I.C.S.E 10, 2018

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CS21BTECH11010

Question: : 3(a) : If $(x+2)$ and $(x+3)$ are factors of x^3+ax+b, find the values of $'a'$ and $'b'$.

Solution: According to the question :

$x+2$ and $x+3$ are factors of x^3+ax+b.
Then, -2 and -3 are solutions of the equation

$$x^3+ax+b=0 \tag{1}$$

On substituting $x=-2$ int the equation (1)

$$\implies (-2)^3+a(-2)+b=0$$

$$\implies 2a-b=-8 \tag{2}$$

On substituting $x=-3$ in the equation (1)

$$\implies (-3)^3+a(-3)+b=0$$

$$\implies 3a-b=-27 \tag{3}$$

The system of equations ,

$$2a-b=-8$$

$$3a-b=-27$$

can be re-written in matrix form as :

$$\begin{pmatrix}2 & -1\\ 3 & -1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}-8\\ -27\end{pmatrix}$$

Let,

$$\mathbf{A}=\begin{pmatrix}2 & -1\\ 3 & -1\end{pmatrix},\mathbf{X}=\begin{pmatrix}a\\ b\end{pmatrix},\mathbf{B}=\begin{pmatrix}-8\\ -27\end{pmatrix}$$

$$\implies \mathbf{AX}=\mathbf{B}$$

Pre-multiplying both sides with $\mathbf{A}^{-1}$:

$$\implies \mathbf{A}^{-1}\mathbf{AX}=\mathbf{A}^{-1}\mathbf{B}$$
$$\implies \mathbf{X}=\mathbf{A}^{-1}\mathbf{B}$$
$$\mathbf{A}=\begin{pmatrix}2 & -1\\ 3 & -1\end{pmatrix}$$
$$\implies |A|=(2\times-1)-(3\times-1)$$
$$\implies |A|=1$$

Since

$$\mathbf{A}^{-1}=\frac{1}{|A|}adj(A)$$

$$\implies \mathbf{A}^{-1}=\begin{pmatrix}-1 & 1\\ -3 & 2\end{pmatrix}$$
$$\implies \mathbf{X}=\begin{pmatrix}-1 & 1\\ -3 & 2\end{pmatrix}\begin{pmatrix}-8\\ -27\end{pmatrix}$$
$$\implies \mathbf{X}=\begin{pmatrix}-19\\ -30\end{pmatrix}$$
$$\implies \begin{pmatrix}a\\ b\end{pmatrix}=\begin{pmatrix}-19\\ -30\end{pmatrix}$$
$$\implies a=-19 \ \ \& \ \ b=-30$$

$\therefore$ The value of $a=-19$ and value of $b=-30$.

Using values of a and b, equation (1) can be re-written as :

$$x^3-19x-30=0 \tag{4}$$

This can be verified by plotting the graph of the equation

$$y=x^3-19x-30 \tag{5}$$

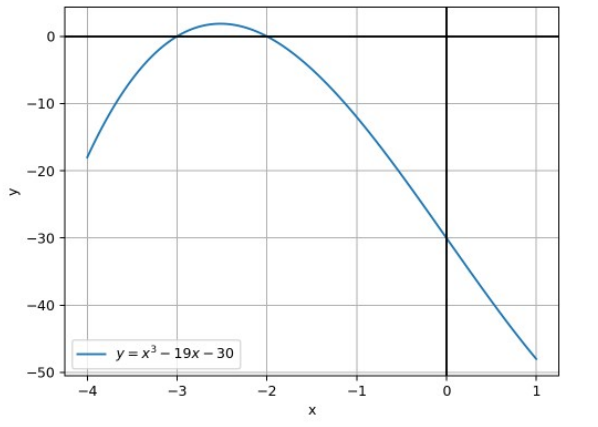

Fig. 1. Graph of equation. 5 intersects X-axis at $x=-3$ & $x=-2$