

# Numbers

## INTRODUCTION

In Hindu Arabic System, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called *digits* to represent any number. This is the *decimal system* where we use the numbers 0 to 9. 0 is called *insignificant digit* whereas 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *significant digits*.

A group of figures, denoting a number is called a *numeral*. For a given numeral, we start from extreme right as Unit's place, Ten's place, Hundred's place and so on.

**Illustration 1:** We represent the number 309872546 as shown below:

Ten Crore $10^8$	Crores $10^7$	Ten Lakhs (million) $10^6$	Lakhs $10^5$	Ten Thousand $10^4$	Thousand $10^3$	Hundred $10^2$	Ten's $10^1$	Units $10^0$
3	0	9	8	7	2	5	4	6

We read it as

'Thirty crores, ninety-eight lakhs, seventy-two thousands five hundred and forty-six.'

In this numeral:

The place value of 6 is  $6 \times 1 = 6$

The place value of 4 is  $4 \times 10 = 40$

The place value of 5 is  $5 \times 100 = 500$

The place value of 2 is  $2 \times 1000 = 2000$  and so on.

The face value of a digit in a number is the value itself wherever it may be.

Thus, the face value of 7 in the above numeral is 7. The face value of 6 in the above numeral is 6 and so on.

## NUMBER SYSTEM

### Natural Numbers

Counting numbers 1, 2, 3, 4, 5, ... are known as *natural numbers*.

The set of all natural numbers can be represented by

$$N = \{1, 2, 3, 4, 5, \dots\}$$

### Whole Numbers

If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5, ... are called *whole numbers*.

The set of whole numbers can be represented by

$$W = \{0, 1, 2, 3, 4, 5, \dots\}$$

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

### Integers

All counting numbers and their negatives including zero are known as *integers*.

The set of integers can be represented by

$$Z \text{ or } I = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

### Positive Integers

The set  $I^+ = \{1, 2, 3, 4, \dots\}$  is the set of all *positive integers*. Clearly, positive integers and natural numbers are synonyms.

### Negative Integers

The set  $I^- = \{-1, -2, -3, \dots\}$  is the set of all *negative integers*. 0 is neither positive nor negative.

### Non-negative Integers

The set  $\{0, 1, 2, 3, \dots\}$  is the set of all *non-negative integers*.

### Rational Numbers

The numbers of the form  $\frac{p}{q}$ , where  $p$  and  $q$  are integers and  $q \neq 0$ , are known as *rational numbers*, e.g.,  $\frac{4}{7}$ ,  $\frac{3}{2}$ ,  $-\frac{5}{8}$ ,  $\frac{0}{1}$ ,  $-\frac{2}{3}$ , etc.

The set of all rational numbers is denoted by  $Q$ .

That is,  $Q = \{x : x = \frac{p}{q}; p, q \in I, q \neq 0\}$

Since every natural number ' $a$ ' can be written as  $\frac{a}{1}$ , every natural number is a rational number. Since 0 can

be written as  $\frac{0}{1}$  and every non-zero integer 'a' can be written as  $\frac{a}{1}$ , every integer is a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or in non-terminating repeating decimals.

For example,  $\frac{1}{5} = 0.2$ ,  $\frac{1}{3} = 0.333\dots$ ,  $\frac{22}{7} = 3.1428714287$ ,  $\frac{8}{44} = 0.181818\dots$ , etc.

The recurring decimals have been given a short notation as

$$\begin{aligned} 0.333\dots &= 0.\overline{3} \\ 4.1555\dots &= 4.0\overline{5} \\ 0.323232\dots &= 0.\overline{32} \end{aligned}$$

### Irrational Numbers

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as *irrational numbers*, e.g.,  $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ , etc.

Note that the exact value of  $\pi$  is not  $\frac{22}{7} \cdot \frac{22}{7}$  is rational while  $\pi$  is irrational number.  $\frac{22}{7}$  is approximate value of  $\pi$ . Similarly, 3.14 is not an exact value of it.

### Real Numbers

The rational and irrational numbers combined together to form *real numbers*, e.g.,  $\frac{13}{21}, \frac{2}{5}, -\frac{3}{7}, \sqrt{3}, 4 + \sqrt{2}$ , etc. are real numbers.

The set of all real numbers is denoted by R.

Note that the sum, difference or product of a rational and irrational number is irrational, e.g.,  $3 + \sqrt{2}, 4 - \sqrt{3}, \frac{2}{5} - \sqrt{5}, 4\sqrt{3}, -7\sqrt{5}$  are all irrational.

### Even Numbers

All those numbers which are exactly divisible by 2 are called *even numbers*, e.g., 2, 6, 8, 10, etc., are even numbers.

### Odd Numbers

All those numbers which are not exactly divisible by 2 are called *odd numbers*, e.g., 1, 3, 5, 7, etc., are odd numbers.

### Prime Numbers

A natural number other than 1, is a *prime number* if it is divisible by 1 and itself only.

For example, each of the numbers 2, 3, 5, 7, etc., are prime numbers.

### Composite Numbers

Natural numbers greater than 1 which are not prime, are known as composite numbers.

For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

#### Notes:

1. The number 1 is neither a prime number nor a composite number.
2. 2 is the only even number which is prime.
3. Prime numbers up to 100 are:  
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, i.e., 25 prime numbers between 1 and 100.
4. Two numbers which have only 1 as the common factor are called *co-primes* or *relatively prime* to each other, e.g., 3 and 5 are co-primes.

Note that the numbers which are relatively prime need not necessarily be prime numbers, e.g., 16 and 17 are relatively prime, although 16 is not a prime number.

## ADDITION AND SUBTRACTION (SHORT-CUT METHODS)

The above method is best illustrated with the help of following example:

**Illustration 2:**  $54321 - (9876 + 8967 + 7689) = ?$

**Step 1** Add 1st column:  $54321$   
 $6 + 7 + 9 = 22$   $9876$   
 To obtain 1 at unit's place add 9  $8967$   
 to make 31. In the answer, write  $7689$   
 9 at unit's place and carry over 3.  $27789$

**Step 2** Add 2nd column:

$$3 + 7 + 6 + 8 = 24$$

To obtain 2 at ten's place, add 8 to make 32. In the answer, write 8 at ten's place and carry over 3.

**Step 3** Add 3rd column:

$$3 + 8 + 9 + 6 = 26$$

To obtain 3 at hundred's place, add 7 to make 33. In the answer, write 7 at hundred's place and carry over 3.

**Step 4** Add 4th column:

$$3 + 9 + 8 + 7 = 27$$

To obtain 4 at thousand's place, add 7 to make 34. In the answer, write 7 at thousand's place and carry over 3.

**Step 5** 5th column:

To obtain 5 at ten-thousand's place add 2 to it to make 5. In the answer, write 2 at the ten-thousand's place.

$$\therefore 54321 - (9876 + 8967 + 7689) = 27789.$$

## MULTIPLICATION (SHORT-CUT METHODS)

1. Multiplication of a given number by 9, 99, 999, etc., that is by  $10^n - 1$

*Method:* Put as many zeros to the right of the multiplicand as there are nines in the multiplier and from the result subtract the multiplicand and get the answer.

**Illustration 3:** Multiply:

- (a) 3893 by 99                      (b) 4327 by 999  
(c) 5863 by 9999

**Solution:** (a)  $3893 \times 99 = 389300 - 3893 = 385407$   
(b)  $4327 \times 999 = 4327000 - 4327 = 4322673$   
(c)  $5863 \times 9999 = 58630000 - 5863 = 58624137$

2. Multiplication of a given number by 11, 101, 1001, etc., that is, by  $10^n + 1$ .

*Method:* Place  $n$  zeros to the right of the multiplicand and then add the multiplicand to the number so obtained.

**Illustration 4:** Multiply:

- (a)  $4782 \times 11$                       (b)  $9836 \times 101$   
(c)  $6538 \times 1001$

**Solution:** (a)  $4782 \times 11 = 47820 + 4782 = 52602$   
(b)  $9836 \times 101 = 983600 + 9836 = 993436$   
(c)  $6538 \times 1001 = 6538000 + 6538 = 6544538$

3. Multiplication of a given number by 15, 25, 35, etc.

*Method:* Double the multiplier and then multiply the multiplicand by this new number and finally divide the product by 2.

**Illustrations 5:** Multiply:

- (a)  $7054 \times 15$                       (b)  $3897 \times 25$   
(c)  $4563 \times 35$

**Solution:** (a)  $7054 \times 15 = \frac{1}{2} (7054 \times 30)$

$$= \frac{1}{2} (211620) = 105810$$

$$(b) 3897 \times 25 = \frac{1}{2} (3897 \times 50) = \frac{1}{2} (194850) = 97425$$

$$(c) 4536 \times 35 = \frac{1}{2} (4536 \times 70) = \frac{1}{2} (319410) = 159705$$

4. Multiplication of a given number by 5, 25, 125, 625, etc., that is, by a number which is some power of 5.

*Method:* Place as many zeros to the right of the multiplicand as is the power of 5 in the multiplier, then divide the number so obtained by 2 raised to the same power as is the power of 5.

**Illustration 6:** Multiply:

- (a)  $3982 \times 5$                       (b)  $4739 \times 25$   
(c)  $7894 \times 125$                       (d)  $4863 \times 625$

**Solution:** (a)  $3982 \times 2 = \frac{39820}{2} = 19910$

$$(b) 4739 \times 25 = \frac{473900}{2^2} = \frac{473900}{4} = 118475$$

$$(c) 7894 \times 125 = \frac{7894000}{2^3} = \frac{7894000}{8} = 986750$$

$$(d) 4863 \times 625 = \frac{48630000}{2^4} = \frac{48630000}{16} = 3039375$$

### Distributive Laws

For any three numbers  $a$ ,  $b$ ,  $c$ , we have

$$(a) a \times b + a \times c = a \times (b + c)$$

$$(b) a \times b - a \times c = a \times (b - c)$$

**Illustration 7:**  $438 \times 637 + 438 \times 367 = ?$ 

**Solution:**  $438 \times 637 + 438 \times 367 = 438 \times (637 + 367)$   
 $= 430 \times 1000$   
 $= 438000$

**Illustration 8:**  $674 \times 832 - 674 \times 632 = ?$ 

**Solution:**  $674 \times 832 - 674 \times 632$   
 $= 674 \times (832 - 632)$   
 $= 674 \times 200 = 134800$

## SQUARES (SHORT-CUT METHODS)

1. To square any number ending with 5.

*Method:*  $(A5)^2 = A(A + 1)/25$

**Illustration 9:**

- (a)  $(25)^2 = 2(2 + 1)/25 = 6/25 = 625$   
 (b)  $(45)^2 = 4(4 + 1)/25 = 20/25 = 2025$   
 (c)  $(85)^2 = 8(8 + 1)/25 = 72/25 = 7225$

2. To square a number in which every digit is one.

*Method:* Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.

**Illustration 10:**

- (a)  $11^2 = 121$   
 (b)  $111^2 = 12321$   
 (c)  $1111^2 = 1234321$   
 (d)  $222^2 = 2^2(111)^2 = 4(12321) = 49284$   
 (e)  $3333^2 = 3^2(1111)^2 = 9(1234321) = 11108889$

3. To square a number which is nearer to  $10x$ .

*Method:* Use the formula:

$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

**Illustration 11:**

- (a)  $(97)^2 = (97 + 3)(97 - 3) + 3^2$   
 $= 9400 + 9 = 9409$   
 (b)  $(102)^2 = (102 - 2)(102 + 2) + 2^2$   
 $= 10400 + 4 = 10404$   
 (c)  $(994)^2 = (994 + 6)(994 - 6) + 6^2$   
 $= 988000 + 36 = 988036$   
 (d)  $(1005)^2 = (1005 - 5)(1005 + 5) + 5^2$   
 $= 1010000 + 25 = 1010025$

### DIVISION

Division is repeated subtraction.

For example, when we divide 63289 by 43, it means 43 can be repeatedly subtracted 1471 times from 63289 and the remainder 36 is left.

$$\begin{array}{r}
 1471 \leftarrow \text{Quotient} \\
 \text{Divisor} \rightarrow 43 \overline{) 63289} \leftarrow \text{Dividend} \\
 \underline{43} \\
 202 \\
 \underline{172} \\
 308 \\
 \underline{301} \\
 79 \\
 \underline{43} \\
 36 \leftarrow \text{Remainder}
 \end{array}$$

Dividend = (Divisor  $\times$  Quotient) + Remainder

or, Divisor =  $\frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

**Illustration 12:** On dividing 7865321 by a certain number, the quotient is 33612 and the remainder is 113. Find the divisor.

**Solution:** Divisor =  $\frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$   
 $= \frac{7865321 - 113}{33612} = \frac{7865208}{33612} = 234$

**Illustration 13:** A number when divided by 315 leaves remainder 46 and the value of quotient is 7. Find the number.

**Solution:** Number = (Divisor  $\times$  Quotient) + Remainder  
 $= (315 \times 7) + 46 = 2205 + 46 = 2251$

**Illustration 14:** Find the least number of 5 digits which is exactly divisible by 632.

**Solution:** The least number of 5 digits is 10000. Dividing this number by 632, the remainder is 520. So, the required number =  $10000 + (632 - 520) = 10112$ .

$$\begin{array}{r}
 15 \\
 632 \overline{) 10000} \\
 \underline{632} \\
 3680 \\
 \underline{3160} \\
 520
 \end{array}$$

**Illustration 15:** Find the greatest number of 5 digits which is exactly divisible by 463.

**Solution:** The greatest number of 5 digits is 99999. Dividing this number by 463, the remainder is 454. So, the required number =  $99999 - 454 = 99545$ .

$$\begin{array}{r} 215 \\ 463 \overline{) 99999} \\ \underline{926} \phantom{00} \\ 739 \phantom{00} \\ \underline{463} \phantom{00} \\ 2769 \phantom{00} \\ \underline{2315} \phantom{00} \\ 454 \phantom{00} \end{array}$$

**Illustration 16:** Find the number nearest to 13700 which is exactly divisible by 235.

**Solution:** On dividing the number 13700 by 235, the remainder is 70. Therefore, the nearest number to 13700, which is exactly divisible by 235 =  $13700 - 70 = 13630$ .

$$\begin{array}{r} 58 \\ 235 \overline{) 13700} \\ \underline{1175} \phantom{00} \\ 1950 \phantom{00} \\ \underline{1880} \phantom{00} \\ 70 \phantom{00} \end{array}$$

### TESTS OF DIVISIBILITY

1. **Divisibility by 2** A number is divisible by 2 if the unit's digit is zero or divisible by 2.

For example, 4, 12, 30, 18, 102, etc., are all divisible by 2.

2. **Divisibility by 3** A number is divisible by 3 if the sum of digits in the number is divisible by 3.

For example, the number 3792 is divisible by 3 since  $3 + 7 + 9 + 2 = 21$ , which is divisible by 3.

3. **Divisibility by 4** A number is divisible by 4 if the number formed by the last two digits (ten's digit and unit's digit) is divisible by 4 or are both zero.

For example, the number 2616 is divisible by 4 since 16 is divisible by 4.

4. **Divisibility by 5** A number is divisible by 5 if the unit's digit in the number is 0 or 5.

For example, 13520, 7805, 640, 745, etc., are all divisible by 5.

5. **Divisibility by 6** A number is divisible by 6 if the number is even and sum of its digits is divisible by 3.

For example, the number 4518 is divisible by 6 since it is even and sum of its digits  $4 + 5 + 1 + 8 = 18$  is divisible by 3.

6. **Divisibility by 7** The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.

For example, consider the number 448. On doubling the unit digit 8 of 448 we get 16.

Then,  $44 - 16 = 28$ .

Since 28 is divisible by 7, 448 is divisible by 7.

7. **Divisibility by 8** A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.

For example, the number 41784 is divisible by 8 as the number formed by last three digits, i.e., 784 is divisible by 8.

8. **Divisibility by 9** A number is divisible by 9 if the sum of its digits is divisible by 9.

For example, the number 19044 is divisible by 9 as the sum of its digits  $1 + 9 + 0 + 4 + 4 = 18$  is divisible by 9.

9. **Divisibility by 10** A number is divisible by 10, if it ends in zero.

For example, the last digit of 580 is zero, therefore, 580 is divisible by 10.

10. **Divisibility by 11** A number is divisible by 11 if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

For example, in the number 38797, the sum of the digits at odd places is  $3 + 7 + 7 = 17$  and the sum of the digits at even places is  $8 + 9 = 17$ . The difference is  $17 - 17 = 0$ , so the number is divisible by 11.

11. **Divisibility by 12** A number is divisible by 12 if it is divisible by 3 and 4.

12. **Divisibility by 18** An even number satisfying the divisibility test of 9 is divisible by 18.

13. **Divisibility by 25** A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero.

For example, the number 13675 is divisible by 25 as the number formed by the last two digits is 75 which is divisible by 25.

14. **Divisibility by 88** A number is divisible by 88 if it is divisible by 11 and 8.

15. **Divisibility by 125** A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero.

For example, the number 5250 is divisible by 125 as 250 is divisible by 125.

## SOME USEFUL SHORT-CUT METHODS

### 1. Test to find whether a given number is a prime

**Step 1** Select a least positive integer  $n$  such that  $n^2 > \text{given number}$ .

**Step 2** Test the divisibility of given number by every prime number less than  $n$ .

**Step 3** The given number is prime only if it is not divisible by any of these primes.

**Illustration 17:** Investigate, whether 571 is a prime number

**Solution:** Since  $(23)^2 = 529 < 571$  and  $(24)^2 = 576 > 571$   
 $\therefore n = 24$

Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Since 24 is divisible by 2, 571 is not a prime number.

**Illustration 18:** Investigate whether 923 is a prime number.

**Solution:** Since  $(30)^2 = 900 < 923$  and  $(31)^2 = 961 > 923$   
 $\therefore n = 31$

Prime numbers less than 31 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Since 923 is not divisible by any of these primes, therefore, 923 is a prime number.

2. The least number, which when divided by  $d_1$ ,  $d_2$  and  $d_3$  leaves the remainders  $r_1$ ,  $r_2$  and  $r_3$  respectively such that  $(d_1 - r_1) = (d_2 - r_2) = (d_3 - r_3)$ , is = (L.C.M. of  $d_1$ ,  $d_2$  and  $d_3$ ) -  $(d_1 - r_1)$  or  $(d_2 - r_2)$  or  $(d_3 - r_3)$ .

**Illustration 19:** Find the least number which when divided by 9, 10 and 15 leaves the remainders 4, 5 and 10, respectively.

**Solution:** Here,  $9 - 4 = 10 - 5 = 15 - 10 = 5$

Also, L.C.M. (9, 10, 15) = 90,

$\therefore$  the required least number =  $90 - 5 = 85$ .

3. A number on being divided by  $d_1$  and  $d_2$  successively leaves the remainders  $r_1$  and  $r_2$ , respectively. If the number is divided by  $d_1 \times d_2$ , then the remainder is  $= (d_1 \times r_2 + r_1)$ .

**Illustration 20:** A number on being divided by 10 and 11 successively leaves the remainders 5 and 7, respectively. Find the remainder when the same number is divided by 110.

**Solution:** The required remainder

$$= d_1 \times r_2 + r_1 = 10 \times 7 + 5 = 75.$$

4. To find the number of numbers divisible by a certain integer.

The method is best illustrated with the help of following example.

**Illustration 21:** How many numbers up to 532 are divisible by 15?

**Solution:** We divide 532 by 15

$$532 = 35 \times 15 + 7$$

The quotient obtained is the required number of numbers. Thus, there are 35 such numbers.

**Illustration 22:** How many numbers up to 300 are divisible by 5 and 7 together?

**Solution:** L.C.M. of 5 and 7 = 35

We divide 300 by 35

$$300 = 8 \times 35 + 20$$

Thus, there are 8 such numbers.

5. Two numbers when divided by a certain divisor give remainders  $r_1$  and  $r_2$ . When their sum is divided by the same divisor, the remainder is  $r_3$ . The divisor is given by  $r_1 + r_2 - r_3$ .

**Illustration 23:** Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

**Solution:** The required divisor

$$= 473 + 298 - 236 = 499.$$

## EXERCISE-I

- $7372 \times 7372 + 7372 \times 628 = ?$   
 (a) 58976000 (b) 58967000  
 (c) 5897600 (d) None of these
- $9999 + 8888 + 777 + ? = 19700$   
 (a) 36 (b) 16  
 (c) 64 (d) 26
- $60 ? 6 \times 111 = 666666$   
 (a) 0 (b) 2  
 (c) 1 (d) 6
- $3149 \times 1 ? 5 = 425115$   
 (a) 3 (b) 2  
 (c) 4 (d) 6
- If the two digits of the age of Mr Manoj are reversed then the new age so obtained is the age of his wife.  $\frac{1}{11}$  of the sum of their ages is equal to the difference between their ages. If Mr Manoj is older than his wife then find the difference between their ages.  
 (a) Cannot be determined  
 (b) 8 years  
 (c) 10 years  
 (d) 9 years  
 (e) 7 years
- If in a long division sum, the dividend is 380606 and the successive remainders from the first to the last are 434, 125 and 413, then divisor is:  
 (a) 451 (b) 843  
 (c) 4215 (d) 3372
- If  $\frac{x}{y} = \frac{3}{4}$ , then the value of  $\left(\frac{6}{7} + \frac{y-x}{y+x}\right)$  equals:  
 (a)  $\frac{5}{7}$  (b)  $1\frac{1}{7}$   
 (c) 1 (d) 2
- The largest natural number by which the product of three consecutive even natural numbers is always divisible, is:  
 (a) 16 (b) 24  
 (c) 48 (d) 96
- Which number should replace both the '\*'s in  $\left(\frac{*}{21}\right) \times \left(\frac{*}{189}\right) = 1$ ?  
 (a) 21 (b) 63  
 (c) 3969 (d) 147
- In a division sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder be 48, then the dividend is:  
 (a) 240 (b) 576  
 (c) 4800 (d) 4848
- What least number must be subtracted from 1294 so that the remainder when divided by 9, 11, 13 will leave in each case the same remainder 6?  
 (a) 0 (b) 1  
 (c) 2 (d) 3
- 24 is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. The first part is:  
 (a) 11 (b) 13  
 (c) 16 (d) 17
- $\frac{1}{4}$  of a number subtracted from  $\frac{1}{3}$  of the same number gives 12. The number is:  
 (a) 144 (b) 120  
 (c) 72 (d) 63
- $\frac{4}{3}$  of a certain number is 64. Half of that number is:  
 (a) 32 (b) 40  
 (c) 80 (d) 16
- A fraction becomes 4 when 1 is added to both the numerator and denominator; and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is:  
 (a) 2 (b) 3  
 (c) 7 (d) 15
- Three numbers are in the ratio 3:4:5. The sum of the largest and the smallest equals the sum of the third and 52. The smallest number is:  
 (a) 20 (b) 27  
 (c) 39 (d) 52
- The sum of three numbers is 68. If the ratio between first and second is 2:3 and that between second and third is 5:3, then the second number is:  
 (a) 30 (b) 20  
 (c) 58 (d) 48
- If 1 is added to the denominator of a fraction, the fraction becomes  $\frac{1}{2}$ . If 1 is added to the numerator, the fraction becomes 1. The fraction is:

## 1.8 Chapter 1

- (a)  $\frac{4}{7}$  (b)  $\frac{5}{9}$   
 (c)  $\frac{2}{3}$  (d)  $\frac{10}{11}$
19.  $\frac{4}{5}$  of a number exceeds its  $\frac{2}{3}$  by 8. The number is:  
 (a) 30 (b) 60  
 (c) 90 (d) None of these
20. What is the sum of all prime numbers from 60 to 80?  
 (a) 361 (b) 341  
 (c) 351 (d) 349
21. The quotient arising from the division of 24446 by a certain divisor is 79 and the remainder is 35, what is the divisor?  
 (a) 39 (b) 309  
 (c) 390 (d) 3009
22. In a division sum, the quotient is 120, the divisor 456 and the remainder 333, find the dividend.  
 (a) 5533 (b) 50553  
 (c) 56053 (d) 55053
23. The quotient arising from a division of a number by 62 is 463 and the remainder is 60, what is the number?  
 (a) 28666 (b) 28766  
 (c) 28576 (d) 28676
24. A number when divided by 221 gives a remainder 43. What remainder will be obtained by dividing the same number by 17?  
 (a) 11 (b) 8  
 (c) 9 (d) 13
25. Which one of the following is the largest prime number of three digits?  
 (a) 997 (b) 999  
 (c) 991 (d) 993
26. When a certain number is multiplied by 7, the product consists entirely of fives; find the least value of such a number.  
 (a) 79365 (b) 78365  
 (c) 77365 (d) 79265
27. In a division sum, the divisor is 10 times the quotient and five times the remainder. What is the dividend, if the remainder is 46?  
 (a) 5636 (b) 5536  
 (c) 5336 (d) 5436
28. Which one of the following is the least number of four digits divisible by 71?  
 (a) 1006 (b) 1065  
 (c) 1094 (d) 1056
29. How many numbers up to 100 are divisible by 7?  
 (a) 14 (b) 107  
 (c) 93 (d) 100
30. How many numbers up to 500 are divisible by 23?  
 (a) 23 (b) 27  
 (c) 21 (d) 19
31. How many numbers up to 200 are divisible 2 and 3 both?  
 (a) 35 (b) 33  
 (c) 29 (d) 27
32. How many numbers between 100 and 300 are divisible by 11?  
 (a) 11 (b) 10  
 (c) 12 (d) 18
33. How many numbers between 150 and 500 are divisible by 2, 3 and 7 together?  
 (a) 9 (b) 8  
 (c) 10 (d) 11
34. The number of five figures to be added to a number of four fives to obtain the least number of six figures exactly divisible by 357 is:  
 (a) 94762 (b) 94802  
 (c) 94485 (d) None of these
35. The nearest figure to 58701 which is divisible by 567 is:  
 (a) 58968 (b) 58434  
 (c) 58401 (d) None of these
36. The digits indicated by \* in 3422213 \*\* so that this number is divisible by 99 are:  
 (a) 1, 9 (b) 3, 7  
 (c) 4, 6 (d) 5, 5
37. The least value to be given to \* so that the number 5 \* 3457 is divisible by 11 is:  
 (a) 2 (b) 3  
 (c) 0 (d) 4
38. The nearest whole number to one million which is divisible by 537 is:  
 (a) 1000106 (b) 999894  
 (c) 1000437 (d) 999563
39. The smallest number between 400 and 500 which is divisible by 9 is:  
 (a) 414 (b) 405  
 (c) 423 (d) None of these



40. Which one of the following is the greatest number of five digits divisible by 231?  
 (a) 99792 (b) 99892  
 (c) 99692 (d) 99972
41. Find the number nearest to 16386 which is exactly divisible by 425.  
 (a) 16575 (b) 16375  
 (c) 16050 (d) 16450
42. Find the least number which must be subtracted from 9269 so that resulting number is exactly divisible by 73?  
 (a) 17 (b) 57  
 (c) 71 (d) 63
43. Find the least number which must be added to 15463 so that the resulting number is exactly divisible by 107?  
 (a) 52 (b) 71  
 (c) 55 (d) 19
44. What is the number just more than 5000 which is exactly divisible by 73?  
 (a) 5001 (b) 5009  
 (c) 5037 (d) 5027
45. The sum of two numbers is 100 and their difference is 37. The difference of their squares is:  
 (a) 37 (b) 100  
 (c) 63 (d) 3700
46. The number of times 79 be subtracted from 50000, so that the remainder be 43759; is:  
 (a) 69 (b) 79  
 (c) 59 (d) None of these
47. The ratio between two numbers is 3:4 and their sum is 420. The greater of the two numbers is:  
 (a) 175 (b) 200  
 (c) 240 (d) 315
48. The difference between the squares of two consecutive numbers is 35. The numbers are:  
 (a) 14, 15 (b) 15, 16  
 (c) 17, 18 (d) 18, 19
49. Three-fourths of one-fifth of a number is 60. The number is:  
 (a) 300 (b) 400  
 (c) 450 (d) 1200
50. The sum of squares of two numbers is 80 and the square of their difference is 36. The product of the two numbers is:  
 (a) 22 (b) 44  
 (c) 58 (d) 116
51. A number when divided by 357 gives a remainder 37. By dividing the same number by 17, the remainder would be:  
 (a) 3 (b) 4  
 (c) 2 (d) None of these
52. The product of two numbers is 120. The sum of their squares is 289. The sum of the two numbers is:  
 (a) 20 (b) 23  
 (c) 169 (d) None of these
53. Three numbers are in the ratio 4:5:6 and their average is 25. The largest number is:  
 (a) 42 (b) 36  
 (c) 30 (d) 32
54. A number exceeds 20% of itself by 40. The number is:  
 (a) 50 (b) 60  
 (c) 80 (d) 320
55. If 16% of 40% of a number is 8, the number is:  
 (a) 200 (b) 225  
 (c) 125 (d) 320
56. 4767 exactly divides \*\*\* 341, the missing digits are:  
 (a) 468 (b) 586  
 (c) 363 (d) None of these
57. A number when divided by a certain divisor left remainder 241, when twice the number was divided by the same divisor, the remainder was 112. Find the divisor.  
 (a) 370 (b) 365  
 (c) 380 (d) 456
58. Two numbers when divided by a certain divisor give remainders 43 and 37 respectively, when their sum is divided by the same divisor, the remainder is 13. Find the divisor.  
 (a) 71 (b) 67  
 (c) 57 (d) 77
59. Two numbers are such that the ratio between them is 3:5; but if each is increased by 10, the ratio between them becomes 5:7. The numbers are:  
 (a) 3, 5 (b) 7, 9  
 (c) 13, 22 (d) 15, 25
60. Divide 50 into two parts so that the sum of their reciprocals is  $\frac{1}{12}$ .  
 (a) 20, 30 (b) 24, 26  
 (c) 28, 22 (d) 36, 14
61. The sum of seven numbers is 235. The average of the first three is 23 and that of last three is 42. The fourth number is:  
 (a) 40 (b) 126  
 (c) 69 (d) 195

## 1.10 Chapter 1

62. The sum of squares of two numbers is 68 and the squares of their difference is 36. The product of the two numbers is:  
 (a) 16 (b) 32  
 (c) 58 (d) 104
63. What is the least value of  $K$  so that the number 6735K1 is divisible by 9?  
 (a) 5 (b) 7  
 (c) 4 (d) 3
64. For what value of  $K$ , the number 7236K2 is divisible by 8?  
 (a) 7 (b) 5  
 (c) 4 (d) 9
65. Find the least values of  $x$  and  $y$  so that the number  $5x423y$  is divisible by 88.  
 (a) 8, 2 (b) 7, 3  
 (c) 9, 4 (d) 6, 5
66. 24 is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. The first part is:  
 (a) 13 (b) 15  
 (c) 17 (d) 19
67. Sum of three numbers is 132. First number is twice the second and third number is one-third of the first. Find the second number.  
 (a) 18 (b) 36  
 (c) 20 (d) 16
68. What least number must be added to 7231 so that the resulting number is exactly divisible by 5 and 9 together?  
 (a) 20 (b) 18  
 (c) 14 (d) 16
69. Find a number nearest to 9231 which is exactly divisible by 3 as well as by 11.  
 (a) 9240 (b) 9340  
 (c) 9540 (d) 9440
70. Find a nearest number to 12199 which is exactly divisible by the product of the first four prime numbers.  
 (a) 12181  
 (b) 12179  
 (c) 11281  
 (d) 11279
71. The sum of squares of two numbers is 90 and the squares of their difference is 46. The product of the two numbers is:  
 (a) 22 (b) 24  
 (c) 26 (d) 28
72. If 40% of a number is 360, what will be 15% of that number?  
 (a)  $20\frac{1}{4}$  (b)  $20\frac{1}{2}$   
 (c)  $22\frac{1}{4}$  (d)  $22\frac{1}{2}$
73. The sum of the digits of a two-digit number is 8. If the digits are reversed the number is increased by 54. Find the number.  
 (a) 17 (b) 19  
 (c) 21 (d) 23

### EXERCISE-2 (BASED ON MEMORY)

1. The sum of the two digits of a two digit number is 12 and the difference between the two digits of the two digit number is 6. What is the two-digit number?  
 (a) 39 (b) 84  
 (c) 93 (d) Cannot be determined  
 (e) None of these  
**[SBI PO, 2008]**
2. Which of the following smallest number should be added to 6659 to make it a perfect square?  
 (a) 230 (b) 65  
 (c) 98 (d) 54  
 (e) None of these  
**[SBI PO, 2008]**
3. Two numbers are such that the sum of twice the first number and thrice the first number and thrice the second number is 36 and the sum of twice the second number is 39. Which is the smaller number?  
 (a) 9 (b) 5  
 (c) 7 (d) 3  
 (e) None of these  
**[SBI PO, 2008]**
4. The difference between a two-digit number and the number obtained by interchanging the two digits of the numbers is 36. What is the difference between the two digits of the number?

- (a) 6 (b) 4  
(c) 3 (d) Cannot be determined  
(e) None of these

[NABARD PO, 2008]

5. In a two digit number the digit in the unit's place is twice the digit in the ten's place and the number obtained by interchanging the digits is more than the original number by 27. What is 50 % of the original number?

- (a) 36 (b) 63  
(c) 48 (d) 18  
(e) None of these

[SBI PO, 2005]

6. 64329 is divided by a certain number. While dividing, the numbers, 175, 114 and 213 appear as three successive remainders. The divisor is successive remainders. The divisor is:

- (a) 184 (b) 224  
(c) 234 (d) 296

[SSC (GL) Prel. Examination, 2007]

7. A 2-digit number is 3 times the sum of its digits. If 45 is added to the number, its digits are interchanged. The sum of the digits of the number is:

- (a) 11 (b) 9  
(c) 7 (d) 5  
(e) None of these

[SSC (GL) Prel. Examination, 2007]

8. A number divided by 56 gives 29 as remainder. If the same number is divided by 8, the remainder will be:

- (a) 7 (b) 6  
(c) 5 (d) 4

[SSC (GL) Prel. Examination, 2007]

9. The total number of integers between 200 and 400, each of which either begins with 3 or ends with 3 or both, is:

- (a) 10 (b) 100  
(c) 110 (d) 120

[SSC (GL) Prel. Examination, 2007]

10. A 4-digit number is formed by repeating a 2-digit number such as 2525, 3232, etc. Any number of this form is always exactly divisible by:

- (a) 7  
(b) 11  
(c) 13  
(d) Smallest 3-digit prime number

[SSC (GL) Prel. Examination, 2005]

11. A number divided by 68 gives the quotient 269 and remainder zero. If the same number is divided by 67, then the remainder is:

- (a) 0 (b) 1  
(c) 2 (d) 3

[SSC (GL) Pref. Examination, 2005]

12. How many numbers less than 1000 are multiples of both 10 and 13?

- (a) 9 (b) 8  
(c) 6 (d) 7

[SSC (GL) Pref. Examination, 2005]

13. If the sum and difference of two numbers are 20 and 8 respectively, then the difference of their squares is:

- (a) 12 (b) 28  
(c) 80 (d) 160

[SSC (GL) Prel. Examination, 2005]

14. The sum of the squares of two positive integer is 100 and the difference of their squares is 28. The sum of the numbers is:

- (a) 12 (b) 13  
(c) 14 (d) 15

[SSC (GL) Prel. Examination, 2005]

15. The smallest number added to 680621 to make the sum a perfect square is:

- (a) 4 (b) 5  
(c) 6 (d) 8

[SSC (GL) Prel. Examination, 2005]

16. What is the least number to be added to 920 to make it a perfect square?

- (a) 41 (b) 31  
(c) 39 (d) 49  
(e) None of these

[Bank of Maharashtra(Specialist Officer), 2006]

17. If the digits of a two-digit number are interchanged, the number so obtained is greater than the original number by 27. If the sum of the two digits of the number is 11, what is the original number?

- (a) 47 (b) 38  
(c) 74 (d) Cannot be determined  
(e) None of these

[Central Bank of India PO, 2006]

18. A two-digit even number has both the digits same and the number is divisible by 4. When divided by 4 the number obtained is an even number. What is the original number?

- (a) 44 (b) 22  
(c) 66 (d) Cannot be determined  
(e) None of these

[IOB PO, 2006]

## 1.12 Chapter 1

19. The sum of the digits of a two-digit number is 81 less than the number. What is the difference between the digits of the number?

(a) 6 (b) 3  
(c) 1 (d) Cannot be determined  
(e) None of these

[IOB PO, 2006]

20. While solving a mathematical problem, Samidha squared a given number and then subtracted 25 from it rather than doing what was required, i.e., first subtracting 25 from the number and then squaring it. But she got the right answer. What was the given number?

(a) 48 (b) 13  
(c) 38 (d) Cannot be determined  
(e) None of these

[IOB PO, 2006]

21. The difference between a two-digit number and the number obtained by interchanging the digits is 18. The sum of the digits is 10 and the digit at the ten's place is bigger than the digit at the unit's place, What is the two-digit number?

(a) 82 (b) 46  
(c) 64 (d) 73  
(e) None of these

[PNB Management Trainee, 2007]

22. When a number is divided by 31 the remainder is 29. When the same number is divided by 16, what will be the remainder?

(a) Data inadequate (b) 13  
(c) 15 (d) 11  
(e) None of these

[IBP Jr. Executive Examination, 2002]

23. If the digit in the units place of a two-digit number is halved and the digit in the tens place is doubled, the number thus obtained is equal to the number obtained by interchanging the digits. Which of the following is definitely true?

(a) Digits in the units place and the tens place are equal.  
(b) Sum of the digits is a two-digit number.  
(c) Digit in the units place is half of the digit in the tens place.  
(d) Digit in the units place is twice the digit in the tens place.  
(e) None of these

[PNB Management Trainee Examination, 2003]

24. Twenty times a positive integer is less than its square by 96. What is the integer?

(a) 24 (b) 20  
(c) 30 (d) Cannot be determined  
(e) None of these

[Bank of Maharashtra PO Examination, 2003]

25. The product of two consecutive even numbers is 9408. The greater number is:

(a) 94 (b) 102  
(c) 104 (d) 98

[IOB Clerk Examination, 2009]

26. If  $\frac{1}{8}$  of  $\frac{2}{3}$  of  $\frac{4}{5}$  of a number is 12, then 30 per cent of the number will be:

(a) 48 (b) 64  
(c) 54 (d) 42

[BSRB Chennai Bank PO, 2000]

27. When any number is divided by 12 then, dividend becomes one-fourth of the other number. By how much per cent is first number greater than the second number?

(a) 200 (b) 150  
(c) 300 (d) Data inadequate

[BSRB Chennai Bank PO, 2000]

28. The sum of the digits of a two digit number is of the difference between the number and the number obtained by interchanging the positions of the digits. What definitely is the difference between the digits of that number?

(a) 5 (b) 9  
(c) 7 (d) Data inadequate

[BSRB Chennai Bank PO, 2000]

29. A number gets reduced to its  $\frac{1}{3}$  when 48 is subtracted from it. What is  $\frac{2}{3}$  of that number?

(a) 24 (b) 72  
(c) 36 (d) 48

[BSRB Bhopal Bank PO, 2000]

30. The sum of three consecutive numbers is given, what is the difference between first and third number?

(a) 1 (b) 3  
(c) Either 1 or 2 (d) 2

[BSRB Bhopal Bank PO, 2000]

31. If the two digits of the age of Mr Manoj are reversed then, the new age so obtained is the age of his wife.

$\frac{1}{11}$  of the sum of their ages is equal to the difference

between their ages. If Mr Manoj is elder than his wife then find the difference between their ages?

- (a) Cannot be determined
- (b) 10 years
- (c) 8 years
- (d) 7 years
- (e) 9 years

**[BSRB Bangalore Bank PO, 2000]**

32. A number is decreased by 4 and divided by 6, the result is 9. What would be the result if 3 is subtracted from the number and then it is divided by 5?

- (a)  $9\frac{2}{5}$
- (b)  $10\frac{1}{5}$
- (c)  $11\frac{2}{5}$
- (d) 11

**[BSRB Delhi Bank PO, 2000]**

33. A two digits number is seven times the sum of its digits. If each digit is increased by 2, the number thus obtained is 4 more than six times the sum of its digits. Find the number.

- (a) 42
- (b) 24
- (c) 48
- (d) Data inadequate

**[BSRB Patna Bank PO, 2001]**

34. If  $A$  and  $B$  are positive integers such that  $9A^2 = 12A + 96$  and  $B^2 = 2B + 3$ , then which of the following is the value of  $5A + 7B$ ?

- (a) 31
- (b) 41
- (c) 36
- (d) 43

**[BSRB Patna Bank PO, 2001]**

35. The digit in the units place of a number is equal to the digit in the tens place of half of that number and the digit in the tens place of that number is less than the digit in units place of half of the number by 1. If the sum of the digits of the number is seven, then what is the number?

- (a) 52
- (b) 16
- (c) 34
- (d) Data inadequate

**[SBI Bank PO, 2001]**

36. Two fifths of one-third of three-sevenths of a number is 15. What is 40 per cent of that number?

- (a) 136
- (b) 140
- (c) 72
- (d) None of these

**[IBPS Jr. Executive Examination, 2002]**

37. If 3167 is added to 4093 and the sum is divided by 145, approximately what will be the outcome?

- (a) 50
- (b) 75
- (c) 60
- (d) 90
- (e) 80

**[IBPS Jr. Executive Examination, 2002]**

38. When a number is divided by 31 the remainder is 29. When the same number is divided by 16, what will be the remainder?

- (a) Data inadequate
- (b) 13
- (c) 15
- (d) 11

**[IBP Jr. Executive Examination, 2002]**

39. Twenty times a positive integer is less than its square by 96. What is the integer?

- (a) 24
- (b) 20
- (c) 30
- (d) Cannot be determined

**[Bank of Maharashtra PO Examination, 2003]**

40. The sum of the squares of two consecutive even numbers is 6500. Which is the smaller number?

- (a) 54
- (b) 52
- (c) 48
- (d) 56

**[Punjab National Bank PO, 2010]**

41. The sum of five consecutive even numbers of set-A is 220. What is the sum of a different set of five consecutive numbers whose second lowest number is 37 less than double of the lowest number of set-A?

- (a) 223
- (b) 225
- (c) 235
- (d) None of these

**[CBI PO, 2010]**

42. The product of two consecutive even numbers is 9408. Which is the greater of the two numbers?

- (a) 96
- (b) 98
- (c) 94
- (d) 92

**[Andhra Bank PO, 2008]**

43. The product of two successive even numbers is 6888. Which is the greater of the two numbers?

- (a) 78
- (b) 82
- (c) 86
- (d) None of these

**[Uttarakhand GBO PO, 2007]**

44. The number obtained by interchanging the digits of a two-digit number is less than the original number

## 1.14 Chapter I

by 63. If the sum of the digits of the number is 11, what is the original number?

- (a) 29 (b) 92  
(c) 74 (d) Cannot be determined

[SBI PO, 2008]

45. A number is of two digits. The position of digits is interchanged and the new number is added to the original number. The resultant number will always be divisible by:

- (a) 8 (b) 9  
(c) 10 (d) 11

[UPPCS, 2012]

46. When  $2^{23}$  is divided by 10, the remainder will be:

- (a) 2 (b) 3  
(c) 4 (d) 8

[SSC (GL), 2011]

47. Find the unit digit in the product  $(4387)^{245} \times (621)^{72}$ .

- (a) 1 (b) 2  
(c) 5 (d) 7

[SSC (GL), 2011]

48. If  $a$  and  $b$  are odd numbers, then which of the following is even?

- (a)  $a + b + ab$  (b)  $a + b - 1$   
(c)  $a + b + 1$  (d)  $a + b + 2ab$

[SSC (GL), 2011]

49.  $2^{16} - 1$  is divisible by:

- (a) 11 (b) 13  
(c) 17 (d) 19

[SSC (GL), 2011]

50. The sum of two numbers is 24 and their product is 143. The sum of their squares is:

- (a) 296 (b) 295  
(c) 290 (d) 228

[SSC (GL), 2011]

51. The unit digit in the sum  $(124)^{372} + (124)^{373}$  is:

- (a) 5 (b) 4  
(c) 2 (d) 0

[SSC (GL), 2011]

52. If the sum of two numbers be multiplied by each number separately, the products so obtained are 247 and 114. The sum of the numbers is:

- (a) 19 (b) 20  
(c) 21 (d) 23

[SSC (GL), 2011]

53. Find a number, one-seventh of which exceeds its eleventh part by 100.

- (a) 1925 (b) 1825  
(c) 1540 (d) 1340

[SSC (GL), 2011]

54.  $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}} = ?$

- (a) 2.3 (b) 3  
(c) 6 (d) 6.3

[SSC (GL), 2011]

55. When 335 is added to 5A7, the result is 8B2. 8B2 is divisible by 3. What is the largest possible value of A?

- (a) 8 (b) 2  
(c) 1 (d) 4

[SSC, 2013]

56. Which one of the following numbers is divisible by 25?

- (a) 303310 (b) 373355  
(c) 303375 (d) 22040

[SSC, 2013]

57. The units digit in  $3 \times 38 \times 537 \times 1256$  is:

- (a) 4 (b) 2  
(c) 6 (d) 8

[SSC, 2013]

58. The last digit, that is, the digit in the unit's place of the number  $[(57)^{25} - 1]$  is:

- (a) 6 (b) 8  
(c) 0 (d) 5

[SSC Assistant Grade III, 2012]

59. A number  $N$  is a positive three-digit number. If  $x$  is in its hundred's place and  $y$  is in its unit's place, then the number  $N - 100x - y$  is always divisible by:

- (a) 8 (b) 9  
(c) 10 (d) 11

[SSC Assistant Grade III, 2012]

60.  $n$  is a whole number which when divided by 4 gives the remainder 3. The remainder when  $2n$  is divided by 4 is:

- (a) 1 (b) 2  
(c) 3 (d) 0

[SSC, 2012]

61. If  $m$  and  $n$  are positive integers and  $(m - n)$  is an even number, then  $(m^2 - n^2)$  will be always divisible by:

- (a) 4 (b) 6  
(c) 8 (d) 12

[SSC, 2012]

62. In a division sum, the divisor is 4 times the quotient and twice the remainder. If  $a$  and  $b$  are respectively the divisor and the dividend, then:

$$(a) \frac{4b-a^2}{a} = 3 \quad (b) \frac{4b-2a}{a^2} = 2$$

$$(c) (a+1)^2 = 4b \quad (d) \frac{a(a+2)}{b} = 4$$

[SSC, 2011]

63. If 73846A is divisible by 11, then the value of A is:

$$(a) 6 \quad (b) 3$$

$$(c) 9 \quad (d) 1$$

[SSC, 2011]

64. The product of two numbers is 1575 and their quotient is  $\frac{9}{7}$ . Then the sum of the numbers is:

$$(a) 74 \quad (b) 78$$

$$(c) 80 \quad (d) 90$$

[SSC, 2011]

65. When  $(67^{67} + 67)$  is divided by 68, the remainder is:

$$(a) 1 \quad (b) 63$$

$$(c) 66 \quad (d) 67$$

[SSC, 2011]

66. The least positive integer that should be subtracted from  $3011 \times 3012$  so that the difference is a perfect square, is:

$$(a) 3009 \quad (b) 3010$$

$$(c) 3011 \quad (d) 3012$$

[SSC, 2011]

67. A number, when divided by 296, gives 75 as the remainder. If the same number is divided by 37 then the remainder will be:

$$(a) 1 \quad (b) 2$$

$$(c) 19 \quad (d) 31$$

[SSC, 2010]

68. The sum and product of two numbers are 12 and 35 respectively. The sum of their reciprocals will be:

$$(a) \frac{1}{3} \quad (b) \frac{1}{5}$$

$$(c) \frac{12}{35} \quad (d) \frac{35}{12}$$

[SSC, 2010]

69. The greatest number among  $\sqrt{5}$ ,  $\sqrt[3]{4}$ ,  $\sqrt[5]{2}$ ,  $\sqrt[7]{3}$  is:

$$(a) \sqrt[3]{4} \quad (b) \sqrt[7]{3}$$

$$(c) \sqrt{5} \quad (d) \sqrt[5]{2}$$

[SSC, 2010]

70. Two numbers are such that their difference, their sum and their product are in the ratio of 1:7:24. The product of the numbers is:

$$(a) 24 \quad (b) 36$$

$$(c) 48 \quad (d) 60$$

[SSC, 2010]

71. Rachita enters a shop to buy ice-creams, cookies and pastries. She has to buy at least 9 units of each. She buys more cookies than ice-creams and more pastries than cookies. She picks up a total of 32 items. How many cookies does she buy?

$$(a) \text{ Either 12 or 13 } \quad (b) \text{ Either 11 or 12 }$$

$$(c) \text{ Either 10 or 11 } \quad (d) \text{ Either 9 or 11 }$$

$$(e) \text{ Either 9 or 10 }$$

[IBPS PO/MT, 2012]

72. The product of three consecutive even number is 4032. The product of the first and the third number is 252. What is five times the second number?

$$(a) 80 \quad (b) 100$$

$$(c) 60 \quad (d) 70$$

$$(e) 90$$

[IBPS PO/MT, 2012]

73. The sum of nine consecutive odd numbers of Set A is 621. What is the sum of a different set of six consecutive even numbers whose lowest number is 15 more than the lowest number of Set A?

$$(a) 498 \quad (b) 468$$

$$(c) 478 \quad (d) 488$$

$$(e) \text{ None of these }$$

[IOB PO, 2011]

74. The sum of five consecutive even numbers of set A is 220. What is the sum of a different set of five consecutive numbers whose second lowest number is 37 less than double of the lowest number of set A?

$$(a) 223 \quad (b) 225$$

$$(c) 235 \quad (d) 243$$

$$(e) \text{ None of these }$$

[Central Bank of India PO, 2010]

75. There are two numbers such that the sum of twice the first number and thrice the second number is 100 and the sum of thrice the first number and twice the second number is 120. Which is the larger number?

$$(a) 32 \quad (b) 12$$

$$(c) 14 \quad (d) 35$$

$$(e) \text{ None of these }$$

[Corporation Bank PO, 2010]

## 1.16 Chapter I

76. The number obtained by interchanging the two digits of a two-digit number is lesser than the original number by 54. If the sum of the two digits of the number is 12, then what is the original number?

(a) 28 (b) 39  
(c) 82 (d) Cannot be determined  
(e) None of these

[IDBI Bank PO, 2009]

77. The difference between a two-digit number and the number obtained by interchanging the two digits of the number is 9. If the sum of the two digits of the number is 15, then what is the original number?

(a) 89 (b) 67  
(c) 87 (d) Cannot be determined  
(e) None of these

[OBC PO, 2009]

78. The difference between a two-digit-number and the number obtained by interchanging the two digits of the number is 9. What is the difference between the two digits of the number?

(a) 3 (b) 2  
(c) 1 (d) Cannot be determined  
(e) None of these

[NABARD Bank Officer, 2009]

ANSWER KEYS													
EXERCISE-I													
1. (a)	2. (a)	3. (a)	4. (a)	5. (d)	6. (b)	7. (c)	8. (c)	9. (b)	10. (d)	11. (b)	12. (b)	13. (a)	
14. (b)	15. (d)	16. (c)	17. (a)	18. (c)	19. (b)	20. (c)	21. (b)	22. (d)	23. (b)	24. (c)	25. (a)	26. (a)	
27. (c)	28. (b)	29. (a)	30. (c)	31. (b)	32. (d)	33. (a)	34. (a)	35. (a)	36. (a)	37. (a)	38. (b)	39. (b)	
40. (a)	41. (d)	42. (c)	43. (a)	44. (c)	45. (d)	46. (b)	47. (c)	48. (c)	49. (b)	50. (a)	51. (a)	52. (b)	
53. (c)	54. (a)	55. (c)	56. (b)	57. (a)	58. (b)	59. (d)	60. (a)	61. (a)	62. (a)	63. (a)	64. (a)	65. (a)	
66. (a)	67. (b)	68. (c)	69. (a)	70. (a)	71. (a)	72. (a)	73. (a)						
EXERCISE-2													
1. (e)	2. (b)	3. (e)	4. (b)	5. (d)	6. (c)	7. (b)	8. (c)	9. (c)	10. (d)	11. (b)	12. (d)	13. (d)	
14. (c)	15. (a)	16. (a)	17. (a)	18. (e)	19. (d)	20. (b)	21. (c)	22. (a)	23. (d)	24. (a)	25. (d)	26. (c)	
27. (d)	28. (a)	29. (d)	30. (d)	31. (e)	32. (d)	33. (a)	34. (b)	35. (a)	36. (d)	37. (a)	38. (a)	39. (a)	
40. (d)	41. (d)	42. (b)	43. (d)	44. (b)	45. (d)	46. (a)	47. (d)	48. (d)	49. (c)	50. (c)	51. (d)	52. (a)	
53. (a)	54. (b)	55. (d)	56. (c)	57. (d)	58. (a)	59. (c)	60. (b)	61. (a)	62. (d)	63. (c)	64. (c)	65. (c)	
66. (c)	67. (a)	68. (c)	69. (c)	70. (c)	71. (c)	72. (a)	73. (e)	74. (e)	75. (a)	76. (e)	77. (d)	78. (c)	



## EXPLANATORY ANSWERS

## EXERCISE-I

1. (a) Given Expression =  $7372 \times (7372 + 628)$   
 $= 7372 \times 8000$   
 $= 58976000.$
2. (a) Let,  $9999 + 8888 + 777 + x = 19700$   
 $\therefore x = 19700 - 19664 = 36.$
3. (a) Let,  $x \times 111 = 666666$   
 $\Rightarrow x = \frac{666666}{111} = 6006 \quad \therefore \text{Missing figure} = 0.$
4. (a) Let,  $3149 \times x = 425115$   
 $\Rightarrow x = \frac{425115}{3149} = 135 \quad \therefore \text{Missing digit} = 3.$
5. (d) Let the age of Mr Manoj be  $(10x + y)$  years.  
 $\therefore$  His wife's age =  $(10y + x)$  years  
Then,  $(10x + y + 10y + x) \frac{1}{11} = 10x + y - 10y - x$   
or,  $x + y = 9x - 9y, \text{ or, } 8x = 10y$   
or,  $\frac{x}{y} = \frac{5}{4}$   
 $\therefore x = 5 \text{ and } y = 4$   
 $[\because \text{any other multiple of 5 will make } x \text{ of two digits}]$   
 $\therefore \text{Difference} = 10x + y - 10y - x$   
 $= 9x - 9y = 9(x - y)$   
 $= 9(5 - 4) = 9 \text{ years.}$
9. (b) Let,  $\frac{x}{21} \times \frac{x}{189} = 1.$   
Then,  $x^2 = 21 \times 189 = 21 \times 21 \times 3 \times 3$   
 $\therefore x = 21 \times 3 = 63.$
10. (d) Let Quotient =  $Q$  and remainder =  $R$   
Then, given  $12Q = 5R$   
Now,  $R = 48 \Rightarrow 12Q = 5 \times 48 \Rightarrow Q = 20$   
 $\therefore \text{Dividend} = 20 \times 240 + 48 = 4848.$
11. (b) The number when divided by 9, 11 and 13 leaving remainder 6 = (L.C.M. of 9, 11, 13) + 6 = 1293  
 $\therefore \text{Required number} = 1294 - 1293 = 1.$
12. (b) Let these parts be  $x$  and  $(24 - x).$   
Then,  $7x + 5(24 - x) = 146 \Rightarrow x = 13$   
So the first part is 13.
13. (a)  $\frac{1}{3}x - \frac{1}{4}x = 12 \Rightarrow \frac{1}{12}x = 12 \Rightarrow x = 144.$
14. (b)  $\frac{4}{5} \times x = 64 \Rightarrow x = \frac{64 \times 5}{4} = 80$   
 $\therefore \frac{1}{2} \times x = \frac{1}{2} \times 80 = 40.$
15. (d) Let the required fraction be  $\frac{x}{y}$   
Then,  $\frac{x+1}{y+1} = 4 \Rightarrow x - 4y = 3$   
and,  $\frac{x-1}{y-1} = 7 \Rightarrow x - 7y = -6$   
Solving these equations, we get  $x = 15, y = 3.$
16. (c) Let the numbers be  $3x, 4x$  and  $5x.$   
Then,  $5x + 3x = 4x + 52 \Rightarrow 4x = 52 \Rightarrow x = 13$   
 $\therefore \text{The smallest number} = 3x = 3 \times 13 = 39.$
17. (a) Let the numbers be  $a, b, c.$   
Then,  $\frac{a}{b} = \frac{2}{3}, \frac{b}{c} = \frac{5}{3} \Rightarrow \frac{a}{b} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}, \frac{b}{c} = \frac{5 \times 3}{3 \times 3} = \frac{15}{9}$   
 $\Rightarrow a:b:c = 10:15:9$   
Let the numbers be  $10x, 15x, 9x,$   
Then,  $10x + 15x + 9x = 68 \Rightarrow 34x = 68 \Rightarrow x = 2$   
 $\therefore \text{Second number} = 15x = 15 \times 2 = 30.$
18. (c) Let the required fraction be  $\frac{x}{y}.$   
Then,  $\frac{x}{y+1} = \frac{1}{2} \Rightarrow 2x - y = 1$   
and,  $\frac{x+1}{y} = 1 \Rightarrow x - y = -1$   
Solving,  $2x - y = 1$  and  $x - y = -1,$   
we get,  $x = 2, y = 3$   
 $\therefore \text{The fraction is } \frac{2}{3}.$
19. (b) Let the number be  $x.$   
Then,  $\frac{4}{5}x - \frac{2}{3}x = 8 \Rightarrow \frac{12x - 10x}{15} = 8 \Rightarrow 2x = 120$   
or,  $x = 60.$
20. (c)  $61 + 67 + 71 + 73 + 79 = 351.$
21. (b) Divisor =  $(24446 - 35) \div 79 = 309.$
22. (d) Dividend =  $456 \times 120 + 33 = 55053.$
23. (b) Number =  $463 \times 62 + 60 = 28766.$
24. (c) Dividing 43 by 17, the remainder is 9.
26. (a)  $\begin{array}{r} 7 \overline{) 555555} \\ \underline{79365} \end{array}$   
 $\therefore \text{The least number is } 79365.$
27. (c) Remainder = 46  
Divisor =  $5 \times 46 = 230$   
Also,  $10 \times \text{quotient} = 5 \times \text{remainder}$   
 $\therefore \text{Remainder} = 2 \times \text{quotient}$   
That is, quotient = 23  
Dividend =  $23 \times 230 + 46 = 5336.$

$$\begin{array}{r} 28. \text{ (b) } 71 \overline{)1000} \text{ (14)} \\ \underline{-71} \phantom{00} \\ 290 \phantom{0} \\ \underline{-284} \phantom{0} \\ 6 \end{array}$$

$\therefore$  Least number =  $1000 - 6 + 71 = 1065$ .

29. (a) Quotient when 100 is divided by 7 is 14.

30. (c) Quotient when 500 is divided by 23 is 21.

31. (b) Quotient when 200 is divided by the L.C.M. of 2 and 3, i.e., 6 is 33.

32. (d) Quotient when 300 and 100 are divided by 11 are 27 and 9.

$\therefore$  Between 300 and 100, there are  $27 - 9$ , i.e., 18 numbers.

33. (a) A number divisible by 2, 3, and 7 is divisible by their L.C.M., i.e., 42. Up to 100, there are 2 numbers divisible by 42. Up to 500, there are 11 numbers divisible by 42.

$\therefore$  Between 100 and 500, there are  $11 - 2$ , i.e., 9 numbers divisible by 42.

34. (a) The least number of six figures is 100000

On dividing 100000 by 357, remainder = 40

$\therefore$  Least number of six figures which is divisible by 357 =  $100000 + (357 - 40) = 100317$

$\therefore$  Required number =  $100317 - 5555 = 94762$ .

35. (a) On dividing 58701 by 567,

$$\text{Remainder} = 300 > \frac{1}{2} (567)$$

$\therefore$  Integer nearest to 58701 and divisible by 567

$$= 58701 + (567 - 300)$$

$$= 58701 + 267 = 58968.$$

36. (a) Let  $x, y$  be the required digits.

The number is to be divisible by 99, i.e., 9 and 11 both.

$\therefore$  Sum of digits is to be divisible by 9, i.e.,

$$3 + 4 + 2 + 2 + 2 + 1 + 3 + x + y = 17 + x + y$$

is to be divisible by 9 and,

$$(y + 3 + 2 + 2 + 3) - (x + 1 + 2 + 4) = 0$$

or, multiple of 11, i.e.,  $y - x + 3 = 0$  or multiple of 11

$$\therefore x = 1, y = 9.$$

37. (a) Let the least value to be given to \* be  $x$

$$\text{Then, } x + 4 + 7 = 5 + 3 + 5$$

$$x = 2.$$

38. (b) On dividing 100000 by 537, remainder

$$= 106 < \frac{537}{2}$$

$\therefore$  Nearest whole number to one million which is divisible by 537 =  $100000 - 106 = 999894$ .

39. (b) On dividing 400 by 9, remainder = 4

$\therefore$  Number nearest to 400 and divisible by 9

$$= 400 + (9 - 4) = 405.$$

40. (a) Greatest number of five digits = 99,999

Dividing this by 231, the remainder = 207

$\therefore$  Required greatest number

$$= 99999 - 207 = 99792.$$

41. (d) Dividing 16386 by 425, the remainder is 361 which is more than half the divisor, therefore, the required number is  $16386 + (425 - 361) = 16450$ .

42. (c) Divide 9269 by 73, the remainder is 71.

$\therefore$  71 is the required least number.

43. (a) Divide 15463 by 107, the remainder is 55, therefore, the number to be added =  $107 - 55 = 52$ .

44. (c) Dividing 5000 by 73, the remainder is 36. The number greater than 5000 is obtained by adding to 5000 the difference of divisor and the remainder.

$$\therefore \text{The required number} = 5000 + (73 - 36) = 5037.$$

45. (d) Let the numbers be  $a$  and  $b$ .

$$\text{Then, } a + b = 100 \text{ and } a - b = 37$$

$$\therefore a^2 - b^2 = (a + b)(a - b) = 100 \times 37 = 3700.$$

46. (b)  $50000 = 79 \times \text{quotient} + 43759$

$$\therefore 50000 - 43759 = 79 \times \text{quotient}$$

$$\text{or, } 6241 = 79 \times \text{quotient}$$

$$\therefore \text{Required number of times} = \frac{6241}{79} = 79.$$

47. (c) Let the numbers be  $3x$  and  $4x$ .

$$\text{Then, } 3x + 4x = 420 \Rightarrow 7x = 420 \Rightarrow x = 60$$

$$\therefore \text{Greater number} = 4 \times 60 = 240.$$

48. (c) Let the numbers be  $x$  and  $(x + 1)$ .

$$\text{Then, } (x + 1)^2 - x^2 = 35 \Rightarrow x^2 + 2x + 1 - x^2 = 35$$

$$\Rightarrow 2x = 34$$

$$\text{or, } x = 17$$

So, the numbers are 17 and 18.

49. (b) Let the number be  $x$ .

$$\text{Then, } \frac{3}{4} \times \frac{1}{5} \times x = 60 \Rightarrow 3x = 60 \times 5 \times 4$$

$$\text{or, } x = 400.$$

50. (a) Let the numbers be  $a$  and  $b$ .

$$\text{Then, } a^2 + b^2 = 80 \text{ and } (a - b)^2 = 36$$

$$(a - b)^2 = 36 \Rightarrow a^2 + b^2 - 2ab = 36$$

$$\Rightarrow 2ab = (a^2 + b^2) - 36 = 80 - 36 = 44$$

$$\Rightarrow ab = 22.$$

51. (a) Let  $K$  be the quotient.

$$\text{Then, number} = 357 \times K + 37$$

$$= 17 \times 21K + 17 \times 2 + 3$$

$$= 17 \times (21K + 2) + 3$$

So, required remainder = 3 and new quotient =  $21K + 2$ .

52. (b) Let the numbers be  $a$  and  $b$ .

$$\text{Then, } (a + b)^2 = (a^2 + b^2) + 2ab$$

$$= 289 + 2 \times 120 = 289 + 240 = 529$$

$$\therefore a + b = \sqrt{529} \Rightarrow a + b = 23.$$

53. (c) Let the numbers be  $4x$ ,  $5x$  and  $6x$ .

$$\text{Then, } \frac{4x+5x+6x}{3} = 25 \quad \text{or, } 15x = 75 \Rightarrow x = 5$$

$\therefore$  The largest number =  $6x = 6 \times 5 = 30$ .

54. (a) Let the required number be  $x$ .

$$\text{Then, } x - \frac{20}{100}x = 40 \quad \text{or, } 5x - x = 200 \quad \text{or, } x = 50.$$

55. (c) Let,  $\frac{16}{100} \times \frac{40}{100} \times x = 8$

$$\text{Then, } x = \frac{8 \times 100 \times 100}{16 \times 40} = 125.$$

56. (b) Last digit of dividend = 1

Last digit of divisor = 7

$\therefore$  Last digit of quotient should be 3

$$4767 \times 3 = 14301$$

$$4767 \times 20 = 95340$$

$$4767 \times 100 = 476700$$

$$\therefore 4767 \times (3 + 20 + 100) = 586341$$

Missing digits are 586.

57. (a) The divisor =  $r_1 + r_2 - r_3$

$$= 241 + 241 - 112 = 370.$$

58. (b) The divisor =  $r_1 + r_2 - r_3 = 43 + 37 - 13 = 67$ .

59. (d) Let the numbers be  $3x$  and  $5x$ .

$$\text{Then, } \frac{3x+10}{5x+10} = \frac{5}{7} \Rightarrow 7(3x+10) = 5(5x+10) \\ \Rightarrow x = 5$$

$\therefore$  The numbers are 15 and 25.

60. (a) Let the numbers be  $x$  and  $(50 - x)$ .

$$\text{Then, } \frac{1}{x} + \frac{1}{50-x} = \frac{1}{12} \Rightarrow \frac{50-x+x}{x(50-x)} = \frac{1}{12}$$

$$\Rightarrow x^2 - 50x + 600 = 0$$

$$\Rightarrow x = 30 \text{ or } 20.$$

$\therefore$  The numbers are 20, 30.

61. (a)  $23 \times 3 + x + 42 \times 3 = 235 \Rightarrow x = 40$

$\therefore$  Fourth number = 40.

62. (a) Let the numbers be  $a$  and  $b$ .

$$\text{Then, } a^2 + b^2 = 68 \text{ and } (a - b)^2 = 36$$

$$\text{Now, } (a - b)^2 = 36 \Rightarrow a^2 + b^2 - 2ab = 36$$

$$\Rightarrow 68 - 2ab = 36$$

$$\Rightarrow 2ab = 32 \Rightarrow ab = 16.$$

63. (a)  $6 + 7 + 3 + 5 + K + 1 = 22 + K$

The least number greater than 22 and divisible by 9 is 27

$$\therefore 27 = 22 + K \Rightarrow K = 5.$$

64. (a) The last three digits  $6K2$  is divisible by 8 if  $K$  is 3 or 7 since 632 and 672 are divisible by 8.

65. (a) Test of 8 is independent of the value of  $x$ . First, apply the test of 8. Last three digit of the given number are  $23y$  which is divisible by 8 if  $y$  is 2. Substitute for  $y$ . The number now becomes  $5x4232$ .

Apply the test of 11.

Sum of the digits at odd and even places is

$$5 + 4 + 3, \text{ i.e., } 12 \text{ and } x + 2 + 2, \text{ i.e., } x + 4$$

$$\therefore x + 4 = 12 \Rightarrow x = 8$$

Hence,  $x = 8$  and  $y = 2$ .

66. (a) Let  $x$  be the first part so that the other part is  $24 - x$ .

$$\therefore 7x + 5(24 - x) = 146 \Rightarrow x = 13.$$

67. (b) Let the second number be  $3x$ , so that the first number is  $6x$  and the third one is  $2x$ .

$$\therefore 6x + 3x + 2x = 132$$

$$\Rightarrow 11x = 132 \quad \text{or} \quad x = 12.$$

$$\text{Second number} = 3x = 3 \times 12 = 36.$$

68. (c) Divide 7231 by 45, the remainder is 31.

$$\therefore \text{Required number} = 45 - 31 = 14.$$

69. (a) A number which is divisible by 3 and 11 is also divisible by 33. Dividing 9331 by 33, the remainder is 24 which is more than half the divisor.

$$\begin{array}{r} 33 \overline{)9331} \quad (279 \\ \underline{66} \\ 263 \\ \underline{231} \\ 321 \\ \underline{297} \\ 24 \end{array}$$

Since the remainder 24 is more than half the divisor 33,

$$\therefore \text{The nearest number} = 9231 + (33 - 24) = 9240.$$

70. (a) Product of first four prime numbers is

$$2 \times 3 \times 5 \times 7 = 210$$

Dividing 12199 by 210, we find the remainder 19, which is less than half the divisor.

$$\begin{array}{r} 210 \overline{)12199} \quad (58 \\ \underline{1050} \\ 1699 \\ \underline{1680} \\ 19 \end{array}$$

$\therefore$  The number nearer to 12199 divisible by 210 is  $12199 - 18 = 12181$ .

71. (a) Let the numbers be  $x$  and  $y$ .

According to the question:

$$x^2 + y^2 = 90 \quad \dots(1)$$

$$\text{and, } (x - y)^2 = 46 \quad \dots(2)$$

From equation (2),

$$(x - y)^2 = 46 \quad \text{or, } x^2 + y^2 - 2xy = 46$$

$$\text{or, } 90 - 2xy = 46 \quad \text{or, } xy = \frac{90-46}{2} = 22$$

$\therefore$  Product of two numbers = 22.

72. (a) Let the number be  $x$ .

Then, we have, 40% of  $x = 360$

$$\therefore x = \frac{360 \times 100}{40} = 900$$

## 1.20 Chapter 1

Now, 15% of  $x = \frac{15}{100} \times 900 = 135$

Again, 15% of 135 =  $\frac{15}{100} \times 135 = 20.25$ .

73. (a) Let the two digit number be  $10x + y$

Then, we have,  $x + y = 8$

...(1)

and,  $10y + x = 10x + y + 54$

or,  $y - x = 6$

...(2)

From equations (1) and (2),

the required number =  $1 \times 10 + 7 = 17$ .

## EXERCISE-2 (BASED ON MEMORY)

1. (e) Let the number be  $10x + y$

$x + y = 12$

and,  $x - y = 6$

$\therefore x = 9$

and,  $y = 3$

or,  $x = 3$

and,  $y = 9$

$\therefore$  Hence the number may be 93 or 39.

$$\begin{array}{r} 2. \text{ (b)} \quad 81 \\ 8 \overline{) 6659} \\ \underline{64} \phantom{00} \\ 259 \\ 161 \overline{) 259} \\ \underline{161} \phantom{00} \\ 98 \end{array}$$

$\therefore$  Required number =  $82 \times 82 - 6659 = 65$

3. (e) Let the numbers be  $x$  and  $y$

$2x + 3y = 36$

$3x + 2y = 39$

$4x + 6y = 72$

$9x + 6y = 117$

$5x = 45$

$\therefore x = 9$

$2 \times 9 + 3y = 36$

$y = \frac{36 - 18}{3} = 6$

$\therefore$  Smaller number is 6

4. (b) Suppose the two-digit number =  $10x + y$

then,  $(10x + y) - 10y + x = 36$

or,  $9(x - y) = 36 \quad \therefore x - y = 4$

5. (d) Let the number be  $(10x + 2x)$

$\therefore [10(2x) + x] - [10x + 2x] = 27$

$\Rightarrow 9x = 27$

$\Rightarrow x = 3$

$\therefore$  The original number = 36

$\therefore$  50% of the original number = 18

10. (d) by 101 which is the smallest 3-digit prime number.

11. (b) The number is  $68 \times 269 = 18292$ . 18292, when divided by 67, leaves a remainder of 1.

12. (d) All multiples of 130, i.e., 130, 260, 390, 520, 650, 780, 910.

13. (d) Let  $x$  and  $y$  be the numbers,

$\therefore x + y = 20, x - y = 8$

$\Rightarrow x = 14, y = 6$

$\therefore x^2 + y^2 = 14^2 + 6^2$

$= (14 + 6)(14 - 6) \Rightarrow 20 \times 8 = 160$ .

14. (c) Let  $x$  and  $y$  be the numbers

Such that  $x^2 + y^2 = 100$

$x^2 - y^2 = 28$

$\Rightarrow x^2 = 64, y^2 = 36$

$\Rightarrow x = 8, y = 6$

$\Rightarrow x + y = 14$

15. (a)  $\sqrt{680625} = 825$

16. (a) We know

$31^2 > 920 > 30^2$  i.e.,  $961 > 920 > 900$

17. (a) We have, difference of the two digits  $\frac{27}{9} = 3$   
Sum of the two digits = 11

Now, the two digits are  $\frac{11+3}{2}$  and  $\frac{11-3}{2}$ , i.e., 7 and 4

Thus, the number is 47 because  $47 < 74$ .

You can check it:  $74 - 47 = 27$ .

18. (e) 88

19. (d)  $10x + y - (x + y) = 81$

or,  $10x + y - x - y = 81$

or,  $9x \Rightarrow 81 \quad \therefore x = 9$

hence, all such numbers are as follows: 90, 91, 92, 93, ... 99.

20. (b) Suppose the number is  $x$ . Then

$x^2 - 25 = (x - 25)^2$

or,  $x^2 - 25 = x^2 - 50x + 625$

or,  $50x = 650$

$\therefore x = 13$ .

21. (c)  $(10x + y) - (10y + x) = 18$

$\Rightarrow 9(x - y) = 18$

$x - y = 2$

Also, given that  $x + y = 10$

- $\therefore x = 6$  and  $y = 4$   
 $\therefore$  the number is 64.
- 22. (a)** Quotient is not given. Hence, remainder cannot be determined.
- 23. (d)** Suppose the two digit number is  $10x + y$   
 Then,  $10y + x = 20x + \frac{y}{2}$   
 or,  $20y + 2x = 40x + y$  or,  $y = 2x$ .
- 24. (a)** Let the positive integer be  $x$ .  
 Now,  $x^2 - 20x = 96$   
 or,  $x^2 - 20x - 96 = 0$   
 or,  $x^2 - 24x + 4x - 96 = 0$   
 or,  $x(x - 24) + 4(x - 24) = 0$   
 or,  $(x - 24)(x + 4) = 0$   
 or,  $x = 24, -4$ .
- 25. (d)**  $x \times (x + 2) = 9408$   
 $x^2 + 2x - 9408 = 0$   
 $x^2 + 98x - 96x - 9408 = 0$   
 $x(x + 98) - 96(x + 98) = 0$   
 $x + 98 = 0, x - 96 = 0$   
 $x = -98, x + 2 = 96 + 2 = 98$   
 greater number  $= x + 2 = 96 + 2 = 98$ .
- 26. (c)** Let the number be  $x$   
 and,  $\frac{1}{8}$  of  $\frac{2}{3}$  of  $\frac{4}{5} \times x = 12$   
 $\therefore \frac{3x}{10} = \frac{3}{10} \times 12 \times 15 = 54$ .
- 27. (d)** Here neither the remainder nor the dividend nor the second number is given, so number cannot be determined.
- 28. (a)** Let the two digit number be  $10x + y$   
 Then,  $x + y = \frac{1}{5}(10x + y - 10y + x)$   
 or,  $x + y = \frac{9}{5}(x - y)$   
 or,  $4x - 14y = 0 \Rightarrow \frac{x}{y} = \frac{7}{2}$   
 Using componendo and dividendo, we have  
 $\frac{x+y}{x-y} = \frac{7+2}{7-2} = \frac{9}{5}$   
 i.e.,  $x - y = 5k$   
 Here  $k$  has the only possible value,  $k = 1$ .  
 Because the difference of two single-digit numbers will always be of a single digit.
- 29. (d)** Let the number be  $x$ .  
 Then,  $x - \frac{x}{3} = 48$   
 $\therefore \frac{2}{3}x = 48$ .
- 30. (d)** Let the three consecutive numbers be  $x, x + 1$  and  $x + 2$  respectively.  
 $\therefore$  Difference between first and third number  
 $= x + 2 - x = 2$ .

- 31. (e)** Let the age of Mr Manoj be  $(10x + y)$  years.  
 $\therefore$  His wife's age  $= (10y + x)$  years  
 Then,  $(10x + y + 10y + x) \frac{1}{11} = 10x + y - 10y - x$   
 or,  $x + y = 9x - 9y$  or,  $8x = 10y$   
 or,  $\frac{x}{y} = \frac{5}{4}$   
 $\therefore x = 5$  and  $y = 4$   
 $[\therefore$  any other multiple of 5 will make  $x$  of two digits]  
 $\therefore$  Difference  $= 10x + y - 10y - x = 9x - 9y$   
 $= 9(x - y) = 9(5 - 4) = 9$  years.
- 32. (d)** Let the number be  $x$   
 $\therefore \frac{x-4}{6} = 9 \Rightarrow x = 58$   
 Again,  $\frac{x-3}{5} = \frac{58-3}{5} = \frac{55}{5} = 11$ .
- 33. (a)** Let the two digit number be  $10x + y$   
 $10x + y = 7(x + y) \Rightarrow x = 2y$  ... (1)  
 $10(x + 2) + y + 2 = 6(x + y + 4) + 4$   
 or,  $10x + y + 22 = 6x + 6y + 28$   
 $\Rightarrow 4x - 5y = 6$  ... (2)  
 Solving equations (1) and (2), we get  $x = 4, y = 2$ .
- 34. (b)**  $9A^2 = 12A + 96 \Rightarrow 3A^2 - 4A - 32 = 0$   
 $\therefore A = \frac{4 \pm \sqrt{16 + 384}}{6} = 4, \frac{-8}{3}$   
 $B^2 = 2B + 3 \Rightarrow B^2 - 2B - 3 = 0$   
 $\therefore B = \frac{2 \pm \sqrt{4 + 12}}{2} = 3, -1$   
 $\therefore 5A + 7B = 5 \times 4 + 7 \times 3 = 20 + 21 = 41$ .
- 35. (a)** Let  $\frac{1}{2}$  of the number  $= 10x + y$  and the number  $= 10V + W$   
 From the given conditions,  
 $W = x$  and  $V = y - 1$   
 Thus, the number  $= 10(y - 1) + x$  ... (1)  
 $\therefore 2(10x + y) = 10(y - 1) + x$   
 $\Rightarrow 8y - 19x = 10$  ... (2)  
 Again, from the question,  
 $V + W = 7 \Rightarrow y - 1 + x = 7$   
 $\therefore x + y = 8$  ... (3)  
 Solving equations (2) and (3), we get  
 $x = 2$  and  $y = 6$   
 $\therefore$  From equation (1)  
 Number  $= 10(y - 1) + x = 52$ .

## 1.22 Chapter I

36. (d) Let the number be 'x'. Then,

$$\frac{2}{5} \times \frac{1}{3} \times \frac{3}{7} \times x = 15$$

$$\text{or, } \frac{2x}{35} = 15 \quad \text{or, } x = \frac{15 \times 35}{2}$$

$$\therefore 40\% \text{ of } x = \frac{2}{5} \times \frac{15 \times 35}{2} = 105.$$

37. (a) Required number =  $\frac{3167 + 4093}{145} = \frac{7260}{145} \approx 50.$

38. (a) Quotient is not given. Hence, remainder cannot be determined.

39. (a) Let the positive integer be x.

$$\text{Now, } x^2 - 20x = 96$$

$$\text{or, } x^2 - 20x - 96 = 0$$

$$\text{or, } x^2 - 24x + 4x - 96 = 0$$

$$\text{or, } x(x - 24) + 4(x - 24) = 0$$

$$\text{or, } (x - 24)(x + 4) = 0$$

$$\text{or, } x = 24, -4.$$

40. (d) Let the two consecutive numbers = x and x + 2 then sum of their square =  $x^2 + (x + 2)^2 = 6500$

$$\Rightarrow x^2 + x^2 + 4x + 4 = 6500$$

$$\Rightarrow 2x^2 + 4x - 6496 = 0$$

$$\Rightarrow x^2 + 2x - 3248 = 0$$

$$\Rightarrow x^2 + 58x - 56x - 3248 = 0$$

$$\Rightarrow x(x + 58) - 56(x + 58) = 0$$

$$(x + 58)(x - 56) = 0$$

$$\therefore x = 56 \text{ or, } -58$$

41. (d) Let the first number is x than five consecutive even numbers are x, x + 2, x + 4, x + 6, x + 8

According to question,

$$x + x + 2 + x + 4 + x + 6 + x + 8 = 220$$

$$\Rightarrow 5x + 20 = 220$$

$$\Rightarrow 5x = 220 - 20$$

$$\Rightarrow x = \frac{200}{5} = 40$$

Again, suppose different set of five consecutive even number's second lowest number = y + 2 which is 37 less than double of the lowest number of Set A

$$= 40 \times 2 - 37 = 43$$

$$\therefore \text{First lowest number} = 43 - 1 = 42$$

$$\text{and, Sum} = 42 + 43 + 44 + 45 + 46 = 220$$

42. (b) Suppose the numbers are x and x + 2

$$\therefore x(x + 2) = 9408$$

$$\Rightarrow x^2 + 2x - 9408 = 0$$

$$\Rightarrow x^2 + 98x - 96x - 9408 = 0$$

$$\Rightarrow x(x + 98) - 96(x + 98) = 0$$

$$\Rightarrow (x - 96)(x + 98) = 0$$

$$\Rightarrow x = 96$$

$$\therefore \text{Largest number} = x + 2 = 98$$

43. (d) Suppose the greater number is x. Then,

$$x(x - 2) = 6888$$

$$\Rightarrow x^2 - 2x - 6888 = 0$$

$$\Rightarrow x^2 - 84x + 82x - 6888 = 0$$

$$\Rightarrow x(x - 84) + 82(x - 84) = 0$$

$$\Rightarrow (x - 84)(x + 82) = 0$$

$$\therefore x = 84 \text{ or } -82$$

So, the greater number is 84.

44. (b) Suppose the number is  $10x + y$

$$\therefore (10x + y) - (10y + x) = 63$$

$$\Rightarrow 9x - 9y = 63$$

$$\Rightarrow x - y = 7$$

$$\text{and, } x + y = 11$$

$$\therefore x = 9$$

$$y = 2$$

So, required number = 92

45. (d) Let the number be  $(10x + y)$ , then

$$10x + y + 10y + x = 11x + 11y$$

$$= 11(x + y)$$

Hence, resultant number will be divisible by 11.

46. (a)  $2^1 = 2$ ;  $2^2 = 4$ ;  $2^3 = 8$ ;  $2^4 = 16$ ;  $2^5 = 32$  i.e., The digits at unit's place repeats itself after power 4.

On dividing 33 by 4, we get 1 as remainder.

Therefore, digit at unit place in the product of  $2 = 2$

Hence, remainder on division by 10 = 2.

47. (d)  $7^1 = 7$ ;  $7^2 = 49$ ;  $7^3 = 343$ ;  $7^4 = 2401$ ;  $7^5 = 16809$  i.e., The digit at unit place repeats itself after power 4. On dividing 343 by 4, we get 1 as remainder.

Therefore, unit's digit in the product of  $(4387)^{245} = (621)^{72} =$  unit's digits in the product of  $(4387)^{245} \times (621)^{72} =$  unit's digits in the product of  $(4387)^1 \times (621)^{72} = 7 \times 1 = 7.$

48. (d) The sum of two odd numbers is even. The product of two odd numbers is also even. Therefore,  $a + b + 2ab =$  Even number.

49. (c)  $2^{16} - 1 = (2^8)^2 - 1$   
 $= (2^8 + 1)(2^8 - 1)$   
 $= (256 + 1)(256 - 1)$   
 $= 257 \times 255 = 65535$

which is exactly divisible by 17.

50. (c) Let the numbers be x and y

$$\text{Given, } x + y = 24$$

$$\text{and, } xy = 143$$

$$\text{So, } x^2 + y^2 = (x + y)^2 - 2xy$$

$$= (24)^2 - 2 \times 143$$

$$= 576 - 286 = 290$$

51. (d)  $4^1 = 4$ ;  $4^2 = 16$ ;  $4^3 = 64$ ;  $4^4 = 256$ ;  $4^5 = 1024$

On dividing 372 by 4, the remainder = 0.

On dividing 373 by 4, the remainder = 1.

So, required unit digit

$$= \text{unit's digit of the sum of } 6 + 4 = 0$$

52. (a) Let the numbers be  $x$  and  $y$ .

$$\therefore (x)(x+y) = 247 \quad \dots(1)$$

$$\text{and, } (y)(x+y) = 114 \quad \dots(2)$$

On adding (1) and (2), we get

$$x^2 + xy + xy + y^2 = 361$$

$$\Rightarrow (x+y)^2 = 361$$

$$\Rightarrow x+y = 19$$

Hence, the sum of numbers is 19.

53. (a) Let the number be  $x$ .

$$\Rightarrow \frac{1}{7}x - \frac{1}{11}x = 100$$

$$\Rightarrow \frac{11x - 7x}{77} = 100$$

$$\Rightarrow 4x = 7700$$

$$\Rightarrow x = 1925.$$

54. (b)  $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}} = ?$

$$\text{Let } x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$$

$$\Rightarrow x = \sqrt{6 + x}$$

On squaring both the sides,

$$x^2 = 6 + x$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow x = 3, -2$$

Since sum of positive integers cannot be negative, so ignore  $-2$ .

55. (d)  $\begin{array}{r} 5 \quad A \quad 7 \\ 3 \quad 3 \quad 5 \\ \hline 8 \quad B \quad 2 \end{array}$

$8B2$  is exactly divisible by 3.

$$\Rightarrow 8 + B + 2 = \text{multiple of } 3$$

$$\therefore B = 2 \text{ or } 5 \text{ or } 8$$

$$\text{Now, } A + 1 + 3 = 8$$

$$\therefore A = 4.$$

56. (c)  $\frac{303375}{25} = \frac{303375 \times 4}{25 \times 4} = \frac{2113500}{100} = 12135$

On multiplying other numbers by 4, the digits at units and tens places will not be zero.

57. (d) Units digit in  $3 \times 38 \times 537 \times 1256$

$$= \text{Units digit in } 3 \times 8 \times 7 \times 6 = 4 \times 2 = 8$$

58. (a)  $7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, 7^5 = 16807, \dots$

The units digit repeats itself after index 4.

Units digit in expansion of  $(57)^{25}$

$$= \text{Units digit in } (57)^1 = 7$$

$$\therefore \text{The required units digit} = 7 - 1 = 6.$$

59. (c) Number  $(N) = 100x + 10z + y$

$$\therefore N - 100x - y$$

$$= 100x + 10z + y - 100x - y$$

$$= 10z$$

60. (b) Required remainder = Remainder obtained on dividing the given remainder by 4 = 2.

**Illustration:** If 19 is divided by 4, remainder = 3

If 38 is divided by 4, remainder = 2.

61. (a)  $m^2 - n^2 = (m - n)(m + n)$

Since  $(m - n)$  is an even number,  $(m + n)$  will also be an even number.

We know that product of two even numbers will always be divisible by 4.

$$[(m - n) \times (m + n) = (2 \times 2)(\dots) = 4(\dots)]$$

62. (d) Since divisor =  $a$  (given)

$$\text{Quotient} = \frac{a}{4}$$

$$\text{Remainder} = \frac{a}{2}$$

$$\therefore \text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\Rightarrow b = \frac{a \times a}{4} + \frac{a}{2} = \frac{a^2 + 2a}{4} = \frac{a(a+2)}{4}$$

$$\Rightarrow 4b = a(a+2)$$

$$\Rightarrow \frac{a(a+2)}{b} = 4.$$

63. (c) A number is exactly divisible by 11, if the difference between the sums of digits at even and odd places be either zero or a multiple of 11.

$$\therefore (A + A + 3) - (6 + 8 + 7) = 0$$

$$\Rightarrow 2A + 3 = 21$$

$$\Rightarrow 2A = 21 - 3 = 18$$

$$\therefore A = \frac{18}{2} = 9$$

64. (c) Let the numbers be  $a$  and  $b$ .

$$\therefore ab = 1575 \text{ and } \frac{a}{b} = \frac{9}{7}$$

$$\therefore ab \times \frac{a}{b} = 1575 \times \frac{9}{7}$$

$$\Rightarrow a^2 = 2025 \Rightarrow a = \sqrt{2025} = 45$$

$$\therefore ab = 1575 \Rightarrow b = \frac{1575}{a} = \frac{1575}{45} = 35$$

$$\therefore a + b = 45 + 35 = 80.$$

65. (c) Remainder when  $(a - 1)^n$  is divided by  $a = (-1)^n$

$$\therefore 67^{67} + 67 = (68 - 1)^{67} \text{ is divided by } 68 = (-1)^{67} = -1$$

$$\therefore \text{Required remainder} = -1 + 67 = 66.$$

66. (c) Expression =  $3011 \times 3012 = 3011(3011 + 1) = (3011)^2 + 3011$

$$\therefore \text{Required answer} = 3011.$$

## 1.24 Chapter 1

67. (a) Let the number be  $x$ . Then,

$$\begin{aligned} x &= 296k + 75 \\ &= (37 \times 8)k + (37 \times 2) + 1 \\ &= 37 \times 8k + 37 \times 2 + 1 \\ &= 37 \times (8k + 2) + 1 \end{aligned}$$

$\therefore$  On dividing by 37 the remainder will be 1.

**Quicker Method:**

Required remainder when  $75 \div 37 = 1$

68. (c) Let the numbers be  $a$  and  $b$ .

$$\begin{aligned} \therefore \text{Sum of reciprocals} &= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} \\ &= \frac{\text{Sum}}{\text{Product}} = \frac{12}{35} \end{aligned}$$

69. (c) Since, 5 is the greatest number among 2, 3, 4 and 5.

Therefore,  $\sqrt{5}$  is the greatest number among  $\sqrt{5}$ ,  $\sqrt[3]{4}$ ,  $\sqrt[5]{2}$ ,  $\sqrt[7]{3}$ .

70. (c) Let the numbers be  $x$  and  $y$ .

$$x + y = 7a; x - y = 1a \text{ and } xy = 24a$$

On solving, we get

$$x = 4a$$

$$y = 3a$$

$$\therefore xy = 12a^2 \Rightarrow 12a^2 = 24a \Rightarrow a = 2$$

$$\therefore \text{Required product} = 24 \times 2 = 48$$

71. (c) Total number of items = 32

Maximum number of ice creams = 9

$$\text{So, } \begin{array}{ccc} 13 & 10 & 9 \\ 12 & 11 & 9 \end{array}$$

Hence number of cookies is either 10 or 11.

Number of pastries is either 13 or 12.

72. (a) Let the three consecutive even numbers be  $2x$ ,  $2x + 2$  and  $2x + 4$ .

$$\text{Then, } (2x)(2x + 2)(2x + 4) = 4032 \quad \dots(1)$$

$$\text{Again, product of first and third number} = 2x \times (2x + 4) = 252 \quad \dots(2)$$

Putting the values of the product of first and third number in equation (1), we have,

$$(2x + 2) \times 252 = 4032$$

$$\text{or, } 2x + 2 = \frac{4032}{252} = 16 \quad \text{or, } 2x = 16 - 2 = 14$$

$$\therefore x = 7$$

Hence, first number =  $7 \times 8 = 14$

$$\text{Second number} = 7 \times 2 + 2 = 16$$

$$\text{and third number} = 7 \times 2 + 4 = 18$$

$$\text{Five times of second number} = 5 \times 16 = 80.$$

73. (e) Average of the nine consecutive odd numbers

$$= \frac{621}{9} = 69$$

So, 69 is the middle number or 5th largest number of Set A.

$$\therefore \text{Smallest number of Set A will be } 69 - 8 = 61$$

$$\text{Lowest number of the set with even numbers} = 61 + 15 = 76$$

Sum of the six consecutive even numbers starting with 76

$$= \frac{6}{2} [(76 \times 2) + (6 - 1) \times 2] = 3[152 + 10] = 486$$

74. (e) Let the first number be  $x$ .

According to the question,

$$x + x + 2 + x + 4 + x + 6 + x + 8 = 220$$

$$\Rightarrow 5x = 220 - 20 = 200 \Rightarrow x = 40$$

$$\text{Second lowest number of Set B} = 40 \times 2 - 37 = 43$$

$$\text{Required sum} = 42 + 43 + 44 + 45 + 46 = 220$$

75. (a) Let the first number be  $f$  and second number  $s$ .

$$\text{Then, } 2f + 3s = 100 \quad \dots(1)$$

$$3f + 2s = 120 \quad \dots(2)$$

Solving (1) and (2), we get

$$F = 32, s = 12.$$

76. (e) Let  $x$  be at the tens place and  $y$  at the units place.

$$\text{Then, } 9(x - y) = 54$$

$$\Rightarrow x - y = 6 \quad \dots(1)$$

$$\text{Also, } x + y = 12 \quad \dots(2)$$

Solving these two equations, we get

$$x = 9 \text{ and } y = 3$$

$$\text{The number is } 10 \times 9 + 3 = 93$$

77. (d)  $9(x - y) = 9 \Rightarrow x - y = 1 \quad \dots(1)$

$$\text{Also, } x + y = 1 \quad \dots(2)$$

$x \rightarrow$  digit at tens place of the number

$y \rightarrow$  digit at units place of the number

Solving these equations,

we get  $x = 8$  and  $y = 7$

Now, the number may be either 87 or 78.

So, our best option is (d) Cannot be determined.

78. (c)  $10x + y - 10y - x = 9$

$$\Rightarrow 9(x - y) = 9 \Rightarrow x - y = 1.$$