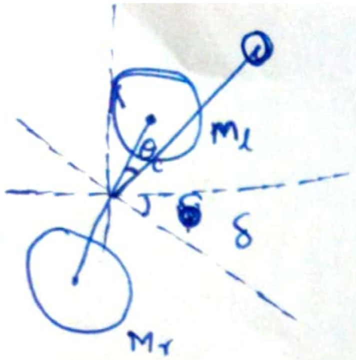
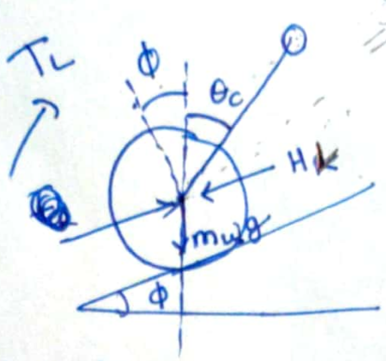
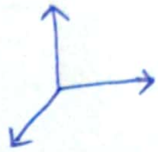




$\boxed{11}$



$$\begin{bmatrix} x \\ \dot{x} \\ \theta \\ \ddot{\theta} \\ \delta \\ \dot{\delta} \end{bmatrix} \begin{array}{l} \} \text{ 1 DOF} \\ \\ \} \ 2^{nd} \text{ DOF} \\ \\ \} \ 3^{rd} \text{ DOF} \\ \\ \end{array}$$

θ_c = tit angle of inverted pendulum.

δ = yaw angle —— ϕ yaw

ϕ = slope angle.

kinematics

$$X_c = X + l\sin(\theta_c+\phi).\cos(\delta)$$

$$Y_c = l\cos(\theta_c+\phi)$$

$$\dot{x}_c = \dot{x} + l\dot{\theta}_c\cos(\theta_c+\phi).\cos(\delta) - l\dot{\delta}\sin(\theta_c+\phi).\sin(\delta)$$

Not sure

$$\dot{y}_c = -l\dot{\theta}_c\sin(\theta_c+\phi)$$

$\langle I \rangle$ $\boxed{\text{Translational kinetic Energy of chassis/body/Pendulum/CG.}}$

$$= \frac{1}{2}M_c\left[(\dot{x}+l\dot{\theta}_c\cos(\theta_c+\phi))^2 + (-l\dot{\theta}_c\sin(\theta_c+\phi))^2\right].$$

$$= \frac{1}{2}M_c\left[\dot{x}^2 + l^2\dot{\theta}_c^2\cos^2(\theta_c+\phi) + 2l\dot{\theta}_c\cos(\theta_c+\phi).\dot{x} + l^2\dot{\theta}_c^2.\sin^2(\phi_c+\phi)\right)$$

$$= \frac{1}{2}M_c\left[\dot{x}^2 + 2\dot{x}.l.\dot{\theta}_c.\cos(\theta_c+\phi) + l^2\dot{\theta}_c^2\right].$$

$\langle II \rangle$ $\boxed{\text{Translational kinetic Energy of wheels}}$

Left wheel.

$$\frac{1}{2}M_W\dot{X}_L^2$$

Right wheel

$$\frac{1}{2}M_W\dot{X}_R^2$$

III (Rotational Kinetic Energy of Body. ($\frac{1}{2} I \omega^2$))

$$= \frac{1}{2} I_c \dot{\theta}_c^2$$

$$= \frac{1}{2} I_c \theta_c^2$$

(I_c = Inertia along wheel axle.)
θ_c = tilt angle of pendulum

(IV) Rotational kinetic Energy of wheels

Left wheel:

$$\frac{1}{2} I_w \dot{\theta}_{WL}^2 = \frac{1}{2} I_w \left(\frac{\dot{x}_L}{r}\right)^2 = \frac{1}{2} \cdot \frac{I_w}{r^2} \cdot \dot{x}_L^2$$

I_w = Inertia of wheel.
r = radius of wheel.

Right wheel

Similarly, $= \frac{1}{2} \cdot \frac{I_w}{r^2} \cdot \dot{x}_R^2$

∴ Total Kinetic Energy: (T).

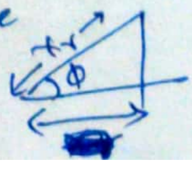
$$T = \frac{1}{2} M_c [\dot{x}^2 + 2\dot{x} l \dot{\theta}_c \cos(\theta_c + \phi) + l^2 \dot{\theta}_c^2]$$
$$+ \frac{1}{2} M_w [\dot{x}_L^2 + \dot{x}_R^2]$$
$$+ \frac{1}{2} I_c \dot{\theta}_c^2$$
$$+ \frac{1}{2} \frac{I_w}{r^2} [\dot{x}_L^2 + \dot{x}_R^2].$$

Potential Energy Of the System: (V)

$$V = M_c g (l \cos\theta_c + r) + \underbrace{M_c g \cdot x_r \sin\phi + 2 M_w g x_r \sin\phi}_{\text{optional term.}}$$

In many derivation this term is NOT considered.

x_r = ~~horizontal~~ distance travelled ON of slope

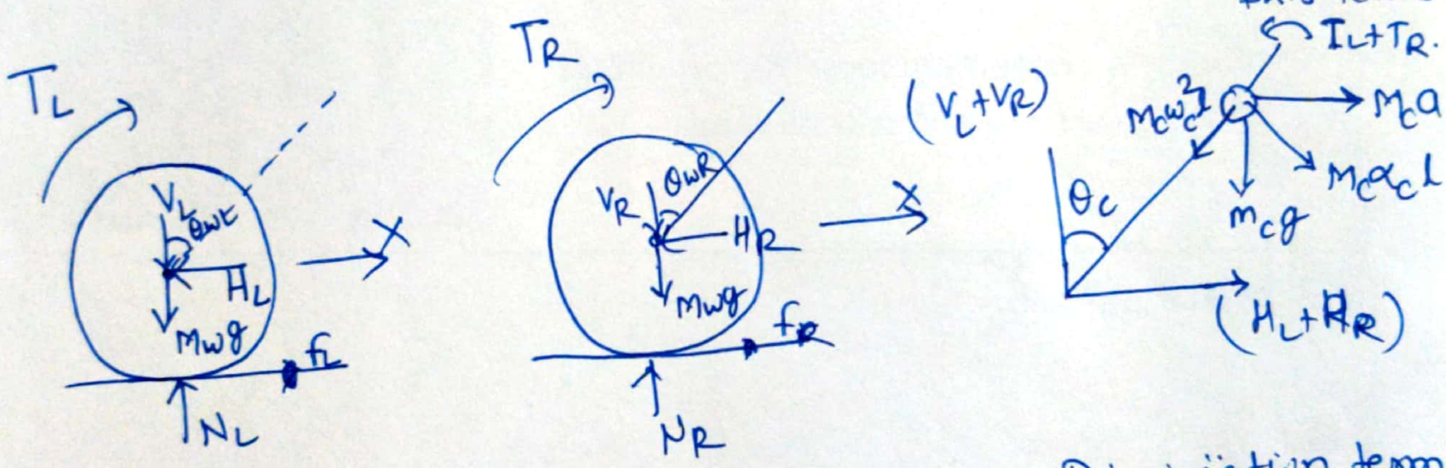


The Lagrangian $\mathcal{L}$

$$\mathcal{L} = \frac{1}{2}\left(M_w + \frac{I_w}{r^2}\right)(\dot{x}_L^2 + \dot{x}_R^2) + \frac{1}{2} M_c \dot{x}^2$$
$$+ \frac{1}{2}(M_c l^2 + I_c)\dot{\theta}_c^2$$
$$+ M_c \dot{x} . l . \dot{\theta}_c . \cos(\theta_c + \phi)$$
$$- M_c g(l \cos\theta_c + r) - (M_c . g . x_r . \sin\phi . + 2 M_w g x_r \sin\phi)$$

optionl term

Let's see about this terms.



Dissipiation term in Lagrange

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q$$

$$R = -K_i \dot{x}_i$$

NOT used for now.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_L}\right) - \frac{\partial L}{\partial x_L} = \frac{T_L}{r} - H_L .$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_R}\right) - \frac{\partial L}{\partial x_R} = \frac{T_R}{r} - H_R$$

$$\left(M_w + \frac{I_w}{r^2}\right)\ddot{x}_L = \frac{T_L}{r} - H_L$$

$$\left(M_w + \frac{I_w}{r^2}\right)\ddot{x}_R = \frac{T_R}{r} - H_R$$

Adding.

$$\left(M_w + \frac{I_w}{r^2}\right)(\ddot{x}_L + \ddot{x}_R) = \left(\frac{T_L + T_R}{r}\right) - (H_L + H_R)$$

$$\boxed{2\ddot{x} = \ddot{x}_L + \ddot{x}_R}$$? How

$$2\left(M_\omega + \frac{I_\omega}{r^2}\right)\ddot{x} = \left(\frac{T_L + T_R}{r}\right) - (H_L + H_R). \quad \text{—— } (*)$$

The equation of motion of chassis displacement is.—
(Only force in Horizontal dir^n on chassis is H_L+H_R).

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = (H_L + H_R)$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{1}{2} M_c \cdot 2\dot{x} + M_c \cdot l \cdot \dot{\theta}_c \cdot \cos(\theta_c + \phi)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{1}{\cancel{2}} M_c \cdot \cancel{2} \cdot \ddot{x} + M_c l \, \ddot{\theta}_c \cdot \cos(\theta_c + \phi) - M_c l \cdot \dot{\theta}_c^2 \cdot \sin(\theta_c + \phi)$$

$$\frac{\partial L}{\partial x} = 0$$

$$M_c \ddot{x} + M_c l \, \ddot{\theta}_c \cos(\theta_c + \phi) - M_c l \cdot \dot{\theta}_c^2 \cdot \sin(\theta_c + \phi) = (H_L + H_R)$$

$$\text{from } (*), \; H_L + H_R = \left(\frac{T_L + T_R}{r}\right) - 2\left(M_\omega + \frac{I_\omega}{r^2}\right)\ddot{x}$$

$$\boxed{\left(M_c + 2M_\omega + \frac{2I_\omega}{r^2}\right)\ddot{x} = \left(\frac{T_L + T_R}{r}\right) - M_c l \, \ddot{\theta}_c \cos(\theta_c + \phi) + M_c l \, \dot{\theta}_c^2 \sin(\theta_c + \phi)}$$

$$q_i = \theta_c \; , \; \dot{q}_i = \dot{\theta}_c$$

$$\frac{\partial L}{\partial \dot{\theta}_c} = (M_c l^2 + I_c)\,\dot{\theta}_c + M_c \cdot \dot{x} \cdot l \cdot \cos(\theta_c + \phi)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_c}\right) = (M_c l^2 + I_c)\,\ddot{\theta}_c + M_c \ddot{x} \cdot l \cdot \cos(\theta_c + \phi) - M_c \cdot \dot{x} \cdot l \cdot \dot{\theta}_c \sin(\theta_c + \phi)$$

$$\frac{\partial L}{\partial \theta_c} = -M_c \dot{x} \cdot l \cdot \dot{\theta}_c \sin(\theta_c + \phi) + M_c g \cdot l \cdot \sin\theta_c$$

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$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_c} - \frac{\partial L}{\partial \theta_c} = -(T_L + T_R)$$

(Note: for previous eq^n of Lagrange we considerd $\left(\frac{T_L+T_R}{r}\right)$ in right side) But Here it's $-(T_L+T_R)$)

$$(M_c l^2 + I_c)\ddot\theta_c + M_c \ddot x l \cos(\theta_c+\phi) - \cancel{M_c \dot x l \dot\theta_c \sin(\theta_c+\phi)}$$
$$+ \cancel{M_c \dot x . l . \dot\theta_c \sin(\theta_c+\phi)} - M_c g l . \sin\theta_c = (T_L+T_R).$$

$$\boxed{(M_c l^2 + I_c)\ddot\theta_c = -(T_L+T_R) + M_c g l \sin\theta_c - M_c \ddot x l \cos(\theta_c+\phi)}$$

for yaw angle δ:

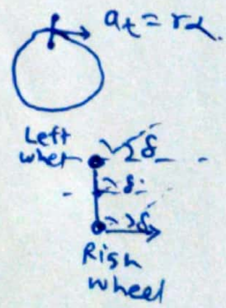
general formula: $\tau = Fr. = I\alpha.$

$$(H_L - H_R)\frac{d}{2} = I_Y \ddot\delta$$

$$H_L - H_R = \frac{2 I_y \ddot\delta}{d}$$

$$a = r\alpha$$
$$\alpha = \frac{a}{r}.$$

$$\ddot\delta = \frac{\ddot x_L - \ddot x_R}{d}$$



from (*)

$$\ddot x_L - \ddot x_R = \frac{\left(\frac{T_L - T_r}{r}\right) - (H_L - H_R)}{\left(M_w + \frac{I_w}{r^2}\right)}$$

$$\ddot{\delta} = \frac{\left(\frac{T_L - T_R}{r}\right) - \left(\frac{2 I_y \ddot{\delta}}{d}\right)}{\left(M_w + \frac{I_w}{r^2}\right) \cdot d} \times \frac{rd}{rd}$$

$$\ddot{\delta} = \frac{(T_L - T_R)\, d}{M_w r d^2 + \frac{I_w}{r} d^2} - \frac{(2 I_y \ddot{\delta})\, r}{M_w r d^2 + \frac{I_w}{r} d^2}$$

$$\ddot{\delta} = \frac{(T_L - T_R)\, d r}{M_w r^2 d^2 + I_w d^2} - \frac{2 I_y \ddot{\delta} r^2}{M_w r^2 d^2 + I_w d^2}$$

$$\ddot{\delta}\left(1 + \frac{2 I_y r^2}{M_w r^2 d^2 + I_w d^2}\right) = \frac{(T_L - T_R)\, d r}{(M_w r^2 d^2 + I_w d^2)}$$

$$\ddot{\delta}\left(\frac{M_w r^2 d^2 + I_w d^2 + 2 I_y r^2}{\cancel{M_w r^2 d^2 + I_w d^2}}\right) = \frac{(T_L - T_R)\, d \cdot r}{\cancel{(M_w r^2 d^2 + I_w d^2)}}$$

$$\boxed{\ddot{\delta} = \frac{(T_L - T_R)\, d \cdot r}{(M_w r^2 d^2 + I_w d^2 + 2 I_y r^2)}}$$

Equation of Motions:

$$\left(2M_w + M_c + \frac{2I_w}{r^2}\right)\ddot{x} = \left(\frac{T_L+T_R}{r}\right) - M_c l \ddot{\theta}_c \cos(\theta_c+\phi) - (M_c+2M_w) g \sin\phi + M_c l \dot{\theta}_c^2 \sin(\theta_c+\phi) \quad -①$$

$$(M_c l^2 + I_c)\ddot{\theta}_c = -(T_L+T_R) + M_c g l \sin\theta_c - M_c \ddot{x} l \cos(\theta_c+\phi) \quad -②$$

$$\ddot{\delta} = \frac{(T_L-T_R)\, d \cdot r}{(M_w d^2 r^2 + I_w d^2 + 2 I_y r^2)} \quad -③$$

Note that $\ddot{x}$ is in terms of $\ddot{\theta}$ in eq^n ① & $\ddot{\theta}$ in terms of $\ddot{x}$ in eq^n ②.. Also note that $\ddot{\delta}$ is independent of other state variable $x, \dot{x}, \theta_c, \dot{\theta}_c$ ----- May be that's why it's control is developed $\tau(\delta)$ differently. (Different Block).

chage $\ddot{\theta}_c$ in eq^n ① with the value of $\ddot{\theta}_c$ in eq②.

$$\ddot{\theta}_c = -\frac{(T_L+T_R)}{(M_c l^2 + I_c)} + \frac{M_c g l \sin\theta_c}{(M_c l^2+I_c)} - \frac{M_c \ddot{x} l \cos(\theta_c+\phi)}{(M_c l^2+I_c)}.$$

Let's make constant. $\lambda = M_c l^2 + I_c$

$$\ddot{\theta}_c = \frac{-(T_L+T_R)}{\lambda} + \frac{M_c g l \sin\theta_c}{\lambda} - \frac{M_c \ddot{x} l \cos(\theta_c+\phi)}{\lambda}.$$

Let's make another constant!

$$2M_w + M_c + \frac{2I_w}{r^2} = \frac{(2M_w r^2 + M_c r^2 + 2I_w)}{r^2} = \frac{\mu}{r^2}.$$

$$\mu = 2M_w r^2 + M_c r^2 + 2I_w$$

(18)

$$\frac{\mu}{r^2}\ddot{x} = \left(\frac{T_L+T_R}{r}\right) + \frac{M_c l (T_L+T_R)\cos(\theta_c+\phi)}{\lambda} - \frac{M_c^2 . l^2 . g . \sin\theta_c . \cos(\theta_c+\phi)}{\lambda}$$

$$+ \frac{M_c^2 l^2 (\ddot{x}) \cos^2(\theta_c+\phi)}{\lambda} - (M_c + 2M_w) g \sin\phi + M_c l \dot{\theta}_c^2 \sin(\theta_c+\phi)$$

$$\left(\frac{\mu}{r^2} - \frac{M_c^2 l^2 . \cos^2(\theta_c+\phi)}{\lambda}\right)\ddot{x} = - - - - .$$

$$\ddot{x} . \left(\frac{\mu\lambda - M_c^2 l^2 . r^2 . \cos^2(\theta_c+\phi)}{\lambda . r^2}\right) = - - - -$$

Let's make another constant. $\beta = M_c l r$.

$$\ddot{x}\left(\frac{\mu\lambda - \beta^2\cos^2(\theta_c+\phi)}{\lambda r^2}\right) = - - - - -$$

~~Let's make another~~

$$\ddot{x} = \frac{\lambda . r . (T_L+T_R)}{\mu\lambda - \beta^2\cos^2(\theta_c+\phi)} + \frac{M_c l r^2 \cos(\theta_c+\phi) . (T_L+T_R)}{\mu\lambda - \beta^2\cos^2(\theta_c+\phi)} - - - - -$$

$$= \underbrace{\left[\frac{\lambda . r . + \beta . r . \cos(\theta_c+\phi)}{\mu\lambda - \beta^2\cos^2(\theta_c+\phi)}\right](T_L+T_R)}_{\text{This is control input term.}} . - - - - - .$$

Let's work on other terms...

$$- \frac{r^2 . M_c^2 . l^2 . g \sin\theta_c \cos(\theta_c+\phi)}{\mu\lambda - \beta^2\cos^2(\theta_c+\phi)} = \frac{\beta^2 . g . \sin\theta_c . \cos(\theta_c+\phi)}{\mu\lambda - \beta^2\cos^2(\theta_c+\phi)}$$

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$$= -\frac{(M_c+2M_w)\cdot g\cdot\lambda\cdot r^2\cdot\sin\phi}{\mu\lambda-\beta^2\cos^2(\theta_c+\phi)} + \frac{\lambda\cdot r^2\cdot M_c\cdot l\cdot\dot{\theta}_c^2\cdot\sin(\theta_c+\phi)}{\mu\lambda-\beta^2\cos^2(\theta_c+\phi)}$$

$$+\frac{\beta\cdot\lambda\cdot r\cdot\dot{\theta}_c^2\sin(\theta_c+\phi)}{\mu\lambda-\beta^2\cos^2(\theta_c+\phi)}$$

Final eqⁿ of $\ddot{x}$ --- NOTE: NOT yet mentioned in terms of states but states (θ is there).

$$\ddot{x} = \frac{\beta\cdot\lambda\cdot r\cdot\dot{\theta}_c^2\sin(\theta_c+\phi) - (M_c+2M_w)g\cdot\lambda\cdot r^2\cdot\sin(\phi) - \beta^2\cdot g\cdot\sin(\theta_c)\cos(\theta_c+\phi)}{(\mu\lambda-\beta^2\cos^2(\theta_c+\phi))}$$

$$+\left[\frac{\lambda\cdot r+\beta\cdot r\cdot\cos(\theta_c+\phi)}{\mu\lambda-\beta^2\cos^2(\theta_c+\phi)}\right](T_L+T_R).$$

From above eqⁿ we can conclude that Linear accelⁿ is dependent on θ_c, ϕ, $\dot{\theta}_c$ & control (T_L+T_R).

Now to find eqⁿ of $\ddot{\theta}$, put eqⁿ ① in eqⁿ ②.

$$\frac{\mu}{r^2}\ddot{x} = \frac{(T_L+T_R)}{r} - M_c\cdot l\,\ddot{\theta}_c\cos(\theta_c+\phi) - (M_c+2M_w)\,g\sin\phi + M_c l\,\dot{\theta}_c^2\sin(\theta_c+\phi)$$

$$\ddot{x} = \frac{(T_L+T_R)\,r}{\mu} - \frac{\beta\cdot r\cdot\ddot{\theta}_c\cos(\theta_c+\phi)}{\mu} - \frac{(M_c+2M_w)\cdot g\cdot r^2\cdot\sin\phi}{\mu}$$

$$+\frac{\beta\cdot r\cdot\dot{\theta}_c^2\sin(\theta_c+\phi)}{\mu}$$

Let's put this value of $\ddot{x}$ in eqⁿ ②.

$$\ddot{\theta}_c = -\frac{(T_L+T_R)}{\lambda} + \frac{M_c g l \cdot \sin\theta_c}{\lambda} - \frac{M_c l \cos(\theta_c+\phi)}{\lambda} \cdot \frac{(T_L+T_R).r}{\mu}$$

$$+ \frac{M_c l \cos(\theta_c+\phi) \cdot \beta . r . \ddot{\theta}_c \cos(\theta_c+\phi)}{\lambda \cdot \mu}$$

$$+ \frac{M_c l \cos(\theta_c+\phi) \; (M_c+2M_w).g.r^2. \sin\phi}{\lambda \cdot \mu}$$

$$- \frac{M_c . l. \cos(\theta_c+\phi) \cdot \beta . r . \dot{\theta}_c^2 \sin(\theta_c+\phi)}{\lambda \cdot \mu}$$

$$-\left(\frac{\beta \cos(\theta_c+\phi)}{\lambda\mu} + \frac{1}{\lambda}\right)(T_L+T_R).$$

$$\ddot{\theta}_c \left(1 - \frac{\beta^2 . \cos^2(\theta_c+\phi)}{\lambda\mu}\right)$$

$$\ddot{\theta}_c \left(\frac{\lambda\mu - \beta^2 \cos^2(\theta_c+\phi)}{\lambda\mu}\right) = \text{-----}$$

$$\ddot{\theta}_c = -\left[\frac{\mu + \beta\cos(\theta_c+\phi)}{\lambda\mu - \beta^2\cos^2(\theta_c+\phi)}\right](T_L+T_R)$$

$$+ \frac{\left(\mu M_c g . l . \sin(\theta_c) + \beta . r . g . (M_c+2M_w) . \sin\phi . \cos(\theta_c+\phi) - \beta^2 . \dot{\theta}_c^2 . \sin(\theta_c+\phi) \cos(\theta_c+\phi)\right)}{\mu\lambda - \beta^2 \cos^2(\theta_c+\phi)}$$

At equilibrium point, the wheel acceleration is zero ($\ddot{x}=0$) 11

$\ddot{X}=0$, Pendulum angular velocity and acceleration are zero

$\dot{\theta}_c=\ddot{\theta}_c=0$.

from eq ① & ②.

eq ① $\Rightarrow$ $0=\left(\frac{T_L+T_R}{r}\right)-(M_c+2M_w)g\sin\phi$

eq ② $\Rightarrow$ $0=-(T_L+T_R)+M_c g l \sin\underline{\underline{\theta_c}} \rightarrow (\theta_R) \rightarrow$ Reference position of pendulum

$$\theta_r=\arcsin\left(\frac{T_L+T_R}{M_c g l}\right)$$

$$\boxed{\theta_r=\arcsin\left(\frac{(M_c+2M_w)\cancel{g}.r.\sin\phi}{M_c g l}\right)}$$

On flat surface, $\phi=0$., $\theta_r=0$, $\theta_c=0$.

on an inclined surface, $\theta_r=$ - - -

$x_1=X$

$x_2=\dot{X}$

$x_3=\theta_c-\theta_r=e_3$ $\quad \theta_c=x_3+\theta_r$

$x_4=\dot{\theta}_c$

$x_5=\delta$

$x_6=\dot{\delta}$

$u_1=T_L$

$u_2=T_R$

$$\theta_c + \phi = \underline{\underline{(x_3 + \theta_r + \phi)}} \qquad \underline{\underline{\theta_c = x_3 + \theta_r}} \qquad 12$$

$$\dot{x}_1 = x_2$$

$$\dot{\theta}_c = x_4$$

$$\dot{x}_2 = \ddot{x} =$$

$$= \frac{\left(\beta.\lambda.r.x_4^2.\sin(x_3+\theta_r+\phi) - (M_c+2M_w)g.\lambda.r^2\sin(\phi) - \beta^2.g.\sin(x_3+\theta_r).\cos(x_3+\theta_r+\phi)\right)}{(\mu\lambda - \beta^2\cos^2(\theta_c+\phi))}$$

$$+ \left[\frac{\lambda.r + \beta.r.\cos(x_3+\theta_r+\phi)}{\mu\lambda - \beta^2\cos^2(x_3+\theta_r+\phi)}\right](u_1+u_2).$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \left[\frac{\left(\mu M_c g.l.\sin(x_3+\theta_r) + \beta.r.g.(M_c+2M_w)\sin\phi.\cos(x_3+\theta_r+\phi) - \beta^2.\underline{(x_4^2)}.\sin(x_3+\theta_r+\phi).\cos(x_3+\theta_r+\phi)\right)}{\mu\lambda - \beta^2\cos^2(\theta_c+\phi)}\right]$$

$$- \frac{\mu + \beta\cos(x_3+\theta_r+\phi)}{\mu\lambda - \beta^2\cos^2(x_3+\theta_r+\phi)}(u_1+u_2).$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = \frac{d.r}{\alpha}(u_1 - u_2) \qquad \boxed{\alpha = M_w d^2 r^2 + I_w d^2 + 2I_y r^2.}$$

very Important Page

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Linear Model: To perform the test of controllability & observability of the model, we need linearized approximation of this non-linear model.

→ To determine the equilibrium point

~~Step 1~~ :

$0 = f(\bar{x}, \bar{u})$ $\bar{x}, \bar{u} \to$ equilibrium point.

$\dot{x} = 0$

$x_2 = 0$

$\dot{x}_2 = 0 = \ddot{x}$ $x_3 + \theta_r + \phi = \theta_c = \cdot\cdot (x_3)$.

equilibrium point $\bar{x}_3 = 0$ $\bar{u}_1 = 0$.

(refer to katriya's report).

$x_3 = \theta_c - \theta_r = e_3$

$x_3 =$ small

$\sin(x_3) = x_3$ $\cos(x_3) = 1$

$\cos(x_3 + \theta_r + \phi) = \cos(\theta_r + \phi)$

$\sin(x_3 + \theta_r) = x_3 \cos(\theta_r) + \sin(\theta_r)$.

Jacobian Matrices/Matrix

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_6} \\ \vdots & \ddots & \\ \frac{\partial f_6}{\partial x_1} & \cdots & \frac{\partial f_6}{\partial x_6} \end{bmatrix}_{\bar{x}, \bar{u}} \quad \text{and} \quad B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_2} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_6}{\partial u_1} & & \frac{\partial f_6}{\partial u_2} \end{bmatrix}_{\bar{x}, \bar{u}}.$$

$$\frac{\partial f_2}{\partial x_3} = \frac{\partial}{\partial x_3}\left[- \frac{\beta^2 g \cdot x_3 \cos(\theta_r) \cdot \cos(\theta_r + \phi)}{\mu\lambda - \beta^2 \cos^2(\theta_c + \phi)} \right]$$

(Note: we are not considering other terms because they will be '0' after derivative or they are not funⁿ of x_3).

$$\boxed{A_{23} = \frac{-\beta^2 \cdot g \cdot \cos(\theta_r) \cdot \cos(\theta_r + \phi)}{\mu\lambda - \beta^2 \cos^2(\theta_r + \phi)}}$$

$$\frac{\partial f_2}{\partial u_2} = \frac{\partial f_2}{\partial u_1} = \boxed{\frac{\lambda . r. + \beta . r. \cos(\theta_r + \phi)}{\mu\lambda - \beta^2 \cos^2(\theta_r + \phi)} = B_{21} = B_{22}}$$

$$\frac{\partial f_4}{\partial x_3} = \frac{d}{dx_3}\left(\mu M_c g l \; x_3 \sin(\theta_r)\right)$$

$$\boxed{= \frac{\mu M_c g l \cos(\theta_r)}{\mu\lambda - \beta^2 \cos^2(\theta_r + \phi)} = A_{43}}$$

$$\dot{\theta}_c = x_4 = 0$$

$$\frac{\partial f_4}{\partial u_2} \; \frac{\partial f_4}{\partial u_1} = \boxed{\frac{\mu + \beta \cos(\theta_r + \phi)}{\mu\lambda - \beta^2 \cos^2(\theta_r + \phi)} = B_{41} = B_{42}}$$

$$\frac{\partial f_6}{\partial x_i} = 0$$

$$\boxed{\frac{\partial f_6}{\partial u_1} = \frac{d . r.}{\alpha} = B_{61} \qquad \frac{\partial f_6}{\partial u_1} = -\frac{d r}{\alpha} = B_{62}}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{21} & B_{22} \\ 0 & 0 \\ B_{41} & B_{42} \\ 0 & 0 \\ B_{61} & B_{62} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Control Design

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1. State feedback. – It is sufficient that system is "controllable"
2. Output feedback → System must be "observable"

Controllability : The system (A,B) is controllable if there exists a (piecewise continuous) control signal $u(t)$ that will take the state of the system from any intital state x_i to any desired final state x_f in a finite interval of time.

Mathematically, controllability theorem states that (A,B) is controllable if only if ...

$$\text{rank}\left[B,\ AB,\ A^2B \ ---\ A^{n-1}B\right] = n = \dim(x).$$

A & B are from Linear Model of wheeled inverted pendulum.

State feedback control system

$$\text{Plant} \quad \dot{x}(t) = f(x(t), u(t))$$

$$\text{State-feedback} \quad u(t) = -K\,x(t).$$

State feedback is:

$$u = -Kx = -\begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

① Define MATLAB function – Non-linear State-space model.

Significance of phase margine & gain margine.

↳ observability used
↳ it reduces above GM, PM.

Observability: All the states are not always measured with sensors

Hence some states are estimated.

Continuous-time linear system:

$$\dot{x} = Ax + Bu$$
$$y = Cx.$$

y = output of the system = Vector of sensor measurements & number of elements in y is equal to the number of sensors being used.

C = output matrix :

Since observability is a sufficient condition for output-feedback design, our goal here is to find an observable system. with the least number of sensor possible.

The system (A, C) is observable if, for any $x(0)$ (Initial State), there is a finite time 'T'. such that $x(0)$ can be determined (uniquely) from $u(t)$ and $y(t)$ for $0 \leq t \leq T$.

Mathematically, observability theorem ..

$$\text{rank} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n = \dim(x)$$

C is determined after sensor selection.

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_L(t) \\ x_R(t) \\ \ddot{\theta}_c(t) \end{bmatrix}$$

Output - feedback control

'^'-Cap denotes estimated states.

Plant : $\dot{x}(t) = f(x(t), u(t))$

Estimator : $\dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) - H[C\hat{x}(t) - y(t)]$

Estimated state-feedback $= u(t) = -K\hat{x}(t)$.

Note that for plant & estimator we use NON-LINER equation of sets.

$K = lqr(A, B, Q, R)$

$H = \underbrace{lqe}_{\text{Linear quadratic estimator.}}(A, eye(4), C, \underbrace{Rw, Rv}_{\text{Noise}})$

LQR = Dynamic System expressed as Linear systems at equillibrium.

Qudratic objective function with states & control i/p

~~For this method~~

Continuous time Linear System

$$\dot{x} = Ax + Bu$$

The cost function of the performance index · is given by

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt.$$

u^* = is the feedback control law that minimizes the value of cost.

$$u^* = -R^{-1} B^T P x = -Kx$$ Determines the location of poles

↳ ?

P is solⁿ of algebric eqⁿ:

$$PA + A^T P + Q - PBR^{-1}B^T P = 0.$$