Set 10 : Modelling epidemics and endemic breakouts of infectious diseases

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This report presents the modelling of epidemics and endemic breakouts of infectious diseases using coupled differential equations. The numerical simulations are carried out using Euler's method, and the results are analyzed to understand the dynamics of the infectious diseases within the population.

I. EPIDEMICS

Consider a population size of N, divided into three classes: the infected class $x(t)$, the susceptible class $y(t)$, and the recovered class $z(t)$, where $x(t)+y(t)+z(t) = N$ (constant). The coupled dynamics of these variables is given by:

$$\dot{x} = Axy - Bx \tag{1}$$
$$\dot{y} = -Axy \tag{2}$$
$$\dot{z} = Bx \tag{3}$$

where A is the infection rate and B is the removal rate ($A, B > 0$). At $t = 0$, $x(0) = x_0$ and $z(0) = 0$, hence $y(0) = y_0 = N - x_0$.

The total number of students in a boarding school is 763. Initially, a single student introduces an infectious disease in this population. Take $A = 2.18 \times 10^{-3}\,\text{day}^{-1}$ and $B = 0.44\,\text{day}^{-1}$.

A. Numerical Integration by Euler's Method

The variables $x(t)$, $y(t)$, and $z(t)$ are integrated using Euler's method with a carefully chosen time step Δt (maybe an hour).

1. Plot of x and y

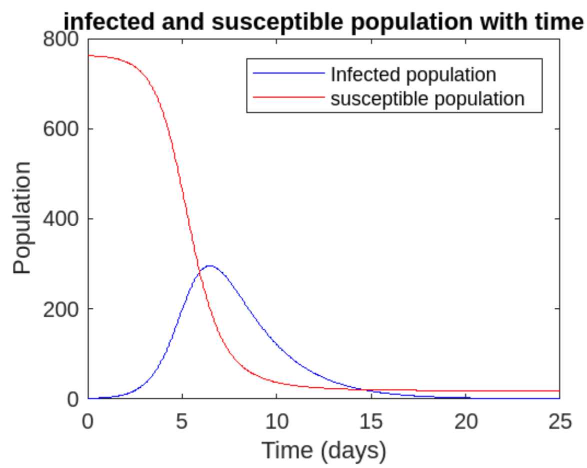

FIG. 1. Plot of x and y with respect to time t (days).

- Time when x reaches its maximum value : 6.4583 days
- $\Delta t = 1 hour$.

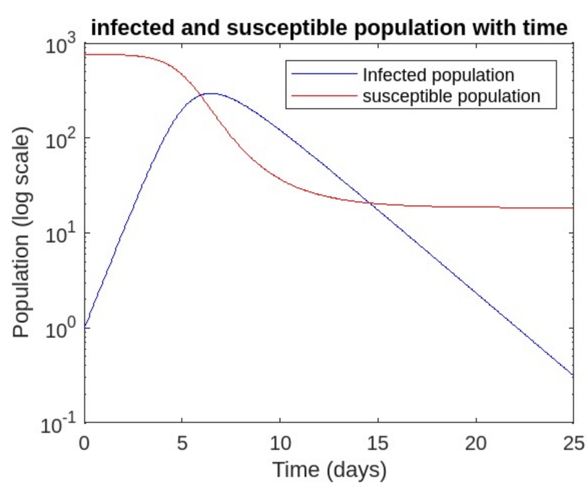

FIG. 2. Logarithmic plot of x and y with respect to time t (days).

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2. *Plot of z*

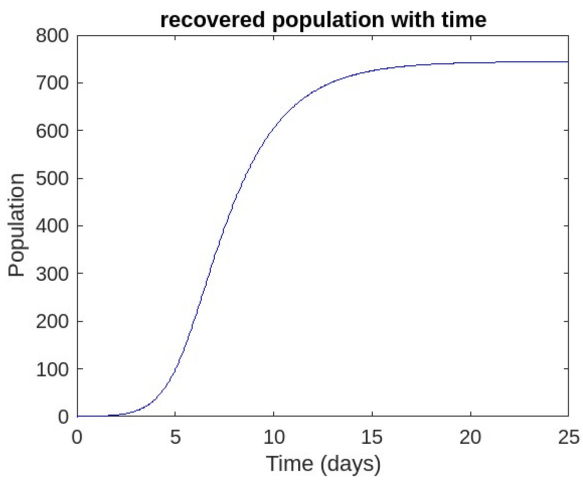

FIG. 3. Plot of z with respect to time t (days).

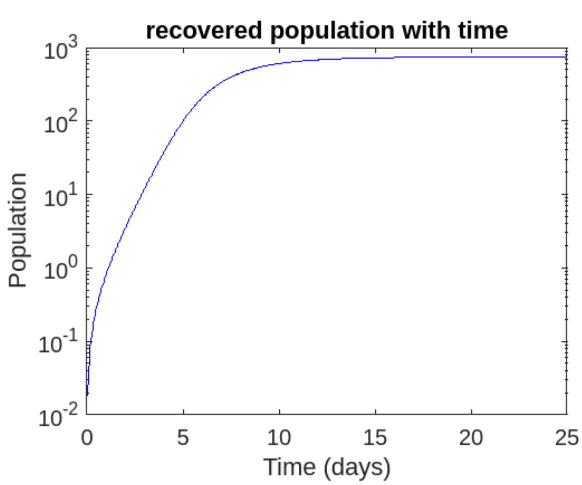

FIG. 4. Logarithmic plot of z with respect to time t (days).

3. *Comparison of x and y*

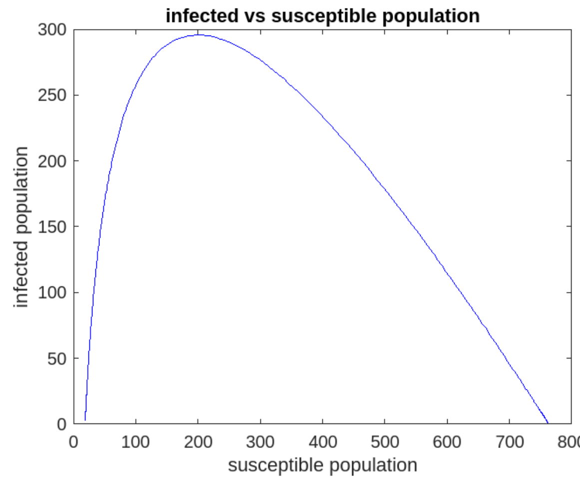

FIG. 5. Plot of x against y.

- Value of B/A = 201 which matches the susceptible population at the maximum infected population in the graph.

- Value of R = 3.7338 i.e R¿1 suggests an epidemic will break out.

II. ENDEMIC DISEASES

For endemic diseases, the total population size N changes over time. Consider the case $N \equiv N(t)$, where the per capita death rate is a and the per capita birth rate is b $(a, b > 0)$. The relevant coupled system of equations is given by:

$$\dot{x} = Axy - Bx - ax \tag{4}$$
$$\dot{y} = bN - Axy - ay \tag{5}$$
$$\dot{z} = Bx - az \tag{6}$$
$$\dot{N} = (b - a)N \tag{7}$$

Consider the case of $a = b = 0.02\,\text{year}^{-1}$ so that $\dot{N} = 0$, i.e., N is fixed. Take $A = 10^{-6}\,\text{year}^{-1}$, $B = 0.333\,\text{year}^{-1}$, $N = 10^6$, $x_0 = 10^5$, and $y_0 = 9 \times 10^5$.

A. Numerical Integration by Euler's Method

The variables $x(t)$, $y(t)$, and $z(t)$ are integrated using Euler's method with a carefully chosen time step Δt (maybe a day).

1. *Plot of x*

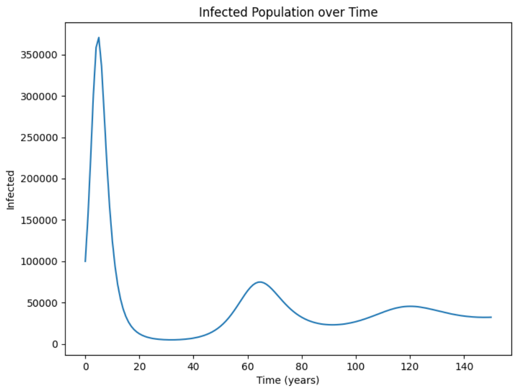

FIG. 6. Plot of x with respect to time t (years), where Δ t = 1 day and $x_0 = 10^5$

- At peak 1 value of time t = 4.2441 years.
- At peak 2 value of time t = 63.8358 years.
- At peak 3 value of time t = 120.7018 years.
- At peak 4 value of time t = 177.6174 years.

2. *Plot of y*

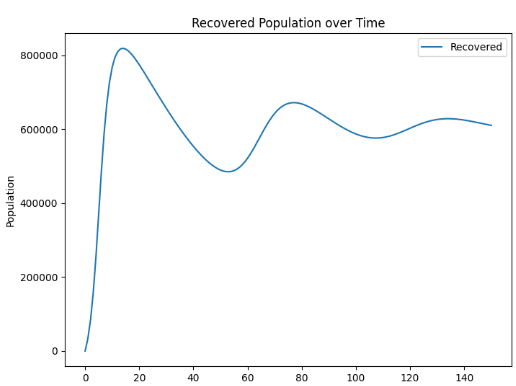

FIG. 7. Plot of y with respect to time t (years), where Δ t = 1 day and $y_0 = 9 \times 10^5$

3. *Plot of z*

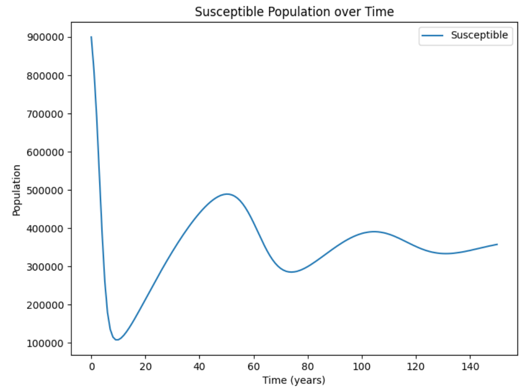

FIG. 8. Plot of z with respect to time t (years), where Δ t = 1 day and $z_0 = 0$