Lab - 3 :Analysis of Modified Logistic Equation and Agricultural Innovation Spread

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We examine the harvesting model analytically and quantitatively in this lab. Additionally, we map the Euler solution to the problem of agricultural improvements.

I. MODEL

The modified logistic equation is as follows

$$\dot{x} = f(x) = rx[1 - (x/k)] - h \tag{1}$$

Where h is the "harvesting" rate.

The x(t) by Euler's method is given by

$$x(n+1) = x(n) + f(x(n))\Delta t \tag{2}$$

The dynamical equation for the spread of agricultural innovations among farmers through personal communications is given by

$$\dot{x} = Cx(n - x) \tag{3}$$

The x(t) for the spread of agricultural innovations among farmers through personal communications is given by

$$x = \frac{Ne^{CNt}}{(N-1) + e^{CNt}} \tag{4}$$

The following is the dynamical equation that describes how agricultural innovations spread among farmers through impersonal communications:

$$\dot{x} = (Cx + C')(n - x) \tag{5}$$

The x(t) for the spread of agricultural innovations among farmers through impersonal communications is given by

$$x = \frac{NC'[1 - e^{-(CN+C')t}]}{C' + CNe^{-(CN+C')t}} \tag{6}$$

Defining X = x/N, T = cNt, A = C/CN and $\dot{X}$ dX/dT,

Recast dynamical equation for the spread of agricultural innovations among farmers through personal communications is given by

$$\dot{X} = X(1 - X) \tag{7}$$

X(T) for the spread of agricultural innovations among farmers through personal communications is given by

$$X = \frac{1}{1 + A^{-1}e^{-T}} \tag{8}$$

Recast dynamical equation for the spread of agricultural innovations among farmers through impersonal communications is given by

$$\dot{X} = (X + A)(1 - X) \tag{9}$$

X(T) for the spread of agricultural innovations among farmers through impersonal communications is given by

$$X = \frac{1 - e^{-(1+A)T}}{1 + A^{-1}e^{-(1+A)T}} \tag{10}$$

The relative error between the analytical solution and the numerical solution is given by,

$$relative\ error = \frac{(analytical\ solution) - (numerical\ solution)}{(analytical\ solution)}$$

II. RESULTS

A. Harvesting Rate

Fig. 1 shows $\dot{x}$ versus x with h = 0, 100, 500.

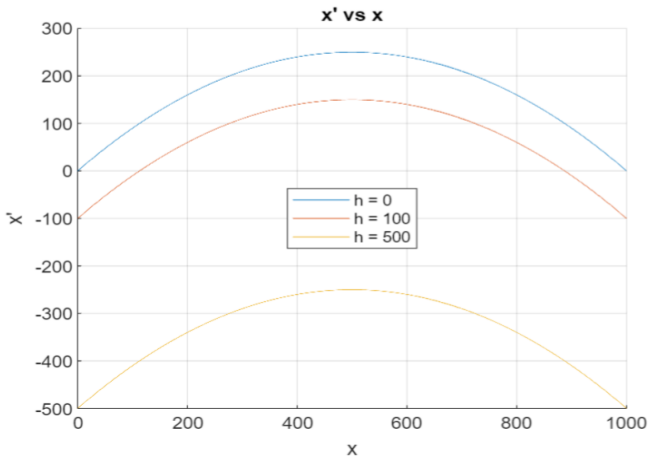

FIG. 1: Here r = 1 and k = 1000.

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Fig. 2 shows x (by Euler's method) versus t with h = 0, 100, 500.

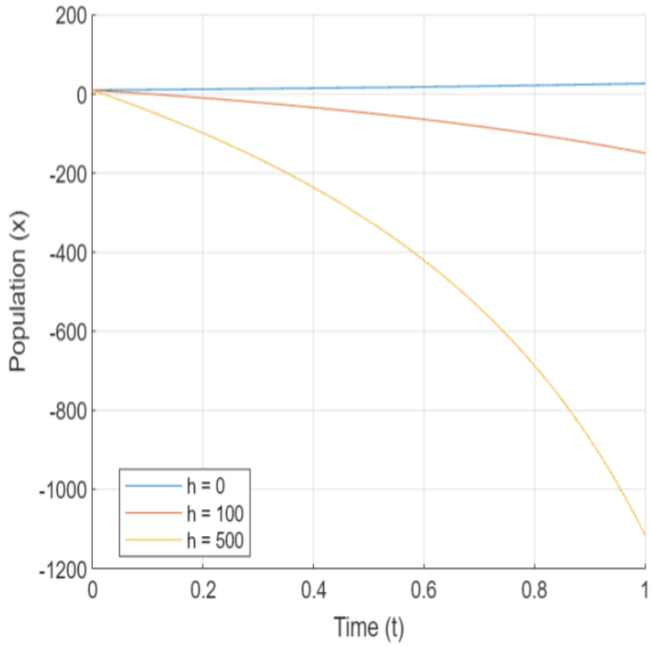

FIG. 2: Here the initial value of x(0) = 10, with $\Delta t = 0.01$

The graph below is linearly increasing, we can say that exact values(analytical solution) is increasing more than numerical values over time.

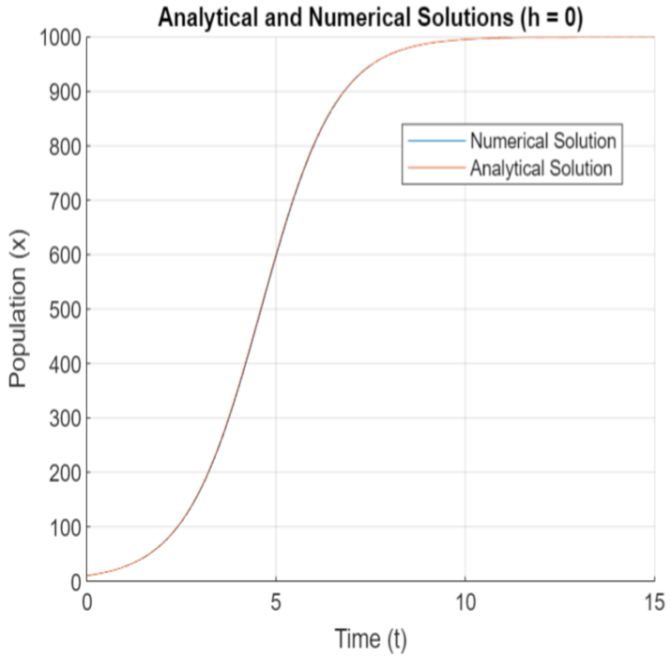

FIG. 3: Here shows comparison between the analytical solution and the numerical solution for h=0, with $\Delta t = 0.01$

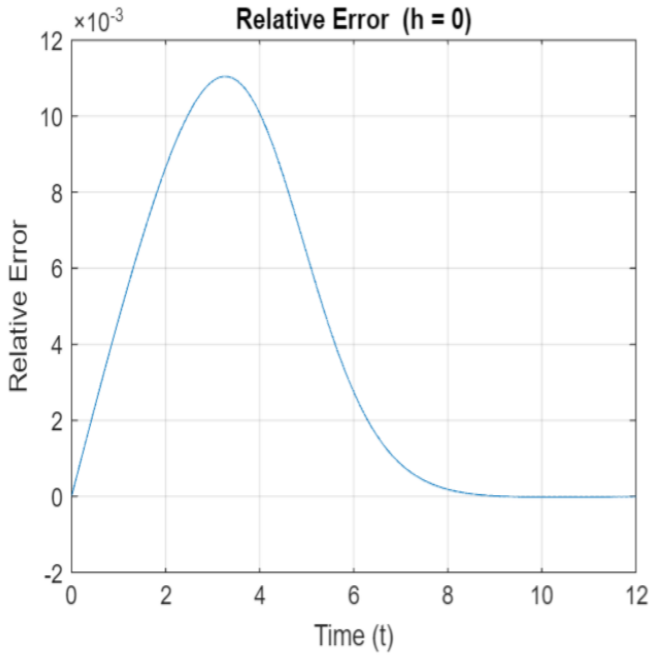

FIG. 4: Here The relative error between the analytical solution and the numerical solution for h=0.

B. Agriculture Innovation

The below graph shows $\dot{X}$ versus X for A = 0, 0.2, 0.5.

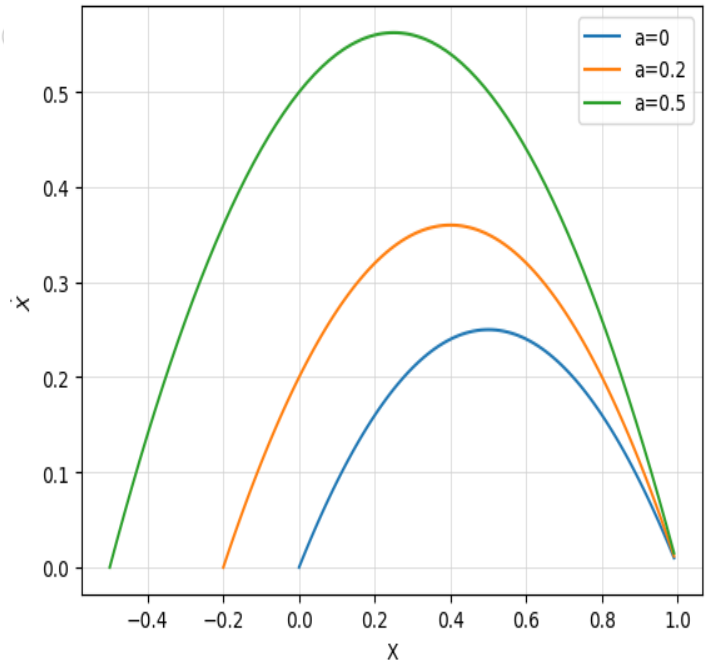

FIG. 5: Plot $\dot{X}$ versus X for A = 0, 0.2, 0.5 and X(0) = 0.0001

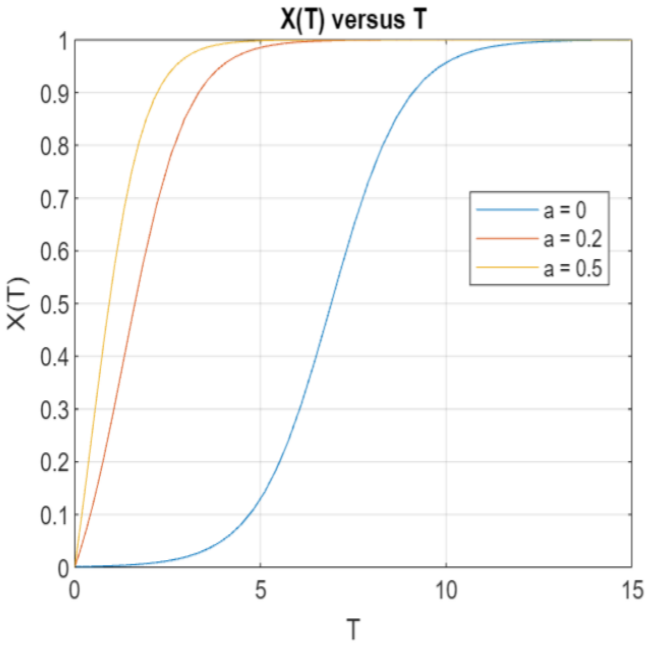

FIG. 6: Plot of the integral solution X(T) for A = 0, 0.2, 0.5 and X(0) = 0.0001

III. CONCLUSION

- As the value of a increase the agricultural evolution will spread more rapidly.
- The error between the analytical and the numerical solution if very minimal for harvesting rate equals to zero.
- We can say that as harvesting rate increases the population decreases initially.