Trigonometric Ratios

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INTRODUCTION

The literal meaning of the word trigonometry is the 'science of triangle measurement'. The word trigonometry is derived from two Greek words trigon and metron which means measuring the sides of a triangle. It had its beginning more than two thousand years ago as a tool for astronomers. The Babylonians, Egyptians, Greeks and the Indians studied trigonometry only because it helped them in unravelling the mysteries of the universe. In modern times, it has gained wider meaning and scope. Presently, it is defined as that branch of mathematics which deals with the measurement of angles, whether of triangle or any other figure.

At present, trigonometry is used in surveying, astronomy, navigation, physics, engineering, etc.

Important Formulae and Results of Trigonometry

I. (i) $180° = \pi$ radians.

(ii) $1° = \dfrac{\pi}{180} = 0.01745$ radians (approximately).

(iii) $\pi = \dfrac{\text{circumference of a cirlce}}{\text{diameter of the circle}}$

$= \dfrac{22}{7} = 3.1416$ (approximately).

(iv) θ (in radian measure) $= \dfrac{l}{r}$.

(v) Each interior angle of a regular polygon of n sides $= \dfrac{n-2}{n} \times 180$ degrees.

II. (i) $\sin\theta \times \text{cosec}\theta = 1$; $\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$;

$\text{cosec}\ \theta = \dfrac{1}{\sin\theta}$.

Also, $-1 \le \sin x \le 1$, $\text{cosec}\ x \le -1$ or $\text{cosec}\ x \ge 1$.

(ii) $\cos\theta \times \sec\theta = 1$; $\cos\theta = \dfrac{1}{\sec\theta}$; $\sec\theta = \dfrac{1}{\cos\theta}$.

Also, $-1 \le \cos x \le 1$, $\sec x \le -1$ or $\sec x \ge 1$.

(iii) $\tan\theta \times \cot\theta = 1$; $\tan\theta = \dfrac{1}{\cot\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$

Also, $-\infty < \tan\theta < \infty$, $-\infty < \cot\theta < \infty$.

(iv) $\sin^2\theta + \cos^2\theta = 1$; $\sin^2\theta = 1 - \cos^2\theta$; $\cos^2\theta = 1 - \sin^2\theta$

(v) $\sin^2\theta = 1 + \tan^2\theta$; $\sec^2\theta - \tan^2\theta = 1$; $\tan^2\theta = \sec^2\theta - 1$.

(vi) $\text{cosec}^2\theta = 1 + \cot^2\theta$; $\text{cosec}^2\theta - \cot^2\theta = 1$; $\cot^2\theta = \text{cosec}^2\theta - 1$.

(vii) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

III. Values of trigonometrical ratios for particular angles

(i)

Angle	sine	cos	tan
0°	0	1	0
$30° = \frac{\pi}{6}$	–	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45° = \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60° = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90° = \frac{\pi}{2}$	1	0	∞
$120° = \frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
$135° = \frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	–1

$150° = \frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
$180° = \pi$	0	–1	0
$270° = \frac{3\pi}{2}$	–1	0	–∞
$360° = 2\pi$	0	1	0

(ii) $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$; $\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$;

$\tan 15° = 2-\sqrt{3}$.

(iii) $\sin 18° = \frac{\sqrt{5}-1}{4} = \cos 72°$;

$\cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72°$.

(iv) $\cos 36° = \frac{\sqrt{5}+1}{4} = \sin 54°$;

$\sin 35° = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54°$.

(v) $\tan 7\frac{1°}{2} = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$;

$\cot 7\frac{1°}{2} = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$.

IV. Signs of trigonometrical ratios

Angle	sin	cos	tan
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$90° - \theta$ or $\frac{\pi}{2} - \theta$	$\cos\theta$	$\sin\theta$	$\cot\theta$
$90° + \theta$ or $\frac{\pi}{2} + \theta$	$\cos\theta$	$-\sin\theta$	$-\cot\theta$
$180° - \theta$ or $\pi - \theta$	$\sin\theta$	$-\cos\theta$	$-\tan\theta$
$180° + \theta$ or $\pi + \theta$	$-\sin\theta$	$-\cos\theta$	$\tan\theta$
$270° - \theta$ or $\frac{3\pi}{2} - \theta$	$-\cos\theta$	$-\sin\theta$	$\cot\theta$
$270° + \theta$ or $\frac{3\pi}{2} + \theta$	$-\cos\theta$	$\sin\theta$	$-\cot\theta$
$360° - \theta$ or $2\pi - \theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$360° + \theta$ or $2\pi + \theta$	$-\sin\theta$	$\cos\theta$	$\tan\theta$

V. Trigonometrical ratios for sum or difference of angles

(i) $\sin(A \pm B) = \sin A \times \cos B \pm \cos A \times \sin B$.

(ii) $\cos(A \pm B) = \cos A \times \cos B \ \sin A \times \sin B$.

(iii) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \times \tan B}$.

(iv) $\cot(A \pm B) = \frac{\cot A \times \cot B \mp 1}{\cot B \pm \cot A}$.

(v) $\tan(A + B + C)$

$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - (\tan A \tan B + \tan B \tan C + \tan C \tan A)}$

(vi) $\sin(A + B) \times \sin(A - B) = \sin^2 A - \sin^2 B$.

(vii) $\cos(A + B) \times \cos(A - B) = \cos^2 A - \sin^2 B$.

VI. Sum or difference of sine or cosine of angles into products

(i) $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$.

(ii) $\sin C - \sin D = 2 \cos \frac{C+D}{2} \cdot \sin \frac{C-D}{2}$.

(iii) $\cos C + \cos D = 2 \cos \frac{C+D}{2} \cdot \cos \frac{C-D}{2}$.

(iv) $\cos C - \cos D = 2 \sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2}$,

VII. Product of sines and cosines of angles into sum or difference of angles

(i) $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$.

(ii) $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$.

(iii) $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$.

(iv) $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$.

VIII. Trigonometrical ratios of multiple angles

(i) $\sin 2A = 2 \sin A \cos A = \frac{2\tan A}{1+\tan^2 A}$.

(ii) $\cos^2 A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$

$= 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$.

(iii) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$.

(iv) $\sin^2 A = \frac{1-\cos 2A}{2}$; $\cos^2 A = \frac{1+\cos 2A}{2}$.

(v) $\tan A = \sqrt{\dfrac{1-\cos 2A}{1+\cos 2A}} = \dfrac{1-\cos 2A}{\sin 2A}$.

(vi) $\sin 3A = 3\sin A - 4\sin^3 A$.

(vii) $\cos 3A = 4\cos^3 A - 3\cos A$.

(viii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$;

$\cot 3A = \dfrac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$.

IX. Trigonometrical ratios of submultiple angles

(i) $\sin A = 2\sin = \dfrac{A}{2}\cos\dfrac{A}{2} = \dfrac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$.

(ii) $\cos A = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2} = 2\cos^2\dfrac{A}{2} - 1$

$= 1 - 2\sin 2\dfrac{A}{2} = \dfrac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$.

(iii) $\tan A = \dfrac{2\tan\frac{A}{2}}{1\tan^2\frac{A}{2}}$.

(iv) $\sin^2\dfrac{A}{2} = \dfrac{1-\cos A}{2}$; $\cos^2\dfrac{A}{2} = \dfrac{1+\cos A}{2}$

(v) $\tan\dfrac{A}{2} = \sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{1-\cos A}{\sin A}$.

(vi) $2\sin\dfrac{A}{2} = \pm\sqrt{1+\sin A} \pm \sqrt{1-\sin A}$.

(vii) $2\cos\dfrac{A}{2} = \pm\sqrt{1+\sin A} \mp \sqrt{1-\sin A}$.

Exercise-I

1. If $\dfrac{1+\cos A}{1-\cos A} = \dfrac{m^2}{n^2}$, $\tan A =$

(a) $\pm\dfrac{2mn}{m^2+n^2}$ (b) $\pm\dfrac{2mn}{m^2-n^2}$

(c) $\dfrac{m^2+n^2}{m^2-n^2}$ (d) None of these

2. If sin 600° cos 30° + cos 120° sin 150° = k, then k =

(a) 0 (b) 1

(c) −1 (d) None of these

3. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta =$

(a) $\sqrt{2}\sin\theta$ (b) $2\sin\theta$

(c) $-\sqrt{2}\sin\theta$ (d) None of these

4. If α lies in the second quadrant, then $\sqrt{\dfrac{1-\sin\alpha}{1+\sin\alpha}} - \sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}} =$

(a) $\tan\alpha$ (b) $2\tan\alpha$

(c) $2\cot\alpha$ (d) $\cot\alpha$

5. If $\cot\theta + \cos\theta = p$ and $\cot\theta - \cos\theta = 9$, then $(p^2 - q^2)^2$ in terms of p and q is:

(a) $16\,pq$ (b) $8\,pq$

(c) $4\,pq$ (d) $12\,pq$

6. If $x = a\operatorname{cosec}^n\theta$ and $y = b\cot^n\theta$, then by eliminating θ

(a) $\left(\dfrac{x}{a}\right)^{\frac{2}{n}} + \left(\dfrac{y}{b}\right)^{\frac{2}{n}} = 1$ (b) $\left(\dfrac{x}{a}\right)^{\frac{2}{n}} - \left(\dfrac{y}{b}\right)^{\frac{2}{n}} = 1$

(c) $\left(\dfrac{x}{a}\right)^{2} - \left(\dfrac{y}{b}\right)^{2} = 1$ (d) $\left(\dfrac{x}{a}\right)^{\frac{1}{n}} - \left(\dfrac{y}{b}\right)^{\frac{1}{n}} = 1$

7. If $\tan\theta = \dfrac{p}{q}$, then $\dfrac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} =$

(a) $\dfrac{(p^2+q^2)}{(p^2-q^2)}$ (b) $\dfrac{(p^2-q^2)}{(p^2+q^2)}$

(c) $\dfrac{(p^2+q^2)}{(p^2-q^2)}$ (d) None of these

8. If $\sin A = \dfrac{3}{5}$, $\tan B = \dfrac{1}{2}$ and $\dfrac{\pi}{2} < A < \pi < B < \dfrac{3\pi}{2}$, the value of $8\tan A - \sqrt{5}$ see B =

(a) $\dfrac{7}{2}$ (b) $\dfrac{5}{2}$

(c) $-\dfrac{5}{2}$ (d) $-\dfrac{7}{2}$

9. If $\sec\theta - \tan\theta = \frac{a+1}{a-1}$, then $\cos\theta =$

(a) $\frac{a^2+1}{a^2-1}$ (b) $\frac{a^2-1}{a^2+1}$

(c) $\frac{2a}{a^2+1}$ (d) $\frac{2a}{a^2-1}$

10. If $\tan 20° = k$, then $\frac{\tan 250° + \tan 340°}{\tan 200° - \tan 110°} =$

(a) $\frac{1+k}{1-k}$ (b) $\frac{1-k}{1+k}$

(c) $\frac{1+k^2}{1-k^2}$ (d) $\frac{1-k^2}{1+k^2}$

11. The value of sin 780°sin 480° + cos 240°cos 300° =

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) 1 (d) None of these

12. If $\tan\theta + \cot\theta = 2$, then $\sin\theta =$

(a) $\pm\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\pm\frac{1}{3}$ (d) None of these

13. If θ is in the first quadrant and $\tan\theta = \frac{3}{4}$, then

$$\frac{\tan\left(\frac{\pi}{2}-\theta\right) - \sin(\pi-\theta)}{\sin\left(\frac{3\pi}{2}+\theta\right) - \cot(2\pi-\theta)} =$$

(a) $\frac{8}{11}$ (b) $\frac{6}{11}$

(c) $\frac{11}{8}$ (d) $\frac{11}{6}$

14. If $\cot 20° = p$, then $\frac{\tan 160° - \tan 110°}{1 + \tan 160° \tan 110°} =$

(a) $\frac{p^2-1}{2p}$ (b) $\frac{p^2+1}{2p}$

(c) $\frac{1-p^2}{2p}$ (d) $\frac{2p}{1+p^2}$

15. If A lies in the second quadrant and B lies in the third quadrant and $\cos A = -\sqrt{\frac{3}{2}}$, $\sin B = -\frac{3}{5}$, then $\frac{2\tan B + \sqrt{3}\tan A}{\cot^2 A + \cos B} =$

(a) $\frac{5}{21}$ (b) $\frac{5}{24}$

(c) $\frac{5}{22}$ (d) None of these

16. The value of $\frac{\sin 150° - 5\cos 300° + 7\tan 225°}{\tan 135° + 3\sin 210°}$ is:

(a) 2 (b) 1

(c) −1 (d) −2

17. If $f(x) = \cos^2 x + \sec^2 x$, its value always is:

(a) $f(x) < 1$ (b) $f(x) = 1$

(c) $2 > f(x) > 1$ (d) $f(x) \geq 2$

18. If $\operatorname{cosec}\theta + \cot\theta = p$, then $\cos\theta =$

(a) $\frac{p^2+1}{p^2-1}$ (b) $\frac{1+p^2}{1-p^2}$

(c) $\frac{p^2-1}{p^2+1}$ (d) $\frac{1-p^2}{1+p^2}$

19. If $\sin\theta = -\frac{7}{25}$ and θ is in the third quadrant, then $\frac{7\cot\theta - 24\tan\theta}{7\cot\theta + 24\tan\theta} =$

(a) $\frac{17}{31}$ (b) $\frac{16}{31}$

(c) $\frac{15}{31}$ (d) None of these

20. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, then $\frac{(m^2-n^2)^2}{mn} =$

(a) 4 (b) 3

(c) 16 (d) 9

21. If $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$ then $(m^2n)^{\frac{2}{3}} + (mn^2)^{\frac{2}{3}} =$

(a) −1 (b) 1

(c) 0 (d) None of these

22. The value of cos 1°cos 2°cos 3°…cos 179° =

(a) 1 (b) −1

(c) 0 (d) None of these

23. Without using trigonometric tables, sin48°sec42° + cos 48°cosec 42° =

(a) 0 (b) 2
(c) 1 (d) None of these

24. tan 5°tan 25°tan 45°tan 65°tan 85° =

(a) −1 (b) 1
(c) $\frac{1}{2}$ (d) None of these

25. $\cos^2 5° + \cos^2 10° + \cos^2 15° + \cdots + \cos^2 90° =$

(a) $8\frac{1}{2}$ (b) $6\frac{1}{2}$
(c) $7\frac{1}{2}$ (d) None of these

26. The value of log tan 1° + log tan 2° +log tan 3° + … + log tan 89° is equal to

(a) 1 (b) 0
(c) 3 (d) None of these

27. log sin 1° log sin 2° log sin 3°…log sin 179° =

(a) 0 (b) 1
(c) $\frac{1}{\sqrt{2}}$ (d) None of these

28. The value of cos 24° + cos 55° + cos 155° + cos 204° is

(a) 1 (b) −1
(c) 0 (d) None of these

29. The value of cos 24° + cos 5° + cos 300° + cos 175° + cos 204° is

(a) 0 (b) $-\frac{1}{2}$
(c) $\frac{1}{2}$ (d) 1

30. $\sin^2\theta = \frac{(x+y)^2}{4xy}$ is possible only when

(a) $x > 0, y > 0, x \neq y$
(b) $x > 0, y > 0, x = y$
(c) None of these

31. If $7\sin^2\theta + 3\cos^2\theta = 4$, then $\tan\theta =$

(a) $\pm\frac{1}{3}$ (b) $\pm\frac{1}{2}$
(c) $\pm\frac{1}{\sqrt{3}}$ (d) $\pm\frac{1}{\sqrt{2}}$

32. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, then $\frac{m^2-1}{n^2-1} =$

(a) $\cos^3\alpha$ (b) $\sin^2\alpha$
(c) $\sin^2\alpha$ (d) $\cos^2\alpha$

33. If $\sec A = a + \left(\frac{1}{4a}\right)$, then sec$A$ + tan$A$ =

(a) $2a$ or $\frac{1}{2a}$ (b) a or $\frac{1}{a}$
(c) $2a$ or $\frac{1}{a}$ (d) a or $\frac{1}{2a}$

34. The value of $\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}$ is:

(a) 0 (b) 1
(c) 2 (d) None of these

35. The value of tan 20° + tan 40° + tan 60° + ⋯ + tan 180° is

(a) 1 (b) −1
(c) 0 (d) None of these

36. If $\cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\alpha = \frac{-3}{5}$, where θ does not lie in the third quadrant and α lies in the third quadrant, $\frac{2\tan\alpha + \sqrt{3}\tan\theta}{\cot^2\theta + \cos\alpha} =$

(a) $\frac{5}{22}$ (b) $\frac{-5}{22}$
(c) $\frac{7}{22}$ (d) None of these

37. The value of cos 24° + cos 55° + cos 125° + cos 204° + cos 300° is:

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) 1 (d) −1

38. $\frac{\cot\theta - \text{cosec}\theta + 1}{\cot\theta + \text{cosec}\theta - 1}$ is equal to:

(a) 1 (b) $\cot\theta + \text{cosec}\theta$
(c) $\text{cosec}\theta - \cot\theta$ (d) None of these

39. If $90° < \alpha < 180°$, $\sin\alpha = \frac{\sqrt{3}}{2}$

and $180° < \beta < 270°$, $\sin\beta = -\frac{\sqrt{3}}{2}$,

then $\frac{4\sin\alpha - 3\tan\beta}{\tan\alpha + \sin\beta} =$

(a) $\frac{2}{3}$ (b) 0
(c) $-\frac{2}{3}$ (d) None of these

40. $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}=$

(a) $2\sin\theta$ (b) $2\cos\theta$
(c) $\dfrac{2}{|\cos\theta|}$ (d) $\dfrac{2}{|\sin\theta|}$

41. If $\sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}}=\operatorname{cosec}\alpha+\cot\alpha$, then the quadrants in which α lies are:

(a) 1, 4 (b) 2, 3
(c) 1, 2 (d) 3, 4

42. If $\operatorname{cosec}\theta-\cot\theta=p$, then the value of $\operatorname{cosec}\theta=$

(a) $\dfrac{1}{2}\left(p+\dfrac{1}{p}\right)$ (b) $\dfrac{1}{2}\left(p-\dfrac{1}{p}\right)$
(c) $p+\dfrac{1}{p}$ (d) $p-\dfrac{1}{p}$

43. The value of $\tan 1°\tan 2° \ldots \tan 89°$ is:

(a) -1 (b) 1
(c) 0 (d) None of these

44. If $\operatorname{cosec}^2\theta=\dfrac{4xy}{(x+y)^2}$, then:

(a) $x=-y$ (b) $x=\dfrac{1}{y}$
(c) $x=y$ (d) None of these

45. The value of $\dfrac{\sin 300°\tan 240°\sec(-420°)}{\cot(-315°)\cos(210°)\operatorname{cosec}(-315°)}$ is:

(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\sqrt{6}$ (d) $\sqrt{8}$

46. The length of an arc which subtends an angle 18° at the centre of the circle of radius 6 cms is:

(a) $\left(\dfrac{\pi}{5}\right)$ cm (b) $\left(\dfrac{2\pi}{5}\right)$ cm
(c) $\left(\dfrac{3\pi}{5}\right)$ cm (d) None of these

47. If x is real and $x+\dfrac{1}{x}=2\cos\theta$, then $\cos\theta=$

(a) $\pm\dfrac{1}{2}$ (b) $\pm\dfrac{1}{3}$
(c) ± 1 (d) None of these

48. Which of the following is correct?

(a) $\sin 1° > \sin 1$ (b) $\sin 1° = \sin 1$
(c) $\sin 1° < \sin 1$ (d) $\sin 1°=\left(\dfrac{\pi}{180}\right)\sin 1$

49. Which one of the following is true?

(a) $\tan 1 = 1$
(b) $\tan 1 = \tan 2$
(c) $\tan 1 < \tan 2$
(d) $\tan 1 > \tan 2$

50. The value of $\cos^2\theta + \sec^2\theta$ is always:

(a) Less than 1
(b) Equal to 1
(c) Lies between 1 and 2
(d) Greater than 2.

51. If $\sin\alpha=\dfrac{2pq}{p^2+q^2}$, then $\sec\alpha-\tan\alpha=$

(a) $\dfrac{p-q}{p+q}$ (b) $\dfrac{pq}{p^2+q^2}$
(c) $\dfrac{p+q}{p-q}$ (d) None of these

52. If $13\sin A=12$, $\dfrac{\pi}{2}<A<\pi$ and $3\sec B=5$, $\dfrac{3\pi}{2}<B<2\pi$ then $5\tan A+3\tan^2 B=$

(a) $\dfrac{20}{3}$ (b) $-\dfrac{20}{3}$
(c) $\dfrac{22}{3}$ (d) $-\dfrac{22}{3}$

53. The value of $\sin 105° + \cos 105°$ is:

(a) $\dfrac{1}{\sqrt{2}}$ (b) $-\dfrac{1}{\sqrt{2}}$
(c) 0 (d) None of these

54. If $\tan A=\dfrac{1}{2}$ and $\tan B=\dfrac{1}{3}$, the value of A + B is:

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{2}$ (d) None of these

55. If $\tan(A-B)=\frac{7}{24}$ and $\tan A=\frac{4}{3}$ where A and B are acute, then $A + B =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) None of these

56. The value of $\frac{(\tan 69°+\tan 66°)}{(1-\tan 69°\tan 66°)}$ is:

(a) 1 (b) 0

(c) 2 (d) −1

57. The value of $\sin^2 75° - \sin^2 15°$ is:

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) None of these

58. If

$\sin\alpha=\frac{8}{17}$, $0<\alpha<90°$ and $\tan\beta=\frac{5}{12}$, $0<\beta<90°$, then $\cos(\alpha - \beta)$ is:

(a) $\frac{210}{221}$ (b) $\frac{171}{221}$

(c) $\frac{220}{221}$ (d) None of these

59. The value of $\sin^2\theta + \sin^2(\theta + 60°) + \sin^2(\theta - 60°) =$

(a) $\frac{1}{2}$ (b) 0

(c) $\frac{3}{2}$ (d) None of these

60. If $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$, then $\alpha+\beta=$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) None of these

61. The value of $\frac{1}{\sin 10°}-\frac{\sqrt{3}}{\cos 10°}=$

(a) 1 (b) 4

(c) 3 (d) None of these

62. The value of $\sqrt{2+\sqrt{2(1+\cos 4A)}}$ is equal to:

(a) $\cos A$ (b) $\sin A$

(c) $2\cos A$ (d) $2\sin A$

63. If $\tan A=\frac{1-\cos B}{\sin B}$, then $\tan 2A =$

(a) $\tan B$ (b) $\cot B$

(c) $2\tan B$ (d) $2\cot B$

64. The value of $\frac{\cos 2\theta}{1-\sin 2\theta}=$

(a) $\tan\left(\frac{\pi}{4}-\theta\right)$ (b) $\cot\left(\frac{\pi}{4}-\theta\right)$

(c) $\tan\left(\frac{\pi}{4}+\theta\right)$ (d) $\cot\left(\frac{\pi}{4}+\theta\right)$

65. The value of $\frac{\tan 40°+\tan 20°}{1-\cot 70°\cot 50°}$ is equal to:

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

66. The value of $\sqrt{3}\operatorname{cosec}20° - \sec 20° =$

(a) 2 (b) 4

(c) 3 (d) None of these

67. The value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is:

(a) 2 (b) 3

(c) 4 (d) None of these

68. $\tan 5x - \tan 3x - \tan 2x$ is equal to:

(a) $\tan 2x \tan 3x \tan 5x$

(b) $\frac{\sin 5x-\sin 3x-\sin 2x}{\cos 5x-\cos 3x-\cos 2x}$

(c) 0

(d) None of these

69. If $\tan A=\frac{n}{n+1}$ and $\tan B=\frac{1}{2n+1}$, the value of $\tan(A + B) =$

(a) − 1 (b) 1

(c) 2 (d) None of these

70. If $\sin A=\frac{1}{\sqrt{10}}$, $\sin B=\frac{1}{\sqrt{5}}$ where A and B are positive and acute, $A + B =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) None of these

71. $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$ is equal to:

(a) $\cot\left(\dfrac{\theta}{2}\right)$ (b) $\tan\left(\dfrac{\theta}{2}\right)$

(c) $\sec\left(\dfrac{\theta}{2}\right)$ (d) $\operatorname{cosec}\dfrac{\theta}{2}$

72. $\tan 7\dfrac{1°}{2}$ is equal to:

(a) $\dfrac{2\sqrt{2}-(1+\sqrt{3})}{\sqrt{3}-1}$ (b) $\dfrac{1+\sqrt{3}}{1-\sqrt{3}}$

(c) $\dfrac{1}{\sqrt{3}}+\sqrt{3}$ (d) $2\sqrt{2}+\sqrt{3}$

73. If $\dfrac{\cos 3A+\sin 3A}{\cos A-\sin A}=1-K\sin 2A$, the value of K is:

(a) –2 (b) 2

(c) 3 (d) 4

74. The value of tan 57° – tan 12° – tan 57°tan 12° =

(a) –1 (b) 1

(c) 0 (d) None of these

75. If 180° < θ < 270°, then the value of $\sqrt{4\sin^4\theta+\sin^2 2\theta}+4\cos^2\left(\dfrac{\pi}{4}-\dfrac{\theta}{2}\right)$ is:

(a) 2 (b) 4

(c) 3 (d) None of these

76. The value of tan100° + tan125° + tan100° tan125° =

(a) $\sqrt{3}$ (b) –1

(c) $\dfrac{1}{\sqrt{3}}$ (d) 1

77. For all θ, the value of $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}=$

(a) $\sec\theta-\tan\theta$ (b) $(\sec\theta+\tan\theta)^2$

(c) $(\sec\theta-\tan\theta)^2$ (d) $\sec\theta+\tan\theta$

78. If $\tan\theta=\dfrac{\cos 15°+\sin 15°}{\cos 15°-\sin 15°}$, then θ =

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{2}$

79. The value of tan56° – tan11° – tan56° tan11° is:

(a) –1 (b) 0

(c) 1 (d) None of these

80. If $A + B$ = 45° and (cot A – 1)(cot + B – 1) = 4K, then K =

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{8}$

(c) $\dfrac{1}{2}$ (d) None of these

EXERCISE-2
(BASED ON MEMORY)

1. $ABCD$ is a rectangle of which AC is a diagonal. The value of $(\tan^2\angle CAD+1)\sin^2\angle BAC$ is:

(a) 2 (b) $\dfrac{1}{4}$

(c) 1 (d) 0

[SSC, 2014]

2. If $\tan x=(\sin 45°)(\cos 45°)+\sin 30°$, then the value of x is:

(a) 30° (b) 45°

(c) 60° (d) 90°

[SSC, 2014]

3. For any real values of θ, $\sqrt{\dfrac{\sec\theta-1}{\sec\theta+1}}=?$

(a) $\cot\theta-\operatorname{coses}\theta$ (b) $\sec\theta-\tan\theta$

(c) $\operatorname{cosec}\theta-\cot\theta$ (d) $\tan\theta-\sec\theta$

[SSC, 2014]

4. If the sum and difference of two angles are $\dfrac{\pi}{4}$ and $\dfrac{\pi}{12}$, respectively, then the values of the angles in degree measure are:

(a) 70°, 65° (b) 75°, 60°

(c) 45°, 90° (d) 80°, 55°

[SSC, 2014]

5. In a $\Delta ABC, \angle B=\dfrac{\pi}{3}, \angle C=\dfrac{\pi}{4}$ and D divides BC internally in the ratio 1:3, then $\dfrac{\sin\angle BAD}{\sin\angle CAD}$ is equal to:

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{6}}$ (d) $\sqrt{6}$

[SSC, 2014]

6. If $\sin 3A = \cos(A - 26°)$, where $3A$ is an acute angle then the value of A is:
(a) 29° (b) 26°
(c) 23° (d) 28°

[SSC, 2014]

7. Value of $\sec^2\theta - \frac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$ is:
(a) 1 (b) 2
(c) −1 (d) 0

[SSC, 2014]

8. If $x = a(\sin\theta + \cos\theta), y = b(\sin\theta - \cos\theta)$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is:
(a) 0 (b) 1
(c) 2 (d) −2

[SSC, 2014]

9. If $\sin 5\theta = \cos 20°(0° < \theta < 90°)$, then the value of θ is:
(a) 4° (b) 22°
(c) 10° (d) 14°

[SSC, 2014]

10. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then the value of $(\cos\theta - \sin\theta)$ is:
(a) $\sqrt{3}\cos\theta$ (b) $\sqrt{3}\sin\theta$
(c) $\sqrt{2}\cos\theta$ (d) $\sqrt{2}\sin\theta$

[SSC, 2013]

11. If x sin 45° = y cosec 30°, then $\frac{x^4}{y^4}$ is equal to:
(a) 4^3 (b) 6^3
(c) 2^3 (d) 8^3

[SSC, 2013]

12. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^{100}\theta + \cot^{100}\theta$ is:
(a) 2 (b) 0
(c) 1 (d) $\sqrt{3}$

[SSC Assistant Grade III, 2013]

13 $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ is equal to:
(a) $1 - \tan\theta - \cot\theta$ (b) $1 + \tan\theta - \cot\theta$
(c) $1 - \tan\theta + \cot\theta$ (d) $1 + \tan\theta + \cot\theta$

[SSC Assistant Grade III, 2013]

14. If $\sec\theta = x + \frac{1}{4x}(0° < \theta < 90°)$, then $\sec\theta + \tan\theta$ is equal to:
(a) $\frac{x}{2}$ (b) $2x$
(c) x (d) $\frac{1}{2x}$

[SSC Assistant Grade III, 2013]

15. The circular measure of an angle of an isosceles triangle is $\frac{5\pi}{9}$. Circular measure of one of the other angles must be:
(a) $\frac{5\pi}{18}$ (b) $\frac{5\pi}{9}$
(c) $\frac{2\pi}{9}$ (d) $\frac{4\pi}{9}$

[SSC Assistant Grade III, 2013]

16. If $x = r\cos\theta\cos\phi$, $y = r\cos\theta\sin\phi$ and $z = r\sin\theta$, then the value of $x^2 + y^2 + z^2$ is:
(a) r^2 (b) r
(c) $\frac{1}{r^2}$ (d) $\frac{1}{r}$

[SSC Assistant Grade III, 2012]

17. If $5\cos\theta + 12\sin\theta = 13$, then $\tan\theta = ?$
(a) $\frac{13}{12}$ (b) $\frac{12}{13}$
(c) $\frac{12}{5}$ (d) $\frac{5}{12}$

[SSC Assistant Grade III, 2012]

18. The value of $\sec^2 12° - \frac{1}{\tan^2 78°}$ is:
(a) 0 (b) 1
(c) 2 (d) 3

[SSC Assistant Grade III, 2012]

19. If $\tan\theta . \cos 60° = \frac{\sqrt{3}}{2}$, then the value of $\sin(\theta - 15°)$ is:
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$
(c) 1 (d) $\frac{1}{\sqrt{2}}$

[SSC Assistant Grade III, 2012]

20. If θ is a positive acute angle and $\tan 2\theta \cdot \tan 3\theta = 1$, then the value of $\left(2\cos^2\frac{50}{2}-1\right)$ is:

(a) $-\frac{1}{2}$ (b) 1
(c) 0 (d) $\frac{1}{2}$

[SSC, 2012]

21. If $\sin 17° = \frac{x}{y}$, then the value of (sec 17° – sin 73°) is:

(a) $\frac{y^2}{x\sqrt{y^2-x^2}}$ (b) $\frac{x^2}{y\sqrt{y^2-x^2}}$
(c) $\frac{x^2}{y\sqrt{x^2-y^2}}$ (d) $\frac{y^2}{x\sqrt{x^2-y^2}}$

[SSC, 2012]

22. In a right-angled triangle *XYZ*, right-angled at *Y*, if $XY = 2\sqrt{6}$ and $XZ - YZ = 2$, then sec *X* + tan *X* is:

(a) $\frac{1}{\sqrt{6}}$ (b) $\sqrt{6}$
(c) $2\sqrt{6}$ (d) $\frac{\sqrt{6}}{2}$

[SSC, 2012]

23. If $0 < \theta < 90°$, the value of $\sin\theta + \cos\theta$ is:

(a) Equal to 1 (b) Greater than 1
(c) Less than 1 (d) Equal to 2

[SSC, 2012]

24. The expression $\frac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°}$ is equal to:

(a) tan 33°cot 57°
(b) tan 57°cot 37°
(c) tan 33°cot 53°
(d) tan 53°cot 37°

[SSC, 2012]

25. The minimum value of $\sin^2\theta + \cos^2\theta + \sec^2\theta + \operatorname{cosec}^2\theta + \tan^2\theta + \cot^2\theta$ is:

(a) 1 (b) 3
(c) 5 (d) 7

[SSC, 2012]

26. If $2\sin\left(\frac{\pi}{2}\right) = x^2 + \frac{1}{x^2}$, then the value of $\left(x - \frac{1}{x}\right)$ is:

(a) –1 (b) 2
(c) 1 (d) 0

[SSC, 2012]

27. If $\sin^2\alpha + \sin^2\beta = 2$, then the value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is:

(a) 1 (b) −1
(c) 0 (d) 0.5

[SSC, 2011]

28. The value of $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$ is:

(a) −1 (b) $\frac{1}{2}$
(c) 0 (d) 1

[SSC, 2011]

29. If $\sin\theta + \cos\theta = \frac{17}{23}, 0 < \theta < 90°$, then the value of $\sin\theta - \cos\theta$ is

(a) $\frac{5}{17}$ (b) $\frac{3}{19}$
(c) $\frac{7}{10}$ (d) $\frac{7}{13}$

[SSC, 2011]

30. If $\tan\theta \cdot \tan 2\theta = 1$, then the value of $\sin^2 2\theta + \tan^2 2\theta$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{10}{3}$
(c) $3\frac{3}{4}$ (d) 3

[SSC, 2011]

ANSWER KEYS

Exercise-1

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (d)	**11.** (a)	**12.** (b)
13. (c)	**14.** (a)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (c)	**21.** (b)	**22.** (c)	**23.** (b)	**24.** (b)
25. (a)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (b)	**31.** (c)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (a)	**36.** (a)
37. (a)	**38.** (c)	**39.** (a)	**40.** (d)	**41.** (a)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (c)	**46.** (c)	**47.** (c)	**48.** (c)
49. (b)	**50.** (d)	**51.** (a)	**52.** (b)	**53.** (a)	**54.** (b)	**55.** (a)	**56.** (d)	**57.** (c)	**58.** (c)	**59.** (c)	**60.** (c)
61. (b)	**62.** (c)	**63.** (d)	**64.** (b)	**65.** (c)	**67.** (b)	**68.** (c)	**69.** (c)	**70.** (c)	**71.** (c)	**72.** (d)	**73.** (a)
74. (b)	**75.** (a)	**76.** (d)	**77.** (d)	**78.** (b)	**79.** (c)	**80.** (c)					

Exercise-2

1. (c)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (c)	**9.** (d)	**10.** (d)	**11.** (a)	**12.** (a)
13. (d)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (c)	**21.** (b)	**22.** (b)	**23.** (b)	**24.** (b)
25. (c)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (d)	**30.** (c)						

EXPLANATORY ANSWERS

Exercise-1

1. **(b)** $n^2 + n^2\cos A = nm - m^2\cos A$

$$\Rightarrow \quad \cos A = \frac{m^2-n^2}{m^2+n^2}$$

$$\sin^2 A = 1-\cos^2 A = 1-\frac{(m^2-n^2)^2}{(m^2+n^2)^2} = \frac{4m^2n^2}{(m^2+n^2)^2}$$

$$\Rightarrow \quad \sin A = \pm\frac{2mn}{m^2+n^2}$$

$$\therefore \quad \tan A = \pm\frac{2mn}{m^2-n^2}.$$

2. **(c)** $K = \sin 240°\cos 30° + \cos 120°\sin 150°$

$= -\sin 60°\cos 30° + (-\cos 60°)(\sin 30°)$

$$= -\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= -\frac{3}{4}-\frac{1}{4} = -1.$$

3. **(a)** Given $\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$= (\sqrt{2}-1)\cos\theta$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}-1}\sin\theta = \frac{(\sqrt{2}+1)\sin\theta}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$= \sqrt{2}\sin\theta + \sin\theta$

$\Rightarrow \quad \cos\theta - \sin\theta = \sqrt{2}\sin\theta.$

4. **(b)** The given expression

$$= \frac{(1-\sin\alpha)-(1+\sin\alpha)}{\sqrt{1-\sin^2\alpha}} = \frac{-2\sin\alpha}{1\cos\alpha 1}$$

$$= \frac{-2\sin\alpha}{-\cos\alpha} \quad \left[\because \quad \frac{\pi}{2} < \alpha < \pi\right]$$

$= 2\tan\alpha.$

5. **(a)** $p^2 - q^2 = 4\cos\theta\cot\theta = 4\dfrac{\cos^2\theta}{\sin\theta}$

$$\Rightarrow \quad (p^2-q^2)^2 = 16\frac{\cos^4\theta}{\sin^2\theta}$$

$$pq = \cot^2\theta - \cos^2\theta = \cos^2\theta\left(\frac{1-\sin^2\theta}{\sin^2\theta}\right) = \frac{\cos^4\theta}{\sin^2\theta}$$

$\therefore \quad (p^2-q^2)^2 = 16pq.$

6. **(b)** $\operatorname{cosec}\theta = \left(\dfrac{x}{a}\right)^{1/n}, \ \cot\theta = \left(\dfrac{y}{b}\right)^{1/n}$

But $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow \left(\frac{x}{a}\right)^{2/n} - \left(\frac{y}{b}\right)^{2/n} = 1.$$

7. (b) $\dfrac{\sin\theta}{\cos\theta} = \dfrac{p}{q} \Rightarrow \dfrac{p\sin\theta}{q\cos\theta} = \dfrac{p^2}{q^2}$

$$\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\sin\theta} = \frac{p^2 - q^2}{p^2 + q^2}.$$

8. (d) $\sin A = \dfrac{3}{5} \Rightarrow \tan A = -\dfrac{3}{4} \left[\because \dfrac{\pi}{2} < A < \pi\right]$

$\tan B = \dfrac{1}{2},\ \sec B = -\dfrac{\sqrt{5}}{2}$

$$\therefore\ 8\tan A - \sqrt{5}\sec B = 8\left(\frac{-3}{4}\right) - \sqrt{5}\left(\frac{-\sqrt{5}}{2}\right)$$

$$= -6 + \frac{5}{2} = -\frac{7}{2}.$$

9. (b) $\sec\theta - \tan\theta = \dfrac{a+1}{a-1}$

$$\Rightarrow \sec\theta + \tan\theta = \frac{a-1}{a+1}$$

$$\Rightarrow \sec\theta + \tan\theta = \frac{a-1}{a+1}$$

Adding, $2\sec\theta = \dfrac{(a+1)^2 + (a-1)^2}{a^2 - 1} = \dfrac{2(a^2+1)}{a^2-1}$

$$\Rightarrow \sec\theta = \frac{a^2+1}{a^2-1}$$

$$\therefore\ \cos\theta = \frac{a^2-1}{a^2+1}.$$

10. (d) $\dfrac{\tan(270° - 20°) + \tan(360° - 20°)}{\tan(180° + 20°) - \tan(90° + 20°)}$

$$= \frac{\cot 20° - \tan 20°}{\tan 20° + \cot 20°} = \frac{\frac{1}{k} - k}{k + \frac{1}{k}} = \frac{1-k^2}{k^2+1}.$$

11. (a) $\sin 780° \sin 480° + \cos 240° \cos 300°$

$= \sin 60° \sin 60° - \cos 60° \cos 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$$

12. (b) $\tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

$$\therefore\ \sin\theta = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

13. (c) $\dfrac{\cot\theta - \sin\theta}{-\cos\theta + \cot\theta}$

$$= \frac{\left(\frac{4}{3}\right) - \left(\frac{3}{5}\right)}{-\left(\frac{4}{5}\right) + \left(\frac{4}{3}\right)} \quad \left[\begin{array}{l} \therefore \theta < 90° \text{ and } \tan\theta = \frac{3}{4} \\ \sin\theta = \frac{3}{5},\ \cos\theta = \frac{4}{5} \end{array}\right]$$

$$= \frac{11}{8}.$$

14. (a) $\dfrac{\tan 160° - \tan 110°}{1 + \tan 160° \tan 110°}$

$$= \frac{\tan(180° - 20°) - \tan(90° + 20°)}{1 + \tan(180° - 20°)\tan(90° + 20°)}$$

$$= \frac{-\tan 20° + \cot 20°}{1 + (-\tan 20°)(-\cot 20°)}$$

$$= \frac{-\frac{1}{p} + p}{\frac{1+1}{p \times p}}$$

$$= \frac{p^2 - 1}{2p}. \quad (\because \cot 20° = p)$$

15. (c) A lies in second quadrant, $\tan A = -\dfrac{1}{\sqrt{3}}$ B lies in third quadrant, $\tan B = \dfrac{3}{4},\ \cos B = -\dfrac{4}{5}$

$$\therefore\ \frac{2\tan B + \sqrt{3}\tan A}{\cot^2 A + \cos B} = \frac{2\left(\frac{3}{4}\right) + \sqrt{3}\left(-\frac{1}{\sqrt{3}}\right)}{(-\sqrt{3})^2 + \left(-\frac{4}{5}\right)}$$

$$= \frac{\frac{3}{2} - 1}{3 - \frac{4}{5}} = \frac{5}{22}.$$

16. (d) $\dfrac{\sin 150° - 5\cos 300° + 7\tan 225°}{\tan 135° + 3\sin 210°}$

$$= \frac{\sin(180° - 30°) - 5\cos(360° - 60°) + 7\tan(180° + 45°)}{\tan(180° - 45°) + 3\sin(180° + 30°)}$$

$$= \frac{\sin 30° - 5\cos 60° + 7\tan 45°}{-\tan 45° - 3\sin 30°} = \frac{\frac{1}{2} - \frac{5}{2} + 7}{-1 - \frac{3}{2}} = -2.$$

17. (d) $f(x) = \cos^2 x + \sec^2 x = (\cos x - \sec x)^2 + 2$

$\Rightarrow f(x) \geq 2.$

18. **(c)** Given $\operatorname{cosec}\theta + \cot\theta = p$

$\Rightarrow \quad \operatorname{cosec}\theta - \cot\theta = \dfrac{1}{p}$

$\Rightarrow \quad \operatorname{cosec}\theta = \dfrac{1}{2}\left(p + \dfrac{1}{p}\right) = \dfrac{p^2+1}{2p}$

$\cot\theta = \dfrac{p^2-1}{2p}$

$\therefore \quad \cos\theta = \dfrac{\cot\theta}{\operatorname{cosec}\theta} = \dfrac{p^2-1}{p^2+1}.$

19. **(a)** Given $180° < \theta < 270° \quad \Rightarrow \quad \tan\theta = \dfrac{7}{24}$

$\therefore \quad \dfrac{7\cot\theta - 24\tan\theta}{7\cot\theta + 24\tan\theta} = \dfrac{7\left(\dfrac{24}{7}\right) - 24\left(\dfrac{7}{24}\right)}{7\left(\dfrac{24}{7}\right) + 24\left(\dfrac{7}{24}\right)}$

$= \dfrac{24-7}{7+24} = \dfrac{17}{31}.$

20. **(c)** $\dfrac{(m^2-n^2)^2}{mn} = \dfrac{(4\tan A\sin A)^2}{\tan^2 A - \sin^2 A}$

$= \dfrac{16\sin^4 A}{\cos^2 A} \cdot \dfrac{\cos^2 A}{\sin^2 A(1-\cos^2 A)}$

$= \dfrac{16\sin^4 A}{\sin^2 A\sin^2 A} = 16.$

21. **(b)** $m = \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$

$n = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

$(m^2 n)^{2/3} + (mn^2)^{2/3}$

$= \left(\dfrac{\cos^4\theta}{\sin^2\theta} \cdot \dfrac{\sin^2\theta}{\cos\theta}\right)^{2/3} + \left(\dfrac{\cos^2\theta}{\sin\theta} \cdot \dfrac{\sin^4\theta}{\cos^2\theta}\right)^{2/3}$

$= \cos^2\theta + \sin^2\theta$

$= 1.$

22. **(c)** $\cos 1°\cos 2°\cos 3° \ldots \cos 179°$

$= \cos 1°\cos 2°\cos 3° \ldots \cos 90° \ldots \cos 178°\cos 179°$

$[\therefore \quad \cos 90° = 0]$

$= 0.$

23. **(b)** $\sin(90° - 42°)\sec 42° + \cos(90° - 42°)\operatorname{cosec} 42°$

$= \cos 42°\sec 42° + \sin 42°\operatorname{cosec} 42°$

$= 1 + 1 = 2.$

24. **(b)** $\tan 5°\tan 25°\tan 45°\tan 65°\tan 85°$

$= \tan 5°\tan 25° \cdot 1 \cdot \tan(90° - 25°)\tan(90° - 50°)$

$= \tan 5°\tan 25° \cdot 1 \cdot \cot 25°\cot 5°$

$= 1.$

25. **(a)** $\cos^2 5° + \cos^2 10° + \cos^2 15° + \cdots + \cos^2 90°$

$= (\cos^2 5° + \cos^2 85°) + (\cos^2 10° + \cos^2 80°) + \cdots + (\cos^2 40° + \cos^2 50°) + \cos^2 45° + \cos^2 90°$

$= (1 + 1 + \ldots \text{ 8 times}) + \dfrac{1}{2} + 0$

$= 8\dfrac{1}{2}.$

26. **(b)** $\log(\tan 1°\tan 2°\tan 3° \ldots \tan 88°\tan 89°)$

$= \log(\tan 1°\tan 89°)(\tan 2°\tan 88°) \ldots \tan 45°$

$= \log(\tan 1°\cot 1°)(\tan 2°\cot 2°) \ldots \tan 45°$

$= \log(1 \cdot 1 \cdot 1 \cdot \ldots 1) = \log 1 = 0.$

27. **(a)** $\log\sin 1° \log\sin 2° \ldots \log\sin 90° \ldots \log\sin 179°$

$= \log\sin 1° \log\sin 2° \ldots (0) \log\sin 91° \ldots \log\sin 179°$

$= 0.$

28. **(c)** $\cos 24° + \cos 55° + \cos 155° + \cos 204°$

$= \cos 24° + \cos 55° + \cos(180° - 25°) + \cos(180° + 24°)$

$= \cos 24° + \cos 55° - \cos 25° - \cos 24° = 0.$

29. **(c)** $\cos 24° + \cos 5° + \cos 300° + \cos 175° + \cos 204°$

$= \cos 24° + \cos 5° + \cos(360° - 60°) + \cos(180° - 5°) + \cos(180° + 24°)$

$= \cos 24° + \cos 5° + \cos 60° - \cos 5° - \cos 24°$

$= \dfrac{1}{2}.$

30. **(b)** $\sin^2\theta \geq 1 \quad \Rightarrow \quad \dfrac{(x+y)^2}{4xy} \geq 1$

$\Rightarrow \quad (x+y)^2 \geq 4xy$

$\Rightarrow \quad (x+y)^2 - 4xy \geq 0$

$\Rightarrow \quad (x-y)^2 \geq 0$

$(x - y)^2 >$ is true.

But $(x - y)^2 = 0$ is true only when $x = y$.

31. **(c)** Dividing by $\cos^2\theta$

$7\tan^2\theta + 3 = 4\sec^2\theta$

$\Rightarrow \quad 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$

$\Rightarrow \quad 3\tan^2\theta = 1$

$\Rightarrow \quad \tan^2\theta = \dfrac{1}{3} \quad \Rightarrow \quad \tan\theta = \pm\dfrac{1}{\sqrt{3}}.$

32. **(d)** $m^2 - 1 = \dfrac{\sin^2\alpha - \sin^2\beta}{\sin^2\beta}$

$n^2 - 1 = \dfrac{\tan^2\alpha - \tan^2\beta}{\tan^2\beta}$

$= \dfrac{\sin^2\alpha\cos^2\beta - \sin^2 13\cos^2\alpha}{\cos^2\alpha\cos^2\beta} \times \dfrac{\cos^2\alpha}{\sin^2\beta}$

$$= \frac{\sin^2\alpha(1-\sin^2\beta)-\sin^2\beta(1-\sin^2\alpha)}{\sin^2\beta\cos^2\alpha}$$

$$= \frac{\sin^2\alpha-\sin^2\beta}{\sin^2\beta\cos^2\alpha}$$

$$\therefore\ \frac{m^2-1}{n^2-1} = \frac{\sin^2\alpha\sin^2\beta}{\sin^2\beta}\times\frac{\cos^2\alpha\sin^2\beta}{\sin^2\alpha-\sin^2\beta}$$

$= \cos^2\alpha.$

33. **(a)** $\tan^2 A = \sec^2 A - 1 = \left(a+\frac{1}{4a}\right)^2 - 1$

$$= \left(a-\frac{1}{4a}\right)^2$$

$$\Rightarrow\ \tan A = \pm\left(a-\frac{1}{4a}\right)$$

$$\therefore\ \sec A + \tan A = a + \frac{1}{4a} + a - \frac{1}{4a}$$

or, $a + \frac{1}{4a} - a + \frac{1}{4a} = 2a$ or $\frac{1}{2a}$.

34. **(c)** $\frac{\sin^3 A+\cos^3 A}{\sin A+\cos A} + \frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}$

$$= \frac{(\sin A+\cos A)(\sin^2 A+\cos^2 A+\sin A\cos A)}{\sin A+\cos A}$$

$$+ \frac{(\cos A-\sin A)(\cos^2 A+\sin^2 A+\sin A\cos A)}{\cos A-\sin A}$$

$= 2(\sin^2 A+\cos^2 A) = 2.$

35. **(c)** tan 20° + tan 40° + tan 60° + … + tan 180°

= tan 20° + tan 40° + ⋯ + tan(180° − 40°) + tan(180° − 20°) + tan 180°

= tan 20° + tan 40° + … + tan(180° − 40°) + tan(180° − 20°) + 0

= tan 20° + tan 40° + … −tan 40° − tan 20°

= (tan 20° − tan 20°) + (tan 40° − tan 40°) + ⋯

= 0 + 0 + … = 0.

36. **(a)** $\cos\theta = -\frac{\sqrt{3}}{2}\cdot 90° < \theta < 180°$

$$\Rightarrow\ \tan\theta = -\frac{1}{\sqrt{3}}$$

and, $\sin\alpha = \frac{-3}{5}$ and $180° < \alpha < 270°$

$$\Rightarrow\ \tan\alpha = \frac{3}{4},\ \cos\alpha = \frac{-4}{5}$$

$$\therefore\ \text{Given expression } = \frac{2\left(\frac{3}{4}\right)+\sqrt{3}\left(-\frac{1}{\sqrt{3}}\right)}{(-\sqrt{3})^2+\left(-\frac{4}{5}\right)}$$

$$= \frac{\frac{1}{2}}{\frac{11}{5}} = \frac{5}{22}.$$

37. **(a)** Given expression

= cos 24° + cos 55° − cos 55° − cos 24° + cos 60°

$= \frac{1}{2}.$

38. **(c)** Given expression

$$= \frac{(\cot\theta-\text{cosec}\theta)+(\text{cosec}^2\theta-\cot\theta)}{\cot\theta+\text{cosec}\theta-1}$$

$$= \frac{(\text{cosec}\theta-\cot\theta)[\text{cosec}\theta+\cot\theta-1]}{\cot\theta+\text{cosec}\theta-1}$$

= cosecθ − cotθ.

39. **(a)** Given $\sin\alpha = \frac{\sqrt{3}}{2}, 90° < \alpha < 180°$

$$\Rightarrow\ \tan\alpha = -\sqrt{3}$$

and, $\sin\beta = -\frac{\sqrt{3}}{2}$, $180° < \beta < 270° \ \Rightarrow\ \tan\beta = \sqrt{3}$

$$\text{Given expression } = \frac{4\left(\frac{\sqrt{3}}{2}\right)-3(\sqrt{3})}{-\sqrt{3}-\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{-\sqrt{3}(2)}{-3\sqrt{3}} = \frac{2}{3}.$$

40. **(d)** Given expression $= \frac{1+\cos\theta+1-\cos\theta}{\sqrt{1-\cos^2\theta}}$

$$= \frac{2}{|\sin\theta|}.$$

42. **(a)** Given cosecθ − cotθ = p

$$\Rightarrow\ \text{cosec}\theta+\cot\theta = \frac{1}{p}$$

$$\Rightarrow\ 2\text{cosec}\theta = p+\frac{1}{p}$$

$$\therefore\ \text{cosec}\theta = \frac{1}{2}\left(p+\frac{1}{p}\right).$$

43. **(b)** Given expression = tan1°tan 2°…tan 45°…tan 88°tan 89°

= tan1°tan 2°…1…tan(90° − 2°)tan(90° − 1°)

= (tan1°cot1°)(tan 2°cot 2°)…1

= 1·1·1·…1 = 1.

44. **(c)** We know that

$$\text{cosec}^2\theta \ge 1\ \Rightarrow\ \frac{4xy}{(x+y)^2} \ge 1$$

$\Rightarrow \quad 4xy-(x+y)^2 \geq 0$

$\Rightarrow \quad -(x-y)^2 \geq 0 \quad \Rightarrow \quad (x-y)^2 \leq 0$

But $(x - y)^2$ cannot be negative

$\therefore \quad (x - y)2 = 0$ is possible only when $x = y$

45. **(c)** Given expression

$$= \frac{\sin(360°-60°)\tan(270°-30°)\sec(360°+60°)}{[-\cot(270°+45°)]\cos(180°+30°)[-\operatorname{cosec}(270°+45°)]}$$

$$= \frac{(-\sin 60°)(\cot 30°)(\sec 60°)}{(\tan 45°(-\cos 30°)(\sec 45°)}$$

$$= \frac{-\left(\frac{\sqrt{3}}{2}\right)(\sqrt{3})(2)}{(1)\left(-\frac{\sqrt{3}}{2}\right)(\sqrt{2})} = \frac{6}{\sqrt{6}} = \sqrt{6}.$$

46. **(c)** We have $l = r\theta$, where θ is in radians

Given: $\theta = 18° = 18° \times \frac{\pi}{180°} = \frac{\pi}{10}$ radians

$\therefore \quad$ Length of the arc $= 6\left(\frac{\pi}{10}\right) = \frac{3\pi}{5}$.

47. **(c)** Given $x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow \quad x^2 - 2x\cos\theta + 1 = 0$

Since x is real, discriminant ≥ 0

$\Rightarrow \quad 4\cos^2\theta - 4 \geq 0$

$\Rightarrow \quad \cos^2\theta \geq 1 \quad \Rightarrow \quad \cos\theta = \pm 1$

As $\cos\theta$ cannot be > 1 or < -1, $\cos\theta = \pm 1$.

48. **(c)** $1^c = \frac{180}{\pi} = \frac{180 \times 7}{22} = 57°$ (approx.)

$\Rightarrow \quad \sin 1 = \sin 57°$

$\therefore \quad \sin 45° < \sin 1 < \sin 60° \quad \Rightarrow \quad \frac{1}{\sqrt{2}} < \sin 1 < \frac{\sqrt{3}}{2}$

$\Rightarrow \quad 0.7 < \sin 1 < 0.8$

Also, $\sin 0° < \sin 1° < \sin 30° \quad \Rightarrow \quad 0 < \sin 1° < 0.5$

$\therefore \quad \sin 1° < \sin 1$.

49. **(d)** $1^c = \frac{180}{\pi} = 57°$ (approx.)

$\Rightarrow \quad \tan 1 = \tan 57° > 0$

Also, $\tan 2 = \tan 114° < 0$

$\therefore \quad \tan 1 > \tan 2$.

50. **(d)** $\cos^2\theta + \sec^2\theta = (\cos\theta - \sec\theta)^2 + 2\cos\theta\sec\theta$

$= (\cos\theta - \sec\theta)^2 + 2$

As $(\cos\theta - \sec\theta)^2$ being a perfect square is always positive, $\cos^2\theta + \sec^2\theta$ is always greater than 2.

51. **(a)** Given $\sin\alpha = \frac{2pq}{p^2+q^2}$

$\Rightarrow \quad \sec\theta = -\frac{p^2+q^2}{p^2-q^2}$, $\tan\alpha = \frac{2pq}{p^2-q^2}$

Given expression $= \frac{p^2+q^2}{p^2-q^2} - \frac{2pq}{p^2-q^2}$

$= \frac{(p-q)^2}{p^2-q^2} = \frac{p-q}{p+q}$.

52. **(b)** Given $\sin A = \frac{12}{13}$, A lies in the second quadrant and $\sec B = \frac{5}{3}$, B lies in the fourth quadrant.

$\Rightarrow \quad \tan A = \frac{12}{5} \tan B = \frac{4}{3}$

Given expression $= 5\left(-\frac{12}{5}\right) + 3\left(\frac{16}{9}\right)$

$= -12 + \frac{16}{3} = -\frac{20}{3}$.

53. **(a)** Give expression

$= \frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$.

54. **(b)** $\tan(A+B) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$

$$= \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$\Rightarrow \quad A + B = \frac{\pi}{4}$.

55. **(a)** $\frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{7}{24} \quad \Rightarrow \quad \frac{\frac{4}{3} - \tan B}{1 + \frac{4}{3}\tan B} = \frac{7}{24}$

$\Rightarrow \quad \tan B = \frac{3}{4}$

$\cot\left(A + B = \frac{\cot A \cot + B - 1}{\cot B + \cot A}\right)$

$= \frac{\left(\frac{3}{4}\right)\left(\frac{4}{3}\right) - 1}{\left(\frac{4}{3}\right) + \left(\frac{3}{4}\right)} = 0$

$\Rightarrow \quad A + B = \frac{\pi}{2}$.

56. (d) Given expression = tan(69° + 66°)

= tan(135°)

= tan(180° − 45°)

= − tan45° = − 1.

57. **(a)** Given expression

= sin(75° + 15°)sin(75° − 15°)

= sin 90°sin 60°

$$=1\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}.$$

58. **(c)** $\cos\alpha=\frac{15}{17}, \cos\beta=\frac{12}{13}, \sin\beta=\frac{5}{13}$

$\therefore\quad \cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$$=\left(\frac{15}{17}\right)\left(\frac{12}{13}\right)+\left(\frac{8}{17}\right)\left(\frac{5}{13}\right)$$

$$=\frac{220}{221}.$$

59. **(c)** Given expression

$$=\sin^2\theta+(\sin\theta\cos 60^\circ+\cos\theta\sin 60^\circ)^2$$
$$+(\sin\theta\cos 60^\circ-\cos\theta\sin 60^\circ)^2$$

$$=\sin^2\theta+2(\sin^2\theta\cos^2 60^\circ+\cos^2\theta\sin^2 60^\circ)$$

$$=\sin^2\theta+\frac{1}{2}\sin^2\theta+\frac{3}{2}\cos^2\theta$$

$$=\frac{3}{2}(\sin^2\theta+\cos^2\theta)=\frac{3}{2}.$$

60. **(c)** $\tan(\alpha+\beta)=\dfrac{\dfrac{m}{m+1}+\dfrac{1}{2m+1}}{1-\left(\dfrac{m}{m+1}\right)\left(\dfrac{1}{2m+1}\right)}$

$$=\frac{2m^2+m+m+1}{2m^2+2m+m+1-m}$$

$$=\frac{2m^2+2m+1}{2m^2+2m+1}=1$$

$$\therefore\quad \alpha+\beta=\frac{\pi}{4}.$$

61. **(b)** Given expression $=\dfrac{\cos10^\circ-\sqrt{3}\sin10^\circ}{\sin10^\circ\cos10^\circ}$

$$=\frac{\left[2\left(\frac{1}{2}\right)\cos10^\circ-\left(\frac{\sqrt{3}}{2}\right)\sin10^\circ\right]}{\left(\frac{1}{2}\right)\sin20^\circ}$$

$$=2(\sin30^\circ\cos10^\circ-\cos30^\circ\sin10^\circ)\times\frac{2}{\sin20^\circ}$$

$$=\frac{4\sin20^\circ}{\sin20^\circ}=4.$$

62. **(c)** Given expression $=\sqrt{2+\sqrt{2(2\cos^2 2A)}}$

$$=\sqrt{2+2\cos2A}$$

$$=\sqrt{4\cos^2 A}=2\cos A.$$

63. **(a)** Given $\tan A=\dfrac{2\sin^2\left(\frac{B}{2}\right)}{2\sin\left(\frac{B}{2}\right)\cos\left(\frac{B}{2}\right)}$

$$=\tan\left(\frac{B}{2}\right)$$

$$\Rightarrow\quad A=\frac{B}{2}\quad\Rightarrow\quad 2A=B$$

$$\Rightarrow\quad \tan2A=\tan B.$$

64. **(b)** Given expression $=\dfrac{\sin\left[\frac{\pi}{2}-2\theta\right]}{1-\cos\left(\frac{\pi}{2}-2\theta\right)}$

$$=\frac{2\sin\left(\frac{\pi}{4}-\theta\right)\cos\left(\frac{\pi}{4}-\theta\right)}{2\sin^2\left(\frac{\pi}{4}-\theta\right)}$$

$$=\cot\left(\frac{\pi}{4}-\theta\right).$$

65. **(a)** Given expression

$$=\frac{\tan40^\circ+\tan20^\circ}{1-\cot(90^\circ-20^\circ)\cot(90^\circ-40^\circ)}$$

$$=\frac{\tan40^\circ+\tan20^\circ}{1-\tan20^\circ\tan40^\circ}$$

$$=\tan(40^\circ+20^\circ)=\tan60^\circ=\sqrt{3}.$$

66. **(b)** Given expression

$$=\frac{\sqrt{3}}{\sin20^\circ}-\frac{1}{\cos20^\circ}=\frac{\sqrt{3}\cos20^\circ-\sin20^\circ}{\sin20^\circ\cos20^\circ}$$

$$=\frac{2\left[\left(\frac{\sqrt{3}}{2}\right)\cos20^\circ-\left(\frac{1}{2}\right)\sin20^\circ\right]}{\frac{1}{2}\sin40^\circ}$$

$$=\frac{2[\cos30^\circ\cos20^\circ-\sin30^\circ\sin20^\circ]}{\frac{1}{2}\sin40^\circ}$$

$$=\frac{4\cos50^\circ}{\sin40^\circ}=\frac{4\sin40^\circ}{\sin40^\circ}=4.$$

67. (c) Given expression

$= (\tan 81° + \tan 9°) - (\tan 63° + \tan 27°)$

$= (\cot 9° + \tan 9°) - (\cot 27° + \tan 27°)$

$= \dfrac{1}{\sin 9° \cos 9°} = \dfrac{1}{\sin 27° \cos 27°}$

$= \dfrac{2}{\sin 18°} - \dfrac{2}{\sin 54°} = \dfrac{2(4)}{\sqrt{5}-1} - \dfrac{2(4)}{\sqrt{5}+1}$

$= \dfrac{8(\sqrt{5}+1-\sqrt{5}+1)}{4} = 4.$

68. (a) We have, $5x = 3x + 2x$

$\Rightarrow \quad \tan 5x = \dfrac{\tan 3x + \tan 2x}{1 - \tan 3x \tan 2x}$

$\Rightarrow \quad \tan 5x - \tan 5x \tan 3x \tan 2x = \tan 3x + \tan 2x$

$\Rightarrow \quad \tan 5x - \tan 3x - \tan 2x = \tan 5x \tan 3x \tan 2x.$

69. (b) $\tan(A+B) = \dfrac{\dfrac{n}{n+1} + \dfrac{1}{2n+1}}{1 - \dfrac{n}{n+1} \times \dfrac{1}{2n+1}}$

$= \dfrac{2n^2 + n + n + 1}{2n^2 + 2n + n + 1 - n}$

$= \dfrac{2n^2 + 2n + 1}{2n^2 + 2n + 1} = 1.$

70. (b) A, B are positive and each less than 90°

$\therefore \quad \cos A = \dfrac{3}{\sqrt{10}}, \ \cos B = \dfrac{2}{\sqrt{5}}$

$\therefore \quad \sin(A+B) = \left(\dfrac{1}{\sqrt{10}}\right)\left(\dfrac{2}{\sqrt{5}}\right) + \left(\dfrac{3}{\sqrt{10}}\right)\left(\dfrac{1}{\sqrt{5}}\right)$

$= \dfrac{5}{\sqrt{50}} = \dfrac{5}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\therefore \quad A + B = \dfrac{\pi}{4}.$

71. (b) Given expression

$= \dfrac{\dfrac{2\sin^2\theta}{2} + \dfrac{2\sin\theta}{2}\dfrac{\cos\theta}{2}}{\dfrac{2\cos^2\theta}{2} + \dfrac{2\sin\theta}{2}\dfrac{\cos\theta}{2}}$

$= \dfrac{\dfrac{2\sin\theta}{2}\left[\dfrac{\sin\theta}{2} + \dfrac{\cos\theta}{2}\right]}{\dfrac{2\cos\theta}{2}\left[\dfrac{\sin\theta}{2} + \dfrac{\cos\theta}{2}\right]}$

$= \tan\dfrac{\theta}{2}.$

72. (a) We have, $\tan A = \dfrac{1 - \cos 2A}{\sin 2A}$. Put $A = 7\dfrac{1}{2}°$

$\tan 7\dfrac{1}{2}° = \dfrac{1 - \cos 15°}{\sin 15°} = \dfrac{1 - \dfrac{\sqrt{3}+1}{2\sqrt{2}}}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \dfrac{2\sqrt{2} - (\sqrt{3}+1)}{\sqrt{3}-1}.$

73. (a) $\dfrac{4\cos^3 A - 3\cos A + 3\sin A - 4\sin^3 A}{\cos A - \sin A}$

$= 1 - K \sin 2A$

$\Rightarrow \quad 1 + 2 \sin 2A = 1 - K \sin 2A$

$\Rightarrow \quad K = -2.$

74. (b) $\tan 45° = \tan(57° - 12°)$

$1 = \dfrac{\tan 57° - \tan 12°}{1 + \tan 57° \tan 12°}$

$\Rightarrow \quad \tan 57° - \tan 12° = 1 + \tan 57° \tan 12°$

$\Rightarrow \quad \tan 57° - \tan 12° - \tan 57° \tan 12° = 1.$

75. (a) Given expression

$= \sqrt{4\sin^4\theta + 4\sin^2\theta\cos^2\theta} + 2\left[1 + \cos\left(\dfrac{\pi}{2} - \theta\right)\right]$

$= \sqrt{4\sin^2\theta(\sin^2\theta + \cos^2\theta)} + 2(1 + \sin\theta)$

$= 2|\sin\theta| + 2 + 2\sin\theta$

$= -2\sin\theta + 2 + 2\sin\theta$

$= 2$(since $180° < \theta < 270° \quad \Rightarrow \quad |\sin\theta| = -\sin\theta$)

76. (d) $\tan 225° = \tan(100° + 125°)$

$1 = \dfrac{\tan 100° + \tan 125°}{1 - \tan 100° \tan 125°}$

$\Rightarrow \quad \tan 100° + \tan 125° + \tan 100° \tan 125° = 1.$

77. (d) Given expression

$= \sqrt{\dfrac{(1+\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)}} = \dfrac{1+\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta.$

78. (b) $\tan\theta = \dfrac{1 + \tan 15°}{1 - \tan 15°} = \dfrac{\tan 45° + \tan 15°}{1 - \tan 45° \tan 15°}$

$= \tan(45° + 15°)$

$$= \tan 60° = \tan\frac{\pi}{3}$$

$$\therefore \quad \theta = \frac{\pi}{3}.$$

79. **(c)** $\tan 45° = \tan(56° - 11°)$

$$1 = \frac{\tan 56° - \tan 11°}{1 + \tan 56° \tan 11°}$$

$1 + \tan 56° \tan 11° = \tan 56° - \tan 11°$

$\therefore \quad \tan 56° - \tan 11° - \tan 56° \tan 11° = 1.$

80. **(c)** $\cot(A + B) = \cot 45°$

$$\Rightarrow \frac{\cot A \cot B - 1}{\cot B + \cot A} = 1$$

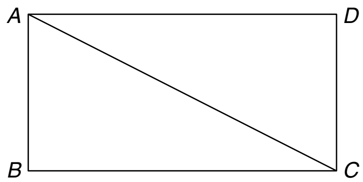
$\Rightarrow \cot A \cot B - \cot A - \cot B + 1 = 1 + 1$

$\Rightarrow (\cot A - 1)(\cot B - 1) = 2$

$$\Rightarrow K = \frac{1}{2}.$$

Exercise-2
(Based on Memory)

1. **(c)**



$\angle ACD = 45°$

$\angle BAC = 45°$

$\therefore \quad (\tan^2 \angle CAD + 1) \cdot \sin^2 \angle BAC$

$= (\tan^2 45° + 1) \sin^2 45°$

$$= (1+1) \times \left(\frac{1}{\sqrt{2}}\right)^2 = 2 \times \frac{1}{2} = 1$$

2. **(b)** $\tan x = \sin 45° \cdot \cos 45° + \sin 30°$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

$\therefore \quad \tan x = \tan 45° \quad \Rightarrow \quad x = 45°$

3. **(c)** $$\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = \sqrt{\frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}} = \sqrt{\frac{\frac{1 - \cos\theta}{\cos\theta}}{\frac{1 + \cos\theta}{\cos\theta}}}$$

$$= \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \sqrt{\frac{(1 - \cos\theta)(1 - \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}}$$

(Rationalising the numerator and the denominator)

$$= \sqrt{\frac{(1 - \cos\theta)^2}{1 - \cos^2\theta}} = \sqrt{\frac{(1 - \cos\theta)}{\sin^2\theta}}$$

$$= \frac{1 - \cos\theta}{\sin\theta} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \operatorname{cosec}\theta - \cot\theta.$$

4. **(b)** Let, the angles be A and B where $A > B$.

$$\therefore \quad A + B = 135° \text{ and } A - B = \frac{\pi}{12} = \frac{\pi}{12} \times \frac{180°}{\pi} = 15°$$

On adding both, we have,

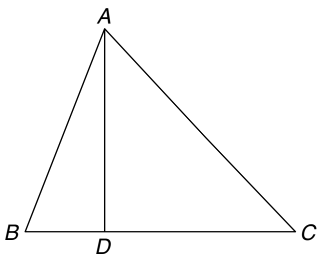
$A + B + A - B = 135° + 15° = 150°$

$$\Rightarrow \quad 2A = 150° \quad \Rightarrow \quad A = \frac{150}{2} = 75°$$

$\therefore \quad A + B = 135°$

$\Rightarrow \quad B = 135° - 75° = 60°$

5. **(c)**



$$\angle B = \frac{\pi}{3}, \angle C = \frac{\pi}{3} \text{ and } \frac{BD}{DC} = \frac{1}{3}$$

From ABD, we have,

$$\frac{BD}{\sin \angle BAD} = \frac{AD}{\sin \angle ABD}$$

$$\Rightarrow \quad \frac{BD}{\sin \angle BAD} = \frac{AD}{\sin\frac{\pi}{3}}$$

$$\Rightarrow \quad \frac{BD}{\sin \angle BAD} = \frac{AD}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \quad AD = \frac{\sqrt{3}}{2} \cdot \frac{BD}{\sin \angle BAD} \qquad \ldots(1)$$

From ADC, we have,

$$\frac{CD}{\sin \angle DAC} = \frac{AD}{\sin \angle ACD}$$

$$\Rightarrow \quad \frac{CD}{\sin \angle DAC} = \frac{AD}{\sin \frac{\pi}{4}}$$

$$\Rightarrow \quad AD = \frac{1}{\sqrt{2}} \cdot \frac{CD}{\sin \angle DAC} \qquad \ldots(2)$$

From equations (1) and (2), we have,

$$\frac{\sqrt{3}}{2} \cdot \frac{BD}{\sin \angle BAD} = \frac{1}{\sqrt{2}} \cdot \frac{CD}{\sin \angle DAC}$$

$$\Rightarrow \quad \frac{\sin \angle BAD}{\sin \angle DAC} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} \times \frac{BD}{CD}$$

$$\Rightarrow \quad \frac{\sin \angle BAD}{\sin \angle DAC} = \frac{\sqrt{3}}{2} \times \sqrt{2} \times \frac{1}{3}$$

$$= \frac{1}{\sqrt{2} \times \sqrt{3}} = \frac{1}{\sqrt{6}}$$

6. **(a)** $\sin 3A = \cos (A - 26°)$

$$\Rightarrow \quad \cos(90° - 3A) = \cos(A - 26°)$$

$$\Rightarrow \quad 90° - 3A = A - 26°$$

$$\Rightarrow \quad 90° + 26° = 3A + A$$

$$\Rightarrow \quad 4A = 116°$$

$$\Rightarrow \quad A = \frac{116°}{4} = 29°$$

7. **(a)** $\sec^2\theta - \dfrac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta}$

$$= \sec^2\theta - \frac{\sin^2\theta(1 - 2\sin^2\theta)}{\cos^2\theta(2\cos^2\theta - 1)}$$

$$= \sec^2\theta - \frac{\sin^2\theta[1 - 2(1 - \cos^2\theta)]}{\cos^2\theta(2\cos^2\theta - 1)}$$

$$= \sec^2\theta - \tan^2\theta \frac{(2\cos^2\theta - 1)}{2\cos^2\theta - 1}$$

$$= \sec^2\theta - \tan^2\theta = 1$$

8. **(c)** $x = a(\sin\theta + \cos\theta)$ and $y = b(\sin\theta - \cos\theta)$

$$= \frac{x}{a} = \sin\theta + \cos\theta \text{ and } \frac{y}{b} = \sin\theta - \cos\theta$$

$$\therefore \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$$

$$= \sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta + \sin^2\theta$$

$$+ \cos^2\theta - 2\sin\theta \cdot \cos\theta$$

$$= 2(\sin^2\theta + \cos^2\theta) = 2$$

9. **(d)** $\sin 5\theta = \cos 20°$

$$\Rightarrow \quad \sin 5\theta = \sin(90° - 20°) = \sin 70°$$

$$\Rightarrow \quad 5\theta = 70°$$

$$\Rightarrow \quad \theta = \frac{70°}{5} = 14°$$

10. **(d)** $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$ $\qquad \ldots(1)$

$\cos\theta - \sin\theta = x$ $\qquad \ldots(2)$

On squaring and adding both the equations, we get,

$\sin^{2\theta} + \cos^{2\theta} + 2\sin\theta.\cos\theta + \cos^{2\theta} + \sin^{2\theta} - 2\sin\theta.\cos\theta = 2\cos^{2\theta} + x^2$

$$\Rightarrow \quad 2 = 2\cos^2\theta + x^2$$

$$\Rightarrow \quad x^2 = 2(1 - \cos^2\theta) = 2\sin^2\theta$$

$$\Rightarrow \quad x = \sqrt{2}\sin\theta$$

11. **(a)** $x \sin 45° = y \operatorname{cosec} 30°$

$$\Rightarrow \quad x \times \frac{1}{\sqrt{2}} = y \times 2$$

$$\Rightarrow \quad \frac{x}{y} = 2\sqrt{2}$$

$$\Rightarrow \quad \frac{x^4}{y^4} = (2\sqrt{2})^4 = 2^4 \times 2^2 = 2^6 = 4^3$$

12. **(a)** $\tan\theta + \cot\theta = 2$

$$\Rightarrow \quad \tan\theta + \frac{1}{\tan\theta} = 2 \quad \Leftrightarrow \quad \tan^2\theta + 1 = 2\tan\theta$$

$$\Rightarrow \quad \tan^2\theta - 2\tan\theta + 1 = 0$$

$$\Rightarrow \quad (\tan\theta - 1)^2 = 0 \quad \Leftrightarrow \quad \tan\theta = 1$$

$$\therefore \quad \cot\theta = \frac{1}{\tan\theta} = 1$$

$$\therefore \quad \tan^{100}\theta + \cot^{100}\theta = 1 + 1 = 2$$

13. **(d)** The given expression $= \dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta}$

$$= \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta} = \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta(1 - \tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)} = \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta} = \tan\theta + \cot\theta + 1$$

14. **(b)** $\sec\theta = x + \frac{1}{4x} = \frac{4x^2 + 1}{4x}$

$\because\ \tan\theta = \sqrt{\sec^2\theta - 1}$

$$= \sqrt{\left(\frac{4x^2 + 1}{4x}\right)^2 - 1} = \sqrt{\frac{(4x^2 + 1)^2 - (4x)^2}{(4x)^2}}$$

$$= \sqrt{\frac{(4x^2 + 1 - 4x)(4x^2 + 1 + 4x)}{(4x)^2}}$$

$$= \sqrt{\frac{[(2x)^2 + (1)^2 - 2\times1\times2x][(2x)^2 + (1)^2 - 2\times1\times2x]}{(4x)^2}}$$

$$= \sqrt{\frac{(2x - 1)^2(2x + 1)^2}{(4x)^2}}$$

$$= \frac{(2x + 1)(2x - 1)}{4x} = \frac{4x^2 - 1}{4x}$$

$$\therefore\ \sec\theta + \tan\theta = \frac{4x^2 + 1}{4x} + \frac{4x^2 - 1}{4x}$$

$$= \frac{4x^2 + 1 + 4x^2 - 1}{4x} = \frac{8x^2}{4x} = 2x$$

15. **(c)** Sum of remaining two angles $= \pi - \frac{5\pi}{9} = \frac{4\pi}{9}$

$\therefore$ Each angle $= \frac{1}{2} \times \frac{4\pi}{9} = \frac{2\pi}{9}$

16. **(a)** $x = r\cos\theta \cdot \cos\phi$

$y = r\cos\theta \cdot \sin\phi$

$z = r\sin\theta$

$$\therefore x^2 + y^2 + z^2 = r^2\cos^2\theta \cdot \cos^2\phi + r^2\cos^2\theta \cdot \sin^2\phi + r^2\sin^2\theta$$

$$= r^2\cos^2\theta(\cos^2\phi + \sin^2\phi) + r^2\sin^2\theta$$

$$= r^2\cos^2\theta + r^2\sin^2\theta$$

$$= r^2(\cos^2\theta + \sin^2\theta) = r^2$$

17. **(c)** $5\cos\theta + 12\sin\theta = 13$

Dividing by $\cos\theta$, we get

$5 + 12\tan\theta = 13\sec\theta$

On squaring, we have

$25 + 144\tan^2\theta + 120\tan\theta = 169\sec^2\theta = 169(1 + \tan^2\theta)$

$\Rightarrow\ 169\tan^2\theta - 144\tan^2\theta - 120\tan\theta = 169 - 25$

$\Rightarrow\ 25\tan^2\theta - 120\tan\theta + 144 = 0$

$\Rightarrow\ (5\tan\theta - 12)^2 = 0$

$\Rightarrow\ 5\tan\theta = 12$

$\Rightarrow\ \tan\theta = \frac{12}{5}$

18. **(b)** $\sec^2 12° - \cot^2 78°$

$= \sec^2 12° - \cot^2(90° - 12°)$

$= \sec^2 12° - \tan^2 12° = 1$

$[\because \sec^2\theta - \tan^2\theta = 1]$

19. **(d)** $\tan\theta \cdot \cos 60° = \frac{\sqrt{3}}{2}$

$\Rightarrow\ \tan\theta \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

$\Rightarrow\ \tan\theta = \sqrt{3} = \tan 60°$

$\Rightarrow\ \theta = 60°$

$\therefore\ \sin(\theta - 15°) = \sin(60° - 15°) = \sin 45° = \frac{1}{\sqrt{2}}$

20. **(c)** $\tan 2\theta \cdot \tan 3\theta = 1$

$\Rightarrow\ \tan 3\theta = \frac{1}{\tan 2\theta} = \cot 2\theta$

$\Rightarrow\ \tan 3\theta = \tan(90° - 2\theta)$

$\Rightarrow\ 3\theta = 90° - 2\theta \Rightarrow 5\theta = 90° \Rightarrow \theta = 18°$

$\therefore\ 2\cos^2\frac{5\theta}{2} - 1 = 2\cos^2 45° - 1 = 2 \times \frac{1}{2} - 1 = 0$

21. **(b)** $\sin 17° = \frac{x}{y}$

$\Rightarrow\ \cos 17° = \sqrt{1 - \sin^2 17°}$

$$= \sqrt{1 - \frac{x^2}{y^2}} = \sqrt{\frac{y^2 - x^2}{y^2}} = \frac{\sqrt{y^2 - x^2}}{y}$$

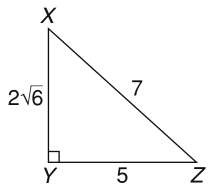
$\therefore\ \sec 17° = \dfrac{y}{\sqrt{y^2 - x^2}}$

$\sin 73° = \sin(90° - 17°) = \cos 17°$

$\therefore\ \sec 17° - \sin 73° = \dfrac{y}{\sqrt{y^2 - x^2}} - \dfrac{\sqrt{y^2 - x^2}}{y}$

$= \dfrac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \dfrac{x^2}{y\sqrt{y^2 - x^2}}$

22. (b)



$XZ - YZ = 2$

$\Rightarrow\ XY^2 + YZ^2 = XZ^2$

$\Rightarrow\ \left(2\sqrt{6}\right)^2 = XZ^2 - YZ^2$

$\Rightarrow\ 24 = (XZ - YZ)(XZ + YZ)$

$\Rightarrow\ XZ + YZ = 12$

Adding both the equations, we have

$2XZ = 14 \Rightarrow XZ = 7 \qquad \therefore\ YZ = 7 - 2 = 5$

$\sec X = \dfrac{7}{2\sqrt{6}}$

$\tan X = \dfrac{5}{2\sqrt{6}}$

$\therefore\ \sec X + \tan X = \dfrac{7}{2\sqrt{6}} + \dfrac{5}{2\sqrt{6}} = \dfrac{12}{2\sqrt{6}} = \sqrt{6}$

23. (b) $Z = \sin\theta + \cos\theta$

$\Rightarrow Z^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta$

$= 1 + 2\sin\theta\cdot\cos\theta$

Now, $\because\ 0 < \theta < 90° \qquad \therefore\ \sin\theta < 1; \cos\theta < 1$

$\Rightarrow\ 2\sin\theta\cdot\cos\theta < 1$

$\therefore\ Z^2 < 2 \quad \Rightarrow\ Z < \sqrt{2} \quad \Rightarrow\ Z < 1.41$

Clearly, the value of $\sin\theta + \cos\theta$ is greater than 1.

24. (b) $\dfrac{\tan 57° + \cot 37°}{\tan 33° + \cot 53°} = \dfrac{\cot 33° + \tan 53°}{\tan 33° + \cot 53°}$

$[\therefore\ \tan(90° - \theta) = \cot\theta, \cot(90° - \theta) = \tan\theta]$

$= \dfrac{\dfrac{1}{\tan 33°} + \tan 53°}{\tan 33° + \dfrac{1}{\tan 53°}}$

$= \dfrac{1 + \tan 53°\cdot\tan 33°}{\tan 33°\cdot\tan 53° + 1} \times \dfrac{\tan 53°}{\tan 33°}$

$= \tan 53°\cdot\cot 33° = \cot 37°\cdot\tan 57°$

25. (d) $\sin^2\theta + \cos^2\theta + \sec^2\theta + \operatorname{cosec}^2\theta + \tan^2\theta + \cot^2\theta$

$= 1 + \sec^2\theta - \tan^2\theta + \operatorname{cosec}^2\theta - \cot^2\theta + 2(\tan^2\theta + \cot^2\theta)$

$= 3 + 2((\tan\theta - \cot\theta)^2 + 2) > 7$; because $(\tan\theta - \cot\theta)^2 > 0$

26. (d) $x^2 + \dfrac{1}{x^2} = 2\sin\left(\dfrac{\pi}{2}\right)$

27. (c)

$\because\ \sin^2\alpha + \sin^2\beta = 2$

$\Rightarrow\ 1 - \cos^2\alpha + 1 - \cos^2\beta = 2$

$\Rightarrow\ \cos^2\alpha + \cos^2\beta = 0$

$\Rightarrow\ \cos\alpha = 0$ and $\cos\beta = 0$

$\Rightarrow\ \alpha = \dfrac{\pi}{2}$ and $\beta = \dfrac{\pi}{2}$

$\therefore\ \cos\left(\dfrac{\alpha + \beta}{2}\right) = \cos\left(\dfrac{\dfrac{\pi}{2} + \dfrac{\pi}{2}}{2}\right) = \cos\left(\dfrac{\pi}{2}\right) = 0$

28. (d) $\cot\dfrac{\pi}{20}\cdot\cot\dfrac{3\pi}{20}\cdot\cot\dfrac{5\pi}{20}\cdot\cot\dfrac{7\pi}{20}\cdot\cot\dfrac{9\pi}{20}$

$= \cot 9°\cdot\cot 27°\cdot\cot 45°\cdot\cot 63°\cdot\cot 81° [\therefore \pi = 180°]$

$= \cot 9°\cdot\cot 27°\cdot\cot 45°\cdot\cot(90° - 27°)\cdot\cot(90° - 9°)$

$= \cot 9°\cdot\cot 27°\cdot\cot 45°\cdot\tan 27°\cdot\tan 9°$

$[\cot(90° - \theta) = \tan\theta]$

$= (\cot 9°\cdot\tan 9°)\cdot(\cot 27°\cdot\tan 27°)\cdot\cot 45° = 1$

$[\therefore \tan\theta\cdot\cot\theta = 1]$

29. (d) $\sin\theta + \cos\theta = \dfrac{17}{23}$ …(1)

Let, $\sin\theta - \cos\theta = x$ …(2)

On squaring and adding both the equations, we have

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta + \sin^2\theta + \cos^2\theta$

$-2\sin\theta\cdot\cos\theta = \left(\frac{17}{13}\right)^2 + x^2$

$\Rightarrow \quad 2(\sin^2\theta + \cos^2\theta) = \frac{289}{169} + x^2$

$\Rightarrow \quad x^2 = 2 - \frac{289}{169} = \frac{338-289}{169} = \frac{49}{169}$

$\Rightarrow \quad x = \sqrt{\frac{49}{169}} = \frac{7}{13}$

30. (c) $\tan\theta\cdot\tan 2\theta = 1$

$\Rightarrow \quad \tan\theta = \frac{1}{\tan 2\theta} = \cot 2\theta$

$\Rightarrow \quad \tan\theta = \tan(90° - 20)$

$\Rightarrow \quad \theta = 90° - 2\theta$

$\Rightarrow \quad 3\theta = 90° \Rightarrow \theta = 30°$

$\therefore \quad \sin^2 2\theta + \tan^2 2\theta = \sin^2 60° + \tan^2 60°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\sqrt{3}\right)^2 = \frac{3}{4} + 3 = 3\frac{3}{4}$