Maths

1). sol

a). Given: n = 56 (Large sample).

$\bar{x}$ => sample mean.

μ => population mean.

z - test statistic is => $\boxed{z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}$

$\bar{x} = 147$, $\mu = 140$,

$\sigma = 16$.

=> $z = \frac{147 - 140}{\frac{16}{\sqrt{56}}} = \frac{7}{2.13809}$

2.43331

$z = 3.2739$.

$\bar{x} = \mu$ | Ho : $\mu = 140$ (mean sales before campaign not effective)

$\bar{x} > \mu$ | H1 : $\mu > 140$ (mean sales after the campaign is effective).

LOS = 5%.

=> Right tailed test

$Z_\alpha = 1.645$.

=> $|Z| > |Z_\alpha|$.

calc. value > Table value.

=> Ho is rejected

=> H1 is accepted

=> The advertisement is effective.

~~Given n = 17 (small sample test)~~

~~1.b). => T-test for one mean.~~

~~$\bar{x} = 155$, $\mu = 145$, $\sigma = 16$.~~

~~The test statistic is => $t = \frac{\bar{x} - \mu}{\sigma/\sqrt{n-1}}$~~

~~$= \frac{155 - 145}{16/\sqrt{16}} = \frac{10}{4} = 2.50 \Rightarrow z$.~~

~~Ho : $\mu = 145$.~~ ~~$z = 2.50\%$.~~

~~H1 : $\mu > 145$ (mean weekly sales after the advertisement campaign is effective).~~

~~LOS => 5%. (Right tailed test~~

~~$Z_\alpha = 1.645$.~~

~~=> $|z| > |Z_\alpha|$)~~

~~=> calc. value > table value~~

~~=> Ho is rejected ; H1 is accepted~~

~~=> The advertisement campaign was effective and successful.~~

1.b) Given: n = 17 (small sample)

=> T- test for one mean.

$\bar{x} = 155$, $\mu = 145$, $\sigma = 16$.

The test statistic is => $t = \frac{\bar{x} - \mu}{\sigma/\sqrt{n-1}}$

$t = \frac{155 - 145}{\frac{16}{\sqrt{16}}} = \frac{10}{4} = 2.50$

=> $z = 2.50\%$.

$\bar{x} = \mu$ | Ho: $\mu = 145$ (Not successful)

$\bar{x} > \mu$ | H_1: $\mu > 145$ (The mean weekly sales increases and advertisement is successful)

(Right tailed)

$t = 2.50$

Degrees of freedom $v = n-1 = 17-1 = 16$

$t_{0.05} = 1.746$

$\Rightarrow |t| > |t_{0.05}|$

$\Rightarrow$ Ho is rejected, H_1 is accepted

$\Rightarrow$ The advertisement is effective

Given: $n = 86$ (Large sample test)

2) a) Z-test (mean test)

Given: $\bar{x} = 1.7$, $\mu = 1.5$, $\sigma = 0.5$

$\bar{x} = \mu$ | Ho: $\mu = 1.5$ (equal to 1.5 litres)

$\bar{x} > \mu$ | H_1: $\mu > 1.5$ (mean household order greater than 1.5 litres)

(Right tailed)

The test statistic is $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$$z = \frac{1.7 - 1.5}{\frac{0.5}{\sqrt{86}}} = \frac{0.2}{0.0539} = 3.710$$

9.2736

$z = 3.710$.

Right tailed test $\Rightarrow$

LOS = 5% $\Rightarrow$ $z_\alpha = 1.645$

$\Rightarrow |z| > |z_\alpha|$

$\Rightarrow$ calc. value > Table value.

$\Rightarrow$ Ho is rejected, H_1 is accepted

$\Rightarrow$ The mean household order is greater than 1.5 litres.

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2) b) Given $n = 8$ (samples)

T-test. Given mean = 30 ($\mu = 30$)

$$\boxed{t = \frac{\bar{x} - \mu}{\sigma/\sqrt{n-1}}}$$

To find $\bar{x}$

$$\boxed{\bar{x} = \frac{\Sigma x}{n}} = \frac{31.1 + 30.7 + 24.3 + 28.1 + 27.9 + 32.2 + 25.4 + 29.1}{8}$$

$$\boxed{\bar{x} = 28.6}$$

mean given, to find S.D

$$\boxed{(S.D)^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} = 824.6 - (28.6)^2$$

$s^2 = \sqrt{6.64}$

$s = 2.57$ (σ)

924.02

Ho: $\bar{x} = \mu$

H_1: $\bar{x} < \mu$ (less than the normal rainfall)

6597.82

$$\Rightarrow t = \frac{28.6 - 30}{\frac{2.57}{\sqrt{8-1}}}$$

$$t = \frac{-1.4}{0.9713} \Rightarrow \boxed{t = -1.441}$$

Degrees of freedom $v = n-1 = 8-1$

$\boxed{v = 7}$

$t_{0.05} = 1.895$.

$\Rightarrow |t| < |t_{0.05}|$

$\Rightarrow$ calc. value < table value.

$1.441 < 1.895$

$\Rightarrow$ H_0 is accepted., H_1 is rejected.

$\Rightarrow$ No evidence to show that avg. rainfall is less than the normal rainfall.

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3). a) Given $n_1 = 250$, $P_1 = \frac{142}{250} = 0.568$

$n_2 = 320$,

$P_2 = \frac{150}{320} = 0.4687$

Large sample

$H_0 : P_1 = P_2$ (new homes storms windows = old homes)

$H_1 : P_1 > P_2$ (new homes storm windows > old homes)

$$\boxed{Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}$$

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2} = \frac{(250)(0.568) + (320)(0.468)}{250 + 320}$$

$P = 0.5118$.

$Q = 1 - P = 0.48814$

$$Z = \frac{0.568 - 0.468}{\sqrt{(0.5118)(0.488)\left(\frac{1}{250} + \frac{1}{320}\right)}}$$

$$= \frac{0.1}{(0.24983)} = \frac{0.1}{\sqrt{0.001780}}$$

~~Z = 56717~~

$$= \frac{0.1}{0.04219}$$

$\boxed{Z = 2.370}$

Right tailed test.

LOS 5% $\Rightarrow Z_\alpha = 1.645$.

$\boxed{\Rightarrow |Z| > |Z_\alpha|}$

$\Rightarrow$ H_0 is ~~acc~~ rejected.

$\Rightarrow$ H_1 is accepted.

3). b) Given $\mu = 7$, $n = 10$. (small sample t-test)

The test statistic is

$\Rightarrow t = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n-1}}$

To find $\bar{x}$:

$\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{7.2 + 7.3 + 7.1 + 6.9 + 6.8 + 6.5 + 6.9 + 6.8 + 7.1 + 7.2}{10}$

$\bar{x} = 6.98 \simeq \boxed{\bar{x} = 7}$

$\cancel{(S.D)^2 = \dfrac{\Sigma x^2}{n} - \left(\dfrac{\Sigma x}{n}\right)^2}$

$\cancel{= 48.7 - (6.98)^2}$

$\cancel{= 48.7 - 48.7204}$

$\cancel{= 0.02}$

$\cancel{= 48.7 - 7^2 = 4}$

$(S.D)^2 = \dfrac{\Sigma x^2}{n} - \left(\dfrac{\Sigma x}{n}\right)^2$

$= 48.7 - (6.98)^2 = \sqrt{0.02}$ (S.D)

$\boxed{S.D = 0.14}$

H_0: $\bar{x} = \mu$ (length equal to 7)

H_1: $\bar{x} \neq \mu$ (length not equal to 7)

(two tailed test)

$t = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n-1}} = \dfrac{6.98 - 7}{\sigma/\sqrt{n-1}} = \dfrac{-0.02}{\frac{0.14}{\sqrt{9}}}$

$= \dfrac{-0.02}{0.046}$

$t = 0.26$

calc. value $= 0.26$. $v = n - 1$, $v = 10 - 1 = 9$.

$t_{0.05}$ (2-tailed) $= 2.262$.

$\Rightarrow 0.26 < 2.262$

$\Rightarrow |t| < |t_\alpha|$

$\Rightarrow$ H_0 is accepted, $\Rightarrow H_1$ is rejected.

$\Rightarrow$ The machine is reliable.

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4). a) Given: $n_1 = 74 + 83 = 157$ (Delhi)

$n_2 = 65 + 107 = 172$.

Large sample (z-test)

$P_1 = \dfrac{74}{157}$ (on time) $= 0.471$

$P_2 = \dfrac{65}{172} = 0.377$.

H_0: $P_1 = P_2$ (trains on time at Delhi = at New Delhi)

H_1: $P_1 \neq P_2$ (trains on time at Delhi ≠ New Delhi)

Note:- asking for any difference in on time arrival.

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.47 - 0.37}{\sqrt{(0.42)(0.57)}}$$

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2} = 0.42 \quad Q = 1-P$$

$Q = 1 - 0.42$

$Q = 0.57.$

$$= \frac{0.47 - 0.37}{\sqrt{(0.42)(0.57)\left(\frac{1}{157} + \frac{1}{172}\right)}}$$

$$= \frac{0.1.0}{\sqrt{(0.2394)(0.01218)}} = \frac{0.1}{\sqrt{0.002915}}$$

$$= \frac{0.1}{0.05399}$$

$\boxed{Z = 1.8}$

LOS = 5%. Z_α = (two tailed)

$Z_\alpha = 1.96.$

$|Z| < |Z_\alpha|$

$1.8 < 1.96.$

=> Ho is accepted, H_1 is rejected

=> There is no significant difference b/w the arrival of time

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4) c) b) sol: $n = 25$ (small sample)

$\bar{x} = 1550, \quad \mu = 1600.$

$S.D = \sigma = 120.$

$$\Rightarrow t = \frac{\bar{x} - \mu}{\sigma/\sqrt{n-1}} = \frac{1550 - 1600}{120/\sqrt{24}} = \frac{-50}{24.48}$$

$Ho: \bar{x} = \mu$

$H_1: \bar{x} < \mu$

Ho => avg mean = company claim

H_1 => avg mean < company claim

$t = -2.04.$

Degrees of freedom => $v = n-1 = 25-1$

$v = 24.$

$t_{0.05} = 1.711$ (one-tailed)

=> $|t| > |t_\alpha|$

=> $2.04 > 1.711.$

=> Ho is rejected

=> H_1 is accepted.

=> The claim of the company cannot be accepted.

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5) a) sol: $n_1 = 250, \quad \bar{x}_1 = 120 \quad \sigma_1 = 12$

$n_2 = 400. \quad \bar{x}_2 = 124. \quad \sigma_2 = 14.$

Two samples and two means are given.

so, Two sample mean test to be used.

$H_0: \bar{x}_1 = \bar{x}_2$ (average of two samples are equal)

$H_1: \bar{x}_1 \neq \bar{x}_2$ (significant difference is there).

Large samples

$$\Rightarrow \boxed{Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$$

$$= \frac{120-124}{\sqrt{\frac{12^2}{250} + \frac{14^2}{400}}} = \frac{-4}{\sqrt{0.576 + 0.49}}$$

$$= \frac{-4}{\sqrt{1.066}} = \frac{-4}{1.032}$$

$Z = 3.8759$

For 1%. Z_α = (two tailed test)

$Z_\alpha = 2.58$.

$\Rightarrow |Z| > |Z_\alpha|$

$\Rightarrow$ H_0 is rejected

$\Rightarrow$ H_1 is accepted.

$\Rightarrow$ There is a significant difference between the two samples.

5) b) Ans. $n_1 = 16$, $\bar{x}_1 = 107$, $\sigma_1 = 9$.

$n_2 = 12$, $\bar{x}_2 = 111$, $\sigma_2 = 10$.

Here, n_1 & n_2 are small samples.

$\rightarrow$ $H_0: \bar{x}_1 = \bar{x}_2$ (t-test)

$H_1: \bar{x}_1 < \bar{x}_2$ (Production rate in latter factory > former factory)

For small sample.

$$\boxed{t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}$$

$$t = \frac{107 - 111}{\sqrt{\frac{16 \times 9^2 + 12 \times 10^2}{(16+12-2)}\left(\frac{1}{16} + \frac{1}{12}\right)}}$$

$$= \frac{-4}{\sqrt{(96)(0.1458)}} = \frac{-4}{\sqrt{14}}$$

$$= \frac{-4}{3.741}$$

$t = -1.069$

Degrees of freedom $\Rightarrow n_1 + n_2 - 2$

$= 28 - 2 = 26$.

$t_{0.05} = 1.706$.

$\Rightarrow |t| < |t|_{0.05}$

$\Rightarrow 1.069 < 1.706$

$\Rightarrow$ Ho is accepted

$\Rightarrow$ H1 is rejected.

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6).a). Gn $\mu = 3.1$, $\sigma = 1.1$. (population mean)

$n = 30$ (Large sample).

$\bar{x} = 2.4$ (sample mean)

$\Rightarrow z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$.

Ho: $\bar{x} = \mu$,

H1: $\bar{x} \neq \mu$ (mean weight of the fish differs from 3.1 kgs)

$\Rightarrow z = \dfrac{2.4 - 3.1}{\frac{1.1}{\sqrt{30}}} = \dfrac{-0.7}{0.200} = 3.5$

$z = 3.5$

LOS = 10% (two tailed test)

$|z_\alpha| = \cancel{1.645}$ 1.615 //.

$|z| > |z_\alpha|$.

Calc. value > table value

$\Rightarrow$ Ho is rejected

$\Rightarrow$ H1 is accepted.

$\Rightarrow$ The mean weight of the fish differs from 3.1kg based on the given sample data in ~~...~~ calculated value.

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6) b) Refer C.W note.

7).a) Gn. sample proportion $p = \dfrac{23}{40} = 0.575$

only one sample ~~...~~. is given so, go for this formula

Gn $n = 40$.

$P \Rightarrow$ population $= \frac{2}{5} = 0.4$

$\Rightarrow$ Ho: $p = P$

H1: $p \neq P$. (two tailed)

$$\boxed{z = \dfrac{p - P}{\sqrt{\frac{PQ}{n}}}} = \dfrac{0.575 - 0.4}{\sqrt{\frac{(0.4)(0.6)}{n}}}$$

0.0774596

$$= \dfrac{0.175}{\sqrt{0.006}} = \dfrac{0.175}{0.0774}$$

$z = 2.259$.

LOS => 5%. two-tailed test

$Z_\alpha = 1.96$.

=> $|z| > |z_\alpha|$

$2.259 > 1.96$.

=> H_0 is rejected

=> H_1 is accepted

=> The new technique is effective

7) b) For Drug A.

$\bar{x}_1 = \frac{\sum x_1}{n_1} = 4.1$,

For Drug B

$\bar{x}_2 = \frac{\sum x_2}{n_2} = 4.528$//.

Ans: $n_1 = 5$, $n_2 = 7$.

$\bar{x}_1 = 4.1$, $\bar{x}_2 = 4.528$.

To find, s_1 and s_2.

$s_1^2 = \frac{\sum x_1^2}{n} - \left(\frac{\sum x_1}{n}\right)^2$

$s_1^2 = 19.358 - (4.1)^2$

$s_1^2 = 2.548$

$s_1 = 1.5962$

$s_2^2 = \frac{\sum x_2^2}{n} - \frac{(\sum x_2)^2}{n_2}$

$= 22.135 - (4.528)^2$

$= 22.135 - 20.502$

$s_2^2 = 1.633 \implies s_2 = 1.2778$

$H_0: \bar{x}_1 = \bar{x}_2$

$H_1: \bar{x}_1 \neq \bar{x}_2$ (differ significantly)

small sample. $n_1 = 5$, $n_2 = 7$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{4.1 - 4.528}{\sqrt{\frac{5 \times (1.59)^2 + 7 \times (1.2778)^2}{5+7-2}\left(\frac{1}{5} + \frac{1}{7}\right)}}$$

$$= \frac{-0.428}{\sqrt{(2.41686)(0.34285)}}$$

$$= \frac{-0.428}{\sqrt{0.82863}} = \frac{0.428}{0.91029}$$

$\boxed{t = 0.47}$

Degrees of freedom $\Rightarrow n_1+n_2-2$

$= 5+7-2$

$= 12-2$

$v = 10$

LOS $= 5\%$.

(two tailed test)

~~$t_{0.05} = 2.2228$~~

~~$t_{0.05} = 2.228$~~

$t_{0.05} = 2.228$ //

$\Rightarrow 0.47 < 2.228$.

calc. value < tabulated value.

$\Rightarrow H_0$ is accepted

$\Rightarrow H_1$ is rejected.

$\Rightarrow$ There is no significant difference.

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8). a) $p \Rightarrow$ sample proportion

$P \Rightarrow$ population proportion.

$\Rightarrow p = \frac{18}{200} = 0.09$

$P = 95\% = \frac{95}{100} = 0.95$

$H_0: p = P$ (atleast (or) equal to 95%)

$H_1: p < P$. (less than 95%)

$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$ $Q = 1-0.95$, $Q = 0.05$

$\Rightarrow z = \frac{0.09-0.95}{\sqrt{\frac{(0.95)(0.05)}{200}}} = \frac{0.09-0.95}{\sqrt{0.0002375}}$

$= \frac{0.09-0.95}{0.01541}$

$= \Rightarrow z = 55.80$.

(Left tailed test)

$z_\alpha = -1.645$

$|z| > |z_\alpha|$

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8). a) $p \Rightarrow$ sample proportion.

$P \Rightarrow$ population proportion

$P = 95\% = 0.95$

$Q = 0.05$.

$H_0: p = P$

$H_1: p < P$ (less than 95%).

$\Rightarrow z = \frac{p-P}{\sqrt{PQ/n}}$

p (sample proportion) $= \frac{200-18}{200} = \frac{182}{200}$ → good equipments should be taken

$p = 0.91$ for spares were conformed to specification) manufacturer claim.

$$z = \frac{0.91 - 0.95}{\sqrt{\frac{(0.95)(0.05)}{200}}}$$

$$= \frac{-0.04}{0.01541}$$

$$z = -2.5957$$

LOS = 5% (two-tailed test)

$z_\alpha = -1.645$

$|z| > |z_\alpha|$

⇒ calc. value > table value

⇒ H_0 is rejected.

⇒ H_1 is accepted.

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8).b) $\bar{x}_1 = 1234$, $\bar{x}_2 = 1036$.

$s_1 = 36$, $s_2 = 40$.

$n_1 = 8$, $n_2 = 7$.

H_0: $\bar{x}_1 = \bar{x}_2$

H_1: $\bar{x}_1 > \bar{x}_2$

(Right tailed test)

Small sample

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\left(\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}}\right)\left(\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)}$$

$$= \frac{198}{\sqrt{\left(\frac{21568}{13}\right)\left(\frac{1}{8} + \frac{1}{7}\right)}} = \frac{198}{21.0807}$$

$t = 9.39$

$V = n_1 + n_2 - 2 = 13$

$t_{0.05} = 1.771$

⇒ $|t| > |t_\alpha|$

⇒ H_0 is rejected

⇒ H_1 is accepted

⇒ Type I bulbs are superior than type II bulbs.

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9) a) Refer C.W. Use two proportions formula.

$$z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}},$$

$$P = \left(\frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}\right)$$

9). b) Sol $n_1 = 12, \quad n_2 = 10.$

$\bar{x}_1 = 5250 \quad \bar{x}_2 = 4900.$

$\sigma_1 = 152, \quad \sigma_2 = 165$

$H_0: \bar{x}_1 = \bar{x}_2$

$H_1: \bar{x}_1 > \bar{x}_2$ (Division I gets more salary than Division II).

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{5250 - 4900}{\sqrt{\frac{12 \times (152)^2 + 10 \times (165)^2}{12 + 10} \times \left(\frac{1}{12} + \frac{1}{10}\right)}}$$

$$= \frac{350}{\sqrt{(24977.18)(0.183333)}}$$

$$= \frac{350}{\sqrt{4579.14}} = \frac{350}{67.669}$$

~~t =~~ $t = 5.1722$ ~~2~~

~~(t = 5.1722)~~

$t = 5.1722$

$v = n_1 + n_2 - 1$

$v = 22 - 1$ LOS => 5% => $t_{0.05}$

$v = 21$ $\quad t_{0.05} = 1.721.$

=> $|t| > |t_{0.05}|$

=> H_0 is rejected

=> H_1 is accepted.

=> Division I gets more salary than Division II.

10). a) Sol $n_1 = 100, \; n_2 = 200.$ (Large samples)

$\bar{x}_1 = 72.4, \quad \bar{x}_2 = 73.9$

$\sigma_1 = 14.8, \quad \sigma_2 = 17.9$

2-sample mean test.

$H_0: \bar{x}_1 = \bar{x}_2$ (two means are equal).

$H_1: \bar{x}_1 \neq \bar{x}_2$ (two means are not equal)

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{72.4 - 73.9}{\sqrt{\frac{(14.8)^2}{100} + \frac{(17.9)^2}{200}}}$$

$$= \frac{-1.5}{\sqrt{3.792}} = \frac{-1.5}{1.947}$$

$z = 0.770.$

~~$z = 0.76$.~~

~~$z = 0.770$ //.~~ Two-tailed test.

$z_\alpha = z_{0.05} = 1.96$.

$\boxed{z = 0.770}$//.

LOS => 10%. => $z_\alpha = z_{0.10} = 1.615$

=> $|z| < |z_\alpha|$.

=> calc. value < Table value

=> H_0 is accepted

=> H_1 is rejected.

=> The two means are equal.

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(10). b) Given: $n_1 = 8$, $n_2 = 6$ (small sample).

$\bar{x}_1 = 166.9$, $\bar{x}_2 = 170.3$

$\sigma_1 = 8.29$, $\sigma_2 = 8.50$.

$H_0: \bar{x}_1 = \bar{x}_2$ (soldier and sailor are equal heights)

$H_1: \bar{x}_1 < \bar{x}_2$ (soldier is shorter than sailors)

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{166.9 - 170.3}{\sqrt{\frac{(8)(8.29)^2 + (6)(8.50)^2}{8+6-2}\left(\frac{1}{8} + \frac{1}{6}\right)}}$$

$= -3.4$

$$\sqrt{(81.9333)(0.29166)}$$

$$= \frac{-3.4}{\sqrt{23.89}} = \frac{-3.4}{4.885}$$

$t = -0.696$.

=> LOS => 5%.

Degrees of freedom $v = n_1 + n_2 - 2$

$v = 12$

$t_{0.05}(v = 12) = 1.782$

=> $|t| < |t_{0.05}|$.

=> H_0 is accepted.

=> H_1 is rejected.

=> The above statement proves that the soldiers are in general shorter than sailors.

1).a). Ans: $n = 500$ (large sample)

$H_0: P = P$ (adults drinks prefer soft drinks makers brand = competitors brand)

$H_1: P > P$ (adults prefer soft drinks makers brand greater than competitors brand)

$p = \frac{\text{soft drinks makers brand}}{\text{total}} = \frac{270}{500} = 0.54.$

$P \Rightarrow$ (population proportion) $= 0.5$ (evenly split in preference)

$Q = 1 - 0.5 = 0.5$

$\Rightarrow z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.50}{\sqrt{\frac{(0.5)(0.5)}{500}}}$

$z = \frac{0.04}{\sqrt{0.0005}} = \frac{0.04}{0.022}$

$z = 1.818$

$\Rightarrow$ Right tailed test

$z_\alpha = z_{0.05} = 1.665$

$\rightarrow |z| > |z_\alpha|$

$\Rightarrow H_0$ is rejected

$\Rightarrow H_1$ is accepted.

$\Rightarrow$ we can conclude that a majority of adults prefer the company's beverage to that of their competitors.

1). b) Ans $n = 8$ (small sample)

$\bar{x} = \frac{\Sigma x}{n} = \frac{12.2 + 11.9 + 12.5 + 12.3 + 11.6 + 11.7 + 12.2 + 12.4}{8}$

$\bar{x} = 12.1$

To find S.D:

$(S.D)^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$= 146.505 - 146.41$

$(S.D)^2 = 0.095$

$S.D = 0.3081$

$\Rightarrow$ For 99% confidence level.

$V = n - 1 = 8 - 1 = 7.$

$\Rightarrow t_{99\%} = 2.998.$

margin of error (ME) $= t \times \frac{s}{\sqrt{n}}$

$= \frac{2.998 \times 0.308}{\sqrt{8}}$

$ME = 0.326.$

99% confidence interval.

$\Rightarrow (\bar{x} - ME, \bar{x} + ME)$

$= (12.1 - 0.326, 12.1 + 0.326)$

$\boxed{99\% = (11.774, 12.426)}$

~~95% confidence interval~~ Int

~~For 95%~~

For 95% confidence interval

$v = n-1 = 8-1 = 7.$

$t_{95\%} = 1.895$

$ME = 1.895 \times \frac{s}{\sqrt{n}}$

$ME = 0.2063$

95% confidence interval

$\Rightarrow (\bar{x} - ME, \bar{x} + ME)$

$\Rightarrow (12.1 - 0.2063, 12.1 + 0.2063)$

$\boxed{\Rightarrow (11.8, 12.3) \Rightarrow 95\%}$

For 95% $\Rightarrow$ (11.8, 12.3)

For 99% $\Rightarrow$ (11.7, 12.45)

1.895
9.365
0.2063

12) $n = 5000$ (large samples)

12).a) H_0: $P = P$ (Proportion of male baby born is same)

H_1: $P \neq P$ (different)

(key word change in question)

$p \Rightarrow$ proportion of male baby born

$p = \frac{52.55}{100} = 0.5255$

$P = \frac{51.46}{100} = 0.5146$

$Q = 1-P = 0.4854$

$\Rightarrow z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.5255 - 0.5146}{\sqrt{\frac{(0.5146)(0.4854)}{5000}}}$

$z = \frac{0.0109}{\sqrt{0.0000499}} = \frac{0.0109}{0.0070638}$

$z = 1.5430$

$z_\alpha = z_{10\%}$ = two tailed test

$z_{10\%} = 1.645$

$\Rightarrow |z| < |z_\alpha|$

$\Rightarrow$ H_0 is accepted

$\Rightarrow$ H_1 is rejected.

$\Rightarrow$ The proportion of male born does not change.

12) b) Ans:- $n = 6$ (small samples)

$$\bar{x_1} = \frac{10.68}{6} = 1.78, \quad \bar{x_2} = \frac{11.79}{6} = 1.965.$$

$$s_1^2 = \frac{\Sigma x_1^2}{n} - (\bar{x_1})^2 = 0.0261.$$

$$s_2^2 = \frac{\Sigma x_2^2}{n} - (\bar{x_2})^2 = 0.0154.$$

In this question, $n_1 = n_2 = n$.

Note:- If $n_1 = n_2 = n$, and if the samples are independent, then the test statistic is given by

$$\boxed{t = \frac{\bar{x_1} - \bar{x_2}}{\sqrt{\frac{s_1^2 + s_2^2}{n-1}}}, \quad v = 2n-2 \text{ ↳ Degrees of freedom.}}$$

$$t = \frac{1.78 - 1.965}{\sqrt{\frac{0.0261 + 0.0154}{5}}} = \frac{-0.185}{\sqrt{0.0083}}$$

$t = -2.03$

$v = 2n - 2 = 12 - 2 = 10$

H_0: $\bar{x_1} = \bar{x_2}$ (Two samples are same)

H_1: $\bar{x_1} \neq \bar{x_2}$ (not significantly differ)

Two tailed test

LOS = 5% ⇒ $t_{0.05}(v = 10) = 2.23$

⇒ $|t| < |t|_{0.05}$ ⇒ H_0 is accepted

⇒ H_1 is rejected.

⇒ There is no significant difference.

13) a) Refer class note.

13) b) Ans $n_1 = ~~4~~, 8, \quad n_2 = 7$

⇒ $\Sigma x_1 = 94, \quad \Sigma x_1^2 = 1138.$

$$s_1^2 = \frac{\Sigma x_1^2}{n} - \left(\frac{\Sigma x_1}{n}\right)^2 = \frac{1138}{8} - \left(\frac{94}{8}\right)^2$$

$s_1^2 = 4.19.$

⇒ $\Sigma x_2 = 73, \quad \Sigma x_2^2 = 785$

$$s_2^2 = \frac{\Sigma x_2^2}{n} - \left(\frac{\Sigma x_2}{n}\right)^2 = \frac{785}{7} - \left(\frac{73}{7}\right)^2$$

$s_2^2 = 3.39.$

$\boxed{s_1^2 = 4.19 \text{ and } s_2^2 = 3.39}$

$$\hat{\sigma}_1^2 = \frac{n_1}{n_1 - 1} s_1^2 = \frac{8}{7} \times 4.19 = 4.79.$$

$$\hat{\sigma}_2^2 = \frac{n_2}{n_2 - 1} s_2^2 = \frac{7}{6} \times 3.39 = 3.96$$

$v_1 = 7, v_2 = 6 \Rightarrow H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$\rightarrow F = \frac{\sigma_1^2}{\sigma_2^2} = \frac{4.79}{3.96}$

~~F = 1.2~~ $F = 1.21$

$\Rightarrow$ LOS $\Rightarrow 5\% \Rightarrow v_1 = n_1 - 1 = 8 - 1 = 7$

$v_2 = n_2 - 1 = 7 - 1 = 6.$

$\Rightarrow |F| < |F|_{0.05}$

$\Rightarrow H_0$ is accepted.

$\Rightarrow H_1$ is rejected

$\Rightarrow$ There is no significant difference.

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14). a). Refer C.W

14). b) Given $n_1 = 13$, $n_2 = 15$.

$\sigma_1^2 = 3.0$, $\sigma_2^2 = 2.5$

$v_1 = n_1 - 1 = 13 - 1 = 12$, $v_2 = n_2 - 1 = 15 - 1 = 14$

$F = \frac{\sigma_1^2}{\sigma_2^2} = \frac{3.0}{2.5} = 1.2.$

$\boxed{H_0 : \sigma_1^2 = \sigma_2^2 \quad H_1 : \sigma_1^2 \neq \sigma_2^2}$

$F_{0.05}(v_1 = 12, v_2 = 14) = 2.53$

$\Rightarrow |F| < |F|_{0.05} \Rightarrow H_0$ is accepted

$\Rightarrow H_1$ is rejected.

$\Rightarrow$ Thus, the two samples could have come from two normal population ~~with the same variance~~ with the same variance.

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15). a) Refer C.W.

15). b) Given 9 : 3 : 3 : 1.

Observed frequencies $882 + 313 + 287 + 118$

Total = 1600

$\Rightarrow$ Total = 16.

Expected frequency $= \frac{9}{16} \times 1600$, $\frac{3}{16} \times 1600$, $\frac{3}{16} \times 1600$, $\frac{1}{16} \times 1600$.

$E_i \Rightarrow 900, 300, 300, 100.$

O_i	882	313	287	118.
E_i	900	300	300	100.

$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} = \frac{18^2}{900} + \frac{13^2}{300} + \frac{13^2}{300} + \frac{18^2}{100}$

$\boxed{\chi^2 = 4.73\%}$ calculated value

$v = n-1 = 4-1 = 3$

From the χ^2 table, $\chi^2_{0.05}(v=3) = 7.82$

$\Rightarrow |\chi^2| < |\chi^2_{0.05}|$

$\Rightarrow$ Ho is accepted | Ho: supports the theory

$\Rightarrow$ H1 is rejected. | H1: not supports the theory

$\Rightarrow$: That is the experimental data supports the theory.

(16). a). Refer tutorial note ::

16b). pending.

(7). a) $\Rightarrow$ Each digit will occur $= \frac{200}{10} = 20$

O_i	18	19	23	21	16	25	22	20	21	15
E_i	20	20	20	20	20	20	20	20	20	20

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$\chi^2 = \frac{2^2}{20} + \frac{1^2}{20} + \frac{9}{20} + \frac{1}{20} + \frac{16}{20} + \frac{25}{20} + \frac{4}{20} + \frac{0}{20} + \frac{1}{20} + \frac{25}{20}$$

$\chi^2 = 4.3\%$ (calc. value)

Table value $v = n-1 = 10-1 = 9$.

$\chi^2_{0.05}(v=9) = 16.919$.

$\Rightarrow |\chi^2| < |\chi^2_{0.05}|$

$\Rightarrow$ Ho is accepted

$\Rightarrow$ H1 is rejected.

19). a) ans $= Q_1 = 2.52, Q_2 = 29.27$

$F_0 = 11.62$

$F_{5\%} = 19.65$

Difference maybe attributed to chance variation,