I.C.S.E 10, 2018

Burra Vishal Mathews CS21BTECH11010

(3)

Question: : 3(a): If (x+2) and (x+3) are factors For given system of equations, of $x^3 + ax + b$, find the values of 'a' and 'b'.

Solution: According to the question:

x+2 and x+3 are factors of x^3+ax+b . Then, -2 and -3 are solutions of the equation

$$x^3 + ax + b = 0 \tag{1}$$

On substituting x = -2 int the equation (1)

$$\implies (-2)^3 + a(-2) + b = 0$$

$$\implies 2a - b = -8 \tag{2}$$

On substituting x = -3 in the equation (1)

$$\implies (-3)^3 + a(-3) + b = 0$$
$$\implies 3a - b = -27$$

The system of equations,

$$2a - b = -8$$

$$3a - b = -27$$

can be re-written in matrix form as:

$$\begin{pmatrix} 2 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -8 \\ -27 \end{pmatrix}$$

Let, ith row is a matrix be represented as R_i . The system of equations,

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

if represented as : $\left(\begin{array}{cc|c} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{array} \right)$

Solution of such system of equations is $x_1 \& y_1$ where matrix is reduced using row transformation to;

$$\left(\begin{array}{cc|c} 1 & 0 & x_1 \\ 0 & 1 & y_1 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 2 & -1 & -8 \\ 3 & -1 & -27 \end{array}\right)$$

Transformation 1: $R_1 \rightarrow R_2 - R_1$

$$\implies \begin{pmatrix} 1 & 0 & -19 \\ 3 & -1 & -27 \end{pmatrix}$$
Transformation 2: $R_2 \rightarrow 3R_1 - R_2$

$$\implies \begin{pmatrix} 1 & 0 & -19 \\ 0 & 1 & -30 \end{pmatrix}$$

$$\therefore \text{ The value of } a = -19 \text{ and value of } b = -30.$$

Using values of a and b, equation (1) can be rewritten as:

$$x^3 - 19x - 30 = 0 (4)$$

This can be verified by plotting the graph of the equation

$$y = x^3 - 19x - 30 (5)$$

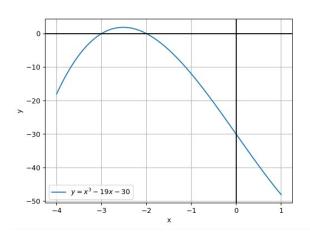


Fig. 1. Graph of equation. (5) intersects X-axis at x = -3 &

Output of python code which checks if both 'a' and 'b' are solutions of the equations (2) and (3)

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Enter the value of 'a': -19
Enter the value of 'b': -30
Given values of 'a' and 'b' are solutions of equations 2a-b-8 & 3a-b-27
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Fig. 2. Output of given code.