Potential Field Control of Multi-Robot System using Dynamic Potential For Navigation

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Abstract

This project aims to demonstrate the coordinated movement of a multi-robot ground vehicle system in negotiating a known static environment littered with obstacles, goals, and battery charging stations. We simulate the behavior of five robots attempting to navigate a mapped landscape containing multiple obstacles, alongside designated home and target locations, employing battery charging stations to optimize energy usage and ensure that the robots do not lose charge. The project employs a Potential Field-Based Control algorithm to govern the collective behavior of the robots, treating each robot akin to a point charge within an environment that mimics a potential field. We designate attractive potential fields toward the target goal position, repulsive potential fields around obstacles, and a variable attractive potential field to the battery charging stations. The potential field of the battery charging stations dynamically adjusts and is inversely proportional to the battery levels of each robot. This encourages the robots to minimize energy use within the field, thereby steering them toward goals or charging stations while circumventing obstacles. The project explores various ways of using parameters such as the strength of attractive potential forces of battery stations, repulsive potential forces from obstacles, and the decay rate of these forces over distance in the potential field control model.

Keywords: potential field, control, multi-robot system, dynamic, attractive, repulsive

1. Introduction

1.1. Motivation

The study of Potential Field-Based Control (PFBC) for multi-robot systems offers several compelling motivations and addresses key challenges in the field of robotics and autonomous systems. the study of PFBC for multi-robot systems is motivated by its potential to provide decentralized, scalable, and adaptable solutions for effective coordination, obstacle avoidance, and energy-efficient navigation in complex environments The seamless navigation of multiple robots through complex terrains poses a significant challenge in various domains, from logistics to rescue missions. The collective behavior of these robots demands sophisticated control mechanisms to efficiently maneuver through environments rife with obstacles while conserving energy. In this context, the integration of Potential Field-Based Control algorithms offers a promising avenue for orchestrating the coordinated movement of these robots.

By exploring the Potential Field-Based Control algorithm in this context, this project endeavors to enhance the collective behavior of multi-robot systems, particularly in negotiating complex environments with energy optimization considerations. The insights garnered from this project can be used for diverse applications, ranging from autonomous logistics such as vehicle parking to search and rescue operations.

1.2. Related Work

Our multi-robot system requires three properties that are crucial to its successful operation. The existence of a local minima in the potential fields, the avoidance of collision between the robots, and ensuring that the robots do not run out of battery by ensuring that the global minima of our potential field switches from the goal to the battery charging station depending on the battery levels of the robot. In the realm of mobile robot path planning and obstacle avoidance, [8] introduces an innovative approach utilizing artificial potential fields. Traditional methods in this domain often face challenges, prompting the proposal of an obstacle avoidance method based on a gravity chain. The conceptualization involves envisioning a rubber band connecting the initial and final points within the obstacle potential field. This rubber band assumes the role of potential field power, and a model is developed to emulate its form. The method generates a steering angle tangent to the rubber band, deviating from the conventional artificial potential field angle. The incorporation of effective obstacle avoidance information into the potential field through a gravity chain addresses prevalent issues associated with artificial potential field methods. Common pitfalls such as convergence to local minima, difficulty reaching the endpoint, and oscillatory movements are mitigated. Simulation results validate the correctness and effectiveness of the proposed method, showcasing its potential to enhance the efficiency of mobile robot path planning and obstacle avoidance strategies.

Challenges in motion coordination, task decomposition, and network communications are addressed through decentralized strategies, ensuring autonomy for large-scale groups. Formation Graphs (FGs) represent inter-agent communications and desired relative positions. Artificial Potential Functions (APFs), incorporating Attractive (APF) and Repulsive Potential Functions (RPF), are key to achieving global convergence and inter-agent collision avoidance. [3] delves into strategies using APFs and FGs for global convergence and non-collision in point or omnidirectional robots. Emphasizing APF modifications for simultaneous convergence and collision avoidance, the review introduces FG concepts, discusses the complexities of APFs and RPFs, and presents contributions like centroid analyses and a new repulsive vector field. Control laws are extended to unicycle-type robots in simulations, with experiments featuring two or three unicycle-like robots using computer vision for position and orientation estimation in the workspace.

[1] is a comprehensive textbook designed for third- and fourth-year undergraduates, offering a unique focus on the computational fundamentals behind the design and control of autonomous robots. The authors employ a class-tested and accessible approach, presenting concepts in a progressive, step-by-step manner. The book covers a broad range of topics, including mechanisms, sensing, actuation, computation, and uncertainty, with a balanced emphasis on both hardware (mechanism, sensor, actuator) and software (algorithms). Rigorous and classroom-tested, the text is suitable for engineering and computer science undergraduates with a sophomore-level understanding of relevant subjects. It features real-world examples, QR codes linking to online resources, and extensive appendices covering project-based curricula and other relevant topics. The book is accompanied by an open-source, platform-independent simulation library and serves as a valuable resource for teaching the complexities of autonomous robotics.

[7] introduces a decentralized framework for controlling and coordinating a group of robots engaged in cooperative manipulation tasks. Leveraging simple potential fields and hierarchical composition, the framework enables decentralized planning and control processes. It empowers the robots to approach, strategically form a trapping configuration around, and efficiently transport an object to a predefined destination. The controllers and planners, rooted in potential fields, demonstrate notable effectiveness in managing complex group interactions, specifically in manipulating and transporting objects within a plane. Theoretical analyses establish the stability of formations under potential field-based controllers in various scenarios. Simulation results confirm the framework's successful application across diverse examples, showcasing resilience to parameter variations. Notably, the decentralized nature of both trajectory generation and estimation/control agent levels suggests scalability to accommodate groups of tens and even hundreds of robots.

2. Methodology

The employed algorithm involves calculating attractive potential fields toward goals, repulsive potential fields around obstacles, and dynamically adjusting attractive potential fields for charging stations based on individual battery levels. In this section we describe our approach to generating the potential field around the obstacles, goals, targets, and the robots themselves in the environment and how they affect robot motion.

2.1. Mathematical Model

For this project, we formulate our mathematical control model that can autonomously switch targets from goals to charging stations in the environment depending on the battery level of the robot while at the same time avoiding obstacles and other robots present in the environment.

We formulate the potential field with a continuous descent towards the goal, introducing pronounced downward spikes precisely at the goal location, creating a distinctive local minimum, as illustrated in Figure 1. Conversely, at obstacle positions, we incorporate potential through upward spikes, guiding the robot to circumvent these hindrances, as depicted in Figure 2. Moreover, our model dynamically calculates and incorporates the potentials emanating from neighboring robots, employing reduced spike radii and magnitudes in comparison to obstacles. This ensures a dynamic avoidance mechanism, significantly reducing the risk of collisions among the robots in motion.

To account for battery charging stations, we introduce additional potential similar to that of the goal at the battery charging stations position. However, this potential is scaled by a parameter inversely proportional to the current battery level. This strategic adjustment becomes more influential at lower battery levels, temporarily overpowering the natural descent toward the goal. Instead, it redirects the robot's trajectory, towards the nearest charging station.

Through this model, our approach dynamically adapts to the changing conditions of energy resources, enabling robots to autonomously re-calibrate their objectives between goals and charging stations while navigating an environment cluttered with obstacles and other robots.

The obstacle potential field is generated in our simulation by using the equation below. This is based on the Gaussian function described in [6] and [5]:

$$O_p(X,Y) = \sum_{i=1}^{n} A \cdot \exp\left(-\frac{(X-x_i)^2 + (Y-y_i)^2}{2\cdot\sigma^2}\right) \tag{1}$$

where

- O_p is the obstacle potential field generated by obstacles at point (X, Y)
- A is the amplitude of the potential field.
- σ^2 is a parameter controlling the spread or reach of the potential field.
- (x_i, y_i) represents the positions of individual obstacles (indexed by i) in the environment.
- n is the total number of obstacles in the environment.

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The obstacle function sums up the contributions of each obstacle to compute the total potential field at the given point (X, Y) . We can see an example of the fields generated by the obstacle function in Figure 2

The goal potential fields for all the goals in the environment are calculated using quadratic functions [9].

$$G_p(X,Y) = \alpha\left((X-x_g)^2 + (Y-y_g)^2\right) - \frac{1}{\left(\sqrt{(X-x_g)^2+(Y-y_g)^2}\right)^2} \tag{2}$$

where

- G_p is the goal potential field generated by goals at point (X, Y)
- α is the amplitude of the potential field.
- σ^2 is a parameter controlling the spread or reach of the potential field.
- X is the X coordinate in a 2D Cartesian system
- Y is the y coordinate in a 2D Cartesian system
- x_g is the x coordinate of the goal position
- y_g is the y coordinate of the goal position

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An example of the field generated by the Goal potential function can be seen in Figure 1

The battery chargers generate a potential field given by the equation:

$$B_i(X,Y) = \gamma_i\left((X-x_{cs})^2 + (Y-y_{cs})^2\right) - \frac{1}{\left(\sqrt{(X-x_{cs})^2+(Y-y_{cs})^2}\right)^2} \tag{3}$$

where γ_i is treated as a variable whose value is calculated as:
$\gamma_i = \frac{\beta}{B_i}$
and x_{cs} is the x coordinate of the charging station y_{cs} is the y coordinate of the charging station

γ_i represents the adjusted gamma value for the ith robot. β is a variable representing a coefficient that influences when the charger is given priority over the goal. B_i denotes the battery level of the ith robot.

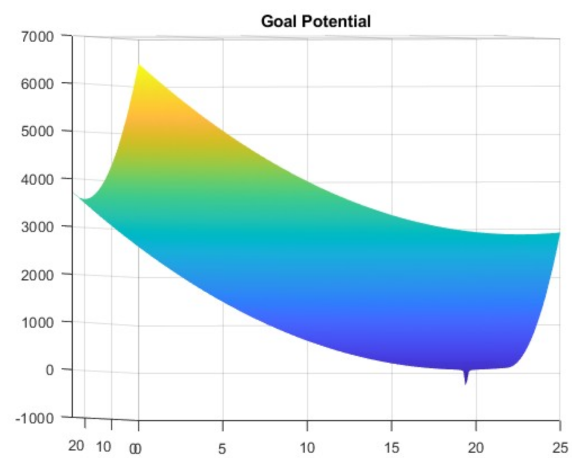

Figure 1. Potential Field Generated by the Goal Function

2.2. Controller Design

The controller is implemented to orchestrate the movement of five robots towards a designated target goal initially. The controller assigns potential fields for obstacles, the target goal, and the charging stations, subsequently combining these fields. Additionally, the controller calculates the potential field generated by other robots and incorporates it for each robot.

During each iteration of the control loop, the robots employ gradient descent on the resultant total potential field to determine their next movement direction. Due to the substantial gradients in the generated potential fields, the controller normalizes the gradient to ensure a maximum movement of 0.5 units per step. This controlled movement guides the robots toward the target goal efficiently.

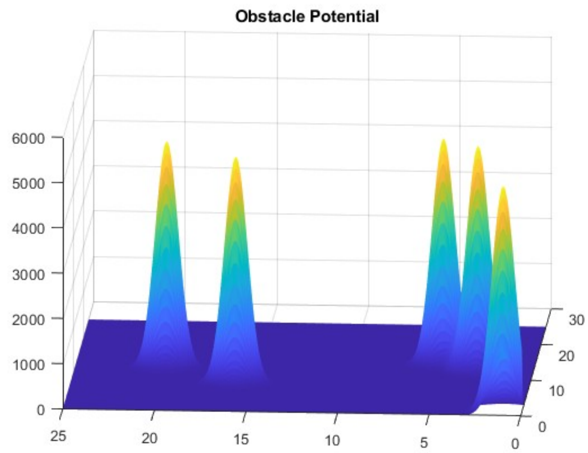

Figure 2. Potential Field Generated by the Obstacle Function

As the robots reach the target goal, the controller recalculates the potential field, considering the home position as the new goal. This adaptive behavior enables the robots to seamlessly transition between the target goal and the home position, effectively managing their battery levels. Over time, as the battery levels decrease, the influence of the charger potential becomes more significant. Consequently, the robots redirect their movement towards the nearest charging station, ensuring their continued operation without exhausting their batteries.

This controller strategy allows the robots to navigate between the target, home, and charging stations, ensuring the battery levels are maintained to prevent operational disruptions. It also ensures that the robots do not collide with each other. Ultimately, this control mechanism enables robust and adaptive navigation while dynamically prioritizing between goal-reaching and battery recharging based on the real-time energy status of the robots.

3. Analysis

3.1. Local Minima

The obstacle potential, by design, needs to repel the robots, and we use a Gaussian function to generate this repulsive force. In the context of potential fields, positive potentials denote repulsive forces. Therefore, the presence of an obstacle is denoted by a positive potential spike in the graph, creating a region of high potential values around the obstacles. Everywhere else, the potential gradually tends to zero, signifying an absence of obstacles.

On the other hand, the goal potential function aims to attract the robots towards the goal position. It adds potential as the distance from the goal position increases. The goal potential is calculated by the quadratic function given below:

$$\alpha\left((X-x_g)^2+(Y-y_g)^2\right) \tag{4}$$

For analyzing the local minima we can calculate the gradient for this part as in [2]

$$2\alpha((X-x_g)+(Y-y_g) \tag{5}$$

This gradient is 0 at $X = x_g$, and $Y = y_g$ and hence there is a local minima the goal position

However, to ensure a definite global minima at the goal location, an additional subtraction of potential is performed at the goal position:

$$\alpha\frac{1}{\left(\sqrt{(X-x_g)^2+(Y-y_g)^2}\right)^2} \tag{6}$$

This configuration ensures that the obstacle potentials and the increasing potential as the distance from the goal increases always contribute positively to the overall potential field. By consistently adding potential in the presence of obstacles and as the robots move away from the goal, the system achieves a local minima at the goal location. Subtracting potential at the goal position further refines this equilibrium point, ensuring a more defined and precise convergence towards the goal.

Consequently, the controller operates within a state of equilibrium at the local/global minima, where the gradient of the potential field becomes zero. This equilibrium enables stable navigation toward the goal while avoiding obstacles by following the gradient descent in the potential field.

$$\frac{\partial G_p}{\partial X} = 2\alpha(X-x_g)+\frac{2(X-x_g)}{\left(\sqrt{(X-x_g)^2+(Y-y_g)^2}\right)^3} \tag{7}$$

$$\frac{\partial G_p}{\partial Y} = 2\alpha(Y-y_g)+\frac{2(Y-y_g)}{\left(\sqrt{(X-x_g)^2+(Y-y_g)^2}\right)^3} \tag{8}$$

3.2. Dynamic Switching of Minima from Goal to Charger Position

The main goal of the project was to create a control method that dynamically switches the heading of the robot from targets to battery charger stations dependent on the battery potential.

Towards this, the battery charger potential is calculated similarly to the goal potential where the goal coordinates are replaced with the battery charger station coordinates. The only difference is that we multiply this with γ.

We can then as per the above statement conclude that for every value of γ the local minima for the battery charger potential is at the battery charging station as per the above proof for the goal potential.

Using this we can use the method used in [4] to prove this dynamic switching.

When we add the two potentials together, the robot will head towards the global minima. We can directly compare the potentials created by the sum of the two potential functions at the minima for each to decide which is the global minima and the condition for which they switch.

$$\begin{aligned} S = \alpha \left((X - x_g)^2 + (Y - y_g)^2 \right) - \\ \frac{1}{\left(\sqrt{(X - x_g)^2 + (Y - y_g)^2}\right)^2}) + \\ \gamma_i \left((X - x_{cs})^2 + (Y - y_{cs})^2 \right) - \\ \frac{1}{\left(\sqrt{(X - x_{cs})^2 + (Y - y_{cs})^2}\right)^2}) \end{aligned} \tag{9}$$

We can then calculate the potentials at x, y as we tend to (x_g, y_g) and (x_{cs}, y_{cs})

$$\begin{aligned} S_{x_g,y_g} = \gamma_i \left((x_g - x_{cs})^2 + (y_g - y_{cs})^2 \right) - \\ \frac{1}{\left(\sqrt{(x_g - x_{cs})^2 + (y_g - y_{cs})^2}\right)^2}) \end{aligned} \tag{10}$$

$$\begin{aligned} S_{x_{cs},y_{cs}} = \alpha \left((x_{cs} - x_g)^2 + (y_{cs} - y_g)^2 \right) - \\ \frac{1}{\left(\sqrt{(x_{cs} - x_g)^2 + (y_{cs} - y_g)^2}\right)^2}) \end{aligned} \tag{11}$$

let us assume $x_g - x_{cs} = r_1$ and $y_g - y_{cs} = r_2$ then

$$S_{x_g,y_g} = \gamma_i (r_1^2 + r_2^2) - \frac{1}{\left(\sqrt{r_1^2 + r_2^2}\right)^2}) \tag{12}$$

and

$$S_{x_{cs},y_{cs}} = \alpha (r_1^2 + r_2^2) - \frac{1}{\left(\sqrt{r_1^2 + r_2^2}\right)^2}) \tag{13}$$

So the goal is the minima when:

$$\begin{aligned} \gamma_i (r_1^2 + r_2^2) - \frac{1}{\left(\sqrt{r_1^2 + r_2^2}\right)^2}) < \\ \alpha (r_1^2 + r_2^2) - \frac{1}{\left(\sqrt{r_1^2 + r_2^2}\right)^2}) \end{aligned} \tag{14}$$

which simplifies to

$$\gamma_i < \alpha_i$$

as we already know that γ is $\frac{\beta}{B_i}$ therefore:

$$\frac{\beta}{B_i} < \alpha_i$$

For the global minima to be at the goal potential:

$$B_i > \frac{\beta}{\alpha} \tag{15}$$

Similarly, the battery charging station is the global minima when:

$$B_i < \frac{\beta}{\alpha} \tag{16}$$

This proves that the global minima changes depending on the Battery Level of the robot and we start moving towards the charger when battery levels deplete at a certain point.

3.3. Analysis of Collision Avoidance Between Robots

Collision avoidance is a crucial feature that needs to be incorporated into the control of multi-robot systems to avoid accidents. To ensure this we have designated potential fields for each of the robots that are deployed in the environment. The potential for each of the robots is calculated using the Gaussian equation and analyzed as in [6], behaving similarly to the one used to calculate the obstacle potential given by:

$$R_{pj}(X,Y) = \frac{1}{5} \sum_{\substack{i=1 \\ i \neq j}}^{n} \cdot A \cdot \exp\left(-\frac{(X - x_i)^2 + (Y - y_i)^2}{\frac{1}{16} \cdot \sigma^2} \right) \tag{17}$$

where

- R_{pj} is the potential field generated by other robots at point (X, Y) for the jth robot.
- σ^2 is a parameter controlling the spread or reach of the potential field.
- (x_i, y_i) represents the positions of the ith individual robot in the environment.
- n is the total number of robots.

The potential field of all robots is similar to the potential field generated by the obstacles but we scale the magnitude of the robot potential fields to a value of $\frac{1}{5}O_p$. The robot potential fields are designed to be repulsive and the potential field of one robot appears as a positive obstacle potential to the other robots. This ensures that the ith robot does not collide with robot j at any given moment during the simulation.

4. Simulation

For our simulation, we set up a square map whose dimensions were set to be 20x20. The map was populated with 5 obstacles, a home position, an initial goal position, and a battery charger location. In Figure 4 we can see that

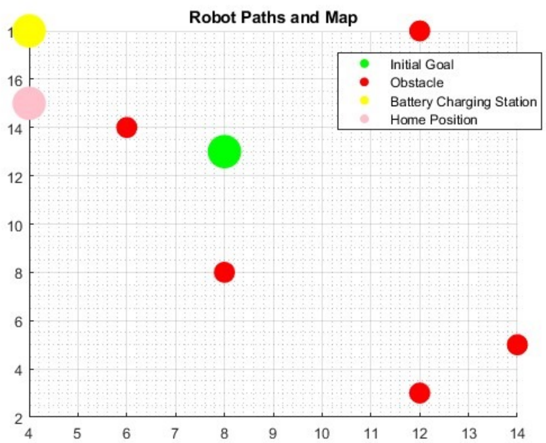

Figure 3. Initial Simulation Setup

we chose five robots to start the simulation. All five robots start at different locations and then make their way to the goal based on the potential fields created by the obstacles and the goal given by the equations (1) and (2) respectively

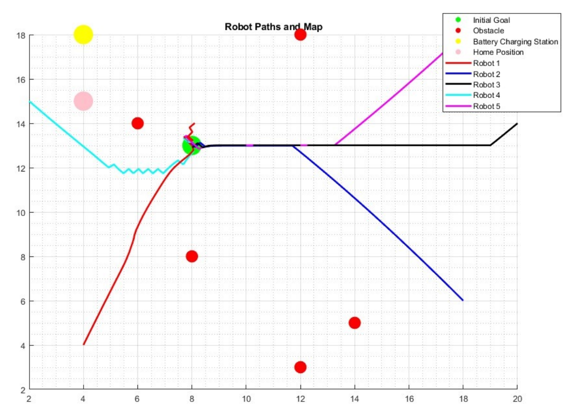

Figure 4. Robots at Goal

As soon as the robots reach the goal position, the home position potential field increases, and the robots are drawn towards the home position and navigate to where the home position is located on the map. We can see the same happen in Figure 5. Meanwhile, depending on the battery levels of each robot, the potential field of the charging station increases for robots as the battery falls below the threshold. We can see in Figure 6 that the black robot has reached the charger and the others are diverting from home towards the charging station. Finally, we can see in Figure 7 the plot of the battery status of all robots. We can also notice that the battery of the robot represented by the black line goes to 100 as it reaches the charging station whereas the battery levels of the other robots are still depleting as they head towards the charging station.

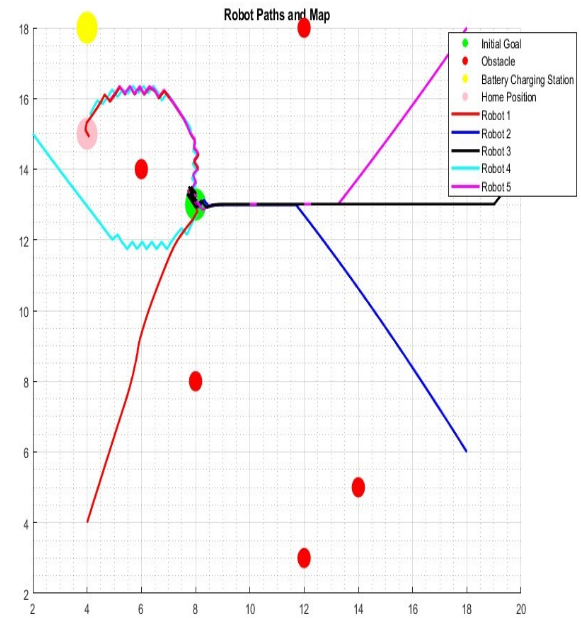

Figure 5. Robots Reached Home

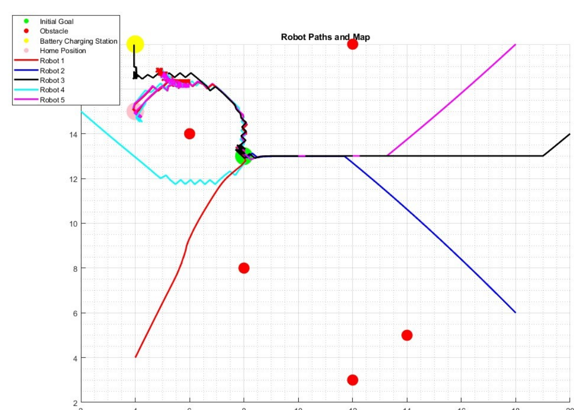

Figure 6. Robot Reached Charging Station

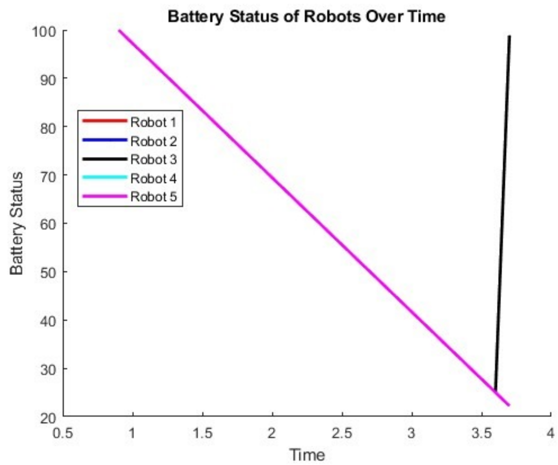

Figure 7. Battery Status of Each Robot

5. Specific Contributions

- New Controller for Dynamic Switching (**Section 2.2**) - ***Everyone***
- Mathematical Model Creation For Potential Generation (**Section 2.1**) - ***Archit and Vishal***
- Mathematical Analysis (Local Minima) (**Section 3.1**) - ***Vishal and Nehal***
- Mathematical Analysis (Dynamic Switching of Minima from Goal to Charger Position) (**Section 3.2**) - ***Archit and Sunder***
- Mathematical Analysis (Robot Collision Avoidance) (**Section 3.3**) - ***Nehal and Sunder***
- Simulation Validation (**Section 4**) - ***Everyone***

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