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SMT. J. J. KUNDALIA COMMERCE COLLEGE

Mathematical and Statistical Foundation of Computer Science

SMT. J. J. KUNDALIA COMMERCE COLLEGE
PRUTHVEERAJ SUMAKEEYA
BCA SEM - 1

Unit - 5

Arithmetic
&
Geometric Progression

Arithmetic & Geometric Progression

- Sequence , Series
- Arithmetic Progression
  - Definition
  - N^{th} Term
  - Sum of n Terms (with proof)
- Geometric Progression
  - Definition
  - N^{th} Term
  - Sum of n Terms (with proof)
- Harmonic Progression
- Relation between AM,GM and HM(Two Numbers)
- Examples

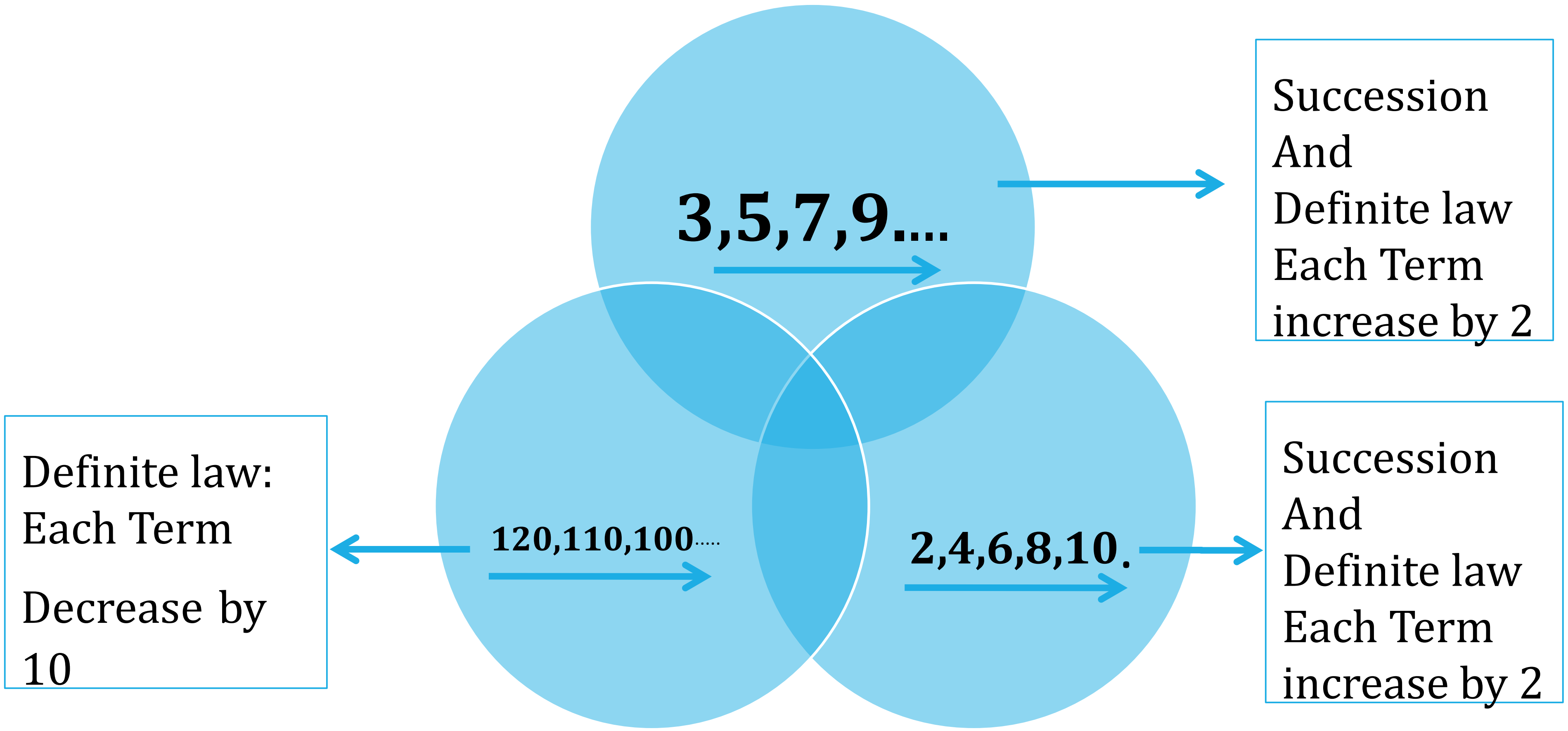
Definition of Progression or Sequence

- "A sequence is a collection of numbers which always stand in the same order and formed in succession according to some definite law".

- Some collection of numbers are given below.

→3, 5, 7, 9....
→120, 110, 100.....
→2, 4, 6, 8, 10...
→ 1/2, 1/4, 1/8.....

Sequence


3,5,7,9....
Succession
And
Definite law
Each Term
increase by 2
120,110,100.....
Definite law:
Each Term
Decrease by
10
2,4,6,8,10.
Succession
And
Definite law
Each Term
increase by 2

Series

- The sum of the terms of any sequence is called a series.
- e.g. : The above sequence or progression can be written in the form of series as below.

→ 3+5+7+9....
→ 120+110+100.....
→ 2+4+6+8+10...
→ 1/2+1/4+1/8....

Series

- The sum of the terms of a sequence cab be done upto any number of terms.
- The sum of first n terms of any sequence is denoted by S_n
- $S_n = \overbrace{\underbrace{T1 + T2 + T_3} + \ldots.. + T_{n-1}} + T_n$

- S_3 = The sum of first 3 Terms.
- S_{10} =The sum of first 10 Terms.
- S_{n-1} = The sum of first (n-1) Terms.

Relation between S_n and S_{n-1}

- $S_n = \underbrace{T1 + T2 + T3 + \ldots\ldots + Tn-1}_{S_{n-1}} + Tn$
- $S_n = S_{n-1} + Tn$
- $Tn = S_n - S_{n-1}$

- Hence The n^{th} term of a Sequence can be obtained by subtracting the sum of (n-1) terms from the sum of n terms.

Arithmetic Progression (A.P)

- A Sequence in which the difference between any two consecutive terms remains constant is called an arithmetic progression.

- The first term of A.P is denoted by **a** and the common difference of any two consecutive terms is denoted by **d**

- In 5,8,11,14,.....
  - a=5, d=3

- In 3,6,9,12.....
  - a =3,d=3

Arithmetic Progression (A.P)

- In 25,20,15,10,..... ..
- In -20,-18,-16,-14,-12......

Arithmetic Progression (A.P)

- $T_1, T_2, T_3, T_4, T_5 \ldots$
- $5, 8, 11, 14, \ldots\ldots$
  - $a=5,\ d=3$
- $T_1=5,\ T_2=8,\ T_3=11,\ T_4=14$
- $d = T_2 - T1 = 8-5 = 3$
- $d = T_3 - T_2 = 11-8 = 3$
- $d = T_4 - T_3 = 14-11 = 3$

Arithmetic Progression (A.P)

- $T_1, T_2, T_3, T_4, T_5 \ldots$
- $2, 4, 8, 16, \ldots\ldots$
- $T_1=2,\ T_2=4,\ T_3=8,\ T_4=16$
- $d = T_2 - T1 = 4-2 = 2$
- $d = T_3 - T_2 = 8-4 = 4$
- $d = T_4 - T_3 = 16-8 = 8$

Some Notations

The following notations are used in an A.P.

- a = The first term of an A.P.
- l = The last term of an A.P.
- d = Common difference
- T_n= n^{th} term of an A.P.
- S_n= Sum of first n terms.
- n = total number of terms

AS per the previous notations

- First term of an A.P. = a
- Second term of an A.P. = $a+d$
- Third term of an A.P. = $a+2d$
- Fourth term of an A.P. = $a+3d$
- Hence the general form of an A.P..

$a, a+d, a+2d, a+3d.....$

The formula for obtaining nth term of an A.P.

The general form of an A.P. is a,a+d,a+2d,a+3d....

- First term = T_1 =a;
- Second term T_2 =a+d = a+(2-1)d;
- Third Term T_3 = a+2d = a+(3-1)d;
- Fourth term = T_4 = a+3d = a+(4-1)d;
- ....
- ....
- ....
- n^{th} term = T_n = a+(n-1)d;

n^{th} Term of an AP

- For an A.P.
  - $T_n = a+(n-1)d;$

- If the a and d are given, any term of an A.P. can be Obtained by the formula $T_n = a+ (n-1)d$
- IF n^{th} term of an A.P. is the last term of that progression then $l = a+(n-1)d;$

Example : Find the required terms of the following A.P

(1)5,8,11,14,........(10^{th} term)

- Here a=5, We want 10^{th} term. So, n = 10.
- $d = T_2 - T_1 = 8 - 5 = 3$
- $d = T_3 - T_2 = 11 - 8 = 3 \rightarrow d = 3$
- $T_n = a + (n-1)d$
- $T_{10} = 5+(10-1)3$
- $T_{10} = 5+(9)3$
- $T_{10} = 5+27$
- $T_{10} = 32$

(2)130, 123, 116, ... (30^{th} term)

- Here $a=130$, We want 30^{th} term. So, $n = 30$
- $d = T_2 - T_1 = 123 - 130 = -7$
- $d = T_3 - T_2 = 116 - 123 = -7 \rightarrow d = -7$
- $Tn = a + (n-1)d$
- $T_{30} = 130 + (30-1)(-7)$
- $T_{30} = 130 + (29)(-7)$
- $T_{30} = 130 + (-203)$
- $T_{30} = 130 - 203$
- $T_{30} = -73$

(3) -12, -8, -4, ...(11^{th} term)

- Here a=-12, We want 11^{th} term. So, n = 11
- $d = T_2 - T_1 = -8 - (-12) = -8 + 12 = 4$
- $d = T_3 - T_2 = -4 - (-8) = -4 + 8 \rightarrow d = 4$
- $Tn = a + (n-1)d$
- $T_{11} = -12 + (11-1)(4)$
- $T_{11} = -12 + (10)(4)$
- $T_{11} = -12 + (40)$
- $T_{11} = 28$

(4) 0.1, 0.5, 0.9, ……(21st term)

- Here a= 0.1 , We want 21st term. So, n = 21
- $d = T_2 - T_1 = 0.5 - 0.1 = 0.4$
- $d = T_3 - T_2 = 0.9 - 0.5 = 0.4 \rightarrow d = 0.4$
- $T_n = a + (n-1)d$
- $T_{21} = 0.1 + (21-1)(0.4)$
- $T_{21} = 0.1 + (20)(0.4)$
- $T_{21} = 0.1 + 8$
- $T_{21} = 8.1$

Ex-2: Obtain the formula for finding n^{th} term of 3,5,7,9,..... and hence find T_{99} and T_{199}

- Here, 3,5,7,9 is an AP
- a=3 and d=2
- $T_n = a+(n-1)d$
- $T_n = 3+(n-1)2$
- $T_n = 3 + 2n - 2$
- $T_n = 2n + 1$

- Here, 3,5,7,9 is an AP
- **a=3 and d=2**
- **1) T_{99}, n=99**
- $T_n = a+(n-1)d$
- $T_n = 3+(99-1)2$
- $T_n = 199$

- Here, 3,5,7,9 is an AP
- **a=3 and d=2**
- **2) T_{199}, n=199**
- $T_n = a+(n-1)d$
- $T_n = 3+(199-1)2$
- $T_n = 399$

Ex-3:The 100^{th} term of an AP is 505 and common difference is 5. Find its first term.

- Here, $T_{100} = 505$
- $n=100$, $d=5$, $a=?$
- $T_n = a+(n-1)d$
- $T_{100} = a + (100-1)(5)$
- $505 = a + (99)(5)$
- $505 = a + 495$
- $505 - 495 = a$
- $a = 10$

Ex-4: The 10th term of an AP is $\frac{5}{2}$ and first term is $\frac{19}{4}$. Find the common difference.

- Here, $T_{10} = \frac{5}{2}$
- $n=10$, $a=\frac{19}{4}$, $d = ?$
- $T_n = a+(n-1)d$
- $T_{10} = a + (10-1)\,d$
- $\frac{5}{2} = \frac{19}{4} + 9d$
- $\frac{5}{2} - \frac{19}{4} = 9d$
-

- $9d = \frac{10}{4} - \frac{19}{4}$
- $9d = \frac{10-19}{4}$
- $9d = -\frac{9}{4}$
- $d = -\frac{1}{4}$

- Common difference $d = -\frac{1}{4}$

Ex-5:The 10th term of an A.P is 32 and its 8th term is 26.Find its 21st term.

- Here, T_{10} = 32 and T8 = 26;
- Put n = 10 and n = 8 in Tn=a+(n-1)d
- T_{10} = a+(10-1)d
- 32 = a +9d
- a+9d = 32..........(i)
- T_8 = a +(8-1)d
- 26 = a+7d.........(ii)
- deduct (ii) from (i)

- $a+9d = 32$
- $a+7d=26$

$2d = 6$

$d=3$

- Put $d=3$ in equation....(i)

$a+9d = 32$

$a+9(3) = 32$

$a=32-27$

$a=5$

- To find T_{21}, Put n =21 in T_n=a+(n-1)d
- T_{21} = 5+(21-1)3
- T_{21} = 5 + (20)3
- T_{21} = 5 + 60
- T_{21} = 65

- To find T_{21}, Put $n = 21$ in $T_n = a + (n-1)d$
- $T_{21} = 5 + (21-1)3$
- $T_{21} = 5 + (20)3$
- $T_{21} = 5 + 60$
- $T_{21} = 65$

Ex-6: There are 60 terms in A.P. Its 21st term is 41 and its 11th term is 21. Find its last term

- Here, $T_{21} = 41$ and $T_{11} = 21$;
- Put n = 60 and n = 21 in $T_n = a+(n-1)d$
- $T_{21} = a+(21-1)d$
- $41 = a + 20d$
- $a+20d = 41$..........(i)
- $T_{11} = a + (11-1)d$
- $21 = a+10d$..........(ii)
- deduct (ii) from (i)

- $a+20d = 41$
- $a+10d = 21$

$10d = 20$

$d = 2$

$d=2$

- Put $d=2$ in equation....(i)

$a+20d = 41$

$a+20(2) = 41$

$a=41-40$

$a=1$

Ex-7: Which term will be 124 in 4,9,14,19,......?

- Here $a=4$, $T_n = 124$, $n=?$
- $d = T_2 - T_1 = 9-4 = 5$
- $d = T_3 - T_2 = 14-9 = 5 \rightarrow d = 5$
- $T_n = a+(n-1)d.$
- $T_n=4+(n-1)d$
- $124 = 4+(n-1)5$
- $124 = 4 + 5n - 5$
- $124 = 5n - 1$

- $124 + 1 = 5n$
- $125 = 5n$
- **$n = 25$**
- **25^{th} term is 124**

Ex-8: The first term is 7 and the last term is 99 of an A.P. of 25 terms. Find its sum of First 25 terms.

- Here n =25, a=7, l=99
- $S_n = \frac{n(a+l)}{2}$
- $S_{25} = \frac{25(7+99)}{2}$
- $S_{25} = \frac{25(106)}{2}$
- $S_{25} = 25(53) = 1325$

Ex-9:Find the sum of first 20 terms in an A.P. 15,18,21,......

- Here a=15,d=3,n=20
- $S_n = \frac{n}{2}[2a+(n-1)d]$
- $S_{20} = \frac{20}{2}[2(15)+(20-1)3]$
- $S_{20} = 10[30+57]$
- $S_{20} = 870$

Ex-10: The sum of first 8 terms of A.P is 124 and the sum of its first 11 terms is 220.Find T_{30} and S_{30}.

- Here S_8=124 and S_{11} = 220 for an A.P
- $S_n = \frac{n}{2}[2a+(n-1)d]$
- Put n=8,
- $S_8 = 8/2\ [2a + (8-1)d]$
- $124 = 4[2a+7d]$
- $2a+7d = 32$ ......(i)

- Now put $n=11$ in $S_n = n/2[2a+(n-1)d]$
- $S_{11} = 11/2[2a +(11-1)d]$
- $220 = 11/2[2a+10d]$
- $220 \times 2/11 = 2a +10d$.......(ii)
- $40= 2a +10d$.......(ii)
- Subtracting equation (ii) from equation (i)
- $2a+7d = 31$

  $2a+10d = 40$

  $-3d = -9$
- $d =3$

- Put d=3 in equation (i),
- $2a+7(3) = 31$
- $2a+21= 31$
- $2a = 10$
- $a=5$
- Put $a=5$, $d=3$ and $n=30$ in $T_n = a+(n-1)d$
- $T_{30} = 5+(30-1)3$

  $= 5+(29)3$

  $=5+87$
- $T_{30}=92$

- Putting a=5,d=3 and n=30 in
- $S_n = n/2\ [2a + (n-1)d]$
- $S_{30} = 30/2\ [2(5) + (30-1)3]$
- $S_{30} = 15[10+(29)3]$
- $S_{30} = 15[10+87]$
- $S_{30} = 15[97]$
- $S_{30} = 1455$

Ex-11 : The sum of how many terms of the sequence 1,3,5,7.......will be 100?

- 1,3,5,7, …… is an A.P , Here $a = 1$
- $d = T2 - T1 = 3 - 1 = 2$
- Let the sum of its first n terms is 100 , $Sn = 100$

- $S_n = \frac{n}{2}[2a+(n-1)d]$
- $100 = \frac{n}{2}[2(1)+(n-1)2]$
- $100 = \frac{n}{2}[2 +2n-2]$

- $100 = \frac{n}{2}[2n]$
- $100 = n[n]$
- $100 = n^2$
- $n = \pm 10$
- $n = 10$ or $n = -10$
- But $n = -10$ is impossible
- $n = 10$
- Hence the sum of first 10 terms of this sequence will be 100.

Ex-12 : The six term of an A.P is 121. Find the sum of its first 11 terms.

- Here $T_6 = 121$
- Put n = 6 in $T_n = a + (n-1)d$
- $T_6 = a + (6-1)d$
- $121 = a + 5d$........ (i)

- Put n = 11 in $S_n = \frac{n}{2}[2a+(n-1)d]$
- $S_n = \frac{11}{2}[2a+(11-1)d]$

- $S_n = \frac{11}{2}[2a+(11-1)d]$
- $S_n = \frac{11}{2}[2a+10d]$
- $S_n = \frac{11}{2} \times 2[a+5d]$
- $S_n = 11[a+5d]$
- $S_n = 11 \times 121$
- $S_n = 1331$

Ex-13: A person has to pay Rs. 3,400 by monthly instalments. If first instalment is Rs. 250 and each instalment is Rs. 20 more than its previous one, in how many instalment this amount would be paid ? Find its last instalment.

Solution: The difference between two consecutive instalment 20 = Constant

- Hence, the instalments form an A. P. Here, a = 250, d= 20
- $S_n = 3400$
- $S_n = \frac{n}{2}[2a + (n-1)d]$
  - $3400 = \frac{n}{2}[2(250) + (n-1)20]$
  - $3400 = \frac{n}{2}[500 + 20n - 20]$
  - $3400 = \frac{n}{2}[480 + 20n]$

- $3400 = n\,[240+10n]$
- $340 = n\,[24+n]$
- $340 = 24n + n^2$
- $n^2 + 24n - 340$
- $(n+34)(n-10)$
- $n=-34$ OR $n=10$, Total terms can never be negative
- So $n = -34$ is not possible
- $n=10$
- Hence, Rs. 3,400 can be paid in 10 monthly instalments.

Example-14 : The age of a person is 51 years. He has 7 sons born at equal interval. Total age of father and sons is 170 years. if the age of the youngest son is 8 years, find the age of the eldest son.

Solution: 7 sons are born at equal interval. Hence their ages form an A.P

- There are 7 terms in this progression
- n = 7. The age of the youngest son is 8 years. a=8
- Age of seven sons + Age of Father = 170
- $S_7 + 51 = 170$
- $S_7 = 119$
- Now $S_n = n/2(a+l)$

- $S_7 = 7/2(8+l)$
- $119 = 7/2\ (8+l)$
- $(8+l) = 119*2\ /7$
- $l = 34 - 8$
- $l = 26$
- The age of the eldest son is 26 years.

Assuming term for an A.P

Even Terms		Odd Terms	
2	a-d, a+d	3	a-d, a, a+d
4	a-3d, a-d, a+d, a+3d	5	a-2d, a-d, a, a+d, a+2d
6	a-5d, a-3d, a-d, a+d, a+3d, a+5d	7	a-3d, a-2d, a-d, a, a+d, a+2d, a+3d

Ex-15: Three numbers are in A. P. Their sum is 15 and product is 105. Find the numbers.

- Three numbers are in A.P
- Suppose the numbers are a-d, a, a+d
- Their sum = $a-d + a + a+d$
- $15 = 3a$
- $5 = a$
- Their Product $=(a-d)(a)(a+d)$
- $105 = (a^2-d^2)a$

- $105 = (a^2-d^2)a$
- $105 = (25-d^2)5$
- $21 = 25-d^2$
- $d^2 = 25-21$
- $d^2 = 4$
- $d = \pm 2$
- **$d = 2$ or $d = -2$**
- **Take $a = 5$ and $d = 2$**
- First number = $a-d = 5-2 = 3$
- Second number = $a = 5$
- Third number = $a+d = 5+2 = 7$
- Three numbers are **3, 5, 7** which are in A.P

- **Take a = 5 and d = -2**
- First number = a-d = 5+2 =7
- Second number = a = 5
- Third number = a+d = 5-2 = 3
- Three numbers are **7, 5, 3** which are in *A*.P
- Required numbers are **3,5,7 or 7,5,3**.

Ex-16: Three numbers are in A.P. Their sum is 15 and the sum of their squares is 93. find the three numbers.

- Three numbers are in A.P
- Suppose the numbers are a-d, a, a+d
- Their sum = a-d + a + a+d
- 15 = a-d + a + a+d
- 15 = 3a
- 5 = a
- a = 5
- The sum of their squares=$(a-d)^2 + (a^2) + (a+d)^2$
- $93 = a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2$
- $93 = 3a^2 + 2d^2$

- $93 = 3(5)^2 + 2d^2$
- $93 = 3(25) + 2d^2$
- $93 = 75 + 2d^2$
- $93 - 75 = 2d^2$
- $18 = 2d^2$
- $9 = d^2$
- $d = \pm 3$

Take $a = 5$ and $d = 3$

- First number = $a-d = 5-3 = 2$
- Second number = $a = 5$
- Third number = $a+d = 5+3 = 8$
- Three numbers are **2, 5, 8** which are in A.P

Take $a = 5$ and $d = -3$

- First number = $a-d = 5+3 = 8$
- Second number = $a = 5$
- Third number = $a+d = 5-3 = 2$
- Three numbers are **8, 5, 2** which are in A.P
- Required numbers are **2, 5, 8 or 8, 5, 2**

Ex-17: Five numbers are in an AP. Their sum is 35 and the product of first and fifth number is 33.Find the numbers.

- Let $a-2d, a-d, a, a+d, a+2d$ are in A.P
- Their sum is 35.
- $35 = a-2d + a-d + a + a+d + a+2d$
- $35 = 5a$
- $7 = a$
- First Number X Fifth Number = 33
- $(a-2d)(a+2d) = 33$

- $a^2 - 4d^2 = 33$
- $49 - 4d^2 = 33$
- $49 - 33 = 4d^2$
- $16 = 4d^2$
- $4 = d^2$
- $d = \pm 2$
- $d = 2$ Or $d = -2$
- Taking $a = 7$ and $d = 2$
- First Number = $a-2d = 7 - 2(2) = 7-4 = 3$
- Second Number = $a-d = 7-2 = 5$

- Third Number = a = 7
- Fourth Number= a+d = 7+2 = 9
- Fifth Number = a+2d = 7+2(2) = 7+4 = 11

- Taking a = 7 or d = -2
- First Number = a-2d = 7 – 2(-2) = 7+4 = 11
- Second Number = a-d = 7-(-2)=7+2 = 9
- Third Number = a = 7
- Fourth Number= a+d = 7+(-2)= 7-2 = 5
- Fifth Number = a+2d = 7+2(-2) = 7-4 = 4
- Therefore. Five numbers are 3, 5, 7, 9, 11 or 11, 9, 7, 5, 3

Geometric Progression (G.P)

- In a sequence, if the ratio of any term to its previous term is constant, it is called Geometric Progression.

OR

- In a sequence, if each term is obtained by multiplying a constant number to its previous term, such sequence is called Geometric Progression.
- The constant ration is called common ratio in Geometric Progression and it is denoted by r.
- The first term of G.P is denoted by a.

The Formula for obtaining n^{th} term of a G.P

- For a G.P, First Term = a and Common ratio = r
- $\frac{T_2}{T_1} + \frac{T_3}{T_2} + \frac{T_4}{T_3} + \cdots \frac{T_n}{T_{n\_1}} = r$
- T1 = a
- T2 = T1.r = ar
- T2 = ar
- T3 = T2.r = ar x r = ar^2

- First Term $T_1 = T_1 = a$
- Second Term $T_2 = T_2 = ar = ar^{2-1}$
- Third Term $T_3 = T_3 = ar^2 = ar^{3-1}$
- Fourth Term $T_4 = T_4 = ar^3 = ar^{4-1}$

. . .

. . .

. . .

- Nth term $T_n = T_n = ar^{n-1}$
- The formula for obtaining nth term of a G.P = $\mathbf{T_n = ar^{n-1}}$

Ex.18: The sum of three numbers in a G.P. is 28 and their product is 512.

- Suppose the numbers are a/r a, ar
- Product of three numbers = 512
- $(a/r)(a)(ar) = 512$
- $a^3 = 512$
- $a=8$
- Also, the sum of three numbers = 28
- $a/r + a + ar = 28$

- $a(1/r + 1 + r) = 28$
- $8(1/r + 1 + r) = 28$
- $2(1 + r + r^2)/r = 7$
- $2 + 2r + 2r^2 = 7r$
- $2 + 2r + 2r^2 - 7r = 0$
- $2 + 2r^2 - 5r = 0$
- $2r^2 - 5r + 2 = 0$
- $2r^2 - 4r - r + 2 = 0$
- $2r(r - 2) -1(r -2) = 0$
- $(r-2) = 0$ or $(2r-1) = 0$

- $(r-2) = 0$ or $(2r-1) = 0$
- $r=2$ or $2r=1$
- $r=2$ or $r=1/2$

- Here **$a=8$ and $r=2$**
- First Term = $a/r = 8/2 = 4$
- Second term = $a = 8$
- Third Term = $ar = (8)(2) = 16$

- Here **$a=8$ and $r=1/2$**
- First Term = $a/r = 8/1/2 = 16$
- Second term = $a = 8$
- Third Term = $ar = (8)(1/2) = 4$

Ex.19: The product of 3 numbers G.P. is 729 and sum of their squares is 819.

- Suppose the numbers are a/r, a, ar
- Product of three numbers = 729
- $(a/r)(a)(ar) = 729$
- $a^3=729$
- $a = 9$
- Sum of their squares = 819
- $(a/r)^2 + a^2 + (ar)^2 = 819$
- $(a^2/r^2) + a^2 + (a^2r^2) = 819$

- $(a^2/r^2) + a^2 + (a^2r^2) = 819$
- $a^2(1/r^2 + 1 + r^2) = 819$
- $(9)^2(1 + r^2 + r^4) = 819r^2$
- $81(1 + r^2 + r^4) = 819r^2$
- $81 + 81r^2 + 81r^4 = 819r^2$
- $81 + 81r^2 + 81r^4 - 819r^2 = 0$
- $81 + 81r^4 - 738r^2 = 0$
- $81r^4 - 738r^2 + 81 = 0$
- $9r^4 - 82r^2 + 9 = 0$
- $9r^4 - 81r^2 - r^2 + 9 = 0$

- $9r^4 - 81r^2 - r^2 + 9 = 0$
- $9r^2(r^2 - 9) - 1(r^2 - 9) = 0$
- $(r^2 - 9)(9r^2 - 1) = 0$
- $(r^2 - 9) = 0$ or $(9r^2 - 1) = 0$
- $r^2 = 9$ or $r^2 = 1/9$
- $r = \pm 3$ or $r = \pm(1/3)$

- (1) $a=9$ & $r= 3$
- (2) $a=9$ and $r= -3$
- (3) $a=9$ and $r= 1/3$
- (4) $a=9$ and $r= -(1/3)$

The formula for obtaining the sum of first n terms in G.P

- The general form of a G.P is a, ar, ar^2, ar^3,……… ar^{n-1}
- Now, $S_n = T_1 + T_2 + T_3 + \ldots\ldots\ldots \ldots \ldots . T_{n-1} + T_n$
- $S_n = a + ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots + ar^{n-2} + ar^{n-1}$ ______(i)
- Multiplying by r on both sides of equation (i)
- $rS_n = ar + ar^2 + ar^3 + \ldots\ldots\ldots\ldots + ar^{n-1} + ar^n$ ____________(ii)
- Subtracting equation (i) from equation (ii)

- $rS_n - S_n = ar^n - a$
- $S_n(r - 1) = ar^n - a$
- $S_n(r - 1) = a(r^n - 1)$
- $S_n = \dfrac{a(r^n - 1)}{(r - 1)}$

- Subtracting equation (ii) from equation (i)
- $S_n - rS_n = a - ar^n$
- $S_n(1 - r) = a(1 - r^n)$
- $S_n = \dfrac{a(1 - r^n)}{(1 - r)}$

- Note : (1) When the value of r is greater than 1

Ex. $r > 1$, the following formula is used for finding Sn

$$S_n = \frac{a(r^n - 1)}{(r - 1)}$$

- (2) When the value of r is less than 1

Ex. $r < 1$, the following formula is used for finding Sn

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

- (3) When $r < 1$ and the value of r^n is almost zero.

$$S_n = \frac{a}{1-r}$$

Ex-20 : Find the sum of 8 term of the following sequence. 2, 6, 18,…….

- We have to find the sum of 8 term $S_8 = ?$
- Here $a = 2$, $r = 3$ and $n = 8$
- As $r > 1$, $S_n = \frac{a(rn-1)}{(r-1)}$
- $S_8 = \frac{2(3^8 - 1)}{(3 - 1)}$

- $S_8 = \dfrac{2(3^8 - 1)}{(3 - 1)}$
- $S_8 = \dfrac{2(6561 - 1)}{2}$
- $S_8 = 6560$

Ex - 21 : Find the sum upto 7 terms. 1024 + 512 + 256 +………..

- Here a = 1024, r = $\frac{1}{2}$ <1 , n = 7
- Sn = $\frac{a(1-r^n)}{(1-r)}$
- S7 = $\frac{1024(1-(\frac{1}{2})^7)}{(1-(\frac{1}{2}))}$

- $S_7 = \dfrac{1024\left(1-\frac{1}{128}\right)}{\frac{1}{2}}$
- $S_7 = \dfrac{2048(128-1)}{128}$
- $S_7 = 16(127)$
- $S_7 = 2032$

Exercise

Find the required terms of the following GPs.

1) 2, 6, 18, 54 ……… (7^{th} term)

2) -48, 24, -12 …………. (10^{th} term)

3) 1/4, 1/2, 1, 2 ……….(8^{th} term)

Find the sum of the following series :

1) 4 + 20 + 100 + ……. (upto 10 terms)

2) 256 + 128 + 64 + …… (upto 10 terms)

Exercise

Ex.1 : The sum of three consecutive terms is 26 and its product is 216.Find the terms.

Ex.2 : The product of 4 numbers in a G.P is 1024. If the product of first and third number is 16. find the four numbers.

Harmonic Progression

- A sequence of numbers are said to form Harmonic Progression when their reciprocals are in Arithmetic Progression.
- For Ex. 1/2, 1/4, 1/6, 1/8…..
- 2,4,6,8….. Are in A.P

Ex.22:Find the 15th term of the series 1/3, 1/7, 1/11, 1/15

- Given sequence 1/3, 1/7, 1/11, 1/15 is in HP
- 3,7,11,15,…..are in A.P
- Here, $a = 3$, $d = 4$, $n = 15$
- nth term of an A.P. is $T_n = a + (n-1)d$
- $T_{15} = 3 + (15-1)4$
- $T_{15} = 3 + (14)4$
- $T_{15} = 3 + 56$

- $T_{15}=3+56$
- $T_{15}=59$.
- 15th term of A.P. is 59.
- 15th term of H.P. is 1/59.

AVERAGES

- ARITHMETIC MEAN
- If three numbers are in A.P. then the middle number is said to be the Arithmetic Mean between the other two.
- if a, A, b are in A.P. then A is Arithmetic Mean between a and b.
- Also, $A - a = b – A$
- $A+A=b+a$
- $2A= b+a$

ARITHMETIC MEAN

- $A = \dfrac{(a+b)}{2}$

GEOMETRIC MEAN

- If three numbers are in G.P. then the middle number is said to be the Geometric Mean between the other two.
- if a, G, b are in G.P. then G is Geometric Mean between a and b.
- Also, $G/a = b/G$
- $G^2 = ba$
- **$G = \sqrt{ba}$**

HARMONIC MEAN

- If three numbers are in H.P. then the middle number is said to be Harmonic Mean between the other two.
- if a, H, b are in H.P. then H is Harmonic Mean between a and b. $1/a$, $1/H$, $1/b$ are in A.P.
- Also $1/H - 1/a = 1/b - 1/H$
- $1/H + 1/H = 1/a + 1/b$
- $2/H = (a+b)/ab$

HARMONIC MEAN

- $2/H = (a+b)/ab$
- $H/2 = ab/(a+b)$
- $\mathbf{H = 2ab/(a+b)}$

Ex.23:For any two real quantities prove that. (1) AM x HM = $(GM)^2$

- (1) AM x HM = $(GM)^2$
- Let a and b are 2 real numbers.
- Then A = a+b / 2, G = √ba , H = 2ab/(a+b)
- L.H.S = AM x GM
- = (a+b / 2)(2ab/(a+b))
- =ab
- R.H.S = $(GM)^2$

- R.H.S = $(GM)^2$
- =(ab)

- L.H.S = R.H.S

Hence proved.

Thank You