Lab - 1 : GDP and Trade Modeling for Top Six Economies using logistic equation

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In this lab, we examined the annual GDP and trade growth of the United States, the United Kingdom, Japan, China, and India. We used the logistic equation to perform modeling. Additionally, we have included the model's statistical analysis.

I. MODEL

The logistic equation exhibits sigmoid growth curves, characterized by an initial rapid growth phase, followed by a gradual leveling off as the value approaches the carrying capacity. This makes it a valuable tool in various fields such as biology, ecology, and economics for modeling population dynamics and predicting long-term trends.

$$\dot{x} = ax - bx^2 \tag{1}$$

With initial condition $x(0) = x_0$ and taking $k = a/b$,

$$x(t) = \frac{kx_0e^{at}}{k + x_0(e^{at} - 1)} \tag{2}$$

Using x = k/2 in Eq. (2) gives the nonlinear time scale

$$t_{nl} = \frac{1}{a}\ln(\frac{k}{x_0} - 1) \tag{3}$$

The relation between GDP and Trade is described by following equation,

$$G(T) \approx T^{\alpha} \tag{4}$$

And for prediction the year when a specific economy reaches the provided value we use the following equation where we will take b = 1960 (according to the data provided)

$$y = b + \frac{1}{a}\ln[\frac{x(k - x_0)}{x_0(k - x)}] \tag{5}$$

II. RESULTS

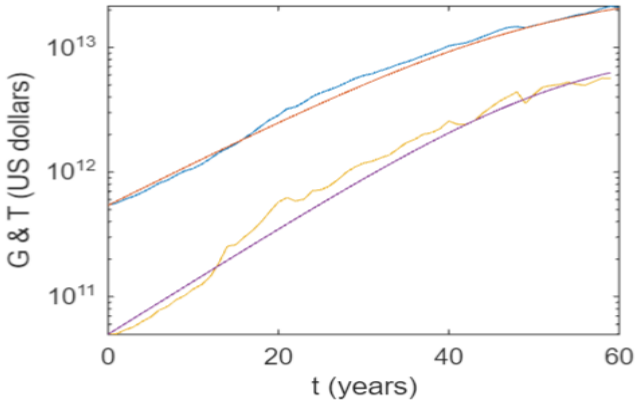

FIG. 1: Figure shows GDP and Trade modeling of USA. Parameters of GDP model are x_{G0} = $ 500 billion , k_G = $30 Trillions , a_G = 0.08 Parameters of Trade model are x_{T0} = $ 50 billion , k_T = $10 Trillions , $a_T = 0.099$.
Statistical measures $\mu_G = 0.11, \sigma_G = 0.094, \mu_T = 0.11, \sigma_T = 0.20$

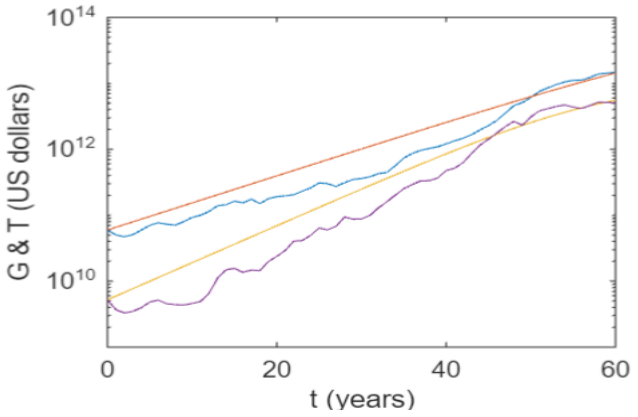

FIG. 2: Figure shows GDP and Trade modeling of China. Parameters of GDP model are x_{G0} = $ 60 billion , k_G = $80 Trillions , a_G = 0.095 Parameters of Trade model are x_{T0} = $ 6 billion , k_T = $10 Trillions , $a_T = 0.13$.
Statistical measures $\mu_G = -0.34, \sigma_G = 0.25, \mu_T = -0.44, \sigma_T = 0.30$

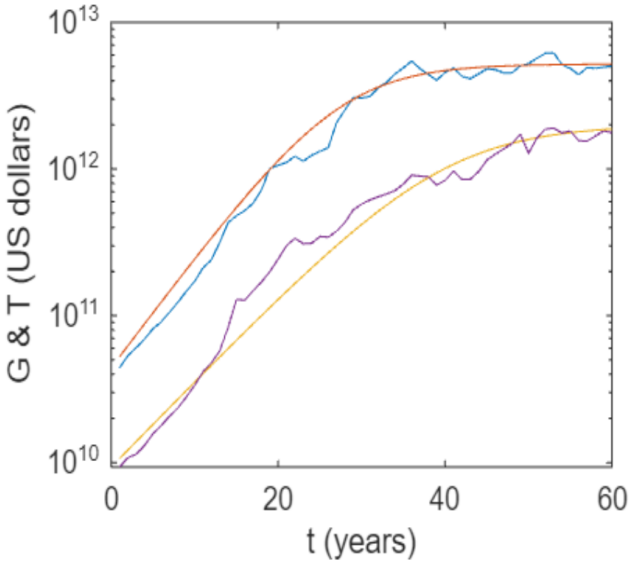

FIG. 3: Figure shows GDP and Trade modeling of Japan. Parameters of GDP model are x_{G0} = \$ 45 billion , k_G = \$5.2 Trillions , a_G = 0.175 Parameters of Trade model are x_{T0} = \$ 10 billion , k_T = \$2 Trillions , $a_T = 0.135$.
Statistical measures $\mu_G = -0.04, \sigma_G = 0.12, \mu_T = 0.19, \sigma_T = 0.36$

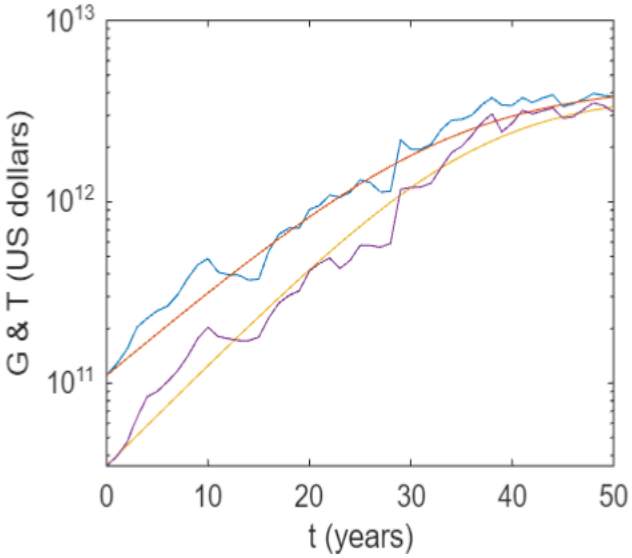

FIG. 4: Figure shows GDP and Trade modeling of Germany. Parameters of GDP model are x_{G0} = \$ 110 billion , k_G = \$4.4 Trillions , a_G = 0.11 Parameters of Trade model are x_{T0} = \$ 35 billion , k_T = \$3.9 Trillions , $a_T = 0.13$.
Statistical measures $\mu_G = 0.11, \sigma_G = 0.18, \mu_T = 0.0056, \sigma_T = 0.23$

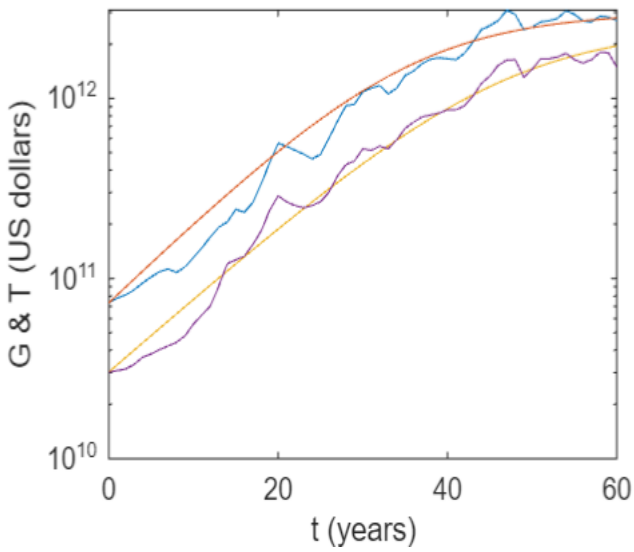

FIG. 5: Figure shows GDP and Trade modeling of UK. Parameters of GDP model are x_{G0} = \$ 73 billion , k_G = \$3 Trillions , $a_G = 0.1$ Parameters of Trade model are x_{T0} = \$ 30 billion , k_T = \$2.5 Trillions , $a_T = 0.09$.
Statistical measures $\mu_G = 0.09, \sigma_G = 0.16, \mu_T = 0.005, \sigma_T = 0.16$

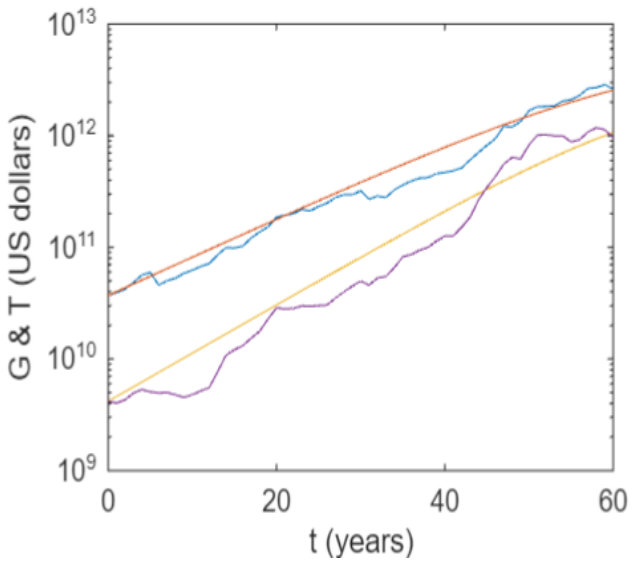

FIG. 6: Figure shows GDP and Trade modeling of India. Parameters of GDP model are x_{G0} = \$ 37 billion , k_G = \$6 Trillions , $a_G = 0.08$ Parameters of Trade model are x_{T0} = \$ 4.2 billion , k_T = \$3 Trillions , $a_T = 0.1$.
Statistical measures $\mu_G = -0.13, \sigma_G = 0.17, \mu_T = -0.18, \sigma_T = 0.34$

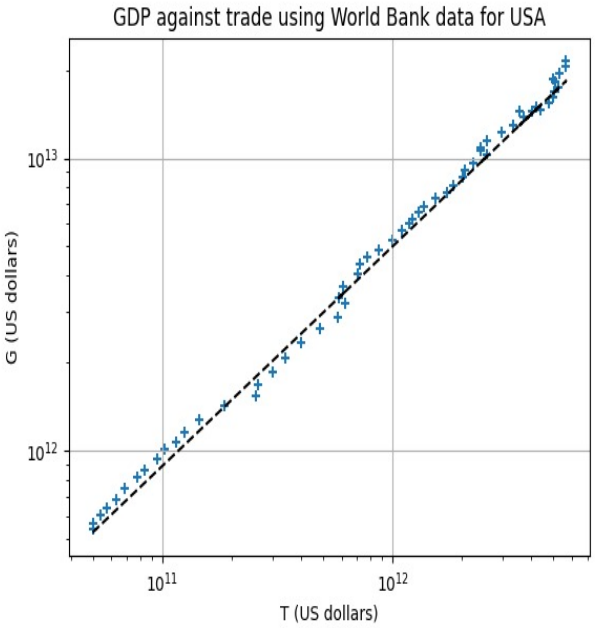

FIG. 7: Figure shows GDP vs Trade plot for USA. Parameter $\alpha = 0.75$. Statistical measures $\mu = 0.0012$ and $\sigma = 0.0023$.

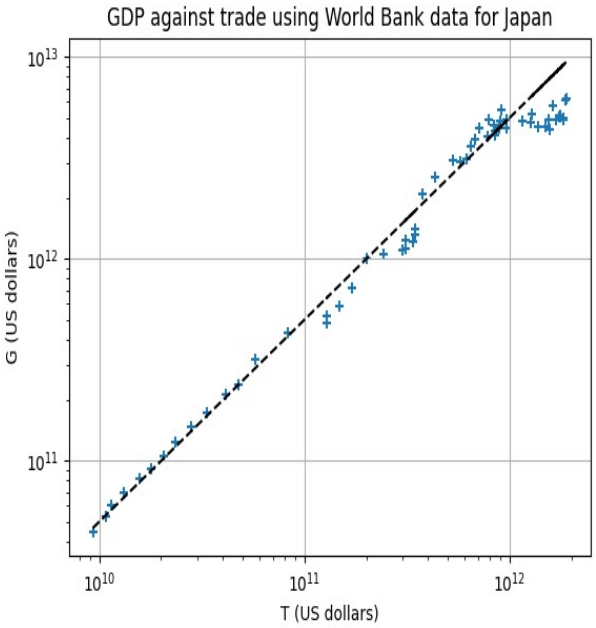

FIG. 9: Figure shows GDP vs Trade plot for Japan. Parameter $\alpha = 0.1$. Statistical measures $\mu = -0.005$ and $\sigma = 0.008$.

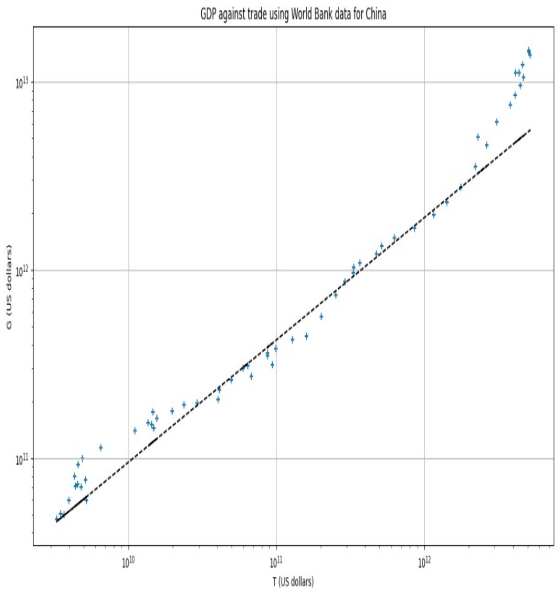

FIG. 8: Figure shows GDP vs Trade plot for China. Parameter $\alpha = 0.65$. Statistical measures $\mu = 0.002$ and $\sigma = 0.01$.

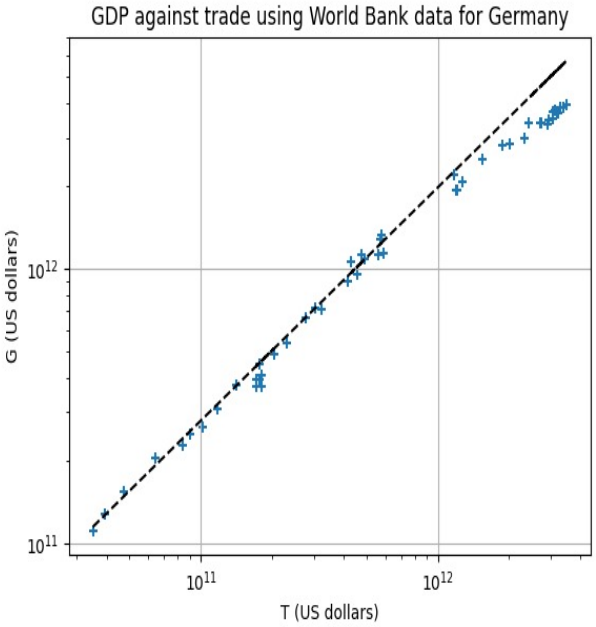

FIG. 10: Figure shows GDP vs Trade plot for Germany. Parameter $\alpha = 0.85$. Statistical measures $\mu = -0.003$ and $\sigma = 0.005$.

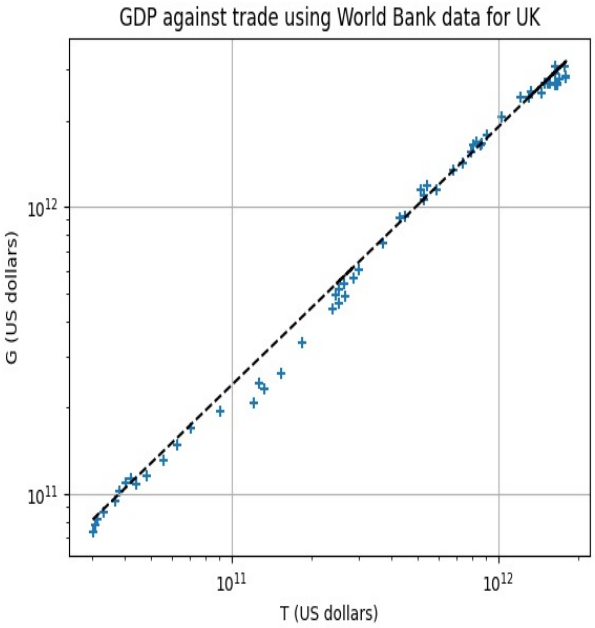

FIG. 11: Figure shows GDP vs Trade plot for UK. Parameter $\alpha = 0.9$. Statistical measures $\mu = -0.0019$ and $\sigma = 0.0032$.

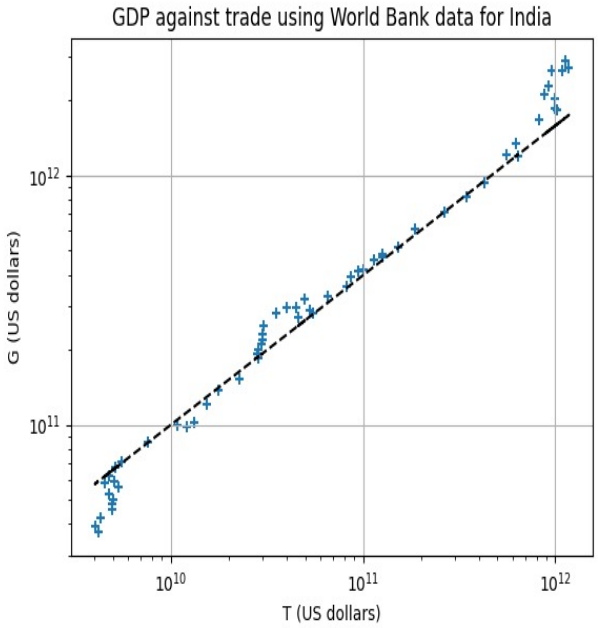

FIG. 13: Figure shows GDP vs Trade plot for India. Parameter $\alpha = 0.6$. Statistical measures $\mu = 0.0015$ and $\sigma = 0.0078$.

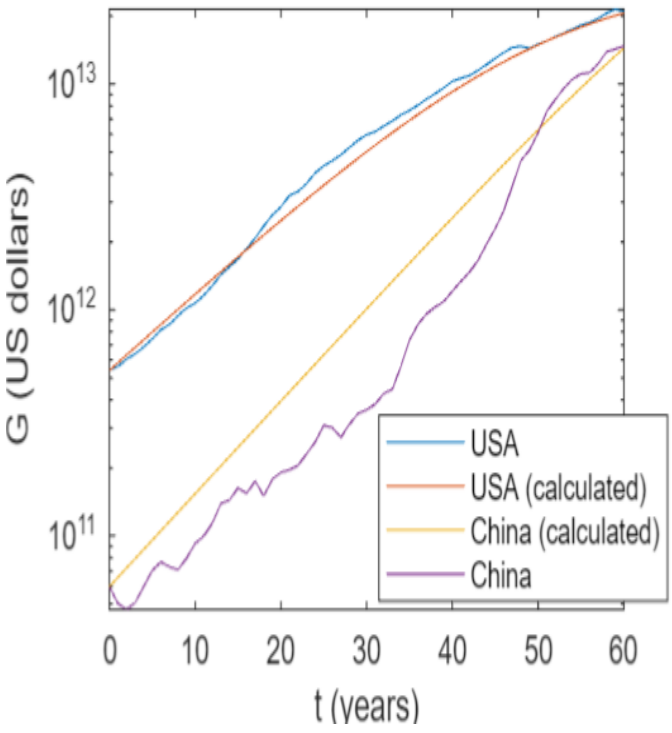

FIG. 12: GDP and Trade comparison of USA and China.

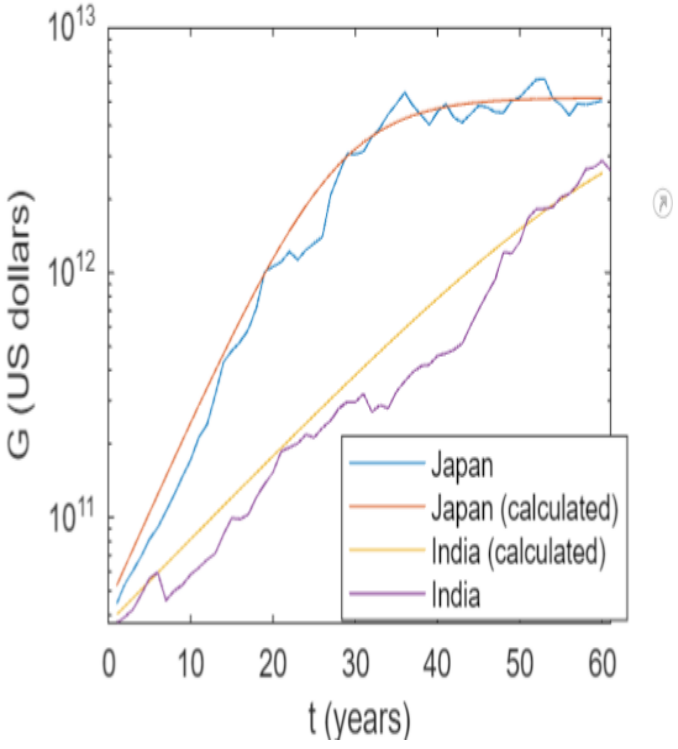

FIG. 14: GDP and Trade comparison of Japan and India.

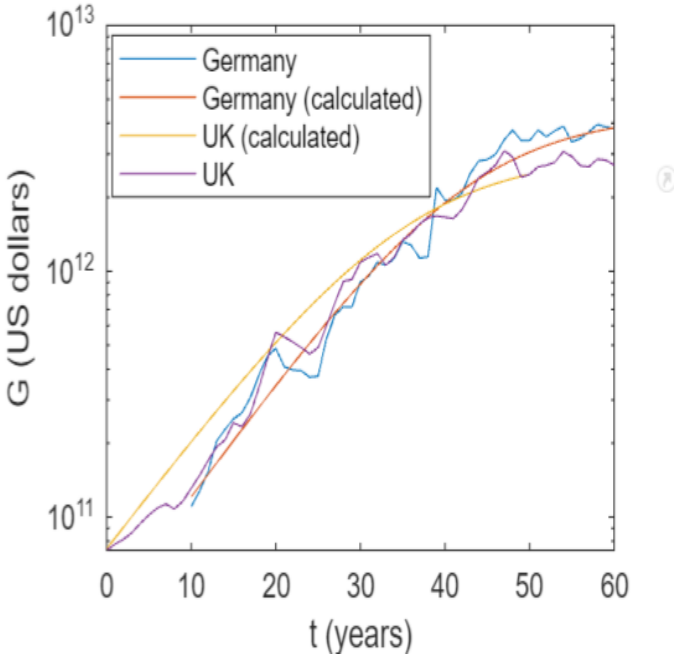

FIG. 15: GDP and Trade comparison of UK and Germany.

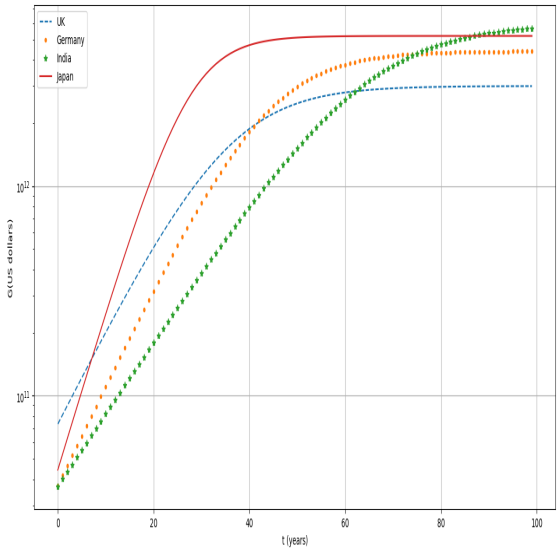

FIG. 16: GDP prediction of UK, Japan, Germany, China and India.

III. CONCLUSION

- When other nations deviate from the model, the GDP and trade growth of Japan and the USA suit the model the best.And also despite the fact that trade and GDP frequently have a positive correlation, their relationship is not strictly linear since a number of factors, including investment, government spending, and domestic consumption, can affect GDP.
- India is expected to overtake the UK by 2020–21, overtake Germany by 2031–32, and overtake Japan by 2047–48 in terms of GDP.
- According to model , India's GDP will be $ 4 trillions in the year 2032-33 and $ 5 trillions in the year 2043-44.

[1] Arnab K. Ray, Abhin Kakkad, Global Dynamics of GDP and Trade , DA-IICT (2022).