UNIT # 12 (PART - I)

MODERN PHYSICS (Atomic and Nuclear physics)

EXERCISE –I

1. $P(D) = 1 - e^{-\lambda t} = 1 - e^{-\lambda \cdot 2/\lambda} = \frac{e^2-1}{e^2}$

2. $T_A = \frac{1}{2}hr,\ T_B = \frac{1}{4}hr;\ T_{A+B} = \frac{\frac{1}{2}\times\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{6}hr$

so, first $\frac{1}{2}$ hr = 1 half lives (by A)

next 1 hr = 4 half lives (by B)

next $\frac{1}{2}$ hr = 3 half lives (by A+B)

thus $N = \frac{N_0}{2^8}$ ($\because$ Total eight half lives)

3. $F = \frac{dp}{dt} \Rightarrow F = \frac{(2P\sin\theta)}{\Delta t}$

$\Rightarrow F = 2np\sin\theta$ [n = number of photon]

$\Rightarrow 1N = 2(n)\frac{h}{\lambda}\sin 30 \Rightarrow n = 10^{27}$

4. In photo electric effect the maximum velocity of e^- will corresponding to KE_{max} & other are less than it.

5. $KE_{max} = h\nu - \phi$

6. For threshold frequency, $h\nu_0 = \phi \Rightarrow \nu_0 = \frac{\phi}{h}$

$\Rightarrow KE_{max} = h\nu - \phi = h(\nu - \nu_0)$

$I_{saturation} \propto n$ where $I = nh\nu$

7. $K_{max} = h\nu - \phi \Rightarrow V_s = \frac{K_{max}}{e} = \frac{4eV}{e} = 4$ volts

8. $\frac{K_1}{K_2} = \frac{h\nu_1 - \phi_1}{h\nu_2 - \phi_2} = \frac{1-0.5}{2.5-0.5} = \frac{1}{4}$

9. $\frac{\phi_1}{\phi_2} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{2.53}{5.06} = \frac{\lambda}{5896} \Rightarrow \lambda = 2948\ \text{Å}$

10. $eV_S = h\nu - \phi$

11. $E_k = h\nu - \phi$

12. $K.E._{max} = \underset{\text{frequency of light}}{h\nu} \quad \underset{\text{depends on properties of cathode}}{-\phi}$

13. $\frac{\phi_1}{\phi_2} = \frac{\lambda_2}{\lambda_1}$

14. $\frac{e(6V_0)}{e(2V_0)} = \frac{\frac{hc}{\lambda}-\phi}{\frac{hc}{2\lambda}-\phi} \Rightarrow \phi = \frac{hc}{4\lambda} \Rightarrow \lambda_0 = 4\lambda$

15. $\because I \propto \frac{1}{r^2}$ $\therefore$ intensity becomes $\frac{1}{4}$th

16. $\frac{v_1}{v_2} = \sqrt{\frac{2h\nu_0 - h\nu_0}{5h\nu_0 - h\nu_0}} = \frac{1}{2} \Rightarrow v_2 = 8\times10^6\ ms^{-1}$

17. $M = M_0e^{-\lambda t} \Rightarrow \frac{M_0}{20} = M_0e^{-\lambda t} \Rightarrow \ell n\left(\frac{1}{20}\right) = -\lambda t$

$\Rightarrow \left(\frac{\ell n(20)}{\ell n2}\right)T_{1/2} = t \Rightarrow \left(\frac{\ell n(2)+\ell n(10)}{\ell n2}\right)T_{1/2} = t$

$\Rightarrow t = 16.42$ days

18. $\frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv^2;\ \frac{4hc}{3\lambda} = \phi_0 + \frac{1}{2}mv_1^2$

$\Rightarrow 0 = \frac{4}{3}\phi_0 - \phi_0 + \frac{4}{3}\left(\frac{1}{2}mv^2\right) - \frac{1}{2}mv_1^2$

$\Rightarrow \frac{1}{2}mv_1^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi_0}{3} \Rightarrow v_1 > v\sqrt{\frac{4}{3}}$

19. (a) $P = \frac{\dot{N}hc}{\lambda}$ (b) $\dot{n} = \frac{0.1}{100}\dot{N}$ (c) $\dot{i} = \dot{n}e$

20. $\frac{K_2}{K_1} = \frac{\frac{hc}{\lambda_2}-\phi}{\frac{hc}{\lambda_1}-\phi} = \frac{2\left(\frac{hc}{\lambda_2}-\phi\right)}{\left(\frac{hc}{\lambda_2}-2\phi\right)} \Rightarrow K_1 < \frac{K_2}{2}$

21. Greater work function greater intercept

22. De broglie waves are independent of shape & size of the object.

23. De broglie waves are probability waves and are applicable for all the objects. Wave nature is observed for the small particles like electrons.

24. $K.E. = q\Delta V$

$\lambda_1 = \frac{h}{p_1} = \frac{h}{\sqrt{2m_1K_1}};\ \lambda_2 = \frac{h}{p_2} = \frac{h}{\sqrt{2m_2K_2}}$

$\because q = q_1 = q_2$ & ΔV is same $\therefore \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$

25. $\lambda = \frac{h}{p}$ (so same)

26. $\lambda = \frac{h}{\sqrt{2mK}}$; $K = \frac{3}{2}kT$

$\therefore \lambda = \frac{h}{\sqrt{2m\frac{3}{2}kT}} = \frac{h}{\sqrt{3mkT}} = \frac{25.15}{\sqrt{T}}$ Å

27. Maximum photon energy = 13.6 eV (emitted)
So K_{max} = 13.6 – 4 = 9.6 eV
Hence stopping potential is – 9.6 V
So – 10V can stop

28. No. of electron that can accommodate in n^{th} shell = $2n^2$
Total number of elements
= $2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2 = 60$

29. $E = \frac{13.6eV}{2^2} = 3.4$ eV

30. $E_4 - E_3 = Z^2\left[\frac{1}{9}-\frac{1}{16}\right] = \frac{7}{144}Z^2$

$E_2 - E_1 = Z^2\left[1-\frac{1}{4}\right] = \frac{3}{4}Z^2$

$E_4 - E_2 = Z^2\left[\frac{1}{4}-\frac{1}{16}\right] = \frac{3}{16}Z^2$

31. $\because \Delta E = h\nu \therefore \nu \propto \left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

$\therefore \nu_{max}$ for $n_2=2$ to $n_1=1$

32. Wave number $\frac{1}{\lambda} = 1.097\times10^7\left[\frac{1}{1^2}-\frac{1}{n^2}\right]$

Now in series limit λ corresponds to $n=\infty$ to $n=1$

$\therefore$ Wave number for series limit = $\frac{1}{\lambda} = 1.097 \quad 10^7$

33. $E = 24.6 + \frac{13.6(2)^2}{1^2} = 79.0$ eV

34. $N = {}^nC_2 = {}^5C_2 = 10$

35. $2\pi r = n(\lambda)$

36. $E = \frac{hc}{\lambda} = \frac{1242eV-nm}{0.021nm} = 59$ keV

37. High atomic no. and high melting point.

38. $\frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

For K_α line, $n_1 = 1$, $n_2 = 2$

$\frac{1}{\lambda} = R(43-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = R(42)^2\left(\frac{3}{4}\right)$ ...(i)

and

$\frac{1}{\lambda'} = R(29-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = R(28)^2\left(\frac{3}{4}\right)$ ...(ii)

Dividing eq. (i) by (ii), we get $\frac{\lambda'}{\lambda} = \frac{9}{4} \Rightarrow \lambda' = \frac{9}{4}\lambda$

39. $\frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

For wavelength of K_α, $n_1 = 1$ to $n_2 = 2$

$\frac{1875R}{4} = R(Z_A-1)^2\left[1-\frac{1}{4}\right]$ ...(i)

and $\frac{675R}{1} = R[Z_B-1]^2\left[1-\frac{1}{4}\right]$ ...(ii)

By solving eq. (i) & (ii) we get
$Z_A = 26$ and $Z_B = 31$
[Four elements lie between these two]

40. $\lambda_{min} \propto \frac{1}{V}$ so $\frac{\lambda}{6.22} = \frac{1}{10} \Rightarrow \lambda_{min} = 0.622$ Å

41. Characteristic X–rays corresponds to the transition of electrons from one shell to another.

43. $\dot{N}$ x (200 10^6 1.6 10^{-19}) = 1000

44. Q = –7(5.6) + [4(7.06)]2 $\Rightarrow$ Q = 17.3 MeV

45. ${}_2^4He \equiv 2n + 2p$
BE = [2(1.0073 + 1.0087) – 4.0015] 931.5 MeV

46. 2 Deuteron $\rightarrow {}_2^4He$
Q = BE of product – BE of reactant
= [28 MeV – 2(2.2 MeV)]
= [28 – 4.4] = 23.6 MeV

47. $K_\alpha = \frac{A-4}{A}Q \Rightarrow 48 = \frac{A-4}{A}50 \Rightarrow A = 100$

48. ${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{\nu}$

49. $\Delta N = N_1 - N_2 = N_0[e^{-\lambda t_1} - e^{-\lambda t_2}]$

50. 90 days $\rightarrow$ 3 half lives, left $\frac{1}{8}$ i.e. 12.5%
Disintegrated = 100 – 12.5 = 87.5%

51. $N = N_0 e^{-\lambda t} \Rightarrow \frac{N_0}{10} = N_0 e^{-\lambda t} \Rightarrow \ell n(1) - \ell n(10) = -\lambda t$

$\Rightarrow t = \left(\frac{\ell n 10}{\ell n 2}\right) T_{1/2} \Rightarrow t = 33$ days

52. 63% or nearly 2/3

53. $R = \lambda N = \lambda\left[\frac{m}{M_w} N_A\right]$

EXERCISE –II

1. (A) $L = \frac{nh}{2\pi}$ (B) $E = \frac{-13.6Z^2}{n^2}$

(C) $v = \frac{c}{137}\frac{Z}{n}$ (D) $K = -\frac{U}{2}$

2. $\frac{1}{\lambda_0} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$, n =4

(a) $\frac{hc}{\lambda_0} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \Rightarrow \frac{1}{\lambda_0} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

or $\frac{hc}{\lambda_0} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} + \frac{hc}{\lambda_3} \Rightarrow \frac{1}{\lambda_0} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$

3. $13.6\left[\frac{1}{1} - \frac{1}{9}\right] = 13.6Z^2\left(\frac{1}{9} - \frac{1}{n^2}\right)$

4. $|\Delta K| = 13.6\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 10.2$ eV

$|\Delta U| = 27.2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 20.4$eV

$\Delta L = \frac{2h}{2\pi} - \frac{h}{2\pi} = \frac{h}{2\pi}$

5. $U = -\frac{ke^2}{3r^3}$; $F = \frac{-\partial U}{\partial r} = \frac{ke^2}{r^4} = \frac{mv^2}{r}$

$mv^2 r^3 = ke^2$; $m^2v^2r^2 = \frac{n^2h^2}{4\pi^2}$

$r = \frac{m}{n^2}$ (constant), Also $V^2 \propto \frac{n^6}{m^4}$

$U = \frac{ke^2}{3\left(\frac{m^3}{n^6}\right)}$, K.E. $= \frac{1}{2}mv^2 \propto \frac{n^6}{m^3}$

$U \propto \frac{n^6}{m^3}$, T.E. $= U + $ K.E. $\propto \frac{n^6}{m^3}$

then total energy $\propto \frac{n^6}{m^3}$

6. $K_{max} = 10.4$eV ; $\phi = 1.7$ eV
$E = 10.4 + 1.7 = 12.1$ eV $\Rightarrow$ n=3 to n=1

$12.1 = \frac{1242.eVÅ}{\lambda}$

7. Stopping potential $\propto$ frequency $\propto \frac{1}{\text{wave length}}$

Saturation current $\propto$ rate of photoelectron emission. Also, $K.E._{max} = h\nu - \phi$, $P = \sqrt{2mKE}$

8. (1) $\lambda_A = \frac{h}{\sqrt{2mT_A}}$ (2) $\lambda_B = \frac{h}{\sqrt{2mT_B}}$

(3) $T_B = T_A - 1.5$ eV (4) $\lambda_B = 2\lambda_A$

9. $K_{max} = \frac{1242}{200}eV - 4.5eV$
$K_{max} = 1.7$eV at cathode
$K_{max} = (1.7 + 2)$ eV at anode
If polarity is revered, no e^- reach at collector.

10. $N = N_0 e^{-\lambda t} \Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$

$\Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} \Rightarrow T_{mean} = \frac{1}{\lambda}$

11. Assuming β to be ${}_{+1}\beta^0$

$${}_{69}E^{172} \xleftarrow{\gamma} {}_{69}D^{172} \xleftarrow{\alpha} {}_{71}C^{176}$$
$$\xleftarrow{{}_{+1}\beta^0} {}_{72}B^{176} \xleftarrow{\alpha} {}_{74}A^{180}$$

12. At t = 0 : $N_1 = N_0$
At time t : $N_2 = N_0 e^{-\lambda t}$
Decayed in time t $(N_1 - N_2) = N_0(1 - e^{-\lambda t})$
Probability that a radioactive nuclei does not decay

in t=0 to t : $\frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

13. Add equation $3\,{}_1^2H \rightarrow {}_2^4He + p + n$

(A) Q=[– [m (${}_2^4He$)+m(p)+m(n)]+3m(${}_1^2H$)] 931 MeV

(B) $10^{16}W = \frac{Q}{t}$

14. A is balanced both in mass number & atomic no.

15. $h\nu = h\nu_0 + eV_s$; $V_s = \frac{(h\nu - h\nu_0)}{e}$; $eV_s = h\nu - \phi$
Here ν=frequency of incident light and
ϕ= properties of emitter

16. $E_1 = \frac{hc}{\lambda}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For second excited state to first excited state

$E_1 = \frac{hc}{\lambda}\left[\frac{1}{4} - \frac{1}{9}\right] \Rightarrow \frac{hc}{\lambda}\left(\frac{5}{36}\right)$

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For first excited state to ground excited state

$$E_2 = \frac{hc}{\lambda}\left[1-\frac{1}{4}\right] \Rightarrow \frac{hc}{\lambda}\left[\frac{3}{4}\right]$$

(A) $\frac{E_1}{E_2} = \frac{5}{27}$ (B) $\frac{E_1}{E_2} = \left(\frac{hc}{\lambda_1}\right)\left(\frac{\lambda_2}{hc}\right) = \frac{\lambda_2}{\lambda_1} = \frac{27}{5}$

(C) $P \propto \frac{1}{\lambda} \Rightarrow \frac{P_1}{P_2} = \frac{5}{27}$

17. P.E. = –2(K.E.)
T.E. = (P.E.) + (K.E.)
T.E. = –2(K.E.) + (K.E.)
T.E. =–(K.E.)–T.E. = K.E. K.E. = 3.4 eV
$\lambda = 6.6 \quad 10^{-10}$ m

18. $122.4 = \frac{13.6Z^2}{1^2} \Rightarrow Z=3;\ 91.8 = 122.4\left[1-\frac{1}{4}\right]$

So an electron of KE 91.8eV can transfer its energy to this atom.

19. Room temperature $\Rightarrow$ n=1
Upon absorption excitations take place to many higher states which upon de-excitation emit all U.V., infrared and visible light.

20. Room temperature $\Rightarrow$ n=1 so lyman series

21. $U = -\frac{27.2eV}{n^2} + C$ $\quad k = \frac{13.6eV}{n^2}$

$E = -\frac{13.6eV}{n^2} + C$ $\quad \Delta E = 13.6eV\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$

here in these questions C = + 27.2 eV

22. We have $r \propto \frac{1}{\mu}$, μ = reduced mass

$\frac{Kq^2}{r} = \frac{mv_e^2}{r_e} = \frac{mv_N^2}{r_N}$; $mv_er_e = \frac{nh}{2\pi}$

23. $K \geq 20.4$ eV for inelastic collision

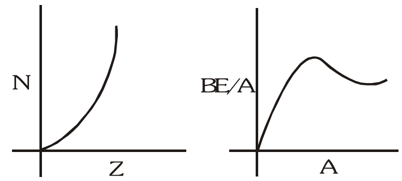
24. α decay : ${}_2He^4$, so both Z & A decreases.
β^+ decay : ${}_{+1}e^0$,
so A will not change but Z will change (decreases)
β^- decay : ${}_{-1}e^0$,
so A will not change but Z will change (increases)
γ decay : no change in A & Z.

25. $V = \frac{12400}{\lambda_{min}} \times 10^{-10}$ volt

$V = \frac{12400 \times 10^{-10}}{66.3 \times 10^{-12}}\text{volt} = \frac{12.4}{66.3} \times 10^4 = 18.75\text{ kV}$

26. $\lambda_{min} \propto \frac{1}{V}$ if V $\uparrow$ then $(\lambda_{min}) \downarrow$
No. of collision per electron increase then intensity increases

27.



28. $\lambda = 0.173$, $N = N_0e^{-\lambda t}$
$\Delta N = (N_0 - N_0e^{-\lambda t})$ = Decayed amount

$\Delta N = N_0\left(1-\frac{1}{e}\right)$

$\Delta N = N_0(1-0.37) = N_0(0.63)$

$\frac{\Delta N}{N_0} = \left[\frac{N_0(0.63)}{N_0}\right] \times 100 = 63\%$

$T_{1/2} = \frac{\log_e 2}{\lambda} = 4\text{year}$

29. $P_r = \frac{n_rhc}{\lambda_r}$, $P_b = \frac{n_bhc}{\lambda_b}$
if $P_r = P_b$ Since $\lambda_r > \lambda_b \Rightarrow n_r > n_b$

30. (a) $P = \frac{\dot{N}hc}{\lambda}$ (b) $i = \dot{n}e$ (c) $\% = \frac{\dot{n}}{\dot{N}} \times 100\%$

31. We have work is done by only electric field. Thus if $\vec{E}\|\vec{v}, |\vec{v}|$ decreases, & thus momentum of electron decreases & vice–versa, while in magnetic field $|\vec{v}|$ remains constant.

32. For electron $\lambda_{db} = \sqrt{\frac{150}{100+50}} = 1Å$

33. In photo electric effect only one to one Interaction.

34. $N = N_0e^{-\lambda_0 t}$, $N' = N_0e^{-10\lambda_0 t} \Rightarrow \frac{1}{e} = e^{-9\lambda_0 t}$

$\Rightarrow t = \frac{1}{9\lambda_0}$

35. $eV_1 = \frac{hc}{\lambda_1} - \phi$, $eV_2 = \frac{hc}{\lambda_2} - \phi$, $eV_3 = \frac{h_0}{\lambda_3} - \phi$

if $2V_2 = V_1 + V_3 \Rightarrow \frac{2}{\lambda_2} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

harmonic progression

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36. $\phi = 5eV - 2eV = 3eV$

$\Rightarrow -V_S = \dfrac{6eV - 3eV}{e} \Rightarrow V_S = -3V$

37. $J_n = \dfrac{nh}{2\pi} \Rightarrow n\lambda_n = 2\pi(r_0 n^2)$

So $J_n \propto \lambda_n \Rightarrow \lambda_n = (2\pi r_0)n$

38. $E_n = -\dfrac{13.6}{n^2}(Z^2)$

$E_{2n} = -\dfrac{13.6}{(2n)^2}Z^2 = -\dfrac{13.6}{4}\left(\dfrac{Z^2}{n^2}\right)$

$E_{2n} - E_n = \left[\left(-\dfrac{13.6}{4}\right)\dfrac{Z^2}{n^2}\right] - \left[-13.6\left(\dfrac{Z^2}{n^2}\right)\right]$

$E_{2n} - E_n = -\dfrac{13.6}{4}\dfrac{Z^2}{n^2} + 13.6\dfrac{Z^2}{n^2}$

$E_{2n} - E_n = \dfrac{Z^2}{n^2}\left(13.6 - \dfrac{13.6}{4}\right) = (10.2)\left(\dfrac{Z^2}{n^2}\right)$

$(E_{2n})(E_n) = \dfrac{(13.6)^2}{4}\left(\dfrac{Z^4}{n^4}\right)$

$$\frac{(E_{2n} - E_n)}{(E_{2n})(E_n)} = \frac{(10.2)\left(\frac{Z^2}{n^2}\right)}{\frac{(13.6)^2}{4}\left(\frac{Z^4}{n^4}\right)} \Rightarrow \frac{(E_{2n} - E_n)}{(E_{2n})(E_n)} \propto \left(\frac{n^2}{Z^2}\right)$$

39. $10 = {}^nC_2 \Rightarrow n = 5$; then 5 orbits are involved upon coming to second excited state so n^{th} excited state is 6^{th} [2^{nd}, 3^{rd}, 4^{th}, 5^{th}, 6^{th}]

40. On coming from $4 \to 1$ energy is greater than, less than or equal to energy corresponding to $2 \to 4$

41. $\dfrac{V^2}{r} = \dfrac{V_0^2}{n^2 r_0 n^2} = \dfrac{V_0^2}{r_0 n^4} \Rightarrow \dfrac{a_1}{a_2} = \dfrac{n_2^4}{n_1^4} = \dfrac{2^4}{3^4} = \dfrac{16}{81}$

42. $T \propto n^3$

$\dfrac{T_i}{T_f} = \dfrac{1}{27} = \left(\dfrac{n}{m}\right)^3 \Rightarrow \dfrac{n}{m} = \dfrac{1}{3}$

43. No of spectral lines

$\dfrac{n(n-1)}{2} = 10 \Rightarrow n^2 - n - 20 = 0 \Rightarrow n=5;\ n=-4$

$\Delta E = 13.6\left[1 - \dfrac{1}{25}\right] = 13.056;\ \lambda = \dfrac{hc}{\Delta E} = 95nm$

44. (a) $\Delta E = 13.6\left(\dfrac{1}{1^2} - \dfrac{1}{5^2}\right) = \dfrac{hc}{\lambda} + \dfrac{1}{2}mv^2$

(b) $0 = mv - \dfrac{h}{\lambda}$

45. Excitation upto n=3 is required so that visible length is emitted upon de-excitation.

So required energy $= 13.6\left(1 - \dfrac{1}{9}\right) = 12.1eV$

46. $\dfrac{r_1}{r_2} = \dfrac{n_1^2}{n_2^2} = \dfrac{1}{4} \Rightarrow \dfrac{n_1}{n_2} = \dfrac{1}{2} \Rightarrow \dfrac{T_1}{T_2} = \dfrac{n_1^3}{n_2^3} = \dfrac{1}{8}$

47. n^{th} excited state $= {}^{n+1}C_2 = \dfrac{n(n+1)}{2} = \Sigma n$

48. $L = \left(\dfrac{nh}{2\pi}\right);\ r = \left(0.53\dfrac{n^2}{Z}\right);\ f \propto \left(\dfrac{Z^2}{n^3}\right)$

(f r L) $\propto Z$ Constant for all orbits.

49. $\lambda_{K\alpha} > \lambda_{K\beta}$ & $\lambda_{K\alpha}, \lambda_{K\beta}$ are type of atom.

50. Energy released $= (BE)_{product} - (BE)_{reactant}$

51. $A_1 = \lambda N e^{-\lambda t_1}$, $A_2 = 2\lambda N e^{-\lambda t_2}$

$$\Rightarrow \frac{2A_1}{A_2} = e^{-\lambda(t_1 - t_2)} \Rightarrow \ell n\frac{2A_1}{A_2} = -\lambda \Delta t \Rightarrow \Delta t = \frac{T\ell n\frac{A_2}{2A_1}}{\ell n 2}$$

52. Radioactivity law is valid for large samples

53. $N_1 = N_0 e^{-\lambda T_1}$; $N_2 = N_0 e^{-\lambda T_2}$

$R_1 = \lambda N_1$; $R_2 = \lambda N_2$

$(N_1 - N_2) = \dfrac{\lambda}{\lambda}(N_1 - N_2) = \dfrac{(R_1 - R_2)}{\lambda}$

$T = \dfrac{\log_e 2}{\lambda}; \lambda = \dfrac{\log_e 2}{T}$

$(N_1 - N_2) = \dfrac{(R_1 - R_2)T}{(\log_e 2)}$

$(N_1 - N_2) \propto (R_1 - R_2)T$

54. Final product state

55. $\dfrac{dN}{dt} = \lambda_1 N_1 + \lambda_2 N_2 \Rightarrow \lambda_1 N_{10} e^{-\lambda_1 t} + \lambda_2 N_{20} e^{-\lambda_2 t}$

EXERCISE –III

Match the column

1. $v \propto \frac{1}{n}, KE \propto \frac{1}{n^2}, J \propto n, \omega = \frac{v}{r}$

but $r \propto n^2$ and $v \propto \frac{1}{n}$ $\therefore$ $\omega \propto \frac{1}{n^3}$

2. For given atomic number, energy and hence frequency of K–series is more than L–series. In one series also β–line has more energy or frequency compared to that of α–line.

3. Consider two equations

$eV_s = \frac{1}{2}mv^2_{max} = h\nu - \phi_0$ ...(i)

no. of photoelectrons ejected/sec $\propto \frac{\text{Intensity}}{h\nu}$ ...(ii)

(A) As frequency is increased keeping intensity constant.

$|V_s|$ will increases, $\frac{1}{2}m(v^2_{max})$ will increase.

(B) As frequency is increased and intensity is decreased.

$|V_s|$ will increase, $\frac{1}{2}m(v^2_{max})$ will increase and saturation current will decrease.

(C) Its work function is increased photo emission may stop.

(D) If intensity is increased and frequency is decreased. Saturation current will increase.

4. (A) In half life active sample reduce = $\frac{R_0}{2}$

$\therefore$ Decay number of nuclei is = $\frac{R_0}{2}$

(B) $N = N_0e^{-\lambda t}$

where λ = decay constant, $\lambda = \frac{\ell n(2)}{t_{1/2}}$

$$N = N_0 e^{-\frac{\ell n2}{t_{1/2}}\frac{t_{1/2}}{\ell n2}} \Rightarrow N = \frac{N_0}{e}$$

$$N_0 - N = N_0\left[\frac{e-1}{e}\right] \Rightarrow \frac{N - N_0}{N_0} = \frac{1-e}{e}$$

(C) $N = \frac{N_0}{(2)^{t/T_{1/2}}} \Rightarrow N = \frac{N_0}{2^{3/2}} \Rightarrow \frac{N_0}{2\sqrt{2}}$

Comprehension–1

1. For Balmer series, n_1 =2 (lower) ; n_2=3,4 (higher)
$\therefore$ In transition (VI), Photon of Balmer series is absorbed.

2. In transition II : E_2 = –3.4 eV, E_4 = –0.85 eV

ΔE = 2.55 eV; $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$ = 487 nm.

3. Wavelength of radiation = 103 nm = 1030Å

$\therefore \Delta E = \frac{12400}{1030\text{Å}} \simeq 12.0\text{eV}$

So difference of energy should be 12.0 eV (approx)
Hence n_1 =1 and n_2=3
(–13.6)eV (–1.51)eV
$\therefore$ Transition is V.

4. For longest wavelength, energy difference should be minimum.So in visible portion of hydrogen atom, minimum energy is in transition VI & IV.

Comprehension–2

1. $i = qf = \left(\frac{2e}{3}\right)\left(\frac{v}{2\pi r}\right) = \frac{ev}{3\pi r}$

2. $M_u = \frac{evr}{3}$

3. $M_{net} = M_u + M_d + M_d = \frac{evr}{3} + \frac{evr}{6} + \frac{evr}{6} = \frac{2evr}{3}$

4. Net magnetic moment in that case will be zero.

Comprehension–3

1. $Q = CV \Rightarrow ne = \frac{\varepsilon_0 A}{d}V$

$$n = \frac{2.85 \times 10^{-12} \times 10}{0.5 \times 10^{-3} \times 1.6 \times 10^{-19}} \times 16$$

n = 8.85 10^9

2. Equivalent resistance

$R = \frac{V}{I} = \frac{16V}{2 \times 10^{-6}A}$ = 8 $10^6\Omega$

3. $P = \frac{nhc}{\lambda}$ where n = number of photons incident per unit time. Also I = ne $\Rightarrow$ $P = \frac{Ihc}{e\lambda}$

$$\lambda = \frac{(2 \times 10^{-6})(6.6 \times 10^{-34})(3 \times 10^8)}{(4 \times 10^{-6})(1.6 \times 10^{-19})}$$

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$= \frac{9.9}{1.6} \times 10^{-7}$ m $= \frac{9900}{1.6}$ Å = 6187 Å

Which came in the range of orange light.

4. $\because$ Stopping potential V_S = 8V and KE = eV_S

$\therefore$ KE = 8eV

Comprehension–4

1. Rate of production of B depends on the decaying rate (A) : $\frac{d_{N_A}}{dt} = -\lambda N_A = \lambda N_1$

B is decaying simultaneously with two rates

$$\frac{d_{N_B}}{dt} = -\lambda_2 N_B = -\lambda_2 N_2$$

$$\frac{d_{N_B}}{dt} = -\lambda_3 N_B = -\lambda_3 N_2$$

Number of nuclei of 'B' is = $\lambda_1 N_1 - \lambda_2 N_2 - \lambda_3 N_2$

2. B will increase when $\lambda N_1 > (\lambda_2 + \lambda_3) N_2$

as initially $N_1 = N_2 = N_0 \Rightarrow \lambda_1 > \lambda_2 + \lambda_3$

3. N_2 = 0, N_1=0 as both will decay completely :

$N_3 = \frac{2N_0\lambda_2}{\lambda_2 + N_3}$ therefore B is incorrect

EXERCISE –IV(A)

1. Number of photons falling/s

$n \propto \frac{1}{r^2}$ [for point source]

So for new distance $n' = \frac{n}{9}$

$$I_s' = \frac{I_s}{9} = \frac{18mA}{9} = 2mA$$

Also saturated current $\propto$ n

V_d is independent of n.

(i) V_S = 0.6V (ii) $I_s = \frac{18 \times (0.2)^2}{(0.6)^2} = 2mA$

2. $E_{330} = \frac{12400}{3300} = 3.7575eV$

$$E_{220} = \frac{12400}{2200} = 5.636eV$$

Let work function be ϕ

eV_0 = (3.7575 – ϕ) e, $2eV_0$ (5.6363–ϕ)e

V_0 = 1.87 V

3.(i) For metal 1 threshold wavelength is λ_1

$\frac{1}{\lambda_1}$ = 0.002 nm^{-1} or λ_1 = 500 nm = 5000Å

For metal 2 threshold wavelength is λ_2

$\frac{1}{\lambda_2}$ = 0.005 nm^{-1} or λ_2 = 200 nm = 2000Å

(ii) $\phi = \frac{hc}{\lambda_0}$ or $\phi \propto \frac{1}{\lambda_0}$ $\therefore \phi_1 : \phi_2 = 2 : 5$

(iii) Metal 1 because λ_1 lies in visible wavelength range.

4. We have eV_0, $\frac{hc}{\lambda} - \phi = h\nu - \phi \Rightarrow V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$

From graph, $\frac{\phi}{e} = 2V \Rightarrow \phi = 2eV$

Also slopes of the graph $= \frac{2}{0.49 \times 10^{15}} = \frac{h}{e}$

$$\Rightarrow h = \frac{2 \times 1.6 \times 10^{-19}}{0.49 \times 10^{15}} = 6.536 \times 10^{-34}\ J/sec$$

5. Maximum kinetic energy of photo electrons

$$K_{max} = \frac{1}{2} mv_{max}^2 = \frac{hc}{\lambda} - \phi_0$$

Now let 3000Å = λ then 6000Å = 2λ

$$\therefore \frac{\frac{hc}{\lambda} - \phi_0}{\frac{hc}{2\lambda} - \phi_0} = \frac{(v_{max})_1^2}{(v_{max})_2^2} = \frac{9}{1}$$

$$\Rightarrow \phi_0 = \frac{7hc}{16\lambda} = \frac{7 \times 6.62 \times 10^{-34} \times 3 \times 10^8}{16 \times 3000 \times 10^{-10} \times 1.6 \times 10^{-19}} = 1.81\ eV$$

6. $KE_{max} = h\nu - \phi$, Also $E_{photo/time} = \frac{\dot{N} hc}{\lambda}$

(i) When intensity of light is decreased numbr of photons decreases but KE_{max} remains same

(ii) When emitting surface is charged, ϕ changes. So KE_{max} changes . If emitter is changed then +ve no. of photoelectrons becomes zero.

(iii) Increasing λ , $\left[\frac{hc}{\lambda} - \phi\right]\downarrow$

7. Energy incident / area of sphere / sec $= \frac{10}{4\pi(0.1)^2}$

Energy incident on atom

$$= \frac{10}{4\pi(0.1)^2} \times \pi \times (0.05 \times 10^{-9})^2 = 6.25 \times 10^{-19} J$$

Energy of one photon $= \frac{hc}{\lambda} = 12.525eV$

No. of photon striking atom/sec

$$= \frac{6.25\times10^{-19}}{12.525\times1.6\times10^{-19}} = 0.3125$$

No. of photons/ area

$$= \frac{10}{4\pi(0.1)^2}\times\frac{1}{12.525\times1.6\times10^{-19}} = \frac{10^{20}\times10\times10}{80\pi}$$

Since n = 1% $n = \frac{10^{20}}{80\pi}$

8. $\lambda_{\text{de-broglie}} = \frac{h}{\sqrt{2mK}} = \frac{h}{mv}$

v= u + at = $0+\left(\frac{eE}{m}\right)t \Rightarrow \lambda = \frac{h}{eEt} \Rightarrow \frac{d\lambda}{dt} = -\frac{h}{eEt^2}$

9. From the figure it is clear that

(P+1). $\lambda/2$ = 2.5 Å

$\therefore$ $\lambda/2$ = (2.5 – 2.0)Å = 0.5 Å

or λ = 1 Å = 10^{-10} m.

de–Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Km}}$$

K = kinetic energy of electron

$$\therefore K = \frac{h^2}{2m\lambda^2} = \frac{(6.63\times10^{-34})^2}{2(9.1\times10^{-31})(10^{-10})^2} = 2.415\times10^{-17}\,J$$

$$= \left(\frac{2.415\times10^{-17}}{1.6\times10^{-19}}\right)eV = 150.8\ eV$$

10.(i) Kinetic energy of electron in the orbits of hydrogen and hydrogen like atoms = |Total energy|

$\therefore$ Kinetic energy = 3.4 eV

(ii) The de–Broglie wavelength is given by

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2Km}}$$

K = kinetic energy of electron

Substituting the values, we have

$$\lambda = \frac{(6.6\times10^{-34}\,J-s)}{\sqrt{2(3.4\times1.6\times10^{-19}\,J)(9.1\times10^{-31}\,kg)}}$$

λ = 6.63 10^{-10} m or λ = 6.63 Å

11. (i) Magnetic moment $= IA = \frac{ehn}{4\pi m}$

Produced is n^{th} orbit

for hydrogen n =1 $\Rightarrow$ Magnetic moment $= \frac{eh}{4\pi m}$

(ii) $\vec{\tau} = \vec{M}\times\vec{B} = \frac{eh}{4\pi m}$ B sin30 = $\frac{ehB}{8\pi m}$

12. $\frac{1}{3}E = \frac{hc}{\lambda} = \lambda = \frac{3hc}{E}$ when E' = E, $\lambda' = \frac{hc}{E} = \frac{\lambda}{3}$

So new wavelength = $\frac{\lambda}{3}$

13. From the given conditions :

$E_n - E_2 = (10.2+17)eV = 27.2eV$

and $E_n - E_3$ = (4.25 +5.95) eV = 10.2 eV

Equation (1) – (2) gives

$E_3 - E_2$ = 17.0 eV or $Z^2(13.6)\left(\frac{1}{4}-\frac{1}{9}\right)$ = 17.0

$\Rightarrow$ $Z^2(13.6)$ (5/36) = 17.0 $\Rightarrow$ Z^2 = 9 $\Rightarrow$ Z = 3

From equation (1) $Z^2(13.6)\left(\frac{1}{4}-\frac{1}{n^2}\right) = 27.2$

$\Rightarrow (3)^2\,(13.6)\left(\frac{1}{4}-\frac{1}{n^2}\right) = 27.2 \Rightarrow \frac{1}{4}-\frac{1}{n^2}$ = 0.222

$\Rightarrow$ $1/n^2$ = 0.0278 $\Rightarrow$ n^2 = 36 $\Rightarrow$ n = 6

14. $\frac{1}{\lambda_B} = R\left[\frac{1}{(2)^2}-\frac{1}{\infty}\right] \Rightarrow \frac{1}{\lambda_B} = \frac{R}{4}$

$\frac{1}{\lambda_p} = R\left[\frac{1}{9}-\frac{1}{16}\right] = \frac{7R}{144}$; $\lambda_B : \lambda_p = \frac{7}{36}$

15. Assuming Bohr's model to be applicable to the He atom too;

Energy of electron = –13.6 4 eV= – 54.4 eV

initial energy of electron = 0

Energy of photon emitted = 544 eV

$$\lambda = \frac{hc}{\Delta E} = 22.8mm$$

16. (i)Operating voltage = 40 kV, 0.5% energy for x ray

$$\therefore \frac{99.5}{100}\times n\times e\times V = 720$$

$$n = \frac{720}{1.6\times10^{-14}\times0.995\times40\times10^3} = 1.1\quad 10^{17}$$

(ii) Velocity of incident e^- $\Rightarrow$ $\frac{1}{2}mv_e^2$ = eV

$$v_e = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times40\times10^3}{9.1\times10^{-31}}}$$

= 1.2 10^8 m/s

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17. Given $\lambda_{K\alpha}$ = 71 pm = 0 71 Å

$$E_K - E_L = \frac{hc}{\lambda_{K\alpha}} = \frac{12400\ eV - Å}{0\cdot71Å} = 17.46\ keV$$

Thus $E_L = E_K$ – 17 46 keV
= 23 32 keV – 17 46 keV = 5 86 keV

18. Total mass annihilated $2m_e = \frac{1}{c^2}MeV$

Total energy produced$=mc^2=\frac{1}{c^2}MeV\times c^2=1MeV$

Energy of 1 photon = 0.5 MeV

$$\lambda = \frac{hc}{E} = \frac{1.2\times10^{-12}}{0.5} = 2.4pm$$

19. $_1^2H + _1^2H \rightarrow _2^4He + Q$

Q = [4 7 – 2 2 1.1] MeV = 23.6 MeV

20. $\because\ \lambda_{min} = \frac{hc}{eV} = \frac{12400}{V}$ Å

At 40 kV : $\lambda_{min} = \frac{12400}{40000}$ = 0.31 Å

Wavelength of K_α is independent of applied potential.

For K_α X–ray : $\frac{3}{4}(13.6)(Z-1)^2 = E = \frac{hc}{\lambda_{K\alpha}}$

$\lambda_{K_\alpha} = \frac{1216}{(Z-1)^2}$ Å and given that

$\lambda_{K_\alpha} = 3\ \lambda_{min} \Rightarrow \frac{1216}{(Z-1)^2}$ = 3 0.31

$$\Rightarrow (Z-1)^2 = \frac{1216}{0.93} \simeq 1308 \Rightarrow Z-1 = 36 \Rightarrow Z = 37$$

21. $N_1 = N_{01}e^{-\lambda t}$ and $N_2 = N_{02}e^{-3\lambda t}$

$\frac{N_1}{N_2} = \frac{N_{01}e^{-\lambda t}}{N_{02}e^{-3\lambda t}} = e^{2\lambda t}$ [$\because N_{01} = N_{02}$, for $\frac{N_1}{N_2} = \frac{e}{1}$]

$\therefore$ 1 = $2\lambda t \Rightarrow t = \frac{1}{2\lambda}$

22. Activity of x = Activity of y or $n_x\ \lambda_x = n_y\ \lambda_y$

$$\Rightarrow n_x\left[\frac{0.693}{(T_{1/2})_x}\right] = n_y\left[\frac{0.693}{(T_{1/2})_y}\right]$$

$$\Rightarrow \frac{n_x}{n_y} = \frac{(T_{1/2})_x}{(T_{1/2})_y} = \frac{3}{27} = \frac{1}{9}$$

23. After t time active fraction

= 1 – 0 36 = 0 64 = $\frac{N}{N_0}$ = $e^{-\lambda t}$

Now at t/2 time, active fraction
= $(e^{-\lambda t})^{1/2}$ = (0.64)½ = 0.8
So decayed fraction in t/2 time is 0 2 or 20%

24. $_{92}^{238}U \rightarrow _{90}^{234}Th + _2^4He$

Δm= (238.05079 – 4.00260 –234.0 4363) u
E = Δmc^2 = 4.24764 MeV
If it emits proton spontaneously, the equation is not balanced in terms of atoms & mass number.

$$_{92}^{238}U \rightarrow _{91}^{237}Pa + _1^4H$$

Δm =(238.05079–237.065121–1.007834)u
= –0.022165 u
$\because$ Δm is negative, so reaction is not spontaneous.

25. Let at t = 0, capacitor starts discharging then at time t, activity of radioactive sample = R = $R_0e^{-\lambda t}$
charge on capacitor = Q = $Q_0\ e^{-t/RC}$

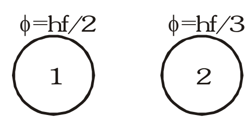
Now $\frac{R}{Q} = \frac{R_0\ e^{-\lambda t}}{Q_0 e^{-t/RC}} = \frac{R_0}{Q_0}\ e^{-t\left(\lambda - 1/Rc\right)}$

It is independent of time if $\lambda = \frac{1}{RC}$

$$R = \frac{1}{\lambda C} = \frac{t_{avg}}{C} = \frac{20\times10^{-3}}{100\times10^{-6}} = 200\ \Omega$$

EXERCISE –IV(B)

1.



Potential at which electron stop coming out

from sphere–1, $V_1 = \frac{\left(hf - \frac{hf}{2}\right)}{e} = \frac{hf}{2e}$

from sphere–2, $V_2 = \left(\frac{hf - \frac{hf}{3}}{e}\right) = \frac{2hf}{3e}$

After connection

(i) V + V = $V_1 + V_2$ (V= final common potential)

$$\Rightarrow 2V = \frac{hf}{2e} + \frac{2hf}{3e} \Rightarrow V = \frac{7hf}{12e}$$

(ii) For sphere–2 : $\frac{k\Delta Q}{R} = \left(\frac{2}{3}\frac{hf}{e} - \frac{7hf}{12e}\right) = \frac{hf}{12e}$

No. of electrons flows $\Delta n = \frac{\Delta Q}{e} = \frac{hfR}{12ke^2}$

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2. Power of source = 2J/sec

Energy of 1 photon = $\frac{hc}{\lambda} = \frac{12400}{6000} = 2.067\text{eV}$

No. of photons emitted /sec

$= \frac{2 \times 10^{19}}{2.067 \times 16} = 6.0474 \times 10^{18}$

No. of photons striking on sphere of 0.6 m

$= \frac{6.0474 \times 10^{18}}{4\pi(0.6)^2} / \text{m}^2$

No. of photons passing through aperture

$= \frac{6.0474 \times 10^{18}}{4\pi(0.6)^2} \times \pi\left(\frac{0.1}{2}\right)^2 = \frac{4.2 \times 10^6}{4}$ photons

No. of photons incident / area on screen (assuming aperture to act as a secondary point source)

$= \frac{4.2 \times 10^{16}}{4 \times 4\pi(5.4)^2}$ = photon flux

= 1.1462 10^{14} photons / m^2 sec

No. of photons incident on detector /sec

= 1.1462 10^{14} $\frac{0.50}{10000}$

Photo current $= \eta \dot{N} e = 2.063 \times 10^{-10}$ A

When the lens of f = –0.6 m is used,

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{-1}{0.3} = v = -0.3\text{m}$

Thus image will be at 0.3 m from the lens in the direction opposite to the screen.

Distance between screen & image = 5.7 m

No. of photons striking lens

= No. of photons striking the aperture

= 1.05 10^{16} photons.

Photons transmitted through the lens

= 0.8 1.05 10^{16}=0.84 10^{16} photon/s

This new situation, A point source emitting 0.84 10^{16} photons/ sec of λ = 6000Å is kept at 5.7 m away from the screen.

Thus photons striking / area of screen

$= \frac{0.84 \times 10^{16}}{4\pi(5.7)^2}$ = photon/sec =2.0574 10^{13}

Electrons emitted=0.9 2.0574 10^{13} $\frac{0.5}{10000}$

Current = 1.4813 10^{-10} A

E_p = 2.067 eV ; ϕ =1eV

K.E.$_{max}$ = 1.067 eV; V_S = 1.067 V

3. Energy of photo with $\lambda_1 = 3000\text{Å} = \frac{hc}{\lambda_1} = 4.14\text{eV}$

Energy of photo with $\lambda_1 = 1650\text{Å} = \frac{hc}{\lambda_2} = 7.53\text{eV}$

Power of source = 5 10^{-3}W

$\dot{N}_2 = \frac{5 \times 10^{-3}}{7.53 \times 10^{-19} \times 1.6} = 4.15 \times 10^{15}$

$\dot{N}_1$ for 1W = $\frac{1}{4.14 \times 1.6 \times 10^{-19}} = 1.5 \times 10^{18}$

Current = 4.8 10^{-3}A

$\Rightarrow \dot{\eta} = \frac{4.8 \times 10^{-3}}{1.6 \times 10^{-19}} = 3 \times 10^{16}$

$\eta = \frac{3 \times 10^{16}}{1.5 \times 10^{18}} = 2 \times 10^{-2} = 2\%$

$\dot{\eta}_2 = 4.15 \times 10^{15} \times 0.02 = 8.3 \times 10^{13}$

Current $= \dot{n} e = 13.3\ \mu\text{A}$

Also $v_{max\ 1650}$ = $2V_{max\ 5000}$

$\Rightarrow KE_{max\ 1650} = 4KE_{max\ 3000}$

⇒4(4.14 – ϕ) = (7.53–ϕ) ⇒16.56 – 7.53 = 3ϕ

$\Rightarrow \phi = \frac{9.03}{3} = 3.01\text{eV}$, $\lambda_{th} = \frac{hc}{E} = 4126\text{Å}$

4. $\frac{hc}{\lambda_1} = \frac{12400}{4000}\text{eV} = 3.1\text{eV}$

$\frac{hc}{\lambda_2} = \frac{12400}{5000}\text{eV} = 2.48\text{eV}$

$\frac{hc}{\lambda_3} = \frac{12400}{6000}\text{eV} = 2.06\text{eV}$

∴ Light having wavelength 6000 Å will not be able to eject electrons.

∴ Photo–current = $(\dot{n}_1 + \dot{n}_2)e$

where $\dot{n}_1$ = no. of photons of light incident having wavelength 4000Å

$\dot{n}_2$ = no. of photons of light incident having wavelength 5000Å. Here

$\dot{n}_1\left(\frac{hc}{\lambda_1}\right) = \dot{n}_2\left(\frac{hc}{\lambda_2}\right) = \dot{n}_3\left(\frac{hc}{\lambda_3}\right) = \frac{I_0}{3} = \frac{3 \times 10^{-3}}{3} = 10^{-3}$

Area intercepted = 2cm^2 = 2 10^{-4}m^2

∴ Photo–current $= (\dot{n}_1 + \dot{n}_2)e$

$= \frac{(10^{-3} \times 2 \times 10^{-4})(5000 + 4000) \times 10^{-10} \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34} \times 3 \times 10^8}$

= 0.144 μA

5. Momentum of particles = $\frac{h}{\lambda_1}$ & $\frac{h}{\lambda_2}$

Since particles are identical, mass of both particles are same.

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$$mv_1 = \frac{h}{\lambda_1} \;\&\; mv_2 = \frac{h}{\lambda_2}\;;\quad \vec{v}_{COM} = \frac{v_1}{2}\tilde{i} + \frac{v_2}{2}\tilde{j}$$

$$\vec{v}_{1COM} = \frac{v_1}{2}\tilde{i} - \frac{v_2}{2}\tilde{j} \Rightarrow \left|\vec{v}_{1COM}\right| = \frac{\sqrt{v_1^2+v_2^2}}{2} = \left|\vec{v}_{2COM}\right|$$

$$\lambda = \frac{h}{mv} = \frac{2h}{m\sqrt{v_1^2+v_2^2}} = \frac{2h}{\sqrt{(mv_1)^2+(mv_2)^2}}$$

$$= \frac{2h}{\sqrt{\left(\frac{h}{\lambda_1}\right)^2+\left(\frac{h}{\lambda_2}\right)^2}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$$

6. (i) $\Delta E = 47.2 = 13.6\ Z^2\left(\frac{1}{4}-\frac{1}{9}\right) \Rightarrow Z = 5$

(ii) $\Delta E = 13.6 \quad 25\left(\frac{1}{9}-\frac{1}{16}\right)$

$$= \frac{13.6\times 25\times 7}{9\times 16}\text{eV} = 16.5\ \text{eV}$$

(iii) $\Delta E_{1\to\infty} = 13.6\ Z^2\text{eV} = \frac{hc}{\lambda}$

$$\Rightarrow 13.6\times 25\text{eV} = \frac{12400\text{eV}-\text{A}}{\lambda} \Rightarrow \lambda = 36.4\text{Å}$$

(iv) $(KE)_{1\text{st orbit}} = \frac{1}{2}mv^2$

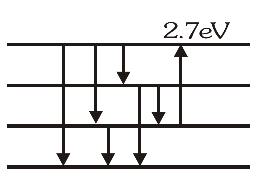
$$= \frac{9.1\times 10^{-31}\times(2.2\times 10^6\times 5)^2}{2\times 1.6\times 10^{-19}} = 344\ \text{eV}$$

$(PE)_{1\text{st orbit}} = -2KE = -688\text{eV}$

$L = \frac{nh}{2\pi} = \frac{h}{2\pi}\quad (n=1)$

(v) Radius $= (R_0)\frac{n^2}{z} = \frac{0.53\times 1^2}{5} = 1.06\quad 10^{-11}\text{m}$

7. Let n = no. of level of excited state
$\Rightarrow {}^nC_2 = 6$ (spectral lines)
$\Rightarrow n = 3$ (3rd excited states)
$\therefore$ No. of excited state level = n+1 = 4



(i) Principal quantum no. of initially excited level B =4

(ii) $2.7 = k\left(\frac{1}{4}-\frac{1}{16}\right) \Rightarrow k = \frac{16}{3}\times 2.7 = 14.4\text{eV}$

$\therefore$ Ionisation energy

$$= k\left(1-\frac{1}{\infty}\right) = k = 14.4\text{eV} = 23.04 \quad 10^{-19}\text{J}$$

(iii) $\Delta E_{max} = 4 \to 1 = 13.5\ \text{eV}$
$\Delta E_{min} = 4 \to 3 = 0.7\ \text{eV}$

8.(i) $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \Rightarrow mvr = \frac{h}{2\pi} \Rightarrow E_{total} = -\frac{e^2}{8\pi\epsilon_0 r}$

(ii) $\frac{dE}{dt} = -P$ (loss of energy per sec)

$$\Rightarrow \frac{d}{dt}\left(-\frac{e^2}{8\pi\epsilon_0 r}\right) = -\frac{P_0}{r^4} \Rightarrow \left(\frac{e^2}{8\pi\epsilon_0 r^2}\right)\frac{dr}{dt} = -\frac{P_0}{r^4}$$

$$\Rightarrow e^2\int_{r_0}^{r} r^2 dr = -8\pi\epsilon_0 P_0\int_0^t dt$$

$$\Rightarrow r^3 = r_0^3 - \frac{6P_0(4\pi\epsilon_0)t}{e^2} \Rightarrow r = r_0\left[1-\frac{3cr_e^2 t}{r_0^3}\right]^{1/3}$$

(iii) For r=0, (to collapse and fall into nucleus)

$$\Rightarrow 1 - \frac{3cr_e^2 t}{r_0^3} = 0$$

$$\Rightarrow t = \frac{r_0^3}{3cr_e^2} = \frac{10^{-30}}{3\times 3\times 10^8\times 9\times 10^{-30}} = \frac{10^{-10}\times 100}{81}\text{sec}$$

9.(i) $mvr_n = \frac{nh}{2\pi}$ and $\frac{mv^2}{r_n} = \frac{4e^2}{4\pi\epsilon_0 r_n^2}$

$$\Rightarrow r_n = \frac{\epsilon_0 h^2}{4\pi e^2}\left(\frac{n^2}{m}\right) = \frac{n^2h^2\epsilon_0}{400\pi me^2}$$

(iii) $E_{n^{th}} = -(13.6)\left(\frac{Z^2 m}{n^2}\right)$

$$\Delta E = 13.6\times 4^2\times 100\left[\frac{1}{2^2}-\frac{1}{4^2}\right] = 408\text{eV}$$

10. Let the age of Earth be t_{year} and initially both were present as N_0;

$$Nu_{238} = \frac{N_0}{2^{t/t_{1/2}}} = \frac{N_0}{2^{t/4.5\times 10^3}}$$

$$Nu_{235} = \frac{N_0}{2^{t/4.3\times 10^8}}$$

$$\frac{Nu_{238}}{Nu_{235}} = \frac{140}{1} = 2^{t[1.18\times 10^{-9}]}$$

t= 6.0418 10^9 years

11. $\xrightarrow{x} A \xrightarrow{\lambda} B$

No. of neucleus disintegrated $= xt - \frac{x}{\lambda}\left(1-e^{-\lambda t}\right)$

For $t = \frac{1}{\lambda}$

Disintegration $= \frac{x}{\lambda}\left[1 - 1 + e^{-1}\right] = \frac{x}{e\lambda}$

Energy released $= \frac{E_0 x}{e\lambda}$

Energy utilized for melting $= 0.5 \times \frac{E_0 x}{e\lambda}$

Mass of ice melted $= \frac{E_0 x}{2e\lambda L_F}$

12. $2\left({}_1^2D\right) \rightarrow {}_1^3T + {}_1^1P$

Mass defect $\Delta M = M_{Product} - M_{Reactant}$

$= \left\{\left(3.016049\right) + 1.00785\right\} - 2\left[2.014102\right]$

$= 4.023899 - 4.028204$

$\Rightarrow \Delta m = 4.3 \quad 10^{-3}$ amu and 1 amu $\rightarrow$ 931.5

$E = \Delta mc^2 = 4.01$ MeV

${}^3T_1 + {}_1^2D \rightarrow {}_2^4He + {}_0^1n$

Δm(mass defect) $= \Delta M_{product} - \Delta M_{Reactant}$

$= [4.002603 + 1.008665] - [3.016049 + 2.014102]$

$= [5.011268 - 5.030151]$

$\Delta m = 0.018883$

$E = \Delta m(931.5) \Rightarrow 17.58$ MeV

$E_{deutron} = \frac{\Delta E_{total}}{3} = 7.2\text{MeV}$

$\frac{\text{Total Energy}}{\text{Total Mass}} \Rightarrow n = \frac{\Delta m_1 + \Delta m_2}{3{}_1^2D}$

$\Rightarrow n = \left(\frac{0.004305 + 0.018883}{3(2.014102)}\right) \times 100$

$= n = 0.384\%$

13. (Take a sample of 10^{20} Cm atoms)

α–decay ${}_{96}Cm^{248} \rightarrow {}_{96}Pu^{244} + {}_2He^4$

Here $\Delta m = (248.\ 072220 - 244.064100 - 4.002603)$ u $= 0.005517$ u

$\Rightarrow E_\alpha = \Delta mc^2 = 0.005517 \quad 931 = 5.136$ MeV

Energy released in the decay of one atom

$E = E_{fission} + E_\alpha = 0.08 \quad 200 + 0.92 \quad 5.136$

$= 20.725$ MeV

Total energy released from the decay of all 10^{20} atoms

$= 20.725 \quad 10^{20}$ MeV $= 3.316 \quad 10^8$J

Power output $= \frac{\text{Total energy released}}{\text{mean life}}$

$= \frac{3.316 \times 10^{-8}}{10^{-13}} = 3.3 \quad 10^{-5}$ watts

14. Let the amount of R_x^{222} be N_0

$N_{7.6} = \frac{N_0}{2^2} = \frac{N_0}{4}$

Nuclei left after 7.6 day $= \frac{N_0}{4}$

Decay constant

$= \frac{\ell n2}{t_{1/2}} = \frac{0.693}{3.8 \times 24 \times 3600}$ /sec

$\lambda = 2.11 \quad 10^{-6}$ sec

Activity $= \frac{N_0}{4}\lambda =$ No. of α particles

No. of neutrons produced

$= \frac{N_0 \times 2.11 \times 10^{-6}}{4 \times 4000} = 1.2 \times 10^6$

$N_0 = 9.095 \quad 10^{15}$ nuclei $= 3.354 \quad 10^{-6}$g

15. $N_1 = M_0 e^{-\lambda t}$

Thus momentum of this mass

$= Mv$, v= velcity of mass at time t $= M_0 e^{-\lambda t} v$

Let at time dt, defect mass is ejected with relative velocity v_0

$v_{eject/gram} = -v_0 + v$

Linear momentum of M at time (t+dt)

$= (M - dm)(v + dv) + dm(v - v_0)$

Since $f_{ext} = 0$

$(M - dm)(v + dv) + dm(v - v_0) = Mv \Rightarrow (M - dm)\,dv = dmv_0$

$\Rightarrow dv = \frac{v_0 dm}{(M - dm)} \Rightarrow \frac{dv}{dt} = \frac{dm}{dt}\left(\frac{v_0}{M - dm}\right) = \frac{r \cdot v_0}{M - rdt}$

$v = v_0 \ell n \frac{M_0}{M_0 - rt}$

But $M_0 - rt = M_0 e^{-\lambda t}$; $v = v_0 \ell n \frac{M_0}{M_0 e^{-\lambda t}} = v_0 \lambda t$

16. Production of radioactive nuclei $= \alpha$/sec

Disintegration $= \lambda N_A$

At any time t $= \frac{dN_A}{dt} = \alpha - \lambda N_A$

$\Rightarrow \int_0^{N_A} \frac{dN_A}{\alpha - \lambda N_A} = \int_0^t dt \Rightarrow N_A = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$

Nuclei disintegrated $= \alpha t - N_A = \alpha\left[t - \frac{1}{\lambda}\left(1 - e^{-\lambda t}\right)\right]$

Total energy produced $= E_0\left[\alpha t - \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)\right]$

Energy used in water heating

$= 0.2E_0\left[\alpha t - \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)\right] = ms\Delta T$

$\Delta T = \frac{0.2E_0\left[\alpha t - \alpha/\lambda\left(1 - e^{-\lambda t}\right)\right]}{mS}$

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EXERCISE –V(A)

ATOMIC STRUCTURE & X-RAY

1. Energy required to ionize a atom from n^{th} orbit is $= + \dfrac{13.6}{n^2} eV$

$$E_2 = \frac{+13.6}{2^2} eV = +3.4 eV$$

2. Energy required to remove an electron from an orbit is + $\dfrac{13.6(Z)^2}{n^2}$ eV . So, to remove the electron from the first excited state of Li^{2+} is

$$E = \frac{+13.6 \times 3^2}{2^2} = 3.4 \quad 9 = +\ 30.6\ eV$$

3. Ionization potential will be lowest for the atom in which the electrons are the farthest from the nucleus. So, the atom with the largest size will have the electron the farthest from the nuclei, hence to remove the electron from this atom will be easiest. So, the atom with least ionization potential is ${}^{133}_{55}Cs$.

4. The wavelengths involved in the spectrum of deuterium $\left({}^2_1D\right)$ are slightly different from that of hydrogen spectrum; because masses of two nuclei are different.

6. The energy of emitted photon is directly proportional to the difference of the two energy levels. This differences is maximum between level (2) and level (1) hence photon for maximum energy will be liberated for this transition only.

7. Whenever an atom gets de-excited from higher to the lower orbit, it emits radiation of a given frequency

$$\Delta E = hf \Rightarrow f \propto \Delta E$$

The photons of highest frequency will be emitted, for the transition in which ΔE is maximum, i.e., from 2 to 1.

8. $\dfrac{k}{r} = \dfrac{mv^2}{r} \Rightarrow mv^2 = k$ (independent or r)

$n\left(\dfrac{h}{2\pi}\right) = mvr \Rightarrow r \propto n$ and $T = \dfrac{1}{2}mv^2$ is independent of n.

10. Rquired energy = 13.6 $(Z)^2 \left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$ eV

$= 13.6\ (3)^2 \left[\dfrac{1}{1^2} - \dfrac{1}{3^2}\right] = 108.8$ eV

11. Number of lines = ${}^nC_2 = {}^4C_2 = \dfrac{(4)(3)}{(2)} = 6$

12. Energy $= \dfrac{L^2}{2I} = \dfrac{(n\hbar)^2}{2(\mu r^2)}$

where μ = reduced mass $= \dfrac{m_1 m_2}{m_1 + m_2}$

so energy $= \dfrac{n^2\hbar^2(m_1 + m_2)}{2m_1 m_2 r^2}$

13. Energy of radiation emitted

$$E = h\nu = E_0 Z^2\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = E_0 Z^2\left[\frac{2n-1}{n^2(n-1)^2}\right]$$

$$h\nu \approx E_0 Z^2\left[\frac{2n}{n^4}\right] \Rightarrow \nu \propto \frac{1}{n^3}$$

PHOTOELECTRIC EFFECT

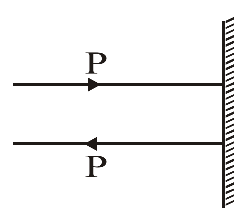
14. Work function $W_0 = hf_0 = \dfrac{hc}{\lambda_0} \Rightarrow \dfrac{W_{Na}}{W_{Cu}} = \dfrac{\lambda_{Cu}}{\lambda_{Na}}$

$$\frac{\lambda_{Na}}{\lambda_{Cu}} = \frac{W_{Cu}}{W_{Na}} = \frac{4.5}{2.3} \approx 2$$

15. Covalent bonding में electron cloud की overlapping region में electron के पाये जाने की probability density ज्यादा होती है जिसे हम wave nature of electron से समझ सकते हैं।

16. $hf_1 = \phi_0 + \dfrac{1}{2}mv_1^2 , hf_2 = \phi_0 + \dfrac{1}{2}mv_2^2 \Rightarrow v_1^2 - v_2^2 = \dfrac{2h}{m}(f_1 - f_2)$

17. Total change in momentum from a reflecting surface = 2P



The momentum of incident radiation $P = \dfrac{2E}{c}$

Hence, momentum transferred $= \dfrac{2E}{c}$

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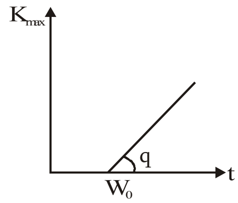
18. According to Einstein's equation of photoelectric effect $hf=W_0+K_{max}$
K_{max} = (h)f–W_0



The slope of K vs f graph is h which is a fundamental constant and same for all metals at all intensifies.

19. W_0=4eV, λ_{max} = ? ; $W_0 = \dfrac{hc}{\lambda_{max}}$

$$4eV = \frac{1240}{\lambda_{max}} nm - eV \quad [\because \text{ hc=1240 nm-eV}]$$

λ_{max} = 310 nm.

20. Photocurrent $\propto$ Intensity

Intensity due to point source at r m is $I \propto \dfrac{1}{r^2}$

As distance is reduced to half, the photocurrent due to point source will increase by a factor of 4.

21. The de-Broglie wavelengths and kinetic energy of a free electron are related as

$$\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{1}{2}} \Rightarrow \lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

22. According to Einstein's theory of photoelectric effect
Energy of incident photon = W_0 + eV_0
W_0 = 6.2 eV; eV_0 = 5 eV
Energy of incident photon = 11.2 eV
The wavelength corresponding to this energy is 110 nm which falls in the ultraviolet region.

23. The photoelectric phenomenon is an instantaneous phenomenon, hence the time taken by an electron to come out of metal is approximately 10^{-10} sec (found experimentally).

24. With the increase in wavelength energy of the photon decreases. Therefore the KE of the electron coming out from the cathode also decreases. Due to which there will be a small decrease in plate current and once λ becomes more than threshold wavelength electron will not come out from the cathode and hence current will become zero.

25. The relation between energy (E) of a photon and momentum (P) associated with the photon is
E = pc

The corresponding momentum $p = \dfrac{E}{c} = \dfrac{hv}{c}$

27. 2d cos i = nλ $\Rightarrow 2d\cos i = \dfrac{h}{\sqrt{meV}} \Rightarrow$V = 50 volt

28. 2d cos i = nλdB

29. $E_k = \dfrac{hc}{\lambda} - \phi_0 \Rightarrow 1.68 = \dfrac{12400}{4000} - \phi_0$
By solving it ϕ_0 = 1.42 eV

31. No. of photons emitting per second from a source of power P is n = (5 10^{24}) Pλ

$$\Rightarrow \text{wavelength emitting } \lambda = \frac{n}{(5 \times 10^{24})P} \left[\text{or } \lambda = \frac{nhc}{P}\right]$$

$$\Rightarrow \lambda = \frac{10^{20}}{5 \times 10^{24} \times 4 \times 10^3} = 0.5 \times 10^{-9} m = 50Å$$

And this wavlength comes in X ray region.

Radioactivity Nuclear Physics

35. $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^n$ where n are is number of half-lives.
If $T_{1/2}$ = 5 years then in 15 years, n=3

$$\Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} \Rightarrow N = \frac{N_0}{8}$$

36. Though the compound can emit all the four particles namely electrons, protons, He^{2+} and neutrons. But the particle neutron can't be deflected in the magnetic field, since it is a neutral particle. Hence, the deflectable particles are protons, electrons and He^{2+}.

38. $U^{238} \rightarrow U^{234} +_2 He^4$
Let recoil speed be V then by COLM

$$234V = 4u \Rightarrow V = \frac{4u}{234}$$

39. Rate of disintegration at any instant is directly proportional to the number of undecayed nuclei at that instant, i.e., $-\dfrac{dN}{dt} = \lambda N$
Where λ = decay constant
We are given that at t=0

$$-\frac{dN}{dt} = \lambda N_0 = 5000 \frac{\text{disintegration}}{\text{minute}}$$

at t =5 minute

$$-\frac{dN}{dt} = \lambda N = 1250 \frac{\text{disintegration}}{\text{minute}}$$

$$N = N_0 e^{-\lambda t}$$

On multiplying both sides by λ we get

$(\lambda N) = (\lambda N_0)e^{-\lambda t} \Rightarrow 1250 = 5000\ e^{-\lambda t} \Rightarrow \frac{1}{4} = e^{-\lambda t}$

Taking logarithm on both sides we get

$\ln 1 - \ln 4 = e^{-\lambda(5)} \Rightarrow 0 - \ln 4 = -5\lambda \Rightarrow \lambda = \frac{1}{5}\ln 2^2$

$\Rightarrow \lambda = 0.4 \ln 2.$

40. 8α have been emitted
$4\beta^-$ have been emitted
$2\beta^+$ have been emitted
α reduces atomic number by 2
β^- increases atomic number by 1
β^+ decreases atomic number by 1
So, $Z_{eff} = 92 - (8\ 2) + (4\ 1) - (2\ 1) = 96 - 18 = 78$

41. A nucleus during decay emits $\alpha(He^{2+})$, β (electrons), γ or neutrino; it does not emit protons during decay.

42. In order to fuse two nuclei against repulsion, the repulsive potential energy has to be supplied by kinetic energy i.e., PE=KE

$\Rightarrow 7.7\quad 10^{-14} = \frac{3}{2}kT$

$\Rightarrow 7.7\quad 10^{-14} = \frac{3}{2}\quad 1.38\quad 10^{-23}T$

$\Rightarrow T = 3.7\quad 10^{9}$ k

43. Applying conservation of momentum, we get

$m_1v_1 = m_2v_2 \Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{1}{2}$

Also, $R \propto m^{1/3} \Rightarrow m \propto R^3$

$\frac{m_1}{m_2} = \frac{1}{2} = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \frac{R_1}{R_2} = \left(\frac{1}{2}\right)^{1/3} = 1 : 2^{1/3}$

44. Given that ${}^2H_1 + {}^2H_1 \rightarrow {}^4He_2 +$ energy
Total B.E. of deuterium nucleus = 2.2 MeV
Total B.E. of He nucleus = 28 MeV
On conservating energy on both sides we get
$(\text{Energy})_{\text{Deutron}}\quad 2 = (\text{Energy})_{\text{He}} +$ Energy released
$\Rightarrow 4.4 = 28 + E$
$\Rightarrow E = 28 - 4.4 = 23.6$ MeV

45. At the closest point of approach
Initial KE = Final PE

$\therefore 5\quad 10^6\quad 1.6\quad 10^{-19} = \frac{kq_1q_2}{r_0}$

$r_0 = \frac{9\times10^9\times92\times1.6\times10^{-19}\times2\times1.6\times10^{-19}}{5\times10^6\times1.6\times10^{-19}}$

$= 5.3\quad 10^{-14}$ m $= 5.3\quad 10^{-12}$ cm.

46. Intensity of gamma radiation when it passes through x is $I = I_0e^{-\mu x}$

$\Rightarrow \frac{I_0}{8} = I_0e^{-\mu(36)}$ ...(i) and $\frac{I_0}{2} = I_0e^{-\mu(x)}$ ...(ii)

$\frac{1}{8} = e^{-36\mu}$ and $\frac{1}{2} = e^{-\mu x} \Rightarrow e^{-3\mu x} = e^{-36\mu} \Rightarrow x = 12$ mm

47. We know that, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$ where, N are the radioactive nuclei left after n-half lives
N_0 are the initial number of nuclei and n is number of half-lives, $\frac{N_0/8}{N_0} = \frac{1}{8} = \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n \Rightarrow n = 3$

Therefore $T_{1/2} = \frac{15}{3} = 5$ min.

48. Radius of a nuclei $\propto$ (Mass Number)$^{1/3}$

$\frac{R_{Te}}{R_{Al}} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3} \Rightarrow R_{Te} = R_{Al}\times\frac{5}{3} = 6$ fermi

49. $X(n,\alpha) \rightarrow {}^7_3Li$

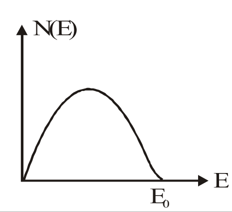
${}^1_0n + {}^A_ZX \rightarrow {}^7_3Li + {}^4_2He$
On conservating atomic number and mass numbers on both sides, we get
A = (7+4) –1 = 10
Z = (3+2) – 0 = 5
Hence, ${}^A_ZX = {}^{10}_5B$

50. Charge on α-particle = 2e
Charge on target nucleus = Ze
When the α-particle is projected towards the target nucleus, then at $r=r_0$, the α-particle comes to momentary rest. This position r_0 from target nucleus is known as distance of closest approach.
Applying law of conservation of energy, we get

$\frac{1}{2}mv^2 = \frac{K(ze)(2e)}{r_0} \Rightarrow r_0 \propto \frac{1}{v^2}, r_0 \propto Ze, r_0 \propto \frac{1}{m}$

51. ${}^7Li_3 + {}^1_1P \rightarrow {}^8_4Be + \gamma$
The particle that will be emitted with Beryllium will be Gamma radiations.

52. The energy spectrum of β-particles emitted from a radioactive source is



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53. In a nuclear reaction the energy remains conserved

$p + {}^{7}_{3}Li \rightarrow 2\left({}^{4}_{2}He\right)$

energy of protons + 7(5.60) =2(4 7.06)

$\Rightarrow$ energy of proton = 17.28 MeV

54. Nuclear binding energy =

[Expected mass of nucleus-Actual mass of nucleus]c^2

Expected mass of nucleus = $8M_p + 9M_n$

Actual mass of nucleus = M_0

Nuclear binding energy = $[8M_p+9M_n-M_0]c^2$

55. Gamma ray is an electromagnetic radiation, due to the emission of gamma ray, neither the mass number not the atomic number changes. Though the daughter nucleus is same as parent nucleus but still there is a difference in the two, i.e., the daughter nucleus so obtained is present in one of the excited states and not in the ground state.

56. Given that

$$\left(T_{1/2}\right)_x = \left(T_{av}\right)_Y \Rightarrow \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y} \Rightarrow \lambda_Y \frac{1}{0.693}\lambda_X$$

$$\Rightarrow \lambda_Y = 1.44\lambda_X \Rightarrow R_Y = 1.44\ R_X$$

As decay rate of Y is more than the element X; hence Y will decay faster than X.

59. Total kinetic energy of products

$$= \text{Total energy released } \frac{p^2}{2m} + \frac{p^2}{2m}$$

$$= (\text{mass defect})\ c^2 \ (\text{where } m = \frac{M}{2} \text{ given})$$

$$\Rightarrow 2\left(\frac{p^2}{2m}\right) = \left[(M + \Delta m) - \left(\frac{M}{2} + \frac{M}{2}\right)\right] \times c^2$$

$$\Rightarrow 2 \times \left[\frac{p^2}{2\left(\frac{M}{2}\right)}\right] = (\Delta m)c^2$$

$$\Rightarrow \frac{2\left(\frac{M}{2}v\right)^2}{M} = (\Delta m)c^2 \Rightarrow v = c\sqrt{\frac{2\Delta m}{M}}$$

60. Because energy is releasing$\Rightarrow$ Binding energy per nucleon of product > that of parent $\Rightarrow E_2 > E_1$.

61. $_{Z}X^{A} \xrightarrow[3({}_{2}\alpha^{4})]{} {}_{Z-6}(\ )^{A-12} \xrightarrow[2({}_{+1}\beta^{0})]{} {}_{Z-8}(\ )^{A-12}$

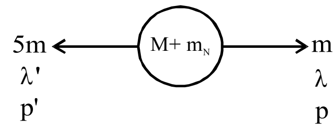
$$\therefore \frac{\text{No. of neutrons}}{\text{No. of protons}} = \frac{(A-12)-(Z-8)}{Z-8} = \frac{A-Z-4}{Z-8}$$

62. $\because \frac{N}{N_0} = \left[\frac{1}{2}\right]^{t/T} \quad \therefore \frac{1}{3} = \left[\frac{1}{2}\right]^{\frac{t_2}{T}} \ \& \ \frac{2}{3} = \left[\frac{1}{2}\right]^{\frac{t_1}{T}}$

$$\Rightarrow \frac{1}{2} = \left[\frac{1}{2}\right]^{(t_2-t_1)\frac{1}{T}} \Rightarrow 1 = \frac{(t_2-t_1)}{T} \Rightarrow T = t_2 - t_1$$

$\Rightarrow t_2 - t_1$ = 20 min.

63.



According to Conservation of linear momentum

P' = P. Therefore λ' = λ

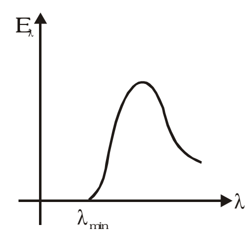
65. Released energy

= [1.6747 10^{-27} – 1.6725 10^{-27} – 9 10^{-31}]

(3 $10^8)^2$ J = 0.73 MeV

EXERCISE –V-B

1. The continuous X–ray spectrum is shown in figure. All wavelengths > λ_{min} are found, where



$$\lambda_{min} = \frac{12375}{V(\text{in volt})}\text{Å}$$

Here, V is the applied voltage.

2. $\lambda(\text{in Å}) = \frac{12375}{W\,(eV)} = \frac{12375}{4.0}\text{Å} \approx 3093\,\text{Å}$

$\Rightarrow \lambda \approx 309.3$ nm $\Rightarrow \lambda = 310$ nm

3. Number of nuclei decreases experientially

$N = N_0 e^{-\lambda t}$ and Rate of decay $\left(-\frac{dN}{dt}\right) = \lambda N$

Therefore, decay process lasts upto t=∞. Therefore a given nucleus may decay at any time after t=0.

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4. Since, the wavelength (λ) is increasing we can say that the galaxy is receding. Doppler effect can be given by–

$$\lambda' = \lambda\sqrt{\frac{c+v}{c-v}} \Rightarrow 706 = 656\sqrt{\frac{c+v}{c-v}} \quad ...(i)$$

$$\Rightarrow \frac{c+v}{c-v} = \left(\frac{706}{656}\right)^2 = 1.16$$

$\therefore$ c + v = 1.16 c – 1.16 v

$$\therefore v = \frac{0.16c}{2.16} = \frac{0.16 \times 3.0 \times 10^8}{2.16} m/s$$

$v \approx 2.2 \quad 10^7$ m/s

If we take the approximation then equation (i) can be written as $\Delta\lambda = \lambda\left(\frac{v}{c}\right)$ ...(ii)

From here $v = \left(\frac{\Delta\lambda}{\lambda}\right).c = \left(\frac{706-656}{656}\right)(3\times10^8)$ m/s

$\Rightarrow v = 0.23 \quad 10^8$ m/s

Which is almost equal to the previous answer.

So, we may use equation (ii) also.

5. Atomic number of neon is 10.

By the emission of two α– particles, atomic number will be reduced by 4. Therefore, atomic number of the unknown element will be Z = 10 – 4 = 6

Similarly mass number of the unknown element will be A = 22 – 2 4 = 14

$\therefore$ Unknown nucleus is carbon (A = 14, Z=6).

6. From law of conservation of momentum,

$P_1 = P_2$ (in opposite directions)

Now de–Broglie wavelength is given by $\lambda = \frac{h}{p}$

[h = Plank's constant]

Since, momentum (p) of both the particles is equal, therefore $\lambda_1 = \lambda_2 \Rightarrow \lambda_1/\lambda_2 = 1$

7. Both the beta rays and the cathode rays are made up of electrons. So, only option (a) is correct.

(b) Gamma rays are electromagnetic waves.

(c) Alpha particles are doubly ionized helium atoms and

(d) Protons and neutrons have approximately the same mass.

8. $(t_{1/2})_x = (t_{mean})_y \Rightarrow \frac{0.693}{\lambda_x} = \frac{1}{\lambda_y} \quad \therefore \lambda_x = 0.693\,\lambda_y$

$\Rightarrow \lambda_x < \lambda_y \Rightarrow$ Rate of decay = λN

Initially number of atoms (N) of both are equal but since $\lambda_y > \lambda_x$, therefore, y will decay at a faster rate than x.

9. Radius of a nucleus is given by

$R = R_0A^{1/3}$ (where $R_0 = 1.25 \quad 10^{-15}$ m)

$= 1.25\ A^{1/3} \quad 10^{-15}$m

Here, A is the mass number and mass of the uranium nucleus will be $m \approx Am_p$

m_p = mass of proton = A(1.67 10^{-27} kg)

$$\therefore \text{Density } \rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{A(1.67\times10^{-27}\text{kg})}{A(1.25\times10^{-15}\text{m})^3}$$

$\Rightarrow \rho \approx 2.0 \quad 10^{17}$ kg/m^3

10. Energy is released in a process when total binding energy of the nucleus (= binding energy per nucleon number of nucleons) is increased or we can say, when total binding energy of products is more than the reactants. By calculation we can see that only in case of option (c), this happens. Given : W $\rightarrow$ 2Y

Binding energy of reactants = 120 7.5 = 900 MeV

and binding energy of products

= 2(60 8.5) = 1020 MeV > 900 MeV.

11. In hydrogen atom $E_n = -\frac{Rhc}{n^2}$. Also $E_n \propto m$

where m is the mass of the electron.

Here, the electron has been replaced by a particle whose mass is double of an electron. Therefore, for this hypothetical atom energy in n[th] orbit will be given by $E_n = -\frac{2Rhc}{n^2}$

The longest wavelength λ_{max} (or minimum energy) photon will correspond to the transition of particle from n=3 to n=2.

$$\therefore \frac{hc}{\lambda_{max}} = E_3 - E_2 = Rhc\left(\frac{1}{2^2}-\frac{1}{3^2}\right) \Rightarrow \lambda_{max} = \frac{18}{5R}$$

12. $v_n \propto \frac{1}{n} \therefore KE \propto \frac{1}{n^2}$ (with positive sign)

Potential energy U is negative and

$$U_n \propto \frac{1}{r_n} \Rightarrow \left[U_n = \frac{-1}{4\pi\epsilon_0}.\frac{Ze^2}{r_n}\right] \propto \frac{1}{n^2}$$

[because $r_n \propto n^2$] (with negative sign)

Similarly total energy $E_n \propto \frac{1}{n^2}$ (with negative sign)

Therefore, when an electron jumps from some excited state to the ground state, value of n will decrease. Therefore, kinetic energy will increase (with positive sign), potential energy and total energy will also increase but with negative sign. Thus, finally kinetic energy will increase, while potential and total energies will decrease.

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13. Minimum wavelength of continuous X–ray spectrum is given by $\lambda_{min}\text{ (in Å)} = \dfrac{12375}{E(\text{in eV})}$

Here, E = energy of incident electrons (in eV)
= energy corresponding to minimum wavelength
$\Rightarrow \lambda_{min}$ of X–rays $\Rightarrow$ E = 80 keV = 80 10^3 eV

$\therefore \quad \lambda_{min}\text{ (in Å)} = \dfrac{12375}{80 \times 10^3} \approx 0.155$

Also the energy of the incident electrons (80 keV) is more than the ionization energy of the K–shell electrons (i.e. 72.5 keV). Therefore, characteristic X–ray spectrum will also be obtained because energy of incident electron is high enough to knock out the electron from K or L–shells.

14. $\dfrac{N_{x_1}(t)}{N_{x_2}(t)} = \dfrac{1}{e} \Rightarrow \dfrac{N_0 e^{-10\lambda t}}{N_0 e^{-\lambda t}} = \dfrac{1}{e}$

(Initially both have same number of nuclei say N_0)
$\Rightarrow e = ^{-\lambda t}/e^{-10\lambda t} \Rightarrow e = e^{9\lambda t}$, $\lambda_{x1} = 10\lambda$ and $\lambda_{x2} = \lambda$

$\Rightarrow 9\lambda\, t = 1 \Rightarrow t = \dfrac{1}{9\lambda}$

15. Energy of infrared radiation is less than the energy of ultraviolet radiation. In options (a), (b) and (c), energy released will be more, while in option (d) only, energy released will be less.

16. Wavelength λ_k is independent of the accelerating voltage (V), while the minimum wavelength λ_c is inversely proportional to V. Therefore, as V is increased, λ_k remains unchanged where as λ_c decreases or $\lambda_k - \lambda_c$ will increase.

17. During β–decay, a neutron is transformed into a proton and an electron. This is why atomic number (Z = number of protons) increases by one and mass number (A = number of protons + neutrons) remains unchanged during beta decay.

18. The total number of atoms can neither remain constant (as in option a) nor can ever increase (as in options b and c). They will continuously decrease with time. Therefore, (d) is the appropriate option.

19. In second excited state n=3,

So, $\quad \ell_H = \ell_{Li} = \left(\dfrac{3h}{2\pi}\right)$

while $\quad E \propto Z^2$ and $Z_H = 1,\ Z_{Li} = 3$
So, $\quad |E_{Li}| = 9\ |E_H|$ or $|E_H| < |E_{Li}|$

20. $i = \dfrac{q}{t} = \dfrac{ne}{t} \Rightarrow n = \dfrac{it}{e}$

Substituting i = 3.2 10^{-3}A
e = 1.6 10^{-19} C and t =1s
we get, n =2 10^{16}

21. $R = R_0\left(\dfrac{1}{2}\right)^n$ ...(i)

Here R = activity of radioactive substance after n half–lives $= \dfrac{R_0}{6}$ (given)

Substituting in equation (i), we get n =4
$\therefore$ t = $(n)t_{1/2}$ = (4) (100 μs) = 400 μs

22. During γ–decay atomic number (Z) and mass number (A) does not change. So, the correct option is (c) because in all other options either Z,A or both is/are changing.

23. $U = eV = eV_0\, \ell n\left(\dfrac{r}{r_0}\right) \Rightarrow \quad |F| = \left|-\dfrac{dU}{dr}\right| = \dfrac{eV_0}{r}$

this force will provide the necessary centripetal force.

Hence, $\dfrac{mv^2}{r} = \dfrac{eV_0}{r} \Rightarrow v = \sqrt{\dfrac{eV_0}{m}}$ ...(i)

Moreover, $mvr = \dfrac{nh}{2\pi}$ ....(ii)

Dividing equation (ii) by (i).

We have : $mr = \left(\dfrac{nh}{2\pi}\right)\sqrt{\dfrac{m}{eV_0}} \Rightarrow r_n \propto n$

24. $(r_m) = \left(\dfrac{m^2}{z}\right)(0.53\text{ Å}) = (n \quad 0.53)\text{Å} \therefore \dfrac{m^2}{z} = n$

m =5 for $_{100}Fm^{257}$ (the outermost shell) and z=100

$\therefore n = \dfrac{(5)^2}{100} = \dfrac{1}{4}$

25. Nuclear density is constant hence, mass ∝ volume $\Rightarrow m \propto V$.

26. Given that $K_1 + K_2$ = 5.5 MeV ...(i)
From conservation of linear momentum, $p_1 = p_2$
$\Rightarrow \sqrt{2K_1(216m)} = \sqrt{2K_2(4m)}$ as $P = \sqrt{2Km}$
$\therefore K_2 = 54\ K_1$ ...(ii)
Solving equation (i) and (ii).
We get K_2 = KE of α–particle = 5.4 MeV

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27. Saturation current is proportional to intensity while stopping potential increases with increase in frequency. Hence, $f_a = f_b$ while $I_a < I_b$

28. $\frac{\lambda_1}{\lambda_2} = \frac{\frac{h}{\sqrt{2mE}}}{\frac{hc}{E}}$ or $\frac{\lambda_1}{\lambda_2} \propto E^{1/2}$

29. Activity reduces from 6000 dps to 3000 dps in 140 days. It implies that half–life of the radioactive sample is 140 days. In 280 days (or two half–lives) activity will remain $\frac{1}{4}$ th of the initial activity. Hence, the initial activity of the sample is

4 6000 dps = 24000dps

30. The first photon will excite the hydrogen atom (in ground state) in first excited state (as $E_2 - E_1 = 10.2eV$). Hence during de–excitation a photon of 10.2eV will be released. The second photon of energy 15eV can ionise the atom. Hence the balance energy i.e.,(15 – 13.6)eV = 1.4 eV is retained by the electron. Therefore, by the second photon an electron of energy 1.4 eV will be released.

31. $\frac{1}{\lambda} \propto (Z-1)^2 \therefore \frac{\lambda_1}{\lambda_2} = \left(\frac{Z_2 - 1}{Z_1 - 1}\right)^2 \Rightarrow \frac{1}{4} = \left(\frac{Z_2 - 1}{11 - 1}\right)^2$

Solving this, we get $Z_2 = 6$

32. $4({}_2He^4) = {}_8O^{16}$

Mass defect

$\Delta m = \{4(4.0026) - 15.9994\} = 0.011$ amu

$\therefore$ Energy released per oxygen nuclei

= (0.011) (931. 48) MeV = 10.24 MeV

33. After two half lives $1/4^{th}$ fraction of nuclei will remain undecayed. $3/4^{th}$ fraction will decay. Hence, the probability that a nucleus decays in two half lives is 3/4.

34. The series in U–V region is Lymen series. Longest wavelength corresponds to minimum energy which occurs in transition from n=2 to n=1.

$$\therefore 122 = \frac{\frac{1}{R}}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} \quad \ldots(i)$$

The smallest wavelength in the infrared region corresponds to maximum energy of Paschen series.

$$\therefore \lambda = \frac{\frac{1}{R}}{\left(\frac{1}{3^2} - \frac{1}{\infty}\right)} \quad \ldots(ii)$$

Solving equation (i) and (ii), we get $\lambda = 823.5$ nm

35. Momentum of striking electrons $p = \frac{h}{\lambda}$

$\therefore$ Kinetic energy of striking electrons

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

This also, maximum energy of X–ray photons.

Therefore, $\frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2} \Rightarrow \lambda_0 = \frac{2m\lambda^2 c}{h}$

36. Rest mass of parent nucleus should be greater than the rest mass of daughter nuclei.

37. As for continuous X–rays $\lambda_{min} = \frac{hc}{eV}$ so cut off wavelength depends on the accelerating potential and is independent of nature of target.

38. Activity = λN & $T_{1/2} = \frac{\ell n 2}{\lambda}$

So $5 = \frac{\ell n2}{T_1}(2N_0)$ & $10 = \frac{\ell n2}{T_2}(N_0) \Rightarrow T_1 = 4T_2$

39. 550 nm light cannot emit electron for metal of work function $\phi = 3eV$, so saturate current decreases for P to Q to R. Also $|V_{sp}| > |V_{sq}| > |V_{sr}|$

40. Energy incident = Power(time)

= (30 mW) (100 ns)

= 3 10^{-9}

So momentum = $\frac{E}{c} = \frac{3\times10^{-9}}{3\times10^{8}} = 10^{-17}$ kg m / sec.

MCQ

1. Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles. ${}^{20}_{1.0}Ne$ is made up of 10 protons plus 10 neutrons.

Therefore, mass of ${}^{20}_{10}Ne$ nucleus,

$$M_1 < 10\,(m_p + m_n)$$

Also, heavier the nucleus, more is the mass defect.

Thus, $20\,(m_n + m_p) - M_2 > 10\,(m_p + m_n) - M_1$

$\Rightarrow 10\,(m_p + m_n) > M_2 - M_1 \Rightarrow M_2 < M_1 + 10(m_p + m_n)$

Now since $M_1 < 10\,(m_p + m_n)$ $\therefore$ $M_2 < 2M_1$

2. Time period,

$T_n = \dfrac{2\pi r_n}{v_n}$ (in n^{th} state) i.e. $T_n \propto \dfrac{r_n}{v_n}$

But $r_n \propto n^2$ and $v_n \propto \dfrac{1}{n}$

Therefore, $T_n \propto n^3$ Given $T_{n1} = 8T_{n2}$

Hence, $n_1 = 2n_2$

3. From the relation,

$eV = \dfrac{hc}{\lambda} - \phi \Rightarrow V = \left(\dfrac{hc}{e}\right)\left(\dfrac{1}{\lambda}\right) - \dfrac{\phi}{e}$

This is equation of straight line.

Slope is $\tan\theta = \dfrac{hc}{e}$

$\phi_1 : \phi_2 : \phi_3 = \dfrac{hc}{\lambda_{01}} : \dfrac{hc}{\lambda_{02}} : \dfrac{hc}{\lambda_{03}} = 1 : 2 : 4$

$\dfrac{1}{\lambda_{01}} = 0.001\text{nm}^{-1} \Rightarrow \lambda_{01} = 10000$ Å

$\dfrac{1}{\lambda_{02}} = 0.002\text{nm}^{-1} \Rightarrow \lambda_{02} = 5000$Å

$\dfrac{1}{\lambda_{03}} = 0.004\text{nm}^{-1} \Rightarrow \lambda_{03} = 2500$ Å

Violet colour has wavelength 4000 Å.

So violet colour can eject photoelectrons from metal–1 and metal–2.

4. (A) For 1 < A < 50, on fusion mass number for compound nucleus is less than 100

$\Rightarrow$ Binding energy per nucleon remains same

$\Rightarrow$ No energy is released

(B) For 51 < A < 100, on fusion mass number for compound nucleus is between 100 & 200

$\Rightarrow$ Binding energy per nucleon increases

$\Rightarrow$ Energy is released.

(C) For 100 < A < 200, on fission, the mass number of product nuclei will be between 50 & 100

$\Rightarrow$ Binding energy per nucleon decreases

$\Rightarrow$ No energy is released

(D) For 200 < A < 260, on fission, the mass number of product nuclei will be between 100 & 130

$\Rightarrow$ Binding energy per nucleon increases

$\Rightarrow$ Energy is released.

5 Angular momentum $= \dfrac{nh}{2\pi} = \dfrac{3h}{2\pi} \Rightarrow n = 3$

Also $r = \dfrac{n^2}{2}a_0 \Rightarrow \dfrac{9a_0}{2} = \dfrac{3^2}{Z}a_0 \Rightarrow Z = 2$

For de-excitation

$\dfrac{1}{\lambda} = Rz^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right] = 4R\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

For n = 3 to n = 1:

$\dfrac{1}{\lambda} = 4R\left[\dfrac{1}{1} - \dfrac{1}{9}\right] \Rightarrow \lambda = \dfrac{9}{32R}$

For n = 3 to n = 2:

$\dfrac{1}{\lambda} = 4R\left[\dfrac{1}{4} - \dfrac{1}{9}\right] \Rightarrow \lambda = \dfrac{9}{5R}$

For n = 2 to n = 1:

$\dfrac{1}{\lambda} = 4R\left[\dfrac{1}{1} - \dfrac{1}{4}\right] \Rightarrow \lambda = \dfrac{3}{R}$

Match the column

2. (i) Energy of capacitor is less $1/2\ cv^2$ therefore p.

(ii) work is done on the gas hence energy increases therefore q.

(iii) When mass decreases its energy increases.

(iv) when current flows energy of magnetic field is generated therefore t.

(B) work is done on the gas

(C) mass is reduced and mass defect is converted into energy.

(D) mass decreases due to mass defect.

Matching List Type

1. Completing reaction in list II

(1) ${}^{15}_{8}O \rightarrow {}^{15}_{7}N + (\beta^+ \text{ decay})$

(2) ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + (\text{Alpha decay})$

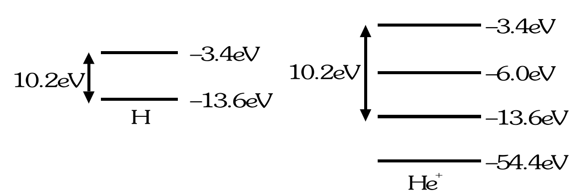
(3) ${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + (\text{Proton emission})$

(4) ${}^{239}_{94}Pu \rightarrow {}^{140}_{57}La + (\text{Fission})$

Comprehension Based Question

Comprehension#1

1.



For H atom ΔE = 10.2 eV, This goes to excite He^+ ion from n = 2 to n = 4

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2. For visible region $\Delta E < 10.2$ eV and in this case $\Delta E = -3.4 - (-6) = 2.6$ eV

$$\lambda = \frac{hc}{\Delta E} = \frac{12400 eV - Å}{2.6 eV} \simeq 4800Å \simeq 4.8 \quad 10^{-7} \text{ m}$$

3. Ratio of kinetic energy $\frac{K_1}{K_2} = \frac{(Z_1/n_1)^2}{(Z_2/n_2)^2}$

Since $n_1 = n_2 = 2$ & $Z_1 = 1$

for H, $Z_2 = 2$ for $He^+ \Rightarrow \frac{K_1}{K_2} = \frac{1}{4}$

Comprehension#2

1. Due to the high temperature developed as a result of collision & fusion causes the core of fusion reactor to plasma.

2. $\left(\frac{3}{2}KT\right)2 = \frac{Kq_1q_2}{r} \quad 3KT = \frac{Kq_1q_2}{r}$

$$T = \frac{1.44 \times 10^{-9}}{4 \times 10^{-15} \times 3 \times 8.6 \times 10^{-5}}$$

$T = 1.39 \quad 10^9$ K $\boxed{1 \times 10^9 < T < 2 \times 10^9 K}$

3. $nt_0 = 8 \quad 10^{14} \quad 9 \quad 10^{-1} = 7.2 \quad 10^{14} > 5 \quad 10^{14}$
$nt_0 = 4 \quad 10^{23} \quad 1 \quad 10^{11} = 4 \quad 10^{34} > 5 \quad 10^{14}$
$nt_0 = 1 \quad 10^{24} \quad 4 \quad 10^{12} = 4 \quad 10^{36} > 5 \quad 10^{14}$

Comprehension#3

1. $E = \frac{p^2}{2m}$ .... (i) $\qquad p = \frac{h}{\lambda}$ ... (ii)

By equation (i) and (ii)

$$E = \frac{h^2}{2m\lambda^2} \Rightarrow \frac{h^2(n^2)}{2m(4a^2)}$$

[$\because \frac{n\lambda}{2}$ = a for stationary wave on string fixed at both end] $E \propto a^{-2}$

2. $E = \frac{h^2}{2m4a^2} \Rightarrow \frac{(6.6 \times 10^{-34})^2}{2(1.0 \times 10^{-30}) \times 4 \times (6.6 \times 10^{-9})^2} \frac{1}{e}$

E = 8 meV

3. $mv = \frac{h}{\lambda} \Rightarrow v = \frac{h}{m\lambda} \Rightarrow \frac{hn}{m(2a)} \Rightarrow v \propto n$

Comprehension 4

1. $E_n = \frac{1}{2}I\omega^2 = \frac{(I\omega)^2}{2I} = \frac{(nh/2\pi)^2}{2I} = \frac{n^2h^2}{8\pi^2 I}$

2. $h\nu = E_2 - E_1 \Rightarrow h\nu = \frac{h^2}{8\pi^2 I}(4-1)$

$$\Rightarrow h\nu = \frac{h^2}{8\pi^2 I}(4-1)$$

$$\Rightarrow I = \frac{3h}{8\pi^2 \nu} = \frac{3 \times 2\pi \times 10^{-34}}{8\pi^2 \times \frac{4}{\pi} \times 10^{11}} = 1.87 \times 10^{-46} kgm^2$$

3. Moment of inertia of CO molecule about centre of mass : $I = \mu r^2$ where $\mu = \frac{m_1 m_2}{m_1 + m_2}$

$$\Rightarrow r = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{I}{\frac{m_1 m_2}{m_1 + m_2}}} = \sqrt{\frac{1.87 \times 10^{-46}}{\frac{12 \times 16 \times 5/3 \times 10^{-27}}{12 + 16}}}$$

$$= 1.3 \times 10^{-10} m$$

Paragraph 5

1. ${}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He + Q$

Total energy released = $(M_{Po} - M_{Pb} - M_{He})C^2$

= [(209.982876) – (205.974455 + 4.002603)] 932 MeV

= [0.005818] 932 MeV = 5.422376 MeV

Kinetic energy of α particle = $\left(\frac{A-4}{A}\right)Q = \left(\frac{206}{210}\right)5.422376 \text{ MeV}$ = 5.319 MeV = 5319 KeV

2. Only in option (C); sum of masses of product is less then sum of mases of reactant

for reaction ${}^{2}_{1}H + {}^{4}_{2}He \rightarrow {}^{6}_{3}Li$. As $M_{Li} < M_{Deutron} + M_{alpha}$

Subjective

1. (i) Let at time t, number of radioactive nuclei are N.
Net rate of formation of nuclei of A

$$\frac{dN}{dt} = \alpha - \lambda N \Rightarrow \frac{dN}{\alpha - \lambda N} = dt \Rightarrow \int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

Solving this equation, we get

$$N = \frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}] \ldots (i)$$

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(ii) (i) Substituting $\alpha = 2\lambda N_0$ and $t = t_{1/2} = \dfrac{\ell n(2)}{\lambda}$ in

equation (i) we get, $N = \dfrac{3}{2}N_0$

(ii) Substituting $\alpha = 2\lambda N_0$

and $t \to \infty$ in equation (i), we get

$$N = \frac{\alpha}{\lambda} = 2N_0 \Rightarrow N = 2N_0$$

2. Given work function W = 1.9 eV

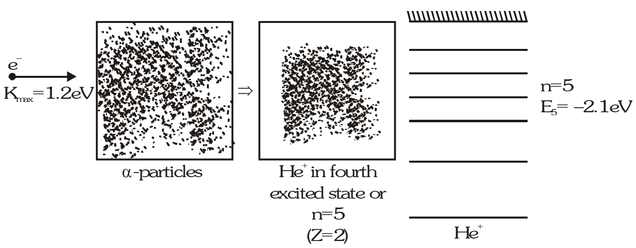
Wavelength of incident light, λ = 400 nm

$\therefore$ Energy of incident light, $E = \dfrac{hc}{\lambda} = 3.1 eV$

(Substituting the values of h, c and λ)

Therefore, maximum kinetic energy of photoelectron $K_{max} = E - W = (3.1 - 1.9) = 1.2$ eV

Now the situation is as shown below :



Energy of electron in 4^{th} excited state of He^+ (n=5)

will be $E_5 = -13.6\dfrac{Z^2}{n^2}$ eV

$$E_5 = -(13.6)\ \frac{(2)^2}{(5)^2} = 2.2 \text{ eV}$$

Therefore, energy released during the combination

= 1.2 – (–2.2) = 3.4 eV

Similarly energies in other energy states of He^+ will

be $E_4 = -13.6\dfrac{(2)^2}{(4)^2} = -3.4$ eV

$$E_3 = -13.6\frac{(2)^2}{(3)^2} = -6.04 \text{ eV}$$

$$E_2 = -13.6\frac{(2)^2}{2^2} = -13.6 \text{eV}$$

The possible transitions are

$\Delta E_{5\to4} = E_5 - E_4 = 1.3 \text{ eV} < 2 \text{ eV}$

$\Delta E_{5\to3} = E_5 - E_3 = 3.84$ eV

$\Delta E_{5\to2} = E_5 - E_2 = 11.5 \text{ eV} > 4 \text{ eV}$

$\Delta E_{4\to3} = E_4 - E_3 = 2.64$ eV

$\Delta E_{4\to2} = E_4 - E_2 = 10.2 \text{ eV} > 4 \text{ eV}$

Hence, the energy of emitted photons in the range of 2eV and 4eV are

3.3 eV during combination and

3.84 eV and 2.64 after combination.

3. Let ground state energy (in eV) be E_1

Then, from the given condition $E_{2n} - E_1 = 204$ eV

$$\Rightarrow \frac{E_1}{4n^2} - E_1 = 204 eV$$

$$\Rightarrow E_1\left(\frac{1}{4n^2} - '1\right) = 204 \text{ eV...(i)}$$

and $E_{2n} - E_n = 40.8 \text{ eV} \Rightarrow \dfrac{E_1}{4n^2} - \dfrac{E_1}{n^2} = 40.8$ eV

$$\Rightarrow E_1\left(\frac{-3}{4n^2}\right) = 40.8 \text{ eV...(ii)}$$

From equation (i) and (ii), $\dfrac{1 - \dfrac{1}{4n^2}}{\dfrac{3}{4n^2}} = 5$

$$\Rightarrow 1 = \frac{1}{4n^2} + \frac{15}{4n^2} \Rightarrow \frac{4}{n^2} = 1 \Rightarrow n=2$$

From equation number (ii),

$$E_1 = -\frac{4}{3}n^2(40.8) \text{ eV} = -\frac{4}{3}(2)^2(40.8) \text{ eV}$$

$\Rightarrow E_1 = -217.6 \text{ eV} \Rightarrow E_1 = -(13.6)Z^2$

$$\therefore Z^2 = \frac{E_1}{-13.6} = \frac{-217.6}{-13.6} = 16 \quad \therefore Z=4$$

$E_{min} = E_{2n} - E_{2n-1}$

$$= \frac{E_1}{4n^2} - \frac{E_1}{(2n-1)^2} = 2 = \frac{E_1}{16} - \frac{E_1}{9} = -\frac{7}{144}E_1$$

$$= -\left(\frac{7}{144}\right)(-217.6) \text{ eV}$$

$\therefore E_{min} = 10.58$ eV

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4. Energy of incident photon,

$E_i = 10.6\text{ eV} = 10.6 \quad 1.6 \quad 10^{-19}\text{J} = 16.96 \quad 10^{-19}\text{ J}$

Energy incident per unit area per unit (intensity) = 2J

$\therefore$ No. of photons incident on unit area in unit time

$$= \frac{2}{16.96 \times 10^{-19}} \Rightarrow 1.18 \quad 10^{18}$$

Therefore, number of photons incident per unit time on given area ($1.0 \quad 10^{-4}\, m^2$)

$$= (1.18 \quad 10^{18})\,(1.0 \quad 10^{-4}) = 1.18 \quad 10^{14}$$

But only 0.83% of incident photons emit photoelectrons

$\therefore$ No. of photoelectrons emitted per second (n)

$$n = \left(\frac{0.53}{100}\right)(1.18 \times 10^{14}) \Rightarrow n = 6.25 \quad 10^{11}$$

$K_{min} = 0$ and $K_{max} = E_i$ – work function

$= (10.6 - 5.6)\text{ eV} = 5.0\text{ eV}$

$\therefore K_{max} = 5.0\text{ eV}$

5.(i) Let at time t=t, number of nuclei of Y and Z are N_Y and N_Z. Then Rate equations of the population of X,Y and Z are

$$\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X \quad \ldots(i)$$

$$\left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y \ldots(ii)$$

$$\Rightarrow \left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y \ldots(iii)$$

(ii) Given $N_Y(t) = \dfrac{N_0\lambda_X}{\lambda_X - \lambda_Y}\left[e^{-\lambda_Y t} - e^{-\lambda_X t}\right]$

For N_Y to be maximum $\dfrac{dN_Y(t)}{dt} = 0$

i.e. $\lambda_X N_X = \lambda_Y N_Y \ldots$ (iv) (from equation (ii))

$$\Rightarrow \lambda_X(N_0 e^{-\lambda_X t}) = \lambda_Y \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$$

$$\Rightarrow \frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1 \Rightarrow \frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

$$\Rightarrow (\lambda_X - \lambda_Y)t\,\ell n\,(e) = \ell n\left(\frac{\lambda_X}{\lambda_Y}\right) \Rightarrow t = \frac{1}{\lambda_X - \lambda_Y}\,\ell n\left(\frac{\lambda_X}{\lambda_Y}\right)$$

Substituting the values of λ_X and λ_Y, we have

$$t = \frac{1}{(0.1 - 1/30)}\,\ell n\left(\frac{0.1}{1/30}\right) = 15\,\ell n(3) \Rightarrow t = 16.48\text{ s.}$$

(iii) The population of X at this moment,

$N_X = N_0 e^{-\lambda X t} = (10^{20})e^{-(0.1)(16.48)} \Rightarrow N_X = 1.92 \quad 10^{19}$

$N_Y = \dfrac{N_X \lambda_X}{\lambda_Y}$ [From equation (iv)]

$$= (1.92 \quad 10^{19})\,\frac{(0.1)}{(1/30)} = 5.76 \quad 10^{19}$$

$N_Z = N_0 - N_X - N_Y = 10^{20} - 1.92 \quad 10^{19} - 5.76 \quad 10^{19}$

$\Rightarrow N_Z = 2.32 \quad 10^{19}$

6.(i) Given mass of α– particle, m=4.002 amu and mass of daughter nucleus M=223.610 amu,

de–Broglie wavelength of α–particle,

$\lambda = 5.76 \quad 10^{-15}$ m

So, momentum of α–particle would be

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.76 \times 10^{-15}}\text{ kg–m/s}$$

$\Rightarrow p = 1.151 \quad 10^{-19}$ kg–m/s

From law of conservation of linear momentum, this should also be equal to the linear momentum the daughter nucleus (in opposite direction).

Let K_1 and K_2 be the kinetic energies of α–particle and daughter nucleus. Then total kinetic energy in the final state is: $K = K_1 + K_2 = \dfrac{p^2}{2m} + \dfrac{p^2}{2M}$

$$= \frac{p^2}{2}\left(\frac{1}{m} + \frac{1}{M}\right) = \frac{p^2}{2}\left(\frac{M + m}{Mm}\right)$$

1 amu = $1.67 \quad 10^{-27}$ kg

Substituting the values, we get

$$K = \frac{(1.151 \times 10^{-19})^2}{2} \times \left(\frac{M + m}{M \times m}\right)$$

$$= \frac{p^2}{2} \times \frac{(4.002 + 223.610)(1.67 \times 10^{-27})}{(4.002 \times 1.67 \times 10^{-27})(223.61 \times 1.67 \times 10^{-27})}$$

$$K = 10^{-12}\text{ J} = \frac{10^{-12}}{1.6 \times 10^{-13}}\text{ MeV} = 6.25\text{ MeV}$$

$\Rightarrow$ K=6.25 MeV

(ii) Mass defect, $\Delta m = \dfrac{6.25}{931.470}$ amu = 0.0067 amu

Therefore, mass of parent nucleus

= mass of α–particle + mass of daughter nucleus+mass defect (Δm)

= (4.002 + 223.610 + 0.0067)amu = 227.62 amu

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7. The reactor produces 1000 MW power or 10^9 W power of 10^9 J/s of power. The reactor is to function for 10yr. Therefore, total energy which the reactor will supply in 10yr is E = (power) (time)

$= (10^9\ J/s)\ (10\ \ 365\ \ 24\ \ 3600\ s) = 3.1536\ \ 10^{17}\ J$

But since the efficiency of the reactor is only 10%, therefore actual energy needed is 10 times of it or $3.1536\ \ 10^{18}$ J. One uranium atom liberates 200MeV of energy or $200\ \ 1.6\ \ 10^{-13}$ or $3.2\ \ 10^{-11}$ J of energy. So, number of uranium atoms needed are

$$n = \frac{3.1536 \times 10^{18}}{3.2 \times 10^{-11}} = 0.9855\ \ 10^{29}$$

or number of kg–moles of uranium needed are

$$n = \frac{0.9855 \times 10^{29}}{6.02 \times 10^{26}} = 163.7$$

Hence, total mass of uranium required is

$$m = (n)M = (163.7)\ (235)\ kg$$

$\Rightarrow m \approx 38470\ kg \Rightarrow m = 3.847\ \ 10^4\ kg$

8.(i) Total 6 lines are emitted.

Therefore $\dfrac{n(n-1)}{2} = 6 \Rightarrow n=4$

So, transition is taking place between m^{th} energy state and $(m + 3)^{th}$ energy state. $E_m = -0.85eV$

$$\Rightarrow -13.6\left(\frac{Z^2}{m^2}\right) = -0.85 \qquad \Rightarrow \frac{Z}{m} = 0.25 \ldots (i)$$

Similarly $E_{m+3} = -0.544$ eV

$$\Rightarrow -13.6\frac{z^2}{(m+3)^2} = -0.544 \Rightarrow \frac{z}{(m+3)} = 0.2 \ ..(ii)$$

Solving equation (i) and (ii) for z and m.

We get m = 12 and z=3

(ii) Smallest wavelength corresponds to maximum difference of energies which is obviously $E_{m+3} - E_m$.

$\therefore \Delta E_{max} = -0.544 - (-0.85) = 0.306$ eV

$$\therefore \lambda_{min} = \frac{hc}{\Delta E_{max}} = \frac{1240}{0.306} = 4052.3\ nm.$$

9. Area of plates A = $5\ \ 10^{-4}\ m^2$

distance between the plates d = 1cm = 10^{-2}m

(i) Number of photoelectrons emitted upto t=10 s are

$$n = \frac{\text{(No. of photons falling in unit area in unit time)}}{10^6} \times (\text{area} \times \text{time})$$

$$= \frac{1}{10^6}\left[(10)^{16}\ \ (5\ \ 10^{-4})\ \ (10)\right] = 5.0\ \ 10^7$$

(ii) At time t=10s charge on plate A,

$q_A = +ne = (5.0\ \ 10^7)\ (1.6\ \ 10^{-19}) = 8.0\ \ 10^{-12}$ C

and charge on plate B,

$q_B = (33.7\ \ 10^{-12} - 8.0\ \ 10^{-12}) = 25.7\ \ 10^{-12}$ C

$\therefore$ Electric field between the plates $E = \dfrac{(q_B - q_A)}{2A\ \epsilon_0}$

$$\Rightarrow E = \frac{(25.7 - 8.0) \times 10^{-12}}{2 \times (5 \times 10^{-4})(8.85 \times 10^{-12})} = 2 \times 10^3 \frac{N}{C}$$

(iii) Energy of photoelectrons at plate A

= E – W = (5–2) eV = 3eV

Increase in energy of photoelectrons

= (eEd) joule = (Ed) eV

= $(2\ \ 10^3)\ (10^{-2})$ eV = 20eV

Energy of photoelectrons at plate B

= (20 + 3)eV = 23 eV

10. $\Delta E = h\nu = Rhc\ (Z-b)^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

For K– series, b=1 $\therefore \nu = Rc(Z-1)^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

Substituting the values,

$$4.2\ \ 10^{18} = (1.1\ \ 10^7)\ (3\ \ 10^8)(Z-1)^2\left(\frac{1}{1} - \frac{1}{4}\right)$$

$\therefore (Z-1)^2 = 1697 \Rightarrow Z - 1 \approx 41 \Rightarrow Z = 42$

11. Maximum kinetic energy of the photoelectrons would be $K_{max} = E - W = (5 - 3)$ eV = 2eV

Therefore, the stopping potential is 2V. Saturation current depends on the intensity of light incident. When the intensity is doubled the saturation current will also become two fold. The corresponding graphs are shown in figure.

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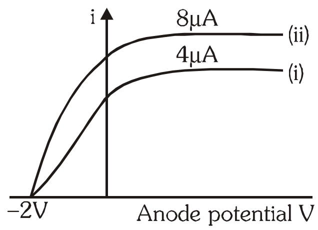

12. Let n_0 be the number of radioactive nuclei at time t=0. Number of nuclei decayed in time t are given by $n_0(1-e^{-\lambda t})$, which is also equal to the number of beta particles emitted during the same interval of time. For the given condition,

$n = n_0(1-e^{-2\lambda})$ ...(i) $\qquad (n + 0.75n) = n_0(1-e^{-4\lambda})$ ...(ii)

Dividing equation (ii) by (i), we get

$$1.75 = \frac{1-e^{-4\lambda}}{1-e^{-2\lambda}} \Rightarrow 1.75 - 1.75\, e^{-2\lambda} = 1 - e^{-4\lambda}$$

$$\therefore\ 1.75\, e^{-2\lambda} - e^{-4\lambda} = \frac{3}{4} \quad \text{...(iii)}$$

Let us take $e^{-2\lambda} = x$

Then the above equation is $x^2 - 1.75x + 0.75 = 0$

$$\Rightarrow x = \frac{1.75 \pm \sqrt{(1.75)^2 - (4)(0.75)}}{2} \Rightarrow x = 1 \text{ and } \frac{3}{4}$$

$\therefore$ From equation (iii) either $e^{-2\lambda} = 1$, $e^{-2\lambda} = \frac{3}{4}$

but $e^{-2\lambda} = 1$ is not accepted because which means $\lambda = 0$. Hence, $e^{-2\lambda} = \frac{3}{4}$

$$\Rightarrow -2\lambda\, \ell n(e) = \ell n\,(3) - \ell n(4) = \ell n(3) - 2\,\ell n(2)$$

$$\therefore\ \lambda = \ell n\,(2) - \frac{1}{2}\ell n(3)$$

Substituting the given values,

$$\ell = 0.6931 - \frac{1}{2}\ (1.0986) = 0.14395\ s^{-1}$$

$$\therefore\ \text{Mean–life } t_{mean} = \frac{1}{\lambda} = 6.947\ s$$

13. Wavelengths corresponding to minimum wavelength (λ_{min}) or maximum energy will emit photoelectrons having maximum kinetic energy.

(λ_{min}) belonging to Balmer series and lying in the given range (450 nm to 750 nm) corresponds to transition from (n=4 to n=2). Here,

$$E_4 = -\frac{13.6}{(4)^2} = -0.85eV \Rightarrow E_2 = \frac{13.6}{(2)^2} = -3.4eV$$

$\therefore\ \Delta E = E_4 - E_2 = 2.55$ eV

K_{max} = Energy of photon – work function

$= 2.55 - 2.0 = 0.55$ eV

14. Let N_0 be the initial number of nuclei of ^{238}U.

After time t, $N_U = N_0\left(\frac{1}{2}\right)^n$

Here n = number of half–lives

$$= \frac{t}{t_{1/2}} = \frac{1.5 \times 10^9}{4.5 \times 10^9} = \frac{1}{3} \Rightarrow N_U = N_0\left(\frac{1}{2}\right)^{\frac{1}{3}}$$

$$\text{and } N_{Pb} = N_0 - N_U = N_0\left[1 - \left(\frac{1}{2}\right)^{1/3}\right]$$

$$\therefore\ \frac{N_U}{N_{Pb}} = \frac{\left(\frac{1}{2}\right)^{1/3}}{1-\left(\frac{1}{2}\right)^{3}} = 3.861$$

15. (i) From the relation $r \propto A^{1/3}$

$$\text{We have } \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3} \Rightarrow \left(\frac{A_2}{4}\right)^{1/3} = (14)^{1/3}$$

$\therefore\ A_2 = 56$

(ii) $Z_2 = A_2$ – no. of neutrons = 56 – 30 = 26

$$\therefore\ f_{ka} = Rc\ (Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3Rc}{4}(Z-1)^2$$

Substituting the given values of R, c and Z.

We get $f_{ka} = 1.55\ \ 10^{18}$ Hz

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16. For $0 \le x \le 1$, PE = E_0

$\therefore$ Kinetic energy K_1 = Total energy – PE

$= 2E_0 - E_0 = E_0 \therefore \lambda_1 = \dfrac{h}{\sqrt{2m(2E_0)}}$ ...(i)

For x > 1, PE = 0

$\therefore$ Kinetic energy K_2 = Total energy $= 2E_0$

$\therefore \quad \lambda_2 = \dfrac{h}{\sqrt{2m(2E_0)}}$ ...(ii)

From equation (i) and (ii), we have $\dfrac{\lambda_1}{\lambda_2} = \sqrt{2}$

17. $\dfrac{h}{\lambda} = p = \sqrt{2mqV}$

$\Rightarrow \dfrac{\lambda_P}{\lambda_\alpha} = \sqrt{\dfrac{m_\alpha q_\alpha}{m_P q_P}} = \sqrt{4 \times 2} = \sqrt{8} \approx 3$

18. $\left|\dfrac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t} \quad \therefore \ell n\left|\dfrac{dN}{dt}\right| = \ell n(N_0\lambda) - \lambda t$

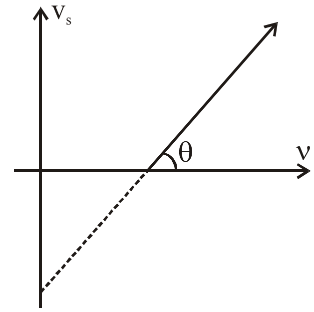
$\therefore \ell n\left|\dfrac{dN}{dt}\right| = \ell n(N_0\lambda) - \lambda t \Rightarrow -\lambda = \text{slope} = -\dfrac{1}{2}\text{year}^{-1}$

$\Rightarrow \lambda = \dfrac{1}{2}\text{yr}^{-1}$

$\therefore t_{1/2} = \dfrac{0.693}{\lambda} = 1.386$ given time is 3 times of $t_{1/2}$

$\therefore$ value of p is 8.

19. $eV_s = h\nu - \phi \Rightarrow V_S = \left(\dfrac{h}{e}\right)\nu - \dfrac{\phi}{e}$



$\Rightarrow$ slope $= \dfrac{h}{e}$ = constant $\Rightarrow$ ratio = 1

20. Given $T_{1/2}$ = 1386 sec.

$$\left|\dfrac{dN}{dt}\right| = 10^3$$

Fraction

$$= \dfrac{N_0 - N}{N_0} = \dfrac{N_0 - N_0 e^{-\lambda t}}{N_0} = 1 - e^{-\lambda t}$$

$$= 1 - e^{\frac{-\ln 2}{1386} \times 80}$$

$$= 1 - e^{\frac{-4}{100}} \simeq 1 - \left[1 - \dfrac{4}{100}\right] = \dfrac{4}{100} \equiv 4\%$$