UNIT # 01 (PART – I)

BASIC MATHEMATICS USED IN PHYSICS, UNIT & DIMENSIONS AND VECTORS

EXERCISE –I

1. Enclosed area : $A = \pi r^2$

so $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

Here r = 8 cm, $\frac{dr}{dt} = 5$ cm/s

$\Rightarrow \frac{dA}{dt} = (2\pi)(8)(5) = 80\pi\ cm^2/s$

2. Slope $\frac{dy}{dx} = 3x^2 - 6x - 9$

if tangent is parallel to the x–axis then $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0$ $\quad\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + +x - 3 = 0$ $\quad\Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = 3$ or $x = -1$ $\quad\Rightarrow y = -20$ or $y = 12$

3. $\because p = t\ell nt$

$\therefore F = \frac{dp}{dt} = \frac{d}{dt}(t\ell nt) = (1)\ell nt + (t)\left(\frac{1}{t}\right) = 1 + \ell nt$

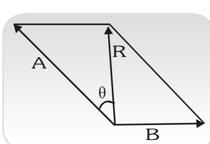
$F = 0 \Rightarrow 1 + \ell nt = 0 \Rightarrow \ell nt = -1 \Rightarrow t = e^{-1} = \frac{1}{e}$

4. Let side of cube be x then $\frac{dx}{dt} = 3$ cm/s

$\because V = x^3 \quad \therefore \frac{dV}{dt} = 3x^2\frac{dx}{dt} = 3 \quad 10^2 \quad 3 = 900\ cm^3/s$

5. Check $\vec{A}.\vec{B} = 0$

6. Let forces be A and B and B < A then A + B = 16



$A\cos\theta = R = 8$ and $A\sin\theta = B$

$\Rightarrow A^2 = 8^2 + B^2$ $\quad\Rightarrow A^2 - B^2 = 64$

$\Rightarrow (A-B)(A+B) = 64$ $\quad\Rightarrow A - B = 4$

$\Rightarrow A = 10N\ \&\ B = 6N$

7. $\sqrt{(0.5)^2 + (-0.8)^2 + c^2} = 1$

$\Rightarrow 0.25 + 0.64 + c^2 = 1$

$\Rightarrow c^2 = 0.11 \quad \Rightarrow c = \pm\sqrt{0.11}$

8. Resultant $= \sqrt{3^2 + 4^2 + 12^2} = \sqrt{5^2 + 12^2} = 13N$

9. Required unit vector

$= \frac{\vec{A} + \vec{B}}{|\vec{A} + \vec{B}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + 2^2}} = \frac{1}{7}(3i + 6\hat{j} - 2\hat{k})$

11. For zero resultant, sum of any two forces ≥ remaining force

13. $\vec{R} = \vec{P} + \vec{Q},\ \vec{R}' = \vec{P} + 2\vec{Q}$

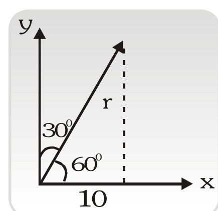
$\because \vec{R}' \cdot \vec{P} = 0 \ \therefore (\vec{P} + 2\vec{Q}) \cdot \vec{P} = 0 \Rightarrow P^2 + 2\vec{Q}.\vec{P} = 0$

$R^2 = P^2 + Q^2 + 2\vec{P}.\vec{Q} = P^2 + Q^2 - P^2 = Q^2 \Rightarrow R = Q$

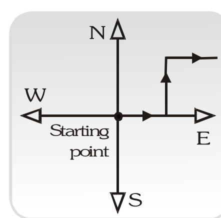
14. $\vec{a} = \vec{c} + \overrightarrow{RP}$ and $\vec{b} = \vec{c} + \overrightarrow{RQ}$ but $\overrightarrow{RP} = -\overrightarrow{RQ}$

$\Rightarrow \vec{a} + \vec{b} = 2\vec{c} + \overrightarrow{RP} + \overrightarrow{RQ} \Rightarrow \vec{a} + \vec{b} = 2\vec{c}$

15. $\cos 60^0 = \frac{10}{r} \Rightarrow r = \frac{10}{1/2} = 20$ units



16.



17. $\hat{v} = \frac{(4-1)\hat{i} + (2+2)\hat{j} + (3-3)\hat{k}}{\sqrt{(4-1)^2 + (2+2)^2 + (3-3)^2}} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$

$\vec{v} = (10)\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right) = 6\hat{i} + 8\hat{j}$

19. Use $R^2 = A^2 + B^2 + 2AB\cos\theta$ or see options

20. Displacement $= \sqrt{12^2 + 5^2 + 6^2}$

$= \sqrt{144 + 25 + 36} = \sqrt{205} = 14.31$ m

21. Required angle $= \frac{2\pi}{12} = \frac{360}{12} = 30^0$

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23. $\because\ |\vec{A}\times\vec{B}|=\sqrt{3}\ \vec{A}\cdot\vec{B}\ \therefore\ AB\sin\theta=\sqrt{3}AB\cos\theta$

$\Rightarrow \tan\theta=\sqrt{3}\ \Rightarrow\ \theta=60$

$|\vec{A}+\vec{B}|=\sqrt{A^2+B^2+2AB\cos 60^\circ}$

$=\sqrt{A^2+B^2+2AB\left(\frac{1}{2}\right)}=\sqrt{A^2+B^2+AB}$

24. $\because\ \vec{A}.\vec{B}$ =ABcosθ

$\therefore$ Projection of $\vec{A}$ on $\vec{B}=A\cos\theta=\frac{\vec{A}.\vec{B}}{B}=\vec{A}.\tilde{B}$

25. $\because \vec{P}+\vec{Q}=\vec{R}\qquad \therefore \vec{Q}=\vec{R}-\vec{P}$

$\Rightarrow Q^2 = R^2 + P^2 - 2RP\cos\theta_1$

$\Rightarrow \cos\theta_1=\frac{1}{2}\ \Rightarrow\ \theta_1=\frac{\pi}{3}$

Now $\because\ \vec{P}+\vec{Q}+\vec{R}=\vec{0}\ \therefore \vec{P}+\vec{R}=-\vec{Q}$

$\Rightarrow P^2 + R^2 + 2PR\cos\theta_2 = Q^2$

$\Rightarrow \cos\theta_2=-\frac{1}{2}\ \Rightarrow \theta_2=\frac{2\pi}{3}$

26. Resultant $=\sqrt{(x^2+y^2)}$

$=\sqrt{(x+y)^2+(x-y)^2+2(x+y)(x-y)\cos\theta}$

$\Rightarrow x^2 + y^2 = 2(x^2+y^2) + 2\ (x^2-y^2)\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2}\left(\frac{x^2+y^2}{y^2-x^2}\right)$

28. Projection on x–y plane $=\sqrt{3^2+1^2}=\sqrt{10}$

29. Velocity of one ball $\vec{v}_1=\tilde{i}+\sqrt{3}\tilde{j}$

Veocity of second ball $\vec{v}_2=2\tilde{i}+2\tilde{j}$

Angle between their path :

$\cos\theta=\frac{\vec{v}_1.\vec{v}_2}{v_1v_2}=\frac{2+2\sqrt{3}}{(2)(2\sqrt{2})}=\frac{1+\sqrt{3}}{2\sqrt{2}}\Rightarrow\theta=15$

31. $|\vec{e}_1-\vec{e}_2|=\sqrt{1^2+1^2-2(1)(1)\cos\theta}=2\sin\frac{\theta}{2}$

33. In a clockwise system $\tilde{k}\times\tilde{j}=\tilde{i}$

34. $\vec{v}=\vec{\omega}\times\vec{r}=\begin{vmatrix}\tilde{i}&\tilde{j}&\tilde{k}\\1&-2&2\\0&4&-3\end{vmatrix}$

$=\tilde{i}\ (6-8)-\tilde{j}\ (-3)+\tilde{k}\,(4)=-2\tilde{i}+3\tilde{j}+4\tilde{k}$

$|\vec{v}|=\sqrt{2^2+3^2+4^2}=\sqrt{4+9+16}=\sqrt{29}$

36. $0.5\ \frac{g}{cc}=0.5\frac{10^{-3}kg}{10^{-6}m^3}=500\frac{kg}{m^3}$

37. $\because\ n_1u_1=n_2u_2\ \therefore n_2=\left(\frac{u_1}{u_2}\right)\ n_1=\left(\frac{M_1L_1T_1^{-2}}{M_2L_2T_2^{-2}}\right)\ (n_1)$

$\Rightarrow n_2=\left[\frac{(1000g)(100cm)(1)^{-2}}{(10g)(10cm)(0.1)^{-2}}\right][1]=10$

1N = 10 Unit of force in new system

So unit of force in new system = 0.1N

OR

As $[F] = MLT^{-2}$ so

unit of force = (10g) (10 cm($0.1s)^{-2}$

$= (10^{-2}\ kg)\ (10^{-1}m)\ 100(s)^{-2}$

$= 0.1\ (kg)\ (m)\ (s)^{-2} = 0.1\ N$

38. αt^2 must be dimensionless

39. Tension→Force but surface tension→Force / length

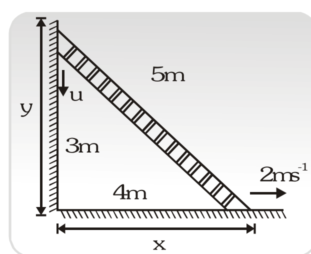
41. $F \equiv MLT^{-2}$, $A \equiv LT^{-2} \Rightarrow L = AT^2$

42. $[a]=\left[\frac{v}{t}\right]=\frac{LT^{-1}}{T}=LT^{-2}$, $[C] = [t] = T$

$[b] = [vt] = LT^{-1}T = L$

EXERCISE –II

1. At any instant $x^2 + y^2 = 5^2$



Differentiating w.r.t. time $2x\frac{2x}{dt}+2y\frac{dy}{dt}=0$

Here $\frac{dx}{dt}=2$, $\frac{dy}{dt}=u\ \Rightarrow u=\frac{8}{3}$ m/s

2. $x^2 + 4 = y \Rightarrow 2xdx = dy$ but $dy = 2dx$

So $2xdx = 2dx \Rightarrow 2x = 2 \Rightarrow x=1 \Rightarrow y = 1^2 + 4 = 5$

3. $I = \frac{2}{5}MR^2 = \frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2 = \frac{8}{15}\pi\rho R^5$

$$\frac{dI}{dt} = \left(\frac{8}{15}\pi\rho\right)(5R^4)\frac{dR}{dt} = \left(\frac{8\pi}{15}\right)\left(\frac{M}{4/3\ \pi R^3}\right)(5R^4)$$

$$\frac{dR}{dt} = 2MR\left(\frac{dR}{dt}\right) = (2)(1)\ (1)\ (2) = 4\ kg\ m^2s^{-1}$$

4. $1\ \text{notwen} = G\frac{1kg \times 1kg}{(1km)^2} = \left(\frac{6.67\times 10^{-11}}{10^6}\right)\left(\frac{kg^2}{m^2}\right)$

$= 6.67\quad 10^{-17}$ newton

5. Length $\ell = \frac{v^2}{a}$, time $t = \frac{v}{a}$

$\Rightarrow$ ratio of unit of length $= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

and ratio of unit of time = 1/3

7. $\because\ n_1u_1 = n_2u_2$

$$\therefore\ \frac{n_2}{n_1} = \frac{u_1}{u_2} = \frac{M_1^{-1}L_1^3T_1^{-2}}{M_2^{-1}L_2^3T_2^{-2}} = \frac{M^{-1}L^3T^{-2}}{M^{-1}(2L)^3\ T^{-2}} = \frac{1}{8}$$

9. $[k] = [\rho]\ [v^2] = [ML^{-3}]\ [L^2T^{-2}] = ML^{-1}T^{-2}$

$= \frac{\text{Force}}{\text{Area}}$ = Modulus of elasticity

10. $\left[\frac{b}{c}\right] = \left[\frac{1/t}{1/x}\right] = \left[\frac{x}{t}\right]$ = wave velocity

11. $P + \frac{aT^2}{V} = (RT+b)\ V^{-c}$

$\Rightarrow P = (RT + b)\ V^{-C} - aT^2\,V^{-1} = AV^m - BV^n$

$\Rightarrow m = -c$ and $n = -1$

12. $\because\ \frac{A}{B} = m \qquad \therefore B = \frac{A}{m}$

$\Rightarrow [B] = \left[\frac{MLT^{-2}}{ML^{-1}}\right] = [L^2T^{-2}]$ = latent heat

14. $C \equiv LT^{-1}$, $G \equiv M^{-1}L^3T^{-2}$, $h \equiv M^1L^2T^{-1} \Rightarrow M = \sqrt{\frac{hc}{G}}$

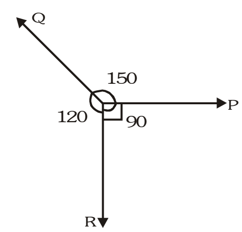
15. $\because 1$ cal $= 4.2$ J $\therefore 1$ cal$=4.2$ kgm^2s^{-2}

$$= \left(4.2\ \alpha^{-1}\beta^{-2}\gamma^2\right)(\alpha\ kg)(\beta m)^2(\gamma s)^{-2}$$

16. Angle between $\vec{a}$ and $\vec{b}$,

$$\cos\theta = \frac{\vec{a}.\vec{b}}{ab} = \frac{-x+2+x+1}{\sqrt{1+4+1}\sqrt{x^2+1^2+(x+1)^2}}$$

$$= \frac{3}{\sqrt{6\left[x^2+1+(x+1)^2\right]}} > 0$$

17. 

$$\frac{P}{\sin 120°} = \frac{Q}{\sin 90°} = \frac{R}{\sin 150°}$$

$$\Rightarrow \frac{P}{\sqrt{3}/2} = \frac{Q}{1} = \frac{R}{1/2}$$

$$\Rightarrow \frac{2P}{\sqrt{3}} = \frac{Q}{1} = \frac{2R}{1} = k\ \text{(constant)}$$

$$\Rightarrow P : Q : R = \frac{\sqrt{3}k}{2} : k : \frac{k}{2} = \sqrt{3} : 2 : 1$$

18. $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{a^2+b^2+c^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})}$

$\because\ \vec{a}\cdot(\vec{b}+\vec{c}) = 0$, $\vec{b}\cdot(\vec{c}+\vec{a}) = 0$ & $\vec{c}\cdot(\vec{a}+\vec{b}) = 0$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = 0$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{a^2+b^2+c^2} = \sqrt{3^2+4^2+5^2} = 5\sqrt{2}$$

19. $\because |\hat{a}+\hat{b}+\hat{c}| = 1$

$\therefore |\hat{a}|^2 + |\hat{b}|^2 + |\hat{c}|^2 + 2(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}) = 1$

$\Rightarrow 1 + 1 + 1 + 2(\cos\theta_1 + \cos\theta_2 + \cos\theta_3) = 1$

$\Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = -1$

20. $\because\ |\vec{a}+\vec{b}| = 1\ \therefore\ 2\cos\frac{\theta}{2} = 1$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2} \Rightarrow \frac{\theta}{2} = \frac{\pi}{3}$$

$|\tilde{a}-\tilde{b}| = 2\sin\frac{\theta}{2} = 2\ \frac{\sqrt{3}}{2} = \sqrt{3}$

21. $a_x = 2a_y$, $\cos\gamma = \frac{a_z}{a} = \cos 135 = -\frac{1}{\sqrt{2}}$

$\Rightarrow a_z = -\frac{a}{\sqrt{2}} = -\frac{5\sqrt{2}}{\sqrt{2}} = -5$

Now $a_x^2 + a_y^2 + a_z^2 = 50 \Rightarrow 4a_y^2 + a_y^2 + 25 = 50$

$\Rightarrow a_y^2 = 5 \Rightarrow a_y = \pm\sqrt{5} \Rightarrow a_x = \pm 2\sqrt{5}$

23. $\because \vec{C} = \vec{A} + \vec{B}\ \therefore C^2 = A^2 + B^2 + 2AB\cos\theta$

If $C^2 < A^2 + B^2$ then $\cos\theta < 0$.

Therefore $\theta > 90^0$

25. Area of triangle $= \frac{1}{2}(\vec{a}\times\vec{b}) = \frac{1}{2}(b\times\vec{c}) = \frac{1}{2}(\vec{c}\times\vec{a})$

26. $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$= (4\tilde{i} - 5\tilde{j} + 5\tilde{k}) + (-5\tilde{i} + 8\tilde{j} + 6\tilde{k}) + (-3\tilde{i} + 4\tilde{j} - 7\tilde{k})$

$+ (12\tilde{i} - 3\tilde{j} - 2\tilde{k}) = 4\tilde{j} + 2\tilde{k}$

$\Rightarrow$ motion will be in y–z plane

28. $\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \tilde{i} & \tilde{j} & \tilde{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = 14\tilde{i} - 38\tilde{j} + 16\tilde{k}$

29. $\vec{r} = a\cos\omega t\tilde{i} + a\sin\omega t\tilde{j}$

velocity $= \vec{v} = \frac{d\vec{r}}{dt} = -\ a\omega\sin\omega t\tilde{i} + a\omega\cos\omega t\tilde{j}$

Acceleration$= \frac{d^2\vec{r}}{dt^2} = -a\omega^2\cos\omega t\ \tilde{i} - a\omega^2\sin\omega t\tilde{j} = -\ \omega^2\vec{r}$

30. $|\vec{A}\cdot\vec{B}| = AB|\cos\theta| = 8$, $|\vec{A}\times\vec{B}| = AB|\sin\theta| = 8\sqrt{3}$

$\Rightarrow |\tan\theta| = \frac{8\sqrt{3}}{8} = \sqrt{3} \Rightarrow \theta = 60, 120$

31. Displacement $d\vec{r} = dx\tilde{i} + dy\tilde{j}$

but 3y + kx =5 so 3dy + kdx = 0

$\Rightarrow d\vec{r} = dx\tilde{i} - \frac{k}{3}dx\tilde{j} = \left(\tilde{i} - \frac{k}{3}\tilde{j}\right)dx$

Work done is zero if $\vec{F}.d\vec{r} = 0$

$(2\tilde{i} + 3\tilde{j})\cdot\left(\tilde{i} - \frac{k}{3}\tilde{j}\right)dx = 0 \Rightarrow (2-k)dx = 0 \Rightarrow k=2$

32. Here α = 45 so inclination of AC with x–axis is 45 . So unit vector along AC

$= \cos 45°\tilde{i} + \sin 45°\tilde{j} = \frac{\tilde{i}+\tilde{j}}{\sqrt{2}}$

33. $(\vec{a} + 3\vec{b}).(7\vec{a} - 5\vec{b}) = 0$

$\Rightarrow 7a^2 - 15b^2 + 16\ \vec{a}\cdot\vec{b} = 0$ ...(i)

and $(\vec{a} - 4\vec{b})\cdot(7\vec{a} - 2\vec{b}) = 0$

$\Rightarrow 7a^2 + 8b^2 - 30\vec{a}\cdot\vec{b} = 0$ ...(ii)

By adding (i) and (ii)

$\Rightarrow -23b^2 + 46\vec{a}\cdot\vec{b} = 0 \Rightarrow 2\vec{a}\cdot\vec{b} = b^2$

So $7a^2 - 15b^2 + 8b^2 = 0 \Rightarrow a^2 = b^2$

$\Rightarrow 2ab\cos\theta = b^2 \Rightarrow 2\cos\theta = 1$

$\Rightarrow \theta = \cos^{-1}(1/2) = 60$

34. For triangle ABC : $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

Now $\overrightarrow{AB} + \overrightarrow{BC} + 2\overrightarrow{CA}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{CA} = \vec{0} + \overrightarrow{CA} = \overrightarrow{CA}$

EXERCISE –III

True /False

2. If $(\vec{A} + \vec{B})\cdot(\vec{A} - \vec{B}) = 0$ then $|\vec{A}| = |\vec{B}|$

3. Two vectors are always coplanar.

Fill in the blanks

1. $W = \vec{F}.\vec{d} = (10\tilde{i} - 3\tilde{j} + 8\tilde{k}).(10\tilde{i} - 2\tilde{j} + 7\tilde{k} - 6\tilde{i} - 5\tilde{j} + 3\tilde{k})$

$= (10\tilde{i} - 3\tilde{j} + 8\tilde{k})\ \ (4\tilde{i} - 7\tilde{j} + 10\tilde{k})$

$= 40 + 21 + 80 = 141$ J

2. Required vector

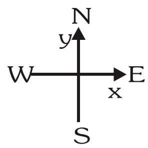
$b\tilde{a} = \left(\sqrt{7^2 + 24^2}\right)\left[\frac{3\tilde{i} + 4\tilde{j}}{\sqrt{3^2 + 4^2}}\right]$

$= (25)\left(\frac{3\tilde{i} + 4\tilde{j}}{5}\right) = 15\tilde{i} + 20\tilde{j}$

3. $\vec{a} \times \vec{b} = \left(x_1\tilde{i} + y_1\tilde{j}\right) \times \left(x_2\tilde{i} + y_2\tilde{j}\right)$

$= x_1y_2\tilde{k} - x_2y_1\tilde{k} = \vec{0} \;\Rightarrow x_1y_2 = x_2y_1$

5. Let unknown displacement be $\vec{s}_3$ then



$2\tilde{i} + 5\left(\cos 37^0\,\tilde{i} - \sin 37^0\,\tilde{j}\right) + \vec{s}_3 = 6\tilde{i} \Rightarrow \vec{s}_3 = 3\tilde{j}$

6. Area $= \dfrac{1}{2}\left|\vec{a} \times \vec{b}\right| = \dfrac{1}{2}\left|\left(\tilde{i} + \tilde{j} + \tilde{k}\right) \times 3\tilde{i}\right|$

$= \dfrac{1}{2}\left|-3\tilde{k} + 3\tilde{j}\right| = \dfrac{1}{2}\left(3\sqrt{2}\right) = \dfrac{3}{\sqrt{2}}$

7. According to question $8\tilde{B} + A\tilde{i} = 2A\tilde{j}$

$\Rightarrow 8\tilde{B} = A\left(2\tilde{j} - \tilde{i}\right) \Rightarrow 8 = A\sqrt{5} \Rightarrow A = \dfrac{8}{\sqrt{5}}$

8. According to question

$(\vec{u} + \vec{v}) \cdot \vec{u} = 0$ and $|\vec{u} + \vec{v}| = \dfrac{v}{2}$

$\Rightarrow u^2 + \vec{u} \cdot \vec{v} = 0$ & $u^2 + v^2 + 2\vec{u} \cdot \vec{v} = \dfrac{v^2}{4}$

$\Rightarrow \dfrac{3}{4}v^2 = u^2 \Rightarrow \cos\theta = -\dfrac{\sqrt{3}}{2} \Rightarrow \theta = 150°$

9. $\left[\dfrac{k_1}{k_2}\right] = \left[\dfrac{1/x}{1/t}\right] = \left[\dfrac{t}{x}\right] = s/m$

10. $T \propto P^a d^b E^c$

$\Rightarrow T = \left(ML^{-1}T^{-2}\right)^a \left(ML^{-3}\right)^b \left(ML^2T^{-2}\right)^c$

$\Rightarrow a + b + c = 0,\ -a - 3b + 2c = 0,\ -2a - 2c = 1$

$\Rightarrow a = \dfrac{-5}{6},\ b = \dfrac{1}{2},\ c = \dfrac{1}{3}$

Comprehension 1

1. $[b] = [V]$

2. $\because \left[\dfrac{a}{V^2}\right] = [P] \quad \therefore [a] = [PV^2]$

3. $[PV] = [RT]$, $[Pb] = [PV] = [RT]$

$\left[\dfrac{a}{V^2}\right] = \left[\dfrac{PV^2}{V^2}\right] = [P] \neq [RT]$ and

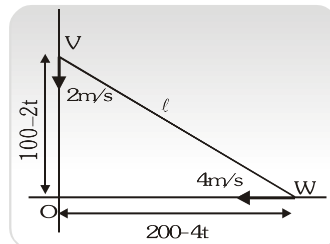
$\left[\dfrac{ab}{V^2}\right] = \left[\dfrac{(PV^2)V}{V^2}\right] = [PV] = [RT]$

4. $\left[\dfrac{ab}{RT}\right] = \left[\dfrac{PV^3}{PV}\right] = \left[V^2\right] = M^0L^6T^0$

5. $[RT] = [PV] = (ML^{-1}T^{-2})(L^3) = ML^2T^{-2} = [\text{Energy}]$

Comprehension 2

1. $\ell = \sqrt{(100 - 2t)^2 + (200 - 4t)^2}$



2. For shortest distance $\dfrac{d\ell}{dt} = 0 \Rightarrow t = 50$ sec

3. $\ell_{min} = \sqrt{(100 - 2 \times 50)^2 + (200 - 4 \times 50)^2} = 0$

Comprehension 3

1. $x = at,\ y = -bt^2 \Rightarrow a^2y + bx^2 = 0$

2. $\dfrac{d\vec{r}}{dt} = a\tilde{i} - 2bt\tilde{j}$ at $t = 0$, $\dfrac{d\vec{r}}{dt} = a\tilde{i}$

3. $\dfrac{d^2\vec{r}}{dt^2} = -2b\tilde{j}$

Comprehension 4

1. Let unit of length, time and mass be L_1, T_1 and M_1 respectively.

According to question

$9.8\ LT^{-2} = 3\ L_1T_1^{-2}$

$\dfrac{1}{2}(272.1)\,(448)^2\ ML^2T^{-2} = 100\ M_1L_1^2T_1^{-2}$

$(272.1)\,(448)\ MLT^{-1} = 10\ M_1L_1T_1$

by solving above equation $L_1 = 153.6\ L$

$= 153.6$ m

2. By solving above equation $T_1 = 6.857T$

$= 6.857$ s

3. By solving above equation $M_1 = 544.2\ M$

$= 544.2$ kg

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EXERCISE –IV(A)

1. $\because$ $A_x = 4$, $A_y = 6$ so $A_x + B_x = 10$ and $A_y + B_y = 9$

(i) $B_x = 10 - 4 = 6\text{m}$ and $B_y = 9 - 6 = 3\text{m}$

(ii) length $= \sqrt{B_x^2 + B_y^2} = \sqrt{36+9} = \sqrt{45}\text{m}$

(iii) $\theta = \tan^{-1}\left(\dfrac{B_y}{B_x}\right) = \tan^{-1}\left(\dfrac{3}{6}\right) = \tan^{-1}\left(\dfrac{1}{2}\right)$

2. (i) Let the angle between $\vec{A}$ and $\vec{B}$ is θ, then

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{\left(2\hat{i}-2\hat{j}-\hat{k}\right)\cdot\left(\hat{i}+\hat{j}\right)}{\left|2\hat{i}+2\hat{j}-\hat{k}\right|\cdot\left|\hat{i}+\hat{j}\right|} = \frac{0}{3\sqrt{2}} = 0$$

$\Rightarrow$ $\theta = 90$

(ii) Resultant

$$\left(\vec{R}\right) = \vec{A}+\vec{B} = \left(2\hat{i}-2\hat{j}-\hat{k}\right)+\left(\hat{i}+\hat{j}\right) = 3\hat{i}-\hat{j}-\hat{k}$$

Projection of resultant on x–axis = 3

(iii) Required vector

$$= \hat{j}-\vec{A} = \hat{j}-\left(2\hat{i}-2\hat{j}-\hat{k}\right) = -2\hat{i}+3\hat{j}+\hat{k}$$

3. (i) Component of $\vec{A}$ along $\vec{B}$ $= \left(\dfrac{\vec{A}.\vec{B}}{B}\right)\hat{B}$

$$= \left(\frac{\vec{A}.\vec{B}}{B}\right)\frac{\vec{B}}{B} = \left[\frac{\left(3\hat{i}+\hat{j}\right).\left(\hat{j}+2\hat{k}\right)}{\sqrt{5}}\right]\frac{\left(\hat{j}+2\hat{k}\right)}{\sqrt{5}} = \frac{1}{5}\left(\hat{j}+2\hat{k}\right)$$

Component of $\vec{A}\perp\vec{B}$

$$= \vec{A}-\left[\frac{\vec{A}\cdot\vec{B}}{\vec{B}}\right]\vec{B} = 3\hat{i}+\hat{j}-\left[\frac{1}{5}\left(\hat{j}+2\hat{k}\right)\right]$$

(ii) Area of the parallelogram

$$= \left|\vec{A}\times\vec{B}\right| = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 0\\ 0 & 1 & 2\end{vmatrix} = \left|2\hat{i}-6\hat{j}+3\hat{k}\right|$$

$$= \sqrt{2^2+(-6)^2+3^2} = 7 \text{ units}$$

(iii) Unit vector perpendicular to both $\vec{A}$ & $\vec{B}$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{\left|\vec{A}\times\vec{B}\right|} = \frac{2\hat{i}-6\hat{j}+3\hat{k}}{7} = \frac{2}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{3}{7}\hat{k}$$

4. Component along the vector $\hat{i}+\hat{j}$

$$= (A\cos\theta)\hat{B} = \frac{(\vec{A}.\vec{B})}{B^2}\vec{B} = \frac{(3\hat{i}+4\hat{j}).(\hat{i}+\hat{j})}{(\sqrt{2})^2}(\hat{i}+\hat{j})$$

$$= \frac{3+4}{2}(\hat{i}+\hat{j}) = \frac{7}{2}(\hat{i}+\hat{j})$$

Component along the vector $\hat{i}-\hat{j}$

$$= (A\cos\theta)\ \hat{B} = \frac{(\vec{A}.\vec{B})}{B^2}\vec{B} = \frac{(3\hat{i}+4\hat{j}).(\hat{i}-\hat{j})(\hat{i}-\hat{j})}{(\sqrt{2})^2}$$

$$= \frac{(3-4)}{2}(\hat{i}-\hat{j}) = -\frac{1}{2}(\hat{i}-\hat{j})$$

6. Let two forces are A and B then

$A + B = P$, $A - B = Q \Rightarrow A = \dfrac{P+Q}{2}$, $B = \dfrac{P-Q}{2}$

Resultant $k = \sqrt{A^2+B^2+2AB\cos\theta}$

$$= \sqrt{\left(\frac{P+Q}{2}\right)^2+\left(\frac{P-Q}{2}\right)^2+2\left(\frac{P+Q}{2}\right)\left(\frac{P-Q}{2}\right)\cos 2\alpha}$$

$$= \sqrt{\frac{P^2}{2}+\frac{Q^2}{2}+\frac{1}{2}(P^2-Q^2)\cos 2\alpha}$$

$$= \sqrt{\frac{P^2}{2}(1+\cos 2\alpha)+\frac{Q^2}{2}(1-\cos 2\alpha)}$$

$$= \sqrt{P^2\cos^2\alpha+Q^2\sin^2\alpha}$$

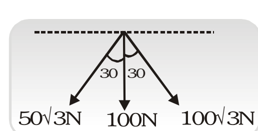
7. $|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$

$$= \sqrt{(10)^2+(6)^2-2(10)(6)\cos 60°} = 2\sqrt{19}$$

$$\tan\alpha = \frac{6\sin 60°}{10-6\cos 60°} = \frac{6\times\sqrt{3}/2}{10-3} = \frac{3\sqrt{3}}{7}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{3\sqrt{3}}{7}\right)$$

8. Resultant force in vertical direction



$= 50\sqrt{3}\cos 30 + 100 + 100\sqrt{3}\cos 30$

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$= 50 \times \dfrac{3}{2} + 100 + 100 \quad \dfrac{3}{2} = 325$ N

Resultant force in horizontal direction

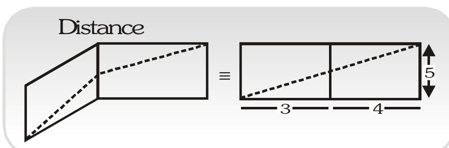
$= 100\sqrt{3}\sin 30 \ - \ 50\sqrt{3}\ \sin 30$

$= 100\dfrac{\sqrt{3}}{2} - \dfrac{50\sqrt{3}}{2} = 25\sqrt{3}$N

so resultant pull $= \sqrt{(325)^2 + (25\sqrt{3})^2}$ = 327.9N

9. $x = at,\ y = -bt^2 = -b\left(\dfrac{x}{a}\right)^2$

11. (i) $|\,\text{displacement}\,| = \sqrt{(3)^2 + (4)^2 + (5)^2} = \sqrt{50}$m



(ii) $L = \sqrt{(7)^2 + (5)^2} = \sqrt{74}$m

12. Let $\vec{c}$ is $= c_x\tilde{i} + c_y\tilde{j}$

then according to question $= \sqrt{c_x^2 + c_y^2} = 5$

$\Rightarrow c_x^2 + c_y^2 = 25$ ....(i)

and $\vec{a}.\vec{c} = 0 \ \Rightarrow 3\ c_x + 4\ c_y = 0$ – ...(ii)

from equation (i) and (ii) $c_x = \pm 4,\ c_y = \mp 3$

14. $\vec{v} = \dfrac{d\vec{r}}{dt} = (6t - 6)\,\hat{i} + (-12t^2)\,\hat{j}$ m/s

$\vec{a} = \dfrac{d\vec{v}}{dt} = (6\hat{i} - 24t\hat{j})$ m/s^2

(i) $\vec{F} = m\vec{a} = 6(6\hat{i} - 24t\hat{j}) = (36\hat{i} - 144t\hat{j})$N

(ii) $\vec{\tau} = \vec{r} \times \vec{F} = \left[(3t^2 - 6t)\hat{i} + (-4t^3)\hat{j}\right] \times \left[36\hat{i} - 144t\hat{j}\right]$

$= [(-144 \quad 3t^2) + (144 \quad 6t^2) + 144\ t^3]\,\hat{k}$

$= (-288\ t^3 + 864\ t^2)\,\hat{k}$

(iii) $\vec{p} = m\vec{v} = 6[(6t - 6)\hat{i} + (-12t^2)\hat{j}]$

$= [36(t-1)\hat{i} - 72t^2\hat{j}]$

(iv) $\vec{L} = \vec{r} \times \vec{p} = \left[(3t^2 - 6t)\,\tilde{i} + (-4t^3\ )\,\tilde{j}\right]$

$\times\left[36(t-1)\,\tilde{i} - 72t^2\tilde{j}\right]$

$= \left[-72t^4 + 288t^3\right]\tilde{k}$

15. $\vec{F}_1 = 5P\hat{j},\ \vec{F}_2 = 4P\hat{i}, \vec{F}_3 = 10P\left(\dfrac{3\hat{i} + 4\hat{j}}{5}\right) = (6\hat{i} + 8\hat{j})P$

$\vec{F}_4 = 15P\dfrac{((-\hat{i} - 3\hat{i}) + (\hat{j} - 4\hat{j}))}{5} = -12P\hat{i} - 9P\hat{j}$

$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 5P\hat{j} + 4P\hat{i} + 6P\hat{i} + 8P\hat{j}$

$-12P\hat{i} - 9P\hat{j} \ = \ -2P\hat{i} + 4P\hat{j}$

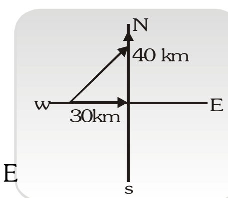
$|\,\vec{F}\,| = P\sqrt{(-2)^2 + (4)^2} = \sqrt{20}P$

$\tan\alpha = \dfrac{4}{-2} \Rightarrow \alpha = \tan^{-1}(-2)$

17. Displacement

$= \sqrt{(30)^2 + (40)^2} = 50$km

$\tan\alpha = \dfrac{40}{30} \Rightarrow \alpha = 53$ N to E



19. Speed $= |\vec{v}| = \sqrt{9 + 25 + 16} = 5\sqrt{2}$m / s

K.E. $= \dfrac{1}{2}mv^2 = \dfrac{1}{2} \times 200 \times 10^{-3} \times 50$ J=5J

20. From graph $\dfrac{dv}{dx} = \dfrac{90 - 50}{40 - 20} = \dfrac{40}{20}\ \dfrac{dv}{dx} = 2$

v (at x = 20) = 50 m/s

$a = v\dfrac{dv}{dx} \Rightarrow a = 50 \quad 2 = 100$ m/s^2

21. (i) $\vec{v} = v_0\tilde{i} + a_0b_0e^{b_0t}\tilde{k}$ (ii) $|\vec{v}| = \sqrt{v_0^2 + a_0^2b_0^2e^{2b_0t}}$

(iii) $\vec{a} = a_0b_0^2e^{b_0t}\tilde{k}$

22. Dimension of α t = $M^0L^0T^0$

$\Rightarrow$ Dimension of α = $M^0L^0T^{-1}$

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Dimension of $\dfrac{v_0}{\alpha} = L^1$

$\Rightarrow$ Dimension of $v_0 = M^0L^1T^{-1}$

23. (i) $c = \dfrac{Q}{m[T_2 - T_1]}$

Dimension of $c = \dfrac{[M^1L^2T^{-2}]}{[M^1L^0T^0][M^0L^0T^0K^1]}$
$= [L^2T^{-2}K^{-1}]$

(ii) $\alpha = \dfrac{\ell_1 - \ell_0}{\ell_0(T_2 - T_1)}$

$\Rightarrow$ Dimension of $\alpha = \dfrac{[M^0L^1T^0]}{[M^0L^1T^0]\,[M^0L^0T^0K^1]}$

(iii) $R = \dfrac{PV}{nT} = \dfrac{\left[M^1L^{-1}T^{-2}\right]\left[L^3\right]}{\left[mol\right]\left[K\right]} = [M^1L^2T^{-2}\ K^{-1}mol^{-1}]$

24. Dimensions of ax = M L T

$\Rightarrow [a] = \dfrac{M^\circ L^\circ T^\circ}{[L]} = L^{-1}$ and $[\phi_0] = [M^1L^2T^{-2}]$

25. $m \propto [v]^k\ [d]^x\ [g]^y$

$[M^1L^0T^0] = [LT^{-1}]^k\ [ML^{-3}]^x\ [LT^{-2}]^y$

$\Rightarrow x = 1,\ k - 3x + y = 0,\ -K - 2y = 0$

$\Rightarrow x = 1,\ y = -3,$ and $K = 6$

26. $R \propto v^a g^b \Rightarrow [L] = [LT^{-1}]^a\ [LT^{-2}]^b$

$\Rightarrow a + b = 1,\ -a - 2b = 0$

$\Rightarrow a = 2,\ b = -1 \ \Rightarrow R \propto \dfrac{v^2}{g}$

27. $[b] = [v] = [L^3]$ dimensions of $\dfrac{a}{RTV} = [M\ L\ T\ ]$

$\Rightarrow [a]$ = dimensions of RTV

$= \left[\dfrac{M^1L^2T^{-2}}{k \times mol}\right]\ [k]\ \ [L^3] = [M^1L^5T^{-2}\ mol^{-1}]$

28. $Y \propto (v)^x(a)^y(F)^z \Rightarrow [M^1L^{-1}T^{-2}] = [LT^{-1}]^x\ [LT^{-2}]^y[MLT^{-2}]^z$
$= [M]^z\ [L]^{x+y+z}\ [T]^{-x-2y-2z}$

$\Rightarrow z = 1,\ x + y + z = -1,\ -x - 2y - 2z = -2$

$\Rightarrow z = 1,\ y = 2,\ x = -4 \Rightarrow Y = F\ a^2v^{-4}$

EXERCISE –IV(B)

1. Surface tension (S)

$= \dfrac{\text{work done}}{\text{Area}} = \dfrac{\text{Energy}}{\text{Area}} = \left[\dfrac{E}{A}\right] = \left[\dfrac{E}{L^2}\right]$

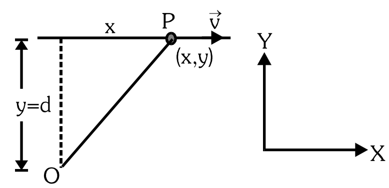
$\because L = vT \quad \therefore S = \dfrac{E}{(vT)^2} = Ev^{-2}\ T^{-2}$

2. Dimension of joule = ML^2T^{-2}

Value of 1 joule in star system

$= (10^{-20})\ (10^{-8})^2(10^{-3})^{-2} = 10^{-30}$ star joule

4. Let $\vec{v} = v\hat{i}$ & $\overrightarrow{OP} = x\hat{i} + y\hat{j} = x\ \hat{i} + d\ \hat{j}$

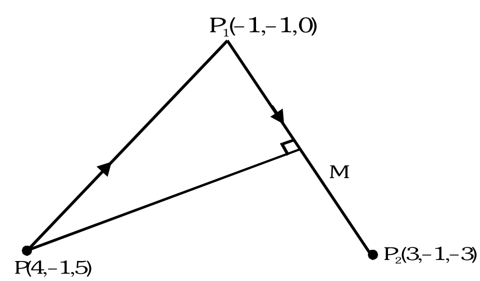


so $\overrightarrow{OP} \times \vec{v} = (x\tilde{i} + d\tilde{j}) \times v\tilde{i} = -dv\tilde{k}$

(d = is constant)

which is independent of position.

5. Vector $\overrightarrow{PP_1} = -5\tilde{i} - 5\tilde{k}$ and $\overrightarrow{P_1P_2} = 4\tilde{i} - 3\tilde{k}$



Let angle between these vectors be θ then

$\cos\theta = \dfrac{\left(-5\tilde{i} - 5\tilde{k}\right)\cdot\left(4\tilde{i} - 3\tilde{k}\right)}{\left(5\sqrt{2}\right)(5)} = -\dfrac{1}{5\sqrt{2}}$

As $PM = PP_1\sin\theta$

so $PM = \left(5\sqrt{2}\right)\left(\dfrac{7}{5\sqrt{2}}\right) = 7$ m

Therefore $t = \dfrac{7\ m}{2\ m/s} = 3.5$ s

6. $\tan\alpha = \dfrac{30}{10\sqrt{3}} \Rightarrow \alpha = 60$

$\tan\beta = \dfrac{20}{20} \Rightarrow \beta = 45 \ \Rightarrow \alpha - \beta = 15$

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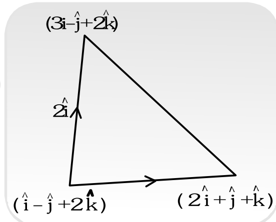
7. Area of triangle



$= \frac{1}{2}|\vec{A}\times\vec{B}| = \frac{1}{2}\left(4\tilde{k}+2\tilde{j}\right)$

$\vec{A} = \left(\tilde{j}+2\hat{k}\right)$

$|A| = \sqrt{5}m^2$

8. By law of reflection $\angle i = \angle r$

$\frac{2-x}{x} = \frac{4}{2} \Rightarrow 4-2x = x \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$

$\vec{A} = \frac{2\tilde{i}}{3}+2\tilde{j}$; $\vec{B} = \frac{4}{3}\tilde{i}-4\tilde{j}$; $\vec{C} = 2\tilde{i}-2\tilde{j}$

$\Rightarrow |A| = \frac{2}{3}\sqrt{10}$, $|B| = \frac{4}{3}\sqrt{10}$, $|C| = 2\sqrt{2}$

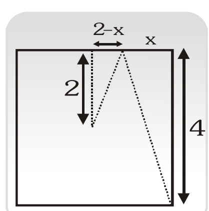
9. $M_1 = \int_0^{L/2} A\left(\rho_0 + kx\right)dx = \left(\rho_0\frac{L}{2}+\frac{kL^2}{8}\right)A$

$M_2 = \int_{L/2}^{L} A\left(2x^2 dx\right) = \frac{2}{3}\left[L^3 - \frac{L^3}{8}\right] = \frac{14L^3}{24}A$

$M_{total} = M_1 + M_2 = \left(\frac{14L^3}{24}+\rho_0\frac{L}{2}+\frac{kL^2}{8}\right)A$

10. $\because$ m = k tan θ

$\therefore$ dm = k sec²θ dθ



$\Rightarrow \frac{dm}{m} = \frac{k\sec^2\theta}{k\tan\theta}\ d\theta$

$\Rightarrow \frac{dm}{m} = \frac{d\theta}{\sin\theta\cos\theta} = \frac{2d\theta}{\sin 2\theta}$

$\Rightarrow$ % error is minimum when sin2θ

has maximum value hence $2\theta = \frac{\pi}{2}$ or $\theta = 45$

11. $\vec{v} = \frac{d\vec{r}}{dt} = 1.2\tilde{i}+1.8t\tilde{j}-1.8t^2\tilde{k}$

At t= 4s, $\vec{v} = 1.2\tilde{i}+7.2\tilde{j}-28.8\tilde{k}$

$P = \vec{F}.\vec{v} = \left(60\tilde{i}-25\tilde{j}-40\tilde{k}\right)\cdot\left(1.2\tilde{i}+7.2\tilde{j}-28.8\tilde{k}\right)$

= 1044W

12. $\vec{v} = A\tilde{i}+\left(3Bt^2-2\right)\tilde{j}+\left(2ct-4\right)\tilde{k}$

At t=2, $A\tilde{i}+(12B-2)\tilde{j}+(4c-4)\tilde{k} = 3\tilde{i}+22\tilde{j}$

Thus, A =3, B=2, C=1

$\therefore\ \vec{v} = 3\tilde{i}+\left(6t^2-2\right)\tilde{j}+\left(2t-4\right)\tilde{k}$

At t=4, $\vec{v} = 3\tilde{i}+(96-2)\tilde{j}+(8-4)\tilde{k} = 3\tilde{i}+94\tilde{j}+4\tilde{k}$

13. $\vec{a} = 5\cos t\tilde{i} - 3\sin t\tilde{j}$

$\Rightarrow \int d\vec{v} = \int 5\cos t\ dt\tilde{i} - \int 3\sin t dt\tilde{j}$

Therefore $\int_{-3}^{v} dv_x = \int_0^t 5\cos t dt \Rightarrow v_x = 5\sin t - 3$

$\frac{dx}{dt} = (5\sin t - 3) \Rightarrow \int_{-3}^{x} dx = \int_0^t \left(5\sin t - 3\right)dt$

$x+3 = 5 - 5\cos t - 3t \Rightarrow x = 2 - 5\cos t - 3t$

Similarly,

$\int_2^v dv_y = -\int_0^t 3\sin t dt$

$\Rightarrow v_y - 2 = 3(\cos t - 1) \Rightarrow v_y = 3\cos t - 1$

$\Rightarrow \int_2^y dy = \int_0^t \left(3\cos t - 1\right)dt$

$\Rightarrow y-2 = 3\sin t - t \Rightarrow y = 2 + 3\sin t - t$

Thus, $\vec{v} = \left(5\sin t - 3\right)\tilde{i} \mid \left(3\cos t - 1\right)\tilde{j}$

and $\vec{s} = \left(2-5\cos t - 3t\right)\tilde{i}+\left(2+3\sin t - t\right)\tilde{j}$

14. $\vec{r} = t\tilde{i}+\frac{t^2}{2}\tilde{j}+t\tilde{k}$

(i) $\vec{v} = \frac{d\vec{r}}{dt} = \tilde{i}+t\tilde{j}+\tilde{k}$ (iii) speed $|\vec{v}| = \sqrt{t^2+2}$

(iii) $\vec{a} = \frac{d\vec{v}}{dt} = \tilde{j}$ (iv) $|\vec{a}| = 1$

(v) $\vec{a}_T = \left(\vec{a}\cdot\vec{v}\right)\vec{v} = \left(\left[\tilde{j}\frac{\left(\tilde{i}+t\tilde{j}+\tilde{k}\right)}{\sqrt{t^2+2}}\right]\right)\frac{\left(\tilde{i}+t\tilde{j}+\tilde{k}\right)}{\sqrt{t^2+2}}$

$\vec{a}_T = \left(\frac{t}{\sqrt{t^2+2}}\right)\vec{v} = \frac{t\left(\tilde{i}+t\tilde{j}+\tilde{k}\right)}{(t^2+2)}$; $|\vec{a}_T| = \frac{t}{\sqrt{t^2+2}}$

As $a_N^2 + a_T^2 = a^2$

so $a_N = \sqrt{a^2 - a_T^2} = \frac{\sqrt{2}}{\sqrt{t^2+2}}$

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EXERCISE –V(A)

1. The dimensions of torque and work are$[ML^2T^{-2}]$

2. As we know that formula of velocity is

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow v^2 = \frac{1}{\mu_0\varepsilon_0} = [LT^{-1}]^2$$

$$\therefore \frac{1}{\mu_0\varepsilon_0} = [L^2T^{-2}]$$

3. Planck's constant (in terms of unit)

(h) = J-s = $[ML^2T^{-2}][T]$ =$[ML^2T^{-1}]$

Momentum (p)

= kg-ms^{-1} = $[M][L][T^{-1}]$=$[MLT^{-1}]$

4. By Newton's formula

$$\eta = \frac{\text{dimensions of force}}{\text{dimensions of area} \quad \text{dimensions of velocity gradient}}$$

$$= \frac{[MLT^{-2}]}{[L^2][T^{-1}]} = [ML^{-1}T^{-1}]$$

5. $\vec{A}\times\vec{B} = \vec{B}\times\vec{A}$

This is only possible if the value of both vectors $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ is zero. This occurs when the angle between $\vec{A}$ and $\vec{B}$ is π.

7. Moment of inertia and moment of a force do not have same dimensions.

8. Dimensions of inductance, i.e. henry are $[ML^2/Q^2]$

10. $F = qvB \Rightarrow B = \frac{(MLT^{-2})}{[C][LT^{-1}]} = MC^{-1}T^{-1}$

11. $F = \frac{1}{4\pi \in_0}\frac{q_1q_2}{r^2} \Rightarrow [MLT^{-2}] = \left[\frac{1}{\in_0}\right] \frac{A^2T^2}{L^2}$

$\Rightarrow [\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

EXERCISE –V(B)

Fill in the blanks :

1. $E = h\nu \Rightarrow [h] = \left[\frac{E}{\nu}\right] = \left[\frac{ML^2T^{-2}}{1/T}\right] = [ML^2T^{-1}]$

2. [X] = [capacitance] = $[M^{-1}L^{-2}T^2Q^2]$ and
[Z] = [Magnetic induction] = $[MT^{-1}Q^{-1}]$
Therefore

$$[Y] = \frac{[X]}{[Z]^2} = \frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2} = [M^{-3}L^{-2}T^4Q^4]$$

3. Electrical conductivity

$$\sigma = \frac{J}{E} \Rightarrow [\sigma] = \left[\frac{I/A}{F/q}\right] = \left[\frac{I/A}{F/It}\right] = \left[\frac{I^2t}{FA}\right]$$

$$= \left[\frac{A^2T}{(MLT^{-2})(L^2)}\right] = [M^{-1}L^{-3}T^3A^2]$$

4. $[P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [PV^2] = [ML^{-1}T^{-2}][L^6] = [ML^5T^{-2}]$

Single Choice

6. $\frac{1}{2}\in_0 E^2$ = $[M^{-1} L^{-3} T^4I^2]$ $[M^2L^2T^{-6}I^{-2}]$= $[M^1L^{-1}T^{-2}]$

7. $\in_0 L\frac{\Delta V}{\Delta t}$ =$[M^{-1}L^{-3}T^4I^2]$ [L] $\frac{[M^1L^2T^{-3}I^{-1}]}{[T^1]}$ = [I]

8. Dimension formula of Boltzman constant

$k \rightarrow [M^1L^2T^{-2}\theta^{-1}]$

$$\frac{\alpha[L^1]}{[M^1L^2T^{-2}\theta^{-1}][\theta^{-1}]} = M^0L^0T^0\theta^0$$

α = $[M^1L^1T^{-2}]$; $\beta = \frac{[M^1L^1T^{-2}]}{[M^1L^{-1}T^{-2}]} = [L^2]$

9. (i) Dipole moment = Charge Length

Dipole moment = $[I^1T^1]$ $[L^1]$ = $[L^1I^1T^1]$

(ii) Electric flux $= \frac{q}{\in_0} = \frac{[I^1T^1]}{[M^{-1}L^{-3}T^4I^2]}$ = $[M^1L^3T^{-3}I^{-1}]$

(iii) Electric field$=\frac{F}{q}$

Electric field $= \frac{[M^1L^1T^{-2}]}{[I^1T^1]}$ = $[M^1L^1T^{-3}I^{-1}]$

MCQ'S

13. $F = \frac{1}{4\pi \epsilon_0}\frac{q_1 q_2}{r^2} \Rightarrow MLT^{-2} = \left[\frac{1}{\epsilon_0}\right]\frac{A^2T^2}{L^2}$

$\Rightarrow \left[\frac{1}{\epsilon_0}\right] = M^1L^3A^{-2}T^{-4} \Rightarrow [\epsilon_0] = M^{-1}L^{-3}A^{+2}T^4$

$\frac{F}{L} = \frac{\mu_0}{4\pi}\frac{i_1 i_2}{r}$

$[ML^{-2}] = \left[\frac{\mu_0 A^2}{L}\right] \Rightarrow [\mu_0] = MLA^{-2}T^{-2}$

14. $e = L\frac{di}{dt} \Rightarrow \frac{\text{volt} - \text{sec}}{\text{Ampere}} = L(\text{Henery})$

$\frac{L}{R}$ = Time constant; [L]= ohm–sec

$\phi = LI \Rightarrow \frac{\text{weber}}{\text{Ampere}} = L(\text{Henery})$

$E = \frac{1}{2} \Rightarrow LI^2 = \frac{\text{Joule}}{(\text{Ampere})^2} = Z(\text{Henery})$

17. Match the Column

(A) $F = \frac{GM_eM_s}{R^2}$

GM_eM_s = F L^2

= Work Metre

= Coulomb Volt Metre

= ML^2T^{-2} Metre = (Kg) (Metre)^3 (S)$^{-2}$

(B) $\frac{3}{2}RT$ = Kinetic energy

$\frac{3RT}{M} = v^2 \Rightarrow (\text{Metre})^2(S)^{-2}$

$\frac{1}{2}QV$ = Energy

$\Rightarrow \frac{QV}{M} = \frac{\text{Energy}}{m} = \frac{(\text{farad})(\text{volt})^2}{\text{kg}}$

(C) $\frac{F^2}{q^2B^2} = \frac{q^2 \in^2}{q^2B^2} \Rightarrow \left(\frac{\in}{B}\right)^2 = v^2 \Rightarrow (r,s)$

(D) $\frac{GMe}{R} = \frac{\text{Work done}}{\text{Mass}} \Rightarrow (\text{Velocity})^2 \Rightarrow (r,s)$

18. Match List I with List II and select the correct answer using the codes given below the lists :

[IIT-JEE 2013]

List I	List II
P. Boltzmann constant	1. $[ML^2T^{-1}]$
Q. Coefficient of viscosity	2. $[ML^{-1}T^{-1}]$
R. Planck constant	3. $[MLT^{-3}K^{-1}]$
S. Thermal conductivity	4. $[ML^2T^{-2}K^{-1}]$

Codes :

	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

Ans. (C)

(P) Boltzmann constant

$$\frac{\text{Energy}}{\text{Temperature}} = \frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$$

(Q) Coefficient of viscosity (η) :

$$\eta = \frac{F\Delta x}{A\ \Delta V},\ [\eta] = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]} = [ML^{-1}T^{-1}]$$

(R) Plank constant (h) :

$$E = hv;\ [h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

(S) Thermal conductivity

$$K = \frac{\Delta Q\ \ell}{\Delta t\ (A\theta)}$$

$$[K] = \frac{[ML^2T^{-2}][L]}{[T][L^2][K]} = MLT^{-3}K^{-1}$$