Probability

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INTRODUCTION

The word *probability* or *chance* is very frequently used in day-to-day life. For example, we generally say, 'He may come today' or 'probably it may rain tomorrow' or 'most probably he will get through the examination'. All these phrases involve an element of uncertainty and probability is a concept which measures this uncertainties. The probability when defined in simplest way is the chance of occurring of a certain event when expressed quantitatively, i.e., probability is a quantitative measure of the certainty.

The probability has its origin in the problems dealing with games of chance such as gambling, coin tossing, die throwing and playing cards. In all these cases the outcome of a trial is uncertain. These days probability is widely used in business and economics in the field of predictions for future.

The following remarks may be important for learning this chapter on probability.

1. **Die:** A die is a small cube used in games of chance. On its 6 faces dots are marked as

   . .. ... : : :.: :::

   Plural of die is dice. The outcome of throwing (or tossing) a die is the number of dots on its uppermost face. An ace on a die means 1 dot.
2. **Cards:** A pack (or deck) of playing cards has 52 cards, divided into 4 suits:

   (i) Spades हुकम (♠) (ii) Clubs चिडी (♣)

   (iii) Hearts पान (♥) (iv) Diamonds ईंट (♦)

   Each suit has 13 cards, nine cards numbered 2 to 10, an Ace (इक्का), a King (बादशाह), Queen (बेगम) and a Jack or Knave (गुलाम). Spades and Clubs are black-faced cards while Hearts and Diamonds are red-faced cards. The Aces, Kings, Queens and Jacks are called *face cards* and other cards are called *number cards*. The Kings, Queens and Jacks are called *court cards*.
3. The number of combinations of n objects taken r at a time ($r \le n$) is denoted by $C(n, r)$ or ${}^nC_r$ and is defined as

$$ {}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots \text{to } r \text{ factors}}{1\cdot 2\cdot 3\ldots r.} $$

Illustration 1: ${}^5C_3 = \dfrac{5\cdot 4\cdot 3}{1\cdot 2\cdot 3} = 10$, ${}^nC_0 = 1$ and ${}^nC_n = 1$.

If $r > \dfrac{n}{2}$, then it is better to simplify ${}^nC_r$ as ${}^nC_{n-r}$.

Illustration 2: ${}^{52}C_{50} = {}^{52}C_{52-50} = {}^{52}C_2 = \dfrac{52.51}{2.1}$

$= 26.51 = 1326.$

When, $r > n$, ${}^nC_r = 0$.

Some Important Terms and Concepts

Random Experiment or Trial: The performance of an experiment is called a *trial*. An experiment is characterised by the property that its observations under a given set of circumstances do not always lead to the same observed outcome but rather to the different outcomes. If in an experiment all the possible outcomes are known in advance and none of the outcomes can be predicted with certainity, then such an experiment is called a *random experiment*.

For example, tossing a coin or throwing a die are random experiments.

Event: The possible outcomes of a trial are called *events*. Events are generally denoted by capital letters A, B, C and so on.

Illustration 3:

(i) When a coin is tossed the outcome of getting a head or a tail is an event.

(ii) When a die is thrown the outcome of getting 1 or 2 or 3 or 4 or 5 or 6 is an event.

Sample Space: The set of all possible outcomes of an experiment is called a *sample space*. We generally denote it by S.

Illustration 4:

(i) When a coin is tossed, $S = \{H, T\}$ where H = head, T = tail.

(ii) When a die is thrown, $S = \{1, 2, 3, 4, 5, 6\}$.

(iii) When 2 coins are tossed simultaneously,

$$S = \{HH, HT, TH, TT\}$$

Equally Likely Events: Events are said to be *equally likely* if there is no reason to expect any one in preference to other. Thus, equally likely events mean outcome is as likely to occur as any other outcome.

Illustration 5: In throwing a die, all the 6 faces (1, 2, 3, 4, 5, 6) are equally likely to occur.

Simple and Compound Events

In the case of *simple events* we consider the probability of happening or non-happening of single events.

Illustration 6: We might be interested in finding out the probability of drawing an ace from a pack of cards.

In the case of *compound events* we consider the joint occurrence of two or more events.

Illustration 7: If from a bag, containing 8 red and 5 green balls, two successive draws of 2 balls are made, we shall be finding out the probability of getting 2 red balls in the first draw and 2 green balls in the second draw. We are thus dealing with a compound event.

Exhaustive Events: It is the total number of all possible outcomes of any trial.

Illustration 8:

(i) When a coin is tossed, either head or tail may turn up and therefore, there are two exhaustive cases.

(ii) There are six exhaustive cases or events in throwing a die.

(iii) If two dice are thrown simultaneously, the possible outcomes are:

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Thus, in this case, there are $36(=6^2)$ ordered pairs. Hence, the number of exhaustive cases in the simultaneous throw of two dice is 36.

(iv) Three dice are thrown, the number of exhaustive cases is 6^3, i.e., 216.

Algebra of Events

If A and B are two events associated with sample space S, then

(i) $A \cup B$ is the event that either A or B or both occur.

(ii) $A \cap B$ is the event that A and B both occur simultaneously.

(iii) $\overline{A}$ is the event that A does not occur.

(iv) $\overline{A} \cap \overline{B}$ is an event of non-occurrence of both A and B, i.e., none of the events A and B occurs.

Illustration 9: In a single throw of a die, let A be the event of getting an even number and B be the event of getting a number greater than 2. Then,

$$A = \{1, 3, 5\},\ B = \{3, 4, 5, 6\}$$

$$\therefore \quad A \cup B = \{1, 3, 4, 5, 6\}$$

$A \cup B$ is the event of getting an odd number or a number greater than 2.

$$A \cap B = \{3, 5\}.$$

$A \cap B$ is the event of getting an odd number greater than 2.

$\overline{A} = \{2, 4, 6\}$ [Those elements of S which are not in A.]

$\overline{A}$ is the event of not getting an odd number, i.e., getting an even number.

$$\overline{B} = \{1, 2\}.$$

$\overline{B}$ is the event of not getting a number greater that 2, i.e., getting a number less than or equal to 2.

$$\overline{A} \cap \overline{B} = \{2\}.$$

$\overline{A} \cap \overline{B}$ is the event of neither getting an odd number nor a number greater than 2.

Mutually Exclusive Events

In an experiment, if the occurrence of an event precludes or rules out the happening of all the other events in the same experiment.

Illustration 10:

(i) When a coin is tossed either head or tail will appear. Head and tail cannot appear simultaneously. Therefore, occurrence of a head or a tail are two mutually exclusive events.

(ii) In throwing a die all the 6 faces numbered 1 to 6 are mutually exclusive since if any one of these faces comes, the possibility of others in the same trial, is ruled out.

Note:

A and B are mutually exclusive events $\Leftrightarrow A \cap B = \phi$, i.e., A and B are disjoint sets.

Illustration 11:

(i) If the random experiment is '*a* die is thrown' and A, B are the events, A: the number is less than 3; B: the number is more than 4, then $A = \{1, 2\}$, $B = \{5, 6\}$. $A \cap B = \phi$, thus A and B are mutually exclusive events.

(ii) If the random experiment is '*a* card is drawn from a well-shuffled pack of cards' and A, B are the events A: the card is Black; B: the card is an ace.

Since a black card can be an ace, $A \cap B \neq \phi$, thus A and B are not mutually exclusive events.

Mutually Exclusive and Exhaustive Events

Events $E_1, E_2, ..., E_n$ are called mutually exclusive and exhaustive if $E_1 \cup E_2 \cup ... \cup E_n = S$, i.e., $\bigcup_{i=1}^{n} E_i = S$ and $E_i \cap E_j = \phi$ for all $i \neq j$.

For example, in a single throw of a die, let A be the event of getting an even number and B be event of getting odd numbers, then

$$A = \{2, 4, 6\}, B = \{1, 3, 5\}$$

$$A \cap B = \phi, A \cup B = \{1, 2, 3, 4, 5, 6\} = S$$

$\therefore$ A and B are mutually exclusive and exhaustive events.

Illustration 12: Two dice are thrown and the sum of the numbers which come up on the dice noted. Let us consider the following events:

A: 'the sum is even'
B: 'the sum is a multiple of 3'
C: 'the sum is less than 4'
D: 'the sum is greater than 11'

Which pairs of these events are mutually exclusive?

Solution: There are $6 \times 6 = 36$ elements in the sample space (Refer to Example 2).

A is the event "the sum is even". It means we have to consider those ordered pairs (x, y) in which $(x + y)$ is even. Thus,

$A = [(1, 1), (2, 2) (1, 3), (1, 5), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6) (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)]$.

Similarly,

$B = [(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)]$

$C = [(1, 1), (2, 1), (1, 2)]$ $D = [(6, 6)]$.

We find that $A \cap B = [(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6)] \neq \phi$

Thus, A and B are not mutually exclusive.

Similarly, $A \cap C \neq \phi$, $A \cap D \neq \phi$, $B \cap C \neq \phi$, $B \cap D \neq \phi$, $C \cap D = \phi$. Thus, C and D are mutually exclusive.

PROBABILITY OF AN EVENT

The probability of an event is defined in the following two ways:

(i) Mathematical (or *a priori*) definition
(ii) Statistical (or empirical) definition.

Mathematical Definition of Probability: Probability of an event A, denoted as $P(A)$, is defined as

$$P(A) = \frac{\text{Number of cases favourable to } A}{\text{Number of possible outcomes}}$$

Thus, if an event A can happen in m ways and fails (does not happen) in n ways and each of $m + n$ ways is equally likely to occur then the probability of happening of the event A (also called success of A) is given by

$$P(A) = \frac{m}{m+n}$$

and that the probability of non-occurrence of the A (also called its failure) is given by

$$P(\text{not } A) \text{ or } P(\bar{A}) = \frac{n}{m+n}$$

If the probability of the happening of a certain event is denoted by p and that of not happening by q, then

$$p + q = \frac{m}{m+n} + \frac{n}{m+n} = 1.$$

Here, p, q are non-negative and cannot exceed unity, i.e., $0 \leq p \leq 1$ and $0 \leq q \leq 1$.

When $p = 1$, then the event is certain to occur.

When $p = 0$, then the event is impossible. For example, the probability of throwing eight with a single die is zero.

Probability as defined above is sometimes called **Priori Probability,** i.e., it is determined before hand, that is, before the actual trials are made.

Illustration 13: A coin is tossed once. What are all possible outcomes? What is the probability of the coin coming up 'tails'?

Solution: The coin can come up either "heads" (H) or "tails" (T). Thus, the set S of all possible outcomes is $S = \{H, T\}$

$$\therefore \quad P(T) = \frac{1}{2}$$

Illustration 14: What is the probability of getting an even number in a single throw of a die?

Solution: Clearly, a die can fall with any of its faces upper most. The number on each of the faces is, therefore, a possible outcome. Thus, there are total 6 outcomes. Since there are 3 even numbers on the die, namely, 2, 4 and 6,

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}.$$

Illustration 15: What is the probability of drawing a 'king' from a well-shuffled deck of 52 cards?

Solution: Well-shuffled ensures equally-likely outcomes. There are 4 kings in a deck. Thus,

$$P(\text{a king}) = \frac{4}{52} = \frac{1}{13}.$$

Odds of an Event

Suppose, there are m outcomes favourable to a certain event and n outcomes unfavourable to the event in a sample space, then odds in favour of the event

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}} = \frac{m}{n}$$

and odds against the event

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcome}} = \frac{n}{m}.$$

If odds in favour of an event A are $a:b$, then the probability of happening of event $A = P(A) = \frac{a}{a+b}$ and probability of not happening of event $A = P(\bar{A}) = \frac{b}{a+b}$.

If odds against happening of an event A are $a:b$, then probability of happening of event $A = P(A) = \frac{b}{a+b}$ and probability of not happening of event$A = P(\bar{A}) = \frac{a}{a+b}$.

Illustration 16: What are the odds in favour of getting a '3' in a throw of a die? What are the odds against getting a '3'?

Solution: There is only one outcome favourable to the event "getting" a 3, the other five outcomes, namely, 1, 2, 4, 5, 6 are unfavourable. Thus,

Odds in favour of getting a '3'

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$$

$$= \frac{1}{5} \text{ or } 1 \text{ to } 5.$$

Odd against getting a '3'

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}$$

$$= \frac{5}{1} \text{ or } 5 \text{ to } 1.$$

Illustration 17: If the odds in favour of an event are 4 to 5, find the probability that it will occur.

Solution: The odds in favour of the event are $\frac{4}{5}$. Thus,

$$\frac{P(A)}{1-P(A)} = \frac{4}{5}, \text{ i.e., } 4[1- P(A)] = 5P(A),$$

i.e., $P(A) = \frac{4}{9}$.

The probability that it will occur $= \frac{4}{9}$.

FUNDAMENTAL THEOREMS ON PROBABILITY

Theorem 1: In a random experiment, if S is the sample space and E is an event, then

(i) $P(E) \geq 0$ (ii) $P(\phi) = 0$ (iii) $P(S) = 1$.

Remarks: It follows from above results that,

(i) probability of occurrence of an event is always non-negative;
(ii) probability of occurrence of an impossible event is 0;
(iii) probability of occurrence of a sure event is 1.

Theorem 2: If E and F are mutually exclusive events, then,

(i) $P(E \cap F) = 0$ and,
(ii) $P(E \cup F) = P(E) + P(F)$.

Notes:

1. For mutually exclusive events E and F, we have
$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F)$.
2. If $E_1, E_2, \ldots, E_k$ are mutually exclusive events, then,
$$P(E_1 \cup E_1 \cup, \cdots, \cup E_k) = P(E_1) + P(E_2) + \cdots + P(E_k).$$

Theorem 3: If E and F are two mutually exclusive and exhaustive events, then $P(E) + P(F) = 1$.

Theorem 4: Let E be any event and $\bar{E}$ be its complementary event, then $P(\bar{E}) = 1 - P(E)$.

Theorem 5: For any two events E and F,

$$P(E - F) = P(E) - P(E \cap F).$$

Theorem 6: (Addition Theorem). For any two events E and F,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Notes:

1. We may express the above results as
$P(E \text{ or } F) = P(E) + (F) - P(E \text{ and } F)$
2. If E and F are mutually exclusive, then
$P(E \cap F) = 0$ and so $P(E \cup F) = P(E) + P(F)$.

Theorem 7: If E_1 and E_2 be two events such that $E_1 \subseteq E_2$, then prove that $P(E_1) \leq P(E_2)$.

Theorem 8: If E is an event associated with a random experiment, then $0 \leq P(E) \leq 1$.

Theorem 9: For any three events E, F, G

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$$

Illustration 18: *A* card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting

(i) a jack or a queen or a king,

(ii) a two of heart or diamond.

Solution:

(i) In a pack of 52 cards, we have:

4 jacks, 4 queens and 4 kings.

Now, clearly a jack and a queen and a king are mutually exclusive events.

$$\text{Also,} P(\text{a jack}) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{a queen}) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{a king}) = \frac{^4C_1}{^{52}C_1} = \frac{4}{52} = \frac{1}{13}.$$

$\therefore$ By the addition theorem of Probability,

P(a jack or a queen or a king) = P(a jack) + P(a queen) + P(a king)

$$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13} = \frac{3}{13}.$$

(ii) P(two of heart or two of diamond)

= P(two of heart) + P(two of diamond)

$$= \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26}.$$

Illustration 19: Find the probability of getting a sum of 7 or 11 in a simultaneous throw of two dice.

Solution: When two dice are thrown we have observed that there are 36 possible outcomes. Now, we can have a sum of 7 as

$1 + 6 = 7$, $2 + 5 = 7$, $3 + 4 = 7$, $4 + 3 = 7$, $5 + 2 = 7$, $6 + 1 = 7$

Thus, the six favourable cases are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

$$\therefore \quad P(\text{a sum of } 7) = \frac{6}{36} = \frac{1}{6}.$$

Again, the favourable cases of getting a sum of 11 are (5, 6), (6, 5)

$$\therefore \quad P(\text{a sum of } 11) = \frac{2}{36} = \frac{1}{18}.$$

Since the events of getting 'a sum of 7' or 'a sum of 11' are mutually exclusive:

$\therefore$ P(a sum of 7 or 11)

= P(a sum of 7) + P(a sum of 11)

$$= \frac{1}{6} + \frac{1}{18} = \frac{4}{18} = \frac{2}{9}.$$

Illustration 20: From a well-shuffled pack of 52 cards, a card is drawn at random, find the probability that it is either a heart or a queen.

Solution: *A*: Getting a heart card *B*: Getting a queen card

$$P(A) = \frac{13}{52},\ P(B) = \frac{4}{52},\ P(A \cap B) = \frac{1}{52}$$

Required probability

$$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$

INDEPENDENT EVENTS

Two event *A* and *B* are said to be independent if the occurrence (or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.

Illustration 21: In the simultaneous throw of 2 coins, 'getting a head' on first coin and 'getting a tail on the second coin are independent events.

Illustration 22: When a card is drawn from a pack of well-shuffled cards and replaced before the second card is drawn, the result of second draw is independent of first draw. We now state, without proof, the theorem which gives the probabilities of simultaneous occurrence of the independent events.

Theorem 10: If *A* and *B* are two independent events, then

$$P(A \text{ and } B) = P(A).\ P(B)$$

Illustration 23: Two dice are thrown. Find dthe probability of getting an odd number on the one die and a multiple of three on the other.

Solution: Since the events of 'getting an odd number' on one die and the event of getting a multiple of three on the other are independent events,

$$P(A \text{ and } B) = P\ (A) \times P(B) \quad (1)$$

Now, $P(A) = P(\text{an odd number}) = \frac{3}{6} = \frac{1}{2}$ [There are 3 odd numbers 1, 3, 5] and $P(B) = P(\text{a multiple of } 3) = \frac{2}{6} = \frac{1}{3}$ [Multiples of 3 are 3 and 6]

$$\therefore \quad \text{From (1), required probability} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.$$

Illustration 24: Arun and Tarun appear for an interview for 2 vacancies. The probability of Arun's selection is 1/3 and that of Tarun's selection is 1/5. Find the probability that

(i) only 1 of them will be selected,

(ii) none of them be selected.

Solution: Let A: Arun is s selected B: Tarun is selected.

Then, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$.

Clearly, 'A' and 'not B' are independent also 'not A' and 'not B' are independent, 'B' and 'not A' are independent.

(i) P (only 1 of them will be selected)

$= P$ (A and not B or B and not A)

$= P(A)\ P(\text{not } B) + P(B)\ P(\text{not } A)$

$$= \frac{1}{3}\left(1-\frac{1}{5}\right)+\frac{1}{5}\left(1-\frac{1}{3}\right)$$

$$= \frac{1}{3}\times\frac{4}{5}\times\frac{1}{5}\times\frac{2}{5}=\frac{4}{15}+\frac{2}{12}$$

$$= \frac{6}{15}=\frac{2}{5}.$$

(ii) P (onle 1 of them be selected)

$= P$ (not A and not B)

$= P(\text{not } A) \times P(\text{not } B)$

$$= \left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{5}\right)$$

$$= \frac{2}{3}\times\frac{4}{5}=\frac{8}{15}.$$

EXERCISE-1

1. In a simultaneous toss of 2 coins, then find the probability of 2 tails.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

2. In a simultaneous toss of 2 coins, then find the probability of exactly 1 tail.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) None of these

3. In a simultaneous toss of 2 coins, then find the probability of no tail.

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) None of these

4. 3 coins are tossed. Find the probability of heads.

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) None of these

5. 3 coins are tossed. Find the probability of exactly 2 heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{1}{8}$ (d) None of these

6. 3 coins are tossed. Find the probability of at least 2 heads.

(a) $\frac{1}{2}$ (b) $\frac{3}{8}$

(c) $\frac{1}{8}$ (d) None of these

7. 3 coins are tossed. Find the probability of atmost 2 heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{7}{8}$ (d) None of these

8. 3 coins are tossed. Find the probability of no heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$

(c) $\frac{1}{2}$ (d) None of these

9. 3 coins are tossed. Find the probability of at least 1 head and 1 tail.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) None of these

10. A coin is tossed 3 times. Find the chance that head and tail show alternately.

(a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) None of these

11. 4 coins are tossed once. Find the probability of 4 tails.

(a) $\frac{1}{16}$ (b) $\frac{5}{16}$

(c) $\frac{9}{16}$ (d) None of these

12. 4 coins are tossed once. Find the operability of exactly 3 tails.

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$

(c) $\frac{5}{16}$ (d) None of these

13. 4 coins are tossed once. Find the probability of exactly 2 tails.

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{3}{8}$ (d) $\frac{5}{16}$

14. 4 coins are tossed once. Find the probability by of at least 1 tail.

(a) $\frac{1}{16}$ (b) $\frac{15}{16}$

(c) $\frac{3}{16}$ (d) None of these

15. In a single throw of 2 dice, find the probability of getting a total of 3 or 5.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

16. In a single throw of 2 dice, find the probability of getting a total of 12.

(a) $\frac{1}{36}$ (b) $\frac{1}{9}$

(c) $\frac{1}{18}$ (d) $\frac{35}{36}$

17. In a single throw of 2 dice, find the probability of getting a total of 11.

(a) $\frac{1}{9}$ (b) $\frac{1}{18}$

(c) $\frac{1}{12}$ (d) $\frac{35}{36}$

18. In a single throw of 2 dice, find the probability of getting a total of 8.

(a) $\frac{5}{36}$ (b) $\frac{1}{18}$

(c) $\frac{1}{12}$ (d) $\frac{31}{36}$

19. In a single throw of 2 dice, the probability of getting a total of 7.

(a) $\frac{5}{36}$ (b) $\frac{1}{18}$

(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

20. In a single throw of 2 dice, what is the probability of a doublet?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{1}{9}$ (d) $\frac{1}{18}$

21. In a single throw of 2 dice, what is the probability of a multiple of 2 on 1 and a multiple of 3 on the other?

(a) $\frac{5}{6}$ (b) $\frac{25}{36}$

(c) $\frac{11}{36}$ (d) $\frac{1}{9}$

22. 2 dice are thrown. Find the probability of getting an odd number on 1 and a multiple of 3 on the other.

(a) $\frac{5}{6}$ (b) $\frac{25}{36}$

(c) $\frac{11}{36}$ (d) $\frac{1}{9}$

23. Doublet of even numbers.

(a) $\frac{1}{36}$ (b) $\frac{1}{18}$

(c) $\frac{1}{12}$ (d) $\frac{1}{9}$

24. A sum less than 6.

(a) $\frac{7}{18}$ (b) $\frac{5}{18}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

25. A sum more than 7.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{5}{12}$

26. A sum greater than 10.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{5}{12}$

27. A sum at least 10.

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

28. An odd number as the sum.

(a) $\frac{1}{36}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

29. An even number as the sum.

(a) $\frac{1}{36}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

30. 6 as the product.

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

31. A multiple of 3 as the sum.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{9}$ (d) $\frac{5}{36}$

32. The product a perfect square (square of a natural number).

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{1}{3}$ (d) $\frac{4}{9}$

33. At least 1 of the 2 numbers as 4.

(a) $\frac{1}{4}$ (b) $\frac{5}{18}$

(c) $\frac{11}{36}$ (d) $\frac{1}{3}$

34. Sum as a prime number

(a) $\frac{5}{12}$ (b) $\frac{1}{2}$

(c) $\frac{7}{12}$ (d) $\frac{3}{4}$

35. In a single throw of 3 dice, find the probability of getting a total of 17 or 18.

(a) $\frac{1}{54}$ (b) $\frac{1}{27}$

(c) $\frac{1}{18}$ (d) None of these

(Q. 36 – 38): In a single throw of 3 dice, then find the probability of getting

36. A total of 5.

(a) $\frac{1}{4}$ (b) $\frac{1}{18}$

(c) $\frac{1}{36}$ (d) $\frac{1}{9}$

37. A total of at most 5.

(a) $\frac{5}{108}$ (b) $\frac{103}{108}$

(c) $\frac{1}{18}$ (d) None of these

38. A total of at least 5.

(a) $\frac{7}{54}$ (b) $\frac{1}{54}$

(c) $\frac{53}{54}$ (d) None of these

39. What is the chance that a leap year, selected at random will contain 53 Sundays?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

40. A card is drawn from a pack of 100 cards numbered 1 to 100. Find the probability of drawing a number which is a square.

(a) $\frac{1}{10}$ (b) $\frac{9}{10}$

(c) $\frac{1}{5}$ (d) $\frac{2}{5}$

41. The letters of word 'SOCIETY' are placed in a row. What is the probability that three come together?

(a) $\frac{3}{7}$ (b) $\frac{2}{7}$

(c) $\frac{1}{7}$ (d) None of these

42. Find the probability that in a random arrangement of letters of the words 'UNIVERSITY' two '*I*'s do not come together.

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

43. If letters of the word PENCIL are arranged in random order, what is the probability that N is always next to E?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

44. 2 dice are thrown. Find the odds in favour of getting the sum 4.

(a) 1:11 (b) 11:1

(c) 4:11 (d) 11:4

45. 2 dice are thrown. Find the odds in favour of getting the sum 5.

(a) 8:1 (b) 1:8

(c) 7:8 (d) 8:7

46. 2 dice are thrown. Find the odds against getting the sum 6.

(a) 5:31 (b) 6:31

(c) 31:5 (d) 31:6

47. What is the probability that 1 card drawn at random from the pack of playing cards may be either a queen or an ace?

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$

(c) $\frac{3}{13}$ (d) None of these

48. In a class of 25 students with roll numbers 1 to 25, a student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either multiple of 5 or 7.

(a) $\frac{6}{25}$ (b) $\frac{4}{25}$

(c) $\frac{8}{25}$ (d) $\frac{7}{25}$

49. An integer is chosen at random from first two hundred natural numbers. What is the probability that the integer chosen is divisible by 6 or 8?

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) None of these

50. 2 dice are together. What is the probability that the sum of 2 numbers is divisible by 3 or by 4?

(a) $\frac{4}{9}$ (b) $\frac{2}{9}$

(c) $\frac{5}{9}$ (d) $\frac{1}{3}$

51. In a simultaneous throw of 2 dice, find $P(A$ or $B)$ if A denotes the event '*a* total of 11 and B denotes the event' 'an odd number on each die'.

(a) $\frac{11}{36}$ (b) $\frac{1}{4}$

(c) $\frac{5}{18}$ (d) $\frac{1}{6}$

52. A box contains 36 tickets numbered 1 to 36, 1 ticket drawn at random. Find the probability that the number on the ticket is either divisible by 3 or is a perfect square.

(a) $\frac{2}{9}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{1}{3}$

53. A drawer contain 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If 1 item is chosen at random, then what is the probability that it is rusted or is a bolt?

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$

(c) $\frac{5}{8}$ (d) None of these

54. 2 unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear.

(a) $\frac{7}{9}$ (b) $\frac{4}{9}$

(c) $\frac{2}{9}$ (d) $\frac{1}{3}$

55. 2 dice are thrown together. What is the probability that the sum of the number on 2 faces is neither 9 nor 11?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

56. A card is drawn from a pack of 52 cards, find the probability of getting spade or ace or red card.

(a) $\frac{9}{13}$ (b) $\frac{4}{13}$

(c) $\frac{11}{13}$ (d) $\frac{10}{13}$

57. A ticket is drawn from two hundred tickets numbered from 1 to 200, find the probability that the number is divisible by 2 or 3 or 5.

(a) $\frac{73}{100}$ (b) $\frac{27}{100}$

(c) $\frac{63}{100}$ (d) None of these

58. A and B are mutually exclusive events of an experiment. If P('not A') = 0.65, $P(A \cup B) = 0.65$ and $P(B) = p$, find the value of p.

(a) 0.70 (b) 0.30
(c) 0.63 (d) 0.35

59. The probability of an event A occurring is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, find the probability that neither A nor B occurs.

(a) 0.2 (b) 0.8
(c) 0.6 (d) None of these

60. The probabilities that a student will receive an A, B, C or D grade are 0.30, 0.38, 0.22 and 0.01, respectively. What is the probability that the student will receive at least B grade?

(a) 0.38 (b) 0.42
(c) 0.68 (d) None of these

61. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting at least 1 contract is 4/5, what is the probability that he will get both?

(a) $\frac{8}{45}$ (b) $\frac{31}{45}$

(c) $\frac{14}{45}$ (d) None of these

62. A card is drawn from an ordinary pack and a gambler bets that it is a spade or an ace. What are the odds against his winning the bet?

(a) 9:4 (b) 4:9
(c) 5:9 (d) 9:5

63. In a race the odds in favour of horses A, B, C and D are 1:3, 1:4, 1:5 and 1:6, respectively. Find the probability that one of them wins the race.

(a) $\frac{221}{420}$ (b) $\frac{391}{420}$

(c) $\frac{331}{420}$ (d) None of these

64. A Chartered Accountant applies for a job in 2 firms X and Y. The ability of his being selected in firm X is 0.7, and being rejected at Y is 0.5 and the probability of at least 1 of his applications being rejected is 0.6. What is the probability that he will be selected in 1 of the firms?

(a) 0.2 (b) 0.8
(c) 0.4 (d) 0.7

65. There are three events A, B, C one of which must and only one can happen, the odds are 8 to 3 against A, 5 to 2 against B, find the odds against C.

(a) 43:34 (b) 34:43
(c) 43:77 (d) 77:43

66. A problem in Statistics is given to four students A, B, C and D. Their chances of solving it are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$, respectively. What is the probability that the problem will be solved?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{5}$ (d) None of these

(Q. 67 – 69): 1 bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. 1 ball is drawn from each bag.

67. Find the probability that both are white.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

68. Find the probability that both are black.

(a) $\frac{5}{24}$ (b) $\frac{19}{24}$

(c) $\frac{11}{24}$ (d) $\frac{1}{24}$

69. Find the probability that 1 is white and 1 is black.

(a) $\frac{11}{24}$ (b) $\frac{13}{24}$

(c) $\frac{1}{2}$ (d) None of these

(Q. 70–77): An urn contains 25 balls numbered 1 to 25. Suppose an odd number is considered a 'success'. 2 balls are drawn from the urn with replacement.

70. Find the probability of getting two successes.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

71. Find the probability of getting exactly one success.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

72. Find the probability of getting at least one success.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

73. Find the probability of getting no success.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

74. Find the probability of getting 3 successes.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) $\frac{26}{27}$ (d) $\frac{7}{27}$

75. Find the probability of getting exactly 2 succeses.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) $\frac{26}{27}$ (d) $\frac{7}{27}$

76. Find the probability of getting at most 2 successes.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) $\frac{26}{27}$ (d) $\frac{7}{27}$

77. Find the probability of getting at least 2 successes.

(a) $\frac{1}{27}$ (b) $\frac{2}{9}$

(c) $\frac{26}{27}$ (d) $\frac{7}{27}$

78. From a pack of cards, two are drawn, the first being replaced before the second is drawn. Find the probability that the first is a diamond and the second is a king.

(a) $\frac{3}{52}$ (b) $\frac{1}{26}$

(c) $\frac{1}{52}$ (d) $\frac{1}{4}$

(Q. 79–82): A husband and wife appear in an interview for 2 vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's is 1/7.

79. What is the probability that only 1 of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

80. What is the probability that both of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

81. What is the probability that none of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

82. What is the probability that at least one of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

(Q. 83–86): Probability that a man will be alive 25 years hence is 0.3 and the probability that his wife will be alive after 25 years hence is 0.4. Find the probability that 25 years hence.

83. Both will be alive.

(a) 0.12 (b) 0.18
(c) 0.28 (d) 0.58

84. Only the man will be alive.

(a) 0.12 (b) 0.18
(c) 0.28 (c) 0.58

85. Only the woman will be alive.

(a) 0.12 (b) 0.18
(c) 0.28 (d) 0.58

86. At least 1 of them will be alive.

(a) 0.12 (b) 0.18
(c) 0.28 (d) 0.58

87. A man speaks truth in 80% of the cases and another in 90% of the cases. While stating the same fact, what is the probability that they contradict?

(a) $\frac{37}{50}$ (b) $\frac{13}{50}$

(c) $\frac{16}{50}$ (d) None of these

88. There are 3 urns *A*, *B* and *C*, *A* contains 4 red balls and 3 black balls. Urn *B* contains 5 red balls and 4 black balls. Urn *C* contains 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the 3 balls drawn consist of 2 red balls and a black ball?

(a) $\frac{47}{42}$ (b) $\frac{25}{42}$

(c) $\frac{19}{42}$ (d) $\frac{23}{42}$

89. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second third and fourth shots are 0.4, 0.3, 0.2 and 0.1, respectively. What is the probability that the gun hits the plane?

(a) 0.4379 (b) 0.6872
(c) 0.6976 (d) None of these

90. *A* can solve 90% of the problems given in a book and *B* solve 70%. What is the probability that at least 1 of them will solve a problem selected at random from the book?

(a) $\frac{3}{100}$ (b) $\frac{97}{100}$

(c) $\frac{83}{100}$ (d) $\frac{17}{100}$

91. *A* and *B* throw a coin alternately till 1 of them gets a head and wins the game. If *A* starts the game, find the probability of winning of *A*.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) None of these

92. 2 persons *A* and *B* throw a die alternately till 1 of them gets a '6' and wins the game. Find the probability of winning of *B*.

(a) $\frac{5}{11}$ (b) $\frac{6}{11}$

(c) $\frac{4}{11}$ (d) $\frac{3}{11}$

93. The letters of the word 'SOCIETY' are placed at row. What is probability that the 3 vowels come together?

(a) $\frac{4}{7}$ (b) $\frac{3}{7}$

(c) $\frac{2}{7}$ (d) $\frac{1}{7}$

94. Find the probability that in a random arrangement of the letters of the word DAUGHTER, the letter D occupies the first place.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

Exercise-2
(Based on Memory)

Directions (Q. 1–2): Study the information carefully to answer the questions that follow:

A basket contains 3 blue, 2 green and 5 red balls.

1. If 3 balls are picked at random, what is the probality that at least on is red?

(a) $\frac{1}{2}$ (b) $\frac{7}{12}$

(c) $\frac{11}{12}$ (d) $\frac{1}{5}$

(e) None of these

[Bank of Maharashtra PO, 2007]

2. If 4 balls are picked at random, what is the probability that 2 are green and 2 are blue?

(a) $\frac{1}{18}$ (b) $\frac{1}{70}$

(c) $\frac{3}{5}$ (d) $\frac{1}{2}$

(e) None of these

[Bank of Maharashtra PO, 2007]

3. Out of 15 students studying in a class, 7 are from Maharashtra, 5 are from Karnataka and 3 are from Goa. 4 students are to be selected at random. What are the chances that at least 1 is from Karnataka?

(a) $\frac{12}{13}$ (b) $\frac{11}{13}$

(c) $\frac{10}{15}$ (d) $\frac{1}{15}$

(e) None of these

[Guwahati PO, 1999]

4. The probability that a teacher will give one surprise test during any class meeting in a week is $\frac{1}{5}$. If a student is absent twice, what is the probability that he will miss at least one test?

(a) $\frac{4}{15}$ (b) $\frac{1}{15}$

(c) $\frac{91}{25}$ (d) $\frac{16}{125}$

(e) None of these

[BSRB Mumbai PO, 1999]

5. A speaks truth in 75% and *B* in 80% cases. In what percentage of case are they likely to contradict each other when narrating the same incident?

(a) 35 (b) 30

(c) 25 (d) 20

(e) None of these

[BSRB Chennai PO, 2000]

6. In a box there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?

(a) $\frac{7}{19}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{9}{21}$

(e) None of these

[IBPS Bank PO Examination, 2002]

7. An urn contains 3 red and 4 green marbles. If 3 marbles are picked at random, then what is the probability that 2 are green and 1 is red?

(a) $\frac{3}{7}$ (b) $\frac{18}{35}$

(c) $\frac{5}{14}$ (d) $\frac{4}{21}$

(e) None of these

[New Indian Insurance PO, 2009]

8. A basket contains 3 blue and 4 red balls. If 3 balls are drawn at random from the basket, then what is the probability that all the 3 are either blue or red ?

(a) 1 (b) $\frac{1}{7}$

(c) $\frac{3}{14}$ (d) None of these

[Bank of India PO, 2010]

9. A bag contains 13 white and 7 black balls. 2 balls are drawn at random. What is the probability that they are of the same colour?

(a) $\frac{41}{190}$ (b) $\frac{21}{190}$

(c) $\frac{59}{190}$ (d) $\frac{99}{190}$

(e) $\frac{77}{190}$

[IBPS PO/MT, 2012])

Direction (Q. 10–14): Study the given information carefully to answer the question that follow.

An urn contains 4 green, 5 blue, 2 red and 3 yellow marbles.

10. If 2 marbles are drawn at random, what is the probability that both are red or at least 1 is red?

(a) $\frac{26}{91}$ (b) $\frac{1}{7}$

(c) $\frac{199}{364}$ (d) $\frac{133}{191}$

(e) None of these

[IBPS PO/MT, 2011]

11. If 3 marbles are drawn at random, what is the probability that at least 1 is yellow?

(a) $\frac{1}{3}$ (b) $\frac{199}{364}$

(c) $\frac{165}{364}$ (d) $\frac{3}{11}$

(e) None of these

[IBPS PO/MT, 2011]

12. If 8 marbles are drawn at random, what is the probability that there are equal numbers of marbles of each colour?

(a) $\frac{4}{7}$ (b) $\frac{361}{728}$

(c) $\frac{60}{1001}$ (d) $\frac{1}{1}$

(e) None of these

[IBPS PO/MT, 2011]

13. If 3 marbles are drawn at random, what is the probability that none is green?

(a) $\frac{2}{7}$ (b) $\frac{253}{728}$

(c) $\frac{10}{21}$ (d) $\frac{14}{91}$

(e) $\frac{30}{91}$

[IBPS PO/MT, 2011])

14. If 4 marbles are drawn at random, what is the probability that 2 are blue and 2 are red?

(a) $\frac{10}{1001}$ (b) $\frac{9}{14}$

(c) $\frac{17}{364}$ (d) $\frac{2}{7}$

(e) None of these

[IBPS PO/MT, 2011]

15. Out of 5 girls and 3 boys, 4 children are to be randomly selected for a quiz contest. What is the probability that all the selected children are girls?

(a) $\frac{1}{14}$ (b) $\frac{1}{7}$

(c) $\frac{5}{17}$ (d) $\frac{2}{17}$

(e) None of these

[SBI Associates Banks PO, 2011]

Directions (Q. 16–18): Study the given information carefully and answer the questions that follow:

A basket contains 4 red, 5 blue and 3 green marbles.

16. If 3 marbles are picked at random, what is the probability that either all are green or all are red?

(a) $\frac{7}{44}$ (b) $\frac{7}{12}$

(c) $\frac{5}{12}$ (d) $\frac{1}{44}$

(e) None of these

[SBI Associate Banks PO, 2010]

17. If 2 marbles are drawn at random, what is the probability that both are red?

(a) $\frac{3}{7}$ (b) $\frac{1}{2}$

(c) $\frac{2}{11}$ (d) $\frac{1}{6}$

(e) None of these

[SBI Associate Banks PO, 2010]

18. If 3 marbles are picked at random, what is the probability that at least 1 is blue?

(a) $\frac{7}{12}$ (b) $\frac{37}{44}$

(c) $\frac{5}{12}$ (d) $\frac{7}{44}$

(e) None of these

[SBI Associate Banks PO, 2010]

Directions (Q. 19–23): Study the following information carefully to answer the questions that follow:

A box contains 2 blue caps, 4 red caps, 5 green caps and 1 yellow cap.

19. If 4 caps are picked at random, what is the probability that none is green?

(a) $\frac{7}{99}$ (b) $\frac{5}{99}$

(c) $\frac{7}{12}$ (d) $\frac{5}{12}$

(e) None of these

[OBC PO, 2009]

20. If 2 caps are picked at random, what is the probability that both are blue?

(a) $\frac{1}{6}$ (b) $\frac{1}{10}$

(c) $\frac{1}{12}$ (d) $\frac{1}{45}$

(e) None of these

[OBC PO, 2009]

21. If 1 cap is picked at random, what is the probability that it is either blue of yellow?

(a) $\frac{2}{9}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{6}{11}$

(e) None of these

[OBC PO, 2009]

22. If 2 caps are picked at random, what is the probability that at least 1 is red?

(a) $\frac{1}{3}$ (b) $\frac{16}{21}$

(c) $\frac{19}{33}$ (d) $\frac{7}{19}$

(e) None of these

[OBC PO, 2009]

23. If 3 caps are picked at random, what is the probability that 2 are red and 1 is green?

(a) $\frac{9}{22}$ (b) $\frac{6}{19}$

(c) $\frac{1}{6}$ (d) $\frac{3}{22}$

(e) None of these

[OBC PO, 2009]

ANSWER KEYS

Exercise-1

1. (b)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (b)	**11.** (a)	**12.** (b)	**13.** (c)
14. (b)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (a)	**21.** (c)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (a)
27. (b)	**28.** (d)	**29.** (c)	**30.** (a)	**31.** (b)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (a)	**38.** (c)	**39.** (b)
40. (a)	**41.** (c)	**42.** (a)	**43.** (a)	**44.** (a)	**45.** (b)	**46.** (c)	**47.** (b)	**48.** (c)	**49.** (a)	**50.** (c)	**51.** (a)	**52.** (b)
53. (c)	**54.** (a)	**55.** (b)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (a)	**60.** (c)	**61.** (c)	**62.** (a)	**63.** (b)	**64.** (b)	**65.** (a)
66. (b)	**67.** (c)	**68.** (a)	**69.** (c)	**70.** (a)	**71.** (b)	**72.** (c)	**73.** (d)	**74.** (a)	**75.** (b)	**76.** (c)	**77.** (d)	**78.** (c)
79. (a)	**80.** (b)	**81.** (c)	**82.** (d)	**83.** (a)	**84.** (b)	**85.** (c)	**86.** (d)	**87.** (b)	**88.** (a)	**89.** (c)	**90.** (b)	**91.** (b)
92. (a)	**93.** (d)	**94.** (a)										

Exercise-2

1. (c)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (e)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (e)	**11** (b)	**12.** (c)	**13.** (e)
14. (a)	**15.** (a)	**16.** (d)	**17.** (e)	**18.** (b)	**19.** (a)	**20.** (e)	**21.** (b)	**22.** (c)	**23.** (d)			

EXPLANATORY ANSWERS

EXERCISE-1

1. **(b)** Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4
There is only one favourable case TT.
$\therefore \quad P(2 \text{ tails}) = \frac{1}{4}$.

2. **(a)** Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4
There are two favourable cases HT, TH.
$\therefore \quad P(\text{exactly 1 tail}) = \frac{2}{4} = \frac{1}{2}$.

3. **(c)** Sample space S = {HH, HT, TH, TT}
Number of exhaustive cases = 4.
There is only one favourable case HH.
$\therefore \quad P(\text{no tails}) = \frac{1}{4}$.

4. **(b)** Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}
Number of exhaustive cases = 8
There is only one favourable case HHH.
$\therefore \quad P(\text{all heads}) = \frac{1}{8}$.

5. **(a)** Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}
Number of exhaustive cases = 8.
There are three favourable cases HHT, HTH, THH.
$\therefore \quad P(\text{exactly 2 heads}) = \frac{3}{8}$.

6. **(a)** Sample space S = {HHH, HHT, HTH, HTT, TTH, THH, TTT}
Number of exhaustive cases = 8
There are four favourable cases HHT, HTH, THH, HHH.
$\therefore \quad P(\text{at least 2 heads}) = \frac{4}{8} = \frac{1}{2}$.

7. (c) Sample space S = {HHH, HHT, HTH, HTT, TTH, THH, TTT}
Number of exhaustive cases = 8.
$P(\text{atmost 2 heads}) = P(\text{not 3 heads})$
$= 1 - P(3 \text{ heads}) = 1 - \frac{1}{8} = \frac{7}{8}$.

8. **(b)** Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}
Number of exhaustive cases = 8
$P(\text{no heads}) = P(\text{all tails}) = \frac{1}{8}$.
($\because$ there is only favourable case ttt).

9. **(c)** There are 6 favourable cases HHT, HTH, HTT, THT, TTH, THH.
Required probability = $\frac{6}{8} = \frac{3}{4}$.

10. **(b)** Sample space S = {HHH, HHT, HTT, HTT, THH, THT, TTH, TTT}
Number of exhaustive cases = 8.
Favourable cases are HTH, THT
Number of favourable cases = 2.
$\therefore \quad$ Required probability = $\frac{2}{8} = \frac{1}{4}$.

11. **(a)** There are $2^4 = 16$ possible outcomes.
Sample space S = {HHHH, HHHT, HHTH, HTHH,THHH, HHTT, HTHT, HTTH, THTH, TTHH, TTHH, TTTH, TTHT, THTT, HTTT, TTTT}
There is only one favourable case TTTT.
$P(4 \text{ tails}) = \frac{1}{16}$.

12. **(b)** There are 4 favourable cases TTTH, TTHT, THTT, HTTT,
$\therefore \quad P(\text{exactly 3 tails}) = \frac{4}{16} = \frac{1}{4}$.

13. **(c)** There are 6 favourable cases HHTT, HTHT, HTTH, THHT, THTH, TTHH.
$\therefore \quad P(\text{exactly 2 tails}) = \frac{6}{16} = \frac{3}{8}$

14. **(b)** P (at least 1 tail) = P(not all heads)
$= 1 - P(\text{all heads})$
$= 1 - \frac{1}{16} = \frac{15}{16}$.

15. **(c)** A total of 3 or 5 may be obtained in 6 ways, viz, (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1).
Number of exhaustive cases = $6 \times 6 = 36$.
$\therefore \quad$ Probability of getting a total of 3 or 5 = $\frac{6}{36} = \frac{1}{6}$.

16. **(a)** A total of 12 may be obtained in 1 way, viz, (6, 6).
$\therefore \quad$ Required probability = $\frac{1}{36}$.

17. **(b)** A total of 11 may be obtained in 2 ways, viz, (5, 6), (6, 5).
$\therefore \quad$ Required probability = $\frac{2}{36} = \frac{1}{18}$.

18. **(a)** A total of 8 may be obtained in 5 ways, viz, (2, 6), (3, 5), (4, 4), (5, 3), 6, 2).
$\therefore \quad$ Required probability = $\frac{5}{36}$.

19. **(c)** A total of 7 may be obtained in 6 ways, viz, (1, 6), (2, 5), (3, 4), (5, 2), (6, 1).

$\therefore$ Required probability $= \frac{6}{36} = \frac{1}{6}$.

20. **(a)** A 'doublet' means that both the dice show the same number on the upper most faces. Therefore, the outcomes, favourable to this event are

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Thus, the number of favourable cases = 6.

Hence, $P(\text{doublet}) = \frac{6}{36} = \frac{1}{6}$.

21. **(c)** In this case, the favourable cases are

(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (3, 4), (3, 6), (6, 2), (6, 4).

Thus, the number of favourable cases = 11.

$\therefore$ Required probability $= \frac{11}{36}$.

22. **(c)** Favourable cases are

(1, 3), (1, 6), (3, 3), (3, 6), (5, 3), (5, 6), (3, 1), (6, 1), (6, 3), (3, 5), (6, 6)

Total Number of exhaustive cases = 6 × 6 = 36

$\therefore$ Required probability $= \frac{11}{36}$.

23. **(c)** A: Getting doublet of even number A =[(2, 2), (4, 4), (6, 6)]

$$n(A) = 3 \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$.

24. **(b)** A: Getting total less than 6

A = [(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (1, 3), (4, 1), (1,4), (3, 2), (2, 3)]

$$n(A) = 10, \; n(A), \; n\ (S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{10}{36} = \frac{5}{18}$.

25. **(d)** A: Getting total more than 7

[(5, 3), (3, 5), (6, 2), (2, 6), (4, 4), (6, 3), (3, 6), (5, 4), (4, 5),

(6, 4), (4, 6), (5, 5), (6, 5), (5, 6), (6, 6)]

$$n\ (A) = 15, \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}$.

26. **(a)** A: Sum greater than 10.

A = [(6, 5), (5, 6), (6, 6)] $n(A) = 3$, $n(S) = 36$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$.

27. **(b)** A: a sum of at least 10 (i.e., 10 or more than 10)

A = [(6, 4), (4, 6), (5, 5), (6, 5), (5, 6), (6, 6)]

$$n(A) = 6, \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$.

28. **(d)** A: Getting sum as an odd number

A = [(1, 2), (2, 1), (1, 4), (4, 1), (2, 3), (3, 2), (4, 3), (3, 4), (6, 1), (1, 6), (5, 2), (2, 5), (5, 4) (4, 5), (6, 3), (6, 5), (5, 6)]

$$n(A) = 18, \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$.

29. **(c)** A: Getting even number as the sum

A = [(1, 1), (1, 3), (3, 1), (2, 2), (3, 3), (4, 2), (2, 4), (5,1), (1, 5), (6, 2), (2, 6), (5, 3), (3, 5), (4, 4), (5, 5), (6, 4), (4, 6), (6, 6)]

$$n(A) = 18, \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$.

30. **(a)** A: Getting 6 as the product

A = [(1, 6), (6, 1), (2, 3), (3, 2)]

$$n(A) = 4, \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$.

31. **(b)** A: Getting a multiple of 3 as the sum

A = [(1, 2), (2, 1), (3, 3), (5, 1), (1, 5), (4, 2), (2, 4), (6,3), (3, 6), (4, 5), (5, 4), (6, 6)]

$$n(A) = 12, \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{12}{36} = \frac{1}{3}$.

32. **(b)** A: Getting the product of two numbers to be a perfect square

A = [(1, 1), (1, 4), (2, 2), (4, 1), (3, 3), (4, 4), (5, 5), (6, 6)]

$$n(A) = 8, \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{8}{36} = \frac{2}{9}$.

33. **(c)** A: Getting at least one of the two number as 4.

A = [(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4 ,2), (4, 3), (4, 5), (4, 6)]

$n(A) = 11$, $n(S) = 36$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{11}{36}$.

34. **(a)** A: Getting sum as a prime number

A = [(1, 1), (1, 2), (2, 1), (2, 3), (3, 2), (4, 1), (1, 4), (4, 3), (3, 4), (6, 1), (1, 6), (2, 5), (5, 2), (6, 5), (5, 6)]

$$n(A) = 11, \; n(S) = 36$$

$\therefore$ Required probability $= P(A) = \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}$.

35. **(a)** Number of exhaustive cases in a single throw of three dice

$= 6 \times 6 \times 6 = 216$

Cases favourable to a total of 17 are (5, 6, 6), (6, 5, 6), (6, 6, 5)

Number of cases favourable to a total of 17 or 18 is 4.

$\therefore$ P(a total of 17 or 18) = $\frac{4}{216} = \frac{1}{54}$.

36. **(c)** Number of exhaustive cases in a single throw of three dice = $6 \times 6 \times 6 = 216$.

Cases favourable to a total of 5 are (1, 2, 2), (2, 1, 2), (2, 2, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1).

$\therefore$ P(a total of 5) = $\frac{6}{216} = \frac{1}{36}$.

37. **(a)** A total of at most 5 means a total 3, 4 or 5.

Cases favourable to a total of 3 are (1, 1, 1)

Cases favourable to a total of 4 are (1, 1, 2), (1, 2, 1), (2, 1, 1)

Cases favourable to a total of 5 are (1, 2, 2), (2, 1, 2), (2, 2, 1),

(1, 1, 3), (1, 3, 1), (3, 1, 11).

Number of cases favourable to a total of 3 or 4 or 5 is 10.

$\therefore$ P(a total of at most 5) = $\frac{10}{216} = \frac{5}{108}$.

38. **(c)** A total of at least 5 means not a total of 3 or 4.

Number of cases favourable to a total of 3 or 4 is 4.

P(a total of 3 or 4) = $\frac{4}{216} = \frac{1}{54}$

$\therefore$ P(a total of at least 5) = P(not a total of 3 or 4)

= 1 – P(a total of 3 or 4)

= $1 - \frac{1}{54} = \frac{53}{54}$.

39. **(b)** We know that a leap year has 366 days and thus a leap year has 52 weeks and 2 days over.

The two over (successive days have the following likely cases:

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Firday and Saturday

(vii) Saturday and Sunday.

$\therefore$ Number of exhaustive cases 'n' = 7.

Out of these, the favourable cases are (i) and (vii)

$\therefore$ Number of favourable cases 'm' = 2

$\therefore$ Probability of having 53 Sundays = $\frac{2}{7}$.

40. **(a)** A: Getting a number which is a square

A = (1, 4, 9, 16, 25, 36, 49, 64, 81, 100)

$n(A) = 10$, $n(S) = 100$

$\therefore$ Required probability = $P(A) = \frac{n(A)}{n(S)} = \frac{10}{100} = \frac{1}{10}$.

41. **(c)** There are 7 letters in the word 'SOCIETY' which can be arranged in 71 ways. Considering the three vowels in the word 'SOCIETY' as one letter, we can arrange 5 letters in a row in 5! ways. Also, three vowels can themselves be arranged in 3! ways.

$\therefore$ The total number of arrangements in which three vowels come together are $5! \times 3!$

Hence, the required probability = $\frac{5! \times 3!}{7!} = \frac{3 \times 2 \times 1}{7 \times 6} = \frac{1}{7}$.

42. **(a)** Out of the letters in the word 'UNIVERSITY' two letters 'I' are alike.

$\therefore$ Number of permutations = $\frac{10!}{2}$ (i)

Number of words in which two 'I' are never together

= Total number of words – Number of words in which two 'I' are together

$= \frac{10!}{2} - 9! = \frac{10! - 2 \cdot 9!}{2} = \frac{9![10-2]}{2} = \frac{9!.8}{2} = 9! \cdot 4$

$\therefore$ Required probability = $\frac{9 \cdot 4}{10!/2} = \frac{9! \cdot 8}{10!} = \frac{8}{10} = \frac{4}{5}$.

43. **(a)** Number of ways in which 6 letters of the word PENCIL can be arranged is $P(6, 6) = 6!$.

If N is next to E, they can be considered as one and the 5 letters can be arranged in $P(5, 5) = 5!$ ways.

$\therefore$ The required probability = $\frac{5!}{6!} = \frac{1}{6}$.

44. **(a)** Let, A be the event of "getting the sum 4".

Then, A = [(1, 32), (3, 1), (2, 2)].

Therefore, 3 favourable outcomes and (36 – 3) = 33 outcomes are unfavourable.

$\therefore$ Odds in favour of sum of 4 = $\frac{3}{33} = \frac{1}{11}$.

45. **(b)** Let, A be the event of "getting a sum of 5".

Then, A = [(1, 4), (4, 1), (2, 3), (3, 2)].

There are 4 favourable outcomes and (36 - 4) = 32 outcomes are unfavourable.

$\therefore$ Odds in favour of sum 5 = $\frac{4}{32} = \frac{1}{8}$.

46. **(c)** Let A be the event "getting the sum 6".

Then, A = [(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)]

There are 5 favourable outcomes and (36 – 5) = 31 outcomes are unfavourable.

$\therefore$ Odds against getting the sum 6 = $\frac{31}{5}$.

47. **(b)** A: Getting a queen B: Getting an ace

$P(A) = \frac{4}{52}$, $P(B) = \frac{4}{52}$, $P(A \cap B) = \frac{0}{52} = 0$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{4}{52} - 0 = \frac{8}{52} = \frac{2}{13}$.

48. **(c)** A: Roll number is multiple of 5 B: Roll number is\ multiple of 7

$A = (5, 10, 15, 20, 25)$ $B = (7, 14, 21)$

$P(A) = \frac{5}{25}$, $P(B) = \frac{3}{25}$, $P(A \cap B) = \frac{0}{25} = 0$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{5}{25} + \frac{3}{25} - 0 = \frac{8}{25}$.

49. **(a)** A: integer chosen is divisible by 6 B: Integer chosen is divisible by 8

$n(A)= 33$, $n(B) = 25$, $n(A \cap B) = 8$, $n(S) = 200$

$P(A) = \frac{33}{200}$, $P(B) = \frac{25}{200}$

$P(A \cap B)= P(A) + A(B) - P(A \cap B)$

$= \frac{33}{200} + \frac{33}{200} - \frac{8}{200} = \frac{50}{200} = \frac{1}{4}$.

50. **(c)** A: Sum of two numbers is divisible by 3

B: Sum of two numbers is divisible by 4

$A = [(1, 2), (2, 1), (3, 3), (5, 1), (1, 5), (2, 4), (4, 2), (5, 4), (4, 5), (6, 3), (3, 6), 6, 6)]$

$B = [(2, 2), (3, 1), (1, 3), (5, 3), (3, 5), (4, 4), (6, 2), (2, 6), (6, 6)]$

$P(A) = \frac{12}{36}$, $P(B) = \frac{9}{36}$, $P(A \cap B) = \frac{1}{36}$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{12}{36} + \frac{9}{36} - \frac{1}{36} = \frac{20}{36} = \frac{5}{9}$.

51. **(a)** A: Getting total of 11 B: Getting odd number one each die

$A = [(6, 5), (5, 6)]$

$B = [1, 1). (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5,3), (5, 5)]$

$P(A) = \frac{2}{36}$, $P(B)$, $P(B) = \frac{9}{36}$, $P(A \cap B) = \frac{9}{36} = 0$

$\therefore$ Required probability $= P(A) + P(B) - P(A \cap B)$

$= \frac{2}{36} + \frac{9}{36} - 0 = \frac{11}{36}$.

52. **(b)** A: Number divisible by 3 B: Number is a perfect square

$A = (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36)$

$B = (1, 4, 9, 16, 25, 36)$

$P(A) = \frac{12}{36}$, $P(B) = \frac{6}{36}$, $P(A \cap B) = \frac{2}{36}$

$\therefore$ Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$= \frac{12}{36} + \frac{6}{36} - \frac{2}{36} = \frac{16}{36} = \frac{4}{9}$.

53. **(c)** A: Getting rusted item B: Getting bolt

$P(A) = \frac{100}{200}$, $P(B) = \frac{50}{200}$, $P(A \cap B) = \frac{125}{200}$

$\therefore$ Required probability $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{100}{200} + \frac{50}{200} - \frac{25}{200} = \frac{125}{200} = \frac{5}{8}$.

54. **(a)** A: Getting, a doublet, B: Getting a total of 10

$A = [(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)]$

$B = [(6, 4), (4, 6), (5, 5)]$

$P(A) = \frac{6}{36}$, $P(B) = \frac{3}{36}$, $P(A \cap B) = \frac{1}{36}$

$\therefore$ Required probability

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left(\frac{6}{36} + \frac{3}{36} - \frac{1}{36}\right)$

$= 1 - \frac{8}{36} = \frac{28}{36} = \frac{7}{9}$.

55. **(b)** A: Getting a total of 9, B: Getting a total of 11

$A = [(5, 4), (4, 5), (6, 3), (3, 6)]$ $B = [(6, 5), (5, 6)]$

$P(A) = \frac{4}{36}$, $P(B) = \frac{2}{36}$, $P(A \cap B) = \frac{0}{36}$

$\therefore$ Required probability $= 1 - P(A \cup B)$

$= [P(A) + P(B) - P(A \cap B)]$

$= 1 - \left(\frac{4}{36} + \frac{2}{36} - 0\right)$

$= 1 - \frac{1}{6} = \frac{5}{6}$.

56. **(d)** A: Getting spade card B: Getting ace card C: Getting red card

$P(A) = \frac{13}{52}$, $P(B) = \frac{4}{52}$, $P(C) = \frac{26}{52}$, $P(A \cap B) = \frac{1}{52}$,

$P(B \cap C) = \frac{2}{52}$, $P(C \cap A) = \frac{0}{52} = 0$,

$P(A \cap B \cap C) = \frac{0}{52} = 0$

$\therefore$ Required probability

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{13}{52}+\frac{4}{52}+\frac{26}{52}-\frac{1}{52}-\frac{2}{52}-0=\frac{40}{52}=\frac{10}{13}$.

57. **(a)** A: Number is divisible by 2 B: Number is divisible buy 3 C: Number is divisible by 5

$P(A) = \frac{100}{200}$, $P(B) = \frac{66}{200}$, $P(C) = \frac{40}{200}$,

$P(B \cap C)\ \frac{33}{200} = P(B \cap C) = \frac{13}{200}$,

$P(C \cap A) = \frac{20}{200}$, $P(A \cap B \cap C) = \frac{6}{200}$

$\therefore$ Required probability

$= P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$-P(C \cap A) + P(A \cap B \cap C)$

$=\frac{100}{200}+\frac{66}{200}+\frac{40}{200}-\frac{33}{200}-\frac{13}{200}-\frac{20}{200}+\frac{6}{200}=\frac{146}{200}=\frac{73}{200}$.

58. **(b)** We know $P(A) = 1 - P(\bar{A})$

$= 1 - 0.65 = 0.35$

and, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow\ 0.65 = 0.35 + p - 0$

[$\because$ A and B are mutually exclusive events]

$\Rightarrow\ p = 0.65 - 0.35 = 0.30$.

59. **(a)** Required probability $= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)$

$= 1 - [0.5 + 0.3 - 0]$

[$\because$ A and B are mutually exclusive events]

$= 1 - 0.8 = 0.2$.

60. **(c)** $P(\text{at least } B \text{ grade}) = P(B \text{ grade}) + P(A \text{ grade})$

$= 0.38 + 0.30 = 0.68$

61. **(c)** A: Contractor will get a plumbing contract

B: Contractor will get an electric contract

$P(A) = \frac{5}{9}$, $P(\bar{B}) = \frac{5}{9}$, $P(A \cup B) = \frac{4}{5}$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow\ \frac{4}{5}=\frac{2}{3} + [1 - P(\bar{B})] - P(A \cap B)$

$\Rightarrow\ \frac{4}{5}=\frac{2}{3} + 1 - \frac{5}{9} - P(A \cap B)$

$\Rightarrow\ (A \cap B) = \frac{2}{3}+1-\frac{5}{9}-\frac{4}{5}$

$= \frac{30+45-25-26}{45}=\frac{14}{45}$.

62. **(a)** Let, A: a spade is drawn and B: an ace is drawn

Probability of winning the bet $= P(A \text{ or } B)$

$= P(A) + P(B) - P(A \text{ and } B)$

$= \frac{13}{52}+\frac{4}{52}-\frac{1}{52}=\frac{16}{52}=\frac{4}{52}$

Probability of losing the bet $= 1- \frac{4}{13}=\frac{9}{13}$

Odds against winning the bet $= \frac{9}{13}:\frac{4}{13} = 9:4$.

63. **(b)** A:A wins the race B:B wins the race C: C wins the race

$P(A) = \frac{1}{1+3}=\frac{1}{4}$, $P(B) = \frac{1}{1+4}=\frac{1}{5}$,

$P(C) = \frac{1}{1+5}=\frac{1}{6}$, $P(D) = \frac{1}{1+6}=\frac{1}{7}$

$\therefore$ Required probability $= P(A)+P(B)+P(C)+P(D)$

$= \frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}=\frac{319}{420}$.

64. **(b)** Let, A and B denote the events that the chartered accountant is selected in firms X and Y, respectively. Then,

$P(A) = 0.7$, $P(\bar{B}) = 0.5$ and $P(\bar{A} \cup \bar{B}) = 0.6$

Now, $P(\bar{A}) = 1 - P(A) = 1 - 0.7 = 0.3$

$P(B) = 1 - P(\bar{B}) = 1- 0.5 = 0.5$

Again, $\overline{A \cap B} = \overline{A \cup B}$ (By De Morgan's law)

$\therefore\ P(A \cap B) = 1 - P\,\overline{A \cap B} = 1- P(\bar{A} \cup \bar{B})$

$\Rightarrow\ P(A \cap B) = 1 - 0.6 = 0.4$

$\therefore\ P(A \cup B) = 0.7 + 0.5 - 0.4 = 0.8$

Hence, the probability that the chartered accountant will be selected in one of the two firms X or Y is 0.8.

65. **(a)** Since odds against the event A are 8:3, the probability of the happening of the event A is given by $P(A) = \frac{3}{8+3}=\frac{3}{11}$.

Similarly, odds against the event B are 5:2, so we have $P(B) = \frac{2}{5+2}=\frac{2}{7}$.

Since the events A, B, C are such that one of them is a must and only one can happen, so the events A, B, C are mutually exclusive and exhaustive and consequently the sum of their probability must be 1.

$\therefore\ P(A) + P(B) + P(C) = 1$ or $\frac{3}{11}+\frac{2}{7} + P(C) = 1$

$\Rightarrow\ P(C) = 1 - \frac{3}{11}-\frac{2}{7}=\frac{34}{77}$.

$\therefore\ P(\bar{C}) = 1 - P(C) = 1 - \frac{34}{77}=\frac{43}{77}$.

$\therefore$ The odds against C are 43:34.

66. **(b)** Probability that A fails to solve the problem is $1 - \frac{1}{3} = \frac{2}{3}$

Probability that B fails to solve the problem is $1 - \frac{1}{4} = \frac{3}{4}$

Probability that C fails to solve the problem is $1 - \frac{1}{5} = \frac{4}{5}$.

Probability that D fails to solve the problem is $1 - \frac{1}{6} = \frac{5}{6}$.

Since the events are independent, the probability that all the four students fail to solve the problem is $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{1}{3}$.

$\therefore$ The probability that the problem will be solved $= 1 - \frac{1}{3} = \frac{2}{3}$.

67. **(c)** Probability of drawing a white ball from the first bag $= \frac{4}{5} = \frac{2}{3}$. Probability of drawing a white ball from the second bag $= \frac{3}{8}$.

Since the events are independent, the probability that both the balls are white $= \frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$.

68. **(a)** Probability of drawing a black ball from first bag $= \frac{2}{6} = \frac{1}{3}$

Probability of drawing a black ball from the second bag $= \frac{5}{8}$.

$\therefore$ Probability that both balls are black $= \frac{1}{3} \times \frac{5}{8} = \frac{5}{24}$.

69. **(c)** The event 'one is white and one is black' is the same as the event 'either the first is white and the second is black or the first is black and the second is white',

$\therefore$ The probability that one is white and one is black $= \frac{2}{3} \times \frac{5}{8} \times \frac{1}{3} \times \frac{3}{8} = \frac{12}{24}$.

70. **(a)** Success: Getting odd number $p = \frac{13}{25}$

$\Rightarrow \quad q = 1 - p = 1 - \frac{13}{25} = \frac{12}{25}$

P(two successes) $= pp = \frac{13}{25} \times \frac{13}{25} = \frac{169}{625}$.

71. **(b)** P(exactly one success) $= pq + qp$

$= \frac{13}{25} \times \frac{12}{25} \times \frac{12}{25} \times \frac{13}{25} = \frac{156+156}{625} = \frac{312}{625}$.

72. **(c)** P(at one success) = 1 P(no success)

$= 1 - qq = 1 - \left(\frac{12}{25}\right)\left(\frac{12}{25}\right) = 1 - \frac{144}{625}$

$= \frac{625-144}{625} = \frac{481}{625}$.

73. **(d)** P(no success) $= qq = \frac{12}{25}\left(\frac{12}{25}\right) = \frac{144}{625}$.

74. **(a)** Success: getting 5 or 6 $p = \frac{2}{6} = \frac{1}{3}$

$\Rightarrow \quad q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

P(3 successes) $= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$.

75. **(b)** P(exactly 2 successes) $= ppq + pqp + qpp$

$= \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}$

$= \frac{2+2+2}{27} = \frac{6}{27} = \frac{2}{9}$.

76. **(c)** P(at most 2 successes) $= P$(no success) $+ P$(1 success) $+ P$(2 successes)

$= 1 - P$(3 successes) $= 1 - pp$

$= 1 - \frac{1}{3}\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = 1 - \frac{1}{27} = \frac{26}{27}$.

77. **(d)** P(at least 2 successes) = + P(3 successes)

$= ppq + pqp + qpp + ppp$

$= \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$

$= \frac{2+2+2+1}{27} = \frac{7}{27}$.

78. **(c)** A: First card is diamond card B: Second card is king card

$P(A) = \frac{13}{52} = \frac{1}{4}$, $P(B) = \frac{4}{52} = \frac{1}{13}$

$\therefore$ Required probability $= P(A)\, P(B) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$.

79. **(a)** A: Husband selected; B: Wife selected

$P(A) = \frac{1}{7} \quad \Rightarrow \quad (\bar{A}) = 1 - P(A) = 1 - \frac{1}{7} = \frac{6}{7}$

$P(B) = \frac{6}{7} \quad \Rightarrow \quad P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{5} = \frac{4}{5}$

P(only one of them will be selected)

$= P(A)\, P(\bar{B}) + P(B)\, P(\bar{A})$

$= \frac{1}{7}\left(\frac{4}{5}\right) + \frac{1}{5}\left(\frac{6}{7}\right) = \frac{4+6}{35} = \frac{10}{35} = \frac{2}{7}$.

80. **(b)** P(both of them will be selected)

$= P(A) \times P(B) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$.

81. **(c)** P(none of them will be selected)

$= P(\bar{A})\, P(\bar{B}) = \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$.

82. **(d)** P(at least one of them will be selected)

$= 1 - P(\bar{A}) \times P(\bar{B})$

$= 1 - \frac{6}{7} \times \frac{4}{5} = 1 - \frac{24}{35} = \frac{11}{35}$.

83. **(a)** A: Husband will be alive 25 years hence

B: Wife will be alive 25 years hence

$P(A) = 0.3 \Rightarrow P(\bar{A}) = 1 - P(A) = 1.03 = 0.7$

$P(B) = 0.4 \Rightarrow P(\bar{B}) = 1 - P(B) = 1 - 04 = 0.6$

Required probability $= P(A)\ P(B) = (0.3)\ (0.4) = 0.12$.

84. **(b)** Required probability $= P(A)\ P(\bar{B}) = (0.3)\ (0.6) = 0.18$.

85. **(c)** Required probability $= P(B)\ P(\bar{A}) = (0.4)\ (0.7) = 0.28$.

86. **(d)** Required probability $= 1 - P(\bar{A})\ P(\bar{B})$

$= 1 - (0.7)\ (0.6)$

$= 1 - 0.42 = 0.58$.

87. **(b)** Let, the two men be A and B. A:A speaks truth; B:B speaks truth

$P(A) = \frac{80}{100} \Rightarrow P(\bar{A}) = 1 - P(A) = 1 - \frac{80}{100} = \frac{20}{100}$

$P(B) = \frac{90}{100} \Rightarrow P(\bar{B}) = 1 - P(B) = 1 - \frac{90}{100} = \frac{10}{100}$

$\therefore$ Required probability $= P(A)\ P(\bar{B}) + P(B)\ P(\bar{A})$

$= \frac{80}{100} \times \frac{10}{100} + \frac{90}{100} \times \frac{20}{100}$

$= \frac{8+18}{100} = \frac{26}{100} = \frac{13}{50}$.

88. **(a)** A: Getting a red ball from urn A

B: Getting a black ball from urn A

C: Getting a red ball from urn B

D: Getting a black ball fron urn C

E: Getting a red ball from urn C

F: Getting a black ball from urn C

$P(A) = \frac{4}{7}$, $P(B) = \frac{3}{7}$, $P(C) = \frac{5}{9}$,

$P(D) = \frac{4}{9}$, $P(E) = \frac{4}{8}$. $P(F) = \frac{4}{8}$

Required probability $= P(A)P(C)P(F) + P(A)P(D)\ P(E) + P(B)P(C)P(E)$

$= \frac{4}{5} \times \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{4}{8} + \frac{3}{7} \times \frac{5}{9} \times \frac{4}{8}$

$= \frac{80+64+60}{504} = \frac{204}{504} = \frac{17}{42}$.

89. **(c)** A: Plane is hit by the first shot

B: Plane is hit by the second shot

C: Plane is hit by the third shot

D: Plane is hit by the fourth shot

$P(A) = 0.4 \Rightarrow P(\bar{A}) = 1 - P(A) = 1 - 0.4 = 0.6$

$P(B) = 0.3 \Rightarrow P(\bar{B}) = 1 - P(B) = 1 - 0.3 = 0.7$

$P(C) = 0.2 \Rightarrow P(\bar{C}) = 1 - P(C) = 1 - 0.2 = 0.8$

$P(D) = 0.1 \Rightarrow P(\bar{D}) = 1 - P(D) = 1 - 0.1 = 0.9$

$\therefore$ Required probability

$= 1 - P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})$

$= 1 - (0.6)\ (0.7)\ (0.8)\ (0.9)$

$= 1 - 0.3024 = 0.6976$.

90. **(b)** A:A solves the problem; B:B solves the problem.

$P(A) = \frac{90}{100} \Rightarrow P(\bar{A}) = 1 - P(A) = 1 - \frac{90}{100} = \frac{10}{100}$

$P(B) = \frac{70}{100} \Rightarrow P(\bar{B}) = 1 - P(B) = 1 - \frac{70}{100} = \frac{30}{100}$

Required probability $= 1 - P(\bar{A})P(\bar{B})$

$= 1 - \frac{10}{100} \times \frac{30}{100} = 1\frac{3}{100}$

$= \frac{97}{100}$.

91. **(b)** Success: Getting head

$\therefore\ p = \frac{1}{2}$

$\Rightarrow\ q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$.

Let, A start the game. A can win the game in 1st, IIIrd, Vth, ... throws. Then

$P(A$ winning$) = p + qqp + qqqqp + \ldots$

$= p[1 + q^2 + q^4 + \ldots]$

$= p\frac{1}{1-q^2} = \frac{1}{2}\frac{1}{1-(1/2)^2}$

$= \frac{4}{2(3)} = \frac{2}{3}$.

92. **(a)** Success: Getting 6

$p = \frac{1}{6} \Rightarrow q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$.

A can win the game in 1st, IIIrd, Vth ... throws

$P(A$ winning$) = p + qqp + qqqqp + \ldots$

$= p[1 + q^2 + q^4 + \ldots] = p\frac{1}{1-q^2}$

$= \frac{1}{6}\frac{1}{1-(5/6)^2} = \frac{1}{6}\frac{36}{36-25} = \frac{6}{11}$

$\therefore\ P(B$ winning$) = 1 - \frac{6}{11} = \frac{5}{11}$.

93. **(d)** A: Three vowels come together

$n(A) = 5!3!$, $\quad n(S) = 7!$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5!\,3!}{7!}$

$= \dfrac{3 \times 2}{7 \times 6} = \dfrac{1}{7}$.

94. (a) $A:D$ occupies the first place

$n(A) = 7!$, $n(S) = 8!$

$\therefore$ Required probability $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{7!}{8!} = \dfrac{1}{8}$.

Exercise-2
(Based on Memory)

1. (c) P (at least one red) 1 − P (no red)

$= 1 - \dfrac{^5C_3}{^{10}C_3} = 1 - \dfrac{10}{120} = 1 - \dfrac{1}{12} = \dfrac{11}{12}$

2. (b) P (2 green + 2 blue)

$= \dfrac{^2C_2 \times {^2C_2}}{^{10}C_3} = \dfrac{1 \times 3}{210} = \dfrac{1}{70}$

3. (b) Number of ways of selecting 4 students out of 15 students $= {^{15}C_4} = \dfrac{15 \times 14 \times 3 \times 12}{1 \times 2 \times 3 \times 4} = 1365$.

The number of ways of selecting 4 students in which no student belongs to Karnataka $= {^{10}C_4}$

$\therefore$ No of ways of selecting at least 1 student from Karnataka $= {^{15}C_4} - {^{10}C_4} = 1155$.

$\therefore$ Required probability $= \dfrac{1155}{1365} = \dfrac{77}{91} = \dfrac{11}{13}$.

4. (b) The probabilty that a student is absent in the class $= \dfrac{2}{6} = \dfrac{1}{3}$.

$\therefore$ The probability of missing his test will be

$= \dfrac{1}{5} \times \dfrac{1}{3} = \dfrac{1}{15}$.

5. (a)

6. (e) Clearly, the ball picked up must be blue, which can be picked in $^7C_1 = 7$ ways.

One ball can be picked from total (8 + 7 + 6 = 21) in $^{21}C_1$ ways.

$\therefore$ Required probability $= \dfrac{7}{21} = \dfrac{1}{3}$.

7. (b) $n(S)$ = Number of ways to select 3 marbles out of 7 marbles $= {^7C_3}$

$= \dfrac{7 \times 6 \times 5}{1 \times 2 \times 3} = 35$

$n(E)$ = Probability that 2 are green and 1 is red

$= {^4C_2} \times {^3C_1} = \dfrac{4 \times 3}{1 \times 2} \times 3 = 18$

Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{18}{35}$

8. (d) Probability to be a Blue $= \dfrac{^3C_3}{^7C_3}$

Probability to be a Red $= \dfrac{^4C_3}{^7C_3}$

Required probability $= \dfrac{^3C_3}{^7C_3} + \dfrac{^4C_3}{^7C_3} = \dfrac{2}{35}$

9. (d) Total numbers of balls = 13 + 7 = 20

Number of sample space $= n(S) = {^{20}C_2} = 190$

Number of events $= n(E) = {^{13}C_2} + {^7C_2} = 78 + 21 = 99$

$\therefore$ $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{99}{190}$

10. (e) Total number of marbles in the urn

= 4 + 5 + 2 + 3 = 14

Total number of possible outcomes

= Selection of 2 marbles out of 14 marbles

$= {^{14}C_2} = \dfrac{14 \times 13}{1 \times 2} = 91$

Total number of favourable cases

$= {^2C_2} + {^2C_1} + {^{12}C_1} = 1 + 2 \times 12 = 25$

$\therefore$ Required probability $= \dfrac{25}{91}$

11. (b) Total number of possible outcomes

$= {^{14}C_3} = \dfrac{14 \times 13 \times 12}{1 \times 2 \times 3} = 364$

When no marbles is yellow, favourable number of cases

$= {^{11}C_3} = \dfrac{11 \times 10 \times 9}{1 \times 2 \times 3} = 165$

$\therefore$ Probability that no marble is yellow $= \dfrac{165}{364}$

$\therefore$ Required probability = (Probability that at least 1 is yellow) = (1 − Probability that no marble is yellow)

$= 1 - \dfrac{165}{364} = \dfrac{364 - 165}{364} = \dfrac{199}{364}$

12. (c) Total possible outcomes $= {^{14}C_8} = {^{14}C_6}$ $[\because {^nC_r} = {^nC_{n-r}}]$

$= \dfrac{14 \times 13 \times 12 \times 11 \times 10 \times 9}{1 \times 2 \times 3 \times 4 \times 5 \times 6} = 3003$

Total number of favourable cases

$= {}^4C_2 \times {}^5C_2 \times {}^2C_2 \times {}^3C_2 = 6 \times 10 \times 1 \times 3 = 180$

$\therefore$ Required probability $= \dfrac{180}{3003} = \dfrac{60}{1001}$

13. **(e)** Total number of possible outcomes

$= {}^{14}C_3 = \dfrac{14 \times 13 \times 12}{1 \times 2 \times 3} = 364$

Now, according to the question, no marble should be green.

$\therefore$ Total number of favourable outcomes

= Selection of 3 marbles out of 5 blue, 2red and 3 yellow marbles

$= {}^{10}C_3 = \dfrac{10 \times 9 \times 8}{1 \times 2 \times 3} = 120$

$\therefore$ Required probability $= \dfrac{120}{364} = \dfrac{30}{91}$

14. **(a)** Total number of possible outcomes

$= {}^{14}C_4 = \dfrac{14 \times 13 \times 12 \times 11}{1 \times 2 \times 3 \times 4} = 1001$

Total number of favourable cases

$= {}^5C_2 \times {}^2C_2 = 10 \times 1 = 10$

$\therefore$ Required probability $= \dfrac{10}{1001}$

15. **(a)** Total number of ways of selecting 4 children out of 8

$= {}^8C_4 = \dfrac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} = 70$

Number of ways of selecting 4 girls out of $5 = {}^5C_4 = 5$

Required probability $= \dfrac{5}{70} = \dfrac{1}{14}$

16. **(d)** $n(s)$ = Total possible outcomes

= Number of ways of picking 3 marbles out of 12

$= {}^{12}C_3 = \dfrac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$

$n(E)$ = Favourable no. of cases

$= {}^3C_3 + {}^4C_3 = 1 + 4 = 5$

Required probability $= \dfrac{n(E)}{n(s)} = \dfrac{5}{220} = \dfrac{1}{44}$.

17. **(e)** $n(s)$ = Total possible outcomes

$= {}^{12}C_2 = \dfrac{12 \times 11}{1 \times 2} = 66$

$n(E)$ = Favourable number of cases

$= {}^4C_2 = \dfrac{4 \times 3}{1 \times 2} = 6$

Required probability $= \dfrac{n(E)}{n(s)} = \dfrac{6}{66} = \dfrac{1}{11}$.

18. **(b)** $n(s)$ =Total possible outcomes

$= {}^{12}C_3 = 220$

$n(E)$ =Favourable number of cases

= Number of ways of picking 3 marbles (none is blue)out of 7

$= {}^7C_3 = \dfrac{7 \times 6 \times 5}{1 \times 2 \times 3} = 35$

Required probability $= 1 - \dfrac{35}{220} = 1 - \dfrac{7}{44} = \dfrac{37}{44}$.

19. **(a)** Required probability is when all caps chosen are blue, red or yellow. Which is equal to

$\dfrac{{}^7C_4}{{}^{12}C_4} = \dfrac{7!\,4!\,8!}{4!\,3!\,12!} = \dfrac{7}{99}$.

20. **(e)** Required probability

$= \dfrac{{}^2C_2}{{}^{12}C_2} = \dfrac{2!}{2!\,0!} / \dfrac{12!}{10!\,2!} = \dfrac{1}{66}$

21. **(b)** Required probability

$= ({}^2C_1 + {}^1C_1) / {}^{12}C_1 = \dfrac{3}{12} = \dfrac{1}{4}$

22. **(c)** Required probability

$= \dfrac{1}{{}^{12}C_2}({}^4C_1 \times {}^8C_1 + {}^4C_2) = \dfrac{38}{12 \times 11} \times 2 = \dfrac{19}{33}$

23. **(d)** Required probability

$= ({}^4C_2 \times {}^5C_1) / {}^{12}C_3 = \dfrac{3}{22}$