Set 9 - Competitive Exclusion and Predator-Prey Dynamics

Vishal Dhoriya(202101446)* and Akhil Patoliya(202101505)†
Dhirubhai Ambani Institute of Information & Communication Technology,
Gandhinagar, Gujarat 382007, India
CS302, Modelling and Simulation

This report explores competitive exclusion and predator-prey dynamics through mathematical modeling. The principles are illustrated through numerical simulations using Euler's method and various initial conditions.

I. INTRODUCTION

The dynamics of ecological systems are often modeled using differential equations, allowing us to study the interactions between species. Competitive exclusion refers to the principle where similar species compete for resources, leading to the extinction of one species. Predator-prey dynamics involve the interaction between predator and prey populations. In this report, we investigate these phenomena through mathematical models and numerical simulations.

II. COMPETITIVE EXCLUSION

A. Model Description

The growth of two species, X and Y, is modeled by the following coupled equations:

$$\dot{x} = Ax - Bx^2 - \alpha xy \tag{1}$$
$$\dot{y} = Cy - Dy^2 - \beta xy \tag{2}$$

where $x(t)$ and $y(t)$ represent the population densities of species X and Y, respectively, and A, B, C, D, α, and β are positive constants.

B. Numerical Simulation

We perform numerical simulations using Euler's method with the given parameters and initial conditions.

1. *Case A*

We start with $x(0) = 0.5$, $y(0) = 1.5$, $B = D = 0$, $\alpha = 0.05289$, and $\beta = 0.00459$.

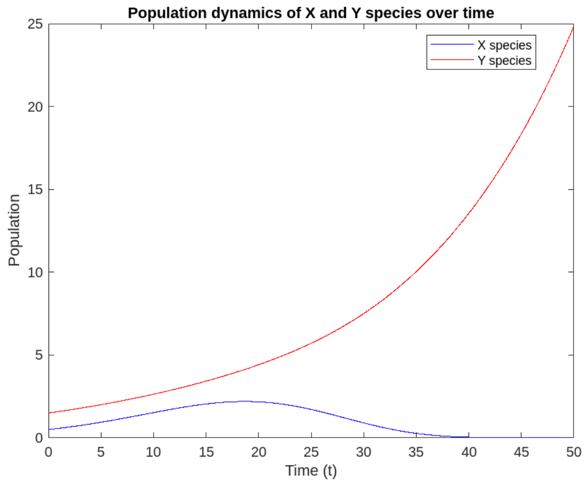

FIG. 1: Plot of $x(t)$ and $y(t)$ with initial conditions specified in Case A.

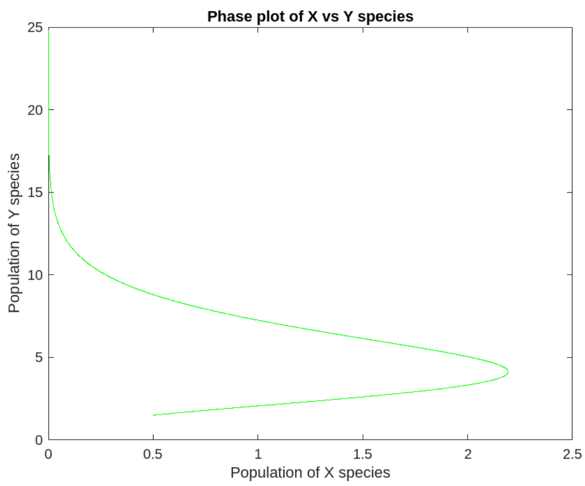

FIG. 2: Plot of $x(t)$ against $y(t)$ with initial conditions specified in Case A.

*Electronic address: 202101446@daiict.ac.in
†Electronic address: 202101505@daiict.ac.in

2. *Case B*

We take $x(0) = y(0) = 0.5$, $B = 0.017$, and $D = 0.010$ using the previous values of α and β.

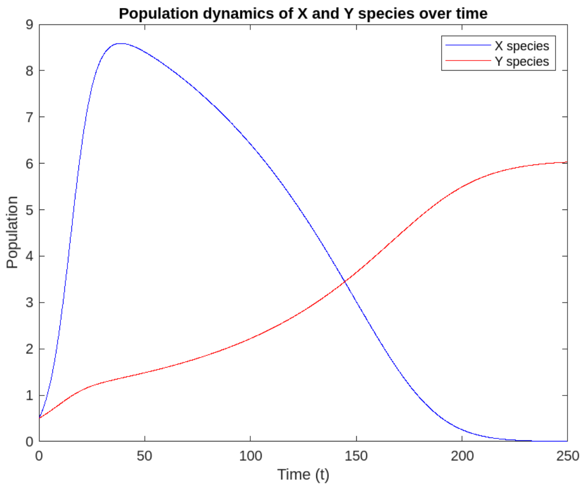

FIG. 3: Plot of $x(t)$ and $y(t)$ with initial conditions specified in Case B.

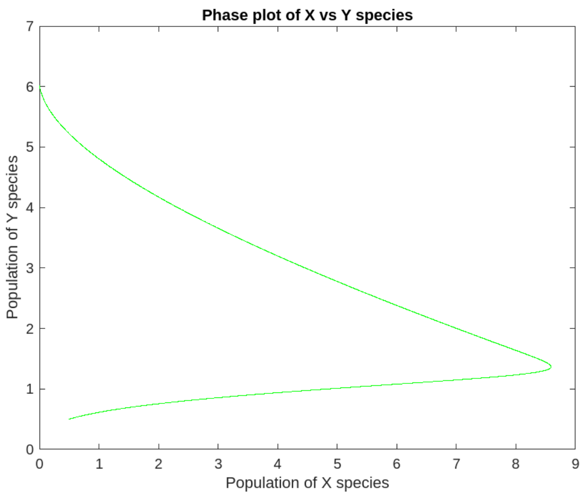

FIG. 4: Plot of $x(t)$ against $y(t)$ with initial conditions specified in Case B.

- Maximum value of x: 8.58
- Maximum value of y: 6.028

3. *Case C*

For no competition, we set $\alpha = \beta = 0$ with the second set of initial conditions and the values of B and D.

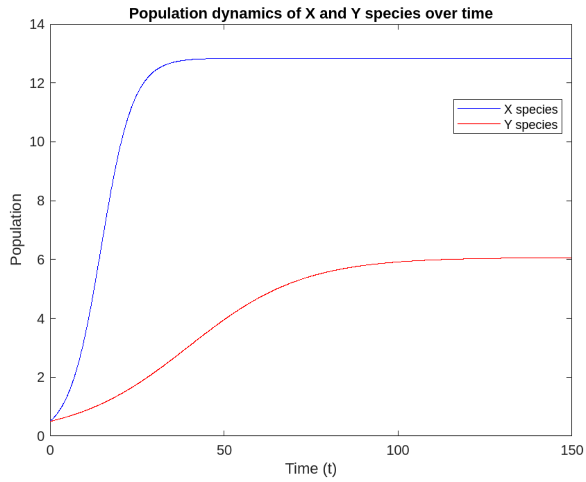

FIG. 5: Plot of $x(t)$ and $y(t)$ with initial conditions specified in Case C.

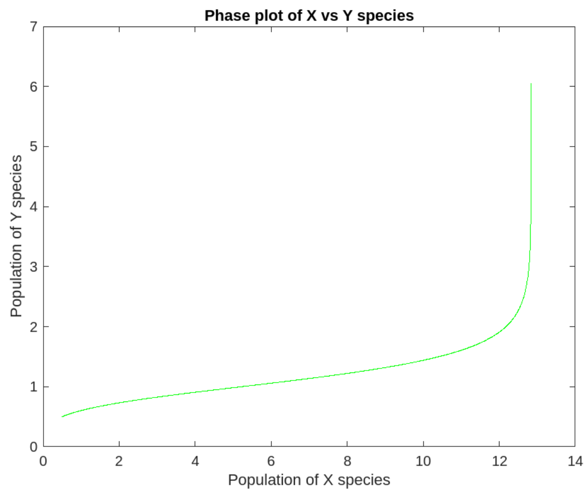

FIG. 6: Plot of $x(t)$ against $y(t)$ with initial conditions specified in Case C.

- Maximum value of x: 12.83
- Maximum value of y: 6.06

III. PREDATOR-PREY DYNAMICS

A. Model Description

The interaction between a prey species X and a predator species Y is modeled by the following coupled equations:

$$\dot{x} = Ax - Bxy - \epsilon x \tag{3}$$
$$\dot{y} = -Cy + Dxy - \epsilon y \tag{4}$$

where $x(t)$ and $y(t)$ represent the population densities of species X (prey) and Y (predator), respectively, and A, B, C, D, and ϵ are constants.

B. Numerical Simulation

We perform numerical simulations using Euler's method with the given parameters and initial conditions.

1. *Case A*

Starting with $x(0) = 200$, $y(0) = 80$, and $\epsilon = 0$.

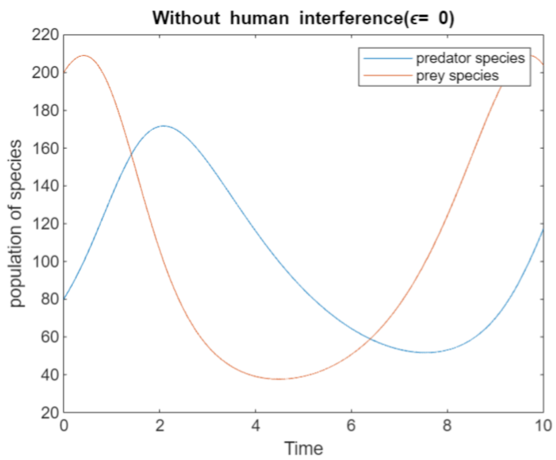

FIG. 7: Plot of $x(t)$ and $y(t)$ with initial conditions specified in Case A.

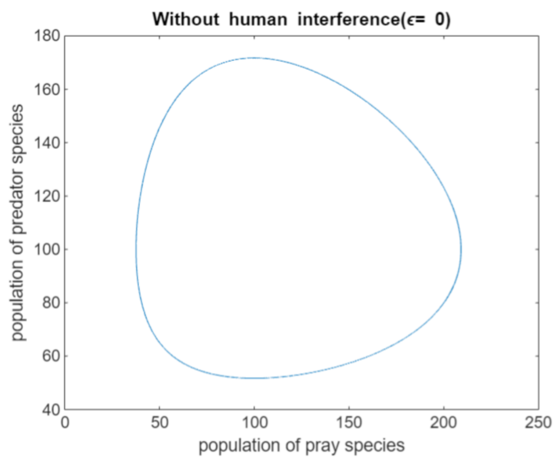

FIG. 8: Plot of $x(t)$ against $y(t)$ with initial conditions specified in Case A.

- Maximum value of y: 171.79

2. *Case B*

When human interference occurs, we choose $\epsilon = 0.1$.

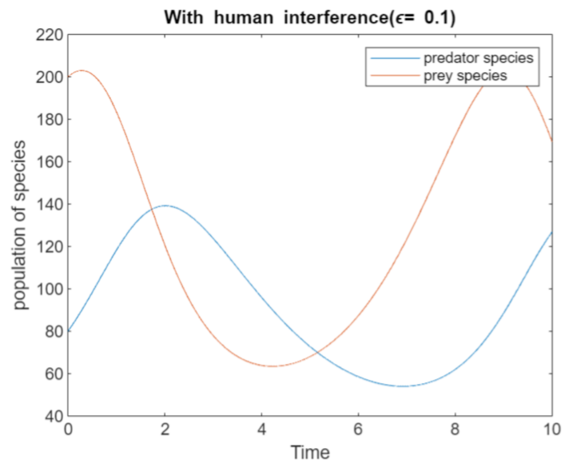

FIG. 9: Plot of $x(t)$ and $y(t)$ with initial conditions specified in Case B.

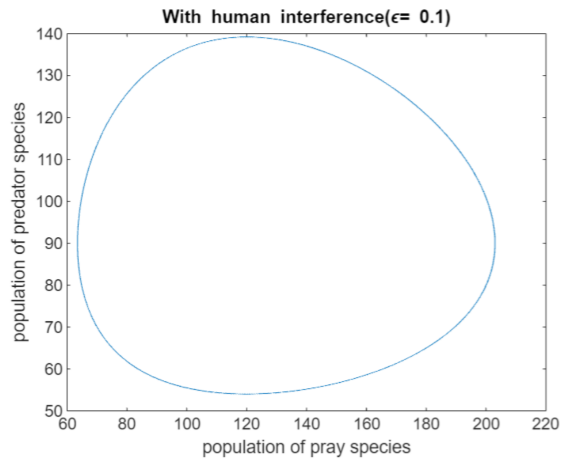

FIG. 10: Plot of $x(t)$ against $y(t)$ with initial conditions specified in Case B.

- Maximum value of y: 139.24

3. Case C

Taking $x(0) = 200$ but $y(0) = 0$ (no predator), we integrate $x(t)$ and plot its logarithm against t.

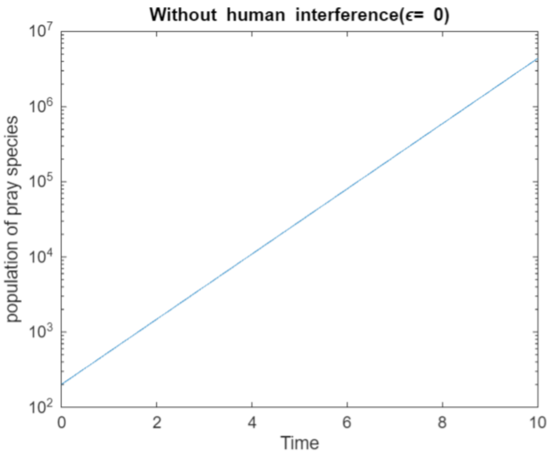

FIG. 11: Plot of the logarithm of $x(t)$ with initial conditions specified in Case C.

The log-linear plot is observed to be linearly increasing, suggesting that the population of prey grows exponentially over time.

4. Case D

Taking $y(0) = 80$ but $x(0) = 0$ (no prey), we integrate $y(t)$ and plot its logarithm against t.

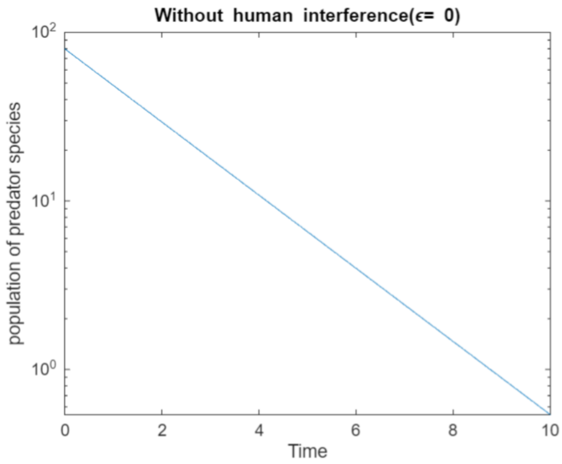

FIG. 12: Plot of the logarithm of $y(t)$ with initial conditions specified in Case D.

The log-linear plot is observed to be linearly decreasing, suggesting that the population of predator decreases exponentially over time.