Tutorial-2

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(1)
```
void fun(int n){
    int j=1, i=0;
    while (i<n){
        i=i+j;
        j++;
    }
}
```

for
j=1 i=1;
j=2 i=1+2;
j=3 i=1+2+3;
} m levels

for (i)

$\because 1+2+3+\ldots<n$

$\because 1+2+3+m<n$

$\because \frac{m(m+1)}{2}<n$

$m \approx \sqrt{n}$

$\because$ by summation method

$\Rightarrow \sum_{i=1}^{m} 1 \Rightarrow 1+1+\ldots \sqrt{n}$ times

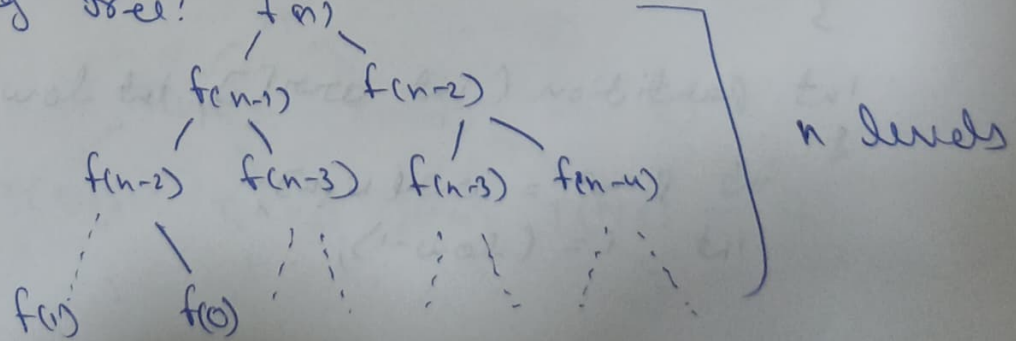
$\therefore \boxed{T(n)=\sqrt{n}}$

(2) for fibonacci series

$f(n) = f(n-1) + f(n-2)$

$f(0)=0 \quad f(1)=1$

by forming tree:



∴ At every function call we get 2 func^{n} calls

∴ for n levels:

we have = 2×2 .... n times

∴ $\boxed{T(n) = 2^n}$

Maximum Space:

considering recursive stack:

no. of calls maximum = n

for each call we have space complexity O(1)

∴ $\boxed{T(n) = O(n)}$

without considering recursive stack:

or each call we have time complexity O(1)

$\boxed{T(n) = O(1)}$

Sol^{n} → (3) (i) nlogn

quick sort

```
void quicksort(int arr[], int low, int high)
{
    if (low < high)
    {
        int pi = position(arr, low, high);
        quicksort(arr, low, pi-1);
        quicksort(arr, pi+1, high);
    }
}
int Partition(int arr[], int low, int high)
{
    int pivot = arr[high];
    int i = (low-1);
```

```
for (int j = low; j <= high-1; j++)
{
    if (arr[j] < pivot)
    {
        i++;
        swap(&arr[i], &arr[j]);
    }
}
swap(&arr[i+1], &arr[high]);
return i+1;
}
```

(ii) n^3

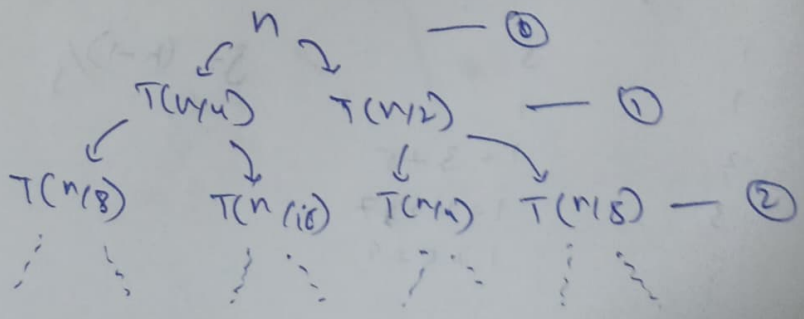
multiplication of two sq. matrix

```
for(i=0; i<r1; i++)
    for(j=0; j<c2; j++)
        for(k=0; k<c1; k++)
        {
            res[i][j] += a[i][k] * b[k][j];
        }
```

(iii) log(log n)

```
for(i=2; i<n; i = i*i)
{
    count++;
}
```

Sol" → (4) $T(n) = T(n/4) + T(n/2) + cn^2$



At level:

$0 \rightarrow cn^2$

$1 \rightarrow \frac{n^2}{4^2} + \frac{n^2}{2^2} = \frac{5n^2}{16}$

$2 \rightarrow \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$

$\vdots$

$\text{max levels} = \frac{n}{2^k} = 1$

$\Rightarrow k = \log_2 n$

$\therefore T(n) = c\left(n^2 + (5/16)n^2 + (5/16)^2 n^2 \ldots + (5/16)^{\log n} n^2\right)$

$T(n) = cn^2\left[1 + \left(\frac{5}{16}\right) + \left(\frac{5}{16}\right)^2 + \ldots + \left(\frac{5}{16}\right)^{\log n}\right]$

$T(n) = cn^2 \times 1 \times \left(\frac{1 - \left(\frac{5}{16}\right)^{\log n}}{1 - 5/16}\right)$

$= cn^2 \times \frac{11}{5} \times \left(1 - \left(\frac{5}{16}\right)^{\log n}\right)$

$\therefore \boxed{T(n) = O(n^2 c)} \rightarrow \boxed{O(cn^2)}$

Soln → (5)

```
int fun(int n)
{
    for(i=1; i<=n; i++)
        for(j=1; j<n; j+=i)
            // O(1)
    }
```

for	i	j	$j = (n-1)/i$ times
1	1	1	
2	2	1+3+5	
3	3	1+4+7	
⋮	⋮	1+5+9	
n	n	⋮	

$\sum_{i=1}^{n} \frac{n-1}{i}$

$\because T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-1)}{3} \cdots + \frac{(n-1)}{n}$

$T(n) = n\left[1 + \frac{1}{2} + \frac{1}{3} \cdots + \frac{1}{n}\right] - 1\times\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$

$= n\log n - \log n$

$\therefore$ **$T(n) = O(n\log n)$**

Soln → (6)

```
for( i=2; i<=n; i = pow(i,k))
{
    O(1)
}
```

for i: $2^1, 2^k, 2^{k^2}, 2^{k^3}, \ldots, 2^{k^m}$

where, $2^{k^m} \quad 2 = n$

$k^m = \log_2 n$

$m = \log_k \log_2 n$

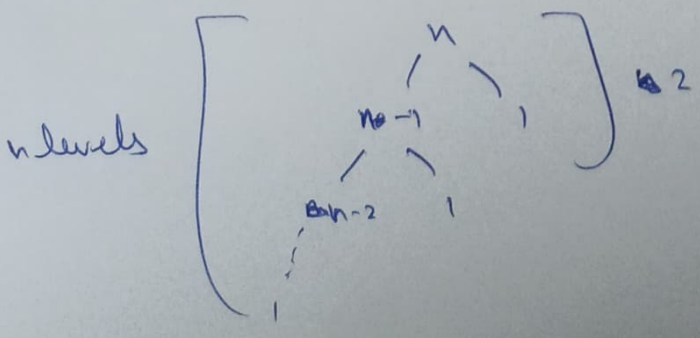
$\therefore \sum_{i=1}^{m} 1$

$1 + 1 + 1 \cdots m$ times

$T(n) = O(\log_k \log n)$

Soln (7) Given algo divides array in 99% & 1% (satisfy algo)

$\therefore T(n) = T(n-1) + O(1)$



'n' work is done at each level for varying

$T(n) = (T(n-1) + T(n-2) + \dots + T(1) + O(1)) \times n$

$= n \times n$

$\therefore \boxed{T(n) = O(n^2)}$

lowest height $= 2$

highest " $= n$

$\boxed{\because \text{diff.} = n-2}$ $n > 1$

The given algorithm produces linear result.

Sol^n → (8) Considering for large value of 'n'

(a) $100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n$
$< \log(n!) < n^2 < 2^n < 4^n < 2^{2^n}$

(b) $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n$
$< n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

(c) $96 < \log_8 n < \log 2n < 5n < n\log_4 n < n\log_2 n <$
$\log(n!) < 8n^2 < 7n^3 < n! < 8^{2^n}$