Races and Games of Skill 14

INTRODUCTION

Race: A contest of speed in running, driving, riding sailing or rowing over a specified distance is called *race*.

Race Course: The ground or path on which contests are arranged is called a *race course*.

Starting Point: The point from where a race begins is called the *starting point*.

Winning Post (or Goal): The point where the race finishes is called the *winning post* or *finishing point* or *goal*.

Dead-heat Race: If all the persons contesting a race reach the finishing point exactly at the same time, then the race is called a *dead-heat race*.

Winner: The person who first reaches the finishing point is the *winner*.

Suppose, A and B are two contestants in a race. We give here certain statements and their corresponding mathematical meanings, which are frequently used:

Statements	Mathematical Interpretation
1. A beats B by t seconds	A finishes the race t seconds before B finishes.
2. A gives B a start of t seconds	A starts t seconds after B starts from the same point.
3. A gives B a start of x metres	While A starts at the starting point, B starts x metres ahead from the starting point at the same time.
4. Game of 100	A game in which the participant scoring 100 points first is the winner.
5. In a game of 100, 'A can give B 20 points'	While A scores 100 points, B scores only (100–20) or 80 points.

SOME USEFUL SHORTCUT METHODS

1. If A is n times as fast as B and A gives B a start of x metres, then the length of the race course, so that both A and B reach the winning post at the same time, must be

$$x\left(\frac{n}{n-1}\right) \text{ m}$$

Illustration 1: A is $1\frac{2}{3}$ times as fast as B. If A gives B a start of 60 m. How long should the race course be so that both of them reach at the same time?

Solution: Here, $n = \frac{5}{3}$ and $x = 60$.

$\therefore$ Length of the race course $= x\left(\frac{n}{n-1}\right)$

$$= 60\left(\frac{5/3}{5/3-1}\right) = 60\left(\frac{5}{5-3}\right) = 150 \text{ m.}$$

2. If A can run x m race in t_1 seconds and B in t_2 seconds, where $t_1 < t_2$, then A beats B by a distance

$$\frac{x}{t_2}\times(t_2 - t_1) \text{ m}.$$

Illustration 2: A can run 100 m in 27 seconds and B in 30 seconds. By what distance A beats B?

Solution: A beasts B by a distance

$$= \frac{x}{t_2}\times(t_2 - t_1)$$

$$= \frac{100}{30}(30-27) = \frac{300}{30} = 10 \text{ m.}$$

Exercise-1

1. In a Km race A beats B by 25 m or 5 sec. Then find out the time taken by A to complete the race.

   (a) 3 minutes 15 seconds (b) 4 minutes 20 seconds
   (c) 2 minutes 30 seconds (d) 5 minutes 10 seconds

2. In a race of 300 m, A beats B by 15 m or 5 sec. A's time over the course is:

   (a) 100 seconds   (b) 95 seconds
   (c) 105 seconds   (d) 90 seconds

3. A can run 500 m in 30 seconds and B in 35 seconds. How many meter start can A give to B in a Km race so that the race may end in a dead-heat?

   (a) $139\dfrac{5}{7}$ (b) $138\dfrac{5}{7}$

   (c) $142\dfrac{6}{7}$ (d) $140\dfrac{5}{7}$

4. A runs $1\dfrac{3}{8}$ times as fast as B. If A gives B a start of 120 m and they reach the goal at the same time, the goal is at a distance of:

   (a) 360 m   (b) 440 m
   (c) 460 m   (d) 380 m

5. In a game of 100 points, A can give B 20 points and C 28 points. Number of points B can give C in a game of 100 points is:

   (a) 10   (b) 90
   (c) 15   (d) 85

6. A's speed is $1\dfrac{1}{2}$ times of B's. In a race A gives B a start of 300 m. How long should the race course be so that both reach the winning post simultaneously?

   (a) 700 m   (b) 900 m
   (c) 800 m   (d) 850 m

7. In a race of 600 m, A can beat B by 60 m and in a race of 500 m, B can beat C by 50 m. By how many m will A beat C in a race of 400 m?

   (a) 364 m   (b) 254 m
   (c) 324 m   (d) 354 m

8. In a 100 m race, A runs at 5 Km/h. A gives B a start of 8 m and still beats him by 8 sec. Find out the speed of B.

   (a) 6.14 Km/h   (b) 4.14 Km/h
   (c) 3.14 Km/h   (d) 2.14 Km/h

9. In a game, A can give B 20 points, A can give C 32 points and B can give C 15 points. How many points make the game?

   (a) 100   (b) 200
   (c) 300   (d) 400

10. At a game of billiards, A can give B 6 points in 50 and he can give C 13 in 65. In a game of 55, number of points B can give C is:

    (a) 3   (b) 4
    (c) 5   (d) 8

11. In a Km race A can beat B by 80 m and B can beat C by 60 m. In the same race, A can beat C by:

    (a) 135.2 m   (b) 130.5 m
    (c) 142 m   (d) 132.5 m

12. In a game of 90 points, A can give B 15 points and C 30 points. How many points can B give C in a game of 100 points?

    (a) 140   (b) 20
    (c) 50   (d) 30

13. In a race of 600 m, A can beat B by 60 m and in a race of 500 m, B can beat C by 50 m. By how many meter will A beat C in a race of 400 m?

    (a) 78 m   (b) 56 m
    (c) 76 m   (d) 86 m

14. In a game A can give B 25 points in 75 and C 18 points in 90. How many points can C give B in a game of 120?

    (a) 20   (b) 30
    (c) 40   (d) 50

15. A and B run a 5 Km race on a round course of 400 m. If their speeds be in the ratio 5:4, then how often does the winner pass the other?

    (a) $4\dfrac{1}{2}$ times   (b) $3\dfrac{1}{2}$ times

    (c) $2\dfrac{3}{4}$ times   (d) $2\dfrac{1}{2}$ times

16. In a 500 m race, the ratio of speeds of two contestants A and B is 3:4. A has a start of 140 m. Then, A wins by:

    (a) 60 m   (b) 40 m
    (c) 20 m   (d) 10 m

17. In a Km race A beats B by 100 m and C by 200 m. By how many can B beat C in a race of 1350 m?

(a) 150 m   (b) 120 m

(c) 1200 m   (d) 210 m

18. Two boys, A and B, runs at $4\frac{1}{2}$ and 6 Km an hour. A having 190 m start. The course being 1 Km, B wins by a distance of:

(a) 60 m   (b) 65 m

(c) 45 m   (d) 75 m

19. A and B runs a Km race. If A gives B a start of 50 m, A wins by 14 sec and, if A gives B a start of 22 sec Km, B wins by 20 m. The time taken by A to run a Km is:

(a) 100 sec   (b) 120 sec

(c) 105 sec   (d) 125 sec

20. A and B take part in a 100 m race. A runs at 5 Km/h. A gives B a start of 8 m and still beat him by 8 sec. Speed of B is:

(a) 5.15 Km/h   (b) 4.14 Km/h

(c) 4.25 Km/h   (d) 4.4 Km/h

EXERCISE-2
(BASED ON MEMORY)

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1. A runner runs $1\frac{1}{4}$ laps of a 5 lap race. What fractional part of the race remains to be run?

(a) $\dfrac{15}{4}$ (b) $\dfrac{4}{5}$

(c) $\dfrac{5}{6}$ (d) $\dfrac{2}{3}$

[SSC (GL) Prel. Examination, 2000]

2. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet next at the starting point again?

(a) 46 minutes 12 seconds

(b) 45 minutes

(c) 42 minutes 36 seconds

(d) 26 minutes 18 seconds

[SSC (GL) Prel. Examination, 2003]

3. From a point on a circular tract 5 Km long A, B and C started running in the same direction at the same time with speeds of $2\frac{1}{2}$ Km/h, 3 Km/h and 2 Km/h. respectively. Then on the starting point all three will meet again after:

(a) 30 hours   (b) 6 hours

(c) 10 hours   (d) 15 hours

[SSC (GL) Prel. Examination, 2003]

4. A can run a distance of 1 Km in 3 minutes 10 seconds and B can run the same distance in 3 minutes 20 seconds. If they start together, by what distance will A beat B?

(a) 50 m   (b) 30 m

(c) 36 m   (d) 60 m

[SI of Police Rec. Examination, 1997]

5. X and Y start from the same point and run around a circular stadium, whose circumference is 4200 m, at the rate of 500 m and 700 m per minute respectively, in the opposite directions. They will meet each other in:

(a) 3.5 minutes   (b) 6.0 minutes

(c) 8.4 minutes   (d) 21 minutes

[SSC (GL) Prel. Examination, 1999]

6. The respective ratio between the speeds of a car, a train and a bus is 5:9:4. The average speed of the car, the bus and the train is 72 Km/h together. What is the average speed of the car and the train together?

(a) 82 Km/h   (b) 78 Km/h

(c) 84 Km/h   (d) Cannot be determined

[Punjab and Sindh Bank PO, 2010]

7. Four runners started running simultaneously from a point on a circular track. They took 200 seconds, 300 seconds, 360 seconds and 450 seconds to complete one round. After how much time do they meet at the starting point for the first time?

(a) 1800 seconds   (b) 3600 seconds

(c) 2400 seconds   (d) 4800 seconds

[SSC (GL) Examination, 2011]

8. In a 100 m race, Kamal defeats Bimal by 5 seconds. If the speed of Kamal is 18 Km/h, then the speed of Bimal is:

(a) 15.4 Km/h   (b) 14.5 Km/h

(c) 14.4 Km/h   (d) 14 Km/h

[SSC (GL) Examination, 2010]

9. Raju runs 1250 meter on Monday and Friday. Another days he runs 1500 meter except for Sunday (He does not run on Sunday). How many kilometer will he run in 3 weeks (first day starting from Monday)?

(a) 12.5 Km   (b) 20.5 Km
(c) 8.5 Km   (d) 25.5 Km
(e) None of these

[IOB PO, 2011]

ANSWER KEYS
EXERCISE-1
1. (a)   **2.** (b)   **3.** (c)   **4.** (b)   **5.** (a)   **6.** (b)   **7.** (c)   **8.** (b)   **9.** (a)   **10.** (c)   **11.** (a)   **12.** (b)   **13.** (c)   **14.** (a)   **15.** (d)   **16.** (c)   **17.** (a)   **18.** (a)   **19.** (a)   **20.** (b)
EXERCISE-2
1. (a)   **2.** (a)   **3.** (c)   **4.** (a)   **5.** (a)   **6.** (c)   **7.** (a)   **8.** (c)   **9.** (d)

EXPLANATORY ANSWERS

EXERCISE-1

1. **(a)** B runs 25 m in 5 seconds.

   $\therefore$ B's time to cover 1 Km $= \dfrac{5}{25} \times 100$

   $= 200$ sec

   A's time to cover one Km $= 200 - 5$

   $= 195$ seconds

   $= 3$ minutes 15 seconds.

2. **(b)** 15 m is covered by B in 5 sec

   $\therefore$ 300 m is covered by B in $\dfrac{5}{15} \times 300 = 100$ seconds

   $\therefore$ A takes $100 - 5 = 95$ seconds.

3. **(c)** Time taken by A to run 1 Km

   $= 30 \times 2 = 60$ seconds.

   Time taken by B to run 1 Km $= 35 \times 2 = 70$ seconds.

   $\therefore$ A can give B a start of $(70 - 60) = 10$ seconds.

   In 35 sec B runs 500 m

   $\therefore$ In 10 sec B runs $= \dfrac{500}{35} \times 10 = \dfrac{1000}{7} = 142\dfrac{6}{7}$ m

   So, A can give B a start of $142\dfrac{6}{7}$ metres in a Km race.

4. **(b)** The speed of A and B are in the ratio 11:8.

   Let, speeds be $11s$ and $8s$ (in m/sec).

   Let, race be of x m.

   Then, time taken by A to run x m is same as that of B to run $(x - 120)$ m.

   $\therefore$ $\dfrac{x}{11s} = \dfrac{x-120}{8s}$ $\therefore$ $3x = 11 \times 120$

   $\therefore$ $x = 440$.

5. **(a)** A scores 100 while B scores $100 - 20 = 80$ and C scores $100 - 28 = 72$.

   $\therefore$ While B scores 80, C scores 72.

   $\therefore$ While B scores 100, C scores $\dfrac{72}{80} \times 100 = 90$.

   $\therefore$ B can give C $100 - 90 = 10$ points.

6. **(b)** A's speed:B's speed

   $= 1.\dfrac{1}{2}:1 = \dfrac{3}{2}:1 = 3:2$

   It means that in a race of 3 m, A gains $(3 - 2)$.

   $= 1$ m over B.

   1 m is gained by A in a race of 3 m.

   $\therefore$ 300 m is gained by A in a race of

   $= \dfrac{3}{1} \times 300 = 900$ m.

7. **(c)** Clearly, if A runs 600 metres, B runs $= 540$ m.

   $\therefore$ If A runs 400 m, B runs $= \left(\dfrac{540}{600} \times 400\right)$

   $= 360$ m

   Again, when B runs 500 m, C runs $= 450$ m

   $\therefore$ When B runs 360 m, C runs

   $= \left(\dfrac{450}{500} \times 360\right)$ m $= 324$ m.

8. **(b)** Time taken by A to cover 100 m

$= 100 \div \left(5 \times \dfrac{5}{18}\right) = 72$ seconds.

∴ B covers $(100 - 8)$ or, 92 m in $(72 + 8)$ or, 80 seconds.

∴ Speed of B $= \dfrac{92}{80} \times \dfrac{16}{5} = 4.14$ Km/h.

9. **(a)** Suppose, x points make the game.

Clearly, when A scores x points, B scores $(x - 20)$ points and C scores $(x - 32)$ points.

Now, when B scores x points, C scores $(x - 15)$ points.

When B scores $(x - 20)$ points,

C scores $\left[\dfrac{(x-15)}{x} \times (x-20)\right]$ points

∴ $\dfrac{(x-15)(x-20)}{x} = x - 20$ or, $x = 100$

Hence, 100 points make the game.

10. **(c)** In a game of 50.

While A scores 50, B scores $50 - 6 = 44$ and in a game of 65.

While A scores 65, C scores $65 - 13 = 52$.

∴ While A scores 50, C scores $\dfrac{52}{65} \times 50 = 40$.

∴ While B scores 44, C's score $= 40$.

While B scores 55, C's score $= \dfrac{40}{44} \times 55 = 50$.

∴ In a game of 55,

B can give C $55 - 50 = 5$ points.

11. **(a)** While A runs 1000 m, B runs $1000 - 80 = 920$ m and while B runs 1000 m, C runs $1000 - 60 = 940$ m.

∴ While B runs 920 m; C runs

$\dfrac{940}{1000} \times 920 = \dfrac{4324}{5}$ m

∴ While B runs 920 m; C runs

$\dfrac{940}{1000} \times 920 = \dfrac{4324}{5}$ m

∴ While A runs 1000 m, C runs $\dfrac{4324}{5}$ m

∴ A can beat C by $1000 - \dfrac{4324}{5} = \dfrac{676}{5} = 135\dfrac{1}{5}$ m.

12. **(b)** A:B:C $= 90{:}75{:}60$

B:C $= \dfrac{75}{60} = \dfrac{70 \times \dfrac{100}{75}}{60 \times \dfrac{100}{75}} = \dfrac{100}{80}$

Hence, in a game of 100 points, B can give C$(100 - 80)$ $= 20$ points.

13. **(c)** If A runs 600 m, B runs $600 - 60$ or, 540 m.

If A runs 400 m, B runs $= \dfrac{540 \times 400}{600} = 360$ m.

Now, when B runs 500 m, C runs $500 - 50 = 450$ m.

∴ When B runs 360 m, C runs

$= \dfrac{450 \times 360}{500} = 324$ m

∴ A beats C by $400 - 324 = 76$ m.

14. **(a)** When A scores 75 points, B scores 50 points.

When A scores 90 points, C scores $(90 - 18) = 72$ points.

∴ When A scores 75 points, C scores $= \dfrac{72}{90} \times 75 = 60$ points.

A:B:C $= 75{:}50{:}60$

C:B $= \dfrac{60}{50} = \dfrac{120}{100}$

So, in a game of 120, C can give B $(120 - 100) = 20$ points.

15. **(d)** When A makes 5 rounds, B makes 4 rounds.

In order to pass each other, the difference in number of rounds made by each must be one. Here, A passes B each time, when A makes 5 rounds.

Distance covered by A in 5 rounds $= \dfrac{5 \times 400}{100} = 2$ Km.

In covering 2 Km, A passes B 1 time.

∴ In covering 5 Km, A passes B

$= \dfrac{5}{2} = 2\dfrac{1}{2}$ times.

16. **(c)** To reach the winning post, A will have to cover a distance of $(500 - 140)$,

i.e., 360 m.

While A covers 3 m, B covers 4 m.

While A covers 360 m, B covers $\dfrac{4}{3} \times 360 = 480$ m.

So, A reaches the winning post while B remains 20 m behind.

∴ A wins by 20 m.

17. **(a)** While A runs 1000 m, B runs $1000 - 100 = 900$ m and C runs $1000 - 200 = 800$ m.

∴ While B runs 900 m, C runs 800 m.

∴ While B runs 1350 m; C runs $\dfrac{800}{900} \times 1350$ $= 1200$ m

∴ B can beat C by $1350 - 1200 = 150$ m.

18. **(a)** Speeds (in m/sec) of A and B are

$\dfrac{9}{2} \times \dfrac{5}{18} = \dfrac{5}{4}$ and $6 \times \dfrac{5}{18} = \dfrac{5}{3}$, respectively.

A has a start of 190 m. So, A has to run $1000 - 190 = 810$ m, while B 1000 m.

Time taken by B to cover 1000 m $= \dfrac{3}{5} \times 1000$

$= 600$ seconds.

In this time, A covers $\dfrac{5}{4} \times 600 = 750$ m.

So, B reaches the winning post while A remains $810 - 750 = 60$ m behind.

$\therefore$ B wins by 60 m.

19. (a) Let, times (in sec) taken by A and B to run a Km, be x and y, respectively.

When B gets a start of 50 m, B runs.

$1000 - 50 = 950$ m while A runs 1000 m.

$\therefore \quad \dfrac{950}{100} y - x = 14,$

i.e., $0.95y - x = 14$ ...(1)

and, when B gets a start of 22 seconds, A runs for $(y - 22)$ seconds, while B runs for y seconds.

$\therefore \quad 1000 - \dfrac{1000}{x}(y - 22) = 20$

i.e., $50y - 49x = 1100$.   ...(2)

Multiplying Eq. (1) by 49 and subtract from Eq. (2)

$3.45y = 414$

$\therefore \quad y = 120$ sec.

$\therefore \quad (1) \Rightarrow x = 0.95 \times 120 - 14 = 100$ seconds.

20. (b) A's speed $= \left(5 \times \dfrac{5}{18}\right)$ m/sec $= \dfrac{25}{18}$ m/sec

Time taken by A to cover 100 m $= \left(100 \times \dfrac{18}{25}\right)$ sec $= 72$ seconds.

$\therefore$ B covers 92 m in $72 + 8 = 80$ seconds.

B's speed $= \left(\dfrac{92}{80} \times \dfrac{18}{5}\right)$ Km/h $= 4.14$ Km/h.

EXERCISE-2
(BASED ON MEMORY)

1. (a) $5 - \dfrac{5}{4} = \dfrac{15}{4}$.

2. (a) L.C.M. of 252, 308 and 198 = 2772 seconds
= $46 \times 60 + 12 = 46$ min 12 seconds.

3. (c) A will reach at starting point in $\dfrac{5 \times 2}{5} = 2$ hours

B will reach at starting point in $\dfrac{5}{3}$ hours

C will reach at starting point in $\dfrac{5}{2}$ hours

Then, on the starting point all three will meet after the

L.C.M. of 2, $\dfrac{5}{3}, \dfrac{5}{2} = \dfrac{10}{1} = 10$ hours.

6. (c) Total speed of car, bus and train $= 72 \times 3 = 216$ Km

Speed of car and train $= \dfrac{5+9}{5+9+4} \times 216 = 168$ Km

Average $= \dfrac{168}{2} = 84$ Km

7. (a) Required = L.C.M. of 200, 300, 350 and 450 s
= 1800 s

8. (c) Time taken by Kamal to run 100 m

$= \dfrac{100}{18 \times \dfrac{5}{18}} = 20$ s

Therefore, time taken by Bimal to run 100 m
= 20 + 5 = 25 s

Hence, Bimal's speed

$= \dfrac{100}{25} = 4$ m/sec

$= \dfrac{4 \times 18}{ }$ Km/h = 14.4 Km/h

9. (d) Raju runs on Monday and Friday $= 1250 \times 2 = 2500$ m

On Tuesday, Wednesday, Thursday and Saturday,

Raju runs $1500 \times 4 = 6000$ m

In 1 week Raju runs $6000 + 2500 = 8500$ m

In 3 weeks Raju runs $3 \times 8500 = 25500$ m = 25.5 K