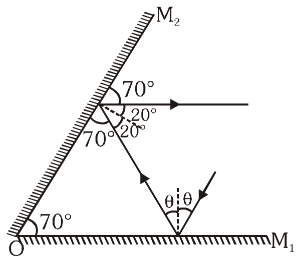
UNIT # 11 (PART - I)

RAY OPTICS AND OPTICAL INSTRUMENTS

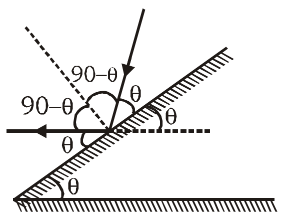
EXERCISE -I

1.

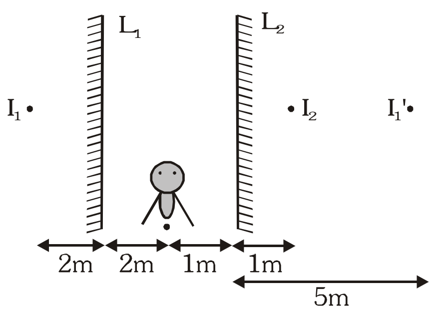


$90 - \theta = 180 - 70 - 70 \Rightarrow \theta = 50$

2.

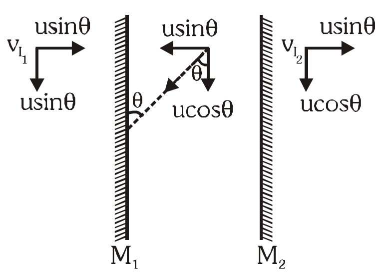


3.



Second Image I_1' (image of I_1) = 5 +1 = 6 m [from person]

4.



$\Rightarrow v_{I_1I_2} = v_{I_2} - v_{I_1} = 0$

5. $\vec{v}_M = 4\hat{i} + 5\hat{j} + 8\hat{k}$, $\vec{v}_0 = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\because$ Plane of the mirror is xy

$\therefore$ $\vec{v}_I = 3\hat{i} + 4\hat{j}$ and for $\tilde{k}v_{IZ}$

$= 2\ v_{MZ} - v_{OZ} = 2 \quad 8 - 5 = 11$

$\Rightarrow \vec{v}_I = 3\hat{i} + 4\hat{j} + 11\hat{k}$

6. Image formation by concave mirror when object is real & placed at focus f, forms image at infinity but not in other cases.

7. For M_1 : u = – 10 cm

f = – 20 cm; h_0 = – 0.1 cm

$v = \dfrac{200}{-10+20} = \dfrac{200}{10} \Rightarrow v = 20$ cm

$m = -\dfrac{v}{u} \Rightarrow \dfrac{h_i}{h_o} = \dfrac{+20}{+10}$

$\Rightarrow h_i = 2 \quad -0.1 = -0.2$; $h_i = -2$mm

For $m_2 = \dfrac{-20}{-10} = 2 \Rightarrow h_i = 0.2$ cm = 2 mm

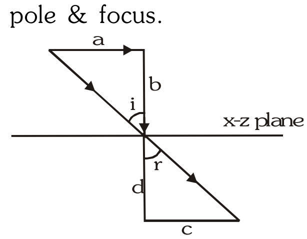
Distance between two images = 2 mm

8. As convex mirror is placed at origin & reflecting surface towards –ve x–axis.

For u < 0 $\Rightarrow$ Real objects.

For all cases of real objects, the image formation towards right of the origin v is +ve and also between pole & focus.

9.

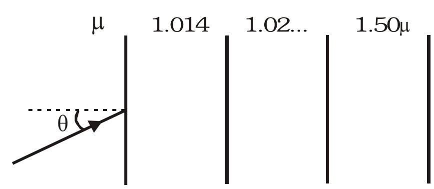


From Snell's law : 1.5 sin i = 2 sin r

$1.5 \dfrac{a}{\sqrt{a^2+b^2}} = 2 \dfrac{c}{\sqrt{c^2+d^2}} \Rightarrow \dfrac{a}{c} = \dfrac{4}{3}$

$\left[\because \sqrt{a^2+b^2} = \sqrt{c^2+d^2} = 1(\text{unit vector})\right]$

10.



For composite transparent II^r slabes

$= \mu\sin\theta =$ constant $\mu\sin\theta = 1.6\ \mu \sin x$

$\sin x = 5/8 \sin\theta$.

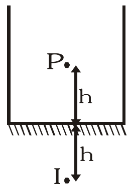
11. Initially observer is mirror for image formation of object O. As O is kept at the bottom, then it sees somewhat above the bottom.

$\therefore$ Distance = $5 + \dfrac{10}{\mu}$

$= 5 + \dfrac{10 \times 3}{4} = 12.5$ cm (in front)

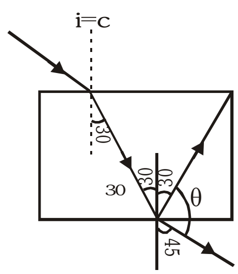
Hence image from the mirror after reflection is also at 12.5 cm.

12. Image formation by mirrorat a depth 'h' from mirror.The distance (apparent) between these two will be = $\frac{2h}{\mu}$



13. Image position coincides with the object when an object is placed on centre of curvature i.e. R = 0.5m when transparent liquid is filled then phenomenon occurs when the pin is placed 0.4 m from mirror i.e. effective distance must be 0.5 again $0.2 + \ \mu\ 0.2 = 0.5$; $\mu = 3/2$

14. From Snell's law $\sqrt{2}$ sin i = 1 sin 90



i' = 45

$\therefore$ Again 1. sin 45 = $\sqrt{2}$ sin r

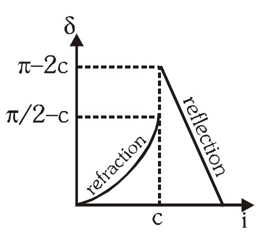
r = 30 & $\sqrt{2}$ sin 30 = 1 sin r'

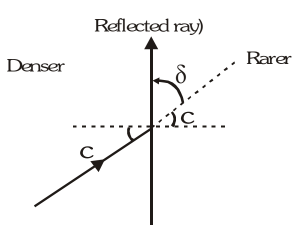
r' = 45

Hence θ = 180 – (30+45) = 105

15. When a ray enters into different medium & again reenter in a same medium there is no deviation due to refraction (overall) but bottom silvered glass surface provides 180 deviation from the reflection.

16. Maximum possible deviation = $\delta = \pi/2 - C$





17. As given : r + r' = 90

u_2 sinr = $u_1$sin r'

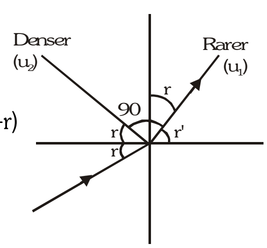
$\Rightarrow u_2$ sin r = u_1 sin (90–r)

u_2 sin C = u_1 sin 90

$\frac{\mu_1}{\mu_2} = \tan r$; $\sin C = \frac{\mu_1}{\mu_2}$

(C: Critical angle)

$\therefore$ sin C = tan r; C = $\sin^{-1}$ (tan r)



18. An object is seen when reflected light from object enters into eyes when refractive index exactly same then no reflection from the object.

19. As Speed of light in vacuum / air

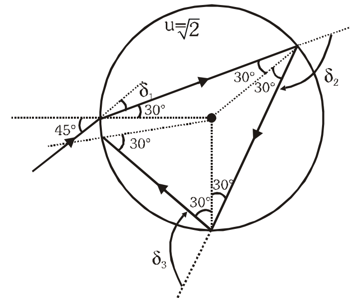
μ = Speed of light in medium

$\mu = \frac{c}{v} \Rightarrow$ velocity is different in different medium

From $v = n\lambda$

But frequency is the fundamental property. It never change by changing the medium hence λ is also changed as v is change.

20.



From Snell's law

1. sin 45 = $\sqrt{2}$ sin r ; r = 30

δ_1 = 45 – 30 $\circlearrowleft$ = 15 $\circlearrowleft$

δ_2 = 180 – 2 30 = 120 $\circlearrowleft$

δ_3 = 180 – 2 30 = 120 $\circlearrowleft$

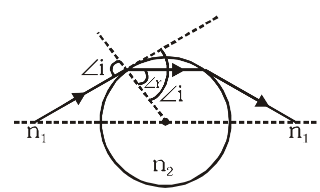
& from Snell's law (refraction)

$\sqrt{2}$ sin 30 = 1. sin r; r = 45

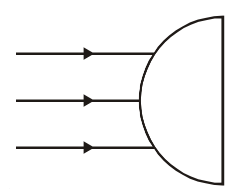
δ_4 = r – i = 45 – 30 = 15 $\circlearrowleft$

so $\delta_{net} = \delta_1 + \delta_2 + \delta_3 + \delta_4$ = 270

21. $n_1 > n_2$ But at here i > r not possible



22. (i) For (i) $\frac{u_2}{v_1} - \frac{1}{\infty} = \frac{u_2 - 1}{6}$



$v_1 = \frac{3}{2}$ 6 2 = 18 cm

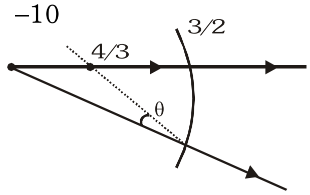
For plane surface : $I_1 \rightarrow$ object

$d_{apperent} = \frac{n_2}{n_1}$ $d_{actual} = \frac{n_2}{n_1}$ (18–R)

$= \frac{1}{3/2}$ $(18-6)$ = 8 cm

23. $\dfrac{3/2}{v} - \dfrac{4/3}{u} = \dfrac{3/2 - 4/3}{-10}$

$\dfrac{3}{2v} = \dfrac{1}{-60} - \dfrac{4}{3u}$



So $v = \dfrac{2}{3} \times -(ve)$

So for any value of u, v is (–ve). So image is virtual.

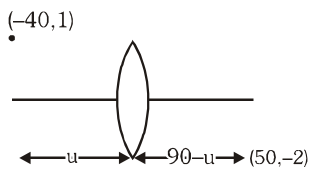
24. As given $u_2 > u_1 > u_3$

$\because u_2 > u_1 \Rightarrow$ lens act as a concave lens for upper half portion of lens & $u_3 < u_1$

$\Rightarrow$ Lower half portion act as a convex lens.

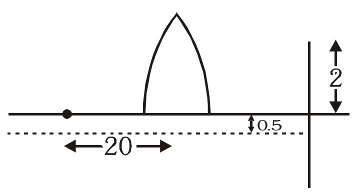
25. $m = \dfrac{-2}{1} = -2 = \dfrac{v}{u} \Rightarrow |v| = |2u|$

$(90-u) = 2u \Rightarrow u = 30$cm



Hence lens location = –40 + 30 = – 10 cm

26. u = – 30 cm; h_0 = + 0.5 cm; f = 20 cm



$v = \dfrac{uf}{u+f} = \dfrac{-30 \times 20}{-30+20} = 60$ cm

$m = \dfrac{v}{u} = \dfrac{+60}{-30} = -2$

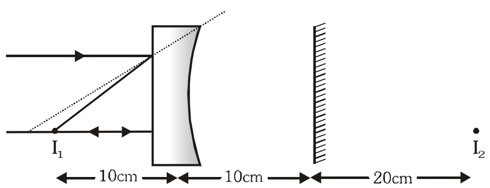
$h_i = mh_o = -2 \quad 0.5 = -$ 1cm from P axis

1+ 0.5 = 1.5 cm below xy.

27. From figure 1st behave as diverging and 2nd behave as converging

In 1st incident ray appear to pass at 5 cm distance point and after refraction it becomes parallel to principle axis i.e. the focal length f = – 5 cm and for second f = 5 cm

28.



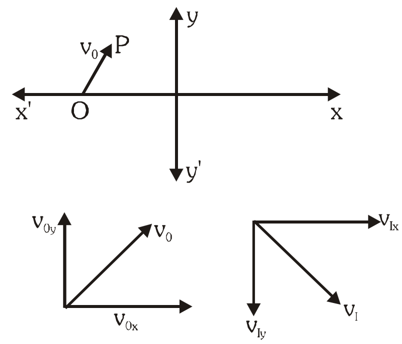
Final image (by lens) object I_2

u = – 30, f = – 10

$v = \dfrac{uf}{u+f} = \dfrac{300}{-30-10} = \dfrac{300}{-40} = -7.5$

$\therefore$ 2.5 cm in front of mirror.

29.



(i) $v_{I_x} = m^2 v_{0x}$ (same direction)

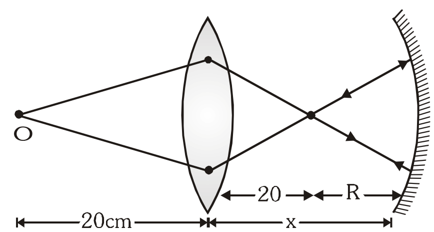
(ii) $\because$ m = –ve, $m = \dfrac{v}{u}$,

(v = –ve, u = +ve) then $v_{Iy} = -ve$

30. $\dfrac{1}{v} - \dfrac{1}{-20} = \dfrac{1}{10} \Rightarrow v = 20$ cm

$\therefore$ R for mirror = 2f = 2 60 = 120 cm

$\therefore$ x =20 + R = 20 + 120 = 140 cm



31. Concave lens behaves as divergent lens and diverges the beam of light as passes from lens. When the screen is brought close towards lens it receive more number of rays hence light intensity increases.

33. $Am = A_1; \; A\left(\dfrac{1}{m}\right) = A_2$

$\Rightarrow A^2 = A_1A_2 \Rightarrow A = \sqrt{A_1A_2}$

34. $X = u_2 - u_1 = 24$

$m = \dfrac{v_1}{u_1} = 3 \Rightarrow v_1 = 3u_1$ or $u_2 = 3u_1$

$3u_1 - u_1 = 24 \Rightarrow u_1 = 12$ cm

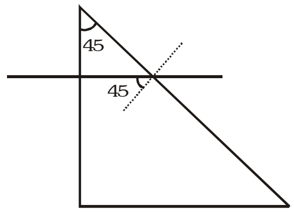
Distance between object & screen is

$D = v_1 + u_1 = 3u_1 + u_1 = 4 \quad 12 = 48$ cm

Hence $f = \dfrac{D^2 - X^2}{4D} = \dfrac{(48)^2 - (24)^2}{4 \times 48}$

$$= \frac{(48+24)(48-24)}{4\times 48} = \frac{72\times 24}{4\times(48)^2} = 9 \text{ cm}$$

35. $\mu_{red} \sin i = 1 \sin r$



$1.39 \quad \frac{1}{1.414} = \sin r$

$\sin r < 1 \Rightarrow$ possible
(refraction is possible)

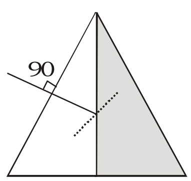
$\mu_{green} \quad \sin i \Rightarrow 1.44 \quad \frac{1}{1.41} > 1 \Rightarrow$ not possible

Same for blue $1.47 \quad \frac{1}{1.41} > 1$

$\Rightarrow$ It also not possible.

36. $\mu \sin A = 1 \quad \sin 90 \quad \mu = \frac{1}{\sin A}$

$\& \ \mu \sin \frac{A}{2} = 1 \quad \sin e = \frac{1}{\sin A} \quad \frac{A}{2} = \sin e$



$$\sin e = \frac{\sin\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} \Rightarrow C = \sin^{-1}\left[\frac{1}{2}\sec\frac{A}{2}\right]$$

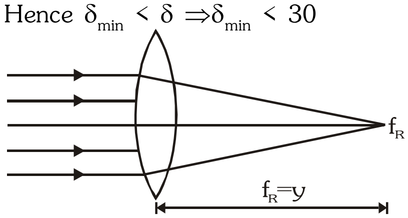
37. $\delta_{min} = 2i - A;\ 60 = 2 \quad 60 - Å;\ A = 60$

$$\mu = \frac{\sin\left(\frac{A+\delta_{min.}}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{60+60}{2}\right)}{\sin\frac{60}{2}}$$

$$\mu = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

38. $i = 50$, equilateral prism $A = 60$; $e = 40$
Total angle of deviation
$\delta = i + e - A = 50 + 40 - 60;\ \delta = 30$
Hence $\delta_{min} < \delta \Rightarrow \delta_{min} < 30$

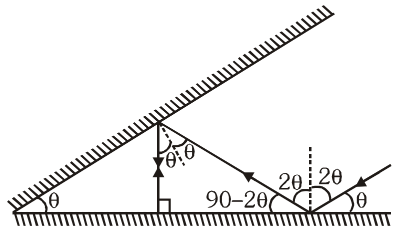
39.



$f \propto \lambda$; $\lambda_R > \lambda_g > \lambda_B$ (VIBGYOR)
$y > z > x$.

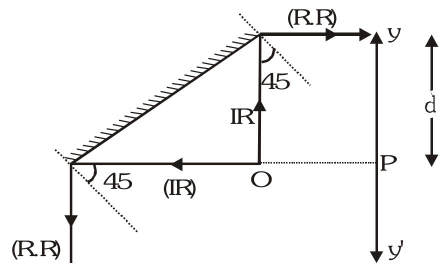
EXERCISE -II

1.



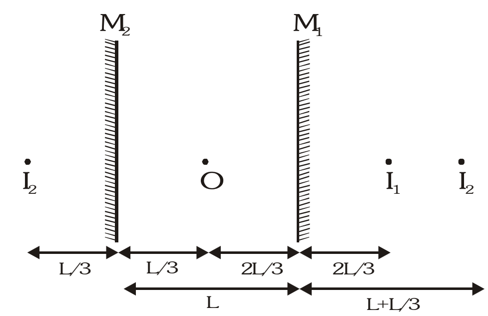
$\Delta OAB \Rightarrow \theta + \theta + 90 + \theta = 180$; $\Rightarrow \theta = 30$

2.



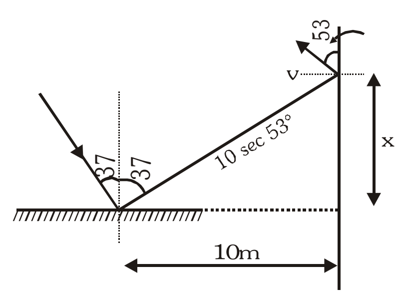
Field of view of image is decided by the reflected rays. As we can see that the reflected rays from the extreme end points of mirror at a height 'd' from P and below P all points on PY'. Hence (C).

3.



Distance between any to images must be 2nL/3

4.



As we know the $\omega_{reflected\ ray} = 2\ \omega_{mirror}$

$\omega_R = 2 \quad 2\pi \quad \frac{9}{\pi} = 18 \quad 2 = 36 \ \therefore\ v_S \cos 37 = v$

$v_S = v \sec 53 = R\omega_R \sec 53 = 10\ (\sec 53)^2 \quad 18$

$= 10 \left(\frac{5}{3}\right)^2 \quad 18 \quad 2 = 1000$ m/s

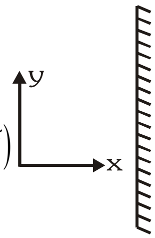
5. $v_I = 2v_M - v_0$

$= 2 \quad 0 - \left(5\tilde{i} + (6+2t)\tilde{j}\right)$

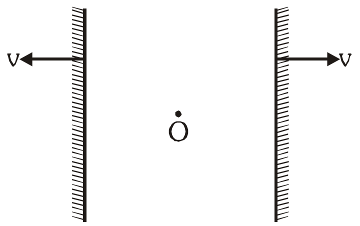
$v_I = -5\tilde{i} + 10\tilde{j}$; $v_{I0} = v_I - v_0$

hence $v_{Io} = -5\tilde{i} + 10\tilde{j} - \left(5\tilde{i} + 10\tilde{j}\right)$

[at t = 2 sec] = $-10\tilde{i}$



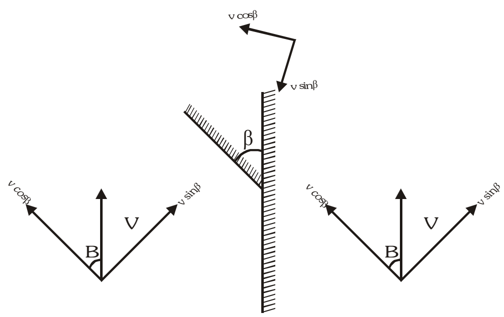
6.



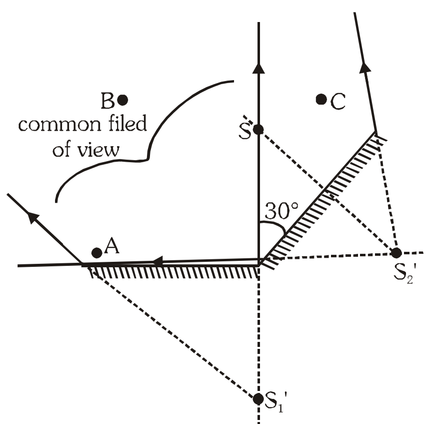
$v_{IG} = 2v_{MG} - v_{OG}$

$v_{IG_1} = 2v$, $v_{IG_2} = 4V$ .... $v_{IGn} = 2nv$

7.



8. Hence we can see from the images velocity in mirror 1 & II[nd] $v_{I2} = 2\,v\sin\beta$



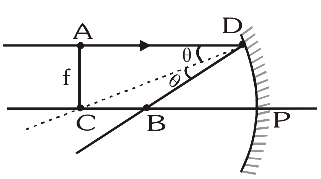
9. If the angle between mirrors is θ then the deviation from two plane mirrors will be = $2\pi - 2\theta$

$300 = 2\pi - 2\theta \Rightarrow \theta = 30$

Hence $\dfrac{360}{30} = 12$

Number of images = m–1 = 12 –1 = 11

10.



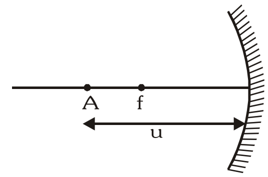
In ΔACD : $\sin\theta = \dfrac{f}{2f} \Rightarrow \theta = 30°$

$PB = R - \dfrac{R}{2}\sec\theta = R - \dfrac{R}{\sqrt{3}}$

$PC = R \Rightarrow CB = PC - PB = R - R + \dfrac{R}{\sqrt{3}}$

$= \dfrac{R}{\sqrt{3}} = \dfrac{2f}{\sqrt{3}} \quad \therefore \quad \dfrac{CB}{f} = \dfrac{2}{\sqrt{3}}$

11.



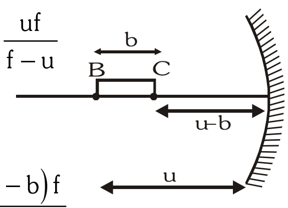
$u = -u$, $f = -f$ $\quad v_A = \dfrac{uf}{f-u}$

Image of last end of rod $v_B = -f$

Length = $v_B - v_A = -f - \dfrac{uf}{f-u} = \dfrac{-f^2 + uf - uf}{f-u}$

Length = $\dfrac{f^2}{u-f}$. Hence [D]

12. $u = -u$, $f = -f$ $\quad v_B = \dfrac{uf}{f-u}$

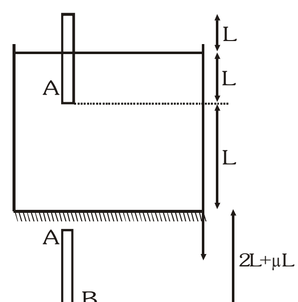


$v_C = \dfrac{(u-b)f}{f-(u-b)}$

$v_B - v_C = \dfrac{uf}{f-u} - \dfrac{(u-b)f}{f-u+b}$

$= \dfrac{ub}{f-u} - \dfrac{uf}{f-u+b} + \dfrac{bf}{f-u+b} = b\left(\dfrac{f}{u-f}\right)^2$

13. When observer in rarer medium and object is in denser medium then the length of immersed portion will be shorter (apparent)



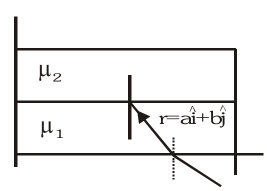
Hence length of image of rod in water

= 2L + μL –L = L + μL

Hence length of image of rod seen by observer in air is = $\dfrac{L + \mu L}{\mu} = \dfrac{L}{\mu} + L$. Hence [B]

14. For parallel composite slab

$\mu \sin\theta$ = constant



$\dfrac{\mu_1 \times a}{\sqrt{a^2+b^2}} = \dfrac{\mu_2 \times c}{\sqrt{c^2+d^2}}$

$\because |\tilde{r}_1| = |\tilde{r}_2| = 1$

$\therefore \left(\sqrt{a^2+b^2} = \sqrt{c^2+d^2} = 1\right) \quad \therefore \mu_1 a = \mu_2 c$

15. For parallel composite slab usinθ = constant

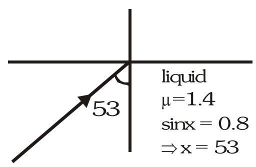
$\mu_0 \quad \sin i = \mu(n) \quad \sin 90$

$\mu_0 \sin 30 \ = \mu_0 + \dfrac{\mu_0}{\mu n - 18}$

$\dfrac{1}{2} = 1 + \dfrac{1}{4n-18} \Rightarrow \dfrac{1}{18-4n} = \dfrac{1}{2} \Rightarrow n = 4$

$\Rightarrow$ upper layer of n=3

16. From Snell's law $\quad 1.4 \quad \sin x = 1 \sin r$

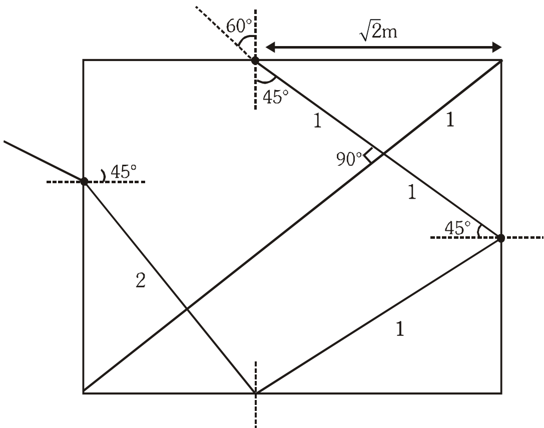


$1.4 \quad 0.8 = \sin r > 1$ (which is not possible)

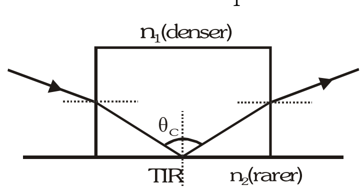
$\Rightarrow$ Ray will not refract but it has total internal reflection. Hence (C).

17. Critical angle $\dfrac{3}{2}$ sin c = 1; sin c = $\dfrac{2}{3}$

Geometrical path length travelled by the light in the slab will be =2 +1 + 2 +1 =6m



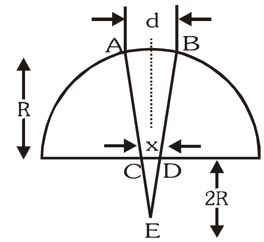
18. $n_1 \sin\theta_C \ = n_2 \Rightarrow \sin\theta_C = \dfrac{n_2}{n_1}$



$1 \quad \sin\theta = n_1 \sin(90-\theta_C) = n_1\cos\theta_C = n_1\sqrt{1-\dfrac{n_2^2}{n_1^2}}$

$\theta = \sin^{-1}\left(\sqrt{n_1^2-n_2^2}\right)$; $\sin\theta < \sqrt{n_1^2-n_2^2}$

19. $\mu_1 = 1;\ \mu_2 = 3/2$



$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{\mu} = \dfrac{\mu_2-\mu_1}{R}$

$\dfrac{3}{2v} - \dfrac{1}{-\infty} = \dfrac{3/2\ -1}{R} = \dfrac{1}{2R}$

v = 3R

ΔABE & ΔCDE are similar $\dfrac{d}{3R} = \dfrac{x}{2R}$

$x = \dfrac{2}{3}d$

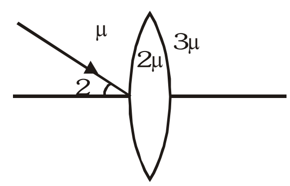
20. $m_1 = \dfrac{1}{u_1-f}\, m_2 = \dfrac{1}{u_2-f};\ \dfrac{m_1}{m_2} = -1$

$\Rightarrow u_2 - f = -(u_1 - f) \Rightarrow u_1 + u_2 = 2f$

$\Rightarrow f = \dfrac{u_1+u_2}{2}$

21. From Snell's law $\therefore\ \mu \sin 2 \ = 2\mu \sin r$

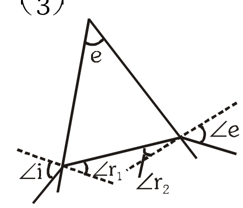
$\because\ \theta = 2$ (small angle)



$\sin 2 \ \simeq 2 \ ;\ \mu \quad 2 \ = 2\mu\ r$

$r = 1$ & $2\mu \quad 1 \ = 3\mu\ r';\ r' = \left(\dfrac{2}{3}\right)^0$

22. $2\alpha = (2\alpha + e) - \alpha$



$e = \alpha,\ r_1 + r_2 = \alpha$

$\sin 2\alpha = \mu \sin r_1$,

$\mu \sin(\alpha - r_1) = \sin\alpha$

$m(\sin\alpha.\cos r_1 - \cos\alpha \sin r) = \sin\alpha$

$$\mu\left[\sin\alpha\sqrt{1-\frac{\sin^2 2\alpha}{\mu^2}} - \frac{\cos\alpha\sin 2\alpha}{\mu}\right] = \sin\alpha$$

$$\sin\alpha\sqrt{\mu^2-\sin^2 2\alpha} - \cos\alpha\sin 2\alpha = \sin\alpha$$

$$\sqrt{\mu^2-\sin^2 2\alpha} - 2\cos^2\alpha = 1$$

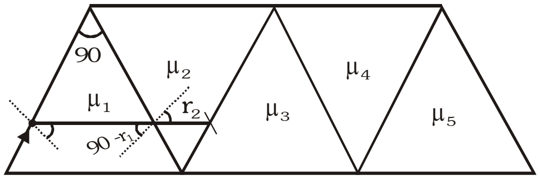
$\mu^2 - \sin^2 2\alpha = 1 + 4\cos^4\alpha + 4\cos^2\alpha$

$\mu^2 = 1 + 4\cos^4\alpha + 4\cos^2\alpha + 4\sin^2\alpha\cos^2\alpha$

$\mu^2 = 1 + 4\cos^2\alpha\,(\sin^2\alpha + \cos^2\alpha)$

$$\cos\alpha = \sqrt{\frac{\mu^2-1}{8}}$$

23.



$1 \sin 90 = \mu_1 \sin r_1$; $\mu_2 \sin r_2 = \mu_1 \cos r_1$

$\mu_2 \cos r_2 = \mu_3 \sin r_3$

$\mu_4 \sin r_4 = \mu_3 \cos r_3$; $\mu_4 \cos r_4 = \mu_5 \sin r_5$

$1. \sin 90 = \mu_5 \cos r_5$

Square and add $1 + \mu_2^2 + \mu_4^2 = \mu_1^2 + \mu_3^2 + \mu_5^2$

24. $\delta_{min} = 38$; $\delta = 44$; $i = 42 \Rightarrow e = 62$

$\delta = i + e - A$; $44 = 42 + 62 - A \Rightarrow A = 60$

$\therefore \delta_{min} = 2i - A$; $38 = 2i - 60 \Rightarrow i = 49$

25. $\delta_2 < \delta_1$

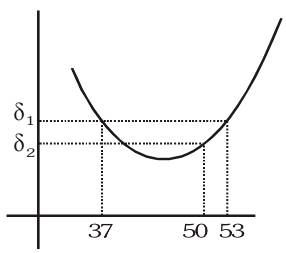
$\delta_1 = i + e - A$

$= 53 + 37 - A = 90 - A$

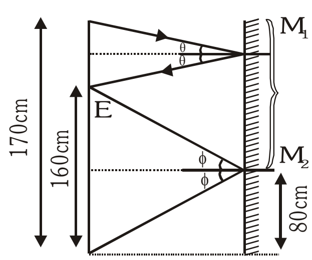
$\delta_2 < 90 - Å$

$50 + e - A < 90 - A$

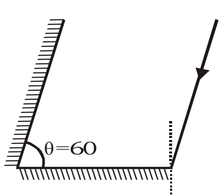
$e < 40$



26. Required length of mirror = M_1M_2 So the mirror should be the length of half of the size of man = $\frac{170}{2}$ = 85 cm & it is placed such that its lower edge at a half the height of eye level height.



27. If the angle between mirrors be $-\theta$ then the angle of deviation after two reflections is $\delta = 2\pi - 2\theta$

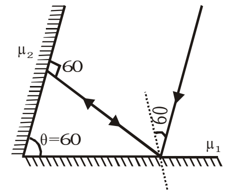


$240 = 2 \quad 180 - 2\theta$

$2\theta = 360 - 240$

$2\theta = 120 \quad (\theta = 60)$ Hence [A].

$\therefore m = \frac{360°}{60} = 6$ (even)

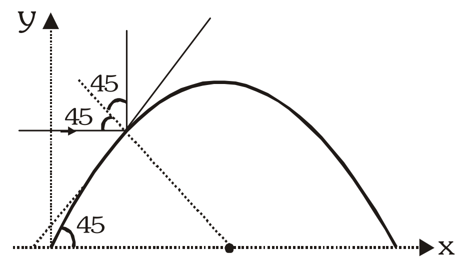


Number of images = n = m–1 = 6–1 =5

[Hence (B) & (C).

Reflected ray from 'M_1' strikes $\perp^r$ on mirror 'M_2'. Hence retrace its path. Hence [D].

28.



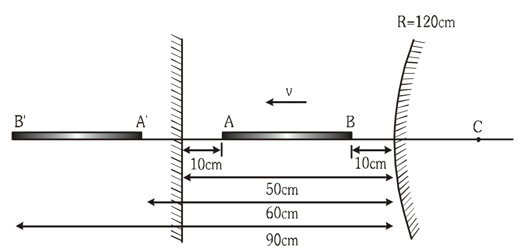
$y = \frac{2}{\pi} \sin \pi x$; $\frac{dy}{dx} = \frac{2}{\pi} \cos\pi \, x\pi \Rightarrow \frac{dy}{dx} = \pm 1$

$\tan 45 = 1 = 2\cos\pi x$

$\cos \pi x = \frac{1}{2} \Rightarrow \pi x = \frac{n\pi}{3} \quad \therefore x = \frac{1}{3}$ and $\frac{2}{3}$

$\therefore y = \frac{2}{\pi} \frac{\sqrt{3}}{2} \left(\because \sin \pi \times \frac{1}{3} = \frac{\sqrt{3}}{2} \right) = \frac{\sqrt{3}}{\pi}$

29.



$u_1 = -60$ & $u_2 = -90$

$-\frac{1}{60} + \frac{1}{v_1} = \frac{1}{60} \Rightarrow v_1 = 30$

$-\frac{1}{90} + \frac{1}{v_2} = \frac{1}{60} \Rightarrow v_2 = 36$

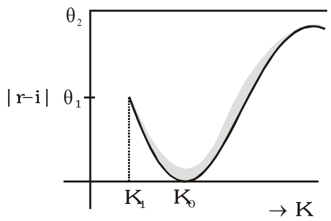
$v_2 - v_1 = 36 - 30 = 6\text{cm}$

$m = \frac{v_2 - v_1}{v_2 - v_1} = \frac{6}{30} = \frac{1}{5}$

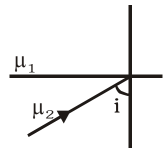
for mirror $v_I = -m^2 v_0$

convex mirror makes virtual image.

30.



$\delta = 0$ at $K_0 \Rightarrow K_0 = \dfrac{\mu_1}{\mu_2} = 1$



$\theta_2 = ?$; $K = \dfrac{\mu_1}{\mu_2}$; μ_1 :denser; K : increases

$r=0, \quad \delta = \dfrac{\pi}{3} - 0, \quad \Rightarrow \quad \theta_2 = \dfrac{\pi}{3}$

$\theta_1 = ?$ $\qquad K = \dfrac{\mu_1}{\mu_2} \downarrow$

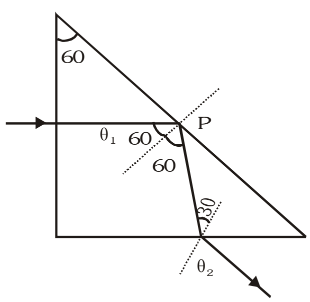
μ_2 :denser; μ_1 : rarer; K : decreases

$\mu_2 \sin i = \mu_1 \sin\dfrac{\pi}{2}$

$\sin\dfrac{\pi}{3} = \dfrac{\mu_1}{\mu_2} = \dfrac{\sqrt{3}}{2} = K_1$

At k_1, $r = \dfrac{\pi}{2}$; so, $\theta_1 = \dfrac{\pi}{2} - \dfrac{\pi}{3} = \dfrac{\pi}{6}$

31. $\theta_1 = 60$



$\dfrac{5}{3}\sin 30 = \dfrac{4}{3}\sin\theta_2$

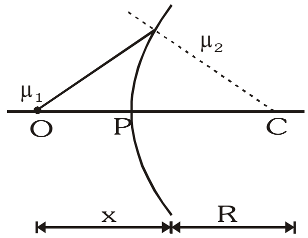
$\sin\theta_2 = \dfrac{5}{8}$

$\theta_2 = \sin^{-1}\dfrac{5}{8}$

Also (C), TIR is at P ceases if

$\dfrac{5}{3}\sin 60 = \mu \sin 90$; $\dfrac{5\times\sqrt{3}}{3\times 2} = \mu$

32.



Real image can't be formed always virtual.

33. $v < 0$

$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$; $\dfrac{\mu_2}{v} = \left(\dfrac{\mu_2 - \mu_1}{R} + \dfrac{\mu_1}{u}\right) < 0$

(A) Virtual image is formed for any position of 0 if $\mu_2 < \mu_1$.

(B) Virtual image can be formed if $x > R$ & $\mu_2 < \mu_1$.

34. $v > 0$

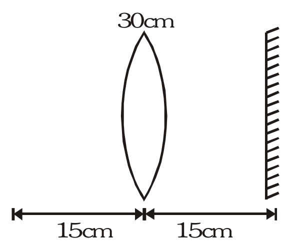
$\therefore \dfrac{\mu_2}{v} = \dfrac{\mu_2 - \mu_1}{-R} + \dfrac{\mu_1}{u} > 0$ and if $\mu_2 < \mu_1$

$\dfrac{\mu_1 - \mu_2}{R} - \dfrac{\mu_1}{x} > 0$; $\dfrac{\mu_1 - \mu_2}{R} > \dfrac{u_1}{x}$

$\therefore x > \dfrac{\mu_1 R}{\mu_1 - \mu_2}$ $\therefore$ (B) if $\mu_2 < \mu_1$,

Then virtual image is formed of $x < \dfrac{\mu_1 R}{\mu_1 - \mu_2}$

35.



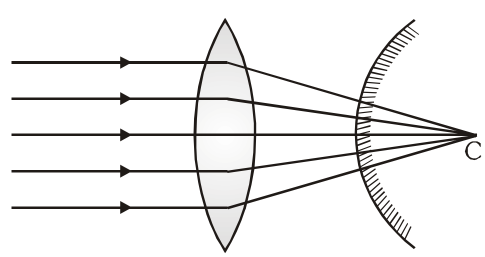
$\dfrac{1}{v} = \dfrac{1}{30} - \dfrac{1}{15} \Rightarrow v = -30$ cm

$\dfrac{1}{v} - \dfrac{1}{-60} = \dfrac{1}{30} \Rightarrow v = 60$ cm

(B) Final image is at 60 cm from lens towards left of it.

(C) final image is real

36.



$d = |f_1| - 2|f_2|$

38. $m_1 m_2 = 1$ & $d > 4f$ (uv method)

40. Height of object

$= \sqrt{\text{H of image 1} \times \text{H of Image 2}} = \sqrt{I_1 I_2} = \sqrt{9 \times 4}$

Height of object = 6 cm

$m_1 = \frac{I_1}{O} = \frac{v_1}{u} \Rightarrow \frac{9}{6} = \frac{v_1}{u_1}$

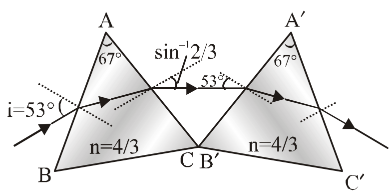
$u_2 \equiv v_1 = \frac{3}{2} v_1 \therefore X = \frac{3}{2} u_1 - u_1 = \frac{u_1}{2} = X$

$D = 90 = u_2 + u_1 = \frac{3}{2} u_1 + u_1$

$u_1 = 36 \Rightarrow X = \frac{36}{2} = 18$

$\therefore$ Distance between two position of lens = X = 18

Focal length $f = \frac{D^2 - V^2}{4D} = \frac{90^2 - 18^2}{4 \times 90} = 21.6$ cm

41. 

43. $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ (condition for achromatism)

$\omega_1 : \omega_2 = 2 : 1$ (Given)

$\Rightarrow f_1 : f_2 = -2 : 1$ ......(i) (for fullfill condition)

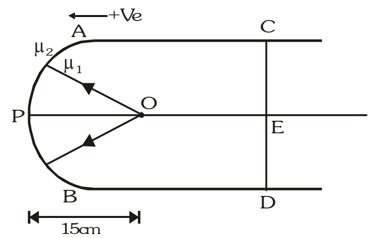
$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{-10} \Rightarrow \frac{1}{f_1}\left(1 + \frac{f_1}{f_2}\right) = \frac{1}{-10}$ ...(ii)

by (i) & (ii) $f_1 = 10$cm & $f_2 = -5$cm

EXERCISE -III

Fill in the blanks

2. Rays starting from O will suffer single refraction from spherical surface APB. Therefore, applying



$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$; $\frac{1.0}{v} - \frac{2.0}{-15} = \frac{1.0 - 2.0}{-10}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{7.5}$ or v = – 30 cm

Therefore, image of O will be formed at 30 cm to the right of P. Note that image will be virtual. There will be no effect of CED.

3. Rays falls normally on the face AB. Therefore it will pass undeviated through AB.

$\therefore r_2 = 90 - 60 = 30$

$\mu = \sqrt{2} = \frac{\sin i_2}{\sin r_2} \Rightarrow i_2 = 45$

Deviation = $i_2 - r_2 = 45 - 30 = 15$

(Deviation at face AC only)

4. $P = 2P_L + P_M$

II case : $\frac{1}{f} = \frac{2}{f_\ell} + \frac{1}{f_m} = \frac{2(\mu - 1)}{R} + \frac{2}{R}$

$\Rightarrow \frac{1}{f} = \frac{2\mu}{R} \Rightarrow F = \frac{R}{2\mu} = 7$ ...(i)

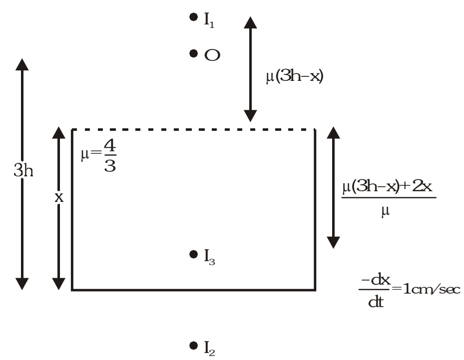
I case : $P = 2P_L + P_M \Rightarrow \frac{1}{F} = \frac{2}{f_L} + \frac{1}{f_M}$

$\frac{1}{f} = \frac{2(\mu - 1)}{R} + \frac{1}{\infty} \Rightarrow F = \frac{R}{2(\mu - 1)} = 20$...(ii)

By Dividing (i) and (ii)

$\frac{R}{2\mu} \times \frac{2(\mu - 1)}{R} = \frac{7}{20} \Rightarrow 20\mu - 20 = 7\mu$

$\Rightarrow 13\mu = 20 \Rightarrow \mu = \frac{20}{13}$

5. 

Apparent distance of image by mirror from bird

$$= \frac{\mu(3h-x)+2x}{\mu} + (3h-x)$$

$$= \frac{2\mu(3h-x)}{\mu} + \frac{2x}{\mu} \Rightarrow 2\,(3h-x) + \frac{2x}{\mu}$$

$\therefore$ Velocity of image

$$= 2\left(0-\frac{dx}{dt}\right) + \frac{2}{\mu}\frac{dx}{dt} = 2\,[0-(-1)] + \frac{2}{\mu}\quad(-1)$$

$$= 2 \quad 1 + \frac{3}{2}\,(-1) = \frac{1}{2} \text{ cm/sec.}$$

Match the column

1. **For (A)** : All reflected rays pass through the two mirror upto infinite reflections.
   For (B) : After IInd reflection ray will not pass through any mirror
   For (C) : Only rays from object will be reflected by the mirrors and no reflected rays will be further reflected.
   For (D) : Similarly as C

2. $m = \frac{f}{f-u}$

   (A) Convex mirror u = –ve, f = +ve
   $m = \frac{f}{f-(f)} = \frac{1}{2}$

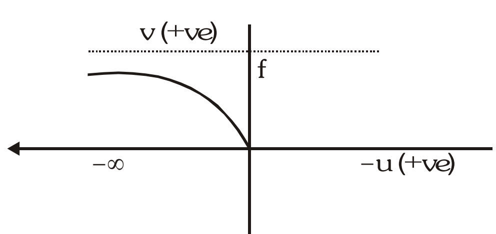
   (B) Concave $\rightarrow$ $m = \frac{-f}{-f-(-f)} = \infty$

   (C) Concave mirror u = – R (2f)

   $$m = \frac{-f}{-f-(-2f)} = \frac{-f}{f} = -1$$

   (D) Convex mirror u = – 2f

   $$m = \frac{f}{f-(-2f)} = \frac{f}{3f} = \frac{1}{3}$$

3. 

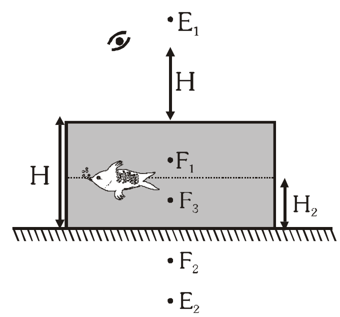
(A) Convex mirror $\Rightarrow$ Real object
$\rightarrow$ virtual image between focus pole

(B) We can see from (v–u) graph, the image distance v between pole & focus for all position of real objects.

(C) $v_{Ix} = -m^2 v_{o_x}$; $v_{Ix} = \left(\frac{f}{f-x}\right)^2 u$

$v_{Ix} = \left(\frac{R}{R-2x}\right)^2 u$; $x \rightarrow$ object distance

(D) Mirror is convex

4. 

E_1 at $\left(\frac{H}{2} + \mu H\right) \Rightarrow H\left(\mu + \frac{H}{2}\right)$

E_2 at $\left(\mu H + H + \frac{H}{2}\right) \Rightarrow H\left(\mu + \frac{3}{2}\right)$

F_1 at $\left(\frac{H}{2\mu} + H\right) \Rightarrow H\left(\frac{1}{2\mu} + 1\right)$

F_2 at $\left(\frac{3H}{2\mu} + H\right) = H\left(\frac{1+3}{2\mu}\right)$

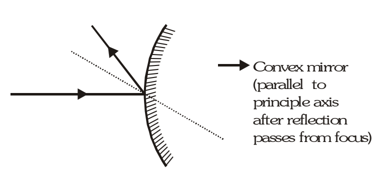
5. $m = \frac{v-f}{f} = \frac{v}{f} - 1$ ($\therefore$ y = mx + c)

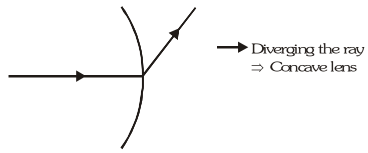
$m = \text{slope} = \frac{1}{f}$ (magnitude of slope of line)

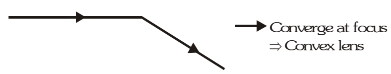
Intercept on y axis c = –1 (unity)

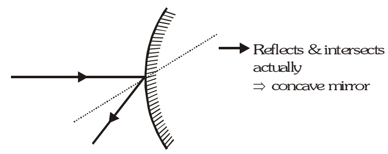
Intercept on x-axis $\Rightarrow$ m = 0

$\Rightarrow 0 = \frac{v}{f} - 1 \Rightarrow v = f$ (focal length)

7. (A) 

(B) 

(C) 

(D) 

8. (A) When object is at focus, image is also real from equi biconvex lens.

$$\because \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For equi biconvex f= $\dfrac{R}{2(\mu-1)}$ ...(i) As $\mu\uparrow$ f $\downarrow$

Hence final image position v decreases

hence m = $\dfrac{v}{u}$ also decrease

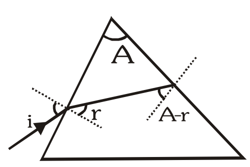
(B) From (i) R $\uparrow$ $\Rightarrow$ f $\uparrow$
Radius of curvature increases i.e. now the object is not placed at focus and it is case when object is between focus and optical centre. Hence image formation is virtual.

(C) Glass slab provides shift of the object in the incident ray direction. So for lens object position becomes between focus and optical centre.

(D) When medium is changed then relative μ.

9. $\lambda_r > \lambda_y > \lambda_g > \lambda_b \Rightarrow \mu_r < \mu_y < \mu_g < \mu_b$
$\Rightarrow C_r > C_y > C_g > C_b$ (C : Critical angle)

10. Critical angle



2 sin θ_C = 1. sin 90

$\theta_C = \sin^{-1}\dfrac{1}{2} = \dfrac{\pi}{6}$

1. sin 90 = 2 sin r

sin r = $\dfrac{1}{2}$ $\Rightarrow$ r = $\dfrac{\pi}{6}$

$\therefore$ 0 < A – r < θ_C for all rays refraction

(A) A = 15 $\Rightarrow$ A – R = – ve hence (a)
(B) A = 45 $\Rightarrow$ 45– 30 = 15 < 30 for same rays
45 – 0 = 45 > 30 for same rays
(C) A = 70 : A–r 70 – 30 = 40 > θ_C(30)
70 - 0 = 70 > θ_C All rays reflected back
(D) A = 50 : A–r = 50 – 30
= 20 < θ_C(30) Refrection
50 - 0 = 50 > θ_C (Reflection)

Comprehension # 2

1. In this case x, H and θ remains same so required length of mirror = $\dfrac{H(H+x)\sin\theta}{H+2x}$

$$= \frac{1(1+1)\sin 45^0}{1+2\times 1} = \frac{\sqrt{2}}{3}\text{ m}$$

2. In this case only x will change so equired length of mirror = $\dfrac{1(1+2)\sin 45^0}{1+2\times 2} = \dfrac{3\times\frac{1}{\sqrt{2}}}{5} = \dfrac{3}{5\sqrt{2}}$ m

3. In this case x = 1–1 = 0 so length of mirror = $\dfrac{1(1+0)\sin 45^0}{1+2\times 0} = \dfrac{1}{\sqrt{2}}$

Comprehension # 3

1. On squeezing the lens, the focal length of lense decreases.

2. As v decreases, the magnification is also decreases.

3. As turnip away from the lens, distance increases i.e. u increases hence for same v, f has to decreases so we should decrease the squeeze of lense.

Comprehension # 4

1. u = –25, f = f, v = 2.5

$$\Rightarrow 2.5 = \frac{-25\times f}{-25+f} \Rightarrow f = \frac{25}{11}$$

2. Maximum focal length = distance of retina from eye – lens = 2.5 cm

3. Normal person power of eye–lens

P = $\dfrac{100}{2.5}$ = 40 D; u = – 100 cm, v = 2.5

$$\therefore 2.5 = \frac{-100\times f}{-100+f} \Rightarrow f = \frac{100}{41}$$

$$\therefore \text{ Power} = \frac{100}{\frac{100}{41}} = 41\text{D}$$

$\therefore$ Required power of lens = 40 = 41 + P_2
P_2 = – 1D

4. Minimum focal length of eye lense from Q.1= $\frac{25}{11}$

$\therefore$ Power = $\frac{100}{25/11} = \frac{100\times 11}{25} = 44D$

And u = – 100 cm, v = 2.5, f= $\frac{100}{41}$,

Power = 41 D

$\therefore$Required power lens = 44D = 41 D + P_2

P_2 = 3D

5. For an average grown up person minimum distance of object should be around 25 cm So (D).

Comprehension # 5

1. The focal length varies with wave length.

2. The combination will be free from chromatic observation if dF = 0. Hence [A].

3. Condition of achromatism

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{\omega_1}{\omega_2} = \frac{-f_1}{f_2}$$

$$\frac{\omega}{\omega'} = \frac{\omega_0}{2\omega_0} = \frac{-f_1}{f_2} \Rightarrow f_2 = -2f_1. \text{ Hence [D]}$$

4. $\omega_1 = 0.02$, $\omega_2 = 0.04$ [Dispersive power]

$$\therefore \frac{\omega_1}{\omega_2} = -\frac{-f_1}{f_2} \Rightarrow \frac{0.02}{0.04} = \frac{-f_1}{f_2} \Rightarrow f_2 = -2f_1$$

$$\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

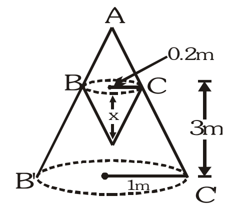
$$\frac{1}{40} = \frac{1}{20} - \frac{1}{40} = \frac{2-1}{40} = \frac{1}{40}$$

Hence f_1 = 20, f_2 = – 40. Hence [A]

5. This aberration related with lenses not for mirrors.

EXERCISE -IV (A)

1. ΔABC & $\Delta AB'C$



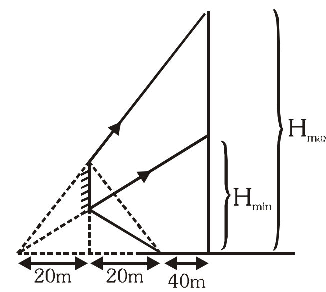
$$\frac{x}{0.20} = \frac{x+3}{1}$$

x = 0.2x + 0.6

$0.8x = 0.6 \Rightarrow x = \frac{3}{4}$ cm

x = 75 cm

2. $\frac{H_{min}}{80cm} = \frac{40cm}{20cm}$

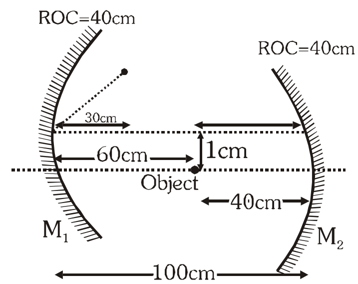


$\Rightarrow H_{min}$ = 160 cm

& $\frac{H_{min}}{80cm} = \frac{80cm}{20cm}$

$\Rightarrow H_{max}$ = 320 cm

3.



f= –20 cm; $\frac{1}{v} + \frac{1}{(-60)} = \frac{1}{(-20)}$

$$\frac{1}{v} = \frac{1}{60} - \frac{1}{20} = \frac{-40}{60\times 20} \Rightarrow v = -30$$

$$m = -\frac{v}{u} = \frac{(-30\times -1)}{(-60)} = \frac{-1}{2}$$

so image coordinates are (–30, 3/2)

At m_2 :

$$\frac{1}{v} + \frac{1}{(-70)} = \frac{1}{-20} \Rightarrow \frac{1}{v} = \frac{1}{70} - \frac{1}{20} = \frac{-50}{70\times 20}$$

$$v = \frac{-140}{5} = -28 \text{ cm}$$

So magnification = $\frac{-v}{u} = \frac{(-28)(-1)}{(-70)} = -\frac{2}{5}$

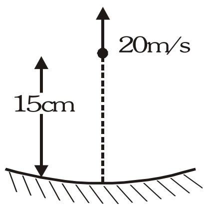
Object length = $\frac{3}{2}$cm

image length = $\left(\frac{3}{2}\right)\left(\frac{2}{5}\right) = \frac{3}{5}$cm below axis of M_2 : $y = \left(-\frac{3}{5}\right)$

x coordinate = (40–28) = 12

so coordinates = $\left(12, -\frac{3}{5}\right)$

4. Speed of ballon = 20m/s



$d = 20 \quad 4 - \frac{1}{2} \quad 10 \quad 16$

d = 0

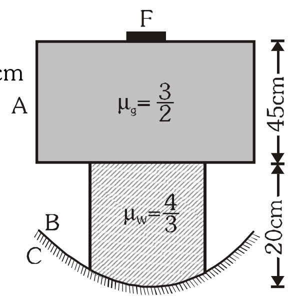
u =–15, R=–20, f=–10

$v = \frac{ub}{u-f} = \frac{150}{-5} = -30$

$m = -\frac{v}{u} = \frac{30}{-15} = -2,\ v_0 = -20$

$v_I = -\ m^2 v_0 = -4 \quad -\ 20 = 80$

5. $x = 45\left(\frac{4}{3}\right) \times \frac{2}{3} = 40\text{cm}$



So total distance = 60 cm

f = – 20 cm

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} + \frac{1}{(-60)} = \frac{1}{-20}$

$\Rightarrow \frac{1}{v} = \frac{1}{60} - \frac{1}{20} = \frac{-40}{60 \times 20} \Rightarrow v = -30$

So image after reflection from mirror will be at d = 30 cm. So 10 cm above 'A'
After refraction at A : It will be at

$\frac{10}{(4/3)}(3/2) = \frac{90}{8} = \frac{45}{4}\text{cm}$

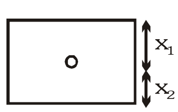
so distance from 'B' layer is

$\left(45 - \frac{45}{4}\right) = 45 \times \frac{3}{4}$

So final image after refraction at 'A' will be at :

$45 \times \frac{3}{4} \times \frac{2}{3} = \frac{45}{2}$ from 'A' surface

6. $\frac{x_1}{\mu} = 6 \ \therefore\ x_1 = 6 \quad \frac{3}{2}$

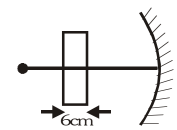


and $\frac{x_2}{\mu} = 4,\ x_2 = 4 \quad \frac{3}{2}$

$\therefore\ t = x_1 + x_2 = 6 + 9 = 15\text{cm}$

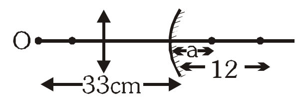
7. Shift due to glass slab

$= 6\left(1 - \frac{1}{3/2}\right) = 6 \quad \frac{1}{3} = 2\text{cm}$



t = thickness of glass slab; μ = 3/2
Reflected final image concides with the object when the effective distance of the object in equal to radius of curvature of mirror u = 40 + 2 = 42 cm.

8.



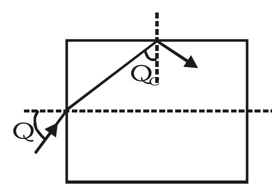
$x = t\left(1 - \frac{1}{\mu}\right) = 3\text{cm}$

For mirror :
Object– I_1 : u = –30, f=20

$v = \frac{-30 \times 20}{-30-20} = 12\ \text{cm} = 33+9 = 42\ \text{cm}$

from object

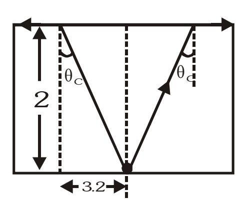
9.



$\sin\theta_c = \frac{\mu_R}{\mu_P} = \frac{3\sqrt{3}}{4} \times \frac{2}{3} = \frac{\sqrt{3}}{2} \Rightarrow Q_C = 60$

$\frac{3\sqrt{3}}{4}\sin\theta = \frac{3}{2}\sin 30 \Rightarrow \sin\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\frac{1}{\sqrt{3}}$

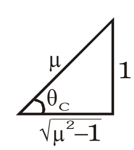
10. Let refrective index = μ



From snell's law $\mu \sin\theta_C = 1.\sin 90 = \sin\theta_C = \frac{1}{\mu}$

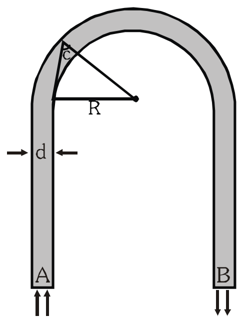
$\tan\theta_C = \frac{1}{\sqrt{\mu^2 - 1}} = \frac{3.2}{2}$

$\mu^2 - 1 = \left(\frac{10}{16}\right)^2;\ \mu = \frac{\sqrt{356}}{16} = 1.17$



11. $\sin c = \frac{R}{R+d} = \frac{1}{1.5} \Rightarrow \frac{R}{R+d} > \frac{1}{1.5}$

$$\Rightarrow \frac{1}{1+d/R} > \frac{1}{1.5} \Rightarrow 1.5 > 1+\frac{d}{R} \Rightarrow \frac{d}{R} < \frac{1}{2}$$

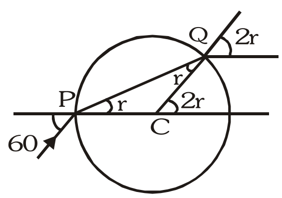


12. $v_B = 16$ cm/sec; $\mu = \frac{4}{3}$ v_f 4 cm/sec

$\vec{v}_{Bf} = \vec{v}_B - \vec{v}_f \Rightarrow -16 = \vec{v}_B - 4$; $\vec{v}_B = -12$cm/sec

$\frac{v_{app}}{n_{obse}} = \frac{v_{act}}{n_{object}} \Rightarrow \frac{12}{4} = \frac{v_{act}}{1}$; $v_{act} = 9$ cm/s

13.



From snell's law

1. sin 60 = μ sin r (at Pt 'P')

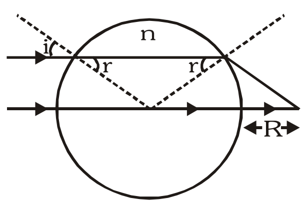
again μ sin r = 1. sin 2 r (at Pt 'Q')

$\therefore$ sin 2r = sin 60

r = 30

$\therefore$ 1. sin 60 = μ sin 30 $\Rightarrow \mu = \sqrt{3}$

14.



From (1) surface : $\frac{n}{v} - \frac{1}{-\infty} = \frac{n-1}{R}$; $v = \frac{nR}{n-1}$

$$\Rightarrow \frac{1}{R} - \frac{n}{v-2R} = \frac{1-n}{-R}, \ \frac{2}{R} = \frac{n}{R} + \frac{n}{v-2R}$$

$$\frac{2}{R} = \frac{n(v-2R+R)}{R(v-2R)} \Rightarrow 2v - 4R = nv - nR$$

On putting the value of v

$$2\frac{nR}{n-1} - 4R = \frac{n \times nR}{(n-1)} = -nR$$

$$2nR - 4nR + 4R = n^2R - n^2R + nR, \ n = \frac{4}{3}$$

15. From lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

we have $\frac{1}{0.3} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$

(Here R_1 = R and R_2 = –R) $\therefore$ R = 0.3

Now applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ at air glass surface, we get $\frac{3/2}{v_1} - \frac{1}{-(0.9)} = \frac{3/2-1}{0.3}$

$\therefore$ v_1 = 2.7m

i.e., first image I_1 will be formed at 2.7m from the lens. This will act as the virtual object for glass water surface.

Therefore, applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ at glass water surface,

we have $\frac{4/3}{v_2} - \frac{3/2}{2.7} = \frac{4/3 - 3/2}{-0.3}$ $\therefore$ v_2 = 1.2 m

i.e. second image I_2 is formed at 1.2m from the lens or 0.4 m from the plane mirror. This will act as a virtual object for mirror. Therefore, third real image I_3 will be formed at a distance of 0.4 m in front of the mirror after reflection from it. Now this image will work as a real object for water-glass interface.

Hence applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

we get $\frac{3/2}{v_4} - \frac{4/3}{-(0.8-0.4)} = \frac{3/2 - 4/3}{0.3}$

$\therefore$ v_4 = 0.54 m i.e. fourth image is formed to the right of the lens at a distance of 0.54 m from it. Now finally applying the same formula for glass-air surface.

$\frac{1}{v_5} - \frac{3/2}{-0.54} - \frac{1-3/2}{-0.3}$ $\therefore$ v_5 = –0.9 m

i.e., position of final image is 0.9 m relative to the lens (rightwards) or the image is formed 0.1 m behind the mirror.

16.

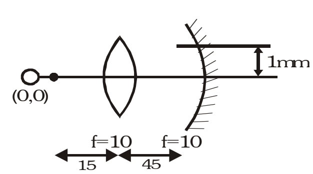


Image formed by the lens $v = \frac{-15 \times 10}{-15+10} = 30$

(Right from lens)

Reflection from mirror u =–15

$$v = \frac{-15 \times -10}{-15+10} = -30\ ;\ m = -\frac{(-30)}{-15} = -2$$

Co–ordinate of Image = (30,0.3cm)

Now again taking refraction from lens)

$$v = \frac{-15 \times 10}{-15+10} = 30 \text{ (left from lens)} \quad m = \frac{30}{-15} = -2$$

$\therefore$ Final co–ordinate of image = (–15cm, –6mm)

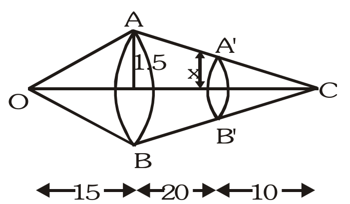
17. For real image $m = \dfrac{-f}{f+16}$

Virtual $m = \dfrac{f}{f+6}$

$\therefore \dfrac{-f}{f+16} = \dfrac{f}{f+6} \Rightarrow f + 16 = -f-6$

$2f = -22 \Rightarrow f = 11cm$

18.

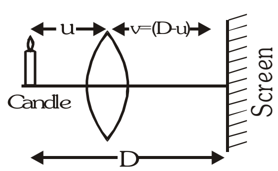


$$v = \frac{uf}{u+f} = \frac{-15 \times 10}{-15+10} = 30$$

ΔABC & $\Delta A'B'C$ are similar $\dfrac{x}{10} = \dfrac{1.5}{30} \Rightarrow x = \dfrac{1}{2}$

$\therefore$ Area $= \pi x^2 = \pi \quad \dfrac{1}{4} = \dfrac{\pi}{4} cm^2$

19. Given– Distance between two position of the lens for image is formed



$x = u_2 - u_1 = 0.8$ m (Given)

$D = u_1 + v_1$

$m_1 = \dfrac{v_1}{u_1} = 3 \Rightarrow v_1 = 3u_1$

$\dfrac{v_2}{u_2} = \dfrac{1}{3} \Rightarrow v_2 = \dfrac{u_2}{3}$

$x = v_1 - u_1 = 3u_1 - u_1 = 0.8$

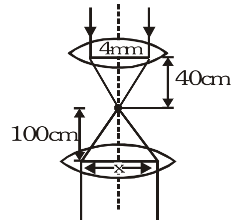
$u_1 = 0.4$

$\therefore v_1 = 3 \quad 0.4 = 1.2$

$D = 1.2 + 0.4 - 1.6$

$$f = \frac{D^2 - x^2}{4D} = \frac{(1.6)^2 - (0.8)^2}{4 \times 1.6} = 0.3m = 30cm$$

20.



From similar triangles $\dfrac{0.4}{40} = \dfrac{x}{100}$; x = 1 cm

22. A=60

$i + e - 60 = 23$ & $e = i + 23$

$2i + 23 - 60 = 23$

$i = 30$

$e = 53 \quad \Rightarrow \dfrac{1}{2} = \mu \sin r_1$

$$\Rightarrow \mu \sin(60 - r_1) = \sin 53° = \frac{4}{5}$$

$$\Rightarrow \mu\left(\frac{\sqrt{3}}{2} \cdot \cos r_1 - \frac{1}{2} \cdot \sin r_1\right) = \frac{4}{5}$$

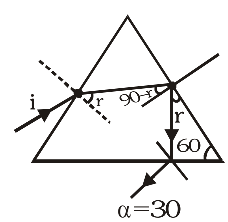
$$\Rightarrow \mu\left(\frac{\sqrt{3}}{2}\sqrt{1-\sin^2 r_1} - \frac{1}{2}\sin r_1\right) = \frac{4}{5}$$

$$\Rightarrow \mu\left(\frac{\sqrt{3}}{2}\sqrt{1-\frac{1}{4\mu^2}} - \frac{1}{2} \times \frac{1}{2\mu}\right) = \frac{4}{5}$$

$$\Rightarrow \sqrt{3}\sqrt{4\mu^2 - 1} - 1 = \frac{4}{5} \times 4 \Rightarrow 4\mu^2 = \left(\frac{21}{5}\right)^2 \frac{1}{3} + 1$$

$$\Rightarrow \mu = \sqrt{\frac{1}{4}\left\{\frac{441}{25 \times 3} + 1\right\}} = \sqrt{\frac{129}{75}} = \frac{\sqrt{43}}{5}$$

23. For critical angle



$\mu \sin(90-r) = 1. \sin 90$

$\cos r = \dfrac{1}{\mu}$

$90 - (120 - r) = (r - 30)$

From snell's law

$\mu \sin(r - 30) = 1 \sin 30$

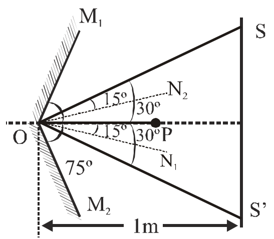
$\mu [\sin r \cos 30 - \cos r \sin 30] = \dfrac{1}{2}$

$$\mu\left[\frac{\sqrt{\mu^2-1}}{\mu} \times \frac{\sqrt{3}}{2} - \frac{1}{\mu} \times \frac{1}{2}\right] = \frac{1}{2}$$

$$\sqrt{u^2 - 1} \times \frac{\sqrt{3}}{2} = 1 \Rightarrow \mu^2 - 1 = \frac{4}{3} \Rightarrow \mu = \left(\frac{7}{3}\right)^{1/2}$$

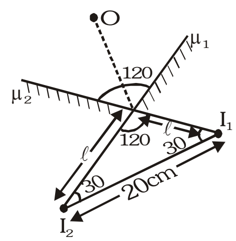
EXERCISE -IV (B)

1. $\frac{AO}{1m} = \tan 30°, \frac{OB}{1m} = \tan 30°$



$\Rightarrow$ AO + OB = AB = 2 tan30 = $\frac{2}{\sqrt{3}}$ m

2. From sine rule



$$\frac{20}{\sin 120°} = \frac{\ell}{\sin 30°} \Rightarrow \ell = 20 \times \frac{\sin 30°}{\cos 30°} = 20 \times \frac{1}{\sqrt{3}}$$

3. (a) $v_{Im} = -\left(\frac{v^2}{u^2}\right) v_{Jm}$

v_{Im} = –1cm/sec $\Rightarrow$ $1m/s = \left(-\frac{1}{100}\right) v_{Jm}$

v_{Jm} = 100cm/sec = 1m/sec, v_m = 20 m/sec

$\Rightarrow v_{Jm} = v_J - v_m$

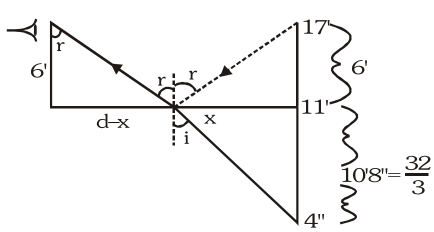
$\Rightarrow v_J = v_{Jm} + v_m$ = 1 + 20 = 21m/sec

(b) $m = \frac{v}{u} \Rightarrow \frac{dm}{dt} = \frac{d}{dt}\left(\frac{f}{u+f}\right)$

$$\Rightarrow \frac{dm}{dt} = \frac{f}{(u+f^2)} \times -\frac{du}{dt} \Rightarrow \frac{dm}{dt} = \left(\frac{f}{u+f}\right)^2 \frac{1}{f} \times \left(-\frac{du}{dt}\right)$$

$$\frac{dm}{dt} = \left(\frac{1}{10}\right)^2 \times \frac{1}{10} \times (-1) = -10^{-3}$$

4.



$$\frac{4}{3}\frac{x}{\sqrt{x^2 + \left(\frac{32}{3}\right)^2}} = \frac{d-x}{\sqrt{36 + (d-x)^2}}$$

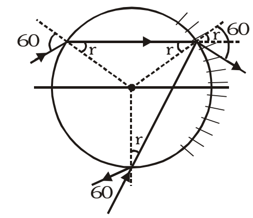
$$16x^2[36 + (d-x)^2] = 9(d-x)^2\left[x^2 + \frac{1024}{9}\right]$$

$$16 \quad 36x^2 + 16x^2(d-x)^2 = 9(d-x)^2x^2 + 1024(d-x)^2$$

$$16 \quad 36\frac{d^2}{4} + 16\frac{d^2}{4} \times \frac{d^2}{4} = 9\frac{d^2}{4} \times \frac{d^2}{4} + 1024\frac{d^2}{4}$$

$$d = \sqrt{\frac{4}{7} \times 448} = 16$$

5. As given in question

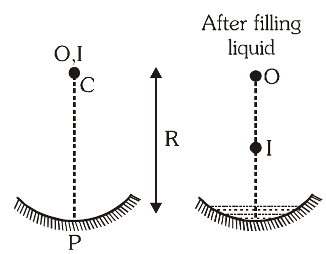


$$(60-r)+(60-r)=\frac{1}{3}(60-r)+(180-2r)+(60-r)$$

$$\Rightarrow 120-2r=100-\frac{4r}{3} \Rightarrow \frac{2r}{3}=20 \Rightarrow r=30$$

$\therefore$ 1 sin 60 = μ sin30 $\Rightarrow$ u = $\sqrt{3}$

6. **Case- I**



$$\frac{1}{f'} = \frac{1}{u} + \frac{1}{v} = \frac{1}{-60} - \frac{1}{30}$$

$$\Rightarrow f' = -\frac{30 \times 60}{90}$$

$\Rightarrow$ f' = –20cm

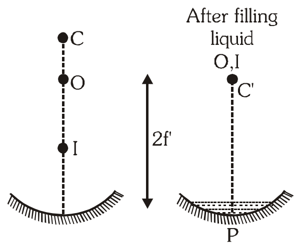
$$\frac{1}{f'} = \frac{1}{f_m} - \frac{2}{f_\ell}, \ \frac{1}{f_\ell} = (\mu - 1)\left(\frac{1}{60} - \frac{1}{\infty}\right)$$

$$\frac{1}{-20} = \frac{1}{-30} - \frac{2}{60}(\mu - 1)$$

$$\Rightarrow \frac{2(\mu-1)}{60} = \frac{1}{20} - \frac{1}{30}$$

$$\Rightarrow \mu - 1 = \frac{3}{2} - 1 \Rightarrow \mu = 1.5$$

Case-II



When there is liquid in the mirror u = 2 f' &u = f' when there is no liquid in the mirror

$$\frac{1}{-u}-\frac{1}{u-30}=-\frac{1}{30}y \Rightarrow \frac{u(u-30)}{2u-30}=30$$

$\Rightarrow u^2 - 10x + (30)^2 = 0$

$$u=\frac{90\pm\sqrt{(10)^2-4(30)^2}}{2}\ ;\ u=45\pm15\sqrt{5}$$

But there u is +ve so that

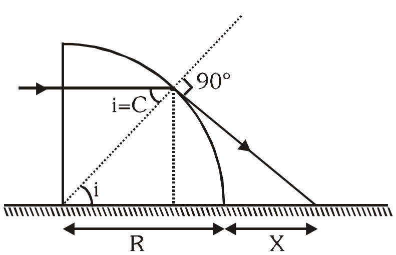
$$\Rightarrow u=45+15\sqrt{5}\ \ \&\ u=2f'\Rightarrow f'=\frac{45}{2}+\frac{15\sqrt{5}}{2}$$

$$\frac{1}{f'}=\frac{1}{f_M}-\frac{2}{f_\ell}-\frac{1}{\frac{15}{2}\left(3+\sqrt{5}\right)}=\frac{1}{-30}+\frac{2(\mu-1)}{60}$$

$$\Rightarrow(\mu-1)=1-\frac{4}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}$$

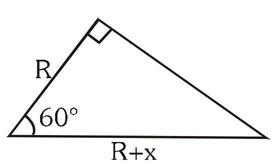
$$\Rightarrow \mu-1=\sqrt{5}-2\ \ \Rightarrow \mu-1=\sqrt{5}-1$$

7.



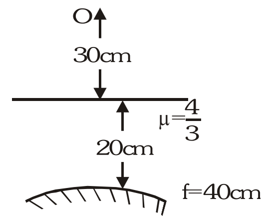
$$\text{SinC}=\frac{1}{\mu}=\frac{1}{2/\sqrt{3}}$$

$$\Rightarrow C=\text{Sin}^{-1}\frac{\sqrt{3}}{2}=60°$$



$$\cos 60°=\frac{R}{R+x}\Rightarrow x=R, x=5\text{cm}$$

8. Image formed by partial reflection from water surface at a distance 30cm below the air water surface.



$\because$ water surface behaves like a plane mirror secondly image formed by refreaction from water surface followed by reflection from mirror and again refrection out of the water surface.

$\therefore$ Distance of the object from mirror

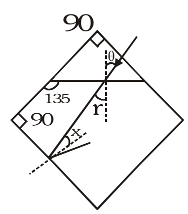
$$= u = 20+\frac{4}{3}\ \ 30 \Rightarrow u = -60$$

$$\therefore v=\frac{-60\times40}{-60-40}\Rightarrow v = 24 \text{ cm}$$

Now refrecation from water surface

$$=\frac{1}{v}-\frac{4/3}{(20+24)}=\frac{1-4/3}{\infty}\Rightarrow v = -33 \text{ cm}$$

9. 90– x – 360 –[135 + 90 + 90–r] $\Rightarrow$ x = 45 – r



For totally internal reflection

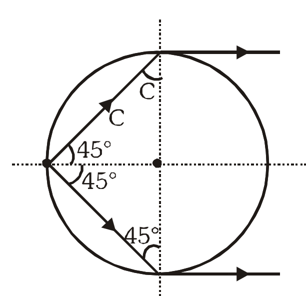
2 sin (45–r) = 1 sin 90

$\Rightarrow$ sin (45–r) = sin30 $\Rightarrow$ r=15

from snell'a law 1 sinθ = 2sin15

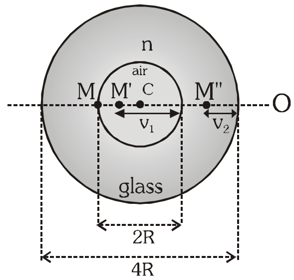
$$\Rightarrow \sin v = 2\ \frac{\sqrt{3}-1}{2\sqrt{2}}\Rightarrow \theta=\sin^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)$$

10. $$\text{Sinc}=\frac{1}{\mu}\Rightarrow C=\sin^{-1}\frac{1}{\sqrt{2}}=45°$$



Required area as shown in figure $=\frac{4\pi R^2}{2}=2\pi R^2$

11. $\dfrac{n}{v_1}+\dfrac{1}{2R}=\dfrac{n-1}{-R}\Rightarrow v_1=-\dfrac{2nR}{(2n-1)}$



Now u_2 = R-V_1= $R+\dfrac{2nR}{(2n-1)}=\dfrac{(4n-1)}{(2n-1)}R$

$$\frac{1}{v_2}+\frac{h}{\left(\frac{4n-1}{2n-1}\right)R}=\frac{1-n}{-2R}\Rightarrow v_2=-\left(\frac{4n-1}{3n-1}\right)\times 2R$$

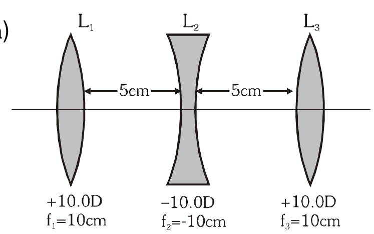
Total shift = 3R − $|v_2|$

$= 3R\ -\dfrac{4n-1}{3n-1}\times 2R\ =\ =\left(\dfrac{n-1}{3n-1}\right)R$

12. $\dfrac{I}{0}=m=\dfrac{v}{u}=\dfrac{f}{f+u}$; rel = $\dfrac{2dI}{dt}=\left[-\dfrac{f_0}{(f+u)^2}\dfrac{du}{dt}\right]\times 2$

$$=\frac{-40\times\frac{1}{2}}{10\times 10}\times 4\times 2\ =\frac{-8}{5}\text{ cm/sec}$$

13. (a)



When a parallel beam comes from '∞' from left, after refraction from L_1 it forms image at 10 cm from L_1.
This image will behave as a object for 'L_2'
So object distance form 'L_2' = + 5 cm

So image formed at v : $\dfrac{1}{v}-\dfrac{1}{5}=\dfrac{1}{-10}$

$$\Rightarrow\frac{1}{v}=\frac{1}{5}-\frac{1}{10}=\frac{1}{10}\Rightarrow v=10\text{cm}$$

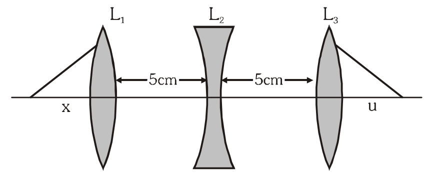
Therefore L_2 will form a image at +10 cm from L_2 on the right side. This image will act as object for L_3
Object distance for L_3 = + 5 cm
final image at v = ?

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}-\frac{1}{(+5)}=\frac{1}{(10)}\Rightarrow\ \frac{1}{v}=\frac{1}{5}+\frac{1}{10}$$

$$\Rightarrow\ \frac{1}{v}=\frac{3}{10}\Rightarrow v=\frac{10}{3}\text{cm}\ \text{ from } L_3$$

Ray converges at d= $\dfrac{10}{3}$cm from L_3

(b)



$$\frac{1}{v}+\frac{1}{x}=\frac{1}{f}\Rightarrow v=\frac{fx}{x-f}\Rightarrow 5-\frac{fx}{x-f}\Rightarrow\frac{1}{v_1}-\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{v_1}+\left(\frac{1}{5-\frac{fx}{x-f}}\right)=\frac{1}{-f}\Rightarrow\ \frac{1}{v_1}=-\frac{1}{f}-\left(\frac{1}{5-\frac{fx}{x-f}}\right)$$

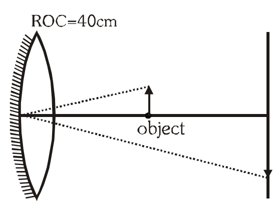
$$5-v_1=u\Rightarrow\left(5-\frac{fu}{u-f}\right)=v_1\Rightarrow\frac{5u-5f-fu}{u-f}$$

$$\frac{1}{v_1}=\frac{u-f}{5u-5f-fu}+\frac{1}{f}+\frac{f}{\left(5-\frac{fx}{x-f}\right)}=\frac{-u+f}{5u-5f-fx}$$

$$\frac{1}{f}+\frac{x-f}{(5x-5f-fx)}=\frac{f-u}{5x-5f-fu}$$

Putting the value after solving $x=\dfrac{50}{3}$cm

14. Initially the system will act as simple concave mirror given



v = −60, m=−2 ⇒ u = −30

$$\Rightarrow\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\ \Rightarrow\ \frac{1}{-60}-\frac{1}{30}=\frac{1}{f}$$

f = − 20 cm ⇒ R = 40 cm
In second case : lens + mirror system
Then P_{meq} = P_L + P_L + P_M
P_{meq} = equivalent power of the mirror
P_L = power of the lens
P_M = power of the mirror

$$P_{meq}=\frac{1}{f_L}+\frac{1}{f_L}+\frac{-1}{f_M}\quad\because\frac{1}{f_L}=\left(\frac{1}{3}\right)\frac{2}{R}$$

$$\frac{1}{f_L}=\left(\frac{1}{3}\right)\frac{2}{R}=\frac{2\times 2}{3R}+\left(\frac{-2}{-R}\right)\left(\frac{3}{3}\right)$$

$$P_{meq}=\frac{10}{3R}=\frac{-1}{f_m}\Rightarrow f_{meq}=\frac{-3R}{10}$$

f_{eqm} = − 12 cm

Now if object is placed at distance = x then

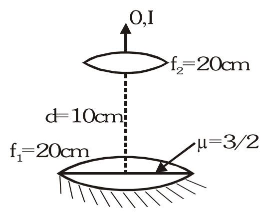
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_{eq}}\ ,\ (\because\ v=-60\ )$$

$$\frac{1}{-60}+\frac{1}{u}=\frac{-1}{12}\Rightarrow\frac{1}{u}=\frac{1}{60}-\frac{1}{12}=\frac{-4}{50}$$

$\Rightarrow$ u=–15 cm

So object displaced = 15 cm

15.



$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}=\frac{2}{f_L}+\frac{1}{f_m}=\frac{2}{20}+\frac{2}{120}$$

$$f_2=\frac{120}{14}=\frac{60}{7}$$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{120}-\frac{1}{u}=\frac{1}{20}\Rightarrow u=\frac{120}{7}$$

$$\left(\frac{120}{7}-10=\frac{-50}{7}\right)\Rightarrow\frac{7}{50}+\frac{1}{u}=\frac{1}{20}\ \therefore\ u=\frac{100}{19}\text{ cm}$$

16. (i) Given that r_2=C $\Rightarrow r_1=45°-C$ & sin e = n sin (45 –C)

sin e = n (sin 45   cos C - cos 45 sin C)

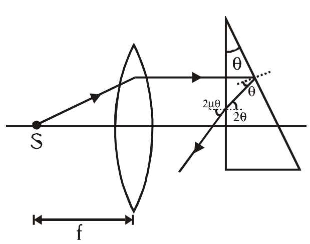
$$\Rightarrow e=\sin^{-1}\frac{1}{\sqrt{2}}\left(\sqrt{n^2-n_1^2}-n_1\right)$$

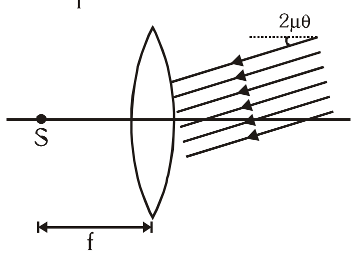
(ii) to pass refracted ray undeviated

r_2 = 0 $\Rightarrow$ r_1 = 45 and sin e = n sin 45

$$\Rightarrow e=\sin^{-1}\left(\frac{1.352}{\sqrt{2}}\right)=\sin^{-1}(0.956)$$

17.





Now $(2\mu\theta)f=d\Rightarrow\mu=\frac{d}{2\theta f}$

EXERCISE -V-A

1. The formula for the number of images formed by two inclined mirrors; inclined at angle θ is

$$N=\frac{360}{\theta}-1$$

$$\text{If }\frac{360}{\theta}=\text{even}$$

whereas $N=\frac{360}{\theta}$ If $\frac{360}{\theta}$ = odd

For θ = 60 , N = $\frac{360}{60}$ =6

Hence, N = 6 –1=5

3. The working of optical fibres is based on the phenomenon of total internal reflection.

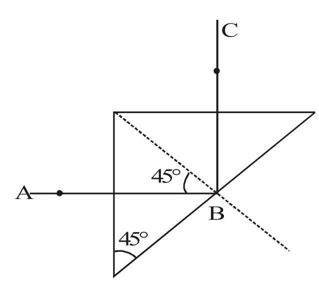
4. The resolving power of an astronomical telescope is = $\frac{D}{1.22\lambda}$ where D is the diameter of the objective lens. In order to improve the resolution of an astronomical telescope; one has to increase the diameter, i.e., linear aperture of the astronomical telescope.

5. The image formed by the objective of a compound microscope is real and enlarged as object lies between focus and centre of curvature.

6. The number of images formed when the angle between the mirrors is 90

N = $\frac{360}{60}$ –1=4–1=3

7. As ray AB is incident at an angle of 45 and for this ray to suffer total internal reflection.



Here 45 > θ_C $\Rightarrow$ sin 45 > $\sin\theta_C$

$$\Rightarrow\frac{1}{\sqrt{2}}>\frac{1}{n}\Rightarrow n>\sqrt{2}$$

8. A silvered lens behaves as a mirror. For the image to be of same size as of object must the object be held at the centre of curvature of mirror when

$$\frac{1}{f_{\text{silvered lens}}}=\frac{1}{f_1}+\frac{1}{f_m}+\frac{1}{f_1}=\frac{2}{f_1}+\frac{1}{f_m}$$

$$\frac{1}{f_1}=\frac{\mu-1}{R},\ \frac{1}{f_m}=\frac{2}{R}\Rightarrow f=\frac{R}{2\mu}$$

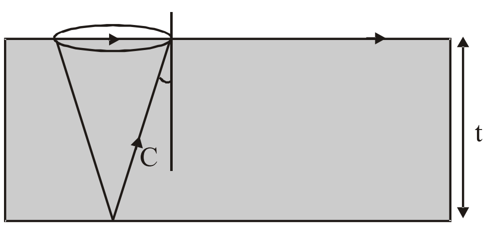
Therefore $\frac{1}{f}=\frac{2(\mu-1)}{R}+\frac{2}{R}=\frac{2\mu}{R}$

The radius of curvature of this mirror

$$=2f=\frac{R}{\mu}=\frac{30}{1.5}=20\ \text{cm}$$

9. Radius of circle

$$=t\tan c=\frac{t}{\sqrt{\mu^2-1}}=\frac{12}{\sqrt{\frac{16}{19}-1}}=\frac{36}{\sqrt{7}}\text{cm}$$



10. Let dots can be resolved by eye at maximum distance R then $\frac{1.22\lambda}{D}=\frac{d}{R}$

$$\Rightarrow R=\frac{Dd}{1.22\lambda}=\frac{(3\times10^{-3})(1\times10^{-3})}{1.22\times500\times10^{-9}}=4.92\ \text{m}\approx 5\text{m}$$

11. Power of lens $=\mu_2-\mu_1\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

In air power is – 5D

$$\therefore\ -5=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\ \ldots\text{(i)}$$

In other medium power is P

$$P=(1.5-1.6)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\ \ldots\text{(ii)}$$

Dividing (ii) by (i) $\frac{P}{-5}=\frac{-0.1}{1}\times\frac{1}{0.5}\Rightarrow P=1D$

12. The relation between angle of minimum deviation and refractive index is D=(μ–1)A

13. $P_1=\frac{1}{f_1}=-15D$, $P_2=\frac{1}{f_2}=5D$

When thin lenses are kept in contact with each other, the effective focal length of the combination will be

$$\frac{1}{f_{eff}}=\frac{1}{f_1}+\frac{1}{f_2}=-15+5=-10$$

$$\Rightarrow f_{eff}=-\frac{1}{10}=-0.1\text{m}=-10\text{cm}$$

15. By using Snell's law we get $\mu=\sqrt{1+\sin^2\theta}$

$$\Rightarrow \frac{2}{\sqrt{3}}=\sqrt{1+\sin^2\theta}\ \Rightarrow\ \sin^2\theta=\frac{1}{3}$$

$$\sin\theta=\frac{1}{\sqrt{3}}\ \Rightarrow\ \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

16. Incident angle from given plane

$$\theta_1=\cos^{-1}\left(\frac{\vec{A}\ \vec{k}}{A}\right)$$

$$=\cos^{-1}\left(\frac{-10}{20}\right)=\cos^{-1}\left(\frac{-1}{2}\right)=\left(\frac{\pi}{3}\right)$$

Now by snells law $n_1\sin\theta_1=n_2\sin\theta_2$

$$\sin\theta_2=\left(\frac{n_1}{n_2}\right)\sin\theta_1=\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right)=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta_2=45$$

17. Velocity of image $=-m^2V_0=-\left(\frac{f}{f-u}\right)^2V_0$

$$=-\left(\frac{20}{20-(-280)}\right)^2(15)=-\frac{1}{15}\ \text{m/s}$$

18. Focal length of a lens is given by

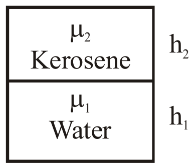
$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f}\propto\mu\propto\frac{1}{\lambda}\ \Rightarrow\ f\propto\lambda$$

$$\lambda_R>\lambda_B\ \Rightarrow\ f_R>f_B$$

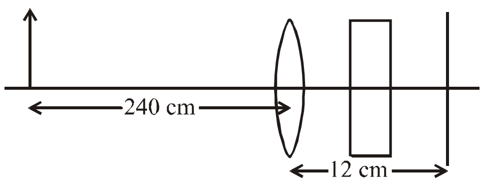
Focal length will increase

19. Apparent shift = $h\left(1-\frac{1}{\mu}\right)$



Total apparent shift = $h_1\left(1-\frac{1}{\mu_1}\right)+h_2\left(1-\frac{1}{\mu_2}\right)$

20. $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{12}-\frac{1}{(-240)} \Rightarrow f=\frac{240}{21}$



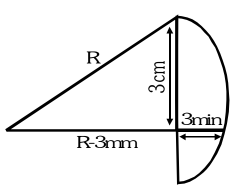
Shift = $h\left(1-\frac{1}{\mu}\right)=1\left(1-\frac{1}{1.5}\right)=\frac{1}{3}$ cm

To get image at film, lens should form image at

distance = $12-\frac{1}{3}=\frac{35}{3}$; $\frac{1}{v}-\frac{1}{u}$

$\Rightarrow \frac{21}{240}=\frac{3}{35}-\frac{1}{u} \Rightarrow u = -560$ cm

22. From trignometry



$R^2 = (3)^2 + (R-0.3)^2 \Rightarrow R \approx 15$ cm

$\mu_{lens}=\frac{3\times10^8}{2\times10^8}=1.5$

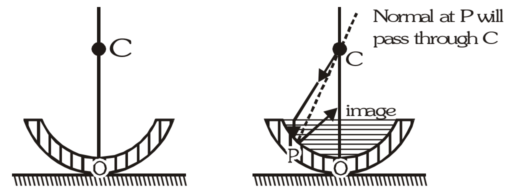
$\therefore$ from Lens maker formula,

$\frac{1}{f}=\left(\frac{\mu_L}{\mu_s}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=0.5\left(\frac{1}{15}\right)=\frac{1}{30}$

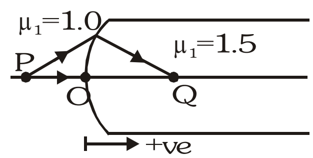
f = 30cm

EXERCISE -V-B

1. The ray diagram is shown in figure. Therefore, the image will be real and between C and O.



2. Let us say PO = OQ = X



Applying $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

Substituting the values with sign

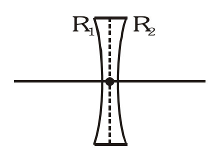
$$\frac{1.5}{+X}-\frac{1.0}{-X}=\frac{1.5-1.0}{+R}$$

(Distances are measured from O and are taken as positive in the direction of ray of light)

$$\therefore \frac{2.5}{X}=\frac{0.5}{R} \qquad \therefore X = 5R$$

3. $R_1 = -R$, $R_2 = +R$, $\mu_g = 1.5$ and $\mu_m = 1.75$

$\therefore \frac{1}{f}=\left(\frac{\mu_g}{\mu_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$



Substituting the values,

we have $\frac{1}{f}=\left(\frac{1.5}{1.75}-1\right)\left(\frac{1}{-R}-\frac{1}{R}\right)=\frac{1}{3.5R}$

$\therefore$ f = +3.5 R

Therefore, in the medium it will behave like a convergent lens of focal length 3.5R. It can be understood as, $\mu_m > \mu_g$, the lens will change its behaviour

4. The lens Maker's formula is :

$$\frac{1}{f}=\left(\frac{n_L}{n_m}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

where n_L = Refractive index of lens and

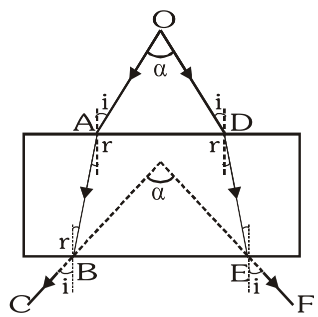
n_m = Refractive index of medium.

In case of double concave lens, R_1 is negative and R_2 is positive.

Therefore, $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ will be –ve.

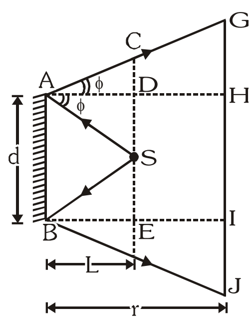
For the lens to be diverging in nature, focal length f should be negative or $\left(\frac{n_L}{n_m}-1\right)$ should be positive or $n_L > n_m$ but since $n_2 > n_1$ (given), therefore the lens should be filled with L_2 and immersed in L_1.

5.



Divergence angle will remain unchanged because in case of a glass slab every emergent ray is parallel to the incident ray. However, the rays are displaced slightly towards outer side.
(In the figure OA|| BC and OD|| EF)

6. The ray diagram will be as follows :



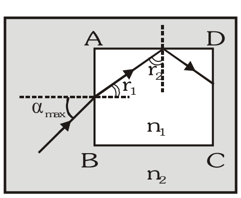
$HI = AB = d;\ DS = CD = \frac{d}{2}$

Since, $AH = 2AD \quad \therefore\ GH = 2CD = 2\ \frac{d}{2} = d$

Similarly, $IJ = d$

$\therefore\quad GJ = GH + HI + IJ = d + d + d = 3d$

7. Rays come out only from CD, means rays after refraction from AB get total internally reflected at AD.
From the figure :



$r_1 + r_2 = 90 \quad \therefore \quad r_1 = 90 - r_2$

$(r_1)_{max} = 90 - (r_2)_{min}$ and $(r_2)_{min} = \theta_C$
[for total internal reflection at AD]

where $\sin\theta_C = \frac{n_2}{n_1}$ or $\theta_C = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\therefore\ (r_1)_{max} = 90 - \theta_C$

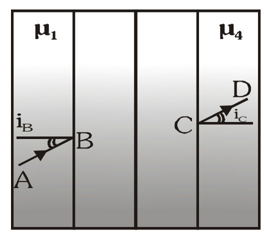
Now applying Snell's law at face AB :

$$\frac{n_1}{n_2} = \frac{\sin\alpha_{max}}{\sin(r_1)_{max}} = \frac{\sin\alpha_{max}}{\sin(90°-\theta_C)} = \frac{\sin\alpha_{max}}{\cos\theta_C}$$

or $\sin\alpha_{max} = \frac{n_1}{n_2}\cos\theta_C$

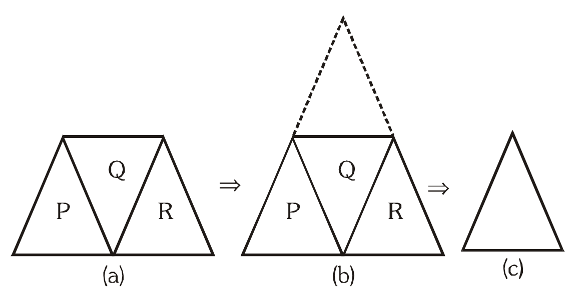
$$\therefore\ \alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2}\cos\theta_C\right] = \sin^{-1}\left[\frac{n_1}{n_2}\cos\sin^{-1}\left(\frac{n_2}{n_1}\right)\right]$$

8. Applying Snell's law at B and C,



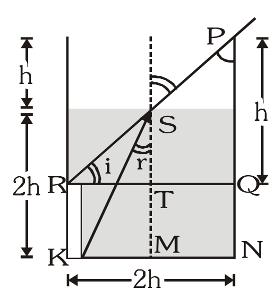
$\mu \sin i$ = constant or $\mu_1 \sin i_B = \mu_4 \sin i_C$
But $AB||CD \therefore\ i_B = i_C$ or $\mu_1 = \mu_4$

9. Figure (a) is part of an equilateral prism of figure (b) as shown in figure which is a magnified image of figure (c).



Therefore, the ray will suffer the same deviation in figure (a) and figure (c).

10.



$PQ = QR = 2h\ \therefore\ \angle i = 45$

$\therefore$ ST = RT = h = KM = MN

So, KS = $\sqrt{h^2+(2h)^2}$ = $h=\sqrt{5}$

$\therefore \sin r = \frac{h}{h\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin 45°}{1/\sqrt{5}} = \sqrt{\frac{5}{2}}$

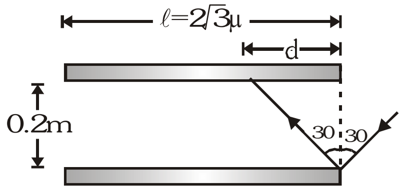
11. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For no dispersion

$d\left\{\frac{1}{f}\right\}=0 \Rightarrow d\mu\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\}=0 \Rightarrow R_1 = R_2$

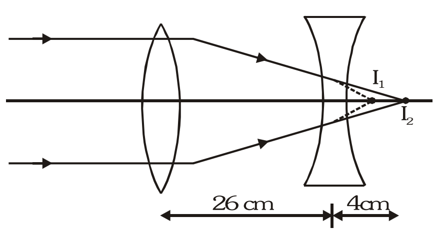
12. d=0.2 tan 30 $= \frac{0.2}{\sqrt{3}}$

$\frac{\ell}{d} = \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$



Therefore, maximum number of reflection are 30.

13. Image formed by convex lens at I_1 will act as a virtual object for concave lens. For concave lens

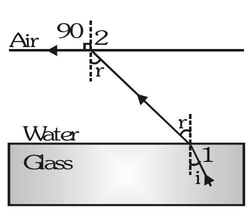


$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{4} = \frac{1}{-20} \Rightarrow$ v = 5 cm

Magnification for concave lens $m = \frac{v}{u} = \frac{5}{4} = 1.25$

As size of the image at I_1 is 2cm. Therefore, size of image at I_2 will be 2 1.25 = 2.5 cm.

14.



Applying Snell's law (μ sin i = constant) at 1 and 2, we have $\mu_1 \sin i_1 = \mu_2 \sin i_2$

Here, $\mu_1 = \mu_{glass}$, i_1 = i; $\mu_2 = \mu_{air}$ = 1 and i_2 = 90

$\therefore \mu_g \sin i = (1)(\sin 90)$ or $\mu_g = \frac{1}{\sin i}$

15. Critical angle $\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$

Wavelength increases in the sequence of VIBGYOR.

According to Cauchy's formula refractive index (μ) decreases as the wavelength increases. Hence the refractive index will increase in the sequence of ROYGBIV. The critical angle θ_C will thus increase in the same order VIBGYOR. For green light the incidence angle is just equal to the critical angle. For yellow, orange and red the critical angle will be greater than the incidence angle. So,these colours will emerge from the glass air interface.

16. During minimum deviation the ray inside the prism is parallel to the base of the prism in case of an equilateral prism.

17. When the object is placed at the centre of the glass sphere, the rays from the object fall normally on the surface of the sphere and emerge undeviated.

18. Distance of object from mirror $= 15 + \frac{33.25}{1.33} = 40$cm

Distance of image from mirror $= 15 + \frac{25}{1.33} = 33.8$cm

For the mirror, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\therefore \frac{1}{-33.8} + \frac{1}{-40} = \frac{1}{f}$ $\therefore$ f = –18.3 cm

$\therefore$ Most suitable answer is (c)

19. Let focal length of convex lens is +f, then length of concave lens would be $-\frac{3}{2}$f. From the given condition. $\frac{1}{30} = \frac{1}{f} - \frac{2}{3f} = \frac{1}{3f}$ $\therefore$ f = 10 cm

Therefore, focal length of convex lens = +10cm and that of concave lens =–15cm

20. Refraction from lens : $\frac{1}{v_1} - \frac{1}{-20} = \frac{1}{15}$

$\therefore$ v = 60cm + ve direction

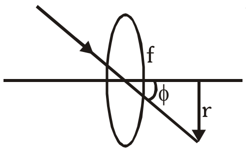
i.e. first image is formed at 60 cm to the right of lens system.

Reflection from mirror : After reflection from the mirror, the second image will be formed at a distance of 60 cm to the left of lens system.

Refraction from lens : $\frac{1}{v_3} - \frac{1}{60} = \frac{1}{15} \leftarrow$ +ve direction or v_3 = 12cm

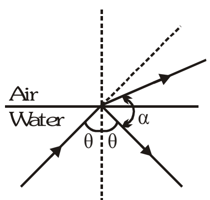
Therefore, the final image is formed at 12 cm to the left of the lens system.

21.

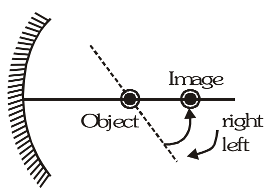


$r = f\tan\theta \Rightarrow r \propto f \therefore \pi r^2 \propto f^2$

22. Since $\theta < \theta_c$, both reflection and refraction will take place. From the figure we can see that angle between reflected and refracted rays α is less than $180 - 2\theta$.



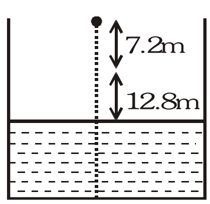
23. Since object and image move in opposite direction, the positioning should be as shown in the figure. Object lies between focus and centre of curvature $f < x < 2f$



24. In the position of minimum deviation angle of refraction $r = \frac{A}{2}$

25. $n_0\sin\theta = \left(\frac{n_0}{8}\right)\sin 90 \Rightarrow \sin\theta = \frac{1}{8} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{8}\right)$

26. $v^2 - u^2 = 2as$; $v^2 = 2(g)(7.2)$; $v = 12$ m/s

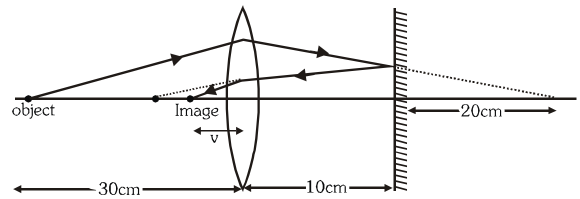


$X_{b/f} = \mu x$

$$\frac{dX_{(B/f)}}{dt} = \mu\frac{dx}{dt} \Rightarrow \frac{4}{3}\ (12\text{m/s}) \Rightarrow 16 \text{ m/s}$$

27. $\frac{1}{v} + \frac{1}{30} = \frac{1}{15} \Rightarrow \frac{1}{v} = \frac{1}{30}$ and $\frac{1}{v} - \frac{1}{10} = \frac{1}{15}$

$\Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{10} \Rightarrow v = 6$



28. Here normal is along $\tilde{j}$

Angle between incident ray and normal

$$\cos\theta = \frac{\frac{1}{2}\left(\tilde{i} + \sqrt{3}\tilde{j}\right)\cdot\tilde{j}}{(1)(1)} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$$

29. Real image $\Rightarrow$ v = +8 m

$m = \frac{v}{u} = -\frac{1}{3} \Rightarrow u = -24$ m

$f = \frac{uv}{u - v} = \frac{(-24)(8)}{-24 - 8} = 6$ m

for plano convex lens

$\frac{1}{f} = (u - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) \Rightarrow \frac{1}{6} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R}\right) \Rightarrow R = 3$ m

30. (P) at prism surface ray moving towards normal so ($\mu_2 > \mu_1$)at block surface ray moving away from normal so ($\mu_3 < \mu_2$)

(Q) No deflection of ray on both surface; so $\mu_1 = \mu_2 = \mu_3$

(R) At prism surface ray moving away from normal so $\mu_2 < \mu_1$. At block surface ray movign away from normal so $\mu_3 < \mu_2$ but since on total internal reflection not takes place on prism surface

$\mu_1 \sin 45 < \mu_2 \sin 90 \Rightarrow \mu_1 < \sqrt{2}\mu_2$

(S) Total internal reflection takes place

so $\mu_1\sin 45 > \mu_2 \sin 90 \Rightarrow \mu_1 > \sqrt{2}\mu_2$

MCQ

1. For total internal reflection to take place :

Angle of incidence, i > critical angle, θ_c

$\Rightarrow \sin i > \sin\theta_c \Rightarrow \sin 45 > \frac{1}{n} \Rightarrow \frac{1}{\sqrt{2}} > \frac{1}{n}$

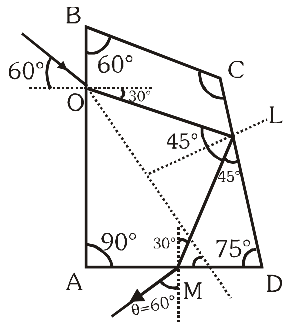
$\Rightarrow n < \sqrt{2} \Rightarrow n > 1.414$

Therefore, possible values of n can be 1.5 or 1.6 in the given options.

2. At point O (1) sin60 = $\sqrt{3}\sin r \Rightarrow r = 30$

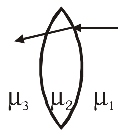
At point L : TIR occurs : because $\sqrt{3}$ sin 45 > 1

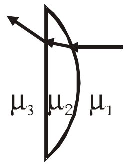
At point M : $\sqrt{3}\sin(30) = (1)\sin\theta \Rightarrow \theta = 60°$

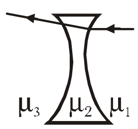


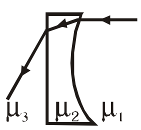
Match the column

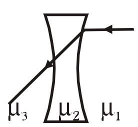
2. (A)–pr, (B) –qst, (C) –prt, (D) –qs

For (p) :  $\Rightarrow \mu_2 > \mu_1$ and $\mu_2 = \mu_3 \Rightarrow \mu_3 > \mu_1$

For (q) :  $\Rightarrow \mu_1 > \mu_2 > \mu_3$

For (r) :  $\Rightarrow \mu_3 = \mu_2 > \mu_1$

For (s) :  $\Rightarrow \mu_3 < \mu_2 < \mu_1$

For (t) :  $\Rightarrow \mu_2 = \mu_3 < \mu_1$

SUBJECTIVE

1. $n_1 = 1.20 + \dfrac{10.8 \times 10^4}{\lambda^2}$ and

$n_2 = 1.45 + \dfrac{1.80 \times 10^4}{\lambda^2}$ Here, λ is in nm.

(a) The incident ray will not deviate at BC if $n_1 = n_2$

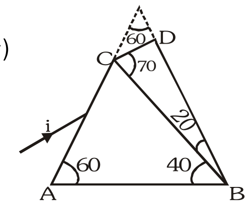
$$\Rightarrow 1.20 + \frac{10.8 \times 10^4}{\lambda_0^2} = 1.45 + \frac{1.80 \times 10^4}{\lambda_0^2} (\lambda = \lambda_0)$$

$$\Rightarrow \frac{9 \times 10^4}{\lambda_0^2} = 0.25 \Rightarrow \lambda_0 = \frac{3 \times 10^2}{0.5} \text{ or } \lambda_0 = 600 \text{ nm}$$

(b) The given system is a part of an equilateral prism of prism angle 60 as shown in figure.

At minimum deviation

$$r_1 = r_2 = \frac{60°}{2} = 30° = r(\text{say})$$



$\therefore n_1 = \dfrac{\sin i}{\sin r}$

$\therefore \sin i = n_1 \sin 30$

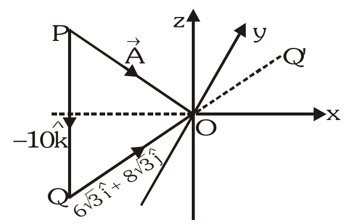
$$\sin i = \left\{1.20 + \frac{10.8 \times 10^4}{(600)^2}\right\}\left(\frac{1}{2}\right) = \frac{1.5}{2} = \frac{3}{4}$$

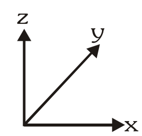
$(\lambda = \lambda_0 = 600 \text{ nm})$

$\Rightarrow i = \sin^{-1}(3/4)$

2. Incident ray

$$\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k} = (6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}) + (-10)\hat{k}$$

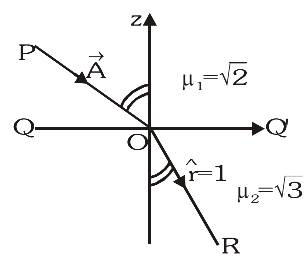


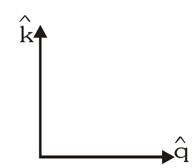


$\vec{QO} + \vec{PQ}$ (As shown in figure)

Note that $\vec{QO}$ is lying on x-y plane.

Note, QQ' and Z-axis are mutually perpendicular. Hence, we can show them in two-dimensional figure as below.





Vector $\vec{A}$ makes an angle i with z-axis, given by

$$i = \cos^{-1}\left\{\frac{10}{\sqrt{(10)^2 + (6\sqrt{3})^2 + (8\sqrt{3})^2}}\right\} = \cos^{-1}\left\{\frac{1}{2}\right\}$$

i = 60

Unit vector in the direction of QOQ' will be

$$\hat{q} = \frac{6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2}} = \frac{1}{5}(3\hat{i} + 4\hat{j})$$

Snell's law gives $\dfrac{\sqrt{3}}{\sqrt{2}} = \dfrac{\sin i}{\sin r} = \dfrac{\sin 60°}{\sin r}$

$\therefore \sin r = \dfrac{\sqrt{3}/2}{\sqrt{3}/\sqrt{2}} = \dfrac{1}{\sqrt{2}} \quad \therefore r = 45$

Now, we have to find a unit vector in refrated ray's direction OR. Say it is $\tilde{r}$ whose magnitude is 1.

Thus, $\tilde{r} = (1\sin r)\tilde{q} - (1-\cos r)\tilde{k} = \dfrac{1}{\sqrt{2}}[\tilde{q} - \tilde{k}]$

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{5}(3\tilde{i} + 4\tilde{j}) - \tilde{k}\right]$$

$$\tilde{r} = \frac{1}{5\sqrt{2}}(3\tilde{i} + 4\tilde{j} - 5\tilde{k})$$

3. Applying $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

First on plane surface

$$\frac{1.5}{AI_1} - \frac{1}{-mR} = \frac{1.5-1}{\infty} = 0$$

$\therefore$ AI_1 = –(1.5 mR)

Then on curved surface

$$\frac{1}{\infty} - \frac{1.5}{-(1.5mR + R)} = \frac{1-1.5}{-R}$$

[$v = \infty$ because final image is at infinity]

$$\Rightarrow \frac{1.5}{(1.5m+1)R} = \frac{0.5}{R}$$

$\Rightarrow 3 = 1.5m + 1 \Rightarrow \dfrac{3}{2}m = 2 \Rightarrow m = \dfrac{4}{3}$

4. (a) Rays coming from object AB first refract from the lens and then reflect from the mirror.

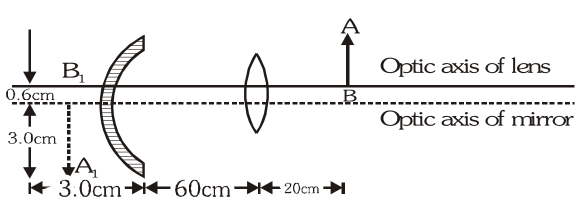
Refraction from the lens :

u =–20 cm, f = +15cm

Using lens formula $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$; $\dfrac{1}{v} - \dfrac{1}{-20} = \dfrac{1}{15}$

$\therefore$ v = +60cm and linear magnification,

$m_1 = \dfrac{v}{u} = \dfrac{+60}{-20} = -3$ i.e., first image formed by the lens will be 60cm from it (or 30cm from mirror) towards left and 3 times magnified but inverted. Length of first image A_1, B_1 would be 1.2 3 = 3.6 cm (inverted).



Reflection from mirror: Image formed by lens (A_1,B_1) will behave like a virtual object for mirror at a distance of 30cm from it as shown. Therefore u=+30cm, f=–30cm

Using mirror formula,

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ or $\dfrac{1}{v} + \dfrac{1}{30} = -\dfrac{1}{30}$ $\therefore$ v =–15cm

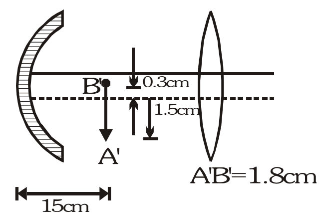
& linear magnification, $m_2 = -\dfrac{v}{u} = -\dfrac{-15}{+30} = +\dfrac{1}{2}$

i.e. final image A'B' will be located at a distance of 15cm from the mirror (towards right) and since magnification is +½ , length of final image would be

$$3.6 \times \frac{1}{2} = 1.8\text{cm}$$

$\therefore$ A'B' = 1.8 cm

Point B_1 is 0.6 cm above the optic axis of mirror, therefore, its image B' would be $(0.6)\left(\dfrac{1}{2}\right)$ = 0.3cm above optic axis. Similarly, point A_1 is 3 cm below the optic axis, therefore, its image A' will be $3 \times \dfrac{1}{2}$ =1.5 cm below the optic axis as shown below:



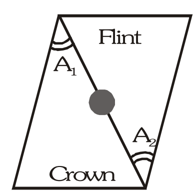
Total magnification of the image.

$m = m_1 \quad m_2 = (-3)\left(+\dfrac{1}{2}\right) = -\dfrac{3}{2}$

$\therefore$ A'B' = (m)(AB) $= \left(-\dfrac{3}{2}\right)(1.2) = -1.8$ cm

Note that, there is no need of drawing the ray diagram if not asked in the questions.

5. (i) When angle of prism is small and angle of incidence is also small, the deviation is given by $\delta = (\mu - 1)A$



Net deviation by the two prism is zero.

So, $\delta_1 + \delta_2 = 0$ or $(\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0$ ...(i)

Here, μ_1 and μ_2 are the refractive indices for crown and flint glasses respectively.

Hence, $\mu_1 = \dfrac{1.51 + 1.49}{2}$ = 1.5

and $\mu_2 = \dfrac{1.77 + 1.73}{2} = 1.75$

A_1= Angle of prism for crown glass = 6

Substituting the values in Eq. (i) we get

(1.5 – 1)(6) + (1.75 – 1)A_2 = 0

This gives A_2 = –4 . Hence, angle of flint glass prism is 4 (Negatice sign shows that flint glass prism is inverted with respect to the crown glass prism.)

(ii) Net dispersion due to the two prisms is

$$= (\mu_{b_1} - \mu_{r_1})A_1 + (\mu_{b_2} - \mu_{r_2})A_2$$

= (1.51 – 1.49) (6) + (1.77 – 1.73) (–4) = –0.04

6. For refraction at first surface

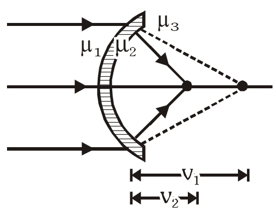
$$\frac{\mu_2}{v_1}-\frac{\mu_1}{-\infty}=\frac{\mu_2-\mu_1}{+R} \quad ...(i)$$

For refraction at second surface,

$$\frac{\mu_3}{v_2}-\frac{\mu_2}{v_1}=\frac{\mu_3-\mu_2}{+R} \quad ...(ii)$$

Adding equation (i) and (ii), we get

$$\frac{\mu_3}{v_2}=\frac{\mu_3-\mu_1}{R} \text{ or } v_2=\frac{\mu_3 R}{\mu_3-\mu_1}$$



Therefore, focal length of the given lens system is

$$\frac{\mu_3 R}{\mu_3-\mu_1}$$

7. Applying Snell's law on face AB,

(1) sin 45 $=(\sqrt{2})$ sinr $\therefore$ $\sin r=\frac{1}{2}$ or r = 30

i.e., ray becomes parallel to AD inside the block.

Now applying, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ on face CD,

$$\frac{1.514}{OE}-\frac{\sqrt{2}}{\infty}=\frac{1.514-\sqrt{2}}{0.4}$$

Solving this equation, we get OE = 6.06 m

8. Differentiating the lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

with respect to time,

we get $-\frac{1}{v^2}.\frac{dv}{dt}+\frac{1}{u^2}.\frac{du}{dt}=0$ (as f = constant)

$$\therefore \left(\frac{dv}{dt}\right)=\left(\frac{v^2}{u^2}\right)\frac{du}{dt} \quad ...(i)$$

Further, substituting proper values in lens formula, we have

$\frac{1}{v}+\frac{1}{0.4}=\frac{1}{0.3}$ (u=–0.4m, f=0.3m) or v = 1.2 m

Putting the values in equation (i)

Magnitude of rate of change of position of image = 0.09 m/s

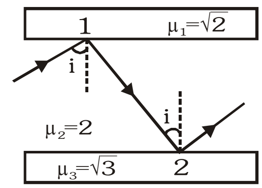
Lateral magnification

$$m=\frac{v}{u} \quad \therefore \frac{dm}{dt}=\frac{u.\frac{dv}{dt}-v.\frac{du}{dt}}{u^2}$$

$$=\frac{(-0.4)(0.09)-(1.2)(0.01)}{(0.4)^2}=-\ 0.3/s$$

$\therefore$ Magnitude of rate of change of lateral magnification = 0.3 /s.

9. Critical anglest at 1 and 2



$$\theta_{C_1}=\sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=45°$$

$$\theta_{C_2}=\sin^{-1}\left(\frac{\mu_3}{\mu_2}\right)=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=60°$$

Therefore, minimum angle of incidence for total internal reflection to take place on both slabs should be 60 .

i_{min} = 60

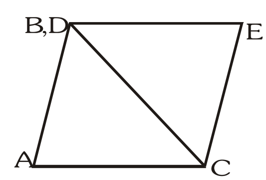
10. (a) At minimum deviation, r_1 = r_2 = 30

$\therefore$ From Snell's law $\mu=\frac{\sin i_1}{\sin r_1}$ or $\sqrt{3}=\frac{\sin i_1}{\sin 30°}$

$\therefore \sin i_1=\frac{\sqrt{3}}{2}$ or i_1 = 60

(b) In the position shown net deviation suffered by the ray of light should be minimum.

Therefore, the II[nd] prism should be rotated by 60 (anticlockwise)



11. $m=\frac{f}{f+u} \Rightarrow m_{25}=\frac{20}{20-25}=-4$ and

$$m_{50}=\frac{20}{20-50}=-\frac{2}{3} \Rightarrow \frac{m_{25}}{m_{50}}=\frac{4\times 3}{2}=6$$

12. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow f=\frac{R}{2}=-10m$

1[st] condition $\frac{-3}{25}+\frac{1}{u_1}=\frac{-1}{10} \Rightarrow u_1=50m$

2[nd] condition $\frac{-7}{50}+\frac{1}{u_2}=-\frac{1}{10} \Rightarrow u_2=25m$

Velocity $=\frac{\Delta u}{\Delta t}=\frac{25}{30}\times\frac{18}{5}=3km/h$