Progression

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INTRODUCTION

In this chapter, we shall be concerned with the study of sequences, i.e., special types of functions whose domain is the set N of natural numbers. We shall study particular types of sequences called *arithmetic* sequences, *geometric* sequences and *harmonic* sequences and also their corresponding series.

Premiums on life insurance, fixed deposits in a bank, loan instalments payments, disintegration or decay of radioactive materials and the like are some of the examples where the concept of sequence and series is used.

SEQUENCE

A *sequence* is a function whose domain is the set N of natural numbers and range, a subset of real numbers or complex numbers.

A sequence whose range is a subset of real numbers is called a *real sequence*. Since we shall be dealing with real sequences only, we shall use the term 'sequence' to denote a 'real sequence'.

Notation

The different terms of a sequence are usually denoted by $a_1, a_2, a_3, \ldots$ or by $t_1, t_2, t_3, \ldots$. The subscript (always a natural number) denotes the position of the term in the sequence. The number occurring at the nth place of a sequence, i.e., t_n is called the *general term* of the sequence.

Note:

A sequence is said to be *finite* or *infinite* (according as finite or infinite number of terms it has.)

PROGRESSIONS

If the terms of a sequence follow certain pattern, then the sequence is called a *progression*.

Illustration 1: Consider the following sequences:

(i) 3, 5, 7, 9, ..., 21.

(ii) 8, 5, 2, – 1, – 4, ...

(iii) 2, 6, 18, 54, ..., 1458.

(iv) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

(v) 1, 4, 9, 16, ...

We observe that each term (except the first) in (i) is formed by adding 2 to the preceding term; each term in (ii) is formed by subtracting 3 from the preceding term; each term in (iii) is formed by multiplying the preceding term by 3; each term in (iv) is formed by dividing the preceding term by 2; each term in (v) is formed by squaring the next natural number. Thus, each of (i) to (v) is a progression. Moreover, (i) and (iii) are finite sequences, whereas (ii), (iv) and (v) are infinite sequences.

However, to define a sequence we need not always have an explicit formula for the nth term. For example, for the infinite sequence 2, 3, 5, 7, 11, 13, 17, ... of all positive prime numbers, we may not be able to give an explicit formula for the nth term.

SERIES

By adding or subtracting the terms of a sequence, we obtain a *series*. A series is finite or infinite according as the number of terms in the corresponding sequence is finite or infinite.

Illustration 2: The following are the series corresponding to the sequences, in illustration 1.

(i) 3 + 5 + 7 + 9 + ... + 21.

(ii) 8 + 5 + 2 + (– 1) + ...

(iii) 2 + 6 + 18 + 54 + ... + 1458.

(iv) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

(v) 1 + 4 + 9 + 16 + ...

ARITHMETIC PROGRESSION (A.P.)

A sequence whose terms increase or decrease by a fixed number is called an *arithmetic progression*. The fixed number is called the *common difference* of the A.P.

In an A.P., we usually denote the first term by a, the common difference by d and the nth term by t_n. Clearly, $d = t_n - t_{n-1}$. Thus, an A.P. can be written as

$$a,\ a + d,\ a + 2d,\ ...,\ a + (n - 1)\, d,\ ...$$

Illustration 3: Consider the series: 1, 3, 5, 7, 9, ...

Here, 2nd term − 1st term = 3rd term − 2nd term = 4th term − 3rd term = ... = 2.

Hence 1, 3, 5, 7, ... are in A.P. whose first term is 1 and common difference is 2.

Illustration 4: The series: 5, 3, 1, –1, –3, –5, –7, ... is in A.P. whose first term is 5 and common difference is –2.

Notes:

- A sequence $t_1, t_2, t_3, t_4, \ldots$ will be in A.P. if $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \ldots$, i.e., $t_n - t_{n-1}$ = constant, for $n \geq 2$.
- Three numbers a, b, c are in A.P. if and only if $b - a = c - b$, i.e., if and only if $a + c = 2b$.
- Any three numbers in an A.P. can be taken as $a - d$, a, $a + d$. Any four numbers in an A.P. can be taken as $a - 3d$, $a - d$, $a + d$ and $a + 3d$.
  Similarly, five numbers in an A.P. can be taken as $a - 2d$, $a - d$, a, $a + d$ and $a + 2d$.

GENERAL TERM OF AN A.P.

Let a be the first term and d be the common difference of an A.P. Then, the A.P. is a, $a + d$, $a + 2d$, $a + 3d$,... . We also observe that,

t_1, the first term, is $a = a + (1 - 1)\, d$;
t_2, the second term, is $a + d = a + (2 - 1)\, d$;
t_3, the third term, is $a + 2d = a + (3 - 1)\, d$;
t_4, the fourth term, is $a + 3d = a + (4 - 1)\, d$;
t_n, the nth term, is $a + (n - 1)\, d$.

Thus, the formula, $t_n = a + (n - 1)\, d$ gives the general term of an A.P.

Notes:

- If an A.P. has n terms, then the nth term is called the *last term* of A.P. and it is denoted by l. Therefore, $l = a + (n - 1)\, d$.
- If a is the first term and d the common difference of an A.P. having m terms, then nth term from the end is $(m - n + 1)$th term from the beginning.
  $\therefore$ nth term from the end $= a + (m - n)\, d$.

Illustration 5: A sequence $< t_n >$ is given by the formula $t_n = 10 - 3n$. Prove that it is an A.P.

Solution: We have,

$t_n = 10 - 3n \Rightarrow t_{n+1} = 10 - 3(n + 1) = 7 - 3n$.

$\therefore\ t_{n+1} - t_n = (7 - 3n) - (10 - 3n) = -3$,

Which is independent of n and hence a constant. Therefore, the given sequence $< t_n >$ is an A.P.

Illustration 6: Find the nth term and 19th term of the sequence 5, 2, – 1, – 4, ...

Solution: Clearly, the given sequence is an A.P. with $a = 5$ and $d = -3$.

$\therefore\ t_n = a + (n - 1)\, d = 5 + (n - 1)(-3) = -3n + 8$.

For the 19th term, putting $n = 19$, we get $t_{19} = -3.19 + 8 = -49$.

Sum of n terms of an A.P.

The sum of n terms of an A.P. with first term 'a' and common difference 'd' is given by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Note:

- If S_n is the sum of n terms of an A.P. whose first term is 'a' and last term is l, then
  $S_n = \frac{n}{2}(a + l)$.
- If common difference d, number of terms n and the last term l, are given then
  $S_n = \frac{n}{2}[2l - (n - 1)\, d]$
- $t_n = S_n - S_{n-1}$.

Illustration 7: Find the sum of the series

.5 + .51 +.52 + ... to 100 terms.

Solution: The given series is an A.P. with first term, $a = .5$ and common difference, $d = .51 - .5 = .01$.

$\therefore$ Sum of 100 terms

$$= \frac{100}{2}[2 \times \cdot 5 + (100 - 1) \times .01]$$
$$= 50\,(1 + 99 \times .01) = 50\,(1 + .99)$$
$$= 50 \times 1.99 = 99.5.$$

Illustration 8: Find the sum of 20 terms of an A.P., whose first term is 3 and the last term is 57.

Solution: We have, $a = 3$, $l = 57$, $n = 20$.

$$\therefore\ S_n = \frac{n}{2}(a + l),\quad \therefore\ S_{20} = \frac{20}{2}(3 + 57) = 600.$$

Hence, the sum of 20 terms is 600.

GEOMETRIC PROGRESSION

A sequence (finite or infinite) of non-zero numbers in which every term, except the first one, bears a constant ratio with its preceding term, is called a *geometric progression*, abbreviated as G.P.

Illustration 9: The sequences given below:

(i) 2, 4, 8, 16, 32, ...

(ii) 3, –6, 12, –24, 48, ...

(iii) $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{108}, \frac{1}{324}, \ldots$

(iv) $\frac{1}{5}, \frac{1}{30}, \frac{1}{180}, \frac{1}{1080}, \frac{1}{6480}, \ldots$

(v) $x, x^2, x^3, x^4, x^5, \ldots$ (where x is any fixed real number),

are all geometric progressions. The ratio of any term in (i) to the preceding term is 2. The corresponding ratios in (ii), (iii), (iv) and (v) are $-2, \frac{1}{3}, \frac{1}{6}$, and x respectively. The ratio of any term of a G.P. to the preceding term is called the *common ratio* of the G.P. Thus, in the above examples, the common ratios are $2, -2, \frac{1}{3}, \frac{1}{6}$ and x respectively.

Note:

In a G.P., any term may be obtained by multiplying the preceding term by the common ratio of the G.P. Therefore, if any one term and the common ratio of a G.P. be known, any term can be written out, i.e., the G.P. is then completely known.

In particular, if the first term and the common ratio are known, the G.P. is completely known. The first term and the common ratio of a G.P. are generally denoted by a and r, respectively.

GENERAL TERM OF A G.P.

Let, a be the first term and r ($\neq 0$) be the common ratio of a G.P. Let $t_1, t_2, t_3, \ldots, t_n$ denote Ist, 2nd, 3rd, ... , nth terms, respectively. Then, we have

$$t_2 = t_1 r,\ t_3 = t_2 r,\ t_4 = t_3 r,\ \ldots,\ t_n = t_{n-1} r.$$

On multiplying these, we get

$t_2 t_3 t_4 \ldots t_n = t_1 t_2 t_3 \ldots t_{n-1} r^{n-1} \Rightarrow t_n = t_1 r^{n-1}$; but $t_1 = a$.

$\therefore$ General term $= t_n = ar^{n-1}$.

Thus, if a is the first term and r the common ratio of a G.P. then the G.P. is $a, ar, ar^2, \ldots, ar^{n-1}$ or $a, ar, ar^2, \ldots$ according as it is fnite or infinite.

Cor. If the last term of a G.P. consisting of n terms is denoted by l, then $l = ar^{n-1}$.

Notes:

- If a is the first term and r the common ratio of a finite G.P. consisting of m terms, then the nth term from the end is given by ar^{m-n}.
- The nth term from the end of a G.P. with the last term l and common ratio r is l/r^{n-1}.
- Three numbers in G.P. can be taken as a/r, a, ar; four numbers in G.P. can be taken as a/r^3, a/r, ar, ar^3; five numbers in G.P. can be taken as a/r^2, a/r, a, ar, ar^2, and so on...
- Three numbers a, b, c are in G.P. if and only if $b/a = c/b$, i.e., if and only if $b^2 = ac$.

Illustration 10: Find the nth term and 12th term of the sequence −6, 18, −54, ...

Solution: The given sequence is a G.P. with $a = -6$ and $r = -3$.

$\therefore\ t_n = ar^{n-1} = (-6)(-3)^{n-1} = (-1)^n \cdot 6 \cdot 3^{n-1}$

For the 12th term, putting $n = 12$, we get

$$t_{12} = (-1)^{12} \cdot 6 \cdot 3^{11} = 2 \cdot 3^{12}.$$

Sum of n terms of a G.P.

The sum of first n terms of a G.P. with first term a and common ratio r is given by $S_n = \frac{a(r^n - 1)}{r - 1}$

Notes:

(i) When $r = 1$

$$S_n = a + a + \ldots \text{ up to } n \text{ terms} = na$$

(ii) It l is the last term of the G.P., then

$$S_n = \frac{lr - a}{r - a},\ r \neq 1$$

Sum of an infinite G.P. when | r | < 1

The sum of an infinite G.P. with first term a and common ratio r is $S_\infty = \frac{a}{1 - r}$; when $|r| < 1$, i.e., $-1 < r < 1$.

Illustration 11: Find the sum of 8 terms and n terms of the sequence 9, −3, 1, −1/3, ...

Solution: The given sequence is a G.P. with $a = 9$ and $r = -\frac{1}{3}$.

We know that

$$S_8 = 9\frac{1-(-1/3)^8}{1-(-1/3)} == \frac{1-1/3^8}{4/3} = \frac{27}{4}\left(1 - \frac{1}{3^8}\right)$$

$$= \frac{27}{4}\frac{3^8-1}{3^8} = \frac{1}{4}\frac{6561-1}{3^5} = \frac{6560}{4\times 243} = \frac{1640}{243}.$$

Also, $S_n = 9\ \dfrac{1-(-1/3)^n}{1-(-1/3)} = 9\dfrac{1-(-1)^n/3^n}{4/3}$.

$$= \frac{27}{4}\frac{3^n-(-1)^n}{3^n} = \frac{3^n-(-1)^n}{4\cdot 3^{n-3}}.$$

Illustration 12: Find the sum of the infinite sequence $7, -1, \frac{1}{7}, -\frac{1}{49}, ...$

Solution: The given sequence is a G.P. with $a = 7$ and $r = -\frac{1}{7}$, so $|r| = \left|-\frac{1}{7}\right| = < 1$.

$$\therefore \quad S = \frac{7}{1-(-\frac{1}{7})} = \frac{7}{8/7} = \frac{49}{8}. \qquad \left(\because S = \frac{a}{1-r}\right)$$

HARMONIC PROGRESSION

A sequence of non-zero numbers $a_1, a_2, a_3, ...$ is said to be a *harmonic* progression (abbreviated as H.P.) if the sequence $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ...$ is an A.P.

Illustration 13: The sequence $1, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}, ...$ is a H.P. The sequence obtained by taking reciprocals of its corresponding terms, i.e., 1, 4, 7, 10, ... is an A.P.

A general H.P. is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, ...$

***n*th term of an H.P.**

*n*th term of H.P.

$$= \frac{1}{n\text{th term of the corresponding A.P.}}$$

Notes:

- Three numbers a, b, c are in H.P. if and only if $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., i.e.,
  $$\frac{1}{a} + \frac{1}{c} = 2\cdot\frac{1}{b} \quad \text{or} \quad b = \frac{2ac}{a+c}.$$
- No term of H.P. can be zero.
- There is no general formula for finding the sum to n terms of H.P.
- Reciprocals of terms of H.P. are in A.P. and then properties of A.P. can be used.

Illustration 14: Find the 100th of the squence

$$1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, ...$$

Solution: The sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, ...$ is an H.P.

Corresponding A.P. is 1, 3, 5, 7, ...

Now, for the corresponding A.P., first term $a = 1$, $d = 2$.

$\therefore$ 100th term of the corresponding A.P.

$= a + (100 - 1)d$

$= 1 + (100 - 1)\ 2 = 199$

Hence, the 100th term of the given sequence $= \frac{1}{199}$.

Some Special Sequences

1. The sum of first n natural numbers
   $$\Sigma n = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$
2. The sum of squares of first n natural numbers Σn^2
   $$= 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}.$$
3. The sum of cubes of first n natural numbers $\Sigma n^3 =$
   $$1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Note:

If nth term of a sequences is

$$T_n = an^3 + bn^2 + cn + d$$

then the sum of n terms is given by

$$S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d,$$

which can be evaluated using the above results.

Illustration 15: Find $2^2 + 4^2 + 6^2 + ... + (2n)^2$.

Solution: nth term of the given series is $(2n)^2$. Then, $T_n = 4n^2$.

$$\therefore \quad S_n = 4\ \Sigma\ n^2 = \frac{4n(n+1)(2n+1)}{6}$$

$$\therefore \quad S_n = \frac{2n(n+1)(2n+1)}{3}.$$

Illustration 16: Sum the series $1^2 + (1^2+2^2) + (1^2+2^2+3^2) + ...$ to n terms.

Solution: Here, $T_n = (1^2 + 2^2 + 3^2 + \dots n^2)$

$$= \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(2n^2+3n+1)}{6}$$

$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n.$$

$$\therefore \quad S_n = \Sigma\left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n\right)$$

$$= \frac{1}{3}\Sigma n^3 + \frac{1}{2}\Sigma n^2 + \frac{1}{6}\Sigma n$$

$$= \frac{1}{3}\frac{n^2(n+1)^2}{4} + \frac{1}{2}\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}[n(n+1) + 2n + 1 + 1]$$

$$= \frac{n(n+1)}{12}(n^2 + 3n + 2)$$

$$= \frac{n(n+1)}{12}(n+1)(n+2)$$

$$= \frac{n}{12}(n+1)^2(n+2).$$

Exercise-I

1. Determine 25th term of an A.P. whose 9th term is -6 and common difference is $\frac{5}{4}$.

(a) 16 (b) 18
(c) 12 (d) 14

2. Which term of the A.P. 5, 13, 21, ... is 181?

(a) 21st (b) 22nd
(c) 23rd (d) 24th

3. Find the nth term of the series:

$\frac{1}{n} + \frac{n+1}{n} + \frac{2n+1}{n} + \dots$

(a) $\frac{3+n^2+n}{n}$ (b) $\frac{1+n^2-n}{n}$
(c) $\frac{2+n^2-n}{n}$ (d) None of these

4. If the pth term of an A.P. is q and the qth term is p, then its rth term is:

(a) $p + q - r$ (b) $p - q - r$
(c) $r + q + p$ (d) None of these

5. Determine k so that $\frac{2}{3}$, k and $\frac{5}{8}k$ are the three consecutive terms of an A.P.

(a) $\frac{16}{33}$ (b) $\frac{14}{33}$
(c) $\frac{12}{33}$ (d) $\frac{18}{33}$

6. Determine k, so that $k + 2$, $4k - 6$ and $3k - 2$ are three consecutive terms of an A.P.

(a) 5 (b) 7
(c) 9 (d) 3

7. The ratio of the 7th to the 3rd term of an A.P. is 12:5. Find the ratio of 13th to the 4th term.

(a) 8:5 (b) 9:4
(c) 7:3 (d) 10:3

8. If 7 times the 7th term of an A.P. is equal to 11 times its 11th term, then the 18th term of the A.P. is:

(a) 1 (b) 2
(c) 0 (d) 3

9. The 4th term of an A.P. is equal to 3 times the first term and the seventh term exceeds twice the third term by 1. Find the first term and the common difference.

(a) 3, 2 (b) 5, 2
(c) 7, 3 (d) 9, 3

10. If the 9th term of an A.P. is 99 and 99th term is 9, find 108th term.

(a) 0 (b) 2
(c) 4 (d) 6

11. If the pth, qth and rth terms of an A.P. are a, b, c, respectively, find the value of:

$a(q - r) + b(r - p) + c(p - q)$

(a) 2 (b) 1
(c) 0 (d) 3

12. A body falls 16 m in the first second of its motion, 48 m in the second, 80 m in the third, 112 m in the fourth and so on. How far does it fall during the 11th second of its motion?

(a) 338 m (b) 340 m
(c) 334 m (d) 336 m

13. A ball rolling up an incline covers 36 meters during the first second, 32 m during the second, 28 m during the next and so on. How much distance will it travel during the 8th second?

(a) 8 m (b) 6 m
(c) 7 m (d) 9 m

14. Determine the sum of the first 35 terms of an A.P. if $t_2 = 2$ and $t_7 = 22$.

(a) 2510 (b) 2310
(c) 2710 (d) 2910

15. If the 5th and the 12th terms of an A.P. are 30 and 65, respectively, then what is the sum of the first 20 terms?

(a) 1175 (b) 1250
(c) 1150 (d) 1350

16. If the 12th term of an A.P. is −13 and the sum of the first four terms is 24, then what is the sum of the first 10 terms?

(a) 0 (b) 2
(c) 1 (d) 4

17. The sum of a series in A.P. is 525. Its first term is 3 and last term is 39. Find the common difference.

(a) 3/2 (b) 3/3
(c) 2/3 (d) 1/3

18. Find the common difference of an A.P. whose first term is 100 and the sum of whose first six terms is five times the sum of the next six terms.

(a) −15 (b) −10
(c) −20 (d) −5

19. How many terms are there in an A.P. whose first and fifth terms are −14 and 2, respectively and the sum of terms is 40?

(a) 15 (b) 5
(c) 10 (d) 20

20. Sum the series

51 + 50 + 49 + ... + 21

(a) 1116 (b) 1122
(c) 1128 (d) 1124

21. The sum of p terms of an A.P. is $3p^2 + 4p$. Find the nth term.

(a) $5n + 2$ (b) $6n + 1$
(c) $8n + 3$ (d) $7n + 3$

22. How many terms of the A.P. 1, 4, 7, ... are needed to give the sum 715?

(a) 33 (b) 22
(c) 24 (d) 27

23. Find the sum of the first hundred even natural numbers divisible by 5.

(a) 50575 (b) 50560
(c) 50500 (d) 50505

24. Find the sum of all integers between 50 and 500 which are divisible by 7.

(a) 17966 (b) 1177996
(c) 17766 (d) 17696

25. Find the sum of the numbers of three digits divisible by 7.

(a) 70334 (b) 70338
(c) 70336 (d) 70332

26. Find the sum of all odd numbers of four digits which are divisible by 9.

(a) 2754000 (b) 2754004
(c) 2754008 (d) 2754000

27. Which term of the geometric sequence

$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, ...$ is $\frac{1}{19683}$?

(a) 9 (b) 7
(c) 11 (d) 13

28. Find the 10th term of the geometric series 5 + 25 + 125 + ...,

(a) 5^{10} (b) 5^9
(c) 5^{11} (d) 5^8

29. Write down the 20th term of the G.P. 1, −1, 1, −1, ...

(a) 1 (b) −1
(c) + 1 (d) None of these

30. Write down the 5th term of the series $\frac{1}{4} - \frac{1}{2} + 1$ ...

(a) 6 (b) 8
(c) 4 (d) 10

31. The 5th term of a G.P. is 2, find the product of first 9 terms.

(a) 508 (b) 512
(c) 504 (d) 516

32. What term of progression

18, −12, 8, ... is $\frac{512}{729}$?

(a) 15 (b) 18
(c) 9 (d) 12

33. The 3rd term of a G.P. is the square of the first term. If the second term is 8, determine the 6th term.

(a) 136 (b) 132
(c) 128 (d) 124

34. If 4th and 8th terms of a G.P. are 24 and 384, respectively, then find out the first term and common ratio.

(a) 2, 3 (b) 5, 3
(c) 3, 2 (d) None of these

35. The first term of a G.P. is 1. The sum of the third and fifth terms is 90. Find the common ratio of the G.P.

(a) 2, −3 (b) 3, −3
(c) 1, −3 (d) 5, −3

36. For what value of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.?

(a) 1, −2 (b) 1, −3
(c) 1, −5 (d) 1, −1

37. A person has two parents (father and mother), four grandparents, eight great grandparents and so on. Find the number of ancestors the person has up to the 10th generation.

(a) 1028 (b) 1024
(c) 1030 (d) 1026

38. In a G.P, the first term is 7, the last term 448 and the sum 889. Find the common ratio.

(a) 4 (b) 6
(c) 8 (d) 2

39. The sum of first three terms of a G.P. is to the sum of first six terms is 125:152. Find the common ratio of G.P.

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

40. Evaluate $\sum_{j=1}^{11}(2+3^j)$

(a) $22+\frac{3}{2}(3^{11}-1)$ (b) $11+\frac{3}{2}(3^{11}-1)$
(c) $22+\frac{3}{2}(3^{10}-1)$ (d) None of these

41. The sum of the first two terms of a G.P. is 36 and the product of the first and the third terms is 9 times the second term, then find the sum of the first 8 terms.

(a) $\frac{3480}{81}$ (b) $\frac{3280}{81}$
(c) $\frac{3680}{81}$ (d) $\frac{3880}{81}$

42. The common ratio of a G.P. is $-\frac{4}{5}$ and the sum to infinity is $\frac{80}{9}$. Find the first term.

(a) 14 (b) 16
(c) 14 (d) 10

43. Sum the series to infinity

$\frac{3}{4}-\frac{5}{4^2}+\frac{3}{4^3}-\frac{5}{4^4}+\frac{3}{4^5}-\frac{5}{45}+\ldots$

(a) $\frac{8}{15}$ (b) $\frac{7}{17}$
(c) $\frac{7}{15}$ (d) $\frac{8}{17}$

44. The product $(32)\,(32)^{1/6}\,(32)^{1/36}\ldots\infty$ is equal to:
(a) 16 (b) 64
(c) 32 (d) 0

45. Find the 9th term of the H.P. 6, 4, 3, ...
(a) $\frac{7}{5}$ (b) $\frac{6}{5}$
(c) $\frac{5}{6}$ (d) None of these

46. Find the nth term of the H.P. whose first two terms are 6 and 3, respectively.

(a) $\frac{6}{n}$ (b) $\frac{7}{n}$
(c) $\frac{5}{n}$ (d) $\frac{8}{n}$

47. If $x > 1$, $y > 1$, $z > 1$ are in G.P., then

$\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in

(a) A.P. (b) H.P.
(c) G.P. (d) None of these

48. $\frac{2}{5}+\frac{3}{5^2}+\frac{2}{5^3}+\frac{3}{5^4}+\ldots\infty$

(a) $\frac{17}{24}$ (b) $\frac{15}{24}$
(c) $\frac{13}{24}$ (d) $\frac{11}{24}$

49. If the first term of a G.P. is 729 and 7th term is 64, determine S_7.
(a) 2259 (b) 3059
(c) 2059 (d) 2459

50. If a be the first term of a G.P., l the nth term and P the product of first n terms then P =

(a) $(al)^{n/2}$ (b) $(a-l)^{n/2}$
(c) $(a+l)^{n/2}$ (d) None of these

EXERCISE-2
(BASED ON MEMORY)

1. The sum $5 + 6 + 7 + 8 + \cdots + 19$ is equal to

(a) 150 (b) 170
(c) 180 (d) 190

[SSC (GL) Prel. Examination, 2005]

2. Given that $1^2 + 2^2 + 3^2 + \cdots + 20^2 = 2870$, the value of $(2^2 + 4^2 + 6^2 + \cdots + 40^2)$ is

(a) 11480 (b) 5740
(c) 28700 (d) 2870

[SSC (GL) Prel. Examination, 2005]

3. Which term of the series $72 + 63 + 54 + \cdots$. is zero?

(a) 11th (b) 10th
(c) 9th (d) 8th

[SSC (GL) Prel. Examination, 2000]

4. What is the 507th term of the sequence:

1, −1, 2, −2, 1, −1, 2, −2, 1,... ?

(a) −1 (b) 1
(c) −2 (d) 2

[SSC (GL) Prel. Examination, 2000]

5. If the 4th term of an A.P. is 14 and the 12 term is 70, then the first term is:

(a) −10 (b) −7
(c) 7 (d) 10

[SSC (GL) Prel. Examination, 2000]

6. $[14^2 + 15^2 + \cdots + 30^2]$ is equal to:

(a) 6836 (b) 8336
(c) 8336 (d) 8636

[SSC (GL) Prel. Examination, 2000]

7. If $1^3 + 2^3 + \cdots + \cdots 10^3 = 3025$,

then $4 + 32 + 108 + \cdots + 4000$ is equal to:

(a) 12000 (b) 12100
(c) 12200 (d) 12400

[SSC (GL) Prel. Examination, 2002]

8. If $1^2 + 2^2 + 3^2 + \cdots + 10^2 = 385$, then

$3^2 + 6^2 + 9^2 + \cdots + 30^2$ is equal to

(a) 3465 (b) 2310
(c) 1155 (d) 770

[SSC (GL) Prel. Examination, 2002]

9. If $1^2 + 2^2 + 3^2 + \cdots + x^2 = \frac{x(x+1)(2x+1)}{6}$, then

$1^2 + 3^2 + 5^2 + \cdots + 19^2$ is equal to

(a) 1330 (b) 2100
(c) 2485 (d) 2500

10. If $1^3 + 2^3 + \cdots + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + \cdots + (0.99)^3$ is close to:

(a) 0.2695 (b) 2.695
(c) 3.695 (d) 0.3695

[SSC (GL) Prel. Examination, 2002]

11. If $\log 2$, $\log(2^x - 1)$ and $\log(2^x + 3)$ (all to the base 10) be three consecutive terms of an Arithmetic Progression, then the value of x is equal to:

(a) 0 (b) 1
(c) $\log_2 5$ (d) $\log_{10} 2$

12. $(1^2 + 2^2 + 3^2 + \cdots + 10^2)$ is equal:

(a) 380 (b) 385
(c) 390 (d) 392

[SSC (GL), 2010]

13. The sum of the series $(1 + 0.6 + 0.06 + 0.006 + 0.0006 + \cdots)$ is:

(a) $1\frac{2}{3}$ (b) $1\frac{1}{3}$

(c) $2\frac{1}{3}$ (d) $2\frac{2}{3}$

[SSC (GL), 2010]

14. The 9th term of the sequence, 0, 3, 8, 15, 24, 35, ⋯ is

(a) 63 (b) 70
(c) 80 (d) 99

[SSC (GL), 2010]

15. Find the sum of all positive multiples of 3 less than 50.

(a) 400 (b) 404
(c) 408 (d) 412

[SSC, 2014]

16. Find the sum of $\left(1-\frac{1}{n+1}\right)+\left(1-\frac{2}{n+1}\right)+\left(1-\frac{3}{n+1}\right)+\cdots+\left(1-\frac{n}{n+1}\right)$.

(a) n (b) $\frac{1}{2}n$

(c) $(n + 1)$ (d) $\frac{1}{2}(n+1)$

[SSC, 2013]

17. If a clock strikes appropriate number of times at each hour, how many times will it strike in a day?

(a) 300 (b) 156

(c) 68 (d) 78

[SSC, 2013]

18. Terms a, 1, b are in arithmetic progression and terms 1, a, b are in geometric progression. Find a and b (given $a \neq b$).

(a) 2, 4 (b) −2, 1

(c) 4, 1 (d) −2, 4

[SSC Assistant grade III, 2013]

19. The sum $11^2 + 12^2 + \cdots$ v $+ 21^2 = ?$

(a) 2926 (b) 3017

(c) 3215 (d) 3311

[SSC, 2012]

20. $\left[\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{99\times 100}\right]$ is equal to:

(a) $\frac{1}{9900}$ (b) $\frac{99}{100}$

(c) $\frac{100}{99}$ (d) $\frac{1000}{99}$

[SSC, 2010]

21. The sum of all the digits of the numbers from 1 to 100 is:

(a) 5050 (b) 903

(c) 901 (d) 900

[SSC, 2010]

ANSWER KEYS

Exercise-1

1. (c)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (a)	**11.** (c)	**12.** (d)	**13.** (a)
14. (b)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (b)	**19.** (c)	**20.** (a)	**21.** (b)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (a)
27. (a)	**28.** (a)	**29.** (b)	**30.** (c)	**31.** (b)	**32.** (c)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (d)	**37.** (b)	**38.** (d)	**39.** (c)
40. (a)	**41.** (b)	**42.** (b)	**43.** (c)	**44.** (b)	**45.** (b)	**46.** (a)	**47.** (b)	**48.** (c)	**49.** (c)	**50.** (a)		

Exercise-2

1. (d)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (b)	**11.** (c)	**12.** (b)	**13.** (a)
14. (c)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (b)	**21.** (a)					

EXPLANATORY ANSWERS

Exercise-1

1. **(d)** Let, a be the first term and d the common difference of an A.P.

Then, $a_n = a + (n - 1)\, d$

$$a_9 = a + (9 - 1)\left(\frac{5}{4}\right)$$

$\Rightarrow$ $a_9 = a + 10$

$\Rightarrow$ $-6 = a + 10$

$\Rightarrow$ $a = -6 - 10 = -16$

$\therefore$ $a_{25} = -16 + (25 - 1)\frac{5}{4} = -16 + 30 = 14.$

2. **(c)** Here, first term $a = 5$

Common difference $d = 8$

Let, 181 be the nth, i.e., $a_n = 181$

$\therefore$ $181 = 5 + (n - 1)8$ or, $176 = (n - 1)8$

$\therefore \quad n - 1 = 176 \div 8 = 22$

$\therefore \quad n = 23$

Hence, 181 is the 23rd term.

3. **(b)** Here $a_1 = \dfrac{1}{n}$, $d = \dfrac{n+1}{n} - \dfrac{1}{n} = 1$

$\therefore \quad a_n = a + (n - 1)d$

$\therefore \quad a_n = \dfrac{1}{n} + n - 1 = \dfrac{1+n^2-n}{n}$.

4. **(a)** Let, 'a' be the first term and d, the common difference

$\therefore \quad a_p = q \;\Rightarrow\; a + (p - 1)\, d = q \qquad \ldots(1)$

$a_q = p \;\Rightarrow\; a + (q - 1)\, d = p \qquad \ldots(2)$

Subltracting (2) from (1)

$(p - q)\, d = q - p = -(p - q)$

$\therefore \quad d = -1$

$\therefore$ From (1), $a + (p - 1)(-1) = q$

i.e., $a - p + 1 = q$

$\therefore \quad a = p + q - 1$

$\therefore \quad a_r = a + (r - 1)d$

$= (p + q - 1) + (r - 1)(-1)$

$= p + q - 1 - r + 1$

$= p + q - r.$

5. **(a)** $\because \; \dfrac{2}{3}, k, \dfrac{5}{8} k$ are in A.P.

$\therefore \quad k - \dfrac{2}{3} = \dfrac{5}{8}k - k \;\Rightarrow\; \dfrac{5k}{8} - 2k = \dfrac{-2}{3}$

$\Rightarrow\; \dfrac{-11k}{8} = \dfrac{-2}{3} \Rightarrow k = \dfrac{16}{33}$

6. **(d)** $\because \; k + 2, 4k - 6, 3k - 2$ are in A.P.

$\therefore \quad (4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$

$\Rightarrow \quad 3k - 8 = -k + 4$

$\therefore \quad 4k = 12 \quad \therefore \quad k = 3.$

7. **(d)** Let, a be the first term and d the common difference of the A.P. Then,

$\dfrac{a+6d}{a+2d} = \dfrac{12}{5} \;\Rightarrow\; 5a + 30d = 12a + 24d$

$\Rightarrow\; -7a + 6d = 0$

$\Rightarrow\; a = \dfrac{6}{7}d$

$\therefore \quad \dfrac{\text{13th term}}{\text{4th term}} = \dfrac{a+12d}{a+3d} = \dfrac{\frac{6}{7}d + 12d}{\frac{6}{7}d + 3d}$

$= \dfrac{90}{27} = \dfrac{10}{3}$.

8. **(c)** Let, a be the first term and d, the common difference of an A.P.

$\therefore \quad a_7 = a + 6d$

$a_{11} = a + 10d \quad \because \quad 7a_7 = 11a_{11}$

$\therefore \quad 7(a + 6d) = 11(a + 10d)$

$\Rightarrow \quad 7a + 42d = 11a + 110d$

$\Rightarrow \quad -4a = 68d$

$\therefore \quad a = -17d \qquad \ldots(1)$

Now, $a_{18} = a + 17d = -17d + 17d$ [Using (1)]

$= 0.$

9. **(a)** Let, a be the first term and d the common difference of A.P.

$\because \quad a_4 = 3a, \quad \therefore \quad a + 3d = 3a$

$\Rightarrow \quad 2a = 3d, \quad \therefore \quad a = \dfrac{3}{2}d \qquad \ldots(1)$

Also, $a_7 = 2a_3 + 1$

$\Rightarrow \quad a + 6d = 2(a + 2d) + 1$

$\Rightarrow \quad a + 6d = 2a + 4d + 1$

$\therefore \quad a = 2d - 1 \qquad \ldots(2)$

From (1) and (2), $\dfrac{3}{2}d = 2d - 1 \Rightarrow 2d - \dfrac{3}{2}d = 1$

$\therefore \quad \dfrac{d}{2} = 1 \;\Rightarrow\; d = 2 \;\therefore\; a = \dfrac{3}{2}d = \dfrac{3}{2} \times 2 = 3.$

10. **(a)** Let, a be the first term and d the common difference of A.P.

$\because \quad a_9 = 99$

$\therefore \quad a + 8d = 99 \qquad \ldots(1)$

Also, $a_{99} = 9 \quad \therefore \quad a + 98d = 9 \qquad \ldots(2)$

Subtracting (2) from (1)

$-90\, d = 90$

$\therefore \quad d = -1 \qquad \ldots(3)$

Substituting this value of d in (1)

$a + 8(-1) = 99 \Rightarrow a = 99 + 8 = 107 \qquad \ldots(4)$

$\therefore \quad a_{108} = a + (108 - 1)\, d = 107 + 107(-1) = 0.$

11. **(c)** Let, A be the first term and D, the common difference of A.P.

$a_p = a, \quad \therefore \quad A + (p - 1)\, D = a \qquad \ldots(1)$

$a_q = b, \quad \therefore \quad A + (q - 1)\, D = b \qquad \ldots(2)$

$a_r = c, \quad \therefore \quad A + (r - 1)\, D = c \qquad \ldots(3)$

$\therefore \quad a(q - r) + b(r - p) + c(p - q)$

$= [A + (p - 1)\, D](q - r) + [A + (q - 1)\, D](r - p)$
$\quad + [A + (r - 1)\, D](p - q)$

$= (q - r + r - p + p - q)\, A + [(p - 1)(q - r)$
$\quad + (q - 1)(r - p) + (r - 1)(p - q)]D$

$= OA + OD = 0.$

12. **(d)** The distances through which the body falls in first, second, third, fourth, ... seconds form an A.P. 16 + 48 + 80 + 112 + ⋯

Here, $a = 16$, $d = 32$

Distance through which it falls in 11th second

$= 11$ the term of the A.P.

$= a + 10d = 16 + 10\ (32)$

$= 16 + 320 = 336$ m.

13. **(a)** Distance covered during the first second 36 m

Distance covered during the 2nd second = 32m

Distance covered during the 3rd second = 28m

The distance covered form an A.P.

$= 36 + 32 + 28 + \cdots$ in which

$a = 36$, $d = -4$

$\therefore$ Distances covered in 8th second

$= 8$th term of the A.P.

$= a + 7d = 36 + 7(-4)$

$= 36 - 28 = 8$ m.

14. **(b)** Let, a and d be the first term and the common difference, respectively.

$\because\ t_n = a + (n-1)d$,

$\therefore\ t_2 = a + (2-1)\ d = 2 \;\Rightarrow\; a + d = 2 \quad \ldots(1)$

and, $t_7 = a + (7-1)\ d = 22 \;\Rightarrow\; a + 6d = 22 \quad \ldots(2)$

Subtracting (1) from (2), we get

$5d = 20 \Rightarrow d = 4$

$\therefore\ a + 4 = 2 \;\Rightarrow\; a = 2 - 4 = -2$ [Using (1)]

Now, $S_n = \dfrac{n}{2}\ [2a + (n-1)d]$

$\therefore\ S_{35} = \dfrac{35}{2}\ [-4 + (35-1)4]$

$= \dfrac{35}{2} \times 132, = 35 \times 66 = 2310.$

15. **(c)** Let, a be the first term and d the common difference of an A.P., then

$a_5 = a + 4d = 30 \quad \ldots(1)$

$a_{12} = a + 11d = 65 \quad \ldots(2)$

Subtracting (1) from (2), we get

$7d = 35 \;\Rightarrow\; d = 5$

$\therefore$ From (1)

$a + 4\ (5) = 30 \;\Rightarrow\; a = 30 - 20 = 10$

Now, $S_n = \dfrac{n}{2}\ [2a + (n-1)d]$

$\therefore\ S_{20} = \dfrac{20}{2}[2 \times 10 + (20-1)5]$

$= 10\ [20 + 95] = 1150.$

16. **(a)** Let, a be the first term and d the common difference of the A.P.

Then, nth term $= a + (n-1)d$

$\therefore\ a_{12} = a + 11d = -13 \quad \ldots(1)$

Again, $S_n = \dfrac{n}{2}\ [\ 2a + (n-1)d]$

$\therefore\ S_4 = 2(2a + 3d)$

But $S_4 = 24$

$\therefore\ 2(2a + 3d) = 24$

$\Rightarrow\ 2a + 3d = 12 \quad \ldots(2)$

Multiplying (1) by 2, we get

$2a + 22d = -26 \quad \ldots(3)$

Subtracting (2) from (3)

$19d = -38 \;\Rightarrow\; d = -2$

Substituting $d = -2$ in (1), we get

$a + 11 \times (-2) = -13 \;\Rightarrow\; a = -13 + 22 = 9$

$\therefore\ S_{10} = \dfrac{10}{2}\ [2 \times 9 + (10-1) \times -2]$

$= 5\ (18 - 18) = 0.$

17. **(a)** If n be the number of terms, then

$a_n = a + (n-1)d,$

where a is the first term and d the common difference.

$\therefore\ 39 = 3 + (n-1)d$

or, $(n-1)d = 36 \quad \ldots(1)$

Also, $S_n = \dfrac{n}{2}[a_1 + a_n]$

$\therefore\ 525 = \dfrac{n}{2}[\ 3 + 39] \;\Rightarrow\; 1050 = n\ (42)$

or, $n = \dfrac{1050}{42} = 25$

Putting $n = 25$ in (1), we get

$(25-1)d = 36 \;\Rightarrow\; d = 36 \div 24 = \dfrac{3}{2} = 1\dfrac{1}{2}.$

18. **(b)** Here, $a = 100$

Let, d be the common difference

Now, $a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

$= 5(a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12})$

$\Rightarrow\ 6\left(\dfrac{a_1 + a_6}{2}\right) = 5 \times 6\left(\dfrac{a_7 + a_{12}}{2}\right)$

$\Rightarrow\ a_1 + a_6 = 5\ (\ a_7 + a_{12})$

$\Rightarrow\ a + a + 5d = 5[a + 6d + a + 11d]$

$\Rightarrow\ 2a + 5d = 10a + 85d$

$\Rightarrow\ 80d = -8a$ or, $d = \dfrac{-a}{10}$

$\therefore\ d = \dfrac{-100}{10} = -10.$

19. **(c)** Here, $a = -14$

Let, d be the common difference

$a_5 = 2 \;\Rightarrow\; a + 4d = 2 \;\Rightarrow\; -14 + 4d = 2$

$\therefore\ d = 4$

Let 40 be the sum of n terms of this A.P.

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad 40 = \frac{n}{2}[2(-14) + (n-1)4] \quad \Rightarrow \quad 80 = n(4n - 32)$

or, $4n^2 - 32n - 80 = 0 \quad \Rightarrow \quad n^2 - 8n - 20 = 0$

$\Rightarrow \quad (n+2)(n-10) = 0$

$\therefore \quad n = 10$ or, -2. But $n \neq -2$.

Hence, the required number of terms are 10.

20. **(a)** Here, $a = 51$, $d = -1$, $a_n = 21$

Now, $a_n = a_1 + (n-1)d$

or, $21 = 51 - n + 1 \quad \therefore \quad n = 52 - 21 = 31$

Now, $S_n = \frac{n}{2}(a_1 + a_n) = \frac{31}{2}(51 + 21)$

$= \frac{31}{2} \times 72 = 31 \times 36 = 1116.$

21. **(b)** Here, $S_p = 3p^2 + 4p$ Putting $p = n$, we have

$S_n = 3n^2 + 4n$

Changing n to $(n-1)$, we get

$S_{n-1} = 3(n-1)^2 + 4(n-1)$

$= 3(n^2 - 2n + 1) + 4n - 4$

$= 3n^2 - 2n - 1$

$\therefore \quad a_n = S_n - S_{n-1}$

$= 3n^2 + 4n - 3n^2 + 2n + 1 = 6n + 1.$

22. **(b)** Here, $a = 1$, $d = 3$,

Let, 715 be the sum of n terms of this A.P.

That is, $S_n = 715$

$\therefore \quad \frac{n}{2}[2a + (n-1)d] = 715$

Putting values, of a and d

$\frac{n}{2}[2 \times 1 + (n-1) \times 3] = 715$

$\Rightarrow \quad \frac{n}{2}[2 + 3n - 3] = 715$

$\Rightarrow \quad 3n^2 - n - 1430 = 0$

$\therefore \quad n = \frac{1 \pm \sqrt{1 - 4(3)(-1430)}}{2 \times 3}$

$= \frac{1 \pm \sqrt{17161}}{6} = \frac{1 \pm 131}{6}$

$\therefore \quad n = \frac{1 + 131}{6} = 22, \; n = \frac{1 - 131}{6} = \frac{-65}{3}$

But $n \neq \frac{-65}{3} \quad \therefore \quad n = 22.$

23. **(c)** Even natural numbers which are divisible by 5 are 10, 20, 30, 40,....

They form an A.P. with $a = 10$, $d = 10$

$S_{100} = \frac{100}{2}[2 \times 10 + (100 - 1) \times 10]$

$= 50(20 + 990) = 50(1010) = 50500.$

24. **(d)** The first integer, after 50 which is divisible by 7 is 56 and the last integer before 500 which is divisible by 7 is 497.

$\therefore$ The sequence of integers between 50 and 500 which are divisible by 7 is 56, 63, 70, ..., 497

It is an A.P. with

$a = 56$, $d = 7$

$a_n = 497 = a + (n-1)d$

$\therefore \quad 497 = 56 + (n-1) \times 7$

$\therefore \quad 7n = 497 + 7 - 56$

or, $\quad 7n = 448$

or, $\quad n = 488 \div 7 = 64$

Required sum $= \frac{n}{2}(a_1 + a_n) = \frac{64}{2}(56 + 497)$

$= 32 \times (553) = 17696.$

25. **(c)** The least and the greatest number of three digits divisible by 7 are 105 and 994, respectively.

$\therefore$ It is required to find the sum of

$105 + 112 + 119 + \ldots + 994$

Here, $a = 105$, $d = 7$, $a_n = 994$

Then, $n = ?$, $S_n = ?$

Now, $a_n = a + (n-1)d$

$\Rightarrow \quad 994 = 105 + (n-1) \times 7$

$\Rightarrow \quad 994 - 105 = 7(n-1)$

$\Rightarrow \quad 889 = 7(n-1)$ or, $(n-1) = 127 \quad \Rightarrow \quad n = 128$

Also, Sum $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{128}{2}[2 \times 105 + (128 - 1) \times 7]$

$= 64[210 + 889]$

$= 64 \times 1099 = 70336.$

26. **(a)** The odd numbers of four digits which are divisible by 9 are 1017, 1035, ..., 9999

These are in A.P. with common difference 18.

Hence, nth term $= a_n = a + (n-1)d$

$a = 1017$, $d = 18$, $l = 9999$

$\therefore \quad 9999 = 1017 + (n-1) \times 18$

$\Rightarrow \quad 18n = 9999 - 999 = 9000$

$\therefore \quad n = 9000 \div 18 = 500$

$\therefore \quad S_n = \frac{n}{2}(a_1 + a_n) = \frac{500}{2}(1017 + 9999)$

$= 250 \times 11016 = 2754000.$

27. **(a)** Let, nth term of the given sequence be $\frac{1}{19683}$. Then,

$$a_n = ar^{n-1} \Rightarrow \frac{1}{19683} = \frac{1}{3}\left(\frac{1}{3}\right)^{n-1},$$

$$\Rightarrow \left(\frac{1}{3}\right)^8 = \left(\frac{1}{3}\right)^{n-1}$$

$$\Rightarrow n - 1 = 8 \Rightarrow n = 9.$$

28. **(a)** The given geometric series is

$5 + 25 + 125 + \cdots$

a = lst term = 5, r = common ratio = 5

$n = 10$

$\therefore$ 10th term $= a_{10} = ar^{10-1}$

$= (5)(5)^9 = 5^{10}$.

29. **(b)** Here, $a = 1$, $r = -1$

Since $a_n = ar^{n-1}$

$\therefore \quad a_{20} = ar^{19} = 1 \times (-1)^{19} = -1$.

30. **(c)** Here, $a = \frac{1}{4}$, $r = -2$. $\because \quad a_n = ar^{n-1}$

$\therefore \quad a_5 = \frac{1}{4} \times (-2)^4 = \frac{1}{4} \times 16 = 4.$

31. **(b)** Let, a be the first term and r the common ratio

$\therefore \quad a_5 = 2 \Rightarrow ar^4 = 2$ …(1)

Now, product of first 9 terms

$= a \times ar \times ar^2 \times \ldots \times ar^8$

$= a^9\, r^{1+2+\cdots+8} = a^9\, r^{36}$

$= (ar^4)^9 = 2^9 = 512.$

32. **(c)** Here $a = 18$, $r = \frac{-2}{3}$

Let, $\frac{512}{729}$ be the nth term so that $a_n = \frac{512}{729}$

Since $a_n = ar^{n-1}$

$$\therefore \quad \frac{512}{729} = 18\left(\frac{-2}{3}\right)^{n-1}$$

$$\Rightarrow \left(\frac{-2}{3}\right)^{n-1} = \frac{512}{729 \times 18} = \frac{256}{9 \times 729}$$

$$= \frac{2^8}{3^2 \times 3^6} = \left(\frac{-2}{3}\right)^8$$

$\therefore \quad n - 1 = 8 \Rightarrow n = 9$

Hence, $\frac{512}{729}$ is the 9th term of progression.

33. **(c)** Let, a be the first term and r be the common ratio of G.P.

We have $a_3 = (a_1)^2 \Rightarrow ar^2 = a^2$

$\Rightarrow r^2 = a$ …(1)

Also, $a_2 = 8 \Rightarrow ar = 8$ …(2)

Multiplying (1) and (2), we get

$$ar^3 = 8 \times a \therefore r^3 = 8 \Rightarrow r = 2$$

From (1) $a = (2)^2 = 4$ $[\because \quad a = r^2]$

Hence, $a_6 = ar^5 = (4)\,(2)^5 = 4 \times 32 = 128$.

34. **(c)** Let a be the first term and r the common ratio. Then,

$\because$ 4th term = 24 $\Rightarrow ar^3 = 24$ …(1)

and, 8th term = 384 $\Rightarrow ar^7 = 384$ …(2)

Dividing (2) by (1), we get

$$\frac{ar^7}{ar^3} = \frac{384}{24} \Rightarrow r^4 = 16 = (2)^4 \Rightarrow r = 2$$

Substituting $r = 2$ in (i), we get

$$a(2)^3 = 24 \Rightarrow a = 24 \div 8 = 3$$

Hence, first term = 3 and common ratio = 2.

35. **(b)** Let, r be the common ratio of G.P.

First term, $a = 1$

Now, $a_3 = ar^2 = r^2$ $(\because \quad a = 1)$

and, $a_5 = ar^4 = r^4$ But $a_3 + a_5 = 90$

$\Rightarrow r^2 + r^4 = 90 \Rightarrow r^4 + r^2 - 90 = 0$

$\Rightarrow (r^2 + 10)\,(r^2 - 9) = 0$

$\therefore \quad r^2 - 9 = 0$ $[\because \quad r^2 + 10 \neq 0]$

$\therefore r = \pm 3$.

36. **(d)** $\frac{-2}{7}, x, \frac{-7}{2}$, are in G.P.

$$\Rightarrow \frac{x}{-2/7} = \frac{-7/2}{x} \Rightarrow x^2 = \frac{-7}{2} \times \frac{-2}{7}$$

$\therefore \quad x^2 = 1 \Rightarrow x = +1$.

37. **(b)** We have 2, 4, 8, ... 10 terms which are in G.P.

Here, $a = 2$, $r = 2$

$\therefore \quad a_n = ar^{n-1}$

$\Rightarrow a_{10} = 2(2)^{10-1} = 2^{10} = 1024$.

Hence, the number of ancestors the person has up to 10th generation = 1024.

38. **(d)** Here, $a = 7$, $l = a_n = 448$, $S_n = 889$

Let, r be the common ratio

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a - lr}{1-r}$$

$$\therefore \quad 889 = \frac{7 - 448r}{1-r} \Rightarrow 889 - 889r = 7 - 448r$$

$$\Rightarrow 889 - 7 = 889r - 448r$$

$$\Rightarrow 882 = 441r \Rightarrow r = 2.$$

39. **(c)** Here, $\frac{S_3}{S_6} = \frac{125}{152}, \frac{a(r^3-1)(r-1)}{a(r^6-1)(r-1)} = \frac{125}{152}$

$$\Rightarrow \frac{r^3-1}{r^6-1} = \frac{125}{152}, \quad \therefore \quad \frac{r^3-1}{(r^3-1)(r^3+1)} = \frac{125}{152}$$

$$\Rightarrow \frac{1}{r^3+1} = \frac{125}{152}, \quad \therefore \quad 152 = 125r^3 + 125$$

$\Rightarrow\ 125r^3 = 27, \ \Rightarrow\ r^3 = \dfrac{27}{125}$.

or, $r^3 = \left(\dfrac{3}{5}\right)^3, \ \therefore\ r = \dfrac{3}{5}$.

Hence, the common ratio of G.P. is $\dfrac{3}{5}$.

40. **(a)** $(2 + 3^1) + (2 + 3^2) + (2 + 3^3) + \cdots + (2 + 3^{11})$
$= (2 + 2 + 2 + \cdots$ up to 11 terms)
$+ (3 + 3^2 + 3^3 + \cdots$ up to 11 terms)

$= 11 \times 2 + \dfrac{3(3^{11}-1)}{3-1} = 22 + \dfrac{3}{2}(3^{11} - 1)$.

41. **(b)** Let, a be the first term and r the common ratio of G.P.

Given: $a_1 + a_2 = 36 \ \Rightarrow\ a + a\,r = 36$
$\Rightarrow\ a\,(1 + r) = 36$ …(1)

Also, $a_1\,a_3 = 9a_2 \ \Rightarrow\ a\,ar^2 = 9 \times ar$
$\Rightarrow\ ar = 9$ …(2)

Subtracting (2) from (1), $a = 27$

From (2), $27\,r = 9 \ \Rightarrow\ r = \dfrac{1}{3}$

$$\therefore\ S_8 = \frac{27\left[1-\left(\frac{1}{3}\right)^8\right]}{1-1/3} = \frac{3\times 27}{2}\left(1-\frac{1}{6561}\right)$$

$$= \frac{81}{2}\times\frac{6560}{6561} = \frac{3280}{81}.$$

42. **(b)** $S_\infty = \dfrac{a}{1-r} \Rightarrow \dfrac{80}{9} = \dfrac{a}{1-\left(\frac{-4}{5}\right)} \Rightarrow \dfrac{80}{9} = \dfrac{a}{a/5}$

$\Rightarrow\ a = \dfrac{80}{9}\times\dfrac{9}{5} = 16$

Hence, the first term is 16.

43. **(c)** $\left(\dfrac{3}{4}+\dfrac{3}{4^3}+\dfrac{3}{4^5}+\ldots \text{ to } \infty\right)-\left(\dfrac{5}{4^2}+\dfrac{5}{4^4}+\dfrac{5}{4^6}+\cdots \text{ to } \infty\right)$

$$= \frac{\frac{3}{4}}{1-\left(\frac{1}{4}\right)^2} - \frac{\frac{5}{4}^2}{1-\left(\frac{1}{4}\right)^2}$$

$$= \frac{3}{4}\times\frac{16}{15} - \frac{5}{16}\times\frac{16}{15} = \frac{4}{5}-\frac{1}{3} = \frac{12-5}{15} = \frac{7}{15}.$$

44. **(b)** $(32)\,(32)^{1/6}\,(32)^{1/36} \cdots \infty$

$$= (32)^{1+\frac{1}{6}+\frac{1}{36}+\ldots+\infty} = (32)^x,$$

where $x = 1+\dfrac{1}{6}+\dfrac{1}{36}+\cdots\infty = \dfrac{a}{1-r}$

$$= \frac{1}{1-\frac{1}{6}} = \frac{6}{5}$$

$\therefore$ Product $= (32)^x = (32)^{6/5} = (2^5)^{6/5} = 2^6 = 64$.

45. **(b)** The given sequence is 6,4,3, ... which is a H.P.
The sequence of reciprocals of its terms is

$\dfrac{1}{6}, \dfrac{1}{4}, \dfrac{1}{3}$ ... which is an A.P.

Here, $a = \dfrac{1}{6}$, $d = \dfrac{1}{4} - \dfrac{1}{6} = \dfrac{1}{12}$

$\therefore\ a_9$ of A.P. $= a + 8d$

$$= \frac{1}{6}+8\times\frac{1}{12} = \frac{1}{6}+\frac{4}{6} = \frac{5}{6}$$

$\therefore$ 9th term of H.P. is $\dfrac{6}{5}$.

46. **(a)** First term of H.P. = 6 and second term of H.P. = 3

$\therefore$ First and second terms of corresponding A.P. are $\dfrac{1}{6}$ and $\dfrac{1}{3}$.

$\therefore\ a = \dfrac{1}{6}$ and $d = \dfrac{1}{3} - \dfrac{1}{6} = \dfrac{1}{6}$

nth term of A.P. $= \dfrac{1}{6} + (n - 1)\,\dfrac{1}{6}$

$$= \frac{1+n-1}{6} = \frac{n}{6}$$

Hence, nth term of H.P. $= \dfrac{n}{6}$.

47. **(b)** $\because\ x, y, z$ are in G.P.

$\therefore\ y^2 = xz$

Taking log on both sides

$2 \log y = \log x + \log z$

$\Rightarrow\ 2 + 2 \log y = (1 + \log x) + (1 + \log z)$

$\Rightarrow\ 2\,(1 + \log y) = (1 + \log x) + (1 + \log z)$

$\Rightarrow\ 1 + \log x,\ 1 + \log y,\ 1 + \log z$, are in A.P.

$\Rightarrow\ \dfrac{1}{1+\log x}, \dfrac{1}{1+\log y}, \dfrac{1}{1+\log z}$ are in H.P.

48. **(c)** $\dfrac{2}{5}+\dfrac{3}{5^2}+\dfrac{2}{5^3}+\dfrac{3}{5^4} + \cdots \infty$

$$= \left(\frac{2}{5}+\frac{2}{5^3}+\frac{2}{5^5}+\cdots\infty\right)+\left(\frac{3}{5^2}+\frac{3}{5^4}+\cdots\infty\right)$$

$$= \frac{2/5}{1-\frac{1}{5^2}} + \frac{3/5^2}{1-\frac{1}{5^2}} = \frac{25}{24}\left(\frac{2}{5}+\frac{3}{25}\right)$$

$$= \frac{25}{24}\times\frac{13}{25} = \frac{13}{24}.$$

49. **(c)** $a = 729$. Let, r be the common ratio

Now, $a_7 = 64 \Rightarrow ar^6 = 64 \Rightarrow 729 \times r^6 = 64$

$$\therefore \quad r^6 = \frac{64}{729} = \left(\frac{2}{3}\right)^6 \quad \therefore \quad r = \frac{2}{3}$$

$$\therefore \quad S_7 = \frac{a(1-r^7)}{1-r} = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-2/3} = \frac{3\times 729}{1}\left[1-\frac{128}{2187}\right]$$

$$= 2187 \times \frac{2059}{2187} = 2059.$$

50. **(a)** If r is the common ratio of G.P., then

$l = a\, r^{n-1}$ …(1)

The first n terms of the G.P. are

$a, ar, ar^2, ar^3, \ldots, ar^{n-1}$

$P = a \times ar \times ar^2 \times ar^3 \times \ldots \times ar^{n-1}$

$= a^n \times r^{1+2+3+\cdots+(n-1)}$

$= a^n \times r^{(n-1)\times\frac{n}{2}} = (a^2)^{n/2} \times (r^{n-1})^{n/2}$

$= (a^2\, r^{n-1})^{n/2} = (a.\, ar^{n-1})^{n/2} = (al)^{n/2}.$

Exercise-2
(Based on Memory)

1. **(c)** $S_n = \frac{n}{2}(a_1 + a_n) = \frac{15}{2}(5+19) = \frac{15}{2}\times 24 = 180$

2. **(a)** Required sum is

$2^2(1^2 + 2^2 + 3^2 + \cdots + 20^2)$

$= 4 \times 2870$

$= 11480$

3. **(c)** The given series is an A.P.

nth term of A.P. $= a_n = a + (n-1)\, d$

where a = 1st term, d = common difference

So, $72 + (n-1)(-9) = 0$

or, $72 - 9n + 9 = 0 \Rightarrow 9n = 81 \Rightarrow n = 9.$

5. **(b)** Let, the first term of the A.P. be a and the common difference be d

According to question,

$a + 3d = 14$ …(1)

and, $a + 11d = 70$ …(2)

On solving (1) and (2), we get

$d = 7, a = -7.$

6. **(d)** $14^2 + 15^2 + \cdots + 30^2$

$= (1^2 + 2^2 + 3^2 + \cdots + 30^2) - (1^2 + 2^2 + \cdots + 13^2)$

$$= \frac{30(30+1)(2\text{x }30+1)}{6} - \frac{13(13+1)(2\text{x }13+1)}{6}$$

$$= \frac{30\times 31\times 61}{6} - \frac{13\times 14\times 27}{6} = 8636.$$

7. **(b)** $4 + 32 + 108 + \cdots + 4000$

$= 4\,(1 + 8 + 27 + \cdots + 1000)$

$= 4\,(1^3 + 2^3 + 3^3 + \cdots + 10^3)$

$= 4 \times 3025 = 12100.$

8. **(a)** $3^2 + 6^2 + 9^2 + \cdots + 30^2$

$= 3^2[1^2 + 2^2 + 3^2 + \cdots + 10^2]$

$= 9 \times 385 = 3465.$

9. **(a)** $(1^2 + 2^2 + 3^2 + \cdots + 19^2) - (2^2 + 4^2 + 6^2 + \cdots + 18^2)$

$$= \frac{19\times 20\times 39}{6} - 2^2(1^2 + 2^2 + 3^2 + \cdots + 9^2)$$

$$= \frac{19\times 20\times 39}{6} - 2^2 \times \frac{9\times 10\times 19}{6}$$

$$= \frac{19\times 20\times 39 - 4\times 9\times 10\times 19}{6}$$

$$= \frac{14820 - 6840}{6} = 1330.$$

10. **(b)** $1^3 + 2^3 + 3^3 + \ldots + 9^3 = 2025.$

Now, $(0.11)^3 + (0.22)^3 + (0.33)^3 + \ldots + (0.99)^3$

$= (0.11)^3\, [1^3 + 2^3 + 3^3 + \ldots + 9^3]$

$= (0.11)^3 \times 2025.$

$= 2025 \times 0.001331 = 2.695.$

11. **(c)** $\log 2, \log(2x - 1)$,

$\log(2x + 3)$ are in A.P.

$\Rightarrow \quad 2[\log(2^x - 1)] = \log 2 + \log(2^x + 3)$

$= \log[2 \times (2^x + 3)]$

$\Rightarrow \quad \log(2^x - 1)^2 = \log[2^{x+1} + 6)]$

$\Rightarrow \quad (2^x - 1)^2 = 2^{x+1} + 6 = 2^x \times 2 + 6$

Let $2^x = y$

$(y-1)^2 = 2y + 6$

$\Rightarrow \quad y^2 - 2y + 1 = 2y + 6$

$\Rightarrow \quad y^2 - 4y - 5 = 0$

$\Rightarrow \quad (y-5)(y+1) = 0$

$\Rightarrow \quad y = 5, -1.$

If $y = 5 \quad \Rightarrow \quad 2x = 5$

$\Rightarrow \quad x\log 2 = \log 5$

$\Rightarrow \quad x = \dfrac{\log 5}{\log 2} \quad \Rightarrow \quad x = \log_2 5.$

12. **(b)** $1^2 + 2^2 + 3^2 \cdots x^2 = x\dfrac{(x+1)(2x+1)}{6}$

$$1^2 + 2^2 + 3^2 \cdots 10^2 = 10\frac{(10+1)(20+1)}{6}$$

$$= \frac{10\times 11\times 21}{6} = \frac{2310}{6} = 385$$

13. **(a)** $1 + 0.6 + 0.06 + 0.006 + \cdots$

$= 1 +$ G.P. with a $= 0.6$ and

$$r = \frac{1}{10} = 1 + \frac{0.6}{1-\frac{1}{10}}$$

$$= 1+\frac{0.6}{0.9} = 1\frac{2}{3}$$

14. **(c)** The sequence is

$+3, +5, +7, +9, +11 \ldots$

7th term $= 35 + 13 = 48$

8th term $= 48 + 15 = 63$

9th term $= 63 + 17 = 80$

15. **(c)** Sum of all multiples of 3 up to 50

$$= 3+6+\cdots+48$$
$$= 3(1+2+3+\cdots+16)$$
$$= \frac{3\times 16(16+1)}{2} = 3\times 8\times 17 = 408$$

$$\left[\because \quad 1+2+3+\cdots+n = \frac{n(n+1)}{2}\right]$$

16. **(b)** $\left(1-\dfrac{1}{n+1}\right)+\left(1-\dfrac{2}{n+1}\right)+\left(1-\dfrac{3}{n+1}\right)+\cdots\left(1-\dfrac{n}{n+1}\right)$

$$= n-\left(\frac{1}{n+1}+\frac{2}{n+1}+\frac{3}{n+1}+\cdots+\frac{n}{n+1}\right)$$
$$= n-\frac{1+2+3+\cdots+n}{n+1}$$
$$= n-\frac{n(n+1)}{2(n+1)} = n-\frac{n}{2} = \frac{n}{2}$$

17. **(b)** Required answer $= 2(1 + 2 + 3 + \cdots + 12)$

$$= 2\times\frac{12\times 13}{2} = 156.$$

18. **(d)** $a, 1, b$ are in AP.

$\therefore \; 1 = \dfrac{a+b}{2}$

$\Rightarrow \quad a + b = 2 \qquad \ldots(1)$

Again, $1, a, b$ are in GP.

$\therefore \quad a^2 = b \qquad \ldots(2)$

Now, putting the value of b from equation (2) in equation (1), we have

$a + a^2 = 2$

$\Rightarrow \quad a^2 + a - 2 = 0$

$\Rightarrow \quad a^2 + 2a - a - 2 = 0$

$\Rightarrow \quad a(a+2) - 1(a+2) = 0$

$\Rightarrow \quad (a-1)(a+2) = 0 \quad \Rightarrow \quad a = -2, 1$

$\therefore \quad b = 4.$

19. **(a)** $\because \quad 1^2+2^2+3^2+\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$\therefore \quad 11^2 + 12^2 + \cdots + 21^2$

$= (1^2 + 2^2 + 3^2 + \cdots + 21^2) - (1^2 + 2^2 + \cdots + 10^2)$

$$= \frac{21(21+1)(42+1)}{6} - \frac{10\times 11\times 21}{6}$$

$$= \frac{21\times 22\times 43}{6} - \frac{10\times 11\times 21}{6}$$

$= 3311 - 385 = 2926$

20. **(b)** $\because \dfrac{1}{1\times 2} = \dfrac{1}{1}-\dfrac{1}{2}$

$$\frac{1}{2\times 3} = \frac{1}{2}-\frac{1}{3}$$

$$\frac{1}{3\times 4} = \frac{1}{3}-\frac{1}{4}$$

$$\frac{1}{4\times 5} = \frac{1}{4}-\frac{1}{5}$$

$\therefore \quad$ Given expression

$$= \left[\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{99\times 100}\right]$$

$$= \left[\begin{array}{l}\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots \\ +\left(\frac{1}{98}-\frac{1}{99}\right)+\left(\frac{1}{99}-\frac{1}{100}\right)\end{array}\right]$$

$$= 1-\frac{1}{100} = \frac{99}{100}$$

21. **(a)** Required sum $= \sum n = \dfrac{n(n+1)}{2}$

$$= \frac{100\times 101}{2} = 50\times 101 = 5050$$