

Simplification

4

SIMPLE ARITHMETIC OPERATIONS

It is a common need to simplify the expressions formulated according to the statements of the problems related to practical life. To do this, it is essential to follow in sequence the mathematical operations given by the term, 'BODMAS'.

BODMAS

Each letter of the word BODMAS stands as follows:

B for Bracket : $\{ \{ (-) \}$

There are four brackets, namely, – i.e., bar, $()$, $\{ \}$ and $[]$. They are removed, strictly in the order –, $()$, $\{ \}$ and $[]$.

O for Of : of

D for Division : \div

M for Multiplication : \times

A for Addition : $+$

S for Subtraction : $-$

The order of various operations in exercises involving brackets and fractions must be strictly performed according to the order of the letters of the word BODMAS.

Note:

Here, $-\overline{5-8} = -(-3) = 3$.

Illustration 1: Simplify

$$8\frac{1}{2} - \left[3\frac{1}{5} \div 4\frac{1}{2} \text{ of } 5\frac{1}{3} + \left\{ 11 - \left(3 - 1\frac{1}{4} - \frac{5}{8} \right) \right\} \right]$$

Solution: Given expression

$$\begin{aligned} &= \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \text{ of } \frac{16}{3} + \left\{ 11 - \left(3 - \frac{5}{4} - \frac{5}{8} \right) \right\} \right] \\ &= \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \text{ of } \frac{16}{3} + \left\{ 11 - \left(3 - \frac{5}{8} \right) \right\} \right] \end{aligned}$$

$$\begin{aligned} &= \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \text{ of } \frac{16}{3} + \left\{ 11 - \frac{19}{8} \right\} \right] \\ &= \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \text{ of } \frac{16}{3} + \frac{69}{8} \right] \\ &= \frac{17}{2} - \left[\frac{16}{5} \div \frac{9}{2} \times \frac{15}{3} + \frac{69}{8} \right] \\ &= \frac{17}{2} - \left[\frac{16}{5} \div \frac{24}{1} + \frac{69}{8} \right] \\ &= \frac{17}{2} - \left[\frac{16}{5} \times \frac{1}{24} + \frac{69}{8} \right] = \frac{17}{2} - \left[\frac{16}{120} + \frac{69}{8} \right] \\ &= \frac{17}{2} - \left[\frac{16+1035}{120} \right] = \frac{17}{2} - \frac{1051}{120} \\ &= \frac{1020-1051}{120} = -\frac{31}{120} \end{aligned}$$

Illustration 2: Simplify

$$5\frac{1}{3} - \left\{ 4\frac{1}{3} - \left(3\frac{1}{3} - 2\frac{1}{3} - \frac{1}{3} \right) \right\}$$

Solution: Given expression

$$\begin{aligned} &= \frac{16}{3} - \left\{ \frac{13}{3} - \left(\frac{10}{3} - \frac{7}{3} - \frac{1}{3} \right) \right\} \\ &= \frac{16}{3} - \left\{ \frac{13}{3} - \left(\frac{10}{3} - \frac{6}{3} \right) \right\} = \frac{16}{3} - \left\{ \frac{13}{3} - \frac{4}{3} \right\} \\ &= \frac{16}{3} - \left\{ \frac{9}{3} \right\} = \frac{16}{3} - \frac{9}{3} = \frac{7}{3} = 2\frac{1}{3} \end{aligned}$$

Use of Algebraic Formulae

The following formulae are sometimes found useful in dealing with the simplifications:

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
4. $(a + b)^2 - (a - b)^2 = 4ab$
5. $a^2 - b^2 = (a + b)(a - b)$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
 $= a^3 + b^3 + 3ab(a + b)$
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
 $= a^3 - b^3 - 3ab(a - b)$
8. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
10. $\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} = (a + b + c)$.
11. $a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$.

Illustration 3: Simplify the following

(i) $0.32 \times 0.32 + 0.64 \times 0.68 + 0.68 \times 0.68$

Solution: Given expression

$$\begin{aligned}
 &= 0.32 \times 0.32 + 2 \times 0.32 \times 0.68 + 0.68 \times 0.68 \\
 &= (0.32)^2 + 2 \times 0.32 \times 0.68 + (0.68)^2 \\
 &= (0.32 + 0.68)^2 \quad [\because a^2 + 2ab + b^2 = (a + b)^2] \\
 &= 1^2 = 1.
 \end{aligned}$$

(ii) $2.45 \times 2.45 - 0.9 \times 2.45 + 0.45 \times 0.45$

Solution: Given expression

$$\begin{aligned}
 &= 2.45 \times 2.45 - 2 \times 2.45 \times 0.45 + 0.45 \times 0.45 \\
 &= (2.45)^2 - 2 \times 2.45 \times 0.45 + (0.45)^2 \\
 &= (2.45 - 0.45)^2 \quad [\because a^2 - 2ab + b^2 = (a - b)^2] \\
 &= (2)^2 = 4.
 \end{aligned}$$

(iii) $\frac{7 \times \{(146 + 92)^2 + (146 - 92)^2\}}{(146)^2 + (92)^2}$

Solution: Given expression

$$\begin{aligned}
 &= \frac{7 \times \{(146)^2 + (92)^2\}}{(146)^2 + (92)^2} \\
 &\quad [\because (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)] \\
 &= 14.
 \end{aligned}$$

(iv) $\frac{(0.345 + 0.255)^2 - (0.345 - 0.255)^2}{0.345 \times 1.02}$

Solution: Given expression

$$\begin{aligned}
 &= \frac{(0.345 + 0.255)^2 - (0.345 - 0.255)^2}{4 \times 0.345 \times 0.255} \\
 &= \frac{4 \times 0.345 \times 0.255}{4 \times 0.345 \times 0.255} \quad [\because (a + b)^2 - (a - b)^2 = 4ab] \\
 &= 1.
 \end{aligned}$$

(v) $\frac{0.682 \times 0.682 - 0.318 \times 0.318}{0.682 - 0.318}$

Solution: Given expression

$$\begin{aligned}
 &= \frac{(0.682)^2 - (0.318)^2}{0.682 - 0.318} \\
 &= (0.682 + 0.318) \quad \left[\because \frac{a^2 - b^2}{a - b} = a + b \right] \\
 &= 1.
 \end{aligned}$$

(vi) $\frac{(3.29)^2 - (0.81)^2}{4}$

Solution: Given expression

$$\begin{aligned}
 &= \frac{(3.29)^2 - (0.81)^2}{3.29 + 0.81} \\
 &= (3.29 - 0.81) \quad \left[\because \frac{a^2 - b^2}{a + b} = a - b \right] \\
 &= 2.48.
 \end{aligned}$$

(vii) $(2.35)^3 + 1.95 \times (2.35)^2 + 7.05 \times (0.65)^2 + (0.65)^3$

Solution: Given expression

$$\begin{aligned}
 &= (2.35)^3 + 3 \times 0.65 \times (2.35)^2 \\
 &\quad + 3 \times 2.35 \times (0.65)^2 + (0.65)^3 \\
 &= (2.35 + 0.65)^3 \\
 &\quad [\because a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3] \\
 &= (3)^3 = 27.
 \end{aligned}$$

(viii) $\frac{(4.32)^3 - 0.96 \times (4.32)^2 + 12.96 \times (0.32)^2 - (0.32)^3}{4 \times 4 \times 4}$

Solution: Given expression

$$\begin{aligned}
 &= \frac{(4.32)^3 - 3 \times 0.32 \times (4.32)^2 + 3 \times 4.32 \times (0.32)^2 - (0.32)^3}{4 \times 4 \times 4} \\
 &= \frac{(4.32 - 0.32)^3}{4^3} \quad [\because a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3] \\
 &= \left(\frac{4}{4} \right)^3 = 1.
 \end{aligned}$$

(ix) $\frac{885 \times 885 \times 885 + 115 \times 115 \times 115}{885 \times 885 + 115 \times 115 - 885 \times 115}$

Solution: Given expression

$$\begin{aligned}
 &= \frac{(885)^3 + (115)^3}{(885)^2 + (115)^2 - 885 \times 115}
 \end{aligned}$$

$$= (885 + 115) \left[\because \frac{a^3 + b^3}{a^2 - ab + b^2} = a + b \right]$$

$$= 1000.$$

$$(x) \frac{0.62 \times 0.62 \times 0.62 - 0.41 \times 0.41 \times 0.41}{0.62 \times 0.62 + 0.62 \times 0.41 + 0.41 \times 0.41}$$

Solution: Given expression

$$= \frac{(0.62)^3 - (0.41)^3}{(0.62)^2 + 0.62 \times 0.41 + (0.41)^2}$$

$$= (0.62 - 0.41) \left[\because \frac{a^3 - b^3}{a^2 + ab + b^2} = a - b \right]$$

$$= 0.21.$$

$$(xi) \frac{(2.3)^3 + (1.5)^3 + (1.2)^3 - 3 \times 2.3 \times 1.5 \times 1.2}{2.3 \times 2.3 + 1.5 \times 1.5 + 1.2 \times 1.2 - 2.3 \times 1.5 - 2.3 \times 1.2 - 1.5 \times 1.2}$$

Solution: Given expression

$$= \frac{(2.3)^3 + (1.5)^3 + (1.2)^3 - 3 \times 2.3 \times 1.5 \times 1.2}{(2.3)^2 + (1.5)^2 + (1.2)^2 - 2.3 \times 1.5 - 2.3 \times 1.2 - 1.5 \times 1.2}$$

$$= (2.3 + 1.5 + 1.2)$$

$$\left[\because \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - ac - bc} = a + b + c \right]$$

$$= 5.$$

Surds and Indices

a^n is called, 'a **surd**' if n is a fraction and a^n is called, 'an **index**' if n is an integer. a is called, 'the **base**'.

SOME USEFUL SHORT-CUT METHODS

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = (a^n)^m = a^{mn}$
- $\left(\frac{a}{b}\right)^{\frac{m}{n}} = \left(\frac{b}{a}\right)^{\frac{m}{n}}$
- $a^m \div b^{-n} = a^m \times b^n$
- $(\sqrt[n]{a})^n = a$, where ' n ' is a +ve integer and ' a ' a +ve rational number.
- $\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab}$, where ' n ' is a +ve integer and ' a ', ' b ' are rational numbers.
- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, where ' n ' is a +ve integer and ' a ', ' b ' are rational numbers.
- $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$, where ' m ', ' n ' are +ve integers and ' a ' is a +ve rational number.
- $\sqrt[m]{\sqrt[n]{(a^k)^m}} = \sqrt[n]{a^k} = \sqrt[mn]{a^{km}}$, where ' m ', ' n ', ' k ' are +ve integers and ' a ' is a +ve rational number.
- $\sqrt{a} \times \sqrt{a} = a$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$
- $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$
- $a + \sqrt{b} = c + \sqrt{d} \Rightarrow a = c$ and $b = d$.

$$16. \frac{1}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{a} + \sqrt{b}}{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})} = \frac{\sqrt{a} + \sqrt{b}}{a - b}$$

$$17. \frac{1}{\sqrt{a} + \sqrt{b}} = \frac{\sqrt{a} - \sqrt{b}}{(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})} = \frac{\sqrt{a} - \sqrt{b}}{a - b}$$

18. If $x = n(n + 1)$, then

$$(a) \sqrt{x - \sqrt{x - \sqrt{x - \dots}}} = n \text{ and}$$

$$(b) \sqrt{x + \sqrt{x + \sqrt{x + \dots}}} = (n + 1)$$

Illustration 4: Find the value of $(243)^{0.8} \div (243)^{0.4}$.

Solution: $(243)^{0.8} \div (243)^{0.4} = (243)^{0.8-0.4}$

$$[\because a^m \div a^n = a^{m-n}]$$

$$= (243)^{0.4}$$

$$= (3^5)^{\frac{2}{5}} = 3^2 = 9.$$

Illustration 5: Find the value of $(27)^{2/3} \div (64)^{-4/3}$

Solution: $(27)^{2/3} \div (64)^{-4/3} = (3^3)^{2/3} \times (64)^{4/3}$

$$[\because a^m \div b^{-n} = a^m \times b^n]$$

$$= 3^2 \times (4^3)^{4/3}$$

$$= 9 \times (4^4) = 9 \times 256 = 2304.$$

Illustration 6: Find the value of $(-3)^{(-2)^{(-4)}}$

Solution: $(-3)^{(-2)^{(-4)}} = \left(-\frac{1}{3}\right)^{(2)^{(-4)}} = \left(\frac{1}{9}\right)^{(-2)^{(-4)}}$

$$= (9)^{(2)^{(-4)}}$$

$$= (81)^{-4} = \left(\frac{1}{81}\right)^4$$

$$= \left(\frac{1}{3^4}\right)^4 = \frac{1}{3}.$$

Illustration 7: Find the value of x if $\sqrt[5]{2x-7} - 3 = 0$.

Solution: We have

$$\begin{aligned}\sqrt[5]{2x-7} - 3 &= 0 \Rightarrow \sqrt[5]{2x-7} = 3 \\ \Rightarrow (\sqrt[5]{2x-7})^5 &= 3^5 \\ \Rightarrow 2x - 7 &= 243 \quad [\because (\sqrt[n]{a})^n = a] \\ \Rightarrow 2x &= 250 \text{ or, } x = 125.\end{aligned}$$

Illustration 8: Find the value of $\sqrt[3]{64} \times \sqrt[5]{512}$.

Solution: $\sqrt[3]{64} \times \sqrt[5]{512}$

$$= \sqrt[3]{64 \times 512} \quad [\because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}]$$

$$= \sqrt[3]{8^2 \times 8^3} = \sqrt[3]{8^5} = 8. \quad [\because \sqrt[n]{a^n} = a]$$

Illustration 9: Find the value of $\sqrt[3]{\sqrt[2]{729}}$.

Solution: $\sqrt[3]{\sqrt[2]{729}} = \sqrt[6]{729} \quad [\because \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}]$

$$= \sqrt[6]{3^6} = 3. \quad [\because \sqrt[n]{a^n} = a]$$

Illustration 10: Find the value of $\frac{\sqrt[7]{(21)^5}}{\sqrt[5]{(7^5)^3}}$.

Solution: Given expression

$$= \frac{\sqrt[7]{(21)^7}}{\sqrt[5]{(7^5)^5}} \quad [\because \sqrt[m]{\sqrt[n]{(a^p)^m}} = \sqrt[n]{a^p}]$$

$$= \frac{21}{7} = 3. \quad [\because \sqrt[n]{a^n} = a]$$

Illustration 11: Find the value of $\sqrt{5} \times \sqrt{125}$.

Solution: $\sqrt{5} \times \sqrt{125} = \sqrt{625} \quad [\because \sqrt{a} \times \sqrt{b} = \sqrt{ab}]$

$$= 25.$$

Illustration 12: Simplify each of the following by rationalizing the denominators.

(i) $\frac{1}{2+\sqrt{3}}$ (ii) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Solution: (i) $\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}.$$

(ii) $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{4^2 \times 3} + \sqrt{3^2 \times 2}}$

$$= \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} = \frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$$

$$= \frac{(7\sqrt{3}-5\sqrt{2})(4\sqrt{3}-3\sqrt{2})}{(4\sqrt{3}+3\sqrt{2})(4\sqrt{3}-3\sqrt{2})}$$

$$= \frac{7\sqrt{3} \times 4\sqrt{3} - 7\sqrt{3} \times 3\sqrt{2} - 5\sqrt{2} \times 4\sqrt{3} + 5\sqrt{2} \times 3\sqrt{2}}{(4\sqrt{3})^2 - (3\sqrt{2})^2}$$

$$= \frac{28\sqrt{3} \times 3 - 21\sqrt{3} \times 2 - 20\sqrt{2} \times 3 + 15\sqrt{2} \times 2}{16 \times 3 - 9 \times 2}$$

$$= \frac{28 \times 3 - 21 \times \sqrt{6} - 20\sqrt{6} + 15 \times 2}{48 - 18}$$

$$= \frac{84 - (21 \times 20)\sqrt{6} + 30}{30} = \frac{114 - 41\sqrt{6}}{30}$$

Illustration 13: If a and b are rational numbers, find the values of a and b in the following equation.

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}.$$

Solution: $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{3+2+2\sqrt{3} \times \sqrt{2}}{3-2} = \frac{5+2\sqrt{6}}{1}$$

$$= 5 + 2\sqrt{6}$$

$$\therefore \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6} \Rightarrow 5 + 2\sqrt{6} = a + b\sqrt{6}.$$

On equating rational and irrational parts, we get

$$a = 5 \text{ and } b = 2.$$

Illustration 14: Find the value of

$$\left(\sqrt{72+\sqrt{72+\sqrt{72+\dots\infty}}}\right) \div \left(\sqrt{12-\sqrt{12-\sqrt{12-\dots\infty}}}\right)$$

Solution: Since $72 = 9 \times 8$,

$$\text{therefore, } 72 + \sqrt{72+\sqrt{72+\dots\infty}} = 9$$

$$\text{Also, since } 12 = 4 \times 3$$

$$\text{therefore, } \sqrt{12-\sqrt{12-\sqrt{12-\dots\infty}}} = 3.$$

$$\text{Thus, the given expression} = \frac{9}{3} = 3.$$

Fractions

1. Continued Fraction

$$\text{Fractions of the form } 7 + \frac{2}{5 + \frac{3}{4 + \frac{2}{3 + \frac{1}{4}}}}$$

are called, 'continued fractions'.

To simplify a continued fraction, we start from the bottom and work upwards.

Illustration 15: Simplify $3 - \frac{1}{4 + \frac{7}{9 - \frac{5}{6 + \frac{2}{3}}}}$.

Solution: $3 - \frac{1}{4 + \frac{7}{9 - \frac{5}{6 + \frac{2}{3}}}} = 3 - \frac{1}{4 + \frac{7}{9 - \frac{15}{9 - \frac{20}{6 + \frac{2}{3}}}}}$

[Multiply the numerator and denominator of the lowest term $\frac{5}{6 + \frac{2}{3}}$ by 3 to get $\frac{15}{20}$.]

$$= 3 - \frac{1}{4 + \frac{7}{9 - \frac{3}{4}}} = 3 - \frac{1}{4 + \frac{28}{33}}$$

[Multiply the numerator and denominator of the lowest term $\frac{7}{9 - \frac{3}{4}}$ by 4 to get $\frac{28}{33}$.]

$$= 3 - \frac{33}{160}$$

[Multiply the numerator and denominator of the term $\frac{1}{4 + \frac{28}{33}}$ by 33 to get $\frac{33}{160}$.]

$$= \frac{480 - 33}{160} = \frac{447}{160} = 2\frac{127}{160}$$

2. Comparison of Fractions

The following points are found useful while comparing two or more fractions.

(a) If the denominators of the fractions are same, the largest is one whose numerator is the largest.

Illustration 16: Which is the largest fraction among the following?

$$\frac{3}{8}, \frac{7}{8} \text{ and } \frac{5}{8}$$

Solution: $\frac{7}{8}$.

(b) If the numerators of the fractions are same, the largest is one whose denominator is the smallest.

Illustration 17: Which is the largest fraction among the following?

$$\frac{5}{2}, \frac{5}{7} \text{ and } \frac{5}{9}$$

Solution: $\frac{5}{2}$

(c) If neither the numerators nor the denominators of the fractions are same, then they are converted into equivalent fractions of the same denominator by taking the L.C.M. of the denominators of the given fractions. Then the fractions are compared according to (1).

Illustration 18: Which is the largest fraction among the following?

$$\frac{1}{2}, \frac{2}{3}, \frac{4}{5} \text{ and } \frac{5}{8}$$

Solution: L.C.M. of 2, 3, 5 and 8 = 120.

$$\frac{1}{2} = \frac{1 \times 60}{2 \times 60} = \frac{60}{120}$$

Then,

$$\frac{2}{3} = \frac{2 \times 40}{3 \times 40} = \frac{80}{120}$$

$$\frac{4}{5} = \frac{4 \times 24}{5 \times 24} = \frac{96}{120}$$

$$\text{and, } \frac{5}{8} = \frac{5 \times 15}{8 \times 15} = \frac{75}{120}$$

Now, the denominator of these fractions are same and the largest numerator is 96. Hence, the largest fraction

is $\frac{96}{120}$, that is, $\frac{4}{5}$.

(d) Two fractions can also be compared by cross-multiplication method.

Illustration 19: Which is greater $\frac{6}{13}$ or $\frac{7}{5}$?

Solution: **Step 1.** By cross-multiplying the two given fractions

$$\frac{6}{13} \times \frac{5}{7},$$

we get $6 \times 7 = 42$ and $13 \times 5 = 65$.

Step 2. Since 65 is greater than 42 and in 65, the numerator of $\frac{5}{7}$ is included, $\therefore \frac{5}{7}$ is greater than $\frac{6}{13}$.

(e) If the difference of the numerator and denominator of each of the given fractions be same, then the fraction of the largest numerator is the smallest.

Illustration 20: Which of the following fraction is the largest?

$$\frac{2}{3}, \frac{3}{4}, \frac{5}{6} \text{ and } \frac{9}{10}$$

Solution: In each of the given fractions, the difference between the numerator and denominator is same and the largest numerator is 9. The largest fraction is $\frac{9}{10}$.

(f) In the given fractions, $\frac{x}{y}, \frac{x+a}{y+b}, \frac{x+2a}{y+2b}, \dots, \frac{x+na}{y+nb}$, where $a < b$

(a) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}} > \text{first fraction}$, the last value is the greatest.

(b) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}} < \text{first fraction}$, the last value is the least.

(c) If $\frac{\text{Increase in Numerator}}{\text{Increase in Denominator}} = \text{first fraction}$, all values are equal.

Illustration 21: Which one the following fractions is the greatest?

$$\frac{3}{8}, \frac{4}{11}, \frac{5}{14}, \frac{6}{17}, \frac{7}{20}$$

Solution: Since, $\frac{\text{increase in numerator}}{\text{increase in denominator}} = \frac{1}{3}$ is less than the first fraction $\frac{3}{8}$, therefore, the first fraction $\frac{3}{8}$ is the greatest.

Illustration 22: Which of the following fractions is the least?

$$\frac{2}{5}, \frac{4}{11}, \frac{6}{17}, \frac{8}{23}$$

Solution: Since, $\frac{\text{increase in numerator}}{\text{increase in denominator}} = \frac{2}{6} = \frac{1}{3}$ is less than the first fraction $\frac{2}{5}$, therefore, the last fraction $\frac{8}{23}$ is the least.

3. Inserting a fraction between two given fractions.

To insert a fraction between two given fractions

$\frac{a_1}{b_1}$ and $\frac{a_2}{b_2}$, the following steps may be useful.

Step 1 The numerators of the two given fractions are added to get the numerator of the resulting fraction, that is, $a_1 + a_2$.

Step 2 The denominators of the two given fractions are added to get the denominator of the resulting fraction, that is, $b_1 + b_2$.

Step 3 The resulting fraction = $\frac{a_1 + a_2}{b_1 + b_2}$.

Illustration 23: Insert a fraction between $\frac{2}{5}$ and $\frac{4}{7}$.

Solution: By using the given method,

$$\frac{2}{5}, \frac{2+4}{5+7}, \frac{4}{7} = \frac{2}{5}, \frac{6}{12}, \frac{4}{7} \text{ or, } \frac{2}{5}, \frac{1}{2}, \frac{4}{7}.$$

Illustration 24: Insert three fractions between $\frac{5}{7}$ and $\frac{9}{11}$.

Solution: By using the given method,

$$\frac{5}{7}, \frac{5+9}{7+11}, \frac{9}{11} = \frac{5}{7}, \frac{14}{18}, \frac{9}{11}$$

$$\text{or, } \frac{5}{7}, \frac{7}{9}, \frac{9}{11}.$$

Further,

$$\frac{5}{7}, \frac{5+7}{7+9}, \frac{7}{9}, \frac{7+9}{9+11}, \frac{9}{11} = \frac{5}{7}, \frac{12}{16}, \frac{7}{9}, \frac{16}{20}, \frac{9}{11}$$

$$\text{or } \frac{5}{7}, \frac{3}{4}, \frac{7}{9}, \frac{4}{5}, \frac{9}{11}.$$

Thus, the three fractions inserted between $\frac{5}{7}$ and $\frac{9}{11}$ are $\frac{3}{4}, \frac{7}{9}$ and $\frac{4}{5}$.

EXERCISE-I

1. Simplify:

$$\frac{3}{10} \div \frac{3}{7} \text{ of } \left(2\frac{3}{10} + 2\frac{3}{5} \right) + \frac{1}{5} \div 1\frac{2}{5} - \frac{2}{7}$$

- (a) 1 (b) 2
(c) 0 (d) 3

$$2. 1 + 1 \div \left\{ 1 + 1 \div \left(1 - \frac{1}{3} \right) \right\} = ?$$

- (a) $\frac{7}{5}$ (b) $\frac{2}{3}$
(c) $\frac{4}{5}$ (d) None of these

$$3. 48 \div 12 \times \left(\frac{9}{8} \text{ of } \frac{4}{3} + \frac{3}{4} \text{ of } \frac{2}{3} \right) = ?$$

- (a) 9 (b) 12
(c) 15 (d) None of these

4. Simplify:

$$2 \div [2 + 2 \div \{2 + 2 \div (2 + 2 \div 3)\}]$$

- (a) 13/15 (b) 17/15
(c) 11/15 (d) None of these

$$5. 7\frac{1}{2} - \left[2\frac{1}{4} \div \left\{ 1\frac{1}{4} - ? \left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right] = 3$$

- (a) $\frac{1}{4}$ (b) $\frac{3}{4}$
(c) $\frac{4}{3}$ (d) None of these

$$6. \text{ The simplification of } \frac{0.8 \times 0.8 \times 0.8 - 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 + 0.8 \times 0.5 + 0.5 \times 0.5} \text{ gives:}$$

- (a) 0.8 (b) 0.4
(c) 0.3 (d) 0.13

$$7. \text{ The simplification of } \left[\frac{1}{2} + \frac{1}{2} \left\{ \frac{3}{4} - \frac{1}{2} \left(\frac{7}{8} - \frac{3}{4} \right) \right\} \right] \text{ yields:}$$

- (a) $\frac{27}{16}$ (b) $\frac{27}{32}$
(c) $\frac{27}{64}$ (d) $\frac{107}{112}$

$$8. \text{ Simplify: } 1 - [2 - \{5 - (4 - \overline{3 - 2})\}]$$

- (a) 1 (b) 2
(c) 3 (d) 4

$$9. 3 \div \left[(8 - 5) \div \left\{ (4 - 2) \div \left(2 + \frac{8}{13} \right) \right\} \right] = ?$$

- (a) $\frac{33}{71}$ (b) $\frac{55}{17}$
(c) $\frac{13}{17}$ (d) None of these

$$10. \frac{69842 \times 69842 - 30158 \times 30158}{69842 - 30158} = ?$$

- (a) 100000 (b) 69842
(c) 39684 (d) 30158

$$11. \text{ Simplify } \frac{2\frac{1}{7} - 2\frac{1}{2}}{2\frac{1}{4} + 1\frac{1}{7}} \div \frac{1}{2 + \frac{1}{2 + \frac{1}{2 - \frac{1}{2}}}}$$

- (a) $-\frac{1}{2}$ (b) $-\frac{1}{8}$
(c) $-\frac{1}{6}$ (d) $-\frac{1}{4}$

$$12. \text{ The value of } \frac{2.75 \times 2.75 \times 2.75 - 2.25 \times 2.25 \times 2.25}{2.75 \times 2.75 + 2.75 \times 2.25 + 2.25 \times 2.25} \text{ is:}$$

- (a) 0.30 (b) 0.50
(c) 3.00 (d) 5.00

$$13. \frac{\frac{1}{2} \div 4 + 20}{\frac{1}{2} \times 4 + 20} = ?$$

- (a) $\frac{81}{88}$ (b) $2\frac{3}{11}$
(c) $\frac{161}{176}$ (d) 1

$$14. \text{ Evaluate } \frac{0.53 \times 0.53 - 2 \times 0.53 \times 0.41 + 0.41 \times 0.41}{0.53 \times 0.41}$$

- (a) 0.16 (b) 0.8
(c) 0.12 (d) None of these

$$15. \text{ The value of } \frac{9^2 \times 18^4}{3^{16}} \text{ is:}$$

- (a) $\frac{2}{3}$ (b) $\frac{4}{9}$
(c) $\frac{16}{81}$ (d) $\frac{32}{243}$

16. The simplification of $1 + \frac{1}{2 + \frac{1}{1 - \frac{1}{3}}}$ yields the result:
- (a) $\frac{2}{7}$ (b) $\frac{7}{9}$
 (c) $\frac{9}{7}$ (d) $\frac{13}{7}$
17. $108 + ?$ of $\frac{1}{3} + \frac{2}{5} \times 3\frac{3}{4} = 10\frac{1}{2}$
- (a) 15 (b) 63
 (c) 24 (d) 36
18. The value of $1 + 1 + \frac{1}{4 \times 3} + \frac{1}{4 \times 3^2} + \frac{1}{4 \times 3^3}$ up to four places of decimals is:
- (a) 1.1202 (b) 1.1203
 (c) 1.1204 (d) None of these
19. $\frac{3}{48}$ is what part of $\frac{1}{12}$?
- (a) $\frac{3}{7}$ (b) $\frac{1}{12}$
 (c) $\frac{3}{4}$ (d) None of these
20. The simplification of $1 + \frac{1}{1 + \frac{1}{1 - \frac{2}{3}}}$ yields the result
- (a) $\frac{7}{4}$ (b) $\frac{4}{5}$
 (c) $\frac{5}{4}$ (d) None of these
21. Which of the following fractions is less than $\frac{7}{8}$ and greater than $\frac{1}{3}$?
- (a) $\frac{1}{4}$ (b) $\frac{23}{24}$
 (c) $\frac{11}{12}$ (d) $\frac{17}{24}$
22. How many $\frac{1}{8}$'s are there in $37\frac{1}{2}$?
- (a) 300 (b) 400
 (c) 500 (d) Cannot be determined.
23. In a college, $\frac{1}{5}$ th of the girls and $\frac{1}{8}$ th of the boys took part in a social camp. What part of the total number of students in the college took part in the camp?
- (a) $\frac{13}{40}$ (b) $\frac{13}{80}$
 (c) $\frac{2}{13}$ (d) Data inadequate
24. $\left\{ 7\frac{1}{2} + \frac{1}{2} \div \frac{1}{2} \text{ of } \frac{1}{4} - \frac{2}{5} \times 2\frac{1}{3} \div 1\frac{7}{8} \text{ of } \left(1\frac{2}{5} - 1\frac{1}{3} \right) \right\} = ?$
- (a) $3\frac{1}{5}$ (b) $2\frac{1}{24}$
 (c) $4\frac{1}{30}$ (d) None of these
25. When simplified, the product $\left(2 - \frac{1}{3} \right) \left(2 - \frac{3}{5} \right) \left(2 - \frac{5}{7} \right) \dots \left(2 - \frac{999}{1001} \right)$ is equal to:
- (a) $\frac{991}{1001}$ (b) $\frac{1001}{13}$
 (c) $\frac{1003}{3}$ (d) None of these
26. The value of $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6}$ is:
- (a) $\frac{7}{30}$ (b) $\frac{1}{3}$
 (c) $\frac{13}{30}$ (d) None of these
27. $15\frac{15}{3} \times 3\frac{1}{6} + 6\frac{1}{3} = 11\frac{7}{18} + ?$
- (a) $39\frac{5}{9}$ (b) $137\frac{4}{9}$
 (c) $29\frac{7}{9}$ (d) None of these
28. $3 \div \left[(8-5) \div \left\{ (4-2) \div \left(2 + \frac{8}{13} \right) \right\} \right] = ?$
- (a) $\frac{13}{17}$ (b) $\frac{68}{13}$
 (c) $\frac{17}{13}$ (d) $\frac{13}{68}$

29. If the numbers $\frac{3}{5}, \frac{2}{3}, \frac{3}{4}$ are given, then we can say that:
- (a) $\frac{3}{4} > \frac{3}{5} > \frac{2}{3}$ (b) $\frac{2}{3} > \frac{3}{5} > \frac{3}{4}$
 (c) $\frac{3}{4} > \frac{2}{3} > \frac{3}{5}$ (d) $\frac{3}{5} > \frac{2}{3} > \frac{3}{4}$
30. $\frac{(272-32)(124+176)}{17 \times 15 - 15} = ?$
- (a) 0 (b) 2.25
 (c) 300 (d) None of these
31. If $\frac{a}{b} = \frac{1}{2}$, then $\frac{3a+2b}{3a-2b}$ is equal to:
- (a) 3 (b) -3
 (c) -5 (d) -1
32. $(20 \div 5) \div 2 + (16 \div 8) \times 2 + (10 \div 5) \times (3 \div 2) = ?$
- (a) 9 (b) 12
 (c) 15 (d) 18
33. $\frac{5}{6} \div \frac{6}{7} \times ? - \frac{8}{9} \div 1\frac{3}{5} + \frac{3}{4} \times 3\frac{1}{3} = 2\frac{7}{9}$
- (a) $\frac{7}{6}$ (b) $\frac{6}{7}$
 (c) 1 (d) None of these
34. $4\frac{1}{2} + 3\frac{1}{6} + ? + 2\frac{1}{3} = 13\frac{2}{3}$
- (a) $3\frac{2}{5}$ (b) $1\frac{2}{5}$
 (c) $4\frac{1}{5}$ (d) $4\frac{1}{6}$
35. The simplification of $\frac{0.67 \times 0.67 \times 0.67 - 0.001}{0.67 \times 0.67 + 0.067 + 0.01}$ gives:
- (a) 0.57 (b) 0.66
 (c) 0.68 (d) 0.77
36. In a certain college, the number of girls is twice the number of boys. $\frac{1}{5}$ th of the girls and $\frac{1}{8}$ th of the boys took part in a social camp. What part of the total number of students took part in the camp?
- (a) $\frac{7}{40}$ (b) $\frac{7}{80}$
 (c) $\frac{2}{12}$ (d) $\frac{1}{24}$
37. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The fraction is:
- (a) $\frac{8}{27}$ (b) $2\frac{2}{3}$
 (c) $1\frac{1}{3}$ (d) None of these
38. Which of the following numbers is the greatest?
- (a) $(0.3)^2$ (b) $1 \div 0.3$
 (c) $\frac{1}{8}$ (d) $\sqrt{0.49}$
39. What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of $\frac{1}{12}$ of all the three fractions?
- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$
 (c) $\frac{1}{4}$ (d) $\frac{1}{6}$
40. A person was to multiply a fraction by $\frac{6}{7}$. Instead, he divided and got an answer which exceeds the correct answer by $\frac{1}{7}$. The correct answer is:
- (a) $\frac{6}{13}$ (b) $\frac{36}{91}$
 (c) $\frac{7}{13}$ (d) None of these
41. $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} + \frac{1}{\sqrt{2}-2} = ?$
- (a) 2 (b) 4
 (c) 0 (d) Cannot be determined
42. The value of $\frac{1}{2} + \frac{1}{2.3} + \frac{1}{2.3.4} + \frac{1}{2.3.4.5}$ is correct to three places of decimal:
- (a) 0.713 (b) 0.715
 (c) 0.717 (d) 0.718
43. $\frac{? \div 12}{0.2 \times 3.6} = 2$
- (a) 17.82 (b) 17.22
 (c) 17.28 (d) 17.12
44. $\sqrt{? \times 7} \times 18 = 84$
- (a) 3.11 (b) 3.12
 (c) 3.13 (d) 3.14

4.10 Chapter 4

45. The difference between the sum of $1\frac{3}{4}$, $2\frac{1}{3}$, $3\frac{5}{12}$, $5\frac{1}{5}$ and $2\frac{1}{6}$ and the nearest whole number is:
- (a) $\frac{2}{15}$ (b) $\frac{13}{15}$
 (c) $\frac{11}{60}$ (d) None of these
46. $\left(2\frac{3}{x}\right) \times \left(y\frac{1}{2}\right) = \frac{3}{4}$, find the values of x and y .
 (a) (3, 19) (b) (3, 14)
 (c) (14, 3) (d) (24, 6)
47. If we multiply a fraction by itself and divide the product by the square of its reciprocal, the fraction obtained is $3\frac{13}{81}$. The original fraction is:
 (a) $\frac{16}{9}$ (b) $\frac{8}{9}$
 (c) $\frac{4}{3}$ (d) $\frac{1}{3}$
48. If $x \times y = (x + 2)^2(y - 2)$, then $7 \times 5 = ?$
 (a) 234 (b) 243
 (c) 343 (d) 423
49. If m and n are whole numbers such that $m^n = 121$, then $(m - 1)^{n+1} = ?$
 (a) 10 (b) 10^2
 (c) 10^3 (d) 10^4
50. Between two fractions $\frac{1}{2}$ and $\frac{1}{8}$, how many fractions are there in all?
 (a) Four (b) Zero
 (c) Sixteen (d) Infinite
51. A boy was asked to multiply a given number by $8/17$. Instead, he divided it by $\frac{8}{17}$ and got the result 225 more than what he should have got if he had multiplied the number by $8/17$. The given number was:
 (a) 8 (b) 17
 (c) 64 (d) 136
52. The value of $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{9.10}$ is:
 (a) $\frac{11}{10}$ (b) $\frac{8}{9}$
 (c) $\frac{9}{10}$ (d) $\frac{25}{1089}$
53. $\frac{\sqrt{1296}}{?} = \frac{?}{2.25}$
 (a) 6 (b) 3
 (c) 9 (d) 12
54. If we multiply a fraction by itself and divide the product by its reciprocal, the fraction thus obtained is $18\frac{26}{27}$. The original fraction is:
 (a) $\frac{8}{27}$ (b) $2\frac{2}{3}$
 (c) $1\frac{1}{3}$ (d) None of these
55. If $\frac{a}{a+b} = \frac{17}{23}$, what is $\frac{a+b}{a-b}$ equal to?
 (a) $\frac{11}{23}$ (b) $\frac{17}{32}$
 (c) $\frac{23}{11}$ (d) $\frac{23}{17}$
56. A tin of oil was $\frac{4}{5}$ the full when 6 bottles of oil were taken out. Again, 4 bottles of oil were poured into it, it was $\frac{3}{4}$ full. How many bottles of oil it may contain?
 (a) 10 (b) 20
 (c) 30 (d) 40
57. In an examination, a student was asked to find $\frac{3}{14}$ of a certain number. By mistake, he found $\frac{3}{4}$ of it. His answer was 150 more than the correct answer. The given number is:
 (a) 180 (b) 240
 (c) 280 (d) 290
58. The value of $\left(2 - \frac{1}{3}\right)\left(2 - \frac{3}{5}\right)\left(2 - \frac{5}{7}\right) \dots \left(2 - \frac{999}{1001}\right)$ is:
 (a) $\frac{1003}{3}$ (b) $\frac{1003}{1001}$
 (c) $\frac{1}{1001}$ (d) None of these
59. If $\sqrt{2^n} = 64$, then find the value of n .
 (a) 8 (b) 10
 (c) 12 (d) 16

60. If $10^{2y} = 25$, then what is the value of 10^y ?

- (a) -5 (b) 5
(c) $\frac{1}{25}$ (d) $\sqrt{\frac{1}{25}}$
(e) None of these

61. $11\frac{1}{3} \times 4\frac{8}{10} \div ? = 22\frac{2}{3}$

- (a) 2.4 (b) 4.2
(c) 2.6 (d) 2.8

62. Simplify:

$$\frac{a^{1/2} + a^{-1/2}}{1-a} + \frac{1-a^{-1/2}}{1+\sqrt{a}}$$

- (a) $\frac{a}{a-1}$ (b) $\frac{a-1}{2}$
(c) $\frac{2}{a-1}$ (d) $\frac{2}{1-a}$

63. If $a^2 + b^2 = 45$ and $ab = 18$, find $\frac{1}{a} + \frac{1}{b}$.

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) Cannot be determined

64. If $\frac{a^2 + b^2}{c^2 + d^2} = \frac{ab}{cd}$, then find the value of $\frac{a+b}{a-b}$ in terms of c and d only.

- (a) $\frac{c+d}{cd}$ (b) $\frac{cd}{c+d}$
(c) $\frac{c-d}{c+d}$ (d) $\frac{c+d}{c-d}$

65. $(1.06 + 0.04)^2 - ? = 4 \times 1.06 \times 0.04$

- (a) 1.04 (b) 1.4
(c) 1.5 (d) Cannot be determined

66. The highest score in an inning was $\frac{2}{9}$ of the total score and the next highest was $\frac{2}{9}$ of the remainder.

These scores differ by 8 runs. What was the total score in the innings?

- (a) 162 (b) 152
(c) 142 (d) 132

67. Simplify $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$

- (a) $17\frac{1}{8}$ (b) $17\frac{3}{8}$
(c) $11\frac{7}{8}$ (d) $17\frac{7}{8}$

68. $\frac{\frac{64}{8} - \frac{9}{3}}{\frac{121}{8} + \frac{64}{3}} = ?$

- (a) $\frac{88}{31}$ (b) $\frac{31}{88}$
(c) $\frac{41}{99}$ (d) $\frac{99}{41}$

69. When $\frac{1}{4}$ of a number is subtracted from $\frac{1}{3}$ of the same number, the remainder obtained is 12. The number is:

- (a) 144 (b) 72
(c) 120 (d) 63
(e) None of these

70. What is the difference between the largest and the smallest fractions:

$$\frac{5}{8}, \frac{21}{35}, \frac{9}{16} \text{ and } \frac{6}{7}?$$

- (a) $\frac{33}{112}$ (b) $\frac{11}{37}$
(c) $\frac{13}{41}$ (d) $\frac{9}{35}$
(e) None of these

71. If a man spends $\frac{5}{6}$ part of money and, again earns $\frac{1}{2}$ part of the remaining money, what part of his money is with him now?

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$
(e) $\frac{1}{3}$

4.12 Chapter 4

72. Manmohan spends $\frac{1}{5}$ part of his money as pocket money and $\frac{4}{5}$ of the remainder in other affairs. If he is left with ₹48 per month, what is his monthly income?
- (a) ₹360 (b) ₹400
(c) ₹320 (d) ₹300
(e) None of these
73. If the difference between $\frac{4}{5}$ part and $\frac{3}{4}$ part of a number is 4, what is the number?
- (a) 60 (b) 100
(c) 80 (d) 40
(e) None of these
74. If $\frac{2}{3}$ part of a number is 96, what is the value of $\frac{3}{4}$ part of the same number?
- (a) 48 (b) 192
(c) 108 (d) 72
(e) None of these
75. A man completes $\frac{2}{15}$ of his journey by aeroplane, $\frac{2}{5}$ by train and the rest by taxi. What part of his journey does he complete by taxi?
- (a) $\frac{8}{15}$ (b) $\frac{7}{15}$
(c) $\frac{9}{15}$ (d) None of these
76. If $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{70}\right) = \frac{x}{70}$, then what is the value of x?
- (a) 69 (b) 35
(c) 20 (d) 15
(e) 1
77. The value of $\frac{1.073 \times 1.073 - 0.927}{1.073 - 0.927} + \frac{(3^4)^4 \times 9^5}{(27)^7 \times (3)^9}$ is:
- (a) 2 (b) $\frac{1}{9}$
(c) $2\frac{1}{9}$ (d) $3\frac{1}{9}$
(e) None of these.
78. The value of $\frac{2^{1/2} \cdot 3^{1/3} \cdot 4^{1/4}}{10^{-1/5} \cdot 5^{3/5}} \div \frac{3^{4/3} \cdot 5^{-7/5}}{4^{-3/5} \cdot 6}$ is:
- (a) 5 (b) 6
(c) 10 (d) 15
(e) None of these

EXERCISE 2 (BASED ON MEMORY)

1. If the numerator of a fraction is increased by 200% and the denominator of the fraction is increased by 150%, the resultant fraction is $\frac{7}{10}$. What is the original fraction?
- (a) $\frac{3}{4}$ (b) $\frac{7}{12}$
(c) $\frac{7}{11}$ (d) $\frac{9}{11}$
(e) None of these
[Bank of Maharashtra PO, 2008]
2. What will come in place of questions mark (?) in the following equation?
 $16^{7.5} \div 8^{3.5} \div 2^{7.5} = ?$
- (a) 8^4 (b) 16^4
(c) 2^{15} (d) 2^{27}
(e) None of the above
[SBI PO, 2005]
3. $(a)^c \times (b)^a \times ? = 0$
- (a) 1 (b) -1
(c) 0 (d) $(c)^b$
(e) None of these
[SBI PO, 2005]
4. $\left[\sqrt[3]{250047} + (56)^2\right] \div 7 = ?$
- (a) 547 (b) 475
(c) 455 (d) 521
(e) None of these
[Andhra Bank PO, 2006]

5. $[(\sqrt{529} \times 36) \div 48] \times ? = 5847.75$

- (a) 346 (b) 317
(c) 339 (d) 325
(e) None of these

[Andhra Bank PO, 2006]

6. $(16\% \text{ of } 480) + (? \% \text{ of } 978) = 653.82$

- (a) 48 (b) 57
(c) 61 (d) 63
(e) None of these

[Andhra Bank PO, 2006]

7. $(0.56\% \text{ of } 225) \times (3.25\% \text{ of } 430) = ?$

- (a) 17.6085 (b) 15.3195
(c) 15.6175 (d) 15.6175
(e) None of these

[Andhra Bank PO, 2006]

8. The difference between $\frac{3}{5}$ of $\frac{2}{3}$ of a number and $\frac{2}{5}$ of $\frac{1}{4}$ of the same number is 288. What is the number?

- (a) 960 (b) 850
(c) 895 (d) 955
(e) None of these

[Andhra Bank PO, 2006]

9. By how much is $\frac{3}{4}$ of 52 less than $\frac{2}{3}$ of 99?

- (a) 27 (b) 33
(c) 39 (d) 66
(e) None of these

[IBBI Bank Officers, 2007]

10. If the numerator of a fraction is increased by 400% and the denominator is increased by 500%. The resultant fraction is $\frac{15}{22}$. What was the original fraction?

- (a) $\frac{9}{11}$ (b) $\frac{5}{11}$
(c) $\frac{7}{11}$ (d) $\frac{11}{13}$

(e) None of these

[Corporation Bank PO, 2007]

11. Which number should replace both the questions marks in the following equation?

$$\frac{?}{928} = \frac{58}{?}$$

- (a) 212 (b) 227
(c) 232 (d) 247
(e) None of these

[Andhra Bank PO, 2007]

12. $3^{3.5} \times 21^2 \times 42^{2.5} \div 2^{2.5} \times 7^{3.5} = 21^?$

- (a) 8 (b) 10
(c) 12.5 (d) 6.5
(e) None of these

[Corporation Bank PO, 2006]

13. If $x = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$ and $y = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$, then $(x + y)$ equals:

- (a) 8 (b) 16
(c) $2\sqrt{15}$ (d) $2(\sqrt{5} + \sqrt{3})$

[SSC (GL) Prel. Examination, 2005]

14. $\left[\frac{1}{\sqrt{2} + \sqrt{3} - \sqrt{5}} + \frac{1}{\sqrt{2} - \sqrt{3} - \sqrt{5}} \right]$ in simplified form equals:

- (a) 1 (b) $\sqrt{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) 0

[SSC (GL) Prel. Examination, 2005]

15. If * represents a number, then the value of * in $5\frac{3}{*} \times 3\frac{1}{2} = 19$ is:

- (a) 7 (b) 4
(c) 6 (d) 2

[SSC (GL) Prel. Examination, 2005]

16. $\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{100}\right)$ results in:

- (a) 0.01 (b) 0.001
(c) 1 (d) 0.1

[SSC (GL) Prel. Examination, 2005]

17. $\left(\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}\right)^2 + \left(\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}\right)^2$ is equal to:

- (a) 64 (b) 62
(c) 66 (d) 68

[SSC (GL) Prel. Examination, 2000]

18. If 25^{25} is divisible by 26, the remainder is:

- (a) 1 (b) 2
(c) 24 (d) 25

[SSC (GL) Prel. Examination, 2000]

19. $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$

- (a) $6^{2/3}$ (b) 6
(c) $3^{1/3}$ (d) 3

[SSC (GL) Prel. Examination, 2000]

20. First 100 multiples of 10 are multiplied together. The number of zeros at the end of the product must be:

- (a) 125 (b) 120
(c) 111 (d) 110

[SSC (GL) Prel. Examination, 2000]

21. $(16)^{0.16} \times (16)^{0.04} \times (2)^{0.2}$ is equal to:

- (a) 1 (b) 2
(c) 4 (d) 16

[SSC (GL) Prel. Examination, 2000]

22. A number divided by 68 gives the quotient 269 and remainder zero. If the same number is divided by 67, the remainder is:

- (a) 0 (b) 1
(c) 2 (d) 3

[SSC (GL) Prel. Examination, 2000]

23. The product of two fractions is $\frac{14}{15}$ and their quotient is $\frac{35}{24}$. The greater fraction is:

- (a) $\frac{7}{4}$ (b) $\frac{7}{6}$
(c) $\frac{7}{3}$ (d) $\frac{4}{5}$

[SSC (GL) Prel. Examination, 2002]

24. Which of the following numbers is least?

$(0.5)^2$, $\sqrt{0.49}$, $\sqrt[3]{0.008}$, 0.23

- (a) $(0.5)^2$ (b) $\sqrt{0.49}$
(c) $\sqrt[3]{0.008}$ (d) 0.23

[SSC (GL) Prel. Examination, 2002]

25. Find the value of * in the following:

$$1\frac{2}{3} \div \times \frac{*}{7} = 1\frac{1}{4} \times \frac{2}{3} \div \frac{1}{6}$$

- (a) $\frac{1}{6}$ (b) 0.6
(c) 0.006 (d) 6

[SSC (GL) Prel. Examination, 2002]

26. A certain amount of money is distributed among A, B and C. A gets $\frac{3}{16}$ and B gets $\frac{1}{4}$ of the whole amount. If C gets ₹81, then B gets:

- (a) ₹30 (b) ₹36
(c) ₹32 (d) ₹40

[SSC (GL) Prel. Examination, 2002]

27. The greatest number which when divides 989 and 1327 leave remainders 5 and 7, respectively, is:

- (a) 8 (b) 16
(c) 24 (d) 32

[SSC (GL) Prel. Examination, 2002]

28. If * means adding 6 times the second number to the first number then $(1 * 2) * 3$ equals:

- (a) 121 (b) 31
(c) 93 (d) 91

[SSC (GL) Prel. Examination, 2003]

29. Find the value of $\frac{2}{1 + \frac{1}{1 - \frac{1}{2}}} \times \frac{3}{\frac{5}{6} \text{ of } \frac{3}{2} \div 1\frac{1}{4}}$

- (a) 6 (b) 8
(c) 4 (d) 2

[SSC (GL) Prel. Examination, 2003]

30. If $x * y = x^2 + y^2 - xy$, the value of $9 * 11$ is:

- (a) 93 (b) 103
(c) 113 (d) 121

[SSC (GL) Prel. Examination, 2003]

31. A man spends $\frac{1}{3}$ of his income on food, $\frac{2}{5}$ of his income on house rent and $\frac{1}{5}$ of his income on clothes. If he still has ₹400 left with him, his income is:

- (a) ₹4000 (b) ₹5000
(c) ₹6000 (d) ₹7000

[SSC (GL) Prel. Examination, 2003]

32. If $a * b = 2a + 3b$, then the value of $2 * 3 + 3 * 4$ is:

- (a) 24 (b) 31
(c) 32 (d) 34

[SSC (GL) Prel. Examination, 2002]

33. The simplified value of $\left[\sqrt[3]{\sqrt{2^9}} \right]^4 \times \left[\sqrt[6]{\sqrt{2^9}} \right]^4$ is:

- (a) 2^{16} (b) 2^{12}
(c) 2^8 (d) 2^4

[SBI PO Examination, 2000]

34. The value of the following is:

$$\sqrt{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}}$$

- (a) 10 (b) 8
(c) 6 (d) 4

[Bank of Baroda PO, 1999]

35. What will come in place of the question mark (?) in the following equation?

$$25^{7.5} \times 5^{2.5} \div 125^{1.5} = 5 ?$$

- (a) 16 (b) 17.5
(c) 8.5 (d) 13
(e) None of these

[PNB Management Trainee Examination, 2003]

36. Two-fifths of one-third for three-sevenths of a number is 15. What is 40 per cent of that number?

- (a) 136 (b) 140
(c) 72 (d) 84
(e) None of these

[IBPS Jr. Executive Examination, 2002]

37. By how much is two-fifths of 200 greater than three-fifths of 125?

- (a) 15 (b) 3
(c) 5 (d) 30
(e) None of these

[Canara Bank PO, 2003]

38. Which of the following has fractions in ascending order?

(a) $\frac{2}{3}, \frac{3}{5}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$ (b) $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$

(c) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{3}{5}, \frac{2}{3}$ (d) $\frac{3}{5}, \frac{2}{3}, \frac{9}{11}, \frac{7}{9}, \frac{8}{9}$

(e) $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}, \frac{2}{3}, \frac{3}{5}$

[NABARD Asst. Manager Examination, 2002]

39. What should come in place of the question mark (?) in the following equation:

$$47^{7.5} \div 47^{3/2} \times 47^{-3} = (\sqrt{47})$$

- (a) 3 (b) $2\frac{1}{2}$
(c) 6 (d) 3.5
(e) None of these

[BSRB Patana PO, 2001]

40. What should come in the place of the question mark (?) in the following equation?

$$\frac{(7 \times ?)^2}{49} = \sqrt{81}$$

- (a) 9 (b) 2
(c) 4 (d) None of these

[BSRB Delhi PO, 2000]

41. $\frac{(10008.99)^2}{10009.001} \times \sqrt{3589} \times 0.4987 = ?$

- (a) 3000 (b) 300000
(c) 3000000 (d) 5000
(e) 9000000

[BSRB Bhopal PO, 2000]

42. In a set of 5 numbers the sum of two of these numbers is 6 more than the sum of the remaining three numbers, whereas the sum of these two numbers is two times one of those numbers. What definitely is one of those two numbers?

- (a) 18 (b) 12
(c) 16 (d) Data inadequate
(e) None of these

[BSRB Chennai PO, 2000]

43. Multiply the difference between the two lowest numbers with the difference between the two highest numbers in the following sequences:

$$89 \ 7 \ 91 \ 72 \ 31 \ 25 \ 18 \ 89 \ 16 \ 58 \ 38 \ 42 \ 86$$

- (a) 18 (b) 77
(c) 81 (d) 16
(e) None of these

[NABARD, 1999]

44. $\frac{1}{5}$ of a number is equal to $\frac{5}{8}$ of the second number. If 35 is added to the first number it becomes 4 times of second number. What is the value of the second number?

- (a) 125 (b) 70
(c) 40 (d) 25
(e) None of these

[NABARD, 1999]

45. At the first stop on his route, a driver unloaded two-fifths of the packages in his van. After he unloaded another three packages at his next stop, half of the original number of packages in the van remained. How many packages were in the van before the first delivery?

4.16 Chapter 4

- (a) 10 (b) 25
(c) 30 (d) 36

46. Which of the given numbers is the greatest?

- (a) $6\sqrt{5}$ (b) $8\sqrt[3]{2}$
(c) $2\sqrt[3]{130}$ (d) $\sqrt[3]{900}$

47. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is:

- (a) 3 (b) 2
(c) 1 (d) None of these.

48. $\sqrt[3]{\frac{72.9}{0.4096}}$ is equal to:

- (a) 0.5625 (b) 5.625
(c) 182 (d) 13.6

[SSC (GL) Prel. Examination, 2000]

49. If the square root of 5 is 2.236, then the square root of 80 equals = 2.236 times of:

- (a) 2 (b) 2.5
(c) 4 (d) 5

[SSC (GL) Prel. Examination, 2000]

50. The digit in the unit's place in the cube root of 21952 is:

- (a) 8 (b) 6
(c) 4 (d) 2

[SSC (GL) Prel. Examination, 2000]

51. Given, $\sqrt{5} = 2.2361$, $\sqrt{3} = 1.7321$, then $\frac{1}{\sqrt{5}-\sqrt{3}}$ is equal to:

- (a) 1.984 (b) 1.9841
(c) 1.98 (d) 2

[SSC (GL) Prel. Examination, 2000]

52. The square root of $(272^2 - 128^2)$ is:

- (a) 256 (b) 200
(c) 240 (d) 144

[SSC (GL) Prel. Examination, 2000]

53. The square root of 0.9 is equal to:

- (a) 0.3 (b) 0.03
(c) 0.94 (d) 0.81

[SSC (GL) Prel. Examination, 2000]

54. The square root of $\frac{0.342 \times 0.684}{0.000342 \times 0.000171}$ is:

- (a) 250 (b) 2500
(c) 2000 (d) 4000

[SSC (GL) Prel. Examination, 2002]

55. If cube root of 175616 is 56, then the value of $\sqrt[3]{175.616} + \sqrt[3]{0.175616} + \sqrt[3]{0.000175616}$ is equal to:

- (a) 0.168 (b) 62.16
(c) 6.216 (d) 6.116

[SSC (GL) Prel. Examination, 2002]

56. Given $\sqrt{2} = 1.414$, then the value of $\sqrt{8} + \sqrt[3]{32} - \sqrt[3]{128} + \sqrt[4]{50}$ is:

- (a) 8.484 (b) 8.526
(c) 8.426 (d) 8.876

[SSC (GL) Prel. Examination, 2003]

57. $\sqrt{\sqrt[3]{0.004096}}$ is equal to:

- (a) 4 (b) 0.4
(c) 0.04 (d) 0.004

[SSC (GL) Prel. Examination, 2003]

58. If $\sqrt{15} = 3.88$, then what is the value of $\sqrt{\frac{5}{3}}$.

- (a) $1.29\bar{3}$ (b) 1.2934
(c) 1.29 (d) 1.295

[SSC (GL) Prel. Examination, 2003]

59. If the square root of 5625 is 75, then $\sqrt{5625} + \sqrt{56.25} + \sqrt{0.5625} =$

- (a) 9 (b) 83.25
(c) 82.80 (d) 8.325

[SSC (GL) Prel. Examination, 2002]

60. What approximate value should come in place of the question mark (?)?

$$36.0001 \div 5.9998 \times \sqrt{?} = 108.0005$$

- (a) 18 (b) 16
(c) 256 (d) 325

[Bank of Maharashtra PO Examination, 2003]

61. $\sqrt{10000} + \frac{3.001}{4.987}$ of 1891.992 = ?

- (a) 2500 (b) 1230
(c) 1640 (d) 1525

[Canara Bank PO Examination, 2003]

62. $1.\overline{27}$ in the form $\frac{p}{q}$ is equal to:

- (a) $\frac{127}{100}$ (b) $\frac{73}{100}$
(c) $\frac{14}{11}$ (d) $\frac{11}{14}$

[SSC (GL) Examination, 2010]

63. If $2p + \frac{1}{p} = 4$ the value of $p^3 + \frac{1}{8p^3}$ is:

- (a) 4 (b) 5
(c) 8 (d) 15

[SSC (GL) Examination, 2010]

64. $(0.1 \times 0.01 \times 0.001 \times 10^7)$ is equal to:

- (a) 100 (b) $\frac{1}{10}$
(c) $\frac{1}{100}$ (d) 10

[SSC (GL) Examination, 2010]

65. Simplified form of $\left[\left(\sqrt[5]{x^{-3/5}}\right)^{-5/3}\right]^5$ is:

- (a) x^5 (b) x^{-5}
(c) x (d) $\frac{1}{x}$

[SSC (GL) Examination, 2010]

66. $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{25}\right)$ is equal to:

- (a) $\frac{2}{25}$ (b) $\frac{1}{25}$
(c) $1\frac{19}{25}$ (d) $\frac{1}{325}$

[SSC (GL) Examination, 2010]

67. The value of $\frac{3\sqrt{2}}{\sqrt{3} + \sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6} + \sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}}$ is:

- (a) 4 (b) 0
(c) 12 (d) $3\sqrt{6}$

[SSC (GL) Examination, 2011]

68. The value of $\frac{2\frac{1}{3} - 1\frac{2}{11}}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}$ is:

- (a) $\frac{38}{109}$ (b) $\frac{109}{38}$
(c) 1 (d) $\frac{116}{109}$

[SSC (GL) Examination, 2011]

69. The value of $3 + \frac{1}{\sqrt{3}} + \frac{1}{3 + \sqrt{3}} + \frac{1}{\sqrt{3} - 3}$ is:

- (a) $3 + \sqrt{3}$ (b) 3
(c) 1 (d) 0

[SSC (GL) Examination, 2011]

70. If $x + \frac{2}{3 + \frac{4}{5 + \frac{7}{6}}} = 10$ then the value of x is:

- (a) $\frac{1276}{135}$ (b) $\frac{53}{6}$
(c) 4.35 (d) 9

[SSC (GL) Examination, 2011]

71. The value of $3 + \frac{3}{3 + \frac{1}{3 + \frac{1}{3}}}$

- (a) $\frac{40}{11}$ (b) $\frac{43}{11}$
(c) $\frac{46}{11}$ (d) $\frac{41}{11}$

[SSC (GL) Examination, 2011]

72. If $x = \frac{\sqrt{5} + 1}{\sqrt{5} - 1}$, then, the value of $5x^2 - 5x - 1$ is:

- (a) 0 (b) 3
(c) 4 (d) 5

[SSC (GL) Examination, 2011]

73. If $\frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} - \sqrt{3-x}} = 2$, then x is equal to:

- (a) $\frac{5}{12}$ (b) $\frac{12}{5}$
(c) $\frac{5}{7}$ (d) $\frac{7}{5}$

[SSC (GL) Examination, 2010]

74. The number 0.121212 ... in the form $\frac{p}{q}$ is equal to:

- (a) $\frac{4}{11}$ (b) $\frac{2}{11}$
(c) $\frac{4}{33}$ (d) $\frac{2}{33}$

[SSC (GL) Examination, 2010]

75. $3\frac{3}{4} + 4\frac{2}{5} - 3\frac{1}{8} = ?$

- (a) $4\frac{1}{40}$ (b) $5\frac{1}{40}$
(c) $6\frac{1}{40}$ (d) $5\frac{3}{40}$

[Bank of Baroda PO Examination, 2010]

76. $\sqrt{5^2 \times 14 - 6 \times 7 + (4)^2} = 18$

- (a) 1 (b) 3
(c) 4 (d) None of these.

[Bank of Baroda PO Examination, 2010]

77. 67.99% of 1401 - 13.99% of 1299 = ?

- (a) 700 (b) 720
(c) 770 (d) 800

[Bank of Baroda PO Examination, 2011]

78. $\left(\frac{24}{9}\right)^2 \times \frac{399}{39} \div \frac{41}{899} = ?$

- (a) 1600 (b) 1650
(c) 1700 (d) 1550

[Bank of Baroda PO Examination, 2011]

79. $(15 \times 0.40)^4 \div (1020 \div 30)^4 \times (27 \times 8)^4 = (3 \times 2)^{?+5}$

- (a) 8 (b) 3
(c) 12 (d) 16

[Bank of Baroda PO Examination, 2011]

80. $3\frac{1}{4} + 2\frac{1}{2} - 1\frac{5}{6} = \frac{(?)^2}{10} + 1\frac{5}{12}$

- (a) 25 (b) $\sqrt{5}$
(c) 625 (d) 5

[Bank of Baroda PO Examination, 2011]

81. $92 \times 576 \div 2\sqrt{1296} = (?)^3 + \sqrt{49}$

- (a) 3 (b) $(9)^2$
(c) 9 (d) 27

[Bank of Baroda PO Examination, 2011]

82. $\frac{1}{6}$ of (92)% of $1\frac{1}{23}$ of (650) = 85 + ?

- (a) 18 (b) 21
(c) 19 (d) 28

[Bank of Baroda PO Examination, 2011]

83. Seema bought 20 pens, 8 packets of wax colours, 6 calculators, and 7 pencil boxes. The price of a pen is ₹7, a packet of wax colour is ₹22, a calculator is ₹175, and a pencil box is ₹14 more than the combined price of one pen and one packet of wax colours. How much amount did Seema pay to the shopkeeper?

- (a) ₹1491 (b) ₹1725
(c) ₹1667 (d) ₹1527

[IBPS Bank PO Examination, 2011]

84. If $a^2 + b^2 + c^2 = ab + bc + ac$, then the value of $\frac{a+c}{b}$ is:

- (a) 0 (b) 2
(c) 1 (d) -1

[SSC Examination, 2014]

85. If $ab + bc + ca = 0$, then the value of $\left(\frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab}\right)$ is:

- (a) 0 (b) 1
(c) 3 (d) $a + b + c$

[SSC Examination, 2014]

86. If $(2 + \sqrt{3})a = (2 - \sqrt{3})b = 1$, then the value of $\frac{1}{a} + \frac{1}{b}$ is:

- (a) 1 (b) 2
(c) $2\sqrt{3}$ (d) 4

[SSC Examination, 2014]

87. If $3x + \frac{3}{x} = 1$, then $x^3 + \frac{1}{x^3} + 1$ is:

- (a) 0 (b) $\frac{1}{27}$
(c) $\frac{5}{27}$ (d) $\frac{28}{27}$

[SSC Examination, 2014]

88. The value of $\frac{1}{a^2 + ax + x^2} - \frac{1}{a^2 - ax + x^2} +$

$\frac{2ax}{a^4 + a^2x^2 + x^4}$ is:

- (a) 2 (b) 1
(c) -1 (d) 0

[SSC Examination, 2014]

105. If $x\left(3 - \frac{2}{x}\right) = \frac{3}{x}$, $x \neq 0$, then the value of $x^2 + \frac{1}{x^2}$ is:

- (a) $2\frac{1}{3}$ (b) $2\frac{2}{3}$
(c) $2\frac{4}{9}$ (d) $2\frac{5}{9}$

[SSC Assistant Grade III, 2012]

106. If $x^2 + y^2 + z^2 + 2 = 2(y - x)$, then value of $x^3 + y^3 + z^3$ is equal to:

- (a) 0 (b) 1
(c) 2 (d) 3

[SSC Assistant Grade III, 2012]

107. If $a^3b = abc = 180$, and a, b, c are positive integers, then the value of c is:

- (a) 110 (b) 1
(c) 4 (d) 25

[SSC, 2012]

108. If $\left(x + \frac{1}{x}\right)^2 = 3$, then the value of $(x^{72} + x^{66} + x^{54} + x^{36} + x^{24} + x^6 + 1)$ is:

- (a) 1 (b) 2
(c) 3 (d) 4

[SSC, 2012]

109. If $a + b + c = 0$, then the value of $\frac{a^2 + b^2 + c^2}{a^2 - bc}$ is:

- (a) 0 (b) 1
(c) 2 (d) 3

[SSC, 2012]

110. If $n = 7 + 4\sqrt{3}$, then the value of $\left(\sqrt{n} + \frac{1}{\sqrt{n}}\right)$ is:

- (a) $2\sqrt{3}$ (b) 4
(c) -4 (d) $-2\sqrt{3}$

[SSC, 2012]

111. If $a + b + c = 6$, $a^2 + b^2 + c^2 = 14$ and $a^3 + b^3 + c^3 = 36$, then the value of abc is:

- (a) 3 (b) 6
(c) 9 (d) 12

[SSC, 2012]

112. If a, b are rational numbers and $(a-1)\sqrt{2} + 3 = b\sqrt{2} + a$, the value of $(a+b)$ is:

- (a) -5 (b) 3
(c) -3 (d) 5

[SSC, 2012]

113. If $\left(x + \frac{1}{x}\right)^2 = 3$, then the value of $x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$ is:

- (a) 0 (b) 1
(c) 84 (d) 206

[SSC, 2012]

114. The value of $\frac{(81)^{3.6} \times (9)^{2.7}}{(81)^{4.2} \times (3)}$ is:

- (a) 3 (b) 6
(c) 9 (d) 8.2

[SSC, 2011]

115. While selling, a businessman allows 40% discount on the marked price and there is a loss of 30%. If it is sold at the marked price, profit percent will be:

- (a) 10% (b) 20%
(c) $16\frac{2}{3}\%$ (d) $16\frac{1}{3}\%$

[SSC, 2011]

116. If $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then the value of $(a - b + c)$ is:

- (a) -1 (b) 3
(c) 1 (d) -2

[SSC, 2011]

117. If $x^2 + 3x + 1 = 0$, then the value of $x^3 + \frac{1}{x^3}$ is:

- (a) -18 (b) 18
(c) 36 (d) -36

[SSC, 2011]

118. If $x^a \cdot x^b \cdot x^c = 1$, then the value of $a^3 + b^3 + c^3$ is:

- (a) 9 (b) abc
(c) $a + b + c$ (d) $3abc$

[SSC, 2011]

119. If $a + \frac{1}{a} + 2 = 0$, then the value of $\left(a^{37} - \frac{1}{a^{100}}\right)$ is:

- (a) 0 (b) -2
(c) 1 (d) 2

[SSC, 2011]

120. If $x + \frac{1}{16x} = 1$, then the value of $64x^3 + \frac{1}{64x^3}$ is:

- (a) 4 (b) 52
(c) 64 (d) 76

121. If a, b, c are three non-zero real numbers such that $a + b + c = 0$; and $b^2 \neq ca$, then the value of $\frac{a^2 + b^2 + c^2}{b^2 - ca}$ is:

- (a) 3 (b) 2
(c) 0 (d) 1 [SSC, 2011]

122. If $a^4 + a^2b^2 + b^2 = 8$ and $a^2 + ab + b^2 = 4$, then the value of ab is:
(a) -1 (b) 0
(c) 0 (d) 1 [SSC, 2011]

123. If $a = 25$, $b = 15$, $c = -10$; then the value of $\frac{a^3 + b^3 + c^3 - 3abc}{(a-b)^2 + (b-c)^2 + (c-a)^2}$ is:
(a) 30 (b) -15
(c) -30 (d) 15 [SSC, 2011]

124. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}}$, then $(x + y - z)^3 + 27xyz$ is equal to:
(a) 0 (b) 1
(c) -1 (d) 27 [SSC, 2010]

125. If $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{\dots}}}}} = (343)^{y-1}$, then y is equal to:
(a) $\frac{2}{3}$ (b) 1
(c) $\frac{4}{3}$ (d) $\frac{3}{4}$ [SSC, 2010]

126. If $a + b + c = 1$ and $ab + bc + ca = \frac{1}{3}$, then $a:b:c$ is:
(a) 1:2:2 (b) 2:1:2
(c) 1:1:1 (d) 1:2:1 [SSC, 2010]

127. If $a^2 + b^2 + \frac{1}{a^2} + \frac{1}{b^2} = 4$, then the value of $a^2 + b^2$ will be:
(a) 1 (b) $1\frac{1}{2}$
(c) 2 (d) $2\frac{1}{2}$ [SSC, 2010]

128. If $\left(x + \frac{1}{x}\right)^2 = 3$, then $\left(x^3 + \frac{1}{x^3}\right)$ is equal to:
(a) 3 (b) 2
(c) 1 (d) 0 [SSC, 2010]

129. $\frac{0.1 \times 0.1 \times 0.1 + 0.02 \times 0.02 \times 0.02}{0.2 \times 0.2 \times 0.2 + 0.04 \times 0.04 \times 0.04}$ is equal to:
(a) 0.125 (b) 0.250
(c) 0.500 (d) 0.855 [SSC, 2010]

130. If $x + \frac{1}{x} = 2$, then the value of $x^{100} + \frac{1}{x^{100}}$ is:
(a) 2 (b) 0
(c) 1 (d) -2 [SSC, 2010]

131. If $x^3 + 3x^2 + 3x = 7$, then x is equal to:
(a) 2 (b) $\sqrt[3]{6}$
(c) 1 (d) -1 [SSC, 2010]

132. If $2x + \frac{2}{x} = 1$, then the value of $x^3 + \frac{1}{x^3}$ is:
(a) $\frac{13}{8}$ (b) $-\frac{11}{8}$
(c) $\frac{11}{8}$ (d) $-\frac{13}{8}$ [SSC, 2010]

133. $\frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}} - \sqrt{16-6\sqrt{7}}}$ is equal to:
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{5}$ [SSC, 2010]

134. If $2x + \frac{1}{3x} = 6$, then $3x + \frac{1}{2x}$ is equal to:
(a) 4 (b) 8
(c) 9 (d) 12 [SSC, 2010]

135. If $x = (\sqrt{2} - 1)^{\frac{1}{2}}$, then the value of $\left(x^2 - \frac{1}{x^2}\right)$ is:
(a) 2 (b) $-2\sqrt{2}$
(c) $2\sqrt{2}$ (d) $-\sqrt{2}$ [SSC, 2010]

136. $\frac{3}{4} \left(1 + \frac{1}{3}\right) \left(1 + \frac{2}{3}\right) \left(1 - \frac{2}{5}\right) \left(1 + \frac{6}{7}\right) \left(1 - \frac{12}{13}\right)$ is equal to:
(a) $\frac{2}{13}$ (b) $\frac{1}{7}$
(c) $\frac{1}{6}$ (d) $\frac{1}{5}$ [SSC, 2010]

4.22 Chapter 4

137. $\frac{(0.87)^3 + (0.13)^3}{(0.87)^2 + (0.13)^2 - (0.87) \times (0.13)}$ is equal to:

- (a) $\frac{1}{2}$ (b) 2
(c) 1 (d) $2\frac{1}{2}$

[SSC, 2010]

138. If $x^2 + y^2 - 2x + 6y + 10 = 0$, then the value of $(x^2 + y^2)$ is:

- (a) 4 (b) 6
(c) 8 (d) 10

[SSC, 2010]

Directions (Q. 139–141): What value will come in place of the question mark (?) in the following questions? (You are not expected to calculate the exact value)

139. $21 + 3.9 \times 2.9 + 8.99 = ?$

- (a) 42 (b) 46
(c) 44 (d) 34
(e) 36

[IBPS PO/MT, 2013]

140. $22.9889 \div ? = 23$

- (a) 23 (b) 1
(c) 23^2 (d) 24
(e) None of these

[IBPS PO/MT, 2013]

141. $10^3 \times 100^3 + 999999999 = 10^? + 10^?$

- (a) 6 (b) 9
(c) 7 (d) 18
(e) 12

[IBPS PO/MT, 2013]

Directions (Q. 142–146): What will come in place of the question mark (?) in the following questions?

142. $4003 \times 77 - 21015 = ? \times 116$

- (a) 2477 (b) 2478
(c) 2467 (d) 2476
(e) None of these

[IBPS PO/MT, 2012]

143. $(4444 \div 40) + (645 \div 25) + (3991 \div 26) = ?$

- (a) 280.4 (b) 290.4
(c) 295.4 (d) 285.4
(e) None of these

[IBPS PO/MT, 2012]

144. $5\frac{17}{37} \times 4\frac{51}{52} \times 11\frac{1}{7} + 2\frac{3}{4} = ?$

- (a) 303.75 (b) 305.75
(c) $303\frac{3}{4}$ (d) $305\frac{1}{4}$
(e) None of these

[IBPS PO/MT, 2012]

145. $\frac{5}{8}$ of 4011.33 + $\frac{7}{10}$ of 3411.22 = ?

- (a) 4810 (b) 4980
(c) 4890 (d) 4930
(e) 4850

[IBPS PO/MT, 2012]

146. $335.01 \times 244.99 \div 55 = ?$

- (a) 1490 (b) 1550
(c) 1420 (d) 1590
(e) 1400

[IBPS PO/MT, 2012]

Directions (Q. 147–154): What will come in the place of question mark (?) in the following questions?

147. $3463 \times 295 - 18611 = ? + 5883$

- (a) 997091 (b) 997071
(c) 997090 (d) 999070
(e) None of these

[IBPS PO/MT, 2011]

148. $(23.1)^2 + (48.6)^2 - (39.8)^2 = ? + 1147.69$

- (a) $(13.6)^2$ (b) $\sqrt{12.8}$
(c) 163.84 (d) 12.8
(e) None of these

[IBPS PO/MT, 2011]

149. $\frac{28}{65} \times \frac{195}{308} \div \frac{39}{44} + \frac{5}{26} = ?$

- (a) $\frac{1}{3}$ (b) 0.75
(c) $1\frac{1}{2}$ (d) $\frac{1}{2}$
(e) None of these

[IBPS PO/MT, 2011]

150. $43931.03 \div 2111.02 \times 401.04 = ?$

- (a) 8800 (b) 7600
(c) 7400 (d) 9000
(e) 8300

[IBPS PO/MT, 2011]

151. $59.88 \div 12.21 \times 6.35 = ?$

- (a) 10 (b) 50
(c) 30 (d) 70
(e) 90

[IBPS PO/MT, 2011]

152. $\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{3}{5}$ of 1715 = ?

- (a) 80 (b) 85
(c) 90 (d) 95
(e) 75

[SBI Associates Banks PO, 2011]

153. $25.05 \times 123.95 + 388.999 \times 15.001 = ?$

- (a) 900 (b) 8950
(c) 8935 (d) 8975
(e) 8995

[SBI Associates Banks PO, 2011]

154. $561 \div 35.05 \times 19.99 = ?$

- (a) 320 (b) 330
(c) 315 (d) 325
(e) 335

[SBI Associates Banks PO, 2011]

Directions (Q. 155–158): What will come in the place of question mark (?) in the following questions?

155. $(21)^2 - 3717 \div 59 = ? \times 8$

- (a) 43.75 (b) 42.25
(c) 45.75 (d) 47.25
(e) None of these

[IOB PO, 2011]

156. $2\frac{1}{8} - 1\frac{1}{16} = ? + 1\frac{1}{32} - 1\frac{9}{64}$

- (a) $2\frac{9}{32}$ (b) $1\frac{9}{64}$
(c) $2\frac{5}{32}$ (d) $1\frac{11}{64}$
(e) None of these

[IOB PO, 2011]

157. $(0.64)^4 \div (0.512)^3 \times (0.8)^4 = (0.8)^{?+3}$

- (a) 5 (b) 12
(c) 0 (d) 6
(e) None of these

[IOB PO, 2011]

158. $\sqrt{15^2 \times 12 \div 9 - 125 + 21} = ?$

- (a) 18 (b) 24
(c) 196 (d) 56
(e) 14

[IOB PO, 2011]

Directions (Q. 159–161): What approximate value will come in the place of the question mark (?) in the following questions? (You are not expected to calculate the exact value.)

159. $7441 \div 34 \times 12 = ? \times 9 + 110$

- (a) 420 (b) 280
(c) 590 (d) 350
(e) 220

[IOB PO, 2011]

160. $\frac{989}{34} \div \frac{65}{869} \times \frac{515}{207} = ?$

- (a) 840 (b) 920
(c) 970 (d) 780
(e) 1000

[IOB PO, 2011]

161. $(32.13)^2 + (23.96)^2 - (17.11)^2 = ?$

- (a) 1270 (b) 1420
(c) 1450 (d) 1360
(e) 1310

[IOB PO, 2011]

Directions (Q. 162–165): What will come in the place of question mark (?) in the following questions?

162. $23 \times 15 - 60 + ? \div 31 = 292$

- (a) 218 (b) 186
(c) 217 (d) 201
(e) None of these

[Indian Bank PO, 2010]

163. $3\frac{3}{4} + 4\frac{2}{5} - 3\frac{1}{8} = ?$

- (a) $4\frac{1}{40}$ (b) $5\frac{1}{40}$
(c) $6\frac{1}{40}$ (d) $5\frac{3}{10}$
(e) None of these

[Indian Bank PO, 2010]

164. $\frac{343 \times 49}{216 \times 16 \times 81} = ?$

- (a) $\frac{7^5}{6^7}$ (b) $\frac{7^5}{6^8}$
(c) $\frac{7^6}{6^7}$ (d) $\frac{7^4}{6^8}$
(e) None of these

[Indian Bank PO, 2010]

165. $\frac{(a-b)^2 - (a+b)^2}{-4a} = \frac{x}{y}$

On simplifying the above mentioned equation, what will be the new equation?

- (a) $xy = b$ (b) $bx = y$
(c) $ab = x$ (d) $yb = x$
(e) $ay = x$

[Corporation Bank PO, 2009]

ANSWER KEYS												
EXERCISE-I												
1. (c)	2. (a)	3. (b)	4. (c)	5. (b)	6. (c)	7. (b)	8. (a)	9. (c)	10. (a)	11. (d)	12. (b)	13. (c)
14. (c)	15. (c)	16. (c)	17. (d)	18. (b)	19. (d)	20. (c)	21. (d)	22. (a)	23. (c)	24. (c)	25. (c)	26. (a)
27. (d)	28. (a)	29. (c)	30. (c)	31. (b)	32. (a)	33. (b)	34. (a)	35. (a)	36. (a)	37. (b)	38. (b)	39. (d)
40. (b)	41. (a)	42. (c)	43. (c)	44. (a)	45. (a)	46. (c)	47. (c)	48. (b)	49. (c)	50. (d)	51. (d)	52. (c)
53. (c)	54. (b)	55. (c)	56. (d)	57. (c)	58. (a)	59. (c)	60. (b)	61. (a)	62. (d)	63. (c)	64. (d)	65. (a)
66. (a)	67. (a)	68. (b)	69. (a)	70. (a)	71. (b)	72. (d)	73. (c)	74. (c)	75. (b)	76. (e)	77. (c)	78. (c)
EXERCISE-2												
1. (b)	2. (a)	3. (c)	4. (e)	5. (c)	6. (e)	7. (a)	8. (a)	9. (a)	10. (a)	11. (c)	12. (a)	13. (a)
14. (c)	15. (a)	16. (a)	17. (b)	18. (d)	19. (d)	20. (c)	21. (b)	22. (b)	23. (b)	24. (c)	25. (d)	26. (b)
27. (c)	28. (b)	29. (d)	30. (b)	31. (c)	32. (b)	33. (d)	34. (d)	35. (d)	36. (e)	37. (c)	38. (b)	39. (c)
40. (c)	41. (b)	42. (d)	43. (a)	44. (c)	45. (c)	46. (a)	47. (b)	48. (c)	49. (c)	50. (a)	51. (b)	52. (c)
53. (c)	54. (c)	55. (c)	56. (a)	57. (b)	58. (a)	59. (b)	60. (d)	61. (b)	62. (c)	63. (b)	64. (d)	65. (c)
66. (a)	67. (b)	68. (a)	69. (b)	70. (a)	71. (b)	72. (c)	73. (b)	74. (c)	75. (b)	76. (d)	77. (c)	78. (d)
79. (b)	80. (d)	81. (c)	82. (c)	83. (c)	84. (b)	85. (a)	86. (d)	87. (b)	88. (d)	89. (c)	90. (a)	91. (d)
92. (a)	93. (b)	94. (a)	95. (c)	96. (a)	97. (d)	98. (d)	99. (c)	100. (d)	101. (b)	102. (b)	103. (d)	104. (c)
105. (c)	106. (a)	107. (b)	108. (a)	109. (c)	110. (b)	111. (b)	112. (d)	113. (a)	114. (c)	115. (c)	116. (c)	117. (a)
118. (d)	119. (b)	120. (b)	121. (b)	122. (d)	123. (d)	124. (a)	125. (c)	126. (c)	127. (c)	128. (d)	129. (a)	130. (a)
131. (c)	132. (b)	133. (a)	134. (c)	135. (a)	136. (b)	137. (c)	138. (d)	139. (a)	140. (b)	141. (b)	142. (d)	143. (b)
144. (b)	145. (c)	146. (a)	147. (a)	148. (c)	149. (d)	150. (a)	151. (c)	152. (b)	153. (c)	154. (a)	155. (d)	156. (d)
157. (c)	158. (e)	159. (b)	160. (c)	161. (e)	162. (c)	163. (b)	164. (a)	165. (d)				

EXPLANATORY ANSWERS

EXERCISE-I

1. (c) Given expression

$$\begin{aligned}
 &= \frac{3}{10} \div \frac{3}{7} \text{ of } \left(\frac{23}{10} + \frac{13}{5} \right) + \frac{1}{5} \times \frac{5}{7} - \frac{2}{7} \\
 &= \frac{3}{10} \div \frac{3}{7} \text{ of } \frac{49}{10} + \frac{1}{5} - \frac{2}{7} = \frac{3}{10} \div \frac{21}{10} - \frac{1}{7} \\
 &= \frac{3}{10} \times \frac{10}{21} - \frac{1}{7} = \frac{1}{7} - \frac{1}{7} = 0.
 \end{aligned}$$

2. (a) Given expression

$$= 1 + 1 \div \left\{ 1 + 1 \div \left(\frac{2}{3} \right) \right\}$$

$$= 1 + 1 \div \left\{ 1 + 1 \times \frac{3}{2} \right\}$$

$$= 1 + 1 \div \left\{ 1 + \frac{3}{2} \right\} = 1 + 1 \div \frac{5}{2}$$

$$= 1 + 1 \times \frac{2}{5} = 1 + \frac{2}{5} = \frac{7}{5}.$$

3. (b) Given expression

$$= 48 \div 12 \times \left(\frac{9}{8} \text{ of } \frac{4}{3} \div \frac{3}{4} \text{ of } \frac{2}{3} \right)$$

$$= \frac{48}{12} \times \left\{ \left(\frac{9}{8} \times \frac{4}{3} \right) \div \left(\frac{3}{4} \times \frac{2}{3} \right) \right\}$$

$$= \frac{48}{12} \times \left(\frac{3}{2} \times 2 \right) = 4 \times 3 = 12.$$

4. (c) Given expression

$$= 2 \div \left[2 + 2 \div \left\{ 2 + 2 \div \left(2 + \frac{2}{3} \right) \right\} \right]$$

$$= 2 \div \left[2 + 2 \div \left\{ 2 + 2 \times \frac{3}{8} \right\} \right]$$

$$= 2 \div \left[2 + 2 \div \frac{11}{4} \right] = 2 \div \left[2 + 2 \times \frac{4}{11} \right]$$

$$= 2 \div \frac{30}{11} = 2 \times \frac{11}{30} = \frac{11}{15}.$$

5. (b) Let the missing figure = x .

$$\frac{15}{2} - \left[\frac{9}{4} \div \left\{ \frac{5}{4} - x \left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right] = 3$$

$$\frac{15}{2} - \left[\frac{9}{4} \div \left\{ \frac{5}{4} - x \right\} \right] = 3$$

$$\frac{15}{2} - 3 = \frac{9/4}{5/4 - x}$$

$$\frac{9}{2} = \frac{9}{5 - 4x} \quad 5 - 4x = 2$$

$$x = \frac{3}{4}.$$

6. (c) We know that $\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$
 \therefore The given expression = $0.8 - 0.5 = 0.3$.

8. (a) Given expression

$$= 1 - [2 - \{5 - (4 - 1)\}]$$

$$= 1 - [2 - \{5 - 3\}]$$

$$= 1 - [2 - 2] = 1 - 0 = 1.$$

9. (c) Given expression

$$= 3 \div \left[3 \div \left\{ 2 \div \frac{34}{13} \right\} \right]$$

$$= 3 \div \left[3 \div \left\{ 2 \times \frac{13}{34} \right\} \right] = 3 \div \left[3 \times \frac{17}{13} \right]$$

$$= 3 \div \frac{51}{13} = 3 \times \frac{13}{51} = \frac{13}{17}.$$

10. (a) Given expression

$$= \frac{(69842)^2 - (30158)^2}{69842 - 30158}$$

$$= \frac{(69842 - 30158)(69842 + 30158)}{69842 - 30158}$$

$$= 100000.$$

11. (d) Given expression

$$= \frac{\frac{15}{7} - \frac{5}{2}}{\frac{9}{4} + \frac{8}{7}} \div \frac{1}{2 + \frac{1}{2 + \frac{2}{3}}}$$

$$= \frac{-5}{14} \times \frac{28}{95} \div \frac{1}{2 + \frac{3}{8}}$$

$$= \frac{-2}{19} \div \frac{8}{19} = \frac{-2}{19} \times \frac{19}{8} = \frac{-1}{4}.$$

12. (b) The given expression

$$= 2.75 - 2.25 = 0.50.$$

13. (c) Given expression

$$= \frac{\frac{1}{2} \times \frac{1}{4} + 20}{2 + 20} \div \frac{161}{8} \times \frac{1}{22} = \frac{161}{176}.$$

14. (c) Given expression

$$= \frac{(0.53)^2 - 2 \times 0.53 \times 0.41 + (0.41)^2}{0.12}$$

$$= \frac{(0.53 - 0.41)^2}{0.12} = \frac{(0.12)^2}{0.12} = 0.12.$$

15. (c) Given expression

$$= \frac{(3^2)^2 \times (3 \times 3 \times 2)^4}{3^{16}}$$

$$= \frac{3^4 \times 3^8 \times 2^4}{3^{16}} = \frac{2^4}{3^4} = \frac{16}{81}.$$

16. (c) Given expression

$$= 1 + \frac{1}{2 + \frac{1}{\frac{2}{3}}} = 1 + 1 + \frac{1}{2 + \frac{3}{2}}$$

$$= 1 + \frac{1}{\frac{7}{2}} = 1 + \frac{2}{7} = \frac{9}{7}.$$

17. (d) Let x be the missing number

$$= 108 \div x \text{ of } \frac{1}{3} + \frac{2}{5} \times 3 \frac{3}{4} = 10 \frac{1}{2}$$

$$= 108 \div \frac{x}{3} + \frac{2}{5} \times \frac{15}{4} = \frac{21}{2}$$

$$= \frac{3 \times 108}{x} + \frac{3}{2} = \frac{21}{2}$$

$$= \frac{3 \times 108}{x} = \frac{21}{2} - \frac{3}{2}$$

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$$= \frac{3 \times 108}{x} = 9$$

$$x = \frac{3 \times 108}{9}$$

$$x = 36.$$

18. (b) Given expression

$$= \frac{108 + 9 + 3 + 1}{108} = \frac{121}{108} = 1.1203.$$

19. (d) Let x of $\frac{1}{12} = \frac{3}{48}$

$$\text{Then, } x = \frac{3}{48} \times 12 = \frac{3}{4}$$

20. (c) Given expression

$$= 1 + \frac{1}{1+3} = 1 + \frac{1}{4} = \frac{5}{4}.$$

21. (d) $\frac{1}{3} = 0.33$ and $\frac{7}{8} = 0.875$

$$\frac{7}{8} = 0.25 \text{ does not lie between } 0.33 \text{ and } 0.875$$

$$\frac{23}{24} = 0.96 \text{ which exceeds } 0.875$$

$$\frac{11}{12} = 0.92 \text{ which exceeds } 0.875$$

$$\frac{17}{24} = 0.708 \text{ which lies between } 0.33 \text{ and } 0.875.$$

22. (a) Number of $\frac{1}{8}$'s = $\frac{75}{2} \div \frac{1}{8} = \frac{75}{2} \times 8 = 300$.

23. (c) Out of 5 girls, 1 took part in the camp.

Out of the 8 boys, 1 took part in the camp.

Out of the 13 students, 2 took part in the camp.

$\therefore \frac{2}{13}$ of total number of students took part in the camp.

24. (c) Given expression

$$= \frac{15}{2} + \frac{1}{2} \div \frac{1}{8} - \frac{2}{5} \times \frac{7}{3} \div \frac{15}{8} \text{ of } \left(\frac{7}{5} - \frac{4}{3} \right)$$

$$= \frac{15}{2} + 4 - \frac{2}{5} \times \frac{7}{3} \div \frac{15}{8} \text{ of } \frac{1}{15}$$

$$= \frac{15}{2} + 4 - \frac{2}{5} \times \frac{7}{3} \div \frac{1}{8}$$

$$= \frac{15}{2} + 4 - \frac{2}{5} \times \frac{7}{3} \times \frac{8}{1}$$

$$= \frac{15}{2} + 4 - \frac{112}{15} = \frac{23}{2} - \frac{112}{15}$$

$$= \frac{121}{30} = 4 \frac{1}{30}.$$

25. (c) Given expression

$$= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \dots \times \frac{1003}{1001} = \frac{1003}{3}.$$

26. (a) Given expression

$$= \frac{1}{6} + \frac{1}{24} + \frac{1}{60} + \frac{1}{120}$$

$$= \frac{20 + 5 + 2 + 1}{120} = \frac{28}{120} = \frac{7}{30}.$$

27. (d) Let, $\frac{47}{3} \times \frac{19}{6} + \frac{19}{3} = \frac{205}{18} + x$

$$\text{Then, } x = \frac{893}{18} + \frac{19}{3} - \frac{205}{18}$$

$$= \frac{893 + 114 - 205}{18} = \frac{802}{18} = 44 \frac{5}{9}.$$

28. (a) Given expression

$$= 3 \div \left[3 \div \left\{ 2 \div \frac{34}{13} \right\} \right]$$

$$= 3 \div \left[3 \div \left(2 \times \frac{13}{34} \right) \right]$$

$$= 3 \div \left[3 \times \frac{17}{13} \right] = 3 \times \frac{13}{51} = \frac{13}{17}.$$

29. (c) $\frac{3}{5} = 0.60$

$$\frac{3}{4} = 0.75$$

$$\frac{2}{3} = 0.66$$

$$\therefore \frac{3}{4} > \frac{2}{3} > \frac{3}{5}.$$

30. (c) Given expression

$$= \frac{240 \times 300}{240} = 300.$$

31. (b) $\frac{3a+2b}{3a-2b} = \frac{3\left(\frac{a}{b}\right)+2}{3\left(\frac{a}{b}\right)-2} = \frac{3\left(\frac{1}{3}\right)+2}{3\left(\frac{1}{3}\right)-2} = \frac{3}{-1} = -3.$

32. (a) Given expression

$$= \frac{4}{2} + 2 \times 2 + 2 \times \frac{3}{2} = 2 + 4 + 3 = 9.$$

33. (b) Let $\frac{5}{6} \div \frac{6}{7} \times x - \frac{8}{9} \div \frac{8}{5} + \frac{3}{4} \times \frac{10}{3} = \frac{25}{9}$. Then,

$$\frac{5}{6} \times \frac{7}{6} \times x - \frac{8}{9} \times \frac{5}{8} + \frac{3}{4} \times \frac{10}{3} = \frac{25}{9}$$

$$\text{or, } \frac{35}{36}x - \frac{5}{9} + \frac{10}{4} = \frac{25}{9} \quad \text{or, } \frac{35}{36}x = \frac{25}{9} + \frac{5}{9} - \frac{5}{2}$$

$$\text{or, } \frac{35}{36}x = \frac{30}{9} - \frac{5}{2} \quad \text{or, } \frac{35}{36}x = \frac{60-45}{18}$$

$$\text{or, } x = \frac{15}{18} \times \frac{36}{35}$$

$$\therefore x = \frac{6}{7}$$

34. (a) Let, $\frac{9}{2} + \frac{19}{6} + x + \frac{7}{3} = \frac{67}{5}$. Then,

$$x = \frac{67}{5} - \left(\frac{9}{2} + \frac{19}{6} + \frac{7}{3} \right)$$

$$= \frac{67}{5} - 10 = \frac{17}{5} = 3\frac{2}{5}$$

35. (a) $0.001 = (0.1)^3$; $0.067 = 0.1 \times 0.67$

The given expression

$$= \frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$$

$$= 0.67 - 0.10 = 0.57.$$

36. (a) Let total number of students = x

$$\text{Number of girl students} = \frac{2x}{3}$$

$$\text{Number of boy students} = \frac{x}{3}$$

Number of girls who took part in the camp

$$= \frac{1}{5} \left(\frac{2x}{3} \right) = \frac{2}{15}x$$

Number of boys who took part in the camp

$$= \frac{1}{8} \left(\frac{x}{3} \right) = \frac{x}{24}$$

Total number of students who took part in the camp

$$= \frac{2}{15}x + \frac{x}{24} = \left(\frac{16+5}{120} \right)x = \frac{7}{40}x$$

37. (b) Let the fraction be $\frac{a}{b}$. Then,

$$\left(\frac{a}{b} \times \frac{a}{b} \right) \div \frac{b}{a} = 18 \frac{26}{27} = \frac{512}{27}$$

$$\text{or } \left(\frac{a}{b} \right)^3 = \left(\frac{8}{3} \right)^3$$

$$\therefore \frac{a}{b} = \frac{8}{3} = 2\frac{2}{3}$$

39. (d) Let $\frac{1}{4} + \frac{1}{6} - x = 3 \times \frac{1}{12}$ then,

$$\frac{1}{4} + \frac{1}{6} - x = \frac{1}{4} \quad \text{or } x = \frac{1}{6}$$

40. (b) Let x be the fraction

$$\frac{7}{6}x - \frac{6}{7}x = \frac{1}{7} \Rightarrow x = \frac{6}{13}$$

$$\text{The correct answer} = \frac{6}{7}x = \frac{6}{7} \times \frac{6}{13} = \frac{36}{91}$$

41. (a) $2 + \sqrt{2} + \left[\frac{\sqrt{2} - 2 + 2 + \sqrt{2}}{(\sqrt{2})^2 - (2)^2} \right]$

$$= 2 + \sqrt{2} + \frac{2\sqrt{2}}{2-4}$$

$$= 2 + \sqrt{2} - \sqrt{2} = 2.$$

42. (c) $\frac{1}{2} = 0.50000 \dots (1)$

$$\frac{1}{2.3} = 0.16667 \dots (2) \quad (\text{divide (1) by 3})$$

$$\frac{1}{2.3.4} = 0.04167 \dots (3) \quad (\text{divide (2) by 4})$$

$$\frac{1}{2.3.4.5} = 0.00833 \dots (4) \quad (\text{divide (3) by 5})$$

Adding, we have 0.71667 or 0.717 up to three places.

43. (c) Putting x in place of?

$$\frac{x \div 12}{0.2 \times 3.6} = 2 \quad \text{or, } x \div 12 = 2 \times 0.2 \times 3.6$$

$$\Rightarrow x = 2 \times 0.2 \times 3.6 \times 12 \quad \text{or, } x = 17.28.$$

44. (a) Substituting x for?, we get

$$\sqrt{x \times 7} \times 18 = 84$$

$$\text{or, } \sqrt{x \times 7} = \frac{84}{18} \quad \text{or, } (\sqrt{x \times 7})^2 = \left(\frac{84}{18} \right)^2$$

$$\text{or, } x \times 7 = \frac{84 \times 84}{18 \times 18} \quad \text{or, } x = \frac{84 \times 84}{18 \times 18 \times 7} = 3.11.$$

45. (a) Sum = $\frac{7}{4} + \frac{7}{3} + \frac{41}{12} + \frac{26}{5} + \frac{13}{6}$

$$= \frac{105 + 140 + 205 + 312 + 130}{60}$$

$$= \frac{892}{60} = 14\frac{13}{15}$$

which is nearer to 15 than 14

$$\text{Difference : } 15 - 14\frac{13}{15} = \frac{2}{15}$$

46. (c) Taking the quotient 2, y and 7, we get $2y = 7$, which gives the quotient as 3

$$\therefore y = 3.5. \text{ Substituting the value of } y, \text{ we get}$$

$$2\frac{3}{x} \times 3\frac{1}{2} = 7\frac{3}{4}$$

$$\text{Now, } \frac{7\frac{3}{4}}{3\frac{1}{2}} = 2\frac{3}{x} \Rightarrow 2\frac{3}{14} = \frac{3}{x}$$

$$\therefore x = 14, y = 3.$$

47. (c) Let x be the fraction

$$x \times x + \left(\frac{1}{x}\right)^2 = 3\frac{13}{81} \Rightarrow x^4 = \frac{256}{81} = \left(\frac{4}{3}\right)^4$$

$$\therefore x = \frac{4}{3}.$$

48. (b) Substituting $x = 7$ and $y = 5$, we get

$$7 \times 5 = (7 + 2)^2(5 - 2) = (9)^2 \times 3 = 243.$$

49. (c) Given that $m^n = 121 \Rightarrow m^n = 11^2$

Hence, $m = 11$, $n = 2$. Substituting these values

$$(m - 1)^{n+1} = (11 - 1)^{2+1} = 10^3 = 1000.$$

51. (d) $x \times \frac{17}{8} - x \times \frac{8}{17} = 225$ or, $\frac{225}{136}x = 225$

$$\therefore x = 136.$$

52. (c) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{9.10}$

$$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{9} - \frac{1}{10}\right) = 1 - \frac{1}{10} = \frac{9}{10}$$

53. (c) Putting x for?

$$\sqrt{1296} \times 2.25 = x^2 \text{ or, } 36 \times 2.25 = x^2$$

$$\text{or, } x = \sqrt{36 \times 2.25} \text{ or, } x = 6 \times 1.5$$

$$\therefore x = 9.$$

54. (b) $x \times x + \frac{1}{x} = 18\frac{26}{27}$ or, $x^3 = \frac{512}{27}$

$$\therefore x^3 = \left(\frac{8}{3}\right)^3 \text{ and so } x = \frac{8}{3} = 2\frac{2}{3}.$$

55. (c) Given that $\frac{a}{a+b} = \frac{17}{23}$

i.e., if $a = 17$, then $a + b = 23$ or, $b = 6$

$$a - b = 17 - 6 = 11$$

$$\therefore \frac{a+b}{a-b} = \frac{23}{11}.$$

57. (c) $\frac{3}{4}x - \frac{3}{14}x = 150$ or, $\frac{15}{28}x = 150$

$$\therefore x = \frac{150 \times 28}{15} = 280.$$

58. (a) Given product

$$= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \dots \times \frac{1003}{1001} = \frac{1003}{3}.$$

59. (c) $\sqrt{2^n} = 64$ or, $(2^n)^{1/2} = 2^6$

$$\Rightarrow \frac{n}{2} = 6 \Rightarrow n = 12.$$

60. (b) $10^y = \sqrt{10^{2y}} = \sqrt{25} = 5.$

61. (a) Putting x for? and solving

$$11\frac{1}{3} \times 4\frac{8}{10} \div x = 22\frac{2}{3}$$

$$\text{or, } 11\frac{1}{3} \times 4\frac{8}{10} = 22\frac{2}{3}x$$

$$\text{or, } x = \frac{11\frac{1}{3} \times 4\frac{8}{10}}{22\frac{2}{3}} \text{ or } x = \frac{1}{2} \times 4\frac{8}{10}$$

$$\therefore x = \frac{1}{2} \times \frac{48}{10} = \frac{24}{10} = 2.4.$$

$$\begin{aligned} 62. (d) & \frac{a^{1/2} + a^{-1/2}}{1-a} + \frac{1-a^{-1/2}}{1+\sqrt{a}} \\ & = \frac{a^{1/2} + a^{-1/2}}{(1+a^{1/2})(1-a^{1/2})} + \frac{1-a^{-1/2}}{1+a^{1/2}} \\ & = \frac{a^{1/2} + a^{-1/2} + (1-a^{-1/2})(1-a^{1/2})}{(1+a^{1/2})(1-a^{1/2})} \\ & = \frac{a^{1/2} + a^{-1/2} + 1 - a^{-1/2} - a^{1/2} + 1}{1-a} \\ & = \frac{2}{1-a}. \end{aligned}$$

$$\begin{aligned} 63. (c) & \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{\sqrt{a^2+b^2+2ab}}{ab} \\ & = \frac{\sqrt{45+2 \times 18}}{18} = \pm \frac{9}{18} = \pm \frac{1}{2}. \end{aligned}$$

$$\begin{aligned} 64. (d) & \frac{a^2+b^2}{c^2+d^2} = \frac{ab}{cd} \text{ or, } \frac{a^2+b^2}{c^2+d^2} = \frac{2ab}{2cd} \\ \text{or, } & \frac{a^2+b^2+2ab}{a^2+b^2-2ab} = \frac{c^2+d^2+2cd}{c^2+d^2-2cd} \end{aligned}$$

[by componendo and dividendo]

$$\text{or, } \left(\frac{a+b}{a-b}\right)^2 = \left(\frac{c+d}{c-d}\right)^2$$

$$\therefore \frac{a+b}{a-b} = \frac{c+d}{c-d}.$$

65. (a) Putting x for? and solving

$$(1.06 + 0.04)^2 - x = 4 \times 1.06 \times 0.04$$

Here, $1.06 = a$ and $0.04 = b$

$$\therefore (a + b)^2 - x = 4ab$$

$$\therefore x = (a + b)^2 - 4ab = (a - b)^2 = (1.06 - 0.04)^2 \\ = (1.02)^2 = 1.0404.$$

66. (a) Let the total score be x runs, such that

$$\frac{2}{9}x - \frac{2}{9} \times \left(x - \frac{2}{9}x\right) = 8 \text{ or, } \frac{2}{9}x - \frac{2}{9} \times \frac{7}{9}x = 8$$

$$\text{or, } \frac{2}{9}x \times \frac{2}{9} = 8 \text{ or, } x = 162.$$

67. (a) $\left(\frac{1}{64}\right)^0 + (64)^{-1/2} + (-32)^{4/5}$

$$= 1 + (8^2)^{-1/2} + (-1 \times 32)^{4/5} \\ = 1 + 8^{-1} + [(-1)^{4/5} \times (32)^{4/5}] \\ = 1 + 8^{-1} + [((-1)^2)^{2/5} \times (2^5)^{4/5}] \\ = 1 + \frac{1}{8} + [1 \times 16] = 17\frac{1}{8}.$$

68. (b) $x = \frac{(64)^2 - 9 \times 121}{121 \times 64} \times \frac{8 \times 11}{(8)^2 + 3 \times 11}$

$$x = \frac{(64)^2 - 3 \times 3 \times 11 \times 11}{11 \times 11 \times 8 \times 8} \times \frac{8 \times 11}{64 + 33}$$

$$\text{or, } x = \frac{(64 + 33)(64 - 33)}{88} \times \frac{1}{64 + 33}$$

$$\text{or, } x = \frac{31}{88}.$$

69. (a) Let the number be 1

$$\therefore \frac{1}{3} \text{ of } 1 = \frac{1}{3} \text{ and, } \frac{1}{4} \text{ of } 1 = \frac{1}{4}$$

$$\therefore \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$

$$\therefore \text{Number } 12 \div \frac{1}{12} = 144.$$

70. (a) L.C.M. of 7, 8, 16 and 35 = 560

$$\therefore \frac{5}{8} = \frac{5 \times 70}{8 \times 70} = \frac{350}{560}$$

$$\frac{21}{35} = \frac{21 \times 16}{35 \times 16} = \frac{336}{560}$$

$$\frac{9}{16} = \frac{9 \times 35}{16 \times 35} = \frac{315}{560}$$

$$\text{and, } \frac{6}{7} = \frac{6 \times 80}{7 \times 80} = \frac{480}{560}$$

\therefore Difference between the largest and the smallest fractions

$$= \frac{6}{7} - \frac{9}{16} = \frac{480}{560} - \frac{315}{560} = \frac{165}{560} = \frac{33}{112}.$$

71. (b) Let the money with the man at first be ₹1

$$\therefore \text{Money spent} = \frac{5}{6} \text{ of } 1 = ₹\frac{5}{6}$$

$$\therefore \text{Remaining money} = 1 - \frac{5}{6} = ₹\frac{1}{6}$$

$$\text{and money earned} = \frac{1}{2} \text{ of } ₹\frac{1}{6} = ₹\frac{1}{12}$$

\therefore Total money with him now

$$= \frac{1}{6} + \frac{1}{12} = ₹\frac{3}{12} = ₹\frac{1}{4}$$

$$\therefore \frac{1}{4} \text{ part of his money is with him now.}$$

72. (d) Let the monthly income of Manmohan be ₹1

$$\therefore \text{Pocket money} = \frac{1}{5} \text{ of } ₹1 = ₹\frac{1}{5}$$

$$\text{Remainder} = 1 - \frac{1}{5} = ₹\frac{4}{5}$$

$$\therefore \text{Other expenses} = \frac{4}{5} \text{ of } ₹\frac{4}{5} = ₹\frac{16}{25}$$

$$\therefore \text{Saving} = \frac{4}{5} - \frac{16}{25} = ₹\frac{4}{25}$$

$$\therefore \text{Monthly income} = 48 \div \frac{16}{25} = ₹300.$$

73. (c) Let the number be 1.

$$\therefore \frac{4}{5} \text{ of } 1 = \frac{4}{5} \text{ and, } \frac{3}{4} \text{ of } 1 = \frac{3}{4}$$

$$\therefore \text{Difference} = \frac{4}{5} - \frac{3}{4} = \frac{1}{20}$$

$$\therefore \text{Number} = \frac{1}{20} \div \frac{1}{20} = 80.$$

74. (c) $\therefore \frac{2}{3} \text{ part} = 96$

$$\therefore \frac{3}{4} \text{ part} = 96 \times \frac{3}{2} \times \frac{3}{4} = 108.$$

75. (b) Journey completed by aeroplane and train

$$= \frac{2}{15} + \frac{2}{5} = \frac{2+6}{15} = \frac{8}{15}$$

$$\therefore \text{Remaining journey} = 1 - \frac{8}{15} = \frac{7}{15}$$

$$\therefore \text{He completed } \frac{7}{15} \text{ part of his journey by taxi.}$$

$$76. (e) \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\dots\left(1 - \frac{1}{70}\right) = \frac{x}{70}$$

$$\therefore \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \dots \times \frac{69}{70} = \frac{x}{70}$$

$$\therefore \frac{1}{70} = \frac{x}{70} \quad \therefore x = 1.$$

$$\begin{aligned} 77. (c) & \frac{1.073 \times 1.073 - 0.927 \times 0.927}{1.073 - 0.927} + \frac{(3^4)^4 \times (9)^6}{(27)^7 \times (3)^9} \\ &= \frac{(1.073)^2 - (0.927)^2}{1.073 - 0.927} + \frac{(3^4)^4 \times (3^2)^6}{(3^3)^7 \times (3)^9} \\ &= \frac{(1.073 + 0.927)(1.073 - 0.927)}{1.073 - 0.927} + \frac{3^{28}}{3^{30}} \end{aligned}$$

$$= 2 + \frac{1}{3^2} = 2 + \frac{1}{9} = 2\frac{1}{9}.$$

$$\begin{aligned} 78. (c) & \frac{2^{1/2} \times 3^{1/3} \times 4^{1/4}}{10^{-1/5} \times 5^{3/5}} \div \frac{3^{4/3} \times 5^{-7/5}}{4^{-3/5} \times 6} \\ &= \frac{2^{1/2} \times 3^{1/3} \times (2^2)^{1/4} \times 10^{1/5}}{5^{3/5}} \div \frac{3^{4/3} \times 4^{3/5}}{5^{7/5} \times 6} \\ &= \frac{2^{1/2} \times 3^{1/3} \times 2^{1/2} \times 2^{1/5} \times 5^{1/5}}{5^{3/5}} \times \frac{5^{7/5} \times 2 \times 3}{3^{4/3} \times 2^{6/5}} \\ &= 2^{\frac{1}{2} + \frac{1}{2} + \frac{1}{5} - \frac{6}{5} + 1} \times 3^{\frac{1}{3} - \frac{4}{3} + 1} \times 5^{\frac{1}{5} - \frac{3}{5} + \frac{7}{5}} \\ &= 2^1 \times 3^0 \times 5^1 = 2 \times 5 = 10. \end{aligned}$$

EXERCISE-2 (BASED ON MEMORY)

$$1. (b) \frac{x+2x}{y+1.5y} = \frac{7}{10} \Rightarrow \frac{3x}{2.5y} = \frac{7}{10} \Rightarrow \frac{x}{y} = \frac{7 \times 2.5}{10 \times 3} = \frac{7}{12}$$

$$2. (a) ? = 16^{7.5} \div 8^{3.5} \div 2^{7.5} \\ = (2)^{30} \div (2)^{10.5} \div (2)^{7.5} = 2^{30-10.5-7.5} = 2^{12} = (8)^4$$

$$3. (c) \therefore (a)^c \times (b)^a \times ? = 0 \quad \therefore ? = 0$$

$$4. (e) 457$$

$$6. (e) 59$$

$$8. (a) \text{ Suppose that the number is } k$$

Then, we have

$$\frac{3}{5} \times \frac{2}{3} \times k - \frac{2}{5} \times \frac{1}{4} \times k = 288$$

$$\text{or, } \frac{2}{5}k - \frac{1}{10}k = 288 \text{ or, } \frac{3}{10}k = 288 \text{ or, } k = \frac{288 \times 10}{3} = 960$$

$$9. (a) \frac{2}{3} \text{ of } 99 - \frac{3}{4} \text{ of } 52 = 66 - 39 = 27$$

$$10. (a) \text{ Suppose original fraction } = \frac{x}{y}.$$

$$\text{Then, } \frac{5x}{6y} = \frac{15}{22} \quad \therefore \frac{x}{y} = \frac{15}{22} \times \frac{6}{5} = \frac{9}{11}$$

$$\begin{aligned} 11. (c) ? &= \sqrt{928 \times 58} = \sqrt{29 \times 32 \times 29 \times 2} \\ &= \sqrt{(29)^2 \times (8)^2} = 29 \times 8 = 232 \end{aligned}$$

$$12. (a) 3^{3.5} \times 21^2 \times 42^{2.5} \div 2^{2.5} \times 7^{3.5}$$

$$\begin{aligned} &= \frac{3^{3.5} \times 21^2 \times 42^{2.5} \times 7^{3.5}}{2^{2.5}} \\ &= \frac{(3^{3.5} \times 7^{3.5}) \times 21^2 \times (21^{2.5} \times 2^{2.5})}{2^{2.5}} \\ &= 21^{3.5} \times 21^2 \times 21^{2.5} = 21^{(3.5+2+2.5)} = 21^8 \end{aligned}$$

$$\begin{aligned} 13. (a) x+y &= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\ &= \frac{(\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}-\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\ &= \frac{5+3+2\sqrt{15}+5+3-2\sqrt{15}}{5-3} = \frac{16}{2} = 8 \end{aligned}$$

$$\begin{aligned} 14. (c) & \frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}} + \frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}} \\ &= \frac{\sqrt{2}-\sqrt{3}-\sqrt{5}+\sqrt{2}+\sqrt{3}-\sqrt{5}}{[(\sqrt{2}-\sqrt{5})+\sqrt{3}]} \times [(\sqrt{2}-\sqrt{5})-\sqrt{3}] \\ &= \frac{2\sqrt{2}-2\sqrt{5}}{(\sqrt{2}-\sqrt{5})^2 - (\sqrt{3})^2} \\ &= \frac{2(\sqrt{2}-\sqrt{5})}{2+5-2\sqrt{10}-3} \end{aligned}$$

$$= \frac{2(\sqrt{2}-\sqrt{5})}{2(2-\sqrt{10})}$$

$$= \frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}(\sqrt{2}-\sqrt{5})} = \frac{1}{\sqrt{2}}$$

15. (a) $5\frac{3}{*} \times 3\frac{1}{2} = 19$

$$\Rightarrow \frac{5*+3}{*} \times \frac{7}{2} = 19$$

$$\Rightarrow \frac{5*+3}{*} = \frac{38}{7} \Rightarrow * = 7$$

16. (a) Given expression $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \dots \times \frac{99}{100}$

$$= \frac{1}{100} = 0.01$$

17. (b) $\left[\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \right]^2 + \left[\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \right]^2$

$$= \left[\frac{(\sqrt{5}+\sqrt{3})(\sqrt{3}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \right]^2 + \left[\frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} \right]^2$$

$$= \left[\frac{5+3+2\sqrt{15}}{5-3} \right]^2 + \left[\frac{5+3-2\sqrt{15}}{5-3} \right]^2$$

$$= [4+\sqrt{15}]^2 + [4-\sqrt{15}]^2$$

$$= 16 + 15 + 8\sqrt{15} + 16 + 15 - 8\sqrt{15}$$

$$= 16 + 15 + 16 + 15 \text{ or } 62.$$

18. (d) $25^{25} = (26-1)^{25}$
 $= 26^{25} + {}^{25}C_1 \times 26^{24} \times (-1)^1$
 $+ {}^{25}C_2 \times 26^{23} \times (-1)^2 + \dots + (-1)^{25}$

[using Binomial theorem]

Now, all the terms are divisible by 26 except the last term $(-1)^{25}$. So, the remainder is $26-1=25$.

19. (d) Let $x = \sqrt[6]{6+\sqrt{6+\dots}}$

On squaring, we get

$$x^2 = 6+x \text{ or, } x^2-x-6=0$$

$$\text{or, } (x-3)(x+2)=0 \text{ or, } x=3, -2$$

But -ve value cannot be accepted.

$$\therefore x=3.$$

21. (b) Given expression $= (2^4)^{0.16} \times (2^4)^{0.04} \times (2)^{0.2}$
 $= 2^{0.64} \times 2^{0.16} \times 2^{0.2}$
 $= 2^1 = 2.$

22. (b) Number $= 269 \times 68 = 18292$
 $= 67 \times 273 + 1.$

23. (b) $\frac{14}{15} \times \frac{35}{24} = \frac{7}{6}$

25. (d) $\frac{5}{3} \div \frac{2}{7} \times \frac{*}{7} = \frac{5}{4} \times \frac{2}{3} \div \frac{1}{6}$

$$\text{or, } \frac{5}{3} \times \frac{7}{2} \times \frac{*}{7} = \frac{5}{4} \times \frac{2}{3} \times \frac{6}{1}$$

$$\text{or, } * = \frac{5}{4} \times \frac{2}{3} \times \frac{6}{1} \times \frac{3}{5} \times \frac{2}{7}$$

$$\text{Hence, } * = 6.$$

26. (b) Suppose that certain amount is ₹x

$$\text{Then, } \left(x - \frac{3}{16}x - \frac{x}{4} \right) = 81$$

$$\text{or, } 16x - 3x - 4x = 81 \times 16$$

$$\text{or, } x = \frac{81 \times 16}{9} = 144$$

$$\text{Hence, B gets} = 144 \times \frac{1}{4} = ₹36.$$

27. (c) Required greatest number is H.C.F. of $(989-5)$ and $(1327-7)$

$$\text{Hence, H.C.F. of } 989 \text{ and } 1320 \text{ is } 24.$$

28. (b) $1 * 2 = 1 + 2 \times 6 = 1 + 12 = 13$

$$(1 * 2) * 3 = 13 * 3 = 13 + 3 \times 6 = 31.$$

29. (d) The given expression

$$= \frac{2}{1+\frac{2}{2-1}} \times \frac{3}{\frac{5}{4} \div \frac{5}{4}} = \frac{2}{3} \times 3 = 2.$$

30. (b) $x * y = x^2 + y^2 - xy$

$$\therefore 9 * 11 = (9)^2 + (11)^2 - 9 \times 11$$

$$= 81 + 121 - 99 = 103.$$

31. (c) Man has $1 - \left(\frac{1}{3} + \frac{2}{3} + \frac{1}{5} \right) = \frac{1}{15}$

$$\therefore \text{Man's income} = 400 \times 15 = ₹6000.$$

32. (b) $2 * 3 + 3 * 4$

$$= [2(2) + 3(3)] + [2(3) + 3(4)]$$

$$= [4 + 9] + [6 + 12] = 31.$$

33. (d) Given expression

$$= \left[\{(2^9)^{1/6}\}^{1/3} \right]^4 \times \left[\{(2^9)^{1/3}\}^{1/6} \right]^4$$

$$= (2^{1/2})^4 \times (2^{1/2})^4 = 2^2 \times 2^2 = 2^4.$$

34. (d) Given expression

$$= \sqrt[10]{\sqrt[25]{108} + \sqrt[154]{15}} = \sqrt[10]{\sqrt[25]{108} + 13}$$

$$= \sqrt[10]{\sqrt[25]{25+11}} = \sqrt[10]{10+6} = \sqrt[16]{16} = 4.$$

4.32 Chapter 4

35. (d) $25^{7.5} \times 5^{2.5} \div 125^{1.5} = 5^?$

or, $5^{2 \times 7.5} \times 5^{2.5} \div 5^{3 \times 1.5} = 5^?$

or, $5^{15} \times 5^{2.5} \div 5^{4.5} = 5^?$

or, $5^{17.5} \times \frac{1}{5^{4.5}} = 5^?$ or, $5^{13} = 5^?$

or, $? = 13$.

36. (e) Let the number be x

Then, $\frac{2}{5} \times \frac{1}{3} \times \frac{3}{7} \times x = 15$

or, $\frac{2x}{35} = 15$ or, $x = \frac{15 \times 35}{2}$

$\therefore 40\% \text{ of } x = \frac{40}{100} \times \frac{15 \times 35}{2} = 105$.

37. (c) Required number

$= \frac{2}{5} \times 200 - \frac{3}{5} \times 125 = 80 - 75 = 5$.

38. (b) $\frac{2}{3} = \frac{2 \times 165}{3 \times 165} = \frac{330}{495}$

$\frac{3}{5} = \frac{3 \times 99}{5 \times 99} = \frac{297}{495}$

$\frac{7}{9} = \frac{7 \times 55}{9 \times 55} = \frac{385}{495}$

$\frac{9}{11} = \frac{9 \times 45}{11 \times 45} = \frac{405}{495}$

$\frac{8}{9} = \frac{8 \times 55}{9 \times 55} = \frac{440}{495}$

Ascending order $\frac{3}{5}, \frac{2}{3}, \frac{7}{9}, \frac{9}{11}, \frac{8}{9}$

39. (c) $((47)^{1/2})^{15} \div ((47)^{1/2})^3 \times ((47)^{1/2})^{-6}$

$= (\sqrt{47})^? = (\sqrt{47})^{15} \div (\sqrt{47})^3 \times (\sqrt{47})^{-6} = (\sqrt{47})^?$

$\Rightarrow (\sqrt{47})^{15-3-6} = (\sqrt{47})^?$

$\therefore ? = 6$.

41. (b) $? = \frac{(10008.99)^2}{10009.001} \times \sqrt{3589} \times 0.4987$

$\approx (10009) \times \sqrt{3600} \times 0.50$

$\approx (10009) \times 60 \times 0.50 \approx 300000$.

43. (a) Difference between two lowest numbers

$= 16 - 7 = 9$

Difference between two highest numbers

$= 91 - 89 = 2$

\therefore Product of these two numbers $= 9 \times 2 = 18$.

44. (c) $\frac{1}{5} I = \frac{5}{8} II \therefore \frac{I}{II} = \frac{25}{8} \dots(1)$

$I + 35 = 4 II$ or, $\frac{25}{8} II + 35 = 4 II$

$\therefore II = 40$.

45. (c) Suppose, there were x packages in the van before delivery.

\therefore After first delivery, the number of packages in the van

$= x - \frac{2}{5}x = \frac{3}{5}x$

After second delivery, the number of packages in the van

$= \frac{3}{5}x - 3 = \frac{3x-15}{5}$

$\therefore \frac{3x-15}{5} = \frac{x}{2}$ (Given)

$\Rightarrow x = 30$.

46. (a) $(6 \times \sqrt{5})^3 = 216 \times 5 = 1080$

$(8 \times \sqrt[3]{2})^3 = 512 \times 2 = 1024$

$(2 \times \sqrt[3]{130})^3 = 8 \times 130 = 1040$

$(\sqrt[3]{900})^3 = 900$.

47. (b) $x = 2 + 2^{2/3} + 2^{1/3}$

$\Rightarrow (x - 2) = 2^{2/3} + 2^{1/3}$

$\Rightarrow (x - 2)^3 = (2^{2/3} + 2^{1/3})^3$

$= 4 + 2 + 3 \times 2^{2/3} \times 2^{1/3} [2^{2/3} + 2^{1/3}]$

$= 6 + 3 \times 2 (x - 2)$

$\Rightarrow (x - 2)^3 = 6 + 6x - 12 = 6x - 6$

$\Rightarrow x^3 - 8 - 6x(x - 2) = 6x - 6$

$\Rightarrow x^3 - 6x^2 + 6x = 2$.

48. (b) $\sqrt[3]{\frac{.9 \times .9 \times .9 \times 100}{.16 \times .16 \times .16 \times 100}} = \frac{.9}{.16} = 5.625$.

49. (c) $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5} = 2.236$.

50. (a) $21952 = 4 \times 4 \times 4 \times 7 \times 7 \times 7$

$\therefore \sqrt[3]{21952} = 4 \times 7 = 28$.

51. (b) $\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{\sqrt{5}+\sqrt{3}}{5-3} = \frac{2.2361+1.7321}{2}$

$= \frac{3.9682}{2} = 1.9841$.

$$\begin{aligned}
 52. \text{ (c) } \sqrt{272^2 - 128^2} &= \sqrt{(272+128)(272-128)} \\
 &= \sqrt{400 \times 144} \\
 &= 20 \times 12 = 240.
 \end{aligned}$$

$$\begin{aligned}
 53. \text{ (c) } \sqrt{0.9} &= \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} \\
 &= \frac{3 \times 3.16}{10} = \frac{9.48}{10} = 0.94.
 \end{aligned}$$

$$\begin{aligned}
 54. \text{ (c) } \frac{0.342 \times 0.684}{0.000342 \times 0.000171} &= \frac{342 \times 684 \times 10^6}{342 \times 171} \\
 &= 4 \times 10^6 \\
 \text{Square root of } 4 \times 10^6 &= 2 \times 10^3 = 2000.
 \end{aligned}$$

$$\begin{aligned}
 55. \text{ (c) } \sqrt[3]{175.616} &= \sqrt[3]{\frac{175616}{10000}} = \frac{56}{10} = 5.6 \\
 \sqrt[3]{0.175616} &= \sqrt[3]{\frac{175616}{1000000}} = \frac{56}{100} = 0.56 \\
 \sqrt[3]{0.000175616} &= \sqrt[3]{\frac{175616}{1000000000}} = \frac{56}{1000} \\
 &= 0.056
 \end{aligned}$$

\therefore Required answer = $5.6 + 0.56 + 0.056 = 6.216$.

$$\begin{aligned}
 56. \text{ (a) } \sqrt{8} + 2\sqrt{32} - \sqrt[3]{128} + \sqrt[4]{50} \\
 &= 2\sqrt{2} + 2 \times 4\sqrt{2} - 3 \times 8\sqrt{2} + 4 \times 5\sqrt{2} \\
 &= 2\sqrt{2} + 8\sqrt{2} - 24\sqrt{2} + 20\sqrt{2} \\
 &= 6\sqrt{2} = 6 \times 1.414 = 8.484.
 \end{aligned}$$

$$\begin{aligned}
 57. \text{ (b) } \sqrt[3]{0.004096} &= \sqrt[3]{((0.16)^3)^{1/3}} \\
 &= \sqrt{0.16} = 0.4.
 \end{aligned}$$

$$58. \text{ (a) } \sqrt{\frac{5}{3}} = \sqrt{\frac{5 \times 3}{3 \times 3}} = \sqrt{\frac{15}{3}} = \frac{3.88}{3} = 1.29\bar{3}.$$

$$\begin{aligned}
 59. \text{ (b) } \sqrt{5625} + \sqrt{56.25} + \sqrt{0.5625} \\
 &= \sqrt{5625} + \sqrt{\frac{5625}{100}} + \sqrt{\frac{5625}{10000}} = 75 + \frac{75}{10} + \frac{75}{100} \\
 &= 75 + 7.5 + 0.75 = 83.25.
 \end{aligned}$$

$$60. \text{ (d) } \frac{36}{6} \times \sqrt{?} = 108$$

$$\text{or, } \sqrt{?} = \frac{108}{6}$$

$$\text{or, } \sqrt{?} = 18$$

$$\text{or, } ? = 324 \approx 325.$$

$$61. \text{ (b) } \sqrt{10000} + \frac{3.001}{4.987} \text{ of } 1891.992 = ?$$

$$\text{or, } ? \approx 100 + \frac{3}{5} \text{ of } 1900 = 100 + 1140 \approx 1230.$$

$$62. \text{ (c) } 1.\overline{27} = 1\frac{27}{99} = 1\frac{3}{11} = \frac{14}{11}$$

$$63. \text{ (b) } 2p + \frac{1}{p} = 4$$

$$\Rightarrow p + \frac{1}{2p} = 2$$

$$\text{Therefore, } \left(p + \frac{1}{2p}\right)^3$$

$$\Rightarrow p^3 + \frac{1}{8p^3} + 3 \times p \times \frac{1}{2p} \left(1 + \frac{1}{2p}\right)$$

$$\Rightarrow 8 = p^3 + \frac{1}{8p^3} + \frac{3}{2} \times 2$$

$$\Rightarrow p^3 + \frac{1}{8p^3} + 3 = 5$$

$$\begin{aligned}
 64. \text{ (d) } 0.1 \times 0.01 \times 0.001 \times 10^7 \\
 &= 10^{-6} \times 10^7 = 10^1 = 10
 \end{aligned}$$

$$\begin{aligned}
 65. \text{ (c) } [(\sqrt[3]{x^{-3/5}})^{-5/3}]^5 &= (x^{-3/5})^{1/5 \times -5/3 \times 5} \\
 &= x^{-3/5 \times -5/3} = x
 \end{aligned}$$

$$\begin{aligned}
 66. \text{ (a) } \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{24}\right) \left(1 - \frac{1}{25}\right) \\
 &= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{23}{24} \times \frac{24}{25} = \frac{2}{25}
 \end{aligned}$$

$$\begin{aligned}
 67. \text{ (b) } \frac{3\sqrt{2}}{\sqrt{3} + \sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6} + \sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}} \\
 &= \frac{3\sqrt{2}(\sqrt{3} - \sqrt{6})}{(\sqrt{3} + \sqrt{6})(\sqrt{3} - \sqrt{6})} - \frac{4\sqrt{3}(\sqrt{6} - \sqrt{2})}{(\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2})} \\
 &\quad + \frac{\sqrt{6}(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} \\
 &= \frac{3\sqrt{6} - 6\sqrt{3}}{(-3)} - \frac{12\sqrt{2} - 4\sqrt{6}}{4} + \frac{3\sqrt{2} - 2\sqrt{3}}{1} \\
 &= \frac{-\sqrt{6} + 2\sqrt{3}}{1} - \frac{3\sqrt{2} - \sqrt{6}}{1} \\
 &= -\sqrt{6} + 2\sqrt{3} - 3\sqrt{2} + \sqrt{6} + 3\sqrt{2} - 2\sqrt{3} = 0
 \end{aligned}$$

$$\begin{aligned}
 68. \text{ (a) } \frac{2\frac{1}{3} - 1\frac{2}{11}}{3 + \frac{1}{3 + \frac{1}{9+1}}} &= \frac{\frac{7}{3} - \frac{11}{11}}{3 + \frac{1}{3 + \frac{3}{10}}} \\
 &= \frac{0}{3 + \frac{1}{3 + \frac{3}{10}}} = 0
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{77-39}{\frac{33}{3+\frac{1}{\frac{33}{10}}}} = \frac{\frac{38}{33}}{3+\frac{10}{33}} \\
 &= \frac{38}{33} \times \frac{33}{109} = \frac{38}{109}
 \end{aligned}$$

$$\begin{aligned}
 69. (b) \quad &3 + \frac{1}{\sqrt{3}} + \left(\frac{1}{3+\sqrt{3}} - \frac{1}{3-\sqrt{3}} \right) \\
 &= 3 + \frac{1}{\sqrt{3}} + \left(\frac{3-\sqrt{3}}{3+\sqrt{3}} - \frac{3-\sqrt{3}}{3-\sqrt{3}} \right) \\
 &= 3 + \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{3} = 3 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \\
 &= 3
 \end{aligned}$$

$$70. (a) \quad x + \frac{2}{3 + \frac{4}{\frac{30+7}{6}}} = 10$$

$$\Rightarrow x + \frac{2}{3 + \frac{4 \times 6}{37}} = 10$$

$$\Rightarrow x + \frac{2}{3 + \frac{24}{37}} = 10$$

$$\Rightarrow x + \frac{2}{\frac{111+24}{37}} = 10$$

$$\Rightarrow x + \frac{2 \times 37}{135} = 10$$

$$\Rightarrow x + \frac{74}{135} = 10$$

$$\Rightarrow 10 - \frac{74}{135} = x$$

$$\Rightarrow x = 10 - \frac{1350-74}{135}$$

$$\Rightarrow x = \frac{1276}{135}$$

$$\begin{aligned}
 71. (b) \quad \text{Expression} &= 3 + \frac{3}{3 + \frac{1}{3 + \frac{1}{3}}} \\
 &= 3 + \frac{3}{3 + \frac{3}{10}} = 3 + \frac{3}{\frac{30+3}{10}}
 \end{aligned}$$

$$= 3 + \frac{30}{23} = 3 + \frac{10}{11}$$

$$= \frac{33+10}{11} = \frac{43}{11}$$

$$72. (c) \quad x = \sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \sqrt{\frac{(\sqrt{5}+1)^2}{5-1}}$$

$$\sqrt{\frac{(\sqrt{5}+1)^2}{4}} = \frac{\sqrt{5}+1}{2}$$

$$\text{Therefore, } 5x^2 - 5x - 1$$

$$= 5 \left(\frac{\sqrt{5}+1}{2} \right)^2 - \frac{5(\sqrt{5}+1)}{2} - 1$$

$$= 5 \left(\frac{5+1+2\sqrt{5}}{4} \right) - \frac{5\sqrt{5}+5}{2} - 1$$

$$= 5 \left(\frac{3+\sqrt{5}}{2} \right) - \frac{5\sqrt{5}+5}{2} - 1$$

$$= \frac{15+5\sqrt{5}-5\sqrt{5}-5-2}{2} = \frac{8}{2} = 4$$

$$73. (b) \quad \frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} - \sqrt{3-x}} = 2$$

$$\Rightarrow \frac{(\sqrt{3+x} + \sqrt{3-x})^2}{(3+x-3+x)} = 2$$

$$\Rightarrow \frac{3+x+3-x+2\sqrt{9-x^2}}{2x} = 2$$

$$\Rightarrow 6+2\sqrt{9-x^2} = 4x$$

$$\Rightarrow 2\sqrt{9-x^2} = 4x-6$$

Squaring both sides

$$4(9-x^2) = 16x^2 + 36 - 48x$$

$$\Rightarrow 36 - 4x^2 = 16x^2 + 36 - 48x$$

$$\Rightarrow 20x^2 = 48x$$

$$\Rightarrow x = \frac{48}{20} = \frac{12}{5}$$

$$74. (c) \quad 0.121212 \dots$$

$$= \frac{12}{99} = \frac{4}{33}$$

$$75. (b) \quad ? = 3\frac{3}{4} + 4\frac{2}{5} - 3\frac{1}{8}$$

$$? = \frac{15}{4} + \frac{22}{5} - \frac{25}{8}$$

$$? = \frac{150+176-125}{40}$$

$$? = \frac{201}{40} = 5\frac{1}{40}$$

$$76. (d) \sqrt{5^2 \times 14 - 6 \times 7 + (4)^?} = 18$$

$$\begin{aligned} \Rightarrow 5^2 \times 14 - 6 \times 7 + (4)^? &= (18)^2 \\ \Rightarrow 25 \times 14 - 42 + (4)^? &= 324 \\ \Rightarrow 350 - 42 + (4)^? &= 324 \\ \Rightarrow 308 + (4)^? &= 324 \\ \Rightarrow (4)^? &= 324 - 308 \\ \Rightarrow (4)^? &= 16 \\ \Rightarrow (4)^? &= (4)^2 \\ \Rightarrow ? &= 2 \end{aligned}$$

$$77. (c) ? = 67.99\% \text{ of } 1401 - 13.99\% \text{ of } 1299$$

$$\begin{aligned} \Rightarrow ? &= 1401 \times \frac{68}{100} - 1300 \times \frac{14}{100} \\ \Rightarrow ? &= 952.68 - 182 \\ \Rightarrow ? &= 770.68 \\ \Rightarrow ? &= 770 \end{aligned}$$

(Approx.)

$$78. (d) ? = \left(\frac{24}{9}\right)^2 \times \frac{399}{39} \div \frac{41}{899}$$

$$\begin{aligned} \Rightarrow ? &= \left(\frac{24}{9}\right)^2 \times \frac{399}{39} \times \frac{899}{41} \\ \Rightarrow ? &= 7.11 \times 10.23 \times 21.92 \\ \Rightarrow ? &= 159.435 = 1550 \end{aligned}$$

(Approx.)

$$79. (b) (3 \times 2)^{?+5} = (15 \times 0.40)^4 \div (1080 \div 30)^4$$

$$\begin{aligned} &\times (27 \times 8)^4 \\ \Rightarrow (3 \times 2)^{?+5} &= (6)^4 \div (36)^4 \times (216)^4 \\ \Rightarrow (6)^{?+5} &= (6)^4 \div (6^2)^4 \times (6^3)^4 \\ \Rightarrow (6)^{?+5} &= (6)^4 \div (6)^8 \times (6)^{12} \\ \Rightarrow (6)^{?+5} &= (6)^{-4} \times (6)^{12} \\ \Rightarrow (6)^{?+5} &= (6)^8 \\ \Rightarrow ? + 5 &= 8 \\ \Rightarrow ? &= 8 - 5 = 3 \end{aligned}$$

$$80. (d) \frac{(?^2)}{10} + 1\frac{5}{12} = 3\frac{1}{4} + 2\frac{1}{2} - 1\frac{5}{6}$$

$$\begin{aligned} \Rightarrow \frac{(?^2)}{10} + \frac{17}{12} &= \frac{13}{4} + \frac{5}{2} - \frac{11}{6} \\ \Rightarrow \frac{(?^2)}{10} &= \frac{13}{4} + \frac{5}{2} - \frac{11}{6} - \frac{17}{12} \\ \Rightarrow \frac{(?^2)}{10} &= \frac{39+30-22-17}{12} \\ \Rightarrow \frac{(?^2)}{10} &= \frac{69-39}{12} \\ \Rightarrow \frac{(?^2)}{10} &= \frac{30}{12} \\ \Rightarrow (?^2) &= \frac{30 \times 20}{12} \end{aligned}$$

$$\Rightarrow (?^2) = 25$$

$$\Rightarrow ? = \sqrt{25} = 5$$

$$81. (c) (?^3) + \sqrt{49} = 92 \times 576 \div 2\sqrt{1296}$$

$$\begin{aligned} \Rightarrow (?^3) + 7 &= 92 \times 576 \div 2 \times 36 \\ \Rightarrow (?^3) + 7 &= 92 \times 576 \div 72 \\ \Rightarrow (?^3) + 7 &= 92 \times 8 \\ \Rightarrow (?^3) + 7 &= 736 \\ \Rightarrow (?^3) &= 736 - 7 = 729 \\ \Rightarrow ? &= \sqrt[3]{729} \\ \Rightarrow ? &= 9 \end{aligned}$$

$$82. (c) 85 + ? = \frac{1}{6} \text{ of } (92)\% \text{ of } 1\frac{1}{23} \text{ of } 650$$

$$\begin{aligned} \Rightarrow 85 + ? &= \frac{1}{6} \times \frac{92}{100} \times \frac{24}{23} \times 650 \\ \Rightarrow 85 + ? &= 104 \\ \Rightarrow ? &= 104 - 85 \\ \Rightarrow ? &= 19 \end{aligned}$$

$$83. (c) \because \text{Price of a pencil box} = 7 + 22 + 14 = ₹43$$

Hence the, required amount Seema paid to the shopkeeper

$$\begin{aligned} &= 20 \times 7 + 8 \times 22 + 6 \times 175 + 7 \times 43 \\ &= 140 + 176 + 1050 + 301 = ₹1667. \end{aligned}$$

$$84. (b) a^2 + b^2 + c^2 = ab + bc + ca$$

$$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$$

On multiplying by 2, we have,

$$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$$

$$\Rightarrow a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ac = 0$$

$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$\Rightarrow a - b = 0$$

$$\Rightarrow a = b; b - c = 0 \Rightarrow b = c \text{ and } c - a = 0$$

$$\Rightarrow c = a$$

$$\therefore \frac{a+c}{b} = \frac{2a}{a} = 2$$

$$85. (a) ab + bc + ca = 0$$

$$\Rightarrow ab + ca = -bc$$

$$\therefore a^2 - bc = a^2 + ab + ac = a(a+b+c)$$

Similarly,

$$b^2 - ac = b(a+b+c), \text{ and}$$

$$c^2 - ab = c(a+b+c)$$

$$\begin{aligned} \therefore \frac{1}{a^2-bc} + \frac{1}{b^2-ca} + \frac{1}{c^2-ab} \\ &= \frac{1}{a(a+b+c)} + \frac{1}{b(a+b+c)} + \frac{1}{c(a+b+c)} \\ &= \frac{1}{(a+b+c)} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = \frac{1}{(a+b+c)} \left(\frac{ac+ca+ab}{abc} \right) \\ &= \frac{1}{a+b+c} \times \frac{0}{abc} = 0 \end{aligned}$$

4.36 Chapter 4

86. (d) $(2 + \sqrt{3})a = (2 - \sqrt{3})b = 1$

$$\Rightarrow a = \frac{1}{2 + \sqrt{3}}$$

$$\therefore \frac{1}{a} = 2 + \sqrt{3}$$

Similarly,

$$b = \frac{1}{2 - \sqrt{3}}$$

$$\frac{1}{b} = 2 - \sqrt{3}$$

$$\therefore \frac{1}{a} + \frac{1}{b} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$$

87. (b) $3x + \frac{3}{x} = 1 \Rightarrow x + \frac{1}{x} = \frac{1}{3}$

On cubing both sides, we have,

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{1}{27}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{1}{3} = \frac{1}{27}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 1 = \frac{1}{27}$$

88. (d) $\frac{1}{a^2 + ax + x^2} - \frac{1}{a^2 - ax + x^2} + \frac{2ax}{a^4 + a^2x^2 + x^4}$
 $= \frac{a^2 - ax + x^2 - a^2 - ax - x^2}{(a^2 + ax + x^2)(a^2 - ax + x^2)} + \frac{2ax}{a^4 + a^2x^2 + x^4}$
 $= \frac{-2ax}{a^4 + a^2x^2 + x^4} + \frac{2ax}{a^4 + a^2x^2 + x^4} = 0$

89. (c) $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$
 $= \frac{1}{2}(333 + 333 + 333)(9 + 1 + 1) = 1000$

90. (a) Quicker Method:

When $x = (a + b + c)^2$, then

$$\frac{x - a^2}{b + c} + \frac{x - b^2}{c + a} + \frac{x - c^2}{a + b}$$

$$= \frac{(a + b + c)^2 - a^2}{b + c} + \frac{(a + b + c)^2 - b^2}{c + a} + \frac{(a + b + c)^2 - c^2}{a + b}$$

$$= \frac{(2a + b + c)(b + c)}{a + b} + \frac{(a + 2b + c)(c + a)}{c + a} + \frac{(a + b + 2c)(a + b)}{a + b}$$

$$= 2a + b + c + a + 2b + c + a + b + 2c$$

$$= 4a + 4b + 4c = 4(a + b + c)$$

91. (d) $(x - a)^3 - \frac{1}{(x - a)^3}$
 $= \left(x - a - \frac{1}{x - a}\right)^3 + 3\left(x - a - \frac{1}{x - a}\right)$

$$= (x - a - x + b)^3 + 3(x - a - x + b)$$

$$= (b - a)^3 + 3(b - a)$$

$$= 5^3 + 3 \times 5 = 125 + 15 = 140$$

92. (a) Let, $x = \sqrt[3]{2^3 \sqrt[3]{4 \sqrt[3]{2^3 \sqrt[3]{4 \dots}}}}$

On squaring, we have, $x^2 = 2^{\frac{2}{3}} \sqrt[3]{4 \sqrt[3]{2^3 \sqrt[3]{4 \dots}}}$

On cubing, we have, $x^6 = 8 \times 4x$

$$\Rightarrow x^5 = 32 = 2^5 \Rightarrow x = 2$$

93. (b) $\frac{3\sqrt{2}}{\sqrt{3} + \sqrt{6}} = \frac{3\sqrt{2}(\sqrt{6} - \sqrt{3})}{(\sqrt{6} + \sqrt{3})(\sqrt{6} - \sqrt{3})}$
 $= \frac{3\sqrt{2}(\sqrt{6} - \sqrt{3})}{6 - 3} = \sqrt{2}(\sqrt{6} - \sqrt{3}) = 2\sqrt{3} - \sqrt{6}$
 $\frac{4\sqrt{3}}{\sqrt{6} + \sqrt{2}} = \frac{4\sqrt{3}(\sqrt{6} - \sqrt{2})}{(\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2})}$
 $= \frac{4\sqrt{3}(\sqrt{6} - \sqrt{2})}{6 - 2} = \sqrt{3}(\sqrt{6} - \sqrt{2}) = 3\sqrt{2} - \sqrt{6}$
 $\frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{6}(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} = \frac{\sqrt{6}(\sqrt{3} - \sqrt{2})}{3 - 2}$

\therefore The given expression

$$= 2\sqrt{3} - \sqrt{6} - 3\sqrt{2} + \sqrt{6} + 3\sqrt{2} - 2\sqrt{3} = 0$$

94. (a) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$
 $\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 3 = 0$
 $\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$
 $\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$
 $\therefore a - 1 = 0 \Rightarrow a = 1$
 $b + 1 = 0 \Rightarrow b = -1$
 $c + 1 = 0 \Rightarrow c = -1$
 $\therefore 4a - 3b + 5c = 4 \times 1 - 3 \times (-1) + 5(-1)$
 $= 4 + 3 - 5 = 2$

95. (c) $2x + \frac{2}{x} = 3 \Rightarrow x + \frac{1}{x} = \frac{3}{2}$

On cubing, we have,

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{27}{8}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{3}{2} = \frac{27}{8}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2} = \frac{27 - 36}{8} = -\frac{9}{8}$$

$$\therefore x^3 + \frac{1}{x^3} + 2 = 2 - \frac{9}{8} = \frac{7}{8}$$

$$\begin{aligned}
 96. \text{ (a)} \quad & a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0 \\
 & \therefore (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 \\
 & = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)
 \end{aligned}$$

$$\begin{aligned}
 & [\text{If } x + y + z = 0, \text{ then, } x^3 + y^3 + z^3 = 3xyz] \\
 & = 3(a + b)(a - b)(b + c)(b - c)(c + a)(c - a)
 \end{aligned}$$

$$97. \text{ (d)} \quad x = \sqrt[3]{5} + 2 \Rightarrow x - 2 = \sqrt[3]{5}$$

On cubing, we have,

$$\begin{aligned}
 & x^3 - 3x^2 \times 2 + 3x(-2)^2 - 2^3 = 5 \\
 & \Rightarrow x^3 - 6x^2 + 12x - 8 = 5 \\
 & \Rightarrow x^3 - 6x^2 + 12x - 13 = 0
 \end{aligned}$$

$$98. \text{ (d)} \quad \frac{1}{3 - \sqrt{8}} = \frac{3 + \sqrt{8}}{(3 - \sqrt{8})(3 + \sqrt{8})}$$

(Rationalizing the denominator)

$$= \frac{3 + \sqrt{8}}{9 - 8} = 3 + \sqrt{8}$$

\therefore The given expression

$$\begin{aligned}
 & = 3 + \sqrt{8} + 3 + \sqrt{8} - 6 - 4\sqrt{2} \\
 & = 6 + 2\sqrt{8} - 6 - 4\sqrt{2} = 2\sqrt{8} - 4\sqrt{2} \\
 & = 2 \times 2\sqrt{2} - 4\sqrt{2} = 0
 \end{aligned}$$

$$99. \text{ (c)} \quad x^2 + \frac{1}{x^2} = 83$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 83$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 83 - 2 = 81 = 9^2$$

$$\Rightarrow x - \frac{1}{x} = 9$$

Cubing both sides, we have

$$\left(x - \frac{1}{x}\right)^3 = 9^3 = 729$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 729$$

$$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times 9 = 729$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 729 + 27 = 756$$

$$100. \text{ (d)} \quad \left(a + \frac{1}{a}\right)^2 = 3 = (\sqrt{3})^2$$

$$\Rightarrow a + \frac{1}{a} = \sqrt{3}$$

$$\text{Cubing both sides, we have } \left(a + \frac{1}{a}\right)^3 = 3\sqrt{3}$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 3\sqrt{3}$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3} \Rightarrow a^3 + \frac{1}{a^3} = 0$$

$$101. \text{ (b)} \quad a = 7 - 4\sqrt{3}$$

$$\therefore \frac{1}{a} = \frac{1}{7 - 4\sqrt{3}} = \frac{1}{7 - 4\sqrt{3}} \times \frac{7 + 4\sqrt{3}}{7 + 4\sqrt{3}} = 7 + 4\sqrt{3}$$

$$\therefore \left(\sqrt{a} + \frac{1}{\sqrt{a}}\right)^2 = a + \frac{1}{a} + 2$$

$$= 7 - 4\sqrt{3} + 7 + 4\sqrt{3} + 2 = 16$$

$$\Rightarrow \sqrt{a} + \frac{1}{\sqrt{a}} = 4$$

$$102. \text{ (b)} \quad \frac{3\sqrt{2}}{\sqrt{6} - \sqrt{3}} = \frac{3\sqrt{2}}{\sqrt{6} - \sqrt{3}} \times \frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} + \sqrt{3}}$$

$$= \frac{3\sqrt{2}(\sqrt{6} + \sqrt{3})}{6 - 3}$$

$$= \sqrt{12} + \sqrt{6} = 2\sqrt{3} + \sqrt{6}$$

$$\frac{4\sqrt{3}}{\sqrt{6} - \sqrt{2}} = \frac{4\sqrt{3}(\sqrt{6} + \sqrt{2})}{6 - 2}$$

$$= \sqrt{18} + \sqrt{6} = 3\sqrt{2} + \sqrt{6}$$

$$\frac{6}{\sqrt{8} + \sqrt{12}} = \frac{6(\sqrt{12} - \sqrt{8})}{12 - 8}$$

$$= \frac{3}{2}(2\sqrt{3} - 2\sqrt{2}) = 3\sqrt{3} - 3\sqrt{2}$$

\therefore Given expression

$$= 2\sqrt{3} + \sqrt{6} - 3\sqrt{2} - \sqrt{6} - 3\sqrt{3} + 3\sqrt{2} = -\sqrt{3}$$

$$103. \text{ (d)} \quad x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= (x + y + z)(x + y + z)^2 - 3xy - 3yz - 3zx$$

$$\Rightarrow x^3 + y^3 + z^3 + 3 = 1[1 - 3(-1)] = 4$$

$$\Rightarrow x^3 + y^3 + z^3 = 1$$

$$104. \text{ (c)} \quad x^2 + y^2 + z^2 = xy + yz + zx$$

$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx = 0$$

$$\Rightarrow (x - y)^2 + (y - z)^2 + (z - x)^2 = 0$$

$$\Rightarrow x - y = 0 \Rightarrow x = y$$

$$y - z = 0 \Rightarrow y = z$$

$$z - x = 0 \Rightarrow z = x$$

$$\therefore \frac{4x + 2y - 3z}{2x} = \frac{4 + 2 - 3}{2} = \frac{3}{2}$$

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105. (c) $3x - 2 = \frac{3}{x} \Rightarrow 3x - \frac{3}{x} = 2$

Dividing both sides by 3, we have

$$x - \frac{1}{x} = \frac{2}{3}$$

On squaring both sides, we get

$$x^2 + \frac{1}{x^2} - 2 = \frac{4}{9}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \frac{4}{9} + 2 = \frac{22}{9} = 2\frac{4}{9}$$

106. (a) $x^2 + y^2 + z^2 + 2 = 2(y - x)$
 $\Rightarrow x^2 + 2x + y^2 - 2y + z^2 + 2 = 0$
 $\Rightarrow (x^2 + 2x + 1) + (y^2 - 2y + 1) + z^2 = 0$
 $\Rightarrow (x + 1)^2 + (y - 1)^2 + z^2 = 0$
 $\Rightarrow x + 1 = 0 \Rightarrow x = -1$
 $y - 1 = 0 \Rightarrow y = 1$
 $z = 0$

$$\therefore x^3 + y^3 + z^3 = -1 + 1 + 0 = 0$$

107. (b) $180 = 2 \times 2 \times 3 \times 3 \times 5$
 $a^3b = abc$
 $\Rightarrow a^2 = bc$
 $a^3b = abc = 180 = 1^2 \times 180 \times 1$
 $= 1^3 \times 180$

$$\Rightarrow c = 1$$

108. (a) $\left(x + \frac{1}{x}\right)^2 = 3$

$$\Rightarrow x + \frac{1}{x} = \sqrt{3}$$

On cubing both sides,

$$\left(x + \frac{1}{x}\right)^3 = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 0 \Rightarrow x^6 + 1 = 0$$

$$\therefore x^{72} + x^{66} + x^{54} + x^{36} + x^{24} + x^6 + 1$$

$$= (x^6)^{12} + (x^6)^{11} + (x^6)^9 + (x^6)^6 + (x^6)^4 + x^6 + 1$$

$$= 1 - 1 - 1 + 1 + 1 + 0 = 1$$

109. (c) $a + b + c = 0$

$$\Rightarrow b + c = -a$$

On squaring both sides,

$$\Rightarrow (b + c)^2 = a^2$$

$$\Rightarrow b^2 + c^2 + 2bc = a^2$$

$$\Rightarrow a^2 + b^2 + c^2 + 2bc = 2a^2$$

$$\Rightarrow a^2 + b^2 + c^2 = 2a^2 - 2bc = 2(a^2 - bc)$$

$$\therefore \frac{a^2 + b^2 + c^2}{a^2 - bc} = 2$$

110. (b) $n = 7 + 4\sqrt{3} = 7 + 2 \times 2 \times \sqrt{3}$

$$= 4 + 3 + 2 \times 2 \times \sqrt{3} = (2 + \sqrt{3})^2$$

$$\therefore \sqrt{n} = 2 + \sqrt{3}$$

$$\therefore \frac{1}{\sqrt{n}} = \frac{1}{2 + \sqrt{3}} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = 2 - \sqrt{3}$$

$$\therefore \sqrt{n} = \frac{1}{\frac{1}{\sqrt{n}}} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$$

111. (b) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$$\Rightarrow 36 = 14 + 2(ab + bc + ca)$$

$$\Rightarrow ab + bc + ca = (36 - 14) \div 2$$

$$\Rightarrow ab + bc + ca = 11 \quad \dots(1)$$

$$\therefore a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\Rightarrow 36 - 3abc = 6(14 - 11)$$

$$\Rightarrow 36 - 3abc = 84 - 66 = 18$$

$$\Rightarrow 3abc = 36 - 18 = 18$$

$$\Rightarrow abc = 6$$

112. (d) $(a - 1)\sqrt{2} + 3 = b\sqrt{2} + a$

$$\Rightarrow a = 3; a - 1 = b$$

$$\Rightarrow 3 - 1 = b \Rightarrow b = 2$$

$$\therefore a + b = 3 + 2 = 5$$

113. (a) $\left(x + \frac{1}{x}\right)^2 = 3 \Rightarrow x + \frac{1}{x} = \sqrt{3}$

One cubing both sides,

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} - 3\sqrt{3} = 0$$

$$\Rightarrow x^6 + 1 = 0$$

$$\therefore x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$$

$$= x^{200}(x^6 + 1) + x^{84}(x^6 + 1) + x^{12}(x^6 + 1) + (x^6 + 1) = 0$$

114. (c) Given expression = $\frac{(81)^{3.6} \times (9)^{2.7}}{(81)^{4.2} \times 3}$

$$= \frac{(3^4)^{3.6} \times (3^2)^{2.7}}{(3^4)^{4.2} \times 3} = \frac{3^{14.4} \times 3^{5.4}}{3^{16.8} \times 3}$$

$$[\therefore (a^m)^n = a^{mn}; a^m \times a^n = a^{m+n}; a^m \div a^n = a^{m-n}]$$

$$= \frac{3^{14.4+5.4}}{3^{16.8+1}} = \frac{3^{19.8}}{3^{17.8}}$$

$$= 3^{19.8-17.8} = 3^2 = 9$$

115. (c) Let the marked price of article be ₹ x and CP be ₹100.

Now, according to the question,

$$\frac{60x}{100} = 100 - 30 = 70$$

$$\Rightarrow 60x = 70 \times 100$$

$$\Rightarrow x = \frac{70 \times 100}{60} = \frac{700}{6} = ₹\frac{350}{3}$$

On selling at marked price, we have

$$\text{profit} = \frac{350}{3} - 100 = \frac{50}{3} = 16\frac{2}{3}$$

$$\therefore \text{profit}\% = 16\frac{2}{3}\%$$

116. (c) $a^2 + b^2 + c^2 = 2a - 2b - 2c - 3$

$$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$$

$$\Rightarrow (a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$$

$$\Rightarrow (a-1)^2 + (b+1)^2 + (c+1)^2 = 0$$

$$\Rightarrow a-1=0 \Rightarrow a=1$$

$$\Rightarrow b+1=0 \Rightarrow b=-1$$

$$\Rightarrow c+1=0 \Rightarrow c=-1$$

$$\therefore a-b+c=1+1-1=1$$

117. (a) $x^2 + 3x + 1 = 0$

On dividing by x , we have

$$x + 3 + \frac{1}{x} = 0 \Rightarrow x + \frac{1}{x} = -3$$

Cubing both sides,

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$\Rightarrow (-3)^3 = x^3 + \frac{1}{x^3} + 3(-3)$$

$$\Rightarrow -27 = x^3 + \frac{1}{x^3} - 9$$

$$\Rightarrow x^3 + \frac{1}{x^3} = -27 + 9 = -18$$

118. (d) $x^a \cdot x^b \cdot x^c = 1 \Rightarrow x^{a+b+c} = 1 = x^0$

$$\Rightarrow a+b+c=0$$

$$\text{Now, } a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

119. (b) $a + \frac{1}{a} + 2 = 0$

$$a^2 + 1 + 2a = 0$$

$$\Rightarrow (a+1)^2 = 0 \Rightarrow a+1=0$$

$$\Rightarrow a = -1$$

$$\therefore a^{37} - \frac{1}{a^{100}} = (-1)^{37} - \frac{1}{(-1)^{100}}$$

$$= -1 - 1 = -2$$

120. (b) $x + \frac{1}{16x} = 1 \Rightarrow 4x + \frac{1}{4x} = 4$

On cubing both sides, we have

$$\left(4x + \frac{1}{4x}\right)^3 = (4x)^3 + \left(\frac{1}{4x}\right)^3 + 3 \times 4x \times \frac{1}{4x} \left(4x + \frac{1}{4x}\right)$$

$$\Rightarrow 64 = 64x^3 + \frac{1}{64x^3} + 3 \times 4$$

$$\Rightarrow 64x^3 + \frac{1}{64x^3} = 64 - 12 = 52$$

121. (b) $a + b + c = 0$

$$\Rightarrow (a+c) = -b$$

On squaring both sides, we have

$$a^2 + c^2 + 2ac = b^2$$

$$\Rightarrow a^2 + c^2 = b^2 - 2ac$$

$$\therefore \frac{a^2 + b^2 + c^2}{b^2 - ca} = \frac{b^2 + b^2 - 2ac}{b^2 - ca} = \frac{2(b^2 - ac)}{b^2 - ac} = 2$$

122. (c) $a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$

$$\Rightarrow 8 = 4(a^2 - ab + b^2)$$

$$\Rightarrow a^2 - ab + b^2 = 2 \quad \dots(1)$$

$$a^2 + ab + b^2 = 4 \quad \dots(2)$$

By equation (2) - (1)

$$a^2 + ab + b^2 - a^2 - ab - b^2 = 4 - 2$$

$$\Rightarrow 2ab = 2 \Rightarrow ab = 1$$

123. (d) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

\therefore Given expression

$$= \frac{\frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]}{(a-b)^2 + (b-c)^2 + (c-a)^2}$$

$$= \frac{1}{2}(a+b+c) = \frac{1}{2}(25+15-10) = 15$$

- 124 (a) $x^{1/3} + y^{1/3} = z^{1/3} \quad \dots(1)$

$$\Rightarrow (x^{1/3} + y^{1/3})^3 = z$$

$$\Rightarrow x + y + 3x^{1/3}y^{1/3}(x^{1/3} + y^{1/3}) = z$$

$$\Rightarrow x + y + 3x^{1/3}y^{1/3}z^{1/3} = z$$

[Using equation (1)]

$$\Rightarrow x + y - z = -3x^{1/3}y^{1/3}z^{1/3}$$

$$\Rightarrow (x + y - z)^3 = -27xyz$$

$$\Rightarrow (x + y - z)^3 + 27xyz = 0$$

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$$\begin{aligned}
 125. (c) \quad & \sqrt{7\sqrt{7\sqrt{7}\dots}} = 7^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots} \\
 & = 7^{\frac{1/2}{1-1/2}} = 7^{\frac{1/2}{1/2}} = 7 \\
 \text{Now, } 7 &= (343)^y = 7^{3y} \\
 \Rightarrow 7^1 &= 7^{3y} \Rightarrow 3y - 3 = 1 \\
 \Rightarrow 3y &= 4 \quad \therefore y = \frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 126. (c) \quad & a + b + c = 1 \\
 \text{Putting } a = b = c &= \frac{1}{3} \\
 \therefore a + b + c &= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \quad \dots(1)
 \end{aligned}$$

$$\begin{aligned}
 \text{and, } ab + bc + ca &= \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{3}{9} \\
 \Rightarrow ab + bc + ca &= \frac{1}{3} \quad \dots(2)
 \end{aligned}$$

On satisfying both the conditions,

$$\begin{aligned}
 a = b = c &= \frac{1}{3} \\
 \therefore a : b : c &= 1 : 1 : 1
 \end{aligned}$$

$$\begin{aligned}
 127. (c) \quad & \because a^2 + b^2 + \frac{1}{a^2} + \frac{1}{b^2} = 4 \\
 \Rightarrow \left(a^2 - 2 + \frac{1}{a^2}\right) + \left(b^2 - 2 + \frac{1}{b^2}\right) &= 0 \\
 \Rightarrow \left(a - \frac{1}{a}\right)^2 + \left(b - \frac{1}{b}\right)^2 &= 0 \\
 \because \text{Square quantities are always positive.} \\
 \therefore \left(a - \frac{1}{a}\right) = 0 \Rightarrow a^2 = 1 \text{ and } \left(b - \frac{1}{b}\right) &= 0. \\
 \Rightarrow b^2 = 1 \\
 \therefore a^2 + b^2 &= 1 + 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 128. (d) \quad & \therefore \left(x + \frac{1}{x}\right)^2 = 3 \\
 \Rightarrow x + \frac{1}{x} &= \sqrt{3} \\
 \Rightarrow \left(x + \frac{1}{x}\right)^3 &= (\sqrt{3})^3 \\
 \Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) &= 3\sqrt{3} \\
 \Rightarrow x^3 + \frac{1}{x^3} + 3(\sqrt{3}) &= 3\sqrt{3} \\
 \therefore x^3 + \frac{1}{x^3} &= 3\sqrt{3} - 3\sqrt{3} = 0
 \end{aligned}$$

$$129. (a) \text{ Let } x = 0.1 \text{ and } y = 0.02$$

$$\begin{aligned}
 \text{Now, given expression} &= \frac{x^3 + y^3}{(2x)^3 + (2y)^3} \\
 &= \frac{x^3 + y^3}{8x^3 + 8y^3} = \frac{1}{8} = 0.125
 \end{aligned}$$

$$130. (a) \quad x + \frac{1}{x} = 2$$

The equation satisfies when we put $x = 1$

$$\text{Let, } x^{100} + \frac{1}{x^{100}} = k \text{ (for all values of } x)$$

Then, it will be same for $x = 1$

$$k = x^{100} + \frac{1}{x^{100}} = 1^{100} + \frac{1}{1^{100}} = 1 + 1 = 2$$

$$\begin{aligned}
 131. (c) \quad & \therefore x^3 + 3x^2 + 3x = 7 \\
 \Rightarrow x^3 + 3x^2 + 3x + 1 &= 8 \\
 \Rightarrow (x + 1)^3 &= (2)^3 \\
 \Rightarrow x + 1 &= 2 \\
 \therefore x &= 1
 \end{aligned}$$

$$132. (b) \quad \because 2x + \frac{2}{x} = 1$$

$$\Rightarrow x + \frac{1}{x} = \frac{1}{2}$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^3 = \left(\frac{1}{2}\right)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{1}{8}$$

$$\therefore x^3 + \frac{1}{x^3} = \frac{1}{8} - \frac{3}{2} = -\frac{11}{8}$$

$$\begin{aligned}
 133. (a) \quad & \therefore \sqrt{16 + 6\sqrt{7}} = \sqrt{9 + 7 + 2\sqrt{9 \times 7}} \\
 &= \sqrt{(\sqrt{9} + \sqrt{7})^2} = \sqrt{9} + \sqrt{7}
 \end{aligned}$$

$$\text{Similarly, } \sqrt{16 - 6\sqrt{7}} = \sqrt{9} - \sqrt{7}$$

$$\therefore \sqrt{16 + 6\sqrt{7}} - \sqrt{16 - 6\sqrt{7}}$$

$$= \sqrt{9} + \sqrt{7} - \sqrt{9} + \sqrt{7}$$

$$= 2\sqrt{7}$$

$$\therefore \text{Given expression} = \frac{\sqrt{7}}{2\sqrt{7}} = \frac{1}{2}$$

$$134. (c) \quad 2x + \frac{1}{3x} = 6 \Rightarrow \frac{6x^2 + 1}{3x} = 6$$

$$\Rightarrow \frac{6x^2 + 1}{3x} \times \frac{3}{2} = 6 \times \frac{3}{2}$$

$$\Rightarrow \frac{6x^2 + 1}{2x} = 9$$

$$\therefore 3x + \frac{1}{2x} = 9$$

$$135. (a) \because x = (\sqrt{2} - 1)^{-1/2}$$

$$\therefore x^2 = (\sqrt{2} - 1)^{-1} = \frac{1}{\sqrt{2} - 1}$$

$$\Rightarrow \frac{1}{x^2} = \frac{1}{(\sqrt{2} - 1)^{-1}} = \sqrt{2} - 1$$

$$\therefore x^2 - \frac{1}{x^2} = \frac{1}{\sqrt{2} - 1} - (\sqrt{2} - 1)$$

$$= \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} - (\sqrt{2} - 1)$$

$$= \sqrt{2} + 1 - \sqrt{2} + 1 = 2$$

$$136. (b) \text{ Given expression } = \frac{3}{4} \times \frac{4}{3} \times \frac{5}{3} \times \frac{3}{5} \times \frac{13}{7} \times \frac{1}{13} = \frac{1}{7}$$

$$137. (c) \text{ Let } 0.87 = a \text{ and } 0.13 = b$$

$$\text{We have, } \frac{a^3 + b^3}{a^2 + b^2 - ab} = a + b$$

$$[\because a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$$

$$\therefore \text{ Given expression } = 0.87 + 0.13 = 1$$

$$138. (d) \therefore x^2 + y^2 - 2x + 6y + 10 = 0$$

$$\Rightarrow x^2 - 2x + 1 + y^2 + 6y + 9 = 0$$

$$\Rightarrow (x - 1)^2 + (y + 3)^2 = 0$$

The value of both $(x - 1)^2$ and $(y + 3)^2$ are square numbers.

They cannot be negative and since when two numbers are positive, on adding we cannot get zero.

\therefore Both have to be zero

$$\Rightarrow (x - 1)^2 = 0$$

$$\Rightarrow x = 1$$

$$\Rightarrow (y + 3)^2 = 0$$

$$\Rightarrow y = -3$$

$$\therefore x^2 + y^2 = 10$$

$$139. (a) ? = 21 + 3.9 \times 2.9 + 8.99 \approx 21 + 4 \times 3$$

$$= 21 + 12 + 9 = 42$$

$$140. (b) 22.9889 \div ? = 23$$

$$\text{or, } \frac{23}{?} = 23 \text{ or, } ? = \frac{23}{23} = 1$$

$$141. (b) 10^7 + 10^7 = 10^3 \times 100^3 + 999999999$$

$$= 10^3 \times 10^6 + 1000000000$$

$$= 10^9 + 10^9$$

$$\text{or, } 2 \times 10^9 = 2 \times 10^9$$

$$\therefore ? = 9$$

$$142. (d) ? \times 116 = 4003 \times 77 - 21015$$

$$\text{or, } ? \times 116 = 308231 - 21015 = 287216$$

$$\text{or, } ? \times 116 = 287216$$

$$\therefore ? = \frac{287216}{116} = 2476$$

$$143. (b) ? = (4444 \div 40) + (645 \div 25) + (3991 \div 26)$$

$$= \frac{4440}{40} + \frac{645}{25} + \frac{3991}{26}$$

$$= 111.1 + 25.8 + 153.5 = 290.4$$

$$144. (b) ? = 5 \frac{17}{37} \times 4 \frac{51}{52} \times 11 \frac{1}{7} + 2 \frac{3}{4}$$

$$= \frac{202}{37} \times \frac{259}{52} \times \frac{78}{7} + \frac{11}{4} = \frac{202}{37} \times \frac{259}{7} \times \frac{3}{2} + \frac{11}{4}$$

$$= 101 \times 3 + \frac{11}{4} = 303 + \frac{11}{4} = \frac{1212 + 11}{4}$$

$$= \frac{1223}{4} = 305.75$$

$$145. (c) ? = \frac{5}{8} \times 4011.33 + \frac{7}{10} \times 3411.22$$

$$= \frac{20056.65}{8} + \frac{23878.54}{10}$$

$$= 2507.08 + 2387.854 = 2507 + 2387 = 4894 \approx 4890$$

$$146. (a) ? = 335.01 \times 244.99 \div 55$$

$$= 335 \times 245 \div 55$$

$$= 335 \times \frac{245}{55} = \frac{82075}{55} = 1422.27 \approx 1490$$

$$147. (a) 3463 \times 295 - 18611 = ? + 5883$$

$$\Rightarrow ? = 1021585 - 18611 - 5883 = 997091$$

$$148. (c) 533.61 + 2361.96 - 1584.04 = ? + 1147.69$$

$$\text{or, } ? = 1311.53 - 1147.69 = 163.84$$

$$149. (d) \frac{28}{65} \times \frac{195}{308} \times \frac{44}{39} + \frac{5}{26} = \frac{4}{13} \times \frac{5}{26} = \frac{8+5}{26} = \frac{13}{26} = \frac{1}{2}$$

$$150. (a) 43931 \div 2111 \times 401 = ?$$

$$\text{or, } ? = 44000 \div 2000 \times 400$$

$$\text{or, } ? = \frac{44000}{2000} \times 400 = 8800$$

$$151. (c) 60 \div 12 \times 6 = 30$$

$$152. (b) \frac{1}{8} \times \frac{2}{3} \times \frac{3}{5} \times 1715 = 85.75 \approx 85$$

$$153. (c) 25 \times 124 + 389 \times 15 = 3100 + 5835 = 8935$$

$$154. (a) \frac{561}{35} \times 20 = 320.5 \approx 320$$

$$155. (d) 441 - \frac{3717}{59} = ? \times 8$$

$$\Rightarrow \frac{441 - 63}{8} = ?$$

$$\therefore ? = 47.25$$

$$156. (d) ? = 2 \frac{1}{8} - 1 \frac{1}{16} - 1 \frac{1}{32} + 1 \frac{9}{64}$$

$$= 2 - 1 - 1 + 1 + \frac{1}{8} - \frac{1}{16} - \frac{1}{32} + \frac{9}{64}$$

$$= 1 + \frac{8 - 4 - 2 + 9}{64} = 1 + \frac{11}{64} = 1 \frac{11}{64}$$

4.42 Chapter 4

$$157. (c) [(0.8)^2]^4 \cdot [(0.8)^3]^3 \times (0.8) = (0.8)^{2+3} (0.8)^{8-9+4} = (0.8)^{7+3}$$

$$3 = ? + 3$$

$$\therefore ? = 0$$

$$158. (e) ? = \sqrt{225 \times \frac{12}{9} - 125 + 21} = \sqrt{300 - 125 + 21} = \sqrt{196} = 14$$

$$159. (b) \frac{7441}{34} \times 12 - 110 = ? \times 9$$

$$\Rightarrow \frac{2626.23 - 110}{9} = ?$$

$$? = 279.5 \approx 280$$

$$160. (c) ? = \frac{989}{34} \times \frac{869}{65} \times \frac{515}{207} = 967.52 \approx 970$$

$$161. (e) ? = (32)^2 + (24)^2 - (17)^2$$

$$\Rightarrow ? = 1024 + 576 - 289$$

$$\Rightarrow ? = 1311 \approx 1310$$

$$162. (c) ? = (292 + 60 - 345) \times 31$$

$$= (352 - 345) \times 31$$

$$= 7 \times 31$$

$$= 217$$

$$163. (b) ? = (3 + 4 - 3) + \left(\frac{3}{4} + \frac{2}{5} - \frac{1}{8} \right)$$

$$= 4 + \frac{30 + 16 - 5}{8 \times 5} = 4 + \frac{41}{40} = 5 \frac{1}{40}$$

$$164. (a) \frac{343 \times 49}{216 \times 16 \times 81} = \frac{7^3 \times 7^2}{6^3 \times (4 \times 9)^2} = \frac{7^5}{6^3 \times 6^4} = \frac{7^5}{6^7}$$

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