Clocks and Calendar

24

CLOCKS

The circumference of a dial of a clock (or watch) is divided into 60 equal parts called **minute spaces.** The clock has two hands—the hour hand and the minute hand. The hour hand (or short hand) indicates time in hours and the minute hand (or long hand) indicates time in minutes. In an hour, the hour hand covers 5 minute spaces while the minute hand covers 60 minute spaces. Thus, in one hour or 60 minutes, the minute hand gains 55 minute spaces over the hour hand.

SOME BASIC FACTS

1. In every hour:
   (a) Both the hands coincide once.
   (b) The hands are straight (point in opposite directions) once. In this position, the hands are 30 minute spaces apart.
   (c) The hands are twice at right angles. In this position, the hands are 15 minute spaces apart.
2. The minute hand moves through 6° in each minute whereas the hour hand moves through $\frac{1^\circ}{2}$ in each minute. Thus, in one minute, the minute hand gains $5\frac{1^\circ}{2}$ than the hour hand.
3. (a) When the hands are coincident, the angle between them is 0°.
   (b) When the hands point in opposite directions, the angle between them is 180°.
   (c) The hands are in the same straight line, when they are coincident or opposite to each other. So, the angle between the two hands is either 0° or 180°.
3. The minute hand moves 12 times as fast as the hour hand.
4. If a clock indicates 6:10, when the correct time is 6, it is said to be 10 minutes **too fast**. And if it indicates 5:50, when the correct time is 6, it is said to be 10 minutes **too slow**.

SOME USEFUL SHORTCUT METHODS

1. The two hands of the clock will be together between H and $(H + 1)$ O'clock at $\left(\frac{60H}{11}\right)$ minutes past H O'clock.

Explanation

At H O'clock the minute hand is $5H$ minute spaces behind the hour hand.

$\because$ The minute hand gains 55 minute spaces in 60 minute,

$\therefore$ The minute hand will gain $5H$ minute spaces in $\frac{60}{55} \times 5H = \frac{60H}{11}$ minutes. Thus, the two hands of clock will be together between H and $(H + 1)$ O'clock at $\left(\frac{60H}{11}\right)$ minutes past H O'clock.

Illustration 1: At what time between 5 and 6 O'clock are the hands of a clock together?

Solution: Here, $H = 5$.

$\therefore \quad \dfrac{60H}{11} = \dfrac{60}{11} \times 5 = \dfrac{300}{11} = 27\dfrac{3}{11}$.

$\therefore$ Hands of a clock are together at $22\dfrac{3}{11}$ minutes past 5 O'clock.

> 2. The two hands of the clock will be at right angles between H and $(H + 1)$ O'clock at $(5H \pm 15)\dfrac{12}{11}$ minutes past H O'clock.

Explanation

At H O'clock, the minute hand will be $5H$ minute spaces behind the hour hand. When the two hands are at right angle, they are 15 minute spaces apart. So there can be two cases:

Case I The minute hand is 15 minute spaces behind the hour hand. In this case, the minute hand will have to gain $(5H - 15)$ minute spaces over the hour hand.

Case II The minute hand is 15 minute spaces ahead of the hour hand. In this case, the minute hand will have to gain $(5H + 15)$ minute spaces over the hour hand. Combining the two cases, the minute hand will have to gain $(5H \mp 15)$ minute spaces over the hour hand.

Now, 55 minute spaces are gained in 60 minutes.

$\therefore$ $(5H \mp 15)$ minute spaces are gained in

$\dfrac{60}{55}(5H \mp 15) = \dfrac{12}{11}(5H \mp 15)$ minutes

So, they are at right angle at $(5H \mp 15)\dfrac{12}{11}$ minutes past H O'clock.

Illustration 2: At what time between 5 and 6 O'clock will the hands of a clock be at right angle?

Solution: Here, $H = 5$

$\therefore \quad (5H \mp 15)\dfrac{12}{11} = (5 \times 5 \mp 15)\dfrac{12}{11} = 10\dfrac{10}{11}$ and $43\dfrac{7}{11}$

$\therefore$ Hands of a clock are at right angle at $10\dfrac{10}{11}$ minutes past 5 and $43\dfrac{7}{11}$ minutes past 5.

> 3. The two hands of the clock will be in the same straight line but not together between H and $(H + 1)$ O'clock at,
>
> $(5H - 30)\dfrac{12}{11}$ minutes past H, when $H > 6$
>
> and, $(5H + 30)\dfrac{12}{11}$ minutes past H, when $H < 6$.

Illustration 3: Find at what time between 2 and 3 O'clock will the hands of a clock be in the same straight line but not together.

Solution: Here, $H = 2 < 6$.

$$\therefore \quad (5H + 30)\frac{12}{11} = (5 \times 2 + 30)\frac{12}{11} = \frac{480}{11}, \text{ i.e., } 43\frac{7}{11}.$$

So, the hands will be in the same straight line but not together at $43\dfrac{7}{11}$ minutes past 2 O'clock.

> 4. Between H and $(H + 1)$ O'clock, the two hands of a clock are M minutes apart at $(5H \mp M)\dfrac{12}{11}$ minutes past H O'clock.

Explanation

At H O'clock, the two hands are $5H$ minute spaces apart.

Case I Minute hand is M minute spaces behind the hour hand. In this case, the minute hand has to gain $(5H - M)$ minute spaces over the hour hand.

Case II Minute hand is M minute spaces ahead of the hour hand. In this case, the minute hand has to gain $(5H + M)$ minute spaces over the hour hand.

Combining the two cases, the minute hand has to gain $(5H \pm M)$ minute spaces over the hour hand.

Now, 55 minute spaces are gained in 60 minutes.

$\therefore$ $(5H \pm M)$ minute spaces are gained in

$\dfrac{60}{55}(5H \mp M) = \dfrac{12}{11}(5H \mp M)$ minutes.

$\therefore$ The hands will be M minutes apart at,

$\dfrac{12}{11}(5H \mp M)$ minutes past H O'clock.

Illustration 4: Find the time between 4 and 5 O'clock when the two hands of a clock are 4 minutes apart.

Solution: Here, $H = 4$ and $M = 4$.

$$\therefore \quad \frac{12}{11}(5H \mp M) = \frac{12}{11}(5 \times 4 \mp 4) = 26\frac{2}{11} \text{ and } 17\frac{5}{11}.$$

$\therefore$ The hands will be 4 minutes apart at $26\dfrac{2}{11}$ minutes past 4 and $17\dfrac{5}{11}$ minutes past 4 O'clock.

5. Angle between the hands of a clock
   (a) When the minute hand is behind the hour hand, the angle between the two hands at M minutes past H O'clock $= 30\left(H - \frac{M}{5}\right) + \frac{M}{2}$ degrees.
   (b) When the minute hand is ahead of the hour hand, the angle between the two hands at M minutes past H O'clock $= 30\left(\frac{M}{5} - H\right) - \frac{M}{2}$ degree.

Illustration 5: Find the angle between the two hands of a clock at 15 minutes past 4 O'clock.

Solution: Here, $H = 4$ and $M = 15$.

$\therefore$ The required angle

$$= 30\left(H - \frac{M}{5}\right) + \frac{M}{2} \text{ degrees}$$

$$= 30\left(4 - \frac{15}{5}\right) + \frac{15}{2} + \frac{75}{2}$$

$$= \frac{75}{2}, \text{ i.e., } 37.5°.$$

6. The minute hand of a clock overtakes the hour hand at intervals of M minutes of correct time. The clock gains or loses in a day by
$$= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right) \text{ minutes.}$$

Illustration 6: The minute hand of a clock overtakes the hour hand at intervals of 65 minutes. How much in day does the clock gain or lose?

Solution: Here, $M = 65$

$\therefore$ The clock gains or, loses in a day by

$$= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right)$$

$$= \left(\frac{720}{11} - 65\right)\left(\frac{60 \times 24}{65}\right)$$

$$= \frac{5}{11} \times \frac{12 \times 24}{13} = \frac{1440}{143}$$

$$= 10\frac{10}{143} \text{ minutes.}$$

Since the sign is +ve, the clock gains by $10\frac{10}{143}$ minutes.

CALENDAR

In this section we shall mainly deal with finding the day of the week on a particular given date. The process of finding it depends upon the number of odd days, which are quite differrent from the odd numbers. So, we should be familiar with **odd days.**

Odd Days

The days more than the complete number of weeks in a given period are called odd days.

Ordinary Year

An ordinary year has 365 days.

Leap Year

That year (except century) which is divisible by 4 is called a leap year, whereas century is a leap year by itself when it is divisible by 400.

For example, 1964, 1968, 1972, 1984, and so on, are all leap years whereas 1986, 1990, 1994, 1998, and so on, are not leap years.

Further, the centuries 1200, 1600, 2000 and so on, are all leap years as they are divisible by 400 whereas 900, 1300, 1500 and so on, are not leap years.

SOME BASIC FACTS

1. An ordinary year has 365 days, i.e., 52 weeks and 1 odd day.
2. A leap year has 366 days, i.e., 52 weeks and 2 odd days.
3. A century has 76 ordinary years and 24 leap years.
   $\therefore$ 100 years = 76 ordinary years + 24 leap years
   = 76 odd days + 24 × 2 odd days
   = 124 odd days = 17 weeks + 5 days
   $\therefore$ 100 years contain 5 odd days.
4. 200 years contain 10 days and therefore 3 odd days.
5. 300 years contain 15 days and therefore 1 odd day.
6. 400 years contain (20 + 1) days and therefore 1 odd day.
7. February in an ordinary year has no odd day, but in a leap year has one odd day.
8. Last day of a century cannot be either Tuesday, Thursday or Saturday.
9. The first day of a century must either be Monday, Tuesday, Thursday or Saturday.

Explanation

Number of odd days in first century = 5

∴ Last day of first century is Friday.

Number of odd days in two centuries = 3

∴ Wednesday is the last day.

Number of odd days in three centuries = 1

∴ Monday is the last day.

Number of odd days in four centuries = 0

∴ Last day is Sunday.

Since the order is continually kept in successive cyles, the last day of a century cannot be Tuesday, Thursday or Saturday. So, the last day of a century should be either Sunday, Monday, Wednesday or Friday. Therefore, the first day of a century must be either Monday, Tuesday, Thursday or Saturday.

SOME USEFUL SHORTCUT METHODS

1. Working rule to find the day of the week on a particular date when reference day is given:

Step I Find the net number of odd days for the period between the reference day and the given date (Exclude the reference day but count the given date for counting the number of net odd days).

Step II The day of the week on the particular date is equal to the number of net odd days ahead of the reference day (if the reference day was before this date) but behind the reference day (if this date was behind the reference day).

Illustration 7: January 11, 1997 was a Sunday. What day of the week was on January 7, 2000?

Solution: Total number of days between January 11, 1997 and January 7, 2000

= (365 − 11) in 1997 + (365 days in 1998)
  + (365 days in 1999) + (7 days in 2000)

= (50 weeks + 4 odd days) + (52 weeks + 1 odd day)
  + (52 weeks + 1 odd day) + (7 odd days)

= 13 days = 1 week + 6 odd days.

Hence, January 7. 2000 would be 6 days ahead of Sunday, i.e., it was on Saturday.

2. Working Rule to find the day of the week on a particular date when no reference day is given

Step I Count the net number of odd days on the given date.

Step II Write:
Sunday for 0 odd day
Monday for 1 odd day
Tuesday for 2 odd days
⋮ ⋮ ⋮
Saturday for 6 odd days.

Illustration 8: What day of the week was on June 5, 1999?

Solution: June 5, 1999 means 1998 years + first five months up to May of 1999 + 5 days of June.

1600 years have 0 odd day.

300 years have 1 odd day.

98 years have 24 leap years + 74 ordinary years

= (24 × 2) + (74 × 1) days

= 122 days = 17 weeks + 3 odd days

Thus, 1998 years have 4 odd days.

January 1, 1999 to May 31 1999, has

= (3 + 0 + 3 + 2 + 3 + 5) 2 odd days

= 16 days = 2 weeks + odd days

Total number of odd days on June 5, 1999

= (4 + 2) odd days = 6 odd days.

Hence, June 5, 1999 was Saturday.

EXERCISE-I

1. At what time between 3 and 4 O'clock are the hands of a clock together?

(a) $15\frac{7}{11}$ minutes past 4

(b) $16\frac{4}{11}$ minutes past 3

(c) $16\frac{2}{11}$ minutes past 2

(d) None of these

2. At what time between 7 and 8 O'clock will the hands of a clock be at right angle?

(a) $19\frac{5}{11}$ minutes past 2

(b) $21\frac{9}{11}$ minutes past 7

(c) 18 minutes past 4

(d) None of these

3. Find at what time between 8 and 9 O'clock will the hands of a clock be in the same straight line but not together?

(a) $11\frac{9}{11}$ minutes past 5

(b) $9\frac{7}{11}$ minutes past 5

(c) $10\frac{10}{11}$ minutes past 8

(d) None of these

4. At what time between 5 and 6 O'clock are the hands of a clock 3 minutes apart?

(a) 24 minutes past 5 (b) 22 minutes past 3

(c) 26 minutes past 4 (d) None of these

5. Find the angle between the two hands of a clock at 30 minutes past 4 O'clock.

(a) 40° (b) 30°

(c) 45° (d) None of these

6. How much does a watch gain or lose per day, if its hands coincide in every 64 minutes?

(a) $32\frac{8}{11}$ minutes gain

(b) $34\frac{2}{11}$ minutes gain

(c) $32\frac{8}{11}$ minutes loss

(d) None of these

7. How often between 11 O'clock and 12 O'clock are the hands of a clock in integral number of minutes apart?

(a) 55 times (b) 56 times

(c) 58 times (d) 60 times

8. Number of times the hands of a clock are in a straight line everyday is:

(a) 44 (b) 24

(c) 42 (d) 22

9. My watch gains 5 seconds in 3 minutes was set right at 7 am. In the afternoon of the same day, when the watch indicates quarter past 4 O'clock, the true time is:

(a) $59\frac{7}{12}$ minutes. past 3

(b) $12\frac{3}{11}$ minutes. past 3

(c) 4 pm

(d) $7\frac{5}{12}$ minutes. past 4

10. My watch gains 5 minutes. in every hour. How many degrees the second hand moves in every minute?

(a) 375° (b) 380°

(c) 390° (d) 365°

11. At what time between 4:30 and 5 will the hands of a clock be in a straight line?

(a) 50 minutes. past 4

(b) 42 minutes. past 4

(c) $54\frac{6}{11}$ minutes. past 4

(d) 46 minutes. past 4

12. Two clocks are set right at 10 am. One gains 20 seconds and the other loses 40 seconds in 24 hours. What will be the true time when the first clock indicates 4 pm on the following day?

(a) $3{:}59\frac{2521}{4321}$ pm (b) $3{:}31\frac{1}{471}$ pm

(c) $3{:}59\frac{7}{12}$ pm (d) $3{:}57\frac{2521}{4321}$ pm

13. A clock takes 9 seconds to strike 4 times. In order to strike 12 times at the same rate, the time taken is:

(a) 27 seconds (b) 36 seconds

(c) 30 seconds (d) 33 seconds

14. How often are the hands of a clock at right angle everyday?
(a) 38 times (b) 44 times
(c) 40 times (d) 48 times

15. A clock is set right at 5 am. The clock loses 16 minutes. in 24 hours. What will be the true time when the clock indicates 10 pm. on the 4th day?
(a) 9 am (b) 11 pm
(c) 11 am (d) 9 pm

16. My watch was 3 minutes slow at 5 pm on Tuesday and it was 5 minutes fast at 11 pm on Wednesday. When did it give correct time?
(a) Wednesday 4:15 am
(b) Wednesday 7:30 am
(c) Tuesday 3:45 pm
(d) None of these

17. How many times do the hands of a clock point towards each other in a day?
(a) 24 (b) 20
(c) 12 (d) 22

18. A man who went out between 3 and 4 and returned between 8 and 9, found that the hands of the watch had exactly changed places. He returned at:
(a) 14 minutes. past 8
(b) $21\frac{1}{13}$ minutes. past 8
(c) $19\frac{2}{13}$ minutes. past 8
(d) $18\frac{6}{13}$ minutes. past 8

19. A clock gains 10 minutes in every 24 hours. It is set right on Monday at 8 am. What will be the correct time on the following Wednesday, when the watch indicates 6 pm?
(a) 5.36 pm. (b) 5.40 pm.
(c) 4.36 pm. (d) None of these

20. If the hands of a clock coincide in every 65 minutes (true time), in 24 hours the clock will gain:
(a) $10\frac{10}{143}$ minutes. (b) $9\frac{12}{143}$ minutes.
(c) $11\frac{12}{143}$ minutes. (d) $12\frac{10}{143}$ minutes.

21. The watch which gains uniformly is 2 minutes. slow at noon on Sunday and is 4 minutes. 48 seconds. fast at 2 pm on the following Sunday. The watch was correct at:
(a) 2 pm on Tuesday
(b) 12 noon on Monday
(c) 1:30 pm on Tuesday
(d) 12:45 pm on Monday

22. A watch which gains uniformly is 6 minutes slow at 4 pm on a Sunday and $10\frac{2}{3}$ minutes fast on the following Sunday at 8 pm. During this period (Day and Time) when was the watch correct?
(a) 2.36 am (b)1.36 am
(c) 2.36 pm (d)1.36 pm

23. If a clock takes 22 seconds to strike 12, how much time will it take to strike 6?
(a) 10 seconds (b) 12 seconds
(c) 14 seconds (d) None of these

24. Mahatma Gandhi was born on October 2, 1869. The day of the week was:
(a) Sunday (b) Monday
(c) Saturday (d) Friday

25. March 5, 1999 was on Friday. What day of the week was on March 5, 2000?
(a) Monday (b) Sunday
(c) Friday (d) Tuesday

26. On what date of August, 1988 did Friday fall?
(a) 5 (b) 4
(c) 14 (d) 17

27. India got independence on August 15, 1947. What was the day of the week?
(a) Monday (b) Friday
(c) Thursday (d) Sunday

28. January 7, 1992 was Tuesday. Find the day of the week on the same date after 5 years, i.e., on January 7, 1997?
(a) Tuesday (b) Wednesday
(c) Saturday (d) Friday

29. Number of times 29th day of the month occurs in 400 consecutive years is:
(a) 4497 (b) 4800
(c) 4400 (d) None of these

30. The first Republic Day of India was celebrated on January 26, 1950. What was the day of the week on that date?
(a) Monday (b) Wednesday
(c) Saturday (d) Thursday

31. In an ordinary year 'March' begin on the same day of the week:
(a) February; November
(b) January; November
(c) February; October
(d) January; September

32. If March 2, 1994 was on Wednesday, January 1994 25, was on:
(a) Wednesday (b) Thursday
(c) Tuesday (d) Monday

33. Calendar for 2000 will serve for also:
(a) 2003 (b) 2006
(c) 2007 (d) 2005

Exercise-2
(Based on Memory)

1. A clock strikes once at 1 O'clock, twice at 2 O'clock, thrice at 3 O'clock and so on. How many times will it strike in 24 hours?
   (a) 78 (b) 136
   (c) 156 (d) 196
   [(GL) Prel. Examination, 2002]

2. The minute hand of a clock is 7 cm long. The area swept by the minute hand in 15 minutes will be:
   (a) 25.6 cm^2 (b) 38.5 cm^2
   (c) 44.0 cm^2 (d) 77.0 cm^2
   [SI Rec. Examination, 1997]

3. If a clock strikes 6 times in five seconds, the number of strikes in 10 seconds is:
   (a) 10 (b) 11
   (c) 9 (d) 8
   [Assstant's Grade Examination, 1998]

4. If the day after tomorrow is Sunday, what day was tomorrow's day before yesterday?
   (a) Friday (b) Thursday
   (c) Monday (d) Tuesday
   [SSC (GL) Examination, 2010]

5. At what time are the hands of clocks together between 6 and 7?
   (a) $32\frac{8}{11}$ minutes past 6
   (b) $34\frac{8}{11}$ minutes past 6
   (c) $30\frac{8}{11}$ minutes past 6
   (d) $32\frac{5}{7}$ minutes past 6
   [SSC (GL) Examination, 2011]

6. Suresh was born on October 4, 1999. Shashikanth was born 6 days before Suresh. The Independence day of that year fell on Sunday. Which day was Shashikanth born?
   (a) Tuesday (b) Wednesday
   (c) Monday (d) Sunday
   [SSC (GL) Examination, 2011]

7. After 9'O clock at what time between 9 pm and 10 pm will the hour and minute hands of a clock point in opposite direction?
   (a) 15 minutes past 9 (b)16 minutes past 9
   (c) $16\frac{4}{11}$ minutes past 9 (d) $17\frac{1}{11}\%$ minutes past 9
   [SSC (GL), 2011]

8. If John celebrated his victory day on Tuesday, January 5, 1965, when will he celebrate his next victory day on the same day?
   (a) January 5, 1970 (b) January 5, 1971
   (c) January 5, 1973 (d) January 5, 1974
   [SSC (GL) Examination, 2011]

9. In the year 1996, the Republic day was celebrated on Friday. On which day was the Independence day celebrated in the year 2000?
   (a) Tuesday (b) Monday
   (c) Friday (d) Saturday
   [SSC (GL) Examination, 2011]

10. A girl was born on September 6, 1970 which happened to be a Sunday. Her birthday would have fallen again on Sunday in:
    (a) 1975 (b) 1977
    (c) 1981 (d) 1982
    [UPPCS Examination, 2012]

11. In every 30 minutes the time of a watch increases by 3 minutes. After setting the correct time at 5 am what time will the watch show after 6 hours?
    (a) 10:54 am (b) 11:30 am
    (c) 11:36 am (d) 11:42 am
    [Corporation Bank PO Examination, 2009]

12. A wall clock gains 2 minutes in 12 hours, while a table clock loses 2 minutes every 36 hours. Both are set right at 12 noon on Tuesday. The correct time when both show the same time next would be:
    (a) 12.30 at night, after 130 days
    (b) 12 noon, after 135 days
    (c) 1.30 at night, after 130 days
    (d) 12 midnight, after 135 days
    [SSC Examination, 2012]

13. In every 30 minutes the time of a watch increases by 3 minutes. After setting the correct time at 5 am, what time will the watch show after 6 hours?
    (a) 10:54 am (b) 11:30 am
    (c) 11:36 am (d) 11:42 am
    (e) 11:38 pm
    [Corporation Bank PO Examination, 2009]

ANSWER KEYS

Exercise-1

1. (c)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (c)	**11.** (c)	**12.** (a)	**13.** (d)
14. (b)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (d)	**19.** (a)	**20.** (a)	**21.** (a)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (b)	**26.** (a)
27. (b)	**28.** (a)	**29.** (a)	**30.** (d)	**31.** (a)	**32.** (c)	**33.** (d)						

Exercise-2

1. (c)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (c)	**11.** (c)	**12.** (b)	**13.** (c)

EXPLANATORY ANSWERS

Exercise-1

1. **(c)** Here, $H = 3$.

$\therefore \quad \frac{60H}{11} = \frac{60}{11} \times 3 = \frac{180}{11} = 16\frac{4}{11}$.

So, the hands of a clock will coincide at $16\frac{4}{11}$ minutes past 3.

2. **(b)** Here, $H = 7$

$\therefore \quad (5H \pm 15)\frac{12}{11} = (5 \times 7 \pm 15)\frac{12}{11}$

$= 21\frac{9}{11}$ and $54\frac{6}{11}$.

$\therefore$ The hands of a clock are at right angle at $21\frac{9}{11}$ minutes past 7 and, $54\frac{6}{11}$ minutes past 7 O'clock.

3. **(c)** Here, $H = 8 > 6$.

$\therefore \quad (5H - 30)\frac{12}{11} = (5 \times 8 - 30)\frac{12}{11} = \frac{120}{11} = 10\frac{10}{11}$.

So, the hands will be in the same straight line but not together at $10\frac{10}{11}$ minutes past 8 O'clock.

4. **(a)** Here, $H = 5$ and $M = 3$.

$\therefore \quad \frac{12}{11}(5H \pm M) = \frac{12}{11}(5 \times 5 \pm 3) = 31\frac{5}{11}$ and 24.

$\therefore$ The hands will be 3 minutes apart at $31\frac{5}{11}$ minutes past 5 and 24 minutes past 5 O'clock.

5. **(c)** Here, $H = 4$ and $M = 30$.

$\therefore$ The required angle $= 30\left(\frac{M}{5} - H\right) - \frac{M}{2}$ degrees

$= 30\left(\frac{30}{5} - 4\right) - \frac{30}{2}$

$= 60 - 15 = 45°$.

6. **(a)** Here, $M = 64$

$\therefore$ The clock gains or loses in a day by

$= \left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right)$

$= \left(\frac{720}{11} - 64\right)\left(\frac{60 \times 24}{64}\right)$

$= \left(65\frac{5}{11} - 64\right) \times \left(\frac{60 \times 24}{64}\right)$

$= \frac{16}{11} \times \frac{60 \times 24}{64} = \frac{360}{11} = 32\frac{8}{11}$ minutes.

Since the sign is +ve, the clock gains by $32\frac{8}{11}$ minutes.

7. **(b)** At 11 O'clock, the hour hand is 4 spaces apart from the minute hand. Since there are 60 spaces in one hour, so $(60 - 4)$ times, i.e., 56 times the hands of the clock are an integral number of minutes apart.

8. **(a)** We know that, any relative position of the hands of a clock is repeated 11 times in every 12 hours.

$\therefore$ In every 12 hours, hands coincide 11 times and are opposite to each other 11 times.

$\therefore$ In every 12 hours, hands are in a straight line $11 + 11 = 22$ times.

$\therefore$ In every 24 hours hands are in a straight line 44 times.

9. **(c)** From 7 am to 4:15 pm, the time is 9 hours 15 minutes, i.e., 555 minutes.

Now, $\frac{37}{12}$ minutes of this watch = 3 minutes of correct watch.

$\Rightarrow$ 555 minutes of this watch = $\left(\frac{3\times 12}{37}\times 555\right)$ minutes on a correct watch = 540 minutes or 9 hours of correct watch.

$\therefore$ Correct time after 7 am is 4 pm.

10. (c) Since minute hand gains 5 minutes in every 60 minutes

$\Rightarrow$ second hand gains 5 seconds in every 60 seconds

$\therefore$ In every 60 seconds true time, it moves 65 seconds or, $65 \times 6° = 390°$.

11. (c) At 4 O'clock hands are 20 minutes spaces apart. At time between 4:30 and 5 the hands will be in straight line when they point in opposite directions and there is a space of 30 minutes. between them. So, to be in this position minute hand has to gain 30 + 20 = 50 minutes. spaces. Minute hand gains 50 minutes in

$$\frac{60}{55}\times 50 = 54\frac{6}{11} \text{ minutes.}$$

$\therefore$ Required time = $54\frac{6}{11}$ minutes past 4.

12. (a) From 10 am to 4 pm on the following day = 30 hours

Now, 24 hours 20 seconds of the first clock = 24 hours of the current clock.

$\therefore$ 1 hour of the first clock = $\frac{24\times 180}{4321}$ hours

$\therefore$ 30 hours of the first clock = $\frac{24\times 180\times 30}{4321}$ hours

Now, $\frac{24\times 180\times 30}{4321}$ hours = 29 hours 59 $\frac{2521}{4321}$ minutes.

$\therefore$ When the first clock indicates 4 pm on the following day the true time will be 3 hours 59 $\frac{2521}{4321}$ minutes.

13. (d) There are 3 intervals when the clock strikes 4

Time taken in 3 intervals = 9 seconds

$\therefore$ Time taken for 1 interval = 3 seconds

In order to strike 12, there are 11 intervals, for which the time taken is 11×3 seconds = 33 seconds.

14. (b) In every hour there are two positions in which hands are at right angle. Each of these positions is repeated 11 times in every 12 hours.

$\therefore$ In every 12 hours, hands are at right angles 11 + 11 = 22 times and in a day hands are at right angles 22 + 22 = 44 times.

15. (b) From 5 am on first day to 10 pm on 4th day is 89 hours. When a clock loses 16 minutes then 23 hours 44 minutes of this clock are the same as 24 hours of correct clock, i.e., $\frac{356}{15}$ hours of this clock = 24 hours of correct clock.

$\therefore$ 89 hours of this clock.

= $\left(\frac{24\times 15}{356}\times 89\right)$ hours of correct clock

= 90 hours of correct clock

$\therefore$ The correct time is 11 pm.

16. (a) Time from 5 pm Tuesday to 11 pm Wednesday = 30 hours

Clock gains 8 minutes in 30 hours

$\therefore$ It gains 3 minutes in $\frac{30}{8}\times 3$ hours

= 11 hours 15 minutes.

$\therefore$ Correct time is 11 hours. 15 minutes after 5 pm.

= 4:15 am on Wednesday.

17. (d) The hands of a clock point towards each other 11 times in every 12 hours. (because between 5 and 7, at 6 O'clock only they point towards each other)

So, in a day the hands point towards each other 22 times.

19. (a) Total number of hours from Monday at 8 am to the following Wednesday at 6 pm.

$24 \times 2 + 10 = 58$ hours

24 hours 10 minutes of this clock are the same as 24 hours of a correct clock.

$\frac{145}{6}$ hours of the incorrect clock = 24 hours of correct clock.

58 hours of the incorrect clock = $\frac{24\times 6}{145}\times 58$ hours of correct clock = $57\frac{3}{5}$ hours of correct clock.

Thus, the correct time on the following Wednesday will be 5:36 pm.

20. (a) The minutes hand gains 60 minutes in $\frac{60}{55}\times 60$

$$= \frac{720}{11} = 65\frac{5}{11} \text{ minutes.}$$

$\therefore$ The hands of a correct clock coincide in every $65\frac{5}{11}$ minutes. But the hands of the clock in question coincide in every 65 minutes.

The clock in question gains $\frac{5}{11}$ minutes in 65 minutes.

$\therefore$ In 24 hours = 24×60 minutes the clock gains

$\frac{5}{11}\times\frac{1}{65}\times 24\times 60 = \frac{1440}{143}$ minutes = $10\frac{10}{143}$ minutes

21. (a) From Sunday noon to the following Sunday at 2 pm, total time = 7 days + 2 hours

= $(7 \times 24 + 2)$ hours = 170 hours.

In this period watch gains 2 + 4 minutes 48 seconds

= $6\frac{48}{60} = 6\frac{4}{5}$ minutes.

$\therefore$ Watch gains $6\frac{4}{5}$ minutes in 170 hours.

$\therefore$ Watch gains 2 minutes in $\frac{170}{34}\times 5\times 2$ = 50 hours i.e., 2 days and 2 hours.

∴ Watch will be correct at 2 pm on Tuesday.

22. **(b)** Total time in hours from Sunday at 4 pm to the following Sunday at 8 am.

$= 6 \times 24 + 16 = 160$ hours

Thus, the watch gains $6 + 10\frac{2}{3} = 16\frac{2}{3}$ minutes in 160 hours

Now, $\frac{50}{3}$ minutes are gained in 160 hours.

∴ 6 minutes are gained in $160 \times \frac{3}{50} \times 6$

$$= \frac{288}{5} \text{ hours} = 57\frac{3}{5} \text{ hours.}$$

or, the watch was correct on Wednesday at 1:36 am.

23. **(a)** In order to hear 12 strikes, there are 11 intervals (12 − 1) and time of each interval is uniform.

Hence, time to hear each strike is $\frac{22}{11} = 2$ seconds

Now, to hear six strikes, there are 6 − 1, i.e., 5 × 2 = 10 seconds.

Hence, it will take 10 seconds for a clock to strike 6.

24. **(c)** 2 October 1869 means

1868 complete years + 9 months + 2 days

1600 years give 0 odd days

200 years give 3 odd days

Number of leap years in 68 years = largest integer less than $\frac{68}{4} = 17$

∴ 68 years contain 17 leap years and 51 non-leap years:

∴ 68 years have 2 × 17 + 51 = 85, i.e., 1 odd day

Also count number of days from January 1, 1869 to October 2, 1869

January	February	March	April	May	June
31	+ 28	+ 31	+ 30	+ 31	+ 30

July	August	September	October	
31	+ 31	+ 30	+ 2	= 275

= 39 weeks + 2 days

∴ This gives 2 odd days

∴ Total number of odd days = 0 + 3 + 1 + 2 = 6

∴ Day on October 2, 1869 was Saturday.

25. **(b)** Year 2000 was a leap year.

Number of days remaining in 1999

= 365 − [31 days of January + 28 days of February + 5 days March].

= 301 days = 43 weeks, i.e., 0 odd day.

Number of days passed in 2000:

January 31 days have 3 odd days.

February 29 days (being leap year) have 1 odd day March 5 days have 5 odd days.

∴ Total number of odd days = 0 + 3 + 1 + 5 + 9 days, i.e., 2 odd days

Therefore, March 5, 2000 would be two days beyond Friday, i.e., on Sunday.

26. **(a)** August 1, 1988 means:

1987 years + 7 months

Number of odd days in 1987 years:

1600 years have 0 odd days

300 years have 1 odd day

87 years have 21 leap years and 66 ordinary years.

So, there are 21 × 2 + 66 × 1 = 108 days, i.e., 15 weeks and 3 odd days.

Number of days between January 1, 1988 to August 1, 1989.

January	February	March	April	May	June	July	August
31	+ 29	+ 31	+ 30	+ 31	+ 30	+ 31	+ 1 =

241 days

i.e., 30 weeks and 4 odd days.

Total number of odd days = 0 + 1 + 3 + 4 = 8 odd days or 1 odd day.

Thus, Friday falls on 5th, 12th, 19th and 26th in August 1988.

27. **(b)** August 15, 1947 = (1600 + 300 + 46) years + January 1 to August 15th, of 1947

= (1600 + 300 + 46) years + 365 − August 16 to December. 31 1947

= (1600 + 300 + 46) years + (365 − 138) days

Number of odd days = 0 + 1 + 1 (from 11 leap years and 35 ordinary years) + 3 = 5 odd days

∴ The day was Friday.

28. **(a)** During the interval we have two leap years as 1992 and 1996 and it contains February of both these years.

∴ The interval has (5 + 2) = 7 odd days or 0 odd day.

Hence, January 7, 1997 was also Tuesday.

29. **(a)** 400 consecutive years contain 97 leap years.

∴ In 400 consecutive years February has 29 days 97 times and the remaining 11 months have 29th day

400 × 11 = 4400 times

∴ 29th day of the month occurs 4400 + 97 = 4497 times.

30. **(d)** Total number of odd days = 1600 years have 0 odd day + 300 years have 1 odd day + 49 years (12 leap years + 37 ordinary years) have 5 odd days + 26 days of January have 5 odd days = 0 + 1 + 5 + 5 = 4 odd days.

So, the day was Thursday.

31. **(a)** In an ordinary year, February has no odd day.

∴ February and March begin on same day of the week.

Also we know that, November and March begin on same day of the week.

32. **(c)** Number of days from January 25, 1994 to March 2, 1994 is

January	February	March	
6	+ 28	+ 2	= 36

∴ Number of odd days = 1

∴ Day on January 25, 1994 is one day before the day on March 2, 1994.

But March 2, 1994 was on Wednesday.

∴ January 25, 1994 was on Tuesday.

33. (**d**) Starting with 2000, count for number of odd days in successive years till the sum is divisible by 7.

2000	2001	2002	2003	2004	
2	+ 1	+ 1	+1	+ 2	= 7

∴ Number of odd days up to 2004 = 0

∴ Calendar for 2000 will serve for 2005 also.

Exercise-2
(Based on Memory)

1. **(c)** $2(1 + 2 + 3 = \ldots \ldots + 12) = \dfrac{2\times12\times(12+1)}{2}$

$= 12 \times 13 = 156.$

2. **(b)** $\dfrac{\pi r^2}{4} = \dfrac{22}{7}\times\dfrac{1}{4} \times (7)^2 = \dfrac{154}{4} = 38.5 \text{ cm}^2.$

4. **(b)** The day after tomorrow is Sunday. Therefore today is Friday.

Hence, the day on tomorrow's day before yesterday is given by:

= Friday – 1 = Thursday

5. **(a)** Required time $= 5 \times 6 \times \dfrac{12}{11}$ minutes past 6

$= 32\dfrac{8}{11}$ minutes past 6.

6. **(a)** Birth date of Sashikant = September 28

Difference in number of days from August 15 to September 28

= 16 + 28 = 44

Number of odd days in 44 days = 2

Birthday of Shashikant = Tuesday.

7. **(c)** At 9 O'clock, the minute hand is 9 × 5 = 45 minutes space behind the hour hand. Hence, the minute hand will have to gain 45 – 30 = 10 minutes.

Therefore, 60 minutes is equal to the gain of 55 minutes spaces.

Hence, gain of 15 minutes spaces equals

$= \dfrac{60}{55}\times15 = \dfrac{180}{11} = 16\dfrac{4}{11}$

Therefore, hour and minute hands of a clock point in opposite direction after 9 O'clock at $16\dfrac{4}{11}$ minutes past 9.

8. **(b)** January 5th, 1965 ⇒ Tuesday

January 5th, 1966 ⇒ Wednesday

January 5th, 1967 ⇒ Thursday

January 5th, 1968 ⇒ Friday

January 5th, 1969 ⇒ Sunday

Since, 1968 is a leap year.

January 5th, 1970 ⇒ Monday

January 5th, 1971 ⇒ Tuesday

9. **(a)** The year 1996 was a leap year and number of days remaining in the year 1996

= 366 – 26 = 340 days

= 48 weeks = 40 odd days

The years 1997, 1998 and 1999 have 3 odd days in total. The year 2000 was also a leap year.

Days till August 15, 2000

= 31 + 29 + 31+ 30 + 31 + 30 + 31 + 15.

= 228 days

$\dfrac{228}{7}$ = 32 weeks 4 odd days

Now, total number of odd days

4 + 4 + 3 = 11 days

$\dfrac{11}{7}$ = 1 week, 4 odd days

Thus, August 15th, 2000 was 4 days beyond Friday i.e., Tuesday.

10. **(c)** Odd number of days from September 6, 1970 to September 6, 1981 = 14

Hence, the Sunday will be on September 6, 1981.

11. **(c)** In every 30 minutes the time of watch increased by 3 minutes = 12 × 3 = 36 minutes

So the time after 6 hours = 5 am + 6 hours + 30 minutes = 11.36 am.

12. **(b)** The wall clock gains 6 minutes in 36 hours, while table watch loses 2 minutes in 36 hours.

∴ Difference of 8 minutes is in $\dfrac{3}{2}$ days

∴ Difference of 12 hours is in

$= \dfrac{3}{2}\times\dfrac{1}{8}\times12\times60 = 135$ days

13. **(c)** In 1 hour it increases by 6 minutes so in 6 hours it increases by 36 minutes.