Heights and Distance

36

INTRODUCTION

Solution of triangles has enormous applications to surveying, navigation, and so on. We shall now consider some simple ones from among them. For this purpose, we need to explain certain terms that are generally used in practical problems.

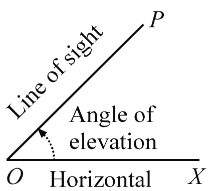


Fig. (a)

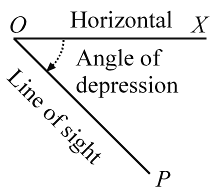


Fig. (b)

1. If OX be a horizontal line through O, the eye of the observer and P be an object in the vertical plane through OX, then $\angle XOP$ is called:

   (i) **the angle of elevation,** if P is *above* OX as shown in Fig. (a); and
   (ii) the **angle of depression,** if P is *below* OX as shown in Fig. (b).

   The straight line OP (joining the eye of the observer to the object) is called the *line of sight* of the observer.

2. Values of the trigonometric ratios for some useful angles:

The values of $\cot\theta$, $\sec\theta$ and $\operatorname{cosec}\theta$ can be found from Table 1, by using the relations $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$.

Table 1

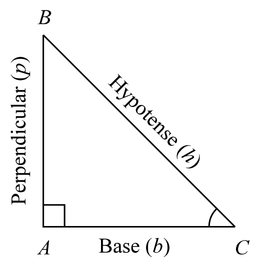
angle (θ) / t-ratio	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

3. **Pythagoras Theorem**

In a right-angled triangle, the square of its hypoteuse is equal to the sum of the squares of its legs (i.e., perpendicular and base).

In other words,

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$



or, $(BC)^2 = (AB)^2 + (AC)^2$ or, $h^2 = p^2 + b^2$.

4. Few important values to memorise:

$$\sqrt{2} = 1.414;\ \sqrt{3} = 1.732;\ \sqrt{5} = 2.236.$$

EXERCISE-1

1. The ratio of the length of a rod and its shadow is $1{:}\sqrt{3}$.

   The angle of elevation of the sum is:

   (a) 30° (b) 45°
   (c) 60° (d) 90°

2. The angle of elevation of moon when the length of the shadow of a pole is equal to its height, is:

(a) 30° (b) 45°
(c) 60° (d) None of these

3. A tower stands on a horizontal plane. A man on the ground 100 m from the base of the tower finds the angle of elevation of the top of the tower to be 30°. What is the height of the tower?

(a) 100 m (b) $100\sqrt{3}$
(c) $100/\sqrt{3}$ (d) None of these

4. When the sun is 30° above the horizontal, the length of shadow cast by a building 50 m high is:

(a) $\frac{50}{\sqrt{3}}$ m (b) $50\sqrt{3}$ m
(c) 25 m (d) $25\sqrt{3}$ m

5. A pole being broken by the wind, the top struck the ground at an angle of 30° and at a distance of 21 m from the foot of the pole. Find out the total height of the pole.

(a) 21 m (b) $21\sqrt{3}$ m
(c) $21/\sqrt{3}$ (d) None of these

6. The upper part of a tree broken by the wind makes an angle of 30° with the ground and the distance from the root to the point where the top of the tree touches the ground is 10 m. What was the height of the tree?

(a) $10\sqrt{3}$ (b) $10/\sqrt{3}$
(c) $20\sqrt{3}$ (d) None of these

7. A tower stands at the end of a straight road. The angles of elevation of the top of the tower from two points on the road 500 m apart are 45° and 60°, respectively. Find out the height of the tower.

(a) $\frac{500\sqrt{3}}{\sqrt{3}-1}$ (b) $5000\sqrt{3}$

(c) $\frac{500\sqrt{3}}{\sqrt{3}+1}$ (d) None of these

8. The shadow of a tower standing on a level plane is found to be 50 m longer when the sun's altitude is 30° than when it is 60°. Find the height of the tower.

(a) $20\sqrt{3}$ m (b) $25/\sqrt{3}$ m
(c) $25\sqrt{3}$ m (d) $20\sqrt{3}$ m

9. In a rectangle, if the angle between a diagonal and a side is 30° and the length of diagonal is 6 cm, the area of the rectangle is:

(a) 9 cm^2 (b) $9\sqrt{3}$ cm^2
(c) 27 cm^2 (d) 36 cm^2

10. The height of a tower is 100 m. When the angle of elevation of the sun changes from 30° to 45°, the shadow of the tower becomes x m smaller. The value of x is:

(a) 100 m (b) $100\sqrt{3}$ m
(c) $100(\sqrt{3} - 1)$ m (d) $\frac{100}{\sqrt{3}}$ m

11. A 20 m high electric pole stands upright on the ground with the help of steel wire to its top and affixed on the ground. If the steel wire makes 60° with the horizontal ground, then find out the length of the steel wire.

(a) $40/\sqrt{3}$ m (b) $40\sqrt{3}$ m
(c) $20/\sqrt{3}$ m (d) $20\sqrt{3}$ m

12. From the top of a lighthouse, 50 m above the sea, the angle of depression of an incoming boat is 30°. How far is the boat from the lighthouse?

(a) $25\sqrt{3}$ m (b) $25/\sqrt{3}$ m
(c) $50\sqrt{3}$ m (d) $50/\sqrt{3}$ m

13. From the top of a 25 m high, cliff the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find out the height of the tower.

(a) 40 m (b) 48 m
(c) 50 m (d) 52 m

14. When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is:

(a) 30° (b) 45°
(c) 60° (d) 75°

15. From the top of 60 m high a lighthouse with its base at sea level, the angle of depression of a boat is 15°. The distance of the boat from the light house is:

(a) $60\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$m (b) $60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m

(c) $30\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$m (d) $30\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m

16. On the level ground, the angle of elevation of the top of the tower is 30°. On moving 20 m nearer, the angle of elevation is 60°. The height of the tower is:

(a) $20\sqrt{3}$ m (b) $10\sqrt{3}$ m
(c) $10(\sqrt{3} - 1)$ m (d) None of these

17. The angle of elevation of the top of a tower from a point 20 m away from its base is 45°. The height of the tower is:

(a) 10 m (b) 20 m
(c) 40 m (d) $20\sqrt{3}$ m

18. At a point, 15 m away from the base of a 15 m high house, the angle of elevation of the top is:

(a) 45° (b) 30°
(c) 60° (d) 90°

19. Angle of depression from the top of a lighthouse of two boats are 45° and 30° due east which are 60 m apart. The height of the light house is:

(a) $60\sqrt{3}$ (b) $30(\sqrt{3}-1)$
(c) $30(\sqrt{3}+1)$ (d) None of these

20. The angle of elevation of the top of a hill from each of the vertices A, B, C of a horizontal triangle is α. The height of the hill is:

(a) $b \tan \alpha \operatorname{cosec} B$ (b) $\frac{a}{2} \tan \alpha \operatorname{cosec} A$

(c) $\frac{c}{2} \tan \alpha \operatorname{cosec} C$ (d) None of these

21. A tower subtends an angle of 30° at a point on the same level as the foot of the tower. At a second point, h m above the first, the depression of the foot of the tower is 60°. The horizontal distance of the tower from the point is:

(a) $h \cot 60°$ (b) $h \cot 30°$

(c) $\frac{h}{2} \cot 60°$ (d) $\frac{h}{2} \cot 30°$

22. If a flag staff of 6 m height placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then the angle that the sun makes with the ground is:

(a) 60° (b) 30°
(c) 45° (d) None of these

23. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 40 m from the bank, he finds the angle to be 30°. The breadth of the river is:

(a) 40 m (b) 60 m
(c) 20 m (d) 30 m

24. The angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole will be:

(a) 30° (b) 60°
(c) 90° (d) 45°

25. The angle of elevation of the top of a TV tower from three points A, B, C in a straight line through the foot of the tower are α, 2α, 3α, respectively. If $AB = a$, the height of the tower is:

(a) $a \tan \alpha$ (b) $a \sin \alpha$
(c) $a \sin 2\alpha$ (d) $a \sin 3\alpha$

26. The angle of elevation of the top of an unfinished tower at a point distant 120 m from its base is 45°. If the elevation of the top at the same point is to be 60°, the tower must be raised to a height:

(a) $120(\sqrt{3}+1)$ m (b) $120(\sqrt{3}-1)$ m

(c) $10(\sqrt{3}+1)$ m (d) None of these

27. A person walking along a straight road towards a hill observes at two points, distance $\sqrt{3}$ Km, the angles of elevation of the hill to be 30° and 60°. The height of the hill is:

(a) $\frac{3}{2}$ Km (b) $\sqrt{\frac{2}{3}}$ Km

(c) $\frac{\sqrt{3}+1}{2}$ Km (d) $\sqrt{3}$ Km

28. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle 30° with the horizontal, then the length of the wire is:

(a) 12 m (b) 10 m
(c) 8 m (d) None of these

29. A man is standing on the 8 m long shadow of a 6 m long pole. If the length of the shadow is 2.4 m, then the height of the man is:

(a) 1.4 m (b) 1.6 m
(c) 1.8 m (d) 2.0 m.

30. The angle of elevation of the top of a tower at a point G on the ground is 30°. On walking 20 m towards the tower, the angle of elevation becomes 60°. The height of the tower is equal to:

(a) $\frac{10}{\sqrt{3}}$ m (b) $20\sqrt{3}$ m

(c) $\frac{20}{\sqrt{3}}$ m (d) $10\sqrt{3}$ m

EXERCISE-2
(BASED ON MEMORY)

1. The top of a 15 m high tower makes an angle of elevation of 60° with the bottom of an electric pole and an angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

(a) 5 m (b) 8 m
(c) 10 m (d) 12 m
(e) None of these

[SBI PO Examination, 1999]

2. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 m from the tower. After 5 second, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming it is running in still water?

(a) 32 Km/h (b) 42 Km/h
(c) 38 Km/h (d) 36 Km/h
(e) 40 Km/h

[SBI Associates PO Examination, 1999]

3. From a point P on a level ground, the angle of elevation of the top of the tower is 30°. If the tower is 100 m high, the distance of the point P from the foot of the tower is:

(a) $100/\sqrt{3}$ m (b) $100\sqrt{3}$ m
(c) $50\sqrt{3}$ m (d) $50/\sqrt{3}$ m

[RRB Gorakhpur ASM Examination, 2002]

4. From a 15 m high bridge on the river, the angle of depression of a boat is 30°. If the speed of the boat be 6 Km/h, then the time taken by the boat to reach exactly below the bridge will be:

(a) $9\sqrt{3}$ second (b) $19/\sqrt{3}$ second
(c) $3\sqrt{3}$ second (d) None of these

[RRB Gorakhpur ASM Examination, 2002]

5. From the top of a tower of height 108 m the angles of depression of two objects on either sides of the tower are 30° and 45°. The distance between the objects are:

(a) $180(3+\sqrt{3})$ m (b) $180(3-\sqrt{3})$ m
c) $180(\sqrt{3}-1)$ m (d) $180(\sqrt{3}+1)$ m

[SSC, 2014]

6. A tower standing on a horizontal plane subtends a certain angle at a point 160 m apart from the foot of the tower. On advancing 100 m towards it, the tower is found to subtend an angle twice as before. The height of the tower is:

(a) 80 m (b) 100 m
(c) 160 m (d) 200 m

[SSC, 2013]

7. The angle of elevation of a tower from a distance 50 m from its foot is 30°. The height of the tower is:

(a) $50\sqrt{3}$ m (b) $\frac{50}{\sqrt{3}}$ m
(c) $75\sqrt{3}$ m (d) $\frac{75}{\sqrt{3}}$ m

[SSC, 2013]

8. $ABCD$ is a rectangle where the ratio of the lengths of AB and BC is 3:2. If P is the midpoint of AB, then the value of sin ($\angle CPB$) is:

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$
(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

[SSC, 2013]

9. $\frac{\sin A}{1+\cos A}+\frac{\sin A}{1-\cos A}$ is $(0° < A < 90°)$.

(a) 2 cosec A (b) 2 sec A
(c) 2 sin A (d) 2 cos A

[SSC, 2013]

10. If $r \sin \theta = 1$, $r\cos\theta=\sqrt{3}$, then the value of $(\sqrt{3}\tan\theta+1)$ is:

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$
(c) 1 (d) 2

[SSC, 2013]

11. The angles of elevation of the top of a tower from the points P and Q, at distances of 'a' and 'b' respectively from the base of the tower and in the same straight line with it are complementary. The height of the tower is:

(a) $\sqrt{ab}$ (b) $\frac{a}{b}$

(c) ab (d) a^2b^2

[SSC Assistant Grade III, 2013]

12. A man from the top of a 100 m high tower sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in m) travelled by the car during this time is:

(a) $100\sqrt{3}$ (b) $\frac{200\sqrt{3}}{3}$

(c) $\frac{100\sqrt{3}}{3}$ (d) $200\sqrt{3}$

[SSC Assistant Grade III, 2012]

13. Two posts are x m apart and the height of one is double that of the other. If from the midpoint of the line joining their feet, an observer finds the angular elevations of their tops to be complementary, then the height (in m) of the shorter post is:

(a) $\frac{x}{2\sqrt{2}}$ (b) $\frac{x}{4}$

(c) $x\sqrt{2}$ (d) $\frac{x}{\sqrt{2}}$

[SSC, 2012]

14. An aeroplane when flying at a height of 5000 m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. The vertical distance between the aeroplanes at that instant is:

(a) $5000\left(\sqrt{3}-1\right)$m (b) $5000\left(3-\sqrt{3}\right)$m

(c) $5000\left(1-\frac{1}{\sqrt{3}}\right)$m (d) 4500 m

[SSC, 2012]

15. The length of a shadow of a vertical tower is $\frac{1}{\sqrt{3}}$ times its height. The angle of elevation of the Sun is:

(a) 30° (b) 45°

(c) 60° (d) 90°

[SSC, 2011]

ANSWER KEYS

Exercise-1

1. (a) **2.** (b) **3.** (c) **4.** (b) **5.** (b) **6.** (a) **7.** (a) **8.** (c) **9.** (b) **10.** (c) **11.** (a) **12.** (c) **13.** (c)
14. (b) **15.** (b) **16.** (b) **17.** (b) **18.** (a) **19.** (c) **20.** (b) **21.** (a) **22.** (a) **23.** (c) **24.** (a) **25.** (c) **26.** (b)
27. (a) **28.** (a) **29.** (c) **30.** (d)

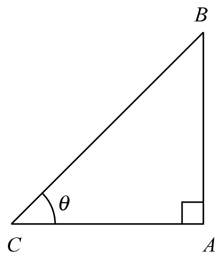
Exercise-2

1. (c) **2.** (a) **3.** (b) **4.** (a) **5.** (d) **6.** (a) **7.** (b) **8.** (d) **9.** (a) **10.** (d) **11.** (a) **12.** (b) **13.** (a)
14. (c) **15.** (c)

EXPLANATORY ANSWERS

EXERCISE-1

1. **(a)** Let, AB be the rod and AC be its shadow.
$\angle ACB = \theta$. Let, $AB = x$.



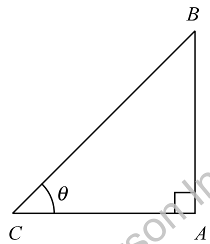
Then, $AC = \sqrt{3}x$.

$\therefore \quad \tan\theta = \dfrac{AB}{AC} = \dfrac{x}{\sqrt{3}x} = \dfrac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = 30°.$

2. **(b)** Let, $AB = x$.
Then, $AC = x$.

$\therefore \quad \tan\ \theta = \dfrac{AB}{AC} = 1$

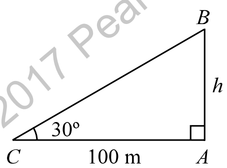
$\Rightarrow \quad \theta = 45°.$



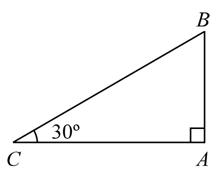
3. **(c)** $\tan\ 30° = \dfrac{h}{100}$

or, $\dfrac{1}{\sqrt{3}} = \dfrac{h}{100}$

or, $h = \dfrac{100}{\sqrt{3}}$ m.



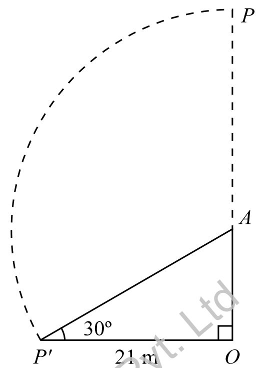
4. **(b)** Let, AB be the building and AC be its shadow.
Then, $AB = 50$ m and $\theta = 30°$.



$\therefore \quad \dfrac{AC}{AB} = \cot 30° = \sqrt{3} \Rightarrow \dfrac{AC}{50} = \sqrt{3}$

$\Rightarrow \quad AC = 50\sqrt{3}$ m.

5. **(b)** Let, OAP be the pole. When broken by wind at A, let its top P strike the ground at P' so that $OP' = 21$ m, $\angle OP'A = 30°$, $AP = AP'$.



We have, $\dfrac{OA}{OP'} = \tan 30° \quad \Rightarrow \quad OA = \dfrac{21}{\sqrt{3}}$.

$\therefore \quad OA = 7\sqrt{3}$.

Also, $\dfrac{AP'}{OP'} = \sec 30°$ or, $\dfrac{AP}{21} = \dfrac{2}{\sqrt{3}}$

$\therefore \quad AP = \dfrac{42}{\sqrt{3}} = 14\sqrt{3} = 14\sqrt{3}$.

Height of the pole $= OP = OA + AP$

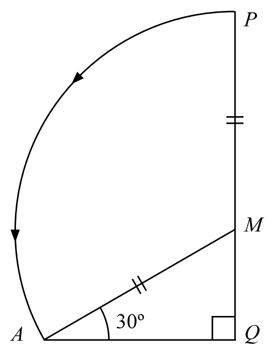
$= 7\sqrt{3} + 14\sqrt{3} = 21\sqrt{3}$.

6. **(a)** Let, QMP be the tree. When broken by the wind, its top P strikes the ground at A such that $\angle QAM = 30°$, $AQ = 10$ m and $MA = MP$.

$\dfrac{MQ}{AQ} = \tan 30° \quad \Rightarrow \quad MQ = \dfrac{10}{\sqrt{3}}$ m

and, $\dfrac{AM}{AQ} = \sec 30° \quad \Rightarrow \quad AM = \therefore\ 10\left(\dfrac{2}{\sqrt{3}}\right) = \dfrac{20}{\sqrt{3}}$.

$\therefore$ Height of the tree:



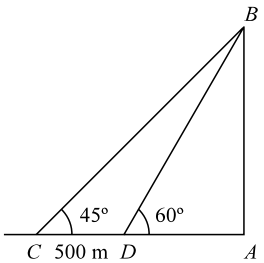
$= QM + MP = QM + AM = \dfrac{10}{\sqrt{3}} + \dfrac{20}{\sqrt{3}}$

$= \dfrac{30}{\sqrt{3}} = 10\sqrt{3} = 10\sqrt{3}$ m.

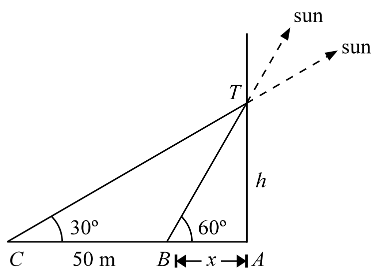
7. **(a)** $CD = AB\ (\cot 45° - \cot 60°)$

or, $AB = \dfrac{CD}{\cot 45° - \cot 60°}$

$= \dfrac{500}{1 - \dfrac{1}{\sqrt{3}}} = \dfrac{500\sqrt{3}}{\sqrt{3} - 1}$ m.



8. **(c)** Let, T be the top of the tower AT. Let, $AT = h$ m. Let, AB and AC be the shadows of the tower when the sun's altitude is 60° and 30°, respectively.



Then, $BC = 50$ m.

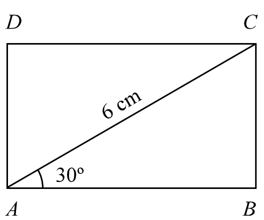
Let, $AB = x$ m. $\dfrac{x}{h} = \cot 60° \Rightarrow x = \dfrac{h}{\sqrt{3}}$ …(1)

Also, $\dfrac{x + 50}{h} = \cot 30° \Rightarrow x + 50 = \sqrt{3}\,h$ …(2)

Subtracting Equation (1) from Equation (2),

$$50 = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)h \Rightarrow h = 25\sqrt{3} \text{ m.}$$

9. **(b)** Let, $ABCD$ be the rectangle in which $\angle BAC = 30°$ and $AC = 6$ cm.

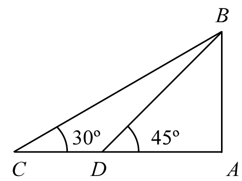


$\dfrac{AB}{AC} = \cos 30° = \dfrac{\sqrt{3}}{2} \Rightarrow AB = 3\sqrt{3}$ cm.

$\dfrac{BC}{AC} = \sin 30° = \dfrac{1}{2} \Rightarrow BC = 3$ cm.

$\therefore$ Area of the rectangle $= AB \times BC = 9\sqrt{3}$ cm^2.

10. **(c)** Let, AB be the tower and AC and AD be its shadows.



Then, $AB = 100$ m.

$\dfrac{AD}{AB} = \cot 45° = 1 \Rightarrow \dfrac{AD}{100} = 1$

$\Rightarrow AD = 100$ m.

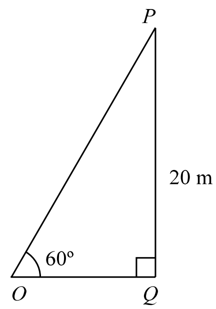
$\dfrac{AC}{AB} = \cot 30° = \sqrt{3} \Rightarrow \dfrac{AC}{100} = \sqrt{3}$

$\Rightarrow AC = 100\sqrt{3}$ m.

$\therefore\ x = AC - AD = 100(\sqrt{3} - 1)$ m.

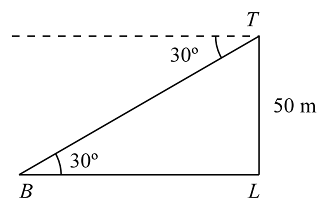
11. **(a)** $\sin 60° = \dfrac{PQ}{OP} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{20}{OP}$

$\Rightarrow OP = 20 \times \dfrac{2}{\sqrt{2}} = \dfrac{40}{\sqrt{3}}$ m.

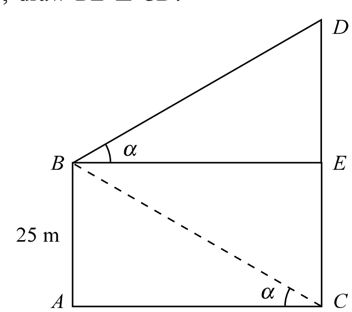


12. **(c)** $\tan 30° = \dfrac{TL}{BL} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{50}{BL}$

$\therefore\ BL = 50\sqrt{3}$ m.



13. **(c)** Let, AB be the cliff and CD be the tower. From B, draw $BE \perp CD$.



$\frac{DE}{BE} = \tan\alpha$ and $\frac{AB}{AC} = \tan\alpha$

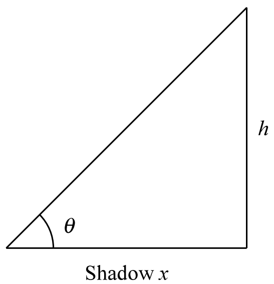
$\therefore \quad \frac{DE}{BE} = \frac{AB}{AC}$

$\therefore \quad DE = AB \qquad (\because\ BE = AC)$

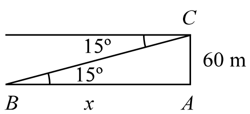
$\therefore \quad CD = CE + DE = AB + AB = 2AB = 50$ m.

14. **(b)** Since $\frac{h}{x} = \tan\theta$ and $h = x$.

$\therefore \quad \tan\theta = 1 \quad \Rightarrow \quad \theta = 45°$.



15. **(b)** Here, B is the position of boat and AC is lighthouse.



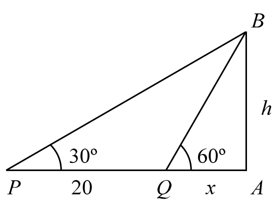
Now, $\frac{AC}{x} = \tan 15° = \tan(45° - 30°)$

$= \frac{1 - \tan 30°}{1 + \tan 30°} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

$\therefore \quad x = 60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ m.

16. **(b)** $\frac{h}{x} = \tan 60° = \sqrt{3}$.

$\therefore \quad h = \sqrt{3}\,x. \qquad \frac{h}{20+x} = \tan 30° = \frac{1}{\sqrt{3}}$

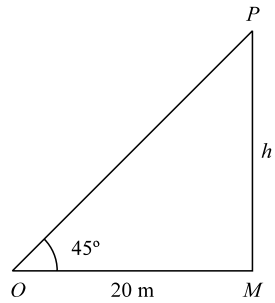


$\therefore \quad \sqrt{3}\,h = 20 + x$

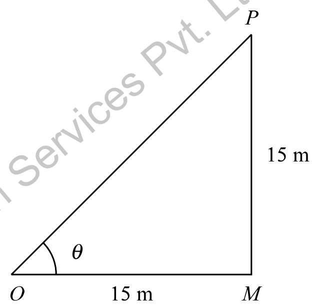
$\therefore \quad \sqrt{3}\,(\sqrt{3}\,x) = 20 + x$ or, $3x = 20 + x$

$\therefore \quad x = 10. \quad \therefore \quad h = 10\sqrt{3}$ m.

17. **(b)** Clearly, $\frac{h}{20} = \tan 45° = 1 \quad \therefore \quad h = 20$ m.

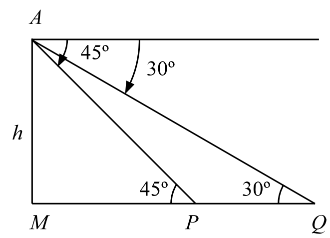


18. **(a)** Let, MP denote the house. Let, θ be the angle of elevation.



$\therefore \quad \tan\theta = \frac{MP}{OM} = \frac{15}{15} = 1. \quad \therefore \quad \theta = 45°$.

19. **(c)** Let, the boats be at P, Q. So that $PQ = 60$ m. Let, MA be the lighthouse.



Let, $h = MA$.

Then, $\frac{h}{MP} = \tan 45° = 1. \quad \therefore \quad h = MP$

Again, $\frac{h}{MP + 60} = \tan 30° = \frac{1}{\sqrt{3}}$.

$\therefore \quad MP + 60 = \sqrt{3}\,h$ or, $h + 60 = \sqrt{3}\,h$

$\therefore \quad (\sqrt{3} - 1)h = 60$

$\therefore \quad h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{2} = 30(\sqrt{3} + 1)$ m

20. **(b)** The distance of the foot from each vertex $= h\cot\alpha$.

$\therefore$ The foot is at the circumcentre of the triangle.

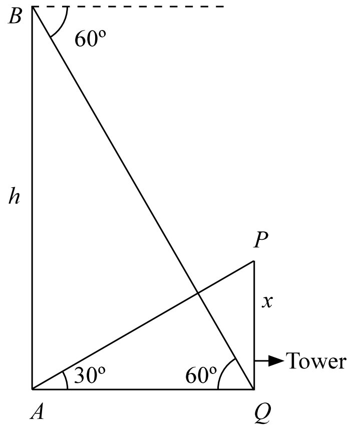
$\therefore \quad R = h\cot\alpha$

$\therefore \quad h = R\tan\alpha = \frac{a}{2\sin\alpha}\tan\alpha = \frac{a}{2}\tan\alpha \cdot \operatorname{cosec}\alpha$.

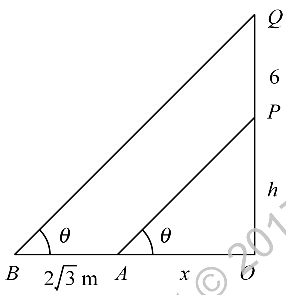
21. **(a)** Let, $PQ = x$ m denote the tower, so that $\angle PAQ = 30°$. Let, $BA = h$ m.

$\therefore \quad \angle BQA = 60°$.

Now, $\dfrac{h}{AQ} = \tan 60° = \sqrt{3}$. $\quad \therefore \quad AQ = \dfrac{h}{\sqrt{3}} = h\cot 60°$.



22. **(a)** Let, OP be the tower of height h m and PQ be the flagstaff of height 6 m. Let, the sun make an angle θ with the ground. Let, $OA = x$ and $AB = 2\sqrt{3}$ be the shadows of the tower and the flagstaff, respectively.



Now, $\tan\theta = \dfrac{h}{x}$.

Also, $\dfrac{h+6}{x+2\sqrt{3}} = \tan\theta$

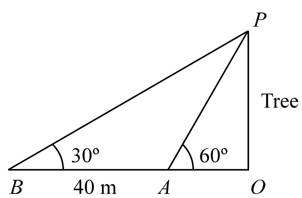
$\therefore \quad \dfrac{h}{x} = \dfrac{h+6}{x+2\sqrt{3}} \quad \Rightarrow \quad hx + 2\sqrt{3}\,h = hx + 6x$

$\Rightarrow \quad 2\sqrt{3}\,h = 6x$

$\Rightarrow \quad \dfrac{h}{x} = \dfrac{6}{2\sqrt{3}} = \sqrt{3}$

$\Rightarrow \quad \tan\theta = \sqrt{3}$. $\quad \therefore \quad \theta = 60°$.

23. **(c)** Let, OA denote the breadth of the river.



$\dfrac{OP}{OA} = \tan 60° = \sqrt{3}$

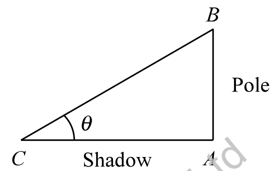
$\therefore \quad OP = \sqrt{3}\,OA$.

Also, $\dfrac{OP}{OA+40} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\therefore \quad OA + 40 = \sqrt{3}\,OP = \sqrt{3}\,(\sqrt{3}\,OA) = 3\,OA$.

$\therefore \quad 2OA = 40 \quad \Rightarrow \quad OA = 20\,\text{m}$.

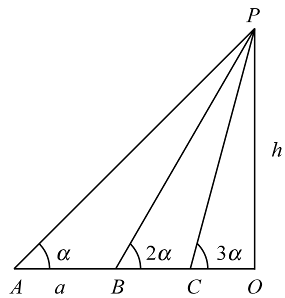
24. **(a)** Given, $AC = \sqrt{3}\,AB$.



$\therefore \quad \cot\theta = \dfrac{AC}{AB} = \sqrt{3} \quad \Rightarrow \quad \tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = 30°$.

25. **(c)** Let, OP be a vertical tower. The elevation of top P from A, B, C are α, 2α, 3α, respectively. $\angle APB = 2\alpha - \alpha = \angle PAB$.



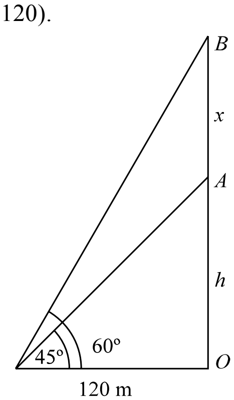
$\dfrac{OP}{BP} = \sin 2\alpha$

$\therefore \quad OP = BP\,\sin 2\alpha = a\sin 2\alpha$.

Thus, height of the tower $= a\sin 2\alpha$.

26. **(b)** $\dfrac{h+x}{120} = \tan 60° = \sqrt{3}$

$h + x = \sqrt{3}\,(120)$.

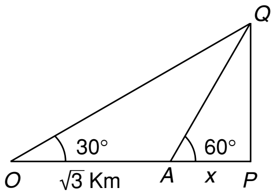


Also, $\dfrac{h}{120} = \tan 45° = 1$.

$\therefore\quad h = 120$ m $\quad\therefore\quad 120 + x = 120\sqrt{3}$.

$\therefore\quad x = 120(\sqrt{3} - 1)$ m.

27. (a) $\dfrac{h}{x} = \tan 60 = \sqrt{3}$. $\quad\therefore\quad h = \sqrt{3}\,x$.



Also, $\dfrac{h}{\sqrt{3}+x} = \tan 30° = \dfrac{1}{\sqrt{3}}$

$\therefore\quad \sqrt{3}\,h = \sqrt{3} + x$

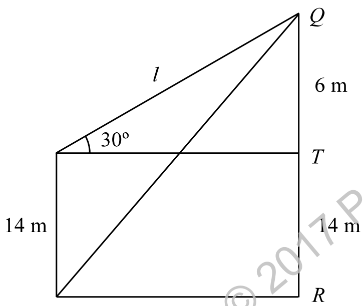
$\therefore\quad \sqrt{3}\,(\sqrt{3}\,x) = \sqrt{3} + x$, or, $3x - x = \sqrt{3}$

$\therefore\quad 2x = \sqrt{3} \quad\therefore\quad x = \dfrac{\sqrt{3}}{2}$

$\therefore\quad h = \sqrt{3}\,.\,\dfrac{\sqrt{3}}{2} = \dfrac{3}{2}$ Km.

28. (a) $\dfrac{6}{l} = \sin 30° = \dfrac{1}{2}$

$\therefore\quad l = 12$ m.



29. (c) Let, h be the height of the man.

$\therefore\quad \dfrac{1}{2} = \dfrac{h}{2.4} \Rightarrow h = \dfrac{3}{4}\,(2.4) = 1.8$ m.

30. (d) Let, $AB = h$ be the height of the tower. Let, $GA = x$.

Then, $\dfrac{h}{x} = \tan 30° = \dfrac{1}{\sqrt{3}}$.

$\therefore\quad h = \dfrac{x}{\sqrt{3}}$.

Also, $\dfrac{h}{x-20} = \tan 60° = \sqrt{3}$.

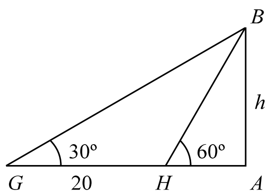
$\therefore\quad h = \sqrt{3}\,(x - 20)$

$\therefore\quad \dfrac{x}{\sqrt{3}} = \sqrt{3}\,(x - 20)$

$\Rightarrow\quad x = 3(x - 20)$

$\quad\quad = 3x - 60$

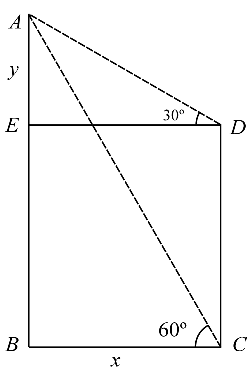
$\Rightarrow\quad 2x = 60 \quad\Rightarrow\quad x = 30.$



$\therefore\quad h = \dfrac{30}{\sqrt{3}} = 10\sqrt{3}$ m.

EXERCISE-2
(BASED ON MEMORY)

1. (c) Let, AB be the tower and CD be the electric pole.



Let, $BC = DE = x$.

Now, $\dfrac{AB}{BC} = \tan 60°$.

$\Rightarrow\quad \dfrac{15}{x} = \sqrt{3} \quad\Rightarrow\quad x = \dfrac{15}{\sqrt{3}}$.

Also, $\dfrac{AE}{DE} = \tan 30°$

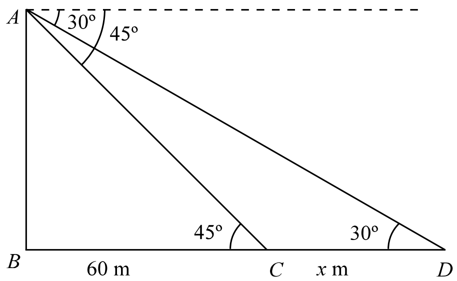
$\Rightarrow\quad \dfrac{y}{15/\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow\quad 3y = 15$ or, $y = 5$.

$\therefore\quad CD = BE = AB - AE = 15 - 5 = 10$ m.

2. **(a)** $\tan 45° = \dfrac{AB}{60} \Rightarrow AB = 60$ m …(1)

$\tan 30° = \dfrac{AB}{60+x}$, or, $AB = \dfrac{1}{\sqrt{3}}(60+x)$ …(2)



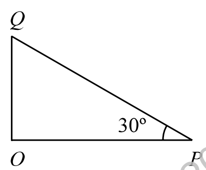
From Equation (1) and Equation (2), $60 + x = 60\sqrt{3}$

$\Rightarrow \quad x = 60(\sqrt{3} - 1) = 43.92$ m $\quad (\because \sqrt{3} = 1.732)$

Speed of the boat $= \dfrac{43.92}{5}$ m/s $= \dfrac{43.92}{5} \times \dfrac{18}{5} = 32$ Km/h.

3. **(b)** Let, OQ be the tower.

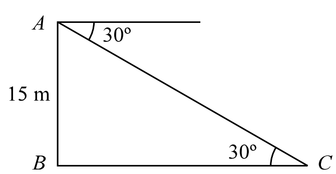
Then, $OQ = 100$ m and $\angle OPQ = 30°$.



In ΔOPQ, $\tan 30° = \dfrac{OQ}{OP} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{100}{OP}$

$\Rightarrow \quad OP = 100\sqrt{3}$ m.

4. **(a)** Let, C be the initial position of the boat, and A be the point on the top of the bridge from where the angle of depression of the boat is 30°.

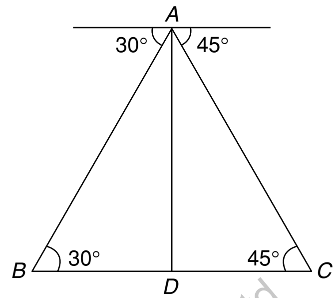


$AB = 15$ m, $BC = \dfrac{AB}{\tan 30°} = \dfrac{15}{1/\sqrt{3}} = 15\sqrt{3}$ m.

Speed of the boat $= 6$ Km/h $= 6 \times \dfrac{5}{18}$ m/s $= \dfrac{5}{3}$ m/s

$\therefore \quad$ Time required $= \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{15\sqrt{3}}{5/3} = 9\sqrt{3}$ second.

5. **(d)**



Let, AD be the tower and B and C be two objects.

$\angle ABD = 30°$ and $\angle ACD = 45°$

$AD = 180$ m

From ΔABD,

$\tan 30° = \dfrac{AD}{BD}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{180}{BD}$

$\Rightarrow \quad BD = 180\sqrt{3}$ m

From ΔADC,

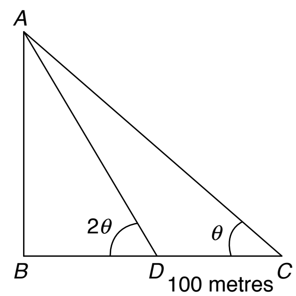
$\tan 45° = \dfrac{AD}{DC}$

$\Rightarrow \quad 1 = \dfrac{180}{DC} \quad \Rightarrow \quad DC = 180$ m

$\therefore \quad BC = BD + DC = 180\sqrt{3} + 180$

$= 180(\sqrt{3}+1)$ m

6. **(a)**



AB = Tower = h m

$CD = 100$ m; $BC = 160$ m

$\angle ACB = \theta \quad \therefore \quad \angle ADB = 26$

In ΔABC,

$$\tan\theta = \frac{AB}{BC} \Rightarrow \tan\theta = \frac{h}{160} \qquad \ldots(1)$$

In ΔABD,

$$\tan 2\theta = \frac{AB}{BD} = \frac{h}{60}$$

$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{h}{60} \Leftrightarrow \frac{2\times\frac{h}{160}}{1-\frac{h^2}{160\times 160}} = \frac{h}{60}$$

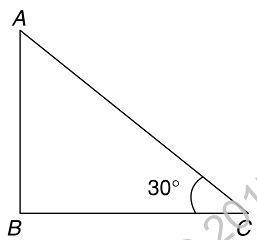
$$\Rightarrow \frac{1}{80\left(1-\frac{h^2}{160\times 160}\right)} = \frac{1}{60}$$

$$\Rightarrow 4\left(1-\frac{h^2}{160\times 160}\right) = 3$$

$$\Rightarrow \frac{h^2}{160\times 160} = 1-\frac{3}{4} = \frac{1}{4} \Leftrightarrow h^2 = 6400$$

$$\Rightarrow h = \sqrt{6400} = 80 \text{ m}$$

7. (b)



AB = Tower = h m

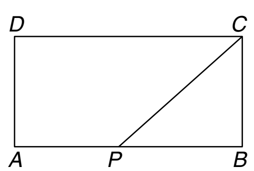
BC = 50 m

$\angle ACB$ = 30°

$$\therefore \tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{50} \Leftrightarrow AB = \frac{50}{\sqrt{3}} \text{ m}$$

8. (d)



AB = $3x$ units

BC = $2x$ units

$$PB = \frac{3}{2}x \text{ units}$$

$$\text{CP} = \sqrt{PB^2 + BC^2} = \sqrt{\frac{9x^2}{4} + 4x^2}$$

$$= \sqrt{\frac{25x^2}{4}} = \frac{5x}{2} \text{ units}$$

$$\therefore \sin\angle CPB = \frac{BC}{CP} = \frac{2x}{\frac{5x}{2}} = \frac{4}{5}$$

9. (a) $\dfrac{\sin A}{1+\cos A} + \dfrac{\sin A}{1-\cos A}$

$$= \frac{\sin A(1-\cos A) + \sin A(1+\cos A)}{(1+\cos A)(1-\cos A)}$$

$$= \frac{\sin A - \sin A\cos A + \sin A + \sin A\cos A}{1+\cos^2 A}$$

$$= \frac{2\sin A}{\sin^2 A} = 2\operatorname{cosec} A$$

10. (d) $r \sin\theta = 1$

$$r\cos\theta = \sqrt{3}$$

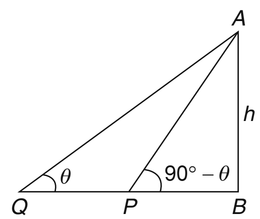
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \tan 30° \Rightarrow \theta = 30°$$

$$\therefore \sqrt{3}\tan\theta + 1 = \sqrt{3}\times\tan 30° + 1$$

$$= \sqrt{3}\times\frac{1}{\sqrt{3}} + 1 = 1 + 1 = 2$$

11. (a)



AB = Tower = h units

$\therefore \angle AQB = \theta \therefore \angle APB = 90° - \theta$

$PB = a$; $BQ = b$

From ΔAQB,

$$\tan\theta = \frac{AB}{BQ}$$

$$\Rightarrow \tan\theta = \frac{h}{b} \qquad \ldots(1)$$

From ΔAPB,

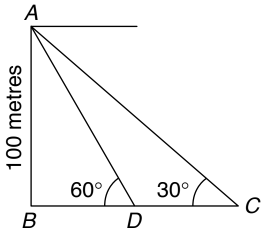
$$\tan(90° - \theta) = \frac{h}{PB}$$

$$\Rightarrow \quad \cot\theta = \frac{h}{a} \qquad \ldots(2)$$

By multiplying both the equations, we have $\tan\theta \cdot \cot\theta = \frac{h}{b} \times \frac{h}{a}$

$$\Rightarrow \quad h^2 = ab \Leftrightarrow h = \sqrt{ab}$$

12. (b)



C = Initial point and

D = Final point

AB = Tower = 100 m

Let CD be x m.

From ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{100}{BD}$$

$$\Rightarrow \quad BD = \frac{100}{\sqrt{3}} \text{ m}$$

From ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

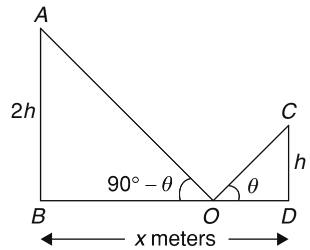
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100}{\frac{100}{\sqrt{3}} + x}$$

$$\Rightarrow \quad \frac{100}{\sqrt{3}} + x = 100\sqrt{3}$$

$$\therefore x = 100\sqrt{3} - \frac{100}{\sqrt{3}}$$

$$= \frac{300 - 100}{\sqrt{3}} = \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3} \text{ m}$$

13. (a) $CD = h$ m, $AB = 2h$ m



$$OB = OD = \frac{x}{2} \text{ m}$$

From ΔOCD, $\tan\theta = \frac{h}{\frac{x}{2}} = \frac{2h}{x} \qquad \ldots(1)$

From ΔOAB, $\tan(90° - \theta) = \frac{AB}{BO}$

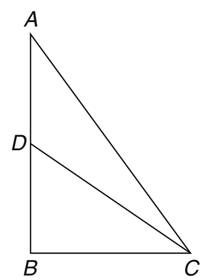
$$\Rightarrow \cot\theta = \frac{2h}{\frac{x}{2}} = \frac{4h}{x} \qquad \ldots(2)$$

Multiplying both equations,

$$\tan\theta \cdot \cot\theta = \frac{2h}{x} \times \frac{4h}{x}$$

$$\Rightarrow \quad x^2 = 8h^2 \qquad \Rightarrow \quad h^2 = \frac{x^2}{8} \Rightarrow h = \frac{x}{2\sqrt{2}} \text{ m}$$

14. (c)



$\angle ACB = 60°$

$\angle DCB = 45°$

$AB = 5000$ m

$AD = x$ m

$\therefore$ From ΔABC, $\tan 60° = \frac{AB}{BC}$

$$\Rightarrow \quad \sqrt{3} = \frac{5000}{BC}$$

$$\Rightarrow \quad BC = \frac{5000}{\sqrt{3}} \text{ m}$$

From ΔDBC, $\tan 45° = \frac{DB}{BC}$

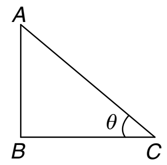
$$\Rightarrow \quad DB = BC = \frac{5000}{\sqrt{3}}$$

$$\therefore \quad AD = AB - BD = 5000 - \frac{5000}{\sqrt{3}}$$

$$= 5000\left(1 - \frac{1}{\sqrt{3}}\right)$$

$$= 5000\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \text{ m}$$

15. (c)



Let, AB be tower and NC be its shadow.

If $AB = x$, then $BC = \frac{x}{\sqrt{3}}$

$$\therefore \quad \tan\theta = \frac{AB}{BC} = \frac{x}{\frac{x}{\sqrt{3}}} = \sqrt{3}$$

$$\therefore \quad \tan\theta = \tan 60°$$

$$\Rightarrow \quad \theta = 60°$$