Set Theory

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INTRODUCTION

The concept of set is fundamental in all branches of mathematics. Sets are the most basic tools of mathematics which are extensively used in developing the foundations of relations and functions, logic theory, sequences and series, geometry, probability theory, etc. In fact, these days most of the concepts and results in mathematics are expressed in the set theoretic language.

The modern theory of sets was developed by the German mathematician Georg Cantor (1845–1918AD). In this chapter, we will study some basic definitions and operations involving sets. We will also discuss the applications of sets.

SET

We observe that in nature, varieties of objects occur in *groups*. These groups are given different names such as, a *collection* of books, a *bunch* of keys, a *herd* of cattle, an *aggregate* of points, etc., depending on the characteristic of objects they represent. In literal sense, all these works have the same meaning. (i.e., a group or a collection). In mathematical language, we call this collection of objects, a *set*. From the above examples, it can be seen that each collection has a well-defined property (characteristic) of its own.

Thus, a *set is a well-defined collection of objects*. When we say well defined, we mean that the objects follow a given rule or rules. With the help of this rule, we will be able to say whether any given object belongs to this set or not. For example, if we say that we have a collection of short students in a class, this collection is not a set as 'short students', is not well defined. However, if we say that we have a collection of students whose height is less than 5 feet, then it represents a set.

It is not necessary that a set may consist of same type of objects, For example, a book, a cup and a plate lying on a table may also form a set, their common property being that they form a collection of objects lying on the table.

Illustration 1: Some other examples of sets are:

(i) The set of numbers 1, 3, 5, 7, 9, 14.
(ii) The set of vowels in the alphabets of English.
(iii) The set of rivers in India.
(iv) The set of all planets.
(v) The set of points on a circle.
(vi) The set of mathematics books in your library.
(vii) The set of even positive integers (i.e., 2, 4, 6, 8, ···).
(viii) The set of multiples of 4 (i.e., 4, 8, 12, ···).
(ix) The set of factors of 12. (i.e., 1, 2, 3, 4, 6, 12).
(x) The set of integers less than zero (i.e., –1, –2, –3, ···).

Notations

Sets are usually denoted by capital letters A, B, C, etc., and their elements by small letters a, b, c, etc.

Let, A be any set of objects and let 'a' be a member of A, then we write $a \in A$ and read it as 'a belongs to A' or 'a is an element of A' or 'a is a member of A'. If a is not an object of A, then we write $a \notin A$ and read it as 'a does not belong to A' or 'a is not an element of A'.

REPRESENTATION OF SETS

There are two ways of expressing a set. These are

1. Tabular form or roster form.
2. Set-builder form or rule method.

Tabular Form or Roster Form

In this method, we list all the members of the set separating them by means of commas and enclosing them in curly brackets {}.

Illustration 2: Let, A be the set consisting of the numbers 1, 3, 4 and 5, then we write $A = \{1, 3, 4, 5\}$.

Notes:

- The order of writing the elements of a set is immaterial. For example, {1, 3, 5}, {3, 1, 5}, {5, 3, 1} all denote the same set.

- An element of a set is not written more than once. Thus, the set {1, 5, 1, 3, 4, 1, 4, 5} must be written as {1, 3, 4, 5}.

Set Builder Form or Rule Method

In this method, instead of listing all elements of a set, we write the set by some special property or properties satisfied by all its elements and write it as

$A = \{x : P(x)\}$ or, $A = \{x \mid x$ has the property $P(x)\}$ and read it as "A is the set of all elements x such that x has the property P". The symbol ':' or '|' stands for 'such that'.

Illustration 3: Let, A be the set consisting of the elements 2, 3, 4, 5, 6, 7, 8, 9, 10. Then, the set A can be written as $A = \{x : 2 \le x \le 10$ and x e $N\}$.

FINITE AND INFINITE SETS

Finite Set

A set having no element or a definite number of elements is called a *finite set*. Thus, in a finite set, either there is nothing to be counted or the number of elements can be counted, one by one, with the counting process coming to an end.

Illustration 4: Each of the following sets is a finite set:

(i) A = the set of prime numbers less than 10
= {2, 3, 5, 7};
(ii) B = the set of vowels in English alphabets
= $\{a, e, i, o, u\}$;
(iii) $C = \{x \mid x$ is divisor of 50$\}$.

Cardinal Number of a Finite Set

The number of distinct elements in a finite set S is called the *cardinal number* of S and is denoted by $n(S)$.

Illustration 5: If $A = \{2, 4, 6, 8\}$ then $n(A) = 4$.

Infinite Set

A set having unlimited number of elements is called an *infinite set*. Thus, in an infinite set, if the elements are counted one by one, the counting process never comes to an end.

Illustration 6: Each of the following sets is an infinite set:

(i) the set of all natural numbers = {1, 2, 3, 4, ⋯).
(ii) the set of all prime numbers = {2, 3, 5, 7, ⋯).
(iii) the set of all points on a given line.
(iv) the set of all lines in a given plane.
(v) $\{x \mid x$ ε R and $0 < x < 1\}$.

EMPTY SET (OR NULL SET)

The set which contains no element is called the *empty set* or the *null set* or *void set*.

The symbol for the empty set or the null set is ϕ. Thus, $\phi = \{\}$, since there is no element in the empty set.

The empty set is a finite set.

Since any object x which is not equal to itself does not exist, the set $A = \{x : x \ne x\}$ is the empty set ϕ.

A set which is not empty, i.e., which has at least one element is called a *non-empty* set or a *non-void* set.

Illustration 7:

(i) The set of natural numbers less than 1 is an empty set.
(ii) the set of odd numbers divisible by 2 is a null set.
(iii) $\{x \mid x \in Z$ and $x^2 = 2\} = \phi$, because there is no integer whose square is 2.
(iv) $\{x \mid x \in R$ and $x^2 = -1\} = \phi$, because the square of a real number is never negative.
(v) $\{x \mid x \in N, 4 < x < 5\}$ is the empty set.
(vi) $\{x \mid x \in Z, -1 < x < 0\}$ is the null set.

The empty set should not be confused with the set {0}. It is the set containing one element, namely 0.

SINGLETON

A set containing only one element is called a *singleton*.

Illustration 8:

(i) The set {0} is a singleton since it has only one element 0.
(ii) The set of even prime numbers is the set {2} which is a singleton.
(iii) $\{x \mid x$ is an integer and $-1 < x < 1\} = \{0\}$ is a singleton.

EQUAL SETS

Two sets A and B are said to be *equal* if they have the same elements and we write $A = B$. Thus, $A = B$ if every element of A is an element of B and every element of B is an element of A.

In symbols, $A = B$ iff $x \in A \Rightarrow x \in B$ and $x \in B \Rightarrow x \in A$. To indicate that two sets A and B are not equal, we will write $A \ne B$.

Illustration 9:

(i) If $A = \{2, 3, 4\}$ and $B = \{x \mid 1 < x < 5, x \in N\}$ then $A = B$.
(ii) If A = the set of letters in the word 'WOLF' and B = the set of letters in the word 'FOLLOW' then $A = B$ as each = {W, O, L, F}, remembering that in a set the repetition of elements is meaningless and order of elements is immaterial.

EQUIVALENT SETS

Two finite sets A and B are said to be *equivalent* if they have the same number of elements, i.e., if we can find a one-to-one correspondence between the elements of the two sets.

The symbol ‘ ~ ’ is used to denote equivalence. Thus, $A \sim B$ is read as “A is equivalent to B”. Two finite sets A and B are equivalent if $n(A) = n(B)$, i.e., if they have the same cardinal number.

Equivalent sets have the same number of elements, not necessarily the same elements. The elements in two equivalent sets may or may not be the same. Thus, equal sets are always equivalent but equivalent sets may or may not be equal.

Illustration 10:

(i) If $A = \{1, 2, 3\}$ and $B = \{2, 4, 6)$ then $A \sim B$.
(ii) If $A = \{a, b, c, d\}$ and $B = \{p, q, r, s\}$ then $A \sim B$.
(iii) If $A = \{3, 5, 7, 9\}$ and $B = \{9, 7, 5, 3\}$ then $A \sim B$.

Also, since A and B have same elements, $\therefore A = B$.

SUBSET OF A SET

If A and B are any two sets, then B is called a *subset* of A if every element of B is also an element of A. Symbolically, we write it as $B \subseteq A$ or $A \supseteq B$

(i) $B \subseteq A$ is read as B is contained in A or B is a subset of.
(ii) $A \supseteq B$ is read as A contains B or A is *super set* of B.

Illustration 11:

(i) The set $A = \{2, 4, 6\}$ is a subset of $B = \{1, 2, 3, 4, 5, 6\}$, since each number 2, 4 and 6 belonging to A, also belongs to B.
(ii) The set $A = \{1, 3, 5\}$ is not a subset of $B = \{1, 2, 3, 4\}$ since $5 \in A$ but $5 \notin B$.
(iii) The set of real numbers is a subset of the set of complex numbers. The set of rational numbers is a subset of the set of real numbers. The set of integers is a subset of the set of rational numbers. Finally, the set of natural numbers is a subset of the set of integers. Symbolically,

$$N \subseteq Z \subseteq Q \subseteq R \subseteq C.$$

Notes:

- If we are to prove that $A \subseteq B$, then we should prove that $x \in A \Rightarrow x \in B$. Symbolically, $A \subseteq B$ if and only if $x \in A \Rightarrow x \in B$.
- If we are to prove that A B, then we should prove that there exists at least one element x such that $x \in A$ but $x \notin B$. Symbolically, A B if and only if there exists $x \in A$ such that $x \notin B$.

Proper Subsets of a Set

A set B is said to be a *proper subset* of the set A if every element of set B is an element of A whereas every element of A is not an element of B.

We write it as $B \subset A$ and read it as “B is a proper subset of A”.

Thus, B is a proper subset of A if every element of B is an element of A and there is at least one element in A which is not in B.

Illustration 12:

(i) If $A = \{1, 2, 5\}$ and $B = \{1, 2, 3, 4, 5\}$. Then A is a proper subset of B.
(ii) The set N of all natural numbers is a proper subset of the set Z of all integers because every natural number is an integer, i.e., $N \subset Z$ but every integer need not be a natural number, i.e., $N \neq Z$.

Note:

If we are to prove that $B \subset A$, then we should prove that $B \subseteq A$ and there exists an element of A which is not in B. Symbolically, $B \subset A$ if and only if $B \subseteq A$ and there exists $x \in A$ such that $x \notin B$.

POWER SET

Elements of a set can also be some sets. Such sets are called *set of sets*. For example, the set $\{\phi, \{1\}, \{2\}, \{3, 4\}\}$ is a set whose elements are the sets ϕ, $\{1\}$, $\{2\}$, $\{3, 4\}$.

The set of all the subsets of a given set A is called the *power set* of A and is denoted by $P(A)$.

Illustration 13:

(i) If $A = \{a\}$, then $P(A) = \{\phi, A\}$.
(ii) If $B = \{2, 5\}$, then $P(B) = \{\phi, \{2\}, \{5\}, B\}$.
(iii) If $S = \{a, b, c\}$, then $P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, S\}$.

Notes:

- Every set is subset of itself.
- Empty set is the subset of every set.
- If a set has n elements, then the number of its subsets is 2^n.

COMPARABLE SETS

If two sets A and B are such that either $A \subset B$ or $B \subset A$, then A and B are said to be *comparable sets*. If neither $A \subset B$ nor $B \subset A$, then A and B are said to be *non-comparable sets*.

Illustration 14:

(i) If $A = \{1, 3, 5\}$ and $B = \{1, 2, 3, 4, 5\}$, then A and B are comparable sets because $A \subset B$.
(ii) If $A = B$, then A and B are comparable sets.

UNIVERSAL SET

If in any discussion on set theory, all the given sets are subsets of a set U, then the set U is called the *universal set*.

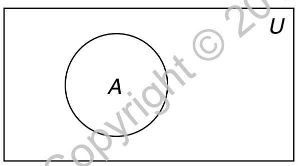
Illustration 15: Let, $A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$, $C = \{3, 5, 7, 11\}$, $D = \{2, 4, 8, 16\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 16\}$ be the given sets. Here the sets A, B, C, D are subsets of the set U. Hence U can be taken as the *universal set*.

VENN DIAGRAMS

In order to visualize and illustrate any property or theorem relating to universal sets, their subsets and certain operations on sets, Venn, a British mathematician developed what are called *Venn diagrams*. He represented a universal set by interior of a rectangle and other sets or subsets by interiors of circles.

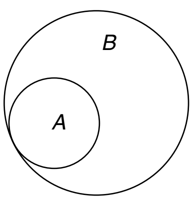
Examples of Certain Relationships Between Sets by Venn Diagrams

1. If U be a set of letters of English alphabets and A, a set of vowels, then $A \subset U$. This relationship is illustrated by Fig. (a).

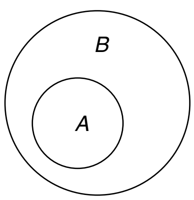


(a)

2. If $A \subset B$ and $A \neq B$, then A and B can be represented by either of the diagrams [Figure (b) and Figure (c)].

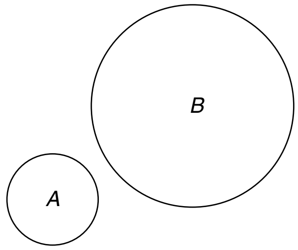


(b)

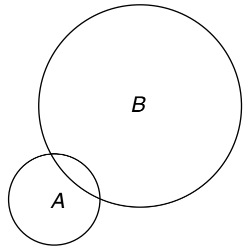


(c)

3. If the sets A and B are not comparable, then neither of A or B is a subset of the other. This fact can be represented by either of the diagrams [Figure (d) and Figure (e)].

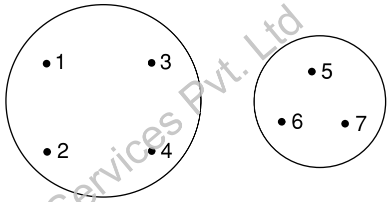


(d)



(e)

4. If $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7\}$, then A and B are disjoint. These can be illustrated by Venn diagram given in Fig. (f).



(f)

COMPLEMENT OF A SET

Let, $A \subset U$ (i.e., A is a proper subset of universal set U). Evidently, U consists of all the elements of A together with some elements which are not in A. Let us now constitute another set consisting of all the elements of U not in A. Naturally, it will form another proper subset of U. We call this subset *the complement of the subset A in U* and denote it by A' or by A^c i.e., $A^c = \{x : x \in U, x \notin A\}$.

Thus, the complement of a given set is a set which contains all those members of the universal set that do not belong to the given set.

Illustration of A′ by Venn Diagram

Let, A be a subset of the universal set U. The shaded area in figure below represents the set A' which consists of those elements of U which are not in A.

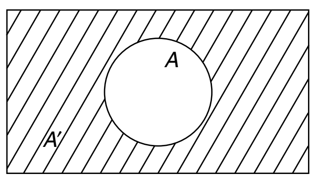


Illustration 16:

(i) If the universal set is $\{a, b, c, d\}$ and $A = \{a, b, d\}$ then $A' = \{c\}$.
(ii) If the universal set $U = \{1, 2, 3, 4, 5, 6\}$ and $A = \{2, 4, 6\}$, then $A' = \{1, 3, 5\}$.
(iii) If $U = N$ and $A = O$ (the set of odd natural numbers), then $A' = E$ (the set of even natural numbers).

(iv) If $U = I$, $A = N$,
then $A' = \{0, -1, -2, -3 \cdots\}$.
(v) If $U = \{1, 2, 3, 4\}$, $A = \{1, 2, 3, 4\}$,
then $A' = \phi$.

Note:

(i) Since $A \subset A$, we get $A' = \phi$.
(ii) $(A')' = A$, i.e., complement of the complement of a set is the set itself.

Operations on Sets

(a) Union of Sets

Let A and B be two given sets. Then the union of A and B is the set of all those elements which belong to either A or B or both.

The union of A and B is denoted by $A \cup B$ and is read as A *union* B. The symbol $\cup$ stands for union. It is evident that union is 'either, or' idea. Symbolically,

$$A \cup B = \{x : \text{either } x \in A \text{ or } x \in B\}.$$

Note:

The union set contains all the elements of A and B, except that the common elements of both A and B are exhibited only once.

Illustration of A ∪ B by Venn Diagram

Let A and B be any two sets contained in a universal set U. Then $A \cup B$ is indicated by the shaded area in the figure below.

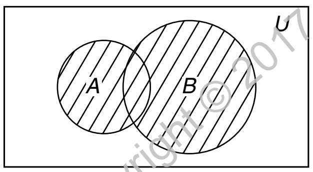


Illustration 17:

(i) Let, $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 6, 7, 9\}$, then,
$A \cup B = \{1, 2, 3, 4, 6, 7, 9\}$.
(ii) If A = O (set of odd natural numbers), $B = E$ (set of even natural numbers), then $A \cup B = N$.
(iii) If A is the set of rational numbers and B the set of irrational numbers, then $A \cup B = R$.
(iv) If $A = \{x : x^2 = 4, x \in I\} = \{2, -2\}$, $B = \{y : y^2 = 9, y \in I\} = \{3, -3\}$, then
$A \cup B = \{-3, -2, 2, 3\}$.
(v) If $A = \{x : 1 < x < 5, x \in N\} = \{2, 3, 4\}$, $B = \{y : 3 < y < 7, y \in N\} = \{4, 5, 6\}$, then $A \cup B = \{2, 3, 4, 5, 6\}$.

Note:

From the definition of the union of two sets A and B, it is clear that

- $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$
- $x \notin A \cup B \Leftrightarrow x \notin A$ and $x \notin B$
- $A \subseteq A \cup B$ and $B \subseteq A \cup B$.

(b) Intersection of Sets

Let A and B be two given sets. Then the intersection of A and B is the set of elements which belong to both A and B. In other words, the intersection of A and B is the set of common members of A and B.

The intersection of A and B is denoted by $A \cap B$ and is read as A *intersection* B. The symbol $\cap$ stands for intersection.

It is evident that intersection is an 'and' idea. Symbolically,

$$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

Note:

From the definition of the intersection of two sets A and B, it is clear that

- $x \in A \cap B \Leftrightarrow x \in A$ and $x \in B$
- $x \notin A \cap B \Leftrightarrow x \notin A$ or $x \notin B$
- $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

Let A and B be any two sets contained in the universal set U. Then $A \cap B$ is indicated by the shaded area, as shown in the figure below.

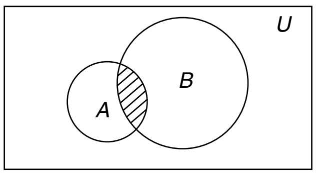
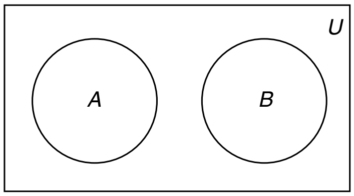


Illustration 18:

(i) If $A = \{1, 2, 3, 6, 9, 18\}$,
and $B = \{1, 2, 3, 4, 6, 8, 12, 24\}$,
then $A \cap B = \{1, 2, 3, 6\}$.
(ii) If A is the set of odd natural numbers and B is the set of even natural numbers, then $A \cap B = \phi$.
[*Intersection of two disjoint sets is empty set*]
(iii) If A and B are sets of points on two distinct concentric circles, then
$A \cap B = \phi$.
(iv) If $A = \{x : 1 < x < 6, x \in N\} = \{2, 3, 4, 5\}$,
$B = \{y : 2 < y < 9, y \in N\}$
$= \{3, 4, 5, 6, 7, 8\}$
then, $A \cap B = \{3, 4, 5\}$.

Disjoint Sets

If $A \cap B = \phi$, then A and B are said to be *disjoint sets*. For example, let $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$. Then, A and B are disjoint sets because there is no element which is common to both A and B. The disjoint sets can be represented by Venn diagram as shown in the figure below.



(c) Difference of Sets

Let A and B be two given sets. The difference of sets A and B is the set of elements which are in A but not in B. It is written as $A - B$ and read as A *difference* B. Symbolically,

$$A - B = \{x : x \in A \text{ and } x \notin B\}.$$

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.

Caution: In general, $A - B \neq B - A$.

Illustration 19:

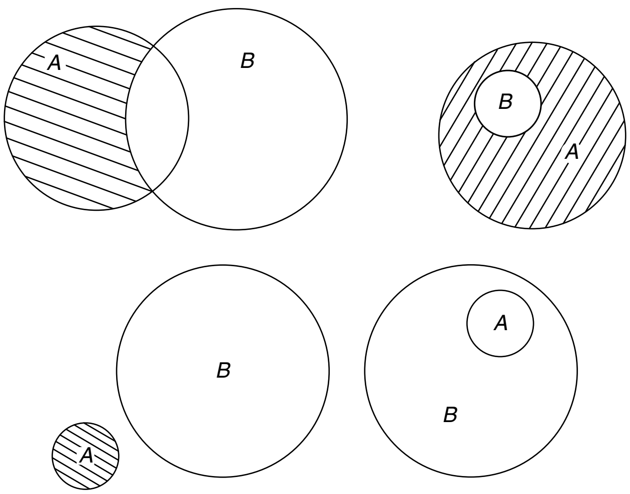
(i) If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 7, 9\}$, then
$A - B = \{1, 3\}$ and $B - A = \{7, 9\}$.
Hence $A - B \neq B - A$.

(ii) If $A = \{12, 15, 17, 20, 21\}$,
$B = \{12, 14, 16, 18, 21\}$
and $C = \{15, 17, 18, 22\}$, then

$$A - B = \{15, 17, 20\}$$
$$B - C = \{12, 14, 16, 21\}$$
$$C - A = \{18, 22\}$$
$$B - A = \{14, 16, 18\}$$
$$A - A = \phi.$$

Illustration of A − B by Venn Diagrams

In the four cases shown by the diagrams below, $A - B$ is given by shaded area.



Applications of Sets

1. If a set S has only a finite number of elements, we denote by $n(S)$ the number of elements of S.

Illustration 20: If $U = \{1, 2, 3, 4, 5\}$, then $n(U) = 5$.

2. For any two sets A and B, with finite number of elements, we have the following formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

3. If A and B are disjoint sets, then

$$n(A \cup B) = n(A) + n(B).$$

Illustration 21: X and Y are two sets such that

$$n(X) = 17,\ n(Y) = 23,\ n(X \cup Y) = 38,$$

find $n(X \cap Y)$.

Solution: $n(X) = 17$, $n(Y) = 23$, $n(X \cup Y) = 38$,
$n(X \cap Y) = ?$
Now, $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$.
Then, $38 = 17 + 23 - n(X \cap Y)$
$\Rightarrow n(X \cap Y) = 17 + 23 - 38 = 2$.

ORDERED PAIR

Let A and B be two non-empty sets. If $a \in A$ and $b \in B$, an element of the form (a, b) is called an *ordered pair*, where 'a' is regarded as 'the first element' and 'b' as the second element. It is evident from the definition that

1. $(a, b) \neq (b, a)$
2. $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$.

Equality of two ordered pairs. Two ordered pairs (a, b) and (c, d) are said to be equal if and only if $a = c$ and $b = d$. The ordered pairs (2, 4) and (2, 4) are equal while the ordered pairs (2, 4) and (4, 2) are different. The distinction between the set {2, 4} and the ordered pair (2, 4) must be noted carefully. We have $\{2, 4\} = \{4, 2\}$ but $(2, 4) \neq (4, 2)$.

CARTESIAN PRODUCT OF SETS

Let, A and B be two non-empty sets. The cartesian product of A and B is denoted by $A \times B$ (read as '*A cross B*') and is defined as the set of all ordered pairs (a, b), where $a\ \varepsilon\ A$ and $b\ \varepsilon\ B$. Symbolically,

$$A \times B = \{(a, b) : a\ \varepsilon\ A \text{ and } b\ \varepsilon\ B\}$$

Illustration 22: Suppose, $A = \{2, 4, 6\}$ and $B = \{x, y\}$ Then,

$$A \times B = \{(2, x), (4, x), (6, x), (2, y), (4, y), (6, y)\}$$
$$B \times A = \{(x, 2), (x, 4), (x, 6), (y, 2), (y, 4), (y, 6)\}$$

Thus, we note that if $A \neq B$, then $A \times B \neq B \times A$.

Illustration 23: Let, $A = \{1, 2, 3\}$ and $B = \phi$. Then, $A \times B = \phi$, as there will be no ordered pair belonging to $A \times B$. Thus, we note that $A \times B = \phi$ if A or B or both of A and B are empty sets.

Illustration 24: Let, $n(A)$ represents the number of elements in set A. In Illustration 22, we can see that $n(A) = 3$, $n(B) = 2$ and $n(A \times B) = 6$. Thus, we note that $n(A \times B = n(A) \times n\ (B)$.

In other works, if a set A has m elements and a set B has n elements, then $A \times B$ has mn elements. Further, it may be noted that $n(A \times B) = n(B \times A)$. This implies that $A \times B$ and $B \times A$ are equivalent sets.

Illustration 25: If there are three sets A, B, C and $a \in A$, $b \in B$, $c \in C$, we form an ordered triplet (a, b, c). The set of all ordered triplets (a, b, c) is called the *cartesian product* of the sets A, B, C. That is,

$$A \times B \times C = \{(a, b, c) : a \in A, b \in B, c \in C\}.$$

Exercise-1

1. Which of the following sets is non-empty?

(a) A = set of odd natural numbers divisible by 2
(b) $B = \{x : x + 5 = 0, x \in N\}$
(c) C = set of even prime numbers
(d) $D = \{x : 1 < x < 2, x \in N\}$,

2. Which of the following sets is finite?

(a) $I = \{x : x \in Z \text{ and } x^2 - 2x - 3 = 0\}$
(b) B = The set of natural numbers which are divisible by 2
(c) C = The set of lines passing through a point.
(d) $D = \{x : x \in Z \text{ and } x > -5\}$

3. Which of the following pairs of sets are not equal?

(a) $A = \{1, 3, 3, 1\}$, $B = \{1, 4\}$
(b) $A = \{x : x + 2 = 2\}$, $B = \{0\}$
(c) $A = \{1, 3, 4, 4\}$, $B = \{3, 1, 4\}$
(d) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$, $B = \left\{\frac{1}{n} : n \in N\right\}$.

4. Which of the following sets is empty?

(a) $A = \{x : x \in n \text{ and } x \le 1\}$
(b) $B = \{x : 3x + 1 = 0, x \in N\}$
(c) $C = \{x : x \text{ is an integer and } -1 < x < 1\}$
(d) D = set of months of the year beginning with F.

5. Which of the following sets is infinite?

(a) $\{x : x$ is a prime number, x is even$\}$
(b) Set of all river in India
(c) Set of all concentric circles

6. Which of the following sets is finite?

(a) The set of the months of a year
(b) $\{1, 2, 3, \cdots\}$
(c) $\{1, 2, 3, \cdots 90, 100\}$
(d) The set of lines which are parallel to x-axis
(e) The set of numbers which are multiples of 5.

7. Which of the following pairs of sets are not equivalent?

(a) $A = \{2, 4, 6, 8\}$, $B = \{u, v, w, x\}$
(b) $A = \{a, b, c\}$, $B = \{\alpha, \beta, \gamma, \delta, \nu\}$
(c) $A = \{\}$, $B = \phi$
(d) $A = \{x : x = 2n, n \in N\}$, $B = \{x : x = 2n + 1, n \in N\}$.

8. Find the cardinal number of the following set: $\{x : x$ is a letter of the word 'ASSASSINATION'$\}$

(a) 4 (b) 6
(c) 8 (d) 2

9. Find the cardinal number of the following set $\{x : x$ is a natural number ≤ 30 and is divisible by 7 or 11$\}$

(a) 4 (b) 8
(c) 6 (d) 2

10. Find the cardinal number of the following set $\{x : x = 2n, n \in N, 4 \le x \le 11\}$

(a) 8 (b) 6
(c) 12 (d) 4

11. Which of the following sets is finite?

(a) $\{x : x \in N$ and x is a prime number$\}$
(b) $\{x : x$ is a quadrilateral on a plane$\}$
(c) $\{x : x \in N \text{ and } x^2 - 25 \le 0\}$
(d) $\{x : x \in N$ and x is a multiple of 3$\}$.

12. For which of the following cases A and B are equivalent?

(a) $A = \{a, b, c, \cdots, z\}$, $B = \{1, 2, 3, \cdots, 24\}$
(b) $A = \left\{\frac{1}{3}, \frac{1}{2}, \frac{3}{5}\right\}$, $B = \left\{x : x = \frac{n}{n+2},\ n \in N\right\}$
(c) $A = \{2, 4, 6\}$, $B = \{(2, 4), (4, 6), (2, 6)\}$
(d) $A = \left\{x : x = \frac{n^3 - 1}{n^3 + 1}, n \in W, n \le 3\right\}$, $B = \left\{0, \frac{7}{9}, \frac{13}{14}\right\}$.

13. In which of the following cases, $A = B$?

(a) $A = \{12, 14, 16\}$, $B = \{16, 18, 20\}$
(b) $A = \phi$, $b = \{\}$
(c) $A = \{x : x \in$ W and $x < 1\}$, $B = \phi$
(d) $A = \{x : x$ is a day of the week beginning with $S\}$, $B = \{$Sunday$\}$.

14. In a class, 50 students play cricket, 20 students play football and 10 play both cricket and football. How many play at least one of these two games?

(a) 60 (b) 45
(c) 55 (d) 65

15. Write down the power set of the set $\{0\}$.

(a) ϕ (b) $\{0\}$
(c) $\{\phi\}$ (d) $\{\phi, \{0\}\}$

16. Find the power set of $A = \{\{a, b\}, c\}$.

(a) $\{\phi, \{a, b\}, \{c\}\}$
(b) $\{A, \{a, b\}, \{c\}\}$
(c) $\{\phi, A, \{a, b\}, \{c\}\}$
(d) None of these

17. Which of the following pairs of sets are comparable?

(a) $A = \{1, 3, 5\}$, and $B = \{3, 2, 5, 6\}$
(b) $A = \{x : x \in N$ and $x \leq 10\}$ and $B = \{1, 2, 3, \cdots, 10, 11\}$
(c) $A = \{1, 2, 3, \{4, 5\}\}$, and $B = \{1, 2, 3, 4, 5\}$.
(d) None of these

18. Let $A = \{\phi, \{\phi\}, 1, \{1, \phi\}, 7\}$. Which of the following is false?

(a) $\phi \in A$ (b) $\{\phi\} \in A$
(c) $\{1\} \in A$ (d) $\{7, \phi,\} \subset A$

19. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements is true?

(a) $\{3, 4\} \subset A$ (b) $\{3, 4\} \in A$
(c) $1 \subset A$ (d) $\{1, 2, 5\} \in A$

20. Let $A = \{1, 3, 5\}$ and $B = \{x : x$ is an odd natural number $< 6\}$. Which of the following is false?

(a) $A \subset B$ (b) $B \subset A$
(c) $A = B$ (d) None of these

21. Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are true?

(a) $\{3, 4\} \subset A$; (b) $\{3, 4\} \in A$;
(c) $\{\{3, 4\}\} \not\subset A$; (d) $\{1, 3, 5\} \subset A$.

22. Write down the power set of $A = \{8, 9\}$.

(a) $\{\phi, \{8\}, \{9\}, \{8, 9\}\}$
(b) $\{\phi, \{8\}, \{9\}\}$
(c) $\{\phi, \{8\}, \{9\}, \{8, 9\}\}$
(d) None of these

23. Write down the power set of $C = \{1, \{2\}\}$.

(a) $\{\phi, \{1\}, \{\{2\}\}\}$
(b) $\{\phi, \{1\}, \{\{2\}\}, \{1, \{2\}\}\}$
(c) $\{\{1\}, \{\{2\}\}, \{1, \{2\}\}$
(d) None of these

24. If $A = \left\{x : x = \dfrac{n-1}{n+1}, n \in W \text{ and } n \leq 10\right\}$; point out the correct statement from the following:

(a) $0 \in A$ (b) $0 \subset A$
(c) $0 \supset A$ (d) $\dfrac{1}{3} \notin A$

25. Which of the following statements is false for the sets A, B and C, where:

$A = \{x | x$ is letter of the word 'BOWL'$\}$
$B = \{x | x$ is a letter of the word 'ELBOW'$\}$
$C = \{x | x$ is a letter of the word 'BELLOW'$\}$
(a) $A \subset B$
(b) $B \supset C$
(c) $B = C$
(d) B is a proper subset of C.

26. Which of the following statements is true?

(a) Every subset of a finite set is finite.
(b) Every subset of an infinite set is infinite.
(c) Every subset of an infinite set is finite.
(d) A proper subset of a finite set is equivalent to the set itself.

27. Let $A = \{x : x \in N \wedge x$ is a multiple of $2\}$;
$B = \{x : x \in N \wedge x$ is a multiple of $5\}$;
$C = \{x : x \in N \wedge x$ is a multiple of $10\}$;
Describe the set $(A \cap B) \cap C$,
(a) A (b) B
(c) $A \cap B$ (d) C

28. Let $A = \{x : x \in N \wedge x$ is a multiple of $2\}$
$B = \{x : x \in N \wedge x$ is a multiple of $5\}$
$C = \{x : x \in N \wedge x$ is a multiple of $10\}$
Describe the set $A \cap (B \cup C)$
(a) A (b) B
(c) C (d) None of these

29. If $U = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $A = \{2, 4, 7\}$, $B = \{3, 5, 7, 9, 11\}$ and $C = \{7, 8, 9, 10, 11\}$, compute: $(A \cap U) \cap (B \cup C)$

(a) $\{7\}$ (b) $\{9\}$
(c) $\{6\}$ (d) $\{5\}$

30. If $U = \{a, b, c, d, e, f\}$, $A = \{a, b, c\}$, find $(U \cup A)'$.

(a) U (b) A
(c) ϕ (d) None of these

31. If $U = \{a, b, c, d, e, f\}$, $A = \{a, b, c\}$, $B = \{c, d, e, f\}$, and $C = \{c, d, e\}$ find $(A \cup B) \cup C$.

(a) A (b) B
(c) C (d) U

32. If $U = \{a, b, c, d, e, f\}$, $A = \{a, b, c\}$, $B = \{c, d, e, f\}$, $C = \{c, d, e\}$, find $(A \cap B) \cup (A \cap C)$.

(a) $\{c\}$ (b) $\{\alpha\}$
(c) $\{b\}$ (d) $\{d\}$

33. Which of the following pairs of sets are disjoint?

(i) $\{1, 2, 3, 4\}$ and $\{x : x$ is a natural number and $4 \leq x \leq 6\}$
(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$
(iii) $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$.

(a) (i) (b) (ii)
(c) (iii) (d) None of these

34. If $U = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, $A = \{3, 5, 7, 9, 11\}$ and $B = \{7, 8, 9, 10, 11\}$, Compute $(A - B)'$.

(a) $\{2, 3, 5, 7, 9, 11, 12\}$
(b) $\{2, 4, 6, 8, 10, 11, 12\}$
(c) $\{2, 4, 6, 8, 9, 10, 11\}$
(d) None of these

35. In a class of 100 students, the number of students passed in English only is 46, in Maths only is 46, in Commerce only is 58. The number who passed in English and Maths is 16, Maths and Commerce is 24 and English and Commerce is 26, and the number who passed in all the subjects is 7. Find the number of the students who failed in all the subjects.

(a) 9 (b) 8
(c) 10 (d) None of these

36. If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements, and Y has 15 elements, how many elements does $X \cap Y$ have?

(a) 5 (b) 7
(c) 9 (d) 11

37. If A and B are two sets such that A has 40 elements, $A \cup B$ has 60 elements and $A \cup B$ has 10 elements, how many elements does B have?

(a) 40 (b) 30
(c) 45 (d) 50

38. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

(a) 52 (b) 32
(c) 42 (d) None of these

39. In a group of 1000 people, there are 750 people who can speak Hindi and 400 who can speak English. How many can speak Hindi only?

(a) 600 (b) 650
(c) 750 (d) 800

40. In a class of 50 students, 35 opted for mathematics and 37 opted for Biology. How many have opted for both Mathematics and Biology? How many have opted for only Mathematics? (Assume that each student has to opt for at least one of the subjects).

(a) 15 (b) 17
(c) 13 (d) 19

41. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

(a) 19 (b) 17
(c) 23 (d) 21

42. In a town with a population of 5000, 3200 people are egg-eaters, 2500 meat eaters and 1500 eat both egg and meat. How many are pure vegetarians?

(a) 600 (b) 800
(c) 900 (d) 850

43. Let $A = \{1, 2\}$, $B = \{2, 3\}$. Evaluate $A \times B$.

(a) $\{(2, 1), (3, 1), (2, 3)\}$
(b) $\{(1, 2), (1, 3), (2, 3)\}$
(c) $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$
(d) None of these

44. If $A = \{a, b\}$, $B = \{2, 3, 5, 6, 7\}$ and $C = \{5, 6, 7, 8, 9\}$, find $A \times (B \cap C)$.

(a) A (b) ϕ
(c) $\{(5, a), (6, a), (7, a), (5, b), (6, b), (7, b)\}$
(d) $\{(a, 5), (a, 6), (a, 7), (b, 5), (b, 6), (b, 7)\}$

45. If $A = \{a, d\}$, $B = \{b, c, e\}$ and $C = \{b, c, f\}$, then $A \times (B \cup C) =$

(a) ϕ
(b) $(A \times B) \cap (A \times C)$
(c) $(A \times B) \cup (A \times C)$
(d) None of these

46. If $A = \{a, d\}$, $B = \{b, c, e\}$ and $C = \{b, c, f\}$, then $A \times (B \cap C) =$

(a) ϕ
(b) $(A \times B) \cap (A \times C)$
(c) $(A \times B) \cup (A \times C)$
(d) None of these

47. If $A = \{a, d\}$, $B = \{b, c, e\}$ and $C = \{b, c, f\}$, then $A \times (B - C) =$

(a) $(A \times B) - (A \times C)$ (b) $(A \times B)$
(c) $A \times C$ (d) None of these

48. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{1, 3, 4\}$ and $D = \{2, 4, 5\}$, then $(A \times B) \cap (C \times D) =$

(a) $(A \cap D) \times (B \cap C)$
(b) $(A \cap C) \times (B \cap D)$
(c) ϕ
(d) None of these

49. The catresian product $A \times A$ has 9 elements and two of them are $(-1, 0)$ and $(0, 1)$. Find the set A?

(a) $A = \{0, 1\}$ (b) $A = \{-1, 0, 1\}$
(c) $A = \{0, -1\}$ (d) $A = \{1, -1,\}$

50. If A and B have n elements in common, how many elements do $A \times B$ and $B \times A$ have in common?

(a) n (b) n^3
(c) n^2 (d) None of these

Exercise-2
(Based on Memory)

1. In an examination, 40% students failed in Hindi, 50% students failed in English. If 21% students failed in both the subjects, find the percentage of those who passed in Hindi.

(a) 31% (b) 40%
(c) 55% (d) 60%

[UPPCS, 2012]

2. In a group of 50 people, 35 speak Hindi, 25 speak both Hindi and English and all the people speak Hindi or English or both. The number of people who speak English only is:

(a) 40 (b) 20
(c) 15 (d) 10

[UPPCS, 2012]

3. In a certain office, 72% of the workers prefer cold drink and 44% prefer tea. If each of them prefers cold drink or tea and 40 like both, then the total number of workers in the office is:

(a) 40 (b) 240
(c) 220 (d) 210

[UPPCS, 2012]

4. In a survey of a town, it was found that 65% of the people surveyed watch the news on T.V. 40% read a newspaper and 25% read a newspaper and watch the news on T.V. What per cent of the people surveyed neither watch the news on T.V. nor read a newspaper?

(a) 5% (b) 10%
(c) 20% (d) 15%

[SSC (GL), 2011]

5. There are 80 families in a small extension area. 20 per cent of these families own a car each. 50 per cent of the remaining families own a motor cycle each. How many families in that extension do not own any vehicle?

(a) 30 (b) 32
(c) 23 (d) 36

[SSC (GL), 2011]

6. Out of 100 families in the neighbourhood, 50 have radios, 75 have TVs and 25 have VCRs. Only 10 families have all three and each VCR owner also has a TV. If some families have radio only, how many have only TV?

(a) 30 (b) 35
(c) 40 (d) 45

[SSC (GL), 2011]

7. If the number of items in a set A is $n(A) = 40$. If $n(B) = 26$ and $n(A \cap B) = 16$ then $n(A \cap B)$ is equal to:

(a) 30 (b) 40
(c) 50 (d) 60

[SSC, 2014]

8. $\sqrt{64009}$ is equal to:

(a) 352 (b) 523
(c) 253 (d) 532

[SSC, 2014]

9. 72% of the students of a certain class took Biology and 44% took Mathematics. If each student took at least one of Biology or Mathematics and 40 students took both of these subjects, the total number of students in the class is:

(a) 200 (b) 240
(c) 250 (d) 320

[SSC, 2010]

10. In an examination, 30% of the total students failed in Hindi, 45% failed in English and 20% failed in both the subjects. Find the percentage of those who passed in both the subjects.

(a) 35.7% (b) 35%
(c) 40% (d) 45%
(e) 44%

[IBPS PO/MT, 2013]

ANSWER KEYS

Exercise-1

1. (c)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (d)	**11.** (c)	**12.** (c)	**13.** (b)
14. (a)	**15.** (d)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (d)	**21.** (b)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (d)	**26.** (a)
27. (d)	**28.** (c)	**29.** (a)	**30.** (c)	**31.** (d)	**32.** (a)	**33.** (c)	**34.** (d)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (a)
40. (c)	**41.** (a)	**42.** (b)	**43.** (c)	**44.** (d)	**45.** (c)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (b)	**50.** (c)		

Exercise-2

1. (d)	**2.** (c)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (d)

EXPLANATORY ANSWERS

Exercise-1

1. (c) (a) As no odd natural number is divisible by 2, the set A is empty.

(b) Since no natural number satisfies the equation $x + 5 = 0$, $\therefore B = \phi$.

(c) Since 2 is an even prime number, i.e., $C = \{2\}$, C is not an empty set.

(d) Since there is no natural number between 1 and 2, D is an empty set.

2. (a) (a) $A = \{x : x \varepsilon Z \text{ and } x^2 - 2x - 3 = 0\} = \{3, -1\}$. $\therefore A$ is a finite set.

(b) B = The set of natural numbers divisible by 2 = $\{2, 4, 6, 8, 10, \cdots\}$. $\therefore B$ is an infinite set.

(c) Since infinite number of lines pass through a point, C is an infinite set.

(d) $D = \{-4, -3, -2, \cdots\}$ Clearly D is an infinite set.

3. (a) (a) $A = \{1, 3\}$; $B = \{1, 4\}$. A and B have different elements, $\therefore A \neq B$.

(b) $A = \{x : x + 2 = 2\} = \{0\}$; $B = \{0\}$. A and B have same elements, $\therefore A = B$.

(c) $A = \{1, 3, 4, 4\} = \{1, 3, 4\}$; $B = \{3, 1, 4\}$. A and B have same elements, $\therefore A = B$.

(d) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \cdots\right\} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$;

$B = \left\{\frac{1}{n} : n \in N\right\} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$.

A and B have same elements, $\therefore A = B$.

14. (a) Given: $n(C) = 50$, $n(F) = 20$, $n(C \cup F) = 10$.

Number of students playing at least one of these two games $= n(C \cup F) = n(C) + n(F) - n(C \cap F)$

$= 50 + 20 - 10 = 60$.

15. (d) Let, $A = \{0\}$. The possible subsets of this set A are ϕ and $\{0\}$, so the power set of the given set A is $P(A) = \{f, \{0\}\}$.

16. (c) Let, $A = \{\{a, b\}, c\}$. To determine $P(A)$: Since A contains two elements $\{a, b\}$, c, $P(A)$ will contain $2^2 = 4$ elements.

The elements of $P(A)$ are ϕ, A, $\{a, b\}$, $\{c\}$.

17. (b) (a) $1 \in A$ but $1 \notin B$ and $6 \in B$ but $6 \notin A$.

$\therefore$ A and B are not comparable.

(b) $A = \{x : x \in N \text{ and } x \leq 10\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ Clearly, $A \subset B \Rightarrow A$ and B are comparable.

(c) $\{4, 5\} \in A$ but $\{4, 5\} \notin B$ and $4 \in B$ but $4 \notin A$.

$\therefore$ A and B are not comparable.

27. (d) $A = \{2, 4, 6, \cdots\}$

$B = \{5, 10, 15, \cdots\}$

$C = \{10, 20, 30, \cdots\}$

$\therefore (A \cap B) = \{2, 4, 6 \cdots\} \cap \{5, 10, 15, \cdots\}$

$= \{10, 20, 30, \cdots\} = C$

$\therefore (A \cap B) \cap C = C \cap C = C$.

28. (c) $B \cup C = \{5, 10, 15, \cdots\} \cup \{10, 20, 30, \ldots\}$

$= \{5, 10, 15, \cdots\}$

$\therefore A \cap (B \cap C) = \{2, 4, 6, \cdots\} \cap \{5, 10, 15, \cdots\}$

$= \{10, 20, 20, \cdots\} = C$.

29. (a) $A \cap U = \{2, 4, 7\}$; $B \cup C = \{3, 5, 7, 8, 9, 10, 11\}$. Then $(A \cap U) \cap (B \cup C) = \{2, 4, 7,\} \cap \{3, 5, 7, 8, 9, 10, 11\} = \{7\}$.

30. (c) $U \cup A = \{a, b, c, d, e, f\} \cup \{a, b, c\}$

$= \{a, b, c, d, e, f\} = U$

$(U \cup A)' = \phi$.

31. (d) $A \cup B = \{a, b, c\} \cup \{c, d, e, f\}$

$= \{a, b, c, d, e, f\}$

$\therefore \quad (A \cup B) \cup C = \{a, b, c, d, e, f\} \cup \{c, d, e\}$
$= \{a, b, c, d, e, f\}$
$= U.$

32. **(a)** $A \cap B = \{a, b, c\} \cap \{c, d, e, f\} = \{c\}$
$A \cap C = \{a, b, c\} \cap \{c, d, e\} = \{c\}$
$\therefore \quad (A \cap B) \cup (A \cap C) = \{c\}.$

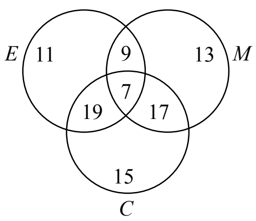
33. **(c)** (a) $\{x : x$ is a natural number and $4 \le x \le 6\}$ $= \{4, 5, 6\}$. Now , $\{1, 2, 3, 4\}$ and $\{4, 5, 6\}$ have one element 4 common. Therefore, the given two sets are not disjoint.

(b) The sets $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$ have one element e as common. Then, the given two sets are not disjoint.

(c) The sets $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$ have no element as common and therefore they are disjoint sets.

34. **(d)** $A - B$ is a set of member which belong to A but do not belong to B
$\therefore \quad A - B = \{3, 5\}$
$\therefore \quad (A - B)' = \{2, 4, 6, 7, 8, 9, 10, 11\}.$

35. **(a)**



No. of students who passed in one or more subjects = 11 + 9 + 13 + 17 + 15 + 19 + 7 = 91

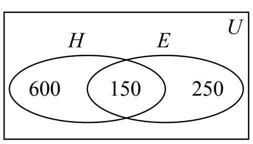
No. of students who failed in all the subjects = 100 − 91 = 9.

36. **(a)** We are given $n(X \cup Y) = 18$, $n(X) = 8$, $n(Y) = 15$. Using the formula
$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$,
we get $n(X \cap Y) = 8 + 15 - 18 = 5$.

37. **(b)** We are given $n(A) = 40$, $n(A \cap B) = 60$ and $n(A \cap B) = 10$. Putting these values in the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ we get $60 = 40 + n(B) - 10 \Rightarrow n(B) = 30$.

38. **(c)** $n(S) = 21$, $n(T) = 32$, $n(S \cap T) = 11$, $n(S \cup T) = ?$
Using $n(S \cup T) = n(S) + n(T) - n(S \cap T)$
$= 21 + 32 - 11 = 42$
Hence, $S \cup T$ has 42 elements.

39. **(a)** Here, $n(H \cup E) = 1000$, $n(H) = 750$, $n(E) = 400$
Using $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

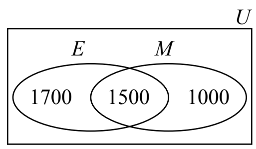


We get $1000 = 750 + 400 - n(H \cap E)$
$\Rightarrow \quad n(H \cap E) = 1150 - 1000 = 150.$
Number of People who can speak Hindi only
$= n(H \cap E') = n(H) - n(H \cap E)$
$= 750 - 150 = 600.$

40. **(c)** Here, $n(M \cup B) = 50$, $n(M) = 35$, $n(B) = 37$, $n(M \cap B) = ?$
Using $n(M \cup B) = n(M) + n(B) - n(M \cap B)$
We get $50 = 35 + 37 - n(M \cap B)$
$\Rightarrow \quad n(M \cap B) = 35 + 37 - 50 = 72 - 50 = 22$
$\therefore$ 22 students have opted for both Mathematics and Biology.
Again number of students who have opted for only Mathematics $= n(M) - n(M \cap B) = 35 - 22 = 13$.

41. **(a)** Let A = set of people who like coffee and B = set of people like tea.
Then, $A \cap B$ = set of people who like at least one of the tow drinks.
And $A \cap B$ = set of people who like both the drinks.
Here, $n(A) = 37$, $n(B) = 52$, $n(A \cup B) = 70$.
Using the result
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
we have $70 = 37 + 52 - n(A \cap B)$
$\Rightarrow \quad n(A \cap B) = 89 - 70 = 19.$
$\therefore$ 19 people like both coffee and tea.

42. **(b)** Let, E be the set of people who are egg-eaters and M be the set of people who are meat-eaters.
We have, $n(E) = 3200$, $n(M) = 2500$, $n(E \cap M) = 1500$.
Using $n(E \cup M) = n(E) + n(M) - n(E \cap M)$
$= 3200 + 2500 - 1500$
$= 5700 - 1500 = 4200.$



$\therefore$ Number of pure vegetarians
$n(U) - n(E \cup M)$
$5000 - 4200 = 800.$

43. **(c)** $A \times B = \{1, 2\} \times \{2, 3\}$
$= \{(1, 2), (1, 3), (2, 2), (2, 3)\}.$

44. **(d)** We have,
$(B \cap C) = \{2, 3, 5, 6, 7\} \cap \{5, 6, 7, 8, 9\}$
$= \{5, 6, 7\}$
$\therefore \quad A \times (B \cap C) = \{a, b\} \times \{5, 6, 7\}$
$= \{(a, 5), (a, 6), (a, 7), (b, 5), (b, 6), (b, 7)\}.$

45. **(c)**
$(B \cup C) = \{b, c, e\} \cup \{b, c, f\} = \{b, c, e, f\}$
$\therefore \quad A \times (B \cup C) = \{a, d\} \times \{b, c, e, f\}$

$= \{(a, b), (a, c), (a, e), (a, f), (d, b), (d, c), (d, e), (d, f)\}$ ...(1)

Also, $(A \times B) = \{(a, b), (a, c), (a, e), (d, b), (d, c), (d, e)\}$

and, $(A \times C) = \{(a, b), (a, c), (a, f), (d, b), (d, c), (d, f)\}$ ...(2)

$\therefore \quad (A \times B) \cup (A \times C) = \{(a, b), (a, c), (a, e), (a, f), (d, b), (d, c), (d, e), (d, f)\}$

From (1) and (2), we have

$A \times (B \cup C) = (A \times B) \cup (A \times C)$.

46. **(b)** $(B \cap C) = \{b, c, e\} \cap \{b, c, f\} = \{b, c\}$

$\therefore \quad A \times (B \cap C) = \{a, d\} \times \{b, c\}$

$= \{(a, b), (a, c), (d, b), (d, c)\}$

Also, $(A \times B) \cap (A \times C)$

$= \{(a, b), (a, c), (a, b), (d, c)\}$

$\therefore \quad A \times (B \cap C) = (A \times B) \cap (A \times C)$.

47. **(a)** $(B - C) = \{b, c, e\} - \{b, c, f\} = \{e\}$

$\therefore \quad A \times (B - C) = \{(a, e), (d, e)\}$ ...(1)

Also, $(A \times B) - (A \times C) = \{(a, e), (d, e)\}$ ...(2)

Hence, from (1) and (2), we have

$A \times (B - C) = (A \times B) - (A \times C)$.

48. **(b)** $(A \times B) = \{1, 2, 3\} \times \{2, 3, 4\}$

$= \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

$(C \times D) = \{1, 3, 4\} \times \{2, 4, 5\}$

$= \{(1, 2), (1, 4), (1, 5), (3, 2), (3, 4), (3, 5), (4, 2), (4, 4), (4, 5)\}$

$\therefore \quad (A \cap B) \cap (C \times D)$

$= \{(1, 2), (1, 4), (3, 2), (3, 4)\}$

Also, $(A \cap C) = \{1, 3\}$ and $(B \cap D)$

$= \{2, 4\}$. Therefore,

$(A \cap C) \times (B \cap D) = \{(1, 2), (1, 4), (3, 2), (3, 4)\}$

Hence, $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

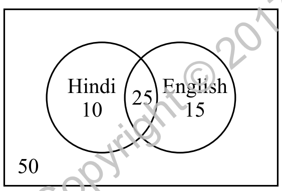
Exercise-2
(Based on Memory)

1. **(d)** In 40% students failed Hindi then 60% students passed in Hindi.

2. **(c)** Number of people speak English only

$= 50 - (10 + 25)$

$= 15$



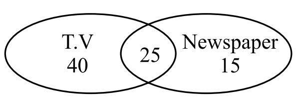
3. **(a)** $72\% + 44\% = 116\%$

$\because \quad 16\% \rightarrow 40$

$\therefore \quad 100\% \rightarrow \frac{40}{16} \times 100 = 250$

Hence, 16% workers are there who prefer both drinks which are 40 in number.

4. **(c)**



Required percentage

$= 100 - (40 + 25 + 15)$

$= 100 - 80 = 20\%$

5. **(b)** 20% of 80 $= \frac{20}{100} \times 80 = 16$

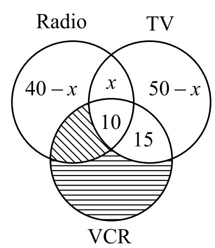
Remaining 50%

$= (80 - 16) \times \frac{50}{100} = 32$

No. of families not owning any vehicle

$= 80 - (32 + 16) = 80 - 48 = 32$

6. **(b)**



$\therefore \quad (40 - x) + x + 10 + 15 + (50 - x) = 100$

$\Rightarrow \quad 115 - 2x + x = 100$

$\therefore \quad x = 115 - 100 = 15$

$\therefore \quad$ Only TV $= 75 - 15 - 10 - 15 = 35$

7. **(c)** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 40 + 26 - 16 = 50$

8. **(c)**

2	$6\,\overline{40}\,\overline{09}$	253
2	4	
45	240	
5	225	
503	1509	
3	1509	
506	×	

$\therefore \quad \sqrt{64009} = 253$

9. **(c)** $n(A)$ = Biology students = 72%

$n(B)$ = Mathematics students = 44%

Total students = $n(A \cup B)$ = 100%

$\therefore$ 100% = 72% + 44% − $n(A \cap B)$

= 116% − $n(A \cap B)$

$\therefore$ $n(A \cap B)$ = 16%

$\Rightarrow$ number of students studying both subjects = 40

$\therefore$ Let the total number of students be x

Now, according to the question,

$$\frac{16x}{100} = 40$$

$\therefore \quad x = 250$

10. **(d)** Let the number of students be 100. Number of students who failed in Hindi is 30%.

$n(H) = 30$

Number of students who failed in English is 45%.

$\therefore \quad n(E) = 45$

Number of students who failed in both the subjects is 20%.

$n(H \cup E) = 20$

Applying the rule,

$n(H \cup \text{E}) = n(H) + n(E) - n(H \cap E)$

$= 30 + 45 - 20 = 55$

Percentage of students who failed in Hindi or English or both the subjects = 55%

Number of students who passed in both the subjects = 100 − 55 = 45%