ELECTROSTATICS

ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the concept of charge.

Types of charge :

(i) Positive charge : It is the deficiency of electrons as compared to proton.

(i) Negative charge : It is the excess of electrons as compared to proton.

SI unit of charge : ampere second i.e. Coulomb Dimension : [A T]

Practical units of charge are ampere hour (=3600 C) and faraday (= 96500 C)

- Millikan calculated quanta of charge by 'Highest common factor' (H.C.F.) method and it is equal to charge of electron.
- 1 C = 3 × 10^9 stat coulomb, 1 absolute - coulomb = 10 C, 1 Faraday = 96500 C.

SPECIFIC PROPERTIES OF CHARGE

- **Charge is a scalar quantity** : It represents excess or deficiency of electrons.
- **Charge is transferable** : If a charged body is put in contact with an another body, then charge can be transferred to another body.
- **Charge is always associated with mass**

  Charge cannot exist without mass though mass can exist without charge.
  - So the presence of charge itself is a convincing proof of existence of mass.
  - In charging, the mass of a body changes.
  - When body is given positive charge, its mass decreases.
  - When body is given negative charge, its mass increases.
- **Charge is quantised**

  The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e. Thus charge q of a body is always given by

  $q = ne$ n = positive integer or negative integer

  The quantum of charge is the charge that an electron or proton carries.

  Note : Charge on a proton = (–) charge on an electron = 1.6 10^{-19} C
- **Charge is conserved**

  In an isolated system, total charge does not change with time, though individual charge may change i.e. charge can neither be created nor destroyed. Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear. No exceptions to the rule have ever been found.
- **Charge is invariant**

  Charge is independent of frame of reference. i.e. charge on a body does not change whatever be its speed.

  Accelerated charge radiates energy

v = 0 (i.e. at rest) $\oplus$ Q produces only $\vec{E}$ (electric field)	v = constant $\oplus\!\rightarrow$ Q produces both $\vec{E}$ and $\vec{B}$ (magnetic field) but no radiation	v ≠ constant (i.e. time varying) $\oplus\!\rightarrow$ Q produces $\vec{E}$, $\vec{B}$ and radiates energy

- **Attraction – Repulsion**

  Similar charges repel each other while dissimilar attract

METHODS OF CHARGING

- **Friction:** If we rub one body with other body, electrons are transferred from one body to the other.

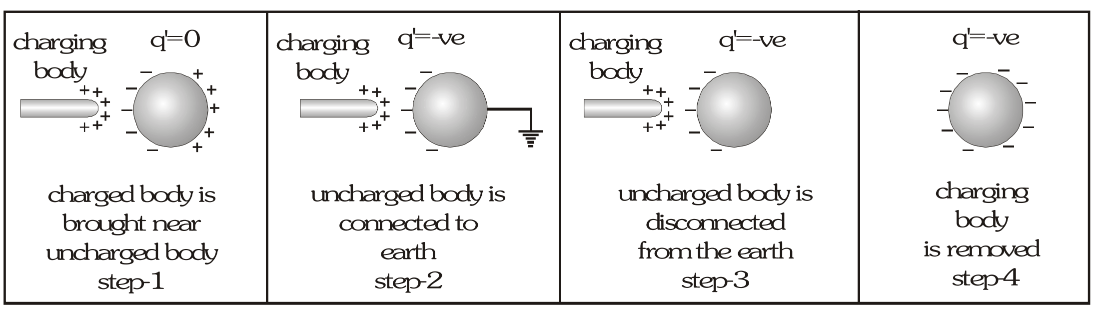
  Transfer of electrons takes places from lower work function body to higher work function body.

Positive charge	Negative charge
Glass rod	Silk cloth
Woollen cloth	Rubber shoes, Amber, Plastic objects
Dry hair	Comb
Flannel or cat skin	Ebonite rod
Note : Clouds become charged by friction	

- **Electrostatic induction**

  If a charged body is brought near a metallic neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'.

  - Charging a body by induction (in four successive steps)



Some important facts associated with induction-

(i) Inducing body neither gains nor loses charge

(ii) The nature of induced charge is always opposite to that of inducing charge

(iii) Induction takes place only in bodies (either conducting or non conducting) and not in particles.

- **Conduction**

  The process of transfer of charge by contact of two bodies is known as conduction. If a charged body is put in contact with uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

  - The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)
  - The charge gained by the uncharged body is always lesser than initial charge present on the charged body.
  - Flow of charge depends upon the potential difference of both bodies. [No potential difference $\Rightarrow$ No conduction].
  - Positive charge flows from higher potential to lower potential, while negative charge flows from lower to higher potential.

GOLDEN KEY POINTS

- Charge differs from mass in the following sense.
  (i) In SI units, charge is a derived physical quantity while mass is fundamental quantity.
  (ii) Charge is always conserved but mass is not (Note : Mass can be converted into energy $E=mc^2$
  (iii) The quanta of charge is electronic charge while that of mass it is yet not clear.
  (iv) For a moving charged body mass increases while charge remains constant.
- True test of electrification is repulsion and not attraction as attraction may also take place between a charged and an uncharged body and also between two similarly charged bodies.
- For a non relativistic (i.e. v << c) charged particle, specific charge $\frac{q}{m}$ =constant
- For a relativistic charged particle $\frac{q}{m}$ decreases as v increases, where v is speed of charged body.

Example

When a piece of polythene is rubbed with wool, a charge of – 2 10^{-7} C is developed on polythene. What is the amount of mass, which is transferred to polythene.

Solution

From Q = ne, So, the number of electrons transferred $n=\frac{Q}{e}=\frac{2\times10^{-7}}{1.6\times10^{-19}}$ = 1.25 10^{12}

Now mass of transferred electrons = n mass of one electron = 1.25 10^{12} 9.1 10^{-31} = 11.38 10^{-19} kg

Example

10^{12} α – particles (Nuclei of helium) per second falls on a neutral sphere, calculate time in which sphere gets charged by 2μC.

Solution

Number of α – particles falling in t second = 10^{12}t

Charge on α – particle = +2e , So charge incident in time t = $(10^{12}t).(2e)$

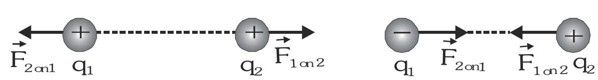
$\because$ Given charge is 2 μC $\therefore$ 2 10^{-6} = $(10^{12}t).(2e)$ $\Rightarrow t=\frac{10^{-18}}{1.6\times10^{-19}}=6.25s$

COULOMB'S LAW

The electrostatic force of interaction between two static point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

If two points charges q_1 and q_2 separated by a distance r. Let F be the electrostatic force between these two charges. According to Coulomb's law.

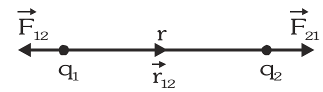
$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$



$F_e=\frac{kq_1q_2}{r^2}$ where $\left[k=\frac{1}{4\pi\epsilon_0}=9\times10^9\frac{Nm^2}{C^2}\right]$ k = coulomb's constant or electrostatic force constant

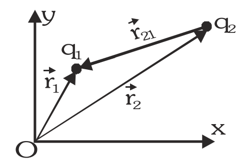
Coulomb's law in vector form

$\overrightarrow{F}_{12}$ = force on q_1 due to q_2 = $\dfrac{kq_1q_2}{r^2}\hat{r}_{21}$



$\overrightarrow{F}_{21} = \dfrac{kq_1q_2}{r^2}\hat{r}_{12}$ (here $\hat{r}_{12}$ is unit vector from q_1 to q_2)

Coulomb's law in terms of position vector



$$\overrightarrow{F}_{12} = \frac{kq_1q_2}{\left|\vec{r}_1 - \vec{r}_2\right|^3}(\vec{r}_1 - \vec{r}_2)$$

Principle of superposition

The force is a two body interaction, i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on a charged particle due to number of point charges is the resultant of forces due to individual point charges, i.e., $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots$

Note : Nuclear force is many body interaction, so principle of superposition is not valid in case of nuclear force.

When a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges individually

$$F = \frac{kq_0q_1}{r_1^2} + \frac{kq_0q_2}{r_2^2} + \cdots + \frac{kq_0q_i}{r_i^2} + \ldots \frac{kq_0q_n}{r_n^2} \text{ in vector form } \overrightarrow{F} = kq_0\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

SOME IMPORTANT POINTS REGARDING COULOMB'S LAW AND ELECTRIC FORCE

- The law is based on physical observations and is not logically derivable from any other concept. Experiments till today reveal its universal nature.
- The law is analogous to Newton's law of gravitation : $F = G\dfrac{m_1m_2}{r^2}$ with the difference that :

  (a) Electric force between charged particles is much stronger than gravitational force, i.e., $F_E >> F_G$. This is why when both F_E and F_G are present, we neglect F_G.

  (b) Electric force can be attractive or repulsive while gravitational force is always attractive.

  (c) Electric force depends on the nature of medium between the charges while gravitational force does not.

  (d) The force is an action–reaction pair, i.e., the force which one charge exerts on the other is equal and opposite to the force which the other charge exerts on the first.
- The force is conservative, i.e., work done in moving a point charge once round a closed path under the action of Coulomb's force is zero.
- The net Coulomb's force on two charged particles in free space and in a medium filled upto infinity are

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \text{ and } F' = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}. \text{ So } \frac{F}{F'} = \frac{\varepsilon}{\varepsilon_0} = K,$$

- Dielectric constant (K) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium filled upto infinity.
- The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its 'centre' as this is true only for spherical charged body, that too for external point.

  Although net electric force on both particles change in the presence of dielectric but force due to one charge particle on another charge particle does not depend on the medium between them.
- Electric force between two charges does not depend on neighbouring charges.

Example

If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them?

Solution

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{r^2} \quad ....(i) \qquad \text{Again, } F' = \frac{1}{4\pi\varepsilon_0}\frac{(2q)(2q)}{(2r)^2} \text{ or } F' = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{4r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = F$$

So, the force will remain the same.

Example

A particle of mass m carrying charge '$+q_1$' is revolving around a fixed charge '$-q_2$' in a circular path of radius r. Calculate the period of revolution.

Solution

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = mr\omega^2 = \frac{4\pi^2 mr}{T^2}$$

$$T^2 = \frac{(4\pi\varepsilon_0)r^2(4\pi^2 mr)}{q_1q_2} \quad \text{or} \quad T = 4\pi r\sqrt{\frac{\pi\varepsilon_0 mr}{q_1q_2}}$$

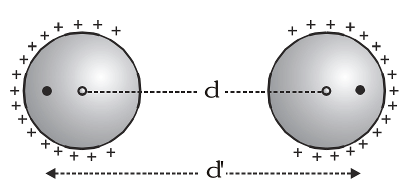
where $\vec{r}$ is the vector drawn from source charge is test charge.

Example

The force of repulsion between two point charges is F, when these are at a distance of 1 m. Now the point charges are replaced by spheres of radii 25 cm having the charge same as that of point charges. The distance between their centres is 1 m, then compare the force of repulsion in two cases.

Solution

In 2nd case due to mutual repulsion, the effective distance between their centre of charges will be increased (d' > d) so force of repulsion decreases as $F \propto \frac{1}{d^2}$

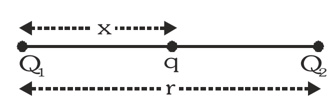


EQUILIBRIUM OF CHARGED PARTICLES

In equilibrium net electric force on every charged particle is zero. The equilibrium of a charged particle, under the action of Colombian forces alone can never be stable.

❒ **Equilibrium of three point charges**

(i) Two charges must be of like nature as $F_q = \frac{KQ_1q}{x^2} + \frac{KQ_2q}{(r-x)^2} = 0$

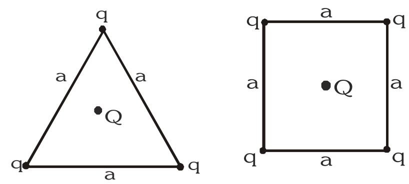


(ii) Third charge should be of unlike nature as $F_{Q_1} = \frac{KQ_1Q_2}{r^2} + \frac{KQ_1q}{x^2} = 0$

Therefore $x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r$ and $q = \frac{-Q_1Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$

❒ Equilibrium of symmetric geometrical point charged system

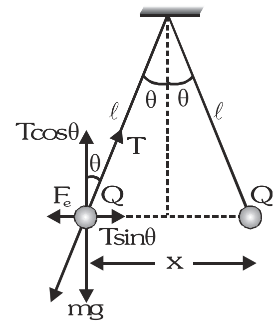
Value of Q at centre for which system to be in state of equilibrium



(i) For equilateral triangle $Q = \dfrac{-q}{\sqrt{3}}$ (ii) For square $Q = \dfrac{-q(2\sqrt{2}+1)}{4}$

❒ Equilibrium of suspended point charge system

For equilibrium position

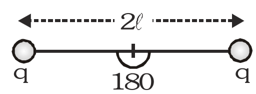


$$T\cos\theta = mg \quad \text{and} \quad T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

- If θ is small then tan

$$\theta \approx \sin\theta = \frac{x}{2\ell} \Rightarrow \frac{x}{2\ell} = \frac{kQ^2}{x^2 mg} \Rightarrow x^3 = \frac{2kQ^2\ell}{mg} \Rightarrow x = \left[\frac{Q^2\ell}{2\pi\epsilon_0\, mg}\right]^{\frac{1}{3}}$$

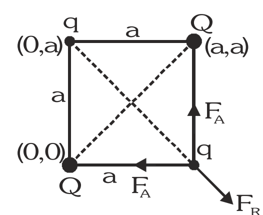
- If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$) then $T = F_e = \dfrac{kq^2}{4\ell^2}$



Example

For the system shown in figure find Q for which resultant force on q is zero.

Solution



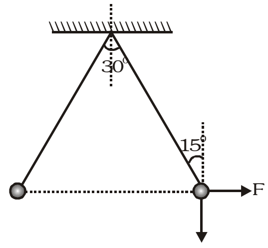
For force on q to be zero, charges q and Q must be of opposite of nature.

Net attraction force on q due to charges Q = Repulsion force on q due to q

$$\sqrt{2}\, F_A = F_R \Rightarrow \sqrt{2}\,\frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2} \Rightarrow q = 2\sqrt{2}\, Q \text{ Hence} \quad q = -2\sqrt{2}\, Q$$

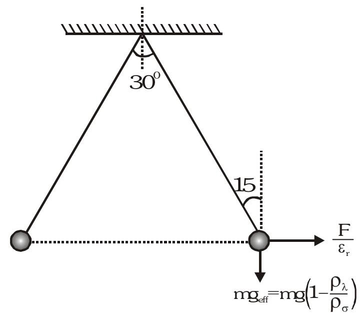
Example

Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30^0 with each other. When suspended in a liquid of density 0.8 g/cc the angle remains same. What is the dielectric constant of liquid. Density of sphere = 1.6 g/cc.



Solution

When set up shown in figure is in air, we have $\tan 15^0 = \dfrac{F}{mg}$ When set up is immersed in the medium as shown in figure, the electric force experienced by the ball will reduce and will be equal to $\dfrac{F}{\varepsilon_r}$ and the effective gravitational force will become $mg\left(1-\dfrac{\rho_\ell}{\rho_s}\right)$ Thus we have $\tan 15^0 = \dfrac{F}{mg\in_r\left(1-\dfrac{\rho_\ell}{\rho_s}\right)} = \dfrac{F}{mg} \Rightarrow \in_r = \dfrac{1}{1-\dfrac{\rho_\ell}{\rho_s}} = 2$



Example

Given a cube with point charges q on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.

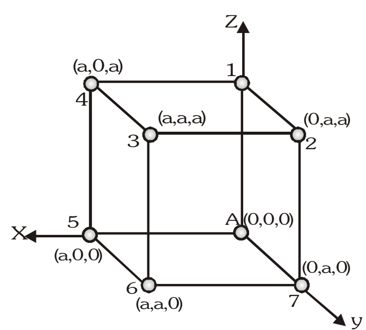
Solution

The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube. For this we use vector form of coulomb's law $\vec{F} = \dfrac{kq_1q_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2)$

From the figure the different forces acting on A are given as $\vec{F}_{A_1} = \dfrac{kq^2(-a\hat{k})}{a^3}$

$$\vec{F}_{A_2} = \frac{kq^2(-a\hat{j}-a\hat{k})}{(\sqrt{2}a)^3},\quad \vec{F}_{A_3} = \frac{kq^2(-a\hat{i}-a\hat{j}-a\hat{k})}{(\sqrt{3}a)^3};\quad \vec{F}_{A_4} = \frac{kq^2(-a\hat{i}-a\hat{k})}{(\sqrt{2}a)^3}$$

$$\vec{F}_{A_5} = \frac{kq^2(-a\hat{i})}{a^3},\quad \vec{F}_{A_6} = \frac{kq^2(-a\hat{i}-a\hat{j})}{(\sqrt{2}a)^3},\quad \vec{F}_{A_7} = \frac{kq^2(-a\hat{j})}{a^3}$$



The net force experienced by A can be given as

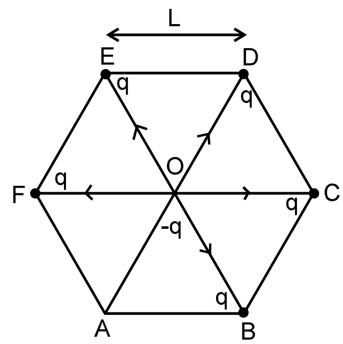
$$\vec{F}_{net} = \vec{F}_{A_1}+\vec{F}_{A_2}+\vec{F}_{A_3}+\vec{F}_{A_4}+\vec{F}_{A_5}+\vec{F}_{A_6}+\vec{F}_{A_7} = \frac{-kq^2}{a^2}\left[\left(\frac{1}{3\sqrt{3}}+\frac{1}{\sqrt{2}}+1\right)(\hat{i}+\hat{j}+\hat{k})\right]$$

Example

Five point charges, each of value +q are placed on five vertices of a regular hexagon of side Lm. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of the hexagon?

Solution

If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on –q at O will be zero (as the forces due to individual charges will balance each other).



Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

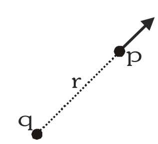
$$\vec{F} + \vec{f} = 0 \Rightarrow \vec{F} = -\vec{f} \Rightarrow F = f = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{q^2}{4\pi\varepsilon_0 L^2}$$

ELECTRIC FIELD

In order to explain 'action at a distance', i.e., 'force without contact' between charges it is assumed that a charge or charge distribution produces a field in space surrounding it. So the region surrounding a charge (or charge distribution) in which its electrical effects are perceptible is called the electric field of the given charge.

Electric field at a point is characterized either by a vector function of position $\vec{E}$ called 'electric intensity' or by a scalar function of position V called 'electric potential'. The electric field in a certain space is also visualized graphically in terms of 'lines of force.' So electric intensity, potential and lines of force are different ways of describing the same field.

Intensity of electric field due to point charge



Electric field intensity is defined as force on unit test charge.

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\tilde{r} = \frac{kq}{r^3}\vec{r}$$

Note : Test charge (q_0) is a fictitious charge that exerts no force on nearby charges but experiences forces due to them.

Properties of electric field intensity :

(i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.

(ii) Electric field due to positive charge is always away from it while due to negative charge always towards it.

(iii) Its unit is Newton/coulomb

(iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

(v) Force on a point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.

$$\vec{E} = q\vec{E}$$

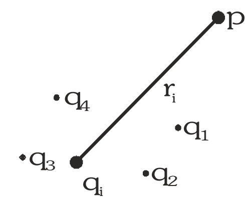
(vi) It obeys the superposition principle that is the field intensity point due to charge distribution is vector sum of the field intensities due to individual charge

GOLDEN KEY POINTS

- Charged particle in an electric field always experiences a force either it is at rest or in motion.
- In presence of a dielectric , electric field decreases and becomes $\frac{1}{\in_r}$ times of its value in free space.
- Test charge is always a unit (+ ve) charge. $\vec{E} = \frac{\vec{F}_{test}}{\text{test charge}}$
- If identical charges are placed on each vertices of a regular polygon, then $\vec{E}$ at centre = zero.

ELECTRIC FIELD INTENSITIES DUE TO VARIOUS CHARGE DISTRIBUTIONS

Due to discrete distribution of charge

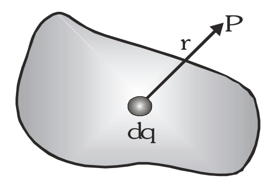


Field produced by a charge distribution for discrete distribution:–

By principle of superposition intensity of electric field due to i[th] charge $\vec{E}_{ip} = \frac{kq}{r_i^3}\vec{r}_i$

$\therefore$ Net electric field due to whole distribution of charge $\vec{E}_p = \sum_{i=1} \vec{E}_i$

Continuous distribution of charge



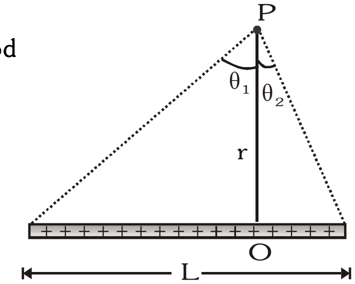
Treating a small element as particle $\vec{E} = \frac{1}{4\pi \in_0}\int \frac{dq}{r^3}\vec{r}$

Due to linear charge distribution $E = k\int_{\ell} \frac{\lambda d\ell}{r^2}$ [λ = charge per unit length]

Due to surface charge distribution $E = k\int_{s} \frac{\sigma ds}{r^2}$ [σ = charge per unit area]

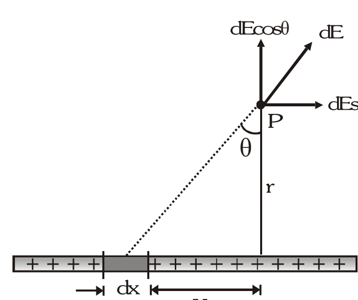
Due to volume charge distribution $E = k\int_{v} \frac{\rho dv}{r^2}$ [ρ = charge per unit volume]

Electric field strength at a general point due to a uniformly charged rod



As shown in figure, if P is any general point in the surrounding of rod, to find electric field strength at P, we consider an element on rod of length dx at a distance x from point O as shown in figure. Now if dE be the electric field at P due to the element, then

$$dE = \frac{kdq}{\left(x^2 + r^2\right)} \quad \text{Here } dq = \frac{Q}{L}dx$$



Electric field strength in x–direction due to dq at P is

$$dE_x = dE\sin\theta = \left[\frac{kdq}{\left(x^2 + r^2\right)}\right]\sin\theta = \frac{kQ\sin\theta}{L\left(x^2 + r^2\right)}dx$$

Here we have $x = r\tan\theta$ and $dx = r\sec^2\theta\, d\theta$

Thus $dE_x = \frac{kQ}{L}\frac{r\sec^2\theta\, d\theta}{r^2\sec^2\theta}\sin\theta$, Strength $= \frac{kQ}{Lr}\sin\theta\, d\theta$

Net electric field strength due to dq at point P in x–direction is

$$E_x = \int dE_x = \frac{kQ}{Lr}\int_{-\theta_2}^{+\theta_1}\sin\theta\, d\theta = \frac{kQ}{Lr}\left[-\cos\theta\right]_{-\theta_2}^{+\theta_1} = \frac{kQ}{Lr}\left[\cos\theta_2 - \cos\theta_1\right]$$

Similarly, electric field strength at point P due to dq in y–direction is $dE_y = dE\cos\theta = \frac{kQ\,dx}{L\left(r^2+x^2\right)}\cos\theta$

Again we have x = r tanθ and dx = r sec² θdθ. Thus we have $dE_y = \frac{kQ}{L}\cos\theta\,\frac{r\sec^2\theta}{r^2\sec^2\theta}d\theta = \frac{kQ}{Lr}\cos\theta\, d\theta$

Net electric field strength at P due to dq in y–direction is

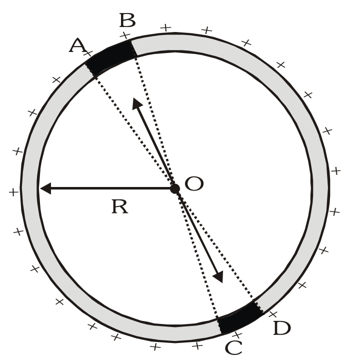
$$E_y = \int dE_y = \frac{kQ}{Lr}\int_{-\theta_2}^{+\theta_1}\cos\theta\, d\theta = \frac{kQ}{Lr}\left[+\sin\theta\right]_{-\theta_2}^{+\theta_1} = \frac{kQ}{Lr}\left[\sin\theta_1 + \sin\theta_2\right]$$

Thus electric field at a general point in the surrounding of a uniformly charged rod which subtend angles θ_1 and θ_2 at the two corners of rod can be given as

in x–direction : $E_x = \frac{kQ}{Lr}\left(\cos\theta_2 - \cos\theta_1\right)$ and in y–direction $E_y = \frac{kQ}{Lr}\left(\sin\theta_1 - \sin\theta_2\right)$

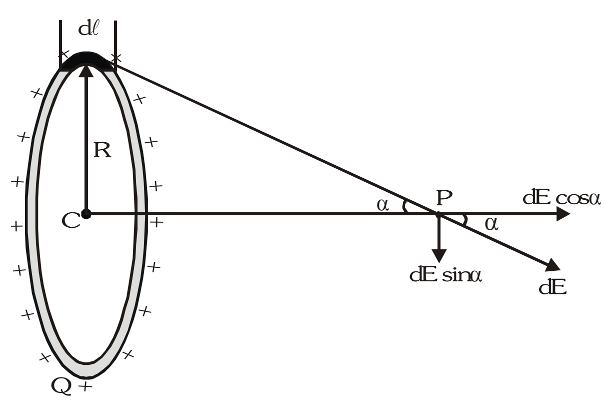
Electric field due to a uniformly charged ring

Case – I : At its centre



Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. The electric field strength at centre due to segment AB is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is E_0=0

Case II : At a point on the axis of Ring



Here we'll find the electric field strength at point P due to the ring which is situated at a distance x from the ring centre. For this we consider a small section of length $d\ell$ on ring as shown. The charge on this elemental section is $dq = \frac{Q}{2\pi R} d\ell$ [Q= total charge of ring]

Due to the element $d\ell$, electric field strength dE at point P can be given as $dE = \frac{Kdq}{\left(R^2+x^2\right)}$

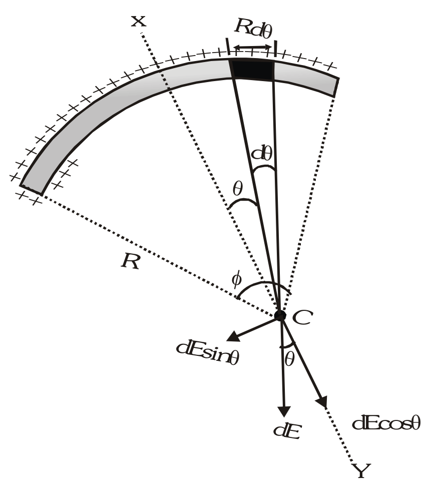
The component of this field strength $dE\sin\alpha$ which is normal to the axis of ring will be cancelled out due to the ring section opposite to $d\ell$. The component of electric field strength along the axis of ring $dE\cos\alpha$ due to all the sections will be added up. Hence total electric field strength at point P due to the ring is

$$E_p = \int dE\cos\alpha = \int_O^{2\pi R} \frac{kdq}{\left(R^2+x^2\right)} \quad \frac{x}{\sqrt{R^2+x^2}} = \int_O^{2\pi R} \frac{kQx}{2\pi R\left(R^2+x^2\right)} d\ell = \frac{kQx}{2\pi R\left(R^2+x^2\right)^{3/2}} \int_O^{2\pi R} d\ell$$

$$= \frac{kQ_x}{2\pi R\left(R^2+x^2\right)^{3/2}}\left[2\pi R\right] = \frac{kQx}{\left(R^2+x^2\right)^{3/2}}$$

Electric field strength due to a charged circular arc at its centre :

Figure shows a circular arc of radius R which subtend an angle ϕ at its centre. To find electric field strength at C, we consider a polar segment on arc of angular width $d\theta$ at an angle θ from the angular bisector XY as shown.



The length of elemental segment is $Rd\theta$, the charge on this element $d\ell$ is $dq = \left(\frac{Q}{\phi}\right).d\theta$

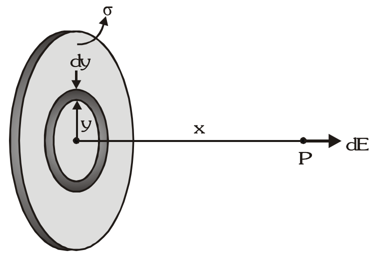
Due to this dq, electric field at centre of arc C is given as $dE = \frac{kdq}{R^2}$

Now electric field component due to this segment $dE\sin\theta$ which is perpendicular to the angular bisector gets cancelled out in integration and net electric field at centre will be along angular bisector which can be calculated by integrating $dE\cos\theta$ within limits from $-\frac{\phi}{2}$ to $\frac{\phi}{2}$. Hence net electric field strength at centre C is $E_C = \int dE\cos\theta$

$$= \int_{-\phi/2}^{+\phi/2} \frac{kQ}{\phi R^2}\cos\theta d\theta = \frac{kQ}{\phi R^2}\int_{-\phi/2}^{+\phi/2}\cos\theta d\theta = \frac{kQ}{\phi R^2}\left[\sin\theta\right]_{-\phi/2}^{+\phi/2} = \frac{kQ}{\phi R^2}\left[\sin\frac{\phi}{2}+\sin\frac{\phi}{2}\right] = \frac{2kQ\sin\left(\frac{\phi}{2}\right)}{\phi R^2}$$

Electric field strength due to a uniformly surface charged disc :

If there is a disc of radius R, charged on its surface with surface charge density σ, we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in figure.



To find electric field at point P due to this disc, we consider an elemental ring of radius y and width dy in the disc as shown in figure. The charge on this elemental ring $dq = \sigma . 2\pi y dy$ [Area of elemental ring $ds = 2\pi y\ dy$]

Now we know that electric field strength due to a ring of radius R, charge Q, at a distance x from its centre on its axis can be given as $E = \dfrac{kQx}{\left(x^2 + R^2\right)^{3/2}}$

Here due to the elemental ring electric field strength dE at point P can be given as

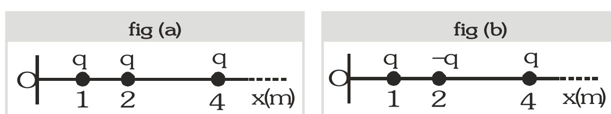
$$dE = \frac{k\,dq\,x}{\left(x^2 + y^2\right)^{3/2}} = \frac{k\sigma 2\pi y\ dy\ x}{\left(x^2 + y^2\right)^{3/2}}$$

Net electric field at point P due to this disc is given by integrating above expression from 0 to R as

$$E = \int dE = \int_0^R \frac{k\sigma 2\pi x y dy}{\left(x^2 + y^2\right)^{3/2}} = k\sigma\pi x \int_0^R \frac{2y\ dy}{\left(x^2 + y^2\right)^{3/2}} = 2k\sigma\pi x \left[-\frac{1}{\sqrt{x^2 + y^2}}\right]_0^R = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

Example

Calculate the electric field at origin due to infinite number of charges as shown in figures below.



Solution

(a) $E_0 = kq\left[\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + - - - -\right] = \frac{kq.1}{(1 - 1/4)} = \frac{4kq}{3}$ $\left[\because\ S_\infty = \frac{a}{1-r}\right.$, $a = 1$ and $\left. r = \frac{1}{4}\right]$

(b) $E_0 = kq\left[\frac{1}{1} - \frac{1}{4} + \frac{1}{16} - - - - -\right] = \frac{kq.1}{\left(1 - (-1/4)\right)} = \frac{4kq}{5}$

Example

A charged particle is kept in equilibrium in the electric field between the plates of millikan oil drop experiment. If the direction of the electric field between the plates is reversed, then calculate acceleration of the charged particle.

Solution

Let mass of the particle = m, Charge on particle = q

Intensity of electric field in between plates = E, Initially mg = qE

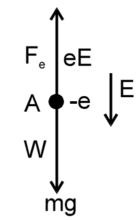
After reversing the field ma = mg + qE $\Rightarrow$ ma = 2mg

$\therefore$ Acceleration of particle $\Rightarrow$ a = 2g

Example

Calculate the electric field intensity E which would be just sufficient to balance the weight of an electron. If this electric field is produced by a second electron located below the first one what would be the distance between them? [Given : e = 1.6×10^{-19} C, m = 9.1 10^{-31} kg and g = 9.8 m/s^2]

Solution



(A)

As force on a charge e in an electric field E

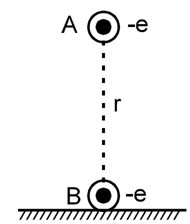
$$F_e = eE$$

So according to given problem

$$F_e = W \Rightarrow eE = mg$$

$$\Rightarrow E = \frac{mg}{e} = \frac{9.1 \times 10^{-31} \times 9.8}{1.6 \times 10^{-19}} = 5.57 \quad 10^{-11}\ \frac{V}{m}$$

As this intensity E is produced by another electron B, located at a distance r below A.



(B)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2} \Rightarrow r = \sqrt{\frac{e}{4\pi\varepsilon_0 E}} \quad \text{So, } r = \left[\frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{5.57 \times 10^{-11}}\right] \simeq 5 \text{ m}$$

Example

A block having mass m = 4 kg and charge q = 50 μC is connected to a spring having a force constant k = 100 N/m. The block lies on a frictionless horizontal track and a uniform electric field E = 5 10^5 V/m acts on the system as shwon in figure. The block is released from rest when the spring is unstretched (at x = 0)

(a) By what maximum amount does the spring expand?

(b) What is the equilibrium position of the block?

(c) Show that the block's motion is simple harmonic and determine the amplitude and time period of the motion.

Solution

(a) As x increases, electric force qE will accelerate the block while elastic force in the spring kx will oppose the motion. The block will move away from its initial position x = 0 till it comes to rest, i.e., work done by the electric force is equal to the energy stored in the spring.

So if x_{max} is maximum stretch of the spring.

$$\frac{1}{2}kx_{max}^2 = (qE)x_{max} \Rightarrow x_{max} = \frac{2qE}{k} \Rightarrow x_{max} = \frac{2 \times (50 \times 10^{-6}) \times (5 \times 10^5)}{100} = 0.5 \text{ m}$$

(b) In equilibrium position $F_R = 0$, so if x_0 is the stretch of the spring in equilibrium position

$$kx_0 = qE \Rightarrow x_0 = (qE/k) = \frac{1}{2}x_{max} = 0.25 \text{ m}$$

(c) If the displacement of the block from equilibrium position (x_0) is x, restoring force will be

$$F = k(x \pm x_0) \mp qE = kx \qquad [\text{ax } kx_0 = qE]$$

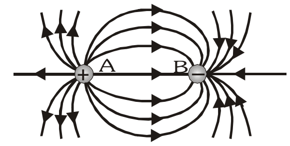
and as the restoring force is linear the motion will be simple harmonic with time period

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4}{100}} = 0.4\pi\,s$$

and amplitude = $x_{max} - x_0$ = 0.5 – 0.25 = 0.25 m

ELECTRIC LINES OF FORCE

Electric lines of electrostatic field have following properties



$q_A > q_B$

(i) Imaginary

(ii) Can never cross each other

(iii) Can never be closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system $(1/\varepsilon_o)$ electric lines are associated with unit charge, so if a body encloses a charge q , total lines of force associated with it (called flux) will be q/ε_o.

(v) Lines of force ends or starts normally at the surface of a conductor.

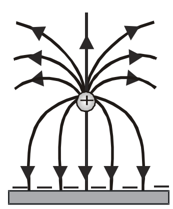
(vi) If there is no electric field there will be no lines of force.

(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity , crowded lines represent strong field while distant lines weak field.

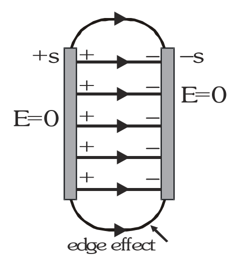
(viii) Tangent to the line of force at a point in an electric field gives the direction of intensity. So a positive charge free to move follow the line of force.

GOLDEN KEY POINTS

- Lines of force starts from (+ve) charge and ends on (–ve) charge.
- Lines of force start and end normally on the surface of a conductor .



fixed point charge near infinite metal plate



- The lines of force never intersect each other due to superposition principle.
- The property that electric lines of force contract longitudinally leads to explain attraction between opposite charges.
- The property that electric lines of force exert lateral pressure on each other leads to explain repulsion between like charges.

Electric flux (ϕ)

The word "flux" comes from a Latin word meaning "to flow" and you can consider the flux of a vector field to be a measure of the flow through an imaginary fixed element of surface in the field.

Electric flux is defined as $\phi_E = \int \vec{E} \cdot d\vec{A}$

This surface integral indicates that the surface in question is to be divided into infinitesimal elements of area $d\vec{A}$ and the scalar quantity $\vec{E} \cdot d\vec{A}$ is to be evaluated for each element and summed over the entire surface.

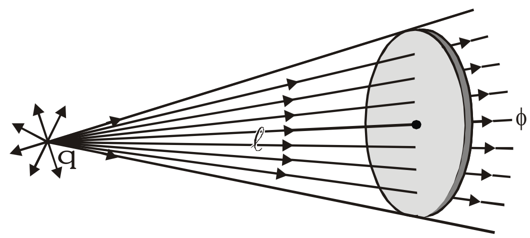
Important points about electric flux :

(i) It is a scalar quantity (ii) **Units** (V–m) and N – m^2/C **Dimensions** : [$ML^3T^{-3}A^{-1}$]

(iii) The value of ϕ does not depend upon the distribution of charges and the distance between them inside the closed surface.

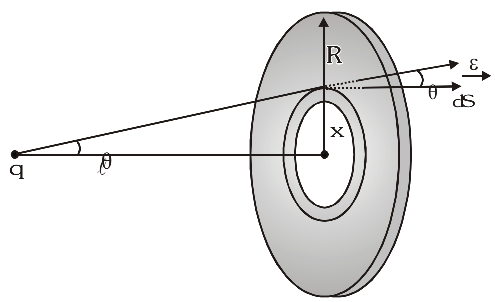
Electric Flux through a circular Disc :

Figure shows a point charge q placed at a distance ℓ from a disc of radius R. Here we wish to find the electric flux through the disc surface due to the point charge q. We know a point charge q originates electric flux in radially outward direction. The flux is originated in cone shown in figure passes through the disc surface.



To calculate this flux, we consider on elemental ring an disc surface of radius x and width dx as shown. Area of this ring (strip) is $dS = 2\pi x\ dx$. The electric field due to q at this elemental ring is given as $E = \dfrac{kq}{\left(x^2 + \ell^2\right)}$

If $d\phi$ is the flux passing through this elemental ring, then



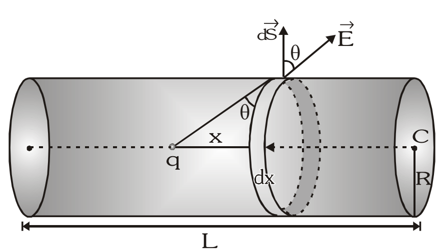
$$d\phi = EdS\cos\theta = \frac{kq}{\left(x^2+\ell^2\right)} \quad 2\pi x\ dx \quad \frac{\ell}{\sqrt{x^2+\ell^2}} = \frac{2\pi kq\ell\ x\ dx}{\left(\ell^2+x^2\right)^{3/2}}$$

$$\phi = \int d\phi = \int_0^R \frac{q\ell}{2\in_0} \frac{x\ dx}{\left(\ell^2+x^2\right)^{3/2}} = \frac{q\ell}{2\in_0}\int_0^R \frac{x\ dx}{\left(\ell^2+x^2\right)^{3/2}} = \frac{q\ell}{2\in_0}\left[-\frac{1}{\sqrt{\ell^2+x^2}}\right]_0^R = \frac{q\ell}{2\in_0}\left[\frac{1}{\ell}-\frac{1}{\sqrt{\ell^2+x^2}}\right]$$

The above result can be obtained in a much simpler way by using the concept of solid angle and Gauss's law.

Electric flux through the lateral surface of a cylinder due to a point charge :

Figure shows a cylindrical surface of length L and radius R. On its axis at its centre a point charge q is placed. Here we wish to find the flux coming out from the lateral surface of this cylinder due to the point charge q. For this we consider an elemental strip of width dx on the surface of cylinder as shown. The area of this strip is dS = $2\pi R.dx$



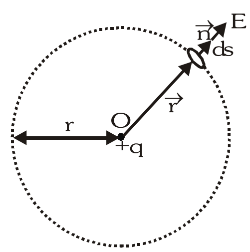
The electric field due to the point charge on the strip can be given as $E = \frac{kq}{\left(x^2 + R^2\right)}$. If $d\phi$ is the electric flux through the strip, then $d\phi = EdS \cos\theta = \frac{Kq}{\left(x^2+R^2\right)} \quad 2\pi\, Rdx \quad \frac{R}{\sqrt{x^2+R^2}} = 2\pi\, KqR^2 \quad \frac{dx}{\left(x^2+R^2\right)^{3/2}}$

Total flux through the lateral surface of cylinder $\phi = \int d\phi = \frac{qR^2}{2\,\epsilon_0} \int_{-L/2}^{+L/2} \frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{q\,\epsilon_0\,\ell}{\sqrt{\ell^2+4R^2}}$

This situation can also be easily handled by using the concepts of Gauss's law.

GAUSS'S LAW

It relates with the total flux of an electric field through a closed surface to the net charge enclosed by that surface and according to it, the total flux linked with a closed surface is $\left(1/\varepsilon_0\right)$ times the charge enclosed by the closed surface i.e., $\int_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$



REGARDING GAUSS'S LAW IT IS WORTH NOTING THAT :

Note :

(i) Flux through gaussian suface is independent of its shape.

(ii) Flux through gaussian suface depends only on charges present inside gaussian surface.

(iii) Flux through gaussian suface is independent of position of charges inside gaussian surface.

(iv) Electric field intensity at the gaussian surface is due to all the charges present (inside as well as out side)

(v) In a close surface incoming flux is taken negative while outgoing flux is taken positive.

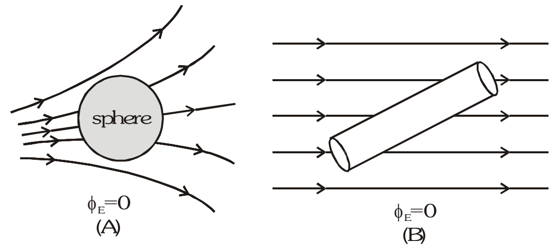
(vi) In a gaussian surface $\phi = 0$ does not employ $E = 0$ but $E = 0$ employs $\phi = 0$.

(vii) Gauss's law and Coulomb's law are equivalent, i.e., if we assume Coulomb's law we can prove Gauss's law and vice–versa. To prove Gauss's law from Coulomb's law consider a hypothetical spherical surface [called Gaussian–surface] of radius r with point charge q at its centre as shown in figure. By Coulomb's law intensity at a point P on the surface will be, $\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\vec{r}$ And hence electric flux linked with area $\vec{ds} \Rightarrow \vec{E}.\vec{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}.\vec{ds}$

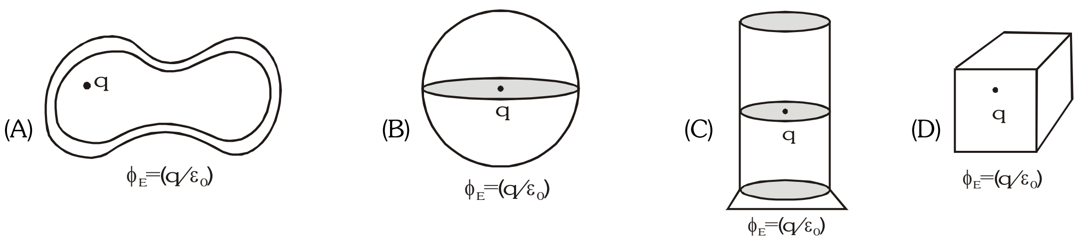
Here direction of $\vec{r}$ and $\vec{ds}$ are same, i.e., $\oint_S \vec{E}.\vec{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\oint_S ds = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\left(4\pi r^2\right) \quad \oint_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$

Which is the required result. Though here in proving it we have assumed the surface to be spherical, it is true for any arbitrary surface provided the surface is closed.

(viii) (a) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non–uniform) total flux linked with it will be zero.



(b) If a closed body encloses a charge q, then total flux

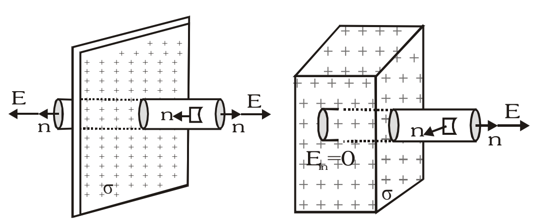


linked with the body will be $\oint_S \vec{E}.\vec{ds} = \dfrac{q}{\epsilon_0}$

From this expression it is clear that the flux linked with a closed body is independent of the shape and size of the body and position of charge inside it.[figure]

Note : So in case of closed symmetrical body with charge at its centre, flux linked with each half will be $\frac{1}{2}(\phi_E) = \left(\frac{q}{2\varepsilon_0}\right)$ and the symmetrical closed body has n identical faces with point charge at its centre, flux linked with each face will be $\left(\frac{\phi_E}{n}\right) = \left(\frac{q}{n\varepsilon_0}\right)$

(ix) Gauss's law is a powerful tool for calculating electric intensity in case of symmetrical charge distribution by choosing a Gaussian– surface in such a way that $\vec{E}$ is either parallel or perpendicular to its various faces. As an example, consider the case of a plane sheet of charge having charge density σ. To calculate E at a point P close to it consider a Gaussian surface in the form of a 'pill box' of cross–section S as shown in figure.



Sheet of charge
(A)

Conductor
(B)

The charge enclosed by the Gaussian–surface = σS and the flux linked with the pill box = ES + 0 + ES = 2ES (as E is parallel to curved surface and perpendicular to plane faces)

So from Gauss's law, $\phi_E = \frac{1}{\varepsilon_0}(q)$, $2ES = \frac{1}{\varepsilon_0}(\sigma S) \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

(x) If $\vec{E}=0$, $\phi = \oint \vec{E}.\vec{ds} = 0$, so q = 0 but if q=0, $\oint \vec{E}.\vec{ds} = 0$ So $\vec{E}$ may or may not be zero.

If a dipole is enclosed by a closed surface then, q =0, so $\oint \vec{E}.\vec{ds} = 0$, but $\vec{E} \neq 0$

Note : If instead of plane sheet of charge, we have a charged conductor, then as shown in figure (B) $E_{in}=0$.

So $\phi_E = ES$ and hence in this case $E=\frac{\sigma}{\epsilon_0}$. This result can be verified from the fact that intensity at the surface of a charged spherical conductor of radius R is, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ with $q = 4\pi R^2\sigma$

So for a point close to the surface of conductor, $E = \frac{1}{4\pi\varepsilon_0 R^2}\left(4\pi R^2\sigma\right) = \frac{\sigma}{\varepsilon_0}$

Example

If a point charge q is placed at the centre of a cube.

What is the flux linked (a) with the cube? (b) with each face of the cube?

Solution

(a) According to Gauss's law flux linked with a closed body is $(1/\varepsilon_0)$ times the charge enclosed and here the closed body cube is enclosing a charge q so, $\phi_T = \frac{1}{\varepsilon_0}(q)$

(b) Now as cube is a symmetrical body with 6–faces and the point charge is at its centre, so electric flux linked with each face will be $\phi_F = \frac{1}{6}(\phi_T) = \frac{q}{6\varepsilon_0}$

Note: (i) Here flux linked with cube or one of its faces is independent of the side of cube.

(ii) If charge is not at the centre of cube (but anywhere inside it), total flux will not change, but the flux linked with different faces will be different.

Example

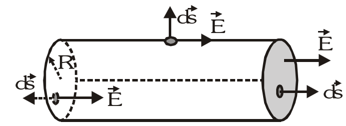
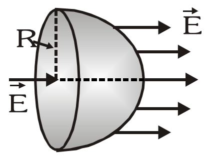
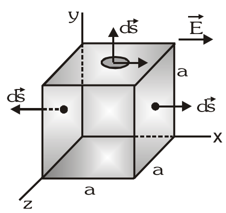
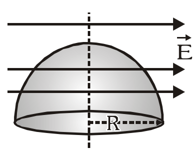
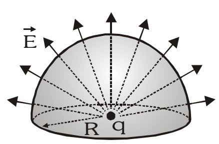
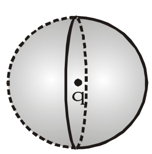
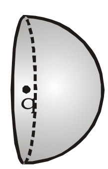
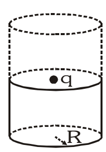
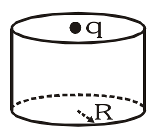
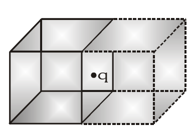
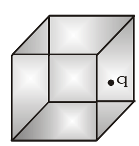
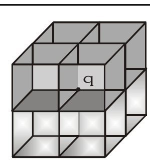
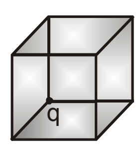
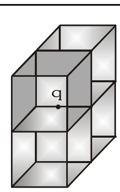
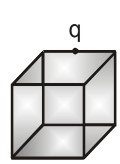
If a point charge q is placed at one corner of a cube, what is the flux linked with the cube?

Solution

In this case by placing three cubes at three sides of given cube and four cubes above, the charge will be in the centre.

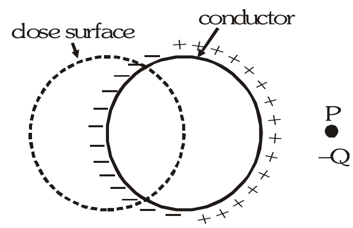
So, the flux linked with each cube will be one–eight of the flux $\frac{q}{\varepsilon_0}$. $\therefore$ Flux associated with given cube = $\frac{q}{8\varepsilon_0}$

FLUX CALCULATION USING GAUSS LAW

	$\phi_{in} = -\pi R^2 E$ and $\phi_{out} = \pi R^2 E \Rightarrow \phi_{total} = 0$
	$\phi_{in} = \phi_{circular} = -\pi R^2 E$ and $\phi_{out} = \phi_{curved} = \pi R^2 E \Rightarrow \phi_{total} = 0$
	$\phi_{in} = -a^2 E$ and $\phi_{out} = a^2 E \Rightarrow \phi_{total} = 0$
	$\phi_{in} = -\frac{1}{2}\pi R^2 E$ and $\phi_{out} = \frac{1}{2}\pi R^2 E \Rightarrow \phi_{total} = 0$
 $E = \frac{kq}{R^2}$ **Note :** here electric field is radial	$\phi = 2\pi R^2 \times \frac{q}{4\pi \epsilon_0 R^2} = \frac{q}{2 \epsilon_0}$
	 $\phi_{hemisphere} = \frac{q}{2 \epsilon_0}$
	 $\phi_{cylinder} = \frac{q}{2 \epsilon_0}$
	 $\phi_{cube} = \frac{q}{2 \epsilon_0}$
	 $\phi = \frac{q}{8 \epsilon_0}$
	 $\phi = \frac{q}{4 \epsilon_0}$

Example

As shown in figure a closed surface intersects a spherical conductor. If a negative charge is placed at point P. What is the nature of the electric flux coming out of the closed surface ?



Solution

Point charge Q induces charge on conductor as shown in figure.

Net charge enclosed by closed surface is negative so flux is negative.

Example

Consider $\vec{E} = 3 \quad 10^3 \hat{i}$ (N/C) then what is the flux through the square of 10 cm side, if the normal of its plane makes 60 angle with the X axis.

Solution

$\phi = ES\cos\theta = 3 \quad 10^3 \quad [10 \quad 10^{-2}]^2 \times \cos 60 = 3 \quad 10^3 \quad 10^{-2} \quad ½ = 15\ Nm^2/C$

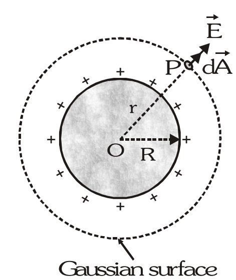
Example

Find the electric field due to an infinitely long cylindrical charge distribution of radius R and having linear charge density λ at a distance half of the radius from its axis.

Solution

$r = \dfrac{R}{2}$ point will be inside so $E = \dfrac{2k\lambda r}{R^2} = \dfrac{2k\lambda}{R^2}\left(\dfrac{R}{2}\right) = \dfrac{\lambda}{4\pi \in_0 R}$

ELECTRIC FIELD DUE TO SOLID CONDUCTING OR HOLLOW SPHERE



- **For outside point (r > R)**

Using Gauss's theorem $\oint \vec{E}.d\vec{s} = \dfrac{\Sigma q}{\in_0}$

$\because$ At every point on the Gaussian surface $\vec{E} \| d\vec{s}$; $\vec{E}.d\vec{s} = E\ ds \cos 0 = E\ ds$

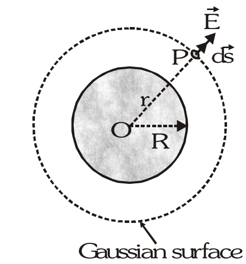
$\therefore \oint E.ds = \dfrac{\Sigma q}{\in_0}$ [E is constant over the gaussian surface] $\Rightarrow E \times 4\pi r^2 = \dfrac{q}{\in_0} \Rightarrow E_p = \dfrac{q}{4\pi \in_0 r^2}$

- **For surface point r = R** : $E_S = \dfrac{q}{4\pi \in_0 R^2}$

- **For Inside point (r < R)** : Because charge inside the conducting sphere or hollow is zero.

(i.e. $\Sigma q = 0$) So $\oint \vec{E}.d\vec{s} = \dfrac{\Sigma q}{\in_0} = 0 \Rightarrow E_{in} = 0$

ELECTRIC FIELD DUE TO SOLID NON CONDUCTING SPHERE



- **Outside (r > R)**

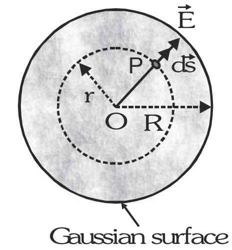
  From Gauss's theorem

$$\oint_s \vec{E}.d\vec{s} = \frac{\Sigma q}{\epsilon_0} \Rightarrow E \times 4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E_P = \frac{q}{4\pi \epsilon_0 r^2}$$

- **At surface (r = R)**

$$E_S = \frac{q}{4\pi \epsilon_0 R^2} \quad \text{Put } r = R$$

- **Inside (r < R) :**



From Gauss's theorem $\oint_s \vec{E}.d\vec{s} = \frac{\Sigma q}{\epsilon_0}$

Where Σq charge contained within Gaussian surface of radius r

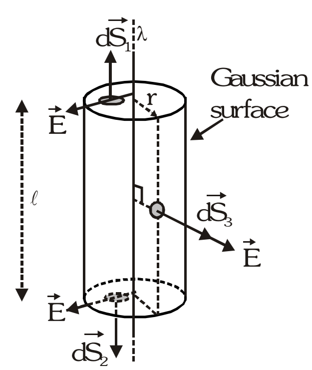
$$E(4\pi r^2) = \frac{\Sigma q}{\epsilon_0} \Rightarrow E = \frac{\Sigma q}{4\pi r^2 \epsilon_0} \quad ...(i)$$

As the sphere is uniformly charged, the volume charge density (charge/volume) ρ is constant throughout the sphere $\rho = \frac{q}{\frac{4}{3}\pi R^3}$ $\Rightarrow$ charge enclosed in gaussian surface $\Sigma q = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{q}{(4/3)\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) \Rightarrow \sum q = \frac{qr^3}{R^3}$

put this value in equation (i) $E_{in} = \frac{1}{4\pi \epsilon_0}\frac{qr}{R^3}$

ELECTRIC FIELD DUE TO AN INFINITE LINE DISTRIBUTION OF CHARGE

Let a wire of infinite length is uniformly charged having a constant linear charge density λ.
P is the point where electric field is to be calculated.
Let us draw a coaxial Gaussian cylindrical surfaces of length ℓ .
From Gauss's theorem



$$\int_{s_1} \vec{E}.d\vec{S}_1 + \int_{s_2} \vec{E}.d\vec{S}_2 + \int_{s_3} \vec{E}.d\vec{S}_3 = \frac{q}{\epsilon_0}$$

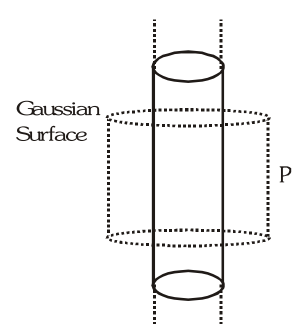
$$\vec{E} \perp d\vec{S}_1 \text{ so } \vec{E}.d\vec{S}_1 = 0 \text{ and } \vec{E} \perp d\vec{S}_2 \text{ so } \vec{E}.d\vec{S}_2 = 0$$

$$E \times 2\pi r\ell = \frac{q}{\epsilon_0} \qquad [\because \vec{E} \parallel d\vec{S}_3]$$

Charge enclosed in the Gaussian surface $q = \lambda\ell$.

So $E \times 2\pi r\ell = \frac{\lambda\ell}{\epsilon_0} \Rightarrow E = \frac{\lambda}{2\pi \epsilon_0 r}$ or $E = \frac{2k\lambda}{r}$ where $k = \frac{1}{4\pi \epsilon_0}$

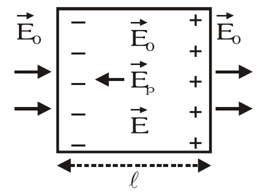
(f) Charged cylindrical nonconductor of infinite length



Electric field at outside point $\vec{E}_A = \frac{2k\lambda}{r}\tilde{r}$ $\quad r > R$

Electric field at inside point $\vec{E}_B = \frac{\lambda \vec{r}}{2\pi \epsilon_0 R^2}$ $\quad r < R$

DIELECTRIC IN ELECTRIC FIELD



Let $\vec{E}_0$ be the applied field, Due to polarisation, electric field is $\vec{E}_p$.

The resultant field is $\vec{E}$. For homogeneous and isotropic dielectric,

the direction of $\vec{E}_p$ is opposite to the direction of $\vec{E}_0$.

So, Resultant field is $E=E_0-E_P$

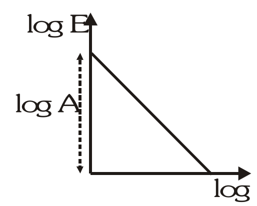
GOLDEN KEY POINTS

- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero ($E \neq 0$ if non zero charge is inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. $x >>> R$.

Example

For infinite line distribution of charge draw the curve between log E and log r.

Solution



$$\because\ E = \frac{\lambda}{2\pi \in_0 r} = \frac{A}{r} \text{ where } A = \frac{\lambda}{2\pi \in_0} = \text{constant}$$

take log on both side log E = log A − log r

Example

A point charge of 0.009 μC is placed at origin.

Calculate intensity of electric field due to this point charge at point $\left(\sqrt{2},\sqrt{7},0\right)$.

Solution

$$\vec{E} = \frac{q\vec{r}}{4\pi \in_0 r^3}\ ;\ \text{where } \vec{r} = x\hat{i}+y\hat{j} = \sqrt{2}\,\hat{i}+\sqrt{7}\,\hat{j}\,,\ \vec{E} = \frac{9\times10^5\times9\times10^{-9}(\sqrt{2}\,\hat{i}+\sqrt{7}\,\hat{j})}{(3)^3} = \left(3\sqrt{2}\,\hat{i}+3\sqrt{7}\,\hat{j}\right)NC^{-1}$$

ELECTROSTATIC POTENTIAL ENERGY

Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles. Electrostatic potential energy is defined in two ways.

(i) Interaction energy of charged particles of a system

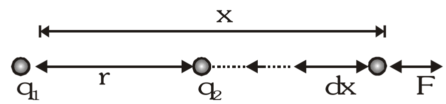
(ii) Self energy of a charged object

- **Electrostatic Interaction Energy**

Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to the given configuration. When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system, hence final potential energy of system will be positive. If the force between the particle is attractive, work will be done by the system and final potential energy of system will be negative.

- **Interaction Energy of a system of two charged particles**

Figure shows two + ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be given as work done in bringing q_2 from infinity to the given separation from q_1.



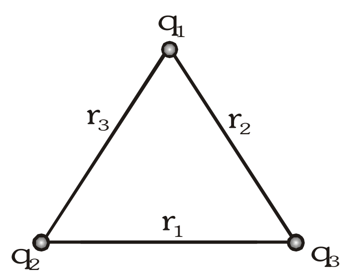
It can be calculated as $W = \int_{\infty}^{r} \vec{F}.\vec{dx} = -\int_{\infty}^{r} \frac{kq_1q_2}{x^2}dx$ [– ve sign shows that x is decreasing]

$$W = \frac{kq_1q_2}{r} = U \text{ [interaction energy]}$$

If the two charges here are of opposite sign, the potential energy will be negative as $U = -\frac{kq_1q_2}{r}$

- **Interaction Energy for a system of charged particles**

When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energies of all the pairs of particles. For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure.



The total interaction energy of this system can be given as $U = \frac{kq_1q_2}{r_3} + \frac{kq_1q_3}{r_2} + \frac{kq_2q_3}{r_1}$

ELECTRIC POTENTIAL

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field potential is defined as the interaction energy of a unit positive charge. If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as $V = \frac{U}{q_0}$ joule/coulomb

Potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces.

Example

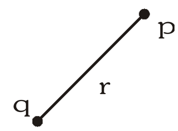
A charge $2\mu C$ is taken from infinity to a point in an electric field, without changing its velocity, if work done against electrostatic forces is $-40\mu J$ then potential at that point is ?

Solution

$$V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20V$$

Note : Always remember to put sign of W and q.

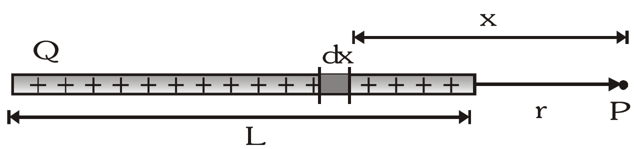
- **Electric Potential due to a point charge in its surrounding :**



The potential at a point P at a distance r from the charge q $V_P = \frac{U}{q_0}$. Where U is the potential energy of charge q_0 at point p, $U = \frac{kqq_0}{r}$. Thus potential at point P is $V_P = \frac{kq}{r}$

Electric Potential due to a charge Rod :

Figure shows a rod of length L, uniformly charged with a charge Q. Due to this we'll find electric potential at a point P at a distance r from one end of the rod as shown in figure.



For this we consider an element of width dx at a distance x from the point P.

Charge on this element is $dQ = \frac{Q}{L}dx$

The potential dV due to this element at point P can be given by using the result of a point charge as

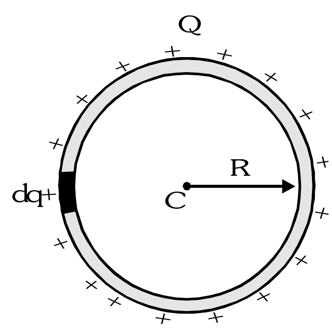
$$dV = \frac{kdq}{x} = \frac{kQ}{Lx}dx$$

Net electric potential at point P : $V = \int dV = \int_r^{r+L} \frac{kQ}{Lx}dx = \frac{kQ}{L}\ln\left(\frac{r+L}{r}\right)$

Electric potential due to a charged ring

Case – I : At its centre

To find potential at the centre C of the ring, we first find potential dV at centre due to an elemental charge dq on ring which is given as $dV = \frac{kdq}{R}$ Total potential at C is $V = \int dV = \int \frac{kdq}{R} = \frac{kQ}{R}$.

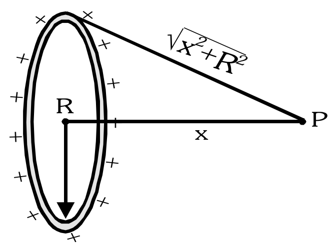


As all dq's of the ring are situated at same distance R from the ring centre C, simply the potential due to all dq's is added as being a scalar quantity, we can directly say that the total electric potential at ring centre is $\frac{kQ}{R}$.

Here we can also state that even if charge Q is non–uniformly distributed on ring, the electric potential C will remain same.

Case II : At a point on axis of ring

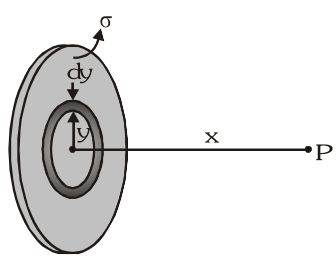
We find the electric potential at a point P on the axis of ring as shown, we can directly state the result as here also all points of ring are at same distance $\sqrt{x^2+R^2}$ from the point P, thus the potential at P can be given as $V_P = \dfrac{kQ}{\sqrt{R^2+x^2}}$



Electric potential due to a uniformly charged disc :

Figure shows a uniformly disc of radius R with surface charge density ρ coul/m². To find electric potential at point P we consider an elemental ring of radius y and width dy, charge on this elemental ring is $dq=\sigma 2\pi y\ dy$.

Due to this ring, the electric potential at point P can be given as $dV = \dfrac{kdq}{\sqrt{x^2+y^2}} = \dfrac{k.\sigma.2\pi y\ dy}{\sqrt{x^2+y^2}}$

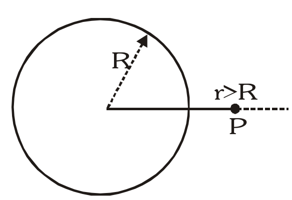


Net electric potential at Point P due to whole disc can be given as

$$V = \int dV = \int_0^R \frac{\sigma}{2\in_0}\frac{y\ dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\in_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\in_0}\left[\sqrt{x^2+R^2}-x\right]$$

ELECTRIC POTENTIAL DUE TO HOLLOW OR CONDUCTING SPHERE

- **At outside sphere**



According to definition of electric potential, electric potential at point P

$$V = -\int_\infty^r \vec{E}.d\vec{r} = -\int_\infty^r \frac{q}{4\pi\in_0 r^2}dr \left[\because\ E_{out} = \frac{q}{4\pi\in_0 r^2}\right];\ V = -\frac{q}{4\pi\in_0}\int_\infty^r \frac{1}{r^2}dr = \frac{q}{4\pi\in_0}\left[\frac{1}{r}\right]_\infty^r = \frac{q}{4\pi\in_0 r}$$

- **At surface**

$$V = -\int_{\infty}^{R} \vec{E}.d\vec{r} = -\int_{\infty}^{R} \frac{q}{4\pi \epsilon_0 r^2} dr \left[\because \ E_{out} = \frac{q}{4\pi\varepsilon_0 r^2}\right]; \ V = -\frac{q}{4\pi \epsilon_0} \int_{\infty}^{R} \left(\frac{1}{r^2}\right) dr = \frac{q}{4\pi \epsilon_0}\left[\frac{1}{r}\right]_{\infty}^{R} \Rightarrow V = \frac{q}{4\pi \epsilon_0 R}$$

- **Inside the surface** :

$\because$ Inside the surface $E = -\dfrac{dV}{dr} = 0 \ \Rightarrow V$ = constant so $V = \dfrac{q}{4\pi \epsilon_0 R}$

ELECTRIC POTENTIAL DUE TO SOLID NON CONDUCTING SPHERE

- **At outside sphere** Same as conducting sphere.
- **At Surface** Same as conducting sphere.

- **Inside the sphere**

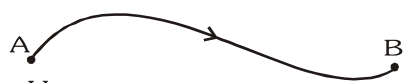
$$V = -\int_{\infty}^{r} \vec{E}.d\vec{r} \quad \Rightarrow \quad V = -\left[\int_{\infty}^{R} E_1 dr + \int_{R}^{r} E_2 dr\right]$$

$$V = -\left[\int_{\infty}^{R}\left(\frac{kq}{r^2}\right)dr + \int_{R}^{r}\left(\frac{kqr}{R^3}\right)dr\right] \quad \Rightarrow \quad V = -\left[kq\left(-\frac{1}{r}\right)_{\infty}^{R} + \frac{kq}{R^3}\left(\frac{r^2}{2}\right)_{R}^{r}\right]$$

$$V = -kq\left[-\frac{1}{R} + \frac{r^2}{2R^3} - \frac{R^2}{2R^3}\right] \quad \Rightarrow \quad V = \frac{kq}{2R^3}\left[3R^2 - r^2\right]$$

Potential Difference Between Two points in electric field

Potential difference between two points in electric field can be defined as work done in displacing a unit positive charge from one point to another against the electric forces.



If a unit +ve charge is displaced from a point A to B as shown work required can be given as $V_B - V_A = -\int_A^B \vec{E}.d\vec{x}$

If a charge q is shifted from point A to B, work done against electric forces can be given as $W = q\,(V_B - V_A)$

If in a situation work done by electric forces is asked, we use $W = q\,(V_A - V_B)$

If $V_B < V_A$, then charges must have tendency to move toward B (low potential point) it implies that electric forces carry the charge from high potential to low potential points. Hence we can say that in the direction of electric field always electric potential decreases.

Example

$1\mu C$ charge is shifted from A to B and it is found that work done by external force is $80\mu J$ against electrostic forces, find $V_A - V_B$

Solution

$W_{AB} = q(V_B - V_A)$

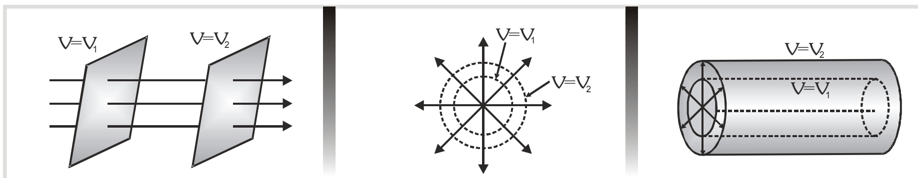
$80\ \mu J = 1\mu C\ (V_B - V_A) \Rightarrow V_A - V_B = -80\ V$

Equipotential surfaces

For a given charge distribution , locus of all points having same potential is called 'equipotential surface'.

- Equipotential surfaces can never cross each other (otherwise potential at a point will have two values which is absurd)

- Equipotential surfaces are always perpendicular to direction of electric field.
- If a charge is moved from one point to the other over an equipotential surface then work done $W_{AB} = -U_{AB} = q(V_B - V_A) = 0 \qquad [\because V_B = V_A]$
- Shapes of equipotential surfaces



for uniform electric field equipotential surfaces are parallel plane

for a point charge, spherical conductor equipotential surfaces are spherical

for a line distribution of charge equipotential surfaces are cylinderical

- The intensity of electric field along an equipotential surface is always zero.

Electric Potential Gradient

The maximum rate of change of potential at right angles to an equipotential surface in an electric field is defined as potential gradient. $\vec{E} = -\vec{\nabla}V = -\text{ grad } V$

Note : Potential is a scalar quantity but the gradient of potential is a vector quantity

In cartesian co–ordinates $\vec{\nabla}V = \left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$

Example

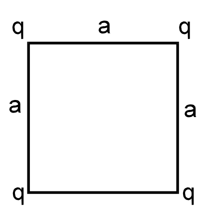
If $V = -5x + 3y + \sqrt{15}\,z$ then find magnitude of electric field at point (x,y,z).

Solution

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\tilde{i} + \frac{\partial V}{\partial y}\tilde{j} + \frac{\partial V}{\partial z}\tilde{k}\right] = -(-5\hat{i} + 3\hat{j} + \sqrt{15}\,\hat{k}) \Rightarrow |\vec{E}| = \sqrt{25+9+15} = \sqrt{49} = 7\,\text{unit}$$

Example

The four charges q each are placed at the corners of a square of side a. Find the potential energy of one of the charges



The electric potential of a point A due to charges B, C and D is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q}{a}$$

$\therefore$ Potential energy of the charge at A is $PE = qV = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{a}$.

Example

A proton moves with a speed of $7.45 \quad 10^5$ m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons.

Given : $\dfrac{1}{4\pi \epsilon_0} = 9 \quad 10^9 \dfrac{N-m^2}{C^2}$; $m_p = 1.67 \quad 10^{-27}$ kg and $e = 1.6 \quad 10^{-19}$ C

Solution

As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closect approach both will move with same velocity. So if v is the common velocity of each particle at closest approach, by 'conservation of momentum'.

$$mu = mv + mv \Rightarrow v = \frac{1}{2}\ u$$

And by conservation of energy $\dfrac{1}{2}\ mu^2 = \dfrac{1}{2}\ mv^2 + \dfrac{1}{2} + \dfrac{1}{4\pi\varepsilon_0}\ \dfrac{e^2}{r}$

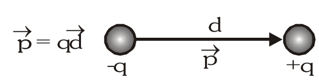
So, $r = \dfrac{4e^2}{4\pi\varepsilon_0 mu^2}$ [as $v = \dfrac{u}{2}$]

And hence substituting the given data,

$$r = 9 \quad 10^9 \quad \frac{4 \times (1.6 \times 10^{-19})^2}{1.67 \times 10^{-27} \times (7.45 \times 10^3)^2} = 10^{-12}\ m$$

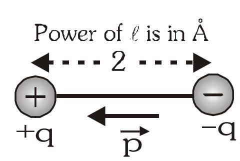
ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a certain distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges, given as



In some molecules, the centres of positive and negative charges do not coincide. This results in the formation of electric dipole. Atom is non – polar because in it the centres of positive and negative charges coincide. Polarity can be induced in an atom by the application of electric field. Hence it can be called as induced dipole.

- **Dipole Moment** : Dipole moment $\vec{p} = q\,\vec{d}$



(i) Vector quantity , directed from negative to positive charge

(ii) **Dimension** : [LTA], **Units** : coulomb metre (or C-m)

(iii) Practical unit is "debye" ≡ Two equal and opposite point charges each having charge 10^{-10} frankline ($\simeq e$) and separation of 1Å then the value of dipole moment $(\vec{p})$ is 1 debye.

$$1 \text{ Debye} = 10^{-10} \quad 10^{-10} \text{ Fr} \quad m = 10^{-20} \quad \frac{C \times m}{3 \times 10^9} \simeq 3.3 \quad 10^{-30}\ C \quad m$$

Example

A system has two charges $q_A = 2.5 \quad 10^{-7}$ C and $q_B = -2.5 \quad 10^{-7}$ C located at points A: (0, 0, – 0.15 m) and B ; (0, 0, + 0.15 m) respectively. What is the total charge and electric dipole moment of the system?

Solution

Total charge = $2.5 \quad 10^{-7} - 2.5 \quad 10^{-7} = 0$

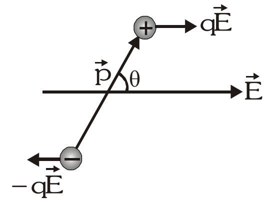
Electric diople moment,

p = Mangitude of either charge   separation between charges

= $2.5 \quad 10^{-7}$ [0.15 + 0.15] C m = $7.5 \quad 10^{-8}$ C m. The direction of dipole moment is from B to A.

- **Dipole Placed in uniform Electric Field**

Figure shows a dipole of dipole moment $\vec{p}$ placed at an angle θ to the direction of electric field. Here the charges of dipole experience forces qE in opposite direction as shown. $\vec{F}_{net} = \left[q\vec{E} + (-q)\,\vec{E}\right] = \vec{0}$



Thus we can state that when a dipole is placed in a uniform electric field, net force on the dipole is zero. But as equal and opposite forces act with a separation in their line of action, they produce a couple which tend to align the dipole along the direction of electric field. The torque due to this couple can be given as

τ = Force   separation between lines of actions of forces = qE   d $\sin\theta$ = pE $\sin\theta$

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{d} \times q\vec{E} = q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

Work done in Rotation of a Dipole in Electric field

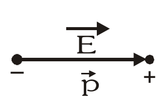
When a dipole is placed in an electric field at an angle θ, the torque on it due to electric field is $\tau = pE\sin\theta$

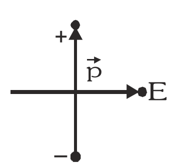
Work done in rotating an electric dipole from θ_1 to θ_2 [uniform field]

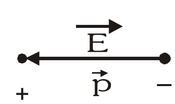
$$dW = \tau\, d\theta \quad \text{so } W = \int dW = \int \tau\, d\theta \text{ and } W_{\theta_1 \to \theta_2} = W = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2)$$

e.g. $W_{0 \to 180} = pE\,[1-(-1)] = 2\,pE \quad W_{0 \to 90} = pE\,(1-0) = pE$

If a dipole is rotated from field direction ($\theta = 0$) to θ then W = pE (1– $\cos\theta$)







$\theta = 0$	$\theta = 90$	$\theta = 180$
τ = minimum = 0	τ = maximum = pE	τ = minimum = 0
W = minimum = 0	W = pE	W = maximum = 2pE

Electrostatic potential energy :

Electrostatic potential energy of a dipole placed in a uniform field is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e., $W_{90°\to\theta} = \int_{90°}^{\theta} pE\sin\theta\, d\theta = -pE\cos\theta = -\vec{p}.\vec{E}$

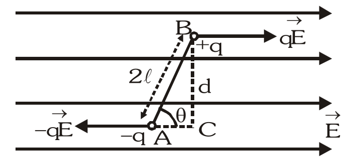
$\vec{E}$ is a conservative field so what ever work is done in rotating a dipole from θ_1 to θ_2 is just equal to change in electrostatic potential energy $W_{\theta_1\to\theta_2} = U_{\theta_2} - U_{\theta_1} = pE\ (\cos\theta_1 - \cos\theta_2)$

Work done in rotating an electric dipole in an electric field

Suppose at any instant, the dipole makes an angle θ with the electric field. The torque acting on dipole. $\tau = qEd = (q\ 2\ell\ \sin\theta)E = pE\sin\theta$ The work done in rotating dipole from θ_1 to θ_2

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$W = pE\ (\cos\theta_1 - \cos\theta_2) = U_2 - U_1 \quad (\because U = -pE\cos\theta)$$



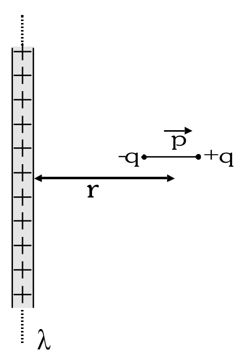
Force on an electric dipole in Non–uniform electric field :

If in a non–uniform electric field dipole is placed at a point where electric field is E, the interaction energy of dipole at this point $U = -\vec{p}.\vec{E}$. Now the force on dipole due to electric field $F = -\dfrac{\Delta U}{\Delta r}$

If dipole is placed in the direction of electric field then $F = -p\dfrac{dE}{dx}$

Example

Calculate force on a dipole in the surrounding of a long charged wire as shown in the figure.



Solution

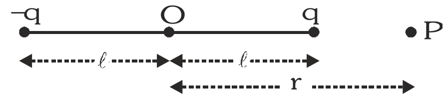
In the situation shown in figure, the electric field strength due to the wire, at the position of dipole as $E = \dfrac{2k\lambda}{r}$

Thus force on dipole is $F = -p.\dfrac{dE}{dr} = -p\left[-\dfrac{2k\lambda}{r^2}\right] = \dfrac{2kp\lambda}{r^2}$

Here –ve charge of dipole is close to wire hence net force an dipole due to wire will be attractive.

ELECTRIC POTENTIAL DUE TO DIPOLE

- **At axial point**



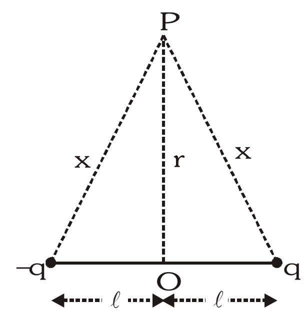
Electric potential due to +q charge $V_1 = \dfrac{kq}{(r-\ell)}$

Electric potential due to –q charge $V_2 = \dfrac{-kq}{(r+\ell)}$

Net electric potential $V = V_1 + V_2 = \dfrac{kq}{(r-\ell)} + \dfrac{-kq}{(r+\ell)} = \dfrac{kq \times 2\ell}{(r^2-\ell^2)} = \dfrac{kp}{r^2-\ell^2}$

If $r >>> \ell \Rightarrow V = \dfrac{kp}{r^2}$

- **At equitorial point**



Electric potential of P due to +q charge $V_1 = \dfrac{kq}{x}$

Electric potential of P due to –q charge $V_2 = -\dfrac{kq}{x}$

Net potential $V = V_1 + V_2 = \dfrac{kq}{x} - \dfrac{kq}{x} = 0 \therefore V = 0$

- **At general point**

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\,.\,\vec{r}}{4\pi\varepsilon_0 r^3} \quad \vec{p} = q\vec{d} \text{ electric dipole moment}$$

Electric field due to an electric dipole

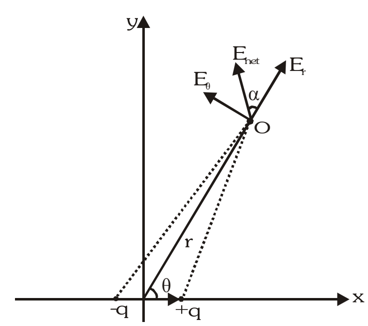


Figure shows an electric dipole placed on x–axis at origin. Here we wish to find the electric field and potential at a point O having coordinates (r, θ). Due to the positive charge of dipole electric field at O is in radially outward direction and due to the negative charge it is radially inward as shown in figure.

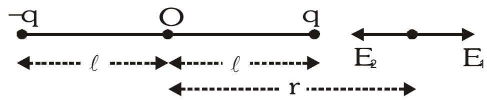
$$E_r = -\frac{\partial V}{\partial r} = \frac{2kp\cos\theta}{r^3} \text{ and } E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{kp\sin\theta}{r^3}$$

Thus net electric field at point O, $E_{net} = \sqrt{E_r^2 + E_\theta^2} = \dfrac{kp}{r^3}\sqrt{1+3\cos^2\theta}$

If the direction of E_{net} is at an angle α from radial direction, then $\alpha = \tan^{-1}\dfrac{E_\theta}{E_r} = \left(\dfrac{1}{2}\tan\theta\right)$

Thus the inclination of net electric field at point O is $(\theta+\alpha)$

- **At a point on the axis of a dipole :**

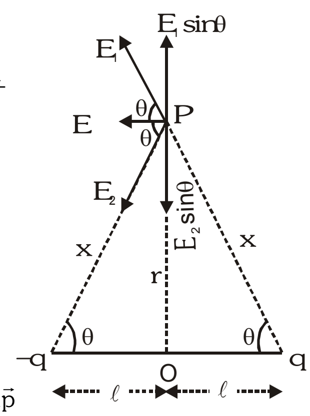


Electric field due to +q charge $E_1 = \dfrac{kq}{(r-\ell)^2}$ Electric field due to –q charge $E_2 = \dfrac{kq}{(r+\ell)^2}$

Net electric field $E = E_1 - E_2 = \dfrac{kq}{(r-\ell)^2} - \dfrac{kq}{(r+\ell)^2} = \dfrac{kq \times 4r\ell}{(r^2-\ell^2)^2}$ [$\because$ p = q 2ℓ = Dipole moment]

$$E = \frac{2kpr}{(r^2-\ell^2)^2} \text{ If } r >>> \ell \text{ then } E = \frac{2kp}{r^3}$$

- **At a point on equitorial line of dipole :**



Electric field due to +q charge $E_1 = \dfrac{kq}{x^2}$; Electric field due to –q charge $E_2 = \dfrac{kq}{x^2}$

Vertical component of E_1 and E_2 will cancel each other and horizontal components will be added So net electric field at P

$E = E_1\cos\theta + E_2\cos\theta$ [$\because$ $E_1 = E_2$]

$E = 2E_1 \cos\theta = \dfrac{2kq}{x^2}\cos\theta \because \cos\theta = \dfrac{\ell}{x}$ and $x = \sqrt{r^2+\ell^2}$

$E = \dfrac{2kq\ell}{x^3} = \dfrac{2kq\ell}{(r^2+\ell^2)^{3/2}} = \dfrac{kp}{(r^2+\ell^2)^{3/2}}$ If r > > > ℓ then $E = \dfrac{kp}{r^3}$ or $\vec{E} = \dfrac{-k\vec{p}}{r^3}$

GOLDEN KEY POINTS

- For a dipole, potential is zero at equatorial position, while at any finite point E ≠ 0
- In a uniform $\vec{E}$, dipole may feel a torque but not a force.
- If a dipole placed in a field $\vec{E}$ (Non-Uniform) generated by a point charge, then torque on dipole may be zero, but F ≠ 0
-

Distribution	Point charge	Dipole
Potential proportional to	r^{-1}	r^{-2}
E proportional to	r^{-2}	r^{-3}

Force between	Point charge	Dipole and point charge	Dipole-dipole
Proportional to	r^{-2}	r^{-3}	r^{-4}

Example

A short electric dipole is situated at the origin of coordinate axis with its axis along x–axis and equator along y–axis. It is found that the magnitudes of the electric intensity and electric potential due to the dipole are equal at a point distant r = $\sqrt{5}$ m from origin. Find the position vector of the point in first quadrant.

Solution

$$\because |E_P| = |V_P| \therefore \frac{kp}{r^3}\sqrt{1+3\cos^2\theta} = \frac{kp\cos\theta}{r^2} \Rightarrow 1+3\cos^2\theta = 5\cos^2\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$$

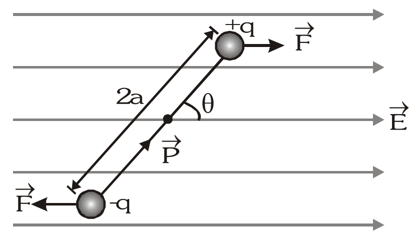
Position vector $\vec{r}$ of point P is $\vec{r} = \dfrac{\sqrt{5}}{2}\left(\tilde{i}+\tilde{j}\right)$

Example

Prove that the frequency of oscillation of an electric dipole of moment p and rotational inertia I for small amplitudes about its equilibrium position in a uniform electric field strength E is $\frac{1}{2\pi}\sqrt{\left(\frac{pE}{I}\right)}$

Solution

Let an electric dipole (charge q and –q at a distance 2a apart) placed in a uniform external electric field of strength E.



Restoring torque on dipole $\tau = -pE\sin\theta = -pE\ \theta$ (as θ is small)

Here – ve sign shows the restoring tendency of torque. $\because \tau = I\alpha \therefore$ angular acceleration = $\alpha = \frac{\tau}{I} = \frac{PE}{I}\theta$

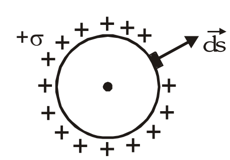
For SHM $\alpha = -\omega^2\theta$ comparing we get $\omega = \sqrt{\frac{pE}{I}}$

Thus frequency of oscillations of dipole $n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{pE}{I}\right)}$

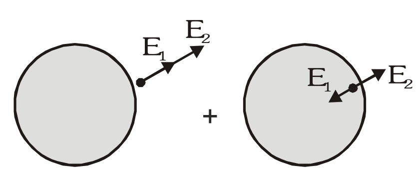
ELECTROSTATIC PRESSURE

Force due to electrostatic pressure is directed normally outwards to the surface .

Force on small element ds of charged conductor



dF = (Charge on ds) x Electric field = (σ ds) $\frac{\sigma}{2\in_0} = \frac{\sigma^2}{2\in_0}ds$



Inside $E_1 - E_2 = 0 \Rightarrow E_1 = E_2$

Just outside $E = E_1 + E_2 = 2E_2 \Rightarrow E_2 = \frac{\sigma}{2\in_0}$

(E_1 is field due to point charge on the surface and E_2 is field due to rest of the sphere).

The electric force acting per unit area of charged surface is defined as electrostatic pressure.

$$P_{eleectrostatic} = \frac{dF}{dS} = \frac{\sigma^2}{2\in_0}$$

Equilibrium of liquid charged surfaces (Soap bubble)

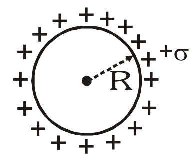
Pressures (forces) act on a charged soap bubble , due to

(i) Surface tension P_T (inward)

(ii) Air outside the bubble P_o (inward)

(iii) Electrostatic pressure P_e (outward)

(iv) Air inside the bubble P_i (outward)



in state of equilibrium inward pressure = outward pressure $P_T + P_o = P_i + P_e$

Excess pressure of air inside the bubble $(P_{ex}) = P_i - P_o = P_T - P_e$

but $P_T = \frac{4T}{r}$ and $P_e = \frac{\sigma^2}{2\in_0} \Rightarrow P_{ex} = \frac{4T}{r} - \frac{\sigma^2}{2\in_0}$ if $P_i = P_o$ then $\frac{4T}{r} = \frac{\sigma^2}{2\in_0}$

Example

Brass has a tensile strength $3.5 \quad 10^8$ N/m^2. What charge density on this material will be enough to break it by electrostatic force of repulsion? How many excess electrons per square Å will there then be? What is the value of intensity just out side the surface?

Solution

We know that electrostatic force on a charged conductor is given by $\frac{dF}{ds} = \frac{\sigma^2}{2\varepsilon_0}$

So the conductor will break by this force if, $\frac{\sigma^2}{2\varepsilon_0} >$ Breaking strength i.e., $\sigma^2 > 2 \times 9 \times 10^{-12} \times 3.5 \times 10^8$

i.e. $\sigma_{min} = \left(3\sqrt{7}\right) \times 10^{-2} = 7.94 \times 10^{-2} \left(C / m^2\right)$

Now as the charge on an electron is $1.6 \quad 10^{-19}$ C, the excess electrons per m^2

Further as in case of a conductor near its surface $E = \frac{\sigma}{\varepsilon_0} = \frac{7.94 \times 10^{-2}}{9 \times 10^{-12}} = 8.8 \times 10^9$ V/m

CONDUCTOR AND IT'S PROPERTIES [FOR ELECTROSTATIC CONDITION]

(i) Conductors are materials which contains large number of free electrons which can move freely inside the conductor.

(ii) In electrostatics, conductors are always equipotential surfaces.

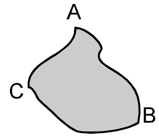
(iii) Charge always resides on outer surface of conductor.

(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.

(v) Electric field is always perpendicular to conducting surface.

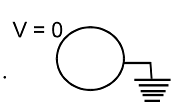
(vi) Electric lines of force never enter into conductors.

(vii) Electric field intensity near the conducting surface is given by formula $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$



$\vec{E}_A = \frac{\sigma_A}{\varepsilon_0}\tilde{n}$; $\vec{E}_B = \frac{\sigma_B}{\varepsilon_0}\tilde{n}$ and $\vec{E}_C = \frac{\sigma_C}{\varepsilon_0}\tilde{n}$

(viii) When a conductor is grounded its potential becomes zero.



(ix) When an isolated conductor is grounded then its charge becomes zero.

(x) When two conductors are connected there will be charge flow till their potential becomes equal.

(xi) Electric pressure at the surface of a conductor is givey by formula $P = \frac{\sigma^2}{2\varepsilon_0}$ where σ is the local surface charge density.

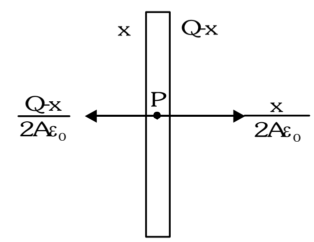
Example

Prove that if an isolated (isolated means no charges are near the sheet) large conducting sheet is given a charge then the charge distributes equally on its two surfaces.

Solution

Let there is x charge on left side of sheet and Q–x charge on right side of sheet.

Since point P lies inside the conductor so $E_P = O$



$$\frac{x}{2A\varepsilon_O} - \frac{Q-x}{2A\varepsilon_O} = 0 \Rightarrow \frac{2x}{2A\varepsilon_O} = \frac{Q}{2A\varepsilon_O} \Rightarrow x = \frac{Q}{2}$$

$$Q - x = \frac{Q}{2}$$

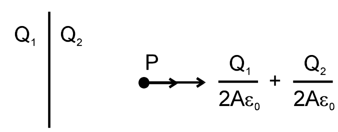
So charge is equally distributed on both sides

Example

If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_O}$, where $Q = Q_1 + Q_2$

Solution

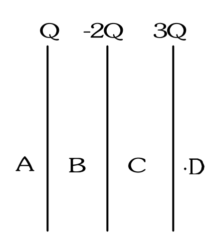
Electric field at point P : $\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q_1+Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$



[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Example

Three large conducting sheets placed parallel to each other at finite distance contains charges Q, –2Q and 3Q respectively. Find electric field at points A, B , C, and D



Sol. $E_A = E_Q + E_{-2Q} + E_{3Q}$. (i) Here E_Q means electric field due to 'Q'.

$$E_A = \frac{(Q-2Q+3Q)}{2A\varepsilon_0} = \frac{2Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$, towards left

(ii) $E_B = \dfrac{Q-(-2Q+3Q)}{2A\varepsilon_0}$, towards right = 0

(iii) $E_C = \dfrac{(Q-2Q)-(3Q)}{2A\varepsilon_0} = \dfrac{-4Q}{2A\varepsilon_0} = \dfrac{-2Q}{A\varepsilon_0}$, towards right $\Rightarrow \dfrac{2Q}{A\,\varepsilon_0}$ towards left

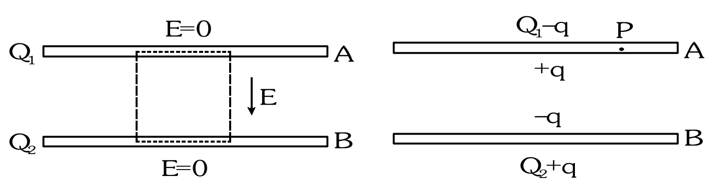
(iv) $E_D = \dfrac{(Q-2Q+3Q)}{2A\varepsilon_0} = \dfrac{2Q}{2A\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$, towards right

Example

Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Prove that the charges on the inner facing surfaces are of equal magnitude and opposite sign. Also find the charges on inner & outer surfaces.

Solution

Consider a Gaussian surface as shown in figure. Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.



The distribution should be like the one shown in figure. To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A. Using the equation $E = \sigma / (2\varepsilon_0)$, the electric field at P

due to the charge $Q_1 - q = \dfrac{Q_1 - q}{2A\varepsilon_0}$ (downward); due to the charge $+ q = \dfrac{q}{2A\varepsilon_0}$ (upward),

due to the charge $- q = \dfrac{q}{2A\varepsilon_0}$ (downward), and due to the charge $Q_2 + q = \dfrac{Q_2 + q}{2A\varepsilon_0}$ (upward).

The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$E_P = \frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}$$

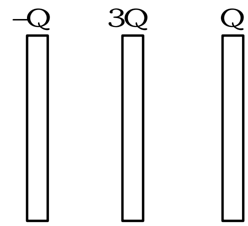
As the point P is inside the conductor, this field should be zero.

Hence, $Q_1 - q - q + q - Q_2 - q = 0 \Rightarrow q = \dfrac{Q_1 - Q_2}{2}$

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost, surfaces get equal charges and the facing surfaces get equal and opposite charges.

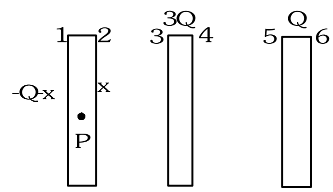
Example

Figure shows three large metallic plates with charges – Q, 3Q and Q respectively. Determine the final charges on all the surfaces.



Solution

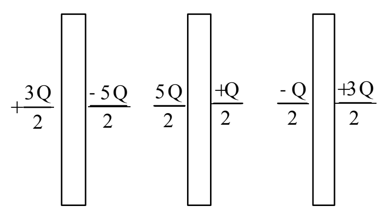
We assume that charge on surface 2 is x. Following conservation of charge, we see that surfaces 1 has charge (– Q – x). The electric field inside the metal plate is zero so fields at P is zero.



Resultant field at P : $E_P = 0 \Rightarrow \dfrac{-Q-x}{2A\varepsilon_0} = \dfrac{x+3Q+Q}{2A\varepsilon_0} \Rightarrow -Q - x = x + 4Q \Rightarrow x = \dfrac{-5Q}{2}$

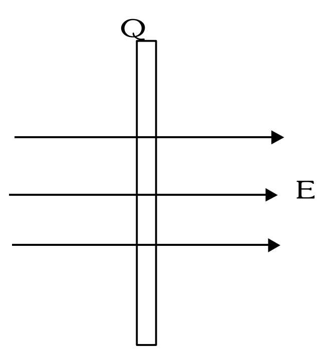
Note : We see that charges on the facing surfaces of the plates are of equal magnitude and opposite sign. This can be in general proved by gauss theorem also. Remember this it is important result.

Thus the final charge distribution on all the surfaces is :



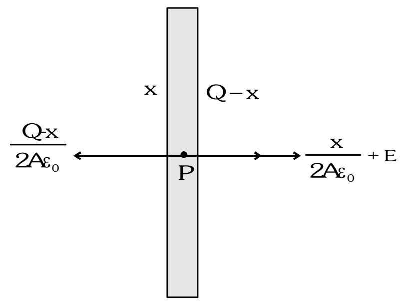
Example

An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces. Also



Solution

Let there is x charge on left side of plate and Q − x charge on right side of plate



$$E_P = 0 \Rightarrow \frac{x}{2A\varepsilon_0} + E = \frac{Q-x}{2A\varepsilon_0} \Rightarrow \frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E \Rightarrow x = \frac{Q}{2} - EA\varepsilon_0$$

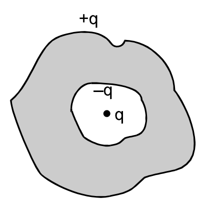
So charge on one side is $\frac{Q}{2} - EA\varepsilon_o$ and other side $\frac{Q}{2} + EA\varepsilon_o$

The resultant electric field on the left and right side of the plate.

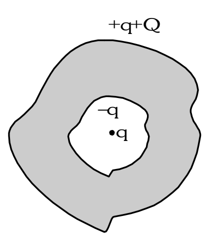
On right side $E = \frac{Q}{2A\varepsilon_0} + E$ towards right and on left side $\frac{Q}{2A\varepsilon_0} - E$ towards left.

SOME OTHER IMPORTANT RESULTS FOR A CLOSED CONDUCTOR.

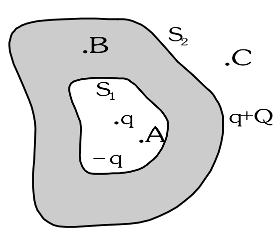
(i) If a charge q is kept in the cavity then −q will be induced on the inner surface and +q will be induced on the outer surface of the conductor (it can be proved using gauss theorem)



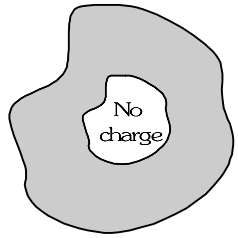
(ii) If a charge q is kept inside the cavity of a conductor and conductor is given a charge Q then −q charge will be induced on inner surface and total charge on the outer surface will be q + Q. (it can be proved using gauss theorem)



(iii) Resultant field, due to q (which is inside the cavity) and induced charge on S_1, at any point outside S_1 (like B,C) is zero. Resultant field due to **q + Q on S_2 and any other charge outside S_2**, at any point inside of surface S_2 (like A, B) is zero

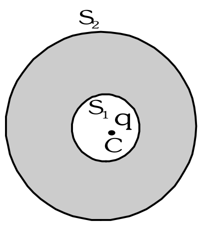


(iv) Resultant field in a charge free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also this result is true. No charge will be induced on the inner most surface of the conductor.



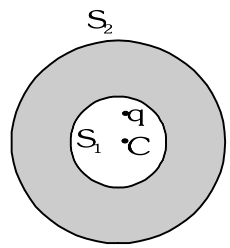
(v). Charge distribution for different types of cavities in conductors

(A)



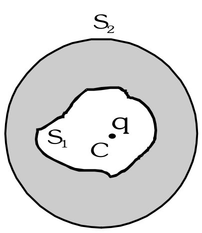
charge is at the common centre
(S_1, S_2 $\rightarrow$ spherical)

(B)



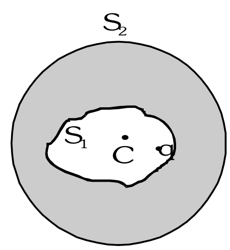
charge is not at the common centre
(S_1, S_2 $\rightarrow$ spherical)

(C)



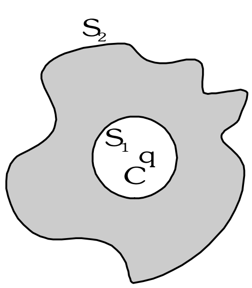
charge is at the centre of S_2
(S_2 $\rightarrow$ spherical)

(D)



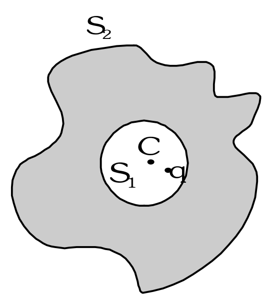
charge is not at the centre of S_2
(S_2 $\rightarrow$ spherical)

(E)



charge is at the centre of
S_1 (Spherical)

(F)



charge not at the centre
of S_1(Spherical)

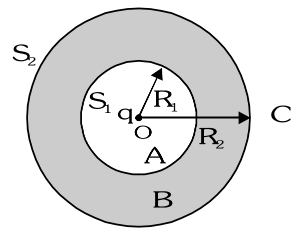
Using the result that $\vec{E}_{res}$ in the conducting material should be zero and using result (iii) We can show that

Case	A	B	C	D	E	F
S_1	Uniform	Nonuniform	Nonuniform	Nonuniform	Uniform	Nonuniform
S_2	Uniform	Uniform	Uniform	Uniform	Nonuniform	Nonuniform

Note : In all cases charge on inner surface S_1 = – q and on outer surface S_2 = q. The distribution of charge on S_1 will not change even if some charges are kept outside the conductor (i.e. outside the surface S_2). But the charge distribution on S_2 may change if some charges(s) is/are kept outside the conductor.

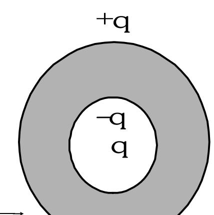
Example

An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q at the centre as shown in figure



(i) Find $\vec{E}$ and V at points A,B and C

(ii) If a point charge Q is kept out side the sphere at a distance 'r' (>>R_2) from centre then find out resultant force on charge Q and charge q.



Solution

At point A : $V_A = \frac{Kq}{OA} + \frac{Kq}{R_2} + \frac{K(-q)}{R_1}$, $\vec{E}_A = \frac{Kq}{OA^3}\overrightarrow{OA}$

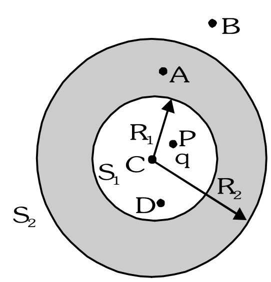
At point B : $V_B = \frac{Kq}{OB} + \frac{K(-q)}{OB} + \frac{Kq}{R_2} = \frac{Kq}{R_2}$, E_B = 0; At point C : $V_C = \frac{Kq}{OC}$, $\vec{E}_C = \frac{Kq}{OC^3}\overrightarrow{OC}$

(ii) Force on point charge Q : $\vec{F}_Q = \frac{KqQ}{r^2}\tilde{r}$ (r = distance of 'Q' from centre 'O')

Force on point charge q: $\vec{F}_q = 0$ (using result (iii) & charge on S_1 uniform)

Example

An uncharged conductor of inner radius R_1 and outer radius R_2 contains a point charge q placed at point P (not at the centre) as shown in figure? Find out the following :



(i) V_C (ii) V_A (iii) V_B (iv) E_A (v) E_B

(vi) force on charge Q if it is placed at B

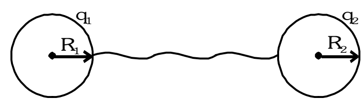
Solution

(i) $V_C = \frac{Kq}{CP} + \frac{K(-q)}{R_1} + \frac{Kq}{R_2}$ (ii) $V_A = \frac{Kq}{R_2}$ (iii) $V_B = \frac{Kq}{CB}$

(iv) E_A = O (point is inside metallic conductor) (v) $E_B = \frac{Kq}{CB^2}\hat{CB}$ (vi) $F_Q = \frac{KQq}{CB^2}\hat{CB}$

(vi) **Sharing of charges :**

Two conducting hollow spherical shells of radii R_1 and R_2 having charges Q_1 and Q_2 respectively and seperated by large distance, are joined by a conducting wire. Let final charges on spheres are q_1 and q_2 respectively.



Potential on both spherical shell become equal after joining, therefore

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \;;\; \frac{q_1}{q_2} = \frac{R_1}{R_2} \;\ldots\text{(i)} \quad \text{and} \quad q_1 + q_2 = Q_1 + Q_2 \quad \ldots\ldots\text{(ii)}$$

from (i) and (ii) $q_1 = \dfrac{(Q_1+Q_2)R_1}{R_1+R_2}$; $q_2 = \dfrac{(Q_1+Q_2)R_2}{R_1+R_2}$

ratio of charges $\dfrac{q_1}{q_2} = \dfrac{R_1}{R_2} ; \dfrac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \dfrac{R_1}{R_2}$

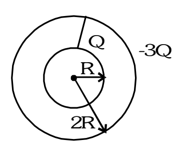
ratio of surface charge densities $\dfrac{\sigma_1}{\sigma_2} = \dfrac{R_2}{R_1}$

Ratio of final charges $\dfrac{q_1}{q_2} = \dfrac{R_1}{R_2}$

Ratio of final surface charge densities. $\dfrac{\sigma_1}{\sigma_2} = \dfrac{R_2}{R_1}$

Example

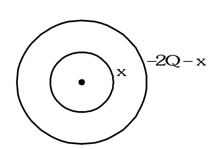
The two conducting spherical shells are joined by a conducting wire and cut after some time when charge stops flowing.Find out the charge on each sphere after that.



Solution

After cutting the wire, the potential of both the shells is equal

Thus, potential of inner shell $V_{in} = \dfrac{Kx}{R} + \dfrac{K(-2Q-x)}{2R} = \dfrac{k(x-2Q)}{2R}$



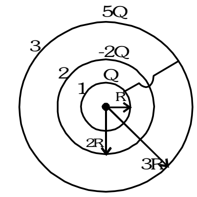
and potential of outer shell $V_{out} = \dfrac{Kx}{2R} + \dfrac{K(-2Q-x)}{2R} = \dfrac{-KQ}{R}$

As $V_{out} = V_{in} \Rightarrow \dfrac{-KR}{R} = \dfrac{K(x-2Q)}{2R} \Rightarrow -2Q = x - 2Q \Rightarrow x = 0$

So charge on inner spherical shell = 0 and outer spherical shell = – 2Q.

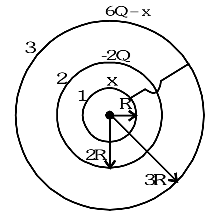
Example

Find charge on each spherical shell after joining the inner most shell and outer most shell by a conducting wire. Also find charges on each surface.



Solution

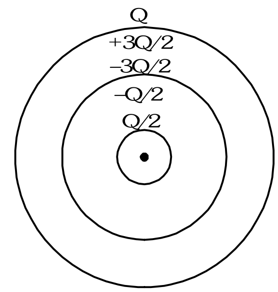
Let the charge on the innermost sphere be x.



Finally potential of shell 1 = Potential of shell 3

$$\frac{Kx}{R}+\frac{K(-2Q)}{2R}+\frac{K(6Q-x)}{3R}=\frac{KQ}{3R}+\frac{k(-2q)}{3R}+\frac{k(5Q)}{3R}$$

$$3x - 3Q + 6Q - x = 4Q \; ; \; 2x = Q \; ; \quad x = \frac{Q}{2}$$



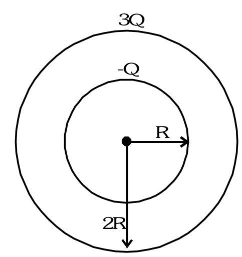
Charge on innermost shell = $\frac{Q}{2}$, charge on outermost shell = $\frac{5Q}{2}$

middle shell = −2Q

Final charge distribution is as shown in figure.

Example

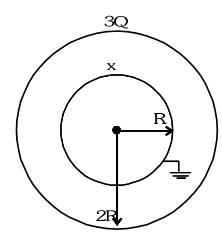
Two conducting hollow spherical shells of radii R and 2R carry charges −Q and 3Q respectively. How much charge will flow into the earth if inner shell is grounded ?



Solution

When inner shell is grounded to the Earth then the potential of inner shell will bcome zero because potential of the Earth is taken to be zero.

$$\frac{Kx}{R}+\frac{K3Q}{2R}=0$$



$x = \frac{-3Q}{2}$, the charge that has increased

$= \frac{-3Q}{2} - (-Q) = \frac{-Q}{2}$ hence charge flows into the Earth = $\frac{Q}{2}$

Example

An isolated conducting sphere of charge Q and radius R is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time what is the amount of heat loss ?

Solution

When two conducting spheres of equal radius are connected charge is equally distributed on them.

So we can say that heat loss of system

$$\Delta H = U_i - U_f = \left(\frac{Q^2}{8\pi\varepsilon_0 R}-0\right)-\left(\frac{Q^2/4}{8\pi\varepsilon_0 R}+\frac{Q^2/4}{8\pi\varepsilon_0 R}\right)=\frac{Q^2}{16\pi\varepsilon_0 R}$$

SOME WORKED OUT EXAMPLES

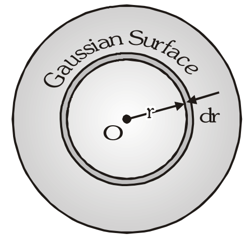
Example#1

For a spherically symmetrical charge distribution, electric field at a distance r from the centre of sphere is $\vec{E} = kr^7\tilde{r}$, where k is a constant. What will be the volume charge density at a distance r from the centre of sphere ?

(A) $\rho = 9k\varepsilon_0 r^6$ (B) $\rho = 5k\varepsilon_0 r^3$ (C) $\rho = 3k\varepsilon_0 r^4$ (D) $\rho = 9k\varepsilon_0 r^0$

Solution **Ans. (A)**

By using Gauss law $\oint \vec{E}\cdot d\vec{S} = \dfrac{q}{\epsilon_0} \Rightarrow (E)(4\pi r^2) = \dfrac{\int \rho(4\pi r^2 dr)}{\epsilon_0}$

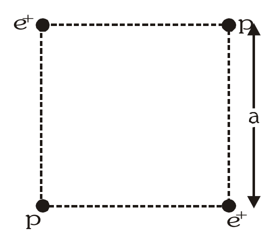


(Note : Check dimensionally that $\rho \propto r^6$)

$$(kr^7)(4\pi r^2) = \frac{\int \rho(4\pi r^2 dr)}{\epsilon_0} \Rightarrow k\varepsilon_0 r^9 = \int \rho r^2 dr$$

Example#2

Two positrons (e^+) and two protons (p) are kept on four corners of a square of side a as shown in figure. The mass of proton is much larger than the mass of positron. Let q denotes the charge on the proton as well as the positron then the kinetic energies of one of the positrons and one of the protons respectively after a very long time will be–

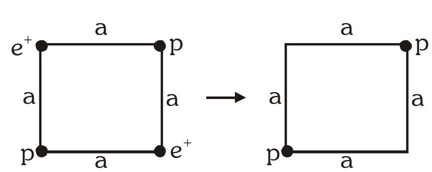


(A) $\dfrac{q^2}{4\pi\epsilon_0 a}\left(1+\dfrac{1}{2\sqrt{2}}\right), \dfrac{q^2}{4\pi\epsilon_0 a}\left(1+\dfrac{1}{2\sqrt{2}}\right)$ (B) $\dfrac{q^2}{2\pi\epsilon_0 a}, \dfrac{q^2}{4\sqrt{2}\pi\epsilon_0 a}$

(C) $\dfrac{q^2}{4\pi\epsilon_0 a}, \dfrac{q^2}{4\pi\epsilon_0 a}$ (D) $\dfrac{q^2}{2\pi\epsilon_0 a}\left(1+\dfrac{1}{4\sqrt{2}}\right), \dfrac{q^2}{8\sqrt{2}\pi\epsilon_0 a}$

Solution **Ans. (D)**

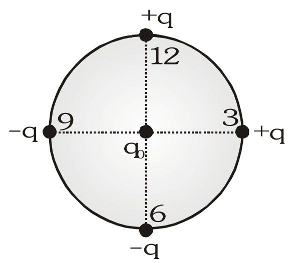
As mass of proton >>> mass of positron so initial acceleration of positron is much larger than proton. Therefore positron reach far away in very short time as compare to proton.



$$2K_{e^+} = \left(\frac{4kq^2}{a}+\frac{2kq^2}{a\sqrt{2}}\right)-\frac{kq^2}{a\sqrt{2}} \Rightarrow K_{e^+} = \frac{q^2}{2\pi\epsilon_0 a}\left(1+\frac{1}{4\sqrt{2}}\right) \text{ and } 2K_p = \frac{kq^2}{a\sqrt{2}} - 0 \Rightarrow K_p = \frac{q^2}{8\sqrt{2}\pi\epsilon_0 a}$$

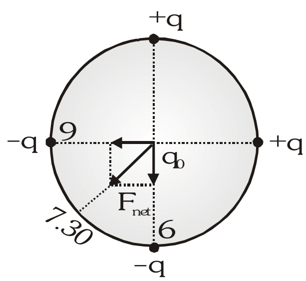
Example#3

Four charges are placed at the circumference of a dial clock as shown in figure. If the clock has only hour hand, then the resultant force on a charge q_0 placed at the centre, points in the direction which shows the time as :–



(A) 1:30 (B) 7:30 (C) 4:30 (D) 10:30

Solution **Ans. (B)**

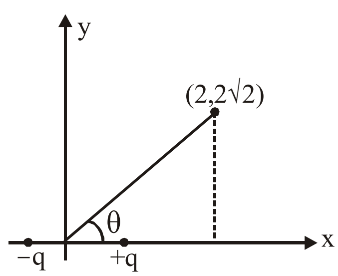


Example#4

A small electric dipole is placed at origin with its dipole moment directed along positive x-axis. The direction of electric field at point (2, 2√2,0) is

(A) along z-axis (B) along y-axis (C) along negative y-axis (D) along negative z-axis

Solution **Ans. (B)**



$\tan\theta = \frac{y}{x} = \sqrt{2}; \cot\theta = \frac{1}{\sqrt{2}}$ Also $\tan\alpha = \frac{\tan\theta}{2} = \frac{1}{\sqrt{2}} = \cot\theta \Rightarrow \theta + \alpha = 90$ i.e., $\vec{E}$ is along positive y-axis.

Example#5

Uniform electric field of magnitude 100 V/m in space is directed along the line y= 3 + x. Find the potential difference between point A (3, 1) & B (1,3).

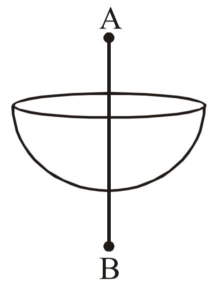
(A) 100 V (B) 200√2V (C) 200 V (D) zero

Solution **Ans. (D)**

Slope of line AB = $\frac{3-1}{1-3} = -1$ which is perpendicular to direction of electric field.

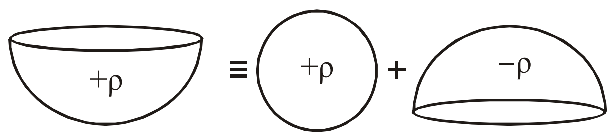
Example#6

The diagram shows a uniformly charged hemisphere of radius R. It has volume charge density ρ. If the electric field at a point 2R distance above its centre is E then what is the electric field at the point which is 2R below its centre?



(A) $\dfrac{\rho R}{6\varepsilon_0}+E$ (B) $\dfrac{\rho R}{12\varepsilon_0}-E$ (C) $\dfrac{-\rho R}{6\varepsilon_0}+E$ (D) $\dfrac{\rho R}{24\varepsilon_0}+E$

Solution **Ans. (B)**



Apply principle of superposition

Electric field due to a uniformly charged sphere = $\dfrac{\rho R}{12\varepsilon_0}$; $E_{\text{resultant}} = \dfrac{\rho R}{12\,\epsilon_0}-E$

Example#7

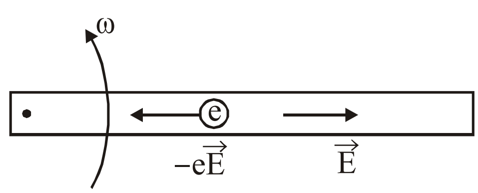
A metallic rod of length l rotates at angular velocity ω about an axis passing through one end and perpendiuclar to the rod. If mass of electron is m and its charge is −e then the magnitude of potential difference between its two ends is

(A) $\dfrac{m\omega^2\ell^2}{(2e)}$ (B) $\dfrac{m\omega^2\ell^2}{e}$ (C) $\dfrac{m\omega^2\ell}{e}$ (D) None of these

Solution **Ans. (A)**

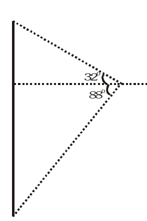
When rod rotates the centripetal acceleration of electron comes from electric field $E=\dfrac{mr\omega^2}{e}$

Thus, $\Delta V = -\int \vec{E}.dr = -\int_0^{\ell}\dfrac{mr\omega^2}{e}dr = \dfrac{m\omega^2\ell^2}{2e}$



Example#8

Consider a finite charged rod. Electric field at Point P (shown) makes an angle θ with horizontal dotted line then angle θ is :-



(A) 60 (B) 28 (C) 44 (D) information insufficient

Solution. **Ans. B**

Required angle = $\dfrac{\theta_2-\theta_1}{2} = \dfrac{88^0-32^0}{2}=\dfrac{56^0}{2} = 28^0$

Example#9

The electric potential in a region is given by the relation V(x) = 4 + 5x². If a dipole is placed at position (–1,0) with dipole moment $\vec{p}$ pointing along positive Y-direction, then

(A) Net force on the dipole is zero.

(B) Net torque on the dipole is zero

(C) Net torque on the dipole is not zero and it is in clockwise direction

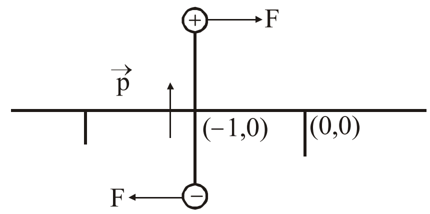
(D) Net torque on the dipole is not zero and it is in anticlockwise direction

Solution **Ans. (AC)**

$V(x) = 4 + 5x^2 \Rightarrow \vec{E} = 10x\hat{i}$

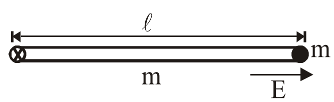
$\therefore$ Net force will be zero and torque not zero

and rotation will be along clockwise direction



Example#10 to 12

A thin homogeneous rod of mass m and length ℓ is free to rotate in vertical plane about a horizontal axle pivoted at one end of the rod. A small ball of mass m and charge q is attached to the opposite end of this rod. The whole system is positioned in a constant horizontal electric field of magnitude $E=\dfrac{mg}{2q}$. The rod is released from shown position from rest.



10. What is the angular acceleration of the rod at the instant of releasing the rod?

(A) $\dfrac{8g}{9\ell}$ (B) $\dfrac{3g}{2\ell}$ (C) $\dfrac{9g}{8\ell}$ (D) $\dfrac{2g}{9\ell}$

11. What is the acceleration of the small ball at the instant of releasing the rod?

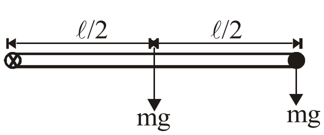
(A) $\dfrac{8g}{9}$ (B) $\dfrac{9g}{8}$ (C) $\dfrac{7g}{8}$ (D) $\dfrac{8g}{7}$

12. What is the speed of ball when rod becomes vertical?

(A) $\sqrt{\dfrac{2g\ell}{3}}$ (B) $\sqrt{\dfrac{3g\ell}{4}}$ (C) $\sqrt{\dfrac{3g\ell}{2}}$ (D) $\sqrt{\dfrac{4g\ell}{3}}$

Solution

10. **Ans. (C)**



By taking torque about hinge $I\alpha = mg\left(\dfrac{\ell}{2}\right)+mg(\ell)$ when $I=\dfrac{m\ell^2}{3}+m\ell^2 \Rightarrow \alpha=\dfrac{9g}{8\ell}$

11. Ans. (B)

Acceleration of ball = $\alpha\ell = \left(\frac{9g}{8\ell}\right)\ell = \frac{9}{8}g$

12. Ans. (C)

From work energy theorem $\frac{1}{2}I\omega^2 = mg\left(\frac{\ell}{2}\right) + mg\ell - qE\ell$

$$\frac{1}{2}\left(\frac{4}{3}m\ell^2\right)\omega^2 = \frac{3}{2}mg\ell - \frac{mg\ell}{2} \Rightarrow \frac{2}{3}m\ell^2\omega^2 = mg\ell \Rightarrow \omega = \sqrt{\frac{3g}{2\ell}}$$

Speed of ball = $\omega\ell = \sqrt{\frac{3g\ell}{2}}$

Example#13

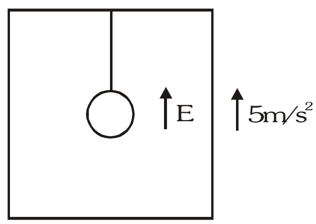
A simple pendulum is suspended in a lift which is going up with an acceleration of 5 m/s^2 . An electric field of magnitude 5 N/C and directed vertically upward is also present in the lift . The charge of the bob is 1 μC and mass is 1 mg . Taking g = π^2 and length of the simple pendulum 1m, find the time period of the simple pendulum (in sec).

Solution **Ans. 2**

$$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

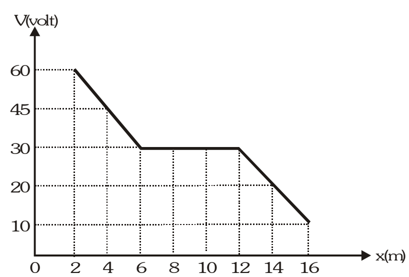
$$g_{eff} = g - \frac{qE}{M} + 5 = 15 - \frac{1\times5\times10^{-6}}{1\times10^{-6}}$$

$g_{eff} = 10 = \pi^2$
T = 2 sec



Example#14

The variation of potential with distance x from a fixed point is shown in figure. Find the magnitude of the electric field (in V/m) at x =13m.



Solution **Ans. 5**

$$E = -\frac{dV}{dx} = \frac{20}{4} = 5 \text{ volt/meter}$$

Example#15

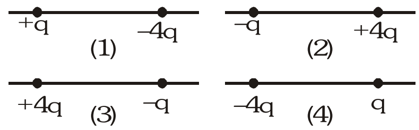
The energy density u is plotted against the distance r from the centre of a spherical charge distribution on a log-log scale. Find the magnitude of slope of obtianed straight line.

Solution. **Ans. 2**

$$u = \frac{1}{2}\epsilon_0E^2 = \frac{1}{2}\epsilon_0\left(\frac{q}{4\pi\epsilon_0 r}\right)^2 = \frac{q^2}{32\pi^2\epsilon_0 r^2} \Rightarrow \log u = \log\left(\frac{q^2}{32\pi^2\epsilon_0 r^2}\right) = \log k - 2\log r$$

Example#16

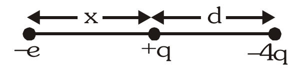
The figure shows four situations in which charges as indicated (q>0) are fixed on an axis. How many situations is there a point to the left of the charges where an electron would be in equilibrium?



Solution Ans. 2

For (1)

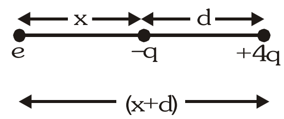
Let the electron be held at a distance x from +q charge.



For equilibrium $\dfrac{q(-e)}{4\pi \in_0 x^2} = \dfrac{(-e)(-4q)}{4\pi \in_0 (x+d)^2}$

We can find value of x for which $F_{net} = 0$ which means that electron will be in equilibrium.

For (2) :



For equilibrium $\dfrac{(-e)(-q)}{4\pi \in_0 x^2} = \dfrac{(-e)4q}{4\pi \in_0 (x+d)^2}$

We can find value of x for which $F_{net} = 0$ which means that electron will be in equilibrium.

In case (3) and (4) the electron will not remain at rest, since it experiences a net non–zero force.

OR

Equilibrium is always found near the smaller charge

Example#17

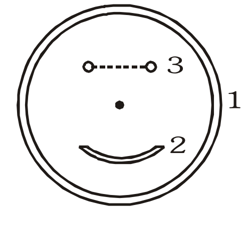
An electric field is given by $\vec{E} = \left(y\hat{i} + x\hat{j}\right)\dfrac{N}{C}$. Find the work done (in J) in moving a 1C charge from $\vec{r}_A = \left(2\hat{i} + 2\hat{j}\right)$ m to $\vec{r}_B = \left(4\hat{i} + \hat{j}\right)$m .

Solution Ans. 0

A= (2,2) and B = (4, 1); $W_{A\to B} = q\left(V_B - V_A\right) = q\int_A^B dV = -\int_A^B q\vec{E}.d\vec{r}$

$$= q\int_A^B \left(y\hat{i} + x\hat{j}\right).\left(dx\hat{i} + dy\hat{j}\right) = q\int_A^B \left(ydx + xdy\right) = -q\int_{(2,2)}^{(4,1)} d\left(xy\right) = -q\left[xy\right]_{(2,2)}^{(4,1)} = -q\left[4-4\right] = 0$$

Example#18



The arrangement shown consists of three elements.

(i) A thin rod of charge –3.0 μC that forms a full circle of radius 6.0 cm.

(ii) A second thin rod of charge 2.0 μC that forms a circular arc of radius 4.0 cm and concentric with the full circle, subtending an angle of 90 at the centre of the full circle.

(iii) An electric dipole with a dipole moment that is perpendicular to a radial line and has magnitude 1.28 10^{-21}C-m.

Find the net electric potential in volts at the centre.

Solution Ans. (0)

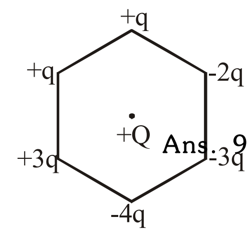
Potential due to dipole at the centre of the circle is zero.

Potentials due to charge on circle = V_1 = $\dfrac{K.(-3\times 10^{-6})}{6\times 10^{-2}}$

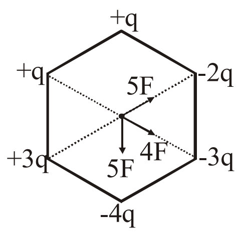
Potential due to arc V_2 = $\dfrac{K.(2\times 10^{-6})}{4\times 10^{-2}}$ Net potential = V_1 + V_2= 0

Example#19

Six charges are kept at the vertices of a regular hexagon as shown in the figure. If magnitude of force applied by +Q on +q charge is F, then net electric force on the +Q is nF. Find the value of n.



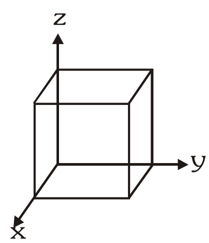
Solution Ans. 9



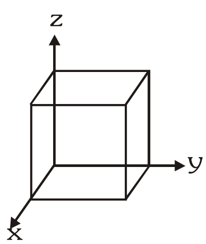
$F_{net} = 9F$

Example#20

Electric field in a region is given by $\vec{E} = -4x\tilde{i} + 6y\tilde{j}$. The charge enclosed in the cube of side 1m oriented as shown in the diagram is given by $\alpha\in_0$. Find the value of α.



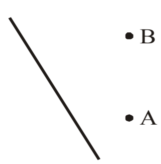
Solution Ans. 2



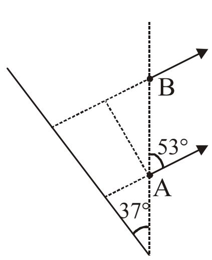
ϕ = (6y) Area − (4x) Area = 6 1 $(1)^2$ − 4 1 $(1)^2$ = 2 therefore $\frac{q}{\varepsilon_0} = 2 \Rightarrow q = 2\varepsilon_0$

Example#21

An infinite plane of charge with $\sigma = 2 \in_0 \frac{C}{m^2}$ is tilted at a 37 angle to the vertical direction as shown below. Find the potential difference, $V_A - V_B$ in volts, between points A and B at 5 m distance apart. (where B is vertically above A).



Solution Ans. 3



$E = \frac{\sigma}{2\varepsilon_0} = 1$ N/C $\therefore V_B - V_A = -\int \vec{E}.d\vec{\ell} = \left(\frac{\sigma}{2 \in_0}\right)(5\cos 53°) = 3V$