Assignment

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CS21BTECH11010

CONTENTS

***Abstract*—This manual provides solutions to the Assignment of Random Numbers**

I. UNIFORM RANDOM NUMBERS

Let U be a uniform random variable between 0 and 1.

I.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/1−URN/code/q1.1.c
wget https://github.com/VishalBurra27/
    randomvar/blob/main/1−URN/code/coeffs.
    h
```

Download the above files and execute the following commands

```
$ gcc q1.1.c −lm
$ ./a.out
```

I.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1}$$

Solution: The following code plots Fig. I.2

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/1−URN/code/q1.2.
    py
```

Download the above files and execute the following commands to produce Fig.I.2

```
$ python3 q1.2.py
```

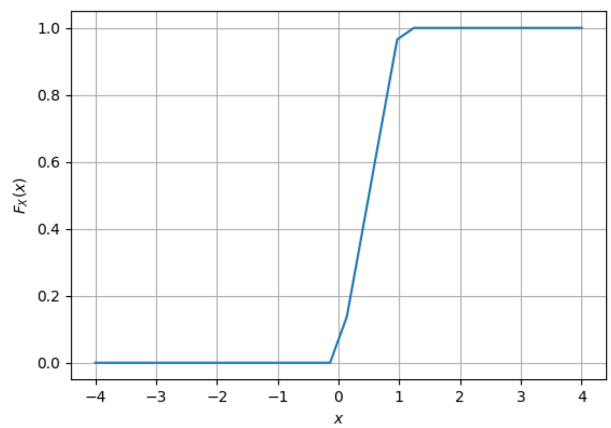

Fig. I.2. The CDF of U

I.3 Find a theoretical expression for $F_U(x)$.
Solution: Given U is a uniform Random Variable

$$p_U(x) = 1 \text{ for} \tag{2}$$

$$F_U(x) = \int_{-\infty}^{\infty} p_U(x)dx \tag{3}$$

$$\boxed{\implies F_U(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x < 1 \\ 1 & x \geq 1 \end{cases}} \tag{4}$$

I.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{5}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{6}$$

Write a C program to find the mean and variance of U.
Solution: Download the following files and execute the C program.

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/1-URN/code/q1.4.c
wget https://github.com/VishalBurra27/
    randomvar/blob/main/2-CLT/code/coeffs.
    h
```

Download the above files and execute the following commands

```
$ gcc q1.4.c -lm
$ ./a.out
```

I.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{7}$$

Solution:

$$\text{var}\left[U\right] = E\left[U - E\left[U\right]\right]^2 \tag{8}$$

$$\implies \text{var}\left[U\right] = E\left[U^2\right] - E\left[U\right]^2 \tag{9}$$

$$E\left[U\right] = \int_{-\infty}^{\infty} x dF_U(x) \tag{10}$$

$$E\left[U\right] = \int_0^1 x \tag{11}$$

$$\implies \boxed{E\left[U\right] = \frac{1}{2}} \tag{12}$$

$$E\left[U^2\right] = \int_{-\infty}^{\infty} x^2 dF_U(x) \tag{13}$$

$$E\left[U^2\right] = \int_0^1 x^2 dF_U(x) \tag{14}$$

$$\implies E\left[U^2\right] = \frac{1}{3} \tag{15}$$

$$\implies \boxed{\text{var}\left[U\right] = \frac{1}{12} = 0.0833} \tag{16}$$

II. Central Limit Theorem

II.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{17}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

Solution: Download the following files and execute the C program.

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/2-CLT/code/q2.1.c
wget https://github.com/VishalBurra27/
    randomvar/blob/main/2-CLT/code/coeffs.
    h
```

Download the above files and execute the following commands

```
$ gcc q2.1.c -lm
$ ./a.out
```

II.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?

Solution: The CDF of X is plotted in Fig. II.2 using the code below

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/2-CLT/code/q2.2.py
```

Download the above files and execute the following commands to produce Fig.II.2

```
$ python3 q2.2.py
```

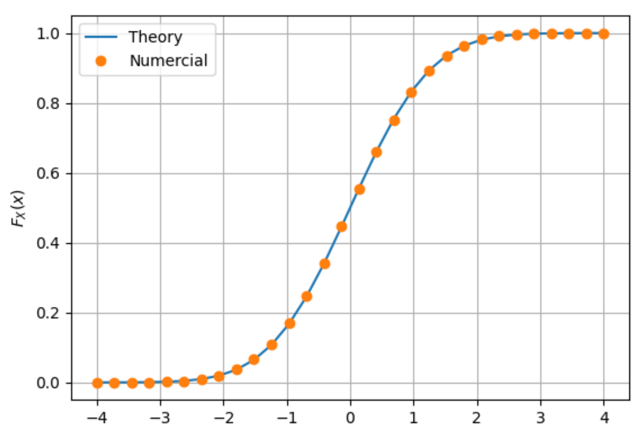

Fig. II.2. The CDF of X

Some of the properties of CDF

a) $\lim_{x\to\infty} F_X(x) = 1$
b) $F_X(x)$ is non decreasing function.
c) Symmetric about one point.

II.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx}F_X(x) \tag{18}$$

What properties does the PDF have?
Solution: The PDF of X is plotted in Fig. II.3 using the code below

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/2-CLT/code/q2.3.py
```

Download the above files and execute the following commands to produce Fig.II.3

```
$ python3 q2.3.py
```

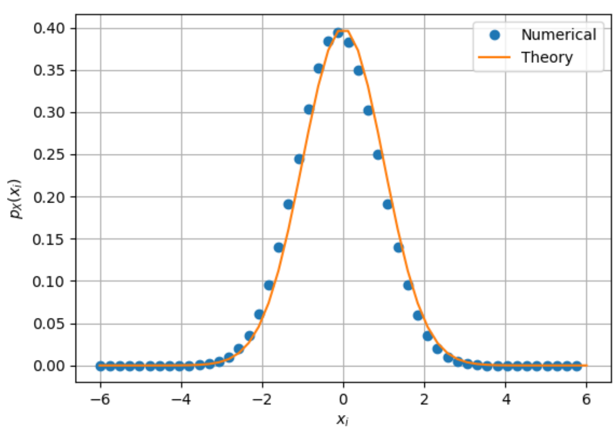

Fig. II.3. The PDF of X

Some of the properties of the PDF:

a) Symmetric about $x = \mu$ in this case
b) Decreasing function for $x > \mu$ and increasing for $x < \mu$ and attains maximum at $x = \mu$
c) Area under the curve is unity.

II.4 Find the mean and variance of X by writing a C program.
Solution: Download the following files and execute the C program.

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/2-CLT/code/q2.4.c
wget https://github.com/VishalBurra27/
    randomvar/blob/main/2-CLT/code/coeffs.
    h
```

Download the above files and execute the following commands

```
$ gcc q2.4.c -lm
$ ./a.out
```

II.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{19}$$

repeat the above exercise theoretically.
Solution:

1) CDF is given by

$$F_X(x) = \int_{-\infty}^{\infty} p_X(x)dx \tag{20}$$

$$\boxed{F_X(x) = 1} \tag{21}$$

2) Mean is given by

$$E(x) = \int_{-\infty}^{\infty} xp_X(x)dx \tag{22}$$

$$\implies \boxed{E(x) = 0} \tag{23}$$

3) Variance is given by

$$\text{var}[U] = E(U^2) - (E(U))^2 \tag{24}$$

$$\implies \boxed{\text{var}[U] = \sqrt{2}} \tag{25}$$

III. From Uniform to Other

III.1 Generate samples of

$$V = -2\ln(1 - U) \tag{26}$$

and plot its CDF.
Solution: Download the following files and execute the C program.

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/3-FUO/code/q3.1.c
wget https://github.com/VishalBurra27/
    randomvar/blob/main/3-FUO/code/coeffs.
    h
```

Download the above files and execute the following commands

```
$ gcc q3.1.c -lm
$ ./a.out
```

The CDF of V is plotted in Fig. III.1 using the code below

```
wget https://github.com/VishalBurra27/
    randomvar/blob/main/3-FUO/code/q3.1.
    py
```

Download the above files and execute the following commands to produce Fig.III.1

```
$ python3 q3.1.py
```

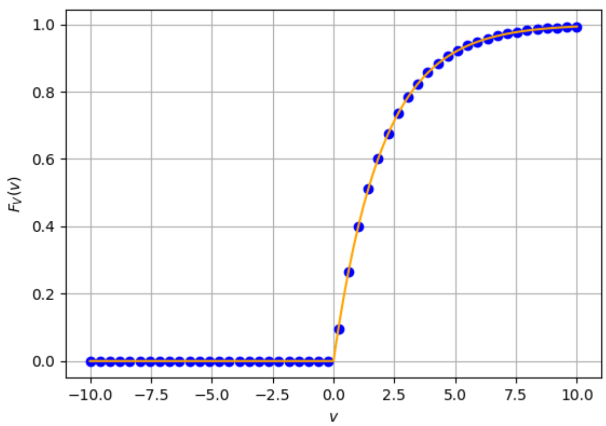

Fig. III.1. The CDF of X

III.2 Find a theoretical expression for $F_V(x)$.
Solution: If Y = g(X), we know that $F_Y(y) = F_X(g^{-1}(y))$, here

$$V = -2\ln(1-U) \tag{27}$$

$$1-U = e^{\frac{-V}{2}} \tag{28}$$

$$U = 1 - e^{\frac{-V}{2}} \tag{29}$$

$$F_V(x) = F_U(1-e^{\frac{-x}{2}}) \tag{30}$$

$$\implies F_V(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{\frac{-x}{2}} & x \geq 0 \end{cases} \tag{31}$$