Q. A principal claims that students have above average IQ. A random sample of 30 students is taken, mean = 112.5. The mean & std dev of population is 100 & 15. Test your hypothesis.

Acceptance Region Method

Sol. (I) $H_0 : \mu \le 100$

$H_A : \mu > 100$

(II) Check for one tailed / two tailed test

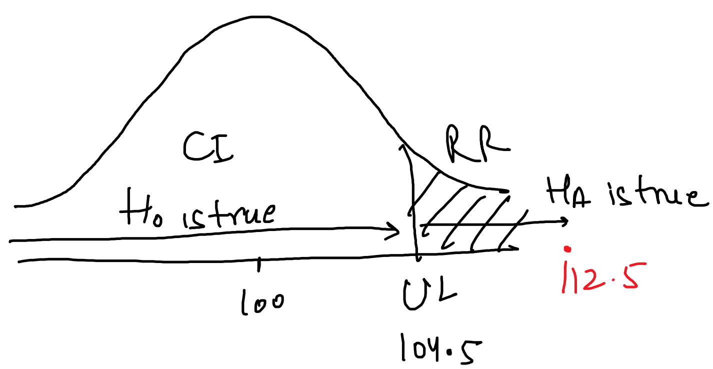
Right tailed test

$\mu = 100$, $\sigma = 15$, $\bar{x} = 112.5$, $(\alpha = 0.05) \xrightarrow{\text{from z-table}} Z\text{-score} = 1.65$

$UL = \mu + z \times \frac{\sigma}{\sqrt{n}}$

$= 100 + 1.65 \times \frac{15}{\sqrt{30}}$

$= 104.5$




Reject $H_0$

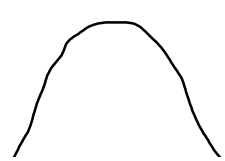
Q2) CV

$\alpha = 0.05 \longrightarrow z = 1.65$ (CV)

$\bar{x} = 112.5$ $\mu = 100$

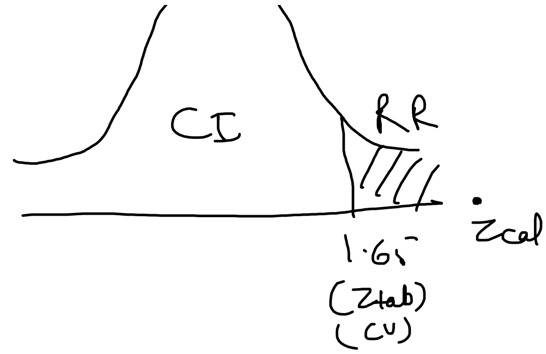
$\sigma = 15$

$\sqrt{n} = 30$



$Z_{cal} = \frac{\bar{x} - \mu}{\ldots} = \frac{112.5 - 100}{\ldots}$

$$Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{112.5 - 100}{15/\sqrt{30}}$$

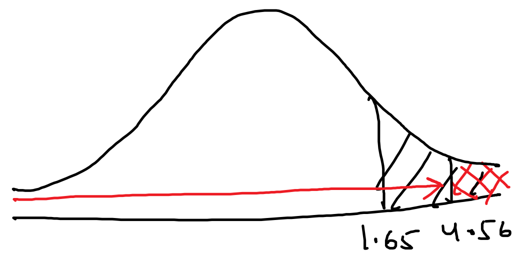



$$= 4.56$$

$Z_{cal} > Z_{tab} \Rightarrow$ Reject $H_0$

p-value $\quad \alpha = 0.05 \quad Z_{tab} = 1.65$

$Z_{cal} = 4.56$




$P(Z_{cal} = 4.56) = 0.9999966$

$P\text{value} = 1 - P(Z_{cal} = 4.56) = 1 - 0.9999966$

$= 0.0000034$

$\alpha = 0.05$

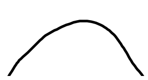
compare p-value with $\alpha$

$p < \alpha \rightarrow$ Reject $H_0$

If sample size $(n < 30) \longrightarrow$ we don't use z-test

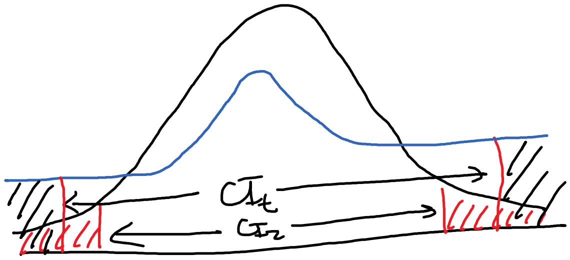
we will t-test.

$A \Rightarrow \downarrow \lambda \times b \uparrow$



$A \rightarrow \downarrow \alpha \times b \uparrow$

cons




t-dist

$$t = \frac{x-\mu}{\sigma/\sqrt{n}}$$

or

$$t = \frac{x-\mu}{s/\sqrt{n}} \quad \sigma \approx s$$

Degrees of freedom: logically independent values

4 = degrees of freedom
↑
4 independent values

$x = \underline{\underline{7}}$

2
3
5
8
x

avg = 5

$$5 = \frac{2+3+5+8+x}{5}$$

$25 = 18 + x$

$x = 7$

if you have n values, $df = n-1$

## Errors

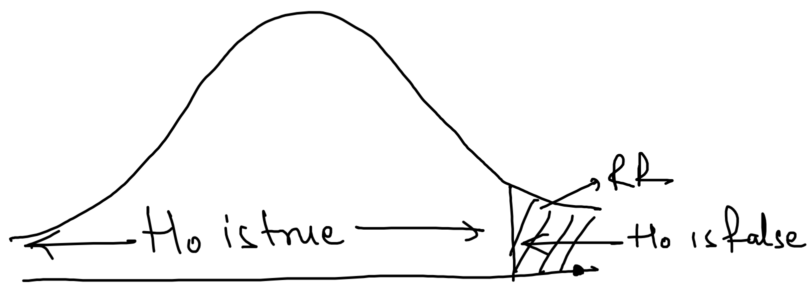
Type-I error

Type II error

| | Actual | |
|---|---|---|
| Predictions | $H_0$ is false, $H_A$ is true | $H_A$ is false, $H_0$ is true |
| $H_A$ is accepted, $H_0$ is rejected | ✓ | Type-I error (FP) |

Predictions | | | error
FP
FN
$H_A$ is rejected
failed to Reject $H_0$ | Type II error | ✓

Quantification of Type I error:




$\alpha = 0.05\%$

$\text{Type I error} = \alpha \Rightarrow 5\%$

Quantification of Type II:

Power of test
↳ ability of test to make right decisions

power of test $\propto$ large no of sample

$\text{Power} = 1 - \beta$

$\boxed{\beta = 1 - \text{Power}}$ → Type II error

Relationship b/w Type I & Type II :

$$\text{Type I} \propto \frac{1}{\text{Type II}}$$ explore :} assignment