EXERCISE - 01

CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. The value of determinant $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$ is equal to -

(A) abc (B) 2abc (C) 0 (D) 4abc

2. If $\begin{vmatrix} \sin 2x & \cos^2 x & \cos 4x \\ \cos^2 x & \cos 2x & \sin^2 x \\ \cos^4 x & \sin^2 x & \sin 2x \end{vmatrix} = a_0 + a_1 (\sin x) + a_2 (\sin^2 x) + \ldots\ldots. + a_n (\sin^n x)$ then the value of a_0 is -

(A) –1 (B) 1 (C) 0 (D) 2

3. The value of the determinant $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$ is equal to -

(A) 0 (B) (a – b)(b – c)(c – a) (C) (a + b)(b + c)(c + a) (D) 4abc

4. For any ΔABC, the value of determinant $\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix}$ is equal to -

(A) 0 (B) 1 (C) sin A sin B sin C (D) sin A + sin B + sin C

5. If $D_p = \begin{vmatrix} p & 15 & 8 \\ p^2 & 35 & 9 \\ p^3 & 25 & 10 \end{vmatrix}$, then $D_1 + D_2 + D_3 + D_4 + D_5$ is equal to -

(A) 0 (B) 25 (C) 625 (D) none of these

6. If A + B + C = π, then $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$ is equal to -

(A) 0 (B) 2 sin B tan A cos C (C) 1 (D) none of these

7. The number of real values of x satisfying $\begin{vmatrix} x & 3x+2 & 2x-1 \\ 2x-1 & 4x & 3x+1 \\ 7x-2 & 17x+6 & 12x-1 \end{vmatrix} = 0$ is -

(A) 3 (B) 0 (C) 1 (D) infinite

8. If a, b, c are pth, qth and rth terms of a GP, then $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is equal to -

(A) 0 (B) 1 (C) log abc (D) pqr

9. If $a_1, a_2, \ldots\ldots. a_n, a_{n+1}, \ldots\ldots$ are in GP and $a_i > 0\ \forall i$, then $\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$ is equal to -

(A) 0 (B) n log a_n (C) n(n + 1) log a_n (D) none of these

10. If $px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix}$ then t is equal to -

(A) 33 (B) 0 (C) 21 (D) none

11. For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is -

(A) 0 (B) log xyz (C) log(x + y + z) (D) logx logy logz

12. If a, b, c > 0 and x, y, z ∈ R, then the determinant $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^y+b^{-y})^2 & (b^y-b^{-y})^2 & 1 \\ (c^z+c^{-z})^2 & (c^z-c^{-z})^2 & 1 \end{vmatrix}$ is equal to -

(A) $a^xb^yc^x$ (B) $a^{-x}b^{-y}c^{-z}$ (C) $a^{2x}b^{2y}c^{2z}$ (D) zero

13. For a non-zero real a, b and c $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = \alpha$ abc, then the values of α is -

(A) –4 (B) 0 (C) 2 (D) 4

14. The equation $\begin{vmatrix} (1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x \end{vmatrix} + \begin{vmatrix} (1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3 \end{vmatrix} = 0$

(A) has no real solution
(B) has 4 real solutions
(C) has two real and two non-real solutions
(D) has infinite number of solutions, real or non-real

15. Let a determinant is given by $A = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$ and suppose determinant A = 6. If $B = \begin{vmatrix} p+x & q+y & r+z \\ a+x & b+y & c+z \\ a+p & b+q & c+r \end{vmatrix}$ then -

(A) det. B = 6 (B) det. B = –6 (C) det. B = 12 (D) det. B = –12

16. If $a \neq b \neq c$ and $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = 0$ then -

(A) a + b+ c = 0
(B) ab + bc + ca = 0
(C) $a^2 + b^2 + c^2 = ab + bc + ca$
(D) abc = 0

17. If a, b, & c are nonzero real numbers, then $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to -

(A) $a^2b^2c^2(a + b + c)$ (B) $abc(a + b + c)^2$ (C) zero (D) none of these

18. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$, then f (100) is equal to - [JEE 98]

(A) 0 (B) 1 (C) 100 (D) –100

19. The value of the determinant $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix}$ is equal to -

(A) $a^3 - b^3$ (B) $a^3 + b^3$ (C) 0 (D) none of these

20. An equilateral triangle has each of its sides of length 6 cm. If (x_1, y_1); (x_2, y_2) & (x_3, y_3) are its vertices then the value of the determinant, $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$ is equal to -

(A) 192 (B) 243 (C) 486 (D) 972

21. If the system of equations $x + 2y + 3z = 4$, $x + py + 2z = 3$, $\mu x + 4y + z = 3$ has an infinite number of solutions, then -

(A) $p = 2, \mu = 3$ (B) $p = 2, \mu = 4$ (C) $3p = 2\mu$ (D) none of these

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

22. If $\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = 0$, then x may be equal to -

(A) a (B) b (C) a + b (D) m

23. If $D(x) = \begin{vmatrix} \sin 2x & e^x \sin x + x\cos x & \sin x + x^2 \cos x \\ \cos x + \sin x & e^x + x & 1 + x^2 \\ e^x \cos x & e^{2x} & e^x \end{vmatrix}$, then the value of $|\ell n \cos (Dx)|$ will be -

(A) independent of x (B) dependent on x (C) 0 (D) non-existent

24. The value of the determinant $\begin{vmatrix} \alpha & \beta & \ell \\ \alpha & x & n \\ \alpha & \beta & x \end{vmatrix}$ is

(A) independent of ℓ (B) independent of n (C) $\alpha(x - \ell)(x - \beta)$ (D) $\alpha\beta(x - \ell)(x - n)$

25. If the system of linear equations $x + ay + az = 0$, $x + by + bz = 0$, $x + cy + cz = 0$ has a non-zero solution then

(A) System has always non-trivial solutions.

(B) System is consistent only when $a = b = c$

(C) If $a \neq b \neq c$ then $x = 0$, $y = t$, $z = -t$ $\forall$ $t \in R$

(D) If $a = b = c$ then $y = t_1$, $z = t_2$, $x = -a(t_1 + t_2)$ $\forall$ $t_1, t_2 \in R$

26. If the system of equations $x + y - 3 = 0$, $(1 + K)x + (2 + K)y - 8 = 0$ & $x - (1 + K)y + (2 + K) = 0$ is consistent then the value of K may be -

(A) 1 (B) $\frac{3}{5}$ (C) $-\frac{5}{3}$ (D) 2

CHECK YOUR GRASP — ANSWER KEY — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	D	A	D	A	D	A	A	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	D	D	D	C	A	C	A	B	D
Que.	21	22	23	24	25	26				
Ans.	D	A,B	A,C	B,C	A,C,D	A,C				

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. Which of the following determinant(s) vanish(es) ?

(A) $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$

(B) $\begin{vmatrix} 1 & ab & \frac{1}{a}+\frac{1}{b} \\ 1 & bc & \frac{1}{b}+\frac{1}{c} \\ 1 & ca & \frac{1}{c}+\frac{1}{a} \end{vmatrix}$

(C) $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$

(D) $\begin{vmatrix} \log_x xyz & \log_x y & \log_x z \\ \log_y xyz & 1 & \log_y z \\ \log_z xyz & \log_z y & 1 \end{vmatrix}$

2. If $f'(x) = \begin{vmatrix} mx & mx-p & mx+p \\ n & n+p & n-p \\ mx+2n & mx+2n+p & mx+2n-p \end{vmatrix}$, then $y = f(x)$ represents -

(A) a straight line parallel to x–axis
(B) a straight line parallel to y–axis
(C) parabola
(D) a straight line with negative slope

3. The determinant $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ is divisible by -

(A) $a + b + c$
(B) $(a + b) (b + c) (c + a)$
(C) $a^2 + b^2 + c^2$
(D) $(a - b)(b - c) (c - a)$

4. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, if -

(A) a, b, c are in AP
(B) a, b, c are in GP
(C) α is a root of the equation $ax^2+bx+c=0$
(D) $(x-\alpha)$ is a factor of $ax^2 + 2bx + c$

5. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$, then the maximum value of $f(x)$ =

(A) 2
(B) 4
(C) 6
(D) 8

6. The parameter on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is-

(A) a
(B) p
(C) d
(D) x

7. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, then f(x) is a polynomial of degree-

(A) 2
(B) 3
(C) 0
(D) 1

8. Given that $q^2 - pr < 0$, $p > 0$, then the value of $\begin{vmatrix} p & q & px+qy \\ q & r & qx+ry \\ px+qy & qx+ry & 0 \end{vmatrix}$ is-

(A) zero (B) positive (C) negative (D) $q^2 + pr$

9. The value of θ lying between $-\frac{\pi}{4}$ & $\frac{\pi}{2}$ and $0 \le A \le \frac{\pi}{2}$ and satisfying the equation

$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix} = 0$ are -

(A) $A = \frac{\pi}{4}, \quad \theta = -\frac{\pi}{8}$ (B) $A = \frac{3\pi}{8} = \theta$ (C) $A = \frac{\pi}{5}, \quad \theta = -\frac{\pi}{8}$ (D) $A = \frac{\pi}{6}, \quad \theta = \frac{3\pi}{8}$

10. The set of equations $x - y + 3z = 2$, $2x - y + z = 4$, $x - 2y + \alpha z = 3$ has -

(A) unique solution only for $\alpha = 0$ (B) unique solution for $\alpha \neq 8$

(C) infinite number of solutions of $\alpha = 8$ (D) no solution for $\alpha = 8$

BRAIN TEASERS — ANSWER KEY — EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,B,C,D	A	A,C,D	B,D	C	B	A	C	A,B,C,D	B,D

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. If a, b, c are sides of scalene triangle, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is positive.

2. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c \equiv (\ell_1 x + m_1 y + n_1)(\ell_2 x + m_2 y + n_2)$, then $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

3. If $x = cy + bz$, $y = az + cx$, $z = bx + ay$, where x, y, z are not all zero, then $a^2 + b^2 + c^2 + 2abc + 1 = 0$.

4. If $\sum_{i=1}^{3} x_i^2 = \sum_{i=1}^{3} y_i^2 = \sum_{i=1}^{3} z_i^2 = 1$ and $\sum_{i=1}^{3} x_i y_i = \sum_{i=1}^{3} y_i z_i = \sum_{i=1}^{3} z_i x_i = 0$ then $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}^2 = 1$

5. Consider the system of equations $a_i x + b_i y + c_i z = d_i$ where i = 1, 2, 3.

If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix} = 0$

then the system of equations has infinite solutions.

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	If the determinant $\begin{vmatrix} a+p & \ell+x & u+f \\ b+q & m+y & v+g \\ c+r & n+z & w+h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then the values of K is	(p)	3
(B)	The values of λ for which the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ & $x + 2y + \lambda z = 12$ is inconsistent	(q)	8
(C)	If x, y, z are in A.P. then the value of the determinant $\begin{vmatrix} a+2 & a+3 & a+2x \\ a+3 & a+4 & a+2y \\ a+4 & a+5 & a+2z \end{vmatrix}$ is	(r)	5
(D)	Let p be the sum of all possible determinants of order 2 having 0, 1, 2 & 3 as their four elements (without repeatition of digits). The value of 'p' is	(s)	0

ASSERTION & REASON

These questions contain, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement - I** : Consider $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Let B_1, B_2, B_3 be the co-factors of b_1, b_2, and b_3 respectively then $a_1B_1 + a_2B_2 + a_3B_3 = 0$

Because

Statement - II : If any two rows (or columns) in a determinant are identical then value of determinant is zero.

(A) A (B) B (C) C (D) D

2. **Statement - I** : Consider the system of equations,

$$2x + 3y + 4z = 5$$
$$x + y + z = 1$$
$$x + 2y + 3z = 4$$

This system of equations has infinite solutions.

Because

Statement - II : If the system of equations is

$$e_1 : a_1x + b_1y + c_1z - d_1 = 0$$
$$e_2 : a_2x + b_2y + c_2z - d_2 = 0$$
$$e_3 : e_1 + \lambda e_2 = 0, \quad \text{where } \lambda \in R \quad \& \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Then such system of equations has infinite solutions.

(A) A (B) B (C) C (D) D

3. **Statement - I** : If a, b, c $\in$ R and $a \neq b \neq c$ and x,y,z are non zero. Then the system of equations

$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0$$

has infinite solutions.

Because

Statement - II : If the homogeneous system of equations has non trivial solution, then it has infinitely many solutions.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

Let $x, y, z \in R^+$ & $D = \begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix}$

On the basis of above information, answer the following questions :

1. If $x \neq y \neq z$ & x, y, z are in GP and D = 0, then y is equal to -

(A) 1 (B) 2 (C) 4 (D) none of these

2. If x, y, z are the roots of $t^3 - 21t^2 + bt - 343 = 0$, $b \in R$, then D is equal to-

(A) 1 (B) 0 (C) dependent on x, y, z (D) data inadequate

3. If $x \neq y \neq z$ & x, y, z are in A.P. and $D = 0$, then $2xy^2z + x^2z^2$ is equal to-

(A) 1 (B) 2 (C) 3 (D) none of these

Comprehension # 2

Consider the system of linear equations

$$\alpha x + y + z = m$$
$$x + \alpha y + z = n$$
and $$x + y + \alpha z = p$$

On the basis of above information, answer the following questions :

1. If $\alpha \neq 1, -2$ then the system has -

(A) no solution (B) infinte solutions
(C) unique solution (D) trivial solution if $m \neq n \neq p$

2. If $\alpha = -2$ & $m + n + p \neq 0$ then system of linear equations has -

(A) no solution (B) infinite solutions (C) unique solution (D) finitely many solution

3. If $\alpha = 1$ & $m \neq p$ then the system of linear equations has -

(A) no solution (B) infinite solutions (C) unique solution (D) unique solution if $p = n$

MISCELLANEOUS TYPE QUESTION — **ANSWER KEY** — EXERCISE-3

- ***True / False***
  1. F 2. T 3. F 4. T 5. F
- ***Match the Column***
  1. (A) → (q); (B) → (p); (C) → (s); (D) → (s)
- ***Assertion & Reason***
  1. A 2. A 3. A
- ***Comprehension Based Questions***
  Comprehension # 1 : 1. A 2. B 3. C
  Comprehension # 2 : 1. C 2. A 3. A

EXERCISE - 04 [A]   CONCEPTUAL SUBJECTIVE EXERCISE

1. Without expanding the determinant prove that :

(a) $\begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} = 0$ (b) $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix} = 0$

2. Prove that :

(a) $\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a & b & c \\ x & y & z \\ yz & zx & xy \end{vmatrix}$ (b) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

3. Prove that : $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

4. Using properties of determinants or otherwise evaluate $\begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$.

5. If $D = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ and $D' = \begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix}$ then prove that $D' = 2D$.

6. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a+b)^3$.

7. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

8. Solve for x, $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$.

9. If $a+b+c=0$, solve for x : $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$.

10. Prove that $\begin{vmatrix} bc & bc'+b'c & b'c' \\ ca & ca'+c'a & c'a' \\ ab & ab'+a'b & a'b' \end{vmatrix} = (ab'-a'b)(bc'-b'c)(ca'-c'a)$.

11. Let the three digit numbers A28, 3B9, and 62C, where A, B, and C are integers between 0 and 9, be divisible by a fixed integer k. Show that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is divisible by k.

12. For a fixed positive integer n, if D = $\begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$ then show that $\left[\frac{D}{(n!)^3} - 4\right]$ is divisible by n.

13. If $D_r = \begin{vmatrix} 2^{r-1} & 2\left(3^{r-1}\right) & 4\left(5^{r-1}\right) \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$ then prove that $\sum_{r=1}^{n} D_r = 0$.

14. Find the value of the determinant $\begin{vmatrix} \frac{1}{z} & \frac{1}{z} & -\frac{(x+y)}{z^2} \\ -\frac{(y+z)}{x^2} & \frac{1}{x} & \frac{1}{x} \\ -\frac{y(y+z)}{x^2 z} & \frac{x+2y+z}{xz} & -\frac{y(x+y)}{xz^2} \end{vmatrix}$

15. Prove that $\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix} = -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$

16. Show that $\begin{vmatrix} a_1 l_1 + b_1 m_1 & a_1 l_2 + b_1 m_2 & a_1 l_3 + b_1 m_3 \\ a_2 l_1 + b_2 m_1 & a_2 l_2 + b_2 m_2 & a_2 l_3 + b_2 m_3 \\ a_3 l_1 + b_3 m_1 & a_3 l_2 + b_3 m_2 & a_3 l_3 + b_3 m_3 \end{vmatrix} = 0$.

17. Solve the following sets of equations using Cramer's rule and remark about their consistency.

(a) $x + y + z - 6 = 0$, $2x + y - z - 1 = 0$, $x + y - 2z + 3 = 0$

(b) $x + 2y + z = 1$, $3x + y + z = 6$, $x + 2y = 0$

(c) $x - 3y + z = 2$, $3x + y + z = 6$, $5x + y + 3z = 3$

(d) $7x - 7y + 5z = 3$, $3x + y + 5z = 7$, $2x + 3y + 5z = 5$

18. Investigate for what values of λ, μ the simultaneous equations $x + y + z = 6$; $x + 2y + 3z = 10$ & $x + 2y + \lambda z = \mu$ have :

(a) A unique solution. (b) An infinite number of solutions. (c) No solution.

19. Find the values of c for which the equations

$$2x + 3y = 0$$
$$(c+2)x + (c+4)y = c+6$$
$$(c+2)^2 x + (c+4)^2 y = (c+6)^2$$

are consistent. Also solve above equations for these values of c.

20. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then prove that $\frac{b^2}{q^2} = \frac{ac}{pr}$.

CONCEPTUAL SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-4(A)

4. -1 8. $x = -1$ or $x = -2$ 9. $x = 0$ or $x = \pm\sqrt{\frac{3}{2}\left(a^2 + b^2 + c^2\right)}$ 14. 0

17. (a) $x = 1$, $y = 2$, $z = 3$; consistent (b) $x = 2$, $y = -1$, $z = 1$; consistent

(c) $x = \frac{13}{3}$, $y = -\frac{7}{6}$, $z = -\frac{35}{6}$; consistent (d) inconsistent

18. (a) $\lambda \neq 3$ (b) $\lambda = 3$, $\mu = 10$ (c) $\lambda = 3$, $\mu \neq 10$

19. $c = -6, -1$, for $c = -6$, $x = 0 = y$ & for $c = -1$, $x = -5$, $y = \frac{10}{3}$

EXERCISE - 04 [B]

BRAIN STORMING SUBJECTIVE EXERCISE

1. Let a, b, c, d be real numbers in G.P. If u, v, w satisfy the system of equations $u + 2v + 3w = 6$, $4u + 5v + 6w = 12$, $6u + 9v = 4$, then show that the roots of the equations

$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2 + [(b-c)^2 + (c-a)^2 + (d-b)^2]\,x + u + v + w = 0$ and $20x^2 + 10(a-d)^2 x - 9 = 0$ are reciprocals of each other. [JEE 99]

2. Prove that $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ -bc+ca+ab & bc-ca+ab & bc+ca-ab \\ (a+b)(a+c) & (b+c)(b+a) & (c+a)(c+b) \end{vmatrix} = 3.(b-c)(c-a)(a-b)(a+b+c)(ab+bc+ca)$

3. If $a^2 + b^2 + c^2 = 1$ then show that the value of the determinant

$\begin{vmatrix} a^2+(b^2+c^2)\cos\theta & ba(1-\cos\theta) & ca(1-\cos\theta) \\ ab(1-\cos\theta) & b^2+(c^2+a^2)\cos\theta & cb(1-\cos\theta) \\ ac(1-\cos\theta) & bc(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix}$ simplifies to $\cos^2\theta$

4. Find the value of the determinant $\begin{vmatrix} \cos(x-y) & \cos(y-z) & \cos(z-x) \\ \cos(x+y) & \cos(y+z) & \cos(z+x) \\ \sin(x+y) & \sin(y+z) & \sin(z+x) \end{vmatrix}$.

5. If $S_r = \alpha^r + \beta^r + \gamma^r$ then show that $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = (\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$.

6. If $ax_1^2 + by_1^2 + cz_1^2 = ax_2^2 + by_2^2 + cz_2^2 = ax_3^2 + by_3^2 + cz_3^2 = d$
and $ax_2x_3 + by_2y_3 + cz_2z_3 = ax_3x_1 + by_3y_1 + cz_3z_1 = ax_1x_2 + by_1y_2 + cz_1z_2 = f$,

then prove that $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = (d-f)\left[\frac{d+2f}{abc}\right]^{1/2}$ $(a, b, c \neq 0)$

7. If $u = ax^2 + 2bxy + cy^2$, $u' = a'x^2 + 2b'xy + c'y^2$, then prove that-

$$\begin{vmatrix} y^2 & -xy & x^2 \\ a & b & c \\ a' & b' & c' \end{vmatrix} = \begin{vmatrix} ax+by & bx+cy \\ a'x+b'y & b'x+c'y \end{vmatrix} = -\frac{1}{y}\begin{vmatrix} u & u' \\ ax+by & a'x+b'y \end{vmatrix}.$$

8. Solve the system of equations : $\left.\begin{array}{l} z + ay + a^2x + a^3 = 0 \\ z + by + b^2x + b^3 = 0 \\ z + cy + c^2x + c^3 = 0 \end{array}\right]$ where $a \neq b \neq c$.

9. If x,y,z are not all zero and if $ax + by + cz = 0$; $bx + cy + az = 0$; $cx + ay + bz = 0$

Prove that $x : y : z = 1 : 1 : 1$ or $1 : \omega : \omega^2$ or $1 : \omega^2 : \omega$.

10. Prove that the system of equations in x and y ; $ax + hy + g = 0$, $hx + by + f = 0$, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = t$ is consistent if $t = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \div \begin{vmatrix} a & h \\ h & b \end{vmatrix}$

BRAIN STORMING SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-4(B)
4. $2\sin(x-y)\sin(y-z)\sin(z-x)$	8. $x = -(a+b+c)$, $y = ab+bc+ca$, $z = -abc$	

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. If a, b, c are pth, qth and rth terms of a GP, and all are positive then $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is equal to- [AIEEE-2002]

(1) 0 (2) 1 (3) log abc (4) pqr

2. If 1, ω, ω^2 are cube roots of unity and $n \neq 3p$, $p \in Z$, then $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$ is equal to- [AIEEE-2003]

(1) 0 (2) ω (3) ω^2 (4) 1

3. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals- [AIEEE-2003]

(1) 1 (2) 0 (3) 2 (4) –1

4. If $a_1, a_2, \ldots\ldots a_n, a_{n+1}, \ldots\ldots$ are in GP and $a_i > 0\ \forall i$, then $\begin{vmatrix} \log a_n & \log a_{n+2} & \log a_{n+4} \\ \log a_{n+6} & \log a_{n+8} & \log a_{n+10} \\ \log a_{n+12} & \log a_{n+14} & \log a_{n+16} \end{vmatrix}$ is equal to- [AIEEE-04,05]

(1) 0 (2) n log a_n
(3) n(n + 1) log a_n (4) none of these

5. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, then f(x) is a polynomial of degree- [AIEEE 2005]

(1) 2 (2) 3 (3) 0 (4) 1

6. The system of equations
$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + \alpha z = \alpha - 1$ has no solution, If α is [AIEEE 2005]

(1) 1 (2) not –2 (3) either -2 or 1 (4) –2

7. If D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0$, $y \neq 0$ then D is- [AIEEE - 2007]

(1) Divisible by both x and y (2) Divisible by x but not y
(3) Divisible by y but not x (4) Divsible by neither x nor y

8. Let A = $\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$, if $|A^2| = 25$ then $|\alpha|$ equals- [AIEEE - 2007]

(1) 5 (2) 5^2 (3) 1 (4) 1/5

9. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that x = cy + bz, y = az + cx and z = bx + ay, then $a^2 + b^2 + c^2 + 2abc$ is equal to [AIEEE - 2008]

(1) 2 (2) –1 (3) 0 (4) 1

10. Let a, b, c be such that $b(a + c) \neq 0$. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$,

then the value of n is :- **[AIEEE - 2009]**

(1) Any odd integer (2) Any integer

(3) Zero (4) Any even integer

11. Consider the system of linear equations :

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has **[AIEEE - 2010]**

(1) Infinite number of solutions (2) Exactly 3 solutions

(3) A unique solution (4) No solution

12. The number of values of k for which the linear equations

$$4x + ky + 2z = 0$$
$$kx + 4y + z = 0$$
$$2x + 2y + z = 0$$

possess a non-zero solution is :- **[AIEEE - 2011]**

(1) 1 (2) zero (3) 3 (4) 2

13. If the trivial solution is the only solution of the system of equations

$$x - ky + z = 0$$
$$kx + 3y - kz = 0$$
$$3x + y - z = 0$$

Then the set of all values of k is: **[AIEEE - 2011]**

(1) {2, –3} (2) R – {2, –3}

(3) R – {2} (4) R – {–3}

PREVIOUS YEARS QUESTIONS — **ANSWER KEY** — EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	1	4	1	1	4	1	4	4	1
Que.	11	12	13							
Ans.	4	4	2							

EXERCISE - 05 [B]

JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. Solve for x the equation $\begin{vmatrix} a^2 & a & 1 \\ \sin(n+1)x & \sin nx & \sin(n-1)x \\ \cos(n+1)x & \cos nx & \cos(n-1)x \end{vmatrix} = 0$

[REE 2001, (Mains), 3 out 100]

2. Test the consistency and solve them when consistent, the following system of equations for all values of λ :

$x + y + z = 1$

$x + 3y - 2z = \lambda$

$3x + (\lambda + 2)y - 3z = 2\lambda + 1$ [REE 2001,(Mains), 5 out 100]

3. Let a, b, c, be real numbers with $a^2 + b^2 + c^2 = 1$, Show that the equation

$\begin{vmatrix} ax - by - c & bx + ay & cx + a \\ bx + ay & -ax + by - c & cy + b \\ cx + a & cy + b & -ax - by + c \end{vmatrix} = 0$ represents a straight line.

[JEE 2001,(Mains), 6 out 100]

4. The number of values of k for which the system of equations

$(k + 1)x + 8y = 4k$

$kx + (k + 3)y = 3k - 1$

has infinitely many solutions is [JEE 2002,(Screening), 3]

(A) 0 (B) 1 (C) 2 (D) infinite

5. The value of λ for which the system of equations $2x - y - z = 12$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ has no solution is [JEE 2004 (Screening)]

(A) 3 (B) –3 (C) 2 (D) –2

6. (a) Consider three point $P = (-\sin(\beta - \alpha), -\cos\beta)$, $Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \pi/4$

(A) P lies on the line segment RQ (B) Q lies on the line segment PR

(C) R lies on the line segment QP (D) P, Q, R are non collinear

(b) Consider the system of equations $x - 2y + 3z = -1$; $-x + y - 2z = k$; $x - 3y + 4z = 1$.

Statement-I : The system of equations has no solution for $k \neq 3$.

and

Statement-II : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$. [JEE 2008, 3+3]

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

7. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$(y + z)\cos 3\theta = (xyz)\sin 3\theta$

$$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is [JEE 2010, 3]

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [B]

1. $x = n\pi,\ n \in I$
2. If $\lambda = 5$, system is consistent with infinite solution given by $z = K$,

$y = \frac{1}{2}(3K + 4)$ and $x = -\frac{1}{2}(5K + 2)$ where $K \in R$

If $\lambda \neq 5$, system is consistent with unique solution given by $z = \frac{1}{3}(1 - \lambda)$; $x = \frac{1}{3}(\lambda + 2)$ and $y = 0$.

4. B 5. D 6. (a) D; (b) A 7. 3