Pipes and Cisterns

11

INTRODUCTION

Pipes are connected to a *tank* or *cistern* and are used to fill or emptying the tank.

Inlet: A pipe connected to a tank or a cistern that fills, it, is known as *inlet*.

Outlet: A pipe connected to a tank or a cistern emptying it is known as *outlet*.

Pipes and Cistern-related mathematical problems are similar to those on 'Time and Work'. The only difference noted is, the work done is in terms of filling or emptying a cistern. The time taken is t by a pipe is the time taken by a pipe or a leak (crack) to fill or emptying a cistern.

Generally, the time taken to fill a cistern is considered as positive. On the other hand, the time taken to empty a cistern is considered as negative. The amount of work done, that is, filling or emptying a cistern is, generally, taken as unity, unless otherwise specified.

SOME BASIC FORMULAE

1. If an inlet can completely fill the empty tank in X hours, the part of the tank filled in 1 hour $= \frac{1}{X}$.
2. If an outlet can empty the full tank in Y hours, the part of the tank emptied in 1 hour $= \frac{1}{Y}$.
3. If both inlet and outlet are open, net part of the tank filled in 1 hour $= \frac{1}{X} - \frac{1}{Y}$.

Illustration 1: A pipe can fill a tank in 5 hours. Find the part of tank filled in one hour.

Solution: The part of the tank filled in 1 hour $= \frac{1}{5}$.

Illustration 2: A pipe can fill a tank in 28 minutes. Find the time in which $\frac{1}{7}$ part of the tank will be filled.

Solution: We have, $\frac{1}{28}$ part of the tank is filled in 1 minute.

$\therefore$ $\frac{1}{7}$ part of the tank is filled in $\frac{28}{7}$ minutes

$= 4$ minutes.

Illustration 3: A pipe can empty a cistern in 40 minutes. Find the time in which $\frac{3}{4}$ part of the cistern will be emptied.

Solution: We have, $\frac{1}{40}$ part of the cistern is emptied in = 1 minute.

$\therefore$ $\frac{3}{4}$ part of the cistern is emptied in

$= 40 \times \frac{3}{4} = 30$ minutes.

Illustration 4: A pipe can empty a cistern in 12 hours. Find the part of the cistern emptied in 4 hours.

Solution: We have, part of the cistern emptied in 1 hour $= \frac{1}{12}$,

$\therefore$ Part of the cistern emptied in 4 hours $= \frac{1}{12} \times 4 = \frac{1}{3}$.

Illustration 5: A tap can fill a cistern is 8 hours and another can empty it in 16 hours. If both the taps are opened simultaneously, find the time (in hours) to fill the cistern.

Solution: Here, $X = 8$ and $Y = 16$.

$\therefore$ Part of the cistern filled in 1 hour

$$= \frac{1}{X} - \frac{1}{Y}$$

$$= \frac{1}{8} - \frac{1}{16} = \frac{1}{16}$$

$\therefore$ Total time taken to fill the cistern = 16 hours.

SOME USEFUL SHORTCUT METHODS

1. Two pipes A, and B, can fill (or empty) a cistern in X and Y hours, while working alone. If both the pipes are opened together, then the time taken to fill (or empty) the cistern is given by

$$\left(\frac{XY}{X+Y}\right) \text{hours.}$$

Explanation

Part of the cistern filled (or emptied) by pipe A alone in 1 hour = $\frac{1}{X}$.

Part of the cistern filled (or emptied) by pipe B alone in 1 hour = $\frac{1}{Y}$.

$\therefore$ Part filled (or emptied) by (A + B) in 1 hour

$$= \frac{1}{X}+\frac{1}{Y}=\frac{X+Y}{XY}$$

Therefore, both the pipes A and B together will fill (or empty) the cistern in $\left(\frac{XY}{X+Y}\right)$ hours.

Illustration 6: Two pipes A and B can fill a cistern in 20 and 30 minutes. If both the pipes are opened simultaneously, how long will it take to fill the cistern?

Solution: Here, $X = 20$ and $Y = 30$.

$\therefore$ Part of the cistern filled by (A + B) in 1 minute

$$= \frac{1}{X}+\frac{1}{Y}=\frac{1}{20}+\frac{1}{30}=\frac{5}{60}=\frac{1}{12}.$$

$\therefore$ Both the pipes A and B together will fill the cistern in 12 minutes.

2. Three pipes A, B and C can fill a cistern in X, Y and Z hours respectively, while working alone. If all the three pipes are opened together, the time taken to fill the cistern is given by

$$\left(\frac{X\times Y\times Z}{XY+YZ+ZX}\right) \text{hours.}$$

Explanation

Part of the cistern filled by A alone in 1 hour = $\frac{1}{X}$

Part filled by B alone in 1 hour = $\frac{1}{Y}$

Part filled by C alone in 1 hour = $\frac{1}{Z}$

All the three pipes are opened.

$\therefore$ Part filled in 1 hour = $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

$$= \frac{XY+YZ+ZX}{XYZ}$$

$\therefore$ The cistern will be filled in $\frac{XYZ}{XY+YZ+ZX}$ hours.

Note:

We can generate more formulae like above by replacing negative sign wherever a pipe starts emptying a cistern instead of the standard positive sign.

Illustration 7: Two pipes, A and B, can separately fill a cistern is 8 hours and 12 hours respectively, while a third pipe C can empty it in 6 hours. In what time will the cistern be full, if all the pipes are opened together?

Solution: Here, $X = 8$, $Y = 12$ and $Z = -6$.

$\therefore$ The cistern will be full in

$$= \left(\frac{8\times12\times-6}{8\times12-12\times6-6\times8}\right) \text{hours}$$

$$= \left(\frac{576}{24}\right) \text{hours or 24 hours.}$$

3. Two pipes, A and B, can fill a cistern in X hours and Y hours, respectively. There is also an outlet C. If all the three pipes are opened together, the tank is full in Z hours. The time taken by C to empty the full tank is given by

$$\left(\frac{XYZ}{XZ+YZ-XY}\right) \text{hours.}$$

Explanation

Part of the tank emptied by C in 1 hour

$$= \left(\left(\frac{1}{X}+\frac{1}{Y}-\frac{1}{Z}\right)\right)$$

$\therefore$ C can empty the full tank in $\left(\frac{XYZ}{XZ+YZ-XY}\right)$ hours.

Illustration 8: Two taps A and B can fill a cistern in 30 minutes and 60 minutes, respectively. There is a third exhaust tap C at the bottom of the tank. If all the taps are opened at the same time, the cistern will be full in 45 minutes. In what time can exhaust tap C empty the cistern when it is full?

Solution: Here, $X = 30$, $Y = 60$ and $Z = 45$.

$\therefore$ Exhaust tap C can empty the cistern in

$$= \left(\frac{XYZ}{XZ + YZ - XY}\right) \text{ minutes}$$

$$= \left(\frac{30 \times 60 \times 45}{30 \times 45 \times 60 \times 45 - 30 \times 60}\right) \text{minutes} = 36 \text{ minutes.}$$

4. A tank takes X hours to be filled by a pipe. But, due to a leak, it is filled in Y hours. The amount of time in which the leak can empty the full tank

$$= \left(\frac{XY}{Y - X}\right) \text{hours.}$$

Illustration 9: A pipe can fill a tank in 12 hours. Due to leakage at the bottom, it is filled in 24 hours. If the tank is full, how much time will the leak take to empty it?

Solution: Here, $X = 12$ and $Y = 24$.

$\therefore$ The time taken by the leak to empty the full tank

$$= \left(\frac{XY}{Y - X}\right) \text{hours} = \left(\frac{12 \times 24}{24 - 12}\right) \text{hours or 24 hours.}$$

5. A cistern has a leak which can empty it in X hours. A pipe which admits Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hours. The capacity of the cistern is

$$\left(\frac{XYZ}{Z - X}\right) \text{ litres.}$$

Illustration 10: A leak at the bottom of a tank can empty the full tank in 6 hours. An inlet pipe fills water at the rate of 4 litres per minute. When the tank is full, the inlet is opened and due to leak, the tank is empty in 8 hours. Find out the capacity of the tank.

Solution: Here, $X = 6$, $Y = 4 \times 60 = 240$ and $Z = 8$.

$\therefore$ The capacity of the tank is

$$= \left(\frac{XYZ}{Z - X}\right) \text{litres} = \left(\frac{6 \times 240 \times 8}{8 - 6}\right) \text{ litres} = 5760 \text{ litres.}$$

6. One fill pipe A is k times faster than the other fill pipe B.
   (a) If B can fill a cistern in x hours, then the time in which the cistern will be full, if both the fill pipes are opened together, is $\left(\frac{x}{k+1}\right)$ hours.
   (b) If A can fill a cistern in y hours, then the time in which the cistern will be full, if both the fill pipes are opened together, is $\left(\frac{k}{k+1}\right) y$ hours.

Illustration 11: One fill pipe A is 10 times faster than the second fill pipe B. If B can fill a cistern in 55 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

Solution: Here, $k = 10$ and $x = 55$.

$\therefore$ The cistern will be full in

$$= \left(\frac{x}{k+1}\right) \text{minutes} = \left(\frac{55}{10+1}\right) \text{minutes or 5 minutes.}$$

Illustration 12: One fill pipe A is 4 times faster than the second fill pipe B. If A can fill a cistern in 15 minutes, then find out the time when the cistern will be full if both fill pipes are opened together.

Solution: Here, $k = 4$ and $y = 15$.

$\therefore$ The cistern will be full in $= \left(\frac{k}{k+1}\right) y$ minutes

$$= \left(\frac{4}{4+1}\right) 15 \text{ minutes} = 12 \text{ minutes.}$$

7. If one fill pipe A is k times faster and takes x minutes less time than the other fill pipe B, then
   (a) the time taken to fill a cistern, if both the pipes are opened together is $\left(\frac{kx}{(k-1)^2}\right)$ minutes.
   (b) A will fill the cistern in $\left(\frac{x}{k-1}\right)$ minutes.
   (c) B will fill the cistern in $\left(\frac{kx}{k-1}\right)$ minutes.

Illustration 13: One fill pipe A is 5 times faster than the second fill pipe B, and takes 32 minutes less than the fill pipe B. When will the cistern be full if both fill pipes are opened together?

Solution: Here, $k = 5$ and $x = 32$.

$\therefore$ The cistern will be full in $= \frac{kx}{(k-1)^2}$ minutes

$$= \frac{5 \times 32}{(5-1)^2} \text{ minutes}$$

$$= 10 \text{ minutes.}$$

EXERCISE-I

1. One tap can fill a cistern in 2 hours and another can empty the cistern in 3 hours. How long will they take to fill the cistern if both the taps are opened?

(a) 6 hours (b) 7 hours
(c) 6.30 hours (d) None of these

2. A tap can fill a tank in 25 minutes and another can empty it in 50 minutes. Find out whether the tank will be filled up or emptied in how many minutes?

(a) The tank is filled up in 50 minutes.
(b) The tank is emptied in 25 minutes.
(c) The tank is filled up in 25 minutes.
(d) None of these

3. A water tank is $\frac{2}{5}$ full. Pipe A can fill the tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, then how long will it take to empty or fill the tank completely?

(a) 6 minutes to fill (b) 6 minutes to empty
(c) 8 minutes to fill (d) None of these

4. Two taps A and B can fill a tank in 10 hours and 15 hours, respectively. If both the taps are opened together, the tank will be full in:

(a) 8 hours (b) 6 hours
(c) 5 hours (d) None of these

5. Two pipes A and B can separately empty a cistern in 12 hours and 15 hours, respectively. In what time will the cistern be emptied, if both the pipes are opened together?

(a) 5 hours 30 minutes (b) 7 hours
(c) 6 hours 40 minutes (d) None of these

6. Two pipes can fill a tank in 10 hours and 12 hours, respectively. While a third pipe emptied the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time the tank will be filled?

(a) 7 hours 30 minutes (b) 6 hours 40 minutes
(c) 8 hours 30 minutes (d) None of these

7. Three pipes A, B and C can fill a cistern in 10, 12 and 15 hours, respectively, while working alone. If all the three pipes are opened together, the time taken to fill the cistern will be:

(a) 4 hours (b) 6 hours
(c) 7 hours (d) None of these

8. Two pipes A and B can fill a cistern in 24 minutes and 30 minutes, respectively. There is also an outlet C. If all the three pipes are opened together, the cistern is full in 20 minutes. How much time will be taken by outlet C to empty the full cistern?

(a) 30 minutes (b) 40 minutes
(c) 45 minutes (d) None of these

9. A cistern is normally filled in 8 hours, but it takes 2 hours longer to fill because of a leak at its bottom. If the cistern is full, the leak will empty it in:

(a) 35 hours (b) 45 hours
(c) 40 hours (d) None of these

10. A cistern has a leak which would empty in 8 hours. A tap is turned on which admits 6 litres a minute into the cistern and it is now emptied in 12 hours. The cistern can hold:

(a) 6840 litres (b) 7860 litres
(c) 8640 litres (d) None of these

11. If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does the faster pipe take to fill the reservoir?

(a) 35 hours (b) 30 hours
(c) 40 hours (d) None of these

12. One fill pipe A is 3 times faster than second fill pipe B and takes 32 minutes less than the fill pipe B. When will the cistern be full if both the pipes are opened together?

(a) 28 mintues (b) 24 minutes.
(c) 30 minutes (d) Data inadequate

13. Two pipes A and B can fill a cistern in 4 minutes and 6 minutes, respectively. If these pipes are turned on alternately for 1 minute each, then how long will it take for the cistern to fill?

(a) 4 m 40 s (b) 3 m 20 s
(c) 4 m 50 s (d) 3 m 30 s

14. There are two taps to fill a tank while a third to empty it. When the third tap is closed, they can fill the tank in 10 minutes and 12 minutes, respectively. If all the three taps be opened, the tank is filled in 15 minutes. If the first two taps are closed, in what time can the third tap empty the tank when it is full?

(a) 7 minutes
(b) 9 minutes and 32 Seconds
(c) 8 minutes and 34 Seconds
(d) 6 minutes

15. Two pipes, A and B, can separately fill a cistern in 15 minutes and 18 minutes, respectively, while a third pipe C can empty it in 6 minutes. Two pipes, A and B, are kept open for 6 minutes in the beginning and, then the third pipe is also opened. In what time will the cistern be emptied?

(a) $16\frac{1}{2}$ minutes (b) 15 minutes

(c) $15\frac{1}{2}$ minutes (d) 16 minutes

16. A reservoir is fitted with two pipes A and B. Pipe A can fill the reservoir 5 hours faster than pipe B. If both the pipe together fill the reservoir in 6 hours, the reservoir will be filled by A alone in:

(a) 10 hours (b) 8 hours
(c) 12 hours (d) 11 hours

17. A cistern is provided by two taps A and B. Tap A can fill it in 20 minutes and tap B in 25 minutes. Both the taps are kept open for 5 minutes and, then the second is turned off. The cistern will be completely filled in another:

(a) 11 minutes (b) 10 minutes
(c) 15 minutes (d) 12 minutes

18. Two pipes, A and B, can separately fill a tank in 6 hours and 8 hours, respectively. Both the pipes are opened together, but $1\frac{1}{2}$ hours later pipe A is turned off. How much time will it take to fill the tank?

(a) 5 hours (b) 6 hours

(c) $4\frac{1}{2}$ hours (d) $5\frac{1}{2}$ hours

19. A cistern has two taps which fill it in 12 minutes and 15 minutes, respectively. There is also a waste pipe in the cistern. When all the pipes are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty a full cistern?

(a) 8 minutes (b) 10 minutes
(c) 12 minutes (d) 16 minutes

20. Two taps can separately fill a cistern in 10 minutes and 15 minutes, respectively. When the waste pipe is open, they can together fill it in 18 minutes. The waste pipe can empty the full cistern in:

(a) 7 minutes (b) 9 minutes
(c) 13 minutes (d) 23 minutes

Exercise-2
(Based on Memory)

1. A cistern has two pipes. One can fill it with water in 8 hours and the other can empty it in 5 hours. In how many hours will the cistern be emptied if both the pipes are opened together when $\frac{3}{4}$ of the cistern is already full of water?

(a) $13\frac{1}{3}$ hours (b) 10 hours

(c) 6 hours (d) $\frac{1}{3}$ hours

[SSC (GL) Prel. Examination, 2007]

2. Tap 'A' can fill a water tank in 25 minutes, tap 'B' can fill the same tank in 40 minutes and tap 'C' can empty that tank in 30 minutes. If all the three taps are opened together, in how many minutes will the tank be completely filled up or emptied?

(a) $3\frac{2}{13}$ (b) $15\frac{5}{13}$

(c) $8\frac{2}{13}$ (d) $31\frac{11}{19}$

(e) None of these

[BSRB Patna PO, 2001]

3. Taps A, B and C are connected to a water tank and the rate of flow of water is 42 litres/h, 56 litres/h and 48 litres/h, respectively. Taps A and B fill the tank while tap C empties the tank. If all the three taps are opened simultaneously, the tank gets completely filled up in 16 hours. What is the capacity of the tank?

(a) 960 litres (b) 2346 litres
(c) 1600 litres (d) 800 litres

[SBI PO, 2001]

4. 20 buckets of water fill a tank when the capacity of each bucket is 13.5 litres. How many buckets will be required to fill the same tank if the capacity of each bucket is 9 litres?

(a) 30 (b) 32
(c) Data inadequate (d) None of these

[IBPS Bank PO, 2002]

5. A pipe of diameter d can drain a certain water tank in 40 minutes. The time taken by a pipe of diameter $2d$ for doing the same job is:

(a) 5 minutes (b) 10 minutes
(c) 20 minutes (d) 50 minutes

[SSC (GL) Prel. Examination, 2000]

6. A tap can empty a tank in one hour. A second tap can empty it in 30 minutes. If both the taps operate simultaneously, how much time is needed to empty the tank?

(a) 20 minutes (b) 30 minutes
(c) 40 minutes (d) 45 minutes

[SSC (GL) Prel. Examination, 2000]

7. Two taps can fill a tank in 4 hours and 6 hours, respectively. A third tap can empty the tank in 3 hours. If all the three taps are opened how much time will it take to fill the tank?

(a) 3 hours (b) 9 hours
(c) 12 hours (d) 24 hours

[SSC (GL) Prel. Examination, 2000]

8. A cistern can be filled with water by a pipe in 5 hours and it can be emptied by a second pipe in 4 hours. If both the pipes are opened when the cistern is full, the time in which it will be emptied is:

(a) 9 hours (b) 18 hours
(c) 20 hours (d) $20\frac{1}{2}$ hours

[SSC (GL) Prel. Examination, 2002]

9. A pump can fill a tank with water in 2 hours. Because of a leak in the tank it was taking $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water off the tank in:

(a) 8 hours (b) 7 hours
(c) $4\frac{1}{3}$ hours (d) 14 hours

[SSC (GL) Prel. Examination, 2002]

10. A cistern is normally filled with water in 10 hours but takes 5 hours longer to fill because of a leak in its bottom. If the cistern is full, then the leak will empty the cistern in:

(a) 20 hours (b) 40 hours
(c) 50 hours (d) 30 hours

[SSC (GL) Prel. Examination, 2002]

11. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minute respectively. There is a third pipe at the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 50 minutes. In how much time can the third pipe alone empty the cistern?

(a) 110 minutes (b) 100 minutes
(c) 120 minutes (d) 90 minutes

[SSC (GL) Prel. Examination, 2003]

12. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?

(a) 4 hours
(b) 4 hours 15 minutes
(c) 3 hours 15 minutes
(d) 3 hours 45 minutes

[SSC (GL) Prel. Examination 2003]

13. A water tank has three taps A, B and C. Tap A, when opened, can fill the water tank alone in 4 hours. Tap B, when opened, can fill the water tank alone in 6 hours, Tap C, when opened, can empty the water tank alone in 3 hours. If tap A, B and C are opened simultaneously, how long will it take to fill the tank completely?

(a) 10 hours (b) 8 hours
(c) 18 hours (d) 12 hours

[Indian Bank PO Examination 2011]

14. A pipe fills a water tank three times faster than another pipe. If the two pipes together can fill the empty tank in 36 minutes, then how much time will the slower pipe alone would take to fill the tank?

(a) 1 hour 21 minutes (b) 1 hour 48 minutes
(c) 2 hours (d) 2 hours 24 minutes

[SSC (GL) Examination 2010]

15. Having the same capacity 9 taps fill up a water tank in 20 minutes. How many taps of the same capacity are required to fill up the same water tank in 15 minutes?

(a) 10 (b) 12
(c) 15 (d) 18

[SSC, 2014]

16. Two pipes A and B can fill a cistern in 3 hours and 5 hours, respectively. Pipe C can empty in 2 hours. If all the three pipes are open, in how many hours the cistern will be full?

(a) 30 hours (b) 10 hours
(c) 15 hours (d) Cannot be filled

[SSC Assistant Grade III, 2013]

17. Two pipes can fill a cistern separately in 24 minutes and 40 minutes respectively. A waste pipe can drain off 30 litres per minute. If all the three pipes are opened, the cistern fills in one hour. The capacity (in litres of the cistern) is:

(a) 800 (b) 400
(c) 600 (d) 500

[SSC Assistant Grade III, 2012]

18. A tank can be filled by pipe A in 2 hours and pipe B in 6 hours. At 10 am pipe A was opened. At what time will the tank be filled if pipe B is opened at 11 am?

(a) 12.45 am (b) 5 pm
(c) 11.45 am (d) 12 pm

[SSC, 2012]

19. A swimming pool has 3 drain pipes. The first two pipes A and B, operating simultaneously, can empty the pool in half the time that C (the 3rd pipe) alone takes to empty it. Pipe A, working alone, takes half the time taken by pipe B. Together they take 6 hours 40 minutes to empty the pool. Time taken by pipe A to empty the pool, in hours, is:

(a) 15 (b) 10
(c) 30 (d) 7

[SSC, 2012]

20. A cistern has 3 pipes A, B and C. A and B can fill it in 3 and 4 hours respectively, and C can empty it in 1 hour. If the pipes are opened at 3 pm, 4 pm and 5 pm respectively on the same day, the cistern will be empty at:

(a) 7:12 pm (b) 7:15 pm
(c) 7:10 pm (d) 7:28 pm

[SSC, 2011]

21. Two pipes can fill an empty tank separately in 24 minutes and 40 minutes respectively and a third pipe can empty 30 gallons of water per minute. If all the three pipes are opened, empty tank becomes full in one hour. The capacity of the tank (in gallons) is:

(a) 800 (b) 600
(c) 500 (d) 400

[SSC, 2010]

22. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in:

(a) $4\frac{1}{2}$ hours (b) 7 hours
(c) 8 hours (d) 14 hours
(e) None of these

[IBPS PO/MT, 2013]

ANSWER KEYS

EXERCISE-1

1. (a) **2.** (a) **3.** (b) **4.** (b) **5.** (c) **6.** (a) **7.** (a) **8.** (b) **9.** (c) **10.** (c) **11.** (b) **12.** (b) **13.** (a)
14. (c) **15.** (a) **16.** (a) **17.** (a) **18.** (b) **19.** (b) **20.** (b)

EXERCISE-2

1. (b) **2.** (d) **3.** (d) **4.** (a) **5.** (b) **6.** (a) **7.** (c) **8.** (c) **9.** (d) **10.** (d) **11.** (b) **12.** (d) **13.** (d)
14. (d) **15.** (b) **16.** (d) **17.** (c) **18.** (c) **19.** (a) **20.** (a) **21.** (b) **22.** (d)

EXPLANATORY ANSWERS

EXERCISE-I

1. (a) Here, $X = 2$ and $Y = 3$.

$\therefore$ Part of the cistern filled in 1 hour

$= \frac{1}{X} - \frac{1}{Y} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$.

$\therefore$ Total time taken to fill the cistern = 6 hours.

2. (a) Here, $X = 25$ and $Y = 50$.

$\therefore$ Part of the tank filled or emptied in 1 minute

$$= \frac{1}{X} - \frac{1}{Y} = \frac{1}{25} - \frac{1}{50} = \frac{1}{50},$$

which is positive, therefore the tank will be filled.

$\therefore$ Total time taken to fill the tank = 50 minutes.

3. (b) Here, $X = 10$ and $Y = 6$.

$\therefore$ Part of the tank filled or emptied in 1 minute

$$= \frac{1}{X} - \frac{1}{Y} = \frac{1}{10} - \frac{1}{6} = -\frac{1}{15},$$

which is negative, therefore the tank will be emptied.

Thus, $\frac{2}{5}$ full of the tank will be emptied in

$= 15 \times \frac{2}{5} = 6$ minutes.

4. (b) Here, $X = 10$ and $Y = 15$.

$\therefore$ The tank will be full in

$= \left(\frac{XY}{X+Y}\right)$ hours

$= \left(\frac{10 \times 15}{10+15}\right)$ hours, or 6 hours.

5. (c) Here, $X = 12$ and $Y = 15$.

$\therefore$ The cistern will be empty in

$= \left(\frac{XY}{X+Y}\right)$ hours

$= \left(\frac{12 \times 15}{12+15}\right)$ hours

$= \frac{20}{3}$ hours or, 6 hours 40 minutes.

6. (a) Here, $X = 10$, $Y = 12$ and $Z = -20$.

$\therefore$ The tank will be full in

$= \left(\frac{X \times Y \times -Z}{XY - YZ - ZX}\right)$ hours

$= \left(\frac{10 \times 12 \times -20}{10 \times 12 - 12 \times 20 - 20 \times 10}\right)$ hours

$= \left(\frac{15}{2}\right)$ hours or, 7 hours 30 minutes.

7. (a) Here, $X = 10$, $Y = 12$ and $Z = 15$.

$\therefore$ Total time taken to fill the cistern

$= \left(\frac{XYZ}{XY + YZ + ZX}\right)$ hours

$= \left(\frac{10 \times 12 \times 15}{10 \times 12 + 12 \times 15 + 10 \times 15}\right)$ hours

$= 4$ hours.

8. (b) Here, $X = 24$, $Y = 30$ and $Z = 20$.

$\therefore$ Total time taken by C to empty the full cistern

$= \left(\frac{XYZ}{XZ + YZ - XY}\right)$ minutes

$= \left(\frac{24 \times 30 \times 20}{24 \times 20 + 30 \times 20 - 24 \times 30}\right)$ minutes

$= 40$ minutes.

9. (c) Here, $X = 8$ and $Y = 8 + 2 = 10$.

$\therefore$ The leak will empty the cistern in

$= \left(\frac{XY}{Y - X}\right)$ hours

$= \left(\frac{8 \times 10}{10 - 8}\right)$ hours, or, 40 hours.

10. (c) Here, $X = 8$, $Y = 6 \times 60 = 360$ and $Z = 12$.

$\therefore$ The capacity of the cistern is

$= \left(\frac{XYZ}{Z - X}\right)$ litres $= \left(\frac{8 \times 360 \times 12}{12 - 8}\right)$ litres

$= 8640$ litres.

11. (b) Let, one pipe take x hours to fill the reservoir. The, other pipe takes $(x - 10)$ hours.

$\therefore \quad \frac{1}{x} + \frac{1}{x-10} = \frac{1}{12}$

$\Rightarrow \quad x(x - 10) = 12(x + x - 10)$

$\Rightarrow \quad x^2 - 34x + 120 = 0$

or, $\quad (x - 30)(x - 4) = 0$

$\therefore \quad x = 30$ or, $x = 4$.

$\therefore$ The faster pipe takes 30 hours to fill the reservoir.

12. (b) Here, $k = 3$ and $x = 32$.

$\therefore$ The cistern will be full in $= \frac{kx}{(k-1)^2}$ minutes

$= \frac{3 \times 32}{(3-1)^2}$ minutes

$= 24$ minutes.

13. **(a)** As the pipes are operating alternately, thus their 2 minutes job is $\frac{1}{4}+\frac{1}{6}=\frac{5}{12}$.

In the next 2 minutes, the pipes can fill another $\frac{5}{12}$ part of cistern. Therefore, in 4 minutes, the two pipes which are operating alternately will fill $\frac{5}{12}+\frac{5}{12}=\frac{10}{12}=\frac{5}{6}$ part.

The part of the cistern left unfilled = $1-\frac{5}{6}=\frac{1}{6}$. Pipe A can fill $\frac{1}{4}$ of the cistern in 1 minute. Pipe A can fill $\frac{1}{6}$ of the cistern in $4\times\frac{1}{6}=\frac{2}{3}$ minutes. Total time taken to fill the cistern $4+\frac{2}{3}=4\frac{2}{3}$ minutes, or, 4 minutes 40 seconds.

14. **(c)** Part emptied by the third pipe in 1 minute

$$=\left(\frac{1}{10}+\frac{1}{12}\right)-\frac{1}{15}=\frac{7}{60}.$$

So, the full tank will be emptied by the third pipe in $\left(\frac{60}{7}\right)$ minute = 8 minute 34 seconds.

15. **(a)** Pipe (A + B)'s 6 minutes job = $6\left(\frac{1}{15}+\frac{1}{18}\right)$

$$=\frac{11}{15}$$

Net work done by the three pipes (A + B + C) in 1 minute

$$=\left(\frac{1}{15}+\frac{1}{18}\right)-\frac{1}{6}=\frac{-4}{90}=\frac{-2}{45}.$$

Net $\frac{2}{45}$ part of the tank is emptied by pipe C in 1 minute.

Net $\frac{11}{15}$ part of the tank is emptied by pipe C in $\frac{45}{2}\times\frac{11}{15}=\frac{33}{2}$ minutes = $16\frac{1}{2}$ minutes.

16. **(a)** Let, A alone can fill the reservoir in x hours.
Then, B can fill the reservoir in $(x + 5)$ hours.

$\therefore \quad \frac{1}{x}+\frac{1}{x+5}=\frac{1}{6}$

$\therefore \quad 6(2x + 5) = x(x + 5)$

or, $x^2 - 7x - 30 = 0$

or, $(x - 10)(x + 3) = 0$

or, $x = 10$ hours.

17. **(a)** Part filled in 1 minute = $\frac{1}{20}+\frac{1}{25}=\frac{9}{100}$.

Part filled in 5 minutes = $\frac{9}{100}\times5=\frac{9}{20}$.

Unfilled part = $1-\frac{9}{20}=\frac{11}{20}$.

This is to be filled by A alone and, hence will be filled in $20\times\frac{11}{20}=11$ minutes.

18. **(b)** Pipe (A + B)'s $1\frac{1}{2}$ hours job = $\frac{3}{2}\left(\frac{1}{6}+\frac{1}{8}\right)$.

Part unfilled = $1-\frac{7}{16}=\frac{9}{16}$.

Pipe B can fill $\frac{1}{8}$ of the tank in = 1 hour.

Pipe B can fill $\frac{9}{16}$ of the tank in = $8\times\frac{9}{16}$

$$=\frac{9}{2}\text{ hours.}$$

Total time taken to fill the tank = $\left(\frac{3}{2}+\frac{9}{2}\right)$ hours = 6 hours.

19. **(b)** Work done by the waste pipe in 1 minute

$$=\left(\frac{1}{12}+\frac{1}{15}\right)-\frac{1}{20}$$

$$=\left(\frac{3}{20}-\frac{1}{20}\right)=\frac{1}{10}$$

$\therefore$ The waste pipe can empty the cistern in 10 minutes.

20. **(b)** Work done by the waste pipe in 1 minute

$$=\left(\frac{1}{10}+\frac{1}{15}\right)-\frac{1}{18}$$

$$=\left(\frac{1}{6}-\frac{1}{18}\right)=\frac{1}{9}$$

$\therefore$ The waste pipe can empty the cistern in 9 minutes.

EXERCISE-2
(BASED ON MEMORY)

1. (b) In one hour, $\frac{1}{5}-\frac{1}{8}=\frac{3}{40}$ of the cistern is emptied.

$\therefore$ Whole cistern is emptied in $\frac{40}{3}$ hours

i.e., $\frac{3}{4}$ of the cistern is emptied in $\frac{40}{3}\times\frac{3}{4}=10$ hours.

3. (d) Capacity of the tank = (42 + 56 − 48) × 16
= 800 litres.

4. (a) Capacity of the tank = 20 × 13.5
= 270 litres

When the capacity of each bucket = 9 litres then the required number of buckets

$= \frac{40}{3} = 30.$

5. (b) Ratio of volumes = $\frac{\pi d^2 h}{\pi\left(\frac{d}{2}\right)^2 h}=\frac{4}{1}$

$\therefore$ A pipe of radius $\frac{d}{2}$ can drain a water tank in 40 minutes.

$\Rightarrow$ A pipe of radius d can drain the same water tank in 10 minutes.

6. (a) In one minute $\frac{1}{60}+\frac{1}{30}=\frac{1}{20}$ of the tank will be empty.

7. (c) In one hour, $\frac{1}{4}+\frac{1}{6}-\frac{1}{3}=\frac{3+2-4}{12}$

$= \frac{1}{12}$ of the tank will be filled.

8. (c) Required time = $\frac{5\times4}{5-4}$ = 20 hours.

9. (d) Required time = $\frac{2\times\frac{7}{3}}{\frac{7}{3}-2}$ = 14 hours.

10. (d) In one hour, $\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$ of the cistern will be empty.

11. (b) Let, the third pipe empty the cistern in K minutes,

i.e., $\frac{1}{60}+\frac{1}{75}-\frac{1}{K}=\frac{1}{50}$

or, $\frac{1}{60}+\frac{1}{75}-\frac{1}{50}=\frac{1}{K}$ or, K = 100 minutes.

12. (d) Half of the tank is filled in $\frac{1}{2}$ × 6 = 3 hours.

Now, we have four taps and each tap can fill the tank in 6 hours.

When all the four taps are opened, then they can fill $\frac{1}{2}$ of the tank in $\frac{6}{4}\times\frac{1}{2}=\frac{3}{2}$ hours = 45 minutes.

$\therefore$ Total time = 3 hours 45 minutes.

13. (d) The required time to fill the tank

$= \frac{1}{\left(\frac{1}{4}+\frac{1}{6}\right)-\frac{1}{3}} = \frac{1}{\frac{5}{12}-\frac{1}{3}} = \frac{1}{\frac{1}{12}} = 12$ hours

14. (d) Let the time taken by the pipe at faster rate to fill the tank be x minutes.

Therefore, $\frac{1}{x}+\frac{1}{3x}=\frac{1}{36}$

$\Rightarrow \frac{3+1}{3x} = \frac{1}{36}$

$\Rightarrow \frac{4}{3x} = \frac{1}{36}$

$\Rightarrow 3x = 4 \times 36$

$\Rightarrow 3x = 144$

$\Rightarrow x = \frac{148}{3} = 48$ minutes

Hence, the time taken by slower pipe
$= 3x = 3 \times 48 = 144$ minutes
= 2 hours 24 minutes.

15. (b) Quicker Method:

$M_1D_1 = M_2D_2$

$\Rightarrow 9\times20 = M_2\times15$

$\Rightarrow M_2 = \frac{9\times20}{15} = 12$ pipes

Note:

Same relation as men and days is applicable here also.

16. (d) Part of cistern filled by three pipes in an hour

$= \frac{1}{3}+\frac{1}{5}-\frac{1}{2}=\frac{10+6-15}{30}=\frac{1}{30}$

Hence, the cistern will be filled in 30 hours.

17. (c) Let the waste pipe drains off the tank in x minutes. According to the question,

$\frac{1}{24}+\frac{1}{40}-\frac{1}{x}=\frac{1}{60}$

$\Rightarrow \frac{1}{x}=\frac{1}{24}+\frac{1}{40}-\frac{1}{60}=\frac{5+3-2}{120}=\frac{1}{20}$

$\Rightarrow x = 20$ minutes

$\therefore$ Capacity of the cistern = 20 × 30 = 600 litres.

18. **(c)** Part of the tank filled in 1 hour by pipe $A = \frac{1}{2}$

Part of the tank filled by both pipes in 1 hour $= \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} = \frac{2}{3}$

$\therefore$ Time taken to fill $\frac{2}{3}$ parts $= 60$ minutes

$\therefore$ Time taken to fill $\frac{1}{2}$ part $= \frac{60 \times 3}{2} \times \frac{1}{2}$

$= 45$ minutes

$\therefore$ The tank will be filled at 11:45 am

19. **(a)** Times taken by pipe B = $2x$ hours

Times taken by pipe A = x hours

$\therefore$ Time taken by pipe $C = \frac{2}{\frac{1}{2x} + \frac{1}{x}} = \frac{2}{\frac{1+2}{2x}}$

$= \frac{4x}{3}$ hours

Now, according to the question,

$\frac{1}{x} + \frac{1}{2x} + \frac{3}{4x} = \frac{1}{6 + \frac{40}{60}} = \frac{1}{6 + \frac{2}{3}}$

$\Rightarrow \quad \frac{4+2+3}{4x} = \frac{3}{20}$

$\Rightarrow \quad 9 \times 20 = 4x \times 3$

$\Rightarrow \quad x = \left(\frac{9 \times 20}{4 \times 3}\right) = 15$ hours.

20. **(a)** Part of the cistern filled in 2 hours by pipe A $= \frac{2}{3}$

Part of the cistern filled in 1 hours by pipe B $= \frac{1}{4}$

$\therefore$ Total part filled $= \frac{2}{3} + \frac{1}{4} = \frac{8+3}{12} = \frac{11}{12}$

When all three pipes are opened, the part filled in one hour $= \frac{1}{3} + \frac{1}{4} - 1 = \frac{4+3-12}{12} = \frac{-5}{12}$

i.e., $\frac{5}{12}$ part will be emptied per hour.

$\therefore$ Time taken to empty $\frac{11}{12}$ part

$= \frac{11}{12} \times \frac{12}{5} = \frac{11}{5}$ hours

$= 2$ hours 12 minutes

$\therefore$ Required time = 5 + 2:12 = 7:12 pm.

21. **(b)** The first pipe will fill $\frac{1}{24}$ part in 1 minute.

Second pipe will fill $\frac{1}{40}$ part in 1 minute.

Both pipes fill the tank in one minute

$= \frac{1}{24} + \frac{1}{40} = \frac{5+3}{120} = \frac{1}{15}$ part

Another third pipe will empty $\frac{1}{x}$ part in one minute.

$\left(\frac{1}{x} = 30 \text{ gallons}\right)$

Required time to fill the tank $= \frac{1}{\left(\frac{1}{15} - \frac{1}{x}\right)}$ minutes.

Now, according to the question,

$\frac{15x}{(x-15)}$ minutes $= 60$ minutes

$\Rightarrow \frac{15x}{x-15} = 60 \Rightarrow x = 4x - 60$

$\Rightarrow \quad x = 20$ minutes

$\because \frac{1}{x} = 30$ gallons

$\Rightarrow \frac{1}{20}$ part $= 30$ gallons

$\therefore$ Total capacity = 600 gallons

22. **(d)** Part of the tank emptied in 1 hour by the leak $= \frac{1}{2} - \frac{3}{7} = \frac{1}{14}$

$\therefore$ The leak will empty the tank in 14 hours.