation $x^2 - 5x + 6 = 0$, construct a quadratic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$.

(a) $6x^2 + 5x - 1 = 0$ (b) $6x^2 - 5x - 1 = 0$
(c) $6x^2 - 5x + 1 = 0$ (d) $6x^2 + 5x + 1 = 0$

29. The roots of $\frac{x+4}{x-4} + \frac{x-4}{x+4} = \frac{10}{3}$ are:

(a) ± 4 (b) ± 6
(c) ± 8 (d) $2 \pm \sqrt{3}$

30. The roots of the equation $ax^2 + bx + c = 0$ will be reciprocal if

(a) $a = b$ (b) $b = c$
(c) $c = a$ (d) None of these

31. Form a quadratic equation whose one root is $3-\sqrt{5}$ and the sum of roots is 6

(a) $x^2 - 6x + 4 = 0$ (b) $x^2 + 6x + 4 = 0$
(c) $x^2 - 6x - 4 = 0$ (d) None of these

32. The value of k for which the roots α, β of the equation: $x^2 - 6x + k = 0$ satisfy the relation $3\alpha + 2\beta = 20$, is

(a) 8 (b) −8
(c) 16 (d) −16

33. Find two consecutive positive odd integers whose squares have the sum 290.

(a) 11, 13 (b) 13, 15
(c) 9, 11 (d) None of these

34. Consider the equation $px^2 + qx + r = 0$, where p, q, r are real. The roots are equal in magnitude but opposite in sign when:

(a) $q = 0, r = 0, p \neq 0$
(b) $p = 0, qr \neq 0$
(c) $r = 0, pr \neq 0$
(d) $q = 0, pr \neq 0$

35. Determine k such that the quadratic equation $x^2 - 2(1+3k)x + 7(3+2k) = 0$ has equal roots

(a) $2, \frac{-10}{9}$ (b) $2, \frac{10}{9}$

(c) $-2, \frac{10}{9}$ (d) $-2, \frac{-10}{9}$

36. If the equations $x^2 + 2x - 3 = 0$ and $x^2 + 3x - k = 0$ have a common root, then the non–zero value of k is:

(a) 1 (b) 2
(c) 3 (d) 4

37. The roots of the equation $4x - 3.2^{x} + {}^{2} + 32 = 0$ would include:

(a) 1, 2 and 3 (b) 1 and 2
(c) 1 and 3 (d) 2 and 3

38. The positive value of m for which the roots of the equation $12x^2 + mx + 5 = 0$ are in the ratio 3:2 is:

(a) $5\sqrt{10}$ (b) $\frac{5}{2}\sqrt{10}$

(c) $\frac{5}{12}$ (d) $\frac{12}{5}$

39. If α, β are the roots of the equation $2x^2 - 3x + 1 = 0$, form an equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

(a) $2x^2 + 5x + 2 = 0$ (b) $2x^2 - 5x - 2 = 0$
(c) $2x^2 - 5x + 2 = 0$ (d) None of these

40. Find the quadratic equation whose roots are reciprocal of the roots of the equation

$3x^2 - 20x + 17 = 0$

(a) $17x^2 - 20x + 3 = 0$
(b) $17x^2 + 20x + 3 = 0$
(c) $17x^2 - 20x - 3 = 0$
(d) None of these

41. If α and β are the roots of the equation

$x^2 - 3\lambda x + \lambda^2 = 0$, find λ if $\alpha^2 + \beta^2 = \frac{7}{4}$

(a) $\pm\frac{1}{2}$ (b) $\pm\frac{\sqrt{7}}{2}$

(c) $\pm\frac{\sqrt{3}}{2}$ (d) None of these

42. If α, β are the roots of the equation $ax^2 + bx + b = 0$, then

$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} =$

(a) 1 (b) 0
(c) 2 (d) 3

43. The expression $x^2 - x + 1$ has:

(a) one proper linear factor
(b) two proper linear factors
(c) no proper linear factor
(d) None of these

44. The length of a rectangular plot is 8 m greater than its breadth. If the area of the plot is 308 m^2, find the length of the plot.

(a) 22 m (b) 18 m
(c) 20 m (d) None of these

45. If α, β are the roots of the equation $x^2 + kx + 12 = 0$ such that $\alpha - \beta = 1$, the value of k is

(a) 0 (b) ± 5
(c) ± 1 (d) ± 7

46. The value of x in the equation

$\left(x+\frac{1}{x}\right)^2-\frac{3}{2}\left(x-\frac{1}{x}\right)=4$ is:

(a) −2 (b) $\frac{1}{2}$

(c) −1 (d) 0

47. If α, β are the roots of the quadratic equation $x^2 - 8x + k = 0$, find the value of k such the $\alpha^2 + \beta^2 = 40$

(a) 12 (b) 14

(c) 10 (d) 16

48. Find the value of k so that the sum of the roots of the equation $3x^2 + (2x + 1)x - k - 5 = 0$ is equal to the product of the roots:

(a) 4 (b) 6

(c) 2 (d) 8

Exercise–2
(Based on Memory)

1. The factors of $(a^2+4b^2+4b-4ab-2a-8)$ are:

(a) $(a-2b-4)(a-2b+2)$

(b) $(a-b+2)(a+4b+4)$

(c) $(a+2b-4)(a+2b+2)$

(d) $(a+2b-1)(a-2b+1)$

[SSC, 2014]

2. Find the value of $\sqrt{30+\sqrt{30+\sqrt{30+\cdots}}}$.

(a) 5 (b) $3\sqrt{10}$

(c) 6 (d) 7

[SSC, 2013]

3. $\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}$ is equal to

(a) 2 (b) 5

(c) 4 (d) 3

[SSC, 2011]

4. The sum of the squares of two natural consecutive odd numbers is 394. The sum of the numbers is:

(a) 24 (b) 32

(c) 40 (d) 28

[SSC, 2011]

Directions (Question. 5–9): In this question two equations numbered I and II are given. You have to solve both the equations and find out the correct option.

5. **I.** $6x^2 + 41x + 63 = 0$

II. $4y^2 + 8y + 3 = 0$

(a) Relationship between x and y cannot be established

(b) $x \geq y$

(c) $x < y$

(d) $x > y$

(e) $x \leq y$

[IBPS PO/MT, 2014]

6. **I.** $x^2 + 10x + 24 = 0$

II. $4y^2 - 17y + 18 = 0$

(a) $x \leq y$

(b) $x \geq y$

(c) Relationship between x and y cannot be established

(d) $x > y$

(e) $x < y$

[IBPS PO/MT, 2014]

7. **I.** $24x^2 + 38x + 15 = 0$

II. $12y^2 + 28y + 15 = 0$

(a) $x \leq y$ (b) $x > y$

(c) $x \geq y$ (d) $x < y$

(e) $x = y$, or Relationship between x and y cannot be established

[IBPS PO/MT, 2014]

8. **I.** $3x^2 - 20x - 32 = 0$

II. $2y^2 - 3y - 20 = 0$

(a) $x < y$

(b) $x \leq y$

(c) $x > y$

(d) Relationship between x and y cannot be established

(e) $x \geq y$

[IBPS PO/MT, 2014]

9. **I.** $x^2 - 20x + 91 = 0$

II. $y^2 - 32y + 247 = 0$

(a) $x > y$
(b) Relationship between x and y cannot be established
(c) $x \geq y$
(d) $x \leq y$
(e) $x < y$

[IBPS PO/MT, 2014]

Directions (Question. 10–14): In each of these questions, two equations (I) and (II) are given. You have to solve both the equations and give answer

(a) If $x < y$ (b) If $x > y$
(c) If $x = y$ (d) If $x \geq y$
(e) If $x \leq y$ or no relationship can be established between x and y.

[IBPS PO/MT, 2013]

10. **I.** $x^2 – 24x + 144 = 0$
II. $y^2 – 26y + 169 = 0$

11. **I.** $2x^2 + 3x – 20 = 0$
II. $2y^2 + 19y + 44 = 0$

12. **I.** $6x^2 + 77x + 121 = 0$
II. $y^2 + 9y - 22 = 0$

13. **I.** $x^2 – 6x = 7$
II. $2y^2 + 13y + 15 = 0$

14. **I.** $10x^2 – 7x + 1 = 0$
II. $35y^2 - 12y + 1 = 0$

[IBPS PO/MT, 2013]

Directions (Question. 15–19): In the following questions, two equations numbered I and II are given. You have to solve both questions and give answer

(a) if $x > y$
(b) if $x \geq y$
(c) if $x < y$
(d) if $x \leq y$
(e) if $x = y$ or the relationship cannot be established.

15. **I.** $x^2 - 19x + 84 = 0$
II. $y^2 - 25y + 156 = 0$

[IOB PO, 2011]

16. **I.** $x^3 - 468 = 1729$
II. $y^2 - 1733 + 1564 = 0$

[IOB PO, 2011]

17. **I.** $\frac{9}{\sqrt{x}} + \frac{19}{\sqrt{x}} = \sqrt{x}$

II. $y^5 - \frac{(2 \times 14)^{11/12}}{\sqrt{y}} = 0$

[IOB PO, 2011]

18. **I.** $\sqrt{784}x + 1234 = 1486$
II. $\sqrt{1089}y + 2081 = 2345$

[IOB PO, 2011]

19. **I.** $\frac{12}{\sqrt{x}} - \frac{23}{\sqrt{x}} = 5\sqrt{x}$

II. $\frac{\sqrt{y}}{12} - \frac{5\sqrt{y}}{12} = \frac{1}{\sqrt{y}}$

[IOB PO, 2011]

Directions (Question. 20–24): In each of these questions, two equations are given. You have to solve these equations and find out the values of x and y and

Give answer
(a) if $x < y$ (b) if $x > y$
(c) if $x \leq y$ (d) if $x \geq y$
(e) if $x = y$

[Andhra Bank PO, 2011]

20. **I.** $4x + 7y = 209$
II. $12x - 14y = - 38$

[Andhra Bank PO, 2011]

21. **I.** $17x^2 + 48x = 9$
II. $13y^2 = 32y - 12$

[Andhra Bank PO, 2011]

22. **I.** $16x^2 + 20x + 6 = 0$
II. $10y^2 + 38y + 24 = 0$

[Andhra Bank PO, 2011]

23. **I.** $8x^2 + 6x = 5$
II. $12y^2 – 22y + 8 = 0$

[Andhra Bank PO, 2011]

24. **I.** $18x^2 + 18x + 4 = 0$
II. $12y^2 + 29y + 14 = 0$

[Andhra Bank PO, 2011]

Directions (Question. 25–29): In the following questions two equations numbered I and II are given. You have to solve both the equations and give answer

(a) $x > y$ (b) $x \geq y$
(c) $x < y$ (d) $x \leq y$
(e) $x = y$ or the relationship cannot be established

[Corporation Bank PO, 2011]

25. **I.** $x^2 - 11x + 24 = 0$
II. $2y^2 - 9y + 9 = 0$

[Corporation Bank PO, 2011]

26. I. $x^3 \times 13 = x^2 \times 247$

II. $y^{1/3} \times 14 = 294 \div y^{2/3}$

[Corporation Bank PO, 2011]

27. I. $\frac{12\times4}{x^{4/7}} - \frac{3\times4}{x^{4/7}} = x^{10/7}$

II. $y^3 + 783 = 999$

[Corporation Bank PO, 2011]

28. I. $\sqrt{500}x + \sqrt{402} = 0$

II. $\sqrt{360}y + (200)^{1/2} = 0$

[Corporation Bank PO, 2011]

29. I. $(17)^2 + 144 \div 18 = x$

II. $(26)^2 - 18 \times 21 = y$

[Corporation Bank PO, 2011]

Directions (Question. 30–34): In each of these questions, two equations are given. You have to solve these equations and find out the values of x and y and give answer

(a) $x < y$ (b) $x > y$
(c) $x \leq y$ (d) $x \geq y$
(e) $x = y$

30. I. $16x^2 + 20x + 6 = 0$

II. $10y^2 + 38y + 24 = 0$

[Punjab and Sind Bank PO, 2011]

31. I. $18x^2 + 18x + 4 = 0$

II. $12y^2 + 29y + 14 = 0$

[Punjab and Sind Bank PO, 2011]

32. I. $8x^2 + 6x = 5$

II. $12y^2 - 22y + 8 = 0$

[Punjab and Sind Bank PO, 2011]

33. I. $17x^2 + 48x = 9$

II. $13y^2 = 32y - 21$

[Punjab and Sind Bank PO, 2011]

34. I. $4x + 7y = 209$

II. $12x - 14y = -38$

[Punjab and Sind Bank PO, 2011]

Directions (Question. 35–39): In the following questions two equations numbered I and II are given. You have to solve both the equations and give answer

(a) if $x > y$ (b) if $x \geq y$
(c) if $x < y$ (d) if $x \leq y$
(e) if $x = y$ or the relationship cannot be established.

[Indian Bank PO, 2010]

35. I. $x^2 - 4 = 0$

II. $y^2 + 6y + 9 = 0$

[Indian Bank PO, 2010]

36. I. $x^2 - 7x + 12 = 0$

II. $y^2 + y - 12 = 0$

[Indian Bank PO, 2010]

37. I. $x^2 = 729$

II. $y = \sqrt{729}$

[Indian Bank PO, 2010]

38. I. $x^4 - 227 = 398$

II. $y^2 + 321 = 346$

[Indian Bank PO, 2010]

39. I. $2x^2 + 11x + 14 = 0$

II. $4y^2 + 12y + 9 = 0$

[Indian Bank PO, 2010]

Directions (Question. 40–44): In the following questions two equations numbered I and II are given. You have to solve both the equations and give answer

(a) If $x > y$ (b) If $x \geq y$
(c) If $x < y$ (d) If $x \leq y$
(e) If $x = y$ or the relationship cannot be established.

40. I. $x^2 - 1 = 0$

II. $y^2 + 4y + 3 = 0$

[Corporation Bank PO, 2009]

41. I. $x^2 - 7x + 12 = 0$

II. $y^2 - 12y + 32 = 0$

[Corporation Bank PO, 2009]

42. I. $x^3 - 371 = 629$

II. $y^3 - 543 = 788$

[Corporation Bank PO, 2009]

43. I. $5x + 2y = 31$

II. $3x + 7y = 36$

[Corporation Bank PO, 2009]

44. I. $2x^2 + 11x + 12 = 0$

II. $5y^2 + 27y + 10 = 0$

[Corporation Bank PO, 2009]

ANSWER KEYS

Exercise–1

1. (c)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (c)	**11.** (d)	**12.** (b)	**13.** (a)
14. (c)	**15.** (b)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (c)	**21.** (d)	**22.** (b)	**23.** (d)	**24.** (a)	**25.** (b)	**26.** (b)
27. (d)	**28.** (c)	**29.** (c)	**30.** (c)	**31.** (a)	**32.** (a)	**33.** (a)	**34.** (d)	**35.** (a)	**36.** (d)	**37.** (d)	**38.** (a)	**39.** (c)
40. (a)	**41.** (a)	**42.** (b)	**43.** (c)	**44.** (c)	**45.** (d)	**46.** (c)	**47.** (a)	**48.** (a)				

Exercise–2

1. (a)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (e)	**7.** (c)	**8.** (d)	**9.** (d)	**10.** (a)	**11.** (d)	**12.** (e)	**13.** (d)
14. (d)	**15.** (d)	**16.** (b)	**17.** (e)	**18.** (a)	**19.** (a)	**20.** (e)	**21.** (a)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (b)	**26.** (c)
27. (d)	**28.** (c)	**29.** (c)	**30.** (b)	**31.** (d)	**32.** (c)	**33.** (a)	**34.** (e)	**35.** (a)	**36.** (b)	**37.** (d)	**38.** (e)	**39.** (c)
40. (b)	**41.** (d)	**42.** (c)	**43.** (a)	**44.** (e)								

EXPLANATORY ANSWERS

Exercise–1

1. (c) $Px^2 + 4x + 1 = 0$

Compare with $Ax^2 + Bx + C = 0$, we get

$$A = P,\ B = 4,\ C = 1$$

For real roots,

$B^2 - 4AC \geq 0 \Rightarrow 16 - 4P \geq 0$

$\Rightarrow 16 \geq 4P \Rightarrow P \leq 4.$

2. (a) The given equation is $2x^2 + Px + 4 = 0$

$\Rightarrow P(x) = 0$ where $P(x) = 2x^2 + Px + 4 = 0$

If 2 is a root of $P(x) = 0$, then $P(2) = 0$

$\Rightarrow 2(2)^2 + P(2) + 4 = 0 \Rightarrow 8 + 2P + 4 = 0$

$\Rightarrow 2P = -12 \Rightarrow P = \frac{-12}{2} = -6$

Hence the given equation is

$2x^2 - 6x + 4 = 0 \Rightarrow 2x^2 - 2x - 4x + 4 = 0$

$\Rightarrow 2x(x-1) - 4(x-1) = 0$

$\Rightarrow 2(x-2)(x-1) = 0$

$\Rightarrow x = 1$ or $x = 2$

Hence second root is 1.

3. (a) The given equation is

$$x^2 - 5x + 6 = 0 \Rightarrow x^2 - 2x - 3x + 6 = 0$$
$$\Rightarrow x(x-2) - 3(x-2) = 0$$
$$\Rightarrow (x-2)(x-3) = 0$$
$$\Rightarrow x - 2 = 0 \text{ or } x - 3 = 0$$

Thus, the other root of the given quadratic equation is 2.

4. (c) $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0$

$\Rightarrow \sqrt{7}x^2 - 13x + 7x - 13\sqrt{7} = 0$

$\Rightarrow x(\sqrt{7}x - 13) + \sqrt{7}(\sqrt{7}x - 13) = 0$

$\Rightarrow (x + \sqrt{7})(\sqrt{7}x - 13) = 0$

$\Rightarrow x + \sqrt{7} = 0$ or $\sqrt{7}x - 13 = 0$

$\Rightarrow x = -\sqrt{7}$ or $x = \frac{13}{\sqrt{7}} = \frac{13\sqrt{7}}{7}$

Thus, the two roots of given quadratic equation are $-\sqrt{7}$ and $\frac{13\sqrt{7}}{7}$.

5. **(a)** The given quadratic equation is

$$3a^2x^2 - abx - 2b^2 = 0 \Rightarrow 3a^2x^2 - 3abx + 2abx - 2b^2 = 0$$
$$\Rightarrow 3ax(ax-b) + 2b(ax-b) = 0$$
$$\Rightarrow (ax-b)(3ax+2b) = 0$$
$$\Rightarrow ax = b \text{ or } 3ax = -2b$$
$$\Rightarrow x = \frac{b}{a} \text{ or } x = -\frac{2b}{3a}.$$

6. **(b)** The given quadratic equation is

$$a^2x^2 - 3abx + 2b^2 = 0 \Rightarrow a^2x^2 - 2abx - abx + 2b^2 = 0$$
$$\Rightarrow ax(ax-2b) - b(ax-2b) = 0$$
$$\Rightarrow (ax-2b)(ax-b) = 0$$
$$\Rightarrow ax - 2b = 0 \text{ or } ax - b = 0$$
$$\Rightarrow x = \frac{2b}{a} \text{ or } x = \frac{b}{a}$$

Thus, the two roots of the given quadratic equation are $\frac{2b}{a}$ and $\frac{b}{a}$.

7. **(b)** Sum of the roots $= \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

Product of the roots $= (\sqrt{2})(2\sqrt{2}) = 4$

Hence the required quadratic equation is x^2 = (sum of the roots) x + (Product of two roots) = 0

$$\Rightarrow x^2 - 3\sqrt{2x} + 4 = 0.$$

8. **(c)** The given quadratic equation is

$$ax^2 + (4a^2 - 3b)x - 12ab = 0$$
$$\Rightarrow ax^2 + 4ax - 3bx - 12ab = 0$$
$$\Rightarrow ax(x+4a) - 3b(x+4a) = 0$$
$$\Rightarrow (ax-3b)(x+4a) = 0$$
$$\Rightarrow ax - 3b = 0 \text{ or } x + 4a = 0$$
$$\Rightarrow x = \frac{3b}{a} \text{ or } x = -4a$$

Thus, the two roots of the given quadratic equation are $-4a$ and $\frac{3b}{a}$.

9. **(a)** The required quadratic equation is x^2 − (sum of the roots) x + (product of the roots) = 0

$$\Rightarrow x^2 - 6x - 16 = 0.$$

10. **(c)** The given quadratic equation is

$3x^2 - 5x + P = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 3$, $b = -5$, $c = P$

If the given quadratic equation has equal roots then its discriminant = 0

$$\Rightarrow b^2 - 4ac = 0 \Rightarrow (-5)^2 - 4(3)(P) = 0$$
$$\Rightarrow 25 - 12P = 0 \Rightarrow P = \frac{25}{12}$$

11. **(d)** Since α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$,

$$\therefore \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$
$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$
$$= \left(-\frac{b}{a}\right)^2 - 2\frac{c}{a} = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}.$$

12. **(b)** Since α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$

$$\therefore \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{a}{c}$$

Now, $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$$= \left(-\frac{b}{a}\right)^3 - 3\frac{c}{a}\left(\frac{-b}{a}\right)$$
$$= \frac{-b^3}{a} + \frac{3bc}{a^2} = \frac{-b^2 + 3abc}{a^3}$$
$$= \frac{b(3ac - b^2)}{a^3}$$

13. **(a)** Since α and β are the roots of the equation $ax^2 + bx + c = 0$

$$\therefore \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$$= \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\frac{c}{a}} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}$$
$$= \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c}{a}} = \frac{b^2 - 2ac}{ac}.$$

14. **(c)** One root is $\sqrt{5}$

So the other root is $-\sqrt{5}$

$\therefore$ Sum of the roots is = 0

and product of the roots $= (\sqrt{5})(-\sqrt{5}) = -5$

$\therefore$ Required equation is x^2 − (sum of the roots)x + (product of the roots) = 0

$$\Rightarrow x^2 - 5 = 0.$$

15. **(b)** The equation $x^2 - px + q = 0$; $p, q \in R$ has no real root if

$B^2 < 4AC$

$\Rightarrow (-p)^2 < 4.1.q \, [\because A = 1, B = -p, C = q]$

$\Rightarrow p^2 < 4q.$

16. **(b)** The given quadratic equation is $x^2 + 5px + 16 = 0(1)$

Comparing it with $ax^2 + bx + c = 0$, we get $a = 1$, $b = 5p$, $c = 16$

If equation (1) has no real roots, then discriminant < 0

$\Rightarrow b^2 - 4ac < 0 \Rightarrow (5p)^2 - 4(1)(16) < 0$

$\Rightarrow 25p^2 - 64 < 0 \Rightarrow 25p^2 < 64$

$\Rightarrow p^2 < \frac{64}{25} \Rightarrow p^2 - \frac{64}{25} < 0$

$\Rightarrow \left(p - \frac{8}{5}\right)\left(p + \frac{8}{5}\right) < 0$

$\Rightarrow$ either $p - \frac{8}{5} > 0$ and $p + \frac{8}{5} < 0$

i.e., $p > \frac{8}{5}$ and $p < -\frac{8}{5}$,

which is not possible

or, $p - \frac{8}{5} < 0$ and $p + \frac{8}{5} > 0$

i.e., $p < \frac{8}{5}$ and $p > -\frac{8}{5}$ i.e., $-\frac{8}{5} < p < \frac{8}{5}$.

17. **(b)** We have $3z^2 + 5z + k$

Here $a = 3$, $b = 5$, $c = k$

$D = b^2 - 4ac = 25 - 12k$

For equal linear factors to exist, $D \geq 0$

i.e., $25 - 12k \geq 0 \Rightarrow 25 \geq 12k$

$\Rightarrow k \leq \frac{25}{12}$

Therefore, the set of real numbers $\leq \frac{25}{12}$ gives the set of value of k for which the given quadratic polynomial can be factored into the product of real linear factors.

18. **(d)** Putting $x = 3$, we get

$27 + 3(k - 1) + 9 = 0$

or $27 + 3k - 3 + 9 = 0$

or $3k = -33$ or $k = -11$.

19. **(a)** The given quadratic equation is $3x^2 - 10x + 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 3$, $b = -10$, $c = 3$

$\therefore$ Sum of the roots $= -\frac{b}{a} = \frac{10}{3}$

$\because$ One root $= \frac{1}{3}$

$\therefore$ The other root $= \frac{10}{3} - \frac{1}{3} = \frac{9}{3} = 3.$

20 . **(c)** $x^4 + 7x^2 + 16 = (x^4 + 8x^2 + 16) - x^2$

$= (x^2 + 4)^2 - x^2$

$= (x^2 + 4 + x)(x^2 + 4 - x)$

$= (x^2 + x + 4)(x^2 - x + 4).$

21. **(d)** $x^2 - 7x + 10 = 0 \Leftrightarrow (x - 5)(x - 2) = 0$

$\Leftrightarrow x = 5, 2$

$x^2 - 10x + 16 = 0 \Leftrightarrow (x - 8)(x - 2) = 0$

$\Leftrightarrow x = 8, 2$

$\therefore$ Common root is 2.

22. **(b)** Here $a = 1$, $b = p$, $c = q$

The roots of the equation $x^2 + px + q = 0$ are equal if

$b^2 - 4ac = 0 \Rightarrow p^2 - 4q = 0 \Rightarrow p^2 = 4q.$

23. **(d)** $x^2 - 6x + 5 = 0 \Leftrightarrow (x - 5)(x - 1) = 0$

$\Leftrightarrow x = 5$ or 1

Also $|x - 3| = 2 \Leftrightarrow x - 3 = 2$ or $-(x - 3) = 2 \Leftrightarrow x = 5$ or $x = 1$

$\therefore x^2 - 6x + 5 = 0$ and $|x - 3| = 2$ are equivalent.

24. **(a)** Let the smaller part be x. Then the larger part $= 16 - x$. Now

$2(16 - x)^2 - x^2 = 164 \Rightarrow 2(256 + x^2 - 32x) - x^2 = 164$

$\Rightarrow x^2 - 64x + 348 = 0$

$\Rightarrow x^2 - 6x - 58x + 348 = 0$

$\Rightarrow x(x - 6) - 58(x - 6) = 0$

$\Rightarrow (x - 6)(x - 58) = 0$

$x = 6$ or $x = 58$

But $m = 58$ is not possible, since sum of the two parts is 16

$\therefore$ $x = 6$, $\therefore$ other part $= 10$.

25. **(b)** The given equation is of the form

$ax^2 + bx + c = 0$

Also, $D = \sqrt{9} = 3$

So, roots are rational

Hence, both the roots must be integers.

26. **(b)** Given equation is

$2x - x^2 = x - 2 \Leftrightarrow x^2 - x - 2 = 0$

$\Leftrightarrow (x + 1)(x - 2) = 0$

$\Leftrightarrow x = 2$ or $= -1$.

27. **(d)** $\log_{10}(x^2 - 6x + 45) = 2$

$\Leftrightarrow x^2 - 6x + 45 = 10^2 = 100$

$\Leftrightarrow x^2 - 6x - 55 = 0$

$\Leftrightarrow (x - 11)(x + 5) = 0$

$\Leftrightarrow x = 11$ or $x = -5$.

28. **(c)** Comparing $x^2 - 5x + 6 = 0$ with

$ax^2 + bx + c = 0$

$a = 1, b = -5, c = 6$

$\therefore \quad \alpha + \beta = \dfrac{-b}{a} = \dfrac{5}{1} = 5$

$\alpha\beta = \dfrac{c}{a} = 6$

Now, we are to form an equation whose roots are $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$.

So the required equation is

x^2 − (sum of roots)x + (Product of roots) = 0

$$x^2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha}\cdot\frac{1}{\beta}\right) = 0$$

$$x^2 - \left(\frac{\alpha + \beta}{\alpha\beta}\right)x + \left(\frac{1}{\alpha\beta}\right) = 0$$

$$x^2 - \frac{5}{6}x + \frac{1}{6} = 0$$

$6x^2 - 5x + 1 = 0$.

29. **(c)** Given equation is: $y + \dfrac{1}{y} = \dfrac{10}{3}$,

where $y = \dfrac{x + 4}{x - 4}$

$\therefore \quad 3y^2 - 10y + 3 = 0 \Rightarrow y = 3, \dfrac{1}{3}$

$\therefore \quad y = \dfrac{x+4}{x-4} = 3$ or, $y = \dfrac{x+4}{x-4} = \dfrac{1}{3}$

$3x + 12 = x - 4$ or, $x + 4 = 3x - 12$

$\Rightarrow \quad x = -8$ or, $x = 8$

30. **(c)** For reciprocal roots, product of roots must be 1

$\therefore \quad \dfrac{c}{a} = \text{I}$ i.e., $c = a$

31. **(a)** Sum of the roots = 6

One root $= 3 - \sqrt{5}$

$\therefore$ the other root $= 6 - (3 - \sqrt{5}) = 3 + \sqrt{5}$

$\therefore$ Product of roots $= (3 - \sqrt{5})(3 + \sqrt{5})$
$= 9 - 5 = 4$

Hence the required equation is

x^2 − (sum of the roots)x + (product of roots) = 0

$\Rightarrow \quad x^2 - 6x + 4 = 0$.

32. **(a)** $\alpha + \beta = 6$ and $3\alpha + 2\beta = 20$

$\Rightarrow \quad \alpha = 4, \; \beta = 2$

Product of the roots = k

So, $k = \alpha\beta = 4 \times 2 = 8$.

33. **(a)** Let, the two consecutive odd positive integers be $2x + 1$ and $2x + 3$ where x is a whole number.

Now,

$(2x + 1)^2 + (2x + 3)^2 = 290$

$\Rightarrow \quad 4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 290$

$\Rightarrow \quad 8x^2 + 16x - 280 = 0$

$\Rightarrow \quad x^2 + 2x - 35 = 0$

$\Rightarrow \quad (x + 7)(x - 5) = 0 \Rightarrow x = 7, -5$

But, $x = -7$ is not possible, since −7 is not a whole number

$\therefore \quad x = 5$.

34. **(d)** Let, the roots be α and $-\alpha$. Then, sum of roots = 0

Also, roots being not equal, discriminant ≠ 0

$\therefore \dfrac{q}{p} = 0$ and $q^2 - 4pr \neq 0$

$\Leftrightarrow q = 0$ and $pr \neq 0$.

35. **(a)** Comparing $x^2 - 2(1 + 3x)x + 7(3 + 2k) = 0$ with

$ax_2 + bx + c = 0$, we get

$a = 1, b = -2(1 + 3k), c = 7(3 + 2k)$

For equal roots $D = b^2 - 4ac = 0$

$\therefore 4(1 + 3k)^2 - 4 \times 1 \times 7(3 + 2k) = 0$

$\Rightarrow \quad 4(1 + k)^2 + 6k) - 84 - 56k = 0$

$\Rightarrow \quad 36k^2 - 32k - 80 = 0$

$\Rightarrow \quad 9k - 8k - 20 = 0$

$$k = \frac{8 \pm \sqrt{64 - 4(9)(-20)}}{2 \times 9}$$

$$= \frac{8 \pm \sqrt{784}}{18} = \frac{8 \pm 28}{18}$$

$$= \frac{36}{18}, \frac{-20}{18} = 2, \frac{-10}{9}.$$

36. **(d)** Let, α be a common root of the given equations.

Then, $\alpha^2 + 2\alpha - 3 = 0$ and $\alpha^2 + 3\alpha - k = 0$

$$\therefore \quad \frac{\alpha^2}{-2k + 9} = \frac{\alpha}{-3 + k} = \frac{1}{3 - 2}$$

So, $\alpha^2 = \dfrac{9 - 2k}{1}$ and $\alpha = \dfrac{k - 3}{1}$

So, $(9-2k)=(k-3)^2$ or $k^2-4k=0$

or, $k(k-4)=0$, so $k=4$.

37. **(d)** Given equation is: $2^{2x} - 3 \,.\, 2^x \times 2^2 + 32 = 0$

or, $2^{2x} - 12 \times 2^x + 32 = 0$

$\Rightarrow y^2-12y+32=0$, where $2^x=y$

$\Rightarrow (y-8)(y-4)=0 \Rightarrow y=8, y=4$

$\therefore\ 2^x=8$ or, $2^x=4$

$\Rightarrow 2^x=2^3$ or, $2^x=2^2$

$\Rightarrow x=3$ or, $x=2$.

38. **(a)** Let, the roots be 3α and 2α

Then, $3\alpha+2\alpha=\dfrac{m}{12}\Rightarrow\alpha=\dfrac{m}{60}$

$$\therefore\ \frac{5}{72}=\left(\frac{m}{60}\right)^2\Leftrightarrow\frac{5}{72}=\frac{m^2}{3600}$$

$$\Leftrightarrow m^2=\frac{3600\times5}{72}=250$$

$$\therefore\ m=\sqrt{250}=5\sqrt{10}.$$

39. **(c)** $\because$ α, β are the roots of the equation $2x^2 - 3x + 1 = 0$

$$\therefore\ \alpha+\beta=\frac{3}{2} \qquad \ldots(1)$$

$$\text{and}\ \alpha\beta=\frac{1}{2} \qquad \ldots(2)$$

We are to form a quadratic equation whose roots are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$ S = sum of the roots

$$=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\left(\frac{3}{2}\right)^2-2\left(\frac{1}{2}\right)}{\frac{1}{2}} \qquad \text{[using (1) and (2)]}$$

$$=\frac{\frac{9}{4}-1}{\frac{1}{2}}=\frac{5}{4}\times\frac{2}{1}=\frac{5}{2}$$

P = Product of the roots

$$=\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=1$$

Hence the required quadratic equation is x^2 – (sum of the roots)x + (Product of the roots) = 0

$$\Rightarrow\ x^2-\frac{5}{2}x+1=0$$

$$\Rightarrow\ 2x^2-5x+2=0.$$

40. **(a)** The given quadratic equation is

$3x^2 - 20x + 17 = 0$ (1)

Compare with $ax^2 + bx + c = 0$, we get

$a = 3,\ b = -20,\ c = 17$

The roots of (1) are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{20\pm\sqrt{400-4(3)(17)}}{2\times3}$$

$$=\frac{20\pm\sqrt{196}}{6}=\frac{20+14}{6},\frac{20-14}{6}$$

$$=\frac{34}{6},\frac{6}{6}=\frac{17}{3},1$$

Hence the roots of (1) are $\dfrac{17}{3}$ and 1. So we have to form an equation whose are $\dfrac{3}{17}$ and 1

Sum of the roots $=\dfrac{3}{17}+1=\dfrac{20}{17}$

Product of the roots $=\dfrac{3}{17}\times1=\dfrac{3}{17}$

Hence, the required equation is

$$x^2-\frac{20}{17}x+\frac{3}{17}=0\Rightarrow17x^2-20x+3=0.$$

41. **(a)** $\because$ α, β are the roots of the equation $x2 - 3lx + l^2 = 0$

$$\alpha+\beta=3\lambda \qquad \ldots(1)$$

$$\text{and}\ \alpha\beta=\lambda^2 \qquad \ldots(2)$$

Now, $\alpha^2+\beta^2=\dfrac{7}{4}$ (given)

$$\Rightarrow\ (\alpha+\beta)^2-2\alpha\beta=\frac{7}{4}$$

$$\Rightarrow(3\lambda)^2-2\lambda^2=\frac{7}{4}\Rightarrow7\lambda^2=\frac{7}{4}$$

$$\Rightarrow\lambda^2=\frac{1}{4}\Rightarrow\lambda=\pm\frac{1}{2}.$$

42. **(b)** $\because$ α, β are the roots of the equation

$$ax^2 + bx + b = 0$$

$$\therefore \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{b}{a}$$

Now, $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{b}{a}} = \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{b}{a}}$

$$= \frac{-\frac{b}{a}}{\sqrt{\frac{b}{a}}} + \sqrt{\frac{b}{a}}$$

$$= -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}} = 0.$$

43. **(c)** Comparing $x^2 - x + 1$with $ax^2 + bx + c$ we have

$a = 1$, $b = -1$, $c = 1$

Here $D = b^2 - 4ac = (-1)^2 - 4(1)(1)$

$= 1 - 4 = -3$

Since $D < 0$, so the given expression has no proper linear factor.

44. **(c)** Let, the breadth of the rectangular plot be x m. Then the length of rectangular plot $= (x + 8)$ m

$\therefore$ Area = Length × Breadth $= x(x + 8)m^2$ But the area of the plot is given to be 308 m^2

$$\therefore \quad x(x+8) = 308 \Rightarrow x^2 + 8x - 308 = 0$$

$$\Rightarrow x^2 + 22x - 14x - 308 = 0$$

$$\Rightarrow x(x+22) - 14(x-22) = 0$$

$$\Rightarrow (x+22)(x-14) = 0$$

$$\Rightarrow x = 14, -22$$

But, $x = -22$ is not possible, since breath cannot be negative

$\therefore \quad x = 14$

Hence the breadth of the rectangular plot = 14 m Length of the rectangular plot = (14 + 8) m = 22 m.

45. **(d)** Let, α, β be the roots of the equation $x^2 + kx + 12 = 0$

$$\therefore \quad \alpha + \beta = -k \text{ and } \alpha\beta = 12$$

Now $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$(1)^2 = k^2 - 4(12) \Rightarrow k^2 = 49 \Rightarrow k = \pm 7.$$

46. **(c)** Put $x - \frac{1}{x} = y$

$$\therefore \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = \left(x + \frac{1}{x}\right)^2 + 4 = y^2 + 4$$

So, given equation becomes

$$\Rightarrow y\left(y - \frac{3}{2}\right) = 0 \Rightarrow y = 0 \text{ or } y = \frac{3}{2}$$

$$\therefore \quad x - \frac{1}{x} = 0 \text{ or } x - \frac{1}{x} = \frac{3}{2}$$

$$\Rightarrow x^2 - 1 = 0 \text{ or } 2x^2 - 3x - 2 = 0$$

$$\Rightarrow x = \pm 1 \quad \text{or } (2x+1)(x-2) = 0$$

or $x = -1/2$ or $x = 2$

47. **(a)** $\because$ α, β are the roots of the equation $x^2 - 8x + 1 = 0$

$$\therefore \quad \alpha + \beta = \frac{-b}{a} = \frac{-(-8)}{1} = 8$$

and $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$\Rightarrow 40 = (8)^2 - 2k \Rightarrow 2k = 24 \Rightarrow k = 12.$$

48. **(a)** The given equation is $3x^2 + (2k + 1)x - k - 5 = 0$

Compare with $ax^2 + bx + c = 0$, we get

$$a = 3, b = 2k + 1, c = -k - 5$$

$\therefore$ Sum of the roots $= \frac{-b}{a} = \frac{-(2k+1)}{3}$

and Product of the roots $= \frac{c}{a} = \frac{-k-5}{3} = \frac{-(k+5)}{3}$

$\because$ Sum of the roots = Product of the roots

$$\therefore \frac{-(2k+1)}{3} = -\frac{(k+5)}{3} \Rightarrow 2k + 1 = k + 5$$

$$\Rightarrow 2k - k = 5 - 1$$

$$\Rightarrow k = 4.$$

Exercise–2
(Based on Memory)

1. (a) $a^2 + 4b^2 + 4b - 4ab - 2a - 8$

$= a^2 + 4b^2 - 4ab - 2a + 4b - 8$

$= (a - 2b)^2 - 2(a - 2b) - 8$

Let, $(a - 2b) = x$

$\therefore$ The given expression $= x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

$= (a - 2b - 4)(a - 2b + 2)$

2. (c) $x = \sqrt{30 + \sqrt{30 + \sqrt{30 + \cdots}}}$

On squaring both sides, we have

$x^2 = 30 + \sqrt{30 + \sqrt{30 + \sqrt{30 + \cdots}}}$

$\Rightarrow x^2 = 30 + x \Leftrightarrow x^2 - x - 30 = 0$

$\Rightarrow x^2 - 6x + 5x - 30 = 0$

$\Rightarrow x(x - 6) + 5(x - 6) = 0$

$\Rightarrow (x - 6)(x + 5) = 0$

$\Rightarrow x = 6$ because $x \neq -5$

3. (d) Let, $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}$

On squaring both sides, we have

$x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}$

$\Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x(x - 3) + 2(x - 3) = 0$

$\Rightarrow (x + 2)(x - 3) = 0$

$\Rightarrow x = 3$ and $x \neq -2$ because numbers are positive.

4. (d) Let, the two natural consecutive odd numbers be n and $(n + 2)$

Now, according to the question,

$\Rightarrow n^2 + (n + 2)^2 = 394$

$\Rightarrow n^2 + n^2 + 4 + 4n = 394$

$\Rightarrow 2n^2 + 4n - 390 = 0$

$\Rightarrow n^2 + 2n - 195 = 0$

$\Rightarrow n^2 + 15n - 13n - 195 = 0$

$\Rightarrow n(n + 15) - 13(n + 15) = 0$

$\Rightarrow (n - 13)(n + 15) = 0$

$\Rightarrow n = 13$ and $n \neq -15$

$\therefore$ numbers are 13 and 15.

$\therefore$ the sum of the numbers $= 13 + 15 = 28$

Quicker Approach:

By mental operation, $13^2 + 15^2 = 169 + 225 = 394$

$\therefore$ Required sum $= 13 + 15 = 28$

5. (c) I. $6x^2 + 41x + 63 = 0$

or, $6x^2 + 27x + 14x + 63 = 0$

or, $3x(2x + 9) + 7(2x + 9) = 0$

or, $(3x + 7)(2x + 9) = 0$

$\therefore x = -\frac{3}{7}, -\frac{2}{9}$

II. $4y^2 + 8y + 3 = 0$

or, $4y^2 + 6y + 2y + 3 = 0$

or, $2y(2y + 3) + l(2y + 3)$

or, $(2y + l)(2y + 3)$

$\therefore y = -\frac{1}{2}, -\frac{3}{2}$

Hence, $x < y$

6. (e) I. $x^2 + 10x + 24 = 0$

or, $x^2 + 6x + 4x + 24 = 0$

or, $x(x + 6) + 4(x + 6) = 0$

or, $(x + 4)(x + 6) = 0$

$\therefore x = -4, -6$

II. $4y^2 - 17y + 18 = 0$

or, $4y^2 - 9y - 8y + 18 = 0$

or, $4y(y - 2) - 9(y - 2) = 0$

or, $(4y - 9)(y - 2) = 0$

$\therefore y = \frac{9}{4}, 2$

Hence, $x < y$

7. (c) I. $24x^2 + 38x + 15 = 0$

or, $24x^2 + 20x + 18x + 15 = 0$

or, $4x(6x + 5) + 3(6x + 5) = 0$

or, $(4x + 3)(6x + 5) = 0$

$\therefore x = -\frac{3}{4}, -\frac{5}{6}$

II. $12y^2 + 28y + 15 = 0$

or, $12y^2 + 18y + 10y + 15 = 0$

or, $6y(2y + 3) + 5(2y + 3) = 0$

or, $(6y + 5)(2y + 3) = 0$

$\therefore \quad y = -\frac{5}{6}, -\frac{3}{2}$

Hence, $x \geq y$

8. **(d)** **I.** $3x^2 - 20x - 32 = 0$

or, $3x^2 - 12x - 8x - 32 = 0$

or, $3x(x - 4) - 8(x - 4) = 0$

or, $(3x - 8)(x - 4) = 0$

II. $2y^2 - 3y - 20 = 0$

or, $2y^2 - 8y + 5y - 20 = 0$

or, $2y(y - 4) + 5(y - 4) = 0$

or, $(2y + 5)(y - 4) = 0$

$\therefore \quad y = 4, -\frac{5}{2}$

Hence no relationship can be established.

9. **(d)** **I.** $x^2 - 20x + 91 = 0$

or, $x^2 - 13x - 7x + 91 = 0$

or, $x(x - 13) - 7(x - 13) = 0$

or, $(x - 7)(x - 13) = 0$

$\Rightarrow \quad x = 13, 7$

II. $y^2 - 32y + 247 = 0$

or, $y^2 - 19y - 13y + 247 = 0$

or, $y(y - 19) - 13(y - 19) = 0$

or, $(y - 13)(y - 19) = 0$

$\Rightarrow \quad y = 13, 19$

Hence, $x \leq y$

10. **(a)** **I.** $x^2 - 24x + 144 = 0$

or, $x^2 - 12x - 12x + 144 = 0$

or, $x(x - 12) - 12(x - 12) = 0$

or, $(x - 12)^2 = 0$

$\therefore \quad x = 12$

II. $y^2 - 26y + 169 = 0$

or, $y^2 - 13y - 13y + 169 = 0$

or, $y(y - 13) - 13(y - 13) = 0$

or, $(y - 13)^2 = 0$

$\therefore \quad y = 13$

Hence, $x < y$

11. **(d)** **I.** $2x^2 + 3x - 20 = 0$

or, $2x^2 + 8x - 5x - 20 = 0$

or, $2x(x + 4) - 5(x + 4) = 0$

or, $(2x - 5)(x + 4) = 0$

or, $x = \frac{5}{2}, -4$

II. $2y^2 + 19y + 44 = 0$

or, $2y^2 + 11y + 8y + 44 = 0$

or, $2y(2y + 11) + 4(2y + 11) = 0$

or, $(y + 4)(2y + 11) = 0$

$\therefore \quad y = -4, -\frac{11}{2}$

Hence, $x \geq y$

12. **(e)** **I.** $6x^2 + 77x + 121 = 0$

or, $6x^2 + 66x + 11x + 121 = 0$

or, $6x(x + 11) + 11(x + 11) = 0$

or, $(6x + 11)(x + 11) = 0$

or, $x = -\frac{11}{6}, -11$

II. $y^2 + 9y - 22 = 0$

or, $y^2 + 11y - 2y - 22 = 0$

or, $y(y + 11) - 2(y + 11)$

or, $(y - 2)(y + 11) = 0$

or, $y = 2, -11$

Hence, no relationship can be established between x and y.

13. **(b)** **I.** $x^2 - 6x = 7$

or, $x^2 - 6x - 7 = 0$

or, $x^2 - 7x + x - 7 = 0$

or, $x(x - 7) + 1(x - 7) = 0$

or, $(x + 1)(x - 7) = 0$

or, $x = -1, 7$

II. $2y^2 + 13y + 15 = 0$

or, $2y^2 + 10y + 3y + 15 = 0$

or, $2y(y + 5) + 3(y + 5) = 0$

or, $(2y + 3)(y + 5) = 0$

or $y = -\frac{3}{2}, -5$

Hence, $x > y$

14. **(d)** **I.** $10x^2 - 7x + 1 = 0$

or, $10x^2 - 5x - 2x + 1 = 0$

or, $5x(2x - 1) - 1(2x - 1) = 0$

or, $(5x - 1)(2x - 1) = 0$

or, $x = \frac{1}{5}, \frac{1}{2}$

II. $35y^2 - 12y + 1 = 0$

or, $35y^2 - 7y + 5y + 1 = 0$

or, $7y(5y - 1) - 1(5y - 1) = 0$

or, $(7y - 1)(5y - 1) = 0$

or, $y = \frac{1}{7}, \frac{1}{5}$

Hence, $x \geq y$

15. (d) I. $x^2 - 19x + 84 = 0$

$\Rightarrow x^2 - 7x - 12x + 84 = 0$

$\Rightarrow (x - 7)(x - 12) = 0$

$\Rightarrow x = 7, 12$

II. $y^2 - 25y + 156 = 0$

$\Rightarrow y^2 - 13y - 12y + 156 = 0$

$\Rightarrow (y - 13)(y - 12) = 0$

$\Rightarrow y = 12, 13$

$\therefore x \leq y$

16. (b) I. $x^3 - 468 = 1729$

$\Rightarrow x^3 = 2197$

$\Rightarrow x = 13$

II. $y^2 - 1733 + 1564$

$\Rightarrow y^2 = 169$

$\Rightarrow y = \pm 13$

$\therefore x \geq y$

17. (e) I. $\frac{9}{\sqrt{x}} + \frac{19}{\sqrt{x}} = \sqrt{x} \Rightarrow 9 + 19 = \sqrt{x} \times \sqrt{x} \Rightarrow x = 28$

II. $y^2 - \frac{(2\times 14)^{11/2}}{\sqrt{y}} = 0 \Rightarrow y^5\sqrt{y} - (2\times 14)^{11/2} = 0$

$\Rightarrow y^{11/2} = (2 \times 14)^{11/2}$

$\Rightarrow y = 2 \times 14 = 28$

$\therefore x = y$

18. (a) I. $\sqrt{784}x + 1234 = 1486$

$\Rightarrow \sqrt{784}x = 252$

$\Rightarrow 28x = 252$

$\Rightarrow x = 9$

II. $\sqrt{1089}y + 2081 = 2345$

$\Rightarrow 33y = 264$

$\Rightarrow y = 8$

$\therefore x > y$

19. (a) I. $\frac{12}{\sqrt{x}} - \frac{23}{\sqrt{x}} = 5\sqrt{x}$

$\Rightarrow 12 - 23 = 5\sqrt{x} \times \sqrt{x}$

$\therefore x = \frac{-11}{5} = -2.2$

II. $\frac{\sqrt{y}}{12} - \frac{5\sqrt{y}}{12} = \frac{1}{\sqrt{y}}$

$\Rightarrow \sqrt{y}\left(\frac{1}{12} - \frac{5}{12}\right) = \frac{1}{\sqrt{y}}$

$\Rightarrow y\left(\frac{-4}{12}\right) = 1$

$\Rightarrow y = \frac{-12}{4} = -3$

$\therefore x > y$

20. (e) $4x + 7y = 209$ …(1)

$12x - 14y = -38$ …(2)

Multiplying (1) by (2):

$8x + 14y = 418$ (3)

Adding (2) and (3):

$20x = 380 \Rightarrow x = 19$

Substituting the value of x in (1), we get

$76 + 7y = 209$

$\Rightarrow 7y = 133 \Rightarrow y = 19$

$\therefore x = y$

21. (a) I. $17x^2 + 48x - 9 = 0$

$\Rightarrow 17x^2 + 51x - 3x - 9 = 0$

$\Rightarrow 17x(x + 3) - 3(x + 3) = 0$

$\Rightarrow (17x - 3)(x + 3) = 0$

$\Rightarrow x = -3, \frac{3}{17}$

II. $13y^2 - 32y + 12 = 0$

$\Rightarrow 13y^2 - 26y - 6y + 12 = 0$

$\Rightarrow 13y(y - 2) - 6(y - 2) = 0$

$\Rightarrow (13y - 6)(y - 2) = 0$

$\Rightarrow y = 2, \frac{6}{13}$

$\therefore x < y$

22. (b) I. $16x^2 + 20x + 6 = 0$

$\Rightarrow 16x^2 + 12x + 8x + 6 = 0$

$\Rightarrow 4x(4x + 3) + 2(4x + 3) = 0$

$\Rightarrow (4x + 2)(4x + 3) = 0$

$\Rightarrow x = -\frac{3}{4}, -\frac{2}{4}$

II. $10y^2 + 38y + 24 = 0$

$\Rightarrow$ $10y^2 + 30y + 8y + 24 = 0$

$\Rightarrow$ $10y(y + 3) + 8(y + 3) = 0$

$\Rightarrow$ $(10y + 8)(y + 3) = 0$

$\Rightarrow$ $y = -3, \ -\dfrac{4}{5}$

$\therefore$ $x > y$

23. **(c)** **I.** $8x^2 + 6x - 5 = 0$

$\Rightarrow$ $8x^2 + 10x - 4x - 5 = 0$

$\Rightarrow$ $2x(4x + 5) - 1(4x + 5) = 0$

$\Rightarrow$ $(2x - 1)(4x + 5) = 0$

$\Rightarrow$ $x = \dfrac{1}{2}, \ -\dfrac{5}{4}$

II. $12y^2 - 22y + 8 = 0$

$\Rightarrow$ $12y^2 - 16y - 6y + 8 = 0$

$\Rightarrow$ $4y(3y - 4) - 2(3y - 4) = 0$

$\Rightarrow$ $(4y - 2)(3y - 4) = 0$

$\Rightarrow$ $y = \dfrac{1}{2}, \ \dfrac{4}{3}$

$\therefore$ $x \le y$

24. **(d)** **I.** $18x^2 + 18x + 4 = 0$

$\Rightarrow$ $18x^2 + 12x + 6x + 4 = 0$

$\Rightarrow$ $6x(3x + 2) + 2(3x + 2) = 0$

$\Rightarrow$ $(6x + 2)(3x + 2) = 0$

$\Rightarrow$ $x = -\dfrac{1}{3}, \ -\dfrac{2}{3}$

II. $12y^2 + 29y + 14 = 0$

$\Rightarrow$ $12y^2 + 21y + 8y + 14 = 0$

$\Rightarrow$ $3y(4y + 7) + 2(4y + 7) = 0$

$\Rightarrow$ $(3y + 2)(4y + 7) = 6$

$\Rightarrow$ $y = -\dfrac{2}{3}, \ -\dfrac{7}{4}$

$\therefore$ $x \ge y$

25. **(b)** **I.** $x^2 - 11x + 24 = 0$

$\Rightarrow$ $x^2 - 8x - 3x + 24 = 0$

$\Rightarrow$ $x(x - 8) - 3(x - 8) = 0$

$\Rightarrow$ $(x - 3)(x - 8) = 0$

$\therefore$ $x = 3$ or, 8

II. $2y^2 - 9y + 9 = 0$

$\Rightarrow$ $2y^2 - 3y - 6y + 9 = 0$

$\Rightarrow$ $y(2y - 3) - 3(2y - 3) = 0$

$\Rightarrow$ $(2y - 3)(y - 3) = 0$

$\therefore$ $y = \dfrac{3}{2}$ or, 3

$\therefore$ $x \ge y$

26. **(c)** **I.** $x^3 \times 13 = x^2 \times 247$

$\Rightarrow$ $\dfrac{x^3}{x^2} = \dfrac{247}{13}$

$\therefore$ $x = 19$

II. $y^{1/3} \times 14 = 249 \div y^{2/3}$

$\Rightarrow$ $y^{\frac{1}{3}} \times y^{\frac{2}{3}} = \dfrac{294}{14}$

$\Rightarrow$ $y^{\frac{1}{3}} \times y^{\frac{2}{3}} = 21$

$\therefore$ $y = 21$

Clearly, $x < y$

27. **(d)** $\dfrac{12 \times 4}{x^{\frac{4}{7}}} - \dfrac{3 \times 4}{x^{\frac{4}{7}}} = x^{\frac{10}{7}}$

$\Rightarrow$ $\dfrac{48}{x^{\frac{4}{7}}} - \dfrac{12}{x^{\frac{4}{7}}} = x^{\frac{10}{7}}$

$\Rightarrow$ $\dfrac{48 - 12}{x^{\frac{4}{7}}} = x^{\frac{10}{7}}$

$\Rightarrow$ $36 = x^{\frac{10}{7} + \frac{4}{7}}$ $\Leftrightarrow$ $36 = x^2$

$\therefore$ $x = \sqrt{36} = \pm 6$

II. $y^3 + 783 = 999$

$\Rightarrow$ $y^3 = 999 - 783$ $\Leftrightarrow$ $y^3 = 216$

$\therefore$ $y = \sqrt[3]{216} \le 6$

Clearly, $x \le y$

28. **(c)** **I.** $\sqrt{500}x + \sqrt{402} = 0$

$\Rightarrow$ $\sqrt{500}x = -\sqrt{402}$

$\therefore$ $x = -\sqrt{\dfrac{402}{500}} = -\sqrt{\dfrac{400}{500}} = -0.9$

II. $\sqrt{360}y + (200)^{1/2} = 0$

$\Rightarrow$ $\sqrt{360}y = -\sqrt{200}$

$\therefore$ $y = -\sqrt{\dfrac{200}{360}} = -0.74$

Clearly, $x < y$

29. **(c)** **I.** $(17)^2 + 144 \div 18 = x$

$\Rightarrow$ $x = 17^2 + 144 \times \dfrac{1}{18}$

$\therefore$ $x = 289 + 8 = 297$

II. $(26)^2 - 18 \times 21 = y$

$\Rightarrow \quad y = 26^2 - 18 \times 21$

$\therefore \quad y = 676 - 378 = 298$

Clearly, $x < y$

30. (b) I. $16x^2 + 20x + 6 = 0$

$\Rightarrow \quad 8x^2 + 10x + 3 = 0$

$\Rightarrow \quad (4x + 3)(2x + 1) = 0$

$\therefore \quad x = -\dfrac{3}{4} \text{ or}, -\dfrac{1}{2}$

II. $10y^2 + 38y + 24 = 0$

$\Rightarrow \quad 5y^2 + 19y + 12 = 0$

$\therefore \quad (y + 3)(5y + 4) = 0$

$\therefore \quad y = -3 \text{ or}, -\dfrac{4}{5}$

Hence, $x > y$

31. (d) I. $18x^2 + 18x + 4 = 0$

$\Rightarrow \quad 9x^2 + 9x + 2 = 0$

$\Rightarrow \quad (3x + 2)(3x + 1) = 0$

$\therefore \quad x = -\dfrac{2}{3} \text{ or}, -\dfrac{1}{3}$

II. $12y^2 + 29y + 14 = 0$

$\Rightarrow \quad (3y + 2)(4y + 7) = 0$

$\therefore \quad y = -\dfrac{2}{3} \text{ or}, -\dfrac{7}{4}$

Hence, $x \geq y$

32. (c) I. $8x^2 + 6x - 5 = 0$

$\Rightarrow \quad (4x + 5)(2x - 1) = 0$

$\therefore \quad x = -\dfrac{5}{4} \text{ or}, \dfrac{1}{2}$

II. $12y^2 - 22y + 8 = 0$

$\Rightarrow \quad 6y^2 - 11y + 4 = 0$

$\Rightarrow \quad (2y - 1)(3y - 4) = 0$

$\therefore \quad y = \dfrac{1}{2} \text{ or}, \dfrac{4}{3}$

Hence, $x \leq y$

33. (a) I. $17x^2 + 48x - 9 = 0$

$\Rightarrow \quad (x + 3)(17x - 3) = 0$

$\Rightarrow \quad x = -3 \text{ or}, \dfrac{3}{17}$

II. $13y^2 - 32y + 12 = 0$

$\Rightarrow \quad (y - 2)(13y - 6) = 0$

$\therefore \quad y = 2 \text{ or}, \dfrac{6}{13}$

Hence, $x < y$

34. (e) I. $4x + 7y = 209$ …(1)

II. $12x - 14y = -38$ …(2)

Now, (1) × 2 + (2), we have

$12x - 14y = -38$

$8x + 14y = 418$

or, $\quad 20x = 380$

$\therefore \quad x = \dfrac{380}{20} = 19$

Now, putting the value of $x = 19$ in equation (1), We have,

$4 \times 19 + 7y = 209$

or, $\quad 7y = 209 - 76 = 133$

$\therefore \quad y = \dfrac{133}{7} = 19$

Hence, $x = y$

35. (a) $x = \pm 2$, $\quad y^2 + 6y + 9 = 0$

$$\begin{array}{cc} 3 & 3 \\ | & | \\ -3 & -3 \end{array}$$

36. (b) $x^2 - 7x + 12 = 0 \quad y^2 + y - 12 = 0$

$$\begin{array}{cccc} -4 & -3 & 4 & -3 \\ | & | & | & | \\ 4 & 3 & -4 & 3 \end{array}$$

37. (d) I. $x = \pm\sqrt{729} = \pm 27$ **II.** $y = 27$

38. (e) I. $x^4 = 398 + 227 = 625$

$\Rightarrow \quad x = \pm 5$

II. $y^2 = (346 - 321) = 25$

$\Rightarrow \quad y = \pm 5$

39. (c) I. $2x^2 + 11x + 14 = 0$ **II.** $4y^2 + 12y + 9 = 0$

$$\begin{array}{cc} 7 & 4 \\ | & | \\ \frac{7}{2} & \frac{4}{2} \\ | & | \\ \frac{-7}{2} & -2 \end{array} \qquad \begin{array}{cc} 6 & 6 \\ | & | \\ \frac{6}{4} & \frac{6}{4} \\ | & | \\ \frac{-3}{2} & \frac{-3}{2} \end{array}$$

40. (b) I. $x^2 - 1 = 0$

or, $\quad x^2 = 1$

or, $\quad x = \pm\sqrt{1} = \pm 1$

II. $y^2+4y+3=0$

or, $y^2+y+3y+3=0$

or, $y(y+1)+3(y+1)=0$

or, $(y+3)(y+1)=0$

or, $y=-3$ or, -1

$\therefore\ x \geq y$

41. (d) I. $x^2-7x+12=0$

or, $x^2-4x-3x+12=0$

or, $x(x-4)-3(x-4)=0$

or, $(x-3)(x-4)=0$

or, $x=3$ or, 4

II. $y^2-12y+32=0$

or, $y^2-8y-4y+32=0$

or, $y(y-8)-4(y-8)=0$

or, $(y-4)(y-8)=0$

or, $y=4$ or, 8

$\therefore\ x \leq y$

42. (c) $x=\sqrt{1000}=10,\ y=\sqrt{1331}=11$

$\therefore\ x < y$

43. (a) Solving these two linear equations, we get $x=5, y=3$.

$\therefore\ x > y$

44. (e) I. $2x^2+11x+12=0$

or, $2x^2+8x+3x+12=0$

or, $2x(x+4)+3(x+4)=0$

or, $(2x+3)(x+4)=0$

$\therefore\ x=-\frac{3}{2}$ or, -4

II. $5y^2+27y+10=0$

or, $5y^2+25y+2y+10=0$

or, $5(y+5)+2(y+5)=0$

or, $(5+2)(y+5)=0$

$\therefore\ y=-\frac{2}{5}$ or, -5