Permutations and Combinations 31

INTRODUCTION

We often come across questions such as the following:

1. In how many ways can 4 bottles be arranged in a row?
2. In how many ways can 5 students be seated at a round table?
3. In how many ways can a group of five people be selected out of a gathering of ten people?
4. In how many ways can 5 maps be selected out of 8 and displayed in a row?

Answers to these questions and many other important and more difficult ones can often be given without actually writing down all the different possibilities. In the present chapter we shall study some basic principles of the art of counting without counting which will enable us to answer such questions in an elegant manner.

FACTORIAL NOTATION

The continued product of first n natural numbers is called n factorial or factorial n and is denoted by $\underline{|n}$ or $n!$

Thus, $\underline{|n}$ or $n!$

$= 1\cdot 2\cdot 3\cdot 4 .... (n-1)n$

$= n\cdot(n-1)\cdot(n-2) .... 3\cdot 2\cdot 1$ (in reverse order)

Notes:

1. When n is a negative integer or a fraction, $n!$ is not defined. Thus, $n!$ is defined only for positive integers.
2. According to the above definition, 0! makes no sense. However, we define $0! = 1$.
3. $n! = n(n-1)!$
4. $(2n)! = 2^n \cdot n!\,[1\cdot 3\cdot 5\cdot 7 .... (2n-1)]$.

Illustration 1: Evaluate

(i) $\dfrac{30!}{28!}$ (ii) $\dfrac{9!}{5!3!}$

(iii) $\dfrac{12!-10!}{9!}$ (iv) $\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!}$.

Solution:

(i) $\dfrac{30!}{28!} = \dfrac{30\times 29\times 28!}{28!} = 30 \times 29 = 870.$

(ii) $\dfrac{9!}{5!3!} = \dfrac{9\times 8\times 7\times 6\times 5!}{5!\times 3\times 2} = 504.$

(iii) $\dfrac{12!-10!}{9!} = \dfrac{12\times 11\times 10!-10!}{9!}$

$= \dfrac{10!}{9!}[132 - 1]$

$= 10 \times 131$

$= 1310.$

(iv) $\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!} = \dfrac{4\times 5}{3!\times 4\times 5}+\dfrac{5}{4!\times 5}+\dfrac{1}{5!}$

$= \dfrac{20}{5!}+\dfrac{5}{5!}+\dfrac{1}{5!}$

$= \dfrac{26}{5!} = \dfrac{13}{60}.$

Illustration 2: Convert into factorials:

(i) 4.5.6.7.8.9.10.11. (ii) 2.4.6.8.10.

Solution: (i) $4\cdot 5\cdot 6\cdot 7\cdot 8\cdot 9\cdot 10\cdot 11$

$= \dfrac{1.2.3.4.5.6.7.8.9.10.11}{1.2.3.}$

$= \dfrac{11!}{3!}.$

(ii) $2\cdot 4\cdot 6\cdot 8\cdot 10$

$= (2\cdot 1)\cdot(2\cdot 2)\cdot(2\cdot 3)\cdot(2\cdot 4)\cdot(2.5)$

$= (2\cdot 2\cdot 2\cdot 2\cdot 2)\cdot(1\cdot 2\cdot 3\cdot 4\cdot 5)$

$= 2^5\cdot 5!.$

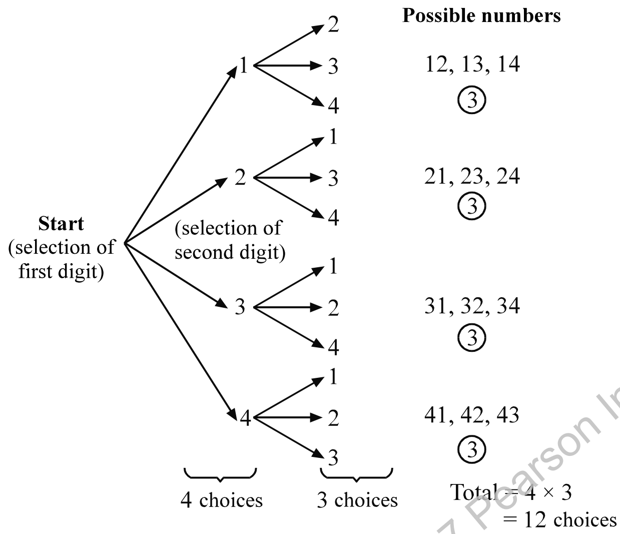
Fundamental Principle of Counting

Multiplication Principle If an operation can be performed in 'm' different ways; following which a second operation can be performed in 'n' different ways, then the 2 operations in succession can be performed in $m \times n$ different ways.

Illustration 3: How many numbers of 2 digits can be formed out of the digits 1, 2, 3, 4, no digit being repeated?

Solution: The first digit can be any 1 of the 4 digits 1, 2, 3, 4, that is the first digit can be chosen in 4 ways. Having chosen the first digit, we are left with 3 digits from which the second digit can be chosen. Therefore, the possible ways of choosing the two digits are

Since the first digit can be chosen in four ways and for each choice of the first digit there are three ways of choosing the second digit, therefore, there are 4×3



ways, that is, 12 ways of choosing both the digits. Thus, 12 numbers can be formed.

Illustration 4: Anu wishes to buy a birthday card for her brother Manu and send it by post. Five different types of cards are available at the card-shop, and four different types of postage stamps are available at the post-office. In how many ways can she choose the card and the stamp?

Solution: She can choose the card in five ways. For each choice of the card she has four choices for the stamp. Therefore, there are 5×4 ways, that is, 20 ways of choosing the card and the stamp.

Illustration 5: Mohan wishes to go from Agra to Chennai by train and return from Chennai to Delhi by air. There are six different trains from Agra to Chennai and five different flights from Chennai to Agra. In how many ways can he perform the journey?

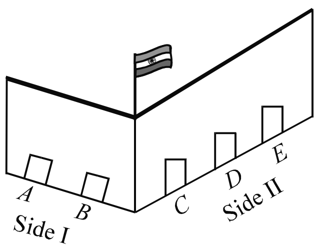
Solution: Since he can choose any one of the six trains for going to Chennai, and for each such choice he has five choices for returning to Agra, he can perform the journey in 6×5 ways, that is, 30 ways.

Addition Principle

If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in 'n' different ways, then either of the two operations can be performed in $(m + n)$ ways.

Note:
he above two principles can be extended for any finite number of operations.

Illustration 6: Suppose there are 5 gates to a stadium, 2 on one side and 3 on the other. Sohan has to go out of the stadium. he can go out from any one of the 5 gates. Thus, the number of ways in which he can go out is 5. Hence, the work of going out through the gates on one side will be done in 2 ways and the work of going out through the gates on other side will be done in 3 ways. The work of going out will be done when Sohan goes out from side I or side II. Thus the work of going out can be done in $(2 + 3) = 5$ ways.



Note:

Addition theorem of counting is also true for more than two opeartions.

Permutation

Each of the different arrangements which can be made by taking some or all of given number of things or objects at a time is called a *permutation.*

Note:

Permutation of things means arrangement of things. The word arrangement is used if order of things is taken into account. Thus, if order of different things changes, then their arrangement also changes.

Notation

Let r and n be positive integers such that $1 \le r \le n$. Then, the number of permutations of n different things, taken r at a time, is denoted by the symbol ^nP_r or $P(n, r)$.

SOME BASIC RESULTS

1. ^nP_r or $P(n, r) = \dfrac{n!}{(n-r)!} = n(n-1)(n-2)$ ...$[n-(r+1)]$, $0 \le r \le n$.

Illustration 7: Evaluate the following:

(i) $P(6, 4)$,
(ii) $P(15, 3)$,
(iii) $P(30, 2)$.

Solution: (i) We have

$$P(6, 4) = \frac{6!}{(6-4)!} = \frac{6.5.4.3.2.1}{2.1} = 360.$$

(ii) We have

$$P(15, 3) = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{(15\cdot 14\cdot 13)\,(12!)}{12!} = 2730.$$

(iii) $P(30, 2) = \dfrac{30!}{(30-2)!} = \dfrac{30!}{28!} = \dfrac{(30.29)\,(28!)}{28!}$

$= 870.$

2. The number of permutations of n things, taken all at a time, out of which p are alike and are of one type, q are alike and are of second type and rest are all different $= \dfrac{n!}{p!\,q!}$.

Illustration 8: There are 5 red, 4 white and 3 blue marbles in a bag. They are drawn one by one and arranged in a row. Assuming that all the 12 marbles ae drawn, determine the number of different arrangements.

Solution: Here, $n = 12$, $p_1 = 5$, $p_2 = 4$ and $p_3 = 3$

$\therefore$ The required number of different arrangements

$$= \frac{n!}{p_1!\,p_2!\,p_3!} = \frac{12!}{5!4!3!}$$

$$= \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1\cdot 3\cdot 2\cdot 1} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7}{2}$$

$$= 990 \times 4 \times 7 = 27720.$$

3. The number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r.

Illustration 9: In how many ways can 5 apples be distributed among 4 boys, there being no restriction to the number of apples each boy may get?

Solution: The required number of ways $= 4^5$.

4. **Permutations under Restrictions**

(a) Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement, is

$$r \cdot {}^{n-1}P_{r-1}.$$

(b) Number of permutations of n different things, taken r at a time, when s particular things are to be always included in each arrangement, is

$$s!\,(r-(s-1)) \cdot {}^{n-s}P_{r-s}.$$

(c) Number of permutations of n different things, taken r at a time, when a particular thing is never taken in each arrangement, is

$$^{n-1}P_r.$$

(d) Number of permutations of n different things, taken all at a time, when m specified things always come together, is $m! \times (n-m+1)!$

(e) Number of permutations of n different things, taken all at a time, when m specified things never come together, is $n! - m! \times (n-m+1)!$.

5. **Circular Permutations**

(a) Number of circular arrangements (permutations) of n different things $= (n-1)!$.

Illustration 10: In how many ways can eight people be seated at a round table?

Solution: Required number of ways $= (8-1)!$

$= 7! = 5040.$

(b) Number of circular arrangements (permutations) of n different things when clockwise and anticlockwise arrangements are not different, that is, when observation can be made from both sides $= \dfrac{1}{2}(n-1)!$.

Illustration 11: Find the number of ways in which n different beads can be arranged to form a necklace.

Solution: Required number of arrangements

$$= \frac{1}{2}(5 - 1)! = \frac{1}{2} \times 4! = 12.$$

Combination

Each of the different groups or selections which can be made by taking some or all of a number of things (irrespective of order) is called a *combination*.

Note:

Combination of things means selection of things. Obviously, in selection of things order of things has no importance. Thus, with the change of order of things selection of things does not change.

Notations The number of combinations of n different things taken r at a time is denoted by ${}^nC_r$ or $C(n, r)$.

$$\text{Thus, } {}^nC_r = \frac{n!}{r!(n-r)!} \qquad (0 \le r \le n)$$

$$= \frac{{}^nP_r}{r!}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r(r-1)(r-2)\ldots 3\cdot 2\cdot 1}$$

If $r > n$, then ${}^nC_r = 0$.

Illustration 12: Evaluate:

(i) ${}^{11}C_3$ (ii) ${}^{10}C_8$ (iii) ${}^{100}C_{98}$

Solution:

(i) $${}^{11}C_3 = \frac{11!}{3!(11-3)!} = \frac{11!}{3!8!}$$

$$= \frac{11 \times 10 \times 9 \times 8!}{3 \times 2 \times 8!} = 165.$$

(ii) $${}^{10}C_8 = \frac{10!}{8!2!} = \frac{10 \times 9 \times 8!}{8! \times 2} = 45.$$

(iii) $${}^{100}C_{98} = \frac{100!}{98!2!} = \frac{100 \times 99 \times 98!}{98! \times 2} = 4950.$$

Some Important Results

1. ${}^nC_r = {}^nC_{n-r}$
2. ${}^nC_0 = {}^nC_n = 1$, ${}^nC_1 = n$
3. If ${}^nC_x = {}^nC_y$ then either $x = y$ or $y = n - x$ that is, $x + y = n$.
4. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
5. $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$.
6. If n is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.
7. If n is odd then the greatest value of ${}^nC_r$ is
$${}^nC_{\frac{n+1}{2}} \text{ or } {}^nC_{\frac{n-1}{2}}.$$
8. $${}^nC_r = \frac{r \text{ decreasing numbers starting with } n}{r \text{ increasing numbers starting with } 1}$$
$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{1\cdot 2\cdot 3\ldots r}.$$
9. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$.
10. ${}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$.
11. Number of combinations of n different things taken r at a time
    (a) when p particular things are always included $= {}^{n-p}C_{r-p}$
    (b) when p particular things are never included $= {}^{n-p}C_r$
    (c) when p particular things are not together in any selection $= {}^nC_r - {}^{n-p}C_{r-p}$.

Illustration 13: In how many ways can 5 members forming a committee out of 10 be selected so that

(i) two particular members must be included.
(ii) two particular members must not be included.

Solution:

(i) When two particular members are included, then, we have to select $5 - 2 = 3$ members out of $10 - 2 = 8$.

$\therefore$ The required number of ways

$$= C(8, 3) = \frac{8!}{3!5!} = \frac{8 \times 7 \times 6}{6} = 56.$$

(ii) When 2 particular members are not included, then, we have to select 5 members out of $10 - 2 = 8$.

$\therefore$ The required number of ways

$$= C(8, 5) = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6}{6} = 56.$$

12. (a) Number of selections of r consecutive things out of n things in a row $= n - r + 1$.
    (b) Number of selections of r consecutive things out of n things along a circle

$$\begin{cases} n, \text{ when } r < n \\ 1, \text{ when } r = n \end{cases}.$$

13. (a) Number of selections of zero or more things out of n different things

$$^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$$

    (b) Number of combinations of n different things selecting at least one of them is

$$^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1.$$

    (c) Number of selections of r things ($r \le n$) out of n identical things is 1.
    (d) Number of selections of zero or more things out of n identical things $= n + 1$.
    (e) Number of selections of one or more things out of n identical things $= n$.
    (f) If out of $(p + q + r + t)$ things, p are alike of one kind, q are alike of second kind, r are alike of third kind and t are different, then the total number of selections is $(p + 1)(q + 1)(r + 1)2^t - 1$.
    (g) The number of ways of selecting some or all out of $p + q + r$ items where p are alike of one kind, q are alike of second kind and rest are alike of third kind is $[(p + 1)(q + 1)(r + 1)] - 1$.

14. (a) Number of ways of dividing $m + n$ different things in two groups containing m and n things, respectively ($m \ne n$):

$$^{m+n}C_m = \frac{(m+n)!}{m!\,n!}.$$

    (b) Number of ways of dividing $m + n + p$ different things in three groups containing m, n and p things, respectively ($m \ne n \ne p$):

$$\frac{(m+n+p)!}{m!\,n!\,p!}.$$

SOME USEFUL SHORTCUT METHODS

1. The number of triangles which can be formed by joining the angular points of a polygon of n sides as vertices are $\frac{n(n-1)(n-2)}{6}$.

Illustration 14: Find the number of triangles formed by joining the vertices of an octagon.

Solution: The required number of triangles

$$= \frac{n(n-1)(n-2)}{6}$$

$$= \frac{8(8-1)(8-2)}{6} = \frac{8\times7\times6}{6} = 56.$$

2. The number of diagonals which can be formed by joining the vertices of a polygon of n sides are $\frac{n(n-3)}{2}$.

Illustration 15: How many diagonals are there in a decagon?

Solution: The required number of diagonals

$$= \frac{n(n-3)}{2} = \frac{10(10-3)}{2}$$

$$= \frac{10\times7}{2} = 35.$$

3. If there are 'm' horizontal lines and 'n' vertical lines then the number of different rectangles formed are given by $(^mC_2 \times {}^nC_2)$.

Illustration 16: In a chess board there are 9 vertical and 9 horizontal lines. Find the number of rectangles formed in the chess board.

Solution: The require number of rectangles.

$$= {}^9C_2 \times {}^9C_2 = 36 \times 36 = 1296.$$

4. These are 'n' points in a plane out of which 'm' points are collinear. The number of triangles formed by the points as vertices are given by $^nC_3 - {}^mC_3$.

Illustration 17: There are 14 points in a plane out of which 4 are collinear. Fine the number of triangles formed by the points as vertices.

Solution: The required number of triangles

$$= {}^{14}C_3 - {}^{4}C_3 = 364 - 4 = 360.$$

> 5. There are 'n' points in a plane out of which 'm' points are collinear. The number of straight lines formed by joining them are given by
> $$({}^{n}C_2 - {}^{m}C_2 + 1).$$

Illustration 18: There are 10 points in a plane out of which 5 are collinear. Find the number of straight lines formed by joining them.

Solution: The required number of straight lines

$$= {}^{n}C_2 - {}^{m}C_2 + 1$$

$$= {}^{10}C_2 - {}^{5}C_2 + 1 = 45 - 10 + 1 = 36.$$

6. If there are 'n' points in a plane and no three points are collinear, then the number of triangles formed with 'n' points are given by $\dfrac{n(n-1)(n-2)}{6}$.

Illustration 19: Find the number of triangles that can be formed with 14 points in a plane of which no three points are collinear.

Solution: The required number of triangles

$$= \frac{n(n-1)(n-2)}{6} = \frac{14\times 13\times 12}{6} = 364.$$

> 7. The number of quadrilaterals that can be formed by joining the vertices of a polygon of n sides are given by $\dfrac{n(n-1)(n-2)(n-3)}{24}$, where $n > 3$.

Illustration 20: Find the number of quadrilaterals that can be formed by joining the vertices of a septagon.

Solution: The required number of quadrilaterals

$$= \frac{n(n-1)(n-2)(n-3)}{24}$$

$$= \frac{7(7-1)(7-2)(7-3)}{24}$$

$$= \frac{7\times 6\times 5\times 4}{24} = 35.$$

> 8. There are n points in a plane and no points are collinear, then the number of straight lines that can be drawn using these 'n' points are given by $\dfrac{n(n-1)}{2}$.

Illustration 21: How many straight lines can be drawn with 18 points on a plane of which no points are collinear?

Solution: The required number of straight lines

$$= \frac{n(n-1)}{2} = \frac{18(18-1)}{2} = \frac{18\times 17}{2} = 153.$$

> 9. In a party every person shakes hands with every other person. If there was a total of H handshakes in the party, then the number of persons 'n' who were present in the party can be calculated from the equation:
> $$\frac{n(n-2)}{2} = H.$$

Illustration 22: In a party every person shakes hands with every other person. If there was a total of 105 handshakes in the party, find the number of persons who were present in the party.

Solution: Let 'n' be the number of persons present in the party.

We have, the equation

$$\frac{n(n-1)}{2} = H$$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 105$$

$$\Rightarrow \quad n(n-1) = 15 \times (15-1) \Rightarrow n = 15.$$

Exercise-1

1. There are 6 candidates for 3 posts. In how many ways can the posts be filled?

(a) 120 (b) 130
(c) 100 (d) 110

2. From among the 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done?

(a) 1360 (b) 1260
(c) 1060 (d) 1160

3. There are 15 buses running between Delhi and Mumbai. In how many ways can a man go to Mumbai and return by a different bus?

(a) 280 (b) 310
(c) 240 (d) 210

4. A teacher of a class wants to set 1 question from each of 2 exercises in a book. If there are 15 and 12 questions in the 2 exercises respectively, then in how many ways can the 2 questions be selected?

(a) 160 (b) 140
(c) 180 (d) 120

5. The students in a class are seated according to their marks in the previous examination. Once, it so happens that four of the students got equal marks and therefore the same rank. To decide their seating arrangement, the teacher wants to write down all possible arrangements one in each of separate bits of paper in order to choose one of these by lots. How many bits of paper are required?

(a) 24 (b) 12
(c) 48 (d) 36

6. For a set of 5 true-or-false questions, no student has written all the correct answers, and no 2 students have given the same sequence of answers. What is the maximum number of students in the class, for this to be possible?

(a) 31 (b) 21
(c) 51 (d) 41

7. A code word is to consist of 2 English alphabets followed by 2 distinct numbers between 1 and 9. For example, CA23 is a code word. How many such code words are there?

(a) 615800 (b) 46800
(c) 719500 (d) 410800

8. There are 6 multiple choice questions on an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next 3 have 5 each?

(a) 6000 (b) 5000
(c) 4000 (d) 8000

9. There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first 2 questions have 3 choices each, the next 2 have 4 choices each and the last two have 5 choices each?

(a) 3450 (b) 3300
(c) 3600 (d) 3400

10. Each section in the first year of plus 2 course has exactly 40 students. If there are 5 sections, in how many ways can a set of 4 student representatives be selected, 1 from each section?

(a) 2560000 (b) 246500
(c) 2240000 (d) 2360000

11. There are 5 letters and 5 directed envelopes. Find the number of ways in which the letters can be put into the envelopes so that all are not put in directed envelopes?

(a) 129 (b) 119
(c) 109 (d) 139

12. There horses H_1, H_2, H_3 entered a field which has 7 portions marked P_1, P_2, P_3, P_4, P_5, P_6 and P_7. If no 2 horses are allowed to enter the same portion of the field, in how many ways can the horses graze the grass of the field?

(a) 195 (b) 205
(c) 185 (d) 210

13. How many different numbers of 2-digits can be formed with the digits 1, 2, 3, 4, 5, 6; no digits being repeated?

(a) 40 (b) 30
(c) 35 (d) 45

14. How many 3-digit odd numbers can be formed from the digits 1, 2, 3, 4, 5, 6 when

(i) repitition of digits is not allowed
(ii) repitition of digits is allowed?

(a) (i) 60, (ii) 108 (b) (i) 50, (ii) 98
(c) (i) 70, (ii) 118 (d) (i) 80, (ii) 128

15. How many 2-digit odd numbers can be formed from the digits 1, 2, 3, 4 5 and 8, if repitition of digits is allowed?

(a) 5 (b) 15
(c) 35 (d) 25

16. How many odd numbers less than 1000 can be formed using the digits 0, 2, 5, 7? (repetition of digits is allowed).

(a) 52 (b) 32
(c) 22 (d) 42

17. How many 3-digit numbers each less than 600 can be formed from the digits 1, 2, 3, 4, 5 and 9, if repetition of digits is allowed?

(a) 180 (b) 160
(c) 165 (d) 185

18. How many words (with or without meaning) of 3 distinct English alphabets are there?

(a) 15600 (b) 14650
(c) 12800 (d) 13700

19. How many numbers are there between 100 and 1000 in which all the digits are distinct?

(a) 548 (b) 648
(c) 748 (d) 756

20. How many integers between 1000 and 10000 have no digits other than 4, 5 or 6?

(a) 91 (b) 51
(c) 81 (d) 71

21. A number lock on a suitcase has 3 wheels each labelled with 10 digits from 0 to 9. If opening of the lock is a particular sequence of 3 digits with no repeats, how many such sequences will be possible?

(a) 720 (b) 760
(c) 680 (d) 780

22. A customer forgets a 4-digit code for an Automatic Teller Machine (A.T.M.) in a bank. However, he remembers that this code consists of digits 3, 5, 6 and 9. Find the largest possible number of trials necessary to obtain the correct code.

(a) 12 (b) 24
(c) 48 (d) 36

23. If $(n + 2)! = 2550(n!)$, find n

(a) 38 (b) 35
(c) 49 (d) 43

24. If $(n + 1)! = 6[(n - 1)!]$, find n

(a) 6 (b) 4
(c) 8 (d) 2

25. If $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{4!(n-4)!}$ are in the ratio 2:1, find the value of n.

(a) 0 (b) 1
(c) 2 (d) 3

26. Find n if ${}^nP_4 = 18.\ {}^{n-1}P_2$.

(a) 4 (b) 8
(c) 6 (d) 12

27. If $P(56, r + 6):P(54, r + 3) = 30800:1$, find r.

(a) 51 (b) 41
(c) 31 (d) 43

28. In how many ways can 10 people line up at a ticket window of a cinema hall?

(a) 3628800 (b) 3482800
(c) 344800 (d) 3328800

29. How many words, with or without meaning, can be formed using all letters of the word EQUATION, using each letter exactly once?

(a) 38320 (b) 39320
(c) 40320 (d) 38400

30. Ten students are participating in a race. In how many ways can the first 3 prizes be won?

(a) 920 (b) 680
(c) 820 (d) 720

31. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

(a) 2880 (b) 2480
(c) 3680 (d) 3280

32. 4 books, 1 each in Chemistry, Physics, Biology and Mathematics are to be arranged in a shelf. In how many ways can this be done?

(a) 12 (b) 36
(c) 24 (d) 48

33. There are 3 different rings to be worn in four fingers with at most 1 in each finger. In how many ways can this be done?

(a) 36 (b) 28
(c) 24 (d) 32

34. In an examination hall, there are 4 rows of chairs. Each row has 8 chairs 1 behind the other. There are 2 classes sitting for the examination with 16 students in each class. It is desired that in each row all students belong to the same class and that no 2 adjacent rows are allotted to the same class. In how many ways can these 32 students be seated?

(a) $2 \times 16! \times 16!$ (b) $2 \times 15! \times 15!$
(c) $2 \times 16! \times 15!$ (d) None of these

35. How many numbers lying between 1000 and 10000. can be formed by using the digits 1, 3, 5, 6, 7, 8, 9, no digits being repeated?
(a) 940 (b) 640
(c) 840 (d) 740

36. How many different numbers of 6 digits can be formed with the numbers 3, 1, 7, 0, 9, 5?
(a) 500 (b) 400
(c) 400 (d) 600

37. How many 3-digit numbers are there, with no digits repeated?
(a) 648 (b) 548
(c) 848 (d) 748

38. If there are 6 periods in each working day of a school, in how many ways can 1 arrange 5 subjects such that each subject is allowed at least 1 period?
(a) 3500 (b) 3600
(c) 3550 (d) 3650

39. 4 alphabets E, K, S and V, one in each, were purchased from a plastic warehouse. How many ordered pairs of alphabets, to be used as initials, can be formed from them?
(a) 18 (b) 12
(c) 14 (d) 16

40. There are 8 students appearing in an examination of which 3 have to appear in a Mathematics paper and the remaining 5 in different subjects. In how many ways can they be made to sit in a row if the candidates in Mathematics cannot sit next to each other?
(a) 14400 (b) 16400
(c) 15400 (d) 17400

41. Find how many words can be formed out of the letters of the word 'ORIENTAL' so that vowels always occupy the odd places.
(a) 576 (b) 578
(c) 676 (d) None of these

42. The number of different 6-digit numbers that are divisible by 10, which can be formed using the digits 1, 2, 7, 0, 9, 5?
(a) 100 (b) 120
(c) 140 (d) 160

43. In how many ways can the letters of the word 'UNIVERSAL' be arranged? In how many of these will E, R, S always occur together?
(a) 32240 (b) 30240
(c) 30240 (d) 31240

44. The principal wants to arrange 5 students on the platform such that the boy SUNIL occupies the second position and such that the girl GITA is always adjacent to the girl NITA. How many such arrangements are possible?
(a) 12 (b) 8
(c) 14 (d) 16

45. In how many different ways, the letters of the work ALGEBRA can be arranged in a row if
(i) The 2 As are together?
(ii) The 2 As are not together?
(a) (i) 720, (ii) 1800
(b) (i) 620, (ii) 1600
(c) (i) 780, (ii) 1860
(d) (i) 720, (ii) 1600

46. In how many ways can 6 apples be distributed among 3 boys, there being no restriction to the number of apples each boy may get?
(a) 729 (b) 739
(c) 759 (d) 749

47. In how many different ways can the letters of the word 'KURUKSHETRA' be arranged?
(a) 4497600 (b) 4979600
(c) 4989600 (d) 4789600

48. In how many different ways can the letters of the word 'ALLAHABAD' be permuted?
(a) 7560 (b) 7840
(c) 7460 (d) 7650

49. How many 3-digit numbers can be formed by using the digits 1, 3, 6 and 8, when the digits may be repeated any number of times?
(a) 48 (b) 64
(c) 80 (d) 32

50. How many 3-digit numbers can be formed by using the digits 0, 2, 3, 6, 8 when the digits may be repeated any number of times?
(a) 110 (b) 120
(c) 100 (d) None of these

51. How many different words can be formed with the letters of the word 'BHARAT'?
In how many of these B and H are never together?
(a) 240, 180 (b) 360, 240
(c) 320, 200 (d) 380, 260

52. How many arrangements can be made of the letters of the word 'ARRANGEMENT'?
(a) 2492800 (b) 249300
(c) 2494800 (d) 2491800

53. If the different permutations of the word EXAMINATION are listed as in a dictionary, how many items are there in this list before the first word starting with E?

(a) 906200 (b) 907200
(c) 908200 (d) 905200

54. How many 5-digit even numbers can be formed using the digits 1, 2, 5, 5, 4?

(a) 16 (b) 36
(c) 24 (d) 48

55. How many numbers greater than a million can be formed with the digits 2, 3, 0, 3, 4, 2, 3?

(a) 360 (b) 240
(c) 480 (d) None of these

56. Find the number of arrangements of the letters of the word 'ALGEBRA' without altering the relative position of the vowels and the consonants.

(a) 80 (b) 48
(c) 64 (d) 72

57. In how many ways can the letters of the word BALLOON be arranged so that two Ls do not come together?

(a) 900 (b) 1200
(c) 800 (d) 600

58. How many different signals can be transmitted by arranging 3 red, 2 yellow and 2 green flags on a pole? [Assume that all the 7 flags are used to transmit a signal.]

(a) 220 (b) 240
(c) 200 (d) 210

59. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1, so that odd digits always occupy the odd places?

(a) 36 (b) 24
(c) 18 (d) 12

60. There are 5 gentlemen and 4 ladies to dine at a round table. In how many ways can they seat themselves so that no 2 ladies are together?

(a) 3280 (b) 2880
(c) 2080 (d) 2480

61. 3 boys and 3 girls are to be seated around a table in a circle. Among the boys, X does not want any girl neighbour and the girl Y does not want any boy neighbour. How many such arrangements are possible?

(a) 6 (b) 4
(c) 8 (d) 2

62. How many different necklaces can be formed with 6 white and 5 red beads?

(a) 18 (b) 24
(c) 21 (d) 27

63. The Chief Ministers of 11 States of India meet to discuss the language problem. In how many ways can they seat themselves at a round table so that the Punjab and Madras Chief Ministers sit together?

(a) 725760 (b) 625760
(c) 925760 (d) 825760

64. If $C(n, 7) = C(n, 5)$, find n

(a) 15 (b) 12
(c) 18 (d) 2

65. If $C(n, 8) = C(n, 6)$, find $C(n, 2)$

(a) 91 (b) 81
(c) 61 (d) 71

66. If the ratio $C(2n, 3):C(n, 3)$ is equal to 11:1, find n.

(a) 6 (b) 9
(c) 12 (d) 18

67. If ${}^{2n}C_r = {}^{2n}C_{r+2}$; find r.

(a) $n - 1$ (b) $n - 2$
(c) $n - 4$ (d) $n - 3$

68. If ${}^{18}C_r = {}^{18}C_{r+2}$; find ${}^{r}C_5$.

(a) 56 (b) 63
(c) 49 (d) 42

69. If $12\ {}^{n}C_2 = {}^{2n}C_3$:find n.

(a) 7 (b) 5
(c) 9 (d) 3

70. Find $\sum_{r=1}^{5} C(5, r)$

(a) 41 (b) 31
(c) 51 (d) 61

71. In how many ways can 5 sportsmen be selected from a group of 10?

(a) 272 (b) 282
(c) 252 (d) 242

72. In how many ways can a cricket team of 11 players be selected out of 16 players, if 2 particular player are always to be included?

(a) 2006 (b) 2004
(c) 2008 (d) 2002

73. In how many ways can a cricket team of 11 players be selected out of 16 players if 1 particular players is to be excluded?

(a) 1565 (b) 1365
(c) 1165 (d) 1265

74. In how many ways can a cricket team of 11 players be selected out of 16 players if 2 particular players are to be included and 1 particular player is to be rejected?

(a) 715 (b) 615
(c) 915 (d) 515

75. A question paper has 2 parts, part A and part B, each containing 10 questions. If the student has to choose 8 from part A and 5 from part B, in how many ways can he choose the question?

(a) 11240 (b) 12240
(c) 13240 (d) 11340

76. In how many ways can a football team of 11 players be selected from 15 players? In how many cases a particular player be included?

(a) 1101 (b) 1011
(c) 1001 (d) 1111

77. How many words, each of 3 vowels and 2 consonants, can be formed from the letters of the word 'INVOLUTE'?

(a) 2280 (b) 2480
(c) 2880 (d) 2680

78. How many lines can be drawn through 21 points on a circle?

(a) 310 (b) 210
(c) 410 (d) 570

79. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls, if each selection consists of 3 balls of each colour.

(a) 3000 (b) 1000
(c) 2000 (d) 4000

80. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 courses are compulsory for every student?

(a) 45 ways (b) 35 ways
(c) 55 ways (d) 65 ways

81. In an examination, Yamini has to select 4 questions from each part. There are 6, 7 and 8 questions in Part I, Part II and Part III, respectively. What is the number of possible combinations in which she can choose the questions?

(a) 39650 (b) 37650
(c) 36750 (d) 38750

82. We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen too. In how many ways can the selection be made?

(a) 24 (b) 32
(c) 16 (d) 40

Exercise-2
(Based on Memory)

1. In how many different ways can the letters of the word 'MIRACLE' be arranged?

(a) 720 (b) 5040
(c) 2520 (d) 40320
(e) None of these

[SBI PO, 2008]

2. In how many different ways can the letters of the word 'BLOATING' be arranged?

(a) 40320 (b) 5040
(c) 2520 (d) 20160
(e) None of these

[Bank of Maharashtra PO, 2008]

3. In how many different ways can the letters of the word 'GAMBLE' be arranged?

(a) 720 (b) 840
(c) 360 (d) 420
(e) None of these

[NABARD PO, 2008]

Directions (4–8): From the following, different committees are to be made as per the requirement given in each question. In how many different ways can it be done?

10 men and 8 women out of which 5 men are teachers, 3 men are doctors and 2 are businessmen. Among the women, 3 are teachers, 2 doctors, 2 researchers and 1 social worker.

4. A committee of 5 in which 3 men and 2 women are there.

(a) 3360
(b) 8568
(c) 4284
(d) 1680
(e) None of these

5. A committee of 4 in which at least 2 women are there.

(a) 75 (b) 150
(c) 214 (d) 20
(e) None of these

6. A committee of 5 in which 2 men teachers, 2 women teachers and 1 doctor are there.

(a) 75 (b) 150
(c) 214 (d) 20
(e) None of these

7. A committee of 7.

(a) 31824 (b) 1200
(c) 9600 (d) 15912
(e) None of these

8. A committee of 3 in which there is no teacher and no doctor.

(a) 100 (b) 120
(c) 10 (d) 12
(e) None of these

[Andhra Bank IT Officer, 2007]

9. In how many different ways can the letters of the word 'FLEECED' be arranged?

(a) 840 (b) 2520
(c) 1680 (d) 49
(e) None of these

[Bank of Maharashtra PO, 2007]

10. In how many ways can 6 people be seated on a sofa if there are only 3 seats available?

(a) 18 (b) 120
(c) 2 (d) 216
(e) None of these

[IDBI Bank Officers' 2007]

11. In how many ways can 5 colours be selected out of 9?

(a) 98 (b) 142
(c) 72 (d) 126
(e) None of these

[Andhra Bank PO, 2007]

12. In how many different ways can the letters of the word 'PADDLED' be arranged?

(a) 910 (b) 2520
(c) 5040 (d) 840
(e) None of these

[Corporation Bank PO, 2006]

13. In how many different ways can 6 children be seated on 6 seats?

(a) 360 (b) 120
(c) 590 (d) 740
(e) None of these

[LIC ADO, 2007]

14. In how many different ways can the letters of the word 'PRIDE' be arranged?

(a) 60 (b) 120
(c) 15 (d) 360
(e) None of these

[Bank of Baroda PO, 2007]

15. In how many different ways can the letters of the word 'WEDDING' be arranged?

(a) 5040 (b) 2500
(c) 2520 (d) 5000
(e) None of these

[OBC PO, 2007]

16. A select group of 4 is to be formed from 8 men and 6 women in such a way that the group must have atleast 1 woman. In how many different ways can it be done?

(a) 364 (b) 1001
(c) 728 (d) 931
(e) None of these

[SBI PO, 2005]

17. On 5 chairs arranged in a row, 5 persons A, B, C, D and E are to be seated in such a way that B and D always sit together (side by side). In how many different ways can it be done?

(a) 120 (b) 48
(c) 60 (d) 24
(e) None of these

[SBI PO, 2005]

18. In how many different ways can the letters of the word 'PRAISE' be arranged?

(a) 720 (b) 610
(c) 360 (d) 210
(e) None of these

[Andhra Bank PO, 2006]

19. 4 boys and 3 girls are to be seated in a row in such a way that no 2 boys sit adjacent to each other. In how many different ways can it be done?

(a) 5040 (b) 30
(c) 144 (d) 72
(e) None of these

[SBI PO, 2005]

20. A committee of 3 members is to be formed out of 3 men and 4 women. In how many different ways can it be done so that at least 1 member is a woman?

(a) 34 (b) 12
(c) 30 (d) 36
(e) None of these

[SBI PO, 2005]

21. A speaks truth in 75% and B in 80% cases. In what percentage of cases are they likely to contradict each other when narrating the same incident?
(a) 35 (b) 30
(c) 25 (d) 20
(e) None of these
[BSRB Chennai PO, 2000]

22. In a box there are 8 red, 7 blue and 6 green balls. 1 ball is picked up randomly. What is the probability that it is neither red nor green?
(a) $\frac{7}{19}$ (b) $\frac{2}{3}$
(c) $\frac{3}{4}$ (d) $\frac{9}{21}$
(e) None of these

23. How many different ways can the letters in the word ATTEND be arranged?
(a) 60 (b) 120
(c) 240 (d) None of these
[Allahabad Bank PO, 2010]

24. In how many different ways can the letters of the word 'CYCLE' be arranged?
(a) 120 (b) 240
(c) 30 (d) None of these
[Punjab National Bank PO, 2010]

25. In how many different ways can the letters of the word DESIGN by arranged so that the vowels are at the 2 ends?
(a) 48 (b) 72
(c) 36 (d) 24
[United Bank of India PO, 2009]

26. In how many different ways can the letters of the word 'SMART' be arranged?
(a) 25 (b) 60
(c) 180 (d) None of these
[IOB PO, 2009]

27. In how many different ways can the letters of the word 'THERAPY' be arranged so that the vowels never come together?
(a) 720 (b) 1440
(c) 5040 (d) 3600
(e) 4800
[IBPS PO/MT, 2012]

28. In how many different ways can the letters of the word 'PRAISE' be arranged?
(a) 720 (b) 610
(c) 360 (d) 210
(e) None of these
[Andhra Bank PO, 2011]

29. In how many different ways can the letters of the word 'PRAISE' be arranged?
(a) 720 (b) 610
(c) 360 (d) 210
(e) None of these
[Punjab and Sind Bank PO, 2011]

30. In how many different ways can the letters of the word'BANKING' be arranged?
(a) 5040 (b) 2540
(c) 5080 (d) 2520
(e) None of these
[Corporation Bank PO, 2010]

31. When all the students in a school are made to stand in rows of 54,30 such rows are formed. If the students are made to stand in rows of 45, how many such rows will be formed?
(a) 25 (b) 42
(c) 36 (d) 32
(e) None of these
[Corporation Bank PO, 2010]

32. In how many different ways can the letters in the word ATTEND be arranged?
(a) 60 (b) 120
(c) 240 (b) 80
(e) None of these
[Allahabad Bank PO, 2010]

Directions (Q. 33–35): Study the given information carefully and answer the questions that follow:
A committee of 5 members is to be formed out of 3 trainees, 4 professors and 6 research associates. In how many different ways can this be done if:

33. The committee should have all 4 professors and 1research associate or all 3 trainees and 2 professors?
(a) 12 (b) 13
(c) 24 (d) 52
(e) None of these
[SBI Associate Banks PO, 2010]

34. The committee should have 2 trainees and 3 research associates?
(a) 15 (b) 45
(c) 60 (d) 9
(e) None of these
[SBI Associate Banks PO, 2010]

35. In how many different ways can the letters of the word 'OFFICES' be arranged?
(a) 2520 (b) 5040
(c) 1850 (d) 1680
(e) None of these
[Indian Bank PO, 2010]

Directions (Q. 36–41): Study the following information carefully to answer the questions that follow:

A committee of 5 members is to be formed out of 5 Professors, 6 Teachers and 3 Readers.

In how many different ways can it be done if:

36. The committee should consist of 2 Professors, 2 Teachers and 1 Reader?

(a) 450 (b) 225
(c) 55 (d) 90
(e) None of these

[IDBI Bank PO, 2009]

37. The committee should include all the 3 Readers?

(a) 90 (b) 180
(c) 21 (d) 55
(e) None of these

[IDBI Bank PO, 2009]

38. In how many different ways can 4 boys and 3 girls be arranged in arow such that all the boys stand together and all the girls stand together?

(a) 75 (b) 576
(c) 288 (d) 24
(e) None of these

[IDBI Bank PO, 2009]

39. In how many different ways can the letters of the word TRUST be arranged?

(a) 240 (b) 120
(c) 80 (d) 25
(e) None of these

[OBC PO, 2009]

40. In how many different ways can the letters of the word 'FINANCE' be arranged?

(a) 5040
(b) 2040
(c) 2510
(d) 4080
(e) None of these

[NABARD Bank Officer 2009]

41. A school team has 8 volleyball players. A 5-member team and a captain will be selected out of these 8 players. How many different selections can be made?

(a) 224 (b) 112
(c) 56 (d) 88
(e) None of these

[Corporation Bank PO, 2009]

ANSWER KEYS

Exercise-1

1. (a)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (c)	**10.** (a)	**11.** (b)	**12.** (d)	**13.** (b)
14. (a)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (b)	**20.** (c)	**21.** (a)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (a)	**26.** (c)
27. (b)	**28.** (a)	**29.** (c)	**30.** (d)	**31.** (a)	**32.** (c)	**33.** (c)	**34.** (a)	**35.** (c)	**36.** (d)	**37.** (a)	**38.** (b)	**39.** (b)
40. (a)	**41.** (a)	**42.** (b)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (b)	**50.** (c)	**51.** (b)	**52.** (b)
53. (b)	**54.** (c)	**55.** (a)	**56.** (d)	**57.** (a)	**58.** (d)	**59.** (c)	**60.** (b)	**61.** (b)	**62.** (c)	**63.** (a)	**64.** (b)	**65.** (a)
66. (a)	**67.** (a)	**68.** (a)	**69.** (b)	**70.** (b)	**71.** (c)	**72.** (d)	**73.** (b)	**74.** (a)	**75.** (d)	**76.** (c)	**77.** (c)	**78.** (b)
79. (c)	**80.** (b)	**81.** (c)	**82.** (c)									

Exercise-2

1. (b)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (e)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (b)	**11.** (d)	**12.** (d)	**13.** (e)
14. (b)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (a)	**21.** (a)	**22.** (e)	**23.** (d)	**24.** (d)	**25.** (a)	**26.** (d)
27. (d)	**28.** (a)	**29.** (a)	**30.** (d)	**31.** (c)	**32.** (e)	**33.** (a)	**34.** (c)	**35.** (a)	**36.** (a)	**37.** (d)	**38.** (c)	**39.** (e)
40. (e)	**41.** (e)											

EXPLANATORY ANSWERS

EXERCISE-1

1. **(a)** The 1st post can be filled up in 6 ways.

The 2nd post can be filled up in 5 way.

and the 3rd post can be filled up in 4 ways.

∴ By the principle of association, the 3 posts can be filled up in $6 \times 5 \times 4 = 120$ ways.

2. **(c)** There are 36 teachers and every 1 has equal chance of being selected as a principal. Hence, the principal can be appointed in 36 ways. When 1 person is appointed as principal, we are left with 35 teachers. Out of these 35 teachers, we can select 1 vice-principal. So, a vice-principal can be selected in 35 ways. Hence, the number of ways in which a principal and vice-principal can be selected $= 36 \times 35 = 1260$.

3. **(d)** The first event of going from Delhi to Mumbai can be performed in 15 ways as he can go by any of the 15 buses. But the event of coming back from Mumbai can be performed in 14 ways (a different bus is to be taken).

Hence, both the events can be performed in

$$15 \times 14 = 210 \text{ ways.}$$

4. **(c)** Since the first exercise contains 15 questions, the number of ways of choosing the first question is 15. Since the second exercise contains 12 questions, the number of ways of choosing the second question is 12. Hence, by the fundamental principle, 2 questions can be selected in $= 15 \times 12 = 180$ ways.

5. **(a)** We are given that 4 students got equal marks. On 1 bit of paper, 1 arrangement of rank is to be written. Let, the students be named as P, Q, R and S.

Now, P can be treated as having rank I in 4 ways Q can be treated as having rank II in 3 ways

R can be treated as having rank III in 2 ways

S can be treated as having rank IV in 1 way.

∴ Total number of bits of paper required for all arrangements $= 4 \times 3 \times 2 \times 1 = 24$.

6. **(a)** Question I can be answered in 2 ways.

Question II can be answered in 2 ways.

Similarly questions III, IV, V each can be answered in 2 ways. Hence, total number of possible different answers $= 2 \times 2 \times 2 \times 2 \times 2 = 32$.

There is only one sequence of all correct answers

Thus, the total number of sequences are $32 - 1 = 31$

[Since no student has written all correct answers]

Now, as no 2 students have given the same sequence of answers, hence the maximum number of students in the class = 31.

7. **(b)** (i) There are in all 26 English alphabets.

We have to choose 2 distinct alphabets.

First alphabet can be selected in 26 ways.

Second alphabet can be selected in 25 ways.

Again, out of 9 digits (1 to 9), first digit can be selected in 9 ways. Second digit can be selected in 8 ways

Thus, the number of distinct codes

$= 26 \times 25 \times 9 \times 8$

$= 46800$.

8. **(d)** Each of the first 3 questions can be answered in 4 ways.

Each of the last 3 questions can be answered in 5 ways.

∴ By the fundamental principle of counting, sequences of answers are

$4 \times 4 \times 4 \times 5 \times 5 \times 5 = 64 \times 125 = 8000$.

9. **(c)** First question can be answered in 3 ways.

Second question can be answered in 3 ways.

3 question can be answered in 4 ways.

4 question can be answered in 4 ways.

5 question can be answered in 5 ways.

6 question can be answered in 5 ways.

Hence, by fundamental principle of counting, the required number of sequences of answers

$= 3 \times 3 \times 4 \times 4 \times 5 \times 5 = 3600$.

10. **(a)** 1 student representative can be selected from section I in 40 ways.

1 student representative can be selected from section II in 40 ways.

1 student representative can be selected from section III in 40 ways.

1 student representative can be selected from section IV in 40 ways

Hence, the number of ways in which a set of 4 student representatives can be selected

$= 40 \times 40 \times 40 \times 40$

$= 2560000$.

11. **(b)** Here, the first letter can be put in any 1 of the 5 envelopes in 5 ways. Second letter can be put in any 1 of the 4 remaining envelopes in 4 ways. Continuing in this way, we get the total number of ways in which 5 letters can be put into 5 envelopes $= 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Since out of the 120 ways, there is 1 one way for putting each letter in the correct envelope. Hence, the number of ways of putting letters all not in directed envelopes $= 120 - 1 = 119$ ways.

12. **(d)** H_1 can graze the grass of either P_1 or P_2 or P_3 ..., or P_7, that is, in 7 ways. After H_1 entered the field, there are 6 portions left for H_2 as no 2 can enter into the same portion of the field. After first 2 entered the field, H_3 can enter the field in 5 ways.

$\therefore$ By the fundamental principle of counting, the 3 horses can graze the grass of the field in $7 \times 6 \times 5 = 210$ ways.

13. **(b)** We have to fill up two places (since numbers are of 2 digits.

The first place can be filled up in 6 ways, as any 1 of the 6 digits can be placed in the first place. The 2nd place can be filled up in 5 ways as no digit is to be repeated. Hence, both places can be filled up in $6 \times 5 = 30$ ways.

14. **(a)**

(i) When repetition of digits is not allowed: Since we have to form a 3-digit odd number, thus the digit at unit's place must be odd. Hence, the unit's place can be filled up by 1, 3 or 5, that is, in 3 ways.

Now, the ten's digit can be filled up by any of the remaining 5 digits in 5 ways and then the hundred's place can be filled up by the remaining 4 digits in 4 ways.

Hence, the number of 3-digit odd numbers that can be formed $= 3 \times 4 \times 5 = 60$.

(ii) When repetition of digits is allowed: Again, the unit's place can be filled up by 1, 3, 5, that is, in 3 ways. But the ten's and hundred's place can be filled up by any of the 6 given digits in 6 ways each. (Since repetition is allowed)

Hence, the number of 3-digit odd numbers that can be formed $= 3 \times 6 \times 6 = 108$.

15. **(b)** The number is odd if 1 or 3 or 5 appears in the unit's place. Therefore, 3 are three ways of filling the unit's place.

Since repetition of digits is allowed, the ten's place can be filled by any of the 5 digits 1, 3, 4, 5 and 8. Hence, number of 2-digit odd numbers $= 3 \times 5 = 15$.

16. **(b)** Since the required numbers are less than 1000, they are 1-digit, 2-digit or 3-digit numbers.

(i) Only 2 1-digit odd numbers are possible, namely 5 and 7.

(ii) For 2-digit odd numbers, the unit's place can be filled up by 5 or 7 in 2 ways and ten's place can be filled up by 2, 5 or 7 (not 0) in 3 ways.

$\therefore$ Possible 2-digit odd numbers $= 2 \times 3 = 6$

(iii) For 3-digit odd numbers, the unit's place can be filled up by 5 or 7 in 2 ways. The ten's place can be filled up by any 1 of the given 4 digits in 4 ways. The hundred's place can be filled up by 2, 5 or 7 (not 0) in 3 ways.

$\therefore$ Possible 3-digit odd number $= 2 \times 4 \times 3 = 24$

$\therefore$ Required number of numbers $= 2 + 6 + 24 = 32$.

17. **(a)** Unit's place can be filled in 6 ways by any 1 of the digits 1, 2, 3, 4, 5 or 9. Also, ten's place can be filled in 6 ways. But hundred's place can be filled only is 5 ways using either 1, 2, 3, 4 or 5 ; 9 cannot be filled in hundred's place as the required number is < 600.

$\therefore$ Required number of numbers $= 6 \times 6 \times 5 = 180$.

18. **(a)** There are 26 distinct English alphabets. First alphabet can be chosen in 26 ways. Second alphabet can be chosen in 25 ways. Third alphabet can be chosen in 24 ways

Number of 3 letter words $= 26 \times 25 \times 24$

$= 15600$.

19. **(b)** Any number between 100 and 1000 is of 3 digits. Since the numbers should have distinct digits, repetition of digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 is not allowed.

Also 0 cannot be placed on the extreme left place. Hundredth place can be filled in 9 ways. Tenth place can be filled in 9 ways. Unit's place can be filled in 8 ways.

$\therefore$ The total 3-digit numbers

$= 9 \times 9 \times 8 = 648$.

20. **(c)** Any number between 1000 and 10000 is of 4 digits. The unit's place can be filled up by 4 or 5 or 6, that is, in 3 ways.

Similarly, the ten's place can be filled up by 4 or 5 or 6, that is, in 3 ways. The hundred's place can be filled up by 4 or 5 or 6, that is in 3 ways and the thousand's place can be filled up by 4 or 5 or 6, that is, in 3 ways.

Hence, the required number of numbers

$= 3 \times 3 \times 3 \times 3 = 81$.

21. **(a)** On first wheel there can be 10 digits. On the second wheel there will be 1 of the 9 digits and on the third wheel there will be 8 digits. Therefore, the number of numbers is $10 \times 9 \times 8 = 720$.

22. **(b)** At the first place he can try any of the 4 digits hence in first trial he tries 4 digits. In the second place he will try 3 remaining digits. Similarly, he will try 2 and 1 digit at the third and fourth places.

Thus, the number of trials is $= 4 \times 3 \times 2 \times 1 = 24$.

23. **(c)** $(n + 2)! = 2550\ (n!)$

$\Rightarrow (n + 2)(n + 1)(n!) = 2550(n!)$

$\Rightarrow (n + 2)(n + 1) = 2550$

$\Rightarrow n^2 + 3n + 2 - 2550 = 0$

$\Rightarrow n^2 + 3n - 2548 = 0$

$\Rightarrow n^2 + 52n - 49n - 2548 = 0$

$\Rightarrow n(n + 52) - 49(n + 52) = 0$

$\Rightarrow (n - 49)(n + 52) = 0$

$\Rightarrow n = 49$ or, $n = -52$

$\Rightarrow n = 49$ as $n = -52$ is rejected being $n \varepsilon N$

$\therefore\ n = 49$.

24. **(d)** $(n + 1)\,! = 6[(n - 1)!]$

$\Rightarrow (n + 1).\ n.\ [(n - 1)!] = 6[(n - 1)!]$

$\Rightarrow n^2 + n = 6 \Rightarrow n^2 + n - 6 = 0$

$\Rightarrow$ $(n - 2)(n + 3) = 0$

$\therefore$ Either $n - 2 = 0$ or $n + 3 = 0$

$\Rightarrow$ $n = 2$ or $n = -3$

n being natural number, so $n \neq -3$, $\therefore$ $n = 2$.

25. **(a)** $\dfrac{n!}{2!(n-2)!} : \dfrac{n!}{4!(n-4)!} = 2:1$

$\Rightarrow$ $\dfrac{n!}{2!(n-2)!} \times \dfrac{4!(n-4)!}{n!} = \dfrac{2}{1} \Rightarrow \dfrac{4!(n-4)!}{2!(n-2)!} = 2$

$\Rightarrow$ $\dfrac{4\times 3\times 2!}{2!} \times \dfrac{(n-4)!}{(n-2)(n-3)(n-4)!} = 2$

$\Rightarrow$ $12 = 2(n - 2)(n - 3)$

$\Rightarrow$ $6 = n^2 - 5n + 6 \Rightarrow n^2 - 5n = 0$

$\Rightarrow$ $n(n - 5) = 0 \Rightarrow n = 0$ or 5.

26. **(c)** ${}^nP_4 = 18 \cdot {}^{n-1}P_2$

$\Rightarrow$ $\dfrac{n!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-1-2)!}$

$\Rightarrow$ $\dfrac{n!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-3)!}$

$\Rightarrow$ $\dfrac{n(n-1)!}{(n-4)!} = 18 \cdot \dfrac{(n-1)!}{(n-3)(n-4)!}$

$\therefore$ $n = \dfrac{18}{n-3}$

i.e., $n^2 - 3n - 18 = 0$ $\quad\Rightarrow (n - 6)(n + 3) = 0$

$\Rightarrow n = 6, -3$

But n cannot be negative

$\therefore$ $n = 6$.

27. **(b)** $P(56, r + 6):P(54, r + 3) = 30800:1$

$\Rightarrow$ $\dfrac{56!}{(50-r)!} : \dfrac{54!}{(51-r)!} = 30800:1$

$\Rightarrow$ $\dfrac{54!.55.56}{(50-r)!} : \dfrac{54!}{(50-r)!.(51-r)} = 30800:1$

$\Rightarrow$ $\dfrac{3080}{1} : \dfrac{1}{51-r} = 30800:1$

$\Rightarrow$ $3080\ (51 - r) = \dfrac{30800}{1}$

$\Rightarrow$ $51 - r = 10 \Rightarrow r = 51 - 10$

$\therefore$ $r = 41$.

28. **(a)** The required number of ways = number of permutations of 10 people taking all 10 at a time.

$= P(10, 10) = 10!$

$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 3628800$.

29. **(c)** The word EQUATION has exactly 8 letters which are all different.

$\therefore$ Number of words that can be formed = number of permutations of 8 letters taken all at a time

$= P(8, 8) = 8!$

$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$.

30. **(d)** Out of 10 students, the first three prizes can be won in

${}^{10}P_3 = \dfrac{10!}{(10-3)!} = \dfrac{10!}{7!}$

$= 10 \times 9 \times 8 = 720$ ways.

31. **(a)** Total number of candidates = 5 + 4 = 9. In the row of 9 positions, the even places are 2nd, 4th, 6th and 8th. Now, number of even places = 4. Number of women to occupy the even places = 4.

$\therefore$ Even places can be filled = $P(4, 4)$ ways

Number of men = 5

$\therefore$ The remaining 5 places can be filled by 5 men

$= P(5, 5)$ ways

$\therefore$ By the fundamental principle of counting:

$\therefore$ The required number of seating arrangements

$= P(4, 4) \times P(5, 5) = 4! \times 5!$

$= 24 \times 120 = 2880$.

32. **(c)** 4 different books can be arranged among themselves, in a shelf, in $P(4, 4)$

$= 4 \times 3 \times 2 \times 1 = 24$ ways.

33. **(c)** Wearing 3 different rings in 4 fingers with at most 1 in each finger is equivalent to arranging 3 different objects in 4 places.

This can be done in $P(4, 3) = 4 \times 3 \times 2 = 24$ ways.

34. **(a)** There are 4 rows of chairs (say I, II, III, IV) consisting of 8 chairs each. It is desired that in each row, all students belong to the same class and no 2 adjacent rows are allotted to same class.

Therefore, 1 class can be seated in either I and III or in II and IV, that is in 2 ways.

Now, 16 students of this class can be arranged in 16 chairs in ${}^{16}P_{16} = 16!$ ways.

16 students of other class can be arranged in remaining 16 chairs in ${}^{16}P_{16} = 16!$ ways

$\therefore$ Total number of ways $= 2 \times 16! \times 16!$.

35. **(c)** A number lying between 1000 and 10000 has four places which can be filled up out of 7 digits in ${}^7P_4 = 7 \times 6 \times 5 \times 4 = 840$ ways.

36. **(d)** The numbers that can be formed, by taking all 6 digits together

$= {}^6P_6 = 6!$

But we have to neglect the numbers which begin with zero. Now, the numbers in which zero comes in the 1st place = 5!

Hence, the required number $= 6! - 5!$

$= 720 - 120 = 600$.

37. **(a)** The required number of 3-digit numbers

= The permutations of the 10 objects 0, 1, 2, 3, 4, 5, 6,7, 8, 9, take 3 at a time, with the condition that 0 is not in the hundred's place.

$= P(10, 3) - P(9, 2)$

$= \frac{10!}{7!} - \frac{9!}{7!}$

$= \frac{10\times9\times8\times7!}{7!} - \frac{9\times8\times7!}{7!}$

$= 10 \times 9 \times 8 - 9 \times 8$

$= 720 - 72 = 648.$

38. **(b)** 6 periods can be arranged for 5 subjects in $P(6, 5)$ ways

$= 6 \times 5 \times 4 \times 3 \times 2 = 720$

1 period is left, which can be arranged for any of the 5 subjects.

∴ 1 left period can be arranged in 5 ways.

∴ The required number of arrangements

$= 720 \times 5 = 3600.$

39. **(b)** The required number of ordered pairs of alphabets, to be used as initials, can be formed $= P(4,2) = \frac{4!}{2!} = 4 \times 3 = 12.$

40. **(a)** Total number of candidates = 8.

5 different subjects candidates can be seated in $P(5, 5) = 5!$ ways.

In between 5 candidates there are six places for 8 Mathematics candidates.

∴ The Mathematics candidates can be seated in $P(6, 3)$ ways

∴ By fundamental principle of counting:

The required number of ways $= 5! \times P(6, 3)$

$= 120 \times \frac{6!}{3!}$

$= 120 \times 6 \times 5 \times 4 = 14400.$

41. **(a)** The vowels in the word 'ORIENTAL' are: O, I, E and A

Total number of letters in the word 'ORIENTAL' = 8.

Number of vowels = 4

O	R	I	E	N	T	A	L
1	2	3	4	5	6	7	8
×		×		×		×	

∵ Vowels occupy odd places, that is 1, 3, 5 and 7. Number of odd places = 4

∴ 4 vowels can be arranged in 4 'X' marked places $= P(4, 4)$ ways $= 4!$ ways

Number of consonants = 4.

∴ 4 consonants can be arranged in four places

$= P(4, 4)$ ways $= 4!$ ways

∴ The required number of words

$= 4! \times 4! = 24 \times 24 = 576.$

42. **(b)** The numbers are divisible by 10 if 0 is in the unit's place.

× × × × × 0

∴ The required numbers which are divisible by 10 = $P(5, 5) = 5! = 120.$

43. **(b)**

(i) The word 'UNIVERSAL' has 9 letters which are all different. These 9 letters can be arranged amongst themselves in 9! ways.

(ii) 3 letters E, R, S can be taken to form 1 block. Thus, 7 letters U, N, I, V, ERS, A, L can be arranged in 7! ways.

Also, 3 letters E, R, S can be arranged amongst themselves in 3! ways.

∴ The total number of arrangements $= 7! \times 3! = 30240.$

44. **(a)** 5 students are to be arranged on a platform. 1 boy SUNIL is fixed at second position.

∴ We have to arrange only 4 students. But GITA is always adjacent to NITA. Considering the 2 girls (NITA and GITA) as 1, the 3 students can be arranged in 3! ways. NITA and GITA themselves can be arranged in 2! ways.

∴ The required number of arrangements

$= 2! \times 3! = (2 \times 1) \times (3 \times 2 \times 1) = 12.$

45. **(a)** ALGEBRA has 7 letters where 2 – A, 1 – L, 1 – G, 1 – E, 1 – B and 1 – R.

(i) Since two A's are always together, we take both the A's as 1 letter.

If P is the number of arrangements, then

$P = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720.$

(ii) Total number of permutations

$q = \frac{7!}{2!} = 7 \times \frac{7\times6\times5\times4\times3\times2\times1}{2\times1} = 2520$

In these permutations, in some permutations, two A's are together while in the rest they are not together.

Hence, the number of permutations in which two A's are not together is

$q - p = 2520 - 720 = 1800.$

46. **(a)** Each apple can be given to any 1 of the 3 boys and this can be done in 3 ways.

$\therefore$ The required number of ways = 3^6 = 729.

47. **(c)** Number of letters in the word 'KURUKSHETRA' is 11 of which 2 are K's, 2 are U's, 2 are R's and remaining are different.

$\therefore$ Required number of permutations = $\frac{11!}{2!2!2!}$

= 4989600.

48. **(a)** The word ALLAHABAD has 9 letters in all. The letter 'A' occurs 4 times, the letter 'L' occurs 2 times and the remaining three letters H, B, D each occur once.

$\therefore$ The require number of permutations

$$= \frac{9!}{4!2!1!1!} = \frac{9\times8\times7\times6\times5\times4!}{4!\times2}$$

= 9 × 8 × 7 × 3 × 5 = 7560.

49. **(b)** There are three places to be filled up to form a 3-digit number. Since any digit may be repeated any number of times, each 1 of three places can be filled up by any of the given 4 digits in 4 ways.

Hence, the number of words that can be formed

= 4^3 = 64.

50. **(c)** There are in all 5 digits. Now, 0 cannot be placed at the hundredth place as in that case the number will not be 3-digited. Thus, hundredth place can be filled up by any of remaining 4 digits in 4 ways.

Since the digits may be repeated any number of times, each of the remaining two places can be filled up by any of the 5 digits in 5 ways each. Thus, the total number of such arrangements = 5^2 = 25.

Hence, the total number of words that can be formed

= 4 × 25 = 100.

51. **(b)** Out of letters in the word 'BHARAT' 2 letters, that is, A's are alike.

$\therefore$ Number of permutations = $\frac{6!}{2!}$ = 360.

Number of words in which B and H are never together = Total number of words − number of words in which B and H are together

$$=360 - \frac{5!}{2!}.\,2 = 360 - 120 = 240.$$

52. **(d)** The given word consists of 11 letters out of which A occurs 2 times, R occurs 2 times, N occurs 2 times and E occurs 2 times and remaining 3 are different.

$\therefore$ Number of arrangements

$$= \frac{11!}{2!2!2!2!} = 2491800.$$

53. **(b)** Starting with A and arranging the other ten letters A, E, I, I, M, N, N, O, T, X (not all distinct, I occurs twice, N occurs twice), there are

$$\frac{10!}{2!2!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{2\times2}$$

= 907200 words.

$\therefore$ The number of items in the list before the first word starting with E is 907200.

54. **(c)** The 5-digit even numbers can be formed out of 1, 2, 5, 5, 4 by using either 2 or 4 in the unit's place. This can be done in 2 ways.

Corresponding to each such arrangement, the remaining four places can be filled up by any of the remaining 4 digits in $\frac{4!}{2!}$ = 12 ways.

[$\because$ 50 occurs twice]

Hence, the total number of words = 2 × 12 = 24.

55. **(a)** A number greater than a million has seven places, and thus all the 7 given digits are to be used. But 2 is repeated twice and 3 is repeated thrice.

$\therefore$ Total number of ways of arranging these 7 digits amongst themselves

$$= \frac{7!}{2!3!} = 420$$

But numbers beginning with zero are no more 7 digited numbers, hence we have to reject those numbers which begin with zero, and such numbers are

$$= \frac{6!}{2!.3!} = 60.$$

Hence, the required number of arrangements,

= 420 − 60 = 360.

56. **(d)** There are 7 letters, of which 2 are A's and the rest are all different. The vowels A, A, E occupy 1st, 4th and 7th places. The number of ways in which they can be arranged in these places is $\frac{3!}{2!}$ = 3.

The consonants L, G, B, R are all different. The number of ways in which they can be arranged in the remaining places in 4!. Since each way of arranging the vowels can be *associated* with each way of arranging the consonants, we find that the total number of arrangements = 3 × 4!, = 3 × 24 = 72.

57. **(a)** There are in all 7 letters in the word BALLOON in which L occurs 2 times and O occurs 2 times.

$\therefore$ The number of arrangements of the 7 letters of the word = $\frac{7!}{2!\times2!}$ = 1260.

If two L's always come together, taking them as 1 letter, we have to arrange 6 letters in which O occurs 2 times.

$\therefore$ The number of arrangements in which the two L's come together

$$= \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360.$$

Hence, the required number of ways in which the two L's do not come together

$= 1260 - 360 = 900$.

58. (d) Here, $n = 3 + 2 + 2 = 7$

$p_1 = 3$, $p_2 = 2$ and $p_3 = 2$

$\therefore$ The required number of different signals

$$= \frac{n!}{p_1!\,p_2!\,p_3!} = \frac{7!}{3!\,2!\,2!} = \frac{7.6.5.4}{2.2}$$

$= 7.6.5 = 210$.

59. (c) The given digits are: 1, 2, 3, 4, 3, 2, 1. Out of these 1, 3, 3, 1 are odd digits.

The odd digits occupy the odd places, that is, they occupy the 1st, 3rd, 5th and 7th place.

$\therefore$ 4 odd places can be filled with 4 odd digits

$$= \frac{4!}{2!\,2!} = \frac{4.3}{2} = 6 \text{ ways.}$$

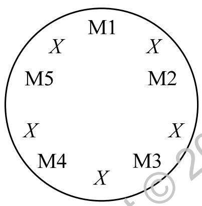
The three even places to be filled with the even digits 2, 4, 2.

These places can be filled $= \dfrac{3!}{2!\,1!} = \dfrac{3.2!}{2!} = 3$ ways

Hence, the required number of numbers

$= 6 \times 3 = 18$.

60. (b) Refer to Fig. Let, us first seat 5 gentlemen on the round table and this can be done in $(5 - 1)! = 24$ ways.



Since no 2 ladies are to sit together, they can occupy the places marked as 'X'.

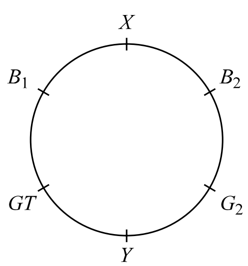
Number of 'X' signs = 5

$\therefore$ 4 ladies can be arranged in five places in

$${}^5P_4 = \frac{5!}{1!} = 120 \text{ ways.}$$

$\therefore$ Required number of ways $= 24 \times 120 = 2880$.

61. (b) Let, B_1, B_2 and X be 3 boys and G_1, G_2, Y be 3 girls. Since X does not want any girl neighbour, B_1, B_2 can be their neightbours.



Similarly, the girl Y does not want any boy neighbour, therefore G_1, G_2 are the only neighbours of Y. Now, B_1, B_2 can arrange themselves in 2! ways and G_1, G_2 can also arrange themselves in 2! ways.

Hence, the required number of permutations

$= 2! \times 2! = 4$.

62. (c) n = Total number of beads $= 6 + 5 = 11$

$P_1 = 6$, $P_2 = 5$

$\therefore$ Number of different necklaces

$$= \frac{1}{2}\frac{(11-1)!}{6!\,5!} = \frac{10!}{2.6!\,5!}$$

$$= \frac{10.9.8.7.6!}{2.6!.5.4.3.2.1} = 3 \times 7 = 21.$$

63. (a) Treat the Punjab and Madras Chief Ministers as one then we have (P, M) + 9 others.

$\therefore$ We have to arrange 10 persons round a table. This can be done in $(10 - 1)! = 9!$ ways.

Corresponding to each of these 9! ways, the Punjab and Madras Chief Ministers can interchange their places in 2! ways. Associating the 2 operations, total number of ways

$= 9!\,2! = (9.8.7.6.4.3.2.1)\,(2.1) = 725760$.

64. (b) We know that $C(n, r) = \dfrac{n!}{r!(n-r)!}$.

Now, $C(n, 7) = C(n, 5)$

$\Rightarrow \dfrac{n!}{7!(n-7)!} = \dfrac{n!}{5!(n-5)!}$

$\Rightarrow 5!(n - 5)! = 7!(n - 7)!$

$\Rightarrow [5!].(n - 5)(n - 6)[(n - 7)!] = 7.6.[5!](n - 7)!$

$\Rightarrow n^2 - 11n + 30 = 42$

$\Rightarrow n^2 - 11n + 30 - 42 = 0$

$\Rightarrow n^2 - 11n - 12 = 0$

$\Rightarrow (n - 12)(n + 1) = 0$

$\Rightarrow n - 12 = 0$ or, $n + 1 = 0$

$\Rightarrow n = 12$ or, $n = -1$.

But $n = -1$ is rejected as n is a non-negative integer,

$\therefore n = 12$.

65. (a) $C(n, 8) = C(n, 6)$

$\Rightarrow \dfrac{n!}{8!(n-8)!} = \dfrac{n!}{6!(n-6)!}$

$\Rightarrow 6!(n - 6)! = 8!(n - 8)!$

$\Rightarrow 6!(n - 6)(n - 7)[(n - 8)!] = 8.7.6!(n - 8)!$

$\Rightarrow n^2 - 13n + 42 = 56$

$\Rightarrow n^2 - 13n + 42 - 56 = 0$

$\Rightarrow n^2 - 13n - 14 = 0$

$\Rightarrow n^2 - 13n - 14 = 0$

$\Rightarrow (n - 14)(n + 1) = 0$

$\Rightarrow n - 14 = 0$ or $n + 1 = 0$

$\Rightarrow$ $n = 14$ or $n = -1$.

But $n = -1$ is rejected as n is a non-negative integer

$\Rightarrow$ $n = 14$

$\therefore$ $C(n, 2) = C(14, 2) = \dfrac{14!}{2!12!} = \dfrac{14.13.12!}{2.12!}$

$= 91$.

66. **(a)** $C(2n, 3):(C, 3) = 11:1$

$\Rightarrow$ $\dfrac{(2n)!}{3!(2n-3)!} : \dfrac{n!}{3!(n-3)!} = 11:1$

$\Rightarrow$ $(2n)(2n-1)(2n-2) : n(n-1)(n-2) = 11:1$

$\Rightarrow$ $2 \times 2(2n-1)(n-1) : (n-1)(n-2) = 11:1$

$\Rightarrow$ $4(2n-1) : n-2 = 11:1$

$\Rightarrow$ $\dfrac{8n-4}{n-2} = \dfrac{11}{1}$

$\Rightarrow$ $11n - 22 = 8n - 4$

$\Rightarrow$ $11n - 8n = 22 - 4$

$\Rightarrow$ $3n = 18$ $\therefore$ $n = 6$.

67. **(a)** ${}^{2n}C_r = {}^{2n}C_{2n-r}$ $[\because {}^nC_r = {}^nC_{n-r}]$

But ${}^{2n}C_r = {}^{2n}C_{r+2}$ [Given]

$\therefore$ ${}^{2n}C_{2n-r} = {}^{2n}C_{r+2}$

$\therefore$ $2n - r = r + 2 \Rightarrow r = n - 1$.

68. **(a)** ${}^{18}C_r = {}^{18}C_{18-r}$ $[\because {}^nC_r = {}^nC_{n-r}]$

But ${}^{18}C_r = {}^{18}C_{r+r}$ [Given]

$\therefore$ ${}^{18}C_{18-r} = {}^{18}C_{r+2}$ or, $18 - r = r + 2 \Rightarrow r = 8$

$\therefore$ ${}^rC_5 = {}^8C_5 = \dfrac{8!}{5!3!} = \dfrac{8\times7\times6\times5!}{3\times2\times1\times5!} = 56$.

69. **(b)** Here, $12 \cdot {}^nC_2 = {}^{2n}C_3$

$\therefore$ $12 \cdot \dfrac{n!}{2!(n-2)!} = \dfrac{(2n)!}{3!(2n-3)!}$ $\left[\because {}^nC_r = \dfrac{n!}{r!(n-r)!}\right]$

or, $12 \cdot \dfrac{n(n-1)(n-2)!}{2!.(n-2)!} = \dfrac{(2n)(2n-1)(2n-2)(2n-3)!}{6(2n-3)!}$

or, $6 \cdot n(n-1) = \dfrac{2n(2n-1)(2n-2)}{6}$

or, $18 \cdot (n-1) = (2n-1)(2n-2)$

or, $9 \cdot (n-1) = (2n-1)(n-1)$

i.e., $9 = 2n - 1 \Rightarrow n = 5$.

70. **(b)** $\displaystyle\sum_{r=1}^{5} C(5, r) = C(5, 1) + C(5, 2) + C(5, 3) + C(5, 4) + C(5, 5)$

$= \dfrac{5!}{1!4!} + \dfrac{5!}{2!3!} + \dfrac{5!}{3!2!} + \dfrac{5!}{4!1!} + \dfrac{5!}{5!0!}$

$= \dfrac{5}{1} + \dfrac{5.4}{2} + \dfrac{5.4}{2} + \dfrac{5}{1} + 1$

$= 5 + 10 + 10 + 5 + 1 = 31$.

71. **(c)** The require number of ways $= C(10, 5)$

$= \dfrac{10!}{5!5!} = \dfrac{10.9.8.7.6}{5.4.3.2} = 3.2.42 = 252$.

72. **(d)** 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \dfrac{16!}{11!5!} = 4368$ ways.

When 2 particular players are always to be included, then 9 more players are to be selected out of remaining 14 players, which can be done in

${}^{14}C_9$ ways $= \dfrac{14!}{9!5!} = 2002$ ways.

73. **(b)** 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \dfrac{16!}{11!5!} = 4368$ ways.

If 1 particular player is to be excluded, then selection is to be made of 11 players out of 15 players and this can be done in ${}^{15}C_{11}$ ways

$= \dfrac{15!}{11!4!} = 1365$ ways.

74. **(a)** 11 players can be selected out of 16 players in ${}^{16}C_{11}$ ways $= \dfrac{16!}{11!5!} = 4368$ ways

If 2 particular players are to be included and one particular player is to be rejected, then we have to select 9 more out of 13 in ${}^{13}C_9$ ways

$= \dfrac{13!}{9!4!} = 715$ ways.

75. **(d)** The required number of ways

$= C(10, 8) . C(10, 5)$

$= \dfrac{10!}{8!2!} \times \dfrac{10!}{5!5!} = \dfrac{10\times9}{2} \times \dfrac{10\times9\times8\times7\times6}{5\times4\times3\times2}$

$= 5 \times 9 \times 3 \times 2 \times 7 \times 6 = 11340$.

76. **(c)** Here, we want to find the number of ways of selecting 11 players out of 15.

$\therefore$ The required number of ways

$= C(15, 11) = \dfrac{15!}{11!4!} = \dfrac{15\times14\times13\times12}{4\times3\times2\times1}$

$= 15 \times 7 \times 13 = 1365$.

A particular player to be included

So, we have to select 10 players out of 14.

$\therefore$ The required number of ways

$= C(14, 10) = \dfrac{14!}{10!4!} = \dfrac{14.13.12.11}{4.3.2.1} = 7.13.11 = 1001$.

77. **(c)** Number of letters in the word = 8

Number of vowels in the word = 4

(1, O, U, E)

Number of consonants in the word = 4

(N, V, L, T)

Out of 4 vowels, we have to select 3.

Out of 4 consonants, we have to select 2.

Also, we have to arrange 3 vowels and 2 consonants.

$\therefore$ The required number of words

$= C(4, 3).C(4, 2).5! = 4 \times \frac{4\times3}{2} = 120 = 2880.$

78. **(b)** We get a line by joining 2 points. If p is the number of lines from 21 points, then,

$p = C(21, 2) = \frac{21!}{2!(21-2)!} = \frac{21\times20(19!)}{2\times1(19!)} = 21 \times 10 = 210$ lines.

79. **(c)** If p is the required number of ways, then

$p = C(6, 3) \times C(5, 3) \times C(5, 3)$

$= \frac{6!}{3!(6-3)!}\times\frac{5!}{3!(5-3)!}\times\frac{5!}{3!(5-3)!}$

$= \frac{6\times5\times4\times3!}{3\times2\times1\times3!}\times\frac{5\times4\times3!}{2\times1\times3!}\times\frac{5\times4\times3!}{2\times1\times3!}$

$= 5 \times 4 \times 5 \times 2 \times 5\times 2 = 2000.$

80. **(b)** Out of available 9 courses, two are compulsory. Hence, the student is free to select 3 courses out of 7 remaining courses: If p is the number of ways of selecting 3 courses out of 7 courses, then

$p = C(7, 3) = \frac{7!}{3!(7-3)!} = \frac{7!}{3!\,4!} = \frac{7\times6\times5\times4!}{3\times2\times1\times4!}$

$= 7 \times 5 = 35$ ways.

81. **(c)** If p is the number of ways of selection, then

$p = C(6, 4) \times C(7, 4) \times (8, 4) = \frac{6!}{4!\,2!}\times\frac{7!}{4!\,3!}\times\frac{8!}{4!\,4!}$

$= \frac{6\times5\times4!}{2\times1\times4!}\times\frac{7\times6\times5\times4!}{3\times2\times1\times4!}\times\frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!}$

$= 15 \times 35 \times 70 = 36750$ ways.

82. **(c)** If A and B both are not selected, then the number of permutations = $C(6, 6) = 1$.

If A and B both are selected, then we are to select 4 persons out of 6 persons.

The number of ways in which this can be done

$= C(6, 4) = \frac{6!}{4!(6-2)!} = \frac{6\times5\times4!}{4!\times2\times1} = 15$

Hence, total number of permutations = 1 + 15 = 16.

Exercise-2
(Based on Memory)

1. **(b)** Required different ways = ${}^7P_7$

$= \lfloor 7 = 5040$

2. **(a)** Required number of ways = 8! = 40320

3. **(a)** Required number of ways = 6! = 720

4. **(a)** Required number of ways

$= {}^{10}C_3 \times {}^8C_2 = 120\times28 = 3360$

5. **(e)** Required number of ways

$= {}^8C_4 \times {}^{10}C_0 + {}^8C_3 \times {}^{10}C_1 + {}^8C_2 \times {}^{10}C_2$

= 70 + 560 + 1200 = 1890

6. **(b)** Required number of ways

$= {}^5C_2 \times {}^3C_2 \times {}^5C_1 = 10\times3\times5 = 150$

7. **(a)** Required number of ways

$= {}^{18}C_7 = \frac{18\times17\times16\times15\times14\times13\times12}{7\times6\times5\times4\times3\times2}$

= 31824

8. **(c)** Required number of ways $= {}^5C_3 = 10$

9. **(a)** Required number of ways $= \frac{7!}{3!} = 840$

10. **(b)** A first, 3 persons can be selected from 6 persons in

${}^6C_3 = \frac{6!}{3!3!} = 20$ ways.

Now, 3 selected persons can sit in 3! = 6 ways.

$\therefore$ Total number of required ways

$= 20\times6 = 120$

11. **(d)** Required number of ways

$$^9C_5 = \frac{9!}{5!(9-5)!} = \frac{6\times7\times8\times9}{2\times3\times4} = 126$$

12. **(d)** The required number of ways $= \frac{7!}{3!} = 840$

13. **(e)** Required number of ways = 6! = 720

14. **(b)** Required number of ways

= 5! = 5 × 4 × 3 × 2 × 1 = 120

15. **(c)** Required number of ways $= \frac{7!}{2!} = 2520$

17. **(b)** Total different ways of sitting

$= {}^4P_4 \times {}^2P_2 = \lfloor 4 \times \lfloor 2 = 48$

18. **(a)** Required number of ways = 6! = 720

19. **(c)** Required different ways

$= \lfloor 3 \times \lfloor 4 = 6 \times 24 = 144$

20. **(a)** Required different ways

$$= {}^7C_3 - {}^3C_3 = \frac{\lfloor 7}{\lfloor 3 \lfloor 4} - \frac{\lfloor 3}{\lfloor 0 \lfloor 3} = 34$$

21. **(a)** Let, the probability of truth spoken by A and B be p_1 and p_2, respectively

$\therefore \quad p_1 = \frac{3}{4}$ and $p_2 = \frac{4}{5}$

They will contradict each other only when 1 speaks truth and the other is lying

i.e., $\frac{3}{4}\times\frac{1}{5}+\frac{4}{5}\times\frac{1}{4} = \frac{3}{20}+\frac{4}{20} = \frac{7}{20} = \frac{35}{100}$, i.e., 35%

22. **(e)** If the ball drawn is neither red nor green, then it must be blue, which can be picked in $^7C_1 = 7$ ways. 1 ball can be picked from the total (8 + 7 + 6 = 21) is $^{21}C_1 = 21$ ways.

$\therefore$ Required probability $= \frac{7}{21} = \frac{1}{3}$.

23. **(d)** There are 6 letters in the word 'ATTEND' whereas, *T* comes 2 times.

So, required number of ways $= \frac{6!}{2!} = \frac{720}{2} = 360$

24. **(d)** CYCLE whereas C comes two times.

So, arrangements are $= \frac{5!}{2!} = \frac{5\times4\times3\times2}{2} = 60$ ways

25. **(a)** The word DESIGN be arranged so that the vowels are at 2 ends then the required number of orders

$= {}^2P_2 \times {}^4P_4$

= 2! × 4!

= 2 × 4 × 3 × 2

= 48

26. **(d)** There are 5 letters in the word SMART.

So, the required number of ways to arrange

$= {}^5P_5 = 5!$

= 5 × 4 × 3 × 2 × 1

= 120

27. **(d)** Total number of letters is 7, and these letters can be arranged in 7! ways

= 1 × 2 × 3 × 4 × 5 × 6 × 7 = 5040 ways.

There are 7 letters in the world THERAPY including 2 vowels. (E, A) and 5 consonants.

Consider 2 vowels as 1 letter.

We have 6 letters which can be arranged in $^6P_6$ = 6 ways.

But vowels can be arranged in 2! Ways.

Hence, the number of ways, all vowels will come together = 6! × 2!

= 1 × 2 × 3 × 4 × 5 × 6 × 2 = 1440

Total number of ways in which vowels will never come together = 5040 – 1440 = 3600

28. **(a)** 6 letters of the word PRAISE can be arranged in 6!ways= 720 ways.

29. **(a)** Required number of ways

$= \lfloor 6 = 6\times5\times4\times3\times2\times1 = 720$

30. **(d)** Required number of ways $= \frac{7!}{2!} = 2520$

31. (c) Total number of students = 54 × 30

When arranged in rows of 45, number of rows

$= \frac{54\times30}{45} = 36$

32. (e) Required number of arrangements $= \frac{6!}{2!}$

$= \frac{6\times5\times4\times3\times2}{2} = 360$

33. **(a)** Number of combinations

$= {}^4C_4 \times {}^6C_1 + {}^3C_3 \times {}^4C_2$

= 1 × 6 + 1 × 6

= 12.

34. **(c)** Number of combinations

= selecting 2 trainees out of 3 and selecting 3 Research Associates out of 6

$= {}^3C_2 \times {}^6C_3 = 3 \times \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 60$

35. **(a)** Required number of ways $= \frac{7!}{2!} = 2520$

36. **(a)** Required number of ways

$= {}^5c_2 \times {}^6c_2 \times {}^3c_1 = 450$

37. **(d)** Required number of ways

$= {}^3c_3 \times {}^{11}c_2 = 1 \times \frac{10 \times 11}{2} = 55$

38. **(c)** Number of groups formed = 2

Number of ways boys could be arranged = 4!

Number of ways girls could be arranged = 3!

Required number of ways = 2 × 4! × 3!

39. (e) Required number of ways $= \frac{5!}{2!} = 60,$

Total number of letters in the word is 5; T is repeated twice.

40 **(e)** N is repeated twice.

Hence the required answer will be $\frac{\lfloor 7}{2} = 2520.$

41. (e) Required number of selections $= {}^8c_1 \times {}^7c_5 = 168$