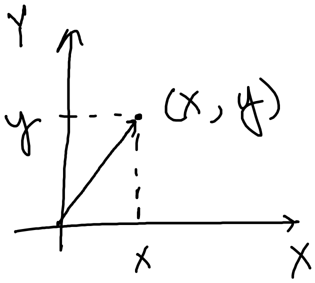
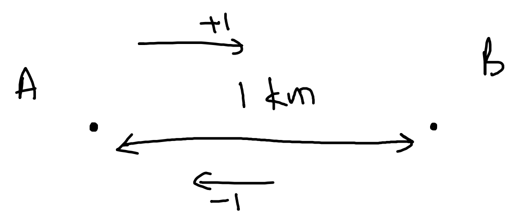
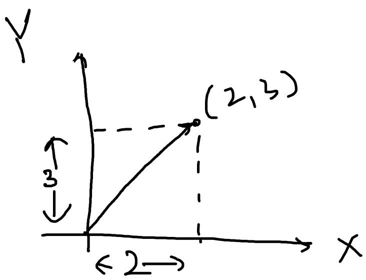
Linear Algebra

Quantities

- Scalar → Magnitude or number
- Vector → Magnitude with direction or number with a sign



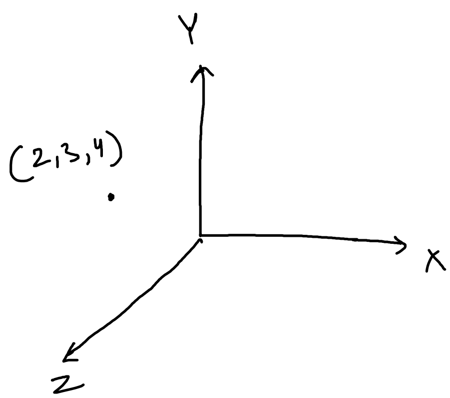




vector = [2 3] ⇒ 2d

Total distance = 2 km ⇒ 1 + 1

Total displacement = +1 − 1 = 0



vector = [2 3 4] ⇒ 3d

vector = [2 3 4 5] ⇒ 4d

vector in nd = [2   3   4   5 - - - - n] = n dimensions.

<u>**Matrix**</u> ⇒ it is a table of numbers.

$$
\begin{array}{c c}
 & \text{columns} \\
 & \downarrow \quad \downarrow \quad \downarrow \\
\text{rows} \; \begin{array}{c}\rightarrow \\ \rightarrow \\ \rightarrow\end{array} & \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}
\end{array}
$$

$$\text{i/p} \longrightarrow \overset{\text{Linear}}{\boxed{\text{Transformation}}} \longrightarrow \text{o/p}$$

<u>**Addition :**</u>

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

<u>Q</u>

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 10 & 11 \end{bmatrix} = \begin{bmatrix} 1+4 & 3+8 \\ 2+10 & 4+11 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 11 \\ 12 & 15 \end{bmatrix}$$

Multiplication

$$\begin{bmatrix} a & b \end{bmatrix}_{1\times\cancel{2}} \begin{bmatrix} c & e \\ d & f \end{bmatrix}_{\cancel{2}\times 2} = \begin{bmatrix} a\times c + b\times d & a\times e + b\times f \end{bmatrix}$$

$$= \begin{bmatrix} ac+bd & ae+bf \end{bmatrix}_{1\times 2}$$

Rule: The no. of columns of first matrix = no of rows of II^{nd} matrix

$$\underset{2\times 2}{\overset{A}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}} \quad \underset{1\times 2}{\overset{B}{\begin{bmatrix} 1 & 2 \end{bmatrix}}} \neq \overset{B}{\underset{1\times 2}{\begin{bmatrix} 1 & 2 \end{bmatrix}}} \overset{A}{\underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}}$$

Q $$\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2\times1+4\times2 & 2\times4+4\times5 \\ 3\times1+6\times2 & 3\times4+6\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 28 \\ 15 & 42 \end{bmatrix}$$

Q $a_{m\times n} \times b_{p\times q} \Rightarrow$ No $\quad n \neq p$

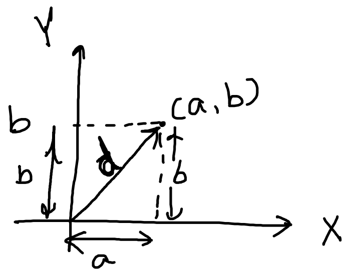
Q $c_{m\times n} \times b_{q\times p}$ (given $q = n$) $\neq c \quad \times b_{q\times p}$

Q $\quad C_{m\times n} \times b_{q\times p}$ (given $q=n$) $\Rightarrow C_{m\times n} \times b_{n\times p}$

$\Rightarrow d_{m\times p}$

Distance

Distance of a point from origin:



By Pythagoras Theorem:

$$H^2 = P^2 + B^2$$

$$d^2 = a^2 + b^2$$

$$\boxed{d = \sqrt{a^2+b^2}} \rightarrow 2d$$

Using LA,

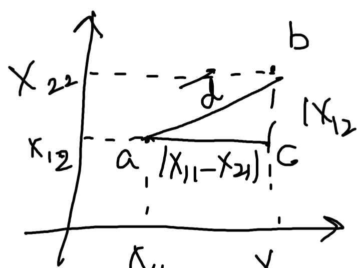
$$d = \sqrt{a^2+b^2+c^2} \rightarrow 3d$$

$$d = \sqrt{a^2+b^2+c^2+ \dots + n^2} \rightarrow nd$$

Distance b/w two points:

$a = [x_{11} \quad x_{12}]$

$b = [x_{21} \quad x_{22}]$



$$d^2 = bc^2 + ac^2 \Rightarrow ac^2 + bc^2$$

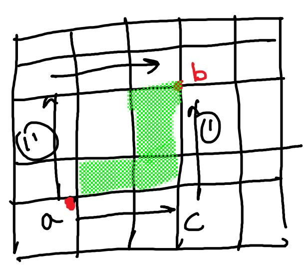
By Pythagoras Theorem:

$$d^2 = (|x_{11} - x_{21}|)^2 + (|x_{12} - x_{22}|)^2$$

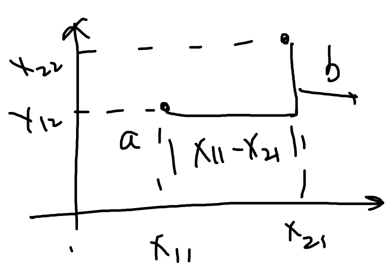
$$d = \sqrt{(|x_{11} - x_{21}|)^2 + (|x_{12} - x_{22}|)^2}$$

$$d = \left[\sum_{i=1}^{n} (x_{1i} - x_{2i})^2 \right]^{1/2} \leftarrow \text{Euclidean distance } (L2\text{-Norm})$$

Manhattan Distance



$ac + cb \Rightarrow a \to b$



$|x_{12} - x_{22}|$

$$d \Rightarrow |x_{11} - x_{21}| + |x_{12} - x_{22}|$$

$$\boxed{d = \sum_{i=1}^{n} |X_{1i} - X_{2i}|}$$

① → 2

Minkowski Distance :

$$d = \left[\sum_{i=1}^{n} |X_{1i} - X_{2i}|^{p} \right]^{1/p}$$

$p = 1, 2, 3 - - - -$

lets put $p=1$,

$$d = \left[\sum_{i=1}^{n} |X_{1i} - X_{2i}| \right] \rightarrow \text{manhattan distance (L-1 Norm)}$$

put $p=2$

$$d = \left[\sum_{i=1}^{n} |X_{1i} - X_{2i}|^{2} \right]^{1/2} \rightarrow \text{euclidean distance (L2 Norm)}$$

Vector Multiplication

✓ dot product → scalar/number

Cross product X → direction

DOT PRODUCT

$$a = [a_1 \quad a_2 \quad a_3 \quad --- \quad a_n]_{1\times n}$$

$$b = [b_1 \quad b_2 \quad b_3 \quad --- \quad b_n]_{1\times n}$$

In Linear Algebra,

$$\text{Vectors} \swarrow \text{(default) Column Vector} \qquad \searrow \text{row Vector}$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}_{\substack{n\times 1 \\ R\times C}} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}_{\substack{n\times 1 \\ R\times C}}$$

$$a = [a_1 \quad a_2 \quad a_3 \quad -- \quad a_n]$$

$$b = [b_1 \quad b_2 \quad b_3 \quad --- \quad b_n]$$

$$a \cdot b = a^T \cdot b$$

$$[\ ]_{1\times 1}$$

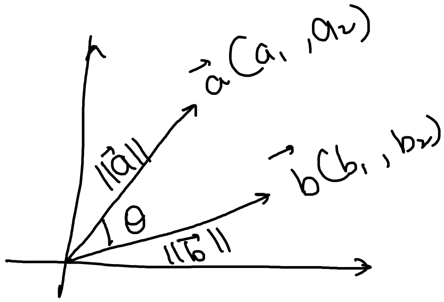
$$= [a_1 \ a_2 \ a_3 \ --- \ a_n]_{1\times \cancel{n}} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}_{\cancel{n}\times 1}$$

$$= [a_1 b_1 + a_2 b_2 + a_3 b_3 + --- + a_n b_n]_{1\times 1} \rightarrow \text{scalar}$$

Angle b/w vectors:

Geometric

Angle b/w vectors:



Geometric

$$a \cdot b = \|\vec{a}\| \, \|\vec{b}\| \cos\theta \quad -(I)$$

$$a \cdot b = a^T b = [a_1 b_1 + a_2 b_2 + \cdots + a_n b_n] \quad -(II)$$

(LA)

Equating (I) & (II) on LHS

$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2}$$

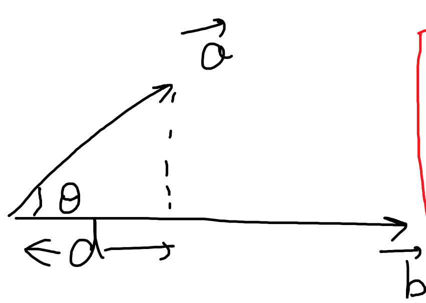
$$\|\vec{a}\| \, \|\vec{b}\| \cos\theta = [a_1 b_1 + a_2 b_2 + \cdots + a_n b_n]$$

$$\|\vec{b}\| = \sqrt{b_1^2 + b_2^2 + \cdots + b_n^2}$$

$$\cos\theta = \frac{[a_1 b_1 + a_2 b_2 + \cdots + a_n b_n]}{\|\vec{a}\| \, \|\vec{b}\|}$$

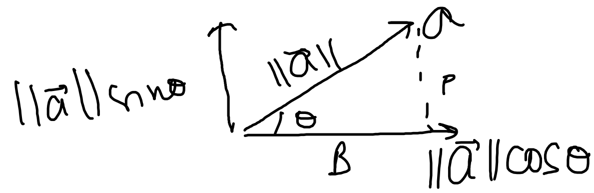
$$\boxed{\theta = \cos^{-1} \frac{\vec{a} \cdot b}{\|\vec{a}\| \, \|\vec{b}\|}}$$

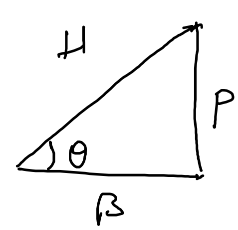
Projection



$$d = \|\vec{a}\| \cos\theta$$

Basics





$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$

$\vec{a} \cdot \vec{b} = d \cdot \|\vec{b}\|$

$\boxed{d = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}}$ → Projection of A on B

$\cos\theta = \dfrac{B}{H}$ $\sin\theta = \dfrac{P}{H}$

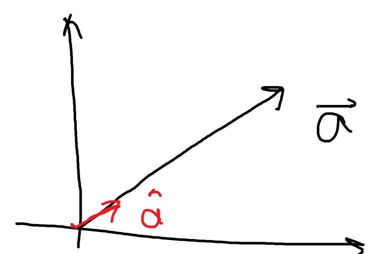
$B = H\cos\theta$ $P = H\sin\theta$

Unit Vector → vector of magnitude ⇒ 1

↳ information regarding direction

$\hat{a} = \dfrac{\cancel{\text{magnitude}} \times \text{direction}}{\cancel{\text{magnitude}}} = \text{direction.}$

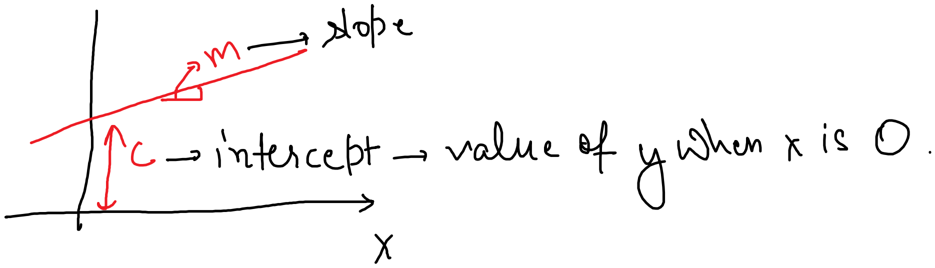
$\boxed{\hat{a} = \dfrac{\vec{a}}{\|\vec{a}\|}}$



Lines

$y = mx + c$ → eqⁿ of line

y



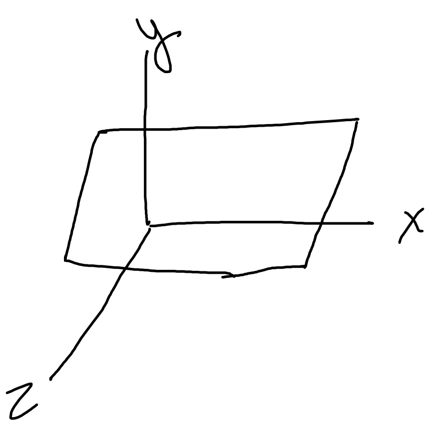
$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta = \frac{dy}{dx}$$

General Eq^n of line ⇒ $ax + by + c = 0$

$$by = -ax - c$$

$$y = \underbrace{-\frac{a}{b}}_{\text{slope}} x \underbrace{- \frac{c}{b}}_{\text{intercept}}$$

Plane:



General Eq^n

$$ax + by + cz + d = 0$$

↓ change coeff.

$$w_1 x + w_2 y + w_3 z + w_0 = 0$$

↓ change axis name

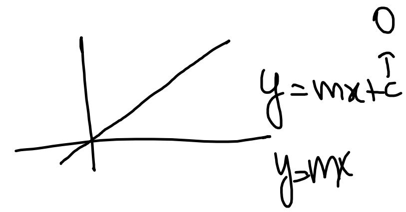
$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$$

Above 3d: Hyperplane: $w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 \cdots + w_n x_n = 0$

Above 3d:

Hyperplane: $w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 - - - + w_n x_n = 0$

$$\boxed{w_0 + \sum_{i=1}^{n} w_i x_i = 0}$$



let say hyperplane passes through origin

$$\boxed{\sum_{i=1}^{n} w_i x_i = 0}$$

$w_i \Rightarrow$ slope (weights)

Eigen Vector & Eigen Value

$$\underbrace{A}_{\text{Matrix (LT)}}\ \underbrace{\vec{x}}_{\text{vector}} = \underbrace{\lambda}_{\text{eigen value}} \vec{x} \longrightarrow \text{eigen vector}$$

eigen vectors: vectors which donot rotate even when linear transformation is applied.

eigen value: (λ): values by vectors get scaled up or down when LT is applied on them.