Modelling and Control of 2-link and 3-link Robotic Manipulator

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NATIONAL INSTITUTE OF TECHNOLOGY KARNATAKA, SURATHKAL

DECLARATION
By the U.G. (B.Tech) Student

We hereby declare that the report of the Mini Project Work entitled "**Modelling and Control of 2-link and 3-link Robotic Manipulator**" which is being submitted to the **National Institute of Technology Karnataka, Surathkal** for the award of the degree of **Bachelors of Technology** in **Electrical and Electronics Engineering Department** is a bonafide report of the work carried out by us. The material contained in this report has not been submitted to any University or Institution for the award of any Degree.

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CERTIFICATE

This is to certify that the Project Work Report entitled **Modelling and Control of 2-link and 3-link Robotic Manipulator** submitted by **Saraswathi Vinod**, Registration Number **16EE140** and **Gayatri Indukumar**, Registration Number **16EE218** in partial fulfillment of the requirements for the award of degree of the **BACHELORS OF TECHNOLOGY** in the Department of **Electrical and Electronics Engineering** is accepted as the record of the work carried out by her.

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Abstract

Robot Manipulators are widely used in industries to perform repetitive tasks. However, despite long years of research manipulator control is still a niche area of research. Non-linear systems like manipulator are multi-input-multi-output and time variant complex problem.

Many different approaches presently followed for the control of manipulator one of them being the classical PID (Proportional Integral Derivative). This report studies control of manipulator using PID. Results of simulations performed for 2-link and 3-link manipulators using MATLAB Simulink, have been presented below.

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Chapter 1

Introduction

For centuries man has been interested in trying to replicate all sorts of creations including themselves. This science of replication is the idea behind robotics. A robot is a machine—especially one programmable by a computer— capable of carrying out a complex series of actions automatically. Robots can be guided by an external control device or the control may be embedded within. Robots may be constructed to take on human form but most robots are machines designed to perform a task with no regard to how they look.

The branch of technology that deals with the design, construction, operation, and application of robots, as well as computer systems for their control, sensory feedback, and information processing is robotics. These technologies deal with automated machines that can take the place of humans in dangerous environments or manufacturing processes, or resemble humans in appearance, behavior, or cognition.

1.1 Why robots?

- Mass Production and Self-Replication
- Mind Transfer from One Robot to Another
- Advanced Intelligence
- Easier to Upgrade
- Dramatically Reduced Energy Needs
- The Potential for Moral Superiority
- Immunity to Damaging and Burdensome Biological Functions
- Technologically Enabled Telepathy
- Dynamic Morphologies

1.2 Robotic Manipulators

Industry-specific robots perform several tasks such as picking and placing objects an example is shown in Fig 1.1(a), movement adapted from observing how similar manual tasks are handled by a fully-functioning human arm. Such robotic arms are also known as robotic manipulators. These manipulators were originally used for applications with respect to bio-hazardous or radioactive materials or for use in inaccessible places.
A robot manipulator is constructed using rigid links connected by joints with one fixed end and one free end to perform a given task (e.g., to move a box from one location to the next). The joints to this robotic manipulator are the movable components, which enables relative motion between the adjoining links. There are also two linear joints to this robotic manipulator that ensure non-rotational motion between the links, and three rotary type joints that ensure relative rotational motion between the adjacent links.

The manipulator can be divided into two parts, each having different functions:

(a) Articulated welding robots used in a factory, a type of industrial robot

Figure 1.1: Industrial Robot

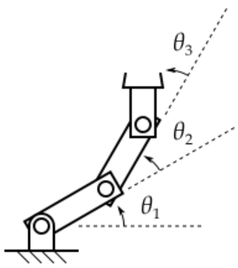

Figure 1.2: 2-Link Manipulator

Arm and Body – The arm and body of the robot consists of three joints connected together by large links. They can be used to move and place objects or tools within the work space.

Wrist – The function of the wrist is to arrange the objects or tools at the work space. The structural characteristics of the robotic wrist includes two or three compact joints. Fig 1.2

Thus by varying the parameters of each link we can make the Manipulator perform any work of our choice with in its workspace.

OBJECTIVE

Modeling a 2-link and 3-link robotic manipulator and simulating with control techniques like PID on MATLAB Simulink.

Chapter 2

Modelling a 2-link manipulator

To model any manipulator, we need two types of equations. Kinematics equations and Dynamics equations. This chapter deals with both the types of equations for a 2-link manipulator.

2.1 Kinematics Equations

Robot kinematics applies geometry to the study of the movement of multi-degree of freedom kinematic chains that form the structure of robotic systems. The emphasis on geometry means that the links of the robot are modeled as rigid bodies and its joints are assumed to provide pure rotation or translation.

Robot kinematics studies the relationship between the dimensions and connectivity of kinematic chains and the position, velocity and acceleration of each of the links in the robotic system, in order to plan and control movement and to compute actuator forces and torques. The relationship between mass and inertia properties, motion, and the associated forces and torques is studied as part of robot dynamics.

Forward Kinematics

Forward kinematics specifies the joint parameters and computes the configuration of the chain.

Inverse Kinematics

Inverse kinematics specifies the end-effector location and computes the associated joint angles.

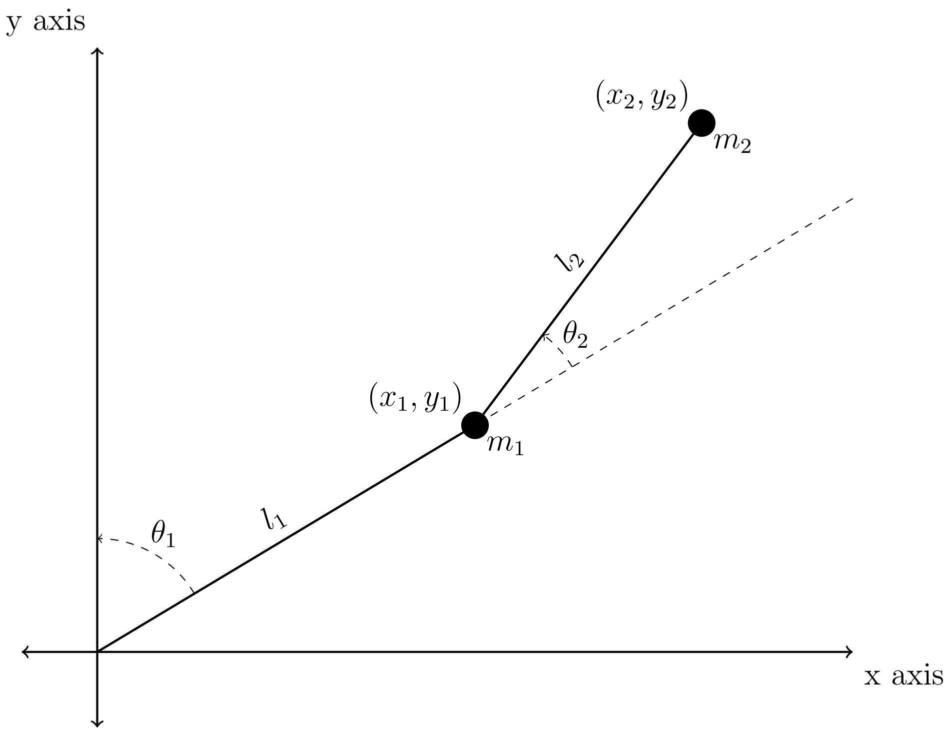

Figure 2.1: 2 link Manipulator

Robot manipulators are created from a sequence of link and joint combinations. The links are the rigid members connecting the joints, or axes. The

axes are the movable components of the robotic manipulator that cause relative motion between adjoining links.
Consider A two-link planar arm as shown in Fig 2.1 where:
$m1$ and $m2$ are mass of joint 2 and end-effector
$a1$ and $a2$ are lengths of link 1 and link 2
θ_1 is angle between y-axis and link 1
θ_2 is angle between link 1 and link 2
(x_2, y_2) is the position of end effector

Kinematics Equations of 2-link manipulator

As we know, we have to find a relation between joint variables θ_1, θ_2 and Cartesian coordinates $(x_1, y_1), (x_2, y_2)$.
Consider $l_1 = a_1$ and $l_2 = a_2$. Using simple trigonometry in Fig 2.1, we get

$$x_1 = l_1 sin(\theta_1) \tag{2.1}$$

$$y_1 = l_1 cos(\theta_1) \tag{2.2}$$

$$x_2 = l_1 sin(\theta_1) + l_2 sin(\theta_1 + \theta_2) \tag{2.3}$$

$$y_2 = l_1 cos(\theta_1) + l_2 cos(\theta_1 + \theta_2) \tag{2.4}$$

This is inverse kinematics where the cartesian coordinates are obtained using joint parameters. These equations will be user to derive dynamic equations.

2.2 Dynamics Equations

For control design purposes, it is necessary to have a mathematical model that reveals the dynamical behavior of a system. Therefore, in this section we derive the dynamical equations of motion for a robot manipulator. Our approach is to derive the kinetic and potential energy of the manipulator and then use Lagrange's equations of motion.

Lagrange Equations of Motion

Lagrange's equation of motion for a conservative system are given by [Marion 1965]

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau \tag{2.5}$$

where q is an n-vector of generalized coordinates q_i, is an n-vector of generalized forces i, and the *Lagrangian* is the difference between the kinetic and potential energies.

$$L = K - P \tag{2.6}$$

In our usage, q will be the joint-variable vector, consisting of joint angles θ_i (in degrees or radians). Then τ is a vector that has components of torque (newton-meters).

Dynamics Equations of 2-link manipulator

To determine its dynamics, examine Fig 2.2, where we have assumed that the link masses are concentrated at the ends of the links.
The joint variable is

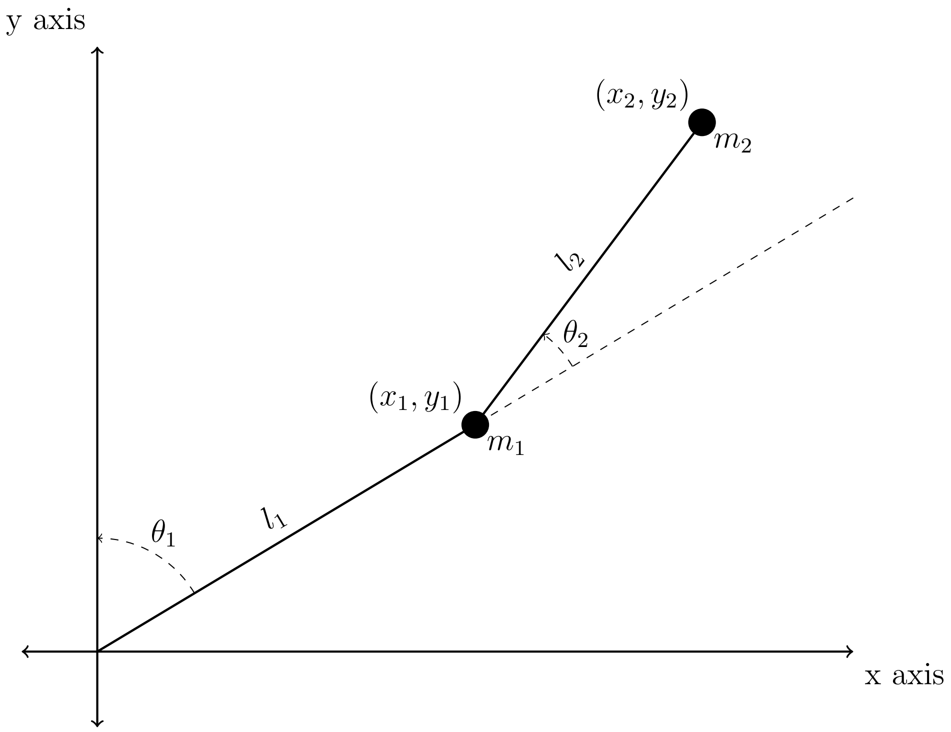

Figure 2.2: 2 link Manipulator

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{2.7}$$

and the generalized force vector is

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{2.8}$$

Kinetic and Potential Energy

For link 1 the kinetic and potential energies are

$$K_1 = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 \tag{2.9}$$

$$P_1 = m_1 g l_1 sin(\theta_1) \tag{2.10}$$

For link 2 we have

$$x_2 = l_1 cos(\theta_1) + l_2 cos(\theta_1 + \theta_2) \tag{2.11}$$

$$y_2 = l_1 sin(\theta_1) + l_2 sin(\theta_1 + \theta_2) \tag{2.12}$$

$$\dot{x}_2 = -l_1\dot{\theta}_1 sin(\theta_1) - l_2(\dot{\theta}_1\dot{\theta}_2) sin(\theta_1 + \theta_2) \tag{2.13}$$

$$\dot{y}_2 = l_1\dot{\theta}_1 cos(\theta_1) + l_2(\dot{\theta}_1\dot{\theta}_2) cos(\theta_1 + \theta_2) \tag{2.14}$$

so that the velocity squared is

$$v_2^2 = \dot{x}_2^2 + \dot{y}_2^2 \tag{2.15}$$

$$v_2^2 = l_1\dot{\theta}_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2l_1l_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2) cos(\theta_2) \tag{2.16}$$

Therefore, the kinetic energy for link 2 is

$$K_2 = \frac{1}{2} m_2 v_2^2 \tag{2.17}$$

$$K_2 = \frac{1}{2} m_2 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 + m_2 l_1 l_2 (\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2) cos(\theta_2) \tag{2.18}$$

The potential energy for link 2 is

$$P_2 = m_2 g y_2 \tag{2.19}$$

$$P_2 = m_2 g[l_1 sin(\theta_1) + l_2 sin(\theta_1 + \theta_2)] \tag{2.20}$$

Lagrange's Equation

$$L = K - P = K_1 + K_2 - P_1 - P_2 \tag{2.21}$$

$$
\begin{aligned}
L = \frac{1}{2}(m_1 + m_2)l_1^2\theta_1^2 + \frac{1}{2}m_2 l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 \\
+m_2 l_1 l_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)cos(\theta_2) \\
-(m_1 + m_2)gl_1 sin(\theta_1) \\
-m_2 g l_2 sin(\theta_1 + \theta_2)
\end{aligned} \tag{2.22}
$$

For the equation 2.5

$$
\begin{aligned}
\frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2)l_1^2\dot{\theta}_1^2 + m_2 l_2^2(\dot{\theta}_1 + \dot{\theta}_2) \\
+m_2 l_1 l_2(2\dot{\theta}_1 + \dot{\theta}_2)cos(\theta_2)
\end{aligned} \tag{2.23}
$$

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2)l_1^2\ddot{\theta}_1 + m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2(2\ddot{\theta}_1 + \theta_2) \\
-m_2 l_1 l_2(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)sin(\theta_2)
\end{aligned} \tag{2.24}
$$

$$
\frac{\partial L}{\partial \theta_1} = -(m_1 + m_2)gl_1 cos(\theta_1) - m_2 g l_2 cos(\theta_1 + \theta_2) \tag{2.25}
$$

$$
\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2(\dot{\theta}_1 + \dot{\theta}_2) + m_2 l_1 l_2\dot{\theta}_1 cos(\theta_2) \tag{2.26}
$$

$$
\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2(\ddot{\theta}_1 + \ddot{\theta}_2) + m_2 l_1 l_2\ddot{\theta}_1 cos(\theta_2) - m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2 sin(\theta_2) \tag{2.27}
$$

$$
\frac{\partial L}{\partial \theta_1} = -m_2 l_1 l_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)sin(\theta_2) - m_2 g l_2 cos(\theta_1 + \theta_2) \tag{2.28}
$$

Finally, according to Lagrange's equation, the arm dynamics are given by the two coupled nonlinear differential equations

$$\begin{aligned} \tau_1 = [(m_1+m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 cos(\theta_2)]\ddot{\theta}_1 \\ +[m_2 l_2^2 + m_2 l_1 l_2 cos(\theta_2)]\ddot{\theta}_2 \\ -m_2 l_1 l_2 (2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2) sin(\theta_2) + (m_1+m_2) g l_1 cos(\theta_1) \\ +m_2 g l_2 cos(\theta_1+\theta_2) \end{aligned} \tag{2.29}$$

$$\begin{aligned} \tau_2 = [m_2 l_2^2 + m_2 l_1 l_2 cos(\theta_2)]\ddot{\theta}_1 \\ +m_2 l_2^2 \ddot{\theta}_2 \\ +m_2 l_1 l_2 \dot{\theta}_1 sin(\theta_2) \\ +m_2 g l_2 cos(\theta_1+\theta_2) \end{aligned} \tag{2.30}$$

So, we can describe the motion of the system by the following equations

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau \tag{2.31}$$

where

$$\begin{cases} q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \\ M(q) = \begin{bmatrix} ((m_1+m_2)l_1^2 + m_2 l_2^2 + 2m_1 l_1 l_2 cos(\theta_2)) & (m_2 l_2^2 + m_2 l_1 l_2 cos(\theta_2)) \\ m_2 l_2^2 + m_2 l_1 l_2 cos(\theta_2) & m_2 l_2^2 \end{bmatrix} \\ V(q,\dot{q}) = \begin{bmatrix} -m_2 l_1 l_2 sin(\theta_2)(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2) \\ -m_2 l_1 l_2 sin(\theta_2)\dot{\theta}_1\dot{\theta}_2 \end{bmatrix} \\ G(q) = \begin{bmatrix} -(m_1+m_2) g l_1 cos(\theta_1) - m_2 g l_2 cos(\theta_1+\theta_2) \\ -m_2 g l_2 cos(\theta_1+\theta_2) \end{bmatrix} \\ \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \end{cases} \tag{2.32}$$

Chapter 3

Modelling a 3-link manipulator

Just like 2-link manipulator, we have to derive kinematics equations for 3-link manipulator.

3.1 Kinematics Equations

Forward Kinematics

Assume $(x_3, y_3, z_3) = (P_x, P_y, P_z)$. Using simple trigonometry in the figure 3.1, we can get the following equations

$$P_x = (l_2 cos(\theta_2) + l_3 cos(\theta_2 + \theta_3)) * sin(\theta_1) \tag{3.1}$$

$$P_y = (l_2 cos(\theta_2) + l_3 cos(\theta_2 + \theta_3)) * cos(\theta_1) \tag{3.2}$$

$$P_z = l_2 sin(\theta_2) + l_3 sin(\theta_2 + \theta_3) + l_1 \tag{3.3}$$

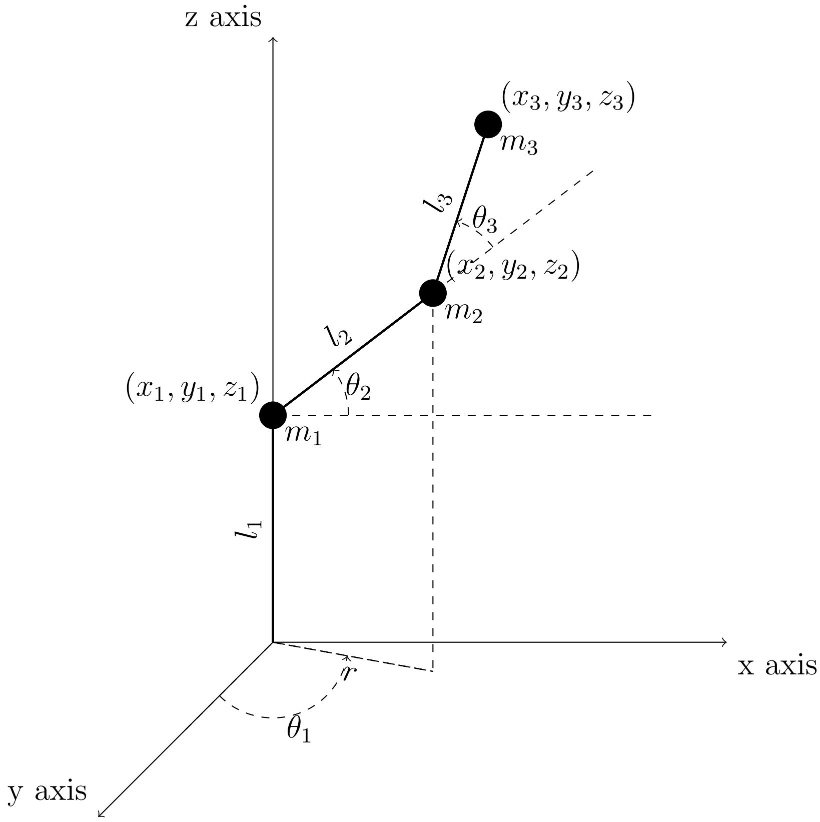

Figure 3.1: 3 link Manipulator

Inverse Kinematics

$$\theta_1 = tan^{-1}\frac{P_y}{P_x} \tag{3.4}$$

$$r^2 = P_x^2 + P_y^2 \tag{3.5}$$

$$D^2 = (P_z - l_1)^2 + r^2 \tag{3.6}$$

$$\theta_3 = cos^1\frac{D^2 - l_3^2 - l_2^2}{2l_2l_3} \tag{3.7}$$

$$\theta_2 = tan^{-1}\frac{r}{P_z - l_1} - tan^{-1}\frac{l_2 + l_3cos(\theta_3)}{l_3sin(\theta_3)} \tag{3.8}$$

Velocity Kinematics

$$\begin{aligned}\dot{x} = (cos(\theta_1) * (l_3cos(\theta_2 + \theta_3) + l_2cos(\theta_2))) * \dot{\theta}_1 \\ -(sin\theta_1 * (l_3sin(\theta_2 + \theta_3) + l_2sin\theta_2)) * \dot{\theta}_2 \\ -(l_3sin(\theta_2 + \theta_3) * sin\theta_1) * \dot{\theta}_3\end{aligned} \tag{3.9}$$

$$\begin{aligned}\dot{y} = (-sin(\theta_1) * (l_3cos(\theta_2 + \theta_3) + l_2cos(\theta_2))) * \dot{\theta}_1 \\ -(cos\theta_1 * (l_3sin(\theta_2 + \theta_3) + l_2sin\theta_2)) * \dot{\theta}_2 \\ -(l_3sin(\theta_2 + \theta_3) * cos\theta_1) * \dot{\theta}_3\end{aligned} \tag{3.10}$$

$$\begin{aligned}\dot{z} = (l_3cos(\theta_2 + \theta_3) + l_2cos\theta_2) * \dot{\theta}_2 \\ +(l_3cos(\theta_2 + \theta_3)) * \dot{\theta}_3\end{aligned} \tag{3.11}$$

3.2 Dynamics Equations

Similar to 2-link manipulator, we will use Lagrangian dynamics to solve the dynamics of this 3-link manipulator shown in 3.2.

Dynamics of the robot manipulators is complex and nonlinear that might make accurate control difficult. The dynamic equations of the robot manipulators are usually represented by the following coupled non-linear differential equations which have been derived from Lagrangians.

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) = \tau \tag{3.12}$$

and we get the following values on solving

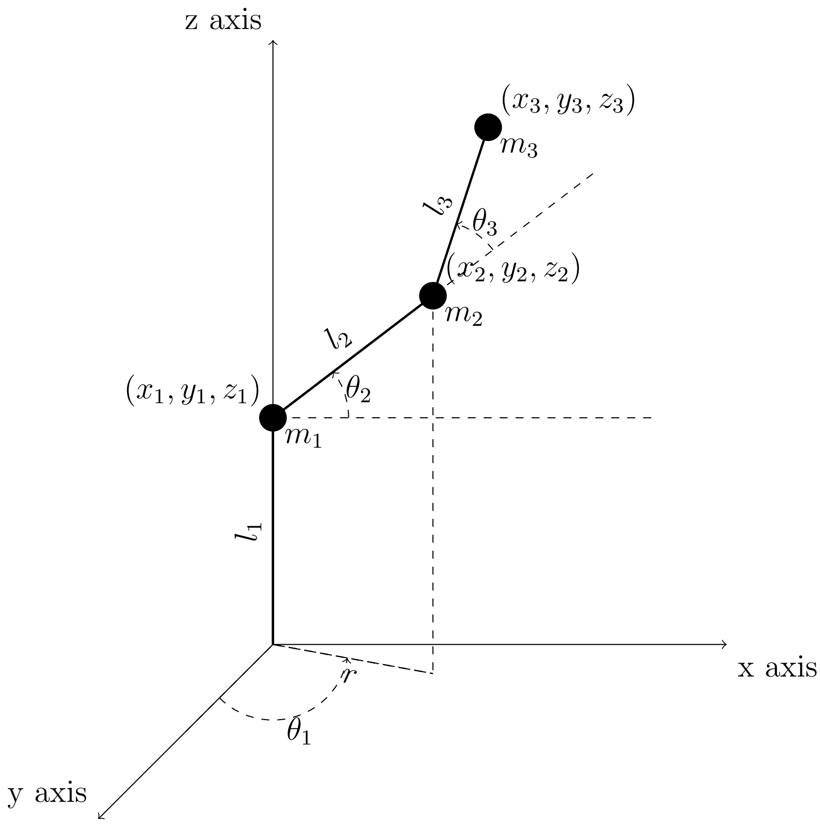

Figure 3.2: 3 link Manipulator

$$
\begin{aligned}
M_{11} &= \frac{1}{2}m_1 r^2 + \frac{1}{3}m_2 l_2^2 cos^2\theta_2 \\
&+ \frac{1}{3}m_3 l_3^2 cos^2(\theta_2+\theta_3) + m_3 l_2^2 cos^2\theta_2 \\
&+ m_3 l_2 l_3 cos(\theta_2+\theta_3) cos\theta_2 \\
M_{12} &= 0 \\
M_{13} &= 0
\end{aligned} \tag{3.13}
$$

$$
\begin{aligned}
M_{21} &= 0 \\
M_{22} &= \frac{1}{3}m_2 l_2^2 + \frac{1}{3}m_3 l_3^2 + m_3 l_2^2 + m_3 l_2 l_3 cos\theta_3 \\
M_{23} &= \frac{1}{3}m_3 l_3^2 + m_3 l_2^2 + \frac{1}{3}m_3 l_2 l_3 cos\theta_3
\end{aligned} \tag{3.14}
$$

$$
\begin{aligned}
M_{31} &= 0 \\
M_{32} &= \frac{1}{3}m_3l_3^2 + m_3l_2^2 + \frac{1}{3}m_2l_2l_3cos\theta_3 \\
M_{33} &= \frac{1}{3}m_3l_3^2
\end{aligned} \tag{3.15}
$$

$$
\begin{aligned}
V_1 &= (\frac{4}{3}m_2l_2^2sin^2(theta_2 + \theta_3) - m_3l_2l_3sin(2\theta_2 + \theta_3))\dot{\theta}_1\dot{\theta}_2 \\
&+ (-\frac{1}{3}m_3l_3^2sin^2(\theta_2 + \theta_3) - m_3l_2l_3cos\theta_2sin(\theta_2 + \theta_3))\dot{\theta}_3\dot{\theta}_1 \\
V_2 &= (-m_3l_2l_3sin\theta_3)\dot{\theta}_2\dot{\theta}_3 - (\frac{1}{2}m_3l_2l_3sin\theta_3)\dot{\theta}_3^2 \\
&+ (\frac{1}{6}m_2l_2^2sin^2\theta_2 + \frac{1}{6}m_3l_3^2sin^2(\theta_2 + \theta_3) \\
&+ \frac{1}{2}m_3l_2^2sin^2\theta_2 + \frac{1}{2}m_3l_2l_3sin(2\theta_2 + \theta_3))\dot{\theta}_1^2 \\
V_3 &= (\frac{1}{2}m_3l_2l_3sin\theta_3)\dot{\theta}_2^2 \\
&+ (\frac{1}{6}m_3l_3^2sin^2(\theta_2 + \theta_3) + \frac{1}{2}m_3l_2l_3cos\theta_2sin(\theta_2 + \theta_3))\dot{\theta}_1^2
\end{aligned} \tag{3.16}
$$

$$
\begin{aligned}
G_1 &= 0 \\
G_2 &= \frac{1}{2}m_2gl_2cos\theta_2 + \frac{1}{2}m_3gl_3cos(\theta_2 + \theta_3) + m_2gl_2cos\theta_2 \\
G_3 &= \frac{1}{2}m_3gl_3cos(\theta_2 + \theta_3)
\end{aligned} \tag{3.17}
$$

where

$$\begin{cases} q = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \\ M(q) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \\ V(q, \dot{q}) = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \\ G(q) = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \\ \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \end{cases} \tag{3.18}$$

and

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) = \tau \tag{3.19}$$

Chapter 4

Designing a controller for the manipulator

Real life problems involve payload masses, wear and tear, friction and other things which are not certainties. So, we must have a controller which will be robust in nature to take care of uncertainties.

Control Theory

Control theory is an interdisciplinary branch of engineering and computational mathematics that deals with the behavior of dynamical systems with inputs, and how their behavior is modified by feedback. The usual objective of control theory is to control a system, often called the plant, so its output follows a desired control signal, called the reference, which may be a fixed or changing value. To do this, a controller is designed, which monitors the output and compares it with the reference. The difference between actual and desired output, called the error signal, is applied as feedback to the input of the system, to bring the actual output closer to the reference.
Autonomous industry came across 1950 onwards. This industry has seen a

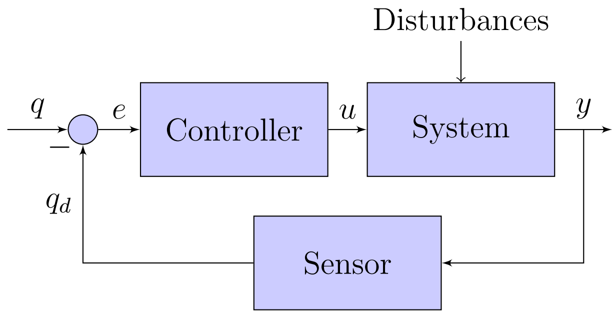

Figure 4.1: A block diagram of a negative feedback control system

great deal of advance in manufacturing and service sector. Control in last 70 years, first half was based on fixed gain controllers. Then they moved on to range of gain controllers. Finally, for last 20 years, researchers have been focusing on Adaptive control, Machine learning and Artificial Intelligence.

This report deals with controlling the Manipulator using a PID controller.

4.1 PID Controller

A Proportional Integral Derivative controller (PID controller) is a control loop feedback mechanism (controller) commonly used in industrial control systems. A PID controller continuously calculates an error value e(t) as the difference between a desired setpoint and a measured process variable and applies a correction based on proportional, integral, and derivative terms (sometimes denoted P, I, and D respectively) which give their name to the controller type.

A PID controller continuously calculates an error value $e(t)$ as the difference between a desired set point and a measured process variable and applies a correction based on proportional, integral, and derivative terms. The controller attempts to minimize the error over time by adjustment of a control

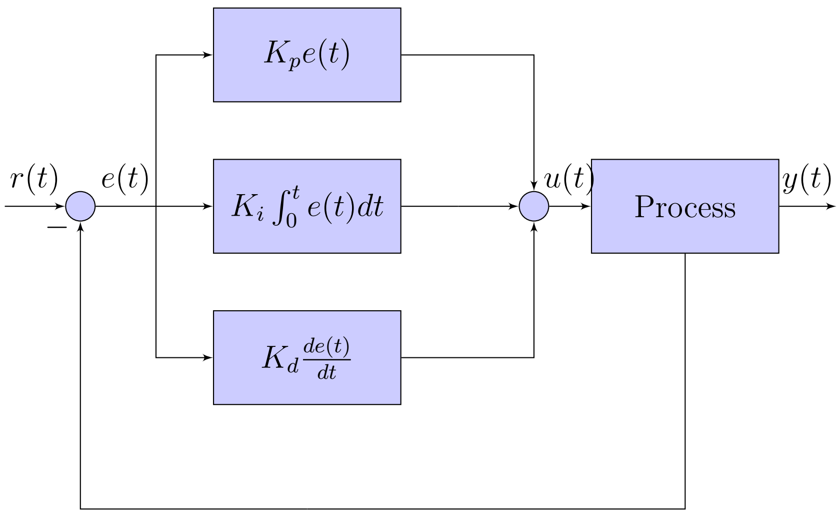

Figure 4.2: PID Controller Block Diagram

variable $u(t)$, such as the position of a control valve, a damper, or the power supplied to a heating element, to a new value determined by a weighted sum:

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \tag{4.1}$$

where K_p, K_i, and K_d all non-negative, denote the coefficients for the proportional, integral, and derivative terms, respectively (sometimes denoted P, I, and D).

From the dynamics equations, we know that:

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) = \tau \tag{4.2}$$

We can have

$$\ddot{q} = M(q)^{-1}[-V(q, \dot{q}) - G(q)] + M(q)^{-1}\tau \tag{4.3}$$

assume

$$U = M(q)^{-1}\tau \tag{4.4}$$

here U is control input. However, the physical torque inputs to the system are

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = M(q) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{4.5}$$

i.e.

$$\tau = M(q)U \tag{4.6}$$

We have initial positions as $\begin{bmatrix} \theta_{1i} \\ \\ \theta_{2i} \end{bmatrix}$ and final positions as $\begin{bmatrix} \theta_{1f} \\ \\ \theta_{2f} \end{bmatrix}$

We have control function U as

$$U = K_p e + K_i \int_0^t e dt + K_d \frac{de}{dt} \tag{4.7}$$

So in our case

$$u_1 = K_{p1}(\theta_{1f} - \theta_1) + K_{d1}\dot{\theta}_1 + K_{i1} \int_0^t e(\theta_1) dt \tag{4.8}$$

$$u_2 = K_{p2}(\theta_{2f} - \theta_2) + K_{d2}\dot{\theta}_2 + K_{i2} \int_0^t e(\theta_2) dt \tag{4.9}$$

Recall: with actual physical torques of

$$\begin{bmatrix} \tau_1 \\ \\ \tau_2 \end{bmatrix} = M(q) \begin{bmatrix} u_1 \\ \\ u_2 \end{bmatrix} \tag{4.10}$$

for the integration, we can say

$$z_1 = \int_0^t e(\theta_1) dt \rightarrow \dot{z}_1 = \theta_{1f} - \theta_1 \tag{4.11}$$

$$z_2 = \int_0^t e(\theta_2) dt \rightarrow \dot{z}_2 = \theta_{2f} - \theta_2 \tag{4.12}$$

Therefore, the complete system of equations are (from Eq 4.3)

$$\begin{cases} \dot{z}_1 = \theta_{1f} - \theta_1 \\ \dot{z}_2 = \theta_{2f} - \theta_2 \\ \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = M(q)^{-1}[-V(q,\dot{q}) - G(q)] \begin{bmatrix} K_{p1}(\theta_{1f} - \theta_1) + K_{d1}\dot{\theta}_1 + K_{i1}z_1 \\ K_{p2}(\theta_{2f} - \theta_2) + K_{d2}\dot{\theta}_2 + K_{i2}z_2 \end{bmatrix} \end{cases} \tag{4.13}$$

Chapter 5

Simulations

5.1 PID Controller

The 2 controller's parameters were tuned to have the best performance. The best values for the parameters was found to be

$$K_{p1} = -200 \quad K_{d1} = -200 \quad K_{i1} = -200 \tag{5.1}$$

$$K_{p2} = -200 \quad K_{d2} = -200 \quad K_{i2} = -200 \tag{5.2}$$

Fig 5.1 is the Simulink model using PID controller for a 2-link manipulator.

Circular path (Fig 5.5, Fig 5.6) and Triangular path (Fig 5.8, Fig 5.9) was used for simulation

Model.JPG

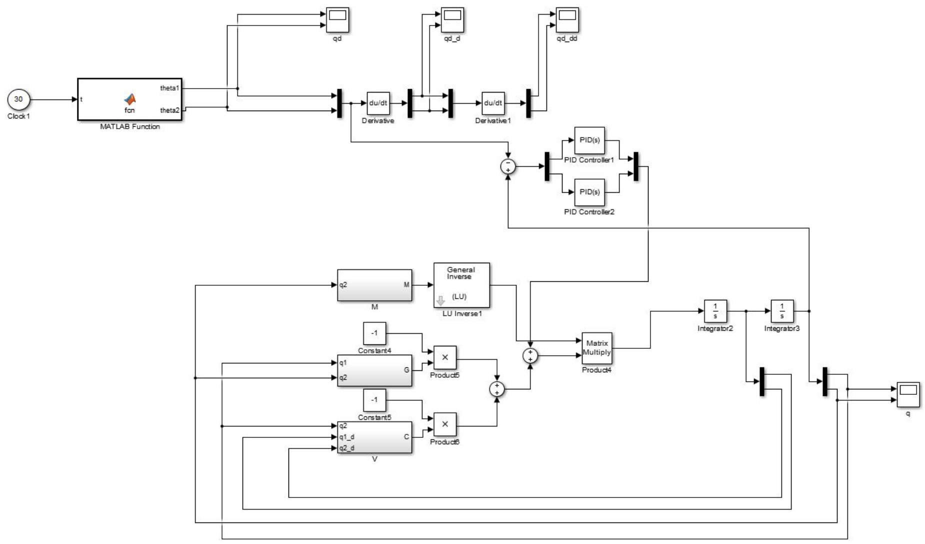

Figure 5.1: 2-Link PID model on Simulink

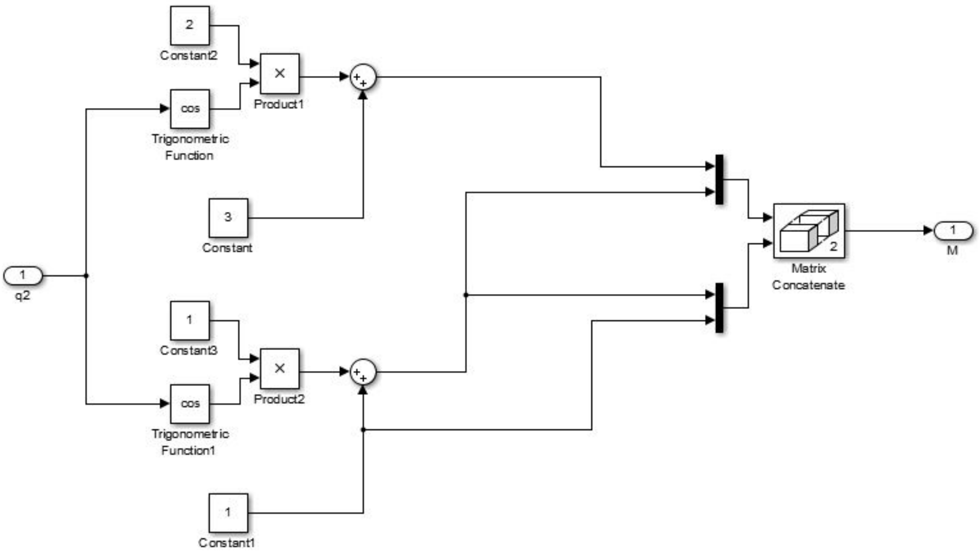

Figure 5.2: The Block representing the Mass Matrix of the 2-Link Manipulator

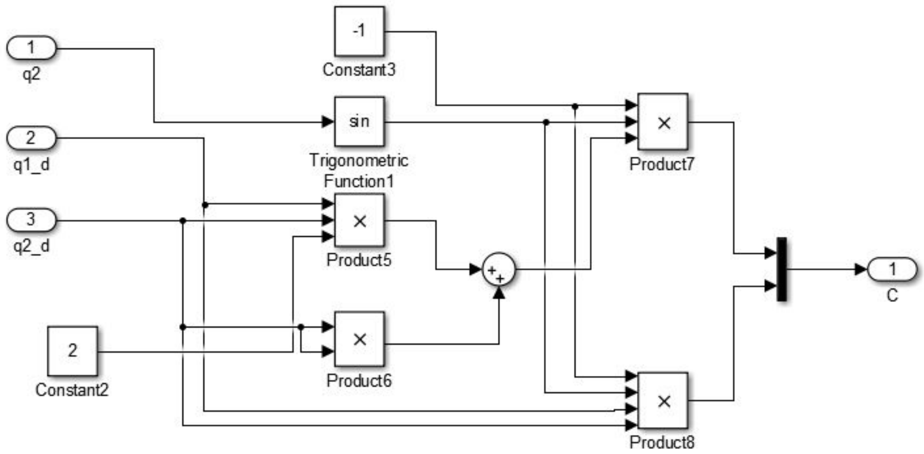

Figure 5.3: The Block representing the Velocity Product term of the 2-Link Manipulator

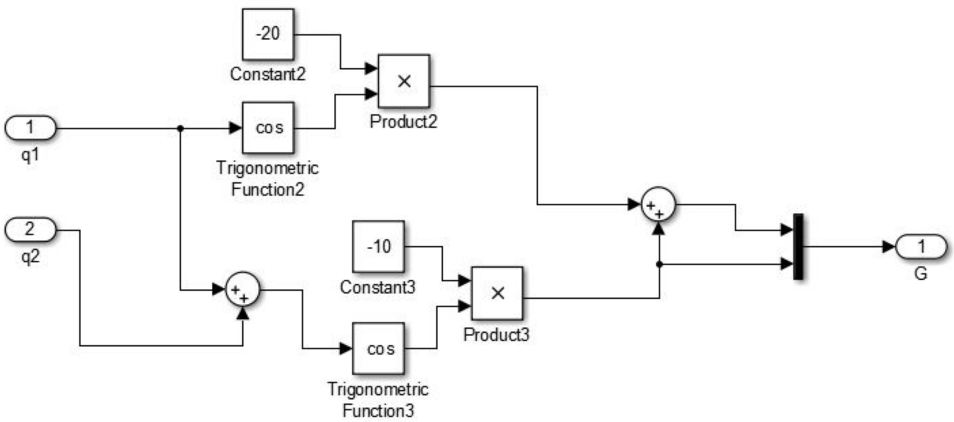

Figure 5.4: The Block representing the Gravity of the 2-Link Manipulator

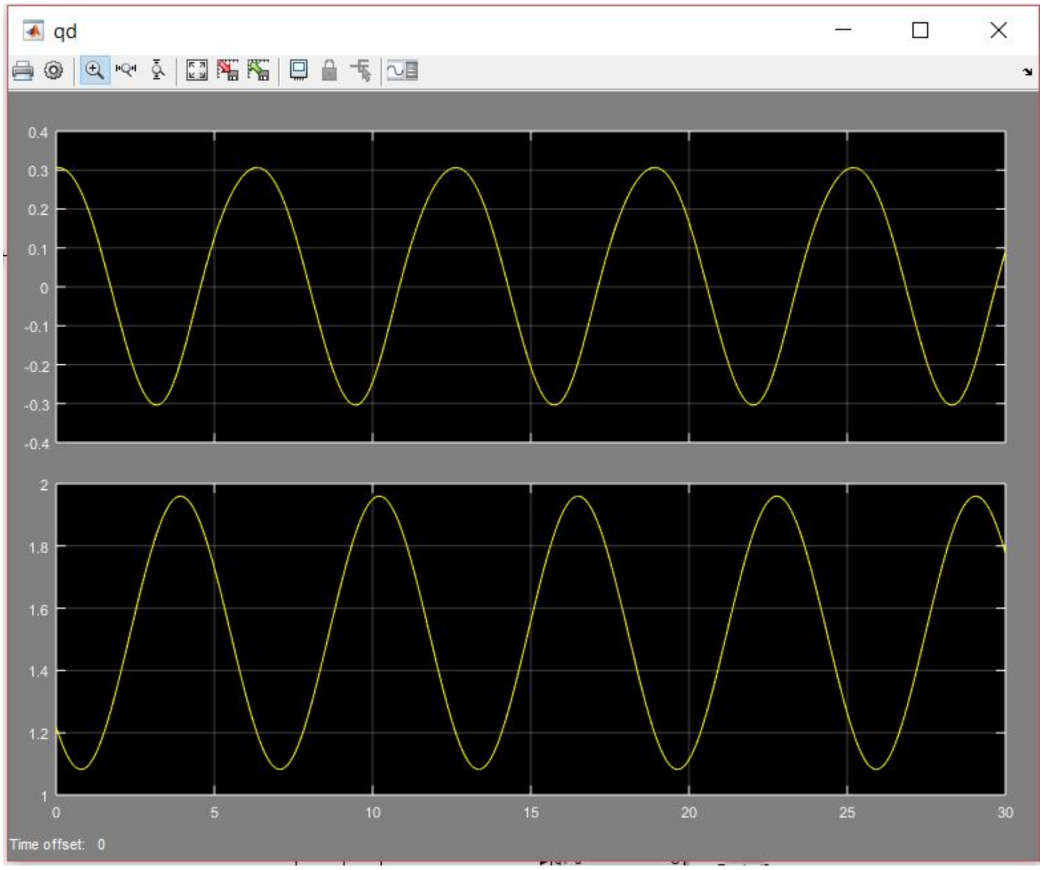

Figure 5.5: Reference theta for circular path using inverse kinematics

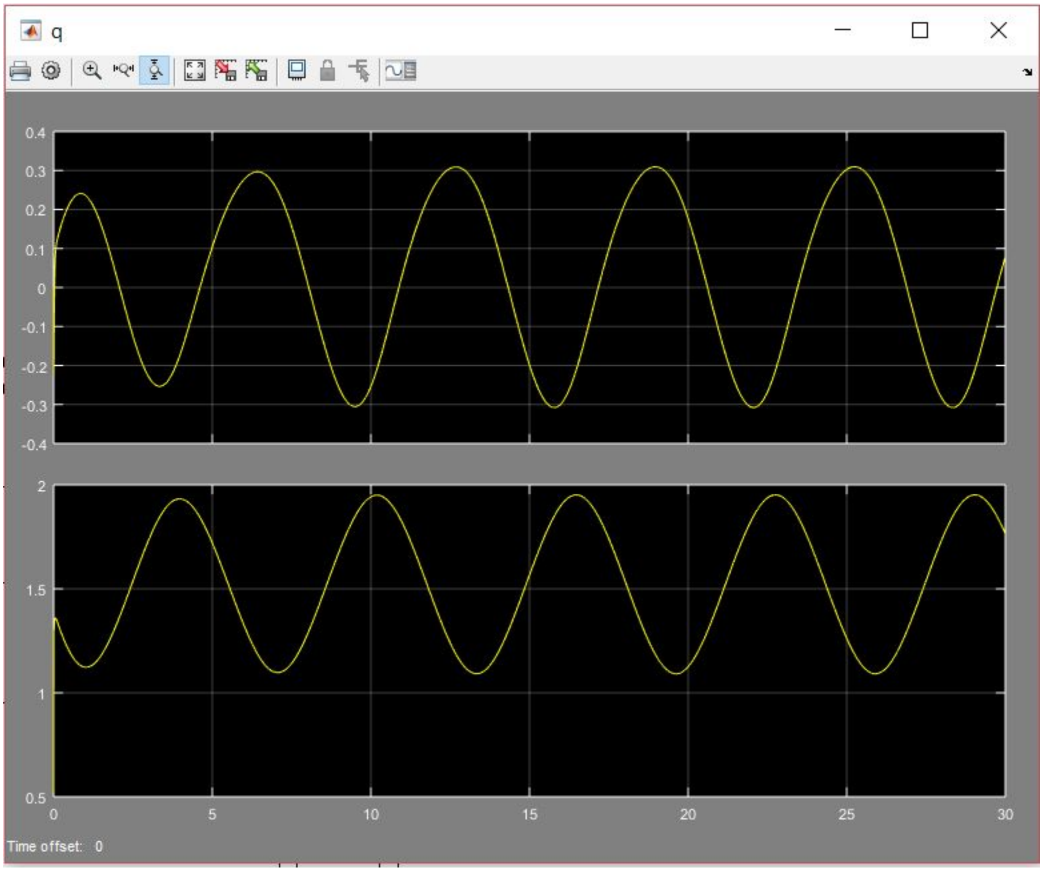

Figure 5.6: Output theta for circular path

manipulator (1).JPG

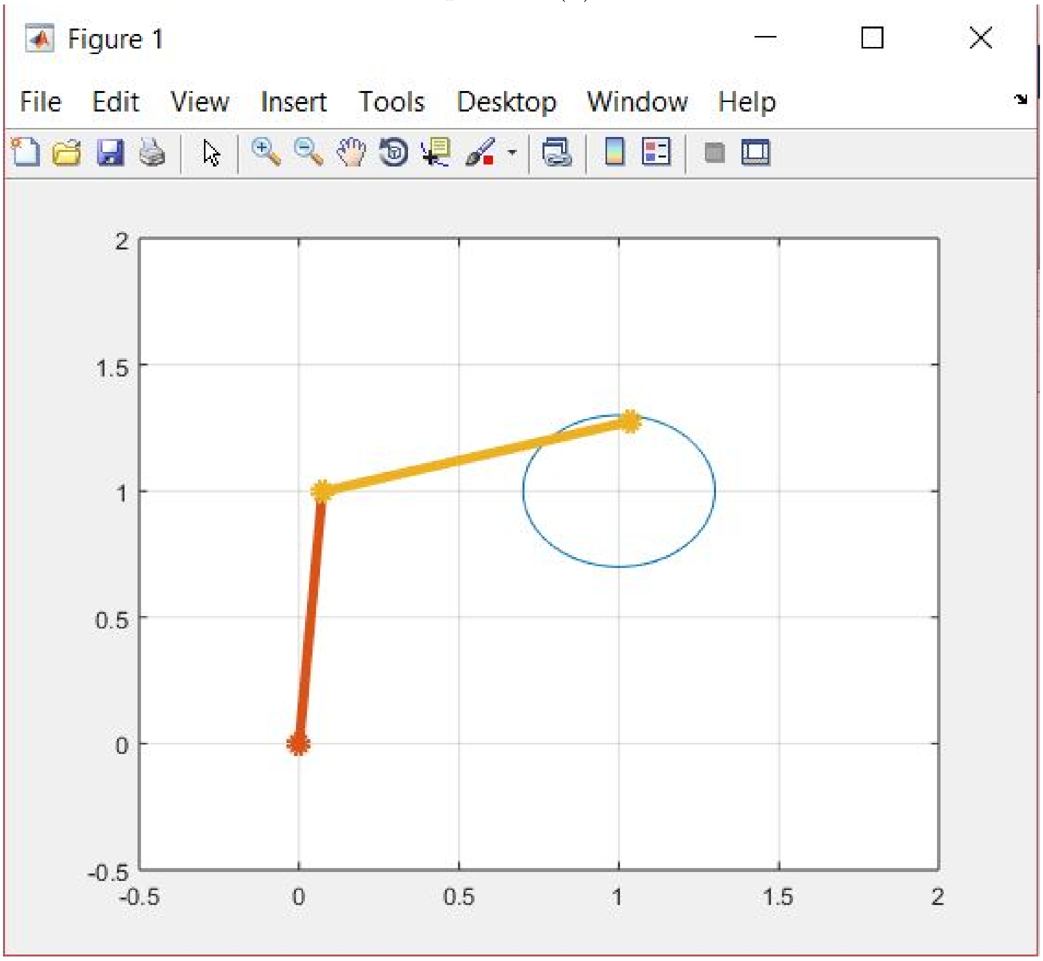

Figure 5.7: Snapshot of animation of the simulation for circular path

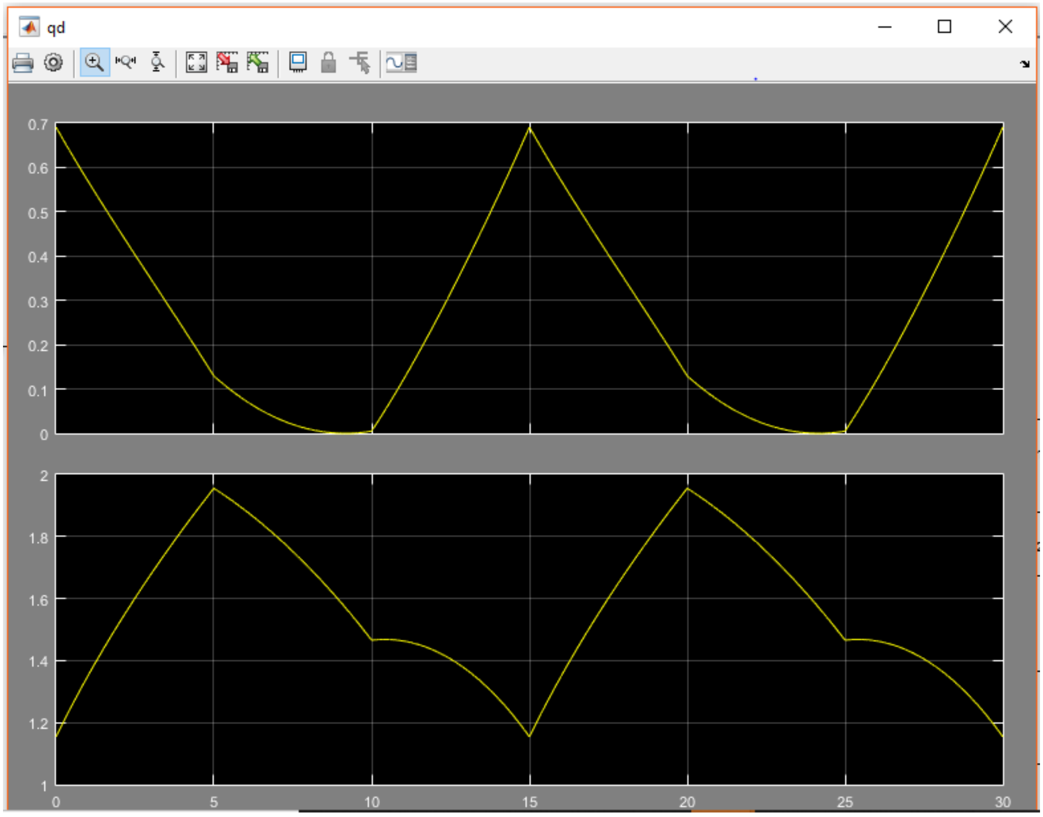

Figure 5.8: Reference theta for triangular path using inverse kinematics

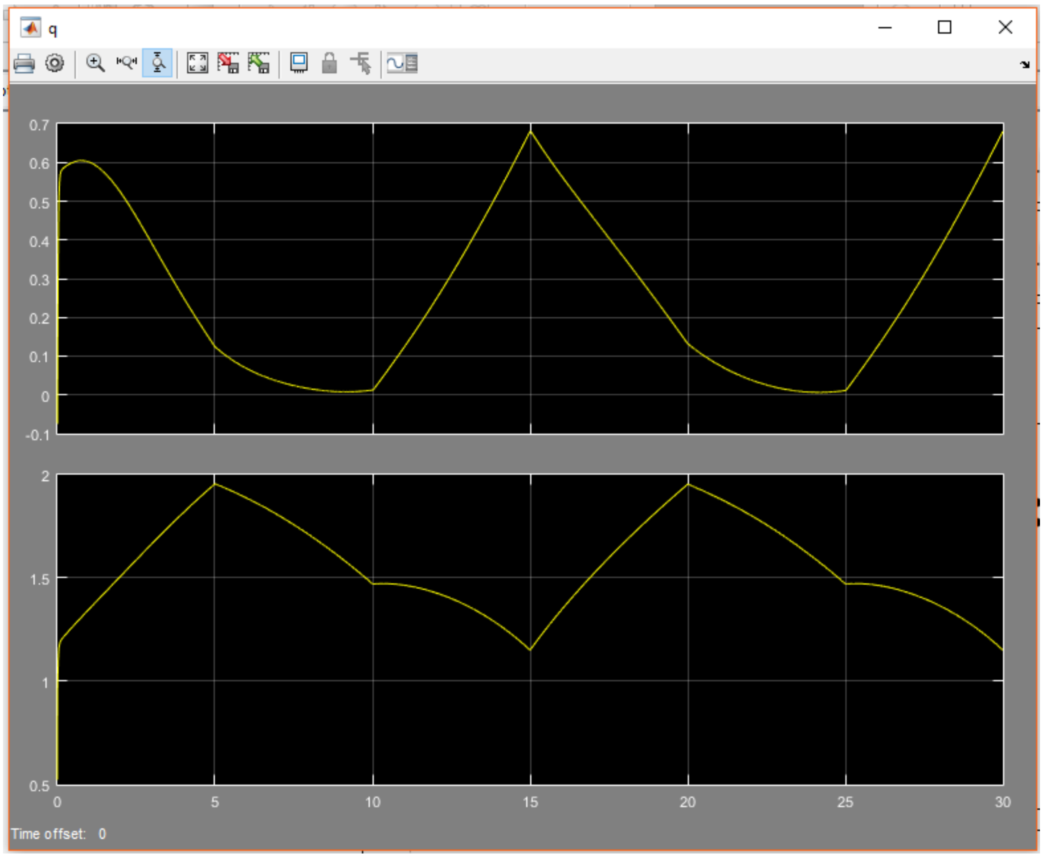

Figure 5.9: Output theta for triangular path

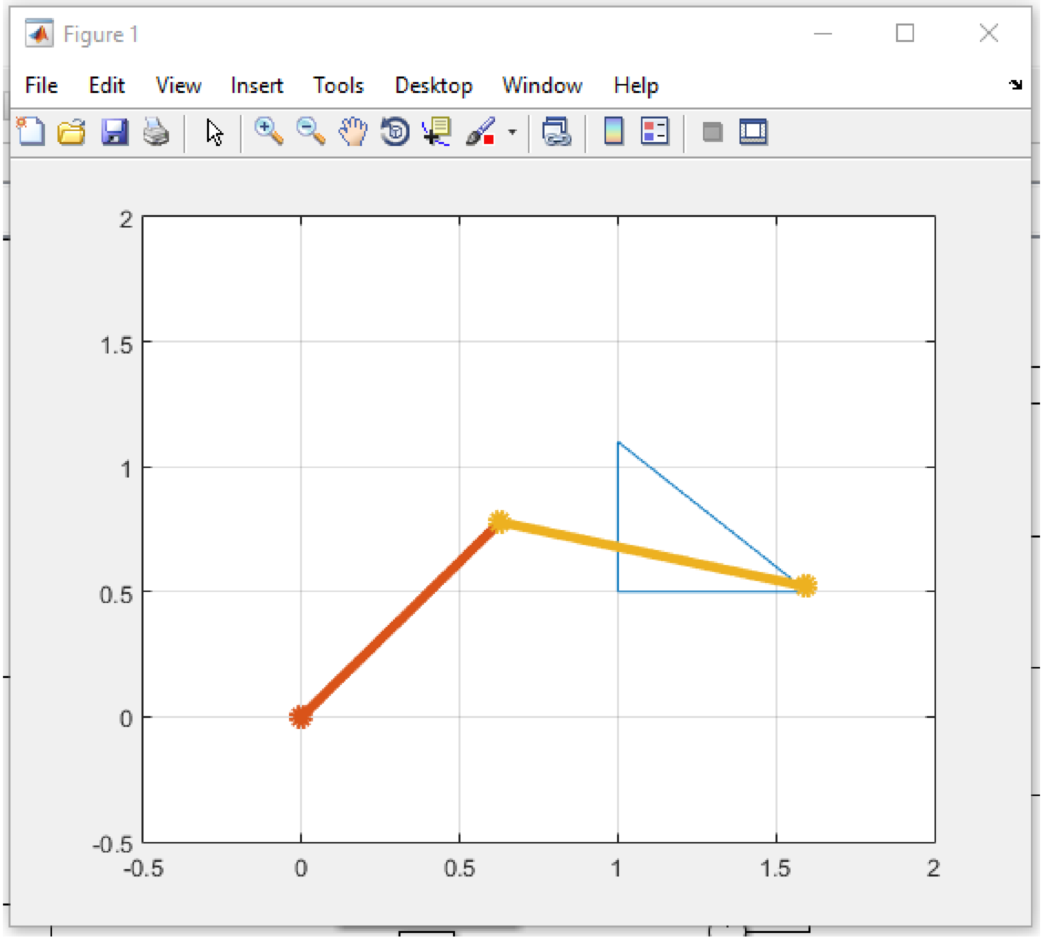

Figure 5.10: Snapshot of animation of the simulation for circular path

5.2 3-link Manipulator with PID

The 3 controller's parameters were tuned to have the best performance. The best values for the parameters was found to be

$$K_{p1} = -0.9 \quad K_{d1} = -4.5 \quad K_{i1} = -0.04 \tag{5.3}$$

$$K_{p2} = 114894673 \quad K_{d2} = 52442 \quad K_{i2} = 38277068912 \tag{5.4}$$

$$K_{p2} = -1303 \quad K_{d2} = -103 \quad K_{i2} = -3294 \tag{5.5}$$

The simulation for 3-link Manipulator using PID controller is shown below (Fig 5.15, Fig 5.16).
The model for the simulation is shown in Fig 5.11.

model.JPG

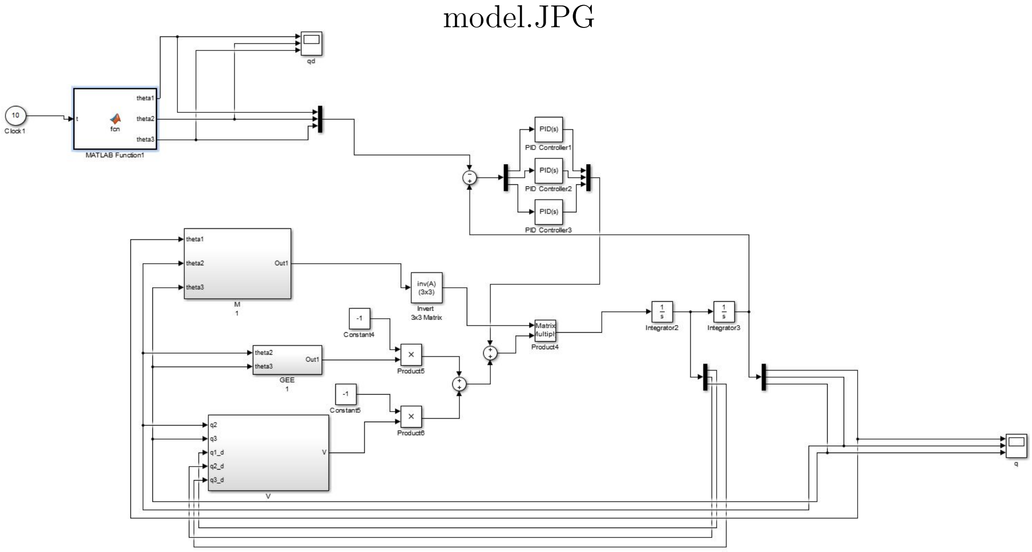

Figure 5.11: The Simulink Model

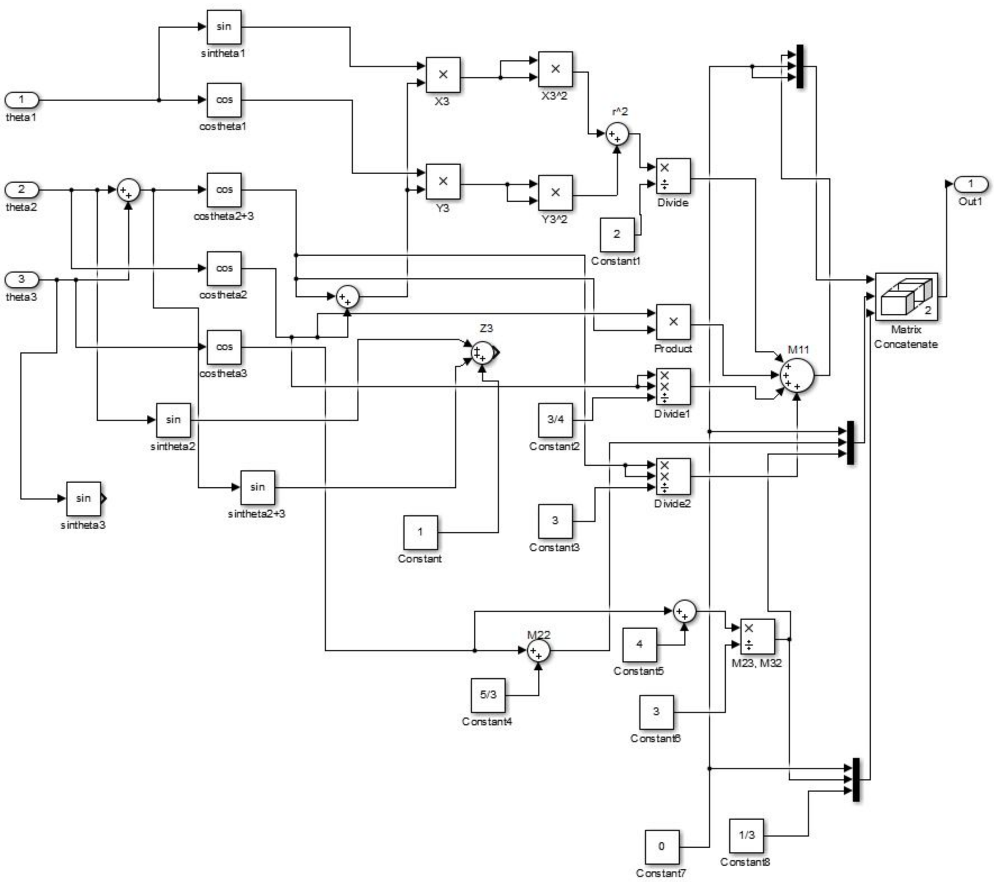

Figure 5.12: The Block representing the Mass Matrix of the 3-Link Manipulator

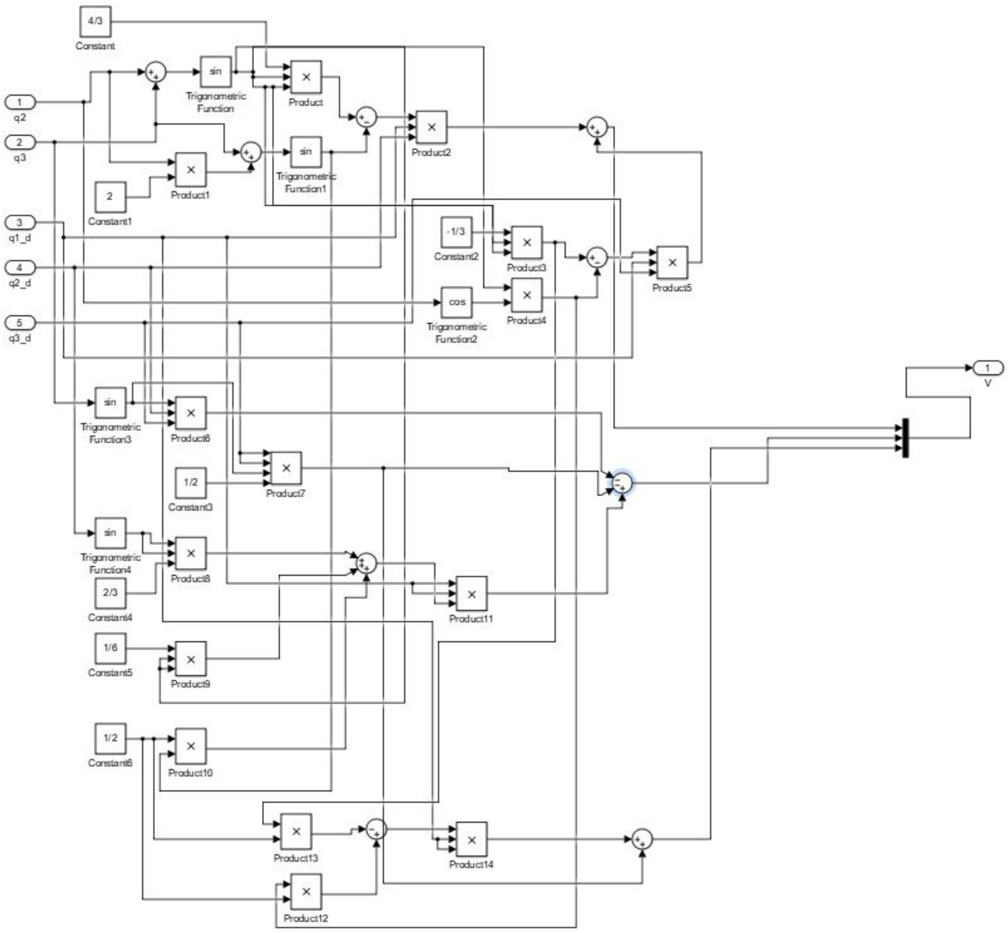

Figure 5.13: The Block representing the Velocity Product term of the 3-Link Manipulator

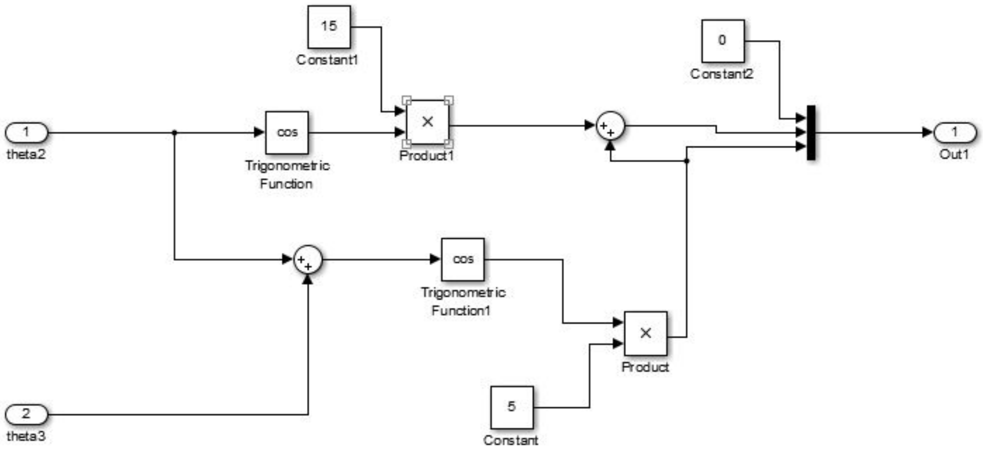

Figure 5.14: The Block representing the Gravity of the 3-Link Manipulator

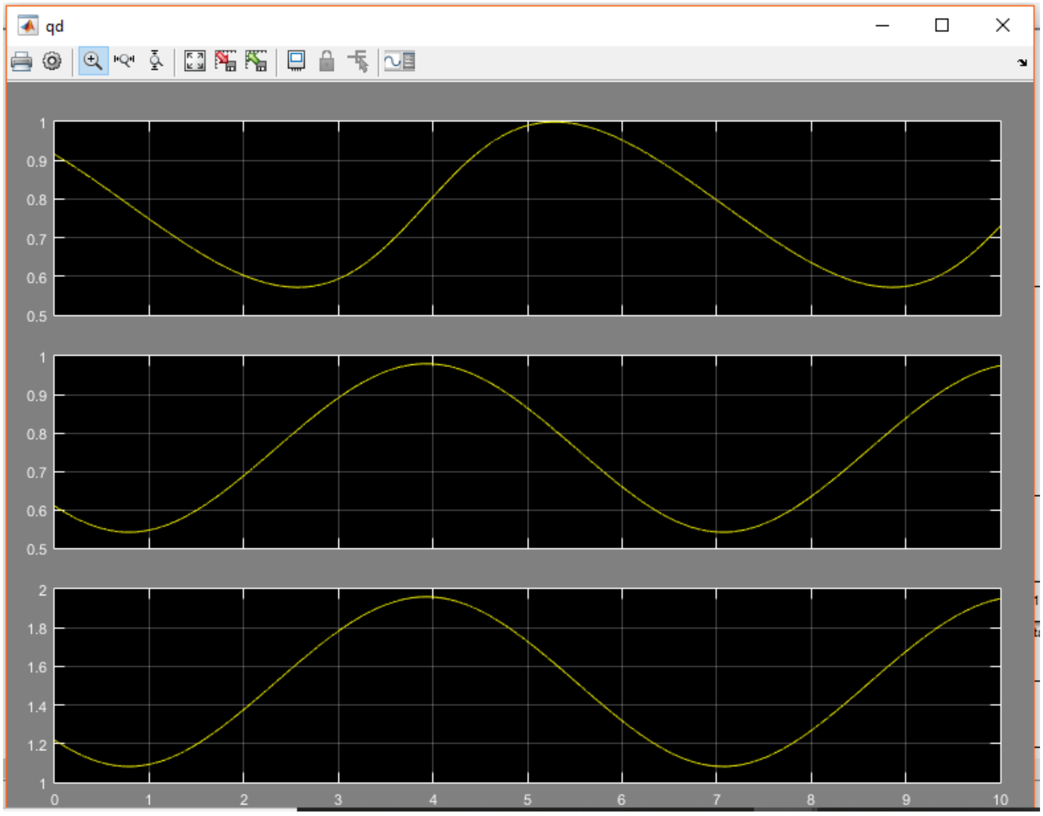

Figure 5.15: reference theta of 3d manipulator

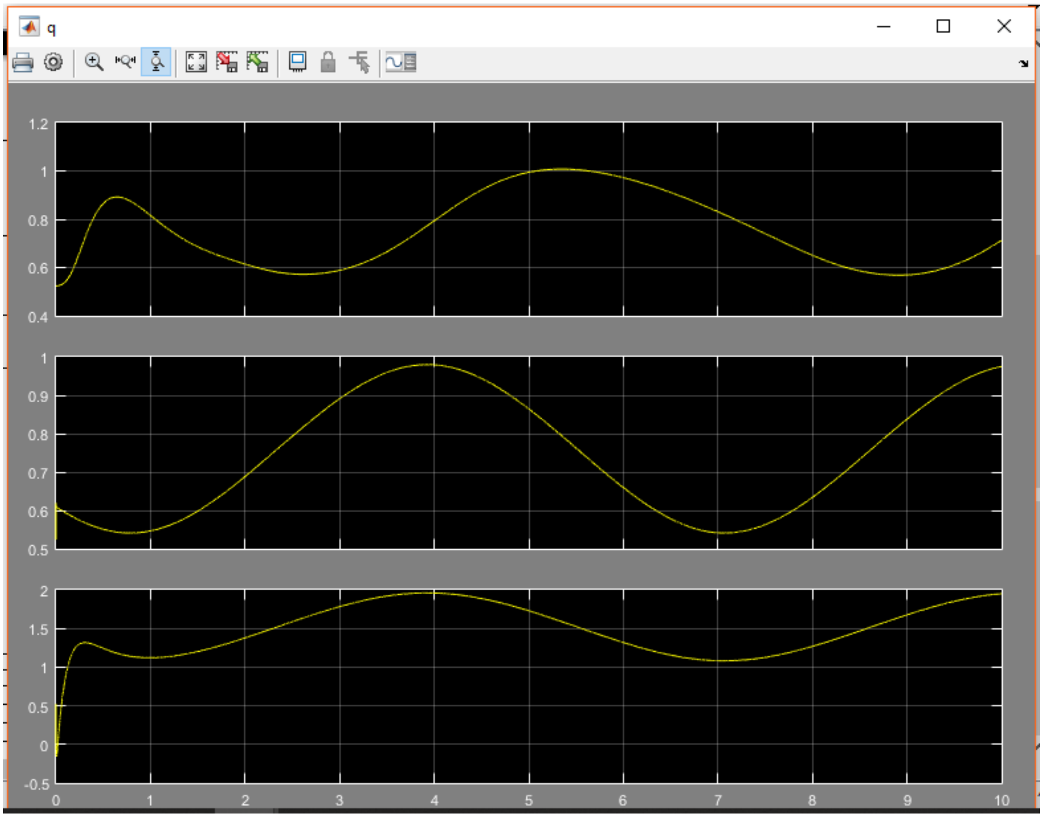

Figure 5.16: output theta of 3d Manipulator

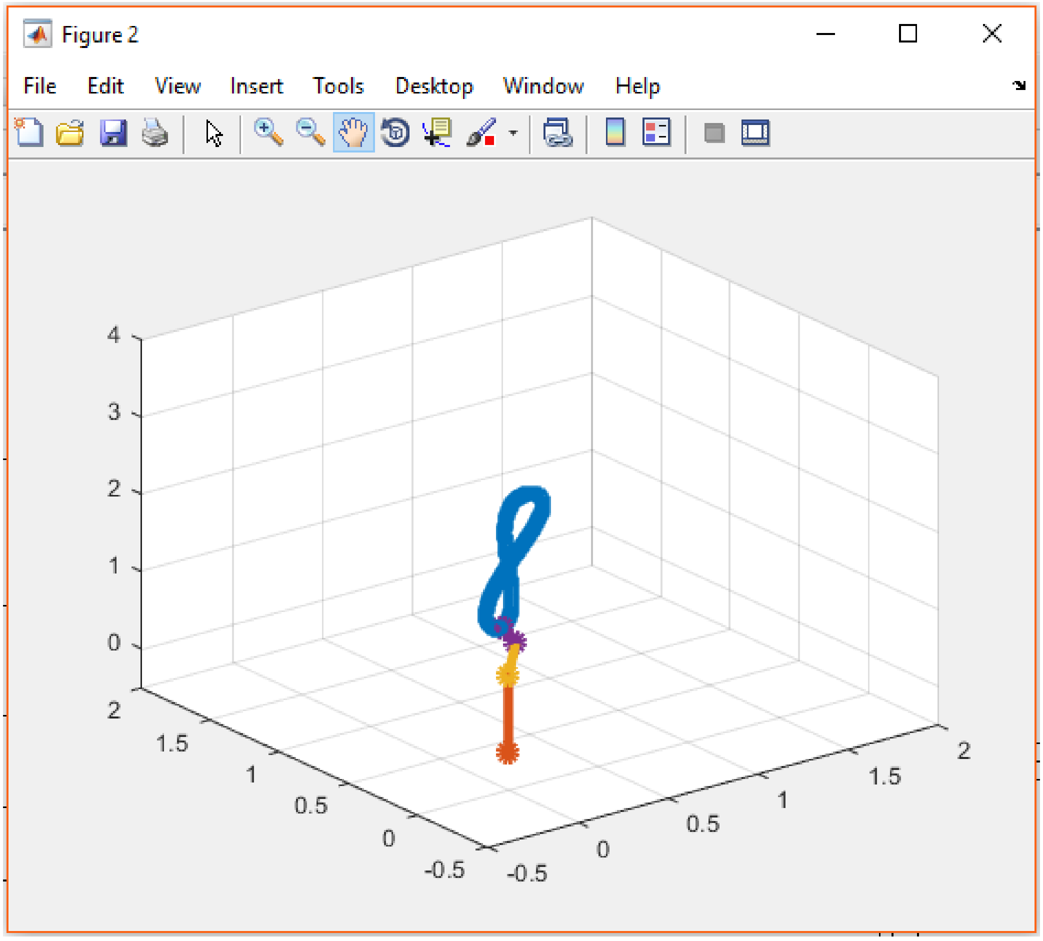

Figure 5.17: Snapshot of animation of the motion of 3-Link Manipulator along a set path

Chapter 6

Observations

1. By tweaking the PID variables, it was observed that the $\dot{q}$ (ω of the joints) was under control and did not reach high values.
2. By studying the PID variables, it was observed that K_p is related to reducing the steady state error of the system and also has an effect on the speed, K_d is related to transient behaviour and K_i is related to overall error cancellation.
3. However, the error is slightly high in case of controller with PID because of the high non linearity of the equation that produces sensitive interaction between controller components and small changes in controller parameters would produce more overshoots and oscillations. The controller is extremely sensitive to slight changes on other variables it is dependant on.

Chapter 7

Conclusion

We successfully modelled a 2-link and 3-link Manipulator on MATLAB Simulink using a PID controller. The results of model were then made into an animation which replicates the motion of the corresponding manipulator in real life.The simulated model was made to run on a variety of predefined paths like circle, triangle etc.

On studying the results of the simulated model we observed that the error in the angles were higher initially due to the presence on transients in the system. Tweaking the PID parameters brings about more accurate and faster results.

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