UNIT # 02 (PART – II)

WORK, POWER, ENERGY AND CONSERVATION LAWS

EXERCISE –I

1. By applying work energy theoram change in kinetic energy = W_g + $W_{ext.P}$

$0 = mg(\ell \cos 37 - \ell \cos 53) + W_{ext\ P}$

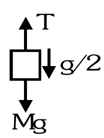
$$= 50 \quad 10 \quad 1\left[\frac{3}{5}-\frac{4}{5}\right] + W_{ext\ P}$$

W_{ext} = 100 joule

2. Work $=\vec{F}.\vec{dr}$, Work $=-\int_0^{\theta}(0.5)(5)Rd\theta \quad \therefore F=mN$

$\Rightarrow$ [work] = (2.5) (R) (2π) = –5 J

3. $W = \vec{f}.\vec{d}$



$$mg - T = \frac{Mg}{2}\ ;\ T = \frac{Mg}{2}$$

$$W = \left(-\frac{Mg}{2}\right)x$$

4. For conservation force work done is independent of the path

$W_{AB} + W_{BC} = W_{AC}$, 3+4 = W_{AC} = 7 J

5. By applying work energy theorem

$$\Delta KE = \vec{f}.\vec{d} = m\left(\frac{v}{t_1}\right)\frac{1}{2}\left(\frac{v}{t_1}\right)t^2 \Rightarrow \Delta K.E. = \frac{mv^2}{2t_1^2}t^2$$

6. Slope of v–t graph ⇒ Acceleration ⇒ –10m/s^2
Area under v–t graph → displacement ⇒ 20 m

work $=\vec{f}.\vec{s}$ = 2 (10) (20) ⇒ –400 J

7. By applying work energy theoram
ΔKE = work done by all the forces

New kinetic energy = $\frac{1}{2}mv_f^2 = \frac{mv^2}{8}$

$$\Rightarrow v_f = \frac{v_0}{2} \Rightarrow v = u - \mu g t_0 \Rightarrow \mu \Rightarrow \frac{v_0}{2gt_0}$$

8. By applying work energy theoram

$$\frac{1}{2}m\frac{v^2}{4} - \frac{1}{2}mv^2 = -\frac{1}{2}kx^2$$

$$\Rightarrow \frac{-3mv^2}{8} = \frac{-1}{2}kx^2\ ;\ k = \frac{3mv^2}{4x^2}$$

9. Total mass ; f ∝ 6m, f = 6m_C (20) = P
To Drive 12m : f ∝ 14m ⇒ f = 14 m_C
(14 m_C) v = 6(m_C) 20 ⇒ 8.57m/s
To drive 6 boggie : force ∝ 8m
force = 8m_C ⇒ P = 8 m_Cv
(8m_C)v = 120m_C ⇒ 15 m/s

10. By applying work energy theoram

$$\frac{1}{2}mv^2 - 0 = W_g + W_{fr}$$

for the second half work energy theorem change in kinetic energy = $W_g + W_{fr}$
0 = 100mg + W_{fr} = –100 mg
As work done for the first half by the gravity is 100mg therefore work done by air resistance is less than 100 mg.

11. $x = 3t - 4t^2 + t^3;\ v = \frac{dx}{dt} = 3 - 8t + 3t^2$

$$a = \frac{dv}{dt} = 0 - 8 + 6t$$

$$W = \int \vec{F}.\vec{dx} = \int_0^4 3(6t-8)(3-8t+3t^2)dt$$

W= 528 mJ

OR

From work energy theorem

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}(3\times10^{-3})$$

$$\left[\left(3-8(4)+3(4)^2\right)-(3)^2\right] = 528\ mJ$$

12. Power = constant, Fv = C

$$mvdv = Cdt \Rightarrow v^2 = \frac{2C}{m}t \Rightarrow v = \sqrt{\frac{2C}{m}t}$$

as $v = \frac{dx}{dt} \Rightarrow \int dx = \sqrt{\frac{2C}{m}}\int\sqrt{t}dt$

$$x = \sqrt{\frac{2C}{m}}\frac{t^{3/2}}{2/3} \Rightarrow x \propto t^{3/2}$$

13. $a_c = k^2 rt^2 \Rightarrow \frac{v^2}{r} = k^2rt^2$

$\Rightarrow v^2 = k^2r^2t^2 \Rightarrow v = krt \Rightarrow a_T = \dfrac{dv}{dt} = kr$

$P = \int m\vec{a}_T.\vec{v} = m(kr).(krt) = mk^2r^2t$

14. P.E. $\rightarrow$ Maximum $\rightarrow$ Unstable equilibrium
P.E. $\rightarrow$ Minimum $\rightarrow$ Stable equilibrium
P.E. $\rightarrow$ Constant $\rightarrow$ Natural equilibrium
$\therefore$ None of these

15. P.E. $\rightarrow$ Maximum $\rightarrow$ Unstable equilibrium
P.E. $\rightarrow$ Minimum $\rightarrow$ Stable equilibrium
P.E. $\rightarrow$ Constant $\rightarrow$ Natural equilibrium

Force $= -\dfrac{dU}{dx} \Rightarrow$ –(slope)

[slope is –ve from E to F]

Force = +ve repulsion

Force = –ve attraction

16. By applying work energy theorem
ΔKE = Work done by all the forces
$0 = W_g + W_{spring} + W_{ext\ agent}$
$-W_g = (W_{spring} + W_{ext\ agent})$
$\Delta U = (W_{spring} + W_{ext\ agent})$ $[\because \Delta U = W_g]$

17. $\Delta U = mgh$
height w.r.t. ground $= (\ell - h)$, $\Delta U = mg(\ell - h)$

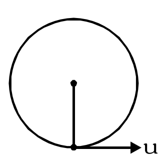
18. By applying work energy theorem

$\Delta K.E = W_S + W_{ext\ agent}$

$0 = -\dfrac{1}{2}Kx^2 + Fx \Rightarrow x = \dfrac{2F}{K}$

Work done $= \dfrac{2F^2}{K}$

19. At lowest point



$T - mg = \dfrac{mu^2}{\ell}$ ....(i)

at highest point T = 0

$mg = \dfrac{mv^2}{\ell}$, $v = \sqrt{g\ell}$ and $v^2 = u^2 + 2as$

$\left(\sqrt{g\ell}\right)^2 = u^2 + 2(-g) \times 2\ell$

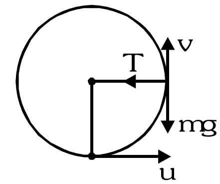
$g\ell = u^2 - 4g\ell$

$u^2 = 5\,g\ell$

Put the value of u^2 in equation (i)

$T - mg = \dfrac{m(5g\ell)}{\ell} \Rightarrow T = 6\ mg$

20. When the string is horizontal



$T = \dfrac{mv^2}{\ell}$ ...(i)

$v^2 = u^2 - 2g\ell$

$v^2 = 5g\ell - 2g\ell = 3g\ell$

So $T = \dfrac{m \cdot 3g\ell}{\ell} = 3mg$

So net force

$= \sqrt{T^2 + (mg)^2} = \sqrt{(3mg)^2 + (mg)^2} = \sqrt{10}\ mg$

21. In case of rod the minimum velocity of particle is zero at highest.

22. As velocity is vector quantity

$\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$ [as $\theta = 90$]

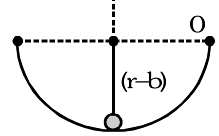
$\Delta v = \sqrt{v_1^2 + v_2^2}$

By applying work energy theorem velocity at z

$\dfrac{1}{2}mv_2{}^2 - \dfrac{1}{2}mu^2 = -\ mgL$

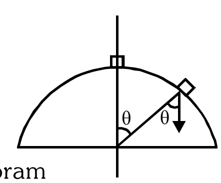
$v_2{}^2 = u^2 - 2gL \Rightarrow \Delta u = \sqrt{2(u^2 - gL)}$

23. By applying work energy theorem $\Delta KE = W_g$



$\dfrac{1}{2}mv^2 = mg(r-b) \Rightarrow v = \sqrt{2g(r-b)}$

24. Net force towards centre equal $= \dfrac{mv^2}{r}$



$mg\cos\theta - N = \dfrac{m_x v^2}{r}$

$v = \sqrt{rg\cos\theta}$

By applying work energy theoram

$\dfrac{1}{2}mrg\cos\theta - 0 = mgr(1 - \cos\theta) = \cos\theta = \dfrac{2}{3}$

25. $\Delta P = \sqrt{P_1^2 + P_2^2 - 2P_1P_2\cos\theta}$

for $\cos\theta$ = maximum $\Rightarrow$ ΔP minimum $\theta = 360$
for $\cos\theta$ = minimum $\Rightarrow$ ΔP maximum $\theta = 180$

26. Tension at any point T = 3mgcosθ

Given 3mg cosθ = 2mg

$$\Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

EXERCISE –II

1. For body B : mg – T = m(2a)

For body A : 2T – mg = ma $\Rightarrow a=\frac{g}{5}$

$a_B = 2a_A$ and $a_A = a$

∴ Velocity of B after travelling distance ℓ

$$= \sqrt{2as} = \sqrt{\frac{4g\ell}{5}}$$

∴ Velocity of A : $v_A = \frac{v_B}{2} = \sqrt{\frac{g\ell}{5}}$

2. COME $\Rightarrow K_1 + U_1 = K_2 + U_2$

$$\Rightarrow 0 + \frac{1}{2}k_1x^2 + \frac{1}{2}k_2x^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}k_1\left(\frac{x}{2}\right)^2 + \frac{1}{2}k_2\left(\frac{x}{2}\right)^2$$

$$\Rightarrow \frac{1}{2}(k_1 + k_2)x^2 = \frac{1}{2}mv^2 + \frac{1}{8}(k_1 + k_2)x^2$$

$$\Rightarrow v = \sqrt{\frac{3}{4}\frac{(k_1+k_2)x^2}{m}}$$

3. Work done against friction = mgh = loss in P.E.

∴ Work done by ext. agent

$=W_f + \Delta PE$

= mgh + mgh = 2mgh

4. COME $\Rightarrow K_1 + U_1 = K_2 + U_2$

$$0 + mg\ell\,(1-\cos 60\,) = \frac{1}{2}mv^2 + 0 \Rightarrow v = \sqrt{g\ell}$$

5. COME : $K_1 + U_1 = K_2 + U_2$

$$0 + mg\,(4R) = \frac{1}{2}mv^2 + mg\,(2R) \Rightarrow mv^2 = 4mgR$$

Forces at position 2 :

$$N=\frac{mv^2}{R} - mg = 4mg - mg = 3\,mg$$

6. $F_{ext} = m_2g - m_1g$ ∴ $P_{inst} = f_{ext}.v = (m_2-m_1)gv$

7. COME : $K_B + U_B = K_C + U_C$

$$\frac{1}{2}mv_0^2 + mgr = \frac{1}{2}mv_C^2 + mg\,r\cos\theta \quad ...(i)$$

Force equation at C

$$\Rightarrow N + \frac{mv_C^2}{r} = mg\cos\theta \quad ...(ii)$$

at C, $N = 0 \Rightarrow \cos\theta = \frac{3}{4}$

8. $W_{man} = \Delta U = U_f - U_i = \left(\frac{m}{2}\right)g\left(\frac{\ell}{4}\right) - \frac{mg\ell}{2} = -\frac{3mg\ell}{8}$

9. At $x = -\sqrt{\frac{2E}{k}}$; $E_{total} = \frac{1}{2}kx^2 = U$ ∴ KE = 0

10. Equation of motion :

$m_A g\sin 37 - T = m_Aa_A$ and $2T - m_Bg = m_Ba_B$

$$a_A = 2a_B = 2\ \frac{g}{12} = \frac{g}{6}$$

$$\therefore v_A = \sqrt{2a_A.s_A} = \sqrt{2\times\frac{g}{6}\times 1} = \sqrt{\frac{g}{3}}$$

$$\therefore v_B = \frac{v_A}{2} = \frac{\sqrt{g}}{2\sqrt{3}}$$

11. COME : $K_B + U_B = K_A + U_A$

$$0 + \frac{1}{2}k\,(13-7)^2 = \frac{1}{2}mv_A^2 + 0$$

$$N_A = \frac{mv_A^2}{R} = \frac{k\times 6^2}{5} = 1440\ N$$

12. $W_f = \Delta KE \Rightarrow \int_r^\infty(-\mu.mg)\,dr = 0 - \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gA}$

13. Conservation of mechanical energy explains the K.E. at A & B are equal.

Acceleration for A = $g\sin\theta_1$

Acceleration for B = $g\sin\theta_2$

$\because \sin\theta_1 > \sin\theta_2$ $\therefore a_1 > a_2$

F_{ext} and displacements are in opposite directions.

14. COME : $K_A + U_A = K_B + U_B$

$$0 + mg\quad 25 = \frac{1}{2}mv_A^2 + mg\times 15 \Rightarrow mv_A^2 = 20mg$$

Forces at B : $N = mg - \frac{mv_A^2}{R} = 0 \Rightarrow R = 20$ m

15. Area of graph

$$= \int P.dx = \int mv.a.dx = \int mv.\left(\frac{vdv}{dx}\right)dx$$

$$= \int_u^v mv^2dv = \frac{m(v^3 - u^3)}{3} = \frac{10.(v^3-1)}{7\times 3}$$

$$= \frac{1}{2}(4+2)\quad 10 \Rightarrow v = 4\ m/s$$

16. Power = $\rho QgH = \rho Av.gH = \rho A\sqrt{2gH}\,.gH$

$= 10^3 \times \frac{\pi d^2}{4} \times \sqrt{2\times10\times40} \times 10 \times 40$ (d= 5 cm)

= 21.5 kW

17. For upward motion : $mgh + fh = \frac{1}{2}m \quad 16^2$

downward motion : $mgh - fh = \frac{1}{2}m \quad 8^2 \Rightarrow h = 8m$

18. $P = \frac{\Delta W}{\Delta t} = \frac{\vec{F}.\Delta\vec{S}}{\Delta t} = \frac{(3\tilde{i}+4\tilde{j})\cdot(8\tilde{i}+6\tilde{j})}{6} = 8W$

19. For equilibrium : $N\cos\theta = mg$ & $N\sin\theta = kx$

$\Rightarrow kx = mg\tan\theta$ (N = normal between m & M)

$\therefore U = \frac{1}{2}kx^2 = \frac{m^2g^2\tan^2\theta}{2k}$

20. $W_g + W_F = \Delta KE \Rightarrow -mgh - f.d = 0 - \frac{1}{2}mv^2$

$-mg\ 1.1 - \mu\ mg\ d = -\frac{1}{2}mv^2$ (μ= 0.6) $\Rightarrow$ d=1.17 m

21. For motion $P \rightarrow 0 \Rightarrow K_O + U_O = K_P + U_P$

For motion $Q \rightarrow 0 \Rightarrow K'_O + U'_O = K_Q + U_Q$

$\Rightarrow K_O = U_P;\quad K'_O = U_2 = 2U_P = 2K_O$

$\Rightarrow t_{Q\rightarrow O} = \sqrt{\frac{2(2h/\sin\alpha)}{g\sin\alpha}} = t_1$

$\Rightarrow t_{P\rightarrow O} = \sqrt{\frac{2(h/\sin\alpha)}{g\sin\alpha}} = t_2 = \sqrt{2}\ t_1$

22. $v = a\sqrt{s} = \frac{ds}{dt} \Rightarrow s = \frac{a^2t^2}{4}$

$\therefore W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}m \times a^2 s = \frac{1}{2}ma^2\frac{(a^2t^2)}{4} = \frac{ma^4t^2}{8}$

23. Maximum elongation in spring $= \frac{2Mg}{K}$

Condition block 'm' to move is

$Kx \geq mg\sin37 + \mu mg\cos37 \Rightarrow M = \frac{3}{5}$

24. COME : $K_1 + U_1 = K_2 + U_2$

$\frac{1}{2}mv_0^2 + 0 = 0 + mg\ell(1-\cos60°) \Rightarrow v_0$ = 7 m/s

25. Conservative forces depends on the end points not on the path. Hence work done by it in a closed loop is zero.

26. For equilibrium, F=0 $\Rightarrow x(3x-2)=0 \Rightarrow x=0 \Rightarrow x=\frac{2}{3}$

27. $v^2 = v_0^2 + 2(-\mu g)L$

For v =0, $v_0 = \sqrt{2\mu gL}$

28. For velocity to maximum acceleration must be zero.

$\Rightarrow mg - kx = ma = 0$

$\Rightarrow x = \frac{mg}{k} = \frac{1\times10}{0.2} = 5cm$

$\therefore$ Height from table = 15 cm

29. $W_N = \Delta KE = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = \frac{1}{2}\quad 1\ (10\sqrt{3})^2 = 150$ J

30. Sum of KE and PE remains constant.

31. $\left(0 - \frac{1}{2}kx^2\right) + (-\mu mgx) = 0 - \frac{1}{2}mv^2 \Rightarrow$ v=8 m/s

32. ΔK.E. = work done by all the forces

ΔK.E. = $m\vec{a}.\vec{s}$

When acceleration is constant

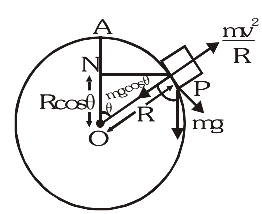
ΔK.E. $\propto t^2$ [as $s = \frac{1}{2}at^2$]

33. $\vec{F} = 3\hat{i} + 4\hat{j}$ is a conservative force ie therefore

$W_1 = W_2$

34. To break off reaction becomes O,

i.e. $mg\cos\theta = \frac{mv^2}{R} \Rightarrow \cos\theta = \frac{v^2}{Rg}$ ....(1)



But from energy considerations

$mgR[1-\cos\theta] = \frac{1}{2}mv^2$

$\Rightarrow v^2 = 2gR(1-\cos\theta)$ using it in (1)

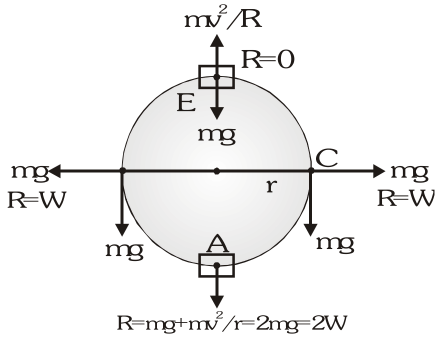
$\cos\theta = 2(1-\cos\theta)$

$\Rightarrow \cos\theta = 2 - 2\cos\theta \Rightarrow \cos\theta = \frac{2}{3}$

So $\sin\theta = \sqrt{1-\frac{4}{9}} = \frac{\sqrt{5}}{3}$

Now tangential acceleration $g\sin\theta = g\frac{\sqrt{5}}{3}$

35. Given $v=\sqrt{gr}$



36. In this case $T = \frac{2\pi r}{u}$ [for 1 resolution]

Also $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

But $t = nT \Rightarrow \sqrt{\frac{2h}{g}} = n\frac{2\pi r}{u} \Rightarrow n = \frac{u}{2\pi r}\sqrt{\frac{2h}{g}}$

37. Given $\frac{1}{2}mv^2 = as^2$....(i)

So $a_r = \frac{v^2}{R} = \frac{2as^2}{mR}$ ....(ii)

Also $a_t = \frac{dv}{dt} = \frac{dv}{ds}.\frac{ds}{dt} = v\frac{dv}{ds}$

But from equation (1) $v = s\sqrt{\frac{2a}{m}}$

put it above $a_t = s\sqrt{\frac{2a}{m}}\left(\sqrt{\frac{2a}{m}}\right) = \frac{2as}{m}$ ....(iii)

So that $a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{2as^2}{mR}\right)^2 + \left(\frac{2as}{m}\right)^2}$

i.e. $a = \frac{2as}{m}\sqrt{1+\left(\frac{s}{R}\right)^2}$

So force $F = ma = 2as\sqrt{1+\left(\frac{s}{R}\right)^2}$

38. Tension will be mg cos θ at extremes but it becomes $mg\cos\theta + \frac{mv^2}{\ell}$.

In the given situation by making diagram, we can shown that $T - Mg\cos\theta = \frac{Mv^2}{L}$ and tangential acceleration = g sinθ.

EXERCISE –III

Match the column

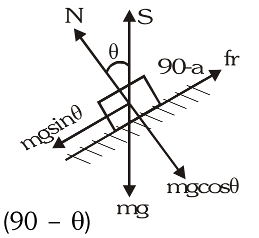
1. W_g = force (displacement in the direction of force)

$W_g = [10 \;\; \frac{1}{2} \;\; 2 \;\; 16] = -160$ joule

$w_N = \vec{N}.\vec{s} = m(g+a)\cos\theta \left(\frac{1}{2}\times 2\times 16\right)\cos\theta$

$= (12)\times\frac{\sqrt{3}}{2}(16)\frac{\sqrt{3}}{2}$

$= 12 \;\; 12 = 144$ J



$W_{fr} = \vec{f_r}.\vec{s}$

$= m(g+a)\sin\theta\,(16)\cos(90-\theta)$

$= (12) \;\; 16 \;\; \frac{1}{4} = 48$ joule

$W_{net} = W_g + W_N + W_{fr} = 32$ joule

2. $f_{conservative} = -\frac{du}{dx} = 30$ Ni

change in kinetic energy =2 [Area under (a–x) graph]

as mass is 1 kg $\Rightarrow$ [80 + 40] = 120,

$KE_{initial} = \frac{1}{2}Mv^2 = 8$ J

(A) KE_f = 128 J

(B) $W_{can} = \vec{f}\times\vec{d} = 30 \;\; 8 \Rightarrow 240$ J

(C) $W_{Net} = \Delta KE = 120$ J

(D) $W_{cons} + W_{ext} = 120$; $W_{ext} = -120$ J

3. By applying conservation of momentum wedge will acquire some velocity $= -\frac{mv_x}{M+m}$ where v_x is velocity of block w.r.t wedge in negative x-direction.

(A) Work done by normal on block is

$$= -\frac{1}{2}M\left(\frac{mv_x}{M+m}\right)^2$$

(B) Work done by normal on wedge is

$$= \frac{1}{2}M\left(\frac{mv_x}{M+m}\right)^2 \text{ is positive.}$$

(C) Net work done by normal is = 0

(D) less than mgh as K.E. is $< \frac{1}{2}m2gh$,

$KE_f >$ KE is positive.

4. For $v \geq \sqrt{5g\ell}$, the bob will complete a vertical circular path.

For $\sqrt{2g\ell} < v < \sqrt{5g\ell}$, the bob will execute projectile motion.

For $v < \sqrt{2g\ell}$, the bob oscillates.

Comprehension#1

1. $W = \vec{f}.d\vec{s} \Rightarrow W = -mg\left(\frac{1}{2}a_0t^2\right)$

2. For the motion of the block in vertical
$mg - N = ma_0,\ N = m(g-a_0)$

$$W_N = -\frac{Na_0t^2}{2} \Rightarrow -\frac{m(g-a_0)a_0t^2}{2}$$

3. For observer A pseudo force on the particle is zero
$W = 0$

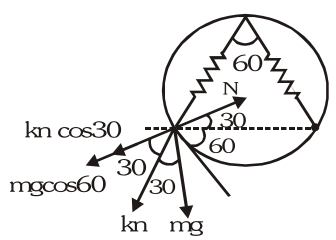
4. $W = \vec{f}_{net}.\vec{ds} \Rightarrow W = ma\frac{1}{2}at^2 \Rightarrow \frac{ma^2t^2}{2}$

5. For observer A the block appears to be stationary
$\therefore$ Displacement is zero hence w =0

Comprehension#2

1. $N - Kx\cos 30 - mg\cos 60 = \frac{Mv^2}{R}$

As velocity of Ring = 0
$N = kx\cos 30 + mg\cos 60$



$$= \frac{(2+\sqrt{3})mg}{\sqrt{3}R}(2-\sqrt{3})R\left(\frac{\sqrt{3}}{2}\right) + \frac{mg\sqrt{3}}{2}$$

$$= \frac{mg}{2} + \frac{mg}{2} = mg$$

2. $f_{net} = (k\cos 60)\,x + mg\cos 30$

$$= \frac{(2+\sqrt{3})mg}{\sqrt{3}R}(2-\sqrt{3})R\frac{1}{2} + \frac{mg\sqrt{3}}{2}$$

$$= \frac{mg}{2}\left[\frac{1}{\sqrt{3}} + \sqrt{3}\right] = \frac{2mg}{\sqrt{3}}$$

$$a_{rev} = 2a\cos 60 = a = \frac{2g}{\sqrt{3}} \text{ horizontal}$$

3. By applying work - energy theoram

$$\frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2;\ \frac{1}{2}mv^2 = \frac{1}{2}\frac{(2+\sqrt{3})mg}{\sqrt{3}g}(2-\sqrt{3})^2R^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{mg}{\sqrt{3}}(2-\sqrt{3})R \Rightarrow v = \sqrt{\frac{gR(2-\sqrt{3})}{\sqrt{3}}}$$

Comprehension#3

1. By applying work energy theoram

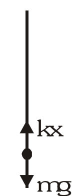
$$\frac{1}{2}Mv^2 - 0 = W_g$$

$$\frac{1}{2}Mv^2 = mg\ell \Rightarrow v = \sqrt{2g\ell}$$

2. $\sqrt{2g\ell} = \sqrt{5g(\ell - x)}$

$$\Rightarrow 2g\ell = 5g(\ell - x) \Rightarrow 5x = 3\ell \Rightarrow x = \frac{3\ell}{5}$$

3. Net force towards the centre will provide the required contripetal force



$$kx - mg = \frac{mv^2}{R}$$

$$kx - mg = \frac{m2g\ell}{\ell}$$

$$\Rightarrow kx = 3mg \Rightarrow x = \frac{3mg}{k}$$

Comprehension#4

1. Particle will have some translatory kinetic energy as well as rotatory energy the whole of the K.E. is converted into potential energy h < 6

2. By applying conservation of mehanical energy

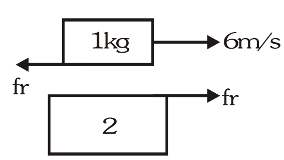
$$\Rightarrow \frac{1}{2}mu^2 = mg(h) \Rightarrow u^2 = 80$$

$$\Rightarrow \frac{1}{2}mu^2\sin^2 30 = mgh \Rightarrow h = 1m$$

Total height = 2 + 1 = 3m

Comprehension#5

1. From the F.B.D. of the blocks :
upper block is –ve and lower block is +ve as



v_{upper} = decreases, v_{lower} = Increases

2. By applying conservation of momentum
$1\ \ 6 + 2\ \ 3 = 3(v) \Rightarrow v = 4m/s$
By applying work energy theoram

$$-\frac{1}{2}(1)(36) + \frac{1}{2}(1)(16) = w_{fr}$$

$\Rightarrow -18 + 8 = W_{fr} \Rightarrow W_{fr} = -10$ J
and Work done on the lower block +10j
$\Rightarrow W_{net} = 0$

Comprehension # 6

1. $u = \frac{A}{r^2} - \frac{B}{r} \Rightarrow \frac{du}{dr} = -\frac{2A}{r^3} + \frac{B}{r^2}$

$f = -\frac{du}{dr} = \frac{2A}{r^3} - \frac{B}{r^2}$, F = 0 $\Rightarrow$ $r = \frac{2A}{B}$

2. As potential is minimum at $r=r_0$ the equilibrium is stable.

3. Given that

$U = \frac{A}{r^2} - \frac{B}{r}$ as $r = \frac{2A}{B}$; $U_i = \frac{AB^2}{4A^2} - \frac{BB}{2A} = \frac{-B^2}{4A}$

$\Rightarrow U_f=0,\ \Delta W = U_f - U_i \Rightarrow \frac{B^2}{4A}$

4. K.E. + P.E. =T.E, $0 + \frac{A}{r^2} - \frac{B}{r} = \frac{-3B^2}{16A}$

By solving the above equation $r = \frac{2r_0}{3}$

Comprehension#7

1. (A) $W_{CL} + W_f = \Delta KE \quad \therefore W_{CL} = \Delta KE - W_f$
   (a) During acceleration motion negative work is done against friction and there is also change is kinetic energy. Hence net work needed is positive.
   (b) During uniform motion work is done against friction only and that is positive.
   (c) During retarded motion, the load has to be stopped in exactly 50 metres. If only friction is considered then the load stops in 12.5 metres which is less than where it has to stop. Hence the camel has to apply some force so that the load stops in 50 m (>12.5 m). Therefore the work done in this case is also positive.

2. $W_{CL}|_{\text{accelerated motion}} = \Delta KE - W_{friction}$
where W_{CL} is work done by camel on load.

$= \left[\frac{1}{2}mv^2 - 0\right] - [-\mu_k mg.50]$

$= \frac{1}{2}\ 1000\ 5^2 + 0.1\ 10\ 1000\ 50$

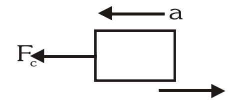
$=1000 \left[\frac{125}{2}\right]$

similarly, $W_{CL}|\text{retardation} = \Delta KE - W_{friction}$

$\left[0 - \frac{1}{2}mv^2\right] - [-\mu_k mg.50] = 1000\left[\frac{75}{2}\right]$

$\therefore \frac{W_{CL}\mid \text{accelerated motion}}{W_{CL}\mid \text{retarded motion}} = \frac{125}{75} = \frac{5}{3} \Rightarrow 5 : 3$

3. Maximum power = F_{max} V
Maximum force applied by camel is during the accelerated motion.
We have $V^2 - U^2 = 2as$, $25 = 0^2 + 2\ a\ 50$
$a = 0.25\ m/s^2$
for accelerated motion



$\therefore F_c - f = ma$
$\therefore F_c = \mu mg + ma$
$= 0.1\ 1000\ 10 + 1000\ 2.5$
$= 1000 + 250 = 1250$ N

This is the critical point just before the point where it attains maximum velocity of almost 5m/s .
Hence maximum power at this point is
$= 1250\ 5 = 6250$ J/s.

4. We have $W = P\Delta T$, $P = 18\ 10^3 V + 10^4$ J/s
$\therefore P_5 = 18\ 10^3\ 5 + 10^4$ J/s and

$\Delta T_5 = \frac{2000m}{5\,m/s} = 400s$

$P_{10} = 18\ 10^3\ 10^4$J/s

and $\Delta T_{10} = \frac{2000m}{10\,m/s} = 200s$

$\therefore \frac{W_5}{W_{10}} = \frac{10^4(9+1)\times 400}{10^4(18+1)\times 200}$

5. The time of travel in accelerated motion = time of travel in retarded motion.

$T_{AB} = T_{CD} = \frac{V}{a} = \frac{5}{0.25} = 20$ sec

Now time for uniform motion $= T_{ac} = \frac{2000}{5} = 400$ s

$\therefore$ Total energy consumed $= \int_0^{440} P\,dt$

$= \int_0^{20} [18.10^3 V + 10^4]dt + \int_{20}^{420} [18.10^3.5 + 10^4]dt$

$+ \int_{420}^{440} [18.10^3 V + 10^4]dt$

$$= \int_0^{20} [18.10^3 V\,dt + \int_0^{20} 10^4\,dt + \left[10^5 t\right]_{20}^{420}$$

$$+ \int_{420}^{440} 18.10^3 V dt + \int_{420}^{440} 10^4 dt$$

Putting Vdt = dx and changing limits appropriately it becomes

$$\int_0^{60} 18.10^3 dx + \left[10^4 t\right]_0^{20} + 10^5 [420 - 20]$$

$$+ \int_{2050}^{2100} 18.10^3 dx + \left[10^4\right]_{420}^{440}$$

$= 18.10^3.50 + 10^4[20]$
$+ 10^5\ 400 + 18.10^3[50] + 10^4[20]$ Joules
$= 90 \quad 10^4 + 20 \quad 10^4 + 400 \quad 10^5$
$+ 90 \quad 10^4 + 20 \quad 10^4$ J $= 4.22 \quad 10^7$ J

Comprehension#8

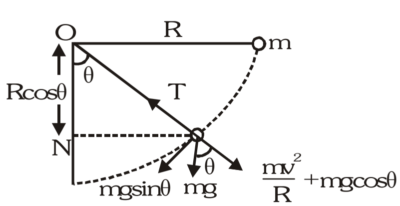
1. By applying work energy theoram change in kinetic energy $= w_S \Rightarrow 0 - \frac{1}{2}mv^2 = W_s$
2. As the kinetic energy of block is decreasing, therefore work done by the normal is $= -\frac{1}{2}mv^2$
3. $W_{net} = -\frac{1}{2}mv^2$
5. $W_{net} = 0$ as for the B change in velocity is zero.
6. As there is no change in kinetic energy stored is due to

Comprehension#9

1. Conservation of mechanical energy can only be applicable in absence of non conservative forces

Comprehension # 10

Balancing the forces $T = \frac{mv^2}{R} + mg\cos\theta$ ....(i)



From energy considerations

$mg\,R\cos\theta = \frac{1}{2}mv^2 \Rightarrow v^2 = 2g\,R\cos\theta$

putting this value in equation (i)

we get $T = 3mg\cos\theta$

Also acceleration $a_{Total} = \sqrt{a_r^2 + a_t^2}$

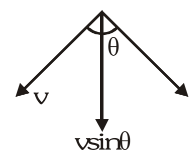
$$= \sqrt{\left(\frac{v^2}{R}\right)^2 + (g\sin\theta)^2} = \sqrt{(2g\cos\theta)^2 + (g\sin\theta)^2}$$

$$= g\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\Rightarrow a_{Total} = g\sqrt{1 + 3\cos^2\theta}$$

Now virtual component of sphere's velocity

$v_y = v\sin\theta = \sqrt{2gR}\sqrt{\cos\theta}\,\sin\theta$



Applying maxima-minima

$$\frac{dv_y}{d\theta} = \sqrt{2gR}\left[\frac{(-\sin\theta)\sin\theta}{2\sqrt{\cos\theta}} + \sqrt{\cos\theta}\cos\theta\right]$$

$$= \sqrt{2gR}\left[\frac{-\sin^2\theta}{2\sqrt{\cos\theta}} + \cos\theta\sqrt{\cos\theta}\right]$$

$$\Rightarrow \frac{\sin^2\theta}{2} = \cos^2\theta \Rightarrow \tan^2\theta = \sqrt{2}$$

$$\Rightarrow \theta = \tan^{-1}\sqrt{2} \Rightarrow \tan\theta = \sqrt{2}$$

So $\sin\theta = \frac{\sqrt{2}}{\sqrt{3}}$ and $\cos\theta = \frac{1}{\sqrt{3}}$

Thus tension $T = 3\,mg\cos\theta$

$= 3mg \quad \frac{1}{\sqrt{3}} = \sqrt{3}\,mg$

Comprehension 11

Using work energy theorom

$$\frac{m \times 2g}{9} \times R\sin\theta + mgR(1 - \cos\theta) = \frac{1}{2}mv^2 \text{ ..(i)}$$

Also $mg\cos\theta = \frac{2mg}{9}\sin\theta + \frac{mv^2}{R}$

$v^2 = gR\cos\theta - \frac{2g}{9}R\sin\theta$ ...(ii)

From equation (i) & (ii)

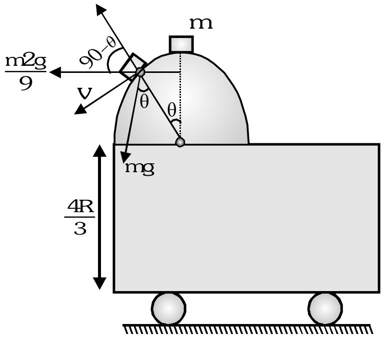
$$\frac{2mg}{9}R\sin\theta + mgR(1-\cos\theta) = \frac{m}{2}\left(gR\cos\theta - \frac{2g}{9}R\sin\theta\right)$$

$\Rightarrow$ 4sinθ + 18 (1–cosθ) = 9cosθ–2sinθ

$\Rightarrow$ 6sinθ + 18–18cosθ = 9 cosθ

$\Rightarrow$ 6 sinθ – 27cosθ + 18 = 0

$\Rightarrow$ 2sinθ – 9 cosθ + 6 = 0



Now let sinθ = x so cosθ = $\sqrt{1-x^2}$

Than 2x – 9 $\sqrt{1-x^2}$ +6 = 0

Solving x = $\frac{3}{5}$ = sinθ so cosθ = $\frac{4}{5}$; θ = 37

Now putting θ=37

in μ = h+ Rcosθ = $\frac{4R}{3} + R \times \frac{4}{5}$

$= \frac{20R+12R}{15} = \frac{32R}{15}$

From equation (ii) v^2 = gRcosθ $\frac{2g}{9}$Rsinθ

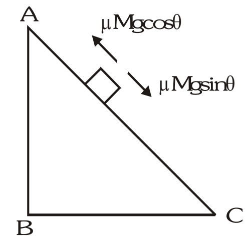
v^2 = gR $\frac{4}{5} - \frac{2g}{9}R \times \frac{3}{5}$

$= gR\left[\frac{4}{5} - \frac{2}{15}\right] = \frac{10gR}{15} = \frac{2gR}{3}$

Now using S = ut + $\frac{1}{2}gt^2$; $\frac{32R}{15} = \sqrt{\frac{2gR}{3}}$ t + $\frac{1}{2}gt^2$

t can be obtained t = $\sqrt{\frac{2R}{g}}$

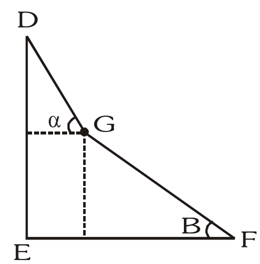
EXERCISE –IV(A)

1. 

W_{Mg} =Mgsinθ AC = Mg AB

W_f = μMgcosθ AC cos 180

= –μMg (BC)



W_{Mg} = Mg(sinα DG + sin β GF) = Mg DE

W_f = – μMg (DG cos α + GF cosβ) = – μMg(EF)

=– μMg BC (∵ BC = EF)

From WET, ΔKE will be same in both cases.

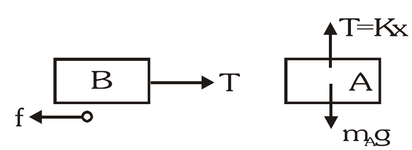
∴ $v_C = v_F$

2. Heat generated = work done against friction

$\Rightarrow$ (μmg) (vt)= (0.2 2 10) 2 5= 40 J

$= \frac{40}{4.2}$ cal = 9.52 cal

3. Blocks are moving with constant speed.



∴ m_Ag = T = kx = f = μm_Bg

$\Rightarrow m_B = \frac{m_A}{\mu} = \frac{2}{0.2}$ =10 kg and x = $\frac{2\times 9.8}{1960}$

∴ Energy stored in spring = $\frac{1}{2}$ kx^2

$= \frac{1}{2}$ 1960 $\left(\frac{19.6}{1960}\right)^2$ = 0.098 J

4. Work done by force = $\int F dx$

$$W = \int_0^{1/2} \pi \sin \pi x dx = \pi \frac{[-\cos \pi x]}{\pi}\Big|_0^{1/2}$$

$$= -\cos\frac{\pi}{2} + \cos 0 = 1J$$

Work done by external agent = – 1 J

5. COME : $K_1 + U_1 = K_2 + U_2$

$$\frac{3mgr}{2} = \frac{1}{2}mv^2 + \frac{1}{2}kr^2 \quad \text{...(i)}$$

Force equation $kr = mg + \frac{mv^2}{r}$

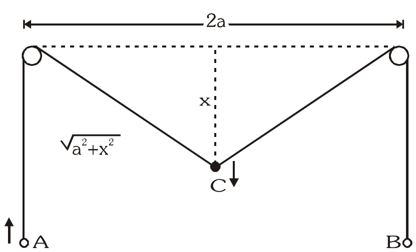
Solving we get, $k = \frac{2mg}{r} = 500$ N/m

6. $a_n = bt^2 = \frac{v^2}{R} \Rightarrow v = \sqrt{bR}\, t \Rightarrow a_t = \sqrt{bR}$

$\therefore$ P = FV = mbRt

$$< P > = \frac{\int_0^t P dt}{\int_0^t dt} = \frac{mbR(t^2/2)}{t} = \frac{mbRt}{2}$$

7. As C falls down, A & B move up.
COME : $K_1 + U_1 = K_2 + U_2$



$$0 + mgx = 0 + 2mg\left(\sqrt{a^2+x^2} - a\right) \Rightarrow x = \frac{4a}{3}$$

8. Potential energy $U = 1\left(\frac{x^2}{2} - x\right) = \frac{x^2}{2} - x$

For minimum U,

$$\frac{dU}{dx} = \frac{2x}{2} - 1 = 0 \text{ and } \frac{d^2U}{dx^2} = 1 = \text{positive}$$

so at x = 1, U is minimum. Hence $U_{min} = -\frac{1}{2}$ J

Total mechanical energy = Max KE + Min PE

$$\Rightarrow \text{Max KE} = \frac{1}{2}mv^2_{max} = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$$

$$\Rightarrow v_{max} = \sqrt{\frac{2}{1}\times\frac{5}{2}} = \sqrt{5}\ ms^{-1}$$

9. Let extension in spring be x_0 due to m_1

then $m_1 g x_0 = \frac{1}{2}kx_0^2 \Rightarrow kx_0 = 2m_1 g$

but $kx_0 \geq mg$ so $2m_1 g \geq mg \Rightarrow m_1 \geq \frac{m}{2}$

therefore minimum value of $m_1 = \frac{m}{2}$

10. $\theta = 3(t + \sin t)$; $\omega = 3 + 3\cos t$; $\alpha = -3\sin t$

$$F = \sqrt{(m\omega^2 R)^2 + (m\alpha R)^2} \quad \left(t = \frac{\pi}{2}\right) = 9\sqrt{10}N$$

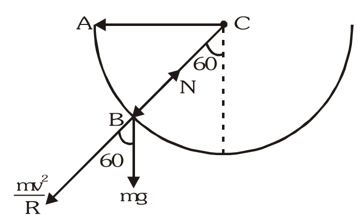
11. COME : $\frac{mv^2}{2} = mgh$

If resultant acceleration, a, makes angle θ with thread, then $a\sin\phi = g\sin\theta$

$$a\cos\phi = \frac{v^2}{\ell} = \frac{2gh}{\ell}$$

$$\therefore \tan\phi = \frac{\ell\sin\theta}{2h} \Rightarrow \phi = \tan^{-1}\left(\frac{\ell\sin\theta}{2h}\right)$$

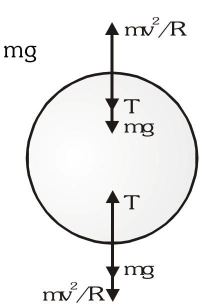
12. COME : $K_1 + U_1 = K_2 + U_2$



$$0 + MgR = \frac{1}{2}mv^2 + \frac{mgR}{2} \Rightarrow v = \sqrt{gR}$$

Forces at B $\Rightarrow N = mg\cos 60 + \frac{mv^2}{R} = \frac{15\sqrt{3}}{2}$

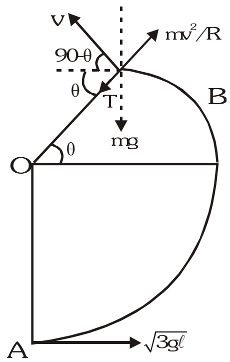
13. $T_{max} = mg + \frac{mv^2}{R}$, $T_{min} = \frac{mv^2}{R} - mg$



$$\frac{T_{max}}{T_{min}} = \frac{mg + \frac{mv^2}{R}}{\frac{mv^2}{R} - mg} = \frac{5}{3} \quad (R=2m)$$

$\Rightarrow v = 4\sqrt{5}$ m/s

14. Here the bob has velocity greater than $\sqrt{2g\ell}$ and smaller than $\sqrt{5g\ell}$. Hence the thread will slack after completing semicircle.



COME : $K_1 + U_1 = K_2 + U_2$

$$\frac{1}{2}m\left(3g\ell\right) + 0 = \frac{1}{2}mv^2 + mg\left(\ell + \ell\sin\theta\right) \text{ ...(i)}$$

Force equation at B :

$$T + mg\sin\theta = \frac{mv^2}{R} \text{ ...(ii)}$$

Solving for T=0, we get $\sin\theta = \frac{1}{3}$ $\therefore$ $v_B = \sqrt{g\ell\sin\theta}$

$\therefore$ The particle will execute projectile motion after tension become zero.

$$\therefore v_{min} = v\sin\theta = \sqrt{\frac{g\ell}{3}} \times \frac{1}{3}$$

15. COME : $K_A + U_A = K_B + U_B$

$$0 + mg(2R) + \frac{1}{2}kR^2 = \frac{1}{2}mv^2 + 0 + 0(\ k = mg/R)$$

$\Rightarrow \frac{mv^2}{R} = 5\ mg$ $\therefore$ Force equation at B

$$\Rightarrow T_B = mg + \frac{mv^2}{R} = 6mg$$

16. For speed u_0, contact at top is lost.

$\Rightarrow N + \frac{mu_0^2}{r} = mg \Rightarrow (N=0)\ u_0 = \sqrt{gr}$

(a) For vertical motion; $t = \sqrt{\frac{2r}{g}}$

$\therefore$ Horizontal distance

$$s = 2u_0.t = 2\sqrt{gr}\ \frac{\sqrt{2r}}{g} = 2\sqrt{2}r$$

(b) COME :

$$\frac{1}{2}\frac{m\left(u_0\right)^2}{3} + mgr = \frac{1}{2}mv^2 + mgr\cos\theta \text{ ...(i)}$$

Force equation : $N + \frac{mv^2}{r} = mg\cos\theta$ ...(ii)

$$\therefore h = r\cos\theta = \frac{19}{27}r$$

(c) $\left|\vec{a}_{net}\right| = \left|\vec{a}_r + \vec{a}_t\right| = \sqrt{\left(g\sin\theta\right)^2 + \left(g\cos\theta\right)^2} = g$

EXERCISE –IV(B)

1. a : Natural length

a : Initial elongation

2a : additional elongation

COME : $\frac{1}{2}k(3a)^2 = mgx \Rightarrow x = \frac{9a}{2}$

(above point of suspension)

2. WET : $W_N + W_{Mg} + W_f + W_{sp} = \Delta KE$

$$0 + 0 - \mu_k.mg\ (2.14 + x) + 0 - \frac{1}{2}kx^2 = 0 - \frac{1}{2}mv^2$$

$\Rightarrow x = 0.1$ m

At x = 1m, $F_{spring} = kx = 2\ \ 0.1 = 0.2$ N

$F_{S.F.} = \mu_S.mg = 0.22\ \ \frac{1}{2}\ \ 10 = 1.1$ N

Hence the block stops after compressing the spring.

$\therefore$ Total distance travelled by block when it stops = 2 + 2.14 + 0.1 = 4.24 m

3. Conservative force, $F = -\frac{dU}{dr} = -\frac{d\left(2r^3\right)}{dr} = -6r^2$

This force supplies the necessary centripetal acceleration.

$$\frac{mv^2}{r} = 6r^2 \Rightarrow \frac{1}{2}mv^2 = 3r^3$$

$E = K + U = 5r^3 = 5\ \ 5\ \ 5\ \ 5 = 625$ J

4. For part AB : (R=4a)

$$\left(\frac{v_0}{4a}\right)t_1 = \frac{\pi}{2} \Rightarrow t_1 = 4\left(\frac{\pi a}{2v_0}\right)$$

For part BC : (R=3a) $\Rightarrow t_2 = 3\left(\frac{\pi a}{2v_0}\right)$

For part CD : (R=2a) : $t_3 = 2\left(\frac{\pi a}{2v_0}\right)$

For part DA : (R=a) =: $t_4 = \left(\frac{\pi a}{2v_0}\right)$

$\therefore\ t = t_1 + t_2 + t_3 + t_4 = \dfrac{5\pi a}{v_0}$

5. At position B;

$mg = T\cos\theta = k.\Delta\ell.\cos\theta$

$$= \frac{2mg}{a}\left[a + \frac{a}{\sin\theta} - a\right]\cos\theta$$

$$= 2mg\cot\theta \Rightarrow \cot\theta = \frac{1}{2}$$

(a) $OB = a\cot\theta = \dfrac{a}{2}$

(b) COME : $K_C + U_C = K_O + U_O$

$$0 + mga + \frac{1}{2}\left(\frac{2mg}{a}\right)\left(\sqrt{2}a\right)^2 = \frac{1}{2}mv^2 + \frac{1}{2}ka^2$$

(i) $\Rightarrow v = 2\sqrt{ga}$

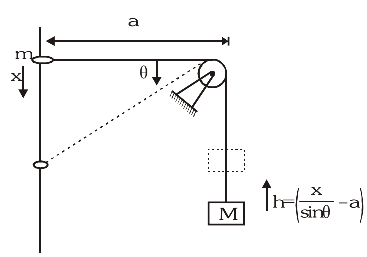
(ii) $K_C + U_C = K_P + U_P$

[P is the point of greatest depth]

$$\Rightarrow mga + \frac{1}{2}\left(\frac{2mg}{a}\right)\left(\sqrt{2}a\right)^2$$

$$= -mgx + \frac{1}{2}\left(\frac{2mg}{a}\right)(a^2 + x^2) \Rightarrow x = 2a$$

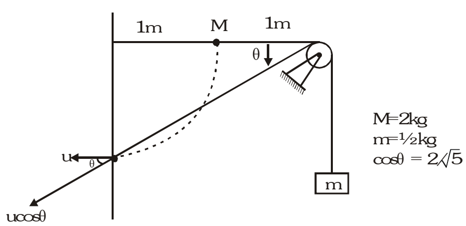
6. COME : $K_i + U_i = K_f + U_f$



$$\Rightarrow 0 + mgx = 0 + Mg\left(\sqrt{a^2 + x^2} - a\right)$$

$$\Rightarrow x = \frac{2mM}{M^2 - m^2}a$$

7. COME : $K_i + U_i = K_f + U_f$



$\Rightarrow 0 + Mg \quad R_1$

$$= 0 + mg\left(\sqrt{5} - 1\right) + \frac{1}{2}Mu^2 + \frac{1}{2}m\left(u\cos\theta\right)^2$$

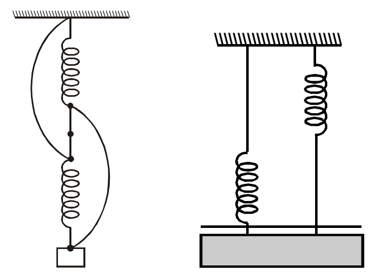
$\Rightarrow u = 3029$ m/s

8. Initial elongation in each spring

$$= \frac{Mg}{2\left(\frac{kx_0}{2}\right)} = \frac{Mg}{kx_0} = 20cm$$

Total initial length of each spring

$= 50 + 20 = 70$ cm



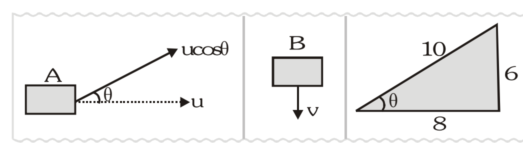
Equilibrium position $= 2\ kx = mg$

$$x = \frac{100}{2\times 500} = 10\ cm$$

and due to inertia it goes

10 cm also up = 20 m

9.



For constant length of string $= v = u\cos\theta$

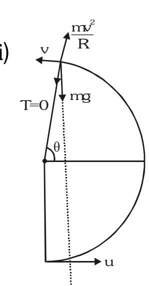
COME :

$$mg\quad 5 = \frac{1}{2}mv^2 + \frac{1}{2}mu^2 \Rightarrow u = \frac{10}{\sqrt{1.64}}$$

$$\therefore\ v = u\cos\theta = \frac{40}{\sqrt{41}}\ m/s$$

10. COME : $\dfrac{1}{2}mu^2 = \dfrac{1}{2}mv^2 + mgL\ (1 + \sin\theta)$....(i)

For equation $\Rightarrow T + mg\sin\theta = \dfrac{mv^2}{L}$ ...(ii)



Since the particle crosses the $\dfrac{L}{8}$ line at its half of its range

$$\therefore\ \frac{v^2\sin\theta.\cos\theta}{g} = L\cos\theta - \frac{L}{8}\quad ...(iii)$$

$\Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = 60°$

From equation (i) $\Rightarrow u = \sqrt{gL\left(2+\dfrac{3\sqrt{3}}{2}\right)}$

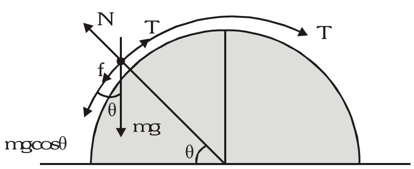
11. WET : $W_{SP} + W_{mg} + W_N + W_f = \Delta KE$

$$\Rightarrow \left[0-\frac{1}{2}k\left(\frac{h}{\sin\theta}\right)^2\right]+\left[mg\sin\theta\times\frac{h}{\sin\theta}\right]+0$$

$$-\mu mgh\cot\theta = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2}{m}\left[mgh-\frac{1}{2}k\left(\frac{h}{\sin\theta}\right)^2-\mu mgh\cot\theta\right]}$$

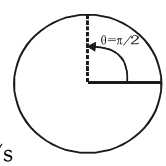
12. WET $\Rightarrow W_{mg} + W_N + W_T + W_f = \Delta KE$



$$-\ mgR+0+W_T+\int_0^{\pi/2}(\mu mg\sin\theta.Rd\theta)\cos 180 = 0$$

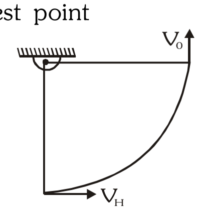
$\Rightarrow W_T = mgR\ (1+\mu)$

13. $\dfrac{1}{2}\alpha t^2 = \dfrac{\pi}{2}$ $(\alpha = \dfrac{\pi}{4}) \Rightarrow t = 2$ sec



$\therefore$ Average velocity $= \dfrac{\sqrt{2}R}{t} = 1$ m/s

14. The string can break at the lowest point



$\therefore T_{max} = mg + \dfrac{mv_H^2}{R}$

$\Rightarrow 45 = 5 + \dfrac{0.5\times v^2}{0.5}$

COME: $v_H^2 = v_0^2 + 2gR$

$v_0^2 = 40 - 2\times 10\times\dfrac{1}{2} = 30$

$\therefore H_{max} = \dfrac{v_0^2}{2g} = \dfrac{30}{2\times 10} = \dfrac{3}{2} = 1.5$ m

EXERCISE –V(A)

1. Spring constant (k)= 800 $\dfrac{N}{m}$

Work done in extending a spring from

X_1 to X_2 = $U_f - U_i$ = $\dfrac{1}{2}kX_2^2 - \dfrac{1}{2}kX_1^2$

$W = \dfrac{1}{2}k\left[X_2^2 - X_1^2\right] = \dfrac{1}{2}$ 800 $[0.15)^2-(0.05)^2]$

$$=400\left[\left(\frac{15}{100}\right)^2-\left(\frac{5}{100}\right)^2\right] = \frac{400}{10000}[225-25]$$

$= \dfrac{400\times 200}{10000} = 8J$

2. k = 5 10^3 N/m

$W = \dfrac{1}{2}k\left[x_2^2 - x_1^2\right]$

$W = \dfrac{1}{2}\times 5\times 10^3\left[\left(10\times 10^{-2}\right)-\left(5\times 10^{-2}\right)^2\right]$

$W = \dfrac{1}{2}\times 5\times 10^3\times 10^{-4}\left[100-25\right]$

$= \dfrac{75\times 5\times 10^{-1}}{2} = \dfrac{75}{4} = 18.75$ N-m

3. Power = FV = constant i.e., mav = k

$\Rightarrow$ av=$k_1 \Rightarrow \left(\dfrac{dv}{dt}\right)v = k_1 \Rightarrow vdv = k_1dt$

On integrating both sides, we get

$\Rightarrow \dfrac{v^2}{2} = k_1t \Rightarrow v^2 = 2k_1t \Rightarrow v = \sqrt{2k_1}\,t^{1/2}$

$\Rightarrow ds = k_2t^{1/2}dt \Rightarrow s = \left(\dfrac{k_2}{3/2}\right)t^{3/2} \Rightarrow s \propto t^{3/2}$

4. Here F $\propto$ x, by using work energy theorem

$\Delta KE = \int F\,dx \Rightarrow \Delta KE \propto \int x\,dx \Rightarrow \Delta KE \propto x^2$

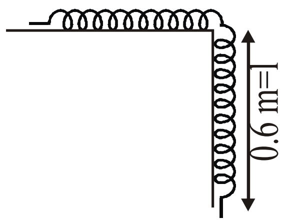
5. Given that acceleration a = $\dfrac{v_1}{t_1}$ ...(i)

Power = Fv   P=(ma)v

P=(ma^2t) [$\because$ v=at]

$P = \left(\dfrac{mv_1^2}{t_1^2}\right)t\left[\text{on replacing a}=\dfrac{v_1}{t_1}\right]$

6. Work done in pulling the hanging part of the chain upon the table= $\frac{mg\ell}{2}$



where m = mass of the hanging part
l = hanging part of chain

$$W = \left(\frac{4}{3}\times 0.6\right)\times\frac{10\times(0.6)}{2} = 3.6\text{ J}$$

7. According to work-energy theorem,
$W=\Delta K$

Case I : $-F\times 3 = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}mv_0^2$

where F is resistive force and v_0 is initial speed.
Case II : Let, the further distance travelled by the bullet before coming to rest is s.

$$\therefore -F(3+s) = K_f - K_i = -\frac{1}{2}mv_0^2$$

$$\Rightarrow -\frac{1}{8}mv_0^2(3+s) = -\frac{1}{2}mv_0^2$$

or $\frac{1}{4}(3+s)=1$ or $\frac{3}{4}+\frac{s}{4}=1$ ors = 1 cm

8. Momentum would be maximum when KE would be maximum and this is the case when total elastic PE is converted into KE.
According to conseration of energy

$$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2$$

$\Rightarrow kL^2 = \frac{(Mv)^2}{M}$ or $MkL^2 = p^2$ ($\because$ p = Mv)

$\Rightarrow p = L\sqrt{Mk}$

9. Applying work-energy theorem at the lowest and highest point, we get
$W_C + W_{NC} + W_{ext} = \Delta K$
$W_C + 0 + 0 = K_f - K_i$

$W_{C(Gravity)} = 0 - \frac{1}{2}\ \ 0.1\ \ 25$

$W_{Gravity} = -1.25$ J

10. $V(x) = \left(\frac{x^4}{4}-\frac{x^2}{2}\right)$

For minimum value of V,

$\frac{dV}{dx} = 0 \Rightarrow \frac{4x^3}{4}-\frac{2x}{4} = 0 \Rightarrow x = 0,\ x = \pm 1$

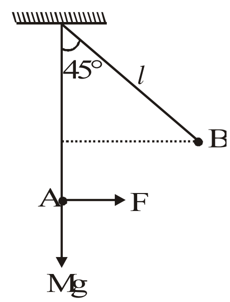
So, V_{min} (x=±1) = $\frac{1}{4}-\frac{1}{2} = \frac{-1}{4}$J

Now, $K_{max} + V_{min}$ = Total mechanical energy

$\Rightarrow K_{max} = \left(\frac{1}{4}\right)+2$ or $K_{max} = \frac{9}{4}$

or $\frac{mv^2}{2} = \frac{9}{4}$ or $v = \frac{3}{\sqrt{2}}$ ms^{-1}

11. Applying work-energy theorem,



Work done by F from A to B
= Work done by Mg from A to B
$\Rightarrow F(\ell\ \sin 45\ )=Mg\ell\ [1-\cos 45\ ]$
$\Rightarrow F=Mg(\sqrt{2}-1)$

12. $a = \frac{F_k}{m} = \frac{15}{2} = 7.5\ m/s^2$.

Now, $ma = \frac{1}{2}kx^2 \Rightarrow 2\times 7.5 = \frac{1}{2}\times 10000\times x^2$

or $x^2 = 3\ \ 10^{-3}$ or x = 0.055 m or x = 5.5 cm

13. Question is somewhat based on approximations. Let mass of athlete is 65 kg.
Approx velocity from the given data is 10 m/s

So, KE = $\frac{65\times 100}{2} = 3250$ J

So, option (d) is the most probable answer.

14. $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

$F = -\frac{dU}{dx} = +12\frac{a}{x^{13}} - \frac{6b}{x^7} = 0 \Rightarrow x = \left(\frac{2a}{b}\right)^{1/6}$

$U(x = \infty) = 0$

$U_{\text{equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a}$

$\therefore U(x = \infty) - U_{\text{equilibrium}} = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$

15. $\frac{1}{2} m\upsilon^2 \propto t$

$\upsilon \propto \sqrt{t} \Rightarrow \frac{d\upsilon}{dt} \propto t^{-\frac{1}{2}}$

$F = ma \propto t^{-\frac{1}{2}} \Rightarrow \propto \frac{1}{\sqrt{t}}$

16. Given same force $F = k_1x_1 = k_2x_2 \Rightarrow \frac{k_1}{k_2} = \frac{x_2}{x_1}$

$W_1 = \frac{1}{2}k_1x_1^2$ & $W_2 = \frac{1}{2}k_2x_2^2$

As $\frac{W_1}{W_2} > 1$ so $\frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} > 1$

$\Rightarrow \frac{Fx_1}{Fx_2} > 1 \Rightarrow \frac{k_2}{k_1} > 1$

$\therefore$ $k_2 > k_1$ statement 2 is true

OR

if $x_1 = x_2 = x$

$\frac{W_1}{W_2} = \frac{\frac{1}{2}K_1x^2}{\frac{1}{2}K_2x^2} = \frac{K_1}{K_2}$

$\therefore \frac{W_1}{W_2} = \frac{K_1}{K_2} < 1$

$\therefore W_1 < W_2$

statement 1 is false

EXERCISE –V(B)

1. Force $= v \times \frac{dm}{dt} = v \frac{d}{dt}$ (volume density)

$= v\frac{d}{dt}(Ax \times \rho) = v \times A\rho\frac{dx}{dt} = A\rho v^2$

$\therefore$ Power = Force velocity

$= (A\rho v^2)(v) = A\rho v^3 \therefore$ Power $\propto v^3$

2. $F = -\frac{dU}{dx}$ $\therefore dU = -Fdx$

$\int dU = -\int_0^x \left(-kx + ax^3\right)dx$ or $U(x) = \frac{kx^2}{2} - \frac{ax^4}{4}$

Let potential energy $U(x) = 0$

$\therefore 0 = \frac{x^2}{2}\left(k - \frac{ax^2}{2}\right)$

x has two roots viz $x = 0$ and $x = \sqrt{\frac{2k}{a}}$.

If $k < \frac{ax^2}{2}$, P.E. will be – ve or

when $x > \sqrt{\frac{2k}{a}}$, P.E. will be negative.

$\because F = -kx + ax^3$ $\therefore$ At x =0, F=0,

Slope of U–x graph is zero at x=0.

Thus P.E. is zero at x=0 and at $x=\sqrt{\frac{2k}{a}}$

Slope of U–x graph, at x=0, is zero.

3. Mechanical energy is conserved in the process. Let x=Maximum extension of the spring.

$\therefore$ Increase in elastic potential energy $= \frac{1}{2}kx^2$

Loss of gravitational potential energy = Mgx

$\therefore Mgx = \frac{1}{2}kx^2$ or $x = \frac{2Mg}{k}$

4. The gravitational field is a conservativefield. In a conservative field, the workdone W does not depend on the path (from A to B). It depends on initial and final points.

$\therefore W_1 = W_2 = W_3$

5. For conservative forces,

$$\Delta U = -\int_0^x F dx = -\int_0^x kx\, dx \text{ or } U(x)-U(0) = -\frac{kx^2}{2}$$

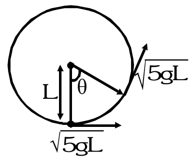
But U(0) = 0, as given in the question,

$$\therefore U(x) = \frac{-kx^2}{2} \text{ or } x^2 = \frac{-2U(x)}{k}$$

It represents a parabola, below x–axis, symmetrical about U–axis, passing through origin.

6. Energy conservation gives

$v^2 = u^2 - 2g(L - L\cos\theta)$

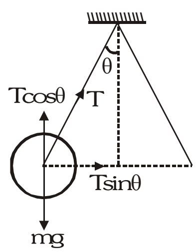


or $\frac{5gL}{4} = 5gL - 2gL\,(1-\cos\theta)$

or $5 = 20 - 8 + 8\cos\theta$ or $\cos\theta$

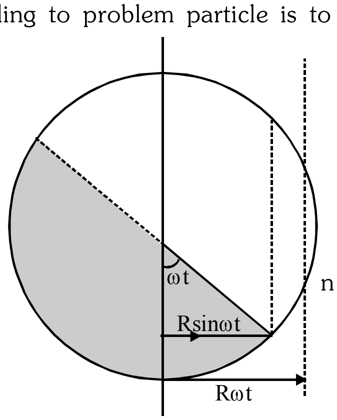
$$= -\frac{7}{8} \Rightarrow \frac{3\pi}{4} < \theta < \pi$$

7. $T\sin\theta = m\omega^2\,(L\sin\theta) \Rightarrow T = m\omega^2 L$



$$\omega_{max} = \sqrt{\frac{T_{max}}{mL}} = \sqrt{\frac{324}{0.5\times 0.5}} = 36 \text{ rad/s}$$

8. According to problem particle is to land on disc.



If we consider a time 't' then x component of displacement is $R\omega t$

$$R\sin\omega t < R\omega t$$

Thus particle P lands in unshaded region.

For Q, x-component is very small and y-component equal to P it will also land in unshaded region.

9. It is a case of uniform circular motion.

Velocity and acceleration keep on changing their directions. Their magnitudes remain constants. Kinetic energy remains constant.

11. (i) For circular motion of the ball, the centripetal force is provided by $(mg\cos\theta - N)$

$$\therefore mg\cos\theta - N = \frac{mv^2}{\left(R+\frac{d}{2}\right)} \quad ...(i)$$

By geometry, $h = \left(R+\frac{d}{2}\right)(1-\cos\theta)$

By conservation of energy,

Kinetic energy= potential energy

$$\frac{1}{2}mv^2 - mg\left(R+\frac{d}{2}\right)(1-\cos\theta) \text{ or}$$

$$v^2 = 2\left(R+\frac{d}{2}\right)(1-\cos\theta)g \quad ...(ii)$$

From (i) & (ii), we get total normal reaction force N.

$N = mg(3\cos\theta - 2)$ ...(iii)

(ii) To find N_A and N_B

For graphs :

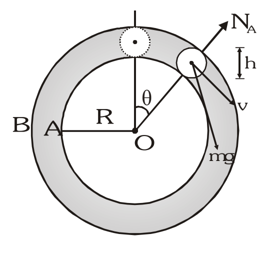
From (iii), at A,

$N_A = mg\,(3\cos\theta - 2)$ ...(iv)

(i) If $N_A = 0$,

i.e. At A, N = 0,

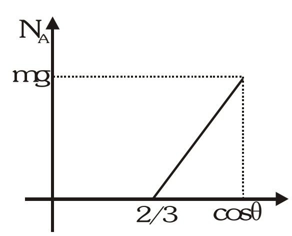
$0 = mg\,(3\cos\theta - 2)$



or $3\cos\theta = 2$ or $\cos\theta = \frac{2}{3}$

When N_A becomes zero, the ball will lose contact with inner sphere A. After this, it makes contact with outer sphere B. When θ – 0, N_A= mg

The N_A versus cos θ graph is a straight line as shown in the figure.



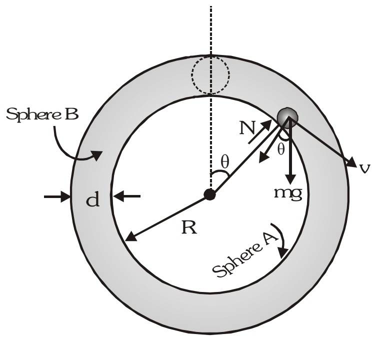
(ii) To find N_B :

Consider : $\cos\theta > \frac{2}{3}$

The ball makes contact with B.

$$N_B - (-mg\cos\theta) = \frac{mv^2}{R+\frac{d}{2}} \text{ or } N_B + mg\cos\theta$$

$$= \frac{mv^2}{R+(d/2)} \quad ...(v)$$



By energy conservation,

$$\frac{1}{2}mv^2 = mg\left[\left(R+\frac{d}{2}\right)-\left(R+\frac{d}{2}\right)\cos\theta\right]$$

or $\dfrac{mv^2}{R+\dfrac{d}{2}} = 2mg\ (1-\cos\theta)$ ...(vi)

From (iv) and (v)

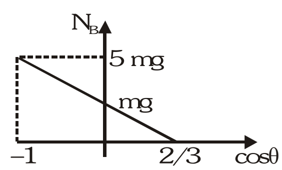
$N_B + mg\cos\theta = 2mg - 2mg\cos\theta$

$N_B = mg(2-3\cos\theta)$ ...(vii)

When $\cos\theta = \dfrac{2}{3}$, $N_B = 0$

When $\cos\theta = -1$, $N_B = 5$ mg.

Thus the $N_B - \cos\theta$ graph is as shown in the figure.



12. $m_1g - T = m_1a$ ...(i)

$T - m_2g = m_2a$ ...(ii)

($m_1 = 0.72$kg; $m_2 = 0.36$ kg)

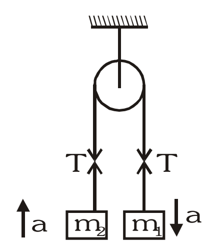
From (i) and (ii) $a = \dfrac{10}{3}$ m/s^2

$d = \dfrac{1}{2}\ \dfrac{10}{3}\ \ 1^2 = \dfrac{5}{3}$ m

$v = 0 + \dfrac{10}{3}\ \ 1 = \dfrac{10}{3}$ m/s

$W_T = 0.36\ \ 10\ \ \dfrac{5}{3} + \dfrac{1}{2}\ \ 0.36\ \ \dfrac{100}{9}$

$W_T = 8$ J



13. By using work energy theorem ($W = \Delta KE$)

$$-\mu mgx - \frac{1}{2}kx^2 = 0 - \frac{1}{2}mV^2$$

$$\Rightarrow V^2 = \frac{1.44}{9} \Rightarrow V = \frac{1.2}{3} = 0.4 = \frac{4}{10} \Rightarrow N = 4$$

14. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is. **[IIT-JEE 2013]**

Ans. (5)

$$P = Fv \Rightarrow \left(mv\frac{dv}{dt}\right) = 0.5$$

$$\int_0^v mvdv = \int_0^5 \frac{1}{2}dt \Rightarrow (0.2)\left(\frac{v^2}{2}\right) = \frac{1}{2}(5)$$

$\Rightarrow v^2 = 25 \Rightarrow v = 5$ m/s

15. The work done on a particle of mass m by a force

$$K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\tilde{i} + \frac{y}{\left(x^2+y^2\right)^{3/2}}\tilde{j}\right]$$

(K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is :- **[IIT-JEE 2013]**

(A) $\dfrac{2K\pi}{a}$ (B) $\dfrac{K\pi}{a}$

(C) $\dfrac{K\pi}{2a}$ (D) 0

Ans. **(D)**

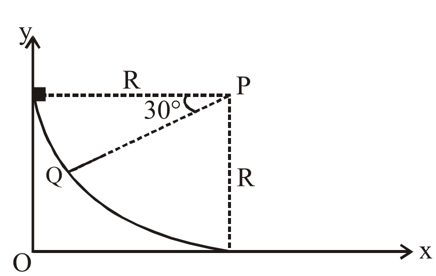
Particle is moving in x-y plane so

$$\vec{r} = x\tilde{i} + y\tilde{j} \Rightarrow \vec{F} = k\left[\frac{x}{r^3}\tilde{i} + \frac{y}{r^3}\tilde{j}\right] = \frac{k}{r^3}\left[x\tilde{i} + y\tilde{j}\right] = \frac{k\vec{r}}{r^3}$$

Force is central (i.e. conservative) so work done by this force in closed loop = 0

Paragraph for Questions 16 and 17

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J. (Take the acceleration due to gravity, g = 10 m s^{-2}) **[IIT-JEE 2013]**



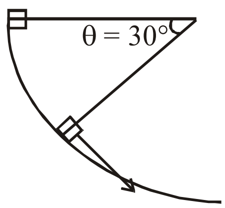
16. The magnitude of the normal reaction that acts on the block at the point Q is

(A) 7.5 N (B) 8.6 N
(C) 11.5 N (D) 22.5 N

Ans. (A)

Work energy principle

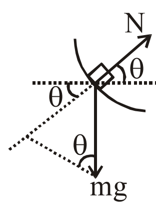
$mgR\sin\theta - W_f = \frac{1}{2}mv^2$ ..... (i)



m = 1 kg
R = 40 m
W_f = 150 s
θ = 30°

$N - mg\ \sin\theta = \frac{mv^2}{R}$

$N = mg\ \sin\ \theta + \frac{mv^2}{R} = 7.5N$



17. The speed of the block when it reaches the point Q is

(A) 5 ms^{-1} (B) 10 ms^{-1}
(C) $10\sqrt{3}ms^{-1}$ (D) 20 ms^{-1}

Ans. (B)

from equation (i)

$v = \sqrt{2\left[gR\sin\theta - \frac{W_f}{m}\right]} = 10\ m/s$