Forecasting_Project

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03/10/2021

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Introduction

This analysis has three parts: Task 1: Forecasting Mortality rate data. Task 2: Forecasting First Flowering day data. Task 3: Forecasting Rank-based Order similarity metric data.

Task 1:

Here, we will forecast the Mortality rate data for the next four weeks using the best fit model. To get this best model we have three approaches.
1. Suitable Distributed Lag and Dynamic LM 2. Smoothing methods 3. State Space models

The best model is the one that has the best MASE score as well as gives the best residual analysis.

Task 2:

Here, we will forecast the First Flowering day data for the next four years using the best fit model. To get this best model we have three approaches.
1. Suitable Distributed Lag and Dynamic LM (With and without slope) 2. Smoothing methods 3. State Space models

The best model is the one that has the best MASE score as well as gives the best residual analysis.

Task 3:

This again has 2 parts.

Part (a)

Part (b)

Part (a):

Here, we will forecast the Rank-based Order similarity metric data for the next three years using the best fit model among Distributed Lag and Dynamic LM The best model is the one that has the best MASE score as well as gives the best residual analysis.

Part (b):

Here, we will forecast the Rank-based Order similarity metric data during the Millennium drought period (1997 - 2009) for the next three years using the best fit model among Distributed Lag and Dynamic LM. The best model is the one that has the best MASE score as well as gives the best residual analysis.

Method

Task 1

The following packages are for all the three parts.

```
library(dplyr)
library(forecast) # Forecasting Functions for Time Series and Linear Models.
[1] - https://cran.r-project.org/web/packages/forecast/index.html
library(dLagM) # Distributed lag model.
library(lmtest) # Testing Linear Regression Models. [2] - https://cran.r-
project.org/web/packages/lmtest/index.html
library(tidyr)
library(tseries) # Time Series Analysis and Computational Finance.[3] -
https://cran.r-project.org/web/packages/tseries/index.html
library(fUnitRoots) # To analyze trends and unit roots in financial time
series. [4] - https://cran.r-project.org/web/packages/fUnitRoots/index.html
library(expsmooth) # Forecasting with Exponential Smoothing. [5] -
https://cran.r-project.org/web/packages/expsmooth/index.html
library(TSA) # Time Series Analysis.
library(urca) # Unit Root and Cointegration Tests. [6] - https://cran.r-
project.org/web/packages/urca/index.html
library(readr)
library(xts)
```

Data

The data here used is the weekly averages of potential effects of both climate and pollution on disease specific mortality between the years 2010-2020.

```
v_Mortality_data <- read.csv("mort.csv", header = TRUE)
head(v_Mortality_data)

##   ï.. mortality  temp chem1 chem2 particle.size
## 1   1     97.85 72.38 11.51  3.37         72.72
## 2   2    104.64 67.19  8.92  2.59         49.60
```

```
## 3   3     94.36 62.94  9.48  3.29         55.68
## 4   4     98.05 72.49 10.28  3.04         55.16
## 5   5     95.85 74.25 10.57  3.39         66.02
## 6   6     95.98 67.88  7.99  2.57         44.01
```

```
# Using str() to check the type of each column.
str(v_Mortality_data)

## 'data.frame':    508 obs. of  6 variables:
##  $ ï..          : int  1 2 3 4 5 6 7 8 9 10 ...
##  $ mortality    : num  97.8 104.6 94.4 98 95.8 ...
##  $ temp         : num  72.4 67.2 62.9 72.5 74.2 ...
##  $ chem1        : num  11.51 8.92 9.48 10.28 10.57 ...
##  $ chem2        : num  3.37 2.59 3.29 3.04 3.39 2.57 2.35 3.38 1.5 2.56
...
##  $ particle.size: num  72.7 49.6 55.7 55.2 66 ...
```

Checking for Missing values.

```
colSums(is.na(v_Mortality_data))

##           ï..     mortality          temp         chem1         chem2
##             0             0             0             0             0
## particle.size
##             0
```

There are no missing values in the data.

Checking the class of v_Mortality_data (It should be a data frame.)

```
class(v_Mortality_data)

## [1] "data.frame"
```

Setting frequency = 365.27/7. Since weekly data.

```
v_Mortality_data_TS <- ts(v_Mortality_data$mortality, start = c(2010, 1),
frequency = (365.27/7))
v_Mortality_temp_data_TS <- ts(v_Mortality_data$temp, start = c(2010, 1),
frequency = (365.27/7))
v_Mortality_chem1_data_TS <- ts(v_Mortality_data$chem1, start = c(2010, 1),
frequency = (365.27/7))
v_Mortality_chem2_data_TS <- ts(v_Mortality_data$chem2, start = c(2010, 1),
frequency = (365.27/7))
v_Mortality_particle.size_data_TS <- ts(v_Mortality_data$particle.size, start
= c(2010, 1), frequency = (365.27/7))
```

Confirming the class of each time series object.

```
class(v_Mortality_data_TS)

## [1] "ts"
```

```
class(v_Mortality_temp_data_TS)

## [1] "ts"

class(v_Mortality_chem1_data_TS)

## [1] "ts"

class(v_Mortality_chem2_data_TS)

## [1] "ts"

class(v_Mortality_particle.size_data_TS)

## [1] "ts"
```

Now let us perform descriptive analysis on each time series object.

Descriptive Analysis

Mortality Rate

```
plot(v_Mortality_data_TS, type = "b", xlab = "weeks", ylab = "Mortality
rate", main = "Time series plot for mortality rate from 2010 to 2020 (508
weeks)", pch = 1)
legend("topright", inset = .03, title = "Rate", legend = "Mortality rate
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

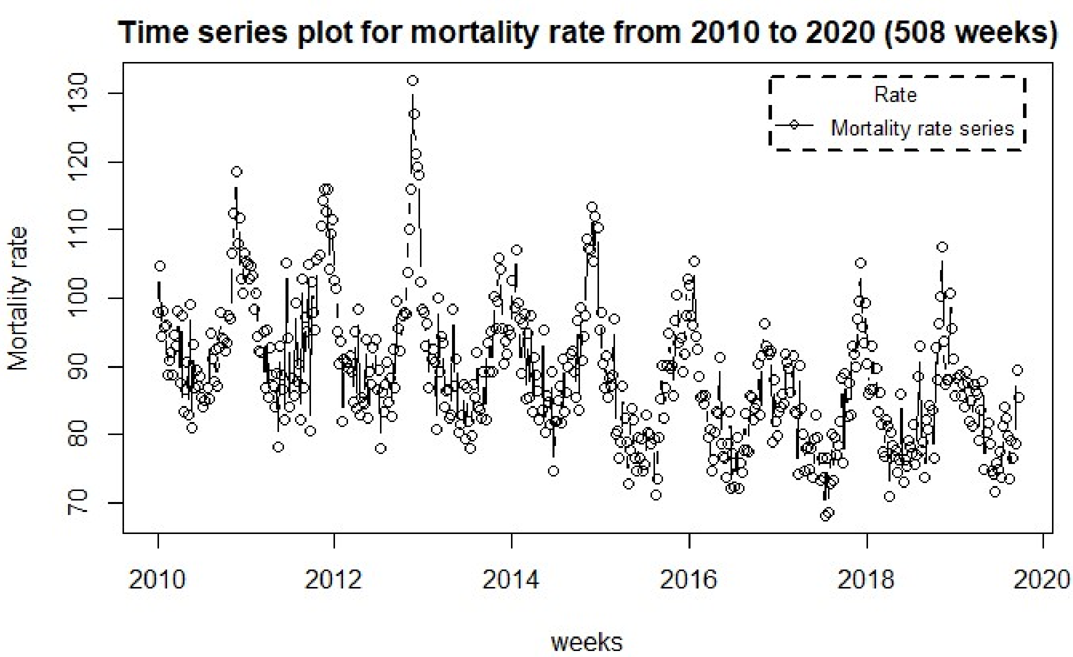


Fig 1.1: Mortality Rate - Time series plot.

```
McLeod.Li.test(y = v_Mortality_data_TS, main = "McLeod-Li Test Statistics for
Mortality Rate.")
```

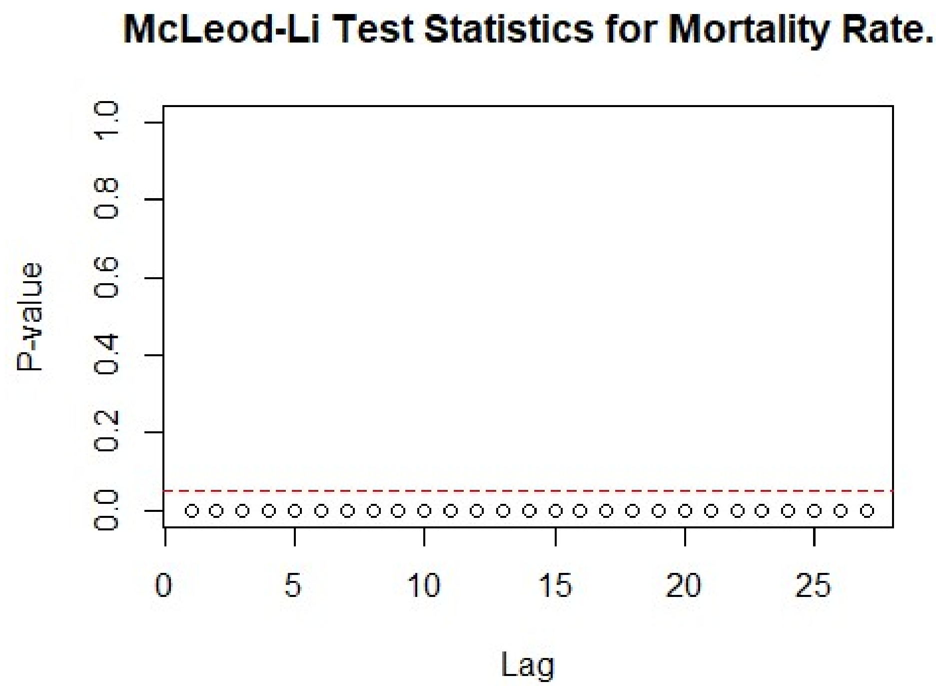


Fig 1.2: McLeod-Li Test Statistics for Mortality Rate.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention at multiple points in the series.
3. From the series plot, we can conclude that there is seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. Therefore, there is no change Autoregressive and moving average behaviour.
6. Also, we cannot see change in variance.

Temperature

```
plot(v_Mortality_temp_data_TS, type = "b", xlab = "weeks", ylab =
"Temperature", main = "Time series plot for temperature from 2010 to 2020
(508 weeks)", pch = 1)
legend("top", inset = .03, title = "Temperature", legend = "Temperature
series", horiz = TRUE, cex = 0.6, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

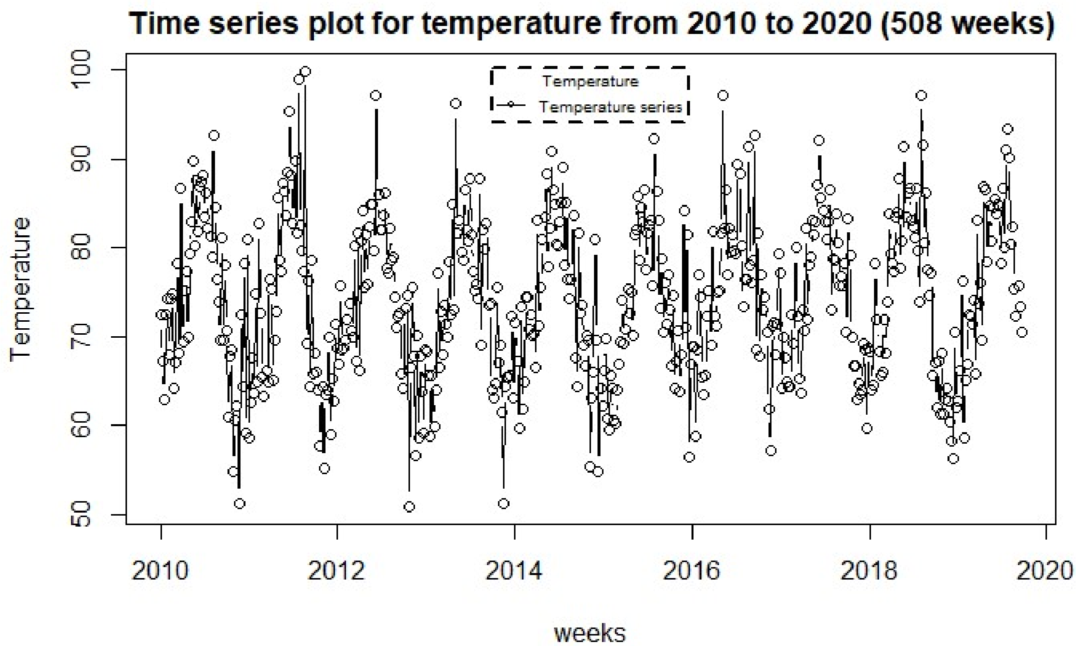


Fig 1.3: Temperature - Time series plot.

```
McLeod.Li.test(y = v_Mortality_temp_data_TS, main = "McLeod-Li Test
Statistics for Temperature")
```

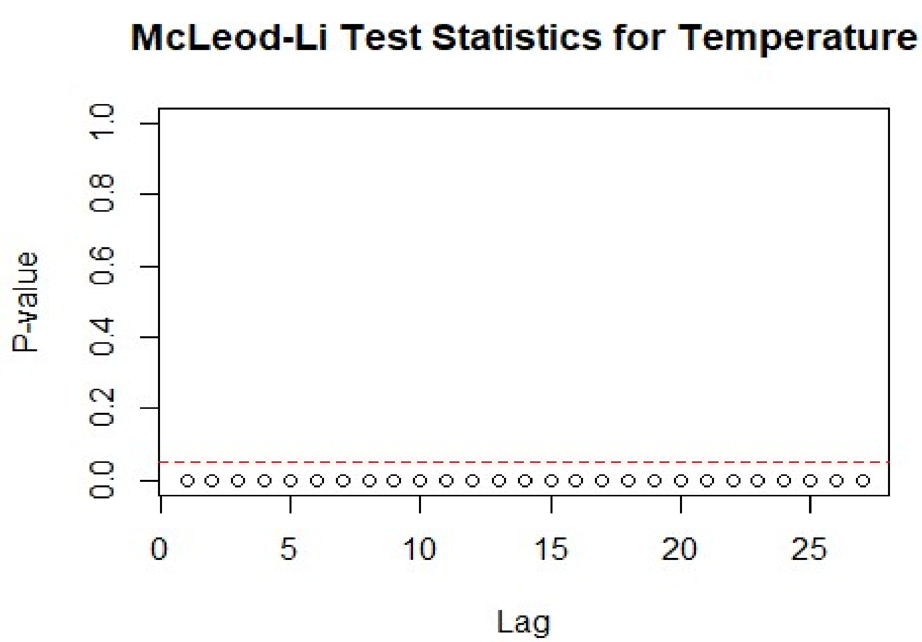


Fig 1.4: McLeod-Li Test Statistics for Precipitation.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention at multiple points in the series.
3. From the series plot, we can conclude that there is seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. Therefore, there is no change Autoregressive and moving average behaviour.
6. Also, we cannot see change in variance.

Chemical Emission 1

```
plot(v_Mortality_chem1_data_TS, type = "b", xlab = "weeks", ylab = "Chemical Emission 1", main = "Time series plot for Chemical Emission 1 from 2010 to 2020 (508 weeks)", pch = 1)
legend("topright", inset = .03, title = "Chemical Emission 1", legend = "Chemical Emission 1 series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

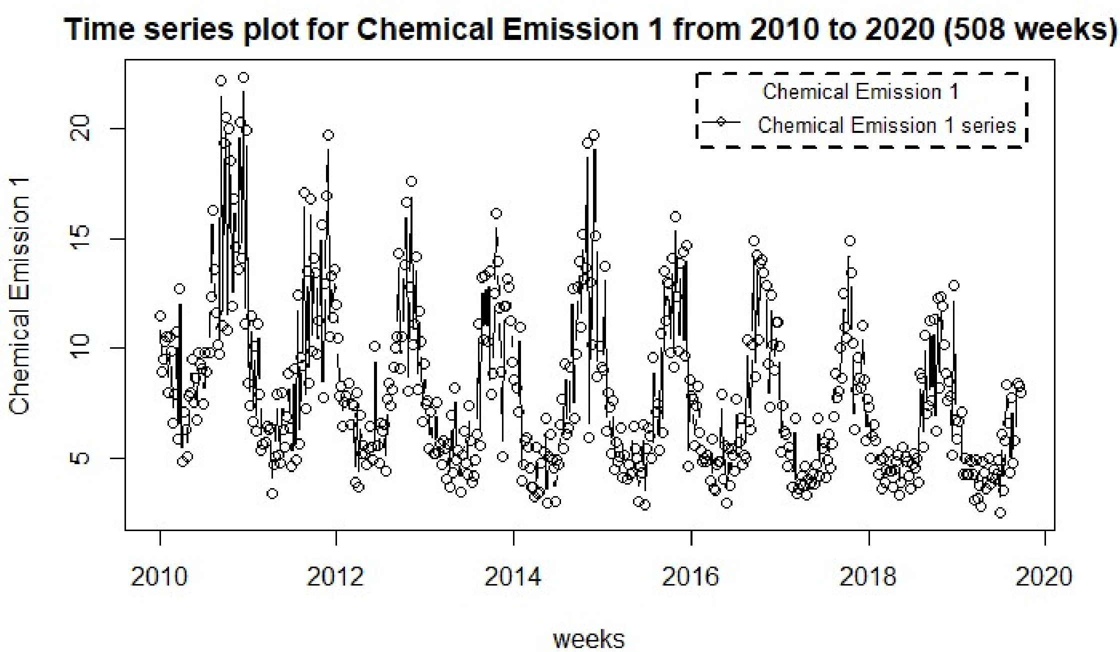


Fig 1.3: Chemical Emission 1 - Time series plot.

```
McLeod.Li.test(y = v_Mortality_chem1_data_TS, main = "McLeod-Li Test Statistics for Chemical Emission 1.")
```

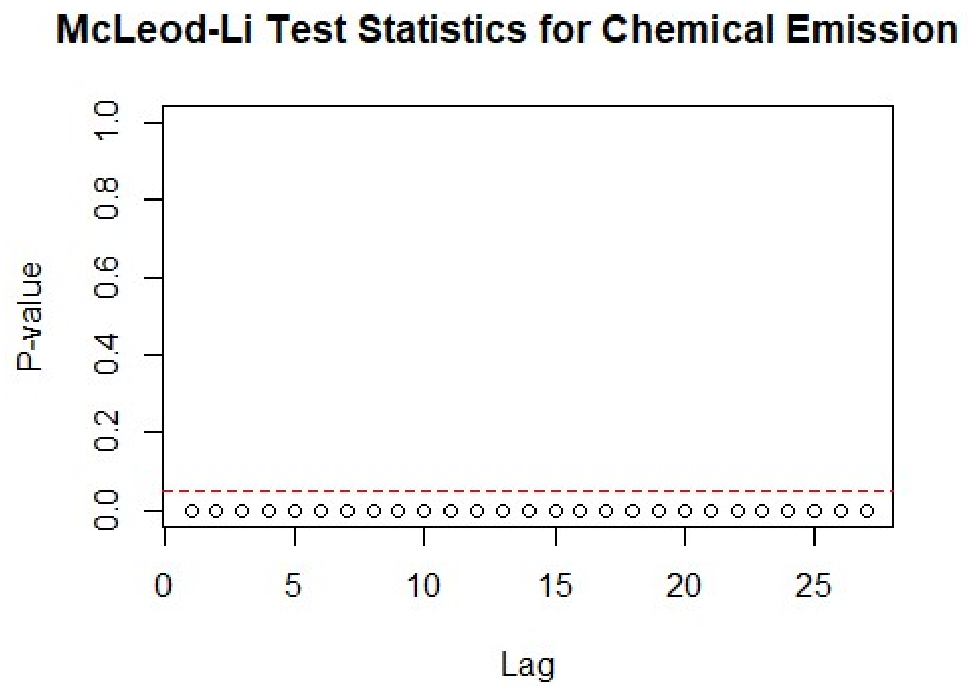


Fig 1.4: McLeod-Li Test Statistics for Chemical Emission 1.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention at multiple points in the series.
3. From the series plot, we can conclude that there is seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. Therefore, there is no change Autoregressive and moving average behaviour.
6. Also, we cannot see change in variance.

Chemical Emission 2

```
plot(v_Mortality_chem2_data_TS, type = "b", xlab = "weeks", ylab = "Chemical
Emission 2", main = "Time series plot for Chemical Emission 2 from 2010 to
2020 (508 weeks)", pch = 1)
legend("topright", inset = .03, title = "Chemical Emission 2", legend =
"Chemical Emission 2 series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2,
box.lwd = 2, pch = 1)
```

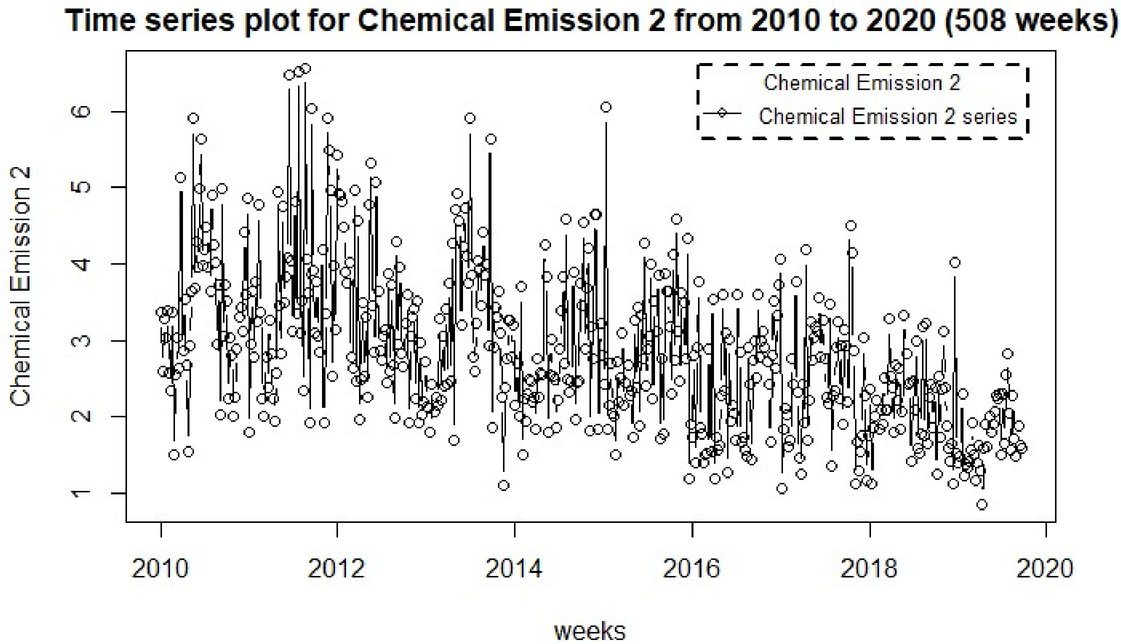


Fig 1.3: Chemical Emission 2 - Time series plot.

```
McLeod.Li.test(y = v_Mortality_chem2_data_TS, main = "McLeod-Li Test
Statistics for Chemical Emission 2.")
```

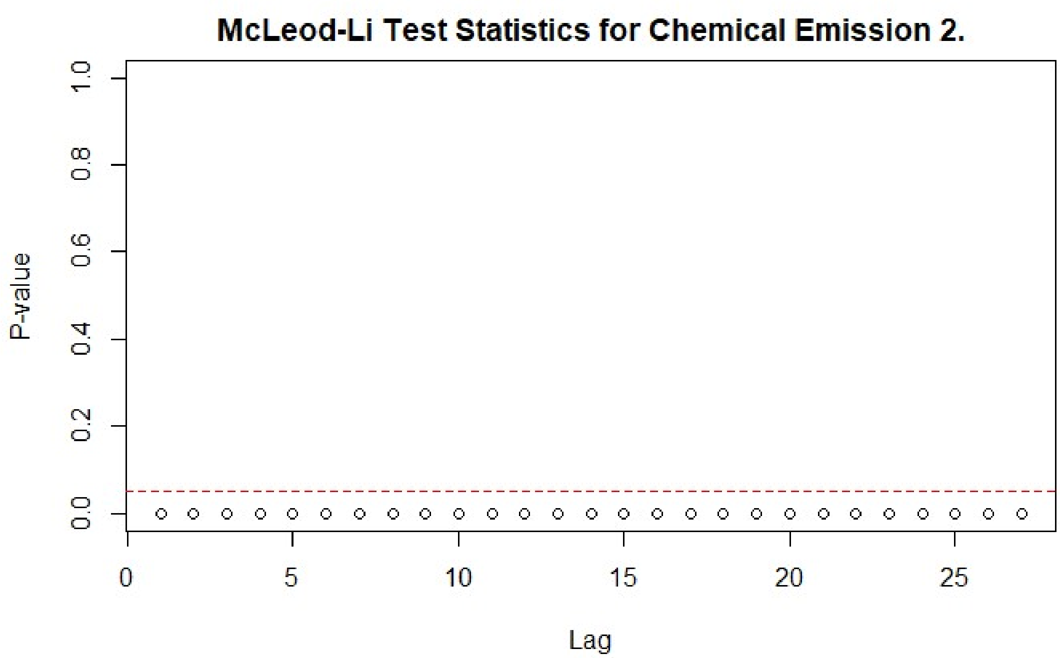


Fig 1.4: McLeod-Li Test Statistics for Chemical Emission 2.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data. But it seems like a downward trend.
2. There is an intervention at multiple points in the series.
3. From the series plot, we can conclude that there is seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. Therefore, there is no change Autoregressive and moving average behaviour.
6. Also, we cannot see change in variance.

Partical size

```
plot(v_Mortality_particle.size_data_TS, type = "b", xlab = "weeks", ylab =
"Partical size", main = "Time series plot for Particle size from 2010 to 2020
(508 weeks)", pch = 1)
legend("topleft", inset = .03, title = "Partical size", legend = "Partical
size", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

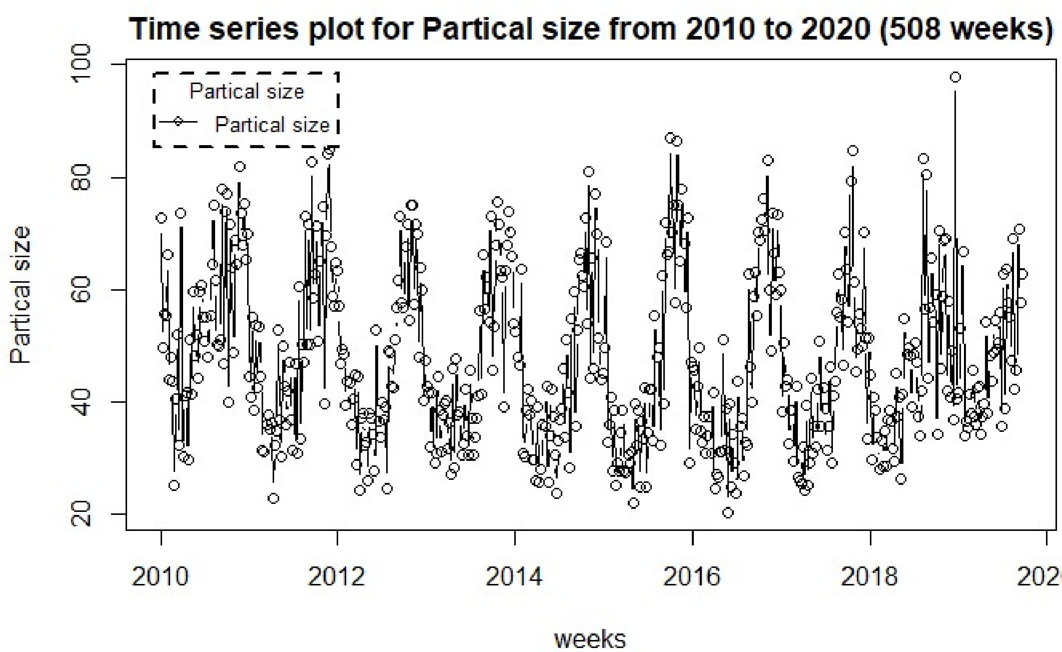


Fig 1.3: Partical size - Time series plot.

```
McLeod.Li.test(y = v_Mortality_particle.size_data_TS, main = "McLeod-Li Test
Statistics for Partical size")
```

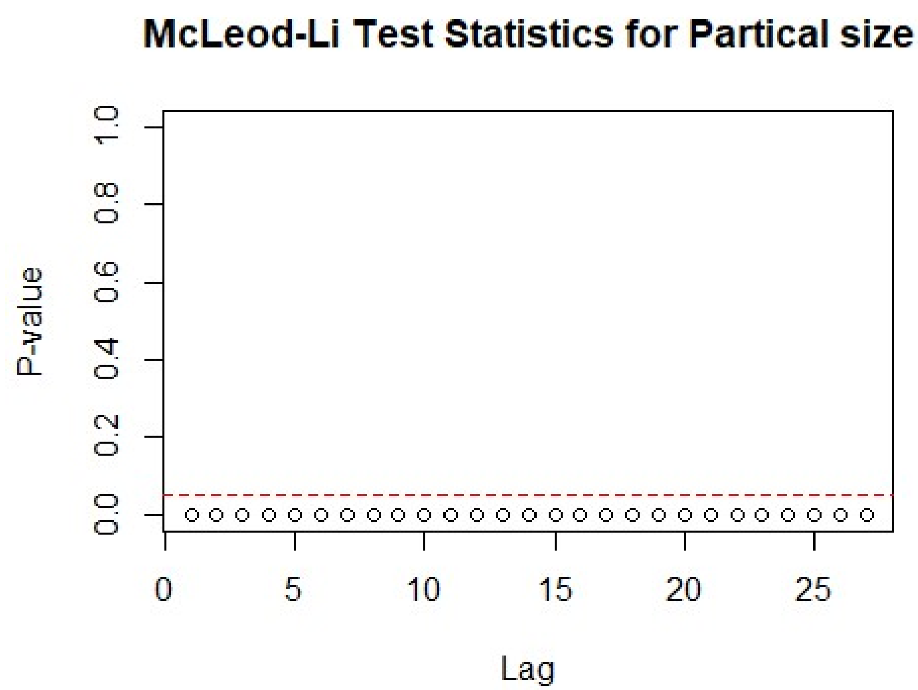


Fig 1.4: McLeod-Li Test Statistics for Partical size.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention at multiple points in the series.
3. From the series plot, we can conclude that there is seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. Therefore, there is no change Autoregressive and moving average behaviour.
6. Also, we cannot see change in variance.

Checking for Stationary in the series

```
# Function to check Stationary on the series.
Stationary_Check <- function(x, m1, m2) {

  # Analysing trends by plotting ACF and PACF.
  par(mfrow = c(1,2))
  acf(x, main = m1)
  pacf(x, main = m2)

  # Lag for ADF test
  d = ar(x)$order

  # Conducting Augmented Dickey-Fuller test.
```

```
  adf.test(x, k = d)
}
```

Checking for Stationary on Mortality Rate series.

```
Stationary_Check(v_Mortality_data_TS, "Mortality Rate - ACF plot", "Mortality
Rate - PACF plot")

## Warning in adf.test(x, k = d): p-value smaller than printed p-value
```

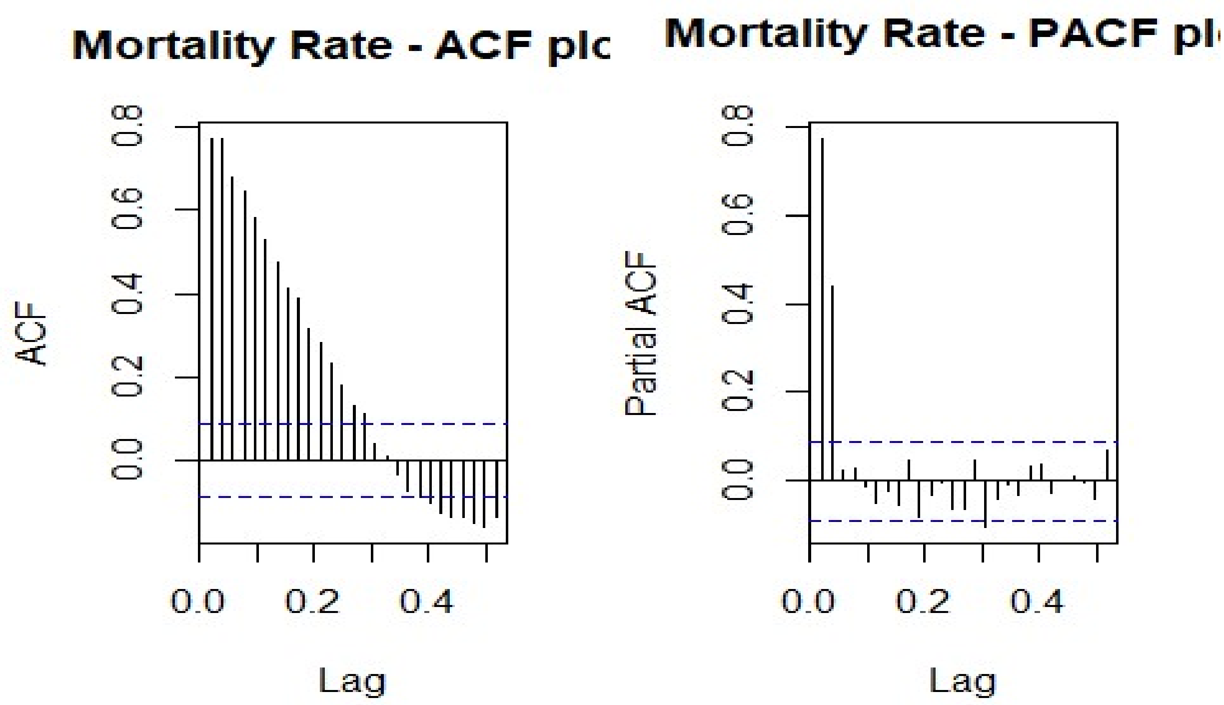


Fig 1.5: Mortality Rate - ACF

Fig 1.6: Mortality Rate - ACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -5.161, Lag order = 2, p-value = 0.01
## alternative hypothesis: stationary
```

The seasonal pattern in the significant lags suggests that there is no trend in the series.
Hypotheses:

H0: The data is not stationary.

HA: The data is stationary.

Interpretations:

p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.
Therefore, the Mortality rate series is Stationary.

Checking for Stationary on Temperature series.

```
Stationary_Check(v_Mortality_temp_data_TS, "Temperature - ACF plot",
"Temperature - PACF plot")

## Warning in adf.test(x, k = d): p-value smaller than printed p-value
```

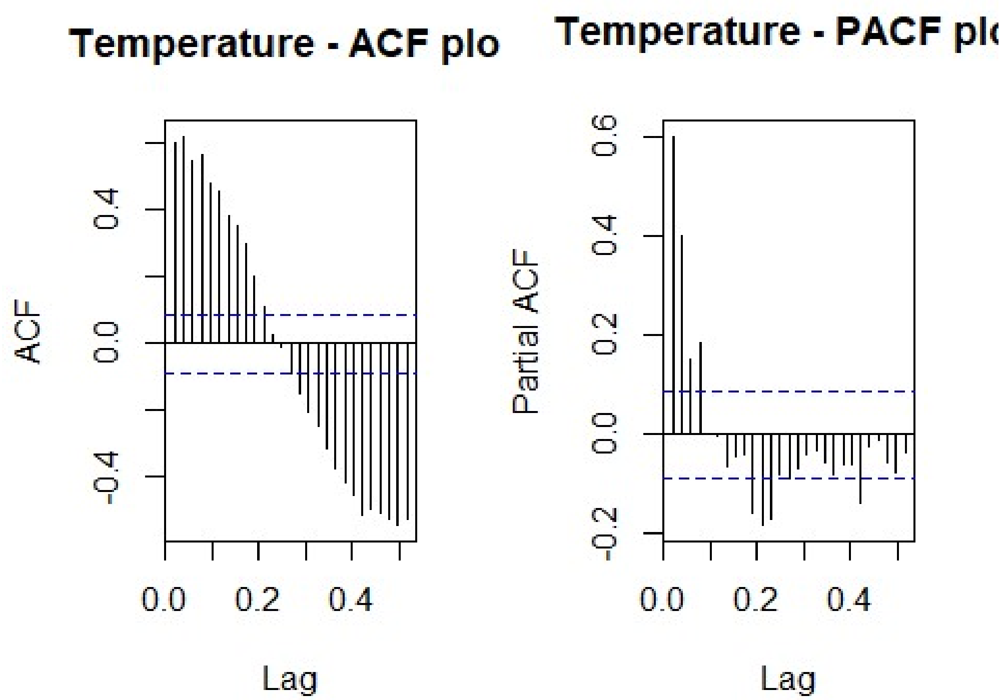


Fig 1.7: Temperature - ACF

Fig 1.8: Temperature - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -8.2554, Lag order = 22, p-value = 0.01
## alternative hypothesis: stationary
```

Fig 1.7: Temperature - ACF Fig 1.8: Temperature - PACF

The seasonal pattern in the significant lags suggests that there is no trend in the series.

Hypotheses:

H0: The data is not stationary.

HA: The data is stationary.

Interpretations:

p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Temperature series is Stationary.

Checking for Stationary on Chemical Emission 1 series.

```
Stationary_Check(v_Mortality_chem1_data_TS, "Chemical Emission 1 - ACF plot",
"Chemical Emission 1 - PACF plot")

## Warning in adf.test(x, k = d): p-value smaller than printed p-value
```

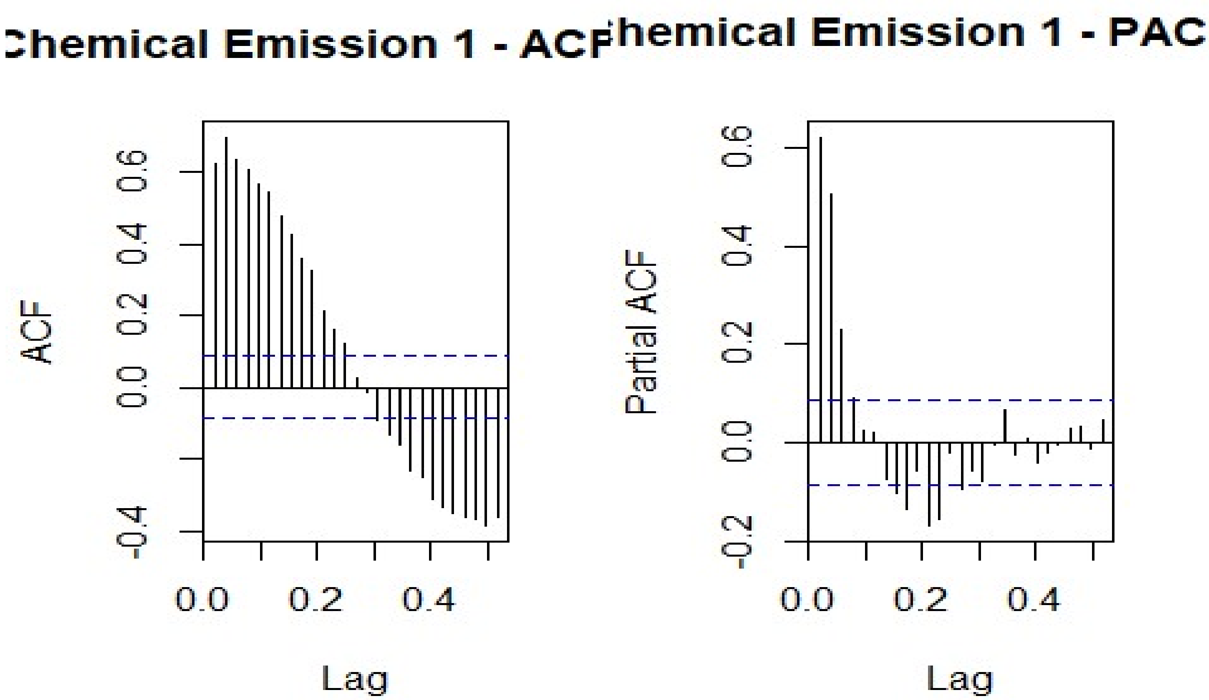


Fig 1.9: Chemical Emission 1 - ACF

Fig 1.10: Chemical Emission 1 - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
```

```
## Dickey-Fuller = -8.1588, Lag order = 16, p-value = 0.01
## alternative hypothesis: stationary
```

The seasonal pattern in the significant lags suggests that there is no trend in the series.

Hypotheses:

H0: The data is not stationary.

HA: The data is stationary.

Interpretations:

p - value : ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Chemical Emission 1 series is Stationary.

Checking for Stationary on Chemical Emission 2 series.

```
Stationary_Check(v_Mortality_chem2_data_TS, "Chemical Emission 2 - ACF plot",
"Chemical Emission 2 - PACF plot")

## Warning in adf.test(x, k = d): p-value smaller than printed p-value
```

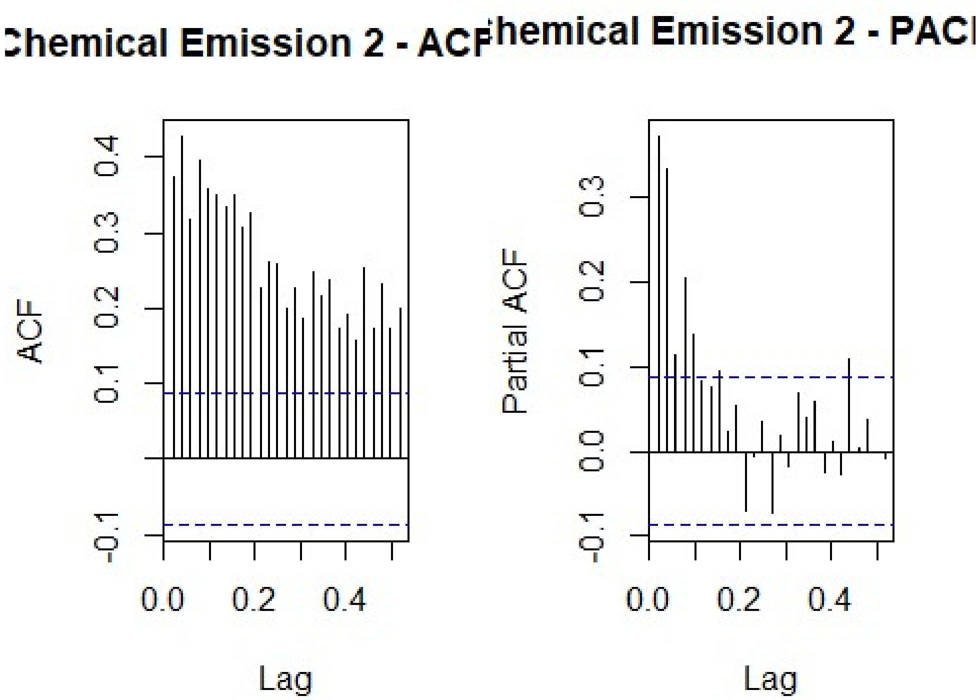


Fig 1.11: Chemical Emission 2 - ACF

Fig 1.12: Chemical Emission 2 - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -5.3362, Lag order = 8, p-value = 0.01
## alternative hypothesis: stationary
```

The decrease in the ACF plot and a high peak in the PACF plot in the beginning, suggests that there is some pattern in the Chemical Emission 2 series.

Hypotheses:

H0: The data is not stationary.

HA: The data is stationary.

Interpretations:

p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Chemical Emission 2 series is Stationary.

Checking for Stationary on Particle Size data.

```
Stationary_Check(v_Mortality_particle.size_data_TS, "Particle Size - ACF plot", "Particle Size - PACF plot")

## Warning in adf.test(x, k = d): p-value smaller than printed p-value
```

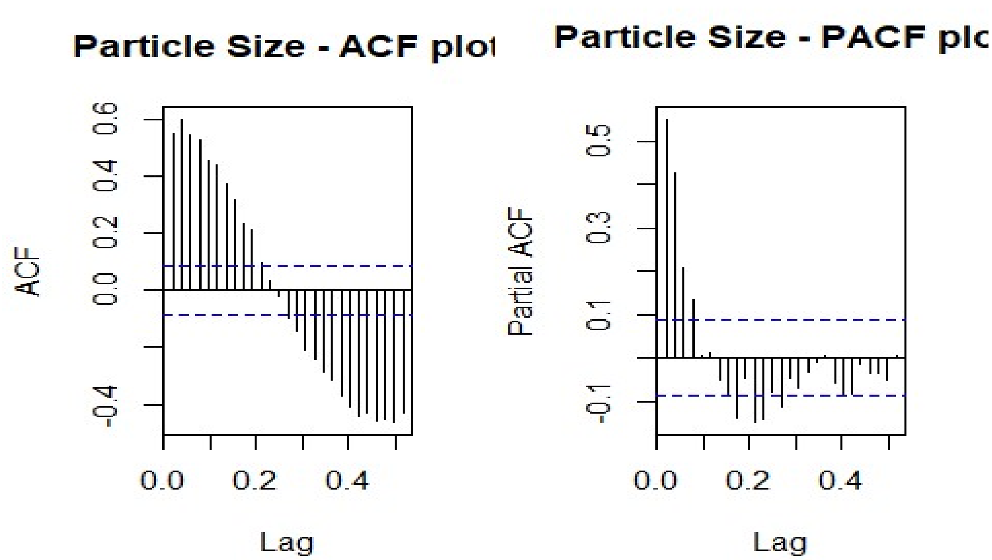


Fig 1.13: Particle size - ACF

Fig 1.14: Particle Size - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -7.2956, Lag order = 14, p-value = 0.01
## alternative hypothesis: stationary
```

The seasonal pattern in the significant lags suggests that there is no trend in the series.

Hypotheses:

> **H_0: The data is not stationary.**
> **H_A: The data is stationary.**

Interpretations:

> p - value : ~ 0.01 < 0.05
>
> p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Particle size series is Stationary.

Therefore, no differentiation is required. As the two series are stationary.

Impact of components on each time series.

The components of a series are usually,

1. Seasonality
2. Trend
3. Remainder

We should decompose the time series into the above components as we can see the impact of these components on the series data.

For this STL decomposition is used, as there is intervention in some of the series. This intervention is might be due to outliers and STL decomposition is robust in the case of outliers.

Decomposing Mortality series into components.

```
v_Mortality_stl_decomp <- stl(v_Mortality_data_TS, t.window = 15, s.window =
"periodic", robust = TRUE)
plot(v_Mortality_stl_decomp, main = "Decomposing Mortality Series into
components")
```

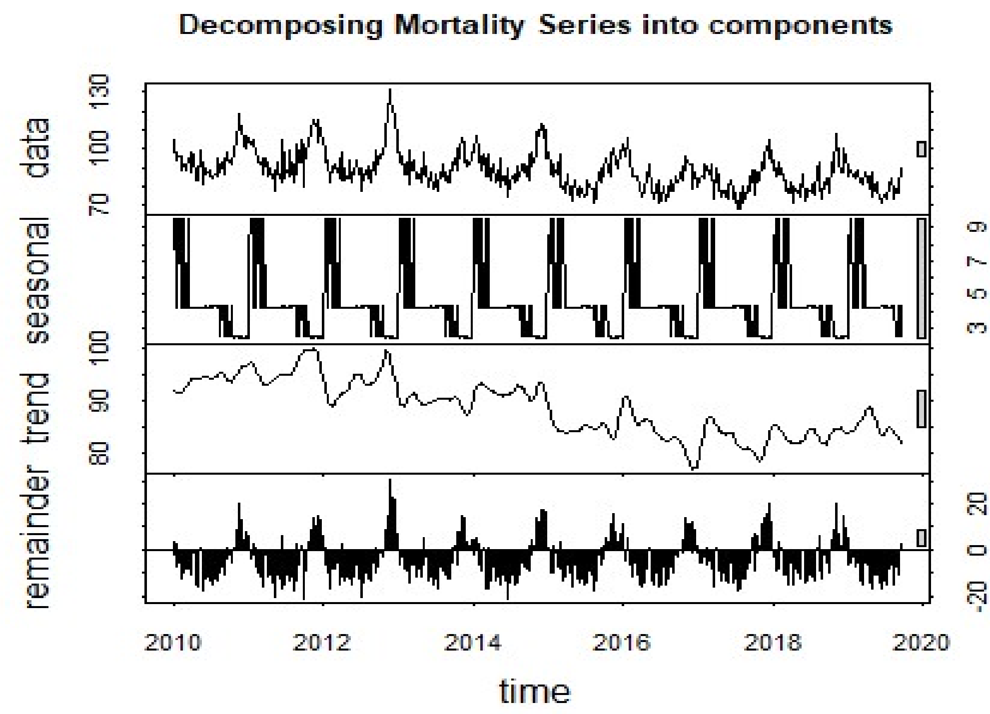


Fig 1.15: Decomposing Mortality series into components - stl decomposition.

1. The seasonality component shows peaks at the same points suggesting some seasonality.
2. The trend in the series data is not shown by the trend component.
3. Remainder component shows a high intervention point around 2013.

Decomposing Temperature series into components.

```
v_temp_stl_decomp <- stl(v_Mortality_temp_data_TS, t.window = 15, s.window =
"periodic", robust = TRUE)
plot(v_temp_stl_decomp, main = "Decomposing Temperature Series into
components")
```

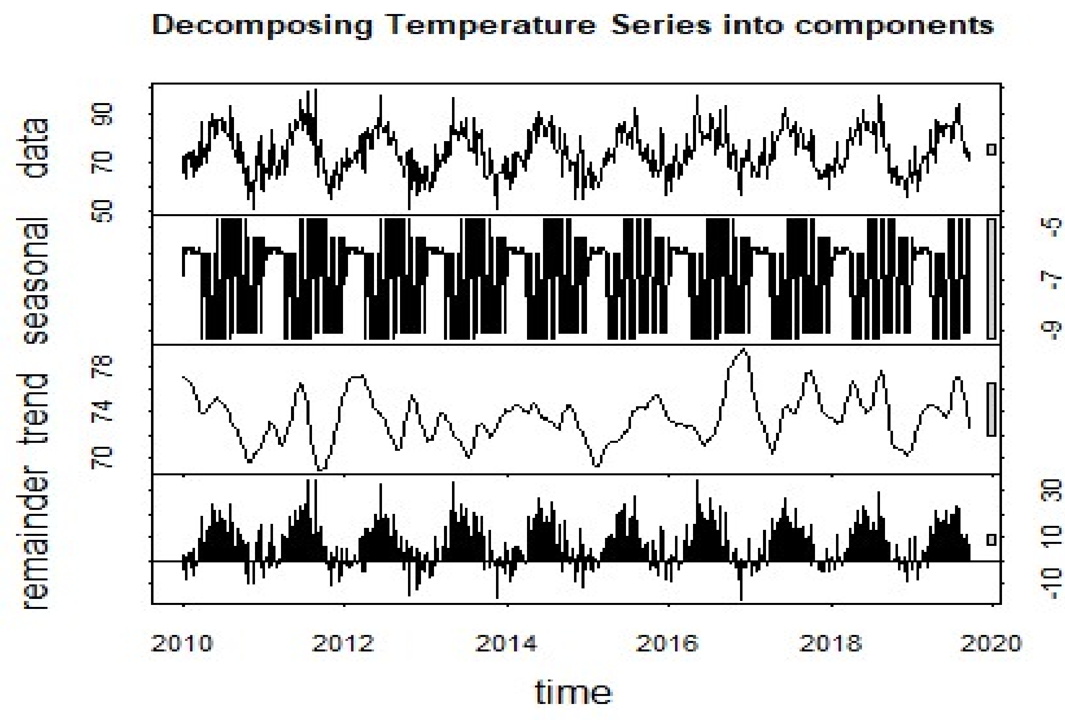


Fig 1.16: Decomposing Temperature series into components - stl decomposition.

1. The seasonal adjusted series is not meaningful in this case as it much deviated from the original series. Suggesting that there is no seasonality in the series. But we observed seasonality in the series. This makes that there is no sense in the seasonality components.
2. The trend in the series data is not shown by the trend component.
3. Remainder component shows no high intervention points.

Decomposing Chemical Emission 1 series into components.

```
v_Chem1_stl_decomp <- stl(v_Mortality_chem1_data_TS, t.window = 15, s.window
= "periodic", robust = TRUE)
plot(v_Chem1_stl_decomp, main = "Decomposing Chemical Emission 1 Series into
components")
```

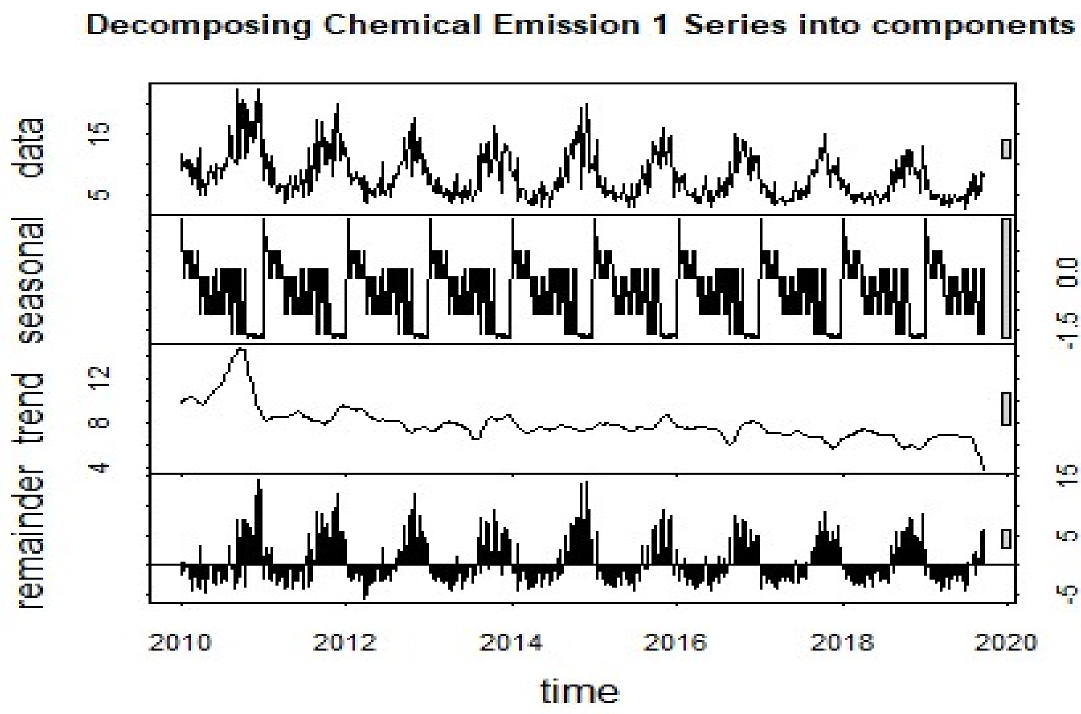


Fig 1.17: Decomposing Chemical Emission 1 series into components - stl decomposition.

1. The seasonal adjusted series is not meaningful in this case as it much deviated from the original series. Suggesting that there is no seasonality in the series. But we observed seasonality in the series. This makes that there is no sense in the seasonality components.
2. The trend in the series data is not shown by the trend component.
3. Remainder component shows a high intervention point at 2011 and 2015.

Decomposing Chemical Emission 2 series into components.

```
v_Chem2_stl_decomp <- stl(v_Mortality_chem2_data_TS, t.window = 15, s.window
= "periodic", robust = TRUE)
plot(v_Chem2_stl_decomp, main = "Decomposing Chemical Emission 2 Series into
components")
```

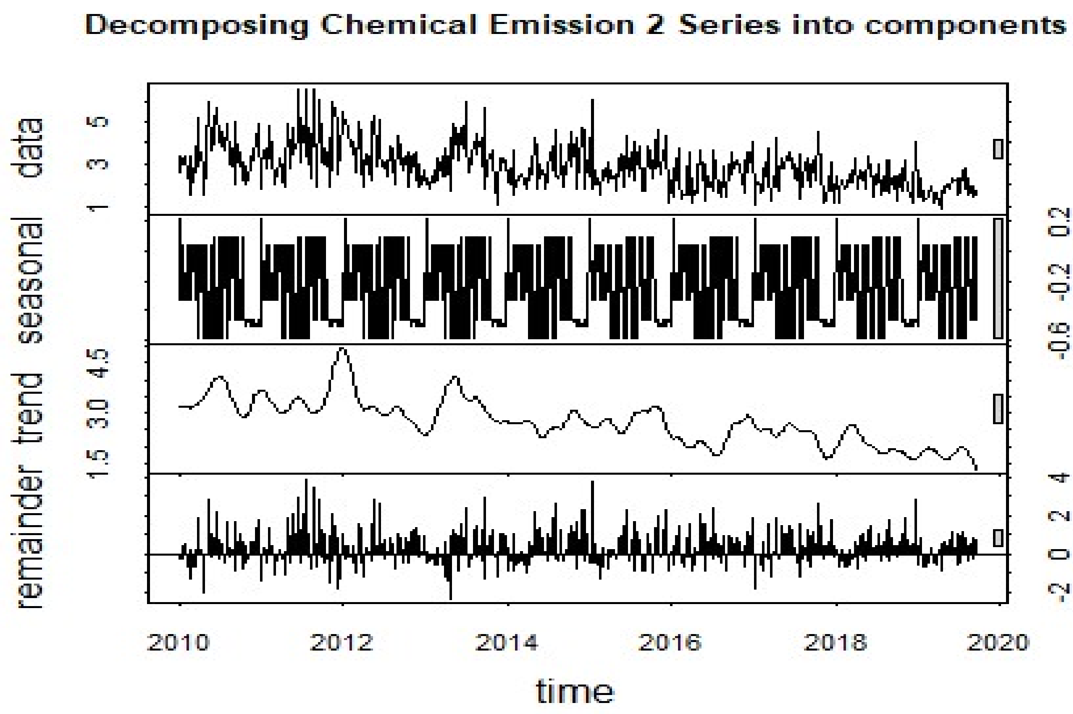


Fig 1.17: Decomposing Chemical Emission 2 series into components - stl decomposition.

1. The seasonal adjusted series is not meaningful in this case as it much deviated from the original series. Suggesting that there is no seasonality in the series. But we observed seasonality in the series. This makes that there is no sense in the seasonality components.
2. The trend in the series data is shown exactly by the trend component.
3. Remainder component shows a high intervention point at multiple points.

Decomposing COPPER price series into components.

```
v_Part_stl_decomp <- stl(v_Mortality_particle.size_data_TS, t.window = 15,
s.window = "periodic", robust = TRUE)
plot(v_Part_stl_decomp, main = "Decomposing Particle price Series into
components")
```

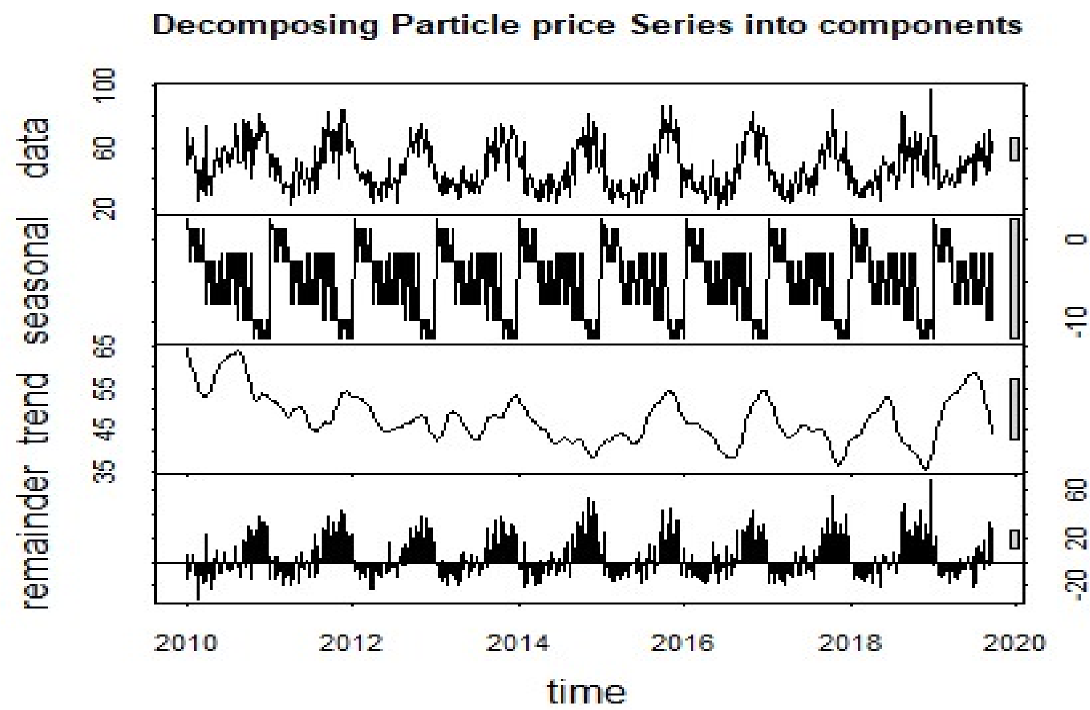


Fig 1.18: Decomposing Particle price series into components - stl decomposition.

1. The seasonal adjusted series is not meaningful in this case as it much deviated from the original series. Suggesting that there is no seasonality in the series. But we observed seasonality in the series. This makes that there is no sense in the seasonality components.
2. The trend in the series data is shown by the trend component.
3. Remainder component shows a high intervention point at multiple points.

Suitable distributed lag models (Multivariate Analysis).

Before this let us find the correlation between the two series.

```
# Calculating the correlation coefficient
cor(v_Mortality_data_TS, v_Mortality_temp_data_TS)

## [1] -0.4386396

cor(v_Mortality_data_TS, v_Mortality_chem1_data_TS)

## [1] 0.5574476

cor(v_Mortality_data_TS, v_Mortality_chem2_data_TS)

## [1] 0.2569989

cor(v_Mortality_data_TS, v_Mortality_particle.size_data_TS)
```

```
## [1] 0.4438713
```

This suggests that Mortality rate has a strong correlation with Chemical emission 1 and Particle size.

As we are going to forecast the Mortality Rate data our dependent variable “y” will be Mortality Rate series object and independent variable “x” will be Chemical emission 1 and Particle size. Since multivariate analysis. For this let us convert the entire data set into time series.

```
v_data_TS <- ts(v_Mortality_data, start = c(2010, 1), frequency = (365.27/7))
cor(v_data_TS)

##                      ï..  mortality        temp       chem1      chem2
## ï..           1.00000000 -0.4587450  0.05951337 -0.32983451 -0.4999335
## mortality    -0.45874498  1.0000000 -0.43863962  0.55744759  0.2569989
## temp          0.05951337 -0.4386396  1.00000000 -0.09785582  0.4043740
## chem1        -0.32983451  0.5574476 -0.09785582  1.00000000  0.5130047
## chem2        -0.49993345  0.2569989  0.40437401  0.51300467  1.0000000
## particle.size -0.07664951  0.4438713 -0.01723095  0.86611747  0.4679340
##              particle.size
## ï..            -0.07664951
## mortality       0.44387133
## temp           -0.01723095
## chem1           0.86611747
## chem2           0.46793404
## particle.size   1.00000000
```

Convert column names.

```
colnames(v_data_TS)<-c("n", "y", "x1", "x2", "x3", "x4")

## Where y -> mortality
## x1 -> temp
## x2 -> chem1
## x3 -> chem2
## x4 -> particle.size
```

Finite distributed lag model

Getting q values for finite distributed lag model based on MASE values.

```
for ( i in 1:10){
  model_1 = dlm(formula=y ~ x2 + x4, data = data.frame(v_data_TS), q = i )
  cat("q = ", i, "AIC = ", AIC(model_1$model), "BIC = ", BIC(model_1$model),
"MASE =", MASE(model_1)$MASE, "\n")
}

## q =  1 AIC =  3543.48 BIC =  3568.851 MASE = 1.145141
## q =  2 AIC =  3490.679 BIC =  3524.491 MASE = 1.087568
## q =  3 AIC =  3464.306 BIC =  3506.551 MASE = 1.054802
```

```
## q =  4 AIC =  3404.234 BIC =  3454.905 MASE = 1.002905
## q =  5 AIC =  3381.397 BIC =  3440.485 MASE = 0.9804769
## q =  6 AIC =  3353.635 BIC =  3421.132 MASE = 0.9585366
## q =  7 AIC =  3321.442 BIC =  3397.341 MASE = 0.9219601
## q =  8 AIC =  3305.874 BIC =  3390.166 MASE = 0.9009314
## q =  9 AIC =  3297.186 BIC =  3389.864 MASE = 0.8920576
## q =  10 AIC =  3293.419 BIC =  3394.473 MASE = 0.8897851
```

As we have the least AIC, BIC and MASE values at q = 10. Let us fit the finite distributed lag model with q = 10.

```
# Finite lag length based on AIC-BIC

finite_dlm_mort = dlm(formula=y ~ x2 + x4, data = data.frame(v_data_TS), q =
10)
summary(finite_dlm_mort)

##
## Call:
## lm(formula = as.formula(model.formula), data = design)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -19.065  -3.835  -0.260   3.552  33.056
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 70.062958   1.493864  46.901  < 2e-16 ***
## x2.t        -0.121965   0.220995  -0.552  0.58128
## x2.1         0.210913   0.221677   0.951  0.34186
## x2.2         0.406731   0.234094   1.737  0.08295 .
## x2.3         0.360535   0.235634   1.530  0.12667
## x2.4         0.647814   0.242696   2.669  0.00786 **
## x2.5         0.004949   0.244396   0.020  0.98385
## x2.6         0.483837   0.243557   1.987  0.04755 *
## x2.7         0.381800   0.239767   1.592  0.11197
## x2.8         0.201629   0.238500   0.845  0.39831
## x2.9         0.175025   0.226108   0.774  0.43927
## x2.10        0.010672   0.225713   0.047  0.96231
## x4.t         0.154497   0.050470   3.061  0.00233 **
## x4.1        -0.080595   0.050770  -1.587  0.11307
## x4.2        -0.061847   0.052011  -1.189  0.23499
## x4.3        -0.096085   0.052257  -1.839  0.06658 .
## x4.4        -0.059814   0.053976  -1.108  0.26836
## x4.5         0.020664   0.054232   0.381  0.70335
## x4.6        -0.057614   0.054019  -1.067  0.28672
## x4.7         0.017066   0.052546   0.325  0.74548
## x4.8         0.034731   0.052547   0.661  0.50896
## x4.9         0.031218   0.050799   0.615  0.53915
## x4.10        0.027749   0.050399   0.551  0.58218
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.444 on 475 degrees of freedom
## Multiple R-squared:  0.6063, Adjusted R-squared:  0.5881
## F-statistic: 33.25 on 22 and 475 DF,  p-value: < 2.2e-16
##
## AIC and BIC values for the model:
##        AIC      BIC
## 1 3293.419 3394.473
```

Hypotheses:

H_0: The data doesn't fit the Finite distributed lag model.
H_A: The data fits the Finite distributed lag model.

Interpretations:

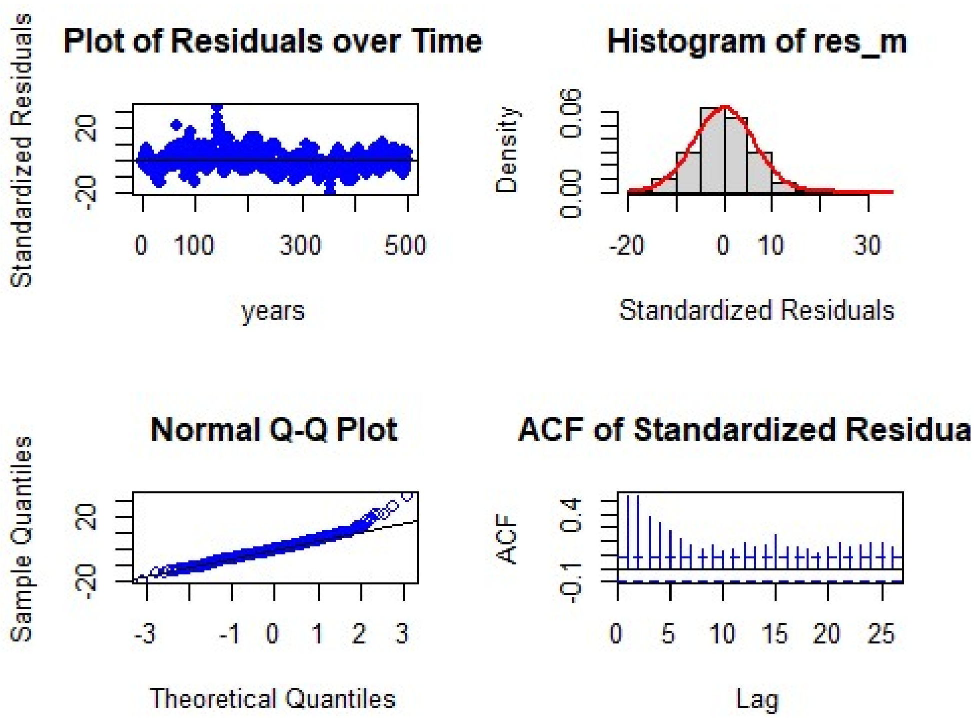
F - statistic is 33.25
R - squared is 0.6063
Adjusted R - squared is 0.5881
Degrees of freedom - DF are (22, 475)
p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Finite distributed lag model.
This model suggests that there is only 58.81% of data variance. Suggesting that the model explains only 58.81% of the trend. Which implies that the model shows some trend.

Now let us check the residual analysis.

Residual analysis

```
# Function for residual analysis.

res_analysis <- function(res_m) {

    par(mfrow = c(2, 2))
    # Scatter plot for model residuals
    plot(res_m, type = "b", pch = 19, col = "blue", xlab = "years", ylab =
"Standardized Residuals", main = "Plot of Residuals over Time")

    abline(h = 0)

    # Standard distribution
    hist(res_m, xlab = 'Standardized Residuals', freq = FALSE)
    curve(dnorm(x, mean = mean(res_m), sd = sd(res_m)), col = "red", lwd = 2,
add = TRUE, yaxt = "n")

    # QQplot for model residuals
```

```
    qqnorm(res_m, col = c("blue"))
    qqline(res_m)

    # Auto-Correlation Plot
    acf(res_m, main = "ACF of Standardized Residuals",col=c("blue"))

    # Shapiro Wilk test
    shapiro.test(res_m)

}

res_analysis(residuals(finite_dlm_mort$model))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.97054, p-value = 1.86e-08
```

Residual Analysis for Finite DLM:

1. The data points are below the line at the start and above the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.

3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise. Also, ACF shows seasonality pattern.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Therefore, Further analysis is needed by adding polynomial to the lag model.

Polynomial distributed lag model

Polynomial distributed lag model makes predictors with only one variable. There fore, let us fit models on the mortality rate separately.

```
y = v_Mortality_data_TS # Independent variable
x1 = v_Mortality_chem1_data_TS # Dependent variable
x2 = v_Mortality_particle.size_data_TS # Dependent variable
```

Polynomial distributed lag model with Chemical emission 1

```
for (i in 1:3){
  model_2 <-  polyDlm(x = as.vector(x1) , y = as.vector(y), q = i , k = i,
show.beta = FALSE)
  cat("q = ", i, "k = ", i, "AIC = ", AIC(model_2$model), "BIC = ",
BIC(model_2$model), "MASE =", MASE(model_2)$MASE, "\n")
}

## q =  1 k =  1 AIC =  3554.683 BIC =  3571.597 MASE = 1.171233
## q =  2 k =  2 AIC =  3505.901 BIC =  3527.034 MASE = 1.122964
## q =  3 k =  3 AIC =  3478.836 BIC =  3504.183 MASE = 1.104185
```

Let us fit a polynomial model of order 3. Since least AIC, BIC and MASE scores.

```
# Ploynomial DLM

PolyDLM_model_mort_chem1 = polyDlm(x = as.vector(x1), y = as.vector(y), q =
3, k = 3, show.beta = TRUE)

## Estimates and t-tests for beta coefficients:
##        Estimate Std. Error t value  P(>|t|)
## beta.0    0.410      0.135    3.03 2.56e-03
## beta.1    0.190      0.134    1.42 1.57e-01
## beta.2    0.764      0.134    5.70 2.07e-08
## beta.3    0.654      0.135    4.84 1.72e-06

summary(PolyDLM_model_mort_chem1)

##
## Call:
## "Y ~ (Intercept) + X.t"
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.8615  -5.3630  -0.4056   4.4528  31.0281
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  72.6825     0.8845  82.170  < 2e-16 ***
## z.t0          0.4103     0.1353   3.032  0.00256 **
## z.t1         -1.1101     0.5367  -2.069  0.03910 *
## z.t2          1.1363     0.4719   2.408  0.01640 *
## z.t3         -0.2464     0.1033  -2.384  0.01750 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.527 on 500 degrees of freedom
## Multiple R-squared:  0.4369, Adjusted R-squared:  0.4324
## F-statistic:    97 on 4 and 500 DF,  p-value: < 2.2e-16
```

Hypotheses:

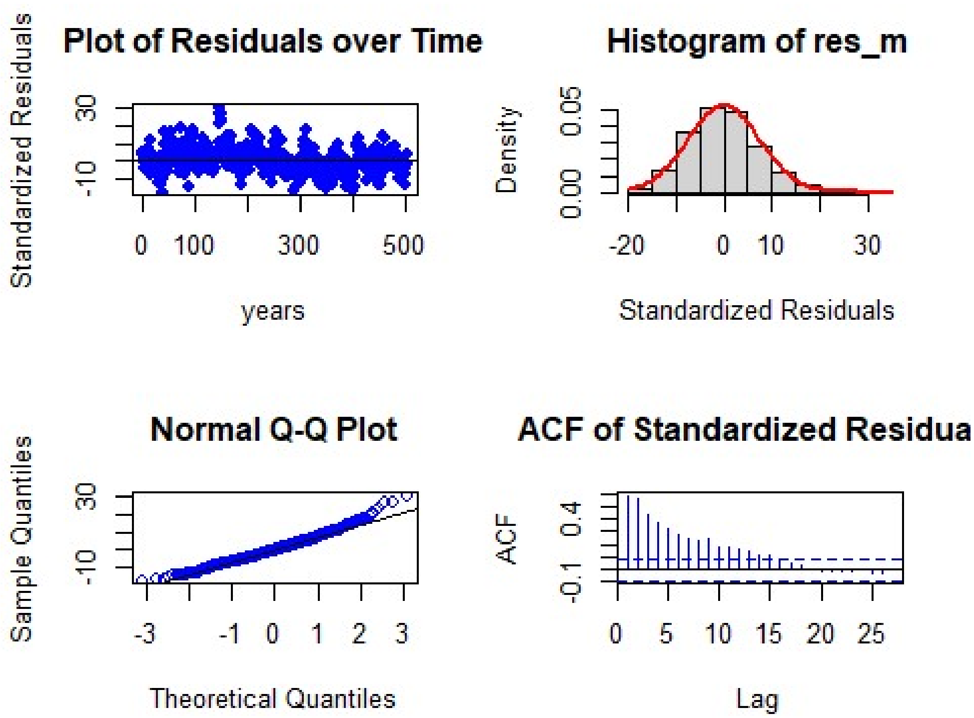
H_0: The data doesn't fit the Polynomial distributed lag model.
H_A: The data fits the Polynomial distributed lag model.

Interpretations:

F - statistic is 97
R - squared is 0.4369
Adjusted R - squared is 0.4324
Degrees of freedom - DF are (4, 500)
p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Polynomial distributed lag model. This model suggests that there is only 43.24% of data variance. Suggesting that the model explains only 43.24% of the trend. Which implies that the model shows some trend.

Residual analysis

```
res_analysis(residuals(PolyDLM_model_mort_chem1$model))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.98453, p-value = 3.338e-05
```

Residual Analysis for Polynomial DLM with Chem1:

1. The data points are above the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise. Also, ACF shows seasonality pattern.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Polynomial distributed lag model with Particle size

```
for (i in 1:3){
  model_2 <-  polyDlm(x = as.vector(x2) , y = as.vector(y), q = i , k = i,
show.beta = FALSE)
  cat("q = ", i, "k = ", i, "AIC = ", AIC(model_2$model), "BIC = ",
```

```
BIC(model_2$model), "MASE =", MASE(model_2)$MASE, "\n")
}

## q =  1 k =  1 AIC =  3653.402 BIC =  3670.316 MASE = 1.306209
## q =  2 k =  2 AIC =  3623.85 BIC =  3644.983 MASE = 1.264552
## q =  3 k =  3 AIC =  3606.089 BIC =  3631.437 MASE = 1.260364
```

Let us fit a polynomial model of order 3. Since least AIC, BIC and MASE scores.

```
# Ploynomial DLM

PolyDLM_model_mort_part = polyDlm(x = as.vector(x2), y = as.vector(y), q = 3,
k = 3, show.beta = TRUE)

## Estimates and t-tests for beta coefficients:
##        Estimate Std. Error t value  P(>|t|)
## beta.0   0.1260     0.0342   3.670 2.65e-04
## beta.1   0.0272     0.0343   0.793 4.28e-01
## beta.2   0.1360     0.0343   3.970 8.33e-05
## beta.3   0.1260     0.0342   3.700 2.42e-04

summary(PolyDLM_model_mort_part)

##
## Call:
## "Y ~ (Intercept) + X.t"
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -21.287  -5.938  -0.093   5.186  32.825
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 68.97944    1.49483  46.145  < 2e-16 ***
## z.t0         0.12568    0.03421   3.673 0.000265 ***
## z.t1        -0.31087    0.14126  -2.201 0.028218 *
## z.t2         0.26679    0.12440   2.145 0.032469 *
## z.t3        -0.05436    0.02728  -1.993 0.046819 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.538 on 500 degrees of freedom
## Multiple R-squared:  0.2756, Adjusted R-squared:  0.2698
## F-statistic: 47.55 on 4 and 500 DF,  p-value: < 2.2e-16
```

Hypotheses:

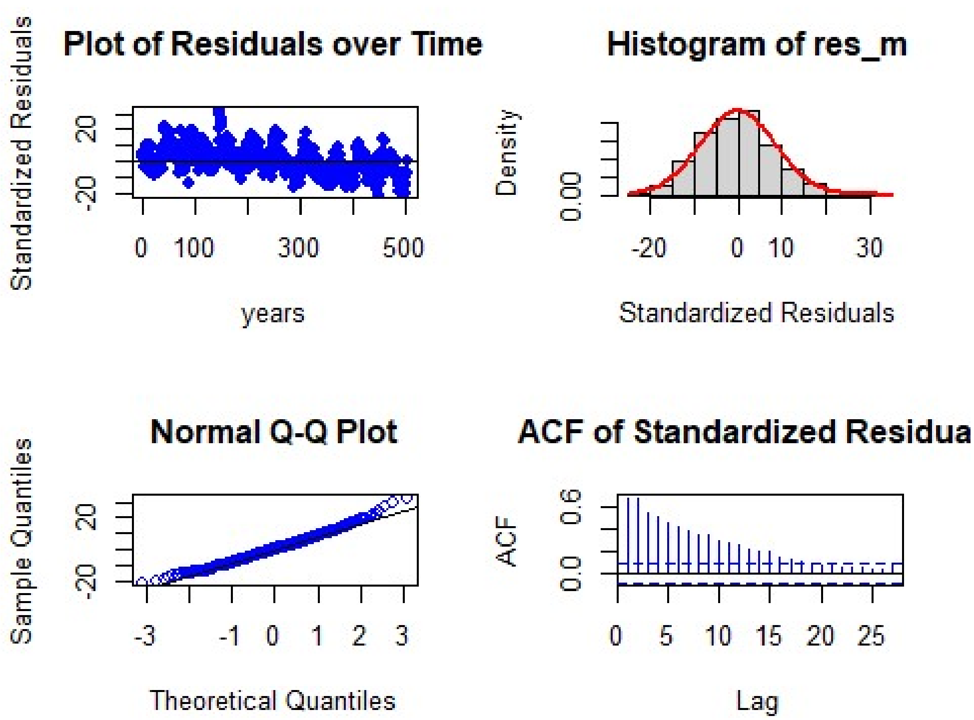
H_0: The data doesn't fit the Polynomial distributed lag model.
H_A: The data fits the Polynomial distributed lag model.

Interpretations:

F - statistic is 47.55
R - squared is 0.2756
Adjusted R - squared is 0.2698
Degrees of freedom - DF are (4, 500)
p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Polynomial distributed lag model. This model suggests that there is only 26.98% of data variance. Suggesting that the model explains only 26.98% of the trend. Which implies that the model shows some trend.

Residual analysis

```
res_analysis(residuals(PolyDLM_model_mort_part$model))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.99092, p-value = 0.003401
```

Residual Analysis for Polynomial DLM with part:

1. The data points are above the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.

3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise. Also, ACF shows seasonality pattern.
5. p - value (0.0034) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Now let us fit Koyck model in the similar war. Since it does not take multiple predictors.

Koyck model

Koyck with Chemical Emission 1

```
# Koyk DLM

Koyck_DLM_mort_chem1 = koyckDlm(x = as.vector(x1) , y = as.vector(y))
summary(Koyck_DLM_mort_chem1)

##
## Call:
## "Y ~ (Intercept) + Y.1 + X.t"
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -20.19302  -4.18513  -0.06397   3.61514  23.08021
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 24.50952    2.55334   9.599  < 2e-16 ***
## Y.1          0.66674    0.03637  18.331  < 2e-16 ***
## X.t          0.63624    0.15311   4.155 3.81e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.891 on 504 degrees of freedom
## Multiple R-Squared: 0.6544,  Adjusted R-squared: 0.653
## Wald test: 443.5 on 2 and 504 DF,  p-value: < 2.2e-16
##
## Diagnostic tests:
## NULL
##
##                             alpha      beta       phi
## Geometric coefficients:  73.5455 0.6362403 0.6667435
```

Hypotheses:

H_0: The data doesn't fit the Koyck distributed lag model.
H_A: The data fits the Koyck distributed lag model.

Interpretations:

Wald test statistic is 443.5
R - squared is 0.6544
Adjusted R - squared is 0.653
Degrees of freedom - DF are (2, 504)
p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Koyck distributed lag model.
This model suggests that there is only 65.3% of data variance. Suggesting that the model explains only 65.3% of the trend. Which implies that the model performs better on the series data when compared to the former model.

Now let us perform residual analysis.

Residual analysis

```
res_analysis(residuals(Koyck_DLM_mort_chem1))

##           2           3           4           5           6           
7
##   9.21436814  -5.94911486   4.08601619  -0.75877704   2.47955863  -
6.56044356
##           8           9          10          11          12          
13
##   0.98640224   2.77128778  -2.16622184   4.08377675   1.52909262   
3.51642996
##          14          15          16          17          18          
19
##  -5.29039308   9.93977670 -10.21228532   3.39046555   3.47145935  -
7.18732006
##          20          21          22          23          24          
25
##  13.24074678 -14.36074231   9.18623077  -4.05857344   0.70471267  -
2.85735853
##          26          27          28          29          30          
31
##   0.01006012  -2.83477705  -3.71123174  -1.22092691  -0.58089444  -
4.99072530
##          32          33          34          35          36          
37
##   2.98715823  -5.36269163  -4.83234266   3.09378729  -5.42633541  -
3.96770828
##          38          39          40          41          42          
43
##   4.63532748  -7.94465483  -6.43482517  -1.74418872  -5.31989236  -
1.26076872
##          44          45          46          47          48          
49
##  -0.04826078   6.72177431   7.49834316  10.47897808  -8.58794435
```

```
6.37878395
##           50           51           52           53           54           
55 
## -10.53019539   0.49850679   2.23672839   4.49484958   5.52965692   
0.92413154 
##           56           57           58           59           60           
61 
##   7.45431750   2.57005145   1.11210238   3.52883875  -2.31295006   
1.26692156 
##           62           63           64           65           66           
67 
##   2.59136926   5.46410994  -5.07511947   8.86458121  -6.69363093   
9.02403867 
##           68           69           70           71           72           
73 
##  -0.54029672   0.18853669  -3.69523140   5.30254314  -8.72577242  
11.16667340 
##           74           75           76           77           78           
79 
##  -1.75557251  -0.43020017  -4.62684179  20.30375745  -4.48044675  -
6.27224218 
##           80           81           82           83           84           
85 
##   1.94635955  -0.86993070   9.73816496  -6.46917249   1.27158653  -
8.38623496 
##           86           87           88           89           90           
91 
##  12.67203413 -10.88441297   6.20214645   0.27617429   6.33465357 -
20.19301935 
##           92           93           94           95           96           
97 
##  15.27755383  -3.56906818  -0.64673653  10.43273428   1.55126385  
10.21599291 
##           98           99          100          101          102          
103 
##   7.88600284   4.40408609  -1.68417403   9.61282047  -6.18617881   
8.08995074 
##          104          105          106          107          108          
109 
##   5.41982559  -3.80902337   1.72085437  -2.07633999  -2.59912956   
4.75341892 
##          110          111          112          113          114          
115 
##  -9.89154749   7.02970654  -0.06396549   0.58221005   2.60149084  -
1.10412680 
##          116          117          118          119          120          
121 
##  -1.66134672   8.86040698  -6.42151375  13.45642292 -10.48127004   
2.22904050
```

```
##          122          123          124          125          126
127
##  -0.17392788   0.50334402   9.69557219  -8.78412369   6.20391232
0.22999553
##          128          129          130          131          132
133
##   3.52683353   3.98950449  -3.43583854   2.87433532 -10.07046325
5.40984341
##          134          135          136          137          138
139
##  -1.34891977   2.39411683   2.49238235  -4.90404611   2.17042746  -
6.57881427
##          140          141          142          143          144
145
##   6.53628267  -6.18532303   8.06740880  -1.15422684  -2.55585628
2.60782266
##          146          147          148          149          150
151
##  -2.19737923   2.64092242   6.09244630   5.26801341  11.59842533
23.08021265
##          152          153          154          155          156
157
##   5.36269639   6.77839941   6.59716609   7.58198344  -5.17687203  -
0.21532920
##          158          159          160          161          162
163
##   2.97990029  -1.64457916   6.21461245  -6.29815320   5.37233309
3.96109295
##          164          165          166          167          168
169
##  -0.35962969  -7.59977170  17.99346771  -5.03055188   6.68192300
2.53784819
##          170          171          172          173          174
175
##  -5.20164898   2.20593202   2.16612035  -4.21440384   0.47922625
13.38453581
##          176          177          178          179          180
181
##  -6.13495064   5.20930299  -7.24124350   0.08737708   0.09317980
4.48289276
##          182          183          184          185          186
187
##  -6.02932828   4.71832306  -4.06759907  -3.69109617   0.52895072
6.56011624
##          188          189          190          191          192
193
##  -1.28062509   3.52954344  -5.38983704  -5.49070739  -4.40979854
1.27307287
##          194          195          196          197          198
```

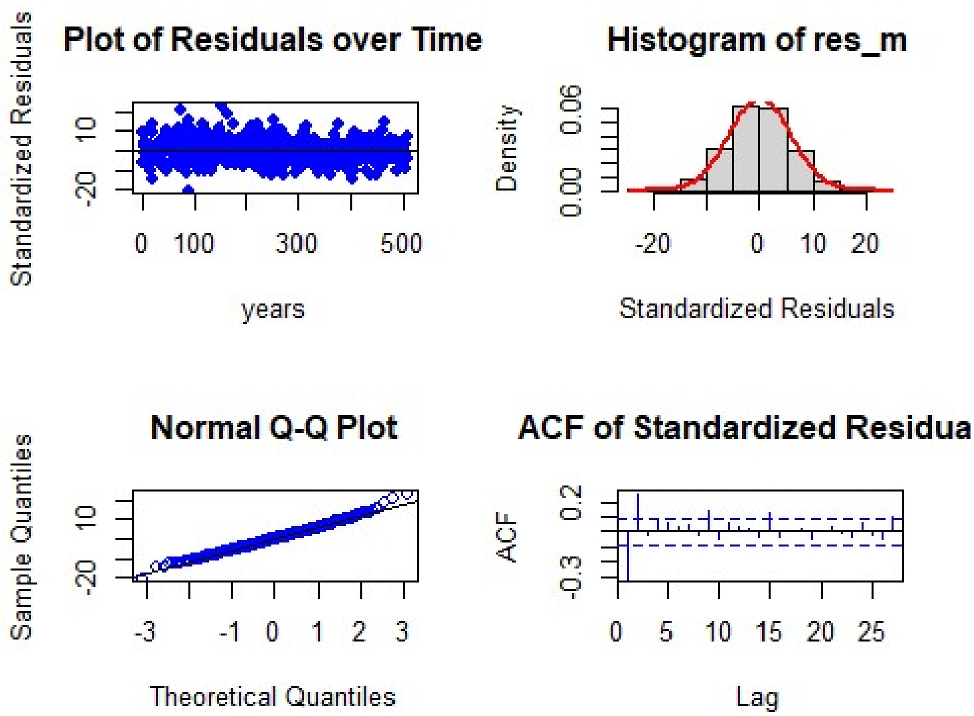
```
199
##  -8.30175940   5.41189141  -2.60995857   5.52381885  -6.77074254  
6.11563176 
##          200          201          202          203          204 
205 
##  -4.68822660   3.83277676   9.23362436   5.90994747  -6.12453642  -
5.26689311 
##          206          207          208          209          210 
211 
##   0.37697139  -3.33724939   2.46937101   0.66583618   9.39759933  
0.53755343 
##          212          213          214          215          216 
217 
##  12.35287789  -3.63972400   3.62637172  -3.02850472   8.71042063  
5.06109414 
##          218          219          220          221          222 
223 
##  -7.86421741  10.59843815  -4.59452514  13.66133639  -8.25448186  
7.86748537 
##          224          225          226          227          228 
229 
##   1.23296754   0.49559914  -3.09769121   1.03482794  10.63065267  -
9.56192989 
##          230          231          232          233          234 
235 
##   4.32221432  -0.77715881  -2.54420907   7.43599229 -11.40720760  
4.55657243 
##          236          237          238          239          240 
241 
##  -0.10916408   1.68650697   0.88520556  -4.27492649   6.23686923  -
5.74485818 
##          242          243          244          245          246 
247 
##   2.01819706   2.12620483  -0.74273636   2.18821130  -2.92419531  -
6.78964454 
##          248          249          250          251          252 
253 
##   6.41655674 -12.40378409   8.70351174  -7.79607543   0.55367669  -
2.24649524 
##          254          255          256          257          258 
259 
##  15.38311436   1.98767554   4.58592644   4.91348081  -4.22151580  
11.49436091 
##          260          261          262          263          264 
265 
##   4.92130789  -8.53354312   3.92365564  -8.09108762  -1.81812185  
4.05721807 
##          266          267          268          269          270 
271
```

```
##  -4.73019543   3.30970972  -1.08868766   3.23616461   9.73740950 -
12.19364495
##          272          273          274          275          276
277
##  -0.57746220  -5.79522153   0.77177722   6.67748312  -2.81116486  -
3.43283395
##          278          279          280          281          282
283
##  -7.09565987   1.97361077   3.32470140  -1.70710190  -6.03036195  -
2.80350073
##          284          285          286          287          288
289
##   2.42259806  -3.83661212  -0.39429129  -4.46919649  -2.28004099
3.90607855
##          290          291          292          293          294
295
##  -3.25029286  -1.17991629  -5.33751597  -2.06589411 -11.18166568  -
1.73461286
##          296          297          298          299          300
301
##  -0.75627228   3.60391001   0.30833984  -9.51384167  -0.90877293  -
0.45974005
##          302          303          304          305          306
307
##   4.05415866  -7.04779346   0.02260254  -9.09204272   7.29299529
5.02668568
##          308          309          310          311          312
313
##  -6.81885254  -1.84522233  -4.15585879  -1.54668677   8.74369458
7.79436201
##          314          315          316          317          318
319
##   3.98658380   0.02996274   3.13853249  12.14491965  -5.75841356
1.18702346
##          320          321          322          323          324
325
##  -1.13656849  -1.11844216   1.04816434  -0.24883784   1.52371716  -
4.47136154
##          326          327          328          329          330
331
##  -0.58370197  -6.09860902  -0.15481282   2.00973954   2.11218027  -
0.16137822
##          332          333          334          335          336
337
##   6.45872202  -9.42300381  -3.74032481  -1.01686762  -5.29017701
2.60225594
##          338          339          340          341          342
343
##   3.01026708 -11.40711264   1.88216372  -8.46207571   3.13138148  -
```

```
9.79266910
##         344          345          346          347          348
349
##   0.30855991  -4.03895378   0.45054246   0.08306385  -6.28355785  -
2.97016989
##         350          351          352          353          354
355
##   2.96336250  -5.70576602  -4.77803025  -4.64103654  -2.53406695
0.68086691
##         356          357          358          359          360
361
## -10.90987870   3.24760686   2.51231548  -1.68091628   1.20545778  -
2.18568399
##         362          363          364          365          366
367
##   0.16681095 -12.88004954   3.81822477  -8.41339114  -5.84305424
2.22409958
##         368          369          370          371          372
373
##  -0.63439180   1.28055255  -1.21353735   8.03093143   1.48047518
1.46263451
##         374          375          376          377          378
379
##  -0.41049564   5.11136490  -4.32764483   0.70392214  -8.18572724
13.49954853
##         380          381          382          383          384
385
##  -3.15506704  -3.41508445  -5.25858517   1.03767018  -0.80624352  -
4.08321395
##         386          387          388          389          390
391
##   0.70736924  -5.21018615   2.73994472   1.20067960  -2.96870485  -
7.72162536
##         392          393          394          395          396
397
##  -0.51382765  -4.33730999  -8.68973234   2.72115534  -9.96392776  -
0.90317195
##         398          399          400          401          402
403
##   2.48664073 -10.16638390   1.27035459  -5.64433417  -3.84086646  -
0.41901828
##         404          405          406          407          408
409
##   1.17694384 -14.60584958   7.20686947 -10.72238200  -0.67207197  -
6.40194430
##         410          411          412          413          414
415
##   9.04246574   0.26102055  -2.02508939   6.94010340   5.27223449
7.17268252
```

```
##          416          417          418          419          420
421
##  -4.33683821   1.47202359   7.57303491  -4.96442319  -2.10130094
1.10402469
##          422          423          424          425          426
427
##   6.46947760  -3.54957633   4.82555968  -4.29277539   3.77907364  -
3.50824187
##          428          429          430          431          432
433
##  -3.88408912  -2.36946240   3.14130488  -0.81558563 -10.49551878
6.02608745
##          434          435          436          437          438
439
##  -2.33269972  -3.11494969  -1.33024307  -4.04568286  -0.22383858
7.09499497
##          440          441          442          443          444
445
## -11.95974424  -0.01786730  -2.80577560   2.25968916  -2.03623411  -
2.75700759
##          446          447          448          449          450
451
##   2.67177156  -6.59728831  -0.43059361   7.05021364   3.98232649 -
12.33203681
##          452          453          454          455          456
457
##  -4.27357184  -7.83258203   3.78725639  -5.59080408   1.12823954  -
4.47126614
##          458          459          460          461          462
463
##  -7.74576605   9.50824906  -3.50956903   5.10584301   4.02668685
9.72615624
##          464          465          466          467          468
469
##  -8.13850940  -4.52153623  -0.04699402  -0.18077296  14.32283276  -
4.39234179
##          470          471          472          473          474
475
##  -1.35784046  -3.35397483   2.35411994   0.73064339  -0.63459247
2.52481794
##          476          477          478          479          480
481
##  -1.48085344   5.97062258   0.55544744  -4.00595554   2.99288716  -
1.87899038
##          482          483          484          485          486
487
##   5.57697345   1.64689153   1.39794362  -4.55425635   3.22803865
4.95551027
##          488          489          490          491          492
```

```
493
## -10.47248794   2.68261110   1.25720734  -2.95710796  -5.19288949  -
2.65960270
##          494          495          496          497          498
499
##  -5.16559836   0.56529598  -1.82200653  -0.93755364  -4.68563497
3.87796566
##          500          501          502          503          504
505
##  -0.04441795  -2.29514986  -2.44379437  -9.14650575   2.52572676  -
4.30699969
##          506          507          508
##  -2.41162959   7.32516359  -3.74267284
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.99486, p-value = 0.08876
```

Residual Analysis for Koyck DLM:

1. The data points are above the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.

3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are no significant lags in Autocorrelation plot suggesting that the stochastic component is white noise.
5. p - value (0.08876) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis cannot be rejected.

Koyck with Particle size

```
# Koyk DLM

Koyck_DLM_mort_part = koyckDlm(x = as.vector(x2) , y = as.vector(y))
summary(Koyck_DLM_mort_part)

##
## Call:
## "Y ~ (Intercept) + Y.1 + X.t"
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -20.8011  -4.3973  -0.1927   3.7232  21.6263
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 19.97832    2.44335   8.177 2.39e-15 ***
## Y.1          0.74335    0.03337  22.273  < 2e-16 ***
## X.t          0.05835    0.03945   1.479     0.14
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.147 on 504 degrees of freedom
## Multiple R-Squared: 0.6236,  Adjusted R-squared: 0.6221
## Wald test: 400.4 on 2 and 504 DF,  p-value: < 2.2e-16
##
## Diagnostic tests:
## NULL
##
##                              alpha       beta       phi
## Geometric coefficients:  77.84216 0.05834985 0.7433483
```

Hypotheses:

H_0: The data doesn't fit the Polynomial distributed lag model.
H_A: The data fits the Polynomial distributed lag model.

Interpretations:

Wald test statistic is 400.4
R - squared is 0.6236 Adjusted
R - squared is 0.6221
Degrees of freedom - DF are (2, 504)

p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Koyck distributed lag model.
This model suggests that there is only 62.21% of data variance. Suggesting that the model explains only 62.21% of the trend. Which implies that the model performs better on the series data when compared to the former model.

Now let us perform residual analysis.

Residual analysis

```
res_analysis(residuals(Koyck_DLM_mort_part))

##            2            3            4            5            6
7
##   9.03089412  -6.65120775   4.71075441  -0.86588011   2.18376634  -
5.48576537
##            8            9           10           11           12
13
##   2.44466431   3.09032190  -2.01888424   5.61940825   0.67856245
4.71688805
##           14           15           16           17           18
19
##  -6.96057045   9.87645373 -11.56371584   3.02637841   3.35428691  -
6.94651296
##           20           21           22           23           24
25
##  14.01660468 -15.42978498   9.97202733  -4.96316374   1.32737661  -
2.81533091
##           26           27           28           29           30
31
##   0.43391490  -3.25479757  -2.69755962  -0.24063861   0.45340819  -
3.01845276
##           32           33           34           35           36
37
##   6.99499610  -3.01445503  -2.78311022   4.47607368  -4.76151925
3.47122208
##           38           39           40           41           42
43
##   6.35263577  -2.90900812  -0.52965168   0.14886345   0.72488175
4.21667368
##           44           45           46           47           48
49
##   1.74406375  10.51612296   9.43360242  10.28121202  -4.52362801
7.66719020
##           50           51           52           53           54
55
##  -4.71735694   0.44417895   7.67930042   3.62811786   4.28952685
1.48304677
##           56           57           58           59           60
61
```

```
##   6.09197239   2.39136049  -0.78373601   4.50622521  -3.05428058
0.16153101
##           62           63           64           65           66
67
##   1.83001806   4.48107222  -5.94768448   8.68733252  -7.53716377
7.84970714
##           68           69           70           71           72
73
##  -1.99704145  -0.86700784  -3.87331350   4.51486714  -9.75658403
11.86420012
##           74           75           76           77           78
79
##  -2.82648206  -1.31032529  -4.79357587  21.44318943  -6.08816718  -
7.83648388
##           80           81           82           83           84
85
##   3.07941476  -1.75201482  12.07366256  -7.89895693   2.24583678  -
7.66143465
##           86           87           88           89           90
91
##  17.53765613 -12.57755952   8.51146185  -0.19938789   9.45160455 -
20.80112197
##           92           93           94           95           96
97
##  18.97538762  -2.50781620  -0.36487578  11.03512597   3.57329579
9.23961449
##           98           99          100          101          102
103
##   8.11495470   6.04039134   1.57073916   8.27233998  -5.47975920
8.58227759
##          104          105          106          107          108
109
##   6.47547562  -3.85998483   1.72083453  -3.05775829  -2.96492291
3.63063824
##          110          111          112          113          114
115
## -10.04503809   7.74028213   0.31356606   0.23906348   2.67546905  -
1.48773432
##          116          117          118          119          120
121
##  -3.16566537   9.38757483  -8.24081104  13.84486046 -12.36760388
2.05037168
##          122          123          124          125          126
127
##  -0.78941167   0.02520035   8.99153675  -9.50818847   5.91209775  -
0.55524759
##          128          129          130          131          132
133
##   4.70470139   2.83523774  -5.03316369   2.63005435 -10.57668494
```

```
5.85125070
##          134          135          136          137          138 
139 
##  -2.04336594   2.56830595   2.82914935  -5.08293345   2.89467687  -
5.98551964 
##          140          141          142          143          144 
145 
##   7.81955485  -5.61200633  10.82512188  -1.82516129  -1.93243690 
4.93403124 
##          146          147          148          149          150 
151 
##   1.27246875   1.60664544   6.92041283   8.67367011  10.82354955 
21.62627580 
##          152          153          154          155          156 
157 
##   4.74714968   3.96748715   5.57321240   6.05323918  -7.81841863  -
0.39557090 
##          158          159          160          161          162 
163 
##   2.22084814  -2.17192253   5.20612659  -7.06071646   4.87051456 
3.01115518 
##          164          165          166          167          168 
169 
##  -0.69344137  -8.90312899  18.14322473  -7.34203611   5.73787401 
0.94076853 
##          170          171          172          173          174 
175 
##  -7.12120833   1.78102185   0.85498945  -4.55980786  -0.19597211 
14.04402169 
##          176          177          178          179          180 
181 
##  -8.20839541   4.23479374  -9.26757281  -0.15352684  -0.11366897 
3.73321131 
##          182          183          184          185          186 
187 
##  -7.24003252   5.18553462  -4.68395027  -4.75255358  -0.19272562 
5.94822438 
##          188          189          190          191          192 
193 
##  -1.95450783   5.33483946  -6.76166517  -2.84355383  -2.95517074 
4.35575318 
##          194          195          196          197          198 
199 
##  -7.18525219   7.97684011  -2.86713825   5.76294566  -5.54741540 
9.69751184 
##          200          201          202          203          204 
205 
##  -3.11070388   4.86718477   8.17672900   3.28841782  -5.67934388  -
4.40353635
```

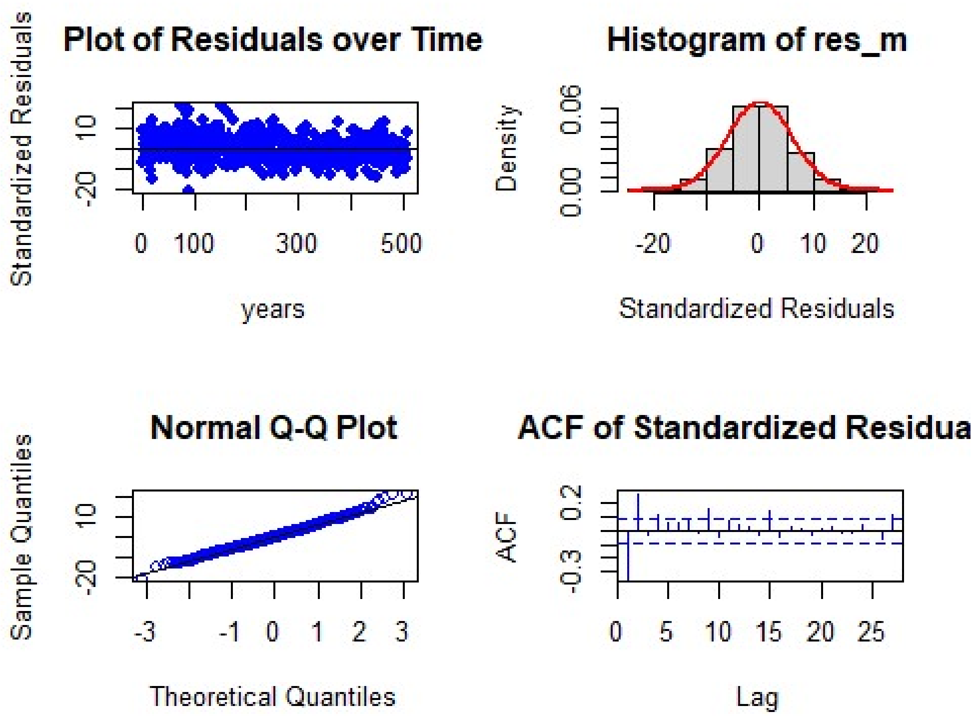
```
##          206          207          208          209          210
211
##   2.03963845  -1.92293123   3.29570042   0.72829335   9.06774260  -
0.38414660
##          212          213          214          215          216
217
##  11.15788691  -4.03384428   1.26861684  -4.70326749   7.87292623
3.55534228
##          218          219          220          221          222
223
##  -9.52648666   9.14247231  -6.70784763  12.23949424 -10.62397455
7.23060393
##          224          225          226          227          228
229
##  -0.58738153  -1.20184515  -4.24344112   0.56914679  10.58858853 -
11.97545396
##          230          231          232          233          234
235
##   3.95411496  -1.38094966  -3.57617393   6.83215212 -13.15671514
4.55341536
##          236          237          238          239          240
241
##  -0.99106993   1.84711306   1.13779888  -4.91714354   7.45066387  -
6.74455422
##          242          243          244          245          246
247
##   2.57874796   2.70285220   1.52288289   1.96776762  -2.35619155  -
4.75330435
##          248          249          250          251          252
253
##   9.43261762 -11.96410233  12.98459559  -6.68438460   3.70299616
2.66774274
##          254          255          256          257          258
259
##  13.64004074   2.63216671   4.70951380   9.31341813  -2.82765455
10.49531762
##          260          261          262          263          264
265
##   4.71536057  -9.28586979   3.98364891  -6.27754971  -2.17959828
4.92329968
##          266          267          268          269          270
271
##  -4.92701550   3.00984255  -0.72954475   2.48224629   9.42494011 -
13.63616315
##          272          273          274          275          276
277
##  -0.94572191  -5.62721394   0.42539392   6.84804700  -4.18334588  -
3.90164834
##          278          279          280          281          282
```

```
283
##  -7.70630032   2.42106166   3.71327840  -2.00592325  -6.84956919  -
3.64740807
##          284          285          286          287          288
289
##   2.06065749  -4.57758030  -0.05870030  -5.63894867  -2.03359509
4.25928241
##          290          291          292          293          294
295
##  -3.70545646  -1.53321817  -4.07710277  -1.82435583 -10.87958174  -
1.10270179
##          296          297          298          299          300
301
##   1.30796253   3.62808410   2.72750134  -8.58007976   1.73482870
0.56860847
##          302          303          304          305          306
307
##   3.80940477  -5.19091479   0.16133518  -6.31820121   8.74344042
4.62153663
##          308          309          310          311          312
313
##  -5.69722586   0.71410372  -4.00003675   1.23137221   7.48972663
7.58886096
##          314          315          316          317          318
319
##   2.92478000  -1.02066889   1.70139318  11.53008412  -6.91913624
0.37304088
##          320          321          322          323          324
325
##  -2.18293010  -2.44587840   0.38643847  -1.17841419   0.67949770  -
5.75666227
##          326          327          328          329          330
331
##  -0.85056946  -6.87381908  -0.67910340   2.12181650   1.76876218  -
0.73508324
##          332          333          334          335          336
337
##   6.69870865 -11.13852481  -3.96717278  -1.68985478  -5.46990380
2.43073217
##          338          339          340          341          342
343
##   2.98625112 -11.70396589   2.32969554  -7.50714799   3.66495704  -
9.20240839
##          344          345          346          347          348
349
##   0.75965618  -3.96551734   0.59366541   1.59847973  -6.56989306  -
2.70629448
##          350          351          352          353          354
355
```

```
##   4.60235065  -1.91080817  -4.46972818  -1.39980947  -1.84311292
3.37154977
##          356          357          358          359          360
361
##  -8.82846745   5.90279110   3.39693857  -2.11055314   0.40197699  -
1.12391942
##          362          363          364          365          366
367
##   0.27002210 -13.14591052   5.15110339  -7.20886580  -4.67997090
1.76272055
##          368          369          370          371          372
373
##  -0.71981806   0.85638225  -1.92216305   7.14172983   0.23743741
0.05718406
##          374          375          376          377          378
379
##  -2.10984999   4.88553204  -6.20189285  -0.36458150  -9.05723551
13.17022454
##          380          381          382          383          384
385
##  -4.54618896  -4.64562412  -6.21733393   0.72332951  -2.40145315  -
4.84243334
##          386          387          388          389          390
391
##   0.13003599  -6.10235044   2.13112772   1.00978577  -4.21125274  -
8.31959469
##          392          393          394          395          396
397
##  -0.19349006  -4.88341803  -8.77903255   3.22354375 -10.09833025  -
0.41737393
##          398          399          400          401          402
403
##   3.25680152  -9.39197642   1.45661207  -5.30734018  -2.23697010
0.89500205
##          404          405          406          407          408
409
##   3.27619303 -13.55510750   9.44508252  -8.13371254   1.18242780  -
5.64354364
##          410          411          412          413          414
415
##   8.59150617  -0.02267756  -1.66977938   7.00800974   4.68942241
7.02184611
##          416          417          418          419          420
421
##  -5.39174061   0.39319352   6.82033030  -5.99166364  -3.05431620
0.52563856
##          422          423          424          425          426
427
##   6.27201021  -4.39100023   3.77220205  -5.42342826   2.54017873  -
```

```
4.42608199
##         428         429         430         431         432
433
##  -4.78648019  -3.06835451   2.90184433  -1.90722004 -11.24101125
5.72994668
##         434         435         436         437         438
439
##  -3.72413409  -3.69508065  -1.81989654  -4.90496776  -1.33163526
6.07721366
##         440         441         442         443         444
445
## -13.64569406  -0.89426770  -4.29316133   1.01997444  -3.40493882  -
3.84499262
##         446         447         448         449         450
451
##   1.41417863  -7.31891034  -1.19294637   8.86723324   2.35626132 -
14.69335989
##         452         453         454         455         456
457
##  -3.64889574  -7.45979819   4.08408684  -4.02948422   1.12972286  -
2.63495933
##         458         459         460         461         462
463
##  -7.62918697  11.88795314  -3.51941672   7.31557310   4.78935507
9.03198719
##         464         465         466         467         468
469
##  -8.66725869  -5.70139827   0.83031216  -0.27088578  13.22647082  -
5.11004863
##         470         471         472         473         474
475
##  -2.19911876  -4.47627117   1.49348856  -0.85797945  -2.17980717
1.52739458
##         476         477         478         479         480
481
##  -3.00611223   4.38354032  -0.84753214  -5.77031917   1.76043490  -
3.98653633
##         482         483         484         485         486
487
##   4.53539425  -0.99463705  -0.30151442  -7.00189183   2.47432558
2.67404850
##         488         489         490         491         492
493
## -12.57464683   1.83261231  -0.47286006  -4.90810033  -7.02386678  -
4.15895257
##         494         495         496         497         498
499
##  -6.54236420  -0.50356746  -3.57244490  -1.95263110  -6.10663490
2.74789134
```

```
##          500          501          502          503          504
505
##   0.23722897  -3.20016967  -4.21576849  -9.82722478   1.98469789  -
4.82530848
##          506          507          508
##  -2.49557004   7.72418440  -4.61924500
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.99482, p-value = 0.08584
```

Residual Analysis for Koyck DLM:

1. The data points are above the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are no significant lags in Autocorrelation plot suggesting that the stochastic component is white noise.
5. p - value (0.08584) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis cannot be rejected.

So far Koyck with Chemical emission 1 is the best model but let us fit ardlDlm model to check whether it fits better than Koyck model or not.

Autoregressive distributed lag model

This again takes multiple predictors.

Getting p and q values for finite distributed lag model based on MASE values.

```
for (i in 1:5){
  for(j in 1:5){
    model_4 = ardlDlm(formula = y ~ x2 + x4, data = data.frame(v_data_TS), p
= i , q = j )
    cat("p = ", i, "q = ", j, "AIC = ", AIC(model_4$model), "BIC = ",
BIC(model_4$model), "MASE =", MASE(model_4)$MASE, "\n")
  }
}

## p =  1 q =  1 AIC =  3232.773 BIC =  3262.373 MASE = 0.8626722
## p =  1 q =  2 AIC =  3117.872 BIC =  3151.685 MASE = 0.771343
## p =  1 q =  3 AIC =  3113.495 BIC =  3151.516 MASE = 0.7702783
## p =  1 q =  4 AIC =  3108.353 BIC =  3150.579 MASE = 0.769892
## p =  1 q =  5 AIC =  3104.428 BIC =  3150.855 MASE = 0.7663933
## p =  2 q =  1 AIC =  3209.942 BIC =  3247.981 MASE = 0.8431273
## p =  2 q =  2 AIC =  3115.262 BIC =  3157.527 MASE = 0.7656794
## p =  2 q =  3 AIC =  3110.805 BIC =  3157.275 MASE = 0.7642383
## p =  2 q =  4 AIC =  3106.131 BIC =  3156.801 MASE = 0.7641585
## p =  2 q =  5 AIC =  3102.548 BIC =  3157.416 MASE = 0.7620808
## p =  3 q =  1 AIC =  3206.975 BIC =  3253.445 MASE = 0.8416035
## p =  3 q =  2 AIC =  3113.362 BIC =  3164.057 MASE = 0.7654371
## p =  3 q =  3 AIC =  3114.739 BIC =  3169.658 MASE = 0.7644454
## p =  3 q =  4 AIC =  3110.085 BIC =  3169.202 MASE = 0.7643283
## p =  3 q =  5 AIC =  3106.499 BIC =  3169.807 MASE = 0.7621898
## p =  4 q =  1 AIC =  3171.64 BIC =  3226.534 MASE = 0.8083768
## p =  4 q =  2 AIC =  3093.839 BIC =  3152.955 MASE = 0.7477227
## p =  4 q =  3 AIC =  3095.396 BIC =  3158.735 MASE = 0.7478399
## p =  4 q =  4 AIC =  3097.377 BIC =  3164.938 MASE = 0.747751
## p =  4 q =  5 AIC =  3093.631 BIC =  3165.381 MASE = 0.7456829
## p =  5 q =  1 AIC =  3167.579 BIC =  3230.887 MASE = 0.8043549
## p =  5 q =  2 AIC =  3087.036 BIC =  3154.565 MASE = 0.7391588
## p =  5 q =  3 AIC =  3088.377 BIC =  3160.128 MASE = 0.7385057
## p =  5 q =  4 AIC =  3090.327 BIC =  3166.297 MASE = 0.7386083
## p =  5 q =  5 AIC =  3092.255 BIC =  3172.446 MASE = 0.7383519
```

(p, q) = (5, 2); (5, 3) has the least AIC, BIC and MASE scores.

Let us fit (5, 2)

```
# ARDLM model
AR_DLM_mort_52 = ardlDlm(formula = y ~ x2 + x4, data = data.frame(v_data_TS),
```

```
p = 5, q = 2)
summary(AR_DLM_mort_52)

##
## Time series regression with "ts" data:
## Start = 6, End = 508
##
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.2494  -3.5098  -0.2163   3.1941  23.3525
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 19.6211638  2.9621009   6.624 9.25e-11 ***
## x2.t         0.0006556  0.1745611   0.004 0.997005
## x2.1         0.4018107  0.1739867   2.309 0.021336 *
## x2.2         0.3500628  0.1810555   1.933 0.053758 .
## x2.3         0.0946475  0.1829440   0.517 0.605141
## x2.4         0.3725505  0.1809623   2.059 0.040051 *
## x2.5        -0.3662787  0.1813384  -2.020 0.043944 *
## x4.t         0.1445148  0.0395899   3.650 0.000290 ***
## x4.1        -0.1515187  0.0398071  -3.806 0.000159 ***
## x4.2        -0.0853990  0.0401870  -2.125 0.034085 *
## x4.3        -0.0327749  0.0399252  -0.821 0.412100
## x4.4         0.0092831  0.0398760   0.233 0.816016
## x4.5         0.0918108  0.0395403   2.322 0.020646 *
## y.1          0.3438820  0.0418985   8.208 2.03e-15 ***
## y.2          0.3712665  0.0397978   9.329  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.119 on 488 degrees of freedom
## Multiple R-squared:  0.7451, Adjusted R-squared:  0.7378
## F-statistic: 101.9 on 14 and 488 DF,  p-value: < 2.2e-16
```

Hypotheses:

H_0: The data doesn't fit the Autoregressive distributed lag model.
H_A: The data fits the Autoregressive distributed lag model.

Interpretations:

F - statistic is 101.9
R - squared is 0.7451
Adjusted R - squared is 0.7378
Degrees of freedom - DF are (14, 488)

p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Autoregressive distributed lag model.
This model suggests that there is only 73.78% of data variance. Suggesting that the model explains only 73.78% of the trend. Which implies that the model shows some trend.

Now let us perform residual analysis.

Residual analysis

```
res_analysis(residuals(AR_DLM_mort_52))

## Time Series:
## Start = 6
## End = 508
## Frequency = 1
##           6           7           8           9          10
11
##  2.18125931 -5.60652325 -2.02357065  2.57044010 -5.69639970
0.56169951
##          12          13          14          15          16
17
##  2.91124606  1.37212879 -0.28115197  4.98076918 -6.27270419  -
2.96589274
##          18          19          20          21          22
23
##  3.26696985 -3.98272547  9.95729405 -7.21933582  3.82166102
0.44677960
##          24          25          26          27          28
29
## -2.07014149 -2.09317167  0.28955780 -1.15526010 -4.30462518  -
1.05369246
##          30          31          32          33          34
35
## -1.37842835 -3.77410679  3.47223279 -0.13611484 -5.40130159  -
0.58149309
##          36          37          38          39          40
41
## -8.18736331 -3.99646189  6.43987433 -6.51537241 -6.51340863  -
7.07637805
##          42          43          44          45          46
47
## -5.01935138 -3.04393017 -4.39097492  5.57987175  6.98208940
6.49161606
##          48          49          50          51          52
53
## -0.86308154  0.13486443 -7.27878136 -7.87860344 -1.83479258
4.73260532
##          54          55          56          57          58
```

```
59
##  -3.03173517   0.88911087   0.50811098   3.64932169  -0.06235321
1.01850161
##           60           61           62           63           64
65
##  -0.68973972  -2.24955085   1.62055876   2.00674587  -4.90469401
6.66668837
##           66           67           68           69           70
71
##  -3.38859084   6.08425164   0.44382899  -1.89671137  -5.07649730
7.03260151
##           72           73           74           75           76
77
##  -4.60166649   7.59324626   3.15362205  -1.34087084  -4.30044518
19.40455530
##           78           79           80           81           82
83
##   3.63608147 -10.95775397  -1.76935404   0.07126750   8.92549264
0.14900717
##           84           85           86           87           88
89
##  -4.33350771  -5.50252118   9.59620055  -3.69120983  -0.22690715
5.42376765
##           90           91           92           93           94
95
##   3.04598498 -14.24942214   6.38706743   3.96596228  -5.12990834
7.73178338
##           96           97           98           99          100
101
##   4.05975520  10.64704635   6.25552423   4.83118092  -1.99216672
7.60720104
##          102          103          104          105          106
107
##  -4.43635996   0.33404171   2.19612166  -4.29273004  -4.77608120  -
3.19672750
##          108          109          110          111          112
113
##  -7.20939569   0.93920764  -8.19692757   3.14855403   3.13993253
0.21395298
##          114          115          116          117          118
119
##   0.72164789  -1.33246798  -3.12789278   7.64672479  -2.20327618
8.80678240
##          120          121          122          123          124
125
##  -5.95122144  -3.51869312   2.01000803   0.85471058   8.76209996  -
5.37102054
##          126          127          128          129          130
131
```

```
##   2.96503616   3.91619612   3.08121846   5.56348217  -3.17309372  
1.22401988 
##          132          133          134          135          136 
137 
##  -9.57872490   2.10519629   3.16664916  -0.51043691   4.56498004  -
2.67173352 
##          138          139          140          141          142 
143 
##   2.34964395  -5.14122957   3.78157058  -2.50069237   7.76472788  
4.11574158 
##          144          145          146          147          148 
149 
##  -3.58110760   1.97731501  -0.87142051   0.74810308   3.24247978  
9.51927061 
##          150          151          152          153          154 
155 
##  10.95927945  23.35246733   9.71457935   0.43051569   3.18439889  
4.75178097 
##          156          157          158          159          160 
161 
##  -9.36956127  -5.93086357  -0.61693381  -3.84391463   4.01093380  -
7.32911311 
##          162          163          164          165          166 
167 
##   2.50257631   4.35719428  -0.42123707  -8.97945107  16.33031088  
1.35168903 
##          168          169          170          171          172 
173 
##   2.21384488   4.48473146  -5.11916968   0.57113729   3.26670539  -
5.16254832 
##          174          175          176          177          178 
179 
##   0.81793608  13.38835606   0.20829419   0.82155257  -5.05278487  -
3.50088383 
##          180          181          182          183          184 
185 
##   0.80987895   5.57545938  -3.31916528   2.09284392  -0.37235296  -
4.97468962 
##          186          187          188          189          190 
191 
##  -0.25569138   6.90410489   1.81476257   5.40750255  -1.99022771  -
6.80417384 
##          192          193          194          195          196 
197 
##  -2.53223447   1.27943959  -3.85542693   0.05675061   2.12110175  
1.59339620 
##          198          199          200          201          202 
203 
##  -4.55464700   4.35239686   0.24360862   2.65988782   7.55099977 
```

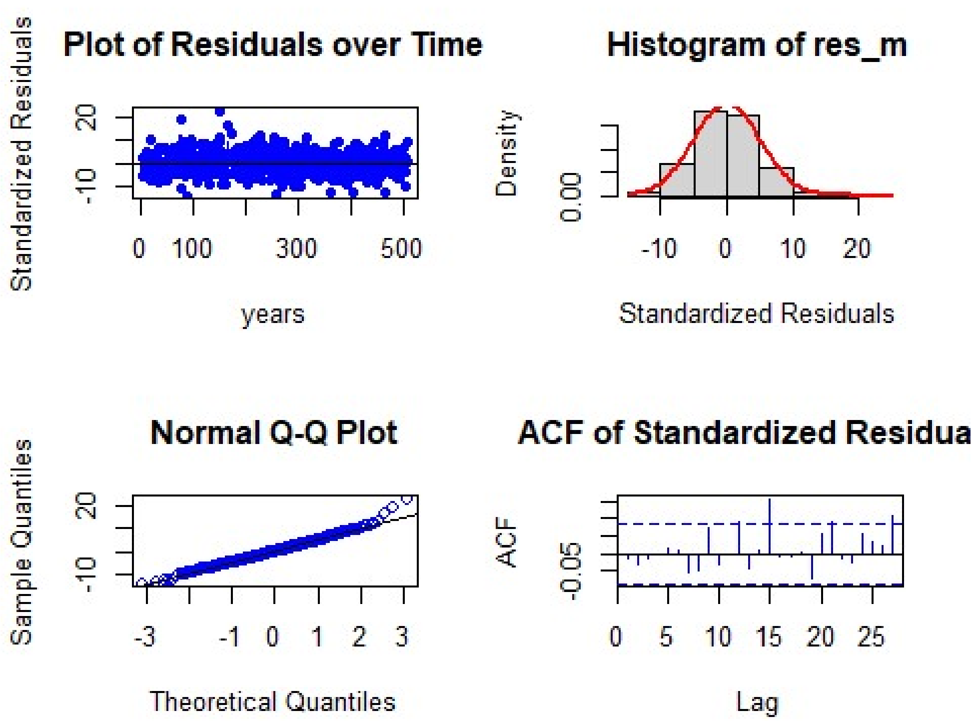
```
5.30533567
##         204         205         206         207         208 
209 
##  -8.34144350  -9.33491188  -1.14093274  -0.07020637   1.38002064 
2.06170738 
##         210         211         212         213         214 
215 
##   7.08162741   1.18597614   8.62107028  -2.60638078   1.16366276  -
4.76411300 
##         216         217         218         219         220 
221 
##   4.99433012   4.41864239  -6.99181838   6.16227405   0.14366735 
10.56176745 
##         222         223         224         225         226 
227 
##  -4.20740544   2.28980098   4.84651262  -0.21631732  -3.66431338 
0.18035953 
##         228         229         230         231         232 
233 
##  12.36007825  -3.87840130  -0.65625413   0.91991437  -3.01611909 
8.73611868 
##         234         235         236         237         238 
239 
##  -7.88439169  -1.07004588   2.02010545   2.87711851   3.90347372  -
1.84517149 
##         240         241         242         243         244 
245 
##   5.15195215  -2.60827541   1.06408057   2.01611644   1.19803207 
4.30334649 
##         246         247         248         249         250 
251 
##  -3.82862197  -7.32261446   4.43779423  -7.37388872   7.49991624  -
5.52474629 
##         252         253         254         255         256 
257 
##  -1.45770756  -0.86140742  11.59820205   3.86626363   3.36064737 
2.88582073 
##         258         259         260         261         262 
263 
##  -0.88032456   3.85131549   6.90804754 -14.00475307  -4.68216787  -
7.75985826 
##         264         265         266         267         268 
269 
##  -3.99136468   3.41680548  -4.56453174  -1.89364977  -0.66733720 
2.62303766 
##         270         271         272         273         274 
275 
##   9.91838770  -9.88236492  -8.26515700  -4.53734794   0.87575511 
9.59557442
```

```
##          276          277          278          279          280
281
##   -0.30029553  -5.44580761  -7.91328190   1.88680522   5.54158714
1.16785221
##          282          283          284          285          286
287
##   -6.83170899  -3.16980736   2.09477623  -1.04509023  -0.20075464  -
1.31735134
##          288          289          290          291          292
293
##   -2.87652392   4.74124607  -0.76989376   0.46943031  -4.64566268  -
1.44110458
##          294          295          296          297          298
299
##  -10.07702278  -1.44401676  -0.72916282   7.34561808   1.94505223  -
5.92158058
##          300          301          302          303          304
305
##   -2.21408896   0.70951536   5.04598377  -2.77257062  -1.36446604  -
9.79110572
##          306          307          308          309          310
311
##    5.36253843   6.58899928  -4.28620161  -3.84162373  -5.47742876  -
3.87173748
##          312          313          314          315          316
317
##    8.12325153   5.20773339   4.57899728  -4.47632181   2.56922245
10.88306678
##          318          319          320          321          322
323
##   -3.59958304  -2.54187011  -1.65270768  -4.17824372  -0.22283688  -
0.64058921
##          324          325          326          327          328
329
##    1.34974866  -3.08343994  -2.73251235  -3.15595662  -2.07362650
4.01481713
##          330          331          332          333          334
335
##    2.94441388   1.04347460   6.52416990  -4.92659506  -7.02002802
1.05115922
##          336          337          338          339          340
341
##   -7.03280029   3.36457336   4.73179503  -7.92983866  -0.92750387  -
6.98710301
##          342          343          344          345          346
347
##    2.42686243  -5.98531907   0.05583307  -3.11337634   0.16949626
0.26213548
##          348          349          350          351          352
```

```
353
##  -1.89430755  -1.85009800   4.82987307  -0.24483711  -5.71752471  -
4.48768719
##          354          355          356          357          358
359
##  -2.95288753   0.52967914  -5.78297329   0.35869976   4.66948306  -
0.98228587
##          360          361          362          363          364
365
##  -0.62089762  -4.41306583  -0.94691460 -13.11568139  -0.18062502  -
3.84985370
##          366          367          368          369          370
371
##  -7.59587973   1.82874552  -1.74821414  -0.03252803  -1.85180450
9.02806984
##          372          373          374          375          376
377
##   3.25242948   0.20935248  -0.66797186   4.12749835  -1.56466132  -
2.19537883
##          378          379          380          381          382
383
##  -7.65022216   9.08990976   3.74814010  -6.52311128  -4.47541220
0.56277604
##          384          385          386          387          388
389
##  -0.85714579  -1.05737901   1.14565683  -2.06728123   1.83373265
3.61125975
##          390          391          392          393          394
395
##   1.05376156  -7.42824531  -0.25808481  -2.98161975  -9.21390459
0.67294358
##          396          397          398          399          400
401
##  -4.84096238  -2.83606975   5.77373457  -7.28466188  -0.91770881
0.24061769
##          402          403          404          405          406
407
##  -1.55892882   2.66229428   1.90064716 -12.73007530   3.77449939  -
5.86060873
##          408          409          410          411          412
413
##  -4.15935391  -3.09557836   8.36132616   2.42181956  -4.59132532
4.18142128
##          414          415          416          417          418
419
##   8.20231753   6.46566379  -1.58449546  -0.92990411   4.71748183  -
4.47516332
##          420          421          422          423          424
425
```

```
##  -5.97093638  -1.18874227   7.07120624  -1.96766655   2.75520313  -
3.72481649
##          426          427          428          429          430
431
##   0.52139622  -2.71463525  -3.99365885  -3.80502414   4.47466951
2.02152076
##          432          433          434          435          436
437
##  -8.73314891   3.92415636  -0.69249786  -1.66948390  -0.49783841  -
0.55900947
##          438          439          440          441          442
443
##  -2.00408885   6.61038385  -6.57196669  -1.80344373   1.43315441
4.09410742
##          444          445          446          447          448
449
##  -0.54215882  -2.14216109   3.71961574  -1.95830112  -1.11945360
9.30559155
##          450          451          452          453          454
455
##   2.59138555  -9.36631547  -6.11654218  -0.95384781   3.49178118  -
2.35329323
##          456          457          458          459          460
461
##  -0.76632112  -2.21054921  -5.52973374   3.20379100   3.68868693
2.26376554
##          462          463          464          465          466
467
##   7.14885809   8.70263919  -2.86038276 -12.91976116  -0.71331752  -
2.14883513
##          468          469          470          471          472
473
##  12.57365600  -4.14082913   1.19756887  -2.78615093   0.78190724  -
2.52391531
##          474          475          476          477          478
479
##   0.95426030   3.98245230  -0.80617398   2.35228029   2.08539788  -
4.62503380
##          480          481          482          483          484
485
##   3.13361869  -0.01398302   4.70232614   3.78357674   2.87337093  -
3.82409649
##          486          487          488          489          490
491
##   1.94898780   5.71460524  -6.73675540   0.43798225   4.43915638  -
1.56628160
##          492          493          494          495          496
497
##  -6.18320648  -1.65615910  -2.88525671   1.54833641   2.89787632  -
```

```
1.92649252
##          498          499          500          501          502
503
##  -0.48262528   1.50262756   6.98569419   0.31160167  -2.19689011 -
10.28253328
##          504          505          506          507          508
##   2.13032081  -0.47552892  -3.61238431   9.91230599   0.68775341
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.98986, p-value = 0.001518
```

Residual Analysis for AR_DLM_mort_52:

1. The data points are below the line at both the start and end of the trend. Randomness is not seen here.
2. From normal distribution curve, the distribution is almost symmetric. This suggests a good fit with a very few data falling outside the normal curve indicating Kurtosis.
3. The data at the tails is deviated but is normal for most part of the line suggesting normality in the trend.
4. There are no significant lags in Autocorrelation plot suggesting that the stochastic component is white noise.

5. p - value (0.001518) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Let us fit (5, 3)

```
# ARDLM model
AR_DLM_mort_53 = ardlDlm(formula = y ~ x2 + x4, data = data.frame(v_data_TS),
p = 5, q = 3)
summary(AR_DLM_mort_53)
```

```
##
## Time series regression with "ts" data:
## Start = 6, End = 508
##
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -14.158  -3.441  -0.156   3.125  23.548
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 19.040440   3.051143   6.240 9.49e-10 ***
## x2.t        -0.009554   0.175093  -0.055 0.956506
## x2.1         0.393798   0.174340   2.259 0.024338 *
## x2.2         0.365912   0.182207   2.008 0.045171 *
## x2.3         0.102061   0.183247   0.557 0.577812
## x2.4         0.366179   0.181205   2.021 0.043847 *
## x2.5        -0.379162   0.182122  -2.082 0.037872 *
## x4.t         0.147885   0.039829   3.713 0.000228 ***
## x4.1        -0.146565   0.040302  -3.637 0.000306 ***
## x4.2        -0.087017   0.040253  -2.162 0.031124 *
## x4.3        -0.039041   0.040704  -0.959 0.337958
## x4.4         0.010466   0.039918   0.262 0.793295
## x4.5         0.092896   0.039578   2.347 0.019318 *
## y.1          0.329671   0.045536   7.240 1.76e-12 ***
## y.2          0.357184   0.043544   8.203 2.11e-15 ***
## y.3          0.034651   0.043395   0.799 0.424967
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.121 on 487 degrees of freedom
## Multiple R-squared:  0.7454, Adjusted R-squared:  0.7376
## F-statistic: 95.07 on 15 and 487 DF,  p-value: < 2.2e-16
```

Hypotheses : H0 : The data doesn't fit the Autoregressive distributed lag model. HA : The data fits the Autoregressive distributed lag model.

Interpretations: F - statistic is 95.07 R - squared is 0.7454 Adjusted R - squared is 0.7376 Degrees of freedom - DF are (15, 487) p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Autoregressive distributed lag model.

This model suggests that there is only 73.76% of data variance. Suggesting that the model explains only 73.76% of the trend. Which implies that the model shows some trend.

Now let us perform residual analysis.

Residual analysis

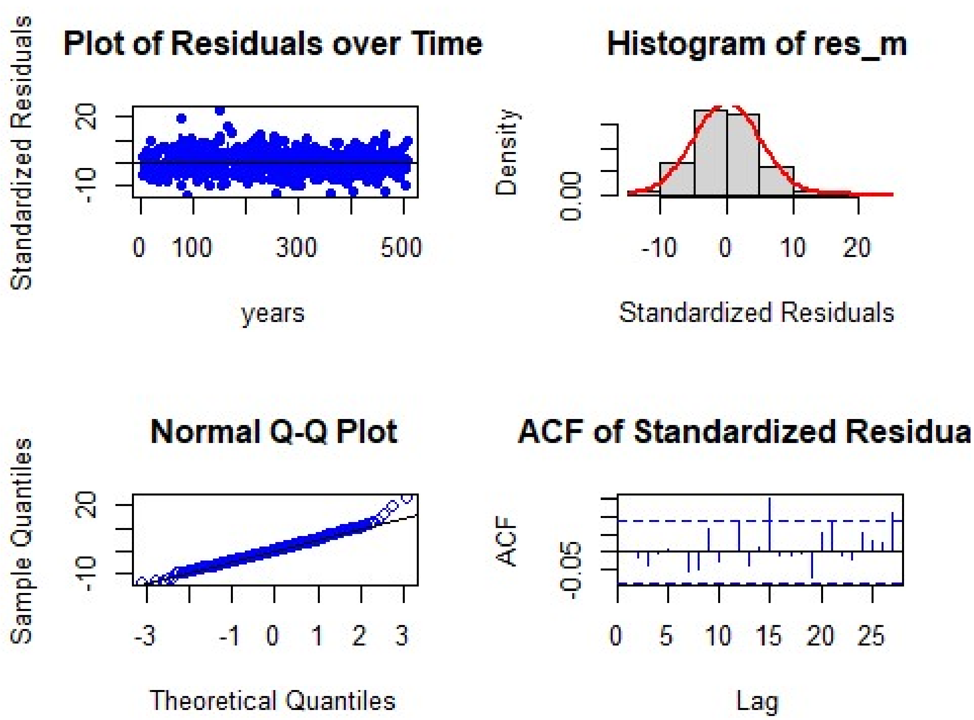
```
res_analysis(residuals(AR_DLM_mort_53))

## Time Series:
## Start = 6
## End = 508
## Frequency = 1
##            6            7            8            9           10
##   2.198622783  -5.653078976  -2.010309452   2.399898622  -5.525722896
##           11           12           13           14           15
##   0.551294106   2.818799923   1.498689813  -0.328021501   4.934136829
##           16           17           18           19           20
##  -6.271450543  -2.928841557   2.928533180  -3.819219609   9.891707083
##           21           22           23           24           25
##  -7.274078949   3.979806659   0.099804594  -1.773896169  -2.300881659
##           26           27           28           29           30
##   0.244475440  -1.183561747  -4.272985999  -1.116122186  -1.377794599
##           31           32           33           34           35
##  -3.705925636   3.444585061  -0.076230973  -5.158293922  -0.672579390
##           36           37           38           39           40
##  -8.160598381  -3.836930892   6.391070909  -6.323248761  -6.382996017
##           41           42           43           44           45
##  -7.324956277  -4.880574352  -2.959941923  -4.341105232   5.776024437
##           46           47           48           49           50
##   7.086674376   6.758227699  -0.677922035   0.074928971  -7.579441309
##           51           52           53           54           55
##  -7.995910653  -2.299767717   4.859438148  -2.869905368   0.971491826
##           56           57           58           59           60
##   0.414446844   3.602722888  -0.086371716   0.776741857  -0.797807728
##           61           62           63           64           65
##  -2.352391416   1.448207524   1.978076613  -4.862081083   6.610297414
##           66           67           68           69           70
##  -3.527355507   6.218143399   0.257105113  -1.693751881  -5.311340617
##           71           72           73           74           75
##   6.869240978  -4.634490328   7.699048321   2.987953720  -0.984676826
##           76           77           78           79           80
##  -4.417594271  19.291876662   3.824166321 -10.415954981  -2.282491911
##           81           82           83           84           85
##  -0.183576907   9.019721965   0.264740125  -4.180360892  -5.761315565
```

```
##            86           87           88           89           90
##   9.480882616 -3.678600008  0.003836582  5.083561655  3.301452301
##            91           92           93           94           95
## -14.157485289  6.180459808  3.626369034 -4.590968611  7.526010043
##            96           97           98           99          100
##   4.124844471 10.915954432  6.306170924  4.913956079 -2.168832813
##           101          102          103          104          105
##   7.406918574 -4.666877969  0.290975844  1.889725007 -4.169278025
##           106          107          108          109          110
##  -5.016611754 -3.539019990 -7.330167620  0.694235690 -8.276854610
##           111          112          113          114          115
##   3.167416805  2.953833931  0.499160754  0.692449767 -1.324678069
##           116          117          118          119          120
##  -3.134850551  7.548052861 -2.113836611  8.975800310 -6.051350067
##           121          122          123          124          125
##  -3.386391241  1.604436373  0.996771393  8.778691263 -5.245014112
##           126          127          128          129          130
##   3.057728760  3.688257590  3.303026719  5.520129608 -3.080840524
##           131          132          133          134          135
##   1.208944986 -9.763159703  2.052558297  2.979121081 -0.244282953
##           136          137          138          139          140
##   4.522561869 -2.589186185  2.407203530 -5.259067699  3.770963222
##           141          142          143          144          145
##  -2.606604803  7.933307126  4.047413538 -3.284926431  1.847049149
##           146          147          148          149          150
##  -0.917341027  0.832096809  3.109408702  9.583486509 11.194666771
##           151          152          153          154          155
##  23.548338814  9.940559525  0.673620262  2.827311803  4.351993215
##           156          157          158          159          160
##  -9.674850411 -6.226717901 -1.210602699 -4.038763668  3.926164179
##           161          162          163          164          165
##  -7.442737578  2.489217795  4.133902996 -0.212492730 -9.057348650
##           166          167          168          169          170
##  16.086161115  1.394979539  2.639604822  4.142474655 -4.995561525
##           171          172          173          174          175
##   0.416196192  3.047796641 -5.079831815  0.759786308 13.239959329
##           176          177          178          179          180
##   0.484837331  1.034829724 -5.317523930 -3.494865792  0.556460809
##           181          182          183          184          185
##   5.621730453 -3.192381170  2.167780897 -0.516898397 -4.815006296
##           186          187          188          189          190
##  -0.394638560  6.859169757  2.025359670  5.622415569 -2.027908951
##           191          192          193          194          195
##  -6.805863159 -2.815315132  1.177802522 -3.705446503  0.113710505
##           196          197          198          199          200
##   2.082106093  1.905763405 -4.482899742  4.370474399  0.136951037
##           201          202          203          204          205
##   2.834141866  7.384106306  5.524003140 -8.085983004 -9.512968188
##           206          207          208          209          210
```

```
##   -1.708382863  -0.296347754   1.365829738   2.079687282   7.240011858
##            211           212           213           214           215
##    1.340973175   8.810093580  -2.658428962   1.213701492  -5.079734561
##            216           217           218           219           220
##    4.930403233   4.214579170  -6.800231645   5.942675045  -0.122362418
##            221           222           223           224           225
##   10.770418773  -4.263865547   2.448307370   4.472617611  -0.013690963
##            226           227           228           229           230
##   -3.757275075  -0.009400148  12.232933585  -3.656895237  -0.511201750
##            231           232           233           234           235
##    0.567261766  -2.908604796   8.675565582  -7.782165234  -0.961056347
##            236           237           238           239           240
##    1.689810506   3.103474473   3.968717804  -1.735081895   5.132157321
##            241           242           243           244           245
##   -2.637089980   1.238434767   1.925431754   1.305106185   4.353051676
##            246           247           248           249           250
##   -3.749723499  -7.355306306   4.162917031  -7.362846191   7.651251008
##            251           252           253           254           255
##   -5.694003238  -1.118512893  -1.031053044  11.738283247   4.023097276
##            256           257           258           259           260
##    3.800726831   2.806395675  -0.824916239   3.567074608   6.722697573
##            261           262           263           264           265
## -13.744644689  -4.939357295  -8.292616670  -4.110210695   3.187491443
##            266           267           268           269           270
##   -4.417708402  -1.841735088  -0.730404424   2.765656550   9.940554830
##            271           272           273           274           275
##   -9.627346990  -8.294515840  -5.077092869   0.857821111   9.619026009
##            276           277           278           279           280
##    0.020357064  -5.242642231  -8.082615401   1.770145727   5.531917183
##            281           282           283           284           285
##    1.460731782  -6.706338140  -3.264253663   1.962027112  -0.909473838
##            286           287           288           289           290
##   -0.062191267  -1.310344377  -2.789493456   4.715323427  -0.661985080
##            291           292           293           294           295
##    0.691730321  -4.684257016  -1.467821786 -10.173284311  -1.435462445
##            296           297           298           299           300
##   -0.838943110   7.613485906   2.178758158  -5.658273998  -2.338939348
##            301           302           303           304           305
##    0.511795459   5.083348829  -2.611995759  -1.169142165  -9.952874881
##            306           307           308           309           310
##    5.261949211   6.467369947  -3.810898186  -3.849326625  -5.692754460
##            311           312           313           314           315
##   -3.903943826   8.022737691   5.471060748   4.997363739  -4.459766304
##            316           317           318           319           320
##    2.471681968  10.682218556  -3.523645614  -2.524914726  -2.056323726
##            321           322           323           324           325
##   -4.262875403  -0.402630564  -0.682384622   1.379993116  -3.116743871
##            326           327           328           329           330
##   -2.769566852  -3.273999824  -2.051801221   3.998256673   3.130323441
```

```
##           331          332          333          334          335
##   1.262420477  6.580210836 -4.873427227 -6.967437510  0.747483663
##           336          337          338          339          340
##  -7.002243999  3.438469600  4.755441686 -7.568849446 -0.910493463
##           341          342          343          344          345
##  -7.174040871  2.540718655 -6.020526935  0.319174558 -3.187131260
##           346          347          348          349          350
##   0.395260599  0.278756748 -1.744066598 -1.804738969  4.799345091
##           351          352          353          354          355
##  -0.096698888 -5.501423498 -4.610022115 -3.072309539  0.537980851
##           356          357          358          359          360
##  -5.599237287  0.408710576  4.541986966 -0.691118432 -0.477369833
##           361          362          363          364          365
##  -4.378576575 -1.014924050 -13.243499841 -0.351681764 -4.150771526
##           366          367          368          369          370
##  -7.357658169  1.700532878 -1.656619025  0.223008437 -1.763826525
##           371          372          373          374          375
##   9.158410898  3.363896780  0.460355282 -0.761510520  4.068601345
##           376          377          378          379          380
##  -1.581627565 -2.151585965 -7.812498259  8.981421278  3.756435018
##           381          382          383          384          385
##  -6.102976939 -4.747830323  0.395411182 -0.895997149 -0.988557493
##           386          387          388          389          390
##   1.166479239 -1.995908051  1.884413046  3.555607039  1.272096013
##           391          392          393          394          395
##  -7.322759959 -0.375831374 -3.073490411 -9.024140668  0.564582742
##           396          397          398          399          400
##  -4.840289309 -2.566144884  5.694677126 -7.040198605 -0.833119482
##           401          402          403          404          405
##   0.048793736 -1.320298645  2.736875596  2.059049402 -12.562645163
##           406          407          408          409          410
##   3.716344240 -5.965098358 -3.903013118 -3.369146290  8.567098137
##           411          412          413          414          415
##   2.647596911 -4.165612566  4.089447829  8.203284118  6.658444257
##           416          417          418          419          420
##  -1.542767554 -0.952259281  4.469740811 -4.493982959 -5.969656037
##           421          422          423          424          425
##  -1.463842029  6.966054399 -1.856129086  2.912140453 -3.857034374
##           426          427          428          429          430
##   0.538701463 -2.869439674 -3.921974539 -3.999216938  4.369527228
##           431          432          433          434          435
##   2.081176980 -8.511651861  3.825156917 -0.826710697 -1.400866659
##           436          437          438          439          440
##  -0.563768469 -0.485462569 -1.965179656  6.533314468 -6.458017155
##           441          442          443          444          445
##  -1.691608762  1.112335009  4.230774435 -0.427436791 -1.996191560
##           446          447          448          449          450
##   3.635861772 -1.869440642 -0.956443108  9.250961972  2.742547837
##           451          452          453          454          455
```

```
##  -9.253448286  -6.381159920  -1.290071441   3.557293319  -2.250575274
##           456           457           458           459           460
##  -0.590343621  -2.158889639  -5.310754821   3.119077724   3.749853420
##           461           462           463           464           465
##   2.651499825   7.197074682   8.801647425  -2.705501436 -12.864354797
##           466           467           468           469           470
##  -1.201056136  -2.320378041  12.669624305  -4.083073366   1.323597056
##           471           472           473           474           475
##  -3.080204014   0.799191521  -2.832484763   0.974262084   4.073250449
##           476           477           478           479           480
##  -0.701385311   2.307052100   2.029980245  -4.509184747   2.975121616
##           481           482           483           484           485
##  -0.156011359   4.783473166   3.734988418   3.024239906  -3.828861410
##           486           487           488           489           490
##   1.844496813   5.511701973  -6.615486628   0.374368387   4.157040853
##           491           492           493           494           495
##  -1.404247181  -6.213368788  -1.860313284  -2.987281502   1.545237257
##           496           497           498           499           500
##   2.919507787  -1.733832316  -0.436953133   1.480634032   7.031833502
##           501           502           503           504           505
##   0.525917673  -2.108455811 -10.421699860   1.944083012  -0.490849977
##           506           507           508
##  -3.341879038   9.739872558   0.872414816
```




```
##
##  Shapiro-Wilk normality test
```

```
##
## data:  res_m
## W = 0.98974, p-value = 0.001387
```

Residual Analysis for AR_DLM_mort_53:

1. The data points are below the line at both the start and end of the trend. Randomness is not seen here.
2. From normal distribution curve, the distribution is almost symmetric. This suggests a good fit with a very few data falling outside the normal curve indicating Kurtosis.
3. The data at the tails is deviated but is normal for most part of the line suggesting normality in the trend.
4. There are no significant lags in Autocorrelation plot suggesting that the stochastic component is white noise.
5. p - value (0.001518) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

So far AR model with order (5, 2) is better model.

Now let us calculate AIC, BIC and MASE scores and store them in a dataframe to check the better model based on MASE score.

```
attr(Koyck_DLM_mort_chem1$model, "class") = "lm"
attr(Koyck_DLM_mort_part$model, "class") = "lm"
attr(AR_DLM_mort_52$model, "class") = "lm"
attr(AR_DLM_mort_53$model, "class") = "lm"

v_model_name <- c("finite_dlm_mort", "PolyDLM_model_mort_chem1",
"PolyDLM_model_mort_part", "Koyck_DLM_mort_chem1", "Koyck_DLM_mort_part",
"AR_DLM_mort_52", "AR_DLM_mort_53")

MASE <- MASE(finite_dlm_mort$model, PolyDLM_model_mort_chem1$model,
PolyDLM_model_mort_part$model, Koyck_DLM_mort_chem1$model,
Koyck_DLM_mort_part$model, AR_DLM_mort_52$model, AR_DLM_mort_53$model)$MASE

aic <- AIC(finite_dlm_mort$model, PolyDLM_model_mort_chem1$model,
PolyDLM_model_mort_part$model, Koyck_DLM_mort_chem1$model,
Koyck_DLM_mort_part$model, AR_DLM_mort_52$model, AR_DLM_mort_53$model)$AIC

bic <- BIC(finite_dlm_mort$model, PolyDLM_model_mort_chem1$model,
PolyDLM_model_mort_part$model, Koyck_DLM_mort_chem1$model,
Koyck_DLM_mort_part$model, AR_DLM_mort_52$model, AR_DLM_mort_53$model)$BIC

v_score <- data.frame(v_model_name, MASE, aic, bic)
colnames(v_score) <- c("MODEL_NAME", "MASE", "AIC", "BIC")
v_score
```

```
##                   MODEL_NAME      MASE      AIC      BIC
## 1           finite_dlm_mort 0.8897851 3293.419 3394.473
## 2 PolyDLM_model_mort_chem1 1.1041854 3478.836 3504.183
## 3  PolyDLM_model_mort_part 1.2603636 3606.089 3631.437
## 4     Koyck_DLM_mort_chem1 0.8708743 3241.979 3258.893
## 5      Koyck_DLM_mort_part 0.9116488 3285.238 3302.152
## 6           AR_DLM_mort_52 0.7391588 3087.036 3154.565
## 7           AR_DLM_mort_53 0.7385057 3088.377 3160.128
```

Comparitively, AR_DLM_solar_53 is the better model in terms of MASE, AIC and BIC scores.

Now let us fit dynamic lm model

Dynamic model

```
v_mort_dyna <- dynlm(y ~ x2 + x4, data = data.frame(v_data_TS))
summary(v_mort_dyna)

##
## Time series regression with "numeric" data:
## Start = 1, End = 508
##
## Call:
## dynlm(formula = y ~ x2 + x4, data = data.frame(v_data_TS))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -19.164  -5.582  -0.935   4.324  39.903
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  79.0240     1.2897  61.273   <2e-16 ***
## x2            1.8404     0.1956   9.410   <2e-16 ***
## x4           -0.1029     0.0486  -2.118   0.0346 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.281 on 505 degrees of freedom
## Multiple R-squared:  0.3168, Adjusted R-squared:  0.3141
## F-statistic: 117.1 on 2 and 505 DF,  p-value: < 2.2e-16
```

Hypotheses:

H_0: The data doesn't fit the Dynamic linear model.
H_A: The data fits the Dynamic linear model.

Interpretations:

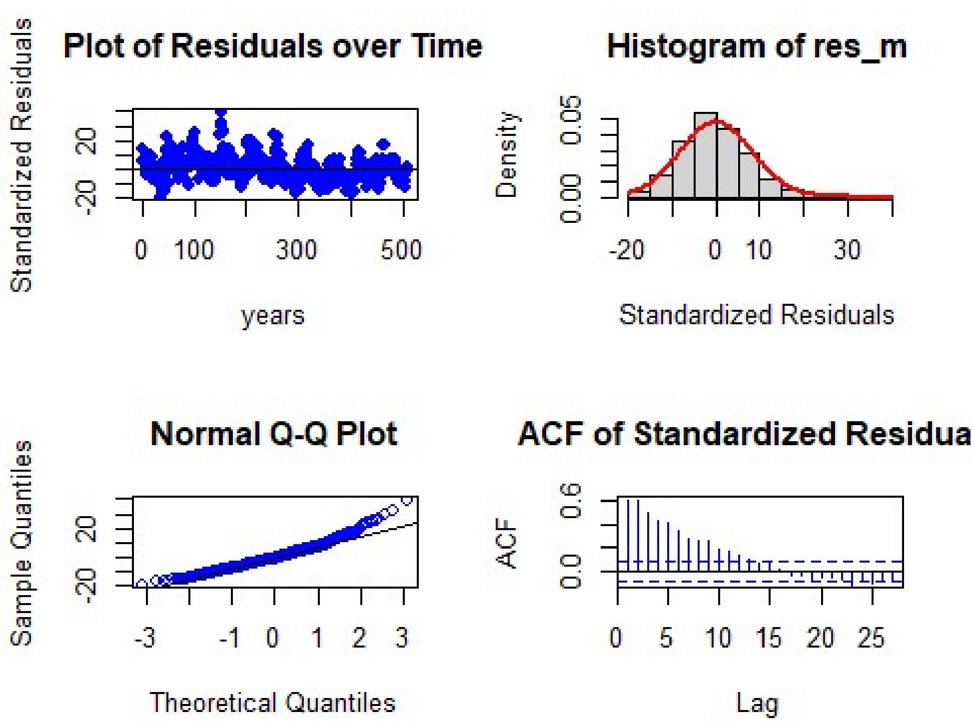
F - statistic is 117.1
R - squared is 0.3168
Adjusted R - squared is 0.3141

Degrees of freedom - DF are (2, 505)
p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Dynamic linear model.
This model suggests that there is only 73.76% of data variance. Suggesting that the model explains only 73.76% of the trend. Which implies that the model shows some trend.

Now let us perform residual analysis.

Residual analysis

```
res_analysis(residuals(v_mort_dyna))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.96747, p-value = 3.518e-09
```

Residual Analysis for v_mort_dyna:

1. The data points are above the line at both the start and end of the trend. Randomness is not seen here.
2. From normal distribution curve, the distribution is almost symmetric. This suggests a good fit with a very few data falling outside the normal curve indicating Kurtosis.

3. The data at the tails is deviated but is normal for most part of the line suggesting normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise. Also, ACF shows seasonality pattern
5. p - value (0.001518) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

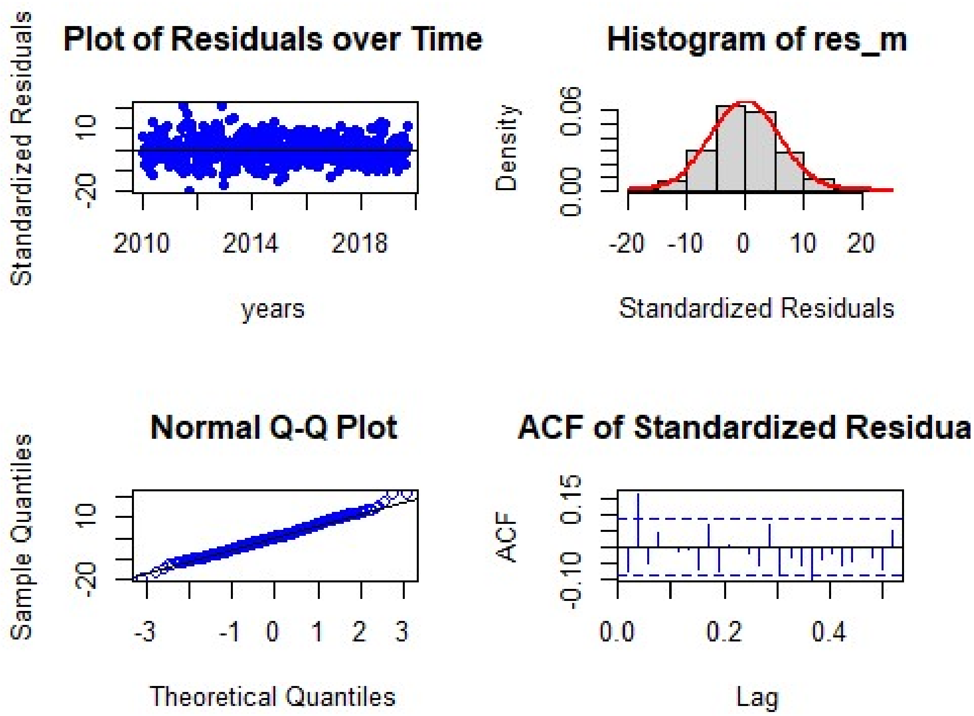
Exponential Smoothing

Since the Seasonality component is week we cannot get additive and multiplicative seasonality. So let us fit with simple seasonality. Since, we need next 4 weeks point forecasts as well as confidence intervals, we used h = 4 (frequency).

```
v_mort_ses <- ses(v_Mortality_data_TS, seasonal = "simple", h = 4)
summary(v_mort_ses)

##
## Forecast method: Simple exponential smoothing
##
## Model Information:
## Simple exponential smoothing
##
## Call:
##  ses(y = v_Mortality_data_TS, h = 4, seasonal = "simple")
##
##   Smoothing parameters:
##     alpha = 0.5111
##
##   Initial states:
##     l = 98.9373
##
##   sigma:  5.8932
##
##      AIC     AICc      BIC
## 4971.256 4971.303 4983.947
##
## Error measures:
##                       ME     RMSE      MAE        MPE     MAPE      MASE
## Training set -0.05496823 5.88156 4.588564 -0.3769165 5.156427 0.6871453
##                     ACF1
## Training set -0.07528577
##
## Forecasts:
##          Point Forecast    Lo 80    Hi 80    Lo 95     Hi 95
## 2019.735       84.66432 77.11191 92.21672 73.11391  96.21472
## 2019.754       84.66432 76.18251 93.14612 71.69251  97.63612
## 2019.774       84.66432 75.34534 93.98329 70.41218  98.91645
## 2019.793       84.66432 74.57742 94.75121 69.23774 100.09089
```

Now let us check the residual analysis.

```
res_analysis(residuals(v_mort_ses))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.99538, p-value = 0.1364
```

Residual Analysis analysis for simple seasonality:

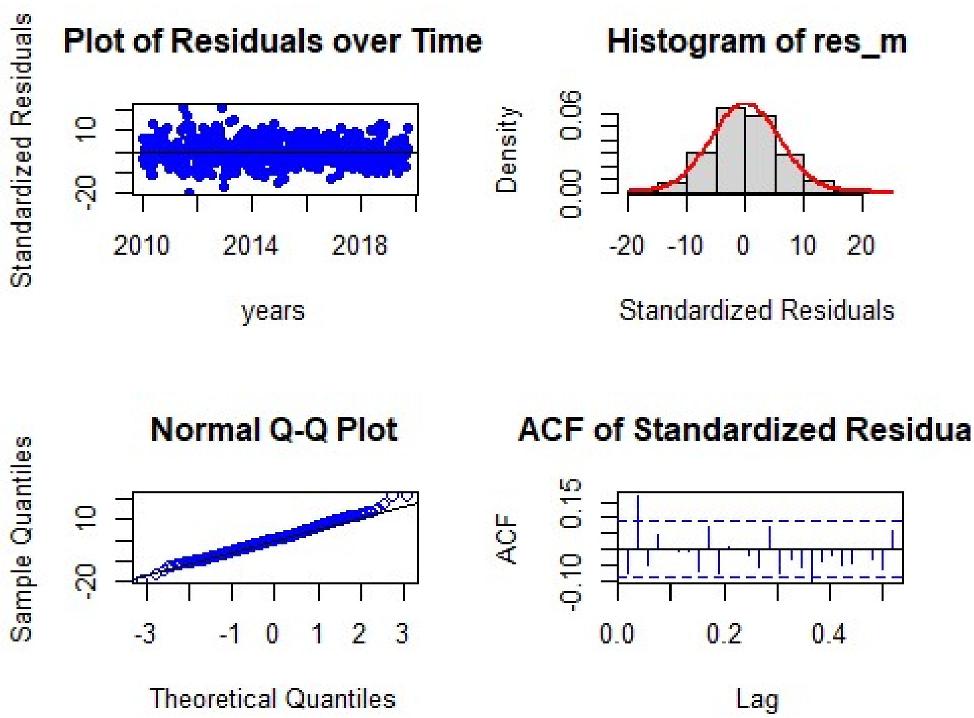
1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (`0.1364`) from Shapiro-Wilk normality test is > 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Now let us fit with damped trend.

```
v_mort_exp <- holt(v_Mortality_data_TS, damped = TRUE, h = 4)
summary(v_mort_exp)

##
## Forecast method: Damped Holt's method
##
## Model Information:
## Damped Holt's method
##
## Call:
##  holt(y = v_Mortality_data_TS, h = 4, damped = TRUE)
##
##   Smoothing parameters:
##     alpha = 0.5082
##     beta  = 1e-04
##     phi   = 0.9119
##
##   Initial states:
##     l = 102.0415
##     b = -1.1632
##
##   sigma:  5.9071
##
##      AIC     AICc      BIC
## 4976.632 4976.800 5002.015
##
## Error measures:
##                       ME     RMSE      MAE        MPE     MAPE     MASE
## Training set -0.02068605 5.877951 4.588308 -0.3396637 5.154426 0.687107
##                    ACF1
## Training set -0.0738815
##
## Forecasts:
##          Point Forecast    Lo 80    Hi 80    Lo 95     Hi 95
## 2019.735       84.64364 77.07339 92.21388 73.06595  96.22133
## 2019.754       84.64444 76.15242 93.13646 71.65702  97.63187
## 2019.774       84.64517 75.32181 93.96854 70.38632  98.90403
## 2019.793       84.64584 74.55921 94.73248 69.21967 100.07202
```

Now let us check the residual analysis.

```
res_analysis(residuals(v_mort_exp))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.99547, p-value = 0.1477
```

Residual Analysis analysis for exponential trend:

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (`0.1477`) from Shapiro-Wilk normality test is > 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Due to week seasonality in the series there is no additive or multiplicative seasonality also there will be no damped in the series.

By exponential smoothing method we got the simple seasonal fit as the best model in terms of MASE and BIC scores.

State Space Model Variations

Let us find the best ets model. Before all let us auto fit the model.

Since, the frequency is greater than 24 we cannot use ets() method. Therefore, stlf() is used.

Since, we need next 4 weeks point forecasts as well as confidence intervals, we used h = 4 (frequency).

```
v_stlf_fit <- stlf(v_Mortality_data_TS, h = 4)
summary(v_stlf_fit)

##
## Forecast method: STL +  ETS(M,N,N)
##
## Model Information:
## ETS(M,N,N)
##
## Call:
##  ets(y = na.interp(x), model = etsmodel, allow.multiplicative.trend =
allow.multiplicative.trend)
##
##   Smoothing parameters:
##     alpha = 0.3015
##
##   Initial states:
##     l = 92.3928
##
##   sigma:  0.0537
##
##      AIC     AICc      BIC
## 4757.163 4757.210 4769.854
##
## Error measures:
##                       ME     RMSE      MAE        MPE     MAPE     MASE
## Training set -0.05169619 4.781323 3.768765 -0.3101758 4.269248 0.564379
##                     ACF1
## Training set -0.014883
##
## Forecasts:
##          Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 2019.735       83.70585 77.88797 89.52373 74.80818 92.60353
## 2019.754       86.99713 80.91984 93.07442 77.70272 96.29155
## 2019.774       89.32161 82.99547 95.64775 79.64662 98.99660
## 2019.793       88.56710 82.00148 95.13272 78.52585 98.60835
```

STL + ETS(M, N, N)

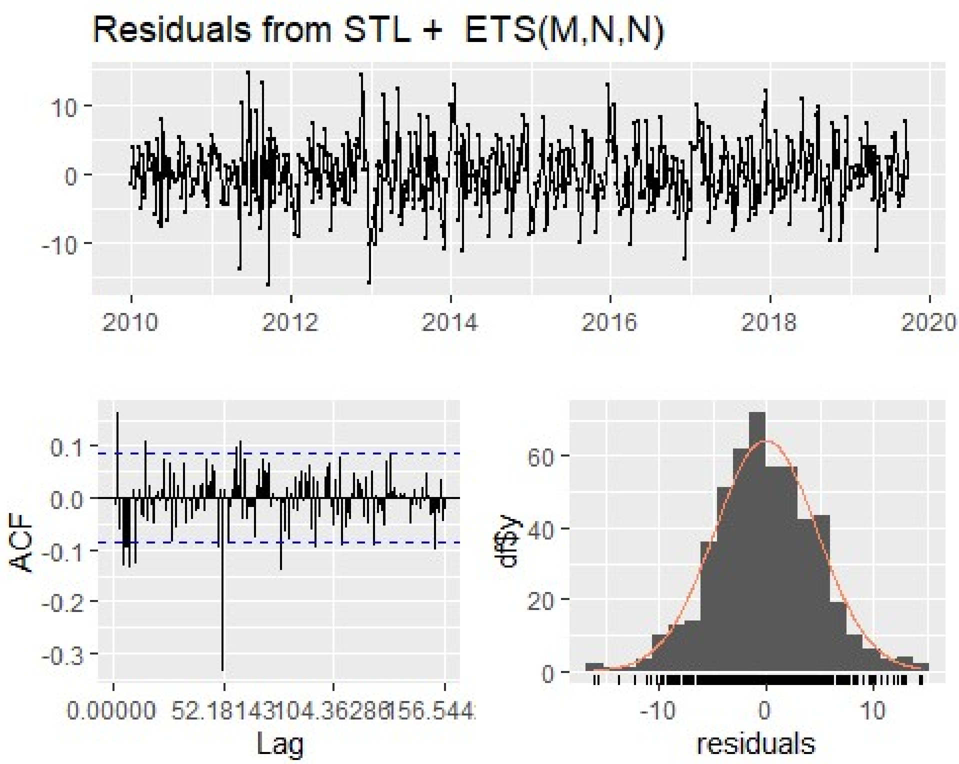
M - Multiplicative errors

N - No trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_stlf_fit)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from STL +  ETS(M,N,N)
## Q* = 245.16, df = 100, p-value = 3.397e-14
##
## Model df: 2.   Total lags used: 102
```

Residual Analysis ETS(M, N, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.

5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Now let us fit ets variable combinations individually.

```
v_ets_fit1 <- ets(v_Mortality_data_TS, model = "ANN")
summary(v_ets_fit1)
```

```
## ETS(A,N,N)
##
## Call:
##  ets(y = v_Mortality_data_TS, model = "ANN")
##
##   Smoothing parameters:
##     alpha = 0.511
##
##   Initial states:
##     l = 98.9364
##
##   sigma:  5.8932
##
##      AIC     AICc      BIC
## 4971.256 4971.303 4983.947
##
## Training set error measures:
##                       ME    RMSE      MAE        MPE     MAPE      MASE
## Training set -0.05497816 5.88156 4.588578 -0.3769459 5.156433 0.6871475
##                     ACF1
## Training set -0.07517189
```

ETS(A, N, N)

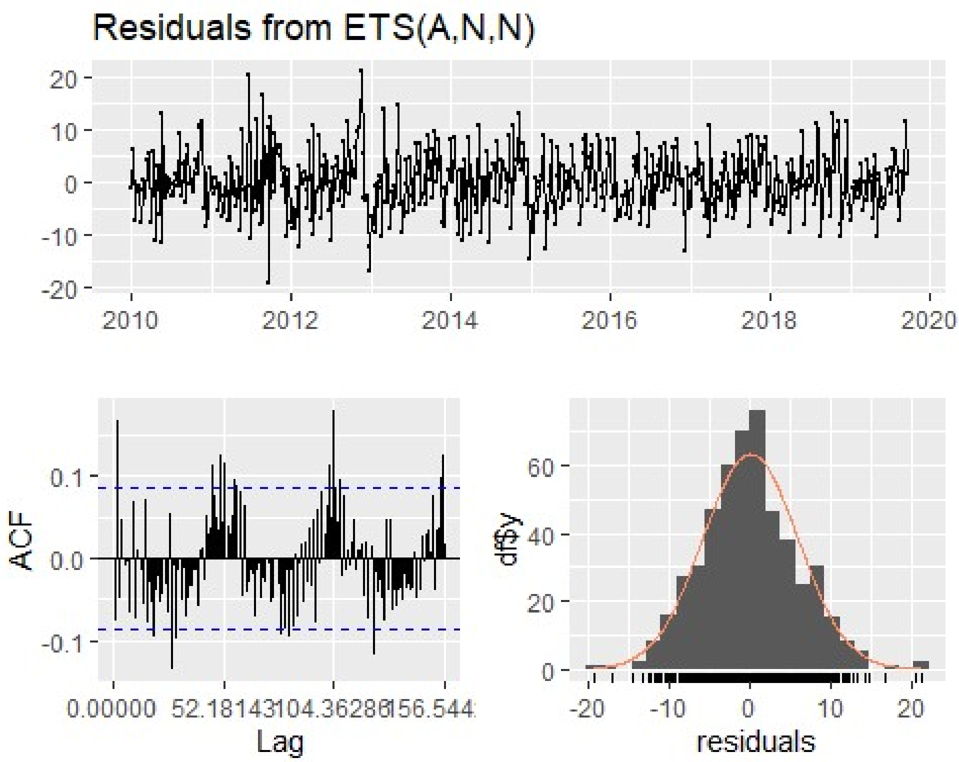
A - Additive errors

N - No trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit1)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(A,N,N)
## Q* = 191.57, df = 100, p-value = 1.006e-07
##
## Model df: 2.   Total lags used: 102
```

Residual Analysis ETS(A, N, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
v_ets_fit2 <- ets(v_Mortality_data_TS, model = "AAN")
summary(v_ets_fit2)
```

```
## ETS(A,A,N)
##
## Call:
##  ets(y = v_Mortality_data_TS, model = "AAN")
##
##   Smoothing parameters:
##     alpha = 0.5122
##     beta  = 1e-04
##
##   Initial states:
##     l = 100.9765
##     b = -0.029
##
##   sigma:  5.906
##
##      AIC     AICc      BIC
## 4975.460 4975.579 4996.612
##
## Training set error measures:
##                        ME    RMSE      MAE        MPE     MAPE      MASE
## Training set -0.004457548 5.88274 4.590352 -0.3178407 5.155916 0.6874132
##                     ACF1
## Training set -0.07651869
```

ETS(A, A, N)

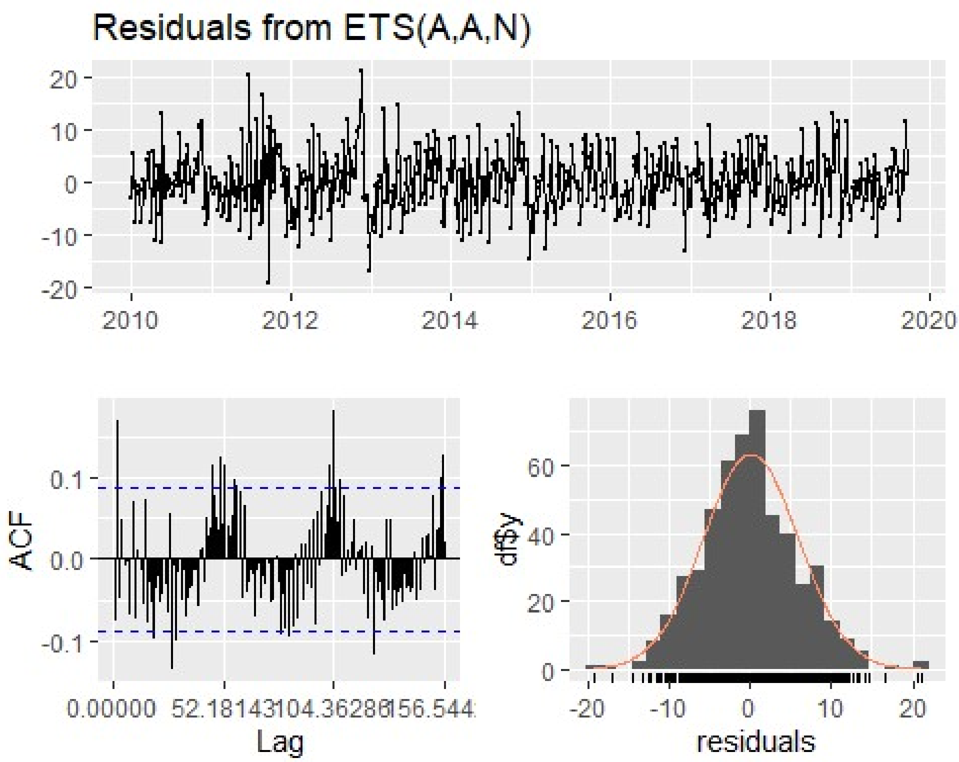
A - Additive errors

A - Additive trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit2)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(A,A,N)
## Q* = 192.28, df = 98, p-value = 4.215e-08
##
## Model df: 4.   Total lags used: 102
```

Residual Analysis ETS(A, A, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
v_ets_fit3 <- ets(v_Mortality_data_TS, model = "MNN")
summary(v_ets_fit3)
```

```
## ETS(M,N,N)
##
## Call:
##  ets(y = v_Mortality_data_TS, model = "MNN")
##
##   Smoothing parameters:
##     alpha = 0.4843
##
##   Initial states:
##     l = 98.5582
##
##   sigma:  0.0656
##
##      AIC     AICc      BIC
## 4954.111 4954.159 4966.803
##
## Training set error measures:
##                       ME    RMSE      MAE        MPE     MAPE      MASE
## Training set -0.05730399 5.88508 4.593891 -0.3849281 5.159608 0.6879431
##                     ACF1
## Training set -0.04372931
```

ETS(A, A, N)

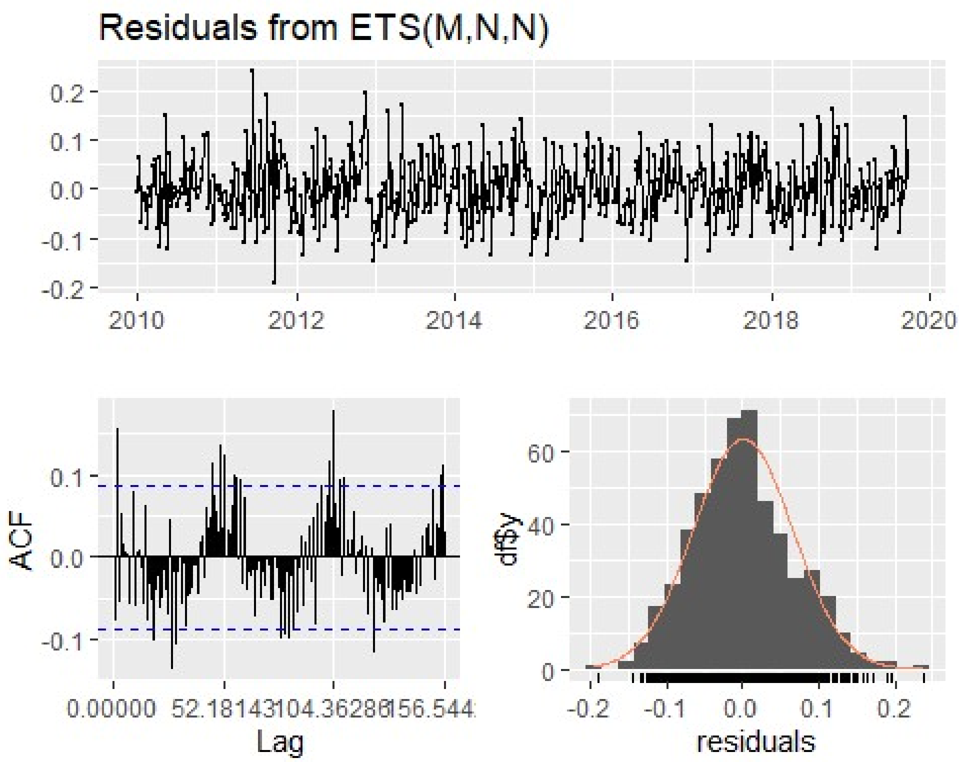
M - Multiplicative errors

N - No trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit3)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(M,N,N)
## Q* = 207.78, df = 100, p-value = 1.514e-09
##
## Model df: 2.   Total lags used: 102
```

Residual Analysis ETS(M, N, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
v_ets_fit4 <- ets(v_Mortality_data_TS, model = "MAN")
summary(v_ets_fit4)
```

```
## ETS(M,Ad,N)
##
## Call:
##  ets(y = v_Mortality_data_TS, model = "MAN")
##
##   Smoothing parameters:
##     alpha = 0.4311
##     beta  = 0.0441
##     phi   = 0.8
##
##   Initial states:
##     l = 101.9204
##     b = -0.7466
##
##   sigma:  0.0657
##
##      AIC     AICc      BIC
## 4957.818 4957.986 4983.201
##
## Training set error measures:
##                      ME     RMSE      MAE        MPE     MAPE      MASE
## Training set -0.041505 5.883233 4.604242 -0.3409889 5.167291 0.6894931
##                      ACF1
## Training set -0.02166677
```

ETS(A, A, N)

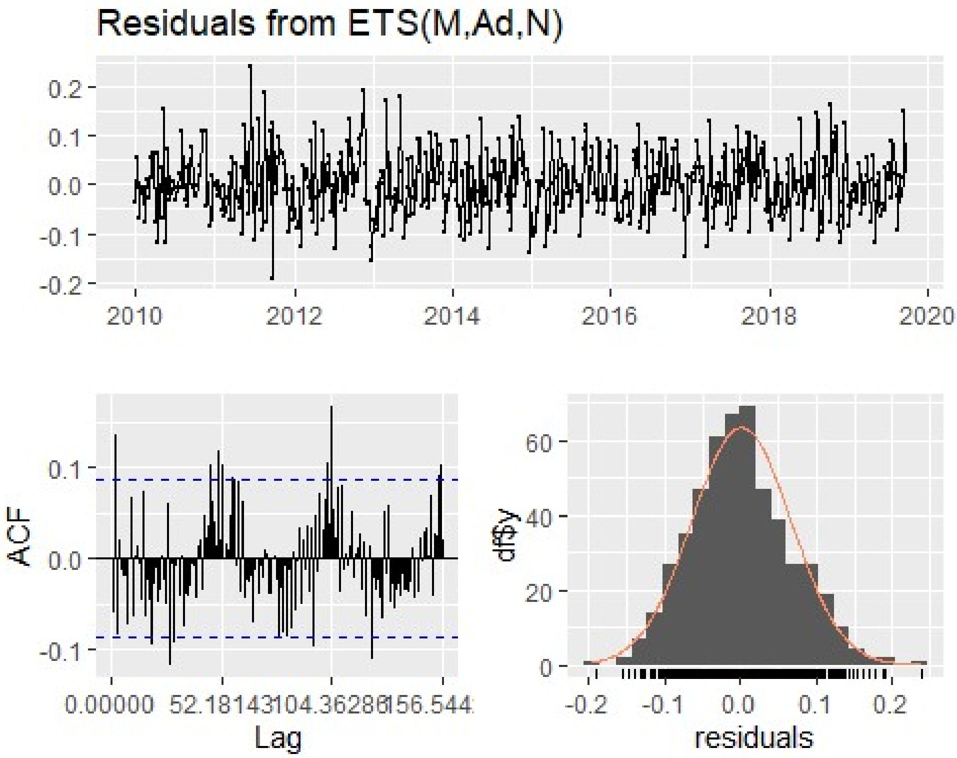
M - Multiplicative errors

A - Additive trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit4)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(M,Ad,N)
## Q* = 160.36, df = 97, p-value = 5.524e-05
##
## Model df: 5.   Total lags used: 102
```

Residual Analysis ETS(M, A, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
v_ets_fit5 <- ets(v_Mortality_data_TS, model = "MMN")
summary(v_ets_fit5)
```

```
## ETS(M,Md,N)
##
## Call:
##  ets(y = v_Mortality_data_TS, model = "MMN")
##
##   Smoothing parameters:
##     alpha = 0.4431
##     beta  = 0.0363
##     phi   = 0.8
##
##   Initial states:
##     l = 100.788
##     b = 0.9747
##
##   sigma:  0.0657
##
##      AIC     AICc      BIC
## 4957.923 4958.091 4983.306
##
## Training set error measures:
##                       ME     RMSE      MAE        MPE     MAPE      MASE
## Training set -0.06310061 5.884085 4.599372 -0.3702585 5.163763 0.6887639
##                     ACF1
## Training set -0.02809165
```

ETS(M, M, N)

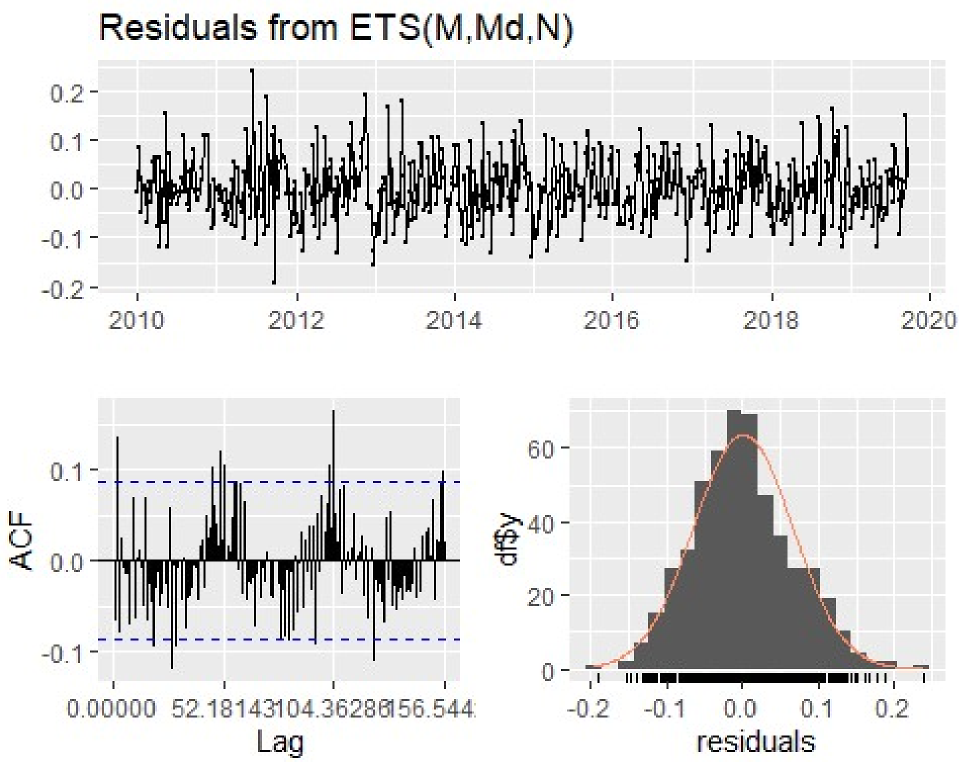
M - Multiplicative errors

M - Multiplicative trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit5)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(M,Md,N)
## Q* = 163.48, df = 97, p-value = 2.833e-05
##
## Model df: 5.   Total lags used: 102
```

Residual Analysis ETS(M, M, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Comparitively, based on AIC, BIC and MASE scores ets(M, M, N) is better.

Among all the methods, ses in simple smoothing.

Forecasting

Let us forecast the mortality rate for the next 4 weeks using the best model.

Forcasting with Smoothing method best model

```
fit <- ses(v_Mortality_data_TS, seasonal = "simple", h = 4)

v_mort_forecasts <- ts.intersect(ts(fit$lower[, 2], start = c(2020),
frequency = 356.27/7), ts(fit$mean, start = c(2020), frequency = 356.27/7),
ts(fit$upper[, 2], start = c(2020), frequency = 356.27/7))
colnames(v_mort_forecasts) <- c("Lower bound", "Point forecast", "Upper
bound")

v_mort_forecasts

## Time Series:
## Start = 2020
## End = 2020.05894405928
## Frequency = 50.8957142857143
##            Lower bound Point forecast Upper bound
## 2020.000    73.11391       84.66432    96.21472
## 2020.020    71.69251       84.66432    97.63612
## 2020.039    70.41218       84.66432    98.91645
## 2020.059    69.23774       84.66432   100.09089
```

Now let us plot the forecast.

```
plot(fit, fcol = "white", main = "Forecast of Mortality rate series for the
next 4 weeks", ylab = "Mortality rate")
lines(fitted(fit), col = "red")
lines(fit$mean, col = "blue", lwd = 2)
legend("top", inset = .03, cex = 0.9, box.lty = 2, box.lwd = 2, pch = 1, lty
= 1, col = c("red", "blue"), c("Data", "Forecasts"))
```

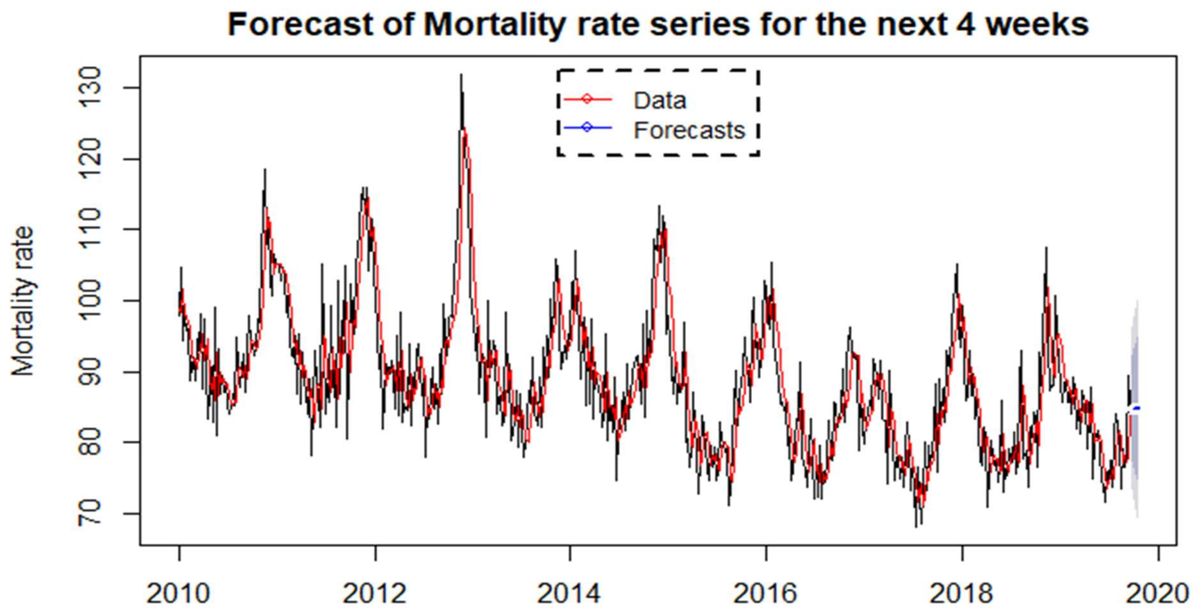


Fig 1.19: Next 4 weeks forecast on the Mortality rate Series with Smoothing method model.

From the four weeks forecast results we can predict that there will be decrease in the mortality rate in the future.

Forcasting with ets method best model

```
fit3 <- ets(v_Mortality_data_TS, model="MMN", damped = T)
fit <- forecast.ets(fit3, h = 4)

v_mort_forecasts <- ts.intersect(ts(fit$lower[, 2], start = c(2020),
frequency = 356.27/7), ts(fit$mean, start = c(2020), frequency = 356.27/7),
ts(fit$upper[, 2], start = c(2020), frequency = 356.27/7))
colnames(v_mort_forecasts) <- c("Lower bound", "Point forecast", "Upper
bound")

v_mort_forecasts

## Time Series:
## Start = 2020
## End = 2020.05894405928
## Frequency = 50.8957142857143
##          Lower bound Point forecast Upper bound
## 2020.000    73.22889       84.62952    95.77543
## 2020.020    73.36310       84.96962    97.26087
## 2020.039    72.18016       85.24268    99.06327
## 2020.059    71.36679       85.46177   100.76502
```

Now let us plot the forecast.

```
plot(fit, fcol = "white", main = "Forecast of Mortality rate series for the
next 4 weeks", ylab = "Mortality rate")
lines(fitted(fit), col = "red")
lines(fit$mean, col = "blue", lwd = 2)
legend("top", inset = .03, cex = 0.9, box.lty = 2, box.lwd = 2, pch = 1, lty
= 1, col = c("red", "blue"), c("Data", "Forecasts"))
```

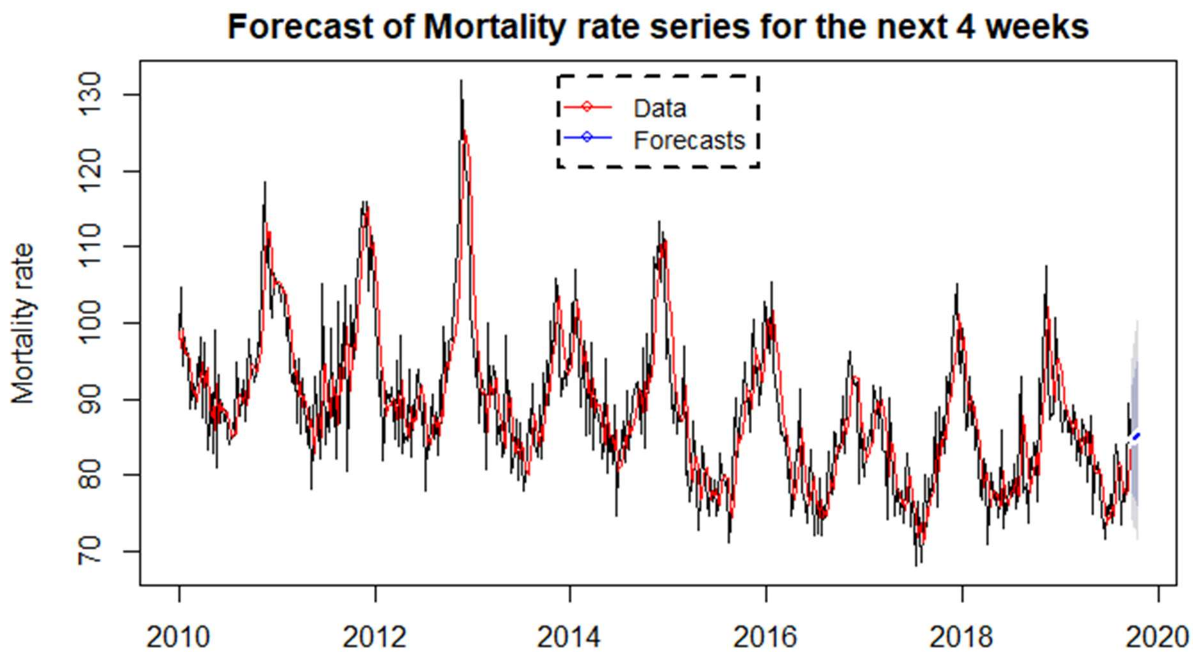


Fig 1.20: Next 4 weeks forecast on the Mortality rate Series with ets method model.

From the four weeks forecast results we can predict that there will be increase in the mortality rate in the future.

Task 2

Data

The data here used is the yearly averaged climate variables measured from 1984 – 2014 (31 years).

```
v_First_Flowering_Day_data <- read.csv("FFD.csv", header = TRUE)
head(v_First_Flowering_Day_data)

##   ï..Year Temperature Rainfall Radiation RelHumidity FFD
## 1    1984    9.371585 2.489344  14.87158    93.92650 217
## 2    1985    9.656164 2.475890  14.68493    94.93589 186
## 3    1986    9.273973 2.421370  14.51507    94.09507 233
## 4    1987    9.219178 2.319726  14.67397    94.49699 222
```

```
## 5    1988   10.202186 2.465301  14.74863    94.08142 214
## 6    1989    9.441096 2.735890  14.78356    96.08685 237
```

```
# Using str() to check the type of each column.
str(v_First_Flowering_Day_data)
```

```
## 'data.frame':    31 obs. of  6 variables:
##  $ ï..Year    : int  1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 ...
##  $ Temperature: num  9.37 9.66 9.27 9.22 10.2 ...
##  $ Rainfall   : num  2.49 2.48 2.42 2.32 2.47 ...
##  $ Radiation  : num  14.9 14.7 14.5 14.7 14.7 ...
##  $ RelHumidity: num  93.9 94.9 94.1 94.5 94.1 ...
##  $ FFD        : int  217 186 233 222 214 237 213 206 188 234 ...
```

Checking for Missing values.

```
colSums(is.na(v_First_Flowering_Day_data))
```

```
##     ï..Year Temperature    Rainfall   Radiation RelHumidity         FFD
##           0           0           0           0           0           0
```

There are no missing values in the data.

Checking the class of v_solar_data. (It should be a data frame.)

```
class(v_First_Flowering_Day_data)

## [1] "data.frame"

v_First_Flowering_Day_Temp_TS <- ts(v_First_Flowering_Day_data$Temperature,
start = 1984, frequency = 1)
v_First_Flowering_Day_Rainfall_TS <- ts(v_First_Flowering_Day_data$Rainfall,
start = 1984, frequency = 1)
v_First_Flowering_Day_Radiation_TS <-
ts(v_First_Flowering_Day_data$Radiation, start = 1984, frequency = 1)
v_First_Flowering_Day_RelHumidity_TS <-
ts(v_First_Flowering_Day_data$RelHumidity, start = 1984, frequency = 1)
v_First_Flowering_Day_data_TS <- ts(v_First_Flowering_Day_data$FFD, start =
1984, frequency = 1)
```

Confirming the class of each time series object.

```
class(v_First_Flowering_Day_Temp_TS)

## [1] "ts"

class(v_First_Flowering_Day_Rainfall_TS)

## [1] "ts"

class(v_First_Flowering_Day_Radiation_TS)

## [1] "ts"
```

```
class(v_First_Flowering_Day_RelHumidity_TS)

## [1] "ts"

class(v_First_Flowering_Day_data_TS)

## [1] "ts"
```

Now let us perform descriptive analysis on each time series object.

Descriptive Analysis

First Flowering Day

```
plot(v_First_Flowering_Day_data_TS, type = "b", xlab = "years", ylab = "First
Flowering Day", main = "Time series plot for yearly First Flowering Day data
from 1984 – 2014 (31 years)", pch = 1)
legend("topright", inset = .03, title = "First Flowering Day", legend =
"First Flowering Day series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2,
box.lwd = 2, pch = 1)
```

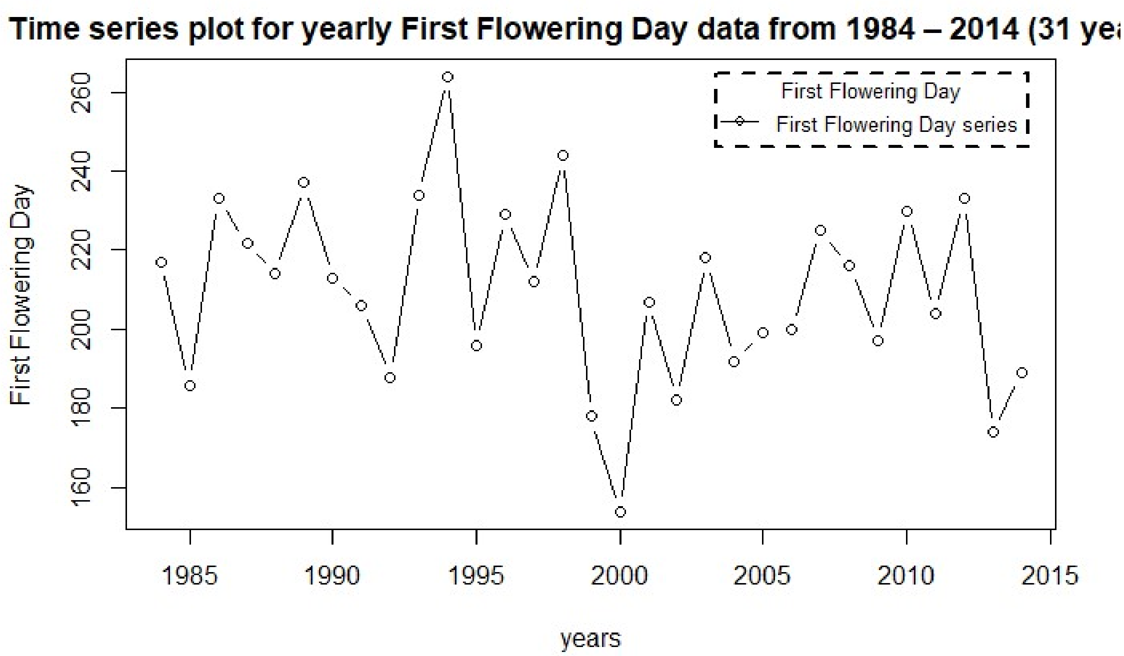


Fig 1.1: First Flowering Day - Time series plot.

```
McLeod.Li.test(y = v_First_Flowering_Day_data_TS, main = "McLeod-Li Test
Statistics for First Flowering Day.")
```

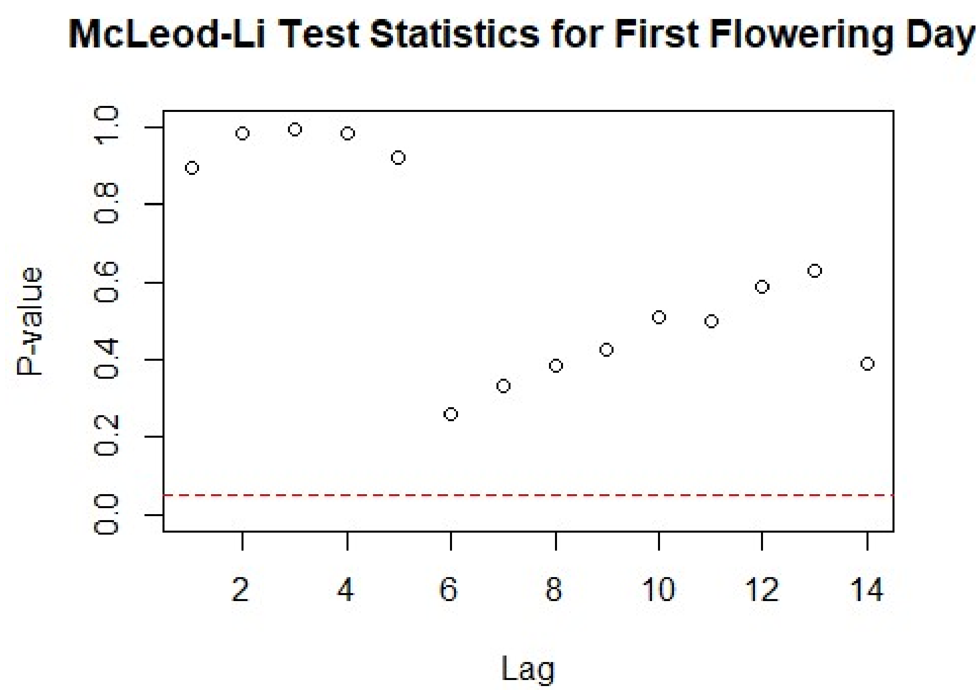


Fig 2.2: McLeod-Li Test Statistics for First Flowering Day.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around multiple years.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Temperature

```
plot(v_First_Flowering_Day_Temp_TS, type = "b", xlab = "years", ylab =
"Temperature", main = "Time series plot for yearly temperature from 1984 -
2014 (31 years)", pch = 1)
legend("top", inset = .03, title = "Temperature", legend = "Temperature
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

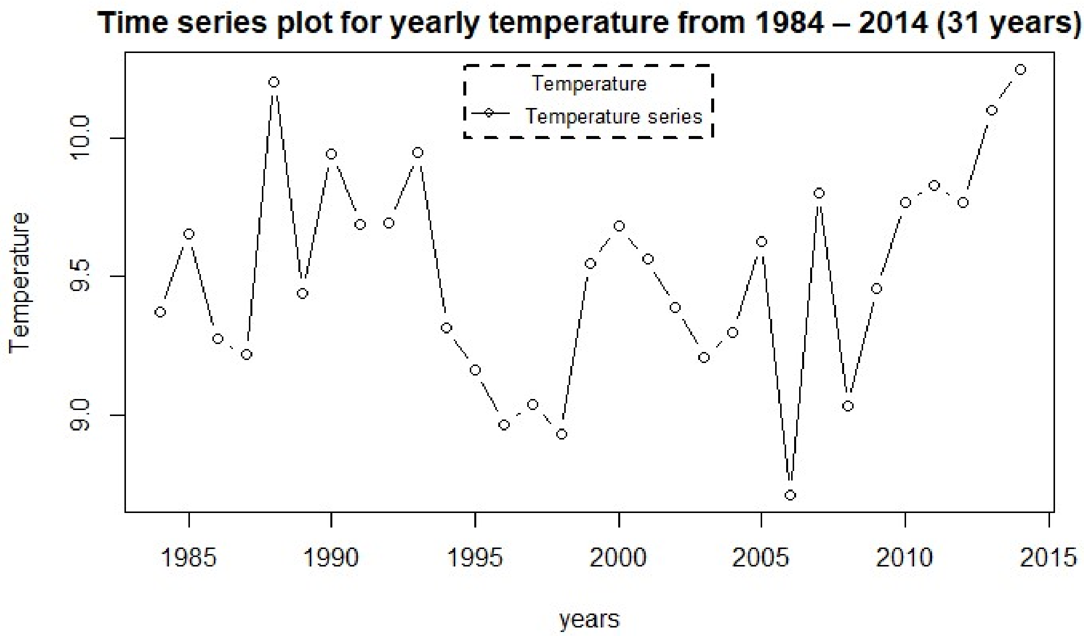


Fig 2.3: Temperature - Time series plot.

```
McLeod.Li.test(y = v_First_Flowering_Day_Temp_TS, main = "McLeod-Li Test
Statistics for Temperature")
```

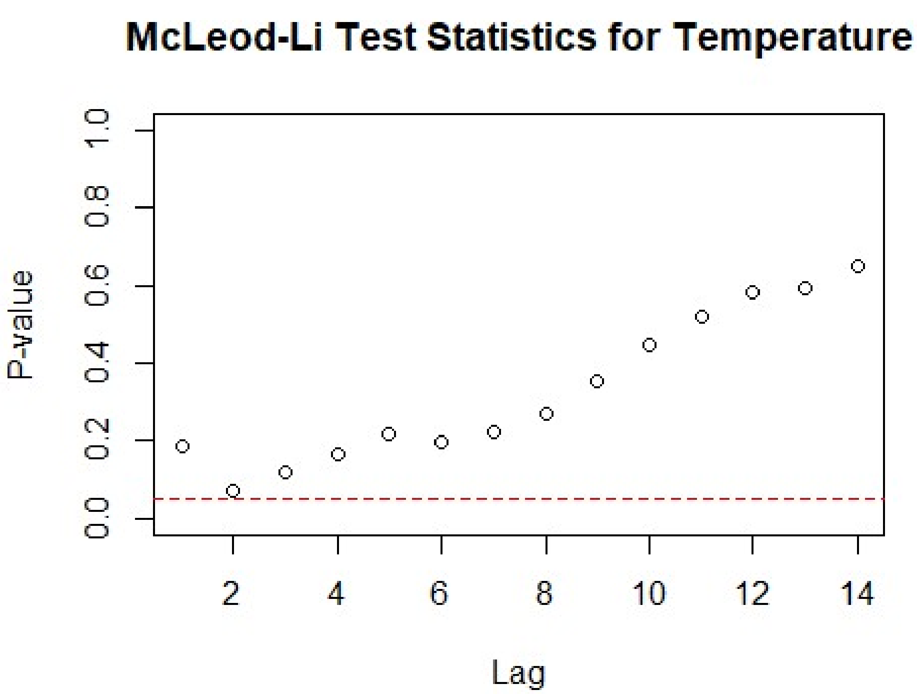


Fig 2.4: McLeod-Li Test Statistics for Temperature

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1996.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Rainfall

```
plot(v_First_Flowering_Day_Rainfall_TS, type = "b", xlab = "years", ylab =
"Rainfall", main = "Time series plot for yearly Rainfall from 1984 – 2014 (31
years)", pch = 1)
legend("bottomleft", inset = .03, title = "Rainfall", legend = "Rainfall
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

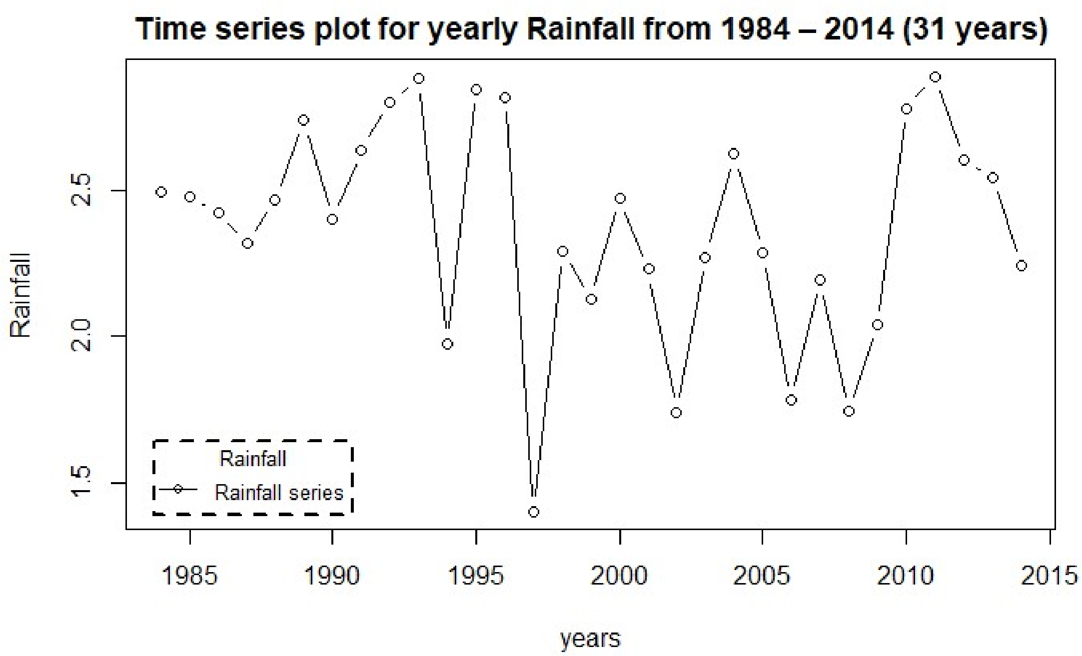


Fig 2.5: Rainfall - Time series plot.

```
McLeod.Li.test(y = v_First_Flowering_Day_Rainfall_TS, main = "McLeod-Li Test
Statistics for Rainfall")
```

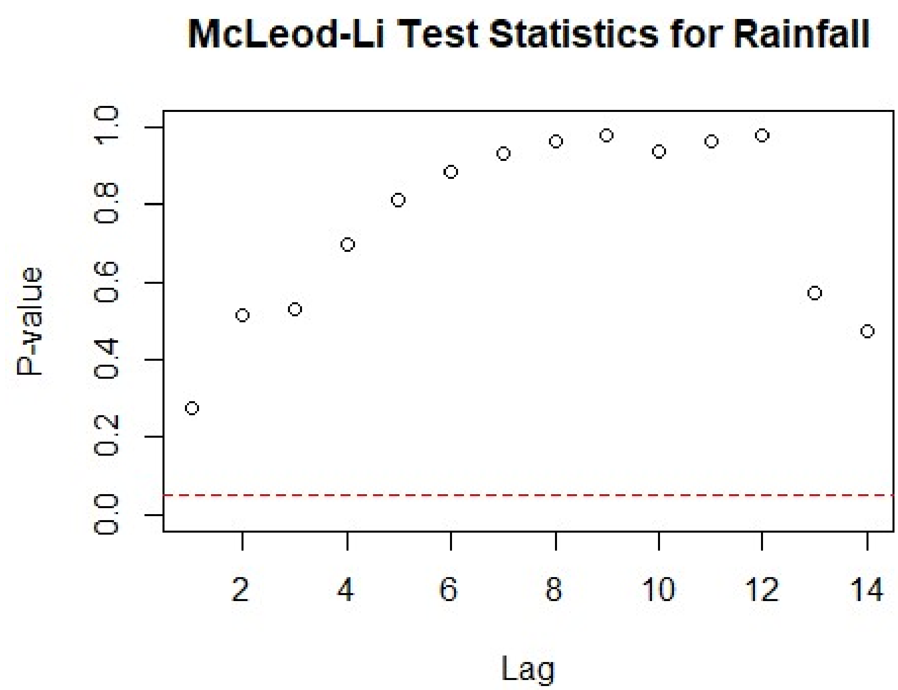


Fig 2.6: McLeod-Li Test Statistics for Rainfall

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1996.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Radiation

```
plot(v_First_Flowering_Day_Radiation_TS, type = "b", xlab = "years", ylab =
"Radiation", main = "Time series plot for yearly Radiation from 1984 – 2014
(31 years)", pch = 1)
legend("topleft", inset = .03, title = "Radiation", legend = "Radiation
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

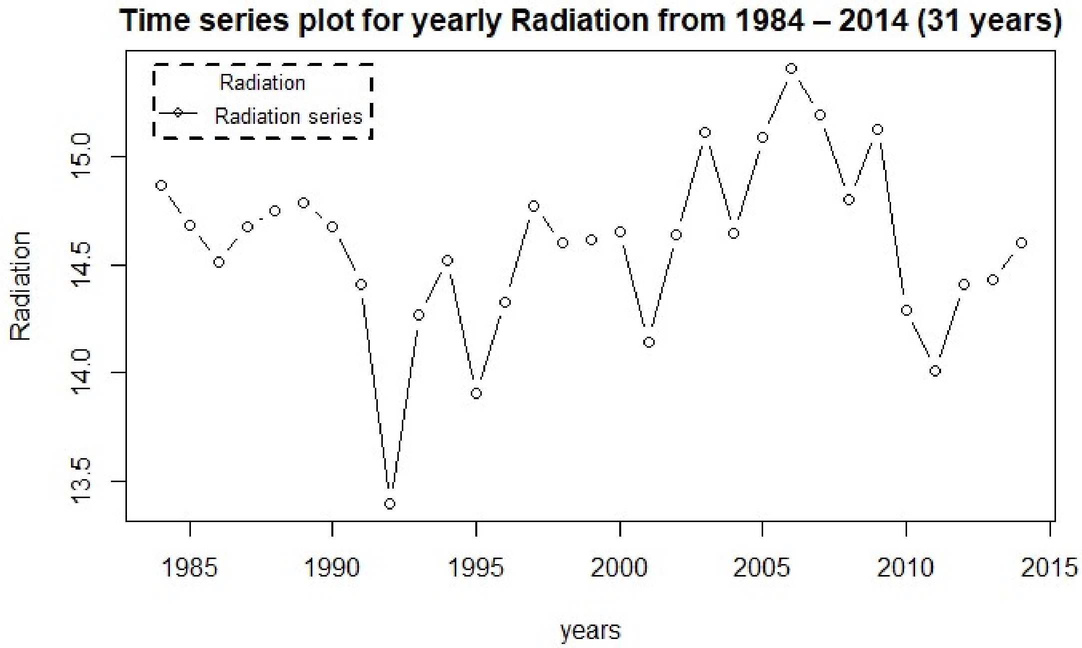


Fig 2.7: Solar radiation - Time series plot.

```
McLeod.Li.test(y = v_First_Flowering_Day_Radiation_TS, main = "McLeod-Li Test Statistics for Radiation.")
```

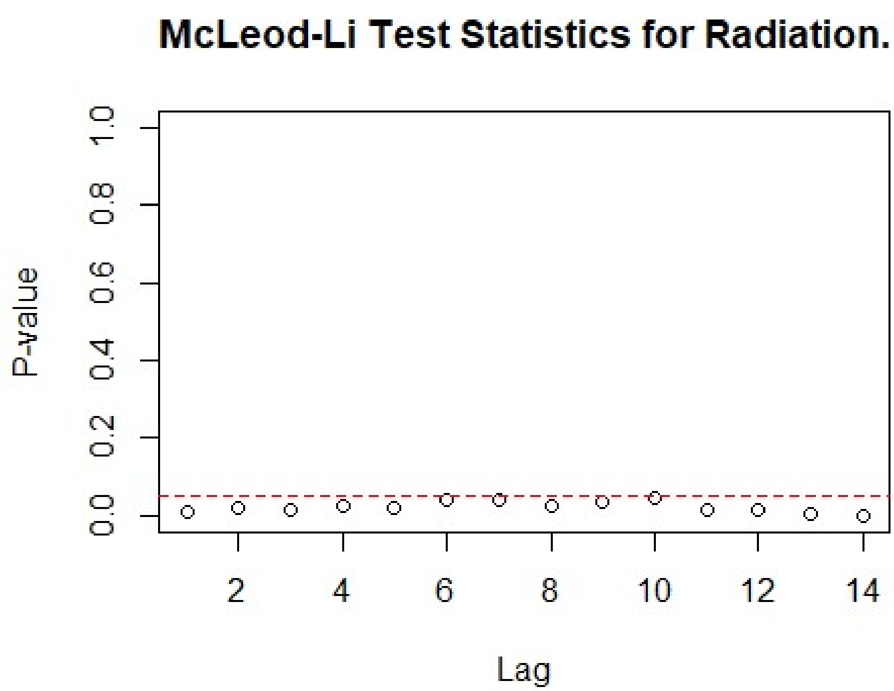


Fig 2.8: McLeod-Li Test Statistics for Radiation.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1992.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Relative Humidity

```
plot(v_First_Flowering_Day_RelHumidity_TS, type = "b", xlab = "years", ylab =
"Relative Humidity", main = "Time series plot for yearly Relative Humidity
from 1984 – 2014 (31 years)", pch = 1)
legend("bottomright", inset = .03, title = "Relative Humidity", legend =
"Relative Humidity series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2,
box.lwd = 2, pch = 1)
```

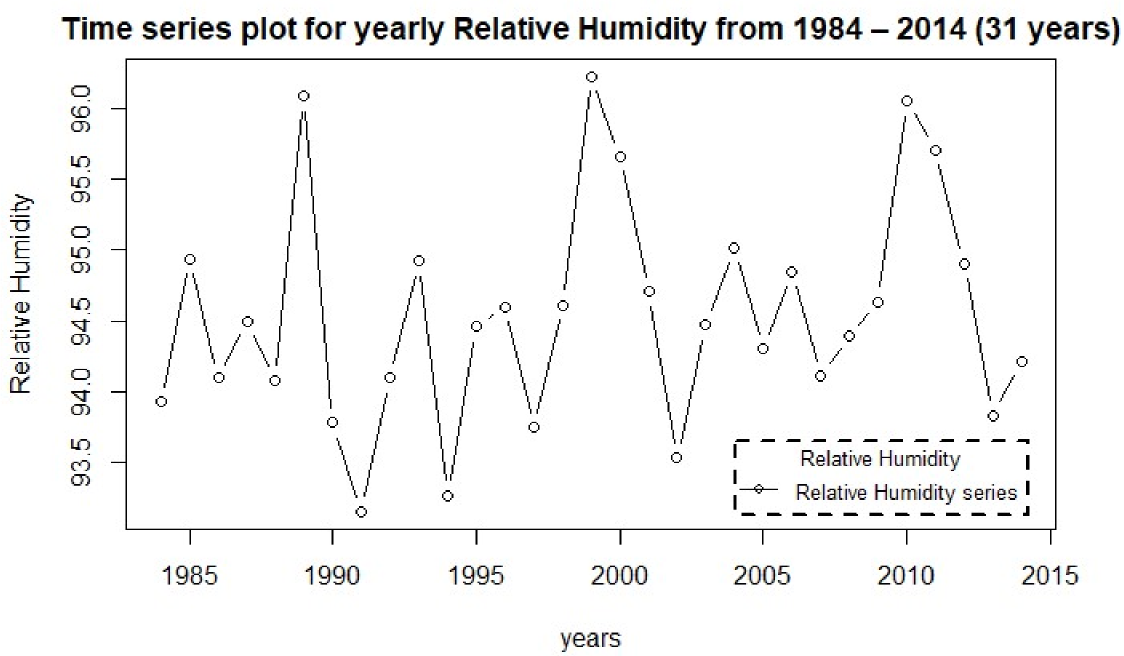


Fig 2.9: Relative Humidity - Time series plot.

```
McLeod.Li.test(y = v_First_Flowering_Day_RelHumidity_TS, main = "McLeod-Li
Test Statistics for Relative Humidity")
```

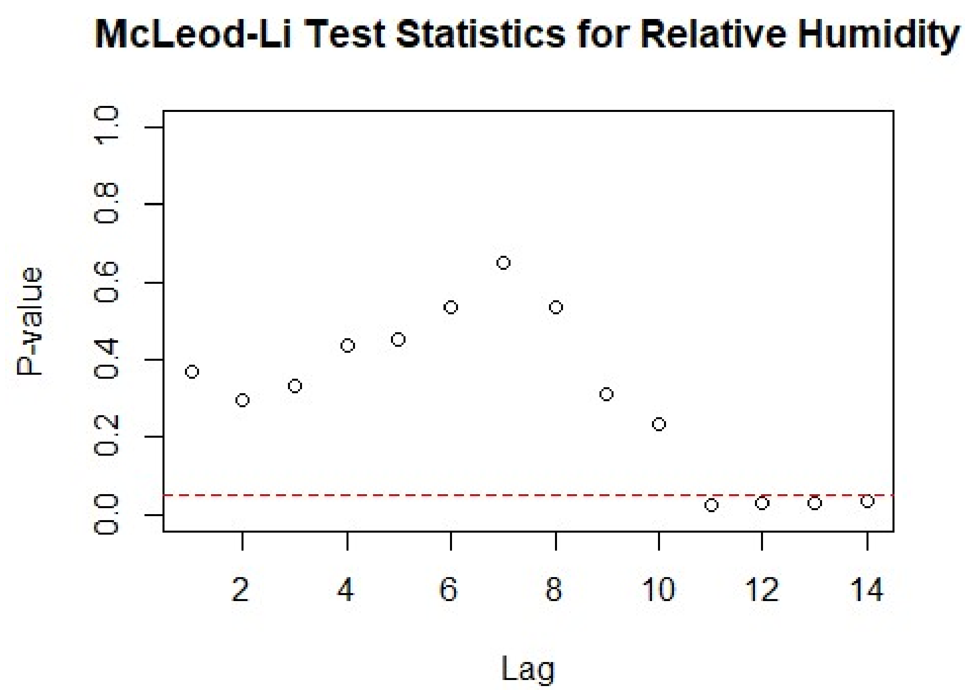


Fig 2.10: McLeod-Li Test Statistics for Relative Humidity.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1989.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Checking for Stationary in the series

Checking for Stationary on First Flowering Day series.

```
Stationary_Check(v_First_Flowering_Day_data_TS, "First Flowering Day - ACF plot", "First Flowering Day - PACF plot")
```

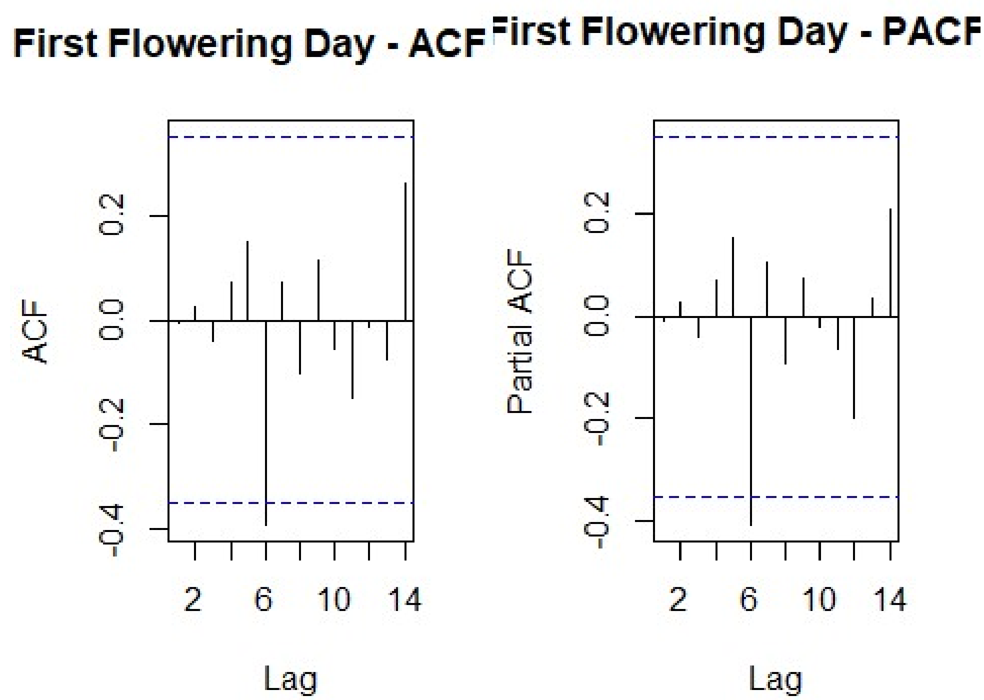


Fig 2.11: First Flowering Day - ACF

Fig 2.12: First Flowering Day - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -5.4552, Lag order = 0, p-value = 0.01
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the First Flowering Day series is Stationary.

Checking for Stationary on Temperature data.

```
Stationary_Check(v_First_Flowering_Day_Temp_TS, "Temperature - ACF plot",
"Temperature - PACF plot")
```

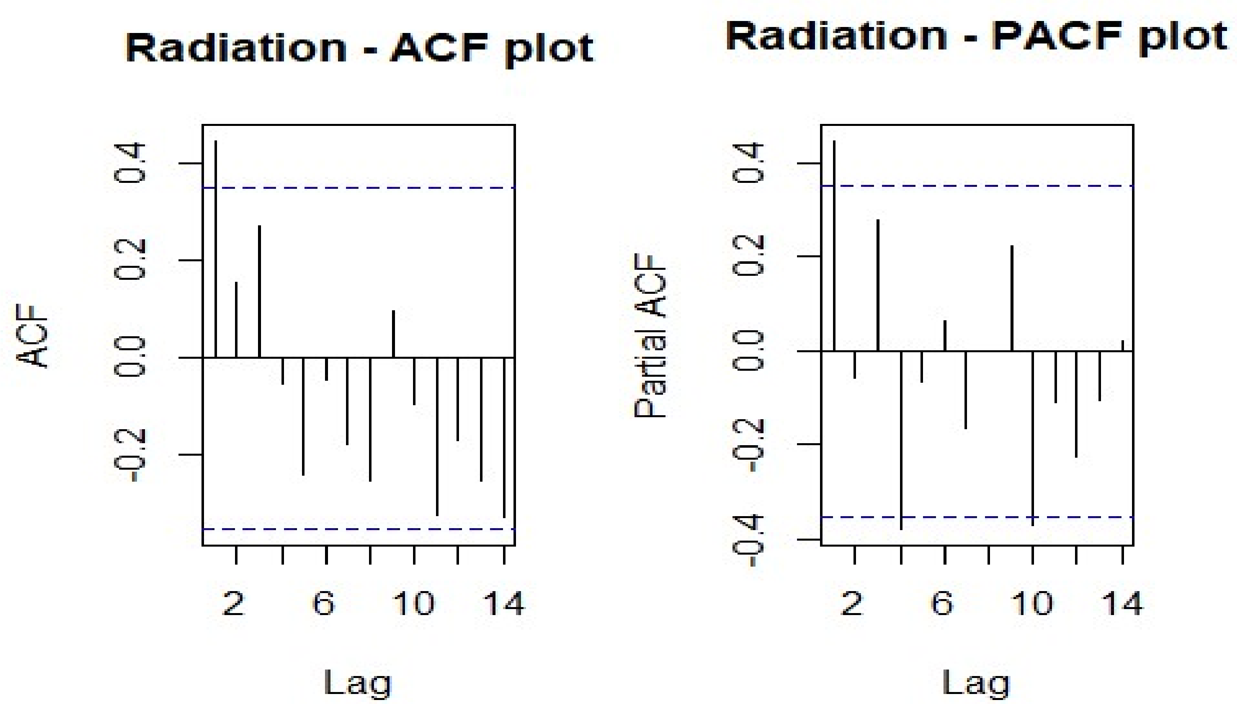


Fig 2.13: Temperature - ACF

Fig 2.14: Temperature - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -1.1484, Lag order = 2, p-value = 0.9002
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: 0.9002 > 0.05

p - value is greater than 0.05 and hence the test is not statistically significant. Therefore, Null hypothesis cannot be rejected i.e., The data is not stationary.

Therefore, the Temperature series is not Stationary.

Checking for Stationary on Radiation data.

```
Stationary_Check(v_First_Flowering_Day_Radiation_TS, "Radiation - ACF plot",
"Radiation - PACF plot")
```

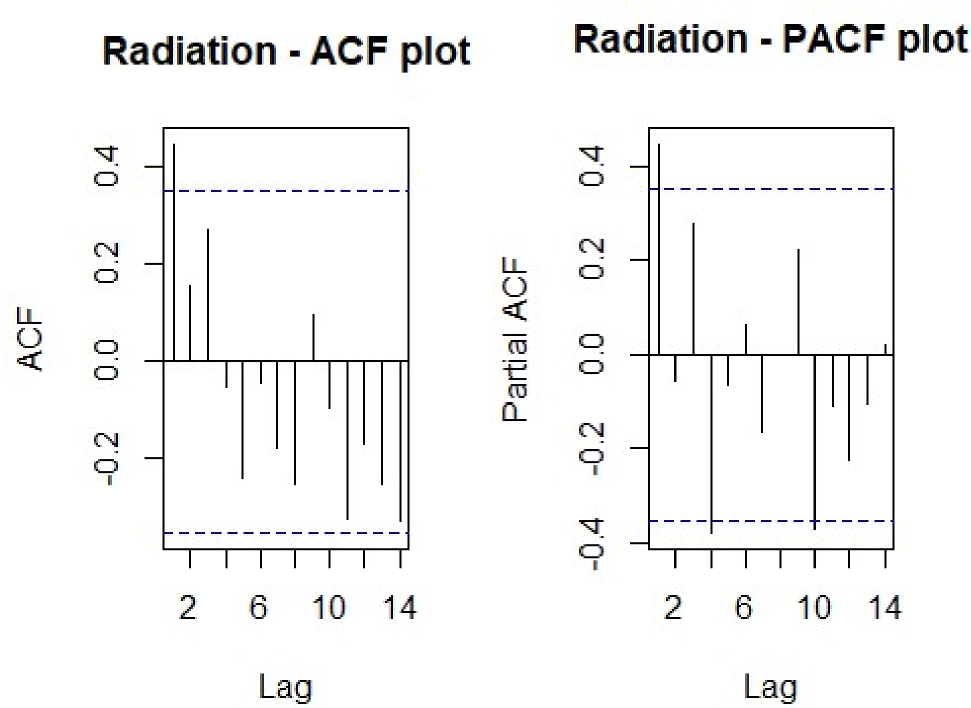


Fig 2.15: Radiation - ACF

Fig 2.16: Radiation - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -2.7317, Lag order = 4, p-value = 0.2911
## alternative hypothesis: stationary
```

The is only one significant lag in the ACF and PACF plot.

Hypotheses:

H0: The data is not stationary.

HA: The data is stationary.

Interpretations:

p - value: 0.2911 > 0.05

p - value is greater than 0.05 and hence the test is not statistically significant. Therefore, Null hypothesis cannot be rejected i.e., The data is not stationary.

Therefore, the Radiation series is not Stationary.

Checking for Stationary on Rainfall data.

```
Stationary_Check(v_First_Flowering_Day_Rainfall_TS, "Rainfall - ACF plot",
"Rainfall - PACF plot")
```

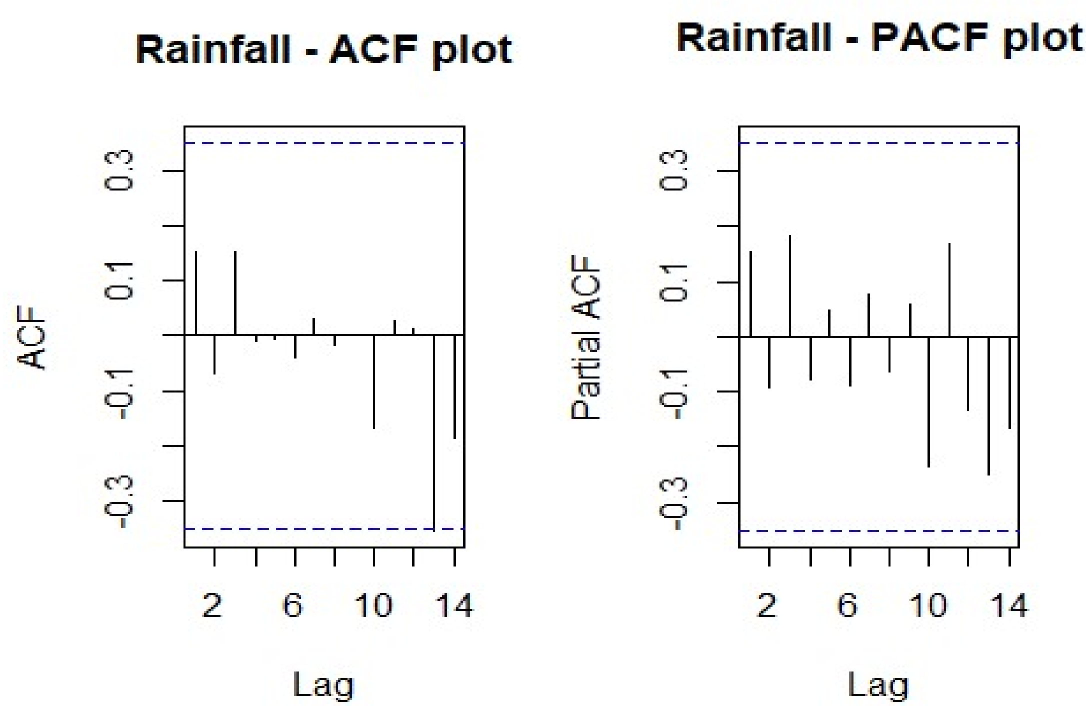


Fig 2.17: Rainfall - ACF

Fig 2.18: Rainfall - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -4.5622, Lag order = 0, p-value = 0.01
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H0: The data is not stationary.
HA: The data is stationary.

Interpretations:

p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Rainfall series is Stationary.

Checking for Stationary on Relative Humidity data.

```
Stationary_Check(v_First_Flowering_Day_RelHumidity_TS, "Relative Humidity - ACF plot", "Relative Humidity - PACF plot")
```

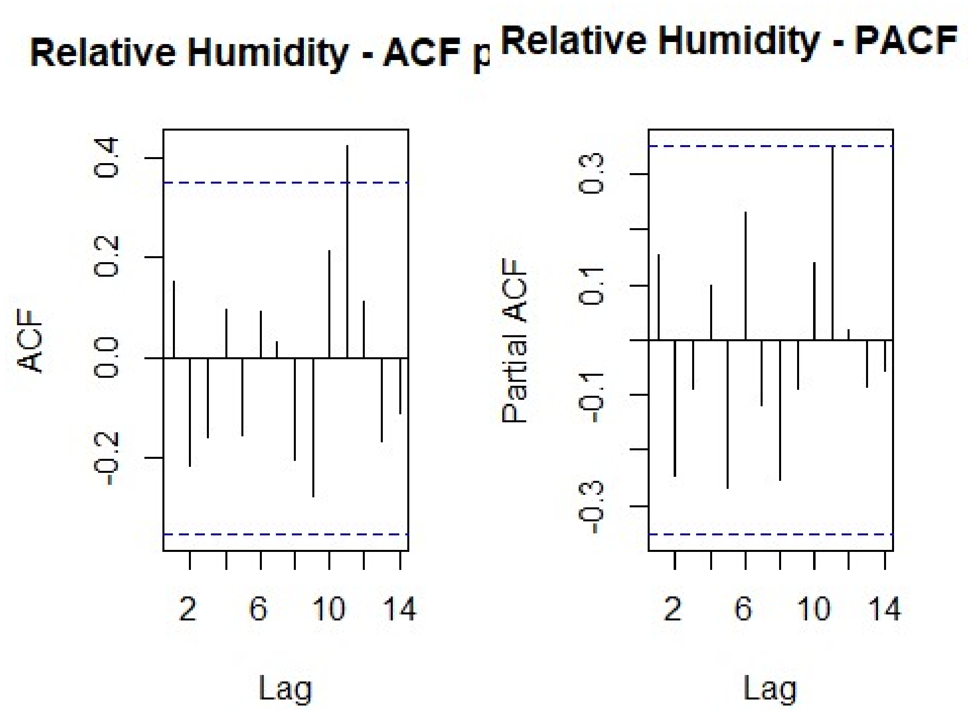


Fig 2.19: Relative Humidity - ACF

Fig 2.20: Relative Humidity - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -4.5749, Lag order = 0, p-value = 0.01
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H0: The data is not stationary.

HA: The data is stationary.

Interpretations:

p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Relative Humidity series is Stationary.

Suitable Distributed lag models (Univariate analysis).

Before this let us find the correlation between the two series.

```
# Calculating the correlation coefficient.
cor(v_First_Flowering_Day_data_TS, v_First_Flowering_Day_Temp_TS)

## [1] -0.2479337

cor(v_First_Flowering_Day_data_TS, v_First_Flowering_Day_Rainfall_TS)

## [1] 0.0506911

cor(v_First_Flowering_Day_data_TS, v_First_Flowering_Day_Radiation_TS)

## [1] 0.04677758

cor(v_First_Flowering_Day_data_TS, v_First_Flowering_Day_RelHumidity_TS)

## [1] -0.1285024
```

This suggests that FFD has a better correlation with Rainfall and Radiation.

As we are going to forecast the FFD data, our dependent variable “y” will be Mortality Rate series object and independent variable “x” will be Rainfall and Radiation.

As we need to check with and without intercept, let us convert the entire data set into time series.

```
v_data_TS_22 <- ts(v_First_Flowering_Day_data, start = 1984, frequency = 1)

cor(v_data_TS_22)

##                 ï..Year  Temperature   Rainfall   Radiation  RelHumidity
## ï..Year       1.0000000  0.148410676 -0.1752091  0.11881829  0.206355767
## Temperature   0.1484107  1.000000000  0.3933255 -0.24096625  0.009646021
## Rainfall     -0.1752091  0.393325545  1.0000000 -0.58131610  0.338461007
## Radiation     0.1188183 -0.240966245 -0.5813161  1.00000000 -0.055209652
## RelHumidity   0.2063558  0.009646021  0.3384610 -0.05520965  1.000000000
## FFD          -0.2329975 -0.247933708  0.0506911  0.04677758 -0.128502440
```

```
##                       FFD
## ï..Year     -0.23299747
## Temperature -0.24793371
## Rainfall     0.05069110
## Radiation    0.04677758
## RelHumidity -0.12850244
## FFD          1.00000000

colnames(v_data_TS_22) <- c("x1", "x2", "x3", "x4", "x5", "y")
```

Finite distributed lag model with Rainfall

With slope.

```
for ( i in 1:10){
  model_1 = dlm(formula=y ~ x3, data=data.frame(v_data_TS_22), q = i )
  cat("q = ", i, "AIC = ", AIC(model_1$model), "BIC = ", BIC(model_1$model),
"MASE =", MASE(model_1)$MASE, "\n")
  }

## q =  1 AIC =  281.9769 BIC =  287.5817 MASE = 0.6758928
## q =  2 AIC =  273.8231 BIC =  280.6596 MASE = 0.6752737
## q =  3 AIC =  265.1863 BIC =  273.1795 MASE = 0.6326564
## q =  4 AIC =  257.086 BIC =  266.1568 MASE = 0.6309338
## q =  5 AIC =  246.1244 BIC =  256.1891 MASE = 0.5808908
## q =  6 AIC =  238.0915 BIC =  249.0614 MASE = 0.5522086
## q =  7 AIC =  231.4148 BIC =  243.1954 MASE = 0.5447815
## q =  8 AIC =  224.8048 BIC =  237.2953 MASE = 0.5553913
## q =  9 AIC =  216.4839 BIC =  229.5764 MASE = 0.5521884
## q =  10 AIC =  197.7206 BIC =  211.2994 MASE = 0.391709
```

As we have the least AIC, BIC and MASE values at q = 10. Let us fit the finite distributed lag model with q = 10.

```
# Finite lag length based on AIC, BIC and MASE

finite_dlm_FFD_slope = dlm(formula = y ~ x3, data=data.frame(v_data_TS_22), q
= 10)
summary(finite_dlm_FFD_slope)

##
## Call:
## lm(formula = as.formula(model.formula), data = design)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -25.4869 -13.6970   0.5488   9.6595  30.6423
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  260.256     89.788   2.899   0.0176 *
```

```
## x3.t          -17.281     13.476  -1.282   0.2317
## x3.1           15.520     12.336   1.258   0.2400
## x3.2          -10.633     12.733  -0.835   0.4253
## x3.3           32.450     12.574   2.581   0.0297 *
## x3.4          -32.881     13.959  -2.356   0.0429 *
## x3.5           40.010     13.844   2.890   0.0179 *
## x3.6           -5.032     13.415  -0.375   0.7163
## x3.7           24.216     14.836   1.632   0.1371
## x3.8          -20.585     14.164  -1.453   0.1801
## x3.9           -8.781     15.032  -0.584   0.5735
## x3.10         -38.573     15.741  -2.450   0.0367 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 22.05 on 9 degrees of freedom
## Multiple R-squared:  0.6663, Adjusted R-squared:  0.2585
## F-statistic: 1.634 on 11 and 9 DF,  p-value: 0.2351
##
## AIC and BIC values for the model:
##        AIC      BIC
## 1 197.7206 211.2994
```

Hypotheses:

H_0: The data doesn't fit the Finite distributed lag model with slope.
H_A: The data fits the Finite distributed lag model slope.

Interpretations:

F - statistic is 1.634
R - squared is 0.6663
Adjusted R - squared is 0.2585
Degrees of freedom - DF are (11, 9)
p - value (0.2351) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model does not fit the Finite distributed lag model with slope.

No residual analysis is required.

Therefore, Further analysis is needed by removing slope to the lag model.

With out slope.

```
for ( i in 1:10){
  model_2 = dlm(formula=y ~ 0 + x3, data=data.frame(v_data_TS_22), q = i )
  cat("q = ", i, "AIC = ", AIC(model_2$model), "BIC = ", BIC(model_2$model),
"MASE =", MASE(model_2)$MASE, "\n")
  }

## q =  1 AIC =  300.1422 BIC =  304.3458 MASE = 0.9477487
## q =  2 AIC =  283.7963 BIC =  289.2655 MASE = 0.8145653
```

```
## q =  3 AIC =  271.2089 BIC =  277.8699 MASE = 0.6998298
## q =  4 AIC =  264.7646 BIC =  272.5397 MASE = 0.7023612
## q =  5 AIC =  249.1985 BIC =  258.0052 MASE = 0.6495448
## q =  6 AIC =  242.314 BIC =  252.065 MASE = 0.6502001
## q =  7 AIC =  233.9019 BIC =  244.5044 MASE = 0.6214299
## q =  8 AIC =  227.4548 BIC =  238.8097 MASE = 0.6177108
## q =  9 AIC =  218.2618 BIC =  230.2633 MASE = 0.6063767
## q =  10 AIC =  209.5666 BIC =  222.1008 MASE = 0.5785768
```

As we have the least AIC, BIC and MASE values at q = 10. Let us fit the finite distributed lag model with q = 10.

```
# Finite lag length based on AIC, BIC and MASE

finite_dlm_FFD_noslope = dlm(formula=y ~ 0 + x3,
data=data.frame(v_data_TS_22), q = 10)
summary(finite_dlm_FFD_noslope)

##
## Call:
## lm(formula = as.formula(model.formula), data = design)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -38.59 -21.00   3.99  17.77  28.44
##
## Coefficients:
##       Estimate Std. Error t value Pr(>|t|)
## x3.t     1.629     15.554   0.105   0.9187
## x3.1    23.569     15.856   1.486   0.1680
## x3.2     2.783     15.647   0.178   0.8624
## x3.3    35.931     16.511   2.176   0.0546 .
## x3.4   -22.958     17.851  -1.286   0.2274
## x3.5    40.349     18.261   2.210   0.0516 .
## x3.6     2.049     17.401   0.118   0.9086
## x3.7    29.584     19.418   1.524   0.1586
## x3.8    -4.658     17.221  -0.270   0.7923
## x3.9     2.018     19.211   0.105   0.9184
## x3.10  -20.819     19.128  -1.088   0.3020
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 29.09 on 10 degrees of freedom
## Multiple R-squared:  0.9907, Adjusted R-squared:  0.9805
## F-statistic: 97.01 on 11 and 10 DF,  p-value: 1.202e-08
##
## AIC and BIC values for the model:
##        AIC      BIC
## 1 209.5666 222.1008
```

Hypotheses:

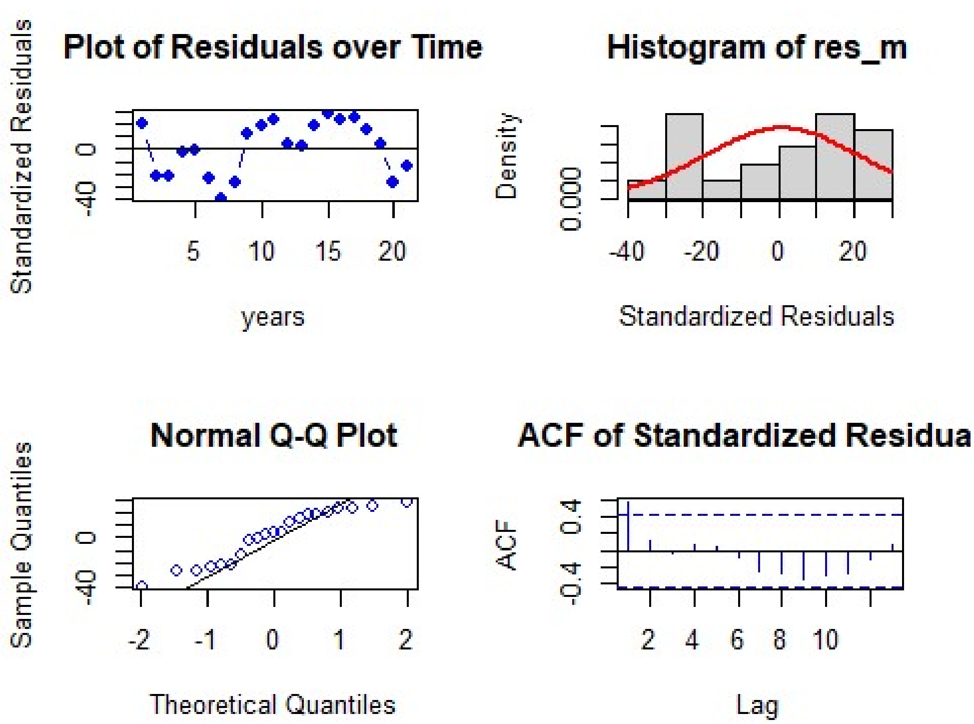
H0: The data doesn′t fit the Finite distributed lag model without slope.
HA: The data fits the Finite distributed lag model with out slepe.

Interpretations:

F - statistic is 97.01
R - squared is 0.9907
Adjusted R - squared is 0.9805
Degrees of freedom - DF are (11, 10)
p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Finite distributed lag model without slope.
This model suggests that there is only 98.05% of data variance. Suggesting that the model explains only 98.05% of the trend. Which implies that the model shows some trend.

Now let us perform residual analysis.

```
res_analysis(residuals(finite_dlm_FFD_noslope$model))
```




```
##
##  Shapiro-Wilk normality test
##
```

```
## data:  res_m
## W = 0.92077, p-value = 0.0899
```

Residual Analysis for Finite DLM:

1. The data points are above the line at the start and below the line at the end of the trend. Randomness is seen in the data. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is not symmetric.
3. QQplot also suggests that there is no normality in the trend.
4. There is only one significant lagin Autocorrelation plot.
5. p - value (0.0899) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

Even though the model fits better it is poor when it comes to residual analysis. Therefore, Further analysis is needed by adding polynomial to the lag model.

```
y = v_First_Flowering_Day_data_TS # Independent variable
x1 = v_First_Flowering_Day_Rainfall_TS # Dependent variable
x2 = v_First_Flowering_Day_Radiation_TS # Dependent variable
```

Polynomial distributed lag model with Rainfall

```
for (i in 1:3){
  model_3 <-  polyDlm(x = as.vector(x1), y = as.vector(y), q = i , k = i,
show.beta = FALSE)
  cat("q = ", i, "k = ", i, "AIC = ", AIC(model_3$model), "BIC = ",
BIC(model_3$model), MASE(model_3)$MASE, "\n")
}

## q =  1 k =  1 AIC =  281.9769 BIC =  287.5817 0.6758928
## q =  2 k =  2 AIC =  273.8231 BIC =  280.6596 0.6752737
## q =  3 k =  3 AIC =  265.1863 BIC =  273.1795 0.6326564
```

Let us fit a polynomial model of order 3. Since least AIC and BIC scores.

```
# Ploynomial DLM

PolyDLM_model_FFD = polyDlm(x = as.vector(x1), y = as.vector(y), q = 3, k =
3, show.beta = TRUE)

## Estimates and t-tests for beta coefficients:
##        Estimate Std. Error t value P(>|t|)
## beta.0    0.784       12.5  0.0629   0.950
## beta.1    4.950       12.5  0.3970   0.695
## beta.2    7.740       12.5  0.6190   0.541
## beta.3   14.900       12.6  1.1800   0.248

summary(PolyDLM_model_FFD)
```

```
##
## Call:
## "Y ~ (Intercept) + X.t"
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -46.70 -13.09   2.31  14.28  45.17
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 142.2479    51.4936   2.762   0.0111 *
## z.t0          0.7836    12.4508   0.063   0.9504
## z.t1          6.7375    55.3454   0.122   0.9042
## z.t2         -3.5211    49.2855  -0.071   0.9437
## z.t3          0.9461    10.8799   0.087   0.9315
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 24.55 on 23 degrees of freedom
## Multiple R-squared:  0.09312,    Adjusted R-squared:  -0.0646
## F-statistic: 0.5904 on 4 and 23 DF,  p-value: 0.673
```

Hypotheses:

H_0: The data doesn't fit the Polynomial distributed lag model.

H_A: The data fits the Polynomial distributed lag model.

Interpretations:

F - statistic is 0.5904

R - squared is 0.09312

Adjusted R - squared is -0.0646

Degrees of freedom - DF are (4, 23)

p - value (0.673) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Polynomial distributed lag model.

Also, this model suggests that there is only -67.3% of data variance.

No residual analysis is required.

Let us fit Koyck model.

Koyck model with Rainfall

```
# Koyk DLM

Koyck_DLM_FFD = koyckDlm(x = as.vector(x1) , y = as.vector(y))
summary(Koyck_DLM_FFD)

##
## Call:
## "Y ~ (Intercept) + Y.1 + X.t"
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -58.691 -21.222   2.697  14.856  68.192
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.266e+02  2.185e+02   0.579    0.567
## Y.1         4.591e-03  2.196e-01   0.021    0.983
## X.t         3.448e+01  8.772e+01   0.393    0.697
##
## Residual standard error: 27.55 on 27 degrees of freedom
## Multiple R-Squared: -0.2505, Adjusted R-squared: -0.3431
## Wald test: 0.07773 on 2 and 27 DF,  p-value: 0.9254
##
## Diagnostic tests:
## NULL
##
##                              alpha     beta         phi
## Geometric coefficients:  127.2254 34.48095 0.004590669
```

Hypotheses:

H0: The data doesn't fit the Polynomial distributed lag model.
HA: The data fits the Polynomial distributed lag model.

Interpretations:

Wald test statistic is 0.07773
R - squared is -0.2505
Adjusted R - squared is -0.3431
Degrees of freedom - DF are (2, 27)
p - value (0.9254) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Koyck distributed lag model.
Also, this model suggests that there is only -34.31% of data variance.

No residual analysis is required.

Let us fit ardlDlm model to check whether it fits better or not.

Autoregressive distributed lag model with Rainfall

with slope

```
for (i in 1:5){
  for(j in 1:5){
    model_4 = ardlDlm(formula=y ~ x3, data=data.frame(v_data_TS_22), p = i ,
q = j )
    cat("p = ", i, "q = ", j, "AIC = ", AIC(model_4$model), "BIC = ",
BIC(model_4$model), "MASE =", MASE(model_4)$MASE, "\n")
```

```
  }
}

## p =  1 q =  1 AIC =  283.9744 BIC =  290.9804 MASE = 0.6760282
## p =  1 q =  2 AIC =  276.7348 BIC =  284.9385 MASE = 0.6838177
## p =  1 q =  3 AIC =  269.3361 BIC =  278.6615 MASE = 0.6676193
## p =  1 q =  4 AIC =  262.6875 BIC =  273.0542 MASE = 0.6489023
## p =  1 q =  5 AIC =  255.5705 BIC =  266.8934 MASE = 0.6396905
## p =  2 q =  1 AIC =  275.8163 BIC =  284.0201 MASE = 0.6749962
## p =  2 q =  2 AIC =  277.8084 BIC =  287.3795 MASE = 0.6762326
## p =  2 q =  3 AIC =  270.6698 BIC =  281.3274 MASE = 0.6621822
## p =  2 q =  4 AIC =  264.2889 BIC =  275.9514 MASE = 0.6506873
## p =  2 q =  5 AIC =  257.2016 BIC =  269.7826 MASE = 0.641259
## p =  3 q =  1 AIC =  267.1614 BIC =  276.4868 MASE = 0.6355497
## p =  3 q =  2 AIC =  269.1262 BIC =  279.7838 MASE = 0.639182
## p =  3 q =  3 AIC =  270.8152 BIC =  282.8051 MASE = 0.6371823
## p =  3 q =  4 AIC =  264.4739 BIC =  277.4323 MASE = 0.6246495
## p =  3 q =  5 AIC =  258.2483 BIC =  272.0874 MASE = 0.6369725
## p =  4 q =  1 AIC =  259.0485 BIC =  269.4152 MASE = 0.6272275
## p =  4 q =  2 AIC =  260.686 BIC =  272.3485 MASE = 0.6099768
## p =  4 q =  3 AIC =  262.1327 BIC =  275.091 MASE = 0.601955
## p =  4 q =  4 AIC =  263.7563 BIC =  278.0105 MASE = 0.6017331
## p =  4 q =  5 AIC =  257.7752 BIC =  272.8724 MASE = 0.6175106
## p =  5 q =  1 AIC =  247.6247 BIC =  258.9476 MASE = 0.5835814
## p =  5 q =  2 AIC =  249.3037 BIC =  261.8846 MASE = 0.5755967
## p =  5 q =  3 AIC =  247.4178 BIC =  261.2568 MASE = 0.5484
## p =  5 q =  4 AIC =  248.4355 BIC =  263.5326 MASE = 0.5256786
## p =  5 q =  5 AIC =  250.0315 BIC =  266.3868 MASE = 0.5306522
```

(p, q) = (5, 4) has the least AIC, BIC and MASE scores.

```
# ARDLM model
AR_DLM_FFD_54_slope = ardlDlm(formula = y ~ x3,
data=data.frame(v_data_TS_22), p = 5 , q = 4)
summary(AR_DLM_FFD_54_slope)

##
## Time series regression with "ts" data:
## Start = 6, End = 31
##
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -27.12 -15.50  -0.65  13.13  35.40
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 120.7626    87.1334   1.386   0.1860
```

```
## x3.t           1.4092    15.0160   0.094   0.9265  
## x3.1          24.4971    15.7233   1.558   0.1401  
## x3.2         -13.2575    16.1170  -0.823   0.4236  
## x3.3          27.1489    13.2278   2.052   0.0580 .
## x3.4         -23.8080    15.4248  -1.543   0.1435  
## x3.5          41.6349    16.3506   2.546   0.0224 *
## y.1            0.2037     0.2245   0.907   0.3785  
## y.2           -0.1554     0.3001  -0.518   0.6120  
## y.3           -0.4561     0.2707  -1.685   0.1127  
## y.4            0.1863     0.2452   0.760   0.4591  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 23.86 on 15 degrees of freedom
## Multiple R-squared:  0.4339, Adjusted R-squared:  0.05646 
## F-statistic:  1.15 on 10 and 15 DF,  p-value: 0.3911
```

Hypotheses:

H0: The data doesn't fit the Autoregressive distributed lag model with slope.
HA: The data fits the Autoregressive distributed lag model with slope.

Interpretations:

F - statistic is 1.15
R - squared is 0.4339
Adjusted R - squared is 0.05646
Degrees of freedom - DF are (2, 27)
p - value (0.3911) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Autoregressive distributed lag model with slope.
Also, this model suggests that there is only 5% of data variance.

No residual analysis is required.

With out slope.

```
for (i in 1:5){
  for(j in 1:5){
    model_5 = ardlDlm(formula=y ~ 0 + x3, data=data.frame(v_data_TS_22), p = 
i , q = j )
    cat("p = ", i, "q = ", j, "AIC = ", AIC(model_5$model), "BIC = ", 
BIC(model_5$model), "MASE =", MASE(model_5)$MASE, "\n")
  }
}
```

```
## p =  1 q =  1 AIC =  283.9744 BIC =  290.9804 MASE = 0.6760282 
## p =  1 q =  2 AIC =  276.7348 BIC =  284.9385 MASE = 0.6838177 
## p =  1 q =  3 AIC =  269.3361 BIC =  278.6615 MASE = 0.6676193 
## p =  1 q =  4 AIC =  262.6875 BIC =  273.0542 MASE = 0.6489023 
## p =  1 q =  5 AIC =  255.5705 BIC =  266.8934 MASE = 0.6396905
```

```
## p =  2 q =  1 AIC =  275.8163 BIC =  284.0201 MASE = 0.6749962
## p =  2 q =  2 AIC =  277.8084 BIC =  287.3795 MASE = 0.6762326
## p =  2 q =  3 AIC =  270.6698 BIC =  281.3274 MASE = 0.6621822
## p =  2 q =  4 AIC =  264.2889 BIC =  275.9514 MASE = 0.6506873
## p =  2 q =  5 AIC =  257.2016 BIC =  269.7826 MASE = 0.641259
## p =  3 q =  1 AIC =  267.1614 BIC =  276.4868 MASE = 0.6355497
## p =  3 q =  2 AIC =  269.1262 BIC =  279.7838 MASE = 0.639182
## p =  3 q =  3 AIC =  270.8152 BIC =  282.8051 MASE = 0.6371823
## p =  3 q =  4 AIC =  264.4739 BIC =  277.4323 MASE = 0.6246495
## p =  3 q =  5 AIC =  258.2483 BIC =  272.0874 MASE = 0.6369725
## p =  4 q =  1 AIC =  259.0485 BIC =  269.4152 MASE = 0.6272275
## p =  4 q =  2 AIC =  260.686 BIC =  272.3485 MASE = 0.6099768
## p =  4 q =  3 AIC =  262.1327 BIC =  275.091 MASE = 0.601955
## p =  4 q =  4 AIC =  263.7563 BIC =  278.0105 MASE = 0.6017331
## p =  4 q =  5 AIC =  257.7752 BIC =  272.8724 MASE = 0.6175106
## p =  5 q =  1 AIC =  247.6247 BIC =  258.9476 MASE = 0.5835814
## p =  5 q =  2 AIC =  249.3037 BIC =  261.8846 MASE = 0.5755967
## p =  5 q =  3 AIC =  247.4178 BIC =  261.2568 MASE = 0.5484
## p =  5 q =  4 AIC =  248.4355 BIC =  263.5326 MASE = 0.5256786
## p =  5 q =  5 AIC =  250.0315 BIC =  266.3868 MASE = 0.5306522
```

(p, q) = (5, 4) has the least AIC, BIC and MASE scores.

```
# Finite lag length based on AIC-BIC

AR_DLM_FFD_54_noslope = ardlDlm(formula=y ~ 0 + x3,
data=data.frame(v_data_TS_22), p = 5 , q = 4)
summary(AR_DLM_FFD_54_noslope)

##
## Time series regression with "ts" data:
## Start = 6, End = 31
##
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -27.12 -15.50  -0.65  13.13  35.40
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 120.7626    87.1334   1.386   0.1860
## x3.t          1.4092    15.0160   0.094   0.9265
## x3.1         24.4971    15.7233   1.558   0.1401
## x3.2        -13.2575    16.1170  -0.823   0.4236
## x3.3         27.1489    13.2278   2.052   0.0580 .
## x3.4        -23.8080    15.4248  -1.543   0.1435
## x3.5         41.6349    16.3506   2.546   0.0224 *
## y.1           0.2037     0.2245   0.907   0.3785
```

```
## y.2          -0.1554     0.3001  -0.518   0.6120
## y.3          -0.4561     0.2707  -1.685   0.1127
## y.4           0.1863     0.2452   0.760   0.4591
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 23.86 on 15 degrees of freedom
## Multiple R-squared:  0.4339, Adjusted R-squared:  0.05646
## F-statistic:  1.15 on 10 and 15 DF,  p-value: 0.3911
```

Hypotheses:

H_0: The data doesn't fit the Autoregressive distributed lag model without slope.
H_A: The data fits the Autoregressive distributed lag model without slope.

Interpretations:

F - statistic is 1.15
R - squared is 0.4339
Adjusted R - squared is 0.05646
Degrees of freedom - DF are (10, 15)
p - value (0.3911) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Autoregressive distributed lag model without slope.
Also, this model suggests that there is only 5.64% of data variance.

No residual analysis is required.

Therefore let us fit models with Radiation.

Finite distributed lag model with Radiation

With slope.

```
for ( i in 1:10){
  model_1 = dlm(formula=y ~ x4, data=data.frame(v_data_TS_22), q = i )
  cat("q = ", i, "AIC = ", AIC(model_1$model), "BIC = ", BIC(model_1$model),
"MASE =", MASE(model_1)$MASE, "\n")
  }

## q =  1 AIC =  281.571 BIC =  287.1758 MASE = 0.6579604
## q =  2 AIC =  273.2458 BIC =  280.0823 MASE = 0.6672639
## q =  3 AIC =  264.666 BIC =  272.6592 MASE = 0.6164754
## q =  4 AIC =  257.9491 BIC =  267.02 MASE = 0.5968786
## q =  5 AIC =  249.3748 BIC =  259.4396 MASE = 0.5870893
## q =  6 AIC =  235.4683 BIC =  246.4382 MASE = 0.5298748
```

```
## q =  7 AIC =  225.705 BIC =  237.4856 MASE = 0.479178
## q =  8 AIC =  219.9335 BIC =  232.424 MASE = 0.4839788
## q =  9 AIC =  211.6427 BIC =  224.7353 MASE = 0.4680972
## q =  10 AIC =  204.3142 BIC =  217.893 MASE = 0.4716663
```

As we have the least AIC, BIC and MASE values at q = 10. Let us fit the finite distributed lag model with q = 10.

```
# Finite lag length based on AIC, BIC and MASE

finite_dlm_FFD_slope_rad = dlm(formula = y ~ x4,
data=data.frame(v_data_TS_22), q = 10)
summary(finite_dlm_FFD_slope_rad)

##
## Call:
## lm(formula = as.formula(model.formula), data = design)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -36.650 -10.774   5.135  12.268  21.533
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  77.1798   518.3355   0.149   0.8849
## x4.t        -14.2644    31.1884  -0.457   0.6583
## x4.1         22.6955    37.2327   0.610   0.5572
## x4.2        -41.5518    22.3579  -1.858   0.0960 .
## x4.3         41.3553    21.3508   1.937   0.0847 .
## x4.4        -23.8377    22.3631  -1.066   0.3142
## x4.5         36.6585    21.2569   1.725   0.1187
## x4.6        -46.5116    22.0486  -2.110   0.0641 .
## x4.7         21.3519    21.6192   0.988   0.3491
## x4.8          0.1483    21.9721   0.007   0.9948
## x4.9         18.1562    27.1471   0.669   0.5204
## x4.10        -5.3017    27.2786  -0.194   0.8502
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 25.8 on 9 degrees of freedom
## Multiple R-squared:  0.5432, Adjusted R-squared:  -0.01504
## F-statistic: 0.9731 on 11 and 9 DF,  p-value: 0.5253
##
## AIC and BIC values for the model:
##        AIC     BIC
## 1 204.3142 217.893
```

Hypotheses:

H_0: The data doesn't fit the Finite distributed lag model with slope.

HA: The data fits the Finite distributed lag model with slope.

Interpretations:

F - statistic is 0.9731
R - squared is 0.5432
Adjusted R - squared is -0.01504
Degrees of freedom - DF are (11, 9)
p - value (0.5253) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model doesn't fits the Finite distributed lag model with slope.

Therefore, no residual analysis is required.

Therefore, Further analysis is needed by removing slope to the lag model.

With out slope.

```
for ( i in 1:10){
  model_2 = dlm(formula=y ~ 0 + x4, data=data.frame(v_data_TS_22), q = i )
  cat("q = ", i, "AIC = ", AIC(model_2$model), "BIC = ", BIC(model_2$model),
"MASE =", MASE(model_2)$MASE, "\n")
  }

## q =  1 AIC =  281.4295 BIC =  285.6331 MASE = 0.6606648
## q =  2 AIC =  274.0228 BIC =  279.492 MASE = 0.6743494
## q =  3 AIC =  264.5047 BIC =  271.1657 MASE = 0.6317954
## q =  4 AIC =  256.7858 BIC =  264.5608 MASE = 0.588564
## q =  5 AIC =  247.3861 BIC =  256.1928 MASE = 0.5877368
## q =  6 AIC =  235.0562 BIC =  244.8072 MASE = 0.5208836
## q =  7 AIC =  224.0442 BIC =  234.6467 MASE = 0.4710983
## q =  8 AIC =  218.1936 BIC =  229.5485 MASE = 0.4792433
## q =  9 AIC =  209.7754 BIC =  221.7769 MASE = 0.4655184
## q =  10 AIC =  202.3659 BIC =  214.9002 MASE = 0.4686756
```

As we have the least AIC, BIC and MASE values at q = 10. Let us fit the finite distributed lag model with q = 10.

```
# Finite lag length based on AIC, BIC and MASE

finite_dlm_FFD_noslope_rad = dlm(formula = y ~ 0 + x4,
data=data.frame(v_data_TS_22), q = 10)
summary(finite_dlm_FFD_noslope_rad)

##
## Call:
## lm(formula = as.formula(model.formula), data = design)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -37.504 -11.163    4.155   12.292   21.823
```

```
##
## Coefficients:
##       Estimate Std. Error t value Pr(>|t|)
## x4.t   -12.329     26.930  -0.458   0.6569
## x4.1    21.788     34.888   0.625   0.5463
## x4.2   -40.600     20.350  -1.995   0.0740 .
## x4.3    41.228     20.264   2.035   0.0693 .
## x4.4   -23.444     21.093  -1.111   0.2924
## x4.5    37.123     19.972   1.859   0.0927 .
## x4.6   -46.252     20.877  -2.215   0.0511 .
## x4.7    21.334     20.535   1.039   0.3233
## x4.8     1.023     20.111   0.051   0.9604
## x4.9    17.627     25.564   0.690   0.5062
## x4.10   -3.306     22.569  -0.147   0.8864
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 24.5 on 10 degrees of freedom
## Multiple R-squared:  0.9934, Adjusted R-squared:  0.9862
## F-statistic: 137.1 on 11 and 10 DF,  p-value: 2.186e-09
##
## AIC and BIC values for the model:
##        AIC      BIC
## 1 202.3659 214.9002
```

Hypotheses:

H_0: The data doesn′t fit the Finite distributed lag model without slope.
H_A: The data fits the Finite distributed lag model without slope.

Interpretations:

F - statistic is 137.1
R - squared is 0.9934

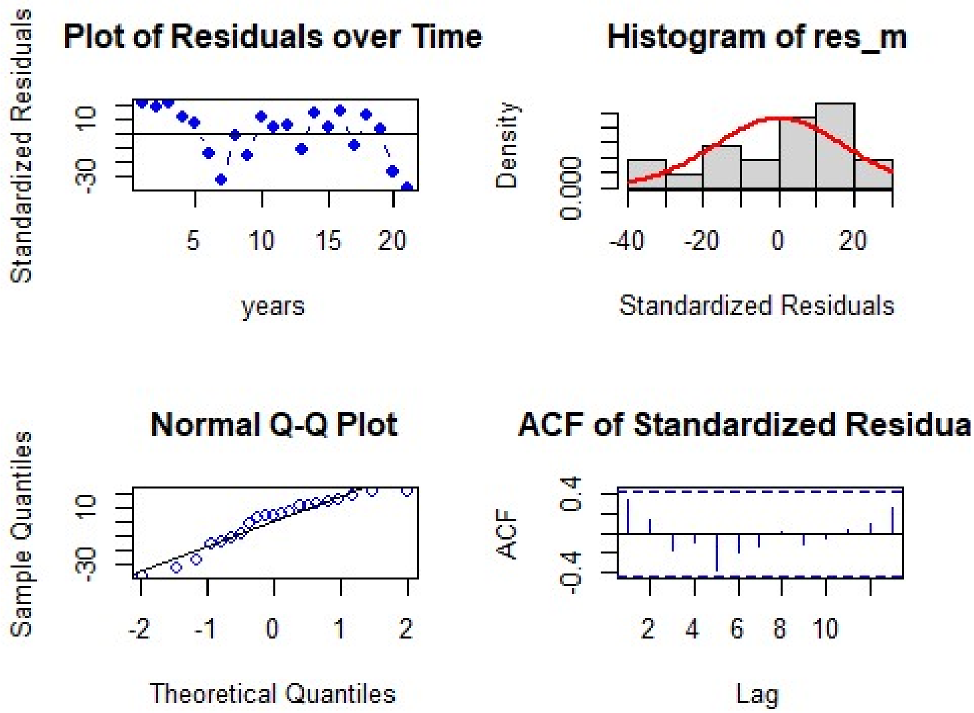
Adjusted R - squared is 0.9862

Degrees of freedom - DF are (11, 10) p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Finite distributed lag model without slope.

This model suggests that there is only 98.62% of data variance. Suggesting that the model explains only 98.62% of the trend. Which implies that the model fits better.

Now let us perform residual analysis.

```
res_analysis(residuals(finite_dlm_FFD_noslope_rad$model))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.91671, p-value = 0.0746
```

Residual Analysis for Finite DLM:

1. The data points are above the line at the start and below the line at the end of the trend. Randomness is seen in the data. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is not symmetric.
3. QQplot also suggests that there is no normality in the trend.
4. There is only one significant lagin Autocorrelation plot.
5. p - value (0.08) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

Even though the model fits better it is poor when it comes to residual analysis. Therefore, Further analysis is needed by adding polynomial to the lag model.

Polynomial distributed lag model with Radiation

```
for (i in 1:3){
  model_3 <-  polyDlm(x = as.vector(x2), y = as.vector(y), q = i , k = i,
show.beta = FALSE)
  cat("q = ", i, "k = ", i, "AIC = ", AIC(model_3$model), "BIC = ",
```

```
BIC(model_3$model), MASE(model_3)$MASE, "\n")
}

## q =  1 k =  1 AIC =  281.571 BIC =  287.1758 0.6579604
## q =  2 k =  2 AIC =  273.2458 BIC =  280.0823 0.6672639
## q =  3 k =  3 AIC =  264.666 BIC =  272.6592 0.6164754
```

Let us fit a polynomial model of order 3. Since least AIC and BIC scores.

```
# Ploynomial DLM

PolyDLM_model_FFD_rad = polyDlm(x = as.vector(x2), y = as.vector(y), q = 3, k
= 3, show.beta = TRUE)

## Estimates and t-tests for beta coefficients:
##        Estimate Std. Error t value P(>|t|)
## beta.0    3.040       13.0  0.2350   0.816
## beta.1    0.551       14.0  0.0395   0.969
## beta.2  -20.700       14.0 -1.4800   0.151
## beta.3   11.700       12.9  0.9040   0.375

summary(PolyDLM_model_FFD_rad)

##
## Call:
## "Y ~ (Intercept) + X.t"
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -57.162 -12.623  -0.322  17.604  37.509
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   288.35     230.78   1.249    0.224
## z.t0            3.04      12.96   0.235    0.817
## z.t1           31.02      69.57   0.446    0.660
## z.t2          -45.57      62.52  -0.729    0.473
## z.t3           12.06      13.82   0.873    0.392
##
## Residual standard error: 24.32 on 23 degrees of freedom
## Multiple R-squared:  0.1098, Adjusted R-squared:  -0.045
## F-statistic: 0.7093 on 4 and 23 DF,  p-value: 0.5939
```

Hypotheses:

H_0: The data doesn't fit the Polynomial distributed lag model.
H_A: The data fits the Polynomial distributed lag model.

Interpretations:

F - statistic is 0.7093
R - squared is 0.1098

Adjusted R - squared is -0.045
Degrees of freedom - DF are (4, 23)
p - value (0.5939) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Polynomial distributed lag model.
Also, this model suggests that there is only -4.5% of data variance.

No residual analysis is required.

Let us fit Koyck model.

Koyck model with Radiation

```
# Koyk DLM

Koyck_DLM_FFD_rad = koyckDlm(x = as.vector(x2) , y = as.vector(y))
summary(Koyck_DLM_FFD_rad)

##
## Call:
## "Y ~ (Intercept) + Y.1 + X.t"
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -55.229 -19.662   3.956  16.232  54.756
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 418.31843  384.12678   1.089    0.286
## Y.1          -0.03254    0.20729  -0.157    0.876
## X.t         -13.87153   25.49529  -0.544    0.591
##
## Residual standard error: 25.54 on 27 degrees of freedom
## Multiple R-Squared: -0.07436,    Adjusted R-squared: -0.1539
## Wald test: 0.1486 on 2 and 27 DF,  p-value: 0.8626
##
## Diagnostic tests:
## NULL
##
##                              alpha      beta         phi
## Geometric coefficients:  405.1353 -13.87153 -0.03254005
```

Hypotheses:

H0: The data doesn't fit the Polynomial distributed lag model.
HA: The data fits the Polynomial distributed lag model.

Interpretations:

Wald test statistic is 0.1486
R - squared is -0.07436
Adjusted R - squared is -0.1539

Degrees of freedom - DF are (2, 27)
p - value (0.8626) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Koyck distributed lag model.
Also, this model suggests that there is only -15.39% of data variance.

No residual analysis is required.

Let us fit ardlDlm model to check whether it fits better or not.

Autoregressive distributed lag model with Radiation

with slope

```
for (i in 1:5){
  for(j in 1:5){
    model_4 = ardlDlm(formula=y ~ x4, data=data.frame(v_data_TS_22), p = i ,
q = j )
    cat("p = ", i, "q = ", j, "AIC = ", AIC(model_4$model), "BIC = ",
BIC(model_4$model), "MASE =", MASE(model_4)$MASE, "\n")
  }
}
```

```
## p =  1 q =  1 AIC =  283.5658 BIC =  290.5718 MASE = 0.6599906
## p =  1 q =  2 AIC =  276.5262 BIC =  284.73 MASE = 0.6718305
## p =  1 q =  3 AIC =  269.1591 BIC =  278.4845 MASE = 0.6531487
## p =  1 q =  4 AIC =  262.6904 BIC =  273.0571 MASE = 0.6386406
## p =  1 q =  5 AIC =  255.6012 BIC =  266.9241 MASE = 0.6196376
## p =  2 q =  1 AIC =  275.24 BIC =  283.4438 MASE = 0.6688985
## p =  2 q =  2 AIC =  277.1882 BIC =  286.7593 MASE = 0.6661641
## p =  2 q =  3 AIC =  269.5463 BIC =  280.204 MASE = 0.646406
## p =  2 q =  4 AIC =  262.9001 BIC =  274.5626 MASE = 0.6221685
## p =  2 q =  5 AIC =  256.4326 BIC =  269.0136 MASE = 0.6306393
## p =  3 q =  1 AIC =  266.5357 BIC =  275.8611 MASE = 0.6144131
## p =  3 q =  2 AIC =  268.3699 BIC =  279.0276 MASE = 0.6149087
## p =  3 q =  3 AIC =  270.1803 BIC =  282.1701 MASE = 0.6220337
## p =  3 q =  4 AIC =  263.6351 BIC =  276.5935 MASE = 0.6016041
## p =  3 q =  5 AIC =  257.3509 BIC =  271.1899 MASE = 0.6164068
## p =  4 q =  1 AIC =  259.9163 BIC =  270.283 MASE = 0.5983539
## p =  4 q =  2 AIC =  261.3427 BIC =  273.0052 MASE = 0.5933603
## p =  4 q =  3 AIC =  263.3127 BIC =  276.2711 MASE = 0.5959315
## p =  4 q =  4 AIC =  265.2261 BIC =  279.4803 MASE = 0.5907476
## p =  4 q =  5 AIC =  258.7707 BIC =  273.8679 MASE = 0.6045879
## p =  5 q =  1 AIC =  251.3746 BIC =  262.6975 MASE = 0.587024
## p =  5 q =  2 AIC =  252.9746 BIC =  265.5556 MASE = 0.5756885
## p =  5 q =  3 AIC =  254.7 BIC =  268.5391 MASE = 0.5539405
## p =  5 q =  4 AIC =  255.7433 BIC =  270.8405 MASE = 0.5278081
## p =  5 q =  5 AIC =  257.5483 BIC =  273.9036 MASE = 0.5314267
```

(p, q) = (5, 4) has the least AIC, BIC and MASE scores.

```
# ARDLM model
AR_DLM_FFD_54_slope_rad = ardlDlm(formula = y ~ x4,
data=data.frame(v_data_TS_22), p = 5 , q = 4)
summary(AR_DLM_FFD_54_slope_rad)

##
## Time series regression with "ts" data:
## Start = 6, End = 31
##
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -47.589  -7.045   0.804  12.449  32.637
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.903e+02  4.908e+02  -0.591    0.563
## x4.t         1.353e+01  1.929e+01   0.702    0.494
## x4.1         9.354e+00  2.091e+01   0.447    0.661
## x4.2        -3.163e+01  1.882e+01  -1.681    0.114
## x4.3         1.114e+01  1.984e+01   0.561    0.583
## x4.4        -4.193e+00  2.204e+01  -0.190    0.852
## x4.5         2.912e+01  2.014e+01   1.446    0.169
## y.1         -8.686e-03  2.456e-01  -0.035    0.972
## y.2          1.181e-01  2.663e-01   0.443    0.664
## y.3          1.556e-01  3.111e-01   0.500    0.624
## y.4          2.052e-01  2.737e-01   0.750    0.465
##
## Residual standard error: 27.46 on 15 degrees of freedom
## Multiple R-squared:  0.2501, Adjusted R-squared:  -0.2498
## F-statistic: 0.5004 on 10 and 15 DF,  p-value: 0.8644
```

Hypotheses:

H_0: The data doesn't fit the Autoregressive distributed lag model with slope.
H_A: The data fits the Autoregressive distributed lag model with slope.

Interpretations:

F - statistic is 0.5004
R - squared is 0.2501
Adjusted R - squared is -0.2498
Degrees of freedom - DF are (10, 15)
p - value (0.8644) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Autoregressive distributed lag model with slope.
Also, this model suggests that there is only -24.98% of data variance.

No residual analysis is required.

With out slope.

```
for (i in 1:5){
  for(j in 1:5){
    model_5 = ardlDlm(formula=y ~ 0 + x4, data=data.frame(v_data_TS_22), p =
i , q = j )
    cat("p = ", i, "q = ", j, "AIC = ", AIC(model_5$model), "BIC = ",
BIC(model_5$model), "MASE =", MASE(model_5)$MASE, "\n")
  }
}

## p =  1 q =  1 AIC =  283.5658 BIC =  290.5718 MASE = 0.6599906
## p =  1 q =  2 AIC =  276.5262 BIC =  284.73 MASE = 0.6718305
## p =  1 q =  3 AIC =  269.1591 BIC =  278.4845 MASE = 0.6531487
## p =  1 q =  4 AIC =  262.6904 BIC =  273.0571 MASE = 0.6386406
## p =  1 q =  5 AIC =  255.6012 BIC =  266.9241 MASE = 0.6196376
## p =  2 q =  1 AIC =  275.24 BIC =  283.4438 MASE = 0.6688985
## p =  2 q =  2 AIC =  277.1882 BIC =  286.7593 MASE = 0.6661641
## p =  2 q =  3 AIC =  269.5463 BIC =  280.204 MASE = 0.646406
## p =  2 q =  4 AIC =  262.9001 BIC =  274.5626 MASE = 0.6221685
## p =  2 q =  5 AIC =  256.4326 BIC =  269.0136 MASE = 0.6306393
## p =  3 q =  1 AIC =  266.5357 BIC =  275.8611 MASE = 0.6144131
## p =  3 q =  2 AIC =  268.3699 BIC =  279.0276 MASE = 0.6149087
## p =  3 q =  3 AIC =  270.1803 BIC =  282.1701 MASE = 0.6220337
## p =  3 q =  4 AIC =  263.6351 BIC =  276.5935 MASE = 0.6016041
## p =  3 q =  5 AIC =  257.3509 BIC =  271.1899 MASE = 0.6164068
## p =  4 q =  1 AIC =  259.9163 BIC =  270.283 MASE = 0.5983539
## p =  4 q =  2 AIC =  261.3427 BIC =  273.0052 MASE = 0.5933603
## p =  4 q =  3 AIC =  263.3127 BIC =  276.2711 MASE = 0.5959315
## p =  4 q =  4 AIC =  265.2261 BIC =  279.4803 MASE = 0.5907476
## p =  4 q =  5 AIC =  258.7707 BIC =  273.8679 MASE = 0.6045879
## p =  5 q =  1 AIC =  251.3746 BIC =  262.6975 MASE = 0.587024
## p =  5 q =  2 AIC =  252.9746 BIC =  265.5556 MASE = 0.5756885
## p =  5 q =  3 AIC =  254.7 BIC =  268.5391 MASE = 0.5539405
## p =  5 q =  4 AIC =  255.7433 BIC =  270.8405 MASE = 0.5278081
## p =  5 q =  5 AIC =  257.5483 BIC =  273.9036 MASE = 0.5314267
```

(p, q) = (5, 4) has the least AIC, BIC and MASE scores.

```
# Finite lag length based on AIC-BIC

AR_DLM_FFD_54_noslope_rad = ardlDlm(formula=y ~ 0 + x4,
data=data.frame(v_data_TS_22), p = 5 , q = 4)
summary(AR_DLM_FFD_54_noslope_rad)

##
## Time series regression with "ts" data:
## Start = 6, End = 31
##
```

```
## Call:
## dynlm(formula = as.formula(model.text), data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -47.589  -7.045   0.804  12.449  32.637
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.903e+02  4.908e+02  -0.591    0.563
## x4.t         1.353e+01  1.929e+01   0.702    0.494
## x4.1         9.354e+00  2.091e+01   0.447    0.661
## x4.2        -3.163e+01  1.882e+01  -1.681    0.114
## x4.3         1.114e+01  1.984e+01   0.561    0.583
## x4.4        -4.193e+00  2.204e+01  -0.190    0.852
## x4.5         2.912e+01  2.014e+01   1.446    0.169
## y.1         -8.686e-03  2.456e-01  -0.035    0.972
## y.2          1.181e-01  2.663e-01   0.443    0.664
## y.3          1.556e-01  3.111e-01   0.500    0.624
## y.4          2.052e-01  2.737e-01   0.750    0.465
##
## Residual standard error: 27.46 on 15 degrees of freedom
## Multiple R-squared:  0.2501, Adjusted R-squared:  -0.2498
## F-statistic: 0.5004 on 10 and 15 DF,  p-value: 0.8644
```

Hypotheses:

H_0: The data doesn't fit the Autoregressive distributed lag model without slope.
H_A: The data fits the Autoregressive distributed lag model without slope.

Interpretations:

F - statistic is 0.5004
R - squared is 0.2501
Adjusted R - squared is -0.2498
Degrees of freedom - DF are (10, 15)
p - value (0.8644) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Autoregressive distributed lag model with slope.
Also, this model suggests that there is only -24.98% of data variance.

No residual analysis is required.

Therefore, Finite DLM without slope w.r.t radiation fits better.

Now let us fit dynamic lm model

Dynamic model

Rainfall

With slope

```
v_FFd_dyna_rain <- dynlm(y ~ x3, data = data.frame(v_data_TS_22))
summary(v_FFd_dyna_rain)

##
## Time series regression with "numeric" data:
## Start = 1, End = 31
##
## Call:
## dynlm(formula = y ~ x3, data = data.frame(v_data_TS_22))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -55.774 -16.601   3.458  17.127  55.805
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  201.919     27.888   7.240 5.68e-08 ***
## x3             3.178     11.627   0.273    0.787
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 23.78 on 29 degrees of freedom
## Multiple R-squared:  0.00257,    Adjusted R-squared:  -0.03182
## F-statistic: 0.07471 on 1 and 29 DF,  p-value: 0.7865
```

Hypotheses:

H_0: The data doesn't fit the Dynamic linear model with slope.
H_A: The data fits the Dynamic linear model with slope.

Interpretations:

F - statistic is 0.07471
R - squared is 0.00257
Adjusted R - squared is -0.03182
Degrees of freedom - DF are (1, 29)
p - value (0.7865) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model doesn't fit the Dynamic linear model with slope.
This model suggests that there is only -3.18% of data variance.

No residual analysis is required.

Therefore, Further analysis is needed by removing slope to the lag model.

With out slope.

```
v_FFd_dyna_rain_noslope <- dynlm(y ~ 0 + x3, data = data.frame(v_data_TS_22))
summary(v_FFd_dyna_rain_noslope)
```

```
##
## Time series regression with "numeric" data:
## Start = 1, End = 31
##
## Call:
## dynlm(formula = y ~ 0 + x3, data = data.frame(v_data_TS_22))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -59.484 -18.121   1.731  22.895  93.445
##
## Coefficients:
##    Estimate Std. Error t value Pr(>|t|)
## x3   86.366      2.934   29.44   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 39.18 on 30 degrees of freedom
## Multiple R-squared:  0.9665, Adjusted R-squared:  0.9654
## F-statistic: 866.6 on 1 and 30 DF,  p-value: < 2.2e-16
```

Hypotheses:

H_0: The data doesn't fit the Dynamic linear model without slope.
H_A: The data fits the Dynamic linear model without slope.

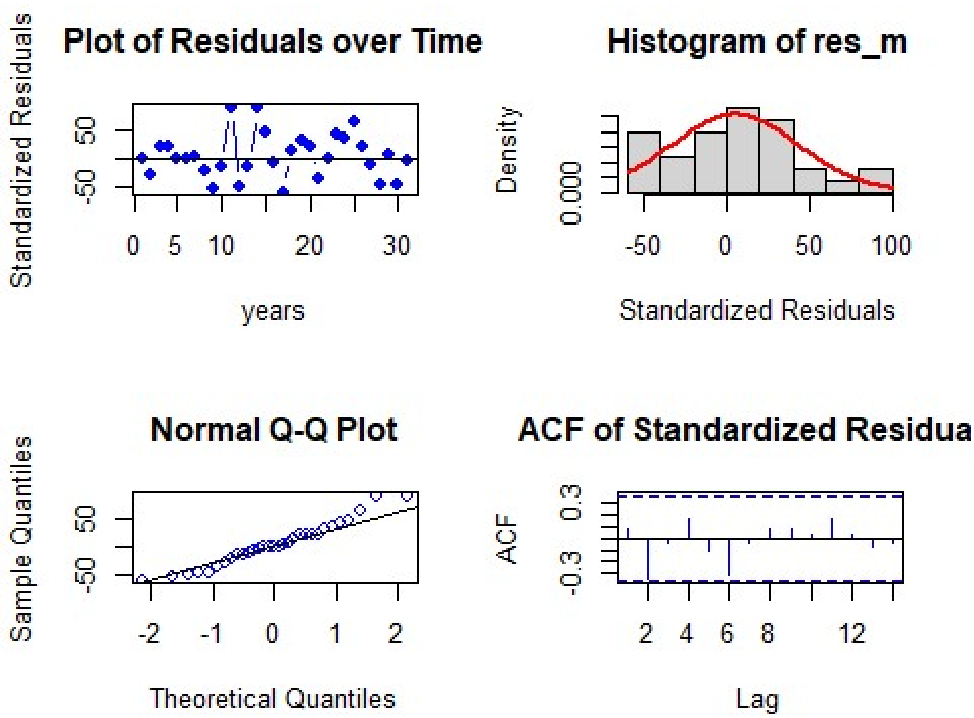
Interpretations:

F - statistic is 866.6
R - squared is 0.9665
Adjusted R - squared is 0.9654
Degrees of freedom - DF are (1, 30)
p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Dynamic linear model.
This model suggests that there is only 96.54% of data variance. Suggesting that the model explains only 96,54% of the trend. Which implies that the model shows some trend.

Now let us perform residual analysis.

Residual analysis

```
res_analysis(residuals(v_FFd_dyna_rain_noslope))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.96579, p-value = 0.4113
```

Residual Analysis for v_FFd_dyna_rain_noslope:

1. The data points are below the line at both the start and end of the trend. Randomness is not seen here.
2. From normal distribution curve, the distribution is almost symmetric. This suggests a good fit with a very few data falling outside the normal curve indicating Kurtosis.
3. The data at the tails is deviated but is normal for most part of the line suggesting normality in the trend.
4. There are no significant lags in Autocorrelation plot suggesting that the stochastic component is white noise.
5. p - value (0.4113) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

Even though the model fits better it is poor when it comes to residual analysis. Let us fit with Radiation.

Radiation

With slope

```
v_FFd_dyna_rad <- dynlm(y ~ x4, data = data.frame(v_data_TS_22))
summary(v_FFd_dyna_rad)

##
## Time series regression with "numeric" data:
## Start = 1, End = 31
##
## Call:
## dynlm(formula = y ~ x4, data = data.frame(v_data_TS_22))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -55.625 -15.751   2.051  17.086  54.734
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  169.916    156.834   1.083    0.288
## x4             2.709     10.744   0.252    0.803
##
## Residual standard error: 23.79 on 29 degrees of freedom
## Multiple R-squared:  0.002188,   Adjusted R-squared:  -0.03222
## F-statistic: 0.0636 on 1 and 29 DF,  p-value: 0.8027
```

Hypotheses:

> **H_0: The data doesn't fit the Dynamic linear model with slope.**
> **H_A: The data fits the Dynamic linear model with slope.**

Interpretations:

> F - statistic is 0.0636
> R - squared is 0.002188
> Adjusted R - squared is -0.03222
> Degrees of freedom - DF are (1, 29)
> p - value (0.8027) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model doesn't fit the Dynamic linear model with slope.
> This model suggests that there is only -3.22% of data variance.

No residual analysis is required.

Therefore, Further analysis is needed by removing slope to the lag model.

With out slope.

```
v_FFd_dyna_rad_noslope <- dynlm(y ~ 0 + x4, data = data.frame(v_data_TS_22))
summary(v_FFd_dyna_rad_noslope)
```

```
##
## Time series regression with "numeric" data:
## Start = 1, End = 31
##
## Call:
## dynlm(formula = y ~ 0 + x4, data = data.frame(v_data_TS_22))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -56.243 -19.058   2.424  17.458  55.657
##
## Coefficients:
##    Estimate Std. Error t value Pr(>|t|)
## x4  14.3455     0.2935   48.87   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 23.86 on 30 degrees of freedom
## Multiple R-squared:  0.9876, Adjusted R-squared:  0.9872
## F-statistic:  2388 on 1 and 30 DF,  p-value: < 2.2e-16
```

Hypotheses:

H_0: The data doesn't fit the Dynamic linear model without slope.
H_A: The data fits the Dynamic linear model without slope.

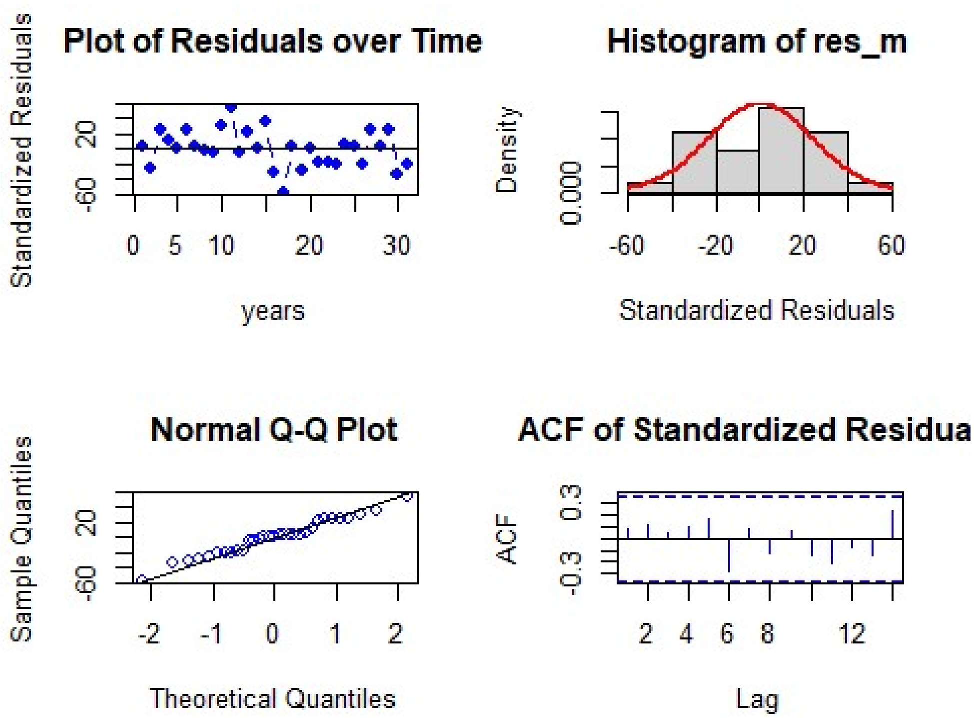
Interpretations:

F - statistic is 2388
R - squared is 0.9876
Adjusted R - squared is 0.9872
Degrees of freedom - DF are (1, 30)
p - value (~ 0.01) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Dynamic linear model without slope.
This model suggests that there is only 98.72% of data variance. Suggesting that the model explains only 98.72% of the trend. Which implies that the model shows some trend.

Now let us perform residual analysis.

Residual analysis

```
res_analysis(residuals(v_FFd_dyna_rad_noslope))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.97519, p-value = 0.6705
```

Residual Analysis for v_FFd_dyna_rad_noslope:

1. The data points are below the line at both the start and end of the trend. Randomness is not seen here.
2. From normal distribution curve, the distribution is almost symmetric. This suggests a good fit with a very few data falling outside the normal curve indicating Kurtosis.
3. The data at the tails is deviated but is normal for most part of the line suggesting normality in the trend.
4. There are no significant lags in Autocorrelation plot suggesting that the stochastic component is white noise.
5. p - value (0.6705) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

Therefore, Dynamic model without slope w.r.t radiation fits better.

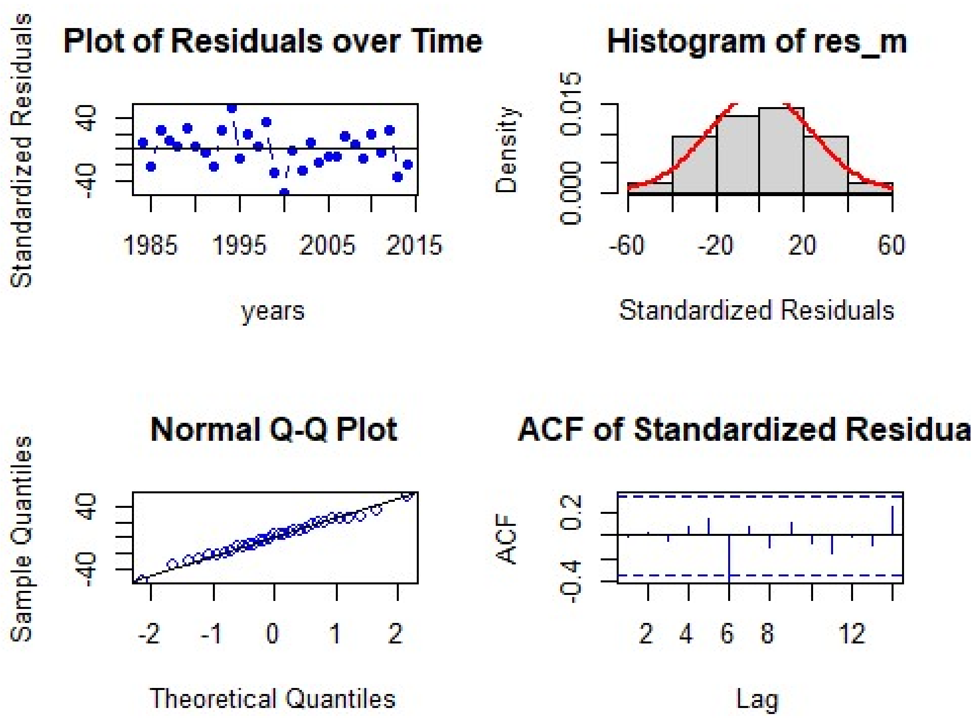
Exponential Smoothing

Since the Seasonality component is week we cannot get additive and multiplicative seasonality. So let us fit with simple seasonality. Since, we need next 3 years point forecasts as well as confidence intervals, we used h = 3 (frequency).

```
fit_ses <- ses(v_First_Flowering_Day_data_TS, seasonal = "simple", h = 4)
summary(fit_ses)

## 
## Forecast method: Simple exponential smoothing
## 
## Model Information:
## Simple exponential smoothing 
## 
## Call:
##  ses(y = v_First_Flowering_Day_data_TS, h = 4, seasonal = "simple") 
## 
##   Smoothing parameters:
##     alpha = 1e-04 
## 
##   Initial states:
##     l = 209.4505 
## 
##   sigma:  23.8149
## 
##      AIC     AICc      BIC 
## 306.9455 307.8343 311.2474 
## 
## Error measures:
##                        ME     RMSE      MAE      MPE     MAPE      MASE
## Training set -0.003678178 23.03386 18.69592 -1.26477 9.173671 0.6614123
##                      ACF1
## Training set -0.006864134
## 
## Forecasts:
##      Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 2015       209.4505 178.9305 239.9705 162.7742 256.1268
## 2016       209.4505 178.9305 239.9705 162.7742 256.1268
## 2017       209.4505 178.9305 239.9705 162.7742 256.1268
## 2018       209.4505 178.9305 239.9705 162.7742 256.1268
```

Now let us check the residual analysis.

```
res_analysis(residuals(fit_ses))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.99329, p-value = 0.9992
```

Residual Analysis analysis for simple seasonality:

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

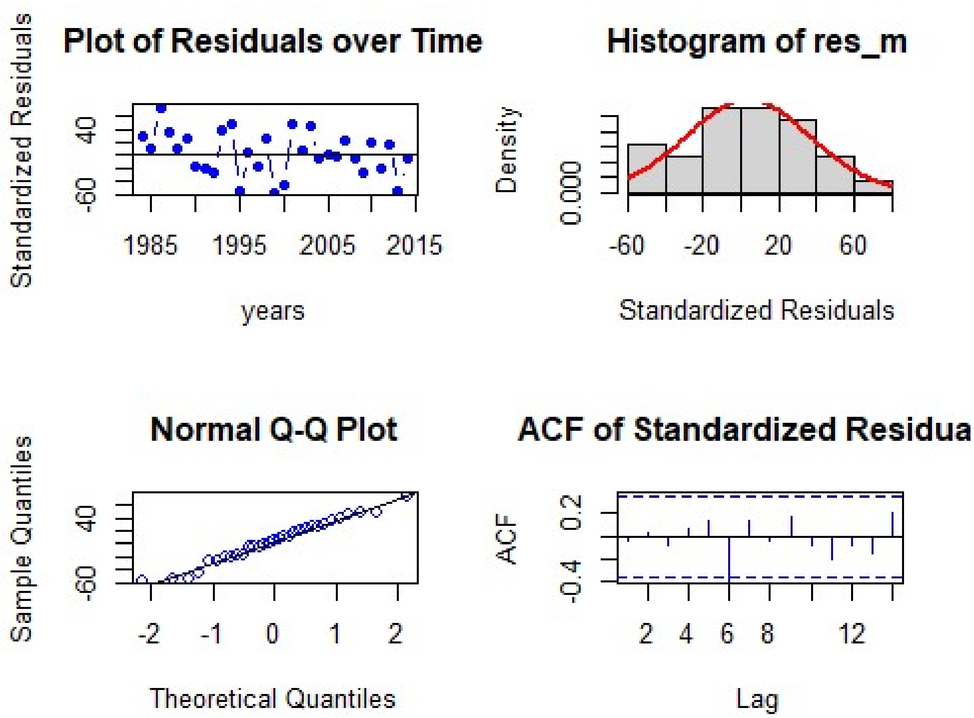
Now let us fit with simple seasonality with both alpha and gamma.

```
fit2 <- holt(v_First_Flowering_Day_data_TS, initial = "simple", h = 4)

#both alpha and beta
summary(fit2)
```

```
##
## Forecast method: Holt's method
##
## Model Information:
## Holt's method
##
## Call:
##  holt(y = v_First_Flowering_Day_data_TS, h = 4, initial = "simple")
##
##   Smoothing parameters:
##     alpha = 0.4947
##     beta  = 0.4105
##
##   Initial states:
##     l = 217
##     b = -31
##
##   sigma:  32.4048
## Error measures:
##                       ME     RMSE      MAE       MPE     MAPE      MASE
## Training set 3.643616 32.40477 26.33733 0.4670524 12.74033 0.9317453
##                     ACF1
## Training set -0.03620177
##
## Forecasts:
##      Point Forecast     Lo 80    Hi 80      Lo 95    Hi 95
## 2015       183.4402 141.91186 224.9686 119.928060 246.9524
## 2016       175.3764 119.36198 231.3909  89.709726 261.0431
## 2017       167.3126  89.06345 245.5618  47.640838 286.9844
## 2018       159.2488  53.12709 265.3705  -3.050347 321.5479
```

Now let us check the residual analysis.

```
res_analysis(residuals(fit2))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.9795, p-value = 0.7985
```

Residual Analysis analysis for simple seasonality:

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

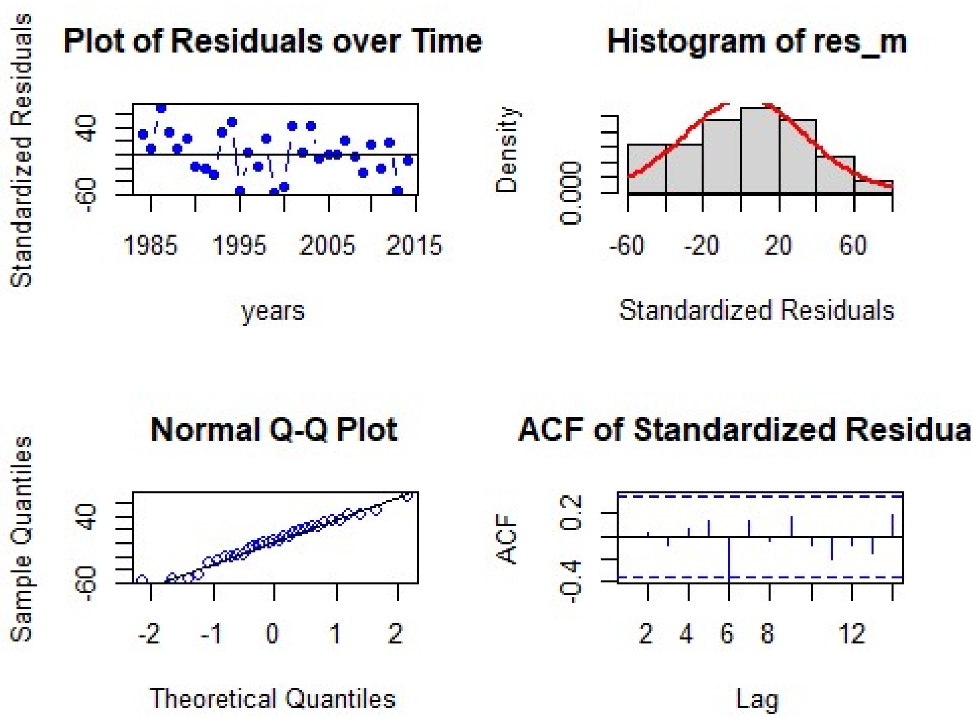
Now let us fit with exponential trend.

```
fit3 <- holt(v_First_Flowering_Day_data_TS, initial="simple", exponential =
TRUE, h = 4)# Fit with exponential trend
summary(fit3)
```

```
##
## Forecast method: Holt's method with exponential trend
##
## Model Information:
## Holt's method with exponential trend
##
## Call:
##  holt(y = v_First_Flowering_Day_data_TS, h = 4, initial = "simple",
##
##  Call:
##      exponential = TRUE)
##
##   Smoothing parameters:
##     alpha = 0.4502
##     beta  = 0.4436
##
##   Initial states:
##     l = 217
##     b = 0.8571
##
##   sigma:  0.1598
## Error measures:
##                    ME     RMSE      MAE        MPE     MAPE      MASE
## Training set 1.966628 31.73873 25.84104 -0.3370878 12.55847 0.9141876
##                     ACF1
## Training set -0.01314563
##
## Forecasts:
##      Point Forecast     Lo 80    Hi 80     Lo 95    Hi 95
## 2015       185.9214 146.63995 223.6395 127.70016 244.4981
## 2016       178.6954 131.37582 230.5629 109.28928 261.5664
## 2017       171.7503 112.46477 244.9998  87.76724 295.4699
## 2018       165.0750  91.71386 268.4287  67.21901 357.7409
```

Now let us check the residual analysis.

```
res_analysis(residuals(fit3))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.97835, p-value = 0.7654
```

Residual Analysis analysis for exponential trend:

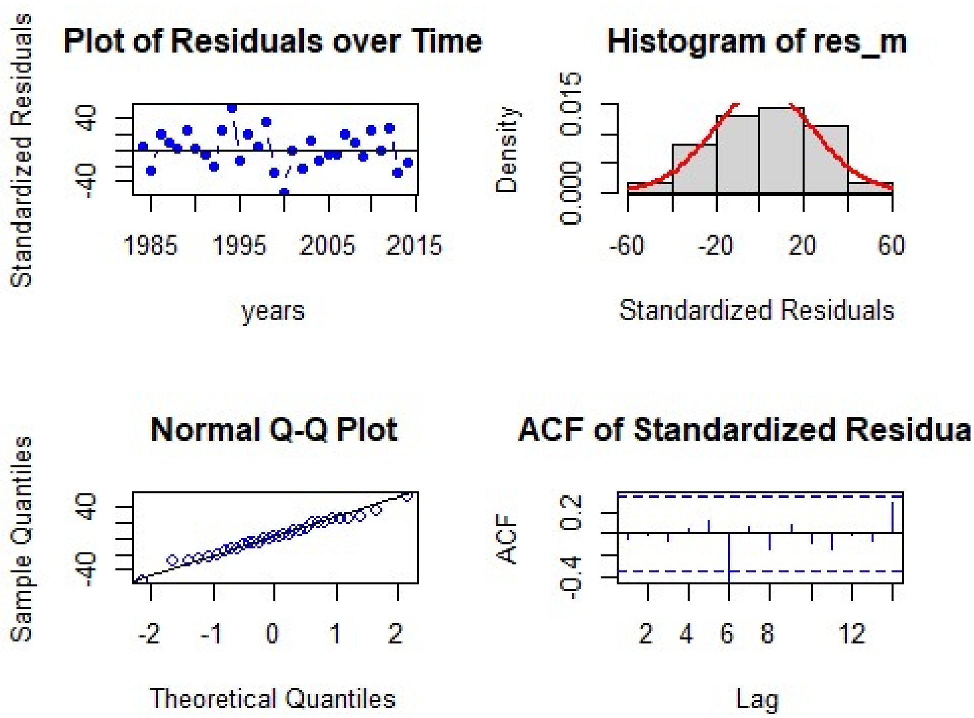
1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
fit4 <- holt(v_First_Flowering_Day_data_TS, damped = TRUE, h = 4)# Fit with
damped trend
summary(fit4)

##
## Forecast method: Damped Holt's method
##
```

```
## Model Information:
## Damped Holt's method
##
## Call:
##  holt(y = v_First_Flowering_Day_data_TS, h = 4, damped = TRUE)
##
##   Smoothing parameters:
##     alpha = 1e-04
##     beta  = 1e-04
##     phi   = 0.9724
##
##   Initial states:
##     l = 213.8104
##     b = -0.5047
##
##   sigma:  24.6844
##
##      AIC     AICc      BIC
## 311.7836 315.2836 320.3875
##
## Error measures:
##                    ME     RMSE      MAE        MPE     MAPE      MASE
## Training set 1.646287 22.60624 17.87068 -0.4309696 8.698589 0.6322173
##                    ACF1
## Training set -0.0440588
##
## Forecasts:
##      Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 2015       203.4051 171.7707 235.0394 155.0245 251.7856
## 2016       203.2069 171.5725 234.8412 154.8263 251.5874
## 2017       203.0142 171.3798 234.6485 154.6336 251.3947
## 2018       202.8268 171.1924 234.4611 154.4462 251.2073
```

Now let us check the residual analysis.

```
res_analysis(residuals(fit4))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.98975, p-value = 0.9885
```

Residual Analysis analysis for exponential trend:

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Due to week seasonality in the series there is no additive or multiplicative seasonality also there will be no damped in the series.

By exponential smoothing method we got the simple seasonal fit as the best model in terms of MASE and BIC scores.

State Space Model Variations

Let us find the best ets model. Before all let us auto fit the model.

```
v_ets_fit <- ets(v_First_Flowering_Day_data_TS)
summary(v_ets_fit)
```

```
## ETS(M,N,N)
##
## Call:
##  ets(y = v_First_Flowering_Day_data_TS)
##
##   Smoothing parameters:
##     alpha = 1e-04
##
##   Initial states:
##     l = 209.448
##
##   sigma:  0.1137
##
##      AIC     AICc      BIC
## 306.9448 307.8337 311.2468
##
## Training set error measures:
##                        ME     RMSE    MAE       MPE     MAPE      MASE
## Training set -0.001214657 23.03386 18.696 -1.263579 9.173601 0.6614151
##                      ACF1
## Training set -0.006864096
```

ETS(M, N, N)

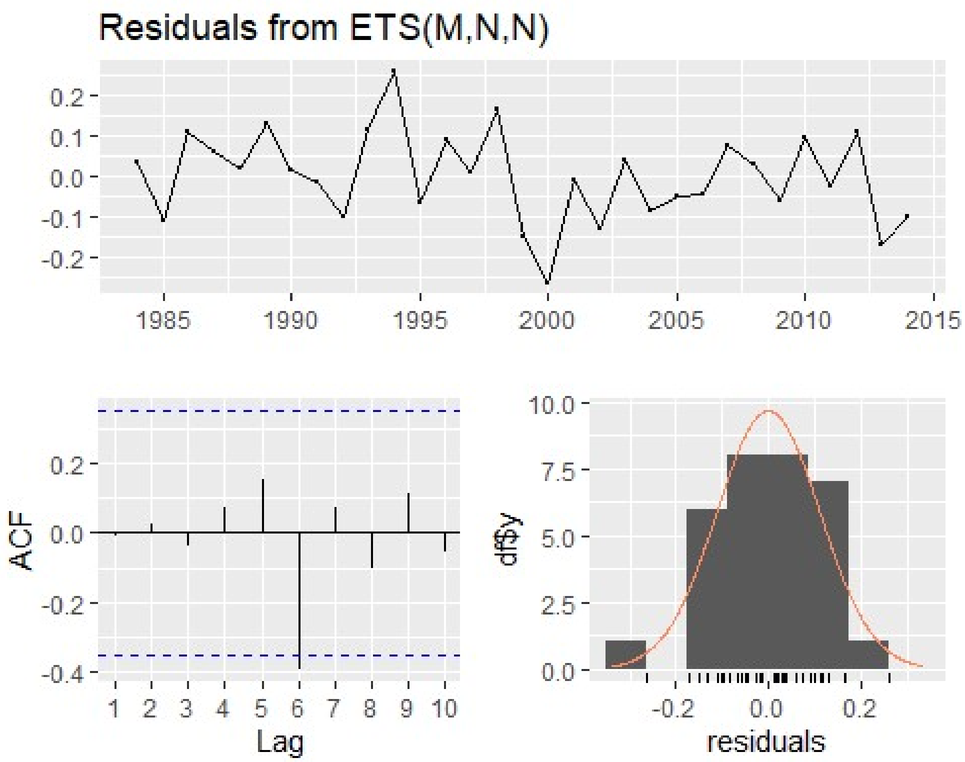
M - Multiplicative errors

N - No trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(M,N,N)
## Q* = 7.489, df = 4, p-value = 0.1122
##
## Model df: 2.   Total lags used: 6
```

Residual Analysis ETS(A, AD, A):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Now let us fit ets variable combinations individually.

```
v_ets_fit1 <- ets(v_First_Flowering_Day_data_TS, model = "ANN")
summary(v_ets_fit1)
```

```
## ETS(A,N,N)
##
## Call:
##  ets(y = v_First_Flowering_Day_data_TS, model = "ANN")
##
##   Smoothing parameters:
##     alpha = 1e-04
##
##   Initial states:
##     l = 209.4516
##
##   sigma:  23.8149
##
##      AIC     AICc      BIC
## 306.9455 307.8343 311.2474
##
## Training set error measures:
##                         ME     RMSE      MAE       MPE     MAPE     MASE
## Training set -0.004800665 23.03386 18.69589 -1.265312 9.173703 0.661411
##                      ACF1
## Training set -0.006864147
```

ETS(A, N, N)

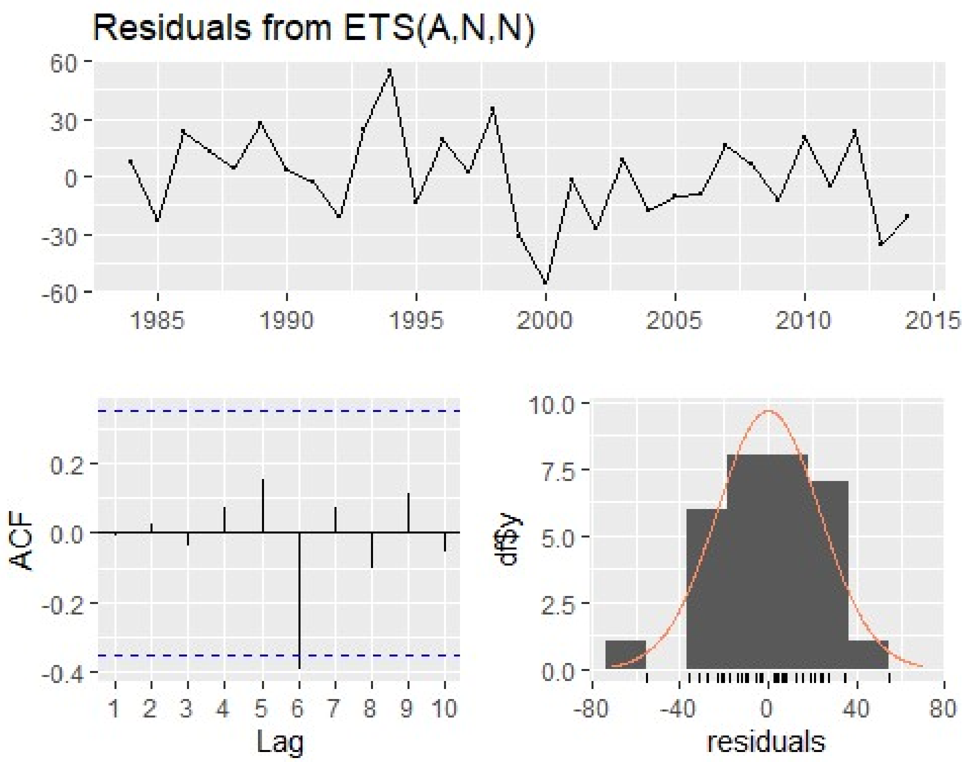
A - Additive errors

N - No trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit1)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(A,N,N)
## Q* = 7.489, df = 4, p-value = 0.1122
##
## Model df: 2.   Total lags used: 6
```

Residual Analysis ETS(A, N, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
v_ets_fit2 <- ets(v_First_Flowering_Day_data_TS, model = "AAN")
summary(v_ets_fit2)
```

```
## ETS(A,A,N)
##
## Call:
##  ets(y = v_First_Flowering_Day_data_TS, model = "AAN")
##
##   Smoothing parameters:
##     alpha = 1e-04
##     beta  = 1e-04
##
##   Initial states:
##     l = 218.839
##     b = -0.5898
##
##   sigma:  24.0066
##
##      AIC     AICc      BIC
## 309.2274 311.6274 316.3973
##
## Training set error measures:
##                      ME     RMSE      MAE       MPE     MAPE      MASE
## Training set 0.04681346 22.40433 17.52335 -1.174704 8.595767 0.6199298
##                     ACF1
## Training set -0.05273509
```

ETS(A, A, N)

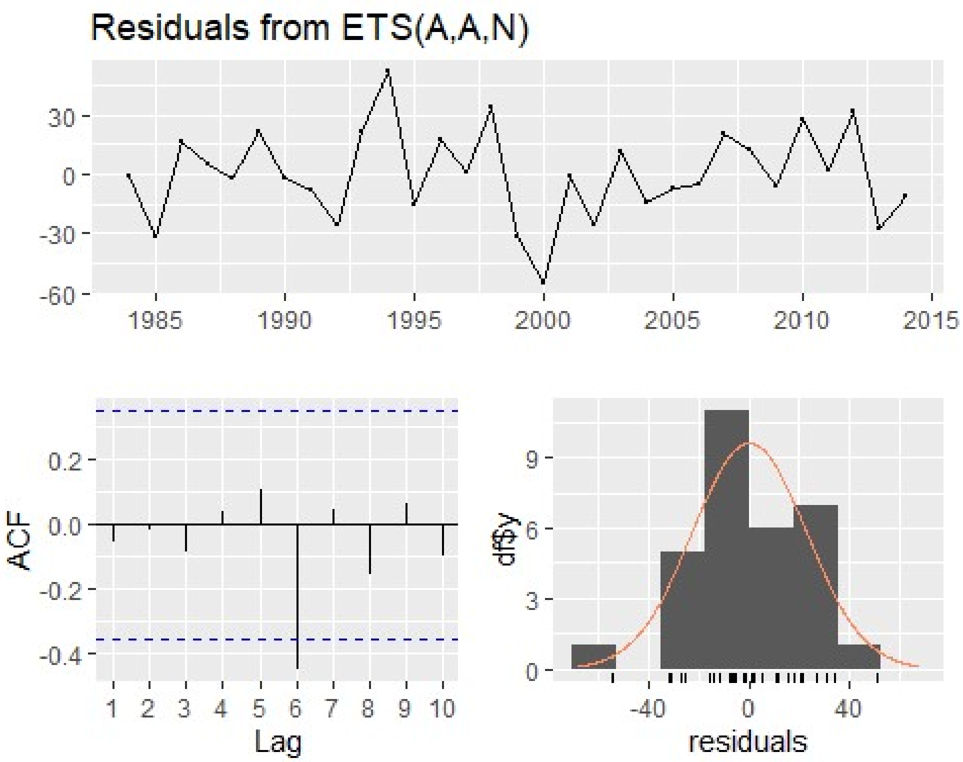
A - Additive errors

A - Additive trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit2)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(A,A,N)
## Q* = 9.1235, df = 3, p-value = 0.02769
##
## Model df: 4.   Total lags used: 7
```

Residual Analysis ETS(A, A, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
v_ets_fit3 <- ets(v_First_Flowering_Day_data_TS, model = "MNN")
summary(v_ets_fit3)
```

```
## ETS(M,N,N)
##
## Call:
##  ets(y = v_First_Flowering_Day_data_TS, model = "MNN")
##
##   Smoothing parameters:
##     alpha = 1e-04
##
##   Initial states:
##     l = 209.448
##
##   sigma:  0.1137
##
##      AIC     AICc      BIC
## 306.9448 307.8337 311.2468
##
## Training set error measures:
##                        ME     RMSE    MAE       MPE     MAPE      MASE
## Training set -0.001214657 23.03386 18.696 -1.263579 9.173601 0.6614151
##                      ACF1
## Training set -0.006864096
```

ETS(A, A, N)

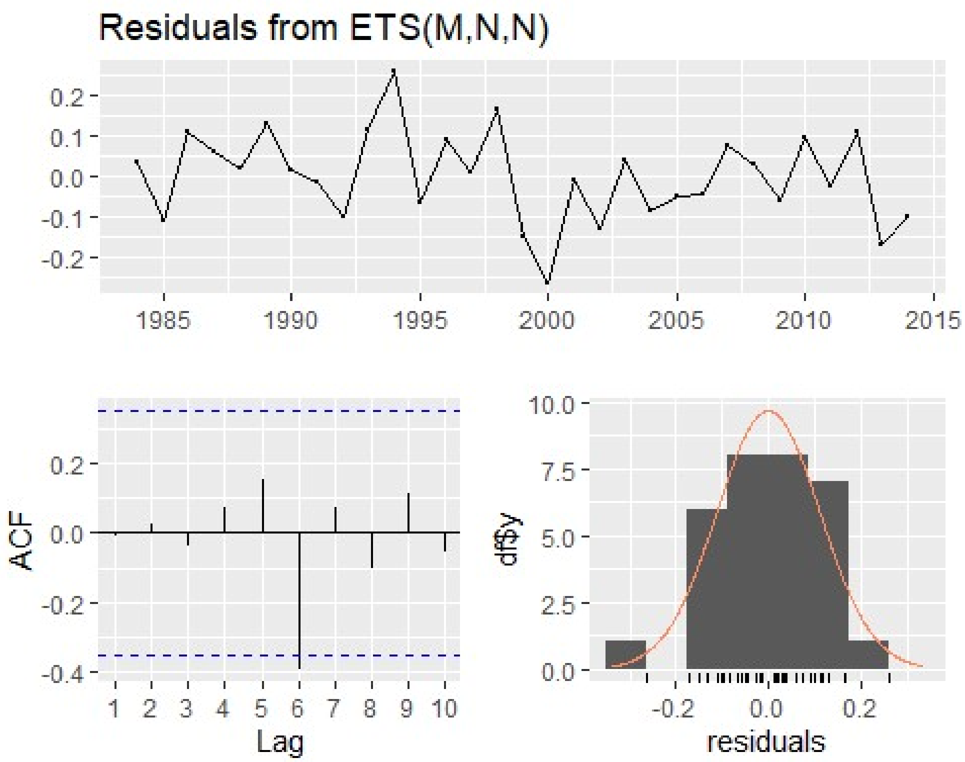
M - Multiplicative errors

N - No trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit3)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(M,N,N)
## Q* = 7.489, df = 4, p-value = 0.1122
##
## Model df: 2.   Total lags used: 6
```

Residual Analysis ETS(M, N, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
v_ets_fit4 <- ets(v_First_Flowering_Day_data_TS, model = "MAN")
summary(v_ets_fit4)
```

```
## ETS(M,A,N)
##
## Call:
##  ets(y = v_First_Flowering_Day_data_TS, model = "MAN")
##
##   Smoothing parameters:
##     alpha = 1e-04
##     beta  = 1e-04
##
##   Initial states:
##     l = 215.3797
##     b = -0.4305
##
##   sigma:  0.1155
##
##      AIC     AICc      BIC
## 309.4179 311.8179 316.5879
##
## Training set error measures:
##                     ME     RMSE      MAE        MPE     MAPE      MASE
## Training set 0.9196759 22.47637 17.60976 -0.7708524 8.605156 0.6229869
##                     ACF1
## Training set -0.04986601
```

ETS(A, A, N)

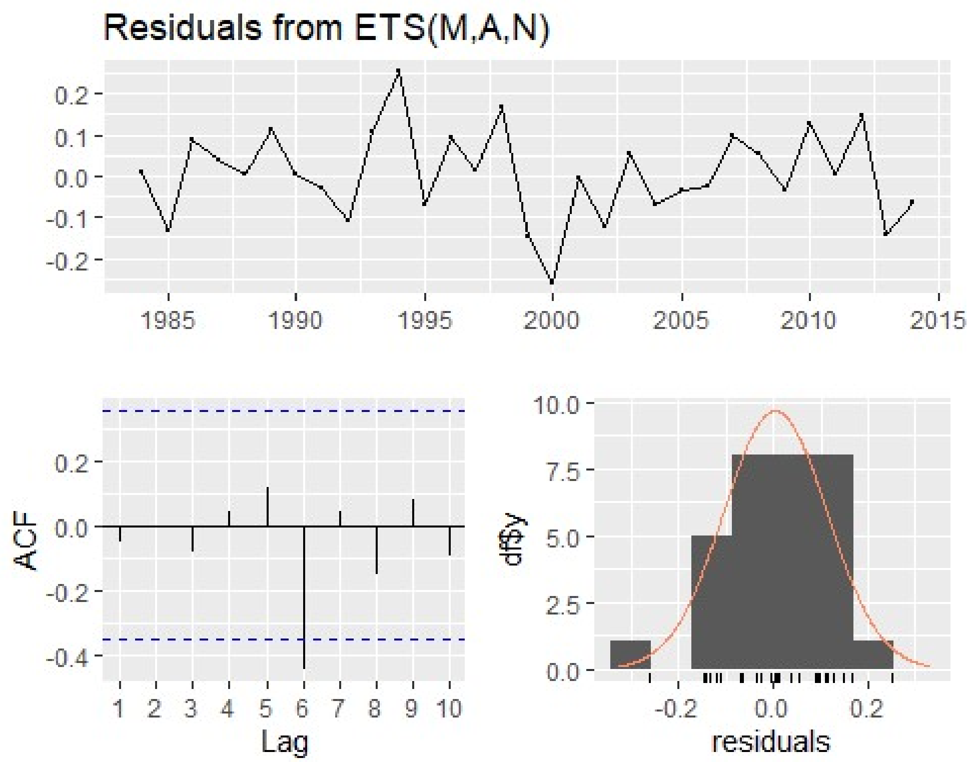
M - Multiplicative errors

A - Additive trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit4)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(M,A,N)
## Q* = 8.9614, df = 3, p-value = 0.02981
##
## Model df: 4.   Total lags used: 7
```

Residual Analysis ETS(M, A, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

```
v_ets_fit5 <- ets(v_First_Flowering_Day_data_TS, model = "MMN")
summary(v_ets_fit5)
```

```
## ETS(M,M,N)
##
## Call:
##  ets(y = v_First_Flowering_Day_data_TS, model = "MMN")
##
##   Smoothing parameters:
##     alpha = 1e-04
##     beta  = 1e-04
##
##   Initial states:
##     l = 216.4878
##     b = 0.9977
##
##   sigma:  0.1153
##
##      AIC     AICc      BIC
## 309.3333 311.7333 316.5032
##
## Training set error measures:
##                     ME   RMSE      MAE        MPE     MAPE      MASE
## Training set 0.8316197 22.443 17.48424 -0.8068944 8.548035 0.6185461
##                     ACF1
## Training set -0.05190951
```

ETS(M, M, N)

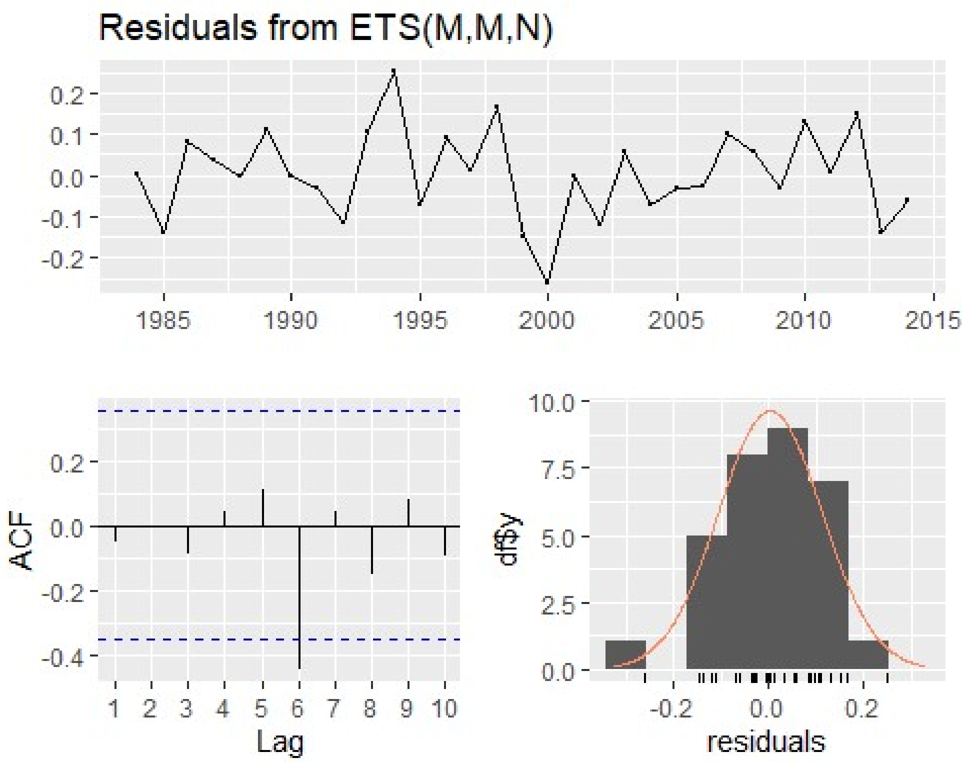
M - Multiplicative errors

M - Multiplicative trend

N - No seasonality.

Let us perform residual analysis on this ETS model.

```
checkresiduals(v_ets_fit5)
```




```
##
##  Ljung-Box test
##
## data:  Residuals from ETS(M,M,N)
## Q* = 9.0495, df = 3, p-value = 0.02864
##
## Model df: 4.   Total lags used: 7
```

Residual Analysis ETS(M, M, N):

1. The data points are below the line at the start and below the line at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (~ 0.01) from Shapiro-Wilk normality test is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Suggesting some normality.

Comparitively, based on MASE and RMSE score ets(M, N, N) is better.

Among all the methods, holt in exponential smoothing with damped trend.

Forecasting

Let us forecast for the next 4 years on First Flowering Day series. From 2015 to 2018. For the optimal model from each method.

Forcasting with Smoothing method best model

```
fit <- holt(v_First_Flowering_Day_data_TS, damped = TRUE, h = 4)

v_FFD_forecasts <- ts.intersect(ts(fit$lower[, 2], start = c(2015), frequency
= 1), ts(fit$mean, start = c(2015), frequency = 1), ts(fit$upper[, 2], start
= c(2015), frequency = 1))
colnames(v_FFD_forecasts) <- c("Lower bound", "Point forecast", "Upper
bound")

v_FFD_forecasts

## Time Series:
## Start = 2015
## End = 2018
## Frequency = 1
##      Lower bound Point forecast Upper bound
## 2015    155.0245       203.4051    251.7856
## 2016    154.8263       203.2069    251.5874
## 2017    154.6336       203.0142    251.3947
## 2018    154.4462       202.8268    251.2073
```

Now let us plot the forecast.

```
plot(fit, fcol = "white", main = "Forecast of First Flowering Day series for
the next 3 years (2015, 2017)", ylab = "First Flowering Day")
lines(fitted(fit), col = "red")
lines(fit$mean, col = "blue", lwd = 2)
legend("bottom", inset = .03, cex = 0.9, box.lty = 2, box.lwd = 2, pch = 1,
lty = 1, col = c("red", "blue"), c("Data", "Forecasts"))
```

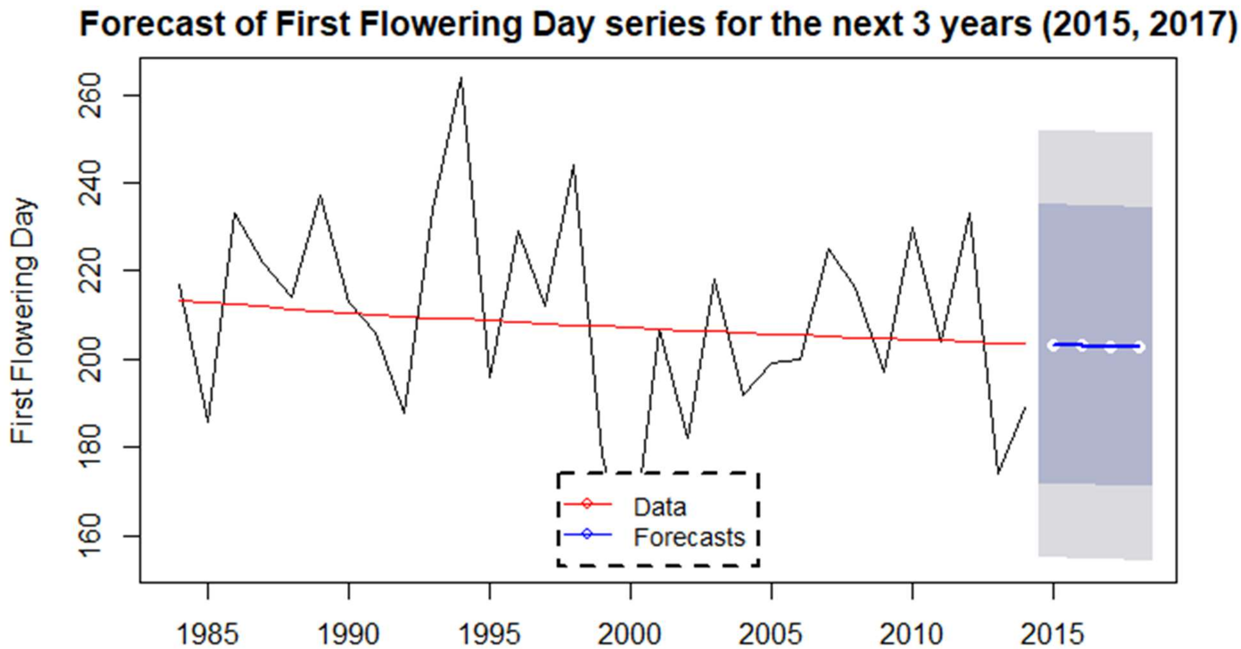


Fig 2.21: Next 4 years forecast on the First Flowering Day Series with Smoothing method model.

From the four year forecast results we can predict that there will be decrease in the First Flowering Day in the future. This suggests that the impact of the chemical components will be less in future.

Forcasting with ets method best model

```
fit3 <- ets(v_First_Flowering_Day_data_TS, model="MMN", damped = T)
fit1 <- forecast.ets(fit3, h = 4)
plot(fit1, fcol = "white", main = "First Flowerind Day series with 4 years
ahead forecasts", ylab = "Radiation")
lines(fitted(fit1), col = "darkgreen")
lines(fit1$mean, col = "darkgreen", lwd = 2)
legend("topleft", lty = 1, col = c("black", "darkgreen"), c("Data",
"Forecasts"))
```

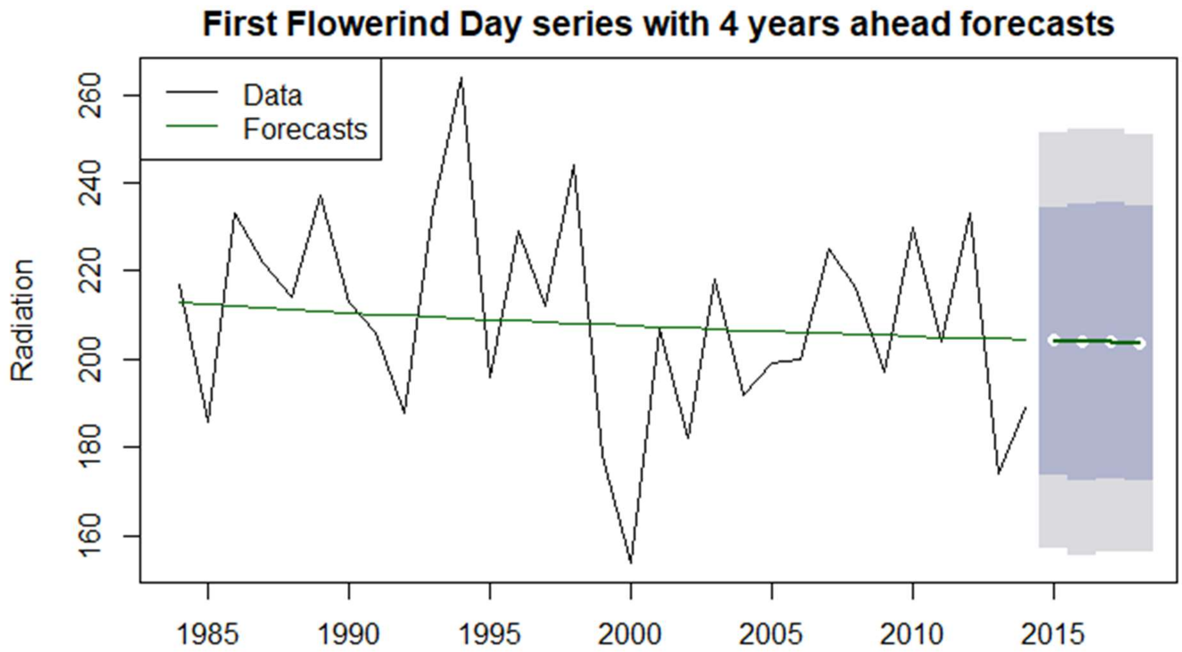


Fig 2.22: Next 4 years forecast on the First Flowering Day Series with ets method model.

```
forecasts <- ts.intersect(ts(fit1$lower[, 2], start = c(2015), frequency =
1), ts(fit1$mean, start = c(2015), frequency = 1), ts(fit1$upper[, 2], start
= c(2015), frequency = 1))
colnames(forecasts) <- c("Lower bound", "Point forecast", "Upper bound")
forecasts

## Time Series:
## Start = 2015
## End = 2018
## Frequency = 1
##      Lower bound Point forecast Upper bound
## 2015    157.2584       204.2685    251.0862
## 2016    154.4180       204.0840    252.5719
## 2017    155.1222       203.9041    249.3414
## 2018    154.7424       203.7289    251.1135
```

From the four year forecast results we can predict that there will be decrease in the First Flowering Day in the future. This suggests that the impact of the chemical components will be less in future.

Task 3

Part (a)

Data

The data here used is the the contemporaneous yearly averaged climate variables measured from 1984 – 2014 (31 years).

```
v_RBO_data <- read.csv("RBO.csv", header = TRUE)
head(v_RBO_data)

##   ï..Year       RBO Temperature Rainfall Radiation RelHumidity
## 1    1984 0.7550088    9.371585 2.489344  14.87158    93.92650
## 2    1985 0.7407520    9.656164 2.475890  14.68493    94.93589
## 3    1986 0.8423860    9.273973 2.421370  14.51507    94.09507
## 4    1987 0.7484425    9.219178 2.319726  14.67397    94.49699
## 5    1988 0.7984084   10.202186 2.465301  14.74863    94.08142
## 6    1989 0.7938803    9.441096 2.735890  14.78356    96.08685

# Using str() to check the type of each column.
str(v_RBO_data)

## 'data.frame':    31 obs. of  6 variables:
##  $ ï..Year    : int  1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 ...
##  $ RBO        : num  0.755 0.741 0.842 0.748 0.798 ...
##  $ Temperature: num  9.37 9.66 9.27 9.22 10.2 ...
##  $ Rainfall   : num  2.49 2.48 2.42 2.32 2.47 ...
##  $ Radiation  : num  14.9 14.7 14.5 14.7 14.7 ...
##  $ RelHumidity: num  93.9 94.9 94.1 94.5 94.1 ...
```

Checking for Missing values.

```
colSums(is.na(v_RBO_data))

##     ï..Year         RBO Temperature    Rainfall   Radiation RelHumidity
##           0           0           0           0           0           0
```

There are no missing values in the data.

Checking the class of v_solar_data. (It should be a data frame.)

```
class(v_RBO_data)

## [1] "data.frame"

v_RBO_Temp_TS <- ts(v_RBO_data$Temperature, start = c(1984), frequency = 1)
v_RBO_Rainfall_TS <- ts(v_RBO_data$Rainfall, start = c(1984), frequency = 1)
v_RBO_Radiation_TS <- ts(v_RBO_data$Radiation, start = c(1984), frequency =
1)
v_RBO_RelHumidity_TS <- ts(v_RBO_data$RelHumidity, start = c(1984), frequency
```

```
= 1)
v_RBO_data_TS <- ts(v_RBO_data$RBO, start = c(1984), frequency = 1)
```

Confirming the class of each time series object.

```
class(v_RBO_Temp_TS)

## [1] "ts"

class(v_RBO_Rainfall_TS)

## [1] "ts"

class(v_RBO_Radiation_TS)

## [1] "ts"

class(v_RBO_RelHumidity_TS)

## [1] "ts"

class(v_RBO_data_TS)

## [1] "ts"
```

Now let us perform descriptive analysis on each time series object.

Descriptive Analysis

Rank-based Order similarity metric

```
plot(v_RBO_data_TS, type = "b", xlab = "years", ylab = "Rank-based Order
similarity metric", main = "Time series plot for yearly Rank-based Order
similarity metric from 1984 - 2014 (31 years)", pch = 1)
legend("topright", inset = .03, title = "Rank-based Order similarity metric",
legend = "Rank-based Order similarity metric series", horiz = TRUE, cex =
0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

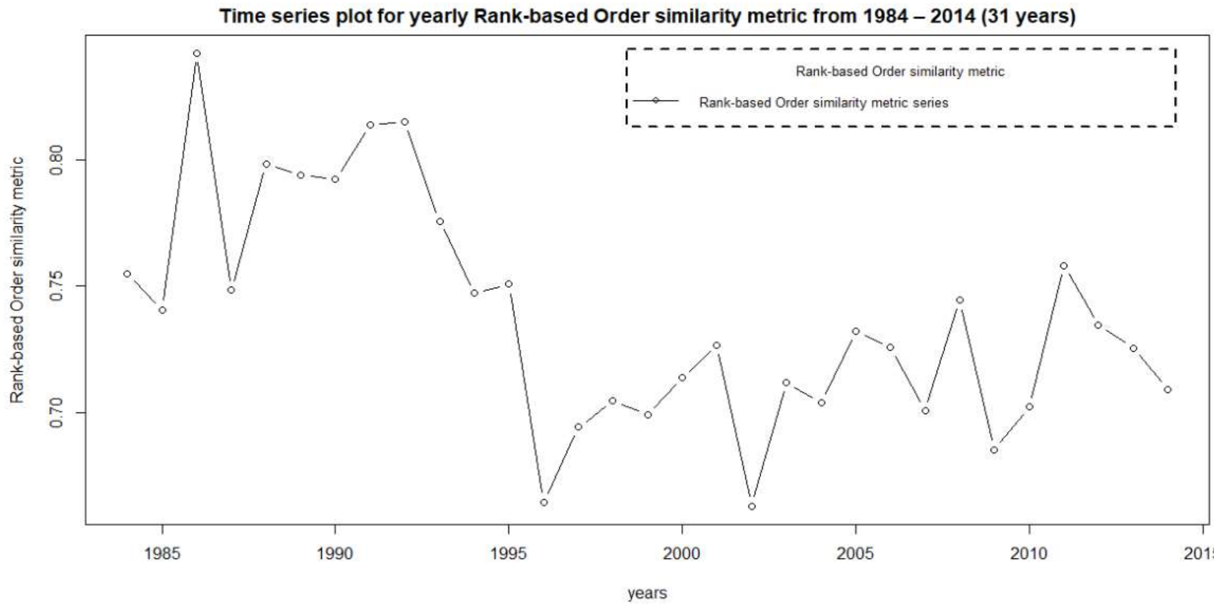


Fig 3.1: Rank-based Order similarity metric - Time series plot.

```
McLeod.Li.test(y = v_RBO_data_TS, main = "McLeod-Li Test Statistics for Rank-based Order similarity metric.")
```

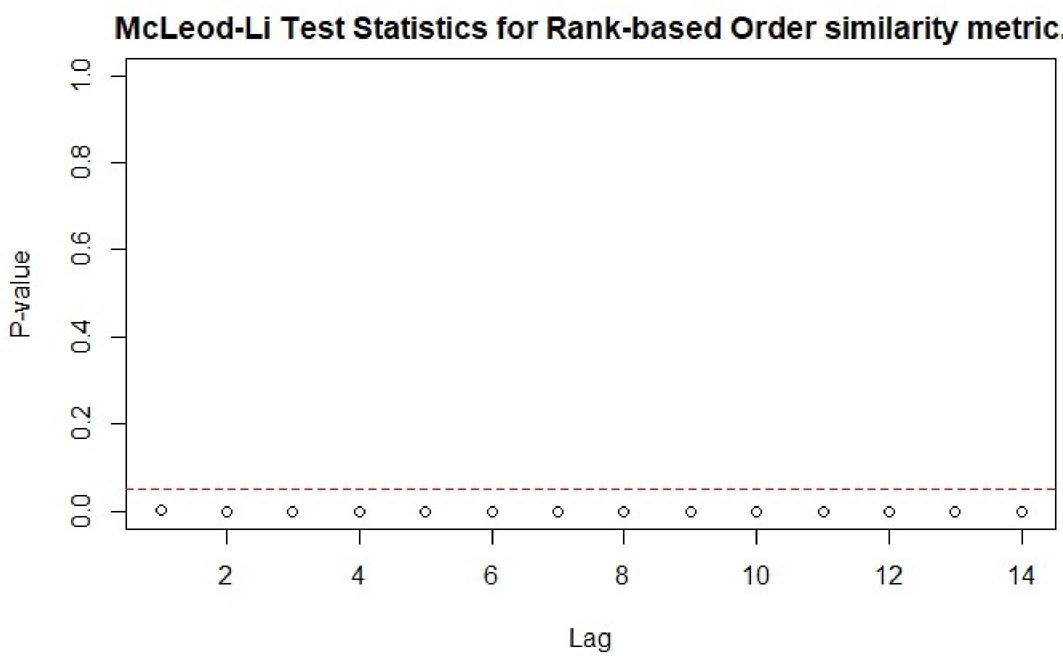


Fig 3.2: McLeod-Li Test Statistics for Rank-based Order similarity metric.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.

2. There is an intervention around multiple years.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Temperature

```
plot(v_RBO_Temp_TS, type = "b", xlab = "years", ylab = "Temperature", main =
"Time series plot for yearly temperature from 1984 - 2014 (31 years)", pch =
1)
legend("top", inset = .03, title = "Temperature", legend = "Temperature
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

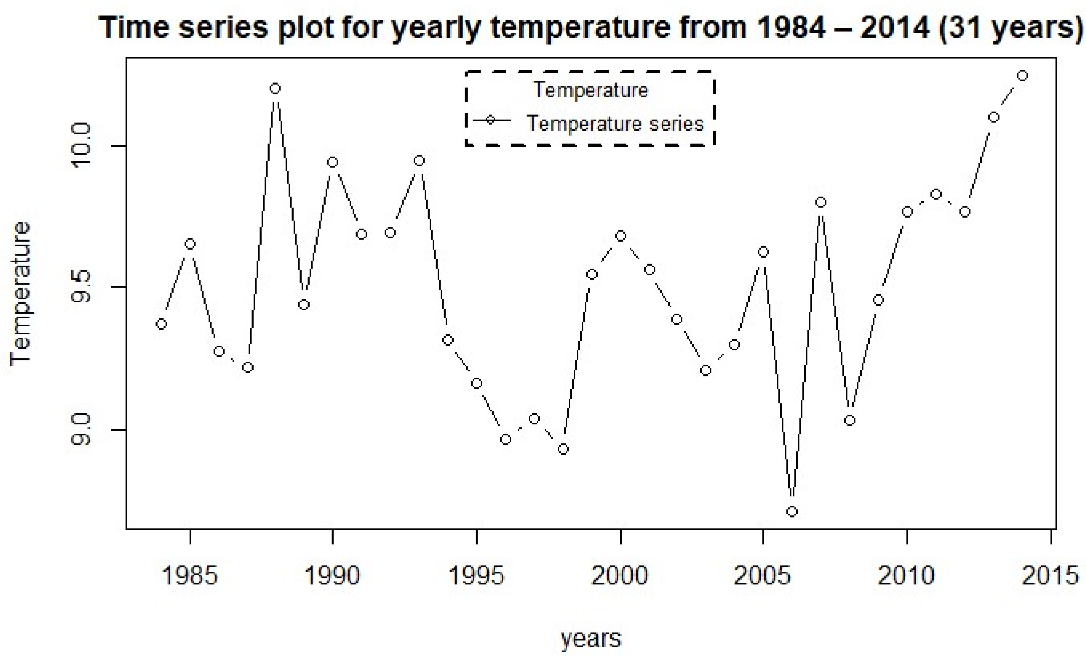


Fig 3.3: Temperature - Time series plot.

```
McLeod.Li.test(y = v_RBO_Rainfall_TS, main = "McLeod-Li Test Statistics for
Temperature")
```

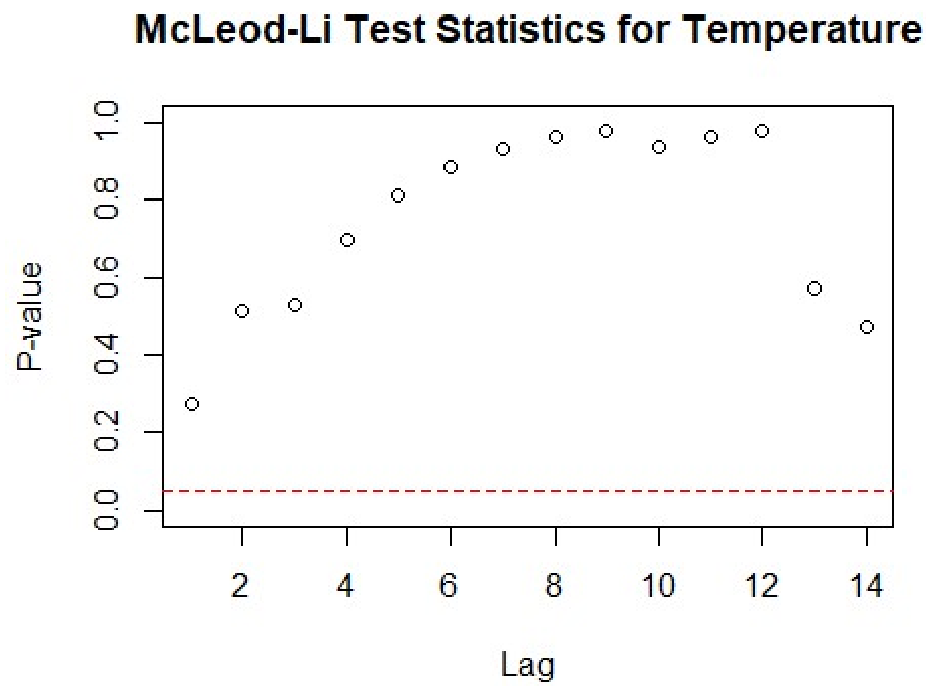


Fig 3.4: McLeod-Li Test Statistics for Temperature

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1996.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Rainfall

```
plot(v_RBO_Rainfall_TS, type = "b", xlab = "years", ylab = "Rainfall", main =
"Time series plot for yearly Rainfall from 1984 - 2014 (31 years)", pch = 1)
legend("bottomleft", inset = .03, title = "Rainfall", legend = "Rainfall
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

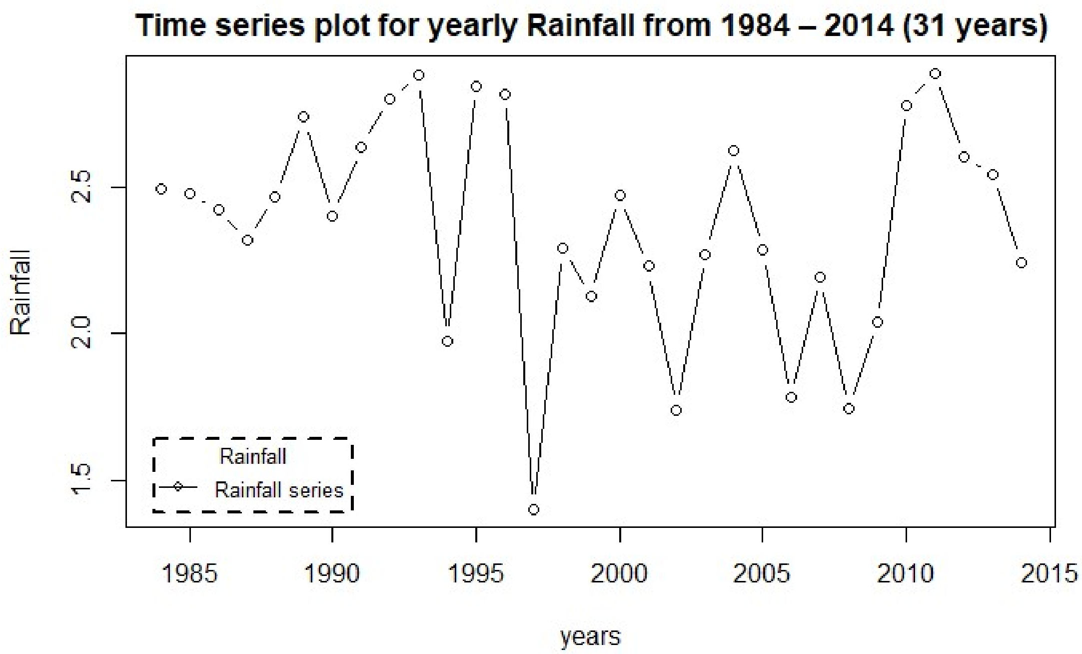


Fig 3.5: Rainfall - Time series plot.

```
McLeod.Li.test(y = v_RBO_Rainfall_TS, main = "McLeod-Li Test Statistics for
Rainfall")
```

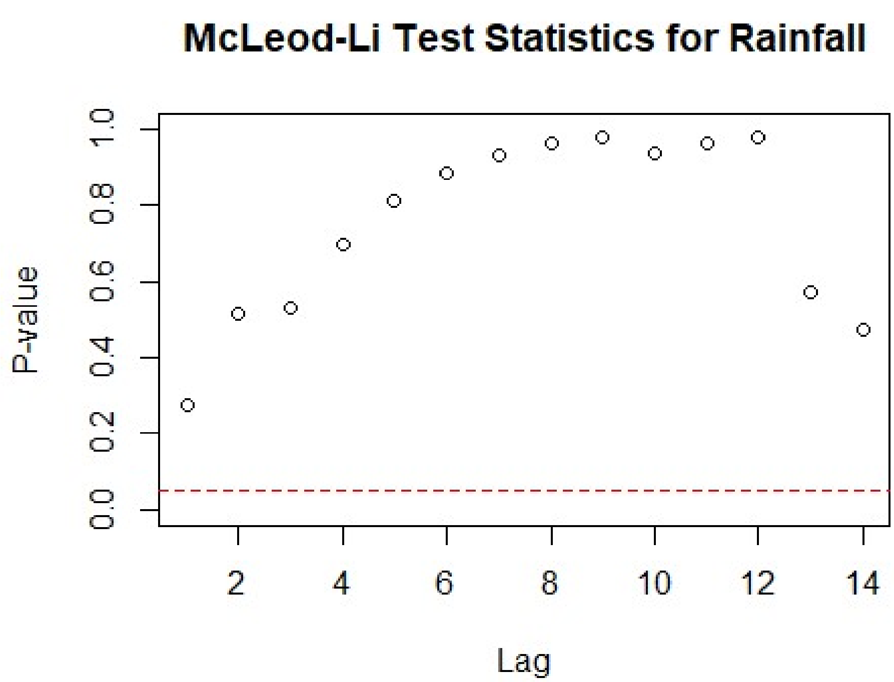


Fig 3.6: McLeod-Li Test Statistics for Rainfall

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1996.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Radiation

```
plot(v_RBO_Radiation_TS, type = "b", xlab = "years", ylab = "Radiation", main
= "Time series plot for yearly Radiation from 1984 – 2014 (31 years)", pch =
1)
legend("topleft", inset = .03, title = "Radiation", legend = "Radiation
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

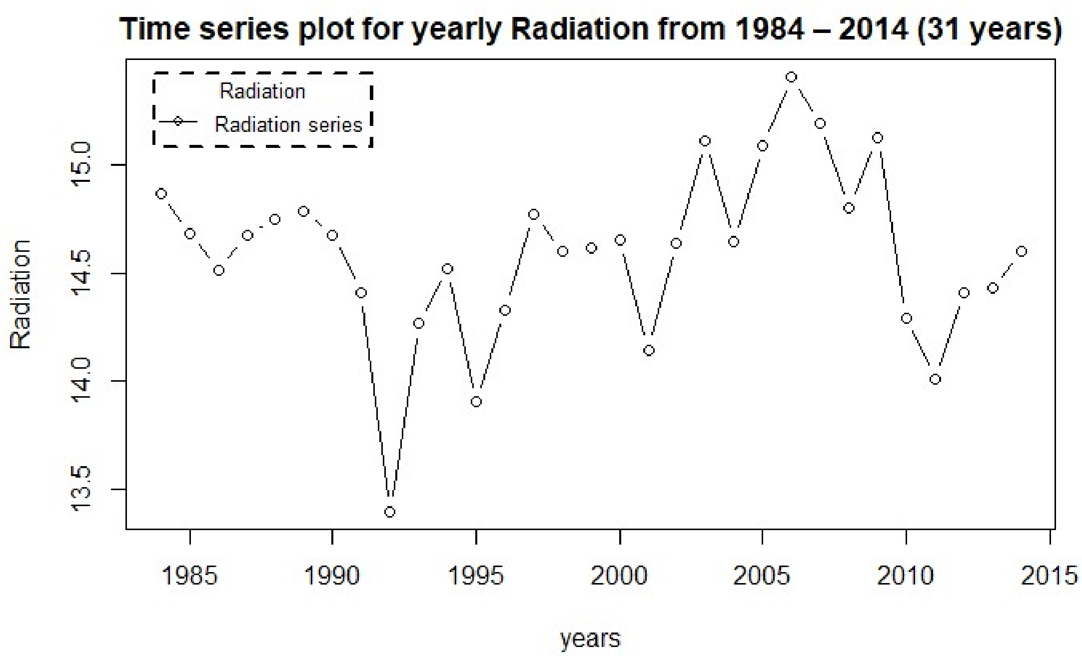


Fig 3.7: Radiation - Time series plot.

```
McLeod.Li.test(y = v_RBO_Rainfall_TS, main = "McLeod-Li Test Statistics for
Radiation")
```

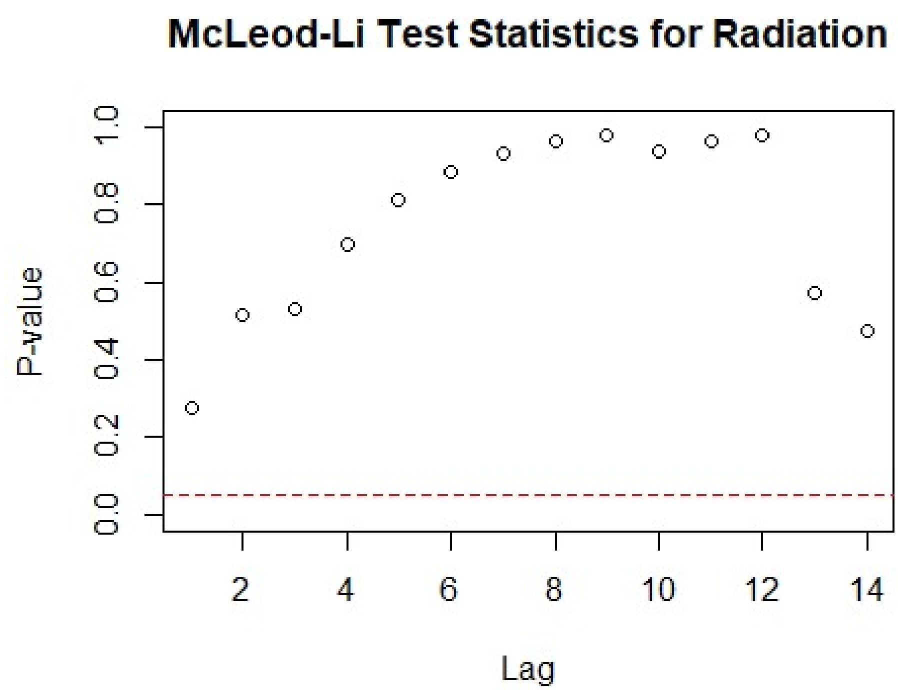


Fig 3.8: McLeod-Li Test Statistics for Radiation

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1992.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Relative Humidity

```
plot(v_RBO_RelHumidity_TS, type = "b", xlab = "years", ylab = "Relative
Humidity", main = "Time series plot for yearly Relative Humidity from 1984 –
2014 (31 years)", pch = 1)
legend("bottomright", inset = .03, title = "Relative Humidity", legend =
"Relative Humidity series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2,
box.lwd = 2, pch = 1)
```

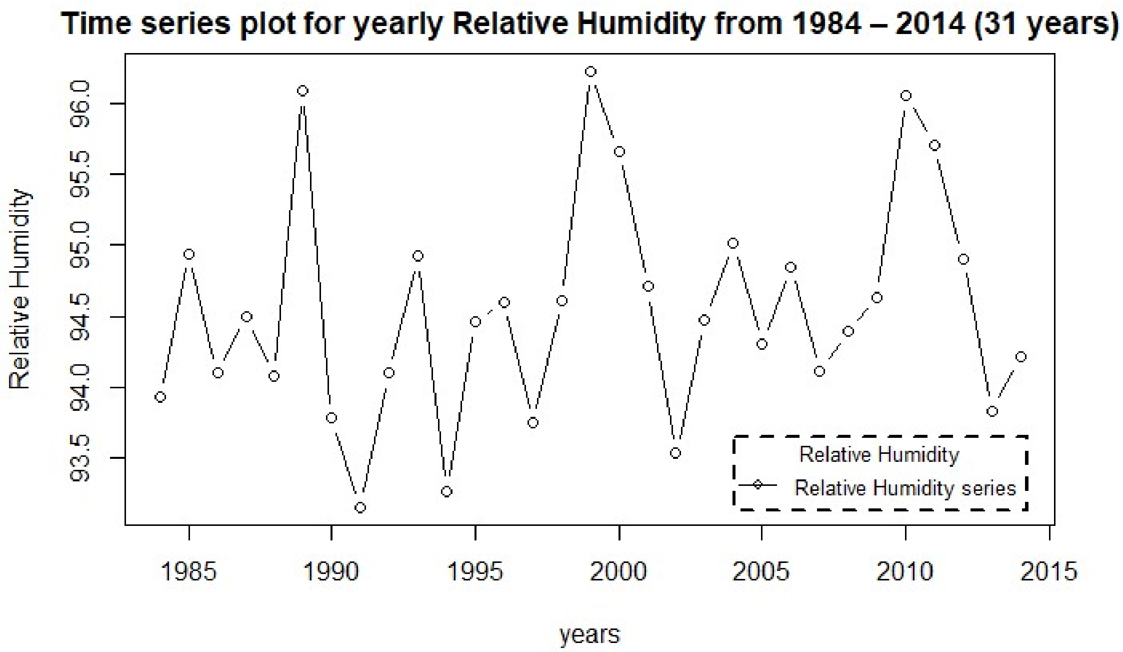


Fig 3.9: Relative Humidity - Time series plot.

```
McLeod.Li.test(y = v_RBO_RelHumidity_TS, main = "McLeod-Li Test Statistics for Relative Humidity")
```

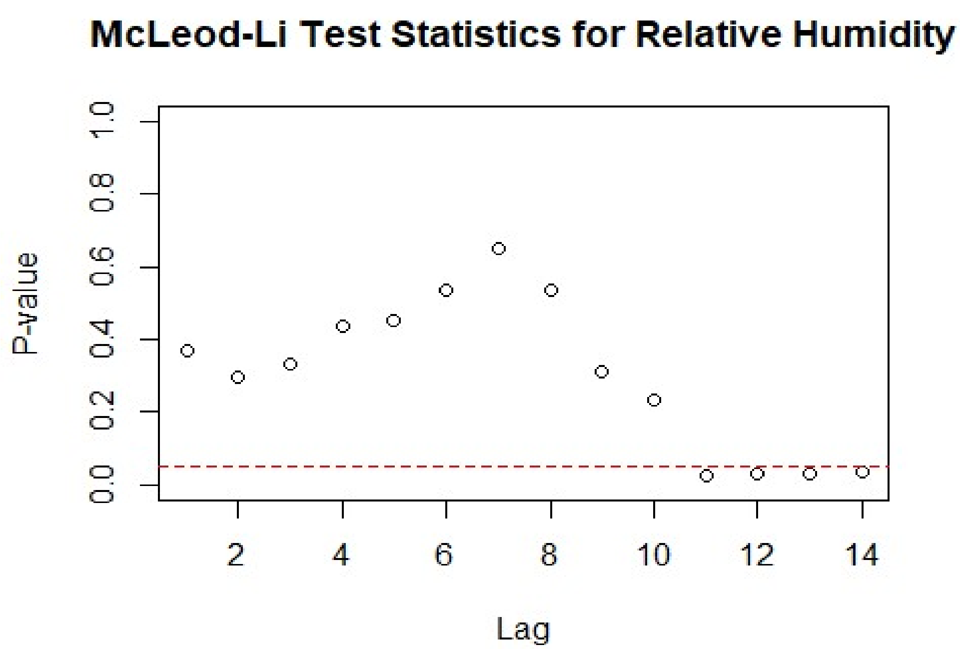


Fig 3.10: McLeod-Li Test Statistics for Relative Humidity.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1989.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Checking for Stationary in the series

Checking for Stationary on Rank-based Order similarity metric series.

```
Stationary_Check(v_RBO_data_TS, "Rank-based Order similarity metric - ACF plot", "Rank-based Order similarity metric - PACF plot")
```

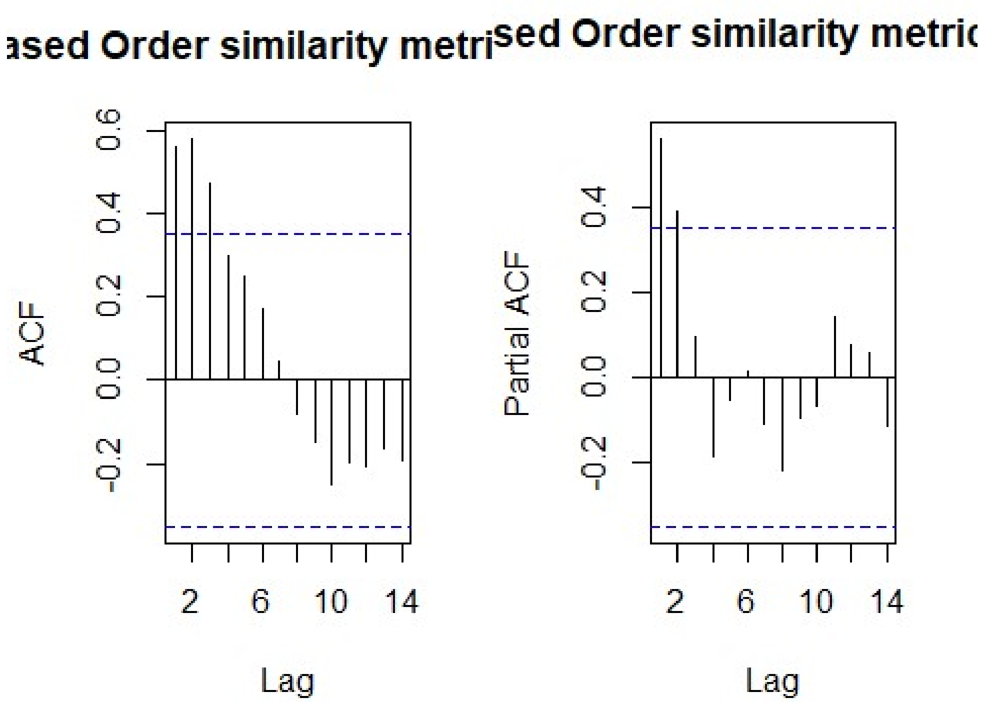


Fig 3.11: Rank-based Order similarity metric - ACF

Fig 3.12: Rank-based Order similarity metric - ACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
```

```
## Dickey-Fuller = -1.4542, Lag order = 2, p-value = 0.7829
## alternative hypothesis: stationary
```

There are significant lags in the ACF and PACF plot suggesting the stochastic component is not white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: 0.7829 > 0.05

p - value is greater than 0.05 and hence the test is not statistically significant. Therefore, Null hypothesis cannot be rejected i.e., The data is not stationary.

Therefore, the Rank - based Order similarity metric series is not Stationary.

Checking for Stationary on Temperature data.

```
Stationary_Check(v_RBO_Temp_TS, "Temperature - ACF plot", "Temperature - PACF plot")
```

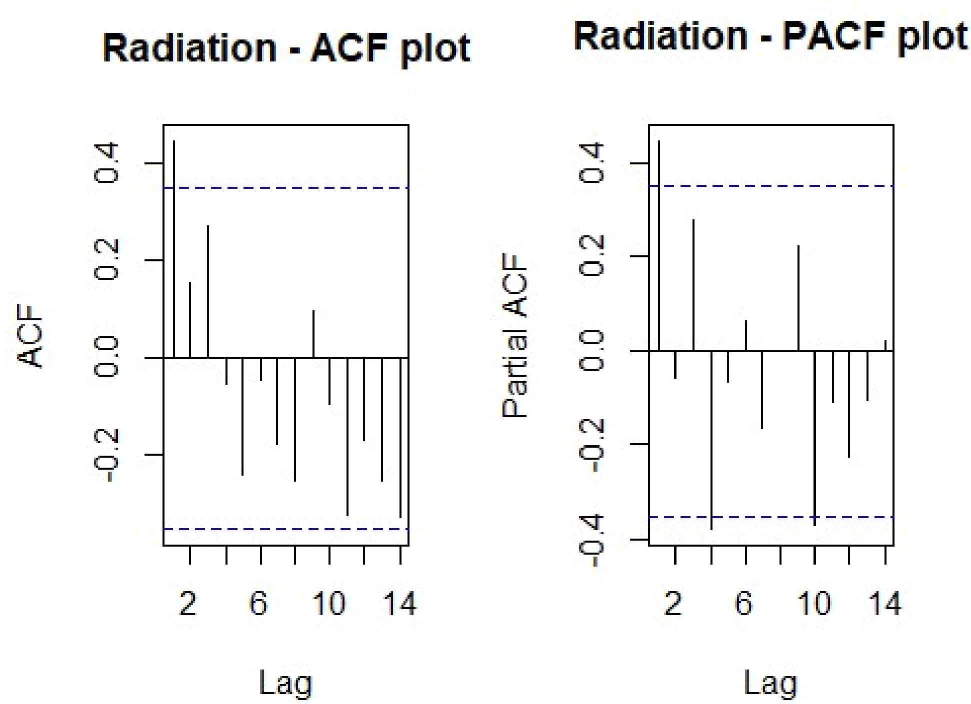


Fig 3.13: Temperature - ACF

Fig 3.14: Temperature - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -1.1484, Lag order = 2, p-value = 0.9002
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: 0.9002 > 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Temperature series is Stationary.

Checking for Stationary on Radiation data.

```
Stationary_Check(v_RBO_Radiation_TS, "Radiation - ACF plot", "Radiation - PACF plot")
```

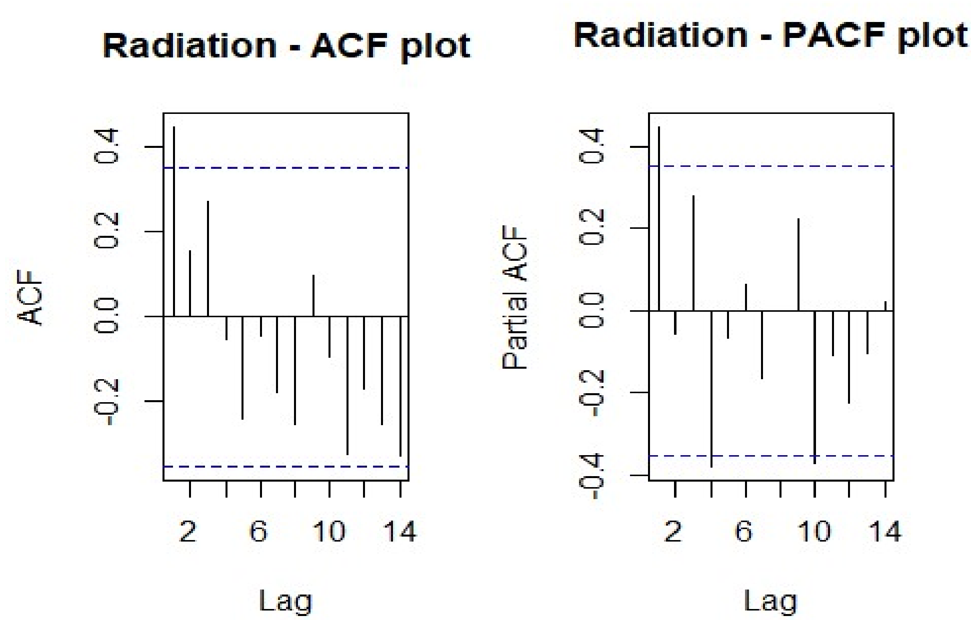


Fig 3.15: Radiation - ACF

Fig 3.16: Radiation - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -2.7317, Lag order = 4, p-value = 0.2911
## alternative hypothesis: stationary
```

The is only one significant lag in the ACF and PACF plot.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: 0.2911 > 0.05

p - value is greater than 0.05 and hence the test is not statistically significant. Therefore, Null hypothesis cannot be rejected i.e., The data is not stationary.

Therefore, the Radiation series is not Stationary.

Checking for Stationary on Rainfall data.

```
Stationary_Check(v_RBO_Rainfall_TS, "Rainfall - ACF plot", "Rainfall - PACF plot")
```

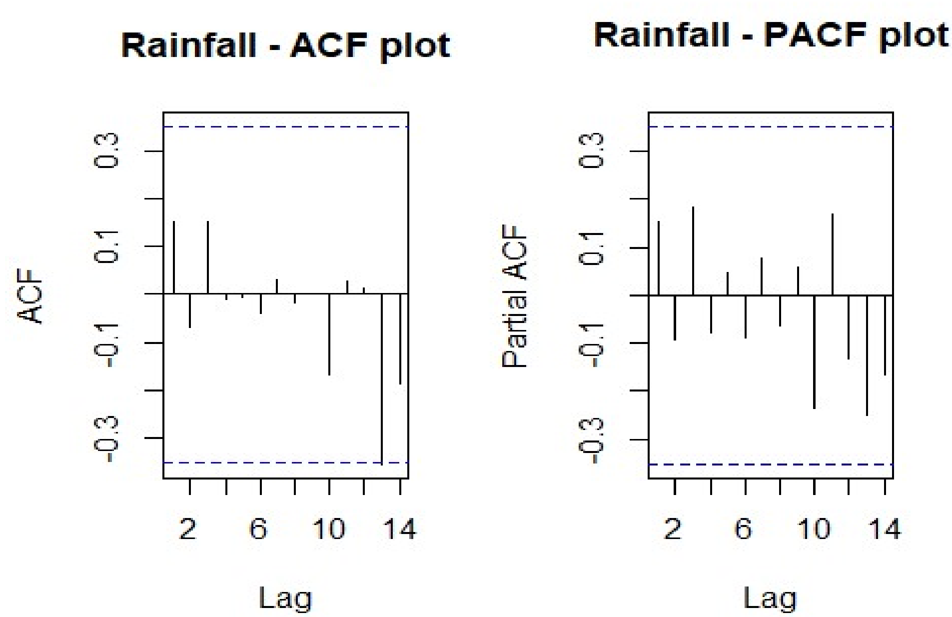


Fig 3.17: Rainfall - ACF

Fig 3.18: Rainfall - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -4.5622, Lag order = 0, p-value = 0.01
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H0: The data is not stationary.
HA: The data is stationary.

Interpretations:

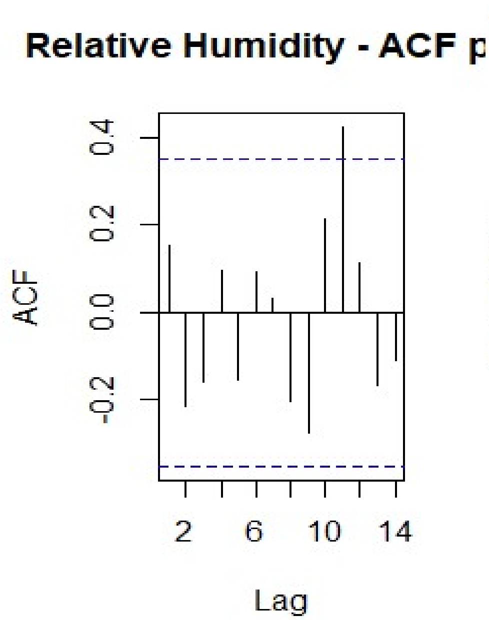
p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Rainfall series is Stationary.

Checking for Stationary on Relative Humidity data.

```
Stationary_Check(v_RBO_RelHumidity_TS, "Relative Humidity - ACF plot",
"Relative Humidity - PACF plot")
```



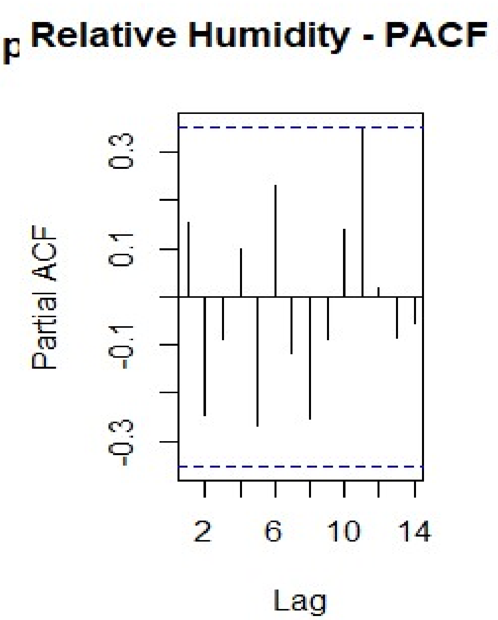


Fig 3.19: Relative Humidity - ACF

Fig 3.20: Relative Humidity - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -4.5749, Lag order = 0, p-value = 0.01
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Relative Humidity series is Stationary.

Suitable distributed lag model.

Before this let us find the correlation between the series.

```
# Calculating the correlation coefficient
cor(v_RBO_data_TS, v_RBO_Temp_TS)

## [1] 0.2610007

cor(v_RBO_data_TS, v_RBO_Rainfall_TS)

## [1] 0.3932282

cor(v_RBO_data_TS, v_RBO_Radiation_TS)

## [1] -0.3173602

cor(v_RBO_data_TS, v_RBO_RelHumidity_TS)

## [1] -0.1776349
```

This suggests that FFD has a better correlation with Rainfall and Radiation.

As we are going to forecast the FFD data, our dependent variable “y” will be Mortality Rate series object and independent variable “x” will be Rainfall and Temperature.

Finite distributed lag model Rainfall

```
x1 = v_RBO_Rainfall_TS # Independent variable
x2 = v_RBO_Temp_TS # Independent variable
y = v_RBO_data_TS # Dependent variable
```

```
for ( i in 1:10){
  model_1 = dlm(x = as.vector(x1) , y = as.vector(y), q = i )
  cat("q = ", i, "AIC = ", AIC(model_1$model), "BIC = ", BIC(model_1$model),
"MASE =", MASE(model_1)$MASE, "\n")
  }

## q =  1 AIC =  -100.898 BIC =  -95.29319 MASE = 0.9417954
## q =  2 AIC =  -96.70956 BIC =  -89.87308 MASE = 0.9993747
## q =  3 AIC =  -97.19966 BIC =  -89.20643 MASE = 0.9796852
## q =  4 AIC =  -90.46187 BIC =  -81.39101 MASE = 1.038827
## q =  5 AIC =  -87.24242 BIC =  -77.17765 MASE = 0.925677
## q =  6 AIC =  -82.31788 BIC =  -71.348 MASE = 0.8543964
## q =  7 AIC =  -77.98405 BIC =  -66.20351 MASE = 0.829337
## q =  8 AIC =  -76.81922 BIC =  -64.32879 MASE = 0.7233794
## q =  9 AIC =  -80.79432 BIC =  -67.70181 MASE = 0.6205897
## q =  10 AIC =  -76.93255 BIC =  -63.35376 MASE = 0.585617
```

As we have the least AIC, BIC and MASE values at q = 10. Let us fit the finite distributed lag model with q = 10.

```
# Finite lag length based on AIC-BIC-MASE

finite_dlm_rain = dlm( x = as.vector(x1) , y = as.vector(y), q = 10)
summary(finite_dlm_rain)

## 
## Call:
## lm(formula = model.formula, data = design)
## 
## Residuals:
##       Min        1Q    Median        3Q       Max 
## -0.050274 -0.013229 -0.001445  0.015071  0.039030 
## 
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  0.709414   0.129781   5.466 0.000397 ***
## x.t          0.016449   0.019478   0.845 0.420260    
## x.1          0.009100   0.017831   0.510 0.622079    
## x.2          0.014628   0.018404   0.795 0.447163    
## x.3         -0.006321   0.018174  -0.348 0.735980    
## x.4         -0.006181   0.020176  -0.306 0.766285    
## x.5          0.004570   0.020010   0.228 0.824453    
## x.6         -0.007054   0.019391  -0.364 0.724424    
## x.7         -0.011836   0.021444  -0.552 0.594424    
## x.8          0.004407   0.020473   0.215 0.834366    
## x.9         -0.021710   0.021728  -0.999 0.343817    
## x.10         0.006802   0.022752   0.299 0.771745    
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03187 on 9 degrees of freedom
## Multiple R-squared:  0.3261, Adjusted R-squared:  -0.4975
## F-statistic: 0.3959 on 11 and 9 DF,  p-value: 0.9251
##
## AIC and BIC values for the model:
##         AIC       BIC
## 1 -76.93255 -63.35376
```

Hypotheses:

H0: The data doesn't fit the Finite distributed lag model.
HA: The data fits the Finite distributed lag model.

Interpretations:

F - statistic is 0.3959
R - squared is 0.3261
Adjusted R - squared is -0.4975
Degrees of freedom - DF are (11, 9)
p - value (0.9251) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model does not fit the Finite distributed lag model with slope.

No residual analysis is required.

Therefore, Further analysis is needed by adding polynomial to the lag model.

Polynomial distributed lag model with Rainfall

```
for (i in 1:3){
  model_3 <-  polyDlm(x = as.vector(x1), y = as.vector(y), q = i , k = i,
show.beta = FALSE)
  cat("q = ", i, "k = ", i, "AIC = ", AIC(model_3$model), "BIC = ",
BIC(model_3$model), "MASE = ", MASE(model_3)$MASE, "\n")
}

## q =  1 k =  1 AIC =  -100.898 BIC =  -95.29319 MASE =  0.9417954
## q =  2 k =  2 AIC =  -96.70956 BIC =  -89.87308 MASE =  0.9993747
## q =  3 k =  3 AIC =  -97.19966 BIC =  -89.20643 MASE =  0.9796852
```

Let us fit a polynomial model of order 3. Since least AIC and BIC scores.

```
# Ploynomial DLM

PolyDLM_model_Rain = polyDlm(x = as.vector(x1), y = as.vector(y), q = 1, k =
1, show.beta = TRUE)

## Estimates and t-tests for beta coefficients:
##        Estimate Std. Error t value P(>|t|)
```

```
## beta.0   0.0420     0.0206    2.04  0.0505
## beta.1   0.0303     0.0206    1.47  0.1520

summary(PolyDLM_model_Rain)

##
## Call:
## "Y ~ (Intercept) + X.t"
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.105903 -0.024178 -0.006166  0.014773  0.099699
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.56594    0.06397   8.847 1.84e-09 ***
## z.t0         0.04199    0.02058   2.040   0.0512 .
## z.t1        -0.01167    0.03126  -0.373   0.7117
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04153 on 27 degrees of freedom
## Multiple R-squared:  0.2156, Adjusted R-squared:  0.1575
## F-statistic: 3.711 on 2 and 27 DF,  p-value: 0.03767
```

Hypotheses:

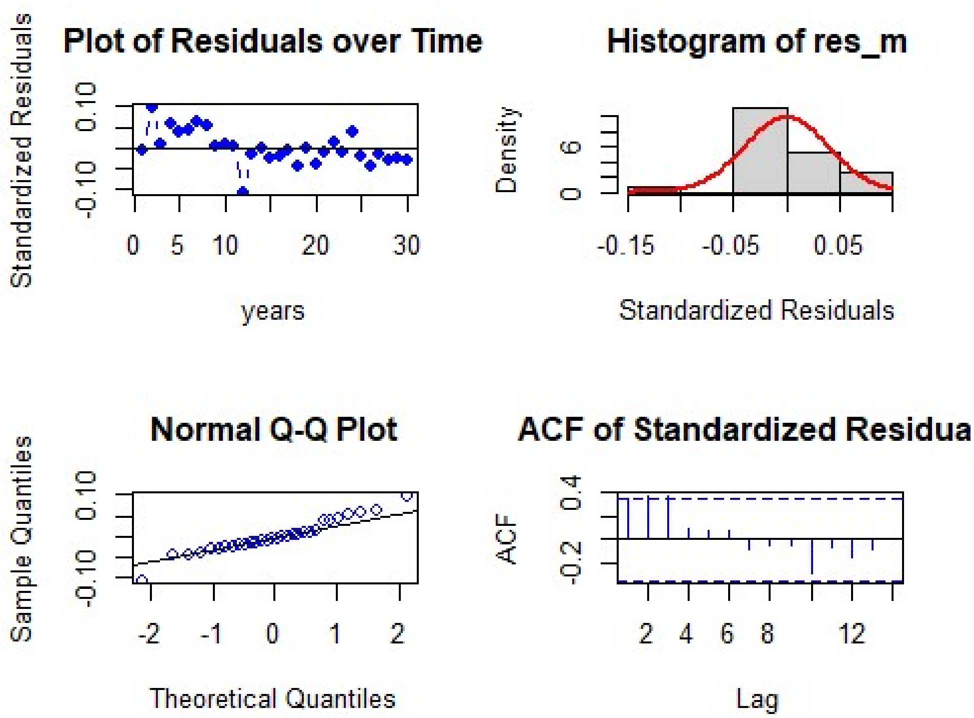
H_0: The data doesn't fit the Polynomial distributed lag model.
H_A: The data fits the Polynomial distributed lag model.

Interpretations:

F - statistic is 3.711
R - squared is 0.2156
Adjusted R - squared is 0.1575
Degrees of freedom - DF are (4, 500)
p - value (0.03767) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Polynomial distributed lag model.
This model suggests that there is only 15.75% of data variance. Suggesting that the model explains only 15.75% of the trend. Which implies that the model shows some trend.

Residual analysis

```
res_analysis(residuals(PolyDLM_model_Rain$model))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.95719, p-value = 0.2621
```

Residual Analysis for Polynomial DLM with part:

1. The data points are both below the line at the start and at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is not symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise. Also, ACF shows seasonality pattern.
5. p - value (0.2621) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

Now let us fit Koyck model.

Koyck model with Rainfall

```
# Koyk DLM
```

```
Koyck_DLM_Rain = koyckDlm(x = as.vector(x1) , y = as.vector(y))
summary(Koyck_DLM_Rain)

##
## Call:
## "Y ~ (Intercept) + Y.1 + X.t"
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.3665 -0.4155 -0.1142  0.3241  1.6012
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.3207     2.4302   0.132    0.896
## Y.1          -6.5147   243.8216  -0.027    0.979
## X.t           2.2101    76.0635   0.029    0.977
##
## Residual standard error: 0.7951 on 27 degrees of freedom
## Multiple R-Squared: -286.5,  Adjusted R-squared: -307.8
## Wald test: 0.01549 on 2 and 27 DF,  p-value: 0.9846
##
## Diagnostic tests:
## NULL
##
##                               alpha    beta       phi
## Geometric coefficients:  0.04267914 2.21011 -6.514689
```

Hypotheses:

H0: The data doesn't fit the Polynomial distributed lag model.
HA: The data fits the Polynomial distributed lag model.

Interpretations:

Wald test statistic is 0.01549
R - squared is -286.5
Adjusted R - squared is -307.8
Degrees of freedom - DF are (2, 27) p - value (0.9846) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Koyck distributed lag model.

No residual analysis is required.

Let us fit ardlDlm model to check whether it fits better or not.

Autoregressive distributed lag model Rainfall

```
for (i in 1:5){
  for(j in 1:5){
    model_4 = ardlDlm(x = as.vector(x1) , y = as.vector(y), p = i , q = j )
    cat("p = ", i, "q = ", j, "AIC = ", AIC(model_4$model), "BIC = ",
BIC(model_4$model), "MASE =", MASE(model_4)$MASE, "\n")
```

```
  }
}

## p =  1 q =  1 AIC =  -105.2619 BIC =  -98.25588 MASE = 0.828275
## p =  1 q =  2 AIC =  -103.3681 BIC =  -95.16429 MASE = 0.8543791
## p =  1 q =  3 AIC =  -106.9248 BIC =  -97.59935 MASE = 0.8322089
## p =  1 q =  4 AIC =  -102.0678 BIC =  -91.70114 MASE = 0.8714349
## p =  1 q =  5 AIC =  -95.80256 BIC =  -84.47969 MASE = 0.8152025
## p =  2 q =  1 AIC =  -99.21505 BIC =  -91.01127 MASE = 0.9189202
## p =  2 q =  2 AIC =  -101.4552 BIC =  -91.88416 MASE = 0.8562257
## p =  2 q =  3 AIC =  -104.9284 BIC =  -94.27076 MASE = 0.8328582
## p =  2 q =  4 AIC =  -100.0892 BIC =  -88.4267 MASE = 0.8706236
## p =  2 q =  5 AIC =  -93.80342 BIC =  -81.22246 MASE = 0.81573
## p =  3 q =  1 AIC =  -102.0287 BIC =  -92.70325 MASE = 0.8852654
## p =  3 q =  2 AIC =  -106.4754 BIC =  -95.8178 MASE = 0.8316226
## p =  3 q =  3 AIC =  -105.1996 BIC =  -93.2098 MASE = 0.8307901
## p =  3 q =  4 AIC =  -99.66585 BIC =  -86.70748 MASE = 0.860244
## p =  3 q =  5 AIC =  -93.30294 BIC =  -79.46387 MASE = 0.8095313
## p =  4 q =  1 AIC =  -96.40802 BIC =  -86.04133 MASE = 0.8956382
## p =  4 q =  2 AIC =  -100.4881 BIC =  -88.82555 MASE = 0.8337316
## p =  4 q =  3 AIC =  -100.0049 BIC =  -87.04657 MASE = 0.7754451
## p =  4 q =  4 AIC =  -98.96532 BIC =  -84.71111 MASE = 0.7942758
## p =  4 q =  5 AIC =  -92.62017 BIC =  -77.52301 MASE = 0.7390848
## p =  5 q =  1 AIC =  -93.55318 BIC =  -82.23031 MASE = 0.7936346
## p =  5 q =  2 AIC =  -94.0473 BIC =  -81.46633 MASE = 0.7842158
## p =  5 q =  3 AIC =  -93.91526 BIC =  -80.0762 MASE = 0.7237105
## p =  5 q =  4 AIC =  -92.68035 BIC =  -77.58319 MASE = 0.7490465
## p =  5 q =  5 AIC =  -90.68282 BIC =  -74.32757 MASE = 0.7469974
```

(p, q) = (5, 3) has the least AIC, BIC and MASE scores.

```
# ARDLM model
AR_DLM_Rain_53 = ardlDlm(x = as.vector(x1) , y = as.vector(y), p = 5, q = 3)
summary(AR_DLM_Rain_53)

##
## Time series regression with "ts" data:
## Start = 6, End = 31
##
## Call:
## dynlm(formula = as.formula(model.text), data = data, start = 1)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.072284 -0.008546  0.000512  0.019051  0.039909
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.184797   0.130413   1.417    0.176
## X.t          0.015255   0.019378   0.787    0.443
```

```
## X.1          0.002519   0.019466   0.129    0.899
## X.2         -0.005928   0.020127  -0.295    0.772
## X.3         -0.022491   0.019505  -1.153    0.266
## X.4         -0.021936   0.020202  -1.086    0.294
## X.5         -0.001398   0.019784  -0.071    0.945
## Y.1          0.319059   0.246953   1.292    0.215
## Y.2          0.275252   0.244529   1.126    0.277
## Y.3          0.255317   0.233870   1.092    0.291
##
## Residual standard error: 0.0332 on 16 degrees of freedom
## Multiple R-squared:  0.592,  Adjusted R-squared:  0.3625
## F-statistic:  2.58 on 9 and 16 DF,  p-value: 0.04715
```

Hypotheses:

H_0: The data doesn't fit the Autoregressive distributed lag model.
H_A: The data fits the Autoregressive distributed lag model.

Interpretations:

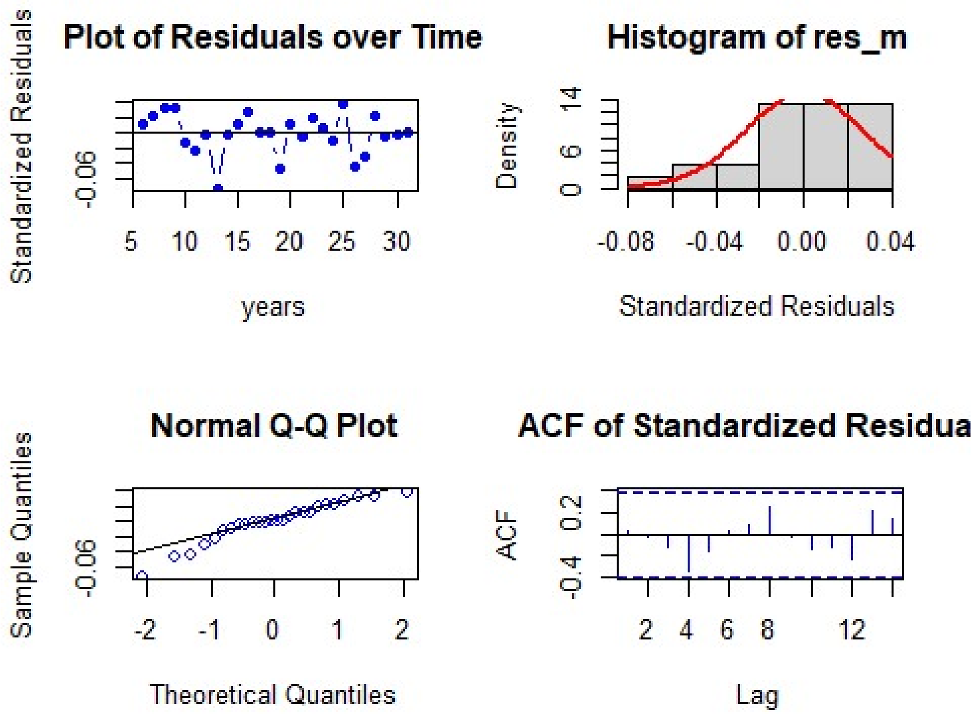
F - statistic is 2.58
R - squared is 0.592
Adjusted R - squared is 0.3625
Degrees of freedom - DF are (9, 16)
p - value (0.04715) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Autoregressive distributed lag model.
This model suggests that there is only 36.25% of data variance. Suggesting that the model explains only 33.25% of the trend. Which implies that the model shows some trend.

Now let us perform residual analysis.

Residual analysis

```
res_analysis(residuals(AR_DLM_Rain_53))

## Time Series:
## Start = 6
## End = 31
## Frequency = 1
##             6             7             8             9            10
##  0.0113163308  0.0235060707  0.0335227623  0.0337536460 -0.0134163971
##            11            12            13            14            15
## -0.0224225605 -0.0013008064 -0.0722835555 -0.0002717885  0.0132324968
##            16            17            18            19            20
##  0.0274869553  0.0024517345  0.0004684645 -0.0462011411  0.0128746231
##            21            22            23            24            25
## -0.0040178536  0.0209900331  0.0068564335 -0.0100556290  0.0399094686
##            26            27            28            29            30
```

```
## -0.0439012239 -0.0306578979  0.0215109321 -0.0036641111 -0.0002427409
##            31
##  0.0005557539
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.93614, p-value = 0.1086
```

Residual Analysis for AR_DLM_Rain_53:

1. The data points are both below the line at the start and at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is not symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise. Also, ACF shows seasonality pattern.
5. p - value (0.1086) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

Now let us fit with Temperature variable.

Finite distributed lag model temperature

```
for ( i in 1:10){
  model_1 = dlm(x = as.vector(x2) , y = as.vector(y), q = i )
  cat("q = ", i, "AIC = ", AIC(model_1$model), "BIC = ", BIC(model_1$model),
"MASE =", MASE(model_1)$MASE, "\n")
  }

## q =  1 AIC =  -101.8617 BIC =  -96.2569 MASE = 0.9239038
## q =  2 AIC =  -95.49894 BIC =  -88.66246 MASE = 1.032564
## q =  3 AIC =  -96.76727 BIC =  -88.77404 MASE = 1.033663
## q =  4 AIC =  -92.75653 BIC =  -83.68567 MASE = 1.005373
## q =  5 AIC =  -91.46337 BIC =  -81.3986 MASE = 0.8594175
## q =  6 AIC =  -85.74127 BIC =  -74.77139 MASE = 0.8103361
## q =  7 AIC =  -82.04015 BIC =  -70.25962 MASE = 0.7518958
## q =  8 AIC =  -83.28717 BIC =  -70.79674 MASE = 0.6497633
## q =  9 AIC =  -88.10651 BIC =  -75.014 MASE = 0.5120906
## q =  10 AIC =  -87.92965 BIC =  -74.35085 MASE = 0.458588
```

As we have the least AIC, BIC and MASE values at q = 10. Let us fit the finite distributed lag model with q = 10.

```
# Finite lag length based on AIC-BIC-MASE

finite_dlm_temp = dlm( x = as.vector(x2) , y = as.vector(y), q = 10)
summary(finite_dlm_temp)

##
## Call:
## lm(formula = model.formula, data = design)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.033097 -0.011942  0.005304  0.008460  0.030820
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.521379   0.537055   0.971   0.3570
## x.t          0.003982   0.016965   0.235   0.8197
## x.1          0.047512   0.018674   2.544   0.0315 *
## x.2         -0.010070   0.019814  -0.508   0.6235
## x.3         -0.024214   0.020051  -1.208   0.2580
## x.4          0.011690   0.021762   0.537   0.6042
## x.5         -0.001764   0.023152  -0.076   0.9409
## x.6          0.017653   0.019245   0.917   0.3829
## x.7          0.015177   0.018673   0.813   0.4373
## x.8         -0.011418   0.020339  -0.561   0.5882
## x.9         -0.036343   0.019400  -1.873   0.0938 .
## x.10         0.008222   0.018870   0.436   0.6733
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.02453 on 9 degrees of freedom
## Multiple R-squared:  0.6008, Adjusted R-squared:  0.1129
## F-statistic: 1.232 on 11 and 9 DF,  p-value: 0.3834
##
## AIC and BIC values for the model:
##         AIC       BIC
## 1 -87.92965 -74.35085
```

Hypotheses:

H_0: The data doesn't fit the Finite distributed lag model.
H_A: The data fits the Finite distributed lag model.

Interpretations:

F - statistic is 1.232
R - squared is 0.6008
Adjusted R - squared is 0.1129
Degrees of freedom - DF are (11, 9)
p - value (0.3834) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model does not fit the Finite distributed lag model.

No residual analysis is required.

Therefore, Further analysis is needed by adding polynomial to the lag model.

Polynomial distributed lag model Temperature

```
for (i in 1:3){
  model_3 <-  polyDlm(x = as.vector(x2), y = as.vector(y), q = i , k = i,
show.beta = FALSE)
  cat("q = ", i, "k = ", i, "AIC = ", AIC(model_3$model), "BIC = ",
BIC(model_3$model), "MASE = ", MASE(model_3)$MASE, "\n")
}

## q =  1 k =  1 AIC =  -101.8617 BIC =  -96.2569 MASE =  0.9239038
## q =  2 k =  2 AIC =  -95.49894 BIC =  -88.66246 MASE =  1.032564
## q =  3 k =  3 AIC =  -96.76727 BIC =  -88.77404 MASE =  1.033663
```

Let us fit a polynomial model of order 3. Since least AIC and BIC scores.

```
# Ploynomial DLM

PolyDLM_model_temp = polyDlm(x = as.vector(x2), y = as.vector(y), q = 1, k =
1, show.beta = TRUE)

## Estimates and t-tests for beta coefficients:
##        Estimate Std. Error t value P(>|t|)
## beta.0   0.0192     0.0202   0.952  0.3490
## beta.1   0.0529     0.0216   2.450  0.0205
```

```
summary(PolyDLM_model_temp)

##
## Call:
## "Y ~ (Intercept) + X.t"
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.075797 -0.024417 -0.002201  0.020077  0.100856
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.05297    0.24450   0.217    0.830
## z.t0         0.01922    0.02019   0.952    0.350
## z.t1         0.03364    0.03291   1.022    0.316
##
## Residual standard error: 0.04087 on 27 degrees of freedom
## Multiple R-squared:  0.2404, Adjusted R-squared:  0.1842
## F-statistic: 4.273 on 2 and 27 DF,  p-value: 0.02442
```

Hypotheses:

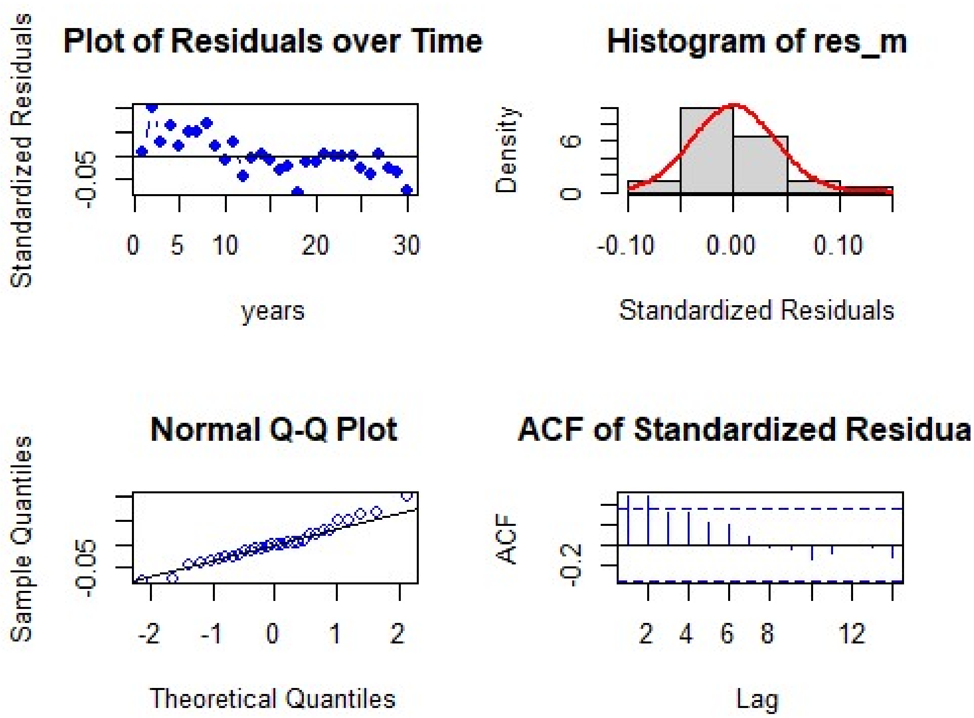
H_0: The data doesn't fit the Polynomial distributed lag model.
H_A: The data fits the Polynomial distributed lag model.

Interpretations:

F - statistic is 4.273
R - squared is 0.2404
Adjusted R - squared is 0.1842
Degrees of freedom - DF are (2, 27)
p - value (0.02442) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Polynomial distributed lag model.
This model suggests that there is only 18.42% of data variance. Suggesting that the model explains only 18.42% of the trend. Which implies that the model shows some trend.

Residual analysis

```
res_analysis(residuals(PolyDLM_model_temp$model))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.9706, p-value = 0.5558
```

Residual Analysis for Polynomial DLM with part:

1. The data points are both below the line at the start and at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is not symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise. Also, ACF shows seasonality pattern.
5. p - value (0.5558) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

```
fit3 <- ets(v_RBO_data_TS, model="MMN", damped = T)
fit1 <- forecast.ets(fit3, h = 3)
```

Now let us fit Koyck model.

Koyck model with temperature

```
# Koyk DLM

Koyck_DLM_temp = koyckDlm(x = as.vector(x2) , y = as.vector(y))
summary(Koyck_DLM_temp)

##
## Call:
## "Y ~ (Intercept) + Y.1 + X.t"
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.15981 -0.04678 -0.01440  0.04750  0.14952
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.2032     1.6184  -0.743    0.464
## Y.1           0.2469     0.4609   0.536    0.597
## X.t           0.1847     0.1947   0.949    0.351
##
## Residual standard error: 0.07557 on 27 degrees of freedom
## Multiple R-Squared: -1.597,  Adjusted R-squared: -1.789
## Wald test: 2.119 on 2 and 27 DF,  p-value: 0.1397
##
## Diagnostic tests:
## NULL
##
##                              alpha      beta      phi
## Geometric coefficients:  -1.597614 0.1847273 0.246894
```

Hypotheses:

H_0: The data doesn't fit the Polynomial distributed lag model.
H_A: The data fits the Polynomial distributed lag model.

Interpretations:

Wald test statistic is 2.119
R - squared is -1.597
Adjusted R - squared is -1.789
Degrees of freedom - DF are (2, 27)
p - value (0.1397) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the data doesn't fit the Koyck distributed lag model.

No residual analysis is required.

Let us fit ardlDlm model to check whether it fits better or not.

Autoregressive distributed lag model with temperature

```
for (i in 1:5){
  for(j in 1:5){
    model_4 = ardlDlm(x = as.vector(x2) , y = as.vector(y), p = i , q = j )
    cat("p = ", i, "q = ", j, "AIC = ", AIC(model_4$model), "BIC = ",
BIC(model_4$model), "MASE =", MASE(model_4)$MASE, "\n")
  }
}
```

```
## p =  1 q =  1 AIC =  -107.8419 BIC =  -100.8359 MASE = 0.8074316
## p =  1 q =  2 AIC =  -105.4398 BIC =  -97.236 MASE = 0.8117033
## p =  1 q =  3 AIC =  -109.6659 BIC =  -100.3404 MASE = 0.8057573
## p =  1 q =  4 AIC =  -102.3384 BIC =  -91.97173 MASE = 0.8610456
## p =  1 q =  5 AIC =  -97.42825 BIC =  -86.10538 MASE = 0.7735245
## p =  2 q =  1 AIC =  -102.1496 BIC =  -93.94579 MASE = 0.9024954
## p =  2 q =  2 AIC =  -104.8752 BIC =  -95.30414 MASE = 0.7913477
## p =  2 q =  3 AIC =  -109.639 BIC =  -98.98132 MASE = 0.790971
## p =  2 q =  4 AIC =  -102.2956 BIC =  -90.63304 MASE = 0.8324633
## p =  2 q =  5 AIC =  -96.20616 BIC =  -83.62519 MASE = 0.7544973
## p =  3 q =  1 AIC =  -106.2352 BIC =  -96.90972 MASE = 0.8876139
## p =  3 q =  2 AIC =  -112.3458 BIC =  -101.6882 MASE = 0.7726235
## p =  3 q =  3 AIC =  -111.2734 BIC =  -99.28358 MASE = 0.7779966
## p =  3 q =  4 AIC =  -103.7311 BIC =  -90.7727 MASE = 0.8253913
## p =  3 q =  5 AIC =  -97.11952 BIC =  -83.28046 MASE = 0.7661286
## p =  4 q =  1 AIC =  -102.0941 BIC =  -91.72737 MASE = 0.8648001
## p =  4 q =  2 AIC =  -105.9038 BIC =  -94.24126 MASE = 0.8119505
## p =  4 q =  3 AIC =  -104.0851 BIC =  -91.12676 MASE = 0.8210751
## p =  4 q =  4 AIC =  -102.0982 BIC =  -87.844 MASE = 0.8188817
## p =  4 q =  5 AIC =  -95.40963 BIC =  -80.31247 MASE = 0.7643666
## p =  5 q =  1 AIC =  -97.85381 BIC =  -86.53094 MASE = 0.7652401
## p =  5 q =  2 AIC =  -99.0233 BIC =  -86.44233 MASE = 0.7549862
## p =  5 q =  3 AIC =  -97.26093 BIC =  -83.42187 MASE = 0.7593949
## p =  5 q =  4 AIC =  -95.45616 BIC =  -80.359 MASE = 0.7544104
## p =  5 q =  5 AIC =  -93.50136 BIC =  -77.1461 MASE = 0.7617211
```

(p, q) = (5, 2) has the least AIC, BIC and MASE scores.

```
# ARDLM model
AR_DLM_temp_52 = ardlDlm(x = as.vector(x2) , y = as.vector(y), p = 5, q = 2)
summary(AR_DLM_temp_52)
```

```
##
## Time series regression with "ts" data:
## Start = 6, End = 31
##
## Call:
## dynlm(formula = as.formula(model.text), data = data, start = 1)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
```

```
## -0.051543 -0.011809  0.002385  0.019521  0.044557
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.093940   0.372713   0.252    0.804
## X.t          0.013231   0.019672   0.673    0.510
## X.1          0.035524   0.022038   1.612    0.125
## X.2         -0.025862   0.023362  -1.107    0.284
## X.3         -0.027595   0.022784  -1.211    0.242
## X.4          0.015043   0.019203   0.783    0.444
## X.5         -0.001256   0.021582  -0.058    0.954
## Y.1          0.403786   0.239703   1.685    0.110
## Y.2          0.343301   0.231245   1.485    0.156
##
## Residual standard error: 0.03034 on 17 degrees of freedom
## Multiple R-squared:  0.638,  Adjusted R-squared:  0.4676
## F-statistic: 3.745 on 8 and 17 DF,  p-value: 0.01057
```

Hypotheses:

H_0: The data doesn't fit the Autoregressive distributed lag model.
H_A: The data fits the Autoregressive distributed lag model.

Interpretations:

F - statistic is 3.745
R - squared is 0.638
Adjusted R - squared is 0.4676
Degrees of freedom - DF are (9, 16)
p - value (0.01057) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Autoregressive distributed lag model.
This model suggests that there is only 46.76% of data variance. Suggesting that the model explains only 46.76% of the trend. Which implies that the model shows some trend.

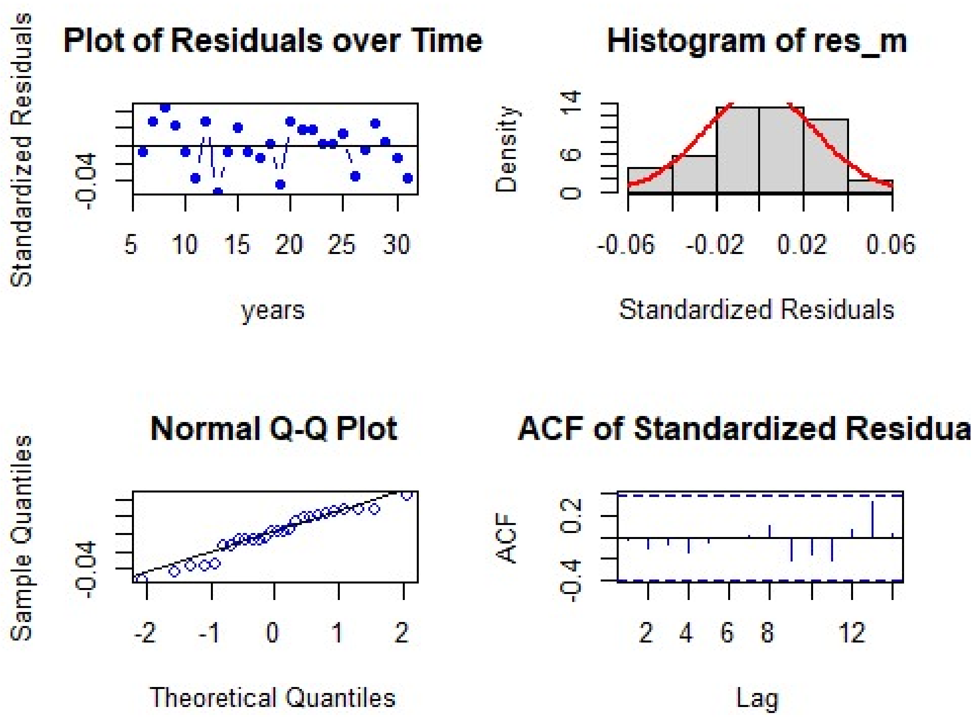
Now let us perform residual analysis.

Residual analysis

```
res_analysis(residuals(AR_DLM_temp_52))
```

```
## Time Series:
## Start = 6
## End = 31
## Frequency = 1
##            6            7            8            9           10
11
## -0.005875304  0.027889332  0.044557126  0.023955102 -0.006835884 -
0.036247181
```

```
##           12           13           14           15           16
17
##  0.028047614 -0.051543021 -0.007389583  0.020090374 -0.006168359 -
0.014619673
##           18           19           20           21           22
23
##  0.002939194 -0.042575018  0.027293790  0.017812015  0.017447685
0.002867739
##           24           25           26           27           28
29
##  0.001903187  0.013329216 -0.035009871 -0.004075594  0.025957684
0.005081968
##           30           31
## -0.013282240 -0.035550300
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.95433, p-value = 0.2922
```

Residual Analysis for AR_DLM_temp_52:

1. The data points are both below the line at the start and at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.

2. From normal distribution curve, the distribution is almost symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are no significant lags in Autocorrelation plot suggesting that the stochastic component is white noise.
5. p - value (0.2922) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

Now let us calculate AIC, BIC and MASE scores and store them in a dataframe to check the better model based on MASE score.

```
attr(AR_DLM_Rain_53$model, "class") = "lm"
attr(AR_DLM_temp_52$model, "class") = "lm"

v_model_name <- c("PolyDLM_model_Rain", "AR_DLM_Rain_53",
"PolyDLM_model_temp", "AR_DLM_temp_52")

MASE <- MASE(PolyDLM_model_Rain$model, AR_DLM_Rain_53$model,
PolyDLM_model_temp$model, AR_DLM_temp_52$model)$MASE

aic <- AIC(PolyDLM_model_Rain$model, AR_DLM_Rain_53$model,
PolyDLM_model_temp$model, AR_DLM_temp_52$model)$AIC

bic <- BIC(PolyDLM_model_Rain$model, AR_DLM_Rain_53$model,
PolyDLM_model_temp$model, AR_DLM_temp_52$model)$BIC

v_score <- data.frame(v_model_name, MASE, aic, bic)
colnames(v_score) <- c("MODEL_NAME", "MASE", "AIC", "BIC")
v_score

##           MODEL_NAME      MASE        AIC       BIC
## 1 PolyDLM_model_Rain 0.9417954 -100.89798 -95.29319
## 2     AR_DLM_Rain_53 0.7237105  -93.91526 -80.07620
## 3 PolyDLM_model_temp 0.9239038 -101.86168 -96.25690
## 4     AR_DLM_temp_52 0.7549862  -99.02330 -86.44233
```

Therefore, AR_DLM_Rain_53 is the better model.

Now let us fit dynamic model.

For dlm model we need some modifications in the series.

```
v_data_TS_33 <- ts(v_RBO_data, start = 1984, frequency = 1)
colnames(v_data_TS_33) <- c("x1", "y", "x2", "x3", "x4", "x5")
```

Dynamic model with Rainfall

```
v_rain_dyna <- dynlm(y ~ x3, data = data.frame(v_data_TS_33))
summary(v_rain_dyna)
```

```
##
## Time series regression with "numeric" data:
## Start = 1, End = 31
##
## Call:
## dynlm(formula = y ~ x3, data = data.frame(v_data_TS_33))
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.094310 -0.027945 -0.003845  0.021733  0.102109
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.62656    0.04891  12.810 1.82e-13 ***
## x3           0.04696    0.02039   2.303   0.0286 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04171 on 29 degrees of freedom
## Multiple R-squared:  0.1546, Adjusted R-squared:  0.1255
## F-statistic: 5.304 on 1 and 29 DF,  p-value: 0.02864
```

Hypotheses:

H_0: The data doesn't fit the Dynamic linear model.
H_A: The data fits the Dynamic linear model.

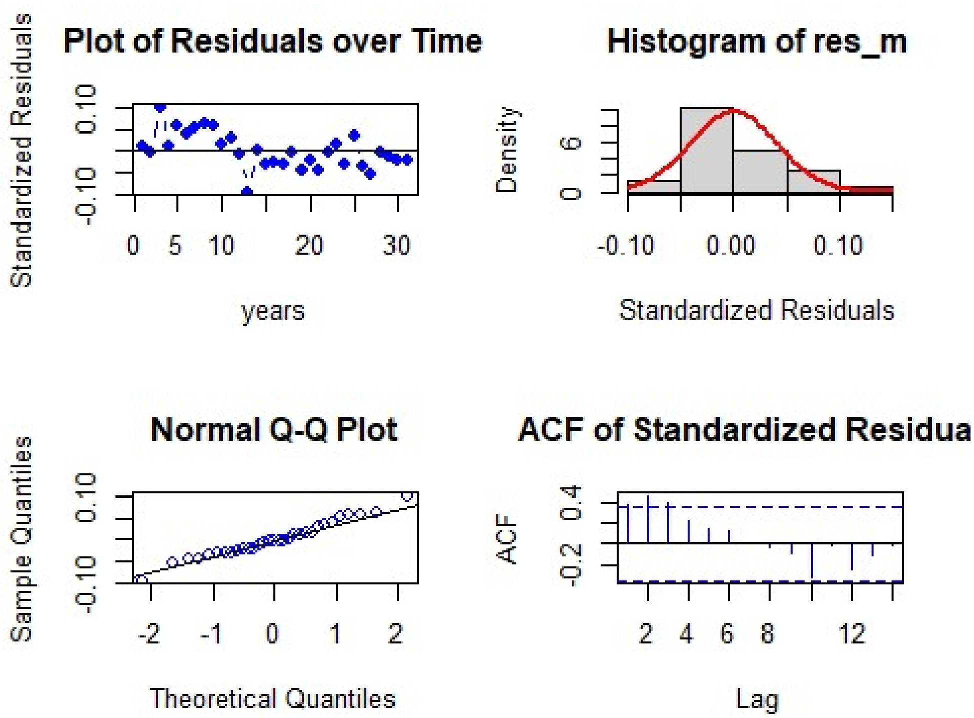
Interpretations:

F - statistic is 5.304
R - squared is 0.1546
Adjusted R - squared is 0.1255
Degrees of freedom - DF are (1, 29)
p - value (0.02864) is < 0.05 and therefore, it is statistically significant. Therefore, Null hypothesis is rejected. Hence, the model fits the Dynamic linear model.
This model suggests that there is only 12.55% of data variance. Suggesting that the model explains only 12.55% of the trend. Which implies that the model shows some trend.

Now let us check residuals.

Residual analysis

```
res_analysis(residuals(v_rain_dyna))
```




```
##
##  Shapiro-Wilk normality test
##
## data:  res_m
## W = 0.9785, p-value = 0.7697
```

Residual Analysis for v_rain_dyna:

1. The data points are above at the start and below at the end of the trend. Randomness is seen to some extent. So, we cannot decide anything at this stage. Further analysis is required.
2. From normal distribution curve, the distribution is not symmetric.
3. The data at the tails is deviated more leaving some part on the line suggesting there is some normality in the trend.
4. There are significant lags in Autocorrelation plot suggesting that the stochastic component is not white noise.
5. p - value (0.7697) from Shapiro-Wilk normality test is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected.

Dynamic model with Temperature

```
v_temp_dyna <- dynlm(y ~ x2, data = data.frame(v_data_TS_33))
summary(v_temp_dyna)

##
## Time series regression with "numeric" data:
```

```
## Start = 1, End = 31
##
## Call:
## dynlm(formula = y ~ x2, data = data.frame(v_data_TS_33))
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.071171 -0.029750 -0.009143  0.022486  0.111775
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.44790    0.19931   2.247   0.0324 *
## x2           0.03048    0.02094   1.456   0.1561
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04379 on 29 degrees of freedom
## Multiple R-squared:  0.06812,    Adjusted R-squared:  0.03599
## F-statistic:  2.12 on 1 and 29 DF,  p-value: 0.1561
```

Hypotheses:

H_0: The data doesn't fit the Dynamic linear model.
H_A: The data fits the Dynamic linear model.

Interpretations:

F - statistic is 2.12
R - squared is 0.06812
Adjusted R - squared is 0.03599
Degrees of freedom - DF are (1, 29)
p - value (0.1561) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model does not fit the Dynamic linear model.

No residual analysis is required.

Forecasting

For forecasting we need the optimal model among each method. We got only 5 fit models with 2 different predictors.

1. "PolyDLM_model_Rain"
2. "AR_DLM_Rain_53"
3. "PolyDLM_model_temp"
4. "AR_DLM_temp_52"
5. "v_rain_dyna"

Among all the polynomial model with temperature has the least MASE score compared.

Therefore using these models to predict RBO series.

Let us forecast for the next 3 years on RBO series. From 2014 to 2016.

```
plot(fit1, fcol = "white", main = "First Flowerind Day series with 3 years
ahead forecasts", ylab = "Radiation")
lines(fitted(fit1), col = "darkgreen")
lines(fit1$mean, col = "darkgreen", lwd = 2)
legend("topleft", lty = 1, col = c("black", "darkgreen"), c("Data",
"Forecasts"))
```

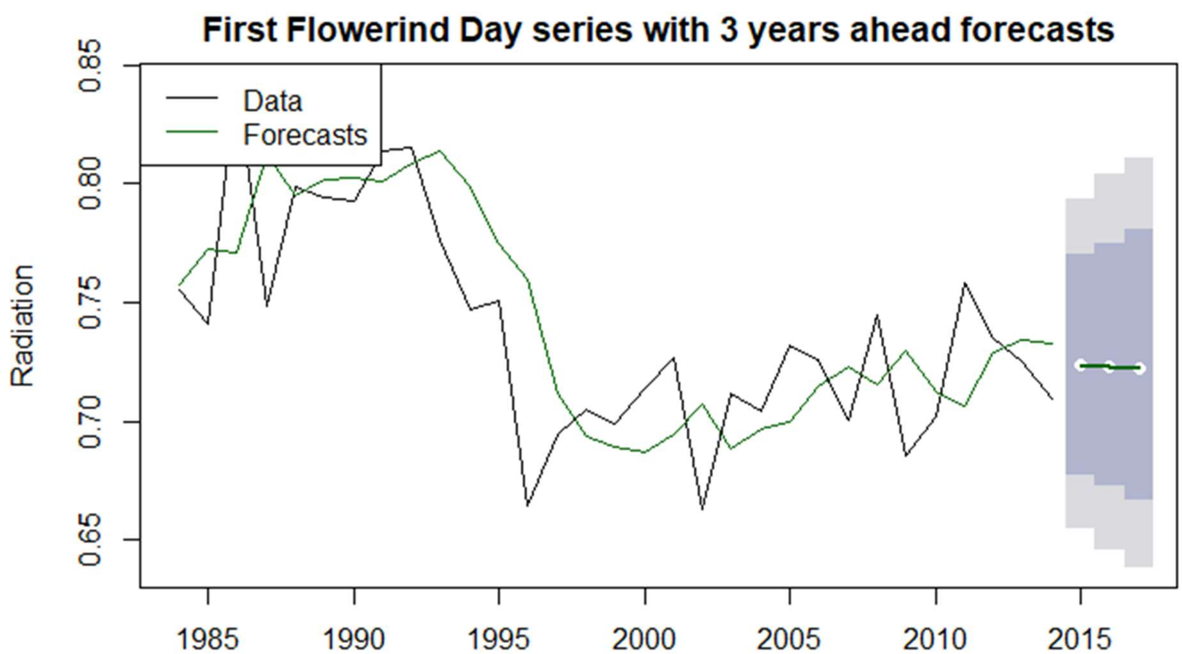


Fig 3.21: Next 3 years forecast on the RBO Series.

```
forecasts <- ts.intersect(ts(fit1$lower[, 2], start = c(2015), frequency =
1), ts(fit1$mean, start = c(2015), frequency = 1), ts(fit1$upper[, 2], start
= c(2015), frequency = 1))
colnames(forecasts) <- c("Lower bound", "Point forecast", "Upper bound")
forecasts

## Time Series:
## Start = 2015
## End = 2017
## Frequency = 1
##      Lower bound Point forecast Upper bound
## 2015   0.6528272      0.7233829   0.7953437
## 2016   0.6443240      0.7226351   0.8007507
## 2017   0.6391059      0.7220372   0.8104688
```

From the three year forecast results we can predict that there will be decrease in the Rank-based Order similarity metric in the future. This suggests that the impact of the chemical components will be less in future.

Part (b)

Data

The data here used is the the contemporaneous yearly averaged climate variables measured from 1984 – 2014 (31 years), particularly during the Millennium Drought (1997 – 2009) (13 years).

To get this, the data is trimmed as below,

```
v_RBO_data_task_b <- tail(v_RBO_data, -13)
v_RBO_data_task_b <- head(v_RBO_data_task_b, -5)
head(v_RBO_data_task_b)

##    ï..Year       RBO Temperature Rainfall Radiation RelHumidity
## 14    1997 0.6941213    9.038356 1.403014  14.77534    93.74685
## 15    1998 0.7045545    8.934247 2.289041  14.60000    94.60822
## 16    1999 0.6992259    9.547945 2.126301  14.61370    96.22603
## 17    2000 0.7137116    9.680328 2.471858  14.65574    95.65738
## 18    2001 0.7267423    9.561644 2.227945  14.14521    94.70712
## 19    2002 0.6629484    9.389041 1.740000  14.63836    93.53233

# Using str() to check the type of each column.
str(v_RBO_data_task_b)

## 'data.frame':    13 obs. of  6 variables:
##  $ ï..Year    : int  1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 ...
##  $ RBO        : num  0.694 0.705 0.699 0.714 0.727 ...
##  $ Temperature: num  9.04 8.93 9.55 9.68 9.56 ...
##  $ Rainfall   : num  1.4 2.29 2.13 2.47 2.23 ...
##  $ Radiation  : num  14.8 14.6 14.6 14.7 14.1 ...
##  $ RelHumidity: num  93.7 94.6 96.2 95.7 94.7 ...
```

Checking for Missing values.

```
colSums(is.na(v_RBO_data_task_b))

##     ï..Year         RBO Temperature    Rainfall   Radiation RelHumidity
##           0           0           0           0           0           0
```

There are no missing values in the data.

Checking the class of v_solar_data. (It should be a data frame.)

```
class(v_RBO_data_task_b)

## [1] "data.frame"
```

```
v_RBO_Temp_task_b_TS <- ts(v_RBO_data_task_b$Temperature, start = c(1997),
frequency = 1)
v_RBO_Rainfall_task_b_TS <- ts(v_RBO_data_task_b$Rainfall, start = c(1997),
frequency = 1)
v_RBO_Radiation_task_b_TS <- ts(v_RBO_data_task_b$Radiation, start = c(1997),
frequency = 1)
v_RBO_RelHumidity_task_b_TS <- ts(v_RBO_data_task_b$RelHumidity, start =
c(1997), frequency = 1)
v_RBO_data_task_b_TS <- ts(v_RBO_data_task_b$RBO, start = c(1997), frequency
= 1)
```

Confirming the class of each time series object.

```
class(v_RBO_Temp_task_b_TS)

## [1] "ts"

class(v_RBO_Rainfall_task_b_TS)

## [1] "ts"

class(v_RBO_Radiation_task_b_TS)

## [1] "ts"

class(v_RBO_RelHumidity_task_b_TS)

## [1] "ts"

class(v_RBO_data_task_b_TS)

## [1] "ts"
```

Now let us perform descriptive analysis on each time series object.

Descriptive Analysis

Rank-based Order similarity metric

```
plot(v_RBO_data_task_b_TS, type = "b", xlab = "years", ylab = "Rank-based
Order similarity metric", main = "Time series plot for yearly Rank-based
Order similarity metric during the Millennium Drought (1997 - 2009) (13
years)", pch = 1)
legend("topleft", inset = .03, title = "Rank-based Order similarity metric",
legend = "Rank-based Order similarity metric series", horiz = TRUE, cex =
0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

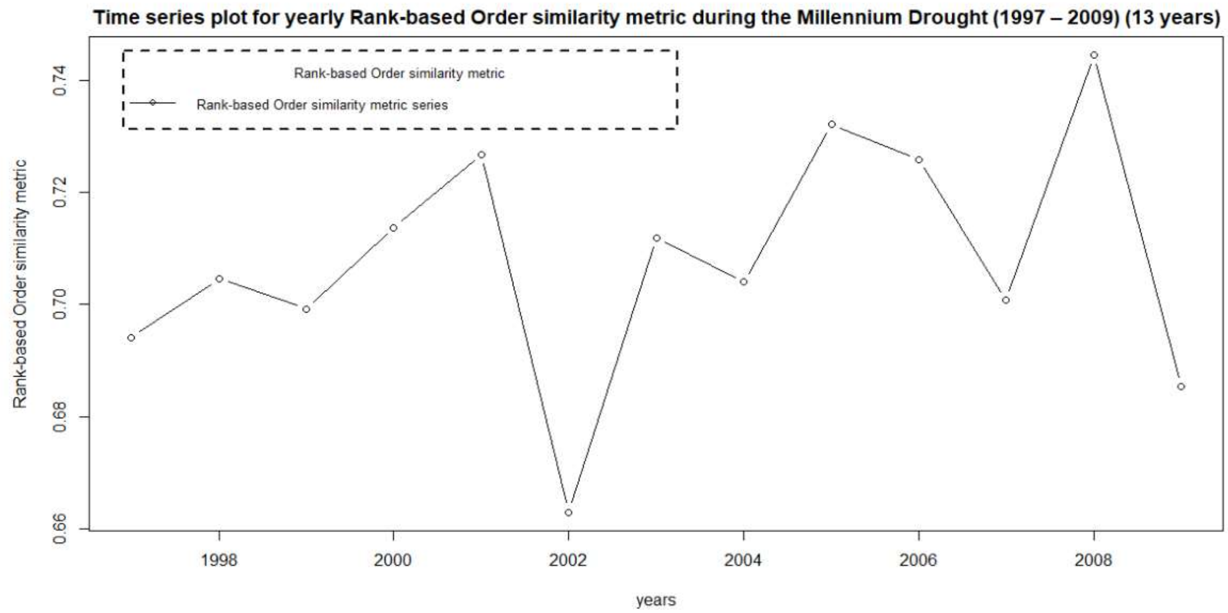


Fig 4.1: Rank-based Order similarity metric - Time series plot.

```
McLeod.Li.test(y = v_RBO_data_task_b_TS, main = "McLeod-Li Test Statistics
for Rank-based Order similarity metric.")
```

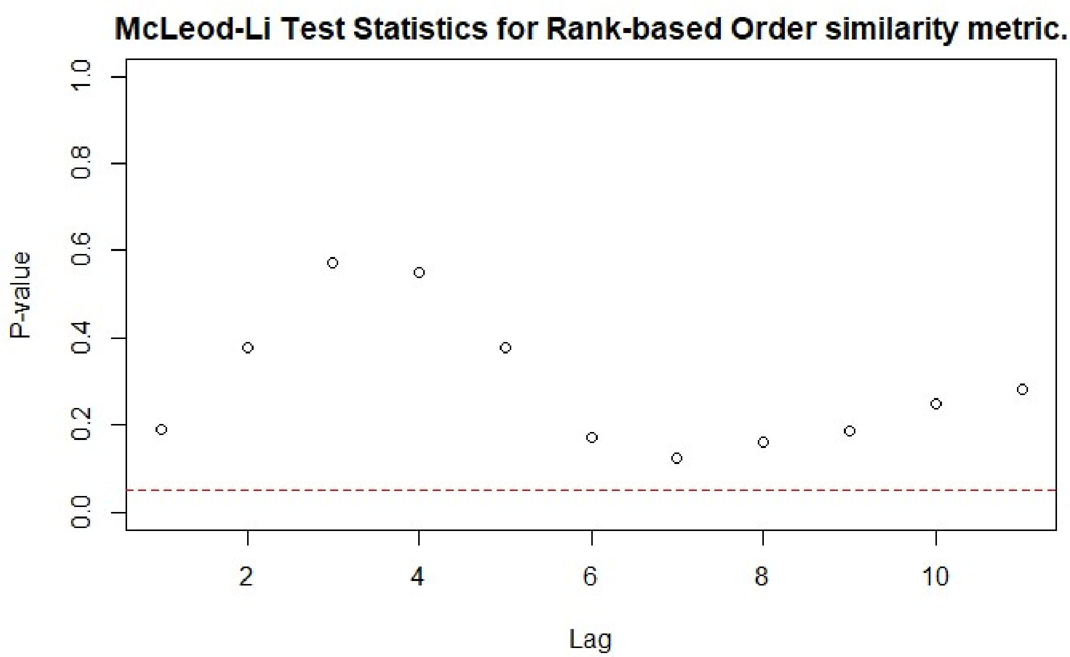


Fig 4.2: McLeod-Li Test Statistics for Rank-based Order similarity metric.

Descriptive analysis

1. From the series plot, we can observe that there is some downward trend in the data.

2. There is an intervention around multiple years.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Temperature

```
plot(v_RBO_Temp_task_b_TS, type = "b", xlab = "years", ylab = "Temperature",
main = "Time series plot for yearly temperature during the Millennium Drought
(1997 – 2009) (13 years)", pch = 1)
legend("top", inset = .03, title = "Temperature", legend = "Temperature
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

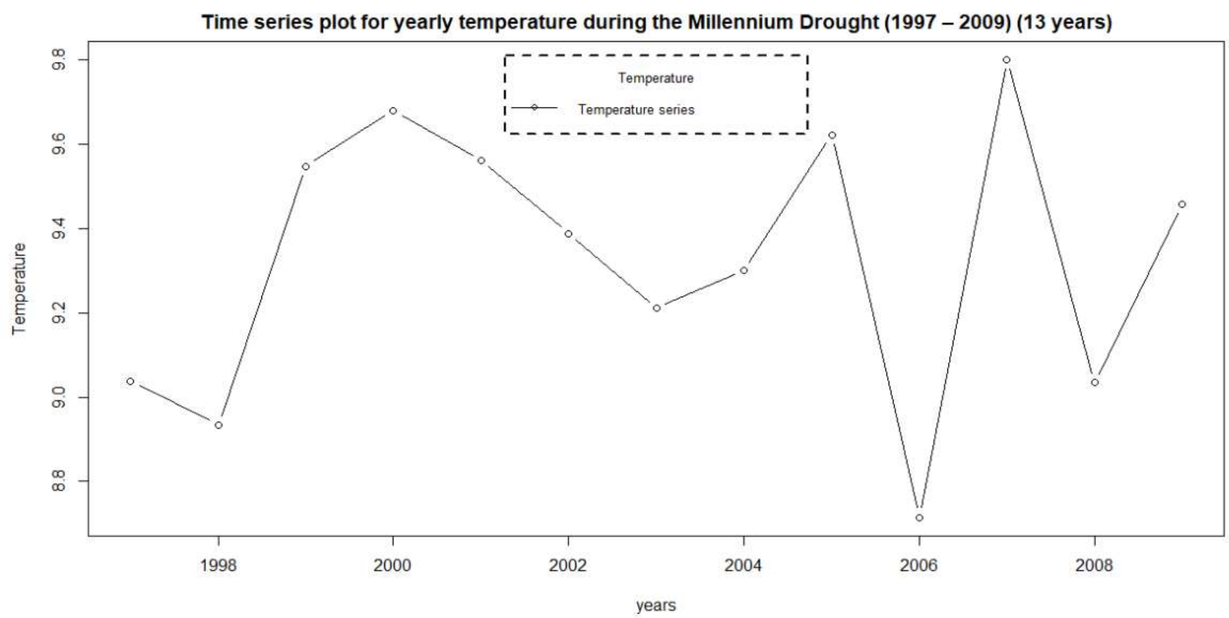


Fig 4.3: Temperature - Time series plot.

```
McLeod.Li.test(y = v_RBO_Temp_task_b_TS, main = "McLeod-Li Test Statistics
for Temperature")
```

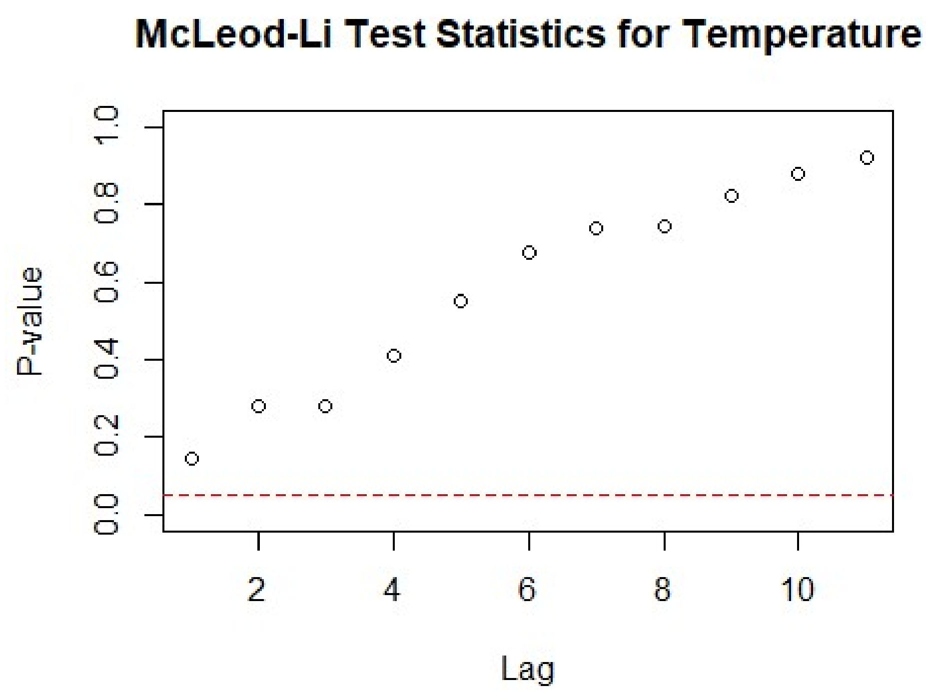


Fig 4.4: McLeod-Li Test Statistics for Temperature

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention at multiple points.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Rainfall

```
plot(v_RBO_Rainfall_task_b_TS, type = "b", xlab = "years", ylab = "Rainfall",
main = "Time series plot for yearly Rainfall during the Millennium Drought
(1997 – 2009) (13 years)", pch = 1)
legend("bottom", inset = .03, title = "Rainfall", legend = "Rainfall series",
horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

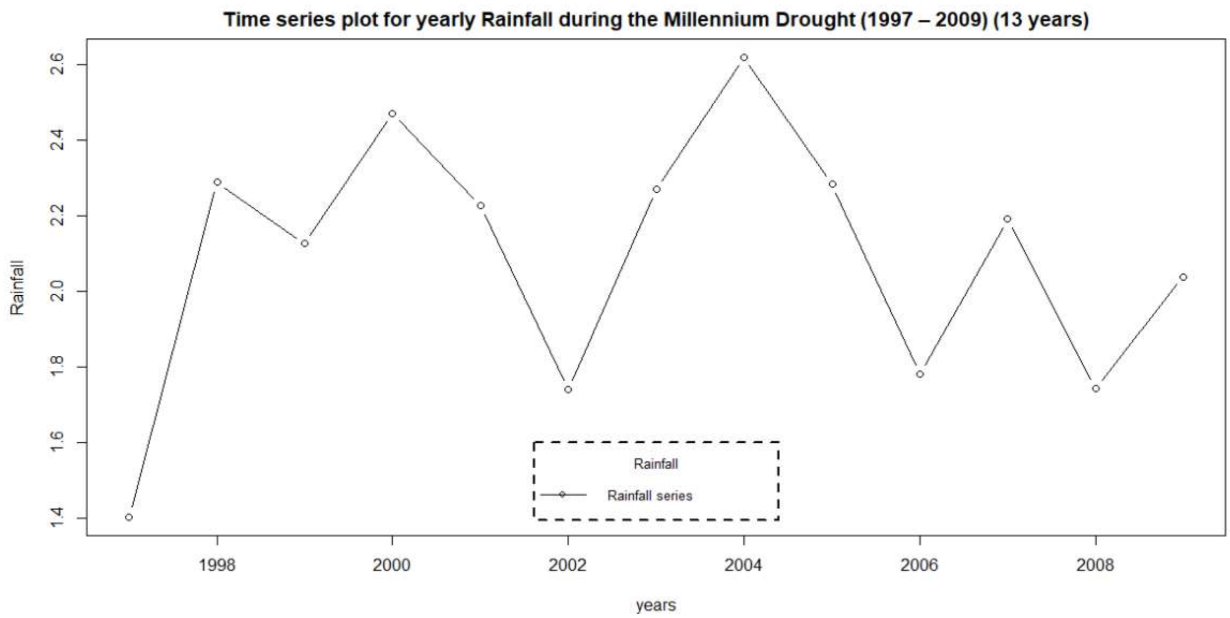


Fig 4.5: Rainfall - Time series plot.

```
McLeod.Li.test(y = v_RBO_Rainfall_task_b_TS, main = "McLeod-Li Test
Statistics for Rainfall")
```

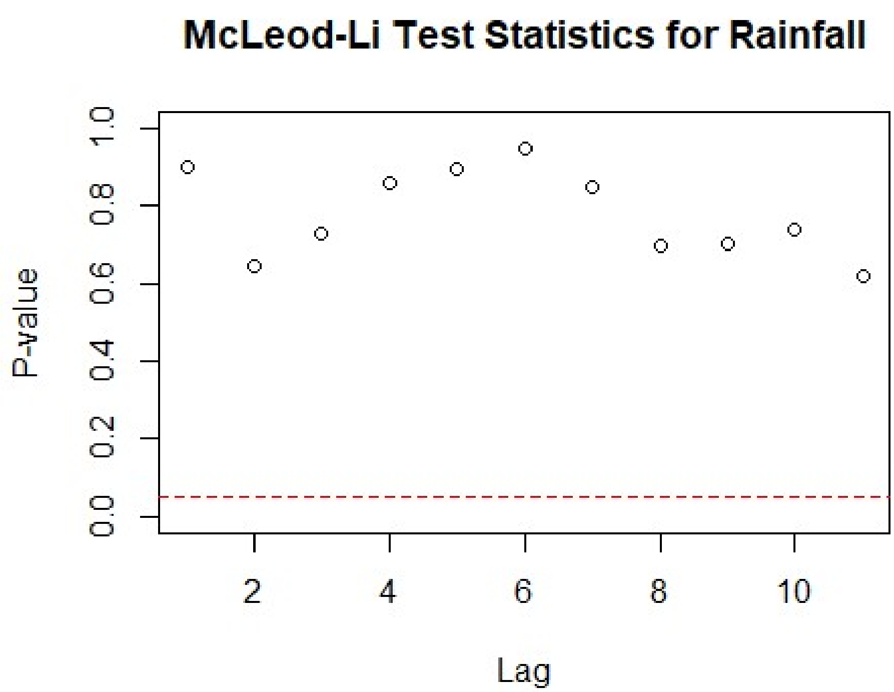


Fig 4.6: McLeod-Li Test Statistics for Rainfall

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 2004.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Radiation

```
plot(v_RBO_Radiation_task_b_TS, type = "b", xlab = "years", ylab =
"Radiation", main = "Time series plot for yearly Radiation during the
Millennium Drought (1997 – 2009) (13 years)", pch = 1)
legend("topleft", inset = .03, title = "Radiation", legend = "Radiation
series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2, box.lwd = 2, pch = 1)
```

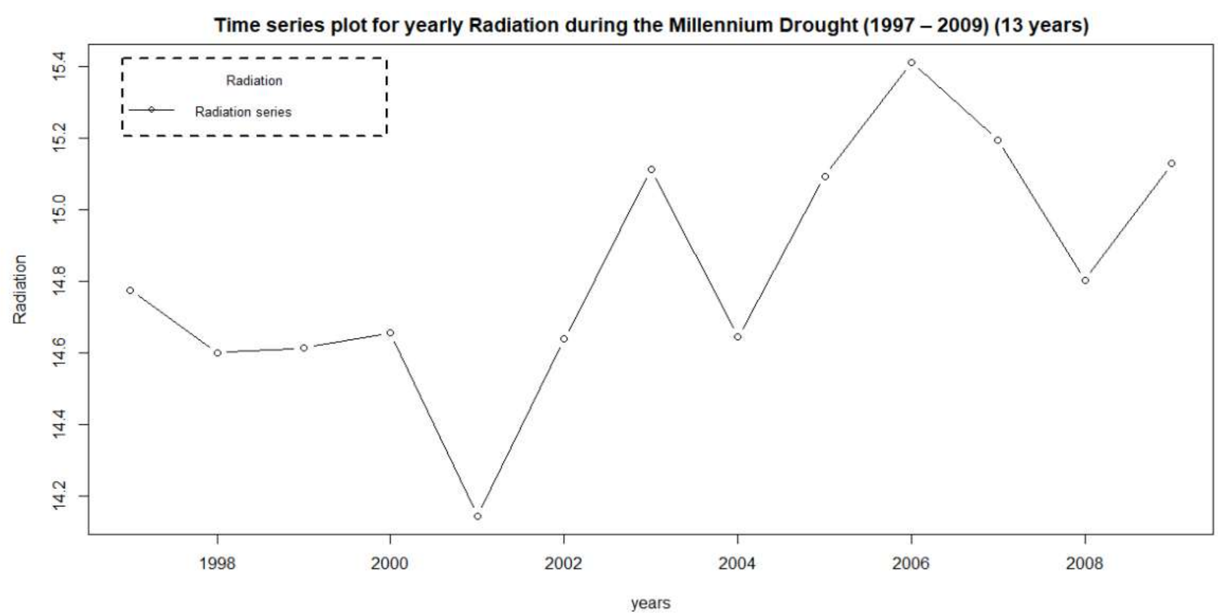


Fig 4.7: Radiation - Time series plot.

```
McLeod.Li.test(y = v_RBO_Rainfall_task_b_TS, main = "McLeod-Li Test
Statistics for Radiation")
```

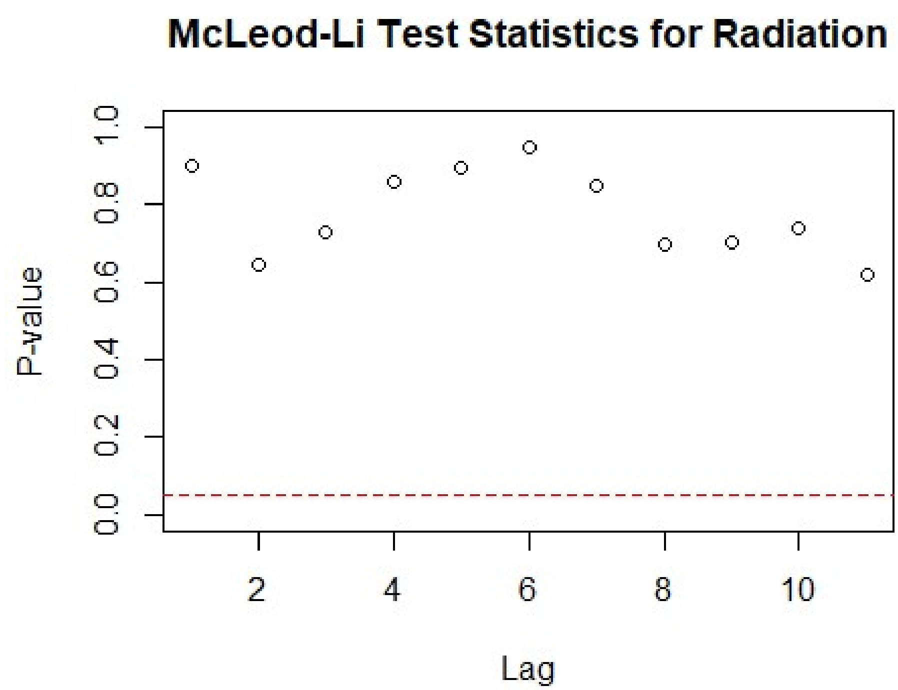


Fig 4.8: McLeod-Li Test Statistics for Radiation

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1992.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Relative Humidity

```
plot(v_RBO_RelHumidity_task_b_TS, type = "b", xlab = "years", ylab =
"Relative Humidity", main = "Time series plot for yearly Relative Humidity
during the Millennium Drought (1997 - 2009) (13 years)", pch = 1)
legend("topright", inset = .03, title = "Relative Humidity", legend =
"Relative Humidity series", horiz = TRUE, cex = 0.8, lty = 1, box.lty = 2,
box.lwd = 2, pch = 1)
```

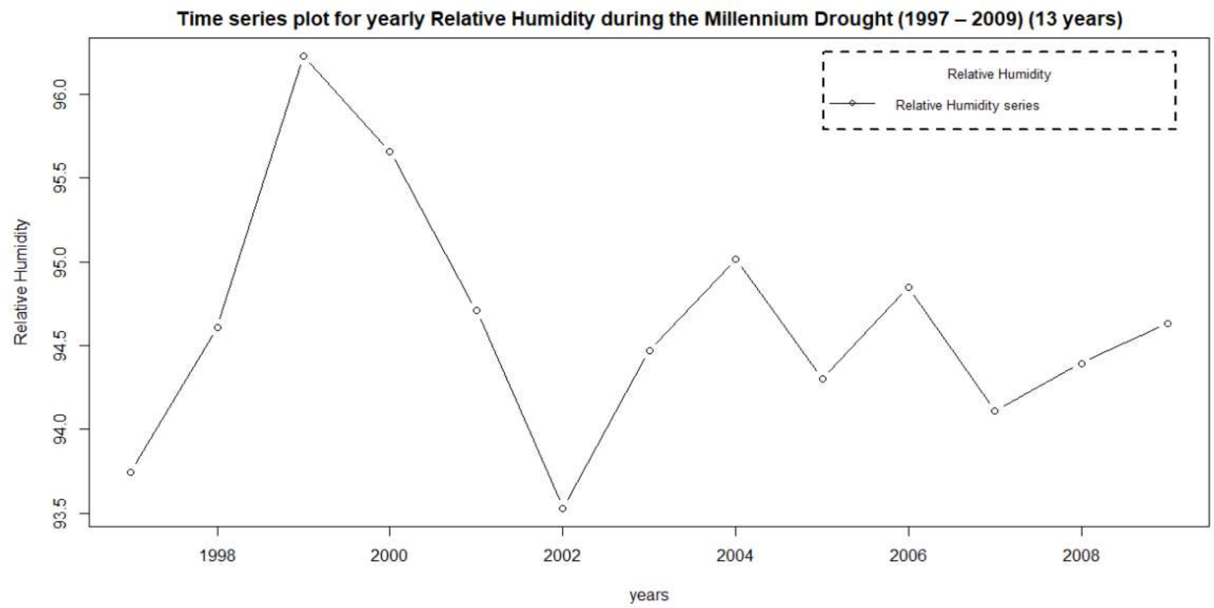


Fig 4.9: Relative Humidity - Time series plot.

```
McLeod.Li.test(y = v_RBO_RelHumidity_TS, main = "McLeod-Li Test Statistics
for Relative Humidity")
```

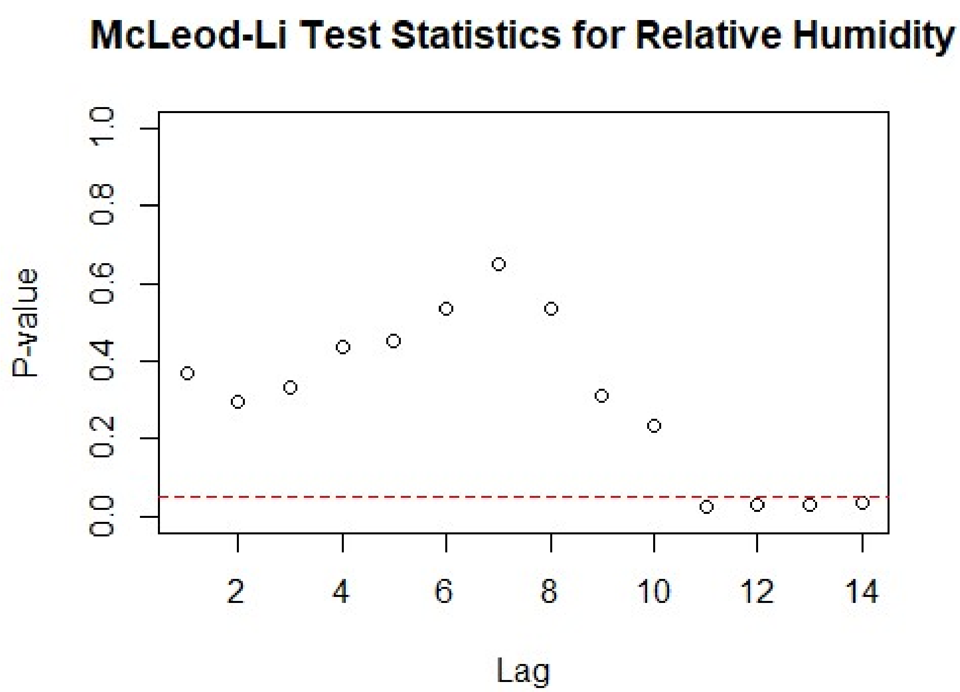


Fig 4.10: McLeod-Li Test Statistics for Relative Humidity.

Descriptive analysis

1. From the series plot, we can observe that there is no trend in the data.
2. There is an intervention around the year 1998.
3. From the series plot, we can conclude that there is no seasonality in the series.
4. There is no consistency across the observed period of time due to higher and lower values.
5. The series shows Autoregressive and moving average behaviour.
6. Also, we can see change in variance.

Checking for Stationary in the series

Checking for Stationary on Rank-based Order similarity metric series.

```
Stationary_Check(v_RBO_data_task_b_TS, "Rank-based Order similarity metric - ACF plot", "Rank-based Order similarity metric - PACF plot")
```

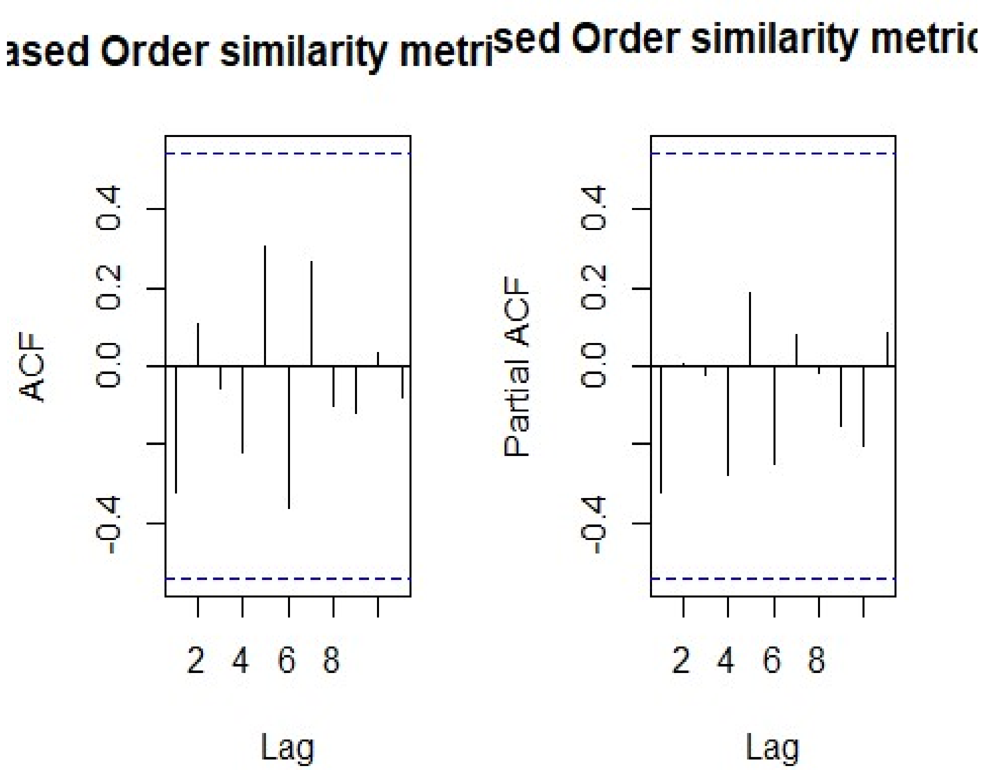


Fig 4.11: Rank-based Order similarity metric – ACF

Fig 4.12: Rank-based Order similarity metric - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -4.7218, Lag order = 0, p-value = 0.01
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value : 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Rank - based Order similarity metric series is Stationary.

Checking for Stationary on Temperature data.

```
Stationary_Check(v_RBO_Temp_task_b_TS, "Temperature - ACF plot", "Temperature - PACF plot")
```

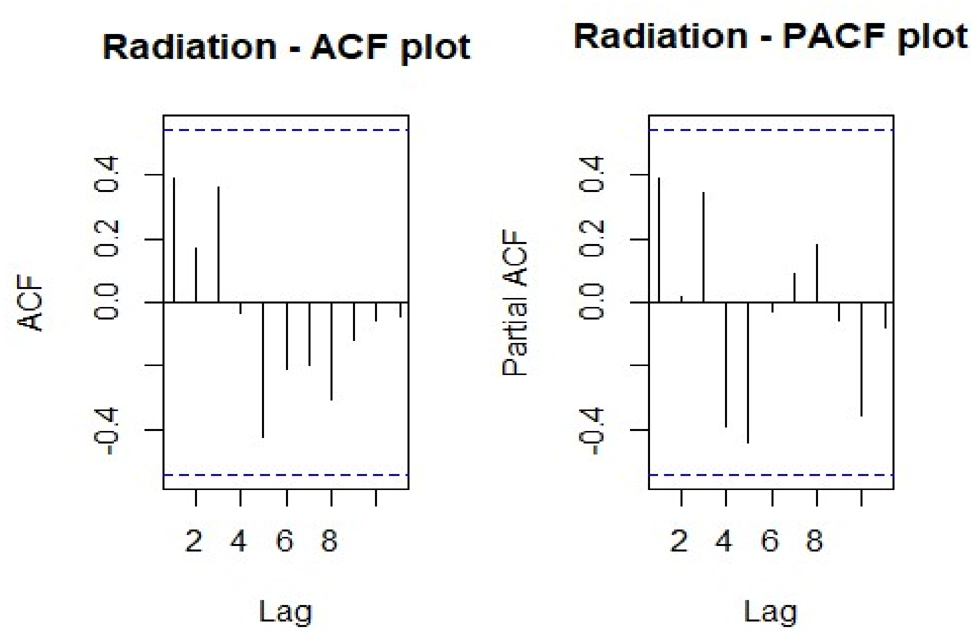


Fig 4.13: Temperature - ACF

Fig 4.14: Temperature - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -4.5475, Lag order = 0, p-value = 0.01
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: $0.01 < 0.05$

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Temperature series is Stationary.

Checking for Stationary on Radiation data.

```
Stationary_Check(v_RBO_Radiation_task_b_TS, "Radiation - ACF plot",
"Radiation - PACF plot")
```

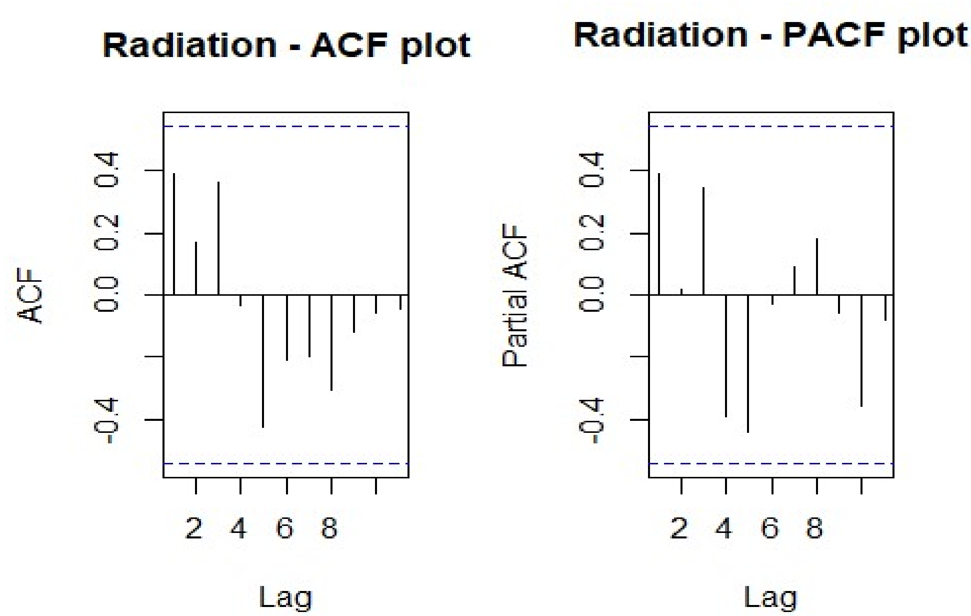


Fig 4.15: Radiation - ACF

Fig 4.16: Radiation - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -2.9296, Lag order = 1, p-value = 0.2182
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: 0.2182 > 0.05

p - value is greater than 0.05 and hence the test is not statistically significant. Therefore, Null hypothesis cannot be rejected i.e., The data is not stationary.

Therefore, the Radiation series is not Stationary.

Checking for Stationary on Rainfall data.

```
Stationary_Check(v_RBO_Rainfall_task_b_TS, "Rainfall - ACF plot", "Rainfall - PACF plot")
```

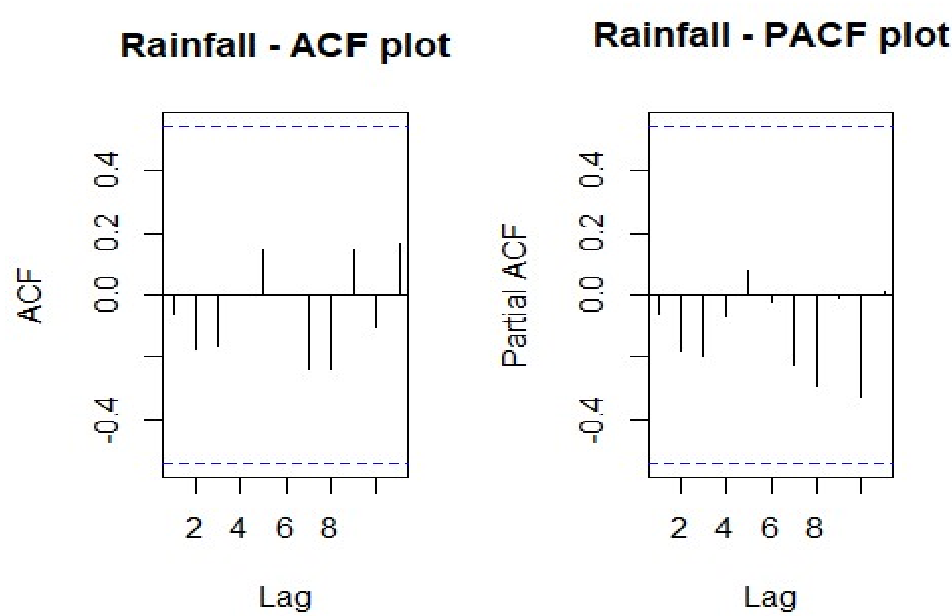


Fig 4.17: Rainfall - ACF

Fig 4.18: Rainfall - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -4.2994, Lag order = 0, p-value = 0.01281
## alternative hypothesis: stationary
```

The are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H0: The data is not stationary.

HA: The data is stationary.

Interpretations:

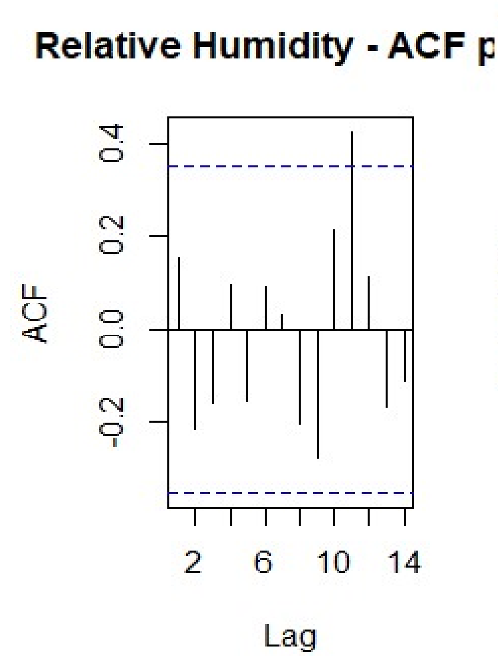
p - value: 0.01281 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Rainfall series is Stationary.

Checking for Stationary on Relative Humidity data.

```
Stationary_Check(v_RBO_RelHumidity_TS, "Relative Humidity - ACF plot",
"Relative Humidity - PACF plot")
```



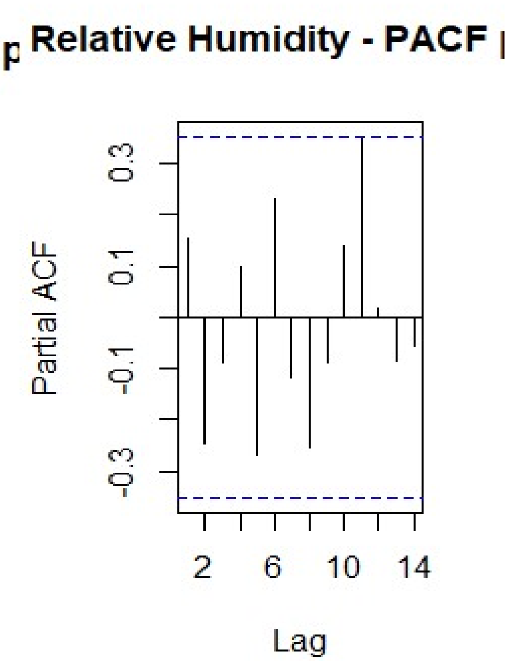


Fig 4.19: Relative Humidity - ACF

Fig 4.20: Relative Humidity - PACF

```
##
##  Augmented Dickey-Fuller Test
##
## data:  x
## Dickey-Fuller = -4.5749, Lag order = 0, p-value = 0.01
## alternative hypothesis: stationary
```

There are no significant lags in the ACF and PACF plot suggesting the stochastic component is white noise.

Hypotheses:

H_0: The data is not stationary.

H_A: The data is stationary.

Interpretations:

p - value: ~ 0.01 < 0.05

p - value is less than 0.05 and hence the test is statistically significant. Therefore, we Null hypothesis can be rejected i.e., The data is stationary.

Therefore, the Relative Humidity series is Stationary.

Suitable distributed lag model.

Before this let us find the correlation between the series.

```
# Calculating the correlation coefficient
cor(v_RBO_data_task_b_TS, v_RBO_Temp_task_b_TS)

## [1] -0.1684678

cor(v_RBO_data_task_b_TS, v_RBO_Rainfall_task_b_TS)

## [1] 0.1669995

cor(v_RBO_data_task_b_TS, v_RBO_Radiation_task_b_TS)

## [1] 0.07287555

cor(v_RBO_data_task_b_TS, v_RBO_RelHumidity_task_b_TS)

## [1] 0.2371467
```

This suggests that RBO has a better correlation with Relative humidity and Rainfall.

As we are going to forecast the RBO data, our dependent variable “y” will be Mortality Rate series object and independent variable “x” will be Relative humidity and Rainfall.

For dlm model we need some modifications in the series.

```
v_data_TS_44 <- ts(v_RBO_data_task_b, start = 1997, frequency = 1)
colnames(v_data_TS_44) <- c("x1", "y", "x2", "x3", "x4", "x5")
```

Dynamic model with Relative humidity

```
v_rain_dyna_taskb <- dynlm(y ~ x5, data = data.frame(v_data_TS_44))
summary(v_rain_dyna_taskb)

##
## Time series regression with "numeric" data:
## Start = 1, End = 13
##
## Call:
## dynlm(formula = y ~ x5, data = data.frame(v_data_TS_44))
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.037403 -0.007743 -0.003386  0.016193  0.038078
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.040585   0.824531   0.049    0.962
## x5          0.007054   0.008713   0.810    0.435
##
## Residual standard error: 0.02177 on 11 degrees of freedom
## Multiple R-squared:  0.05624,    Adjusted R-squared:  -0.02956
## F-statistic: 0.6555 on 1 and 11 DF,  p-value: 0.4353
```

Hypotheses:

H0: The data doesn't fit the Dynamic linear model.
HA: The data fits the Dynamic linear model.

Interpretations:

F - statistic is 0.6555
R - squared is 0.05624
Adjusted R - squared is -0.02956
Degrees of freedom - DF are (1, 11)
p - value (0.4353) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model does not fit the Dynamic linear model.

No residual analysis is required.

Dynamic model with Rainfall

```
v_rel_hum_dyna_taskb <- dynlm(y ~ x3, data = data.frame(v_data_TS_44))
summary(v_rel_hum_dyna_taskb)

##
## Time series regression with "numeric" data:
## Start = 1, End = 13
##
## Call:
```

```
## dynlm(formula = y ~ x3, data = data.frame(v_data_TS_44))
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.041465 -0.009269 -0.005660  0.017174  0.040068
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.68603    0.03981  17.234 2.62e-09 ***
## x3           0.01057    0.01881   0.562    0.586
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0221 on 11 degrees of freedom
## Multiple R-squared:  0.02789,    Adjusted R-squared:  -0.06048
## F-statistic: 0.3156 on 1 and 11 DF,  p-value: 0.5855
```

Hypotheses:

H_0: The data doesn't fit the Dynamic linear model.
H_A: The data fits the Dynamic linear model.

Interpretations:

F - statistic is 0.3156
R - squared is 0.02789
Adjusted R - squared is -0.06048
Degrees of freedom - DF are (1, 11)
p - value (0.5855) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model does not fit the Dynamic linear model.

No residual analysis is required.

Since the model didn't fit on the two variables let us fit the remaining 2 variables also.

Dynamic model with Radiation

```
v_rad_dyna_taskb <- dynlm(y ~ x4, data = data.frame(v_data_TS_44))
summary(v_rad_dyna_taskb)
```

```
##
## Time series regression with "numeric" data:
## Start = 1, End = 13
##
## Call:
## dynlm(formula = y ~ x4, data = data.frame(v_data_TS_44))
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.044285 -0.009024 -0.002502  0.015010  0.036522
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.639791   0.282039   2.268   0.0444 *
## x4          0.004607   0.019011   0.242   0.8130
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02235 on 11 degrees of freedom
## Multiple R-squared:  0.005311,   Adjusted R-squared:  -0.08512
## F-statistic: 0.05873 on 1 and 11 DF,  p-value: 0.813
```

Hypotheses:

> **H0: The data doesn't fit the Dynamic linear model.**
> **HA: The data fits the Dynamic linear model.**

Interpretations:

F - statistic is 0.05873
R - squared is 0.005311
Adjusted R - squared is -0.08512
Degrees of freedom - DF are (1, 11)
p - value (0.813) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model does not fit the Dynamic linear model.

No residual analysis is required.

Dynamic model with Temperature

```
v_temp_dyna_taskb <- dynlm(y ~ x2, data = data.frame(v_data_TS_44))
summary(v_temp_dyna_taskb)

##
## Time series regression with "numeric" data:
## Start = 1, End = 13
##
## Call:
## dynlm(formula = y ~ x2, data = data.frame(v_data_TS_44))
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.044526 -0.007965 -0.002129  0.010852  0.033104
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.81163    0.18269   4.443 0.000991 ***
## x2          -0.01109    0.01957  -0.567 0.582200
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.02209 on 11 degrees of freedom
## Multiple R-squared:  0.02838,	Adjusted R-squared:  -0.05995
## F-statistic: 0.3213 on 1 and 11 DF,  p-value: 0.5822
```

Hypotheses:

H_0: The data doesn't fit the Dynamic linear model.
H_A: The data fits the Dynamic linear model.

Interpretations:

F - statistic is 0.3156
R - squared is 0.02789
Adjusted R - squared is -0.06048
Degrees of freedom - DF are (1, 11)
p - value (0.5855) is > 0.05 and therefore, it is not statistically significant. Therefore, Null hypothesis is not rejected. Hence, the model does not fit the Dynamic linear model.

No residual analysis is required.

To forecast there is predictor variable that is fitted on the dynamic model.

Conclusion

Task 1:

The data columns are converted into the time series objects for each column in the data set. They all have seasonality in their series with no obvious trend, behavior or change in variance and with intervention points.

Mortality rate is stationary and hence it can be directly send to the models as it is a dependent variable. Similarly, all other variables becomes the independent variables or predictors. All the variables are stationary.

Decomposition of components suggested that, the seasonality component is week and the trend component doesn't show any trend with the data.

We got Mortality rate has a strong correlation with Chemical emission 1 and Particle size. Therefore, we use only these variables to fit the models.

Now to find the best model we have three approaches.

1. Suitable Dlag models and dynamic LM: Since, we are doing multivariate analysis it is required to convert the entire data frame into time series. We sent the two correlated variables into the formula. The respective summary and residuals are analysed then.

This approach only supports Finite dlm, AR dlm as well as dynamic model. But for polynomial and Koyck we fit predictors individualls they donot perform multivariate analysis.

Here we got AR_DLM_solar_53 as the better model in terms of residual analysis as well as MASE scores.

Though dynamic model performed better, it suffered with randomness, normality and correlation to some extent.

2. Smoothing methods:

Since the Seasonality component is week we cannot get additive and multiplicative seasonality. The best model here obtained is the simple seasonality model.

3. State space model variations: Here we got ETS(M, N, N) as the best model automatically. it's mase score is high compared to other models but is efficient in case of BIC and AIC. It suffered with randomness, normality and correlation to some extent.

But the MASE scores are shown least for simple seasonal smoothing method. Also, ets(M, M, N) is better in state space variations method.

So we considered these as the best model for forecasting the Mortality rate data for the next 4 weeks.

From the 4 weeks forecast results of smoothing we can predict that there will be decrease in the Mortality rate in the future. This suggests that the impact of the chemical components will be more in future.

Also, from the 4 weeks forecast results of smoothing we can predict that there will be increase in the Mortality rate in the future. This suggests that the impact of the chemical components will be more in future.

But as we forecast with 95% confidence intervals, we cannot consider this as accurate.

Task 2:

The data columns are converted into the time series objects for each column in the data set. They all have some obvious trend, behavior or change in variance and with intervention points leaving with no seasonality.

First Flower Day series is stationary and hence it can be directly send to the models as it is a dependent variable. Similarly, all other variables becomes the independent variables or predictors. All the variables are stationary except Temperature and Radiation but it is not necessary to make them stationary.

We got FFD has a strong correlation with Rainfall and Radiation comparitively. Therefore, we use only these variables to fit the models.

Now to find the best model we have three approaches.

1. Suitable Dlag models and dynamic LM: Since, we are doing univariate analysis we will model predictors individually. Here, we also require to analysis the models by fitting them with and without slope. We sent each correlated variables into the formula. The respective summary and residuals are analysed then.

This approach only supports Finite dlm, AR dlm as well as dynamic model.

Here only three models are fitted.

```
finite dlm with Temperature without slope.
finite dlm with Radiation without slope.
Dynamic model without slope with radiation.
```

Without slope we got best results but they did not perform better with residuals. But dynamic model is better in terms of r squared.

Though dynamic model performed better, it suffered with randomness, normality and correlation to some extent.

2. Smoothing methods:

Since there in no Seasonality in the series cannot use additive and multiplicative seasonality. The best model here obtained is the simple seasonality model.

3. State space model variations: Here we got ETS(M, N, N) as the best model automatically. It's MASE score is high compared to other models but is efficient in case of BIC and AIC. It suffered with randomness, normality and correlation to some extent.

Among all the methods, holt in exponential smoothing with damped trend. When compared with MASE, BIC and AIC score is better in Smoothing method. Also, ets(M, M, N) is better in state space variations method.

So we considered these as the best model for forecasting the First Flower Day data for the next 4 years. From the 4 years forecast results we can predict that there will be decrease in the First Flower Day in the future. This suggests that the impact of the chemical components will be less in future. But as we forecast with 95% confidence intervals we cannot consider this as accurate.

Task 3:

Part (a):

The data columns are converted into the time series objects for each column in the data set. They all have some obvious trend, behavior or change in variance and with intervention points leaving with no seasonality.

Rank-based Order similarity metric series is stationary and hence it can be directly send to the models as it is a dependent variable. Similarly, all other variables becomes the independent variables or predictors. All the variables are stationary except Temperature and Radiation but it is not necessary to make them stationary.

We got Rank-based Order similarity metric has a strong correlation with Rainfall and Temperature comparitively. Therefore, we use only these variables to fit the models.

Now to find the best model we use Suitable Dlag models and dynamic LM:

Since, we are doing univariate analysis we will model predictors individually. The respective summary and residuals are analysed then.

Here, the fitted models among each method with 2 different predictors are

1. “PolyDLM_model_Rain”
2. “AR_DLM_Rain_53”
3. “PolyDLM_model_temp”
4. “AR_DLM_temp_52”
5. “v_rain_dyna”

Among the polynomial models temperature has the least MASE score compared. Whereas, AR_DLM has Rainfall and Dynamic has only one variable fit i.e., with rainfall.

So we considered these as the best models for forecasting the Rank-based Order similarity metric data for the next 3 years. From the 3 years forecast results we can predict that there will be decrease in the Rank-based Order similarity metric in the future. But as we forecast with 95% confidence intervals we cannot consider this as accurate.

Part (b):

The RBO data in Task 3 is filtered from 1997 to 2009 considering the millennium drought period.

The data columns are converted into the time series objects for each column in the data set. They all have some obvious trend, behavior or change in variance and with intervention points leaving with no seasonality.

Rank-based Order similarity metric series is stationary and hence it can be directly send to the models as it is a dependent variable. Similarly, all other variables becomes the independent variables or predictors. All the variables are stationary except for Radiation but it is not necessary to make them stationary.

We got Rank-based Order similarity metric has a strong correlation with Relative humidity and Rainfall comparitively. Therefore, we use only these variables to fit the models.

Now to find the best model we use only dynamic LM.

Since, we are doing univariate analysis we will model predictors individually. The respective summary and residuals are analysed then.

The model is not fit on both Relative humidity and Rainfall. Therefore, model is fitted on the other two variables too.

No model is fitted on the data and hence no forecasting is done.

References

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[7] Week 3, 4, 5 and 6 module tasks.

[8] My previous works for Time series Analysis in my Second year First Sem.