Lab - 1 : Modeling the Growth of IBM through Logistic Function Analysis

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In this lab, we estimate the trajectory of IBM's revenue and human resource growth over time using the logistic equation's integral solution. We then compare our predictions to the actual data by calculating the mean and standard deviation to assess how accurate they were. We use modeling to investigate the relationship between the financial and human resources of multinational corporations such as IBM.

I. MODEL

First-order autonomous dynamical systems have the general form of $\frac{dx}{dt} = f(x)$ where x = x(t), with t being time.A basic model of a nonlinear function is given by

$$\frac{dx}{dt} = ax - bx^2 \qquad (1)$$

with a and b being fixed parameters. This leads to the well-known logistic equation,

$$f(x) = ax - bx^2 \qquad (2)$$

Under the initial condition of x(0) = x_0, and with the definition of k = a/b, the integral solution of Eq. (1) is

$$x(t) = \frac{kx_0 e^{at}}{k + x_0(e^{at} - 1)} \qquad (3)$$

which is the logistic function.Eq.(2) represents the logistic equation whose integral solution Eq.(3) is widely used to predicate human resources, profit and net revenue of the companies like IBM.

II. RESULTS

The following are the results/graphs that we have achieved during our study , figure.1 is for Revenue, figure.2 for Human Resources , figure.3 for Profit and figure.4 for finding the corrleation between the Human resource and Revenue
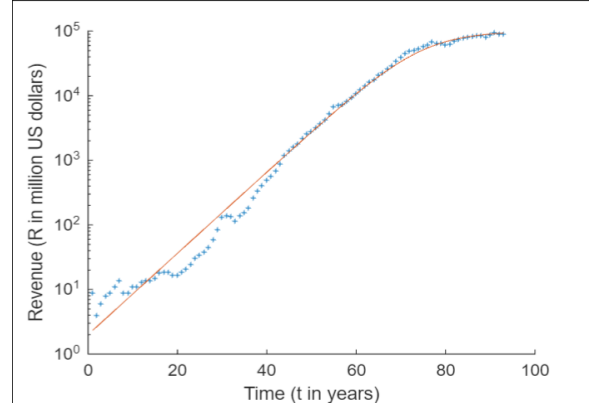

FIG. 1: Shows the net revenue growth of IBM company over 41-60 years. The parameter values to fit the revenue growth are $\rho 1 = 0.145 year^{-1}$ and k_R= 100 billion dollars. The mean $\mu_r = 0.0210$ and standard deviation $\sigma_r = 0.1096$ is found between real and analytical data.
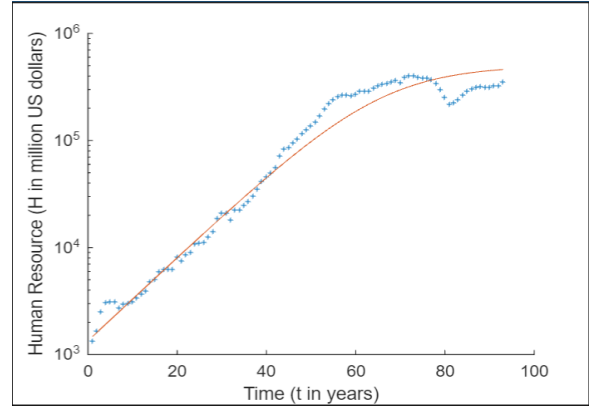

FIG. 2: Shows the growth of human resources of IBM company over 70-75 years. The parameter values to fit the human resource growth are $1 = 0.09\ year^1$ and $k_H = 500000$ The mean $\mu_h = 0.0606$ and standard deviation $\sigma_r = 0.0816$ is found between real and analytical data.
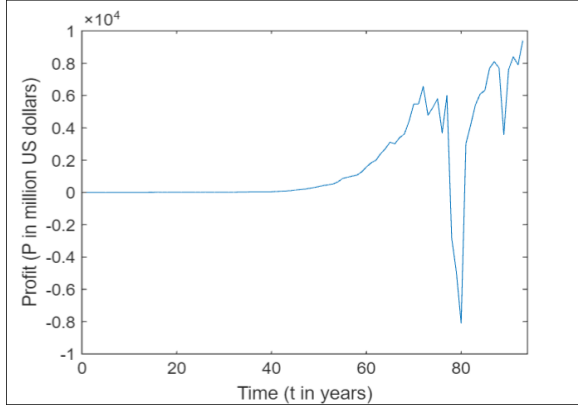
FIG. 3: Shows the profit of IBM company over 75-80 years. Around this time IBM suffered huge losses. Its effect is seen in the loss of human resources around the same period in Fig.2. The net annual earnings (the profit P) of IBM grow steadily up to 75-80 years which is also the same timescale as the above two graphs.
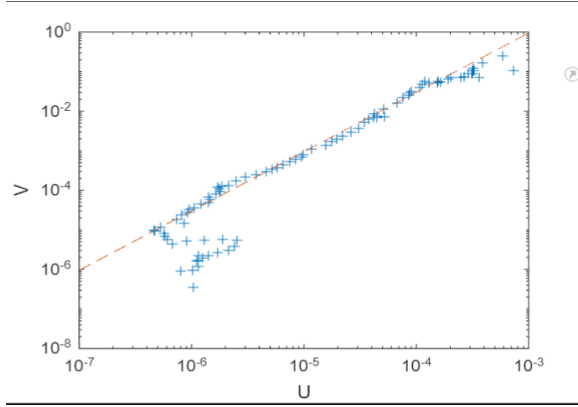

FIG. 4: Shows the correlation of Human resources and Net revenue of IBM over the timescale of 75-80 years. Here Defining $V = R^{-1} - k_r^{-1}$ and $U = H^{-1} - k_h^{-1}$ and $\beta = \frac{\rho_1}{\eta_1}$, the H-R phase solutions are transformed to a compact power-law form as $V = U^{\beta}$. The log-log fits the power law very well except for the bottom left. Log-log plot of correlated growth of H and R, with $\beta = 1.5$ (close to $\beta = 1.6$).

III. CONCLUSIONS

- We used the logistic equation, a simple mathematical model, to predict the growth of IBM's profits and human resources. Our study led us to the conclusion that,while H and R growth are correlated, we cannot say that they are directly proportionate to one another, rather there must be some other factors influencing them also.

- Figure 4 further indicates that revenue and human resources are growing in tandem(or can say are correlated), as they were showing linearly increasing behaviour for (75-80) and after 78–80 years, the value of these two variables is almost at saturation.

- We may conclude that the logistic equation is significantly followed by the annual growth in revenue and the growth in human resources.

[1] Arnab K. Ray, *Logistic modelling of economic dynamics*, DA-IICT (2023).