Set 11: Simulating the time evolution of a diffusive process

Vishal Dhoriya (202101446) and Akhil Patoliya (202101505)
Dhirubhai Ambani Institute of Information & Communication Technology, Gandhinagar, Gujarat 382007, India
CS302, Modeling and Simulation

This paper investigates solutions to diffusion and wave equations, focusing on Gaussian functions. Plots of the functions are generated with varying parameters, and numerical integration is employed to analyze their behavior. Insights gained from these analyses are discussed.

I. THE POINT-SOURCE SOLUTION OF THE DIFFUSION EQUATION

A. Model Equation

The point-source solution of the diffusion equation is given by the Gaussian function

$$\psi(x,t) = (4\pi Dt)^{-1/2} \exp\left(-\frac{x^2}{4Dt}\right) \quad (1)$$

Gaussian function

$$y(x,t) = y_0 \exp\left[-a(x-vt)^2\right] \quad (2)$$

(a solution of the wave equation).

B. Solution

A. Plot ψ versus x in a single graph with three different values of $t > 0$. State the values of t and D in the plot.

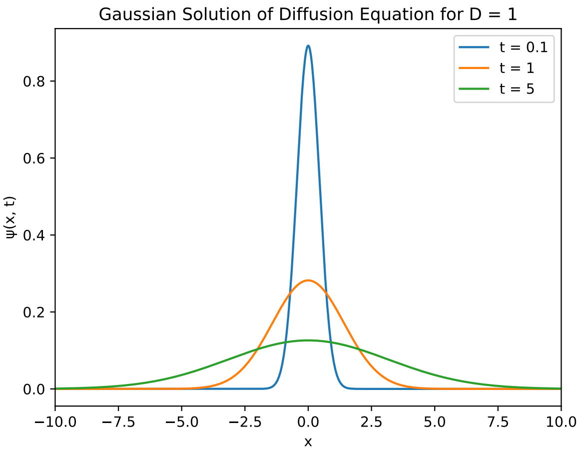

FIG. 1: The plot shows ψ as x grows. To obtain this result we have used the Eq.(1) and put the values of parameters as $D = 1$ and for different $t = 0.1, 1, 5$.

B. Plot $\psi(0,t)$ versus t, starting with a value of t that is slightly greater than zero. Separately plot the same in a log-log graph and note the slope of the plot.

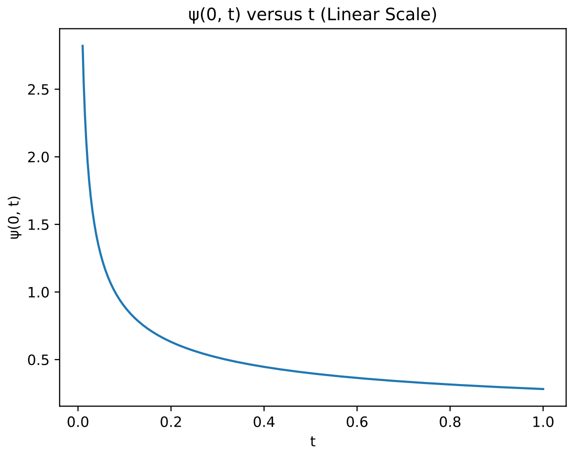

FIG. 2: The plot shows $\psi(0,t)$ as $t > 0$ grows. To obtain this result we have used the Eq.(1) and put the value of parameter as $D = 1$.(linear scale)

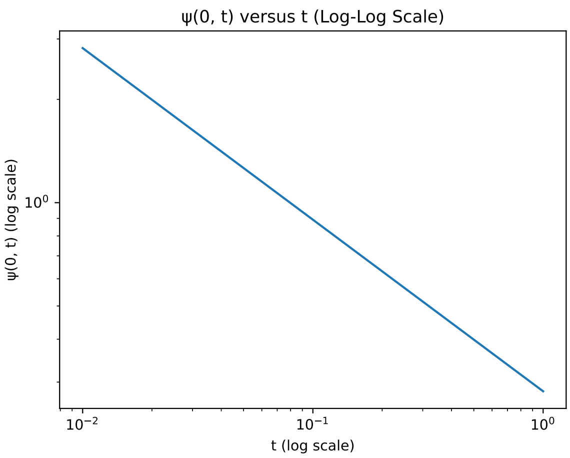

FIG. 3: The plot shows $\psi(0,t)$ as $t > 0$ grows. To obtain this result we have used the Eq.(1) and put the value of parameter as $D = 1$. (Log-log scale), where the slope is -0.5

C. Consider another Gaussian function

$$y(x,t) = y_0 \exp\left[-a(x-vt)^2\right]$$

(a solution of the wave equation). Taking $y_0 = a = v = 1$, plot y versus x for the same three values of t in Part A. The shifting profile of the function simulates a travelling wave pulse (like a tsunami).

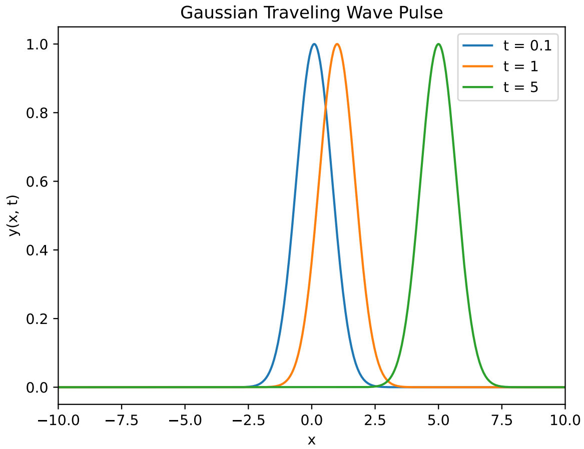

FIG. 4: The plot shows y as x grows. To obtain this result we have used the Eq.(2) and put the value of parameter as $y_0 = a = v = 1$ and $t = 0.1, 1, 5$.

C. Conclusion

- We note that the point-source solution of the ψ vs. t graph flattens as t grows because the diffusion equation is inversely proportional to square root of time ,due to which the value decreases with increase in t.
- In the case of the wave equation, the graph's formed at various t remains the same in shape regardless of the amount of time.
- The graphs overlap on each other for the point-source solution of the diffusion equation at various times, but they move from the origin for the wave equation (a train of pulses may be seen).

II. GENERAL DIFFERENTIAL EQUATION OF DIFFUSION

Model equation

We have integrated the following equation with the initial conditions $\psi(0,0) = 1000$ and $\psi(x \neq 0, 0) = 0$ and got the following results.

$$\psi(i, n+1) = \psi(i, n) + \frac{D(\Delta t)}{(\Delta x)^2}[\alpha(i, n)] \tag{3}$$

$$\alpha(i, n) = \psi(i+1, n) + \psi(i-1, n) - 2\psi(i, n) \tag{4}$$

We took $D = 1$ (or the value of D in the previous question), $\Delta t = 0.0001$, and $\Delta x \geq \sqrt{2D\Delta t}$.

A. Solution

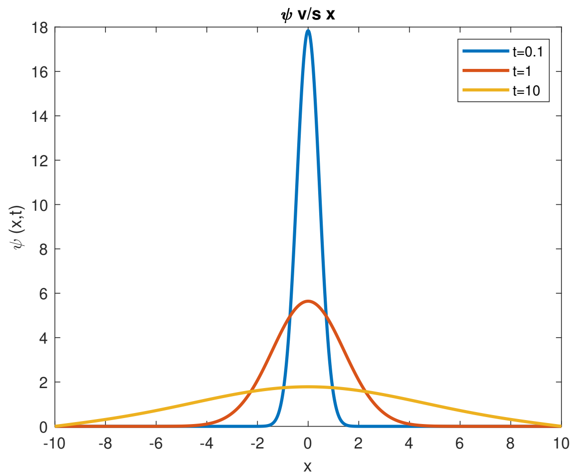

FIG. 5: The plot shows ψ as x grows. To obtain this result we have used the Eq.(3) and Eq.(4) and put the values of parameters as $D = 1$, $\Delta x = 0.02$, $\Delta t = 0.0001$ and for different $t = 0.1, 1, 10$

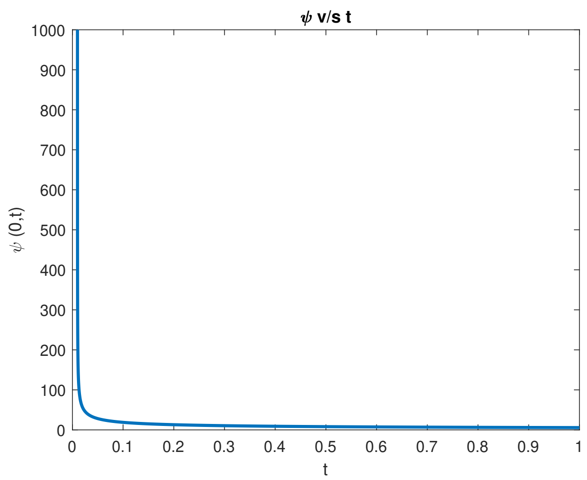

FIG. 6: The plot shows $\psi(0, t)$ as $t > 0$ grows. To obtain this result we have used the Eq.(3) and Eq.(4) and put the values of parameters as $D = 1$, $\Delta x = 0.02$, $\Delta t = 0.0001$. (Linear scale)

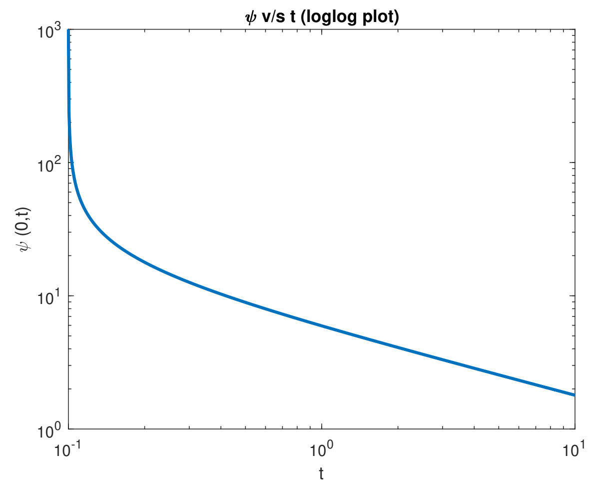

FIG. 7: The plot shows $\psi(0, t)$ as $t > 0$ grows. To obtain this result we have used the Eq.(3) and Eq.(4) and put the values of parameters as $D = 1$, $\Delta x = 0.02$, $\Delta t = 0.0001$.(Log-log scale)

B. Conclusion

- The shape point-source solution of graph of ψ(x,t) vs t is similar in both analytical and numerical solutions.

- In the graph of $\psi(0, t)$ vs t ,we can see a an exponential decrease as the t increase.

- For log-log graph of $\psi(0, t)$ vs t ,there is some noise introduced at the starting of the graph. It can be removed by taking appropriate D.