# Linear Equations

# 27

## LINEAR EQUATION IN ONE VARIABLE

A linear equation in one variable is an equation of the type $ax + b = 0$ or, $ax = c$, where $a$, $b$, $c$ are constants (real numbers), $a \neq 0$ and $x$ is an unknown variable.

The solution of the linear equation $ax + b = 0$ is $x = -\frac{b}{a}$. We also say that $-\frac{b}{a}$ is the root of the linear equation $ax + b = 0$.

For example, the equation $2x + 3 = 0$ is a linear equation in one unknown variable $x$. Its solution or root is $-\frac{3}{2}$.

## LINEAR EQUATION IN TWO VARIABLES

A linear equation in two variables is an equation of the type $ax + by + c = 0$ or $ax + by = d$, where $a$, $b$, $c$ and $d$ are constants, $a \neq 0$, $b \neq 0$.

For example, $3x + 4y + 7 = 0$ and $2x - 3y = 5$ are linear equations in two variables $x$ and $y$.

**Methods of Solving Two Simultaneous Linear Equations**

**1. Method of Substitution**

**Step 1.** Find the value of one variable, say $y$, in terms of the other, i.e., $x$ from either equation.

**Step 2.** Substitute the value of $y$ so obtained in the other equation. Thus, we get an equation in only one variable $x$.

**Step 3.** Solve this equation for $x$.

**Step 4.** Substitute the value of $x$, thus obtained, in step 1 and find the value of $y$.

**Illustration 1:** Solve $2x + 3y = 7$, $3x - y = 5$.

**Solution:** The given equations are

$$x + y = 7 \qquad \ldots(1)$$

and, $3x - 2y = 11$ ...(2)

From Equation (1), we get $y = 7 - x$.

Substituting $y = 7 - x$ in Equation (2), we get

$$3x - 2(7 - x) = 11 \Rightarrow 3x - 14 + 2x = 11$$
$$\Rightarrow 5x = 25 \Rightarrow x = 5.$$

Substituting this value of $x$ in Equation (1), we get $5 + y = 7 \Rightarrow y = 7 - 5$ or $y = 2$.

Hence, $x = 5$, $y = 2$ is the required solution.

**2. Method of Elimination**

**Step 1.** Multiply both the equations by such numbers so as to make the coefficients of one of the two unknowns numerically the same.

**Step 2.** Add or subtract the two questions to get an equation containing only one unknown. Solve this equation to get the value of the unknown.

**Step 3.** Substitute the value of the unknown in either of the two original equations. By solving that the value of the other unknown is obtained.

**Illustration 2:** Solve: $-6x + 5y = 2$, $-5x + 6y = 9$.

**Solution:** The given equations are

$$-6x + 5y = 2 \qquad \ldots(1)$$
$$-5x + 6y = 9 \qquad \ldots(2)$$

Multiplying Equation (1) by 6,

$$-36x + 30y = 12 \qquad \ldots(3)$$

Multiplying Equation (2) by 5,

$$-25x + 30y = 45 \qquad \ldots(4)$$

Subtracting Equation (4) from Equation (3), we get

$$-11x = -33 \text{ or } x = 3.$$

Substituing $x = 3$ in Equation (1), we get

$$-18 + 5y = 2 \text{ or } y = 4.$$

Hence, $x = 3$ and $y = 4$ is the required solution.

**3. Short-cut Method**

Let, the two equations be

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2.$$

The solution is written as

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

i.e., $x = -\dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, $y = -\dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$.

**Illustration 3:** Solve $3x + 2y = -25$, $-2x - y = 10$.

**Solution:** The two equations are

$$3x + 2y = -25$$
$$-2x - y = 10$$

The solution is given by

$$\frac{x}{2\times 10-(-1)\times(-25)} = \frac{y}{(-25)\times(-2)-3\times 10} = \frac{-1}{3\times(-1)-(-2)\times 2}$$

or, $\dfrac{x}{-5} = \dfrac{y}{20} = \dfrac{-1}{1}$

or, $x = 5$, $y = -20$.

## Consistent and Inconsistent Equations

When a system of equations has a solution, the system is called consistent. And when a system of equations has no solution, the system is called inconsistent.

**Test for Consistency**

If we are given two linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$. Then,

(*a*) If $\dfrac{a_1}{a_2} \ne \dfrac{b_1}{b_2}$, the system will have exactly one solution and will be consistent.

**Note:**

The graphs of such equations will have intersecting lines.

(*b*) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$, the system is consistent and has infinitely many solutions.

**Note:**

The graphs of such equations will have coincident lines.

(*c*) If $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \ne \dfrac{c_1}{c_2}$, the system has no solution and is inconsistent.

**Note:**

The graphs of such equations will have parallel lines.

**Illustration 4:** For what values of $k$, will the system of equations $kx + 2y = 5$ and $3x + y = 1$ have a unique solution?

**Solution:** If the given system of equations has a unique solution,

$$\frac{a_1}{a_2} \ne \frac{b_1}{b_2} \Rightarrow \frac{k}{3} \ne \frac{2}{1} \Rightarrow k \ne 6.$$

Hence, for $k \ne 6$, the given system of equations will have a unique solution.

**Illustration 5:** For what value of $k$, the system of equations $3x + 4y = 6$ and $6x + 8y = k$ represent, coincident lines?

**Solution:** If the given system of equations represents coincident lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{6} = \frac{4}{8} = \frac{6}{k}$$
$$\Rightarrow k = \frac{6\times 8}{4} = 12.$$

**Illustration 6:** For what value of $k$ the equations $9x + 4y = 9$ and $7x + ky = 5$ have no solution?

**Solution:** The given system of equations will have no solution

if, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \ne \dfrac{c_1}{c_2} \Rightarrow \dfrac{9}{7} = \dfrac{4}{k} \ne \dfrac{9}{5}$

$\Rightarrow 9k = 28$ or $k = \dfrac{28}{9}$.

## EXERCISE-I

1. Solve: $x + y = 3$, $2x + 3y = 7$.
(a) $x = 2, y = 1$ (b) $x = 1, y = 2$
(c) $x = 3, y = 2$ (d) None of these

2. Solve: $x + y = 7$, $3x - 2y = 11$
(a) $x = 7, y = 3$ (b) $x = 5, y = 2$
(c) $x = 5, y = 3$ (d) None of these

3. Solve: $7x + 11y = 1$, $8x + 13y = 2$
(a) $x = -5, y = 3$ (b) $x = -7, y = 2$
(c) $x = -3, y = 2$ (d) None of these

4. Solve: $8u - 3v = 5uv$ $6u - 5v = -2uv$
(a) $u = \frac{13}{23}, v = \frac{22}{33}$ (b) $u = \frac{11}{5}, v = \frac{13}{22}$
(c) $u = \frac{11}{23}, v = \frac{22}{31}$ (d) None of these

5. Solve: $2(3u - v) = 5uv$, $2(u + 3v) = 5uv$
(a) $u = 2, v = 1$ (b) $u = 3, v = 2$
(c) $u = 4, v = 3$ (d) None of these

6. Solve: $ax + by = a - b$, $bx - ay = a + b$
(a) $x = 2, y = -1$ (b) $x = -2, y = 1$
(c) $x = 1, y = -1$ (d) None of these

7. Solve for $x$ and $y$: $\frac{2x}{a} + \frac{y}{b} = 2, \frac{x}{a} - \frac{y}{b} = 4.$
(a) $x = 2a, y = -2b$ (b) $x = 3a, y = -3b$
(c) $x = 3a, y = -2b$ (d) None of these

8. Given: $4x + \frac{6}{y} = 15$ and $6x - \frac{8}{y} = 14$. Find '$p$' if, $y = px - 2$.
(a) $\frac{5}{3}$ (b) $\frac{7}{3}$
(c) $\frac{4}{3}$ (d) None of these

9. Given: $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$ and $\frac{3}{x} + \frac{2}{y} = 0$. Find '$a$' for which $y = ax - 4$.
(a) 1 (b) 0
(c) 2 (d) None of these

10. If $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$ and $\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$, where $x \neq 0$, $y \neq 0$, the values of $x$ and $y$ are, respectively
(a) 0 and 1 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

11. The number of solutions of the equations $x + \frac{1}{y} = 2$ and $2xy - 3y = -2$ is:
(a) 0 (b) 1
(c) 2 (d) None of these

12. The equations $ax + b = 0$ and $cx + d = 0$ are consistent, if:
(a) $ad = bc$ (b) $ad + bc = 0$
(c) $ab - cd = 0$ (d) $ab + cd = 0$

13. The solution to the system of equations $|x + y| = 1$ and $x - y = 0$ is given by:
(a) $x = y = \frac{1}{2}$ (b) $x = y = -\frac{1}{2}$
(c) $x = 1, y = 0$ (d) $x = y = \frac{1}{2}$ or $x = y = \frac{-1}{2}$

14. In the system of the equations $\frac{1}{x} + \frac{1}{y} = \frac{5}{6}$, $\frac{1}{y} + \frac{1}{z} = \frac{7}{12}$ and $\frac{1}{z} + \frac{1}{x} = \frac{3}{4}$, values of $x$, $y$ and $z$ will be:
(a) 4, 3 and 2 (b) 3, 2 and 4
(c) 2, 3 and 4 (d) 3, 4 and 2

15. If $2x + y = 35$ and $3x + 4y = 65$, then find the value of $\frac{x}{y}$.
(a) 2 (b) 1
(c) 3 (d) None of these

16. I am three times as old as my son. 5 years later, I shall be two and a half times as old as my son. How old am I and how old is my son?
(a) 45 years, 15 years (b) 40 years, 10 years
(c) 60 years, 25 years (d) 50 years, 20 years

17. A man has some hens and cows. If the number of heads be 48 and number of feet equals 140, then the number of hens will be:
(a) 26 (b) 24
(c) 23 (d) 22

18. ₹49 were divided among 150 children. Each girl got 50 paise and a boy 25 paise. How many boys were there?
(a) 100 (b) 102
(c) 104 (d) 105

**19.** Find the condition for the following system of linear equations to have a unique solution:
$ax + by = c$, $lx + my = n$.

(a) $an \neq cl$ (b) $am \neq bl$
(c) $bm \neq al$ (d) None of these

**20.** Find the value of $k$ for which the system:
$kx + 2y = 5$, $3x + y = 1$ has a unique solution.

(a) $k = 6$ (b) $k \neq 9$
(c) $k \neq 6$ (d) None of these

**21.** Find the value of $c$ for which the system
$cx + 3y = c - 3$, $12x + cy = c$ has infinitely many solutions.

(a) 6 (b) 8
(c) 4 (d) None of these

**22.** Find the value of $k$ for which the system of equations $2x + 2y = 5$, $3x + ky = 7$ has no solution.

(a) 5 (b) 7
(c) 3 (d) 9

**23.** Find the value of $k$ for which the system of equations $2x + ky = 1$, $3x - y = 7$ has a unique solution.

(a) $k = -\frac{2}{3}$ (b) $k \neq \frac{2}{3}$

(c) $k \neq -\frac{2}{3}$ (d) $k = \frac{2}{3}$

## EXERCISE-2
## (BASED ON MEMORY)

**1.** An amount of money is to be divided among $P$, $Q$ and $R$ in the ratio of 3:5:7 respectively. If the amount received by $R$ is ₹4000 more than the amount received by $Q$, what will be the total amount received by $P$ and $Q$ together?

(a) ₹8000 (b) ₹12000
(c) ₹16000 (d) Cannot be determined

**[Allahabad Bank PO, 2010]**

**2.** The total marks obtained by a student in Physics, Chemistry and Mathematics together is 120 more than the marks obtained by him in Chemistry. What are the average marks obtained by him in Physics and Mathematics together?

(a) 60 (b) 120
(c) 40 (d) Cannot be determined

**[Allahabad Bank PO, 2010]**

**3.** Deepak has some hens and some goats. If the total number of aminal heads is 90 and the total number of animal feet is 248, what is the total number of goats Deepak has?

(a) 32 (b) 36
(c) 34 (d) Cannot be determined

**[Punjab National Bank PO, 2010]**

**4.** The sum of the 2 digits of a numbers is 15 and the difference between them is 3. What is the product of the 2 digits of the 2 digits number?

(a) 56 (b) 63
(c) 42 (d) None of these

**[Punjab National Bank PO, 2010]**

**5.** If $2x + 3y = 78$ and $3x + 2y = 72$, what is the value of $x + y$?

(a) 36 (b) 32
(c) 30 (d) Cannot be determined

**[Punjab National Bank PO, 2010]**

**6.** There are some parrots and some tigers in a forest. If the total number of animal heads in the forest are 858 and total number of animal legs are 1746, what is the number of parrots in the forest?

(a) 845 (b) 833
(c) 800 (d) None of these

**[Corporation Bank PO, 2010]**

**7.** There are 2 numbers such that the sum of twice the first number and thrice the second number is 100 and the sum of thrice the first number and twice the second number is 120. Which is the larger number?

(a) 32 (b) 12
(c) 14 (d) 35

**[Corporation Bank PO, 2010]**

**8.** The cost of 8 pens and $r$ pencils are ₹176 and the cost of 2 pens and 2 pencils is ₹48. What is the cost of 1 pen?

(a) ₹16 (b) ₹14
(c) ₹12 (d) None of these

**[Andhra Bank PO, 2009]**

**9.** Rubina could get equal number of ₹55, ₹85 and ₹105 tickets for a movie. She spents 2940 for all the tickets. How many of each did she buy?

(a) 12 (b) 14
(c) 16 (d) Cannot be determined

**[IBPS Bank PO, 2011]**

**10.** The difference between a 2 digit number and the number obtained by interchanging the 2 digits of the number is 9. If the sum of the 2 digits of the number is 15, then what is the original number?

(a) 89 (b) 67
(c) 87 (d) Cannot be determined

**[OBC PO, 2009]**

**11.** If $3Y + 9X = 54$ and $\frac{28X}{13Y} = \frac{140}{39}$, then what is the value of $Y - X$?

(a) −1 (b) −2
(c) 2 (d) 1

**[IOB PO, 2009]**

**12.** On a School's Annual Day sweets were to be equally distributed amongst 112 children. But on that particular day, 32 children were absent. Thus the remaining children got 6 extra sweets each. How many sweets was each child originally supposed to get?

(a) 24 (b) 18
(c) 15 (d) Cannot be determined

**[IOB PO, 2009]**

**13.** The difference between a 2-digit number and the number obtained by interchanging the 2 digits of the number is 9. What is the difference between the 2 digits of the number?

(a) 3 (b) 2
(c) 1 (d) Cannot be determined

**[NABARD Bank PO, 2009]**

**14.** Swapana spent ₹44620 on Deepawali Shooping, ₹32764 on buying Laptop and the remaining 32% of the total amount she had as cash with her. What was the total amount?

(a) ₹36416 (b) ₹113800
(c) ₹77384 (d) Cannot be determined

**[SBI PO, 2008]**

**15.** Rohit has some 50 paise coins, some ₹2 coins, some ₹1 coins and some ₹5 coins. The value of all coins is ₹50. Number of ₹2 coins is 5 more than the ₹5 coins. 50 paise coins are double in number than ₹1 coin. Value of 50 paise coins and ₹1 coins is ₹26. How many ₹2 coins does he have?

(a) 4 (b) 2
(c) 7 (d) Cannot be determined

**[Union Bank of India PO, 2011]**

**16.** The cost of 5 chairs and 3 tables is ₹3110. Cost of 1 chair is ₹210 less than the cost of 1 table. What is the cost of 2 tables and 2 chairs?

(a) ₹1660 (b) ₹1860
(c) ₹2600 (d) Cannot be determined

**[Bank of Baroda PO Examination, 2011]**

**17.** In a family of husband, wife and a daughter, the sum of the husband's age, twice the wife's age and thrice the daughters age is 85; while the sum of twice the husband's age, 4 times the wife's age and 6 times the daughter's age is 170. It is also given that the sum of 5 times the husband's age, ten times the wife's age and 15 times the daughter's age equals 450. The number of possible solutions, in terms of the ages of the husband, wife and the daughter, to this problem is:

(a) 0 (b) 1
(c) 2 (d) infinitely many

**18.** The number obtained by interchanging the 2 digits of a 2 digit number is lesser than the original number by 54. If the sum of the 2 digits of the number is 12, then what is the original number?

(a) 28 (b) 39
(c) 82 (d) None of these

**[IDBI PO, 2009]**

**19.** The sum of twice of a number and thrice of 42 is 238. What will be the sum of thrice of that number and twice of 42?

(a) 245 (b) 250
(c) 264 (d) 252

**[Syndicate Bank PO, 2010]**

**20.** A students was asked to divide a number by 6 and add 12 to the quotient. He, however, first added 12 to the number and then divided it by 6, gets 112 as the answer. The correct answer should have been:

(a) 124 (b) 122
(c) 118 (d) 114

**[SSC (GL), 2011]**

**21.** If $4x+5y=83$ and $3x:2y=21:22$, then $(y - x)$ equals:

(a) 3 (b) 4
(c) 7 (d) 11

**[SSC, 2014]**

**22.** The sum of 2 numbers is equal to 20 and their difference is 25. The ratio of the two numbers is:

(a) 9:1 (b) 7:9
(c) 3:5 (d) 2:7

**[SSC, 2014]**

**23.** The value of $k$ for which the graphs of $(k-1)x + y - 2 = 0$ and $(2-k)x - 3y + 1 = 0$ are parallel is:

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) 2 (d) −2

**[SSC, 2011]**

**24.** The graphs of $x + 2y = 3$ and $3x - 2y = 1$ meet the $y$-axis at two points having distance:

(a) $\frac{8}{3}$ units (b) $\frac{4}{3}$ units
(c) 1 units (d) 2 units

**[SSC, 2011]**

## ANSWER KEYS

### Exercise-1

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** (a) | **2.** (b) | **3.** (c) | **4.** (c) | **5.** (a) | **6.** (c) | **7.** (a) | **8.** (c) | **9.** (b) | **10.** (d) | **11.** (a) | **12.** (a) | **13.** (d) |
| **14.** (b) | **15.** (c) | **16.** (a) | **17.** (a) | **18.** (c) | **19.** (b) | **20.** (c) | **21.** (a) | **22.** (c) | **23.** (c) | | | |

### Exercise-2

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** (c) | **2.** (a) | **3.** (c) | **4.** (d) | **5.** (c) | **6.** (d) | **7.** (a) | **8.** (d) | **9.** (a) | **10.** (c) | **11.** (b) | **12.** (c) | **13.** (c) |
| **14.** (b) | **15.** (c) | **16.** (a) | **17.** (a) | **18.** (d) | **19.** (d) | **20.** (b) | **21.** (b) | **22.** (a) | **23.** (a) | **24.** (d) | | |



## EXPLANATORY ANSWERS

### Exercise-1

**1.** **(a)** $x + y = 3$ …(1)

$2x + 3y = 7$ …(2)

$y = 3 - x$ [From Equation (1)];

$\therefore\ 2x + 3(3 - x) = 7 \Rightarrow 2x + 9 - 3x = 7$

$\Rightarrow\ -x = -2$, i.e., $x = 2$

and, $y = 3 - 2 = 1$.

$\therefore\ x = 2$ and $y = 1$.

**2.** **(b)** $x + y = 7$ …(1)

$3x - 2y = 11$ …(2)

$y = 7 - x$ [From Equation (1)];

$\therefore\ 3x - 2(7 - x) = 11 \Rightarrow 3x - 14 + 2x = 11$

$\Rightarrow\ 5x = 25$, i.e., $x = 5$

$\therefore\ x = 5$ and, $y = 2$.

**3.** **(c)** $7x + 11y = 1$ …(1)

$8x + 13y = 2$ …(2)

Multiplying Equation (1) and Equation (2) by 8 and 7 respectively, we have

$56x + 88y = 8$ and $56x + 91y = 14$

Subtracting the second equation from the first, we get $-3y = -6$, i.e., $y = 2$

From Equation (1), $x = \frac{1-11(y)}{7} = \frac{1-11(2)}{7} = -3$

$\therefore\ x = -3$ and $y = 2$.

**4.** **(c)** $8u - 3v = 5uv$,

$6u - 5v = -2uv$

Dividing both equations by $uv$, we get

$\frac{8}{v} - \frac{3}{u} = 5$ and $\frac{6}{v} - \frac{5}{u} = -2$.

Putting $\frac{1}{v} = x$ and $\frac{1}{u} = y$, we get

$8x - 3y = 5$ …(1)

and, $6x - 5y = -2$ …(2)

Multiplying Equation (1) and Equation (2) by 6 and 8, respectively, we have,

$48x - 18y = 30$ and $48x - 40y = -16$

Subtracting the second equation from the first, we get

$22y = 46$, i.e., $y = -\frac{23}{11}$

From Equation (1), $x = \left(\frac{5+3y}{8}\right) = \frac{\left(5+3\left(\frac{23}{11}\right)\right)}{8} = \frac{31}{22}$

$\therefore \quad u = \frac{1}{y} = \frac{11}{23}$ and $v = \frac{1}{x} = \frac{22}{31}$.

**5.** **(a)** $2(3u - v) = 5uv \Rightarrow \frac{6}{v} - \frac{2}{u} = 5$

$2(u + 3v) = 5uv \Rightarrow \frac{2}{v} + \frac{6}{u} = 5$

Putting $\frac{1}{v} = x$ and $\frac{1}{u} = y$, we get

$6x - 2y = 5$ …(1)

$2x + 6y = -2$ …(2)

Multiplying Equation (1), and Equation (2) by 3 and 1 respectively, we have

$18x - 6y = 15$ and $2x + 6y = 5$

Adding these two equations, we get $20x = 20$, i.e., $x = 1$

From Equation (1), $y = \left(\frac{6x-5}{2}\right) = \left(\frac{6\times 1-5}{2}\right) = \frac{1}{2}$

$\therefore \quad u = \frac{1}{y} = 2$ and $v = \frac{1}{x} = 1$

**6.** **(c)** $ax + by = a - b$ and $bx - ay = a + b$

Here, $\frac{a_1}{a_2} = \frac{a}{b}$ and $\frac{b_1}{b_2} = -\frac{b}{a} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, a unique solution exists.

Writing the coefficients in the following array

$b \quad a - b \qquad a \quad b$

$-a \quad a + b \qquad b \quad -a$

we get, $\frac{x}{b(a+b)+a(a-b)} = \frac{y}{b(a-b)-a(a+b)}$

$= \frac{-1}{-a^2-b^2}$

$\Rightarrow \quad \frac{x}{b^2+a^2} = \frac{y}{-b^2-a^2} = \frac{1}{a^2+b^2}$

$\therefore \quad x = 1$ and $y = -1$.

**7.** **(a)** The given equations are

$2\frac{x}{a} + \frac{y}{b} = 2$ …(1)

$\frac{x}{a} - \frac{y}{b} = 4$ …(2)

Adding (1) and (2), we get

$3\frac{x}{a} = 6 \Rightarrow 3x = 6a \Rightarrow x = \frac{6a}{3} = 2a$

Putting $x = 2a$ in (1), we get

$2\frac{2a}{a} + \frac{y}{b} = 2 \Rightarrow 4 + \frac{y}{b} = 2$

$\Rightarrow \frac{y}{b} = 2 - 4 = -2$

$\Rightarrow y = -2b$

Hence, the required solution is $x = 2a$, $y = -2b$.

**8.** **(c)** We have, $4x + \frac{6}{y} = 15$ …(1)

$6x - \frac{8}{y} = 14$ …(2)

Multiplying equation (1) by 3 and equation (2) by 2, we get

$12x + \frac{18}{y} = 45$ …(3)

$12x - \frac{16}{y} = 28$ …(4)

Subtracting equation (4) from equation (3), we get

$\frac{34}{y} = 17 \Rightarrow y = \frac{34}{17} = 2$

Putting $y = 2$ in equation (1), we get

$4x + \frac{6}{2} = 15 \Rightarrow 4x + 3 = 15 \Rightarrow 4x = 15 - 3$

$\Rightarrow 4x = 12 \Rightarrow x = \frac{12}{14} = 3$

Hence, the solution is $x = 3$, $y = 2$. Now,

$y = px - 2 \Rightarrow 2 \Rightarrow p(3) - 2$

$\Rightarrow 3p = 4 \Rightarrow p = \frac{4}{3}$.

**9.** **(b)** We have, $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$ …(1)

$\frac{3}{x} + \frac{2}{y} = 0$ …(2)

Putting $\frac{1}{x} = u$ and $\frac{1}{v} = u$, we get

$$2u+\frac{2}{3}v=\frac{1}{6} \qquad \ldots(3)$$

$3u + 2v = 0$ …(4)

Multiplying equation (4) by $\frac{1}{3}$, we get

$$u+\frac{2}{3}v=0 \qquad \ldots(5)$$

Subtracting equation (5) from equation (3), we get

$$u=\frac{1}{6}$$

Putting $u=\frac{1}{6}$ in (4), we get

$$3\left(\frac{1}{6}\right)+2v-0 \;\Rightarrow\; \frac{1}{2}+2v-0$$

$$\Rightarrow\; 2v=-\frac{1}{2} \;\Rightarrow\; v=-\frac{1}{4}$$

Now, $u=\frac{1}{6} \;\Rightarrow\; \frac{1}{x}=\frac{1}{6} \;\Rightarrow\; x=6$

and, $v=-\frac{1}{4} \;\Rightarrow\; \frac{1}{y}=-\frac{1}{4} \;\Rightarrow\; y=-4$

Hence, the solution is $x = 6$, $y = -4$. Again,

$y = ax - 4 \;\Rightarrow\; -4 \;\Rightarrow\; a(6) - 4$

$\Rightarrow\; 6a = -4 + 4 \;\Rightarrow\; 6a = 0$

$$\Rightarrow\; a=\frac{0}{6}=0.$$

**10.** **(d)** Multiplying each equation throughout by $xy$, we get
$3x + 2y = 9$ and, $9x + 4y = 21$
On solving these equations, we get
$x = 1$, $y = 3$.

**11.** **(a)** First equation gives $\frac{1}{y}=2-x$ or, $y=\frac{1}{2-x}$

second equation is $y(2x - 3) = -2$

or, $\frac{2x-3}{2}=-2 \quad \therefore \quad 2x - 3 = -2(2 - x)$

or, $2x - 3 = -4 + 2x$

This gives $1 = 0$

This is impossible. So, these is no solution at all.

**12.** **(a)** The equations are consistent if

$\frac{a}{c}=\frac{b}{d}$ i.e., if $ad = bc$.

**13.** **(d)** $|x + y| = 1$, Û $x + y = 1$
or, $-(x + y) = 1$ i.e., $x + y = -1$
Solving $x + y = 1$, $x - y = 0$, we get

$x=\frac{1}{2}$ and $y=\frac{1}{2}$

Solving $x + y = -1$ and $x - y = 0$, we get

$x=-\frac{1}{2}$ and $y=\frac{-1}{2}$

$\therefore \quad x=y=\pm\frac{1}{2}.$

**15.** **(c)** We have, $2x + y = 35$ …(1)
$3x + 4y = 65$ …(2)
From equation (1), $y = 35 - 2x$
Putting this value of y in (2), we get
$3x + 4(35 - 2x) = 65 \;\Rightarrow\; 3x + 140 - 8x = 65$
$\Rightarrow\; -5x = 65 - 140 = -75$

$$\Rightarrow\; x=\frac{-75}{-5}=15$$

Substituting this value of $x$ in $y = 35 - 2x$, we get
$y = 35 - 2(15) = 35 - 30 = 5$

$\therefore \quad \frac{x}{y}=\frac{15}{5}=3.$

**16.** **(a)** Let, that I am $x$ years old and my son is $y$ years old. Then, according to the first condition of the problem,
$x = 3y$ …(1)
Five years later, my age = $(x + 5)$ years and my son's age = $(y + 5)$ years.
Then, according to the second condition of the problem,

$$x+5=2\frac{1}{2}(y+5) \qquad \ldots(2)$$

Putting $x = 3y$ from (1), we get

$$3y+5=2\frac{1}{2}(y+5) \;\Rightarrow\; 3y+5=\frac{5}{2}(y+5)$$

$\Rightarrow\; 6y + 10 = 5(y + 5)$
[Multiplying both sides by 2]
$\Rightarrow\; 6y + 10 = 5y + 25$
$\Rightarrow\; 6y - 5y = 25 - 10$
$\Rightarrow\; y = 15$
Putting $y = 15$ in (1) in (2), we get $x = 3(15) = 45$
Hence, I am 45 years old and my son is 15 years old, at present.

**17.** **(a)** Let, there be $x$ hens and $y$ cows
Then, $x + y = 48$ …(1)
and, $2x + 4y = 140$ …(2)
Solving (1) and (2), we get $x = 26$.

**18.** **(c)** Let, the number of boys be $x$ and the number of girls be $y$
Then, $x + y = 150$

$$\frac{25}{100}x+\frac{50}{100}y=49$$

Solving $x + y = 150$ and $x + 2y = 196$, we get $x = 104$.

**19.** **(b)** we have, $ax + by = c$ …(1)

$lx + my = n$ …(2)

Here, $a_1 = a$, $b_1 = b$, $c_1 = c$

$a_2 = l$, $b_2 = m$, $c_2 = n$.

If the given system of linear equations has a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{a}{l} \neq \frac{b}{m} \Rightarrow am \neq bl$$

**20.** **(c)** we have, $kx + 2y = 5$ …(1)

$3x + y = 1$ …(2)

Here, $a_1 = k$, $b_1 = 2$, $c_1 = 5$

$a_2 = 3$, $b_2 = 1$, $c_2 = 1$

The given system of equations will have a unique solution

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ i.e., if, $\frac{k}{3} \neq \frac{2}{1}$

i.e., if $\frac{k}{3} \neq 2$ i.e., if $k \neq 6$.

**21.** **(a)** For the equation, $cx + 3y = c - 3$ and $12x + cy = c$ to have an infinite number of solutions we must have,

$$\frac{c}{12} = \frac{3}{c} = \frac{(c-3)}{c} \Rightarrow \frac{c}{12} = \frac{3}{c} \text{ and } \frac{c}{12} = \frac{(c-3)}{c}$$

$\Rightarrow$ $c^2 = 36$ and $c^2 = 12c - 36$

$\Rightarrow$ $c = \pm 6$ and $(c - 6)^2 = 0$

$\Rightarrow$ $c = \pm 6$ and $c = 6$

$\Rightarrow$ $c = 6$.

**22.** **(c)** For no solution to exist. we need

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{2}{3} = \frac{2}{k}, \text{ i.e., } k = 3.$$

**23.** **(c)** For a unique solution to exist. we require

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{2}{3} \neq \frac{k}{-1}, \text{ i.e., } k \neq -\frac{2}{3}.$$

## Exercise-2
## (Based on Memory)

**1.** **(c)** Let, $P$, $Q$ and $R$ got ₹$3x$, ₹$5x$ and ₹$7x$.

$7x - 5x = 4000$

$x = 2000$

$P + Q = 3x + 5x = 8x =$ ₹16000.

**2.** **(a)** Let, marks got in physics, chemistry and mathematics are $P$, $C$ and $M$.

$P + C + M = C + 120$

$\therefore$ $P + M = 120$

$\frac{P+M}{2} = 60$.

**3.** **(c)** (H) Hens has one head and two feet.

(G) Goats has one head and four feet.

According to question,

H + G = 90 …(1)

2H + 4G = 248 …(2)

Multiplying by 2 in equation (1) and subtract

$$\begin{array}{r} 2H + 2G = 180 \\ 2H + 4G = 148 \\ - \quad - \quad \\ \hline -2G = -80 \therefore G = 34 \end{array}$$

$\therefore$ Number of goats = 34

Put the value of G in Equation (1),

H + 34 = 90

H = 56

**4.** **(d)** $x + y = 15$ …(1)

$x - y = 3$ …(2)

Add Equation (1) and (2),

$x = 9$, $y = 6$

Product of two digits of the number = 9 × 6 = 54

**5.** **(c)** $2x + 3y = 78$ …(1)

$3x + 2y = 72$ …(2)

Multiplying by 2 in Equation (1) and 3 in Equation (2) and subtract.

$$\begin{array}{r} 4x + 6y = 156 \\ 9x + 6y = 216 \\ - \quad - \quad - \\ \hline -5x = -60 \\ x = 12 \end{array}$$

Put the value of $x$ in Equation (1)

$2 \times 12 + 3y = 78$

$y = 18$

Then $x + y = 12 + 18 = 30$

**6.** **(d)** P + T = 858 …(1)

(Because both have one head)

2P + 4T = 1746 …(2)

(Because parrot has two legs and tiger has four legs) Multiply by 2 in Equation (1) and substract

$$\begin{array}{r} 2P+2T=1716 \\ -2P+4T=1746 \\ -\quad -\quad - \\ \hline -2T=-30 \end{array}$$

$\therefore$ T = 15

P = 843

**7.** **(a)** $2x + 3y = 100$ ...(1)

$3x + 2y = 120$ ...(2)

Multiply by 3 in Equation (1) and multiply by 2 in Equation (2) and then subtracted

$$\begin{array}{r} 6x+9y=300 \\ 6x+4y=240 \\ -\quad -\quad - \\ \hline 5y=60 \end{array}$$

$y = 12$

$x = 32$

**8.** **(d)** Let, the cost of one pen is ₹$x$ and the cost of one pencil is ₹$y$.

$8x + 4y = 176$ ...(1)

$2x + 2y = 48$ ...(2)

Multiplying by 2 in Equation (2),

$4x + 4y = 96$ ...(3)

Substracting Equation (3) and (1),

$$\begin{array}{r} 8x+4y=176 \\ 4x+4y=96 \\ -\quad -\quad - \\ \hline 4x=80 \\ x=20 \end{array}$$

So, the cost of one pen = ₹20

**9.** **(a)** Let, total tickets = $x$

Then, $55 \times x + 85 \times x + 105 \times x = 2940$

$\Rightarrow$ $245x = 2940$

$\Rightarrow$ $x = \dfrac{2940}{245}$

$\Rightarrow$ $x = 12$

**10.** **(c)** Let, the number is $10x + y$

(When number at unit place is $y$ and at tens place is $x$)

$(10x + y) - (10y + x) = 9$

$x - y = 1$ ...(1)

$x + y = 15$ ...(2)

Solving Equation (1) and (2) we get,

$x = 8$

$y = 7$

Required number = $10 \times 8 + 7 = 87$

**11.** **(b)** $3Y + 9X = 54$ ...(1)

$\dfrac{28X}{13Y} = \dfrac{140}{39}$

$1820Y - 1092X = 0$ ...(2)

From Equation (1) and (2),

$X = 5$, $Y = 3$

$\therefore$ $Y - X = 3 - 5 = -2$

**12.** **(c)** Let, each child got $x$ sweets.

$\therefore$ $112 \times x = (112 - 32) \times (x + 6)$

$112x = 80 \times (x + 6)$

$112x = 80x + 480$

$112x - 80x = 480$

$32x = 480$

$x = 15$

**13.** **(c)** Let, the number is $10x + y$.

$(10x + y) - (10y + x) = 9$

$\Rightarrow$ $9x - 9y = 9$

$\therefore$ Required difference $x - y = 1$

**14.** **(b)** Let, total amount was ₹$x$.

$\because$ $x - (44620 + 32764) = \dfrac{x \times 32}{100}$

$\Rightarrow$ $x - 77384 = 0.32\,x$

$\Rightarrow$ $x - 0.32x = 77384$

$\Rightarrow$ $0.68x = 77384$

$x = \dfrac{77384}{0.68}$

$= 113800$

**15.** **(c)** Let, 50 paise coins = $2x$ and ₹1 coins = $x$ both are ₹26 then the number of ₹1 coins will be 13 and number of 50 paise coins will be 26. Remaining amount = 50 − 26 = 24. Now if ₹5 coins are $x$ in number then ₹2 coins will be $x + 5$. Then, with the help of hit and trial method ₹5 coins will be in number and ₹2 coins will be $x + 5 = 2 + 5 = 7$ in number.

**16.** **(a)** Let, the cost of one Chair = C

and the cost of one Table = T

Then, 5C + 3T = 3110 ...(1)

and,

T − C = 210 or, T = 210 + C ...(2)

On putting value of T in Equation (1),

5C + 3(210 + C) = 3110

$\Rightarrow$ 5C + 630 + 3C = 3110

$\Rightarrow$ 8C = 3110 − 630

$\Rightarrow$ $C = \dfrac{2480}{8} =$ ₹310

$\therefore$ Cost of one Table (T) = 210 + 310

= 520

Hence, the cost of two Tables and two Chairs.

= 2T + 2C

= 2 × 520 + 2 × 310

= 1040 + 620

= ₹1660

**17.** **(a)** Let, the husband's age be $x$.

Let, the wife's age be $y$.

Let, the daughter's age be $z$.

According to questions,

$x + 2y + 3z = 85$ ...(1)

$2x + 4y + 6z = 170$ ...(2)

$5x + 10y + 15z = 450$ ...(3)

From Equation (2), $x + 2y + 3z = 85$

From Equation (3), $x + 2y + 3z = 90$

From Equation (1), $x + 2y + 3z = 85$

Hence, the above system of equation will give no solution.

**18.** **(d)** Let, required number = $10x + y$

Where $x > y$

According to question,

$(10x + y) - (10y + x) = 54$

$\Rightarrow\ 9x - 9y = 54$

$\Rightarrow\ 9(x - y) = 54$

$\Rightarrow\ x - y = 6$ ...(1)

and, $x + y = 12$ ...(2)

From Equation (1) and (2),

$x - y = 6$

$x + y = 12$

$\underline{2x = 18}$

$x = 9$

Value of $x = 9$, put in Equation (2)

$9 + y = 12$

$y = 12 - 9$

$y = 3$

$\therefore$ Number = $10 \times 9 + 3 = 90 + 3 = 93$

**19.** **(d)** According to the question, number = $x$

$x \times 2 + 42 \times 3 = 238$

$2x = 112$

$x = 56$

Again $3 \times 56 + 42 \times 2 = 168 + 84 = 252$

**20.** **(b)** Let, the number be $x$

Therefore, $\dfrac{x+12}{6} = 112$

$\Rightarrow\ x + 12 = 672$

$\Rightarrow\ x = 672 - 12 = 660$

Hence, correct answer

$= \dfrac{660}{6} + 12 = 110 + 12 = 122.$

**21.** **(b)** $\dfrac{3x}{2y} = \dfrac{21}{22}$

$\Rightarrow\ \dfrac{x}{y} = \dfrac{21}{22} \times \dfrac{2}{3} = \dfrac{7}{11} \quad \Rightarrow\ \dfrac{x}{7} = \dfrac{y}{11} = k$

Now, according to the question,

$4x + 5y = 83$

$\Rightarrow\ 4 \times 7k + 5 \times 11k = 83$

$\Rightarrow\ 28k + 55k = 83$

$\Rightarrow\ 83k = 83 \quad \Rightarrow\ k = 1$

$\Rightarrow\ x = 7$ and $y = 11$

$\therefore\ y - x = 11 - 7 = 4$

**22.** **(a)** Let, numbers be $x$ and $y$.

Now, according to the question,

$x + y = 25$ ...(1)

$x - y = 20$ ...(2)

On adding (1) and (2), we have,

$2x = 45$

$\Rightarrow\ x = \dfrac{45}{2} = 22.5$

From equation (1),

$22.5 + y = 25$

$\Rightarrow\ y = 25 - 22.5 = 2.5$

$\therefore$ Required ratio = 22.5:2.5 = 9:1

**23.** **(a)** The graphs of $(k-1)x + y - 2 = 0$ and $(2 - k)x - 3y + 1 = 0$ are parallel.

$\therefore\ \dfrac{k-1}{2-k} = \dfrac{1}{-3} \quad \Rightarrow\ -3k + 3 = 2 - k$

$\Rightarrow\ -3k + k = 2 - 3 \quad \Rightarrow\ -2k = -1$

$\Rightarrow\ k = \dfrac{1}{2}$

***Note:***

Two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel if,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

**Alternative method:**

$(k-1)x + y - 2 = 0$

$\Rightarrow\ y = (1-k)x + 2$ ...(1)

and, $(2-k)x - 3y + 1 = 0$

$\Rightarrow\ 3y = (2-k)x + 1$

$\therefore\ y = \left(\dfrac{2-k}{3}\right)x + \dfrac{1}{3}$ ...(2)

$\because\ m_1 = m_2$

$\Rightarrow\ 1 - k = \dfrac{2-k}{3} \quad \Rightarrow\ 3 - 3k = 2 - k$

$\therefore\ k = \dfrac{1}{2}$

**24.** **(d)** On $y$-axis, $x = 0$

Putting $x = 0$ in $x + 2y = 3$,

$2y = 3 \Rightarrow y = \frac{3}{2}$

Putting $x = 0$ in $3x - 2y = 1$

$-2y = 1 \Rightarrow y = -\frac{1}{2}$

$\therefore$ Points on $y$-axis are $\left(0, \frac{3}{2}\right)$ and $\left(0, -\frac{1}{2}\right)$.

$\therefore$ Required distance $= \sqrt{(0-0)^2 + \left(\frac{3}{2} + \frac{1}{2}\right)^2}$

$= \sqrt{0+4} = 2$ units

***Note:***

Distance $= \frac{3}{2} + \frac{1}{2} = 2$