

# Newton's Laws of Motion

## Motion

Motion of a body is its movement and is identified by change in either its location or orientation or both, relative to other objects.

### Location

Location of a rigid body tells us where it is placed and can be measured by position coordinates of any particle of the body or its mass center. It is also known as position.

### Orientation

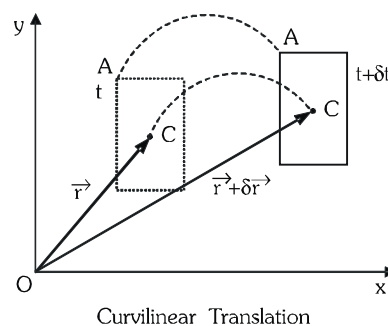
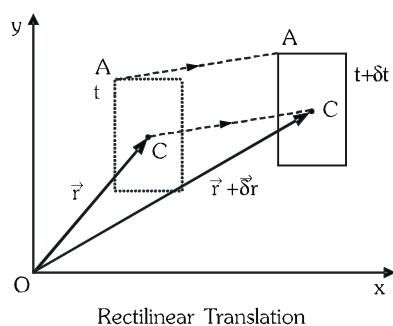
Orientation of a body tells us how it is placed with respect to the coordinate axes. Angles made with the coordinate axes by any linear dimension of the body or a straight line drawn on it, provide suitable measure of orientation.

### Translation and Rotation Motion

If a body changes its location without change in orientation, it is in pure translation motion and if it changes orientation without change in location, it is in pure rotation motion.

### Translation Motion

Let us consider the motion of a plate, which involves only change in position without change in orientation. It is in pure translation motion. The plate is shown at two different instants  $t$  and  $t+\delta t$ . The coordinate axes shown are in the plane of the plate and represent the reference frame. A careful observation makes the following points obvious.



- None of the linear dimension or any line drawn on the body changes its angles with the coordinate. Therefore, there is no rotation motion.
- All the particles of the body including its mass center move on identical parallel trajectories. Here trajectories of corner A and center C are shown by dashed lines.
- All the particles and mass center of the body cover identical segments of their trajectories in a given time interval. Therefore, at any instant of time all of them have identical velocities and accelerations.

Pure translation motion of a body can be represented by motion of any of its particle. This is why, we usually consider a body in pure translation motion as a particle.

## Momentum: Amount of Motion

Amount of motion in a body depends on its velocity and mass.

Linear momentum of a body is defined as product of its mass and velocity. It provides measure of amount of motion.

Linear momentum  $\vec{p}$  of a body of mass  $m$ , moving with velocity by  $\vec{v}$  is expressed by the following equation.

$$\vec{p} = m\vec{v}$$

SI unit of momentum is kg-m/s.

Dimensions of momentum are  $MLT^{-1}$

## Force

The concept of force is used to explain mutual interaction between two material bodies as the action of one body on another in form of push or pull, which brings out or tries to bring out a change in the state of motion of the two bodies. A mutual interaction between two bodies, which creates force on one body, also creates force on the other body. Force on body under study is known as *action* and the force applied by this body on the other is known as *reaction*.

### Contact and Field Forces:

When a body applies force on other by direct contact, the force is known as contact force. When two bodies apply force on each other without any contact between them, the force is known as field force.

When you lift something, you first hold it to establish contact between your hand and that thing, and then you apply the necessary force to lift. When you pull bucket of water out of a well, the necessary force you apply on the rope by direct contact between your hand and the rope and the rope exerts the necessary force on the bucket through a direct contact. When you deform a spring, you have to hold the spring and establish contact between your hand and the spring and then you apply the necessary force. In this way, you can find countless examples of contact forces.

Things left free, fall on the ground, planets orbit around the sun, satellites orbit around a planet due to *gravitational force*, which can act without any contact between the concerned bodies. A plastic comb when rubbed with dry hair, becomes electrically charged. A charged plastic comb attracts small paper pieces without any physical contact due to *electrostatic force*. A bar magnet attracts iron nails without any physical contact between them. This force is known as *magnetic force*. The gravitation, electrostatic and magnetic forces are examples of field forces.

### Basic Characteristics of a Force:

Force is a vector quantity therefore has *magnitude* as well as *direction*. To predict how a force affects motion of a body we must know its magnitude, direction and point on the body where the force is applied. This point is known as a *point of application* of the force. The direction and the point of application of a force both decide *line of action* of the force. Magnitude and direction decide effect on translation motion and magnitude and line of action decides effects on rotation motion.

## Newton's Laws of Motion

Newton has published three laws, which describe how forces affect motion of a body on which they act. These laws are fundamental in nature in the sense that the first law gives concept of force, inertia and the inertial frames; the second law defines force and the third law action and reaction as two aspects of mutual interaction between two bodies.

### The First Law

Every material body has tendency to preserve its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by external forces impressed on it.

- **Inertia**

The tendency of a material body to preserve its present state of uniform motion or of rest is known as inertia of the body. It was first conceived by Galileo.

Inertia is a physical quantity and mass of a material body is measure of its inertia.

- **Inertial Frame of Reference**

The first law requires a frame of reference in which only the forces acting on a body can be responsible for any acceleration produced in the body and not the acceleration of the frame of reference. These frames of reference are known as inertial frames.

### The Second Law

The rate of change in momentum of a body is equal to, and occurs in the direction of the net applied force.

A body of mass  $m$  in translational motion with velocity  $\vec{v}$ , if acted upon with a net external force  $\vec{F}$ , the second law suggests:

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

If mass of the body is constant, the above equation relates the acceleration  $\vec{a}$  of the body with the net force  $\vec{F}$  acting on it.

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\vec{a}$$

The first law provides concept of force and the second law provides the quantitative definition of force, therefore the second law is also valid only in inertial frames.

SI unit of force is newton. It is abbreviated as N. One newton equals to one kilogram-meter per second square.

$$1 \text{ N} = 1 \text{ kg-m/s}^2$$

Dimensions of force are  $\text{MLT}^{-2}$

### The Third Law

Force is always a two-body interaction. The first law describes qualitatively and the second law describes quantitatively what happens to a body if a force acts on it, but do not reveal anything about what happens to the other body participating in the interaction responsible for the force.

The third law accounts for this aspect of the force and states that every action on a body has equal and opposite reaction on the other body participating in the interaction.

## Concept of Free Body Diagram (FBD)

A force on a body can only exist when there is another body to create it, therefore in every physical situation of concern there must be two or more bodies applying forces on each other. On the other hand the three laws of Newton, describe motion of a single body under action of several forces, therefore, to analyze a given problem, we have to consider each of the bodies separately one by one. This idea provides us with the concept of free body diagram.

A free body diagram is a pictorial representation in which the body under study is assumed free from rest of the system i.e. assumed separated from rest of the interacting bodies and is drawn in its actual shape and orientation and all the forces acting on the body are shown.

### How to draw a Free Body Diagram (FBD)

- Separate the body under consideration from the rest of the system and draw it separately in actual shape and orientation.
- Show all the forces whether known or unknown acting on the body at their respective points of application.

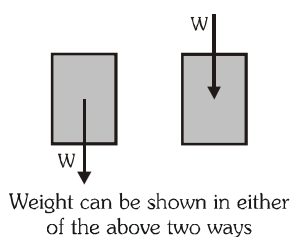
For the purpose count every contact where we separate the body under study from other bodies. At every such point, there may be a contact force. After showing, all the contact forces show all the field forces.

## Various Field Forces

Field forces include the gravitational force (weight) electrostatic forces and magnetic forces, which can easily be identified. At present, we consider only gravitational pull from the earth i.e. weight of the body.

### Weight: The net gravitational pull of the Earth

The gravitational pull from the earth acts on every particle of the body hence it is a distributed force. The net gravitational pull of the Earth on a body may be considered as weight of the body. It is assumed to act on the center of gravity of the body. For terrestrial bodies or celestial bodies of small size, this force can be assumed uniform throughout its volume. Under such circumstances, center of gravity and center of mass coincide and the weight is assumed to act on them. Furthermore, center of mass of uniform bodies lies at their geometrical center. At present, we discuss only uniform bodies and assume their weight to act on their geometrical center. In the figure weight of a uniform block is shown acting on its geometrical centre that coincides with the center of mass and the centre of gravity of the body.

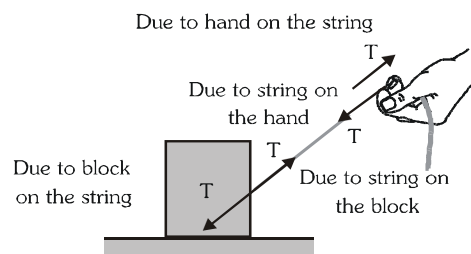


## Various Contact Forces

At every point where a body under consideration is supposed to be separated from other bodies to draw its free-body diagram, there may be a contact force. Most common contact forces, which we usually encounter, are tension force of a string, normal reaction on a surface in contact, friction, spring force etc.

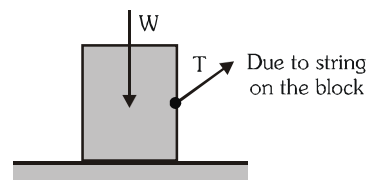
### Tension Force of Strings

A string or similar flexible connecting links as a thread or a chain etc. we use to transmit a force. Due to flexibility, a string can be used only to pull a body connected to it by applying a force always along the string. According to the third law, the connected body must also apply an equal and opposite force on the string, which makes the string taut. Therefore, this force is known as tension force  $T$  of the string. In the given figure is shown a block pulled by a string, which is being pulled by a person.

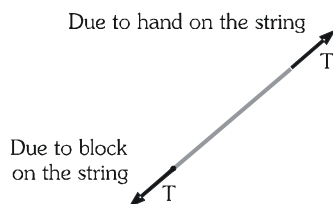


The tension force applied by string on the block and the force applied by the block on the string shown in the figure constitute a third law action-reaction pair. Similarly, tension force applied by the string on hand and force applied by the hand on string is another third law action-reaction pair.

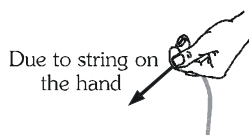
While studying motion of the block, the force applied by the string on it, weight of the block and a reaction from the floor has to be considered. In the figure only weight and tension of string are shown.



To study motion of the string, the force applied by the block on the string and the force applied by the hand on the string must be considered. These forces are shown in the FBD of string.

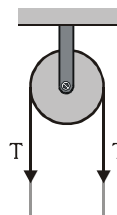


To study conditions of motion of the person, the force applied by the string on the hand has to be considered as shown in the figure.



### String passing over a pulley

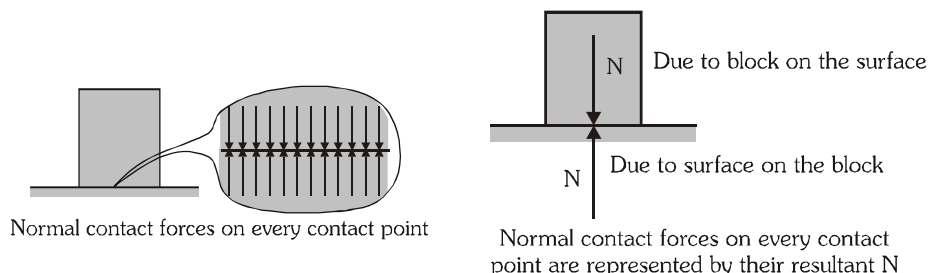
A pulley is a device consisting of a wheel, which can rotate freely on its axle. A single pulley changes direction of tension force. At present for simplicity, we discuss only ideal pulley, which is massless i.e. has negligible mass and rotates on its axle without any friction. An ideal pulley offers no resistance to its rotation, therefore tension force in the string on both sides of it are equal in magnitude. Such a pulley is known as ideal pulley.



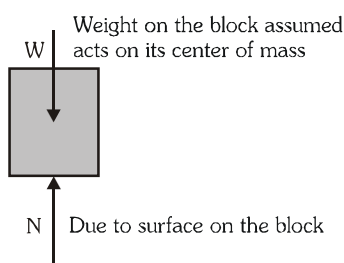
## Normal Reaction

Two bodies in contact, when press each other, must apply equal and opposite forces on each other. These forces constitute a third law action-reaction pair. If surfaces of the bodies in contact are frictionless, this force acts along normal to the surface at the point of contact. Therefore, it is known as normal reaction.

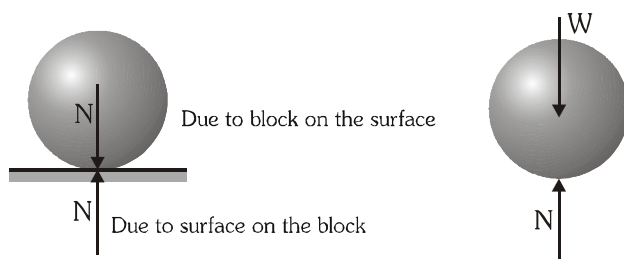
Consider a block of weight  $W$  placed on a frictionless floor. Because of its weight it presses the floor at every point in contact and the floor also applies equal and opposite reaction forces on every point of contact. We show all of them by a single resultant  $N$  obtained by their vector addition.



To apply Newton's laws of motion (NLM) on the block, its weight  $W$  and normal reaction  $N$  applied by the floor on the block must be considered as shown in the following figure. It is the FBD of the block.

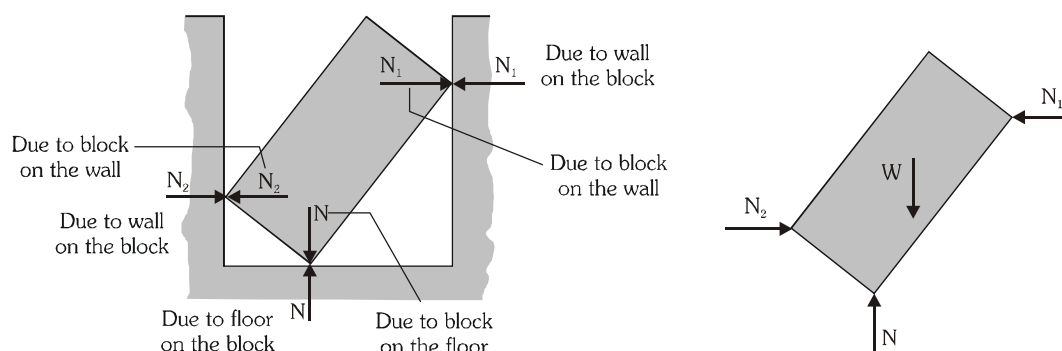


Consider a spherical ball of weight  $W$  placed on a floor. The normal reaction from the floor on the ball and from the ball on the floor makes third law action-reaction pair. These forces are shown in the left figure.



To apply Newton's laws of motion (NLM) on the ball, its weight  $W$  and normal reaction  $N$  applied by the floor on the ball must be considered as shown in the above right figure. It is the FBD of the ball.

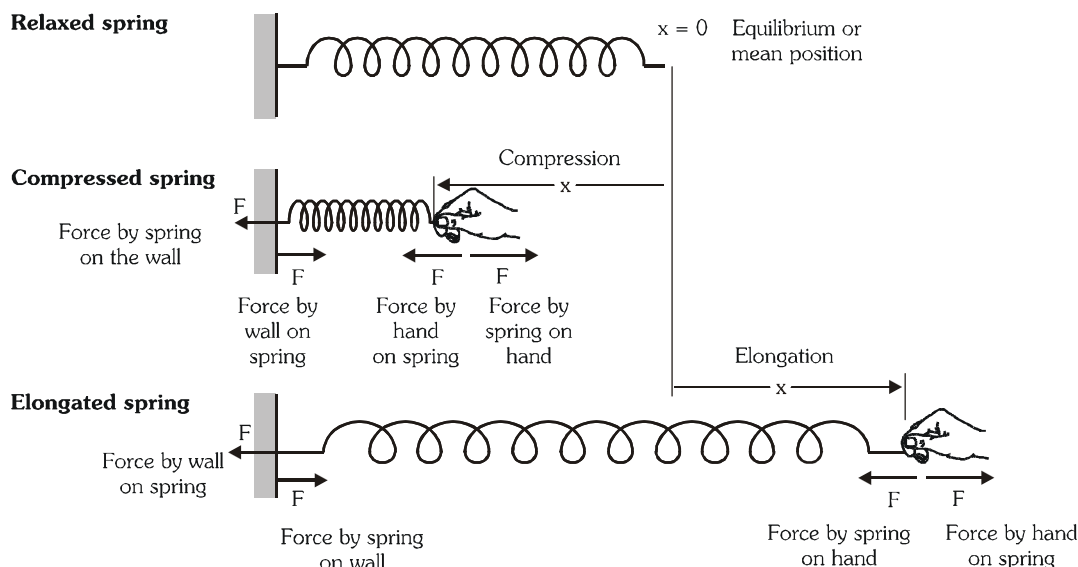
When two surfaces make contact, the normal reaction acts along the common normal and when a surface and a sharp corner make a contact the normal reaction acts along the normal to the surface. Consider a block placed in a rectangular trough as shown in the figure.



To apply Newton's laws of motion (NLM) on the block, its free body diagram (FBD) is shown in the above right figure.

### Spring Force

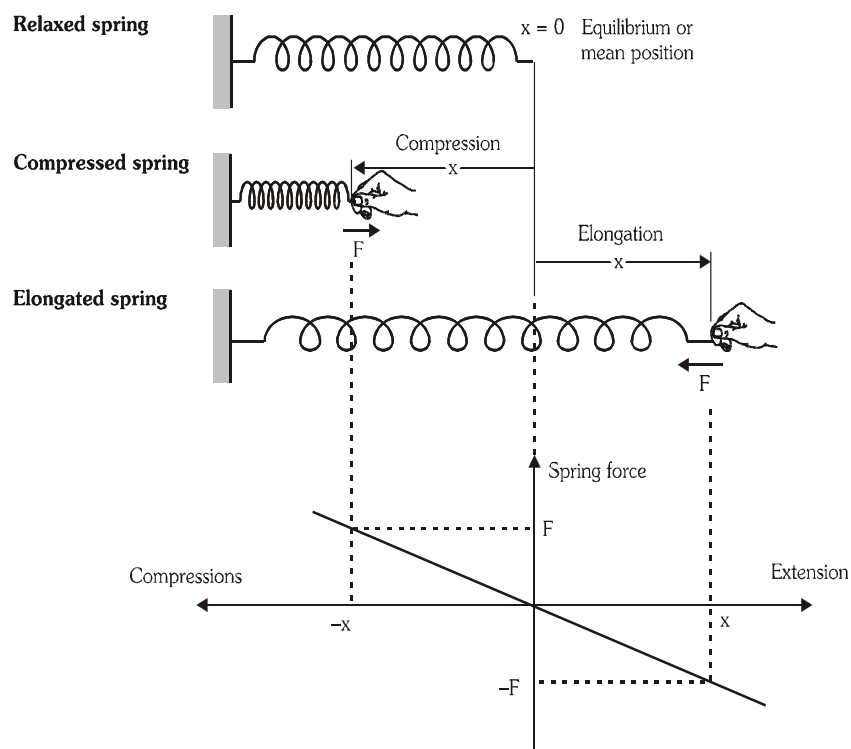
When no force acts on a spring, it is in relaxed condition i.e. neither compressed nor elongated. Consider a spring attached to a fixed support at one of its end and the other end is free. If we neglect gravity, it remains in relaxed state. When it is pushed by a force  $F$ , it is compressed and displacement  $x$  of its free end is called compression. When the spring is pulled by a force  $F$ , it is elongated and displacement  $x$  of its free end is called elongation. Various forces developed in these situations are shown in the following figure.



The force applied by the spring on the wall and the force applied by the wall on the spring make a third law action-reaction pair. Similarly, force by hand on the spring and the force by spring on the hand make another third law action-reaction pair.

### Hooke's Law:

How spring force varies with deformation in length  $x$  of the spring is also shown in the following figure.



The force  $F$  varies linearly with  $x$  and acts in a direction opposite to  $x$ . Therefore, it is expressed by the following equation

$$F = -kx$$

Here, the minus (-) sign represents the fact that force  $F$  is always opposite to  $x$ .

The constant of proportionality  $k$  is known as *force constant of the spring* or simply as *spring constant*. The slope modulus of the graph equals to the spring constant.

SI unit of spring constant is newton per meter or (N/m).

Dimensions of spring constant are  $MT^{-2}$ .

## Translational Equilibrium

A body in state of rest or moving with constant velocity is said to be in translational equilibrium. Thus if a body is in translational equilibrium in a particular inertial frame of reference, it must have no linear acceleration. When it is at rest, it is in *static equilibrium*, whereas if it is moving at constant velocity it is in *dynamic equilibrium*.

### Conditions for translational equilibrium

For a body to be in translational equilibrium, no net force must act on it i.e. vector sum of all the forces acting on it must be zero.

If several external forces  $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_i, \dots$  and  $\vec{F}_n$  act simultaneously on a body and the body is in translational equilibrium, the resultant of these forces must be zero.

$$\sum \vec{F}_i = \vec{0}$$

If the forces  $\vec{F}_1, \vec{F}_2, \dots, \vec{F}_i, \dots$  and  $\vec{F}_n$  are expressed in Cartesian components,

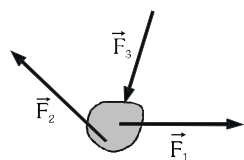
we have :

$$\sum F_{ix} = 0 \quad \sum F_{iy} = 0 \quad \sum F_{iz} = 0$$

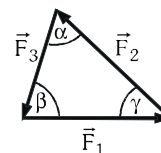
If a body is acted upon by a single external force, it cannot be in equilibrium.

If a body is in equilibrium under the action of only two external forces, the forces must be equal and opposite.

If a body is in equilibrium under action of three forces, their resultant must be zero; therefore, according to the triangle law of vector addition they must be coplanar and make a closed triangle.



$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0} \Rightarrow$$

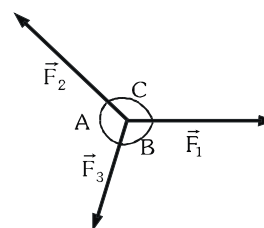
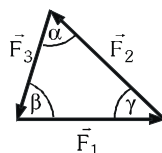


The situation can be analyzed by either graphical method or analytical method.

- Graphical method makes use of sine rule or Lami's theorem.

$$\text{Sine rule : } \frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

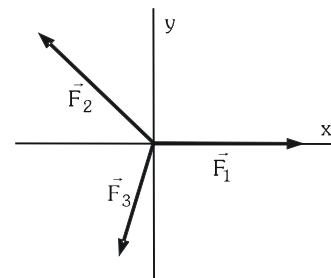
$$\text{Lami's theorem : } \frac{F_1}{\sin A} = \frac{F_2}{\sin B} = \frac{F_3}{\sin C}$$



- Analytical method makes use of Cartesian components. Since the forces involved make a closed triangle, they lie in a plane and a two-dimensional Cartesian frame can be used to resolve the forces. As far as possible orientation of the x-y frame is selected in such a manner that angles made by forces with axes should have convenient values.

$$\sum F_x = 0 \Rightarrow F_{1x} + F_{2x} + F_{3x} = 0$$

$$\sum F_y = 0 \Rightarrow F_{1y} + F_{2y} + F_{3y} = 0$$



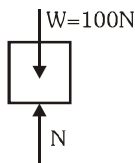
Problems involving more than three forces should be analyzed by analytical method. However, in some situations, there may be some parallel or anti-parallel forces and they should be combined first to minimize the number of forces. This may sometimes lead a problem involving more than three forces to a three-force system.

### Example

Consider a box of mass 10 kg resting on a horizontal table and acceleration due to gravity to be  $10 \text{ m/s}^2$ .

- Draw the free body diagram of the box.
- Find value of the force exerted by the table on the box.
- Find value of the force exerted by the box on the table.
- Are force exerted by table on the box and weight of the box third law action-reaction pair?

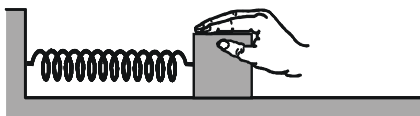
### Solution



- N : Force exerted by table on the box.
- The block is in equilibrium.  $\sum \vec{F} = \vec{0} \Rightarrow W - N = 0 \Rightarrow N = 100 \text{ N}$
- N = 100 N : Because force by table on the box and force by box on table make Newton's third law pair.
- No

### Example

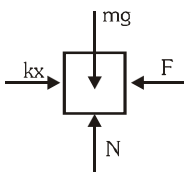
Consider a spring attached at one of its ends to a fixed support and at other end to a box, which rests on a smooth floor as shown in the figure. Denote mass of the box by m, force constant of the spring by k and acceleration due to gravity by g.



The box is pushed horizontally displacing it by distance x towards the fixed support and held at rest.

- Draw free body diagram of the box.
- Find force exerted by hand on the box.
- Write all the third law action-reaction pairs.

### Solution

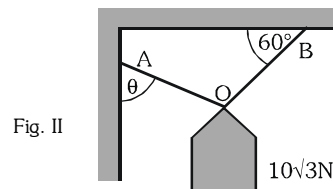
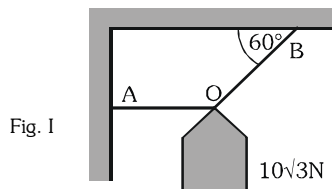


- F is push by hand.
- Since the block is in equilibrium  $\sum F_x = 0 \Rightarrow F = kx$
- Force by hand on box and force by box on hand.
  - Force by spring on box and force by box on spring.
  - Normal reaction by box on floor and normal reaction by floor on box.
  - Weight of the box and the gravitational force by which box pulls the earth.
  - Force by spring on support and force by support on spring.



### Example

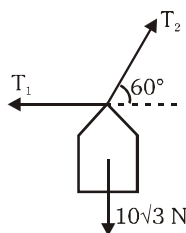
- (a) A box of weight  $10\sqrt{3}$  N is held in equilibrium with the help of two strings OA and OB as shown in figure-I. The string OA is horizontal. Find the tensions in both the strings.



- (b) If you can change location of the point A on the wall and hence the orientation of the string OA without altering the orientation of the string OB as shown in figure-II. What angle should the string OA make with the wall so that a minimum tension is developed in it?

### Solution

- (a) Free body diagram of the box



*Graphical Method* : Use triangle law

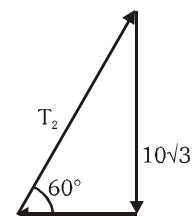
$$\begin{aligned} T_2 \sin 60^\circ &= 10\sqrt{3} \Rightarrow T_2 = 20\text{N} \\ T_1 \tan 60^\circ &= 10\sqrt{3} \Rightarrow T_1 = 10\text{N} \end{aligned}$$

*Analytical Method*: Use Cartesian components

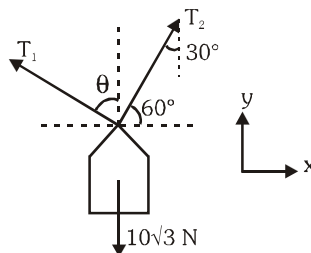
$$\sum F_x = 0 \Rightarrow T_2 \cos 60^\circ = T_1 \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow T_2 \sin 60^\circ = 10\sqrt{3} \quad \dots(ii)$$

From equation (i) & (ii) we have  $T_1 = 10\text{N}$  and  $T_2 = 20\text{N}$



- (b) Free body diagram of the box



*Graphical Method* : Use triangle law

For  $T_1$  to be minimum, it must be perpendicular to  $T_2$ .

From figure  $\theta = 60^\circ$

*Analytical Method* : Use Cartesian components

$$\sum F_x = 0 \Rightarrow T_2 \cos 60^\circ = T_1 \sin \theta \quad \dots(i)$$

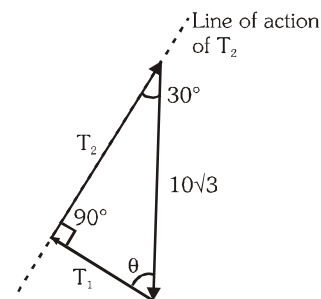
$$\sum F_y = 0 \Rightarrow T_1 \cos \theta + T_2 \sin 60^\circ = 10\sqrt{3} \quad \dots(ii)$$

From equation (i) and (ii), we have 
$$T_1 = \frac{10\sqrt{3}}{\sqrt{3} \sin \theta + \cos \theta}$$

If  $T_1$  is minimum,  $\sqrt{3} \sin \theta + \cos \theta$  must be maximum. Maximum value of  $\sqrt{3} \sin \theta + \cos \theta$  is 2.

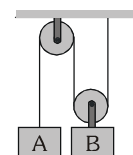
$$\sqrt{3} \sin \theta + \cos \theta = 2$$

Solving the above equation we get  $\theta = 60^\circ$

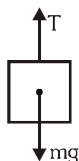


**Example**

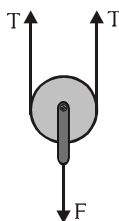
Two boxes A and B of masses  $m$  and  $M$  are suspended by a system of pulleys are in equilibrium as shown. Express  $M$  in terms of  $m$ .

**Solution**

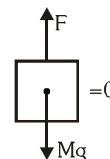
Since tension on both sides of a pulley are equal and string is massless therefore tension everywhere on the string must have same magnitude.



FBD of block A



FBD of pulley



FBD of Block B

$$\text{For equilibrium of block A} \quad \sum \vec{F} = \vec{0} \Rightarrow T = mg \quad \dots(i)$$

$$\text{For equilibrium of pulley attached to block B} \quad \sum \vec{F} = \vec{0} \Rightarrow F = 2T \quad \dots(ii)$$

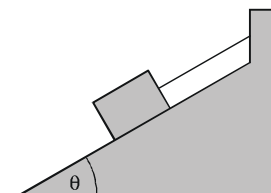
$$\text{For equilibrium of block B} \quad \sum \vec{F} = \vec{0} \Rightarrow F = Mg \quad \dots(iii)$$

From equation (i), (ii) and (iii), we have  $M = 2m$

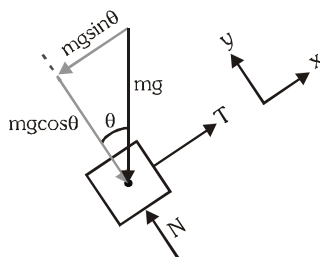
**Example**

A box of mass  $m$  rests on a smooth slope with help of a thread as shown in the figure. The thread is parallel to the incline plane.

- Draw free body diagram of the box.
- Find tension in the thread.
- If the thread is replaced by a spring of force constant  $k$ , find extension in the spring.

**Solution**

- Free body diagram of the block



$$(b) \quad \text{The block is in equilibrium, therefore} \quad \sum F_x = 0 \Rightarrow T = mg \sin \theta \quad \dots(i)$$

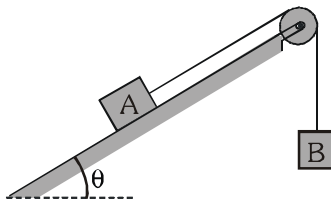
- If the thread is replaced by a spring, spring force must be equal to  $T$ , therefore

$$T = kx \quad \dots(ii)$$

$$\text{From equation (i) and (ii), we have} \quad x = \frac{mg \sin \theta}{k}$$

### Example

Block A of mass  $m$  placed on a smooth slope is connected by a string with another block B of mass  $M$  as shown in the figure. If the system is in equilibrium, express  $M$  in terms of  $m$ .



### Solution

For equilibrium of the block A, net force on it must be zero.

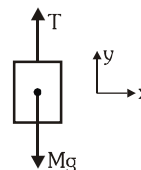
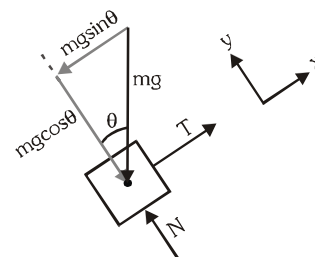
$N$ : Normal reaction from slope

$$\sum F_x = 0 \Rightarrow T = mg \sin \theta \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow N = mg \cos \theta \quad \dots(ii)$$

For equilibrium of block B, the net force on it must be zero.

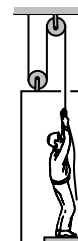
$$\sum F_y = 0 \Rightarrow T = Mg \quad \dots (ii)$$



From equations (i) and (ii), we have  $M = m \sin \theta$

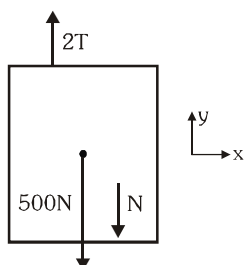
### Example

A 70 kg man standing on a weighing machine in a 50 kg lift pulls on the rope, which supports the lift as shown in the figure. Find the force with which the man should pull on the rope to keep the lift stationary and the weight of the man as shown by the weighing machine.



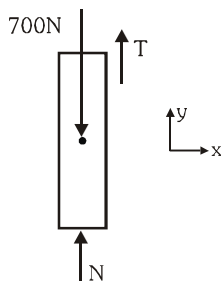
### Solution

Tension magnitude everywhere in the string is same. For equilibrium of the lift.



$$\sum F_y = 0 \Rightarrow 500 + N = 2T \quad \dots(i)$$

To analyse equilibrium of the man let us assume him as a block



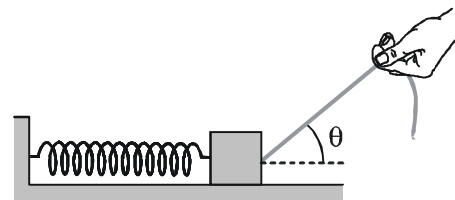
$$\sum F_y = 0 \Rightarrow N + T = 700 \quad \dots(ii)$$

From equations (i) & (ii), we have  $T = 400 \text{ N}$  and  $N = 300 \text{ N}$

Here,  $T$  is the pull of mass and  $N$  is reading of the weighing machine.

**Example**

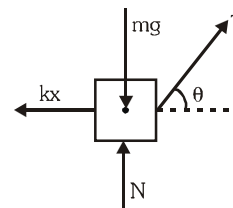
A block of mass  $m$  placed on a smooth floor is connected to a fixed support with the help of a spring of force constant  $k$ . It is pulled by a rope as shown in the figure. Tension force  $T$  of the rope is increased gradually without changing its direction, until the block loses contact from the floor. The increase in rope tension  $T$  is so gradual that acceleration in the block can be neglected.



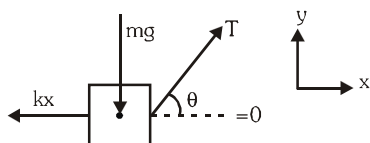
- Well before the block loses contact from the floor, draw its free body diagram.
- What is the necessary tension in the rope so that the block loses contact from the floor?
- What is the extension in the spring, when the block loses contact with the floor?

**Solution**

- Free body diagram of the block, well before it loses contact with the floor.



- When the block is about to leave the floor, it is not pressing the floor. Therefore  $N = 0$  and the block is in equilibrium.



$$\sum F_x = 0 \Rightarrow T \cos \theta = kx \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow T \sin \theta = mg \quad \dots(ii)$$

From equations (ii), we have  $T = mg \csc \theta$

- From equation (i) and (ii), we have  $x = \frac{mg \cot \theta}{k}$

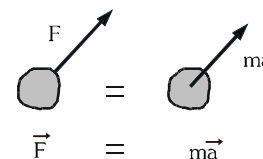
**Dynamics of Particles: Translation motion of accelerated bodies**

Newton's laws are valid in inertial frames, which are un-accelerated frames. At present, we are interested in motion of terrestrial bodies and for this purpose; ground can be assumed a satisfactory inertial frame.

In particle dynamics, according to Newton's second law, forces acting on the body are considered as cause and rate of change in momentum as effect. For a rigid body of constant mass, the rate of change in momentum equals to product of mass and acceleration vector. Therefore, forces acting on it are the cause and product of mass and acceleration vector is the effect.

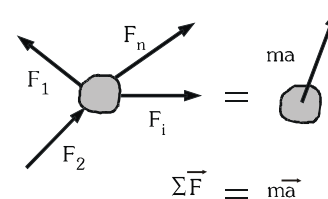
To write the equation of motion it is recommended to draw the free body

diagram, put a sign of equality and in front of it draw the body attached with a vector equal to mass times acceleration produced. In the figure is shown a body of mass  $m$  on which a single force  $\vec{F}$  acts and an observer in an inertial frame of reference observes the body moving with acceleration  $\vec{a}$ .



Acceleration imparted to a body by a force is independent of other forces,

therefore when several forces  $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_n$  act simultaneously on a body, the acceleration imparted to the body is the same as a single force equal to the vector sum of these forces could produce. The vector sum of these forces is known as the net resultant of these forces.



$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = m\vec{a}$$

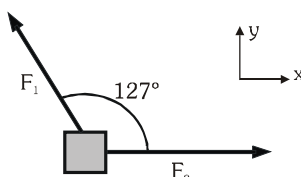
$$\sum \vec{F} = m\vec{a}$$

In Cartesian coordinate system the vector quantities in the above equation is resolved into their components along x, y, and z axes as follows:

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$

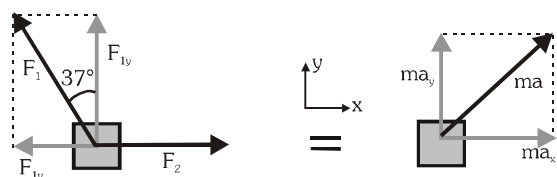
### Example

Two forces  $F_1$  and  $F_2$  of magnitudes 50 N and 60 N act on a free body of mass  $m = 5$  kg in directions shown in the figure. What is acceleration of object with respect to the free space?



### Solution

In an inertial frame of reference with its x-axis along the force  $F_2$ , the forces are expressed in Cartesian components.



$$\vec{F}_1 = (-30\vec{i} + 40\vec{j}) \text{ N and } \vec{F}_2 = 60\vec{i} \text{ N}$$

$$\sum F_x = ma_x \Rightarrow a_x = 6 \text{ m/s}^2$$

$$\sum F_y = ma_y \Rightarrow a_y = 8 \text{ m/s}^2$$

$$\vec{a} = (6\vec{i} + 8\vec{j}) \text{ m/s}^2$$

### Example

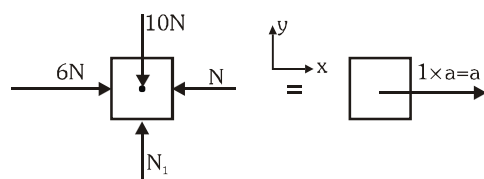
Boxes A and B of mass  $m_A = 1$  kg and  $m_B = 2$  kg are placed on a smooth horizontal plane. A man pushes horizontally the 1 kg box with a force  $F = 6$  N. Find the acceleration and the reaction force between the boxes.



### Solution

Since both the blocks move in contact it is obvious that both of them have same acceleration. Say it is 'a'.

Applying NLM to block A



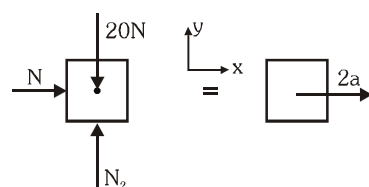
N: Normal reaction from B

$N_1$ : Normal reaction from floor

$$\sum F_x = ma_x \Rightarrow 6 - N = a \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow N_1 = 10 \text{ N} \quad \dots(ii)$$

Applying NLM to block B



N : Normal reaction from A

$N_2$ : Normal reaction from ground

$$\sum F_x = ma_x \Rightarrow N = 2a \quad \dots(iii)$$

$$\sum F_y = 0 \Rightarrow N_2 = 20 \text{ N} \quad \dots(iv)$$

From equations (i) & (iii), we have  $a = 2 \text{ m/s}^2$  and  $N = 4 \text{ N}$

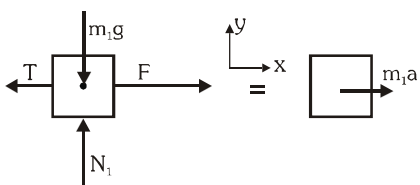
**Example**

Two blocks A and B of masses  $m_1$  and  $m_2$  connected by light strings are placed on a smooth floor as shown in the figure. If the block A is pulled by a constant force  $F$ , find accelerations of both the blocks and tension in the string connecting them.

**Solution**

String connecting the blocks remain taut keeping separation between them constant. Therefore it is obvious that both of them move with the same acceleration. Say it is 'a'.

Applying NLM to block A



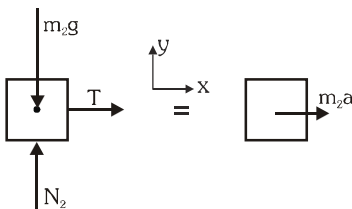
T: Tension of string

$N_1$  : Normal reaction from ground

$$\sum F_x = ma \Rightarrow F - T = m_1 a \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow N_1 = m_1 g \quad \dots(ii)$$

Applying NLM to block B.



T : Tension of string

$N_2$  : Normal reaction from ground.

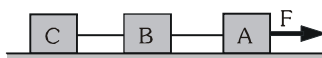
$$\sum F_x = ma_x \Rightarrow T = m_2 a \quad \dots(iii)$$

$$\sum F_y = 0 \Rightarrow N_2 = m_2 g \quad \dots(iv)$$

From equation (i) and (iii), we have  $a = \frac{F}{m_1 + m_2}$  and  $T = \frac{m_2 F}{m_1 + m_2}$

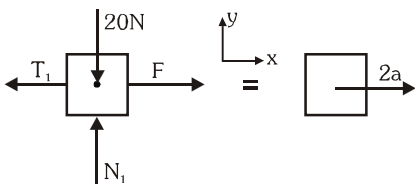
**Example**

Three identical blocks A, B and C, each of mass 2.0 kg are connected by light strings as shown in the figure. If the block A is pulled by an unknown force  $F$ , the tension in the string connecting blocks A and B is measured to be 8.0 N. Calculate magnitude of the force  $F$ , tension in the string connecting blocks B and C, and accelerations of the blocks.

**Solution**

It is obvious that all the three blocks move with the same acceleration. Say it is 'a'.

Applying NLM to the block A.



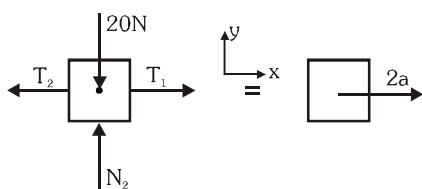
$T_1$ : Tension of string connecting blocks A and B.

$N_1$  : Normal reaction from floor.

$$\sum F_x = ma_x \Rightarrow F - T_1 = 2a \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow N_1 = 20N \quad \dots(ii)$$

Applying NLM to the block B.



$T_1$ : Tension of string connecting blocks A and B.

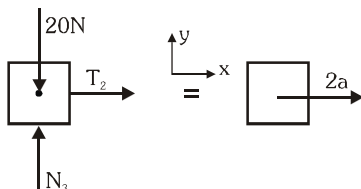
$T_2$ : Tension of string connecting B & C.

$N_1$ : Normal reaction from floor.

$$\sum F_x = ma_x \Rightarrow T_1 - T_2 = 2a \quad \dots(iii)$$

$$\sum F_y = 0 \Rightarrow N_2 = 20N \quad \dots(iv)$$

Applying NLM to the block C.



$T_3$ : Tension of string connecting B & C.

$N_3$ : Normal reaction from floor.

$$\sum F_x = ma_x \Rightarrow T_2 = 2a \quad \dots(v)$$

$$\sum F_y = 0 \Rightarrow N_3 = 20N \quad \dots(vi)$$

From equations (i), (iii) and (v), we have

$$F = 6a \quad \dots(vii)$$

Now using the fact that  $T_1 = 8N$  with equation (i), We have  $a = 2 \text{ m/s}^2$

Now from equation (i) we have  $F = 12 \text{ N}$

From equation (iii), we have  $T_2 = 4 \text{ N}$

### Example

Two blocks A and B of masses  $m_1$  and  $m_2$  connected by uniform string of mass  $m$  and length  $\ell$  are placed on smooth floor as shown in the figure. The string also lies on the floor. The block A is pulled by a constant force  $F$ .



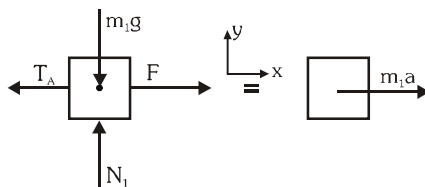
(a) Find accelerations  $a$  of both the blocks and tensions  $T_A$  and  $T_B$  at the ends of the string.

(b) Find an expression for tension  $T$  in the string at a distance  $x$  from the rear block in terms of  $T_A$ ,  $T_B$ ,  $m$ ,  $\ell$  and  $x$ .

### Solution

It is obvious that both the blocks and the whole string move with the same acceleration say it is ' $a$ '. Since string has mass it may have different tensions at different points.

(a) Applying NLM to block A.



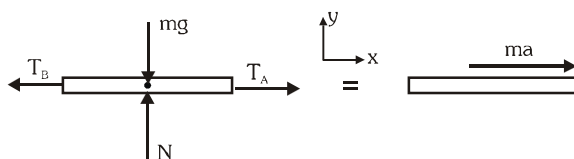
$T_A$ : Tension of the string at end connected to block A.

$N_1$ : Normal reaction of floor

$$\sum F_x = ma_x \Rightarrow F - T_A = m_1a \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow N_1 = m_1g \quad \dots(ii)$$

Applying NLM to the rope



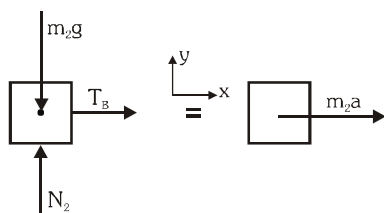
$T_B$ : Tension of string at end connected to block B.

$N$ : Normal reaction of floor

$$\sum F_x = ma_x \Rightarrow T_A - T_B = ma \quad \dots(iii)$$

$$\sum F_y = 0 \Rightarrow N = mg \quad \dots(iv)$$

Applying NLM to the block B



$T_B$  : Tension of string

$N_2$  : Normal reaction from floor

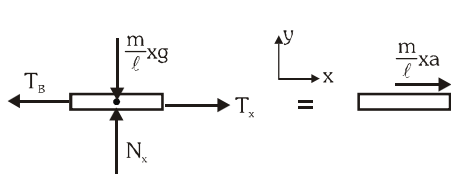
$$\sum F_x = ma_x \Rightarrow T_B = m_2 a \quad \dots(v)$$

$$\sum F_y = 0 \Rightarrow N_2 = m_2 g \quad \dots(vi)$$

From equations (i), (iii) and (v), we have

$$a = \frac{F}{m + m_1 + m_2} \dots(vii) \quad T_A = \frac{(m + m_2)F}{m + m_1 + m_2} \dots(viii) \quad T_B = \frac{m_2 F}{m + m_1 + m_2} \dots(ix)$$

- (b) To find tension at a point  $x$  distance away from block B, we can consider string of length  $x$  or  $\ell - x$ .  
 Let us consider length of string  $x$  and apply NLM.



$\frac{mx}{\ell}$  : mass of length  $x$ .

$T_x$  = Tension at distance  $x$

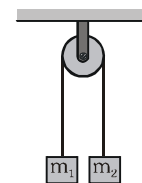
$N_x$  = Normal reaction of floor

$$\sum F_x = ma_x \Rightarrow T_x - T_B = \frac{mx}{\ell} a \quad \dots(x)$$

From equation (vii), (viii), (ix) and (x), we have  $T_x = \left( \frac{m}{\ell} x + m_2 \right) \frac{F}{m + m_1 + m_2}$

### Example

The system shown in the figure is released from rest. Assuming mass  $m_2$  more than the mass  $m_1$ , find the accelerations of the blocks and the tension in the string.



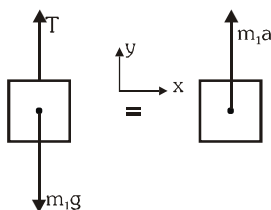
### Solution

It is obvious that both blocks move with same acceleration magnitudes. Say it is 'a'.

Since  $m_2$  is heavier, it moves downwards and  $m_1$  moves upwards.

Tension at both the ends of the string has same magnitude. Say it is 'T'.

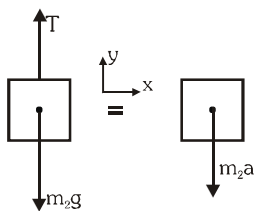
Apply NLM to block A of mass  $m_1$



$$\sum F_y = ma_y \Rightarrow T - m_1 g = m_1 a \quad \dots(i)$$

Apply NLM to block of mass  $m_2$



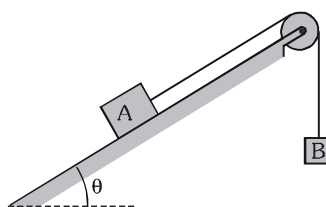


$$\sum F_y = ma_y \Rightarrow m_2g - T = m_2a \quad \dots(ii)$$

From equations (i) & (ii), we have  $a = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) g$ ,  $T = \left( \frac{2m_1m_2}{m_1 + m_2} \right) g$

### Example

Block A of mass  $m$  placed on a smooth slope is connected by a string with another block B of mass  $M (> m \sin \theta)$  as shown in the figure. Initially the block A is held at rest and then let free. Find acceleration of the blocks and tension in the string.



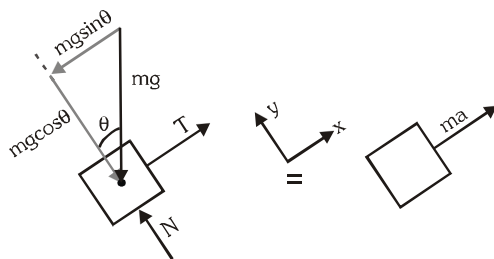
### Solution

Both the blocks must move with the same magnitude of acceleration.

Since  $M > m \sin \theta$ , block B move downward pulling block A up the plane. Let acceleration magnitude is 'a'.

Tension at both the ends of the string is same. Say it is 'T'.

Apply NLM to block A



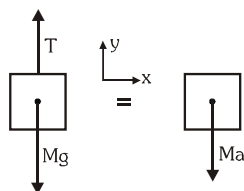
N: Normal reaction from slope

T : Tension of string

$$\sum F_x = ma_x \Rightarrow T - mg \sin \theta = ma \quad \dots(i)$$

$$\sum F_y = 0 \Rightarrow N = mg \cos \theta \quad \dots(ii)$$

Apply NLM to block B



T: Tension of string

$$\sum F_y = ma_y \Rightarrow Mg - T = Ma \quad \dots(iii)$$

From equation (i) & (iii), we have  $a = \frac{(M - m \sin \theta)g}{M + m}$ ,  $T = \frac{(1 + \sin \theta)mMg}{m + M}$

## System of Interconnected bodies

In system of interconnected bodies, several bodies are interconnected in various manners through some sort of physical links. Sometimes these physical links includes ropes and pulleys and sometimes the bodies under investigation are pushing each other through direct contact. In systems consisting of bodies interconnected through ropes and pulleys, relation between their accelerations depends on the arrangement of the ropes and pulleys. In addition, in system where bodies push each other, they affect relation between their accelerations due to their shapes.

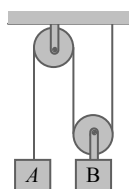
In kinematics while dealing with dependant motion or constrained motion, we have already learnt how to find relations between velocities and accelerations of interconnected bodies.

Analysis of physical situations involving interconnected bodies often demands relation between accelerations of these bodies in addition to the equations obtained by application of Newton's laws of motion. Therefore, while analyzing problems of interconnected bodies, it is recommended to explore first the relations between accelerations and then apply Newton's laws of motion.

In following few examples, we learn how to deal with problems of interconnected bodies.

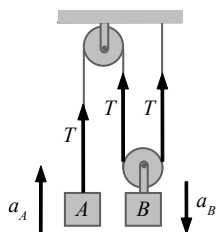
### Example

Two boxes A and B of masses  $m$  and  $M$  interconnected by an ideal rope and ideal pulleys, are held at rest as shown. When it is released, box B accelerates downwards. Find accelerations of the blocks.

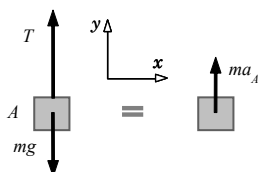


### Solution.

We first show tension forces applied by the string on the box A and the pulley connected to box B. Since the string, as well as the pulleys, both are ideal; the string applies tension force of equal magnitude everywhere. Denoting the tension force by  $T$ , we show it in the adjacent figure.



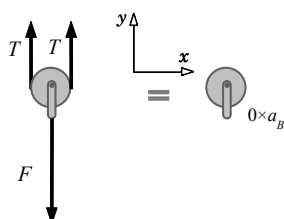
We first explore relation between accelerations  $a_A$  and  $a_B$  of the boxes A and B, which can be written either by using constrained relation or method of virtual work or by inspection.



$$a_A = 2a_B \quad \dots(i)$$

Applying Newton's Laws of motion to box A

$$\sum F_y = ma_y \rightarrow T - mg = ma_A \quad \dots(ii)$$



Applying Newton's Laws of motion to the pulley

$$\sum F_y = ma_y \rightarrow 2T - F = 0 \times a_B$$

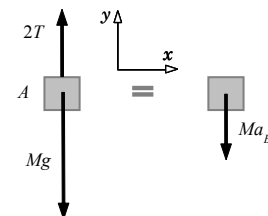
$$F = 2T \quad \dots(iii)$$

Applying Newton's Laws of motion to box B

$$\sum F_y = ma_y \rightarrow Mg - 2T = Ma_B \quad \dots(iv)$$

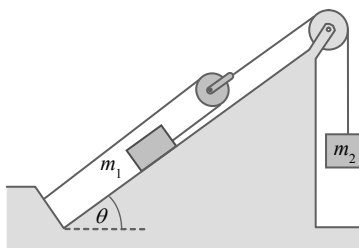
From equations (i), (ii), (iii) and (iv), we have

$$a_A = 2 \left( \frac{M - 2m}{M + 4m} \right) g \text{ and } a_B = \left( \frac{M - 2m}{M + 4m} \right) g$$



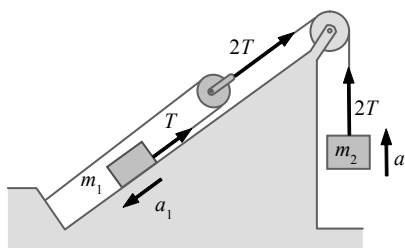
### Example

In the system shown in figure, block  $m_1$  slides down a friction less inclined plane. The pulleys and strings are ideal. Find the accelerations of the blocks.



### Solution

Tension forces applied by the strings are shown in the adjacent figure.

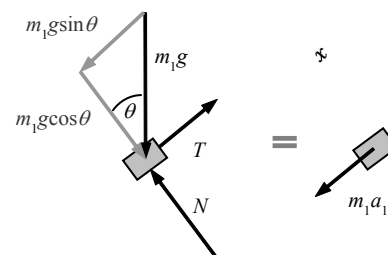


Let the block  $m_1$  is moving down the plane with an acceleration  $a_1$  and  $m_2$  is moving upwards with accelerations  $a_2$ . Relation between accelerations  $a_1$  and  $a_2$  of the blocks can be obtained easily by method of virtual work.

$$a_1 = 2a_2 \quad \dots(i)$$

Applying Newton's laws to analyze motion of block  $m_1$

$$\sum F_x = ma_x \rightarrow m_1 g \sin \theta - T = m_1 a_1 \quad \dots(ii)$$

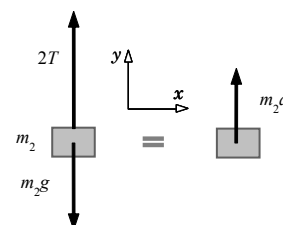


Applying Newton's laws to analyze motion of block  $m_2$

$$\sum F_y = ma_y \rightarrow 2T - m_2 g = m_2 a_2 \quad \dots(iii)$$

From equation (i), (ii) and (iii), we have

$$a_1 = \frac{2(2m_1 \sin \theta - m_2)}{4m_1 + m_2} g \quad a_2 = \frac{2m_1 g \sin \theta - m_2 g}{4m_1 + m_2}$$



## FRICTION

Whenever surfaces in contact are pressing each other slide or tend to slide over each other, opposing forces are generated tangentially to the surfaces in contact. These tangential forces, which oppose sliding or tendency of sliding between two surfaces are called frictional forces. Frictional forces on both bodies constitute third law action-reaction pair.

### Types of Friction

Before we proceed further into detailed account of frictional phenomena, it is advisable to become familiar with different types of frictional forces. All types of frictional phenomenon can be categorized into dry friction, fluid friction, and internal friction.

#### Dry Friction

It exists when two solid un-lubricated surfaces are in contact under the condition of sliding or tendency of sliding. It is also known as Coulomb friction.

#### Fluid Friction

Fluid friction is developed when adjacent layers of a fluid move at different velocities and gives birth to phenomena, which we call viscosity of the fluid. Resistance offered to motion of a solid body in a fluid also comes in this category and commonly known as viscous drag. We will study this kind of friction in fluid mechanics.

#### Internal Friction

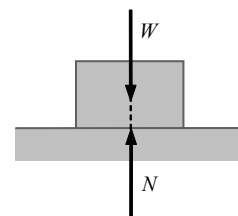
When solid materials are subjected to deformation, internal resistive forces developed because of relative movement of different parts of the solid. These internal resistive forces constitute a system of force, which is defined as internal friction. They always cause loss of energy.

Frictional forces exist everywhere in nature and result in loss of energy that is primarily dissipated in form of heat. Wear and tear of moving bodies is another unwanted result of friction. Therefore, sometimes, we try to reduce their effects – such as in bearings of all types, between piston and the inner walls of the cylinder of an IC engine, flow of fluid in pipes, and aircraft and missile propulsion through air. Though these examples create a negative picture of frictional forces, yet there are other situations where frictional forces become essential and we try to maximize the effects. It is the friction between our feet and the earth surface, which enables us to walk and run. Both the traction and braking of wheeled vehicles depend on friction.

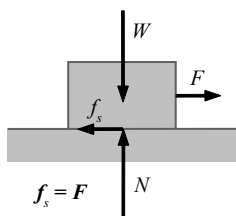
### Types of Dry Friction

In mechanics of non-deformable bodies, we are always concerned with the dry friction. Therefore, we often drop the word “dry” and simply call it friction.

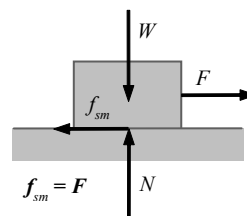
To understand nature of friction let us consider a box of weight  $W$  placed on a horizontal rough surface. The forces acting on the box are its weight and reaction from the horizontal surface. They are shown in the figure. The weight does not have any horizontal component, so the reaction of the horizontal surface on the box is normal to the surface. It is represented by  $N$  in the figure. The box is in equilibrium therefore both  $W$  and  $N$  are equal in magnitude, opposite in direction, and collinear.



Now suppose the box is being pulled by a gradually increasing horizontal force  $F$  to slide the box. Initially when the force  $F$  is small enough, the box does not slide. This can only be explained if we assume a frictional force, which is equal in magnitude and opposite in direction to the applied force  $F$  acts on the box. The force  $F$  produces in the box a tendency of sliding and the friction force is opposing this tendency of sliding. The frictional force developed before sliding initiates is defined as static friction. It opposes tendency of sliding.



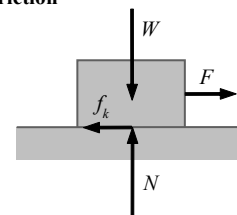
**Static Friction**



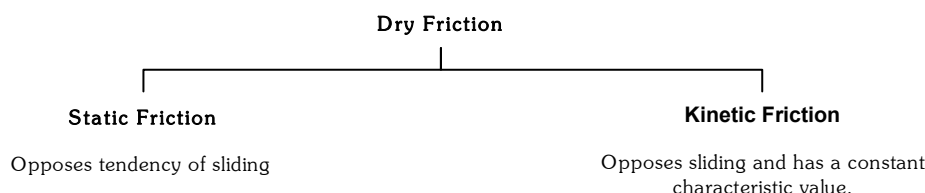
**Limiting friction: The maximum Static Friction**

As we increase  $F$ , the box remains stationary until a value of  $F$  is reached when the box starts sliding. Before the box starts sliding, the static friction increases with  $F$  and counterbalances  $F$  until the static friction reaches its maximum value known as limiting friction or maximum static friction  $f_{sm}$ .

When the box starts sliding, to maintain it sliding still a force  $F$  is needed to overcome frictional force. This frictional force is known as kinetic friction ( $f_k$ ). It always opposes sliding.



**Kinetic friction**



## Laws of Friction

When a normal force  $N$  exists between two surfaces, and we try to slide them over each other, the force of static friction ( $f_s$ ) acts before sliding initiates. It can have a value maximum up to the limiting friction ( $f_{sm}$ ).

$$f_s \leq f_{sm}$$

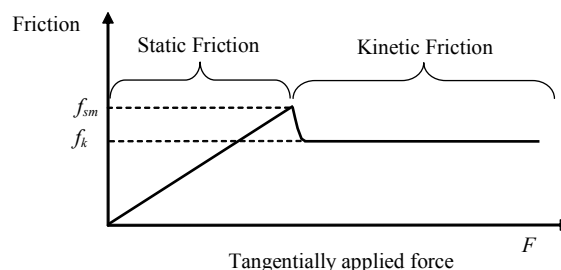
The limiting friction is experimentally observed proportional to the normal reaction between surfaces in contact.

$$f_{sm} = \mu_s N$$

Here  $\mu_s$  is the constant of proportionality. It is known as the coefficient of static friction for the two surfaces involved.

When sliding starts between the surfaces, the frictional force rapidly drops to a characteristic value, which always opposes the sliding. This characteristic frictional force is known as kinetic friction ( $f_k$ ). Kinetic friction is experimentally found proportional to the normal reaction between surfaces in contact.

$$f_k = \mu_k N$$



Here  $\mu_k$  is the constant of proportionality. It is known as the coefficient of kinetic friction for the two surfaces involved.

The frictional forces between any pair of surfaces are decided by the respective coefficients of friction. The coefficients of friction are dimensionless constants and have no units. The coefficient of static friction ( $\mu_s$ ) is generally larger than the coefficient of kinetic friction ( $\mu_k$ ) but never become smaller; at the most both of them may be equal. Therefore, the magnitude of kinetic friction is usually smaller than the limiting static friction ( $f_{sm}$ ) and sometimes kinetic friction becomes equal to the limiting static friction but it can never exceed the limiting friction.

The limiting static friction and the kinetic friction between any pair of solid surfaces follow these two empirical laws.

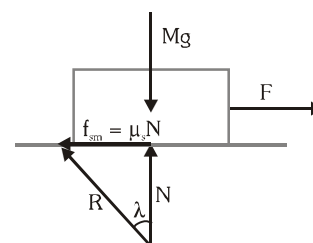
- Frictional forces are independent of measured area of contact.
- Both the limiting static friction and kinetic friction are proportional to the normal force pressing the surfaces in contact.

## Angle of Friction

The angle of friction is the angle between resultant contact force of and normal reaction  $N$ , when sliding is initiating. It is denoted by  $\lambda$

$$\tan \lambda = \frac{f_{sm}}{N} = \frac{\mu_s N}{N} = \mu_s$$

- For smooth surface  $\lambda = 0$

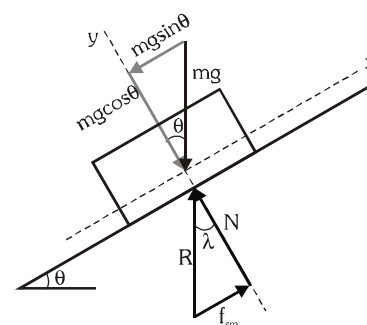


## Angle of Repose ( $\theta$ )

A body is placed on an inclined plane and the angle of inclination is gradually increased. At some angle of inclination  $\theta$  the body starts sliding down the plane due to gravity. This angle of inclination is called angle of repose ( $\theta$ ).

Angle of repose is that minimum angle of inclination at which a body placed on the inclined starts sliding down due to its own weight.

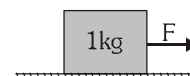
Thus, angle of repose = angle of friction.



### Example

A block of mass 1 kg is at rest on a rough horizontal surface, where coefficients of static and kinetic friction are 0.2 and 0.15. Find the frictional forces if a horizontal force

- (a)  $F = 1 \text{ N}$       (b)  $F = 1.96 \text{ N}$       (c)  $F = 2.5 \text{ N}$  is applied on a block



### Solution

Maximum force of friction is the limiting friction  $f_{sm} = 0.2 \times 1 \times 9.8 \text{ N} = 1.96 \text{ N}$

- (a) For  $F = 1 \text{ N}$ ,  $F < f_{sm}$

So, body is in rest means static friction is present and hence  $f_s = F = 1 \text{ N}$

- (b) For  $F = 1.96 \text{ N}$ ,  $F = f_{sm} = 1.96 \text{ N}$ . The block is about to slide, therefore  $f = 1.96 \text{ N}$

- (c) For  $F = 2.5 \text{ N}$ , So  $F > f_{sm}$

Now body is sliding and kinetic friction acts.

Therefore  $f = f_k = \mu_k N = \mu_k mg = 0.15 \times 1 \times 9.8 = 1.47 \text{ N}$

### Example

Length of a uniform chain is  $L$  and coefficient of static friction is  $\mu$  between the chain and the table top. Calculate the maximum length of the chain which can hang from the table without sliding.

### Solution

Let  $y$  be the maximum length of the chain that can hang without causing the portion of chain on table to slide.

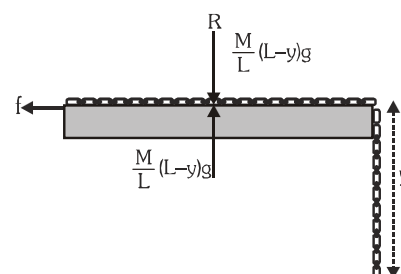
Length of chain on the table =  $(L - y)$

Weight of part of the chain on table =  $\frac{M}{L}(L - y)g$

Weight of hanging part of the chain =  $\frac{M}{L}yg$

For equilibrium with maximum portion hanging, limiting friction = weight of hanging part of the chain

$$\mu \frac{M}{L}(L - y)g = \frac{M}{L}yg \Rightarrow y = \frac{\mu L}{1 + \mu}$$

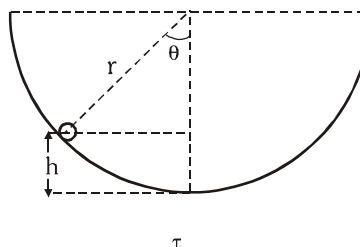
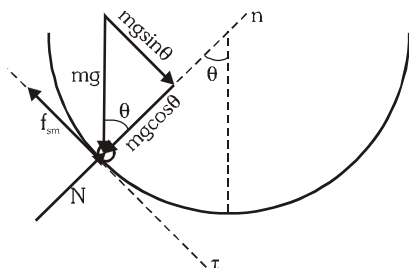


### Example

An insect crawls on the inner surface of hemispherical bowl of radius  $r$ . If the coefficient of friction between an insect and bowl is  $\mu$  and the radius of the bowl is  $r$ , find the maximum height to which the insect can crawl up.

### Solution

The insect can crawl up the bowl till the component of its weight tangent to the bowl is balanced by limiting frictional force.



$$\sum F_n = 0 \rightarrow N = mg \cos \theta \quad \dots(i)$$

$$\sum F_t = 0 \rightarrow f_{sm} = mg \sin \theta \quad \dots(ii)$$

$$\text{Force of limiting friction } f_{sm} = \mu N \quad \dots(iii)$$

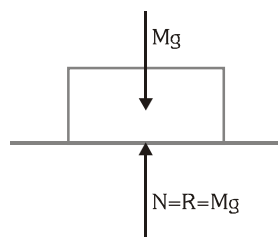
$$\text{From equation (i), (ii) and (iii), } \tan \theta = \mu \quad \dots(iv)$$

$$h = r - r \cos \theta = r \left[ 1 - \frac{1}{\sqrt{1 + \mu^2}} \right]$$

### Example

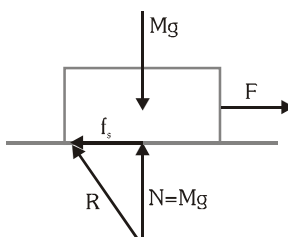
A body of mass  $M$  is kept on a rough horizontal ground (static friction coefficient  $= \mu_s$ ). A person is trying to pull the body by applying a horizontal force  $F$ , but the body is not moving. What is the contact force between the ground and the block.

### Solution



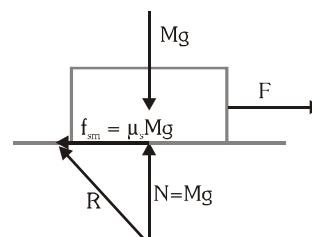
When  $F = 0$

$$R = Mg$$



When  $F < f_{sm}$

$$R = \sqrt{(Mg)^2 + f_s^2}$$



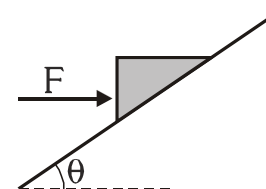
When  $F = f_{sm}$

$$R = Mg \sqrt{1 + \mu_s^2}$$

$$\text{Therefore } Mg \leq R \leq Mg \sqrt{1 + \mu_s^2}$$

### Example

A block rests on a rough inclined plane as shown in fig. A horizontal force  $F$  is applied to it (a) Find the force of normal reaction, (b) Can the force of friction be zero, if yes when? and (c) Assuming that friction is not zero find its magnitude and direction of its limiting value.

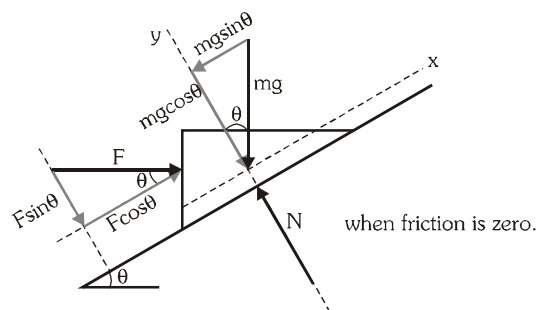


## Solution

$$(a) \sum F_y = 0 \rightarrow N = mg \cos \theta + F \sin \theta$$

$$(b) \sum F_x = 0 \rightarrow F \cos \theta = mg \sin \theta \Rightarrow F = mg \tan \theta$$

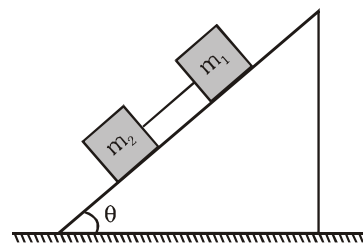
$$(c) \text{Limiting friction } f_{sm} = \mu N = \mu (mg \cos \theta + F \sin \theta);$$



It acts down the plane if body has tendency to slide up and acts up the plane if body has tendency to slide down.

## Example

Two blocks with masses  $m_1 = 1$  kg and  $m_2 = 2$  kg are connected by a string and slide down a plane inclined at an angle  $\theta = 45^\circ$  with the horizontal. The coefficient of sliding friction between  $m_1$  and plane is  $\mu_1 = 0.4$  and that between  $m_2$  and plane is  $\mu_2 = 0.2$ . Calculate the common acceleration of the two blocks and the tension in the string.



## Solution

As  $\mu_2 < \mu_1$ , block  $m_2$  has greater acceleration than  $m_1$  if we separately consider the motion of blocks. But they are connected so they move together as a system with common acceleration.

So acceleration of the blocks :

$$a = \frac{(m_1 + m_2)g \sin \theta - \mu_1 m_1 g \cos \theta - \mu_2 m_2 g \cos \theta}{m_1 + m_2}$$

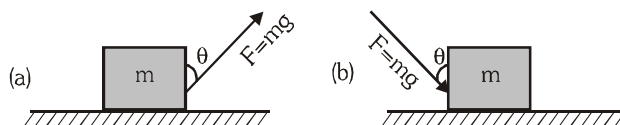
$$= \frac{(1+2)(10)\left(\frac{1}{\sqrt{2}}\right) - 0.4 \times 1 \times 10 \times \frac{1}{\sqrt{2}} - 0.2 \times 2 \times 10 \times \frac{1}{\sqrt{2}}}{1+2} = \frac{22}{3\sqrt{2}} \text{ ms}^{-2}$$

$$\text{For block } m_2 : m_2 g \sin \theta - \mu_2 m_2 g \cos \theta - T = m_2 a \Rightarrow T = m_2 g \sin \theta - \mu_2 m_2 g \cos \theta - m_2 a$$

$$= 2 \times 10 \times \frac{1}{\sqrt{2}} - 0.2 \times 2 \times 10 \times \frac{1}{\sqrt{2}} - 2 \times \frac{22}{3\sqrt{2}} = \frac{2}{3\sqrt{2}} \text{ N}$$

## Example

A block of mass  $m$  rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between block and surface is  $\mu$ . A force  $F = mg$  acting at an angle  $\theta$  with the vertical side of the block. Find the condition for which block will move along the surface.





### Solution

For (a) : normal reaction  $N = mg - mg \cos \theta$  frictional force  $= \mu N = \mu(mg - mg \cos \theta)$

Now block can be pulled when : Horizontal component of force

$\geq$  frictional force i.e.  $mg \sin \theta \geq \mu(mg - mg \cos \theta)$

$$\text{or } 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \geq \mu(1 - \cos \theta)$$

$$\text{or } 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \geq 2\mu \sin^2 \frac{\theta}{2} \quad \text{or } \cot \frac{\theta}{2} \geq \mu$$

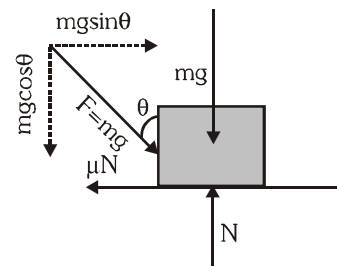
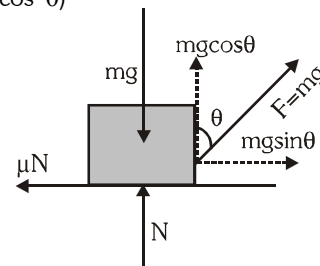
**For (b) :** Normal reaction  $N = mg + mg \cos \theta = mg(1 + \cos \theta)$

Hence, block can be pushed along the horizontal surface when

horizontal component of force  $\geq$  frictional force

i.e.  $mg \sin \theta \geq \mu mg(1 + \cos \theta)$

$$\text{or } 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \geq \mu \times 2 \cos^2 \frac{\theta}{2} \Rightarrow \tan \frac{\theta}{2} \geq \mu$$



### Example

A body of mass  $m$  rests on a horizontal floor with which it has a coefficient of static friction  $\mu$ . It is desired to make the body move by applying the minimum possible force  $F$ . Find the magnitude of  $F$  and the direction in which it has to be applied.

### Solution

Let the force  $F$  be applied at an angle  $\theta$  with the horizontal as shown in figure.

$$\sum F_y = 0 \rightarrow N = mg - F \sin \theta \quad \dots(i) \quad N = mg - F \sin \theta \quad \dots(i)$$

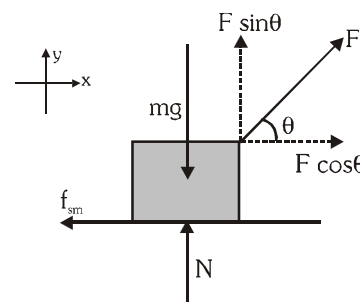
$$\sum F_x = 0 \rightarrow F \cos \theta \geq f_{sm} \Rightarrow F \cos \theta \geq \mu N \quad [\text{as } f_{sm} = \mu N] \quad \dots(ii)$$

Substituting value of  $N$  from equation (i) in (ii),

$$F \geq \frac{\mu mg}{(\cos \theta + \mu \sin \theta)} \quad \dots(iii)$$

For the force  $F$  to be minimum  $(\cos \theta + \mu \sin \theta)$  must be maximum,

maximum value of  $\cos \theta + \mu \sin \theta$  is  $\sqrt{1 + \mu^2}$  so that  $F_{\min} = \frac{\mu mg}{\sqrt{1 + \mu^2}}$  with  $\theta = \tan^{-1}(\mu)$



### Example

A book of 1 kg is held against a wall by applying a force  $F$  perpendicular to the wall. If  $\mu_s = 0.2$ , what is the minimum value of  $F$  ?

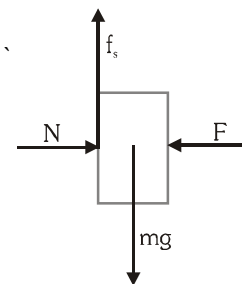
**Solution**

The situation is shown in fig. The forces acting on the book are–

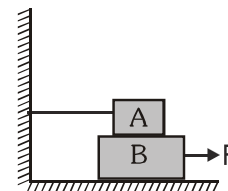
For book to be at rest it is essential that  $Mg = f_s$

But  $f_{s \max} = \mu_s N$  and  $N = F$

$$\therefore Mg = \mu_s F \Rightarrow F = \frac{Mg}{\mu_s} = \frac{1 \times 9.8}{0.2} = 49 \text{ N}$$

**Example**

A is a 100 kg block and B is a 200 kg block. As shown in fig., the block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then calculate the minimum force required to move the block B. ( $g = 10 \text{ m/s}^2$ ).

**Solution**

When B is tied to move, by applying a force  $F$ , then the frictional forces acting on the block B are  $f_1$  and  $f_2$  with limiting values,  $f_1 = (\mu_s)_A m_A g$  and  $f_2 = (\mu_s)_B (m_A + m_B)g$

Then minimum value of  $F$  should be (for just tending to move),

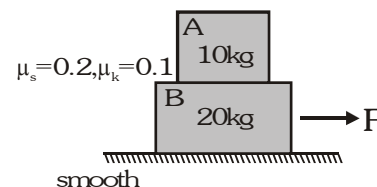
$$F = f_1 + f_2 = 0.2 \cdot 100 g + 0.3 \cdot 300 g = 110 g = 1100 \text{ N}$$

**Example**

In the given figure block A is placed on block B and both are placed on a smooth horizontal plane. Assume lower block to be sufficiently long.

The force  $F$  pulling the block B horizontally is increased according to

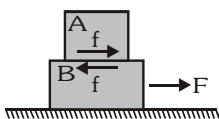
$$\text{law } F = 10t \text{ N}$$



(a) When does block A start slipping on block B? What will be force  $F$  and acceleration just before slipping starts?

(b) When  $F$  is increased beyond the value obtained in part (a), what will be acceleration of A ?

(c) Draw acceleration-time graph.

**Solution****Direction of friction forces**

Block A moves forward always, due to friction, therefore friction on it must be in forward direction.

Friction between two adjacent surfaces are equal and opposite because they make Newton's third law action reaction pair.

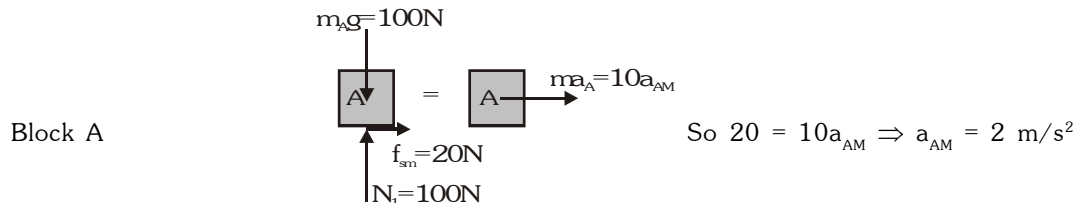
**Range of Value of friction**

Before slipping starts, friction is static  $f_s \leq 20 \text{ N}$

After slipping starts, friction is kinetic  $f_k = 10 \text{ N}$

### Maximum possible acceleration

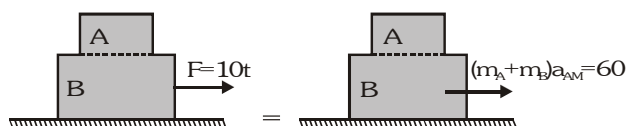
A can accelerate only due to friction, its maximum possible acceleration is  $a_{AM}$  (when  $f_s = f_{sm} = 20 \text{ N}$ )



### Sequence of slipping :

Since ground is smooth, block B first starts slipping on the ground and carries A together with it. When acceleration of A & B becomes equal to  $a_{AM}$ , Block A starts slipping on B.

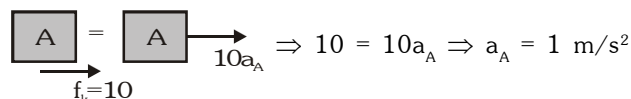
- (a) Just before the moment A starts slipping, both were moving together with acceleration  $a_{AM}$ . Considering them as a one body.



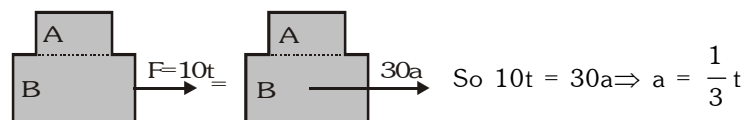
(On a smooth stationary surface we will not show the normal forces i.e. FBD of combined block showing horizontal forces only).

Value of F  $F = 60 \text{ N}$   
and Time  $10t = 60 \Rightarrow t = 6 \text{ s}$

- (b) If F is increased beyond 60 N, A slides and kinetic friction acts on it. Now acceleration of A



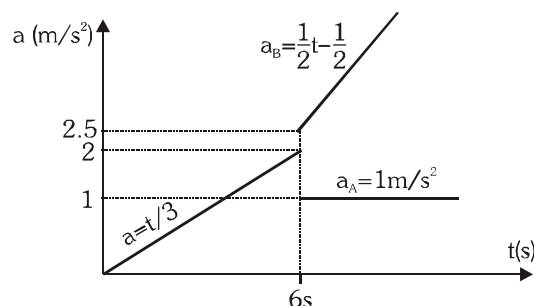
- (c) When  $F \leq 60 \text{ N}$ , both are moving with same acceleration  $a$ . We treat them as one body.



This acceleration increases to  $a_{AM} = 2 \text{ m/s}^2$ , when  $F = 60 \text{ N}$  at  $t = 6 \text{ s}$ . Thereafter A starts slipping and its acceleration provided by kinetic friction, drops to a constant value  $a_A = 1 \text{ m/s}^2$ . However acceleration of B keeps on increasing according to equation

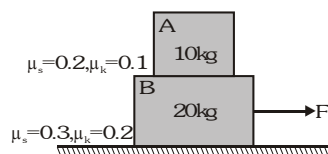


Graph between acceleration and time

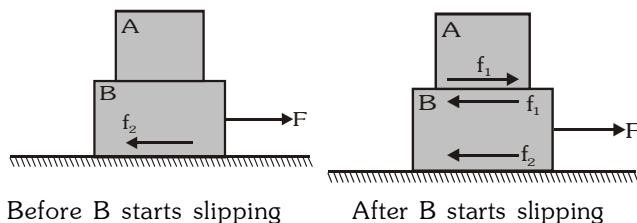


**Example**

Block A is placed on another block B, which rests on a rough horizontal ground. Horizontal force  $F$  pulling the block B is increased gradually.



- Find the maximum value of  $F$  so that no motion occurs.
- Find maximum  $F$  so that A does not slide on B.
- If  $F$  is increased beyond the value obtained in part (b) what are acceleration of both the blocks? Explain your answer in terms of  $F$ .
- If  $F$  is increased according to law  $F = 10t$  N draw  $a-t$  graph

**Solution****Directions of friction forces****Range of values of frictional forces**

$$\begin{aligned}
 f_{1k} &= 10 \text{ N (A is slipping)} \\
 f_{1s} &= 20 \text{ N (A is not slipping)} \\
 f_{2k} &= 60 \text{ N (B is slipping)} \\
 f_{2s} &\leq 90 \text{ N (B is at rest)}
 \end{aligned}$$

**Maximum possible acceleration** : A can move only due to friction. Its maximum possible acceleration is

$$\text{Block A} \quad \begin{array}{c} \text{A} \\ \xrightarrow{f_{1sm}=20} \end{array} = \begin{array}{c} \text{A} \\ \xrightarrow{10a_{Am}} \end{array} \Rightarrow a_{Am} = 2 \text{ m/s}^2$$

**Sequence of slipping** : When  $F \geq f_{2s}$ , block B starts slipping on ground and carries block A together with it till its acceleration reaches value  $a_{Am}$ . Thereafter A also starts slipping on B.

(a)  $F = 90 \text{ N}$

- (b) When A does not slide on B, both move with the same acceleration ( $a_{Am}$ ) and can be treated as one body, which can have maximum acceleration  $a_{Am} = 2 \text{ m/s}^2$ .

$$F - 60 = 60 \Rightarrow F = 120 \text{ N}$$

- (c) When  $F$  is increased beyond  $F = 120 \text{ N}$ , block A starts sliding and friction between A & B drops to  $f_{1k} = 10 \text{ N}$ . Both the blocks now move with different acceleration so we treat them separate bodies. Now acceleration A also drops to a constant value  $a_A$ .

Acceleration of A :

$$\begin{array}{c} \text{A} \\ \xrightarrow{f_{1k}=10} \end{array} = \begin{array}{c} \text{A} \\ \xrightarrow{10a_A} \end{array} \Rightarrow 10 = 10 a_A \Rightarrow a_A = 1 \text{ m/s}^2$$

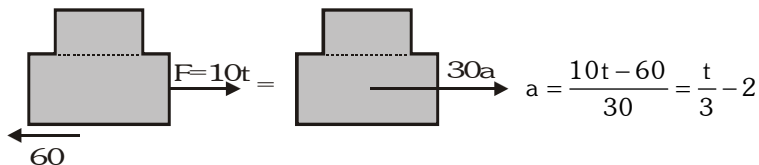
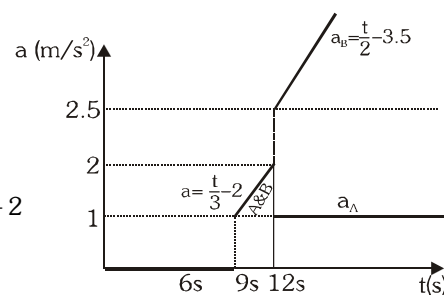
Acceleration of B :

$$\begin{array}{c} \text{B} \\ \xleftarrow{f_{1k}=10} \end{array} \xrightarrow{F} \begin{array}{c} \text{B} \\ \xrightarrow{20a_B} \end{array} \quad \text{So } F - 70 = 20a_B \Rightarrow a_B = \frac{F - 70}{20}$$

- (d) If  $F = 10t$ , values of acceleration of both the blocks in different time intervals are as under:

- $F \leq 90 \text{ N} \Rightarrow t \leq 9 \text{ s} \quad a_A = a_B = 0$
- $90 \text{ N} < F \leq 120 \text{ N} \quad 9 \text{ s} < t \leq 12 \text{ s} \quad a_A = a_B = \frac{t}{3} - 2$

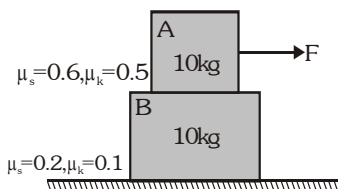
In the interval both the blocks move as one body



- $F > 120 \text{ N} \Rightarrow t > 12 \text{ s} \quad a_A = 1; a_B = \frac{t}{2} - 3.5$

### Example

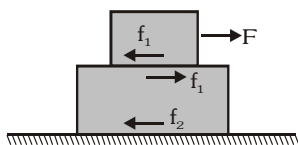
Block A is placed on another block B, which rests on a rough horizontal ground. Horizontal force pulling A is increased gradually



- Find maximum  $F$  so that none of the blocks move. Which block starts sliding first ?
- Express acceleration of each block as function of  $F$  for all positive values of  $F$ .
- If  $F = 10t$  draw  $a-t$  graph

### Solution

Directions of friction forces



Range of values of friction forces

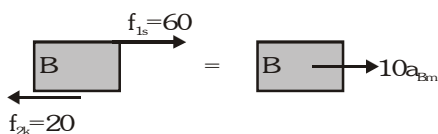
$$f_{1s} \leq 60 \text{ N}$$

$$f_{1k} = 50 \text{ N}$$

$$f_{2s} \leq 40 \text{ N}$$

$$f_{2k} = 20 \text{ N}$$

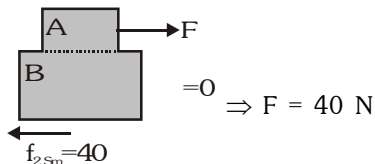
**Maximum possible acceleration of B :** Block B acceleration due to friction only. Its maximum acceleration is



$$60 - 20 = 10a_{Bm} \Rightarrow a_{Bm} = 4 \text{ m/s}^2$$

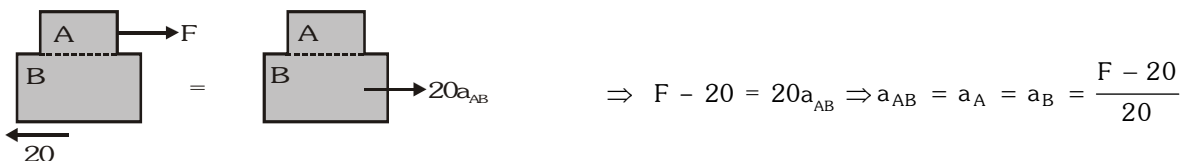
**Sequence of slipping :** Smaller, limiting friction is between B and ground so it will start sliding first. Then both will move together till acceleration B reaches its maximum possible values  $4 \text{ m/s}^2$ . Thereafter A starts sliding on B

- (a) Till the  $F$  reaches the limiting friction between block B and the ground none of the blocks move.



- (b) If  $F \leq 40 \Rightarrow a_A = a_B = 0 \dots(i)$

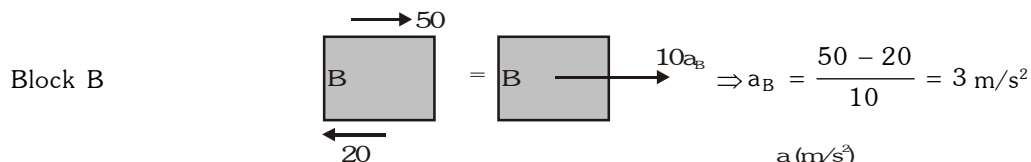
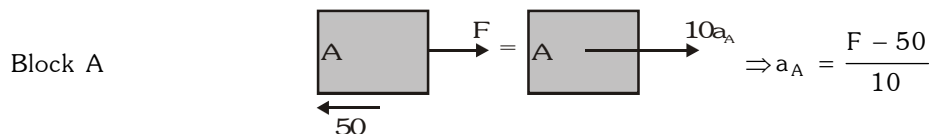
If  $F > 40$  N, block B starts sliding and carries A together with it with the same acceleration till acceleration reach to  $4 \text{ m/s}^2$ . At this moment A starts slipping. Before this moment we may treat both of them as single body.



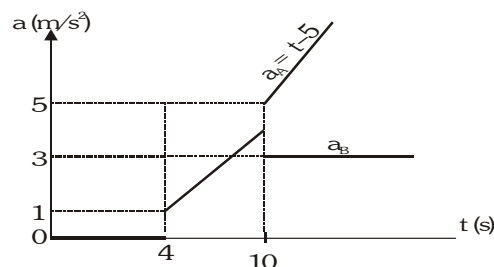
When A starts sliding on B,  $a_A = a_B = 4$ , from the above equation, we have  $F = 100$  N.

When  $F \geq 100$  N block A also starts slipping on B and friction between A & B drops to value 50 N.

Now since they move with different acceleration we treat them separately.

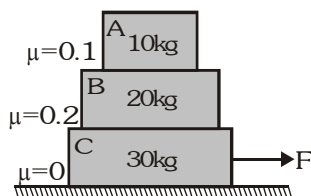


- (c)  $F \leq 40$  N       $t \leq 4$  s       $a_A = a_B = 0$
- $40 \leq F \leq 100$      $4 < t \leq 10$      $a_A = a_B = \frac{t}{2} - 1$
- $100 < F$        $\Rightarrow t > 10$        $a_A = t - 5, a_B = 3 \text{ m/s}^2$



### Example

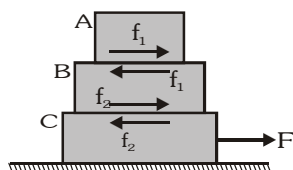
Block A is placed on B and B is placed on block C, which rests on smooth horizontal ground as shown in the figure. Block A is pulled horizontally by a force  $F$  which increases gradually.



- (a) Decide sequence of slipping.
- (b) If  $F$  is increased gradually find acceleration of each block for all values of  $F$ .
- (c) If  $F = 15t$  N, draw  $a-t$  graph.

**Solution**

**Direction of friction forces :**



**Range of values of friction forces**

$$f_1 \leq 10 \text{ N (A does not slides on B)}$$

$$f_1 = 10 \text{ N (A slides on B)}$$

$$f_2 \leq 60 \text{ N (B does not slides on C)}$$

$$f_2 = 60 \text{ N (B slides on C)}$$

**Maximum possible acceleration :** Blocks A and B move due to friction forces only, we find their maximum possible acceleration.

Block A

$$f_{1sm} = 10 \quad a_{Am} = 1 \text{ m/s}^2$$

Block B

$$a_{Bm} = \frac{60 - 10}{20} = 2.5 \text{ m/s}^2$$

**(a) Sequence of slipping**

Since ground is smooth the block C starts sliding first

A starts slipping on B secondly till that moment all the three blocks move with same acceleration, which can achieve maximum value of  $a_{AM} = 1 \text{ m/s}^2$ .

Thirdly B starts sliding on C, till that moment B & C move with the same acceleration  $a_{Bm} = 2.5 \text{ m/s}^2$

**(b)** Before A starts slipping, all the three were moving with the same acceleration  $a_{AM} = 1 \text{ m/s}^2$ . We therefore treat then as a single body.

$$a_{ABC} = \frac{F}{60}$$

When A starts sliding  $a_{ABC} \leq a_{AM} \Rightarrow \frac{F}{60} \leq 1 \Rightarrow F \leq 60 \text{ N}$

When  $F \geq 60 \text{ N}$ , block A starts slipping on B and its acceleration decided by friction  $f_1$ , achieves a constant value  $a_A = 1 \text{ m/s}^2$ .

Now,  $F$  is increased beyond  $60 \text{ N}$  and B and C will continue to move together till their acceleration  $a_{BC}$  becomes  $a_{Bm} = 2.5 \text{ m/s}^2$ , when slipping between B and C starts. Till this moment, we treat B and C as one body.

$$a_{BC} = \frac{F - 10}{50}$$

When slipping between B & C starts :  $a_{BC} = a_{Bm} \Rightarrow \frac{F - 10}{50} = 2.5 \Rightarrow F \leq 135 \text{ N}$

When  $F > 135 \text{ N}$ , block B also starts slipping on C. Now acceleration of A & B achieves the maximum value  $a_{Bm} = 2.5 \text{ m/s}^2$  and acceleration of block C is decided by  $F$ .

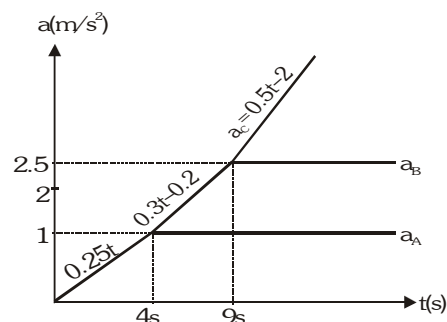


Acceleration of blocks for different values of force.

- $F \leq 60 \text{ N}$   $a_A = a_B = a_C = a_{ABC} = \frac{F}{60}$
- $60 < F \leq 135 \text{ N}$   $a_A = a_{Am} = 1 \text{ m/s}^2$ ,  $a_B = a_C = a_{BC} = \frac{F - 10}{50}$
- $135 < F$   $a_A = a_{Am} = 1$ ,  $a_B = a_{Bm} = 2.5$ , and  $a_C = \frac{F - 60}{30}$

(c) If  $F = 15t$

- $F \leq 60$   $t \leq 4 \text{ s}$   $a_A = a_B = a_C = \frac{F}{60} = \frac{t}{4} = 0.25t$
- $60 < F \leq 135$   $4 < t \leq 9$   $a_A = 1 \text{ m/s}^2$   
 $a_B = a_C = \frac{F - 10}{50} = 0.3t - 0.2$
- $135 < F$   $9 < t$   $a_A = 1 \text{ m/s}^2$   
 $a_B = 2.5 \text{ m/s}^2$   
 $a_C = \frac{F - 60}{30} = 0.5t - 2$



## Inertial and Non-inertial Reference Frames

A body is observed in motion, when it changes its position or orientation relative to another body or another set of bodies. A frame of reference consists of a set of coordinate axes fixed with the body relative to which the motion is observed and a clock. The coordinate axes are required to measure position of the moving body and the clock is required to measure time.

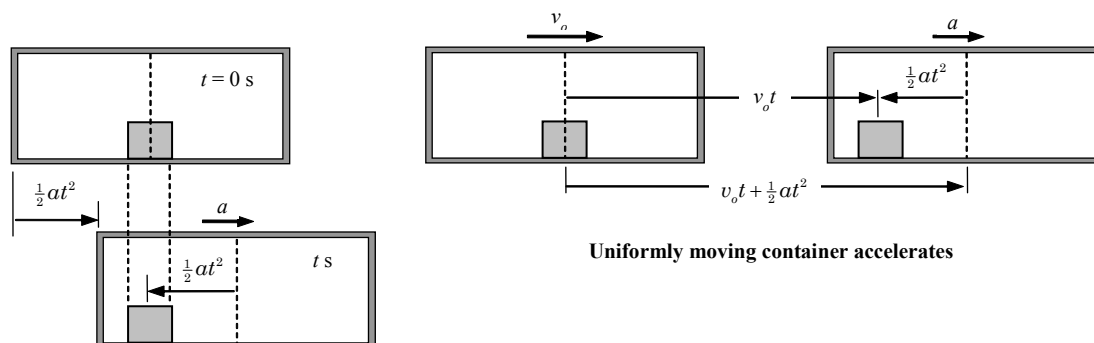
All the kinematical variables position, velocity, and acceleration are measured relative to a reference frame; therefore depend on the state of motion of the reference frame and we say that motion is essentially a relative concept. When the reference frame and a body both are stationary or move identically, the body is observed stationary relative to the reference frame. It is only when the reference frame and the body move in different manner, the body is observed moving relative the reference frame.

Now think about the whole universe where the planets, stars, galaxies and other celestial bodies all are in motion relative to each other. If any one of them can be assumed in state of rest, we can attach a reference frame to it and define motion of all other bodies relative to it. Such a body, which we assume in state of rest with respect to all other bodies in the universe, is known in absolute rest and the reference frame attached to it as most preferred reference frame. Unfortunately, the very notion of the reference frame and the idea motion as a relative concept, make it impossible to find a body anywhere in the universe at absolute rest. Therefore, the idea of absolute rest and a preferred reference frame become essentially meaningless. Now we can have only two categories of reference frames. In one category, we can have reference frames that move with uniform velocities and in the other category; we can have reference frames that are in accelerated motion.

To understand the above ideas let us think an experiment. Consider a closed container on a goods train either at rest or moving with constant velocity  $v_0$  on a level track. The floor of the container is smooth and a block is placed in the center of the container. We observe the situation relative to two reference frames, one fixed with



the ground and other fixed with the container. Relative to the ground frame both the container and the block are at rest or move together with the same velocity and relative to the container frame the block is at rest as shown in the figure.



Container accelerates from rest

Uniformly moving container accelerates

Now let the driver of the train accelerates the container at uniform rate  $a$ . If the train were initially at rest, relative to the ground, the block remains at rest and the container moves forward. Relative to the container the block moves backwards with the same magnitude of acceleration as with the container moves forward. If the train were initially moving uniformly, relative to the ground the block continues to move with the same original velocity and train accelerates and becomes ahead in space. Relative to the container the block appears moving backward with acceleration that is equal in magnitude to the acceleration of the container.

Now consider a man sitting on a fixed chair in the container. He is always stationary relative to the container. If he does not look outside, in no way he can know whether the container is at rest or moving uniformly. However, he can definitely say whether the container accelerates or not, by observing motion of the block relative to the container. Because net forces acting on the block are still zero, therefore observed acceleration of the block can only be due to acceleration of the container as per Newton's laws of motion.

Now we can conclude that there can be only two kinds of reference frames either non-accelerated or accelerated. The reference frames that are non-accelerated i.e. at rest or moving with uniform velocities are known as inertial reference frames and those in accelerated motion as non-inertial reference frames.

## Inertial Reference Frames and Newton's laws of motion

In Newton's laws of motion, force is conceived as two-body interaction that can be the only agent producing acceleration in a body. As far as we observe motion of a body from an inertial frame, any acceleration observed in a body can only be due to some forces acting on the body. All the three laws are in perfect agreement with the observed facts and we say that all the laws hold true in inertial reference frames.

## Non-Inertial Reference Frames and Newton's laws of motion

A body if at rest or in uniform velocity motion relative to some inertial frame net forces acting on it must be zero. Now if motion of the same body is observed relative to a non-inertial frame, it will be observed moving with acceleration that is equal in magnitude and opposite in direction to the acceleration of the non-inertial frame. This observed acceleration of the body is purely a kinematical effect. But to explain this observed acceleration relative to the non-inertial frame according to Newton's laws of motion, we have assumed a force must be acting on the body. This force has to be taken equal to product of mass of the body and opposite of acceleration vector of the non-inertial frame. Since this force is purely an assumption and not a result of interaction of the body with any other body, it is a fictitious force. This fictitious force is known as pseudo force or inertial force.

Until now, we have learnt the idea that how we can apply Newton's laws of motion in non-inertial frame to a body in equilibrium in inertial frame. Now it is turn to discuss how we can apply Newton's laws of motion to analyze motion in non-inertial frame of a body, which is in accelerated motion relative to an inertial frame.

Consider a net physical force  $\vec{F}$  in positive x-direction applied on the box. Here by the term physical force, we refer forces produced by two body interactions. Relative to inertial frame A, the box is observed to have an acceleration  $\vec{a} = \vec{F}/m$  defined by the second law of motion and a force equal in magnitude and opposite in direction to  $\vec{F}$  can be assigned to the body exerting  $\vec{F}$  on the box constituting Newton's third law pair. All the three laws of motion hold equally well in inertial frame A.

Relative to non-inertial frame B, the box is observed moving in x-direction with acceleration  $\vec{a}_B = \vec{a} - \vec{a}_0$ , which can satisfy Newton's second law, only if the fictitious force  $\vec{F}_0 = -m\vec{a}_0$  is assumed acting together with the net physical force  $\vec{F}$  as shown in the figure. Now we can write modified equation of Newton's second law in non-inertial frame.

$$\vec{F} + \vec{F}_0 = m\vec{a}_B \rightarrow \vec{F} - \vec{F}_0 = m\vec{a}_B$$

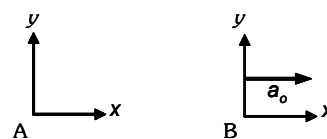
From the above discussion, we can conclude that in non-inertial frames Newton's second law is made applicable by introducing pseudo force in addition to physical forces. The pseudo force equals to the product of mass of the concerned body and the acceleration of the frame of reference in a direction opposite to the acceleration of the frame of reference.

## Practical inertial frame of reference

The definition of an inertial frame of reference is based on the concept of a free body in uniform velocity motion or absolute rest. It is impossible to locate a body anywhere in the universe, where forces from all other bodies exactly balance themselves and lead to a situation of uniform velocity motion according to the first law. Thus, we cannot find anywhere in the universe a body, to which we attach a frame of reference and say that it is a perfect inertial frame of reference. It is the degree of accuracy, required in analyzing a particular physical situation that decides which body in the universe is to be selected a preferred inertial frame of reference.

The earth and other planets of the sun are rotating about their own axis and revolving around the sun and the sun is moving too. In fact, all celestial bodies in the universe the sun, its planets, other stars, our galaxy the Milky Way, and other galaxies are in accelerated motion whose nature is not known exactly. Therefore, none of them can be used as a perfect inertial frame of reference. However, when the acceleration of any one of the above-mentioned celestial bodies becomes negligible as compared to the accelerations involved in a physical situation, a frame of reference attached with the corresponding celestial body may be approximated as an inertial frame of reference and the physical situation under consideration may be analyzed with acceptable degree of accuracy.

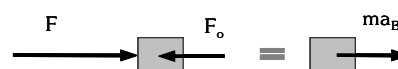
The centripetal acceleration due to rotation of the earth at any point on its surface varies from zero at the poles to a maximum value of approximately  $0.034 \text{ m/s}^2$  at the equator. The physical phenomena, which we usually observe describe motion of a body on or near the earth surface such as motion of transport vehicles, short-



Reference frames A is an inertial frame and B is a non inertial frame.



A net physical force imparts acceleration to the box in inertial frame A.



A net physical force and pseudo force imparts acceleration to the box in non-inertial frame B.

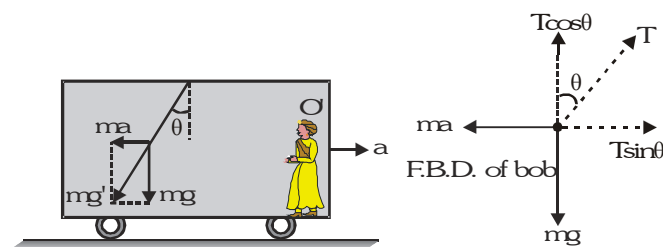
range missiles, an oscillating pendulum etc. In these phenomena, the acceleration due to rotation of the earth may be neglected and a frame of reference attached at any point on the earth surface may be considered as a preferred inertial frame of reference. If the moving body is at considerable distance from the earth as in the case of satellites, long-range missiles etc., the effect of rotation of the earth become significant. For these situations or like ones we can attach the frame of reference at the earth's center and consider it as an inertial frame of reference. In astronomical field and space exploration programs, we require very high accuracy. Therefore, a frame of reference attached to distant stars is used as an inertial frame of reference. These stars are situated at such a vast distance from the earth that they appear as a motionless point source of light thus closely approach to the condition of absolute rest.

### Example

A pendulum of mass  $m$  is suspended from the ceiling of a train moving with an acceleration ' $a$ ' as shown in figure. Find the angle  $\theta$  in equilibrium position.

### Solution

Non-inertial frame of reference (Train)



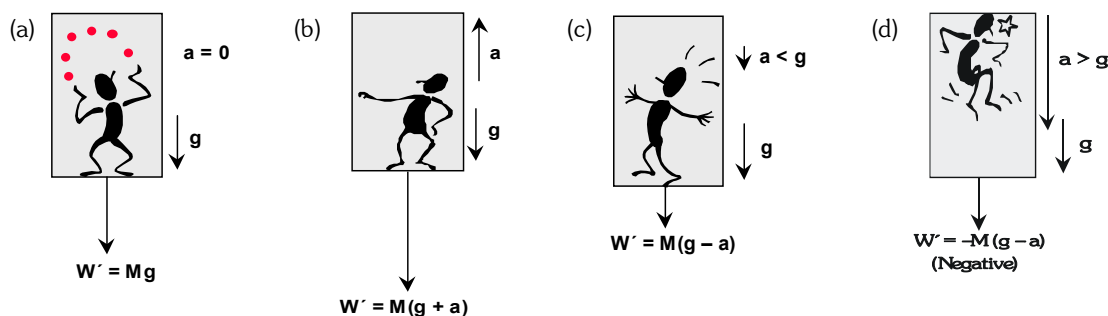
F.B.D. of bob w.r.t. train. (physical forces + pseudo force) : with respect to train, bob is in equilibrium

$$\therefore \Sigma F_y = 0 \Rightarrow T \cos \theta = mg \quad \text{and} \quad \Sigma F_x = 0 \Rightarrow T \sin \theta = ma \Rightarrow \tan \theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1} \left( \frac{a}{g} \right)$$

### Example

The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight  $W = Mg$  be standing in a lift.

We consider the following cases :



### Case

- (a) If the lift moving with constant velocity  $v$  upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift. So apparent weight  $W' = Mg$  Actual weight.

- (b) If the lift is accelerated upward with constant acceleration  $a$ . Then net forces acting on the man are (i) weight  $W = Mg$  downward (ii) fictitious force  $F_0 = Ma$  downward.  
 So apparent weight  $W' = W + F_0 = Mg + Ma = M(g + a)$
- (c) If the lift is accelerated downward with acceleration  $a < g$ . Then fictitious force  $F_0 = Ma$  acts upward while weight of man  $W = Mg$  always acts downward. So apparent weight  $W' = W + F_0 = Mg - Ma = M(g - a)$

#### Special Case :

If  $a = g$  then  $W' = 0$  (condition of weightlessness). Thus, in a freely falling lift the man will experience weightlessness.

- (d) If lift accelerates downward with acceleration  $a > g$ . Then as in Case c. Apparent weight  $W' = M(g - a)$  is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

#### Example

A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with (i) constant velocity, and (ii) constant acceleration?

#### Solution

- (i) In the case of constant velocity of lift, there is no fictitious force; therefore the apparent weight = actual weight. Hence the reading of machine is 50 kgwt.
- (ii) In this case the acceleration is upward, the fictitious force  $ma$  acts downward, therefore apparent weight is more than actual weight i.e.  $W' = m(g + a)$ .

$$\text{Hence scale shows a reading} = m(g + a) = \frac{mg \left(1 + \frac{a}{g}\right)}{g} = \left(50 + \frac{50a}{g}\right) \text{ kg wt.}$$

#### Example

Two objects of equal mass rest on the opposite pans of an arm balance. Does the scale remain balanced when it is accelerated up or down in a lift?

#### Solution

Yes, since both masses experience equal fictitious forces in magnitude as well as direction.

#### Example

A passenger on a large ship sailing in a quiet sea hangs a ball from the ceiling of her cabin by means of a long thread. Whenever the ship accelerates, she notes that the pendulum ball lags behind the point of suspension and so the pendulum no longer hangs vertically. How large is the ship's acceleration when the pendulum stands at an angle of  $5^\circ$  to the vertical?

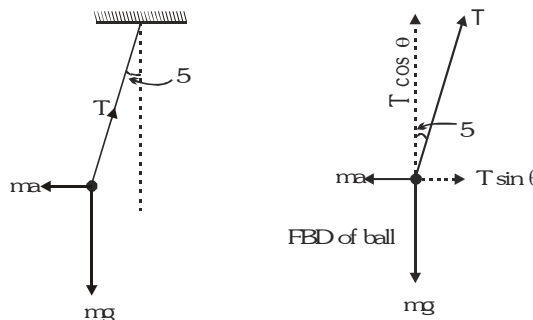
#### Solution

The ball is accelerated by the force  $T \sin 5^\circ$ .

$$\text{Therefore } T \sin 5^\circ = ma$$

$$\text{Vertical component } \Sigma F = 0, \text{ so } T \cos 5^\circ = mg$$

$$\text{By solving } a = g \tan 5^\circ = 0.0875 g = 0.86 \text{ m/s}^2$$



### Example

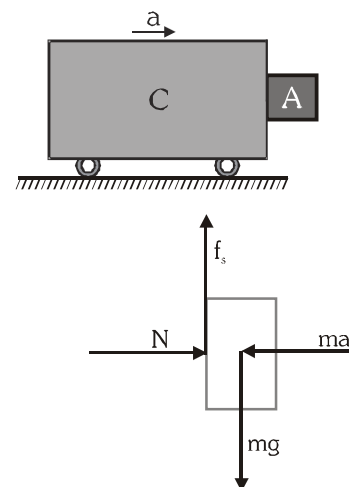
Consider the figure shown here of a moving cart C. If the coefficient of friction between the block A and the cart is  $\mu$ , then calculate the minimum acceleration  $a$  of the cart C so that the block A does not fall.

### Solution

The forces acting on the block A (in block A's frame (i.e. non inertial frame) are :

For A to be at rest in block A's frame i.e. no fall,

$$\text{We require } W = f_s \Rightarrow mg = \mu(ma) \text{ Thus } a = \frac{g}{\mu}$$



### Example

A block of mass 1kg lies on a horizontal surface in a truck, the coefficient of static friction between the block and the surface is 0.6, What is the force of friction on the block. If the acceleration of the truck is  $5 \text{ m/s}^2$ .

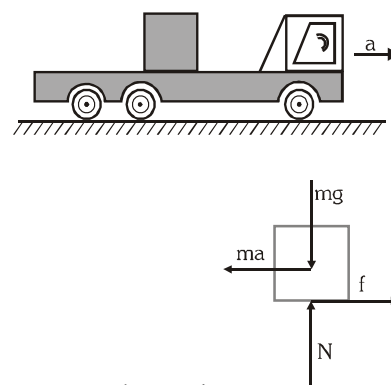
### Solution

Fictitious force on the block  $F = ma = 1 \times 5 = 5 \text{ N}$

While the limiting friction force

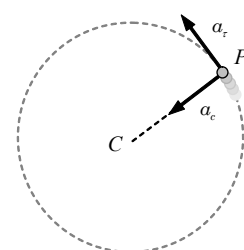
$$F = \mu_s N = \mu_s mg = 0.6 \times 1 \times 9.8 = 5.88 \text{ N}$$

As applied force  $F$  lesser than limiting friction force. The block will remain at rest in the truck and the force of friction will be equal to 5N and in the direction of acceleration of the truck.



### Dynamics of Circular Motion

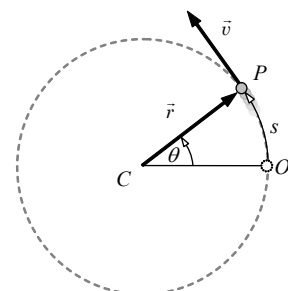
Velocity vector points always tangent to the path and continuously change its direction, as a particle moves on a circular path even with constant speed and give rise to normal component of acceleration, which always points toward the center of the circular path. This component of acceleration is known as centripetal (center seeking) acceleration and denoted by  $a_c$ . Moreover, if speed also changes the particle will have an additional acceleration component along the tangent to the path. This component of acceleration is known as tangential acceleration and denoted by  $a_t$ .



The centripetal and tangential accelerations

The centripetal acceleration accounts only for continuous change in the direction of motion whereas the tangential acceleration accounts only for change in speed.

Consider a particle moving on circular path of radius  $r$ . It passes point O with velocity  $v_o$  at the instant  $t = 0$  and point P with velocity  $v$  at the instant  $t$  traveling distance  $s$  along the path and angular displacement  $\theta$  as shown in the figure. Kinematics of this circular motion is described in terms of linear variables as well as angular variables.



## Kinematics of Circular Motion

Angular Variables	Linear Variables
Angular displacement $\theta$	Distance traveled $s$
Angular velocity $\omega = \frac{d\theta}{dt}$	Speed $v = \frac{ds}{dt}$
Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta}$	Tangential acceleration $a_t = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v \frac{dv}{ds}$
	Centripetal acceleration $a_c = \frac{v^2}{r}$

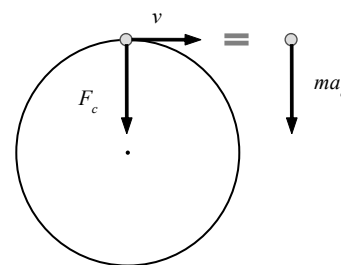
### Relations between angular and linear variable in circular motion

$$\begin{aligned}
 \text{Distance traveled} & \quad s = \theta r \\
 \text{Speed} & \quad v = \omega r \\
 \text{Tangential acceleration} & \quad a_t = ar \\
 \text{Centripetal acceleration} & \quad a_c = \frac{v^2}{r} = \omega^2 r = \omega v
 \end{aligned}$$

### Application of Newton's law in Circular Motion

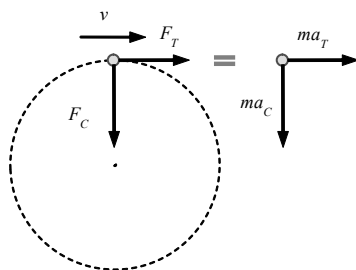
Consider a particle of mass  $m$  moving with uniform speed  $v$  in a circle of radius  $r$  as shown in figure. It necessarily possesses a centripetal acceleration and hence there must be a net force ( $\sum \vec{F} = \vec{F}_c$ ) acting always towards the center according to the second law. This net force  $F_c$  acting towards the center is known as centripetal force.

$$\sum \vec{F} = m\vec{a} \rightarrow F_c = ma_c$$

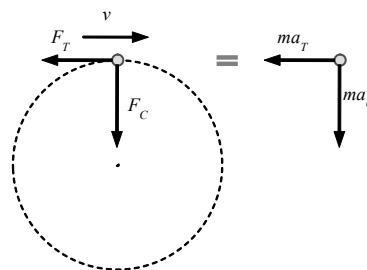


When a particle is whirled with the help of a string in a horizontal circle, the required centripetal force is the tension in the string. The gravitational attraction between a satellite and the earth, between moon and the earth, between the sun and its planets, and the electrostatic attraction between the nucleus and electrons are the centripetal forces and provide the necessary centripetal acceleration.

Now consider a particle moving on circular path with varying speed. The net acceleration has two components, the tangential acceleration and the centripetal acceleration. Therefore, the net force must also have two components, one component in tangential direction to provide the tangential acceleration and the other component towards the center to provide the centripetal acceleration. The former one is known as tangential force and the latter one as centripetal force.



Particle moving with increasing speed.



Particle moving with decreasing speed.

When the particle moves with increasing speed the tangential force acts in the direction of motion and when the particle moves with decreasing speed the tangential force acts in direction opposite to direction of motion.

To write equations according to the second law, we consider the tangential and the radial directions as two

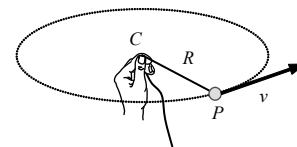
mutually perpendicular axes. The components along the tangential and the radial directions are designated by subscripts T and C.

$$F_C = ma_C$$

$$F_T = ma_T$$

### Example

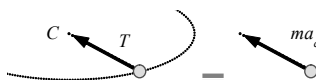
In free space, a man whirls a small stone P of mass  $m$  with the help of a light string in a circle of radius  $R$  as shown in the figure. Establish the relation between the speed of the stone and the tension developed in the string. Also, find the force applied on the string by the man.



### Solution.

The system is in free space therefore no force other than the tension acts on the stone to provide necessary centripetal force. The tension does not have any component in tangential direction therefore tangential component of acceleration is zero. In the adjoining figure it is shown that how tension ( $T$ ) in the string produces necessary centripetal force.

Applying Newton's second law of motion to the stone, we have  $F_C = ma_C \rightarrow T = ma_C = \frac{mv^2}{R}$

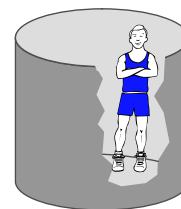


The end where man holds the string is stationary and tension applied by the string to this end is  $T$  towards the stone, therefore the man must apply a force equal to the tension in magnitude but in a direction away from the stone.

$$T = \frac{mv^2}{R}$$

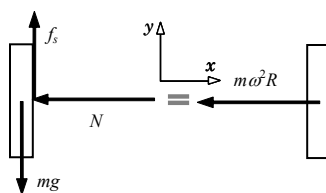
### Example

A boy stands on a horizontal platform inside a cylindrical container of radius  $R$  resting his back on the inner surface of the container. The container can be rotated about the vertical axis of symmetry. The coefficient of static friction between his back and the inner surface of the container is  $\mu_s$ . The angular speed of the container is gradually increased. Find the minimum angular speed at which if the platform below his feet removed, the boy should not fall.



### Solution.

As the container rotates at angular speed  $\omega$  the boy moves in a circular path of radius  $R$  with a speed  $v = \omega R$ . Since the angular speed is increased gradually the angular acceleration can be ignored and hence the tangential acceleration of the boy too. Thus, the boy has a centripetal acceleration of  $\omega^2 R$ , provided by the normal reaction  $N$  applied by the wall of the container. The weight of the boy is balanced by the force of static friction. All these force are shown in the adjoining figure where the boy is shown schematically by a rectangular box of mass  $m$ .



$$\sum F_x = ma_x \rightarrow N = m\omega^2 R \quad \dots(i)$$

$$\sum F_y = ma_y \rightarrow f_s = mg \quad \dots(ii)$$

Since force of static friction cannot be greater than the limiting friction  $\mu_s N$ ,

we have  $f_s \leq \mu_s N \quad \dots(iii)$

From the above equations, the minimum angular speed is  $\omega_{\min} = \sqrt{\frac{g}{\mu_s R}}$

**Example**

A motorcyclist wishes to travel in circle of radius  $R$  on horizontal ground and increases speed at constant rate  $a$ . The coefficient of static and kinetic frictions between the wheels and the ground are  $\mu_s$  and  $\mu_k$ . What maximum speed can he achieve without slipping?

**Solution.**

The motorcyclist and the motorcycle always move together hence they can be assumed to behave as a single rigid body of mass equal to that of the motorcyclist and the motorcycle. Let the mass of this body is  $m$ . The external forces acting on it are its weight ( $mg$ ), the normal reaction  $N$  on wheels from ground, and the force of static friction  $f_s$ . The body has no acceleration in vertical direction therefore; the normal reaction  $N$  balances the weight ( $mg$ ).

$$N = mg \quad \dots(i)$$

The frictional force cannot exceed the limiting friction.

$$f_{sm} \leq \mu_s N \quad \dots(ii)$$

During its motion on circular path, the only external force in horizontal plane is the force of static friction, which is responsible to provide the body necessary centripetal and tangential acceleration. These conditions are shown in the adjoining figure where forces in vertical direction are not shown.

$$F_t = ma_t \quad \dots(iii)$$

$$F_C = ma_C = \frac{mv^2}{r} \quad \dots(iv)$$

The above two forces are components of the frictional force in tangential and normal directions. Therefore, we have

$$f_s = \sqrt{F_t^2 + F_C^2} = m\sqrt{a_t^2 + a_C^2} \quad \dots(v)$$

The centripetal acceleration increases with increase in speed and the tangential acceleration remains constant. Therefore, their resultant increases with speed. At maximum speed the frictional force achieves its maximum value (limiting friction  $f_{sm}$ ), therefore from eq. (i), (ii), (iii), (iv), and (v), we have

$$v = \left[ r^2 \left\{ (\mu_s g)^2 - a^2 \right\} \right]^{\frac{1}{4}}$$

**Circular Turning of Roads**

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways:

- By friction only.    • By banking of roads only.    • By friction and banking of roads both.

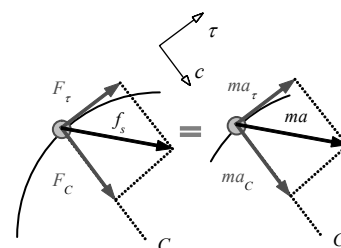
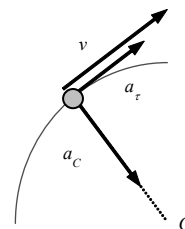
In real life the necessary centripetal force is provided by friction and banking of roads both.

**• By Friction only**

Suppose a car of mass  $m$  is moving at a speed  $v$  in a horizontal circular arc of radius  $r$ . In this case, the necessary centripetal force to the car will be provided by force of friction  $f$  acting towards centre.

$$\text{Thus, } f = \frac{mv^2}{r} \quad \because f_{\max} = \mu N = \mu mg$$

$$\text{Therefore, for a safe turn without sliding } \frac{mv^2}{r} \leq f_{\max} \Rightarrow \frac{mv^2}{r} \leq \mu mg \Rightarrow v \leq \sqrt{\mu rg}$$

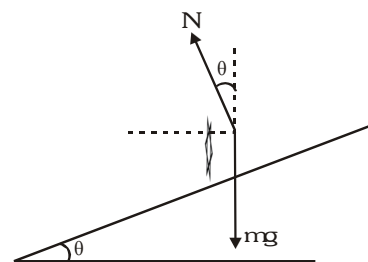




• **By Banking of Roads only**

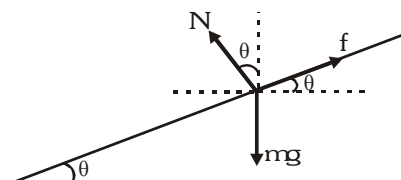
Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.

$$N \sin \theta = \frac{mv^2}{r} \text{ and } N \cos \theta = mg \Rightarrow \tan \theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg \tan \theta}$$



• **Friction and Banking of Road both**

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight ( $mg$ ) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction  $N$  is also fixed (perpendicular to road) while the direction of the third force, i.e., friction  $f$  can be either inwards or outwards while its magnitude can be varied upto a maximum limit ( $f_{\max} = \mu N$ ). So, the magnitude of normal reaction  $N$  and direction plus magnitude of friction  $f$  are so adjusted that the resultant of the three forces mentioned above is  $\frac{mv^2}{r}$  towards the centre.

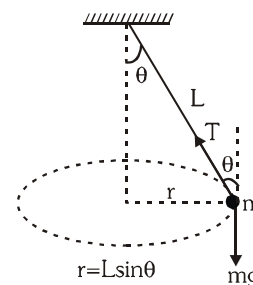


**Conical Pendulum**

If a small particle of mass  $m$  tied to a string is whirled in a horizontal circle, as shown in figure. The arrangement is called the 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,,

$$T \sin \theta = \frac{mv^2}{r} \text{ and } T \cos \theta = mg \Rightarrow v = \sqrt{rg \tan \theta}$$

$$\therefore \text{Angular speed } \omega = \frac{v}{r} = \sqrt{\frac{g \tan \theta}{r}}$$



$$\text{So, the time period of pendulum is } T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r}{g \tan \theta}} = 2\pi \sqrt{\frac{L \cos \theta}{g}}$$

**Example**

Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity =  $10 \text{ m/s}^2$ ]

**Solution**

Here centripetal force is provided by friction so

$$\frac{mv^2}{r} \leq \mu mg \Rightarrow v_{\max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 \text{ ms}^{-1}$$

**Example**

For traffic moving at 60 km/hr, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? ( $g = 10 \text{ m/s}^2$ )

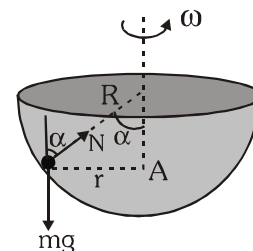
**Solution**

In case of banking  $\tan \theta = \frac{v^2}{rg}$  Here  $v = 60 \text{ km/hr} = 60 \times \frac{5}{18} \text{ ms}^{-1} = \frac{50}{3} \text{ ms}^{-1}$   $r = 0.1 \text{ km} = 100 \text{ m}$

$$\text{So } \tan \theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1} \left( \frac{5}{18} \right)$$

**Example**

A hemispherical bowl of radius  $R$  is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is  $\alpha$ . Find the angular speed at which the bowl is rotating.

**Solution**

$$N \cos \alpha = mg \text{ and } N \sin \alpha = m r \omega^2 \text{ but } r = R \sin \alpha$$

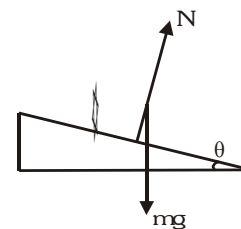
$$\Rightarrow N \sin \alpha = m R \sin \alpha \omega^2 \Rightarrow N = m R \omega^2$$

$$\Rightarrow (m R \omega^2) \cos \alpha = mg \Rightarrow \omega = \sqrt{\frac{g}{R \cos \alpha}}$$

**Example**

A car is moving along a banked road laid out as a circle of radius  $r$ .

- What should be the banking angle  $\theta$  so that the car travelling at speed  $v$  needs no frictional force from the tyres to negotiate the turn?
- The coefficients of friction between tyres and road are  $\mu_s = 0.9$  and  $\mu_k = 0.8$ . At what maximum speed can a car enter the curve without sliding towards the top edge of the banked turn?

**Solution**

$$(a) \quad N \sin \theta = \frac{mv^2}{r} \text{ and } N \cos \theta = mg \Rightarrow \tan \theta = \frac{v^2}{rg}$$

**Note :** In above case friction does not play any role in negotiating the turn.

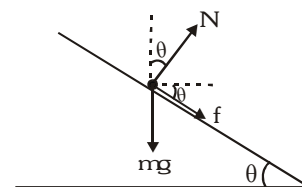
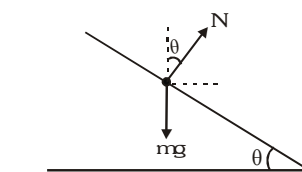
- If the driver moves faster than the speed mentioned above, a friction force must act parallel to the road, inward towards centre of the turn.

$$\Rightarrow F \cos \theta + N \sin \theta = \frac{mv^2}{r} \text{ and } N \cos \theta = mg + f \sin \theta$$

For maximum speed of  $f = \mu N$

$$\Rightarrow N (\mu \cos \theta + \sin \theta) = \frac{mv^2}{r} \text{ and } N (\cos \theta - \mu \sin \theta) = mg$$

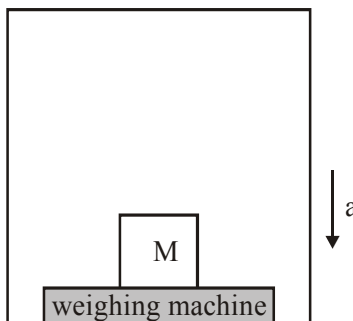
$$\Rightarrow \frac{v^2}{rg} = \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \Rightarrow v = \sqrt{\left( \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} \right) rg}$$



## SOME WORKED OUT EXAMPLES

### Example #1

With what acceleration 'a' shown the elevator descends so that the block of mass M exerts a force of  $\frac{Mg}{10}$  on the weighing machine? [g = acceleration due to gravity]



- (A) 0.3 g                      (B) 0.1 g                      (C) 0.9 g                      (D) 0.6 g

**Solution**

**Ans. (C)**

FBD of block :

$$Mg - N = Ma; \text{ Now according to question } N = \frac{Mg}{10} \text{ so } a = \frac{Mg - \frac{Mg}{10}}{M} = 0.9 g$$

### Example #2

An astronaut accidentally gets separated out of his small spaceship accelerating in inter-stellar space at a constant acceleration of  $10 \text{ m/s}^2$ . What is the acceleration of the astronaut at the instant he is outside the spaceship?

- (A)  $10 \text{ m/s}^2$                       (B)  $9.8 \text{ m/s}^2$                       (C)  $\approx 0 \text{ m/s}^2$                       (D) could be anything

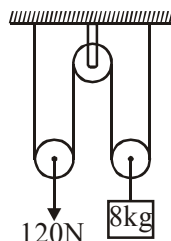
**Solution**

**Ans. (C)**

When the astronaut is outside the spaceship, the net external force (except negligible gravitational force due to spaceship) is zero as he is isolated from all interactions.

### Example #3

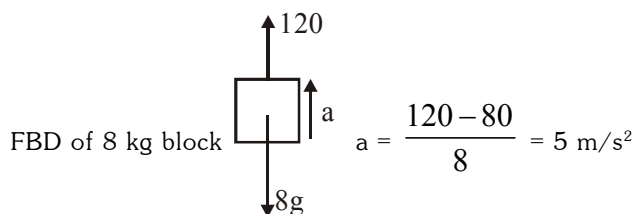
If the string is pulled down with a force of 120 N as shown in the figure, then the acceleration of 8 kg block would be



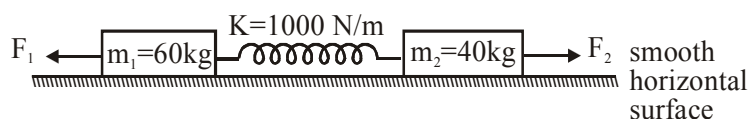
- (A)  $10 \text{ m/s}^2$                       (B)  $5 \text{ m/s}^2$                       (C)  $0 \text{ m/s}^2$                       (D)  $4 \text{ m/s}^2$

Solution

Ans. (B)

**Example #4**

In the shown situation, which of the following is/are possible?



- (A) Spring force = 52 N, if  $F_1 = 40 \text{ N}$  and  $F_2 = 60 \text{ N}$   
 (B) Spring force = 52 N, if  $F_1 = 60 \text{ N}$  and  $F_2 = 40 \text{ N}$   
 (C) Spring force = 0, if  $F_1 = F_2 = 100 \text{ N}$   
 (D) Spring force  $\neq 0$ , if  $F_1 = 0.2 \text{ N}$  and  $F_2 = 0.3 \text{ N}$

Solution

Ans. (A,B,D)

If  $F_1 \neq F_2$ , then system will move with acceleration so spring force  $\neq 0$

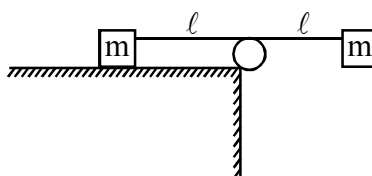
If  $F_1 = 40 \text{ N}$  &  $F_2 = 60 \text{ N}$  then  $a = \frac{F_2 - F_1}{m_1 + m_2} = \frac{20}{100} = \frac{1}{5} \text{ m/s}^2$

and spring force =  $F_1 + m_1 a = 40 + \frac{1}{5}(60) = 52 \text{ N}$

If  $F_1 = 60 \text{ N}$  &  $F_2 = 40 \text{ N}$  then spring force = 52 N

**Example #5**

As shown in figure, the left block rests on a table at distance  $\ell$  from the edge while the right block is kept at the same level so that thread is unstretched and does not sag and then released. What will happen first ?



- (A) Left block reach the edge of the table  
 (B) Right block hit the table  
 (C) Both (A) & (B) happens at the same time  
 (D) Can't say anything

Solution

Ans. (A)

Net force in horizontal direction is more for left block so it will reach the edge of the table first.

**Example #6**

The force exerted by the floor of an elevator on the foot of a person standing there is less than the weight of the person if the elevator is

- (A) going up and slowing down  
 (B) going up and speeding up  
 (C) going down and slowing down  
 (D) going down and speeding up

Solution

Ans. (A,D)

If  $N < mg$  then  $N = m(g - a) \Rightarrow$  elevator is going down with acceleration or elevator is going up with retardation.

**Example #7**

If a body is placed on a rough inclined plane, the nature of forces acting on the body is(are)

- (A) gravitational (B) electromagnetic (C) nuclear (D) weak

**Solution**

**Ans. (A,B)**

When a body is placed on a rough inclined plane, it is acted upon by a reactional force due to plane [electromagnetic in nature], a frictional force due to roughness of plane [ electromagnetic in nature] and a gravitational force (due to its weight).

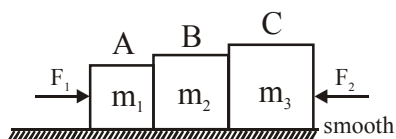
**Example #8**

For shown situation let

$N_1$  = Normal reaction between A & B

$N_2$  = Normal reaction between B & C

Which of the following statement (s) is/are correct ?



- (A) If  $F_1 > F_2$  then  $N_1 \neq N_2$  and  $F_2 < \sqrt{N_1 N_2} < F_1$  (B) If  $F_1 < F_2$  then  $N_2 > N_1$   
(C) If  $F_1 = F_2$  then  $N_1 = N_2$  (D) If  $F_1 = F_2$  then  $N_1 \neq N_2$

**Solution**

**Ans. (A, B, C)**

If  $F_1 > F_2$ , the system moves towards right so  $N_1 < F_1$ ,  $N_2 < N_1$  &  $F_2 < N_2$

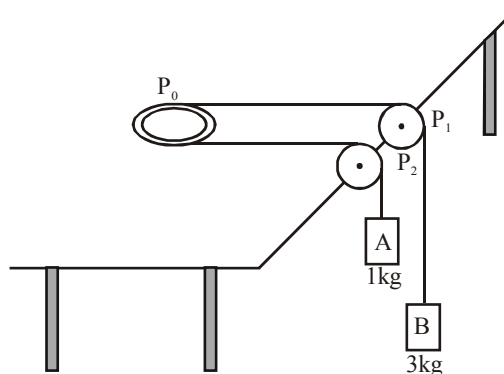
$$F_2 < N_1 \text{ or } N_2 < F_1$$

If  $F_1 < F_2$ , the system moves towards left so  $N_1 < N_2$

If  $F_1 = F_2$ , the system does not move.

**Example #9 to 11**

A smooth pulley  $P_0$  of mass 2 kg is lying on a smooth table. A light string passes round the pulley and has masses 1 kg and 3kg attached to its ends. The two portions of the string being perpendicular to the edge of the table so that the masses hang vertically. Pulleys  $P_1$  and  $P_2$  are of negligible mass. [  $g=10\text{m/s}^2$  ]



9. Tension in string is  
(A) 12 N (B) 6 N (C) 24 N (D) 18 N
10. Acceleration of pulley  $P_0$  is  
(A)  $2 \text{ m/s}^2$  (B)  $4 \text{ m/s}^2$  (C)  $3 \text{ m/s}^2$  (D)  $6 \text{ m/s}^2$
11. Acceleration of block A is  
(A)  $6 \text{ m/s}^2$  (B)  $4 \text{ m/s}^2$  (C)  $3 \text{ m/s}^2$  (D)  $8 \text{ m/s}^2$

**Solution**

9. **Ans. (B)**

Let acceleration of  $P_0 = a_0$ ; acceleration of  $A = a_1$ , acceleration of  $B = a_2$

$$\text{By constraint relations } a_0 = \frac{a_1 + a_2}{2} \dots (i)$$

$$\text{Now for pulley } P_0 : 2T = 2a_0 \Rightarrow T = a_0 \dots (ii)$$

$$\text{For block A : } 1g - T = 1(a_1) \Rightarrow 10 - T = a_1 \dots (iii)$$

$$\text{For block B : } 3g - T = 3(a_2) \Rightarrow 30 - T = 3a_2 \dots (iv)$$

By putting the values of  $a_0$ ,  $a_1$  &  $a_2$  in equation (i)

$$T = \frac{(10 - T) + \left(10 - \frac{T}{3}\right)}{2} \Rightarrow T = 6\text{N}$$

10. **Ans. (D)**

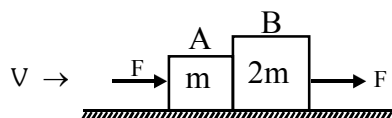
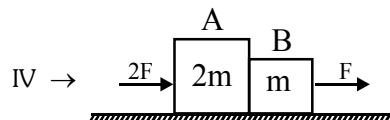
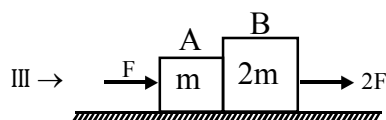
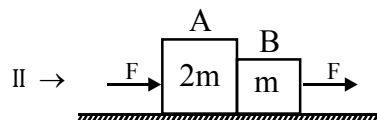
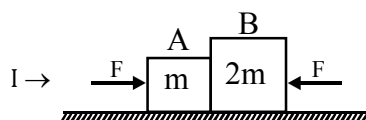
Acceleration of pulley;  $a_0 = T = 6 \text{ m/s}^2$

11. **Ans. (B)**

Acceleration of block A;  $a_1 = 10 - T = 10 - 6 = 4 \text{ m/s}^2$

**Example 12**

Five situations are given in the figure (All surfaces are smooth)



**Column-I**

- (A) Accelerations of A & B are same
- (B) Accelerations of A & B are different
- (C) Normal reaction between A & B is zero
- (D) Normal reaction between A & B is non zero

**Column-II**

- (P) I
- (Q) II
- (R) III
- (S) IV
- (T) V

**Solution**

**Ans. (A) → (P,R,S,T) ; (B) → (Q) ; (C) → (Q,R,S) ; (D) → (P, T)**

I :  $a_A = a_B = 0$  &  $N \neq 0$

II :  $a_A = \frac{F}{2m}$ ,  $a_B = \frac{F}{m}$  &  $N = 0$

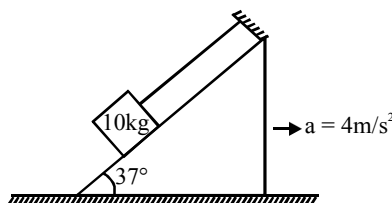
III :  $a_A = \frac{F}{m}$ ,  $a_B = \frac{2F}{2m} = \frac{F}{m}$  &  $N = 0$

IV :  $a_A = \frac{2F}{2m} = \frac{F}{m}$ ,  $a_B = \frac{F}{m}$  &  $N = 0$

V :  $a_A = a_B = \frac{2F}{3m}$  &  $N \neq 0$

**Example #13 to 15**

A body of mass 10 kg is placed on a smooth inclined plane as shown in figure. The inclined plane is moved with a horizontal acceleration  $a$ .



13. The normal reaction between block and inclined plane is :-  
 (A) 92 N (B) 44 N (C) 56 N (D) Can't be determined
14. The tension in thread is :-  
 (A) 92 N (B) 44 N (C) 56 N (D) Can't be determined
15. At what acceleration 'a' will the body lose contact with the inclined plane ?  
 (A) 10 m/s<sup>2</sup> (B) 13.33 m/s<sup>2</sup> (C) 3.33 m/s<sup>2</sup> (D) 6.66 m/s<sup>2</sup>

**Solution**

13. **Ans. (C)**

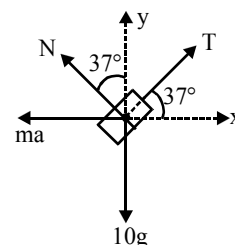
FBD of block w.r.t. inclined plane

$$\sum F_x = 0 \Rightarrow T \cos 37^\circ - N \sin 37^\circ - ma = 0$$

$$\Rightarrow 4T - 3N = 200 \dots\dots(i)$$

$$\sum F_y = 0 \Rightarrow N \cos 37^\circ + T \sin 37^\circ - 10g = 0$$

$$\Rightarrow 4N + 3T = 500 \dots\dots(ii)$$



By solving equation (i) & (ii),  $N = 56$  newton

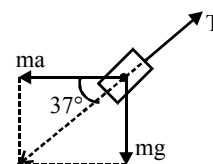
14. **Ans. (A)**

From above equation,  $T = 92$  newton

15. **Ans. (B)**

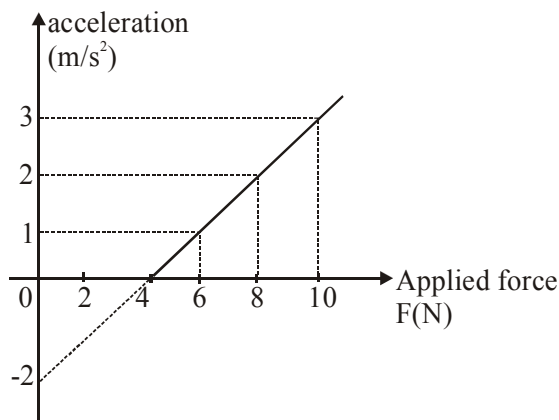
To lose contact,  $N = 0$

$$\tan 37^\circ = \frac{mg}{ma} \Rightarrow \frac{3}{4} = \frac{10}{9} \Rightarrow a = \frac{40}{3} = 13.33 \text{ m/s}^2$$



**Example #16**

A block of unknown mass is at rest on a rough horizontal surface. A force  $F$  is applied to the block. The graph in the figure shows the acceleration of the block w.r.t. the applied force.



The mass of the block and coefficient of friction are ( $g = 10 \text{ m/s}^2$ )

(A) 2 kg, 0.1

(B) 2 kg, 0.2

(C) 1 kg, 0.1

(D) can't be determined

**Solution**

**Ans. (B)**

$$\text{Acceleration of block, } a = \frac{F - \mu mg}{m} \Rightarrow a = \left(\frac{1}{m}\right)F - \mu g$$

$$\text{From graph ; slope} = \frac{1}{m} = \frac{1}{2} \Rightarrow m = 2\text{kg} \text{ and y-intercept; } -\mu g = -2 \Rightarrow \mu = 0.2$$

**Example #17**

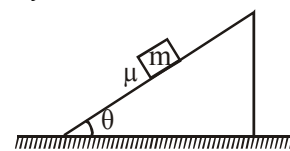
A body is placed on an inclined plane. The coefficient of friction between the body and the plane is  $\mu$ . The plane is gradually tilted up. If  $\theta$  is the inclination of the plane, then frictional force on the body is

(A) constant upto  $\theta = \tan^{-1}(\mu)$  and decreases after that

(B) increases upto  $\theta = \tan^{-1}(\mu)$  and decreases after that

(C) decreases upto  $\theta = \tan^{-1}(\mu)$  and constant after that

(D) constant throughout



**Solution**

**Ans. (B)**

Friction force  $F = mg \sin \theta$  if  $\theta \leq \tan^{-1}(\mu)$  and  $F = \mu mg \cos \theta$  if  $\theta \geq \tan^{-1}(\mu)$

which increases upto  $\theta = \tan^{-1}(\mu)$  and then decreases.

**Example #18**

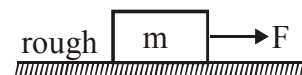
A block is placed on a rough horizontal surface and a horizontal force  $F$  is applied to it as shown in figure. The force  $F$  is increased from zero in small steps. The graph between applied force and frictional force  $f$  is plotted by taking equal scales on axes. The graph is

(A) a straight line of slope 45

(B) a straight line parallel to F-axis

(C) a straight line parallel to f-axis

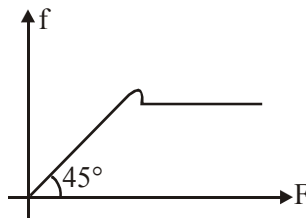
(D) a straight line of slope 45 for small  $F$  and a straight line parallel to F-axis for large  $F$ .





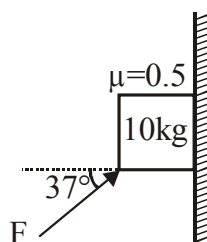
**Solution**

**Ans. (D)**



**Example #19**

A force  $F$  pushes a block weighing  $10\text{ kg}$  against a vertical wall as shown in the figure. The coefficient of friction between the block and wall is  $0.5$ . The minimum value of  $F$  to start the upward motion of block is  $[g = 10\text{ m/s}^2]$



(A)  $100\text{ N}$

(B)  $500\text{ N}$

(C)  $\frac{500}{3}\text{ N}$

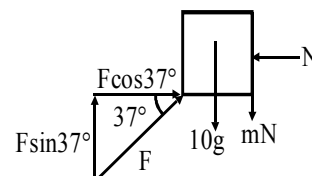
(D) can't be determined

**Solution**

**Ans. (B)**

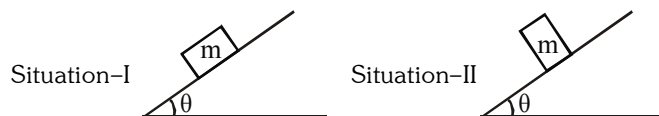
$$N = F \cos 37^\circ = \frac{4}{5}F \text{ and to start upward motion } F \sin 37^\circ = 10g + \mu N$$

$$\frac{3}{5}F = 100 + (0.5)\left(\frac{4}{5}F\right) \Rightarrow F = 500\text{ N}$$



**Example #20**

A block is first placed on its long side and then on its short side on the same inclined plane (see figure). The block slides down in situation II but remains at rest in situation I. A possible explanation is



(A) The normal contact force is less in situation-II.

(B) The frictional force is less in situation-II because the contact area is less.

(C) The shorter side is smoother.

(D) In situation-I, frictional force is more.

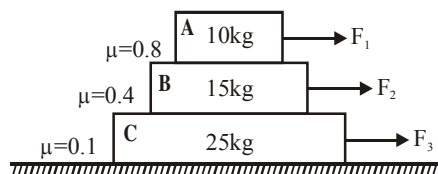
**Solution**

**Ans. (C, D)**

This is due to less frictional force or low friction coefficient.

**Example #21**

For the situation shown in the figure below, match the entries of column I with the entries of column II. [  $g = 10 \text{ m/s}^2$  ]

**Column I**

- (A) If  $F_1 = 160 \text{ N}$ ,  $F_2 = 0$  &  $F_3 = 0$ , then  
 (B) If  $F_1 = 140 \text{ N}$ ,  $F_2 = 0$  &  $F_3 = 0 \text{ N}$ , then  
 (C) If  $F_1 = F_2 = 0$  &  $F_3 = 45 \text{ N}$ , then  
 (D) If  $F_1 = 0$ ,  $F_2 = 160 \text{ N}$  and  $F_3 = 0$ , then

**Column II**

- (P) There is no relative motion between A & B  
 (Q) There is no relative motion between B & C  
 (R) There is a relative motion between A & B  
 (S) There is a relative motion between B & C  
 (T) There is a relative motion between C & ground

**Solution**

**Ans. (A)  $\rightarrow$  (Q,R,T) ; (B)  $\rightarrow$  (Q,R,T) ; (C)  $\rightarrow$  (P,Q) ; (D)  $\rightarrow$  (S,T)**

Maximum frictional force between A and B =  $(0.8)(10)(10) = 80 \text{ N}$

Maximum frictional force between B and C =  $(0.4)(25)(10) = 100 \text{ N}$

Maximum frictional force between C and ground =  $(0.1)(50)(10) = 50 \text{ N}$

Therefore there is no relative motion between B and C if  $F_2 = 0$  &  $F_3 = 0$

To start motion  $(F_1 + F_2 + F_3)_{\min} = 50 \text{ N}$

To start slipping between B and C (if  $F_3 = 0$ ),  $(F_1 + F_2)_{\min} = 150 \text{ N}$

If  $F_2 = F_3 = 0$  then to start slipping between A & B  $F_{1\min} = 87.5 \text{ N}$