EXERCISE - 01 CHECK YOUR GRASP

3. Given expression reduce to

$$\frac{\sin 22\cos 8+\cos 22\sin 8}{\sin 23\cos 7+\cos 23\sin 7} = \frac{\sin 30}{\sin 30}=1$$

4. **Hint** : Since the given triangle is right angled triangle with sides are $(\sin\theta - \cos\theta)$ & $(\cos\theta + \sin\theta)$ so use $H^2 = P^2 + B^2$.

5. $\sin\theta \ \sec^7\theta + \cos\theta \ \text{cosec}^7\theta$

$= \tan\theta \sec^6\theta + \cot\theta \ \text{cosec}^6\theta$

$$= \sqrt{\frac{a}{b}}\left(1+\frac{a}{b}\right)^3 + \sqrt{\frac{b}{a}}\left(1+\frac{b}{a}\right)^3$$

$$= (a+b)^3\left[\frac{\sqrt{a}}{b^{7/2}}+\frac{\sqrt{b}}{a^{7/2}}\right] = \frac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}.$$

8. $$\frac{\sin 2\alpha+\sin 4\alpha-\sin 3\alpha}{\cos 2\alpha+\cos 4\alpha-\cos 3\alpha}$$

$$= \frac{2\sin 3\alpha\cos\alpha-\sin 3\alpha}{2\cos 3\alpha\cos\alpha-\cos 3\alpha}$$

$$= \frac{\sin 3\alpha(2\cos\alpha-1)}{\cos 3\alpha(2\cos\alpha-1)}=\tan 3\alpha$$

9. $\cos 20 + 2\sin^2 55 - \sqrt{2}\sin 65$

$= \cos 20 + 1 - \cos 110 - \sqrt{2}\sin 65$

$= 2\sin 65 \sin 45 + 1 - \sqrt{2}\sin 65 = 1.$

10. Apply C & D

12. $(2\sin 5\alpha\cos 3\alpha)\,2\cos 2\alpha$

$= (\sin 8\alpha+\sin 2\alpha)2\cos 2\alpha$

$= 2\sin 8\alpha\cos 2\alpha+2\sin 2\alpha\cos 2\alpha$

$= \sin 10\alpha+\sin 6\alpha+\sin 4\alpha$

13. $$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]$$

$$-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

$$=3\left[\cos^4\alpha+\sin^4\alpha\right]-2\left[\cos^6\alpha+\sin^6\alpha\right]$$

$$=3[1-2\sin^2\alpha\cos^2\alpha]-2(1-3\sin^2\alpha\cos^2\alpha) = 1$$

14. Use $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

16. $\cot 123 \cot 147 \ \cot 133 \ \cot 137$

$$=\frac{\cos 123^\circ\cos 147^\circ}{\sin 123^\circ\sin 147^\circ}.\frac{\cos 133^\circ\cos 137^\circ}{\sin 133\sin 137^\circ}$$

$$=\frac{\cos 270^\circ+\cos 24^\circ}{\cos 24^\circ-\cos 270^\circ}.\frac{\cos 270^\circ+\cos 24^\circ}{\cos 24^\circ-\cos 270^\circ}=1$$

17. $$\frac{\sin(2A+B)}{\sin B}=\frac{5}{1}$$

$$\Rightarrow \frac{\sin(2A+B)+\sin B}{\sin(2A+B)-\sin B}=\frac{6}{4} \qquad \text{(Apply C \& D)}$$

$$\Rightarrow \tan(A+B)=\frac{3}{2}\tan A$$

18. $$\frac{\sin\left(A+\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)} = k \qquad \text{(Apply C \& D)}$$

$$\Rightarrow \frac{\sin\left(A+\frac{C}{2}\right)+\sin\frac{C}{2}}{\sin\left(A+\frac{C}{2}\right)-\sin\frac{C}{2}} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{2\sin\left(\frac{A+C}{2}\right)\cos\frac{A}{2}}{2\cos\left(\frac{A+C}{2}\right)\sin\frac{A}{2}} = \frac{k+1}{k-1}$$

$$\Rightarrow \frac{\tan\left(\frac{A+C}{2}\right)}{\tan\frac{A}{2}} = \frac{k+1}{k-1} \Rightarrow \frac{1}{\tan\frac{A}{2}\tan\frac{B}{2}} = \frac{k+1}{k-1}$$

$$\Rightarrow \tan\frac{A}{2}\ \tan\frac{B}{2} = \frac{k-1}{k+1}$$

19. $$\frac{1-4\sin 10^\circ\sin 70^\circ}{2\sin 10^\circ} = \frac{1-2(\cos 60^\circ-\cos 80^\circ)}{2\sin 10^\circ}$$

$$= \frac{2\cos 80^\circ}{2\sin 10^\circ} = 1.$$

21. Given $\alpha - \beta = 15$ ....(i)

also $\sin\alpha = \sin\left(\frac{\pi}{2}-2\beta\right)$

$\Rightarrow \alpha = \frac{\pi}{2}-2\beta$ ....(ii)

Now solve (i) & (ii)

23. Use

$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots \sin(\alpha+\overline{n-1}\,\beta)$

$$= \frac{\sin\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

24. $\cot(A+C) = \cot\frac{3\pi}{4} \Rightarrow \frac{\cot A\cot C - 1}{\cot C + \cot A} = -1$

$\Rightarrow 1 - \cot A\cot C = \cot A + \cot C$ .... (i)

Now $(\cot A + 1)(\cot C + 1)$

$= 1 + \cot A + \cot C + \cot A\cot C$

$= 1 + 1 - \cot A\cot C + \cot A\cot C$ (using (i))

$= 2.$

25. $t_1 = 4\sin 63\cos 63 = 2\sin 126 = 2\sin 54$

Now $\log_{2\sin 54}(2\sin 18) = \log_{\frac{\sqrt{5}+1}{2}}\left(\frac{\sqrt{5}-1}{2}\right) = -1.$

26. $\ell = \left(\frac{\frac{\cos^4 x}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}(1-\sin^2 x)}\right)^2 = 1$

and $m = a^{\log_{\sqrt{a}} 2} = a^{\log_a 4} = 4$

so $\ell^2 + m^2 = 17$

29. $\sqrt{\sin\theta + \left(\sqrt{\sin\theta + \sqrt{\sin\theta + \ldots + \infty}}\right)} = (\sec^4\alpha - \sin\theta)$

$\sqrt{\sin\theta + (\sec^4\alpha - \sin\theta)} = \sec^4\alpha - \sin\theta$

$\sin\theta = \sec^4\alpha - \sec^2\alpha = \sec^2\alpha\tan^2\alpha$

(B) $\frac{2(2\sin^2\alpha)}{4(\cos^2\alpha)^2} = \tan^2\alpha\sec^2\alpha$

32. $\cos 4\theta - \cos 4\phi = 2\cos^2 2\theta - 2\cos^2 2\phi$

$= 2(\cos 2\theta + \cos 2\phi)(\cos 2\theta - \cos 2\phi)$

$= 2(2\cos^2\theta - 2\sin^2\phi)\, 2(\cos^2\theta - \cos^2\phi)$

$= 8(\cos\theta + \sin\phi)(\cos\theta - \sin\phi)(\cos\theta - \cos\phi)$

$(\cos\theta + \cos\phi)$

EXERCISE - 02 BRAIN TEASERS

2. $\sqrt{\frac{1-\sin A}{1+\sin A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$

$\Rightarrow \sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$

$\Rightarrow \frac{|1-\sin A|}{|\cos A|} + \frac{\sin A}{\cos A} = \frac{1}{\cos A}$ $(\because 1 - \sin A \geq 0)$

$\Rightarrow \frac{1}{\cos A} = \frac{1}{\cos A}$ when $\cos A > 0$

$\Rightarrow$ A belongs to Ist & IVth quadrant.

3. $\sqrt{2 + 2\sqrt{\cos^2 2\theta}} = \sqrt{2 + 2|\cos 2\theta|}$

$= \sqrt{2 - 2\cos 2\theta}$ $(\pi < 2\theta < \frac{3\pi}{2})$

$= 2|\sin\theta| = 2\sin\theta$

4. $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1+\cot^2\theta}} - 2\tan\theta\cot\theta = -1$

$\Rightarrow 1 + \sin\theta\cos\theta - \cos\theta|\sin\theta| - 1 = 0$

$\Rightarrow \sin\theta \geq 0 \Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right)$

6. (A) $\frac{\cos 2\alpha\,\tan\left(\frac{\pi}{4} + \alpha\right)}{1 + \cos\left(\frac{\pi}{2} - 2\alpha\right)}$

$= \frac{(\cos^2\alpha - \sin^2\alpha)}{1 + \sin 2\alpha}.\tan\left(\frac{\pi}{4} + x\right)$

$= \frac{\cos^2\alpha - \sin^2\alpha}{(\cos\alpha + \sin\alpha)^2}\tan\left(\frac{\pi}{4} + x\right)$

$= \frac{1-\tan\alpha}{1+\tan\alpha}.\frac{1+\tan\alpha}{1-\tan\alpha} = 1.$

(B) $\frac{\sin\alpha}{\sin\alpha - \cos\alpha\tan\frac{\alpha}{2}} - \cos\alpha$

$= \frac{\sin\alpha\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} - \cos\alpha = \frac{\sin\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = 1$

(C) $\frac{1}{\sin^2 2\alpha} + \frac{(\cos^2\alpha - \sin^2\alpha)^2}{\sin^2 2\alpha}$

$= \frac{1 + \cos^2 2\alpha}{\sin^2 2\alpha} \neq 1$

(D) $\frac{(\sin\alpha + \cos\alpha)^2}{(\sin\alpha + \cos\alpha)^2} = 1$

7. Let $y = \frac{\sqrt{3}\sin(\alpha+\beta) - \frac{2}{\cos\pi/6}\cos(\alpha+\beta)}{\sin\alpha}$

$= \frac{\sqrt{3}\sin(\alpha+\beta) - \frac{4}{\sqrt{3}}\cos(\alpha+\beta)}{\sin\alpha}$

$$= \frac{3\sin(\alpha+\beta) - 4\cos(\alpha+\beta)}{\sqrt{3}\sin\alpha}$$

Case I : If β lies in I quadrant i.e. $\tan\beta > 0$.

then

$$y = \frac{3\sin\alpha\left(\frac{3}{5}\right) + 3\cos\alpha\left(\frac{4}{5}\right) - 4\cos\alpha\left(\frac{3}{5}\right) + 4\left(\frac{4}{5}\right)\sin\alpha}{\sqrt{3}\sin\alpha}$$

$$= \frac{5\sin\alpha}{\sqrt{3}\sin\alpha} = \frac{5}{\sqrt{3}}$$

Case II : If β lies in II quadrant i.e. $\tan\beta < 0$

$$\text{then } y = \left(\frac{\frac{7}{5}\sin\alpha + \frac{24}{5}\cos\alpha}{\sqrt{3}\sin\alpha}\right)$$

$$= \frac{\sqrt{3}(7 + 24\cos\alpha)}{15}$$

so none option is correct.

8. Given $A > B$

$$\frac{2\tan x}{1+\tan^2 x} = k \Rightarrow \sin 2x = k$$

since A & B both satisfy the equation

so $\sin 2A = \sin 2B \Rightarrow 2A = \pi - 2B$

$$\Rightarrow A + B = \frac{\pi}{2} \Rightarrow C = \frac{\pi}{2}$$

9. On solving given equation we get $\sin\theta = \pm 4/5$

$$\text{Now use } \sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

11. $\cos(A - B) = \frac{3}{5}$ & $\tan A \tan B = 2$

$$\Rightarrow \cos A \cos B + \sin A \sin B = \frac{3}{5}$$

& $\sin A \sin B = 2\cos A \cos B$, solve both equations

$$\Rightarrow \cos A \cos B = \frac{1}{5} \text{ and } \sin A \sin B = \frac{2}{5}$$

$$\cos(A + B) = \frac{1}{5} - \frac{2}{5} = -\frac{1}{5}.$$

12. $\cos A + \cos B = 1$

$$2\cos\frac{A+B}{2}\cos\frac{A-B}{2} = 1 \Rightarrow \cos\frac{A-B}{2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \cos(A-B) = 2\cos^2\frac{A-B}{2} - 1 = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$\& \quad |\cos A - \cos B| = \left|2\sin\frac{A+B}{2}\sin\frac{B-A}{2}\right| = \sqrt{\frac{2}{3}}$$

13. Given $3\sin^2 A = 1 - 2\sin^2 B$

$\Rightarrow 3\sin^2 A = \cos 2B$ ......(i)

& $3\sin 2A = 2\sin 2B$ ......(ii)

divide (i) by (ii) we get

$\tan A \tan 2B = 1$

$$\text{Now } \tan(A+2B) = \frac{\tan A + \tan 2B}{1 - \tan A \tan 2B}$$

$\Rightarrow A + 2B = \pi/2$

15. Hint : $2\sin 11\ 15' = 2\sqrt{\frac{1-\cos 22.5}{2}}$

23. $$f(x) = \frac{\sin x}{|\sec x|} + \frac{\cos x}{|\operatorname{cosec} x|}$$

$= \sin x\ |\cos x| + \cos x\ |\sin x|$

$\Rightarrow f(x)$ is constant when $\sin x$ and $\cos x$ are of opposite sign, i.e. $f(x)$ is in II[nd] or IV[th] quadrant.

25. $$f_n(\theta) = \frac{\sin\frac{\theta}{2}(1+\cos\theta)(1+\cos 2\theta)(1+\cos 4\theta).....(1+\cos 2^n\theta)}{\cos\frac{\theta}{2}\cos\theta\cos 2\theta\cos 4\theta......\cos 2^n\theta}$$

$$= \frac{\sin\frac{\theta}{2}\left(2\cos^2\frac{\theta}{2}\right)(2\cos^2\theta)(2\cos^2 2\theta)...(2\cos 2^{n-1}\theta)}{\cos\frac{\theta}{2}\cos\theta\cos 2\theta\cos 4\theta...\cos 2^n\theta}$$

$$= \frac{2^{n+1}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\theta\cos 2\theta.....\cos 2^{n-1}\theta}{\cos 2^n\theta}$$

$$= \frac{\sin 2^n\theta}{\cos 2^n\theta} = \tan 2^n\theta$$

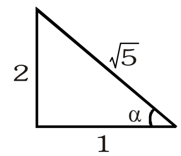
$$f_2\left(\frac{\pi}{16}\right) = \tan 2^2\left(\frac{\pi}{16}\right) = 1$$

$$f_3\left(\frac{\pi}{32}\right) = 1,\ f_4\left(\frac{\pi}{64}\right) = 1,\ f_5\left(\frac{\pi}{128}\right) = 1$$

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

Fill in the blanks :

1. $\dfrac{\cos\alpha}{\sin^3\alpha+\cos^3\alpha}$



$$= \frac{-\frac{1}{\sqrt{5}}}{\frac{-8}{5\sqrt{5}}-\frac{1}{5\sqrt{5}}} = \frac{5}{9}$$

5. $(1 + \sin\alpha - \cos\alpha)^2 + (1 + \cos\alpha - \sin\alpha)^2$

$= 4 - 4\sin\alpha\cos\alpha$

6. $\dfrac{2\cos 4x}{\sin 4x(1+\cos 8x)} = \dfrac{2\cos 4x}{\sin 4x(2\cos^2 4x)}$

$= \dfrac{2}{\sin 8x} = 2 \operatorname{cosec} 8x > 2.$

Match the column :

1. (A) $\dfrac{\cos 10^\circ-\sqrt{3}\sin 10^\circ}{\sin 10^\circ\cos 10^\circ} = \dfrac{2\left(\frac{1}{2}\cos 10^\circ-\frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\frac{2\sin 10^\circ\cos 10^\circ}{2}}$

$= \dfrac{4\cos 70^\circ}{\sin 20^\circ} = 4$

(B) $\dfrac{4\cos 20^\circ\sin 20^\circ-\sqrt{3}\cos 20^\circ}{\sin 20^\circ}$

$= \dfrac{2\sin 40^\circ - \sqrt{3}\cos 20^\circ}{\sin 20^\circ}$

$= \dfrac{2\sin 90^\circ\sin 40^\circ-2\cos 30^\circ\cos 20^\circ}{\sin 20^\circ}$

$= \dfrac{\cos 50^\circ-\cos 130^\circ-\cos 50^\circ-\cos 10^\circ}{\sin 20^\circ}$

$= -\dfrac{2\cos 70^\circ\cos 60^\circ}{\sin 20^\circ} = -1.$

(C) $\dfrac{\cos 40^\circ+\cos 40^\circ-\cos 20^\circ}{\sin 20^\circ}$

$= \dfrac{\cos 40^\circ-2\sin 30^\circ\sin 10^\circ}{\sin 20^\circ}$

$= \dfrac{\cos 40^\circ-\sin 10^\circ}{\sin 20^\circ} = \dfrac{\sin 50^\circ-\sin 10^\circ}{\sin 20^\circ}$

$= \dfrac{2\cos 30^\circ\sin 20^\circ}{\sin 20^\circ} = \sqrt{3}$

(D) $2\sqrt{2}\sin 10 \left[\dfrac{1}{2\cos 5^\circ}+\dfrac{\cos 40^\circ}{\sin 5^\circ}-2\sin 35^\circ\right]$

$= 2\sqrt{2}\sin 10\left[\dfrac{\sin 5^\circ+2\cos 5^\circ\cos 40^\circ-4\sin 35^\circ\sin 5^\circ.\cos 5^\circ}{2\cos 5^\circ\sin 5^\circ}\right]$

$= 2\sqrt{2}$ [sin5 +cos 45 +cos 35 −cos 25 +cos 45]

$= 2\sqrt{2}$ [sin5 + 2 cos 45 − 2 sin 30 sin 5]

= 4.

2. (B) **Hint :** $\log_{\sqrt{5}}\left[2\sin\left(\theta-\frac{\pi}{4}\right)+3\right]$

(C) $7\cos^2\theta + 6\sin\theta\cos\theta - \sin^2\theta$

$= 7\left(\dfrac{1+\cos 2\theta}{2}\right)+3\sin 2\theta-\left(\dfrac{1-\cos 2\theta}{2}\right)$

$= \dfrac{6+8\cos 2\theta+6\sin 2\theta}{2} \Rightarrow \lambda = 8,\ \mu = -2$

(D) **Hint :** $5\cos\theta+3\left(\dfrac{\cos\theta}{2}-\dfrac{\sin\theta\sqrt{3}}{2}\right) + 3$

$= \dfrac{13\cos\theta-3\sqrt{3}\sin\theta}{2} + 3$

Assertion & Reason :

2. $(\sin\theta-1)^2 = 0 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \dfrac{\pi}{2}$

$\Rightarrow$ Statement-I only hold for n = 1 hence false. Statement-II is true.

3. $\cos^2\theta = \dfrac{(x+y)^2}{4xy} = \dfrac{x^2+y^2}{4xy}+\dfrac{1}{2}$

$\dfrac{x^2+y^2}{2} \ge xy$ (AM $\ge$ GM)

$(x - y)^2 \ge 0$ only when x = y

Statement-II is true.

4. If A is obtuse than 0 < B + C < 90

$\Rightarrow \tan(B + C) = \dfrac{\tan B+\tan C}{1-\tan B\tan C} > 0$

as numerator is positive

$1 - \tan B\tan C > 0$

$\tan B\tan C < 1$

Statement II is obviously true & it explain I

5. Statement-I: $\cos\alpha + \cos\left(\alpha+\dfrac{2\pi}{3}\right)+\cos\left(\alpha+\dfrac{4\pi}{3}\right)=0$

$= \cos\alpha + 2\cos(\alpha+\pi)\cos\left(\dfrac{\pi}{3}\right) = 0$

$= \cos\alpha - \cos\alpha = 0$

i.e. a + b + c = 0 (say)

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

hence statement I is true

But statement II is false as vice versa is not true.

Comprehension # 1 :

1. $\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\cos\left(\pi-\dfrac{\pi}{7}\right) = -\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}$

$\Rightarrow \alpha = \dfrac{\pi}{7} = \dfrac{\pi}{2^3-1}$

Then value is $-\left(\frac{-1}{8}\right) = \frac{1}{8}$

2. $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

Put $\cos\frac{7\pi}{15} = -\cos\frac{8\pi}{15}$ and arrange

$= \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\left(-\cos\frac{8\pi}{15}\right)\cos\frac{3\pi}{15}$

$\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}$

$$= -\left(\frac{\sin 2^4\frac{\pi}{15}}{2^4\sin\frac{\pi}{15}}\right)\cos 36 \,.\, \cos 72 \,.\, \frac{1}{2}$$

$$= -\frac{1}{2^4}\left(\frac{\sqrt{5}+1}{4}\times\frac{\sqrt{5}-1}{4}\right)\frac{1}{2} = 1/128.$$

3. $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}(1)\sin\frac{5\pi}{14}\sin\frac{3\pi}{14}\sin\frac{\pi}{14}$

$= \sin^2\frac{\pi}{14}\sin^2\frac{3\pi}{14}\sin^2\frac{5\pi}{14}$

$$= \left[\cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)\right]^2$$

$$= \left[\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right]^2$$

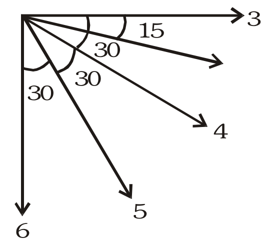
$$= \left[\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right]^2 = \left[\frac{\sin\frac{8\pi}{7}}{2^3\sin\frac{\pi}{7}}\right]^2 = \frac{1}{64}$$

Comprehension # 2 :

1. **Hint** : $1^c = 57$

$\Rightarrow$ sin1 < sin57 $\Rightarrow$ cos1 > cos57

2.



$\Rightarrow$ Answer is 90 – 15 = 75

3. Let the number of side of a polygon be 5x and angle α.

& for other polygon number of side be 4x and angle β.

$$\alpha = \frac{(n-2)\pi}{n} = \frac{(5x-2)\pi}{5x} \;;\quad \beta = \frac{(4x-2)\pi}{4x}$$

$$\alpha - \beta = \frac{\pi}{20} = \frac{\pi}{x}\left(\left(\frac{5x-2}{5}\right)-\left(\frac{4x-2}{4}\right)\right)$$

$\Rightarrow$ x = 2 $\Rightarrow$ side : 10, 8

4. $\frac{4x}{3}\times\frac{90}{100}+3x+\frac{2\pi x}{75}\times\left(\frac{180°}{\pi}\right) = 180°$

$\Rightarrow$ x = 20

Angles are 24 , 60 , 96

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. $\cos(y-z)+\cos(z-x)+\cos(x-y) = -\frac{3}{2}$

$\Rightarrow$ 2cosy cosz + 2siny sinz + 2cosz cosx + 2sinz sinx + 2cosx cosy + 2sinx siny + 3 = 0

$\Rightarrow$ $(\sin x+\sin y+\sin z)^2 + (\cos x+\cos y+\cos z)^2 = 0$

$\Rightarrow$ cos x + cos y + cos z = 0 = sin x + siny + sinz

2. **Hint** : $\cos 2\alpha = \cos[(\alpha+\beta)+(\alpha-\beta)]$

3. $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma)$

$$=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$+2\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$=2\cos\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]$$

$$=4\cos\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\right]$$

4. Hint : $\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)]$

5. (a) $\sin^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\sin^4\frac{5\pi}{16}+\sin^4\frac{7\pi}{16}$

$$=\sin^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\cos^4\frac{3\pi}{16}+\cos^4\frac{\pi}{16}$$

$$=1-2\sin^2\frac{\pi}{16}\cos^2\frac{\pi}{16}+1-2\sin^2\frac{3\pi}{16}\cos^2\frac{3\pi}{16}$$

$$= 2-\frac{1}{2}\left[\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}\right]$$

$$= 2-\frac{1}{2}\left[\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right] = \frac{3}{2}$$

6. $\cos\theta = \dfrac{\cos\alpha - e}{1 - e\cos\alpha}$

$\Rightarrow \cos\theta - e\cos\theta\cos\alpha = \cos\alpha - e$

$\Rightarrow \cos\theta - \cos\alpha = e(\cos\theta\cos\alpha - 1)$

$\Rightarrow \dfrac{\cos\theta - \cos\alpha}{\cos\theta\cos\alpha - 1} = e$

$\Rightarrow \dfrac{\cos\theta\cos\alpha - 1 + \cos\theta - \cos\alpha}{\cos\theta\cos\alpha - 1 - \cos\theta + \cos\alpha} = \dfrac{1+e}{1-e}$

$\Rightarrow \dfrac{(\cos\theta - 1)(\cos\alpha + 1)}{(\cos\theta + 1)(\cos\alpha - 1)} = \dfrac{1+e}{1-e}$

$\Rightarrow \dfrac{-2\sin^2\frac{\theta}{2}2\cos^2\frac{\alpha}{2}}{2\cos^2\frac{\theta}{2}\left(-2\sin^2\frac{\alpha}{2}\right)} = \dfrac{1+e}{1-e}$

$\Rightarrow \dfrac{\tan^2\frac{\theta}{2}}{\tan^2\frac{\alpha}{2}} = \dfrac{1+e}{1-e}$.

8. $\operatorname{cosec}\theta + \cot\theta = \dfrac{1+\cos\theta}{\sin\theta} = \cot\dfrac{\theta}{2}$

$\Rightarrow \operatorname{cosec}\theta = \cot\dfrac{\theta}{2} - \cot\theta$

Now $\operatorname{cosec}\theta + \operatorname{cosec}2\theta + \operatorname{cosec}2^2\theta + \ldots + \operatorname{cosec}2^{n-1}\theta$

$= \cot\dfrac{\theta}{2} - \cot\theta + \cot\theta - \cot 2\theta + \cot 2\theta$

$- \cot 2^2\theta + \ldots + \cot 2^{n-2}\theta - \cot 2^{n-1}\theta$

$= \cot\dfrac{\theta}{2} - \cot 2^{n-1}\theta$.

9. (a) $\sin\alpha + \sin\beta = 2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

$= 2\sin\dfrac{c}{2}\cos\left(\dfrac{\alpha-\beta}{2}\right)$

Its max. value is $2\sin\dfrac{c}{2}$

(c) $\tan\alpha + \tan\beta = \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$

$= \dfrac{2\sin c}{\cos(\alpha+\beta) + \cos(\alpha-\beta)}$

$= \dfrac{\sin c}{\cos^2\frac{c}{2} - \sin^2\left(\frac{\alpha-\beta}{2}\right)}$

This is minimum when denominator is maximum

i.e. when $\sin^2\left(\dfrac{\alpha-\beta}{2}\right)$ is zero

$\Rightarrow \dfrac{2\sin\frac{c}{2}\cos\frac{c}{2}}{\cos^2\frac{c}{2}} = 2\tan\dfrac{c}{2}$

10. (b) $x - y = \dfrac{\pi}{4} \Rightarrow \cot(x-y) = \cot\dfrac{\pi}{4}$

$\Rightarrow \cot x\cot y + 1 = \cot y - \cot x$

$\Rightarrow \cot x(\cot y + 1) = \cot y - 1$

$\Rightarrow \cot x(3 - \cot x) = 1 - \cot x$

$\Rightarrow \cot^2 x - 4\cot x + 1 = 0$

$\Rightarrow \cot x = 2 \pm \sqrt{3} \Rightarrow x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

$\therefore y = x - \dfrac{\pi}{4} = \dfrac{5\pi}{12} - \dfrac{\pi}{4} = \dfrac{\pi}{6}$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

2. Let $y_1 = \dfrac{\cos 3x}{\sin 2x\sin 4x} = \dfrac{2\sin x\cos 3x}{2\sin x\sin 2x\sin 4x}$

$= \dfrac{\sin 4x - \sin 2x}{2\sin x\sin 2x\sin 4x}$

$= \dfrac{\operatorname{cosec}x}{2}(\operatorname{cosec}2x - \operatorname{cosec}4x)$

$y_2 = \dfrac{\cos 5x}{\sin 4x\sin 6x} = \dfrac{\operatorname{cosec}x}{2}[\operatorname{cosec}4x - \operatorname{cosec}6x]$

$y_3 = \dfrac{\cos 7x}{\sin 6x\sin 8x} = \dfrac{\operatorname{cosec}x}{2}[\operatorname{cosec}6x - \operatorname{cosec}8x]$

$y_4 = \dfrac{\cos 9x}{\sin 8x\sin 10x} = \dfrac{\operatorname{cosec}x}{2}[\operatorname{cosec}8x - \operatorname{cosec}10x]$

$\therefore y_1 + y_2 + y_3 + y_4 = \dfrac{\operatorname{cosec}x}{2}[\text{cosce}2x - \operatorname{cosec}10x]$

$= \dfrac{\operatorname{cosec}x[\sin 10x - \sin 2x]}{2\sin 2x\sin 10x}$

$= \operatorname{cosec}x.\ \operatorname{cosec}2x.\ \sin 4x.\ \cos 6x.\ \operatorname{cosec}10x.$

3. $\dfrac{1}{2}\left(\dfrac{p}{\sin 2\beta} - \dfrac{q}{\cos 2\beta}\right) = \dfrac{p\cos 2\beta - q\sin 2\beta}{2\sin 2\beta\cos 2\beta}$

Put $p = \sqrt{p^2+q^2}\sin\alpha$ & $q = \sqrt{p^2+q^2}\cos\alpha$

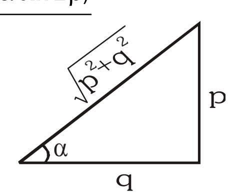
using $\tan\alpha = p/q$

$= \dfrac{\sqrt{p^2+q^2}\sin\alpha\cos 2\beta - \sqrt{p^2+q^2}\cos\alpha\sin 2\beta}{2\sin 2\beta\cos 2\beta}$

$= \sqrt{p^2+q^2}\ \dfrac{(\sin\alpha\cos 2\beta - \cos\alpha\sin 2\beta)}{\sin 4\beta}$

$= \sqrt{p^2+q^2}\ \dfrac{\sin(\alpha - 2\beta)}{\sin 4\beta}$

$= \sqrt{p^2+q^2}$



4. $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right)=\sqrt{\frac{1-\cos\left(\frac{\pi}{2}+y\right)}{1+\cos\left(\frac{\pi}{2}+y\right)}}=\sqrt{\frac{1+\sin y}{1-\sin y}}$

$\Rightarrow \frac{1+\sin y}{1-\sin y}=\left(\frac{1+\sin x}{1-\sin x}\right)^3 \;\Rightarrow\; \frac{\sin y+1}{\sin y-1}=\left(\frac{\sin x+1}{\sin x-1}\right)^3$

Now apply C & D and solve it.

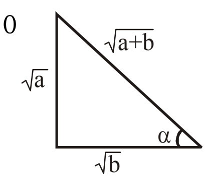
5. $\frac{\sin^4\alpha}{a}+\frac{\cos^4\alpha}{b}=\frac{1}{a+b}$

$\Rightarrow \frac{(a+b)}{a}\sin^4\alpha + \frac{(a+b)}{b}\cos^4\alpha = 1$

$\Rightarrow \sin^4\alpha + \frac{b}{a}\sin^4\alpha + \frac{a}{b}\cos^4\alpha + \cos^4\alpha = 1$

$\Rightarrow 1 - 2\sin^2\alpha\cos^2\alpha + \frac{b}{a}\sin^4\alpha + \frac{a}{b}\cos^4\alpha = 1$

$\Rightarrow \left(\sqrt{\frac{b}{a}}\sin^2\alpha-\sqrt{\frac{a}{b}}\cos^2\alpha\right)^2 = 0$



$\Rightarrow \tan^2\alpha = \frac{a}{b}$

Now $\frac{\sin^8\alpha}{a^3}+\frac{\cos^8\alpha}{b^3}$

$= \frac{a^4}{(a+b)^4}\frac{1}{a^3}+\frac{b^4}{(a+b)^4}\frac{1}{b^3}=\frac{1}{(a+b)^3}$

6. **Hint** : Multiply P by $2\sin\frac{\pi}{19}$ & Q by $2\sin\frac{\pi}{12}$

we get $P=\frac{1}{2}$ & $Q=-\frac{1}{2}$

7. Squaring LHS & RHS

LHS = $16\sin^2 27$

$= 16\left(\frac{1-\cos 54}{2}\right)=8\left(1-\sqrt{1-\sin^2 54}\right)$

Put value of sin54 & solve it

RHS : make it in LHS form

9. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma$

$= \cos^2\alpha+1-\sin^2\beta+\cos^2(\alpha+\beta)$

$= 1+\cos^2\alpha-\sin^2\beta+\cos^2(\alpha+\beta)$

$= 1+\cos(\alpha+\beta)\cos(\alpha-\beta)+\cos^2(\alpha+\beta)$

$= 1+\cos(\alpha+\beta)[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$

$= 1+2\cos\alpha\cos\beta\cos\gamma$

10. If ΔABC is equilateral then $A = B = C = \frac{\pi}{3}$

$\therefore \cot\frac{\pi}{3}+\cot\frac{\pi}{3}+\cot\frac{\pi}{3}=\sqrt{3}$

Now if $\cot A + \cot B + \cot C = \sqrt{3}$

sq. both sides we get

$\Sigma\cot^2 A + 2\Sigma\cot A\cot B = 3(1)$

we know that $\Sigma\cot A\cot B = 1$

$\therefore \Sigma\cot^2 A + 2\Sigma\cot A\cot B = 3(\Sigma\cot A\cot B)$

$\Rightarrow \Sigma\cot^2 A - \Sigma\cot A\cot B = 0$

$\Rightarrow (\cot A-\cot B)^2+(\cot B-\cot C)^2+(\cot C-\cot A)^2 = 0$

$\Rightarrow \cot A = \cot B = \cot C \Rightarrow A = B = C$

$\Rightarrow \Delta ABC$ is equilateral.

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

3. since sum of a number & its reciprocal is always ≥ 2

$\therefore y \geq 2$ But $y = 2$ only when $\theta = 0$

$\therefore y = 2$

Ans. $y > 2$

6. $\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$; $\sin(\alpha-\beta)=\frac{1}{2}=\sin\frac{\pi}{6}$

$\sin(\alpha+\beta) = 1 = \sin\frac{\pi}{2}$; $\alpha-\beta=\frac{\pi}{6}$

$\alpha+\beta=\frac{\pi}{2}$

$\alpha-\beta=\frac{\pi}{6}$

$2\alpha = 120 \Rightarrow \alpha = 60 = \frac{\pi}{3}$

$\beta = 30 = \frac{\pi}{6}$

tan[60 + 2 30] tan[2 60 + 30]

= tan120 tan150 = tan(180 – 60) tan(180 – 30)

= +tan60 tan30 = +1

9. $U^2=a^2+b^2+2\sqrt{(a^2\cos^2\theta+b^2\sin^2\theta)(a^2\sin^2\theta+b^2\cos^2\theta)}$

$= a^2 + b^2 +$

$2\sqrt{a^4\sin^2\theta\cos^2\theta+a^2b^2\cos^4\theta+a^2b^2\sin^4\theta+b^4\sin^2\theta\cos^2\theta}$

$= a^2 + b^2$

$+ 2\sqrt{(a^4+b^4)\sin^2\theta\cos^2\theta+a^2b^2(1-2\sin^2\theta\cos^2\theta)}$

$= a^2 + b^2 +$

$2\sqrt{a^4\sin^2\theta\cos^2\theta-2a^2b^2\sin^2\theta\cos^2\theta+b^4\sin^2\theta\cos^2\theta+a^2b^2}$

$= a^2 + b^2 + 2\sqrt{(a^2-b^2)^2 \sin^2\theta\cos^2\theta + a^2b^2}$

$= a^2 + b^2 + 2\sqrt{a^2b^2 + \frac{(a^2-b^2)^2}{4}.\sin^2 2\theta}$

If $\sin^2 2\theta = 1$

$U^2_{(max.)} = a^2 + b^2 + 2\sqrt{a^2b^2 + \left(\frac{a^2-b^2}{4}\right)^2}$

$= a^2 + b^2 + \sqrt{4a^2b^2 + a^4 + b^4 - 2a^2b^2}$

$= a^2 + b^2 + (a^2 + b^2) = 2(a^2 + b^2)$

if $\sin^2 2\theta = 0$

$U^2_{(min.)} = a^2 + b^2 + 2\sqrt{a^2b^2 + \frac{(a^2-b^2)^2}{4}.0}$

$= a^2 + b^2 + 2ab = (a + b)^2$

$U^2_{(max.)} - U^2_{(min.)} = 2(a^2 + b^2) - (a + b)^2$

$= 2(a^2 + b^2) - (a^2+2ab+b^2) = (a - b)^2$

10. Squaring and adding

$4\cos^2\frac{\alpha-\beta}{2} = \frac{(21)^2 + (27)^2}{(65)^2}$

$\Rightarrow \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}} \qquad \left(\because \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}\right)$

11. $0 < x < \pi$

$\cos x + \sin x = \frac{1}{2}$

$1 + 2\sin x \cos x = \frac{1}{4} \Rightarrow \sin 2x = -\frac{3}{4}$

$\Rightarrow \frac{2\tan x}{1+\tan^2 x} = -\frac{3}{4} \Rightarrow 8\tan x + 3 + 3\tan^2 x = 0$

$\tan x = \frac{-8 \pm \sqrt{64 - 4.3.3}}{2.3} = \frac{-8 \pm \sqrt{64-36}}{6}$

$= \frac{-8 \pm \sqrt{28}}{6} = \frac{-8 \pm 2\sqrt{7}}{6} = \frac{-4 \pm \sqrt{7}}{3}$

13. $A = \sin^2 x + \cos^4 x$

$= \cos^4 x - \cos^2 x + 1$

$= (\cos^2 x)^2 - 2.\ \cos^2 x.\frac{1}{2} + \frac{1}{4} - \frac{1}{4} + 1$

$= (\cos^2 x - \frac{1}{2})^2 + \frac{3}{4}$

Again put $\cos x = 0 \Rightarrow \frac{1}{4} + \frac{3}{4} = 1$

$\cos x = 1 \Rightarrow \frac{1}{4} + \frac{3}{4} = 1$

$\frac{3}{4} \le A \le 1$ Ans.

14. $3\sin P + 4\cos Q = 6$ ..... (1)

$4\sin Q + 3\cos P = 1$ ..... (2)

Squaring (1) and (2). Then adding

$9 + 16 + 24\sin(P + Q) = 37$

$\Rightarrow 24\sin(P + Q) = 12$

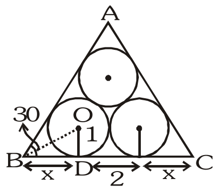
$\Rightarrow \sin(P + Q) = \frac{1}{2} \Rightarrow \sin R = \frac{1}{2}$

$\therefore R = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

But $R = \frac{5\pi}{6}$ does't satisfy the given equation so

$R = \frac{\pi}{6}$

3. In ΔBOD



$\tan 30 = \dfrac{1}{x}$

$\Rightarrow x = \sqrt{3}$

$\Rightarrow BC = 2 + 2\sqrt{3}$

area of $\Delta ABC = \dfrac{\sqrt{3}}{4}(2 + 2\sqrt{3})^2 = 4\sqrt{3} + 6$

4. $\because \quad \theta \in \left(0, \dfrac{\pi}{4}\right) \Rightarrow \tan\theta < 1$ & $\cot\theta > 1$

let $\tan\theta = 1 - x$ and $\cot\theta = 1 + y$

when $x, y > 0$ and are very small

$\therefore t_1 = (1 - x)^{1-x}$, $t_2 = (1 - x)^{1+y}$

$t_3 = (1 + y)^{1-x}$ $t_4 = (1 + y)^{1+y}$

clearly $t_4 > t_3$ & $t_1 > t_2$

also $t_3 > t_1$

Then $t_4 > t_3 > t_1 > t_2$

5. (a) $\dfrac{\tan^4 x}{2} + \dfrac{1}{3} = \dfrac{\sec^4 x}{5}$

put $\tan^2 x = t$

on solving we get $t = 2/3$

$\Rightarrow \sin^2 x = \dfrac{2}{5} \quad \Rightarrow \cos^2 x = \dfrac{3}{5}$.

(b) $$\sqrt{2}\sum_{m=1}^{6}\frac{\sin\left[\left(\theta+\frac{m\pi}{4}\right)-\left(\theta+\frac{m\pi}{4}-\frac{\pi}{4}\right)\right]}{\sin\left(\theta+\frac{m\pi}{4}-\frac{\pi}{4}\right)\sin\left(\theta+\frac{m\pi}{4}\right)}$$

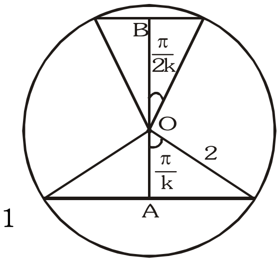
$$\sqrt{2}\sum_{m=1}^{6}\left[\cot\left(\theta+\frac{(m-1)\pi}{4}\right)-\cot\left(\theta+\frac{m\pi}{4}\right)\right]$$

$\Rightarrow \cot\theta + \tan\theta = 4.$

$\Rightarrow \tan\theta = 2 \pm \sqrt{3}$

$\Rightarrow \theta = \dfrac{\pi}{12}$ or $\dfrac{5\pi}{12}$

6. (b) $OA = 2\cos\dfrac{\pi}{k}$



$OB = 2\cos\dfrac{\pi}{2k}$

$2\cos\dfrac{\pi}{k} + 2\cos\dfrac{\pi}{2k} = \sqrt{3} + 1$

$2\cos^2\dfrac{\pi}{2k} - 1 + \cos\dfrac{\pi}{2k} = \dfrac{\sqrt{3}+1}{2}$

Let $\cos\dfrac{\pi}{2k} = t$

$2t^2 + t - 1 - \dfrac{\sqrt{3}+1}{2} = 0$

$\Rightarrow 4t^2 + 2t - (3 + \sqrt{3}) = 0$

$\Rightarrow t = \dfrac{\sqrt{3}}{2}, -\dfrac{1+\sqrt{3}}{2}$

$t = -\dfrac{1+\sqrt{3}}{2}$ (not possible)

$t = \dfrac{\sqrt{3}}{2} = \cos 30° = \cos\dfrac{\pi}{6} \Rightarrow \cos\dfrac{\pi}{2k} = \cos\dfrac{\pi}{6}$

$k = 3$

7. $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\Rightarrow \tan\theta = \sqrt{2} + 1$ ...(i)

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{2}-1} = \sqrt{2} + 1$ ...(ii)

from (i) & (ii)

$\Rightarrow P = Q$