possibilities

# Probability

quantification of probabilities

$$\text{Coin toss} = \{ \underbrace{H^{\to 1/2}, T^{\to 1/2}}_{\text{sample space}} \} \to \text{event}$$

event

$$P(H) = \frac{1}{2}$$

$$P = \frac{\text{event}}{\text{sample space}} = [0 - 1]$$

# Basics

$$\text{event} \le \text{sample space}$$

- Probability is a way of measuring the chance of something happening.
- An event is any occurrence that has a probability attached to it—in other words, an event is any outcome where you can say how likely it is to occur.
- Probability is measured on a scale of 0 to 1. If an event is impossible, it has a probability of 0. If it's an absolute certainty, then the probability is 1.

$$\textbf{Probability} = \frac{\textbf{Number of favorable (desired) outcomes}}{\textbf{Total number of possible outcomes}}$$

# Basics

- Sample Space is set of all possible events or outcomes.

**Probability:**

In a random experiment, let S be the sample space and $E \subseteq S$, then E is an event.

$$P(E) = \frac{\text{number of distinct elements in } E}{\text{number of distinct elements in } S} = \frac{n(E)}{n(S)}$$

$$= \frac{\text{number of outcomes favourable to occurrence of } E}{\text{number of all possible outcomes}}$$

$\{H, T\}$

$P(H) = \frac{1}{2}$ $\quad$ $P(H') = \frac{1}{2} = P(T)$ $\quad$ $\underline{P(H)} + \underline{P(H')} = \frac{1}{2} + \frac{1}{2} = 1$

# Basics

$P(H') = 1 - P(H)$

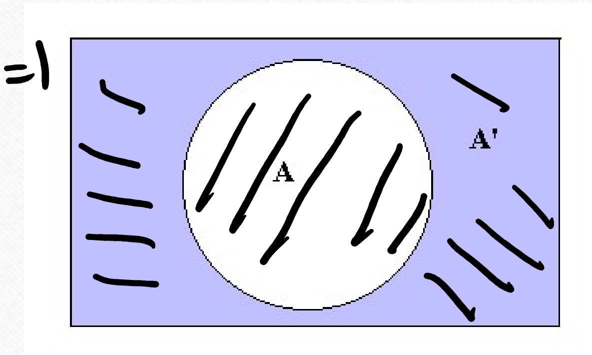
- Complimentary events: Just a way of expressing something didn't happen. A' is the complementary event of A. It's the probability that event A does not occur.

$$P(A) + P(A') = 1$$

$$P(A') = 1 - P(A)$$

$P(A) + P(A') = 1$

$P(A') = 1 - P(A)$




- A' is complement of A.

Q Dice = $\{1, 2, 3, 4, 5, 6\}$

$P(1) = \frac{1}{6}$

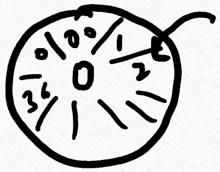
$P(\text{even}) = \frac{3}{6} = \frac{1}{2}$

$P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$

$P(\text{prime}) = \frac{3}{6} = \frac{1}{2}$

$P(n > 4) = \frac{2}{6} = \frac{1}{3}$

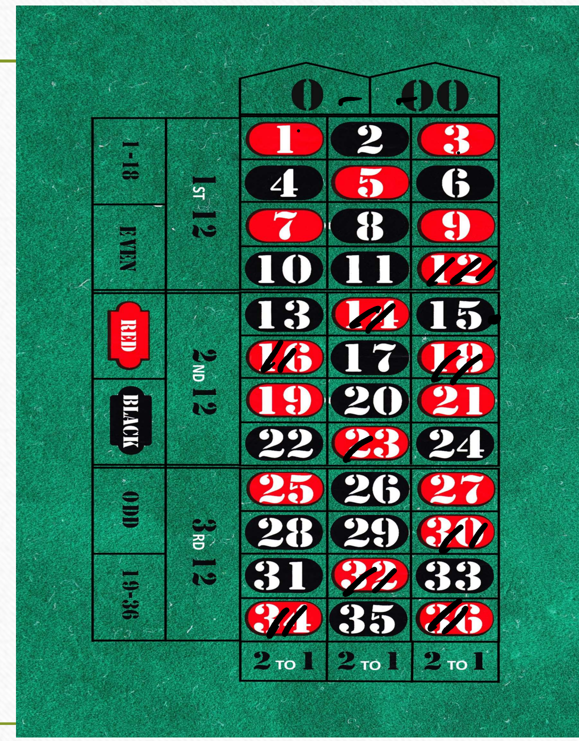
$P(\text{even}) = P(\text{odd}')$




0 & 00 → neither odd nor even

# Question to Play!!

- Find probability of ball landing in 9. $P(9) = \frac{1}{38}$
- What's the probability of getting a black? $P(B) = 18/38$
- What's the probability of getting an even number? $P(E) = 18/38$
- Calculate the probability of getting a black or a red by counting how many pockets are black or red and dividing by the number of pockets.




$P(B \text{ or } R) = \frac{36}{38}$

$\Rightarrow P(B \text{ or } R) = \underset{P(B)}{\frac{18}{38}} + \underset{P(R)}{\frac{18}{38}} = \frac{36}{38}$

$$P(B \text{ or Even}) = P(B) + P(\text{Even}) - P(B \,\&\, \text{Even})$$

$$= \frac{18}{38} + \frac{18}{38} - \frac{10}{38} = \frac{26}{38}$$

* If two events have nothing in common, then simply add probabilities.

* If two events have common values, then remove duplicates by subtracting these value from total sum.

# Can we Add Probabilities?

- Calculate the probability of getting a black or a red by counting how many pockets are black or red and dividing by the number of pockets.

We can find the probability of getting a black or red by adding these two probabilities together.

$$P(B)+P(R) = P(B \text{ or } R)$$

$$18/38 + 18/38 = 36/38$$

- What if we add probabilities of getting black and even together and check it by counting the black and even pockets?

The answer is different as 0.947 and 0.684

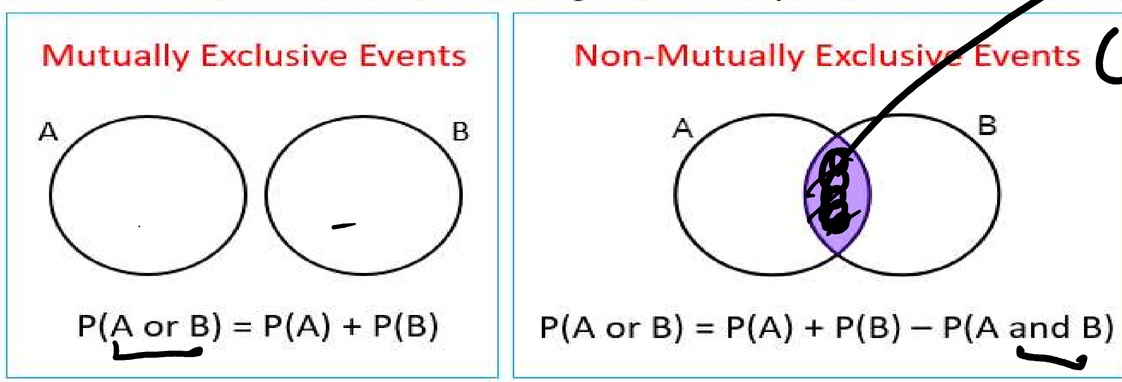
$\frac{36}{38}$ $\frac{26}{38}$

$\{H, T\}$ $P(H)$ $P(T)$

# Mutually & Non Mutually Exclusive Events

- Mutually Exclusive Events: If two events are mutually exclusive, only one of the two can occur. $P(A \cap B) = 0$ as they have no common element.
- Non-Mutually Exclusive Events: If two events intersect, it's possible they can occur simultaneously.

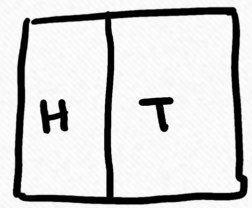
$(A \cap B)$ or $(B \cap A)$




- $A \cup B$ = A or B
- $A \cap B$ = A and B
- If $A \cup B = 1$, then A and B are said to be exhaustive.

# Exhaustive Events.

$P(H \text{ or } T) = P(H) + P(T)$




sample space

$= \frac{1}{2} + \frac{1}{2} = 1$

* $P(H \cup T) = 1$

Q Dice = {1, 2, 3, 4, 5, 6}

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |

$P(1) = \frac{1}{6} + \frac{1}{}$

$$P(1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$= 1$$

Hence, events are exhaustive

Q $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A$ = prime no. $B$ = multiple of 2 $C$ = perfect square

Check whether A, B, C are exhaustive or not?

Sol. $A = \{2, 3, 5, 7\}$ $B = \{2, 4, 6, 8, 10\}$ $C = \{1, \underline{4}, 9\}$

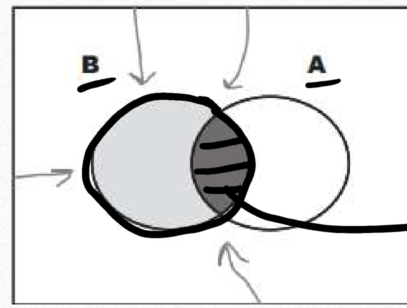
$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = S$

$P(\text{even}/\text{black}) = \frac{10}{18}$ | $P(A|B) \Rightarrow$ Prob. of A given B has already occured!

# Conditional Probability

- Condition Probability measure the probability of one event occurring relative to another occurring.
- P(A|B) = Probability of A given B has already happened.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

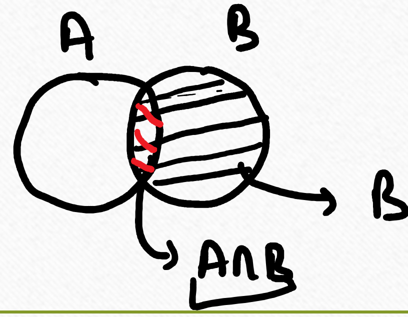



$\frac{(A \cap B)}{B}$

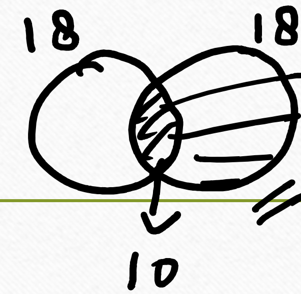
- From roulette, find the probability that the pocket is even, given that we already know it's black.

P(even|black) = 10/18

$A \quad , B$




[18 even]




$\frac{10}{18}$ $\frac{n(E \cap B)}{n(B)}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{n(E \cap B)/n(S)}{n(B)/n(S)} = \frac{\frac{10}{38}}{\frac{18}{38}} = \frac{10}{18}$$

$$P(A) = \frac{n(E)}{n(S)}$$

11:25 am

# Probability Tree

$$P(E/B) = \frac{P(E \cap B)}{P(B)}$$

$\frac{18}{38} = P(B)$ → Black

- $P(E/B) = \frac{10}{18}$ → even $\Rightarrow P(B) \times P(E/B) = P(E \cap B)$
- $P(O/B) = 8/18$ → odd $\Rightarrow P(O \cap B)$

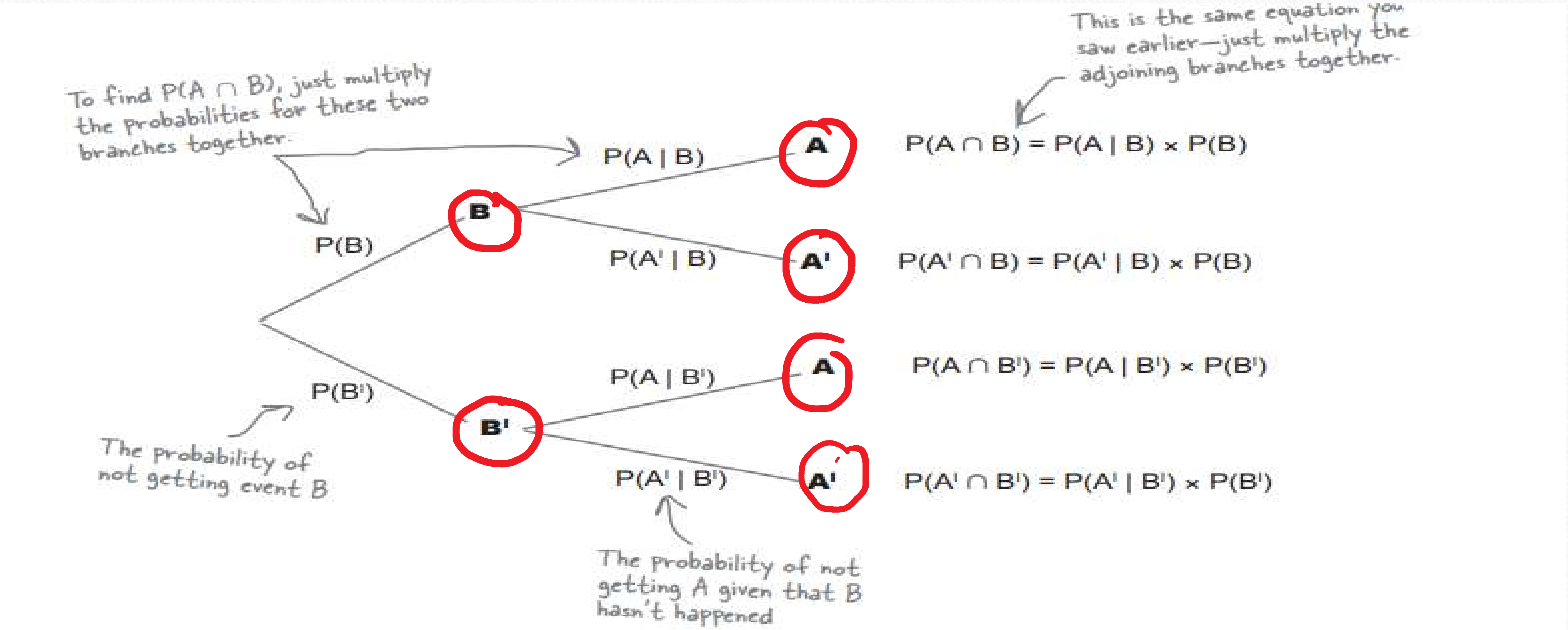
$P(R) = 18/38$ → Red

- $P(E/R) = 8/18$ → even $\Rightarrow P(E \cap R)$
- $P(O/R) = 10/18$ → odd $\Rightarrow P(O \cap R)$

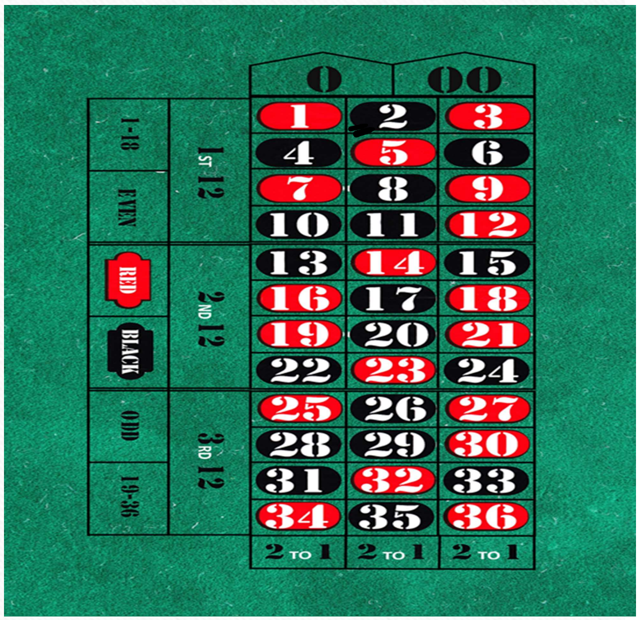
$P(G) = 2/38$ → Green

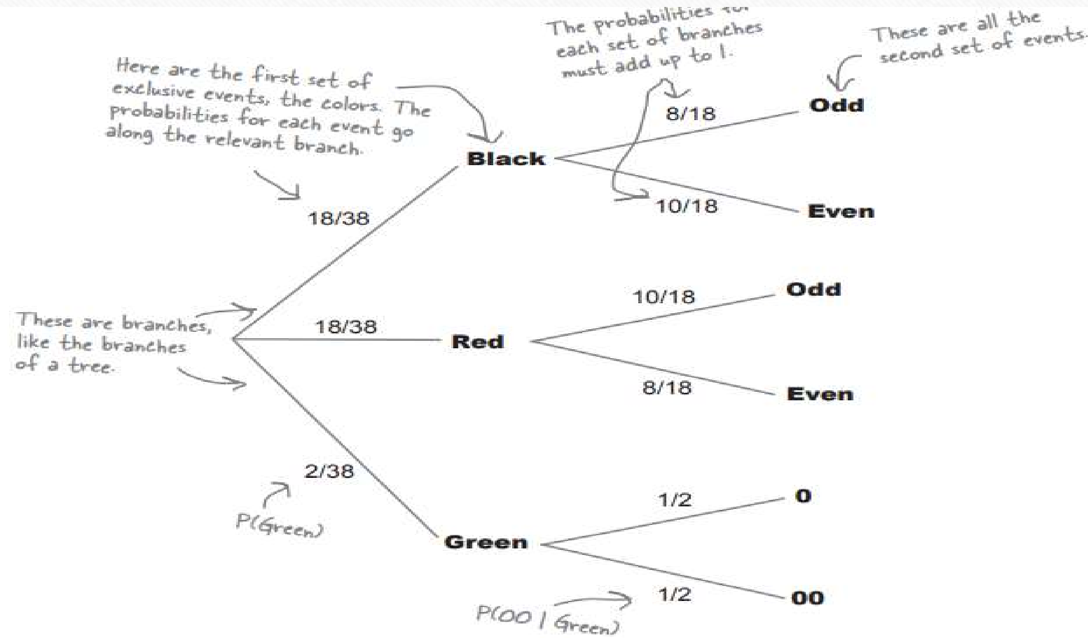
- $P(0/G) = 1/2$ → 0 $\Rightarrow P(0 \cap G)$
- $P(00/G) = 1/2$ → 00 $\Rightarrow P(00 \cap G)$

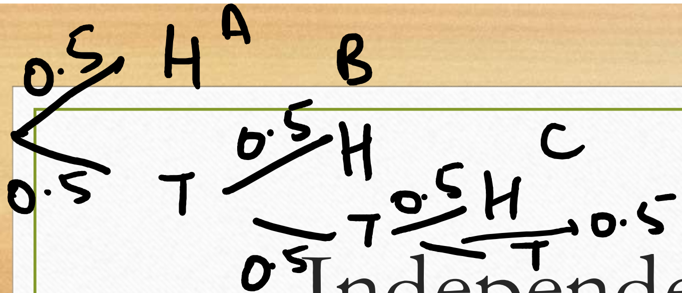
# Probability Tree

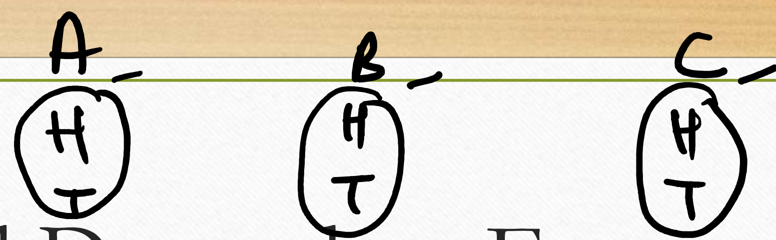


# Complete Solution





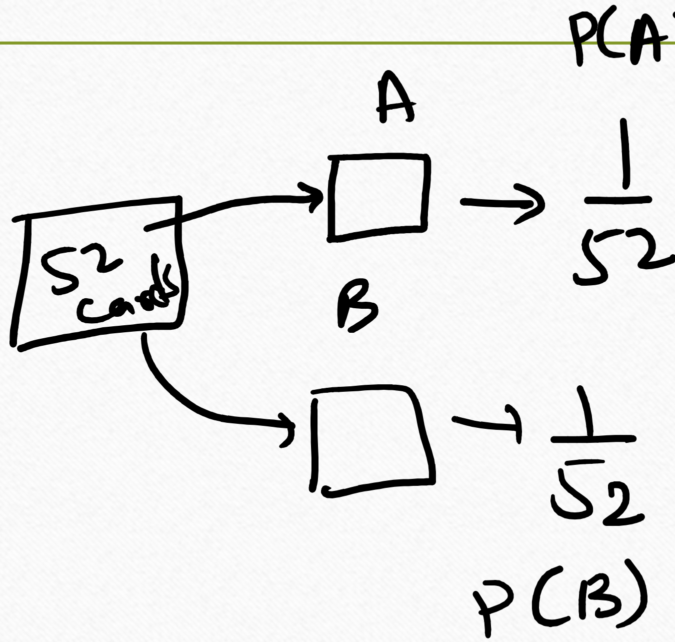

# Independent and Dependent Events

- Dependent Events: Events A and B are said to be dependent if P(A | B) is different from P(A). It's a way of saying that the probabilities of A and B are affected by each other.
- Independent Events: Independent events aren't affected by each other. They don't influence each other's probabilities in any way at all. If one event occurs, the probability of the other occurring remains exactly the same.

If you have two events A and B where P(A | B) = P(A), then the events A and B must be independent.

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ $\quad$ $\mathbf{P(A \mid B) = P(A)}$ $\quad$ $P(A) = \frac{P(A \cap B)}{P(B)}$ $\quad$ $\mathbf{P(A \cap B) = P(A) \times P(B)}$

$P(A/B) = P(A)$, if events are independents




$$P(A/B) = \frac{1}{52} = P(A)$$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ 

if A & B are independent

$P(A|B) = P(A)$

$P(A \cap B) = P(A|B) \times P(B)$

* $P(A \cap B) = P(A) \times P(B)$

↳ Conditions are independent.

$= P(A \cap B) = P(A) \times P(B)$ ✓ $E$ & $F$ are independent

**Q** A dice is thrown, there are two events:

- $E$: No. appearing is a multiple of 3
- $F$: No. appearing is an even n.

$E$ & $F$ are independent or not!

**Sol.** $E: \{\underbrace{3, 6}\}$ $F: (\underbrace{2, 4, 6})$ $E \cap F: \{\underbrace{6}\}$

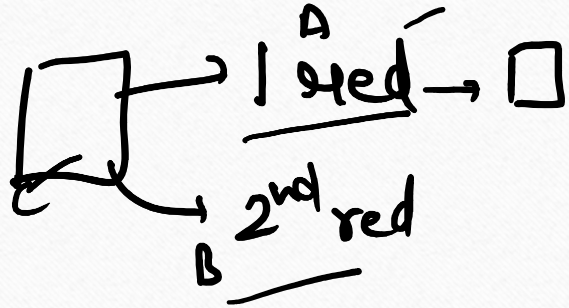
$P(E \cap F) = \frac{1}{6}$ $P(E) = \frac{1}{3}$ $P(F) = \frac{1}{2}$

$\frac{1}{6} = \frac{1}{3} \times \frac{1}{2}$

Q. Two cards are randomly drawn without replacement. Find the probability that both cards are red.

$\boxed{\text{Deck}}$

**Sol.**




51

$$\underset{\substack{\downarrow \\ A \text{ and } B}}{P(\overset{R_1}{A} \cap \overset{R_2}{B})} = \underset{P(R_1)}{\frac{\cancel{26}}{\cancel{52}_2}} \times \underset{P(R_2/R_1)}{\frac{25}{51}} = \frac{25}{102} = P(\text{Red cards})$$

C

52 cards

$\frac{26}{52}$

B(26) R(26) → 1 Red

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B(26) R(25) → 25

$\times \frac{26}{52} \times \frac{25}{51}$