Logarithms

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INTRODUCTION

Logarithm, in Mathematics, is the 'exponent' or 'power' to which a stated number called the *base*, is raised to yield a specific number. For example, in the expression $10^2 = 100$, the logarithm of 100 to the **base** 10 is 2. This is written as $\log_{10} 100 = 2$. Logarithms were originally invented to help simplify the arithmetical processes of multiplication, division, expansion to a power and extraction of a 'root', but they are nowadays used for a variety of purposes in pure and applied Mathematics.

Logarithm

If for a positive real number ($a \neq 1$), $a^m = b$, then the index m is called the logarithm of b to the base a. We write this as

$$\log_a b = m$$

'log' being the abbreviation of the word 'logarithm'. Thus,

$$a^m = b \Leftrightarrow \log_a b = m$$

where, $a^m = b$ is called the *exponential form* and $\log_a b = m$ is called the *logarithmic form*.

Illustration 1: Refer to the following Table

Exponential form	logarithmic form
$3^5 = 243$	$\log_3 243 = 5$
$2^4 = 16$	$\log_2 16 = 4$
$3^0 = 1$	$\log_3 1 = 0$
$8^{1/3} = 2$	$\log_8 2 = \frac{1}{3}$

LAWS OF LOGARITHMS

1. **Product formula**

   The logarithm of the product of two numbers is equal to the sum of their logarithms.
   i.e., $\log_a (mn) = \log_a m + \log_a n$.
   Generalisation: In general, we have
   $\log_a (mnpq...) = \log_a m + \log_a n + \log_a p + \log_a q + ...$

2. **Quotient formula**

   The logarithm of the quotient of two numbers is equal to the difference of their logarithms.
   i.e., $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$,
   where, a, m, n are positive and $a \neq 1$.

3. **Power formula**

   The logarithm of a number raised to a power is equal to the power multiplied by logarithm of the number.
   i.e., $\log_a (m^n) = n \log_a m$,
   where, a, m are positive and $a \neq 1$.

3. **Base changing formula**

   $\log_n m = \frac{\log_a m}{\log_a n}$. So, $\log_n m = \frac{\log m}{\log n}$.
   where, m, n, a are positived and $n \neq 1$, $a \neq 1$.

4. **Reciprocal relation**

   $\log_b a \times \log_a b = 1$,
   where, a, b are positive and not equal to 1.

5. $\log_b a = \frac{1}{\log_a b}$

6. $a^{\log_a x} = x$, where, a and x are positive, $a \neq 1$.

7. If $a > 1$ and $x > 1$, then $\log_a x > 0$.

8. If $0 < a < 1$ and $0 < x < 1$, then $\log_a x > 0$.

9. If $0 < a < 1$ and $x > 1$, then $\log_a x > 0$.

10. If $a > 1$ and $0 < x < 1$, then $\log_a x < 0$.

SOME USEFUL SHORTCUT METHODS

1. Logarithm of 1 to any base is equal to zero.
   i.e., $\log_a 1 = 0$, where $a > 0$, $a \neq 1$.

2. Logarithm of any number to the same base is 1.
   i.e., $\log_a a = 1$, where $a > 0$, $a \neq 1$.

Common Logarithms

There are two bases of logarithms that are extensively used these days. One is base e (e = 2. 71828 approx.) and the other is base 10. The logarithms to base e are called natural logarithms. The logarithms to base 10 are called the common logarithms.

$\log_{10}10 = 1$, since $10^1 = 10$.
$\log_{10}100 = 2$, since $10^2 = 100$.
$\log_{10}10000 = 4$, since $10^4 = 10000$.
$\log_{10}0.01 = -2$, since $10^{-2} = 0.01$.
$\log_{10}0.001 = -3$, since $10^{-3} = 0.001$
and, $\log_{10}1 = 0$, since $10^0 = 1$.

EXERCISE-1

1. Find $\log_{3/2} 3.375$.
(a) 2 (b) 3
(c) 5/2 (d) 17/2

2. If $x = \log_{2a} a$, $y = \log_{3a} 2a$ and $z = \log_{4a} 3a$, find $yz(2 - x)$.
(a) 1 (b) –1
(c) 2 (d) –2

3. $\dfrac{\log x}{l+m-2m} = \dfrac{\log z}{n+l-2m} = \dfrac{\log z}{n+l-2m}$, find $x^2y^2z^2$.
(a) 2 (b) –1
(c) 4 (d) 1

4. If $\log \dfrac{x+y}{5} = \dfrac{1}{2}(\log x + \log y)$, then $\dfrac{x}{y} + \dfrac{y}{x} =$
(a) 20 (b) 23
(c) 22 (d) 21

5. If $\log(x + y) = \log\left(\dfrac{3x-3y}{2}\right)$, then $\log x - \log y =$
(a) log 2 (b) log 3
(c) log 5 (d) log 6

6. If $\log_2 x + \log_4 x + \log_{16} x = 21/4$, then $x =$
(a) 8 (b) 4
(c) 2 (d) 16

7. $7\log\dfrac{16}{15} + 5\log\dfrac{25}{24} + 3\log\dfrac{81}{80} =$
(a) log 2 (b) log 3
(c) log 5 (d) None of these

8. If $0 < a \leq x$, the minimum value of $\log_a x + \log_x a$ is:
(a) 1 (b) 2
(c) 3 (d) 5

9. If $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b}$, then $xyz = x^a \cdot y^b \cdot z^c = x^{b+c} \cdot y^{c+a} \cdot z^{a+b} =$
(a) 1 (b) 0
(c) 2 (d) None of these

10. $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y} =$
(a) 0 (b) 2
(c) 1 (d) None of these

11. If $\log_{10}[98 + \sqrt{x^2 - 12x + 36}] = 2$, then $x =$
(a) 4 (b) 8
(c) 12 (d) 4,8

12. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, then
(a) $xyz = x + y + z + 2$
(b) $xyz = x + y + z + 1$
(c) $x + y + z = 1$
(d) $xyz = 1$.

13. If $a^x = b^y = c^z = d^w$, then $\log_a (bcd) =$
(a) $\dfrac{1}{x}\left(\dfrac{1}{y}+\dfrac{1}{z}+\dfrac{1}{w}\right)$ (b) $x\left(\dfrac{1}{y}+\dfrac{1}{z}+\dfrac{1}{w}\right)$
(c) $\dfrac{y+z+w}{x}$ (d) None of these

14. If $\log_{10} 2 = 0.3010$, then $\log_{10}(1/2) =$
(a) –0.3010 (b) 0.6990
(c) $\bar{1}.6990$ (d) $\bar{1}.3010$

15. If $\log_2(3^{2x-2} + 7) = 2 + \log_2(3^{x-1} + 1)$, then $x =$

(a) 0 (b) 1

(c) 2 (d) 1 or 2

16. If $\log_a b = \log_b c = \log_c a$, then

(a) $a > b \geq c$ (b) $a < b < c$

(c) $a = b = c$ (b) $a < b \leq c$.

17. If $\dfrac{1}{\log_x 10} = \dfrac{2}{\log_a 10} - 2$, then $x =$

(a) $a/2$ (b) $a/100$

(c) $a^2/10$ (d) $a^2/100$

18. If $a^2 + b^2 = c^2$, then $\dfrac{1}{\log_{c+a} b} + \dfrac{1}{\log_{c-a} b} =$

(a) 1 (b) 2

(c) −1 (d) −2

19. If $\log_{10} 87.5 = 1.9421$, then the number of digits in $(875)^{10}$ is:

(a) 30 (b) 29

(c) 20 (d) 19

20. If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$, then the number of zeros between the decimal point and the first significant figure in $(0.0432)^{10}$ is:

(a) 10 (b) 13

(c) 14 (d) 15

21. If $(4.2)^x = (0.42)^y = 100$, then $\dfrac{1}{x} - \dfrac{1}{y} =$

(a) 1 (b) 2

(c) 1/2 (d) −1

22. $\dfrac{\log_9 11}{\log_5 13} - \dfrac{\log_3 11}{\log_{\sqrt{5}} 13} =$

(a) 1 (b) −1

(c) 0 (d) None of these

23. If $\dfrac{\log x}{2} = \dfrac{\log y}{3} = \dfrac{\log z}{5}$, then yz in terms of x is:

(a) x (b) x^2

(c) x^3 (d) x^4

24. If $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$, then $x =$

(a) 1/2 (b) 3/2

(c) 5/2 (d) 1

25. $\dfrac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5} =$

(a) 5 (b) 2

(c) 5/2 (d) 3/2

26. $\log_{10}\tan 40° \cdot \log_{10} 41° \ldots \log_{10}\tan 50° =$

(a) 1 (b) 0

(c) −1 (d) None of these

27. If $\log_8 p = 2.5$, $\log_2 q = 5$, then p in terms of q is

(a) $q\sqrt{q}$ (b) $2q$

(c) q (d) $q/2$

28. If $y = \dfrac{1}{a^{1-\log_a x}}$, $z = \dfrac{1}{a^{1-\log_a y}}$ and $x = a^k$, then $k =$

(a) $\dfrac{1}{a^{1-\log_a z}}$ (b) $\dfrac{1}{1-\log_a z}$

(c) $\dfrac{1}{1+\log_z a}$ (d) $\dfrac{1}{1-\log_z a}$

29. If $\log_e 2 \cdot \log_b 625 = \log_{10}{}^{16} \cdot \log_e 10$, then $b =$

(a) 4 (b) 5

(c) 1 (d) e

30. $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$

(a) $\log 2$ (b) 1

(c) 0 (d) None of these

31. $2^{\log_3 7} - 7^{\log_3 2}$

(a) $\log_2 7$ (b) $\log 7$

(c) $\log 2$ (d) 0

32. If $\log_{30} 3 = a$, $\log_{30} 5 = b$, then $\log_{30} 8 =$

(a) $3(1 - a - b)$ (b) $a - b + 1$

(c) $1 - a - b$ (d) $3(a - b + 1)$

33. If $0 < a < 1$, $0 < x < 1$ and $x < a$, then $\log_a x$:

(a) < 1 (b) > 1

(c) < 0 (d) ≤ 1

34. $\log_5 2$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a prime number

35. $\log_5\left(1+\dfrac{1}{5}\right) + \log_5\left(1+\dfrac{1}{6}\right) + \log_5\left(1+\dfrac{1}{7}\right) + \cdots + \log_5\left(1+\dfrac{1}{624}\right)$

(a) 5 (b) 4

(c) 3 (d) 2

36. If $\log_{10} 2986 = 3.4751$, then $\log_{10} 0.02986 =$

(a) $\bar{1}.2986$ (b) $\bar{2}.4751$

(c) 0.34751 (d) None of these

37. If $\log(2a - 3b) = \log a - \log b$, then $a =$

(a) $\dfrac{3b^2}{2b-1}$ (b) $\dfrac{3b}{2b-1}$

(c) $\dfrac{b^2}{2b+1}$ (d) $\dfrac{3b^2}{2b+1}$

38. If $\log(x-y) - \log 5 - \frac{1}{2}\log x - \frac{1}{2}\log y = 0$, then $\frac{x}{y} + \frac{y}{x} =$

(a) 25 (b) 26
(c) 27 (d) 28

39. If $\log x{:}3 = \log y{:}4 = \log z{:}5$, then $zx =$

(a) $2y$ (b) y^2
(c) $8y$ (d) $4y$

40. If $3 + \log_5 x = 2\log_{25} y$, then $x =$

(a) $y/125$ (b) $y/25$
(c) $y^2/625$ (d) $3 - y^2/25$

41. If $\frac{\log_2 a}{3} = \frac{\log_3 b}{3} = \frac{\log_4 c}{4}$ and $a^{1/2} \cdot b^{1/3} \cdot c^{1/4} = 24$, then

(a) $a = 24$ (b) $b = 81$
(c) $c = 64$ (d) $c = 256$

42. If $\frac{\log_2 x}{3} = \frac{\log_2 y}{4} = \frac{\log_2 z}{5k}$ and $\frac{z}{x^3 y^4} = 1$, then $k =$

(a) 3 (b) 4
(c) 5 (d) −5

43. $\dfrac{3 + \log_{10} 343}{2 + \frac{1}{2}\log\left(\frac{49}{4}\right) + \frac{1}{3}\log\left(\frac{1}{125}\right)} =$

(a) 3 (b) 3/2
(c) 2 (d) 1

44. If $\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2}$, then $x^{a-b} \cdot y^{b-c} \cdot z^{c-a} =$

(a) 0 (b) −1
(c) 1 (d) 2

45. If $3^{x-2} = 5$ and $\log_{10} 2 = 0.20103$, $\log_{10} 3 = 0.4771$, then $x =$

(a) $1\frac{22187}{47710}$ (b) $2\frac{22187}{47710}$
(c) $3\frac{22187}{47710}$ (d) None of these

46. If $\log_2 = 0.30103$ and $\log_3 = 0.4771$, then number of digits in $(648)^5$ is:

(a) 12 (b) 13
(c) 14 (d) 15

47. If $\log x = \frac{\log y}{2} = \frac{\log z}{5}$, then $x^4 \cdot y^3 \cdot z^{-2} =$

(a) 2 (b) 10
(c) 1 (d) 0

48. $\dfrac{\log\sqrt{27} + \log\sqrt{1000} + \log 8}{\log 120}$

(a) 1/2 (b) 1
(c) 3/2 (d) 2

49. For $x > 0$, if $y = \frac{10^{\log_{10}^{x}}}{x^2}$ and $x = y^a$, then $a =$

(a) 1 (b) −1
(c) 0 (d) 2

50. If $x = 100_{4/3}(1/2)$, $y = \log_{1/2}(1/3)$, then

(a) $x > y$ (b) $x < y$
(c) $x = y$ (d) $x \geq y$

Exercise-2
(Based on Memory)

1. If $\log 2 = 0.30103$, the number of digits in 2^{64} is:

(a) 18 (b) 19
(c) 20 (d) 21

[SI of Police Rec. Examination, 1997]

2. If $\log_{10} 2 = 0.301$, then the value of $\log_{10}(50)$ is

(a) 0.699 (b) 1.301
(c) 1.699 (d) 2.301

[SI Rec. (Delhi Police) Examination, 1997]

3. Given that $\log_{10} 2 = 0.3010$, the $\log_2 10$ is equal to:

(a) 0.3010 (b) 0.6990
(c) $\frac{1000}{301}$ (d) $\frac{699}{301}$

[Assistant's Grade Examination, 1997]

4. If $\log 2 = x$, $\log 3 = y$ and $\log 7 = z$, then the value of $\log\left(4.\sqrt[3]{63}\right)$ is:

(a) $-2x + \frac{2}{3}y + \frac{1}{3}z$ (b) $2x + \frac{2}{3}y + \frac{1}{3}z$
(c) $2x + \frac{2}{3}y - \frac{1}{3}z$ (d) $2x - \frac{2}{3}y + \frac{1}{3}z$

[Assistant's Grade Examination, 1998]

5. If $\log_{12} 27 = a$, then $\log_6 16$ is:

(a) $\frac{4(3-a)}{3+a}$ (b) $\frac{4(3+a)}{3-a}$

(c) $\frac{3+a}{4(3-a)}$ (d) $\frac{3-a}{4(3+a)}$

[Assistant's Grade Examination, 1998]

6. If $\log_x 4 = 0.4$, then the value of x is:

(a) 4 (b) 16

(c) 1 (d) 32

[Assistant's Grade Examination, 1998]

7. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is:

(a) 2^{10} (b) 2^{1000}

(c) 2^{100} (d) 2^{10000}

[SSC (GL) Prel. Examination, 1999]

8. If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is:

(a) 0.4471 (b) 1.4471

(c) 2.4471 (d) 1.4471

[SSC (GL) Prel. Examination, 1999]

9. If $\log(0.57) = \bar{1}.756$, then the value of $\log 57 + \log(0.57)^3 + \log\sqrt{0.57}$ is:

(a) 0.902 (b) $\bar{1}.902$

(c) 1.146 (d) $\bar{2}.146$

[SSC (GL) Prel. Examination, 1999]

10. If $\log_{10} 2 = 0.3010$ is given, then $\log_2 10$ is equal to:

(a) 0.3010 (b) 0.6990

(c) $\frac{1000}{301}$ (d) $\frac{699}{301}$

[SSC (GL) Prel. Examination, 2000]

11. If $\log 3 = 0.477$ and $(1000)^x = 3$, then x equals.

(a) 0.159 (b) 10

(c) 0.0477 (d) 0.0159.

[SSC (GL) Prel. Examination, 2000]

12. If $\log 2 = 0.3010$, then $\log 5$ equals.

(a) 0.3010

(b) 0.699

(c) 0.7525

(d) Given $\log_2$, it is not possible to calculate $\log_5$.

[SSC (GL) Prel. Examination, 2000]

13. If $\log 90 = 1.9542$ then $\log 3$ equals.

(a) 0.9771 (b) 0.6514

(c) 0.4771 (d) 0.3181

[SSC (GL) Prel. Examination, 2000]

14. The number of digits in 8^{10} is (when $\log_2 = 0.30103$)

(a) 19 (b) 10

(c) 17 (d) 16

[RRB, Kolkata Supervisor (P.Way) Examination, 2000]

15. If $\log_{10}(x^2 - 6x + 45) = 2$, then the values of x are:

(a) 10, 5 (b) 11, −5

(c) 6, 9 (d) 9, −5

[RRB Allahabad ASM Examination, 2002]

16. If $\log_{10} 2 = 0.30$, then $\log_2 10$ is:

(a) 3.3220 (b) 5

(c) 0.3322 (d) 3.2320

[RRB Allahabd ASM Examination, 2002]

ANSWER KEYS

Exercise-1

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (c)	**11.** (d)	**12.** (a)	**13.** (b)
14. (c)	**15.** (d)	**16.** (c)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (b)	**21.** (c)	**22.** (c)	**23.** (d)	**24.** (b)	**25.** (c)	**26.** (b)
27. (a)	**28.** (b)	**29.** (b)	**30.** (c)	**31.** (d)	**32.** (a)	**33.** (b)	**34.** (c)	**35.** (b)	**36.** (b)	**37.** (a)	**38.** (c)	**39.** (b)
40. (a)	**41.** (d)	**42.** (c)	**43.** (a)	**44.** (c)	**45.** (c)	**46.** (d)	**47.** (c)	**48.** (c)	**49.** (b)	**50.** (b)		

Exercise-2

1. (c)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (c)	**11.** (a)	**12.** (b)	**13.** (c)
14. (b)	**15.** (b)	**16.** (a)										

EXPLANATORY ANSWERS

EXERCISE-1

1. **(b)** $\log_{3/2} 3.375 = x \Rightarrow \left(\frac{3}{2}\right)^x = 3.375$

$\Rightarrow (1.5)^x = (1.5)^3 \Rightarrow x = 3.$

2. **(a)** $yz(2 - x) = 2yz - xyz = 2\log_{4a} 2a - \log_{4a} a$

$= \log_{4a}\left(\frac{4a^2}{a}\right) = 1.$

3. **(d)** Each is equal to k

$\Rightarrow \log x = k\,(l + m - 2n),$

$\log y = k\,(m + n - 2l),\ \log z = k\,(n + l - 2m).$

$\Rightarrow \log xyz = k\,(0) \Rightarrow xyz = e^0 = 1 \Rightarrow x^2y^2z^2 = 1.$

4. **(b)** $\log\left(\frac{x+y}{5}\right) = \frac{1}{2}[\log x + \log y]$

$\Rightarrow x + y = 5\sqrt{xy} \Rightarrow x^2 + y^2 = 23xy$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = 23.$

5. **(c)** $x + y = \frac{3x - 3y}{2} \Rightarrow x = 5y \Rightarrow \frac{x}{y} = 5$

$\Rightarrow \log x - \log y = \log 5.$

6. **(a)** $\log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \frac{21}{4}$

$\Rightarrow \log_2^x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{21}{4} \Rightarrow \log_2 x = 3 \Rightarrow x = 8.$

7. **(a)** $7\log\left(\frac{2^4}{5\times 3}\right) + 5\log\left(\frac{5^2}{2^3\times 3}\right) + 3\log\left(\frac{3^4}{2^4\times 5}\right)$

$= 28\log 2 - 7\log 5 - 7\log 3 + 10\log 5 - 15\log 2$

$-5\log 3 + 12\log 3 - 12\log 2 - 3\log 5 = \log 2.$

8. **(b)** $0 < a \le x$; Min. value of $\log_a x + \log_x a$ is 2 when we put $x = a$.

9. **(a)** $\frac{\log x}{b - c} = \frac{\log y}{c - a} = \frac{\log z}{a - b} = k$ (say)

$\Rightarrow \log x = k\,(b - c),\ \log y = k\,(c - a),\ \log z = k\,(a - b)$

$\Rightarrow \log x + \log y + \log z = 0 \Rightarrow xy\,z = 1.$

Also, $a\log x + b\log y + c\log z = 0 \Rightarrow x^a \,.\, y^b \,.\, z^c = 1.$

Again $(b + c)\log x + (c + a)\log y + (a + b)\log z = 0.$

$\Rightarrow x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = 1.$

$\therefore\ xyz = x^a \cdot y^b \cdot z^c = x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = 1.$

10. **(c)** $x^{\log y - \log z} \cdot y^{\log z - \log x} \cdot z^{\log x - \log y} = k$ (say)

$\Rightarrow (\log y - \log z)\log x + (\log z - \log x)\log y$

$+ (\log x - \log y)\log z = \log k = 0$

$\Rightarrow k = 1.$

11. **(d)** $98 + \sqrt{x^2 - 12x + 36} = 100$

$\Rightarrow \sqrt{x^2 - 12x + 36} = 2$

$\Rightarrow x^2 - 12x + 32 = 0$

$\Rightarrow x = 8, 4.$

12. **(a)** $x = \log_a bc \Rightarrow a^x = bc \Rightarrow a^{x+1} = abc$

$\Rightarrow a = (abc)^{1/x+1}.$

Similarly, $b = (abc)^{1/y+1}$ and $c = (abc)^{1/z+1}$

$\therefore\ abc = (abc)^{\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}}$

$\Rightarrow 1 = \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$\Rightarrow (x + 1)(y + 1)(z + 1) = (y + 1)(z + 1)$

$+ (x + 1)(z + 1) + (x + 1)(y + 1)$

$\Rightarrow xyz = x + y + z + 2.$

13. **(b)** $b^y = a^x \Rightarrow b = a^{\frac{x}{y}}, c = a^{\frac{x}{z}}, d = a^{\frac{x}{w}}$

$\log_a(bcd) = \log_a\left(a^{\frac{x}{y}}.a^{\frac{x}{z}}.a^{\frac{x}{w}}\right) = \frac{x}{y} + \frac{x}{z} + \frac{x}{w} = x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right).$

14. **(c)** $\log_{10}\left(\frac{1}{2}\right) = -\log_{10} 2 = -\,0.3010$

$= 1 - 0.3010 - 1 = \bar{1}.6990.$

15. **(d)** $\log_2(3^{2x-2} + 7) = \log_2^{\,4} + \log_2(3^{x-1} + 1)$

$[\because\ 2 = 2\log_2^{\,2} = \log_2 2^2]$

$\Rightarrow 3^{2x-2} + 7 = 4\,(3^{x-1} + 1)$

$\Rightarrow t^2 + 7 = 4(t + 1)$, where, $3^{x-1} = t$

$\Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1, 3$

When $t = 1 \Rightarrow 3^{x-1} = 1 \Rightarrow x = 1$

When $t = 3 \Rightarrow 3^{x-1} = 3^1 \Rightarrow x = 2.$

16. **(c)** $\log_a b = \log_b c = \log_c a = k$ (say)

$b = a^k,\ c = b^k,\ a = c^k$

$\Rightarrow c = (a^k)^k = a^{k2} = (c^{k2})^k = c^{k3}$

$\Rightarrow k^3 = 1 \Rightarrow k = 1.\ \therefore\ a = b = c.$

17. **(d)** $\log_{10} x = 2\log_{10} a - 2$

$\Rightarrow \log_{10} x = 2\,(\log_{10} a - 1)$

$\Rightarrow \log_{10} x = 2\log_{10}\left(\frac{a}{10}\right) \Rightarrow x = \frac{a^2}{100}.$

18. **(b)** $\log_b(c + a) + \log_b(c - a)$

$= \log_b(c^2 - a^2) = \log_b b^2 = 2.$

19. **(a)** $x = (875)^{10} = (87.5 \times 10)^{10}$

$\therefore \quad \log_{10} x = 10(\log_{10} 87.5 + 1)$

$= 10(1.9421 + 1)$

$= 10(2.9421) = 29.421.$

$\therefore \quad x = \text{Antilog } (29.421).$

$\therefore$ Number of digits in $x = 30$.

20. **(b)** $x = (0.0432)^{10} = \left(\dfrac{432}{10000}\right)^{10} = \left(\dfrac{3^3.2^4}{10^4}\right)^{10}$

$\therefore \quad \log_{10} x = 10\ (3\log_{10} 3 + 4\log_{10} 2 - 4)$

$= 10\ (1.4313 + 1.2040 - 4)$

$= 10\ (-1.3647) = -13.647$

$= \overline{14}.353$

$\therefore \quad x = \text{Antilog } (\overline{14}.353)$

$\therefore$ Number of zeros between the decimal and the first significant figure = 13.

21. **(c)** $(4.2)^x = 100 \quad \Rightarrow \quad (42)^x = 10^{2+x}$

$\Rightarrow \quad 42 = \left(\dfrac{42}{100}\right)^{y} \qquad \cdots(1)$

$\dfrac{2}{x} - \dfrac{2}{y} = 100 \quad \Rightarrow \quad (42)^y = 10^{2+2y}$

$\Rightarrow \quad 42 = 10^{\frac{2}{y}+2} \qquad \cdots(2)$

From (1) and (2), $\dfrac{2}{x} - \dfrac{2}{y} = 1 \Rightarrow \dfrac{1}{x} - \dfrac{1}{y} = \dfrac{1}{2}$.

22. **(c)** $\dfrac{\log_9 11}{\log_5 13} - \dfrac{\log_3 11}{\log_{\sqrt{5}} 13} = \dfrac{\log_3 11}{2.\log_5 13} - \dfrac{\log_3 11}{2.\log_5 13} = 0.$

23. **(d)** $\dfrac{\log x}{2} = \dfrac{\log y}{3} = \dfrac{\log z}{5} = k$ (say)

$\Rightarrow \quad \log x = 2k,\ \log y = 3k,\ \log z = 5k$

$\Rightarrow \quad \log yz = 3k + 5k = 8k;\ \log x^4 = 8k$

$\therefore \quad \log yz = \log x^4 \quad \Rightarrow \quad yz = x^4.$

24. **(b)** $4^x + \dfrac{4^x}{2} = \dfrac{3^x}{\sqrt{3}} + 3^x.\sqrt{3}$

$\Rightarrow \quad 4^x \cdot \dfrac{3}{2} = 3^x \cdot \dfrac{4}{\sqrt{3}} \Rightarrow \left(\dfrac{4}{3}\right)^x = \dfrac{8}{3\sqrt{3}}$

$\Rightarrow \quad \left(\dfrac{4}{3}\right)^x = \left(\dfrac{4}{3}\right)^{3/2} \Rightarrow x = \dfrac{3}{2}.$

25. **(c)** $\dfrac{\log 7^{5/2} + \log 5^{5/2} - \log 2^{5/2}}{\log 17.5}$

$= \dfrac{5(\log 7 + \log 5 - \log 2)}{2\log\left(\dfrac{35}{2}\right)} = \dfrac{5}{2}$

26. **(b)** $\log_{10} \tan 40° \cdot \log_{10} \tan 41° \cdots \log_{10} \tan 50°$

$= 0$, since $\log_{10} \tan 45 = 0$.

27. **(a)** $\log_8 p = \dfrac{5}{2} \quad \Rightarrow \quad p = (8)^{5/2} = 2^{\frac{15}{2}} = (2^5)^{3/2}$

$\log_2 q = 5 \quad \Rightarrow \quad q = 2^5.$

$\therefore \quad p = q^{3/2}.$

28. **(b)** $\log_a y = \dfrac{1}{1-\log_a x},\ \log_a z = \dfrac{1}{1-\log_a y}$

$\therefore \quad \log_a z = \dfrac{1}{1-\left(\dfrac{1}{1-\log_a x}\right)} = \dfrac{1-\log_a x}{-\log_a x}$

$\Rightarrow \quad -\log_a z = -1 + \dfrac{1}{\log_a x}$

$\Rightarrow \quad \dfrac{1}{\log_a x} = 1 - \log_a z$

$\therefore \quad \log_a x = \dfrac{1}{1-\log_a z} \Rightarrow x = \dfrac{1}{a^{1-\log_a^z}} = a^k$ (given)

$\therefore \quad k = \dfrac{1}{1-\log_a z}.$

29. **(b)** $\log_e 2 \cdot 4\log_b 5 = 4 \cdot \log_{10} 2 \cdot \log_e 10 = 4\log_e 2$

$\Rightarrow \quad \log_b 5 = 1 \quad \Rightarrow \quad b = 5.$

30. **(c)** $5^{\sqrt{\log_5 7}} - (7^{\log_7 5})^{\frac{1}{\sqrt{\log_7 5}}}$

$= 5^{\sqrt{\log_5 7}} - \dfrac{1}{5^{\sqrt{\log_7 5}}}$

$\Rightarrow \quad 5^{\sqrt{\log_5 7}} - 5^{\sqrt{\log_5 7}} = 0.$

31. **(d)** $2^{\log 37} - 7^{\log 32} = 2^{\log 27\ .\ \log 32} - 7^{\log 32}$

$= 7^{\log 32} - 7^{\log 32} = 0.$

32. **(a)** $a + b = \log_{30} 15 = \log_{30}\left(\dfrac{30}{2}\right) = 1 - \log_{30} 2$

$\Rightarrow \quad \log_{30} 2 = 1 - a - b.$

$\therefore \quad \log_{30} 8 = 3(1 - a - b).$

33. **(b)** $0 < a < 1,\ 0 < x < 1$ and $x < a$

$\Rightarrow \quad \log_a x > \log_a a \quad \Rightarrow \quad \log_a x > 1.$

34. **(c)** $\log_5 2 = \dfrac{p}{q} \quad \Rightarrow \quad 2 = 5^{p/q} = 2^q = 5^p$

$\Rightarrow$ even number = odd number,

which is a contradiction.

$\therefore \quad \log_5 2$ is an irrational number.

35. **(b)** $\log_5 \dfrac{6}{5} + \log_5 \dfrac{7}{6} + \log_5 \dfrac{8}{7} + \cdots + \log_5 \dfrac{625}{624}$

$= \log_5\left(\dfrac{6}{5}\cdot\dfrac{7}{6}\cdot\dfrac{8}{7}\cdots\dfrac{625}{624}\right) = \log_5\left(\dfrac{625}{5}\right) = 4.$

36. **(b)** $\log_{10}(0.02986) = \log_{10}\left(\dfrac{2986}{100000}\right)$

$= 3.4751 - 5 = -1.5249$

$= \bar{2}.4751.$

37. **(a)** $2a - 3b = \dfrac{a}{b} \Rightarrow 2ab - 3b^2 = a$

$\Rightarrow 3b^2 = a(2b - 1)$

$\Rightarrow a = \dfrac{3b^2}{2b-1}$.

38. **(c)** $(x - y)^2 = 25xy \Rightarrow x^2 + y^2 = 27xy \Rightarrow \dfrac{x}{y}+\dfrac{y}{x} = 27.$

39. **(b)** $\dfrac{\log x}{3}=\dfrac{\log y}{4}=\dfrac{\log z}{5} = k$

$\Rightarrow \log x = 3k;\ \log y = 4k;\ \log z = 5k.$

$\Rightarrow \log(zx) = \log z + \log x = 8k = 2\log y$

$\therefore zx = y^2.$

40. **(a)** $3 + \log_5 x = \log_5 y \Rightarrow \log_5(125x) = \log_5 y \Rightarrow x = \dfrac{y}{125}$.

41. **(d)** $\dfrac{\log_2 a}{3}=\dfrac{\log_3 b}{3}=\dfrac{\log_4 c}{4}=k$

$\Rightarrow a = 2^{2k},\ b = 3^{3k},\ c = 4^{4k}$ and

$a^{1/2}\cdot b^{1/3}\cdot c^{1/4} = 2^k\cdot 3^k\cdot 4^k = 24$

$\Rightarrow 24^k = 24^1 \Rightarrow k = 1.$

$\therefore a = 4,\ b = 27,\ c = 256.$

42. **(c)** $\dfrac{z}{x^3y^4} = 1 \Rightarrow \log_2 z - 3\log_2 x - 4\log_2 y = 0$

$\Rightarrow \log_2 z - \dfrac{3.3}{5k}\cdot\log_2 z - 4\cdot\dfrac{4}{5k}\cdot\log_2 z = 0$

$\Rightarrow 1 - \dfrac{9}{5k}-\dfrac{16}{5k} = 0$

$\Rightarrow 5k - 25 = 0 \Rightarrow k = 5.$

43. **(a)** $\dfrac{3(1+\log_{10}7)}{2+\log\dfrac{7}{2}+\log\dfrac{1}{5}} = \dfrac{3(1+\log_{10}7)}{2+\log\left(\dfrac{7}{10}\right)}$

$= \dfrac{3(1+\log_{10}7)}{1+\log_{10}7} = 3.$

44. **(c)** Each ratio $= k \Rightarrow \log x = k(a^2 + ab + b^2)$

$\Rightarrow (a - b)\log x = k(a^3 - b^3)$

$\Rightarrow \log x^{a-b} = k(a^3 - b^3) \Rightarrow x^{a-b} = e^{k(a^3-b^3)}$

Similarly, $y^{b-c} = e^{k(b^3-c^3)}$, $z^{c-a} = e^{k(c^3-a^3)}$.

$\therefore x^{a-b}\,.\,y^{b-c}\,.\,z^{c-a} = e^0 = 1.$

45. **(c)** $3^{x-2} = 5 \Rightarrow 3^x = 45 = \left(\dfrac{90}{2}\right)$

$\Rightarrow x\log_{10}3 = \log_{10}90 - \log_{10}2$

$= 2\log_{10}3 + 1 - \log_{10}2$

$\Rightarrow x(0.4771) = 1.65317$

$\Rightarrow x = \dfrac{165317}{47710} = 3\dfrac{22187}{47710}$.

46. **(d)** $\log(648)^5 = 5\log(81 \times 8) = 20\log 3 + 15\log 2$

$= 20(0.4771) + 15(0.30103)$

$= 14.05745.$

$\therefore$ Number of digits in $(648)^5$ is 15.

47. **(c)** $\dfrac{\log x}{1}=\dfrac{\log y}{2}=\dfrac{\log z}{5}=k$

$\Rightarrow \log x = k,\ \log y = 2k,\ \log z = 5k.$

$\therefore \log(x^4\,.\,y^3\,.\,z^{-2}) = 4\log x + 3\log y - 2\log z = 0$

$\Rightarrow x^4\,.\,y^3\,.\,z^{-2} = 1.$

48. **(c)** $\dfrac{\log\sqrt{27}+\log\sqrt{1000}+\log 8}{\log 120}$

$= \dfrac{\dfrac{3}{2}(\log 3+\log 10+\log 4)}{\log 3+\log 10+\log 4}=\dfrac{3}{2}$

49. **(b)** $y = \dfrac{10^{\log_{10}x}}{x^2}=\dfrac{1}{x}=\dfrac{1}{y^a} = y^{-a} \Rightarrow a = -1.$

50. **(b)** $x = \log_{4/w3}(1/2) = -\log_{4/3} 2 < 0$

and, $y = \log_{1/2}(1/3) = \log_2 3 > 0 \Rightarrow y > x.$

EXERCISE-2
(BASED ON MEMORY)

1. (c) $\log 2^{64} = 64 \log 2 = 64 \times .30103 = 19.26592.$

$\therefore$ Number of digits in $2^{64} = 19 + 1 = 20.$

2. (c) $\log_{10} 50 = \log_{10}\left(\frac{50 \times 2}{2}\right) = \log_{10} \frac{100}{2}$

$= \log_{10} 100 - \log_{10} 2 = 2 - .301 = 1.699.$

3. (c) $\log_2 10 = \frac{\log 10}{\log 2} = \frac{1}{\log 2} = \frac{1.0000}{.3010}$

$= \frac{10000}{3010} = \frac{1000}{301}$

4. (b) $\log(4 \times \sqrt[3]{63}) = \log(2^2 \times (3 \times 3 \times 7)^{1/3})$

$= \log 2^2 + \log(3 \times 3 \times 7)^{1/3}$

$= 2\log 2 + \frac{1}{3} \log(3^2 \times 7)$

$= 2\log 2 + \frac{1}{3} (\log 3^2 + \log 7)$

$= 2\log 2 + \frac{2}{3} \log 3 + \frac{1}{3} \log 7$

$= 2x + \frac{2}{3} y + \frac{1}{3} z.$

5. (a) $\log_{12} 27 = a \Rightarrow \frac{\log 27}{\log 12} = a$

$\Rightarrow a \log 12 = \log 3^3$

$\Rightarrow a \log(4 \times 3) = 3 \log 3$

$\Rightarrow a (\log 4 + \log 3) = 3 \log 3$

$\Rightarrow a \log 4 + a \log 3 = 3 \log 3$

$\Rightarrow a \log 2^2 = (3 - a) \log 3$

$\Rightarrow 2a \log 2 = (3 - a) \log 3$

$\Rightarrow \frac{\log 2}{\log 3} = \frac{3 - a}{2a}.$

Now, $\log_6 16 = \frac{\log 16}{\log 6} = \frac{\log 2^4}{\log(2 \times 3)}$

$= \frac{4 \log 2}{\log 2 + \log 3} = \frac{4\frac{\log 2}{\log 3}}{\frac{\log 2}{\log 3} + 1} a$

$= \frac{4\left(\frac{3-a}{2a}\right)}{\frac{3-a}{2a} + 1} = \frac{4(3-a)}{3+a}.$

6. (d) $\frac{\log 4}{\log x} = \frac{2}{5} \Rightarrow \frac{2 \log 2}{\log x} = \frac{2}{5}$

$\Rightarrow \log x = 5 \log 2 = \log 2^5 = \log 32$

$\Rightarrow x = 32.$

7. (b) $\log_x^{y} = 100,\ \log_2^{x} = 10$

$\Rightarrow \frac{\log y}{\log x} = 100, \frac{\log x}{\log 2} = 10$

$\Rightarrow \frac{\log y}{\log 2} = 1000 \Rightarrow \log_2 y = 1000$

$\Rightarrow y = 2^{1000}.$

8. (a) $\log_{10} 2.8 = \log_{10} \frac{28}{10} = \log 28 - \log 10$

$= \log(7 \times 4) - \log 10$

$= \log 7 + 2 \log 2 - \log 10$

$= 0.8451 + 2 \times 0.3010 - 1$

$= 0.8451 + 0.6020 - 1 = 0.4471.$

9. (a) $\log\left(\frac{57 \times 100}{100}\right) + 3\log(0.57) + \frac{1}{2}\log(0.57)$

$= \log(0.57) + \log 10^2 + 3 \log(0.57) + \frac{1}{2} \log(0.57)$

$= \left(1 + 3 + \frac{1}{2}\right) \log(0.57) + 2 = (4.5 \times \bar{1}.756) + 2$

$= 4.5 \times (-1 + .756) + 2 = 0.902.$

10. (c) $\log_2 10 = \frac{\log 10}{\log 2} = \frac{1}{.3010} = \frac{1000}{301} a.$

11. (a) $x \log 1000 = \log 3 \Rightarrow 3x = \log 3$

$\Rightarrow x = \frac{\log 3}{3} = \frac{.477}{3} = .159.$

12. (b) $\log 5 = \log \frac{10}{2} = \log 10 - \log 2$

$= 1 - 0.3010 = 0.6990.$

13. (c) $\log 90 = 1.9542$

$\Rightarrow \log(3^2 \times 10) = 1.9542$

$\Rightarrow 2 \log 3 + \log 10 = 1.9542$

$\Rightarrow \log 3 = \frac{.9542}{2} = .4771.$

14. (b) $8^{10} = (2^3)^{10} = 2^{30}.$

Let, $y = 2^{30} \Rightarrow \log y = \log(2)^{30} = 30 \log 2$

$= 30 \times 0.30103 = 9.0309$

$\therefore y = (10)^{9.0309}$, which contains 10 digits.

15. (b) $\log_{10}(x^2 - 6x + 45) = 2$

$\Rightarrow x^2 - 6x + 45 = 10^2$

$\Rightarrow x^2 - 6x + 45 - 100 = 0$

$\Rightarrow x^2 - 6x - 55 = 0$

$\Rightarrow (x + 5)(x - 11) = 0$

$\Rightarrow x = 11 \text{ or } -5.$

16. (a) $\log_{10} 2 = 0.3010$

$\therefore \log_2 10 = \frac{1}{\log_{10} 2} = \frac{1}{0.3010} = 3.3220$