In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
```

In [2]:
```
import warnings
warnings.filterwarnings('ignore')
```

Discrete Probability Distribution: Binomial

- A T-shirt manufacturing company regularly conducts quality checks at specified period on the products it manufactures. Their empirical data of manufacturing report says,4% of their products were defective. Suppose a random sample of 250 T-shirts were picked from the manufacturing unit,what is the probability that,
- a) None of the T-shirts are defective
- b) 5 or fewer T-shirts are defective
- c) 20 or more T-shirts are defective
- d) What is the mean value of this Binomial Distribution
- e) Plot the Binomial Distribution

In [4]:
```
import scipy.stats as stats
```

In [ ]:
```
#a) we need to calculate P(x=0) = n C x*P^x*Q^(n-x)
#Here n=250, x=0, P=defective % = 4%=0.04, Q= Non-defective % = 96% = 0.96
n=250
p=0.04
stats.binom.pmf(0,n,p)
```

Out[61]:
```
3.696649374485171e-05
```

In [ ]:
```
#b) 5 or fewer => P(x<=5)= P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)
stats.binom.cdf(5,n,p)
```

Out[56]:
```
0.06329328366342199
```

In [ ]:
```
#c) 20 or more => P(x>=20)= P(x=20)+........+ all cummulative probabilities in right hand side
1-stats.binom.cdf(19,n,p)
```

Out[57]: 0.002814404938111137

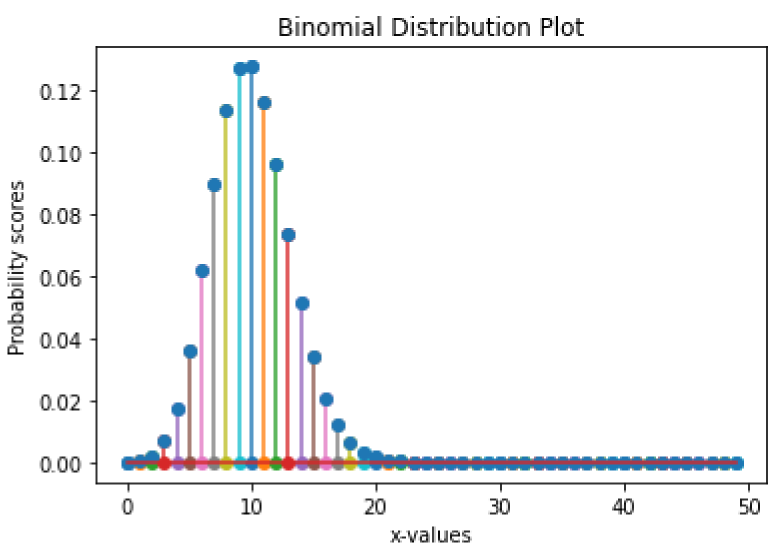
In [ ]:
```
#d)Mean value of Binomial distribution = n*p
#Here, mean implies 4% of 250
0.04*250
```

Out[58]: 10.0

In [ ]:
```
stats.binom.pmf(10,n,p)
```

Out[63]: 0.12768812205208388

In [ ]:
```
#e) to generate the plot,we need to define x-axis
x=np.arange(0,50)
binom_dist=stats.binom.pmf(x,n,p)
plt.stem(x,binom_dist,'o-')
plt.xlabel('x-values')
plt.ylabel('Probability scores')
plt.title('Binomial Distribution Plot')
plt.show()
```



In [ ]:
```
x.dot(binom_dist) #this is analogous to the notion summation of x.p(x)
```
Out[64]:
```
9.999999999999698
```

In [ ]:
```
np.argmax(binom_dist),np.max(binom_dist)
```
Out[19]:
```
(10, 0.12768812205208388)
```

In [ ]:
```
sd=np.sqrt(((x-10)**2).dot(binom_dist))  #more generic formula
sd
```
Out[65]:
```
3.0983866769658617
```

In [ ]:
```
np.sqrt(n*p*(1-p))
```
Out[66]:
```
3.0983866769659336
```

- We could notice,the peak probability score of 12.76% at the mean value (10 T-shirts)

Discrete Probability Distribution: Poisson

A customer care unit of a fiber optic broadband service provider receives on an average 30 emails on a day regarding complaint in their service. To strategically plan the resource of service engineers, the following calculations are performed

---

a) What is the probability of receiving 100 complaints in a given day?

b)What is the probability of receiving 20 or less complaints in a given day?

c)What is the probability of receiving 35 or more complaints in a given day?

d)Plot the poisson distribution

In [ ]:
```
#a) we need to calculate P(x=100)
#Here Lambda = 30 emails (average)
lamb=30
stats.poisson.pmf(100,lamb)
```

Out[68]:
```
5.16758180183853 96e-24
```

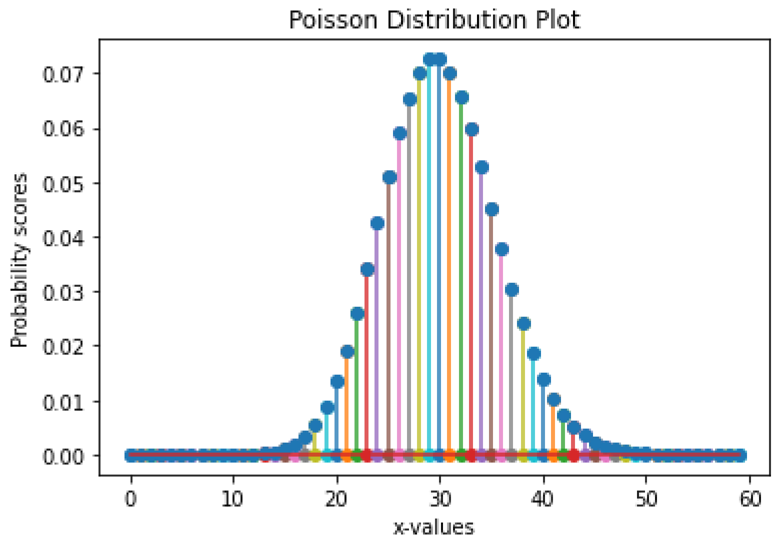
In [ ]:
```
#b)P(x<=20)
stats.poisson.cdf(20,lamb)
```

Out[69]:
```
0.035284618454228846
```

In [ ]:
```
#c) P(x>=35)
1-stats.poisson.cdf(34,lamb)
```

Out[70]:
```
0.20269167451688286
```

In [ ]:
```
x=np.arange(0,60)
pois_dist=stats.poisson.pmf(x,lamb)
plt.stem(x,pois_dist,'o-')
plt.xlabel('x-values')
plt.ylabel('Probability scores')
plt.title('Poisson Distribution Plot')
plt.show()
```



In [ ]:
```
np.argmax(pois_dist)
```

Out[73]: 29

In [ ]:
```
pois_dist[29]
```

Out[74]: 0.07263452647159181

In [ ]:
```
pois_dist[30]
```

Out[75]: 0.07263452647159181

- We can notice the highest probability centered around the mean (ie) 30 complaints

In [ ]:
```
np.sqrt(x.dot(pois_dist)) #cross checking the mean value from histogram perspective x.P(x)
```

Out[80]:
```
5.477220430202007
```

In [ ]:
```
np.sqrt(((x-30)**2).dot(pois_dist))
```

Out[79]:
```
5.477144707167058
```

#Normal Distribution | Value of the random variable | Area under the normal distribution (CDF) | | -------------------------------- | --------------------------| | $\mu - \sigma \leq x \leq \mu + \sigma$ | 0.6828 | | $\mu - 2\sigma \leq x \leq \mu + 2\sigma$ | 0.9545 | | $\mu - 3\sigma \leq x \leq \mu + 3\sigma$ | 0.9974 |

- Based on last 5 year statistics, an app based food delivery company declared its statistics at Chennai, saying the average delivery time to any part of Chennai is 90 min, with a standard deviation of 20 min. Compute the following,
- a) What proportion of delivery were done less than 60 min?
- b) What proportion of delivery were done more than 100 min?
- c) What proportion of delivery were done between 30min to 60min?

In [ ]:
```
#a)
stats.norm.cdf(60,90,20)
#answer is 6.68%
#.cdf by default will count the left hand
```

Out[5]:
```
0.06680720126885807
```

In [5]:
```
#or alternatively we can pass
stats.norm.cdf(-1.5)
#we can convert the value into Z score and pass the value.
```

Out[5]:
```
0.06680720126885807
```

In [9]:
```
1-stats.norm.cdf(60,50,20)
```

Out[9]:
```
0.3085375387259869
```

In [ ]:
```
(60-90)/20
```

Out[8]:
```
-1.5
```

In [6]:
```
#OR alternatively we can pass the
stats.norm.cdf(-1.0)
```

Out[6]:
```
0.15865525393145707
```

In [10]:
```
#b)
1-stats.norm.cdf(100,90,20)
```

Out[10]:
```
0.3085375387259869
```

In [11]:
```
#c)
stats.norm.cdf((60-90)/20)-stats.norm.cdf((30-90)/20)
```

Out[11]:
```
0.06545730323722798
```

- Inverse Survival function gives the value given a probability

In [ ]:
```
#What is the cutoff value for 5% area in standard normal curve?
#2.5% either side of the curve
2.5/100
```

Out[19]:
```
0.025
```

In [12]:
```
stats.norm.isf(0.025) #95% area
```

Out[12]:
```
1.9599639845400545
```

In [ ]:
```
stats.norm.isf(0.005) #99% area
```

Out[10]:
```
2.575829303548901
```

In [13]:
```
stats.norm.isf(0.05) #90% area
```

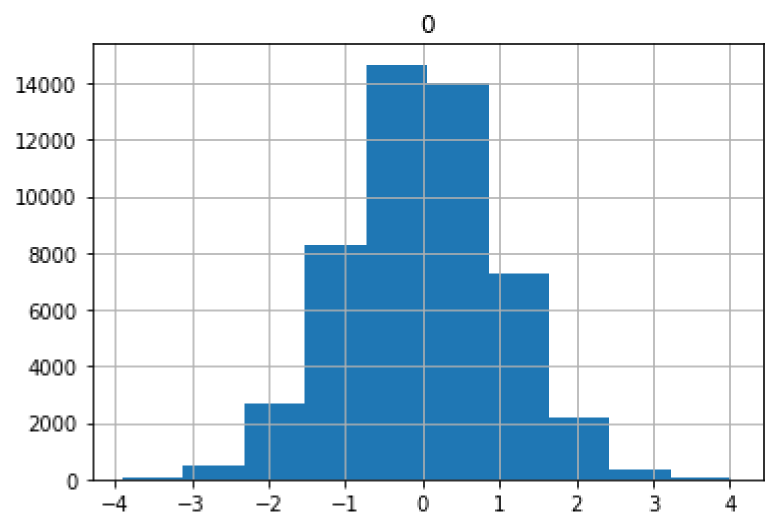
Out[13]:
```
1.6448536269514729
```

In [15]:
```
import random
pop_sal=np.random.randn(50000)
pop_sal
#simulating 50000 of the population
```

Out[15]:
```
array([-0.6889953 ,  0.46889536,  0.23472991, ..., -0.57287622,
        0.41898068,  0.0233784 ])
```

In [16]:
```
pop_sal_df=pd.DataFrame(pop_sal)
pop_sal_df.hist()
```

Out[16]:
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7fcad12d5358>]],
      dtype=object)
```



In [19]:
```
#convert all 50000 samples in real INR units (assume pop_mean(true_mean=6.78LPA,sigma=40k))
sal=pop_sal*40000+678000
sal[:50]#just for display
```

Out[19]:
```
array([650440.18816621, 696755.81443549, 687389.19627987, 577756.73698671,
       680252.23970453, 667314.90670656, 702169.03126286, 604371.99263918,
       714377.71707805, 646829.52412998, 683683.72694913, 691232.142362  ,
       721821.70520121, 656616.62369856, 732765.70370266, 665166.57252265,
       668233.9903018 , 611905.56246847, 661503.73512871, 665964.43539245,
       629421.84009142, 664270.24106678, 695880.24398613, 721860.70182764,
       686795.62710355, 733167.93087388, 686468.0575932 , 666492.27814401,
       678659.88698166, 632251.08709472, 652891.3067268 , 663805.47579432,
       702513.78957679, 765021.64205576, 738109.13540897, 737951.70536068,
       643509.42933873, 696239.88287976, 715224.77919337, 696034.67589335,
       685827.22912836, 714394.02661737, 724932.3531023 , 722378.44347268,
       692728.57689258, 702100.39727511, 681792.80248471, 628897.39377278,
       703697.28426036, 621250.90480723])
```

In [20]:
```
np.mean(sal)
```

Out[20]:
```
678089.8193874795
```

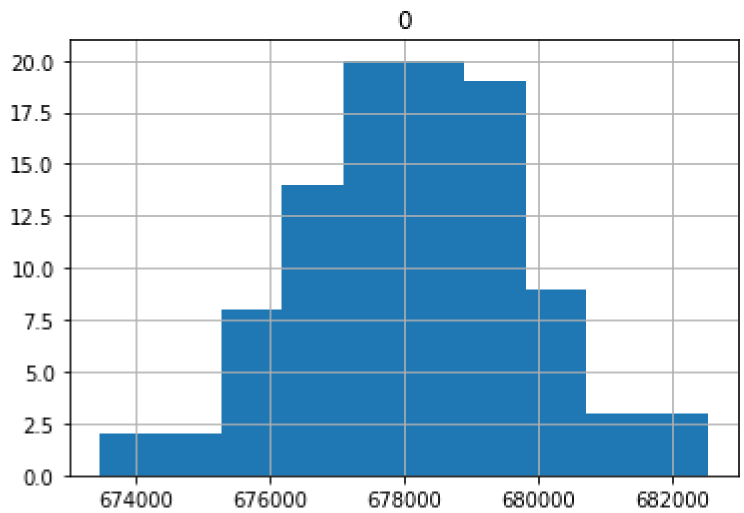
In [21]:
```
#Method-1 (as per the theoretical definition of CLT)
trial=[]
for itr in np.arange(100):
  samp=[]
  for val in np.arange(500):
    samp.append(np.random.choice(sal))
  trial.append(np.mean(samp))
```

In [22]:
```
len(trial)
```

Out[22]:
```
100
```

In [25]:
```
trial_df=pd.DataFrame(trial)
trial_df.hist()
```

Out[25]:
```
array([[&lt;matplotlib.axes._subplots.AxesSubplot object at 0x7fcad11dbd68&gt;]],
      dtype=object)
```



In [24]:
```
np.mean(trial)
```

Out[24]:
```
678166.1314576634
```

In [ ]:
```
np.mean(trial),np.std(trial,ddof=1)
```

Out[30]:
```
(678171.6527113924, 1693.8784830311258)
```

In [26]:
```
#95% CI range estimate of pop_mean is given by
(678171.6527113924-1.96*1693.8784830311258,678171.6527113924+1.96*1693.8784830311258)
```

Out[26]:
```
(674851.6508846513, 681491.6545381334)
```

In [27]:
```
#68% CI range estimate of pop_mean is given by
(678171.6527113924-1*1693.8784830311258,678171.6527113924+1*1693.8784830311258)
```

Out[27]:
```
(676477.7742283612, 679865.5311944236)
```

In [ ]:
```
trial_df=pd.DataFrame(trial)
trial_df.std()
```

Out[24]:
```
0    1693.878483
dtype: float64
```

In [ ]:
```
np.std(trial,ddof=1)
```

Out[26]:
```
1693.8784830311258
```

In [ ]:
```
#(95% range of this sal)
678171.6527113924-1.96*1693.8784830311258
```

Out[31]:
```
674851.6508846513
```

In [ ]:
```
678171.6527113924+1.96*1693.8784830311258
```

Out[32]:
```
681491.6545381334
```

In [ ]:
```
#[6.74Lacs to 6.81Lacs]
```

In [ ]:
```
 #Method-2 (practical approach with approx)
samp=[]
for val in np.arange(500):
  samp.append(np.random.choice(sal))
```

In [ ]:
```
np.mean(samp),np.std(samp,ddof=1)
```

Out[36]: (681015.0281048274, 41018.99285935366)

In [ ]:
```
#Standard Error of Mean in practical approach
sem=np.std(samp,ddof=1)/np.sqrt(500)
sem
```

Out[37]: 1834.425128041865

In [ ]:
```
681015.0281048274-1.96*sem
```

Out[38]: 677419.5548538653

In [ ]:
```
681015.0281048274+1.96*sem
```

Out[39]: 684610.5013557895

In [ ]:
```
#De-moivre's approximation of Discrete Distribution with Standard Normal Distribution
#Generate Binomial distribution with p=0.5, n=25 and calculate P(x<=14)
stats.binom.cdf(14,25,0.5)
```

Out[32]: 0.7878218889236449

In [ ]:
```
#If x is a normal random variable,mean=n*p, sd=npq
x_bar=25*.5
x_bar
```

Out[34]: 12.5

In [ ]:
```
x_var=25*0.5*0.5
x_sd=np.sqrt(x_sd)
```

Out[36]: 2.5

In [ ]:
```
(14-12.5)/2.5
```

Out[37]: 0.6

*In discrete distribution, the histogram bars are centered on the numbers. This means P(x<=14) in Discrete distribution is actually the area under the bars less than x=14.5, we need to account for that extra 0.5, while calculating the same area in continuous distribution

In [ ]:
```
(14.5-12.5)/2.5
```

Out[42]:
```
0.8
```

In [ ]:
```
#we need to calculate the prob P(x<=14) which equals to P(z<=0.6)
stats.norm.cdf(0.6)
```

Out[41]:
```
0.7257468822499265
```

In [ ]:
```
stats.norm.cdf(0.8)
```

Out[43]:
```
0.7881446014166034
```

- Now we have approximated Discrete distribution using Normal Distribution [0.7878 = 0.7881 (approx)]

In [ ]: