"Binomial Distribution"

Q1 What is Garvit's fav colour?

a) Grey b) Black c) white d) Red

P(C) = 0.25

P(NC) = 0.75

Q2 What is MOB of Garvit?

a) Jan b) April c) dec d) march

P(C) = 0.25

P(NC) = 0.75

Q3 What is Garvit's fav hobby?

a) Chess b) reading c) MMA d) Singing

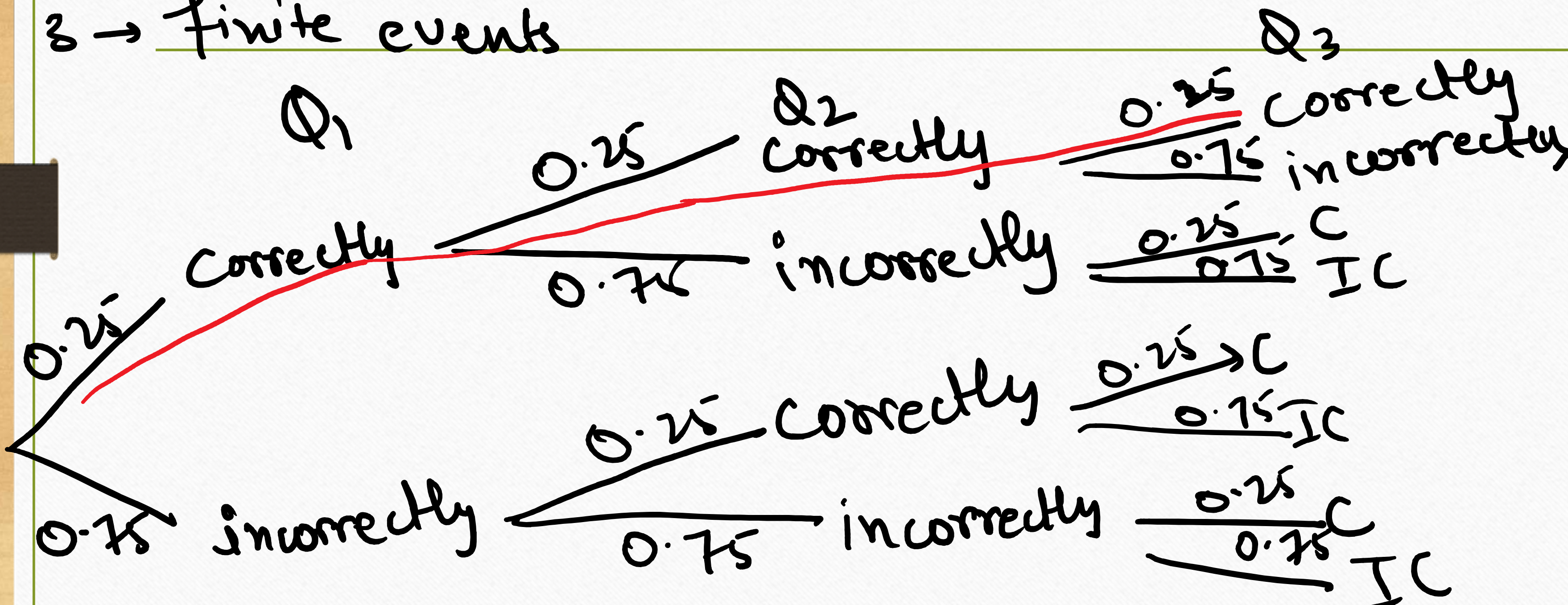
P(C) = 0.25

P(NC) = 0.75

1 → All events are independent of each other

2 → events have only to outcomes → Success / → Failure

3 → finite events



Q. What is probability of answering all questions correctly?

$$0.25 \times 0.25 \times 0.25 = (0.25)^3$$

Q. What is probability of answering 2 questions correctly?

Sol. $0.25 \times 0.25 \times 0.75 + 0.75 \times 0.25 \times 0.25 + 0.25 \times 0.75 \times 0.25$

$= 3 \times (0.25 \times 0.25 \times 0.75) = {}^{3}C_{2} \times (0.25 \times 0.25 \times 0.75)$

Q What is the probability of answering all questions incorrectly?

$0.75 \times 0.75 \times 0.75$

$(0.75)^3$

Q What is the prob. of answering one question correctly?

Sol. $3 \times 0.75 \times 0.75 \times 0.25 = {}^3C_1 \times 0.75 \times 0.25 \times 0.75$

$P(X=0) = (0.75)^3$

$(3-1)$

$P(X=1) = {}^2C_1 \times (0.75)^2 \times 0.25^1$

$P(X=2) = {}^3C_2 \times 0.75^{\overset{3-2}{1}} \times 0.25^2$

$P(X=3) = {}^3C_3 \times 0.75^{\overset{3-3}{0}} \times 0.25^3 = {}^3C_3 \times 0.25^3 = 1 \times 0.25^3$

$P(X=r) = {}^3C_r \times 0.75^{3-r} \times 0.25^r$

p = prob of success   q = prob of failure

events

$P(X = r) = {}^nC_r \times p^r \times q^{n-r}$

RV

n = no. of all events

r = selected events

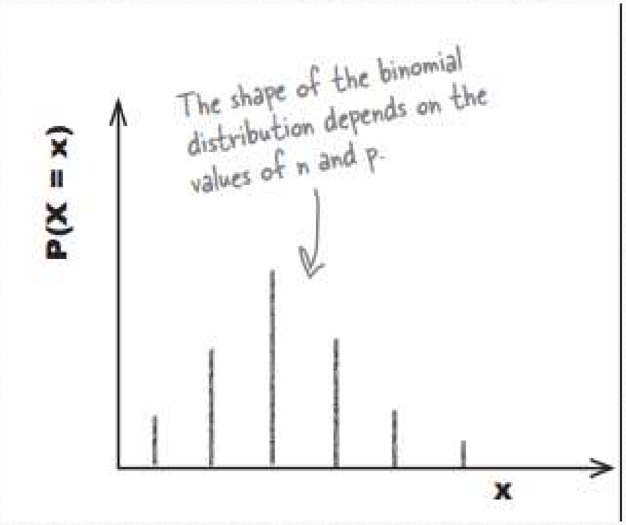
Binomial distribution

Expectation : $E(X) = np$

$Var(X) = npq$

Binomial Distribution

- The exact shape of the binomial distribution varies according to the values of n and p. The closer to 0.5 p is, the more symmetrical the shape becomes. In general it is skewed to the right when p is below 0.5, and skewed to the left when p is greater than 0.5.
- Expectation: $E(x) = np$
- Variance: $V(x) = npq$
- n = number of trials, p = success, q= failure



Q Fan malfunctioned 3 times in previous week. What is the prob. that it will malfunction 0 times in current weeks ?

$\lambda = 3$

Poisson Distribution

$P(X = r) = \frac{e^{-3} \times 3^0}{0!} =$

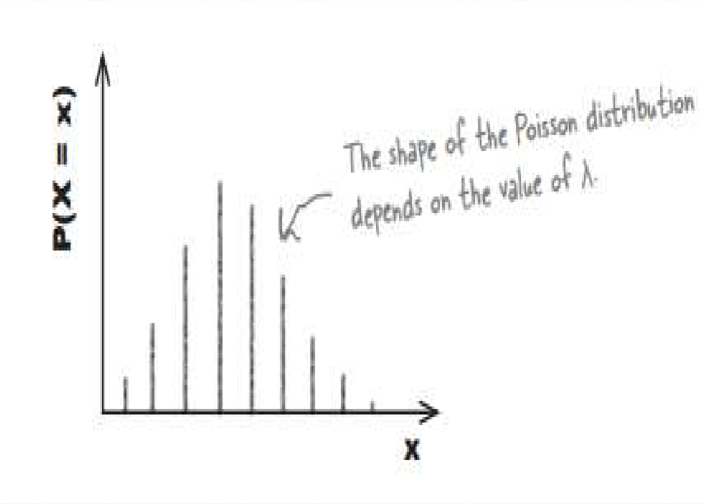
- The Poisson distribution covers situations where: Let's use the variable X to represent the number of occurrences in the given interval.
- If X follows a Poisson distribution with a mean of λ occurrences per interval or rate, we write this as: Individual events occur at random and independently in a given interval. This can be an interval of time or space—for example, during a week, or per mile.
  - The mean number of occurrences in the interval or the rate of occurrences, and it's finite. The mean number of occurrences is normally represented by the Greek letter λ (lambda)

$$P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

e = exponential $= 2.718$

Poisson Distribution

- The shape of the Poisson distribution varies depending on the value of λ. If λ is small, then the distribution is skewed to the right, but it becomes more symmetrical as λ gets larger.
- If λ is an integer, then there are two modes, λ and λ - 1. If λ is not an integer, then the mode is λ.
- $E(X) = \lambda$
- $V(X) = \lambda$



Q You have a m/c that malfunctions 3 times a week on avg. What is the prob. that it will malfunction twice in next week?

Sol.

$$P(X = r) = \frac{e^{-\lambda} \times \lambda^{r}}{r!} = \frac{e^{-3} \times 3^{2}}{2!}$$

$e = 2.718$

$$= 0.224$$

Q Aratrik → prob. of cookie is broken $\underbrace{= 0.1}$,

the boss asked, prob. of $\underbrace{15}$ broken cookies in

a box $\underbrace{\text{100 cookies}}$?

Sol. Whenever you have a very large no. of n in Binomial distribution & prob of success is very small then. $np \simeq \lambda$, $npq \simeq \lambda$

Bino. Dist $\simeq$ Poisson Distribution

$np = 100 \times 0.1 = 10 = \lambda$

$$\frac{e^{-10} \; 10^{15}}{15!}$$

Q A student need to take an exam, but he hasn't revise for it. He will guess all the answers & prob. of getting answer right is 0.05. There are 50 question in total. What is the prob. that he'll 5 question rights?

$\lambda = np$

$$\frac{e^{-\lambda} \times \lambda^{r}}{r!}$$

$\lambda = 50 \times \frac{0.05}{100} = 2.5$

$$P(X=5) = \frac{e^{-2.5} \times 2.5^{5}}{5!}$$

$= 0.067$

→ learn music, coding, puzzles, maths

→ physical exercises.

delay

at 25

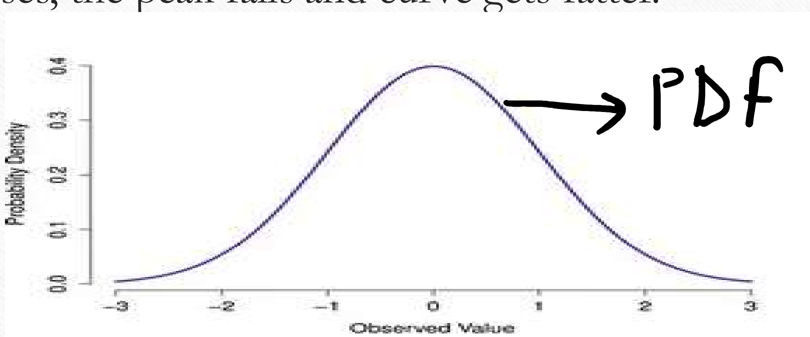
Normal Distribution

- Pdf curve that looks like a bell is normal distribution.
- Its called normal because majority of activities in universe follow this distribution.
- X ~ N(m, v): X follows normal distribution with mean ‘m’ and variance ‘v’.
- As variance increases, the peak falls and curve gets fatter.

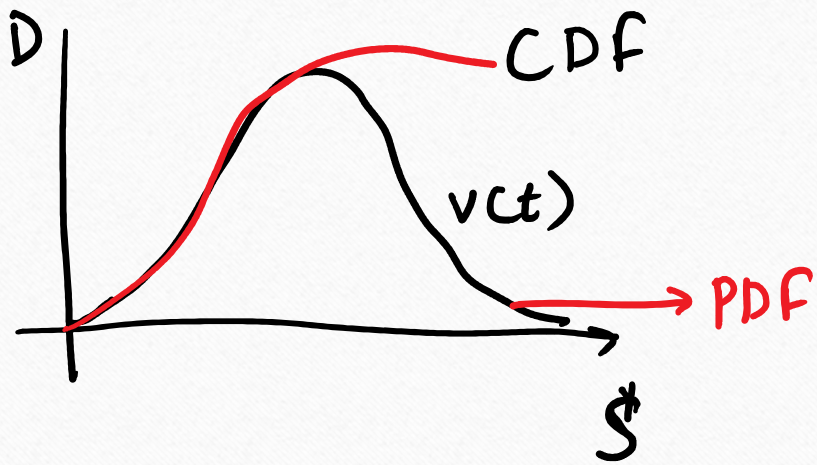
• point

• confidence interval

10 mins



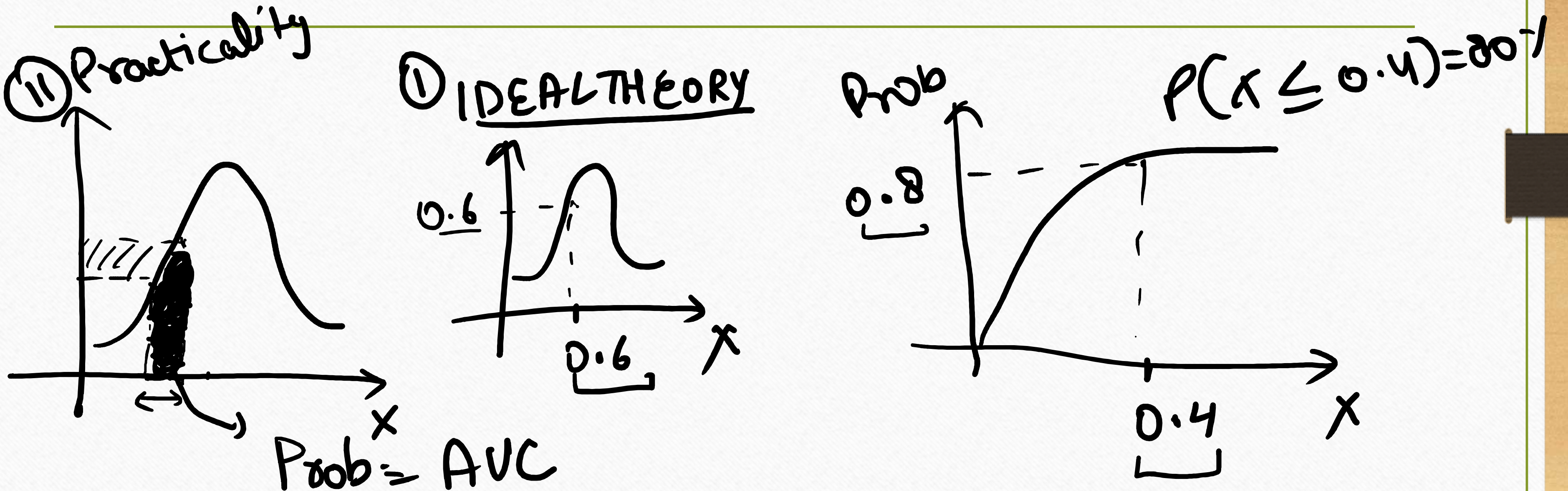
→ PDF



$$CDF = \int PDF$$

PDF $\xrightarrow{\text{integration}}$ CDF

PDF $\xleftarrow[\text{differentiation}]{}$ CDF



$$\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \longrightarrow \text{integral calculator.}$$

Normal Distribution

- PDF equation:

Normal Distribution Formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$\mu =$ mean of x

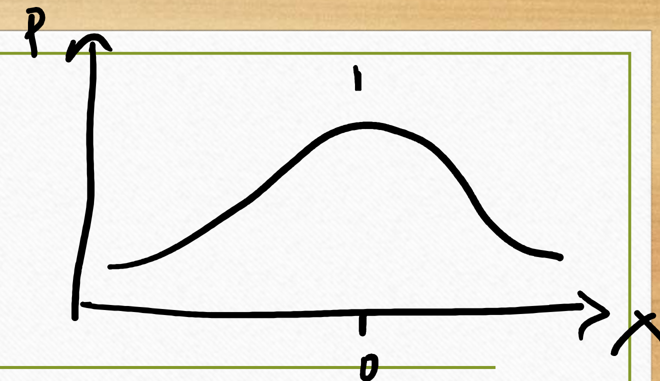
$\sigma =$ standard deviation of x

$\pi \approx$ 3.14159 ...

$e \approx$ 2.71828 ...

CDF equation: it is obtained by integrating pdf equation.

$$PDF = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



ignore all constants , $\mu = 0, \sigma = 1$

all the x

$$PDF = e^{-x^2} = y$$

at $x=0$ $\quad y = e^0 = 1$

at $x=1$ $\quad y = e^{-1} = 0.36$

$\downarrow$ 20 times

at $x=2$ $\quad y = e^{-4} = 0.018$

$\downarrow$ 100 times

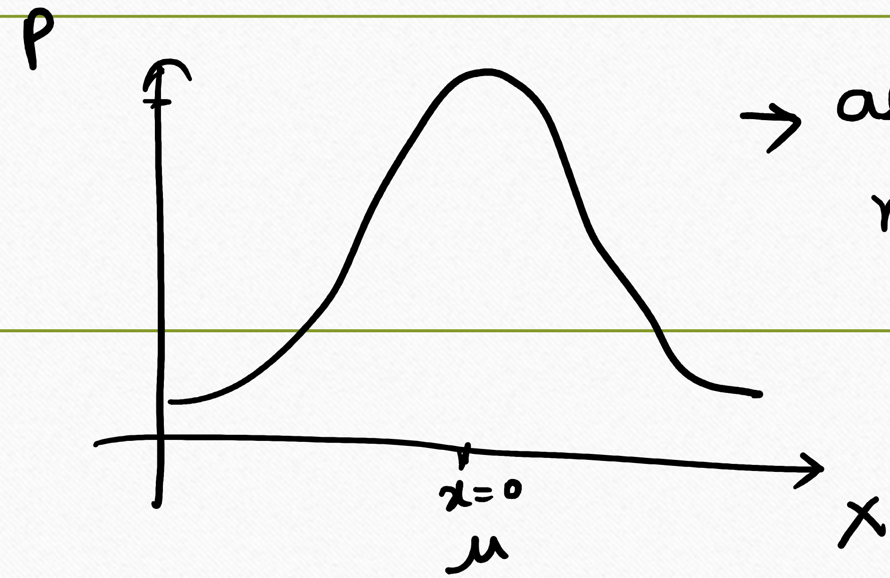
at $x=3$ $\quad y = e^{-9} = 0.00012$

at $x=0$ $\quad y = e^0 = 1$

at $x=-1$ $\quad y = e^{-(-1)^2} = e^{-1} = 0.36$

at $x=-2$ $\quad y = e^{-4} = 0.018$

at $x=-3$ $\quad y = e^{-9} = 0.00012$



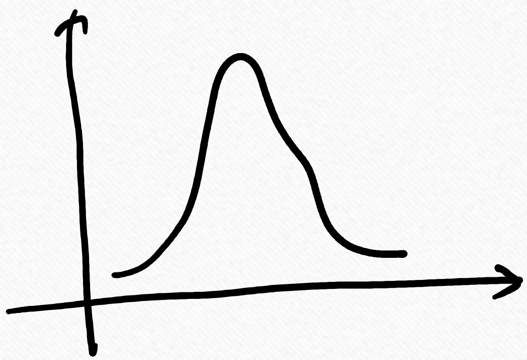
→ as x moves away from mean, then $y\downarrow$

→ as x, moves away from mean, $y\downarrow$ by factor e^{-x^2}.

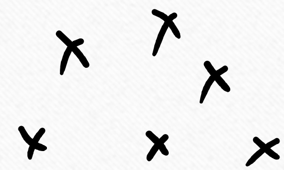
$X \sim N(\mu, \sigma^2) \Rightarrow$ X(RV) follows ND that has mean μ
& variance σ^2.

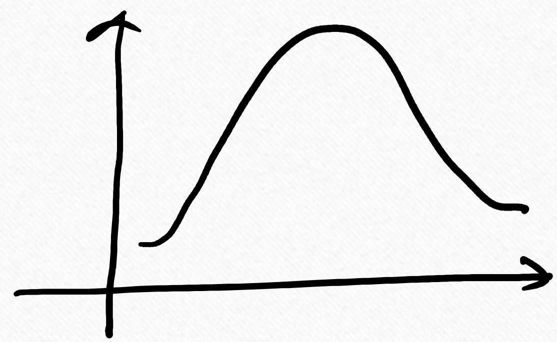
$X \sim N(\mu, \sigma) \Rightarrow$ X(RV) follows ND that has mean μ
& std dev σ.

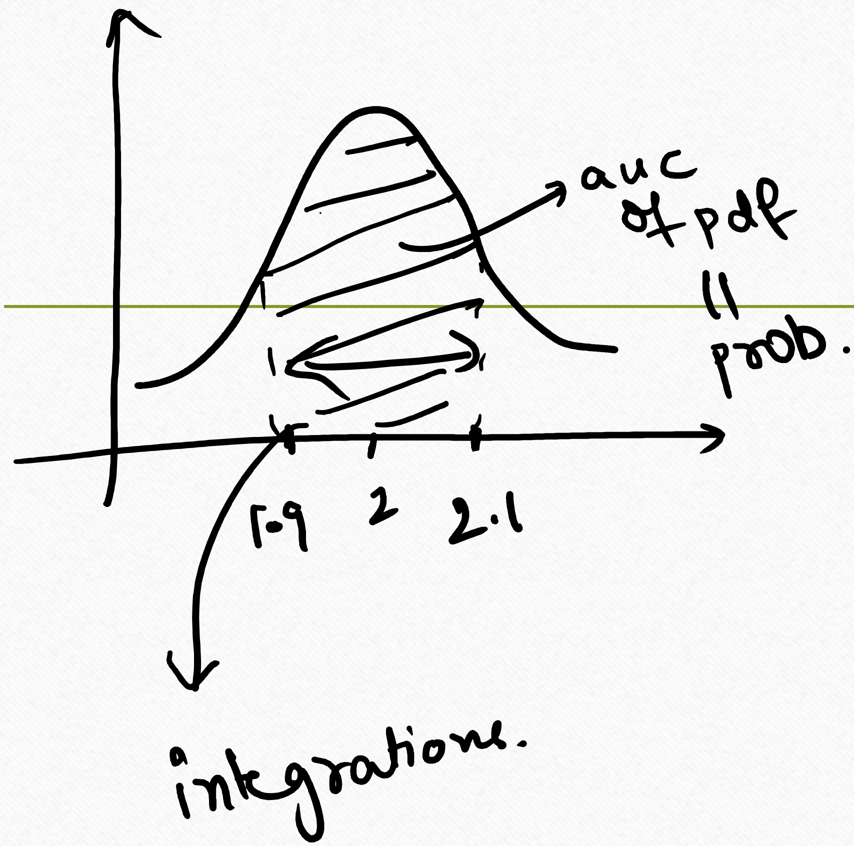
$X \sim N(0,2)$



$X \sim N(0,4)$







P(rainfall = inches)

↳ exactly 2 inches

↓

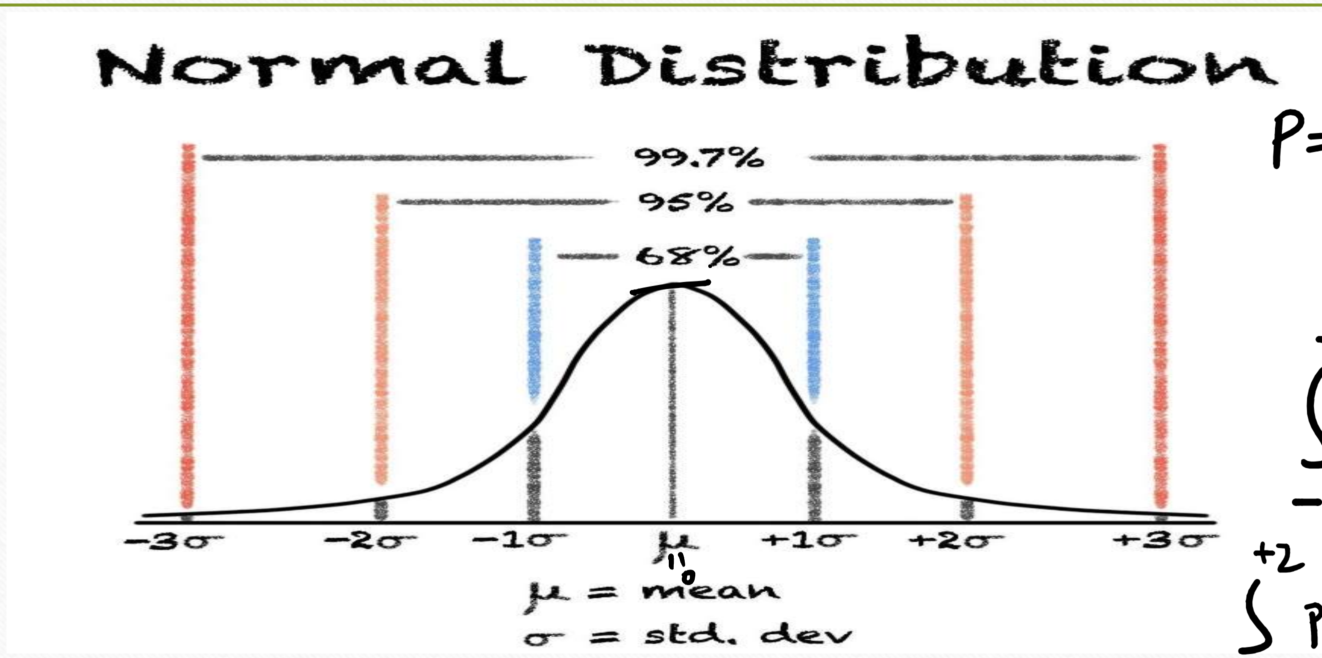
not an extra molecular

$$1.9 < X < 2.1$$

Empirical Formula

$\underbrace{\mu = 0,\ \sigma = 1}$

$$P = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$P = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\int_{-1}^{+1} P = 0.68\%$$

$$\int_{-2}^{+2} P = 0.95$$

Q What % of individuals have salary in range of [20k, 60k] if $\mu = 40k$ & $\sigma = 10k$?
Given salary is normally distribution.

Sol.

$$20k = \overset{\mu}{40k} - \overset{\sigma}{10k} \times 2 \Rightarrow \mu - 2\sigma$$

$$60k = \overset{\mu}{40k} + \overset{\sigma}{10k} \times 2 \Rightarrow \mu + 2\sigma$$

$\Rightarrow 95\%$

CHEBYSHEV'S INEQUALITY

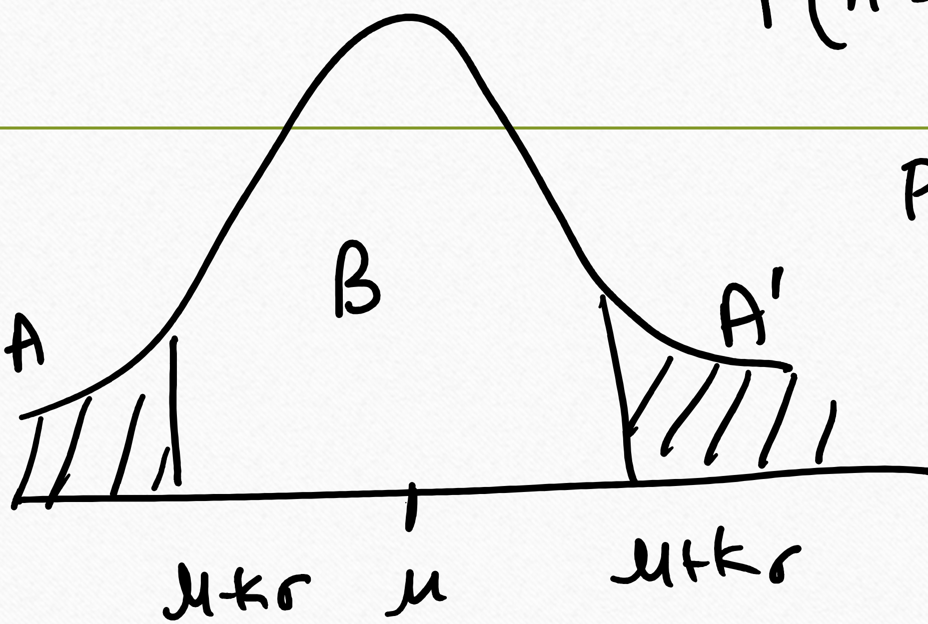
Expression: $P(|x-\mu| \geq k\sigma) \leq \frac{1}{k^2}$ $k \Rightarrow$ arbitary no.

$$P(x \geq \mu + k\sigma \ \& \ x \leq \mu - k\sigma) \leq \frac{1}{k^2}$$

$$P(\mu - k\sigma < X < \mu + k\sigma) > 1 - \frac{1}{k^2}$$

$$P(A+A') \leq \frac{1}{k^2}$$

$$P(A \,\&\, A') \leq \frac{1}{k^2}$$



$$P(B) > 1 - \frac{1}{k^2}$$

$$P(A \,\&\, A') + P(B) = 1$$

$$P(B) = 1 - P(A \,\&\, A')$$

$$= 1 - \frac{1}{k^2}$$

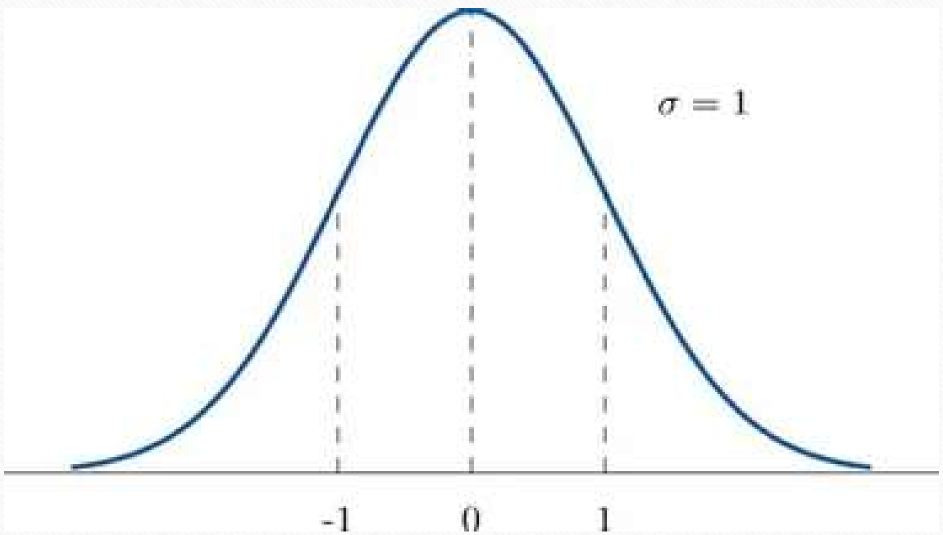
Q What % of individuals have salary in range of
$\Rightarrow$ [20k, 60k] given $\mu = 40k$, $\sigma = 10k$?

Sol. $P(20k < X < 60k) = 1 - \frac{1}{k^2}$

$$P(\overset{\mu - \sigma \times k}{40k - 10k \times 2} < X < \overset{\mu + \sigma \times k}{40k + 10k \times 2}) = 1 - \frac{1}{2^2}$$

$$= 1 - \frac{1}{4} = \frac{3}{4} = 75\%$$

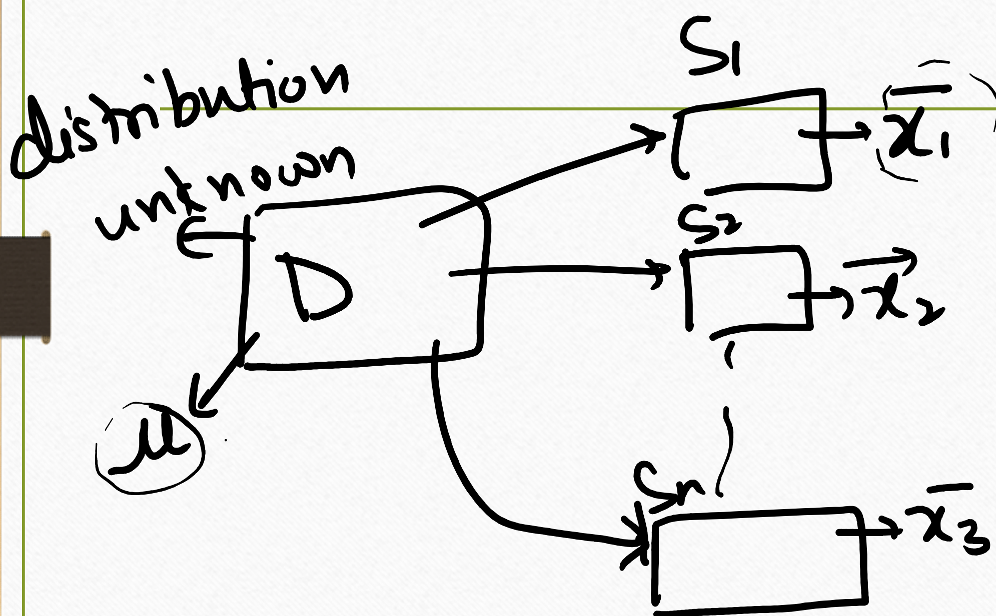
Standard Normal Distribution

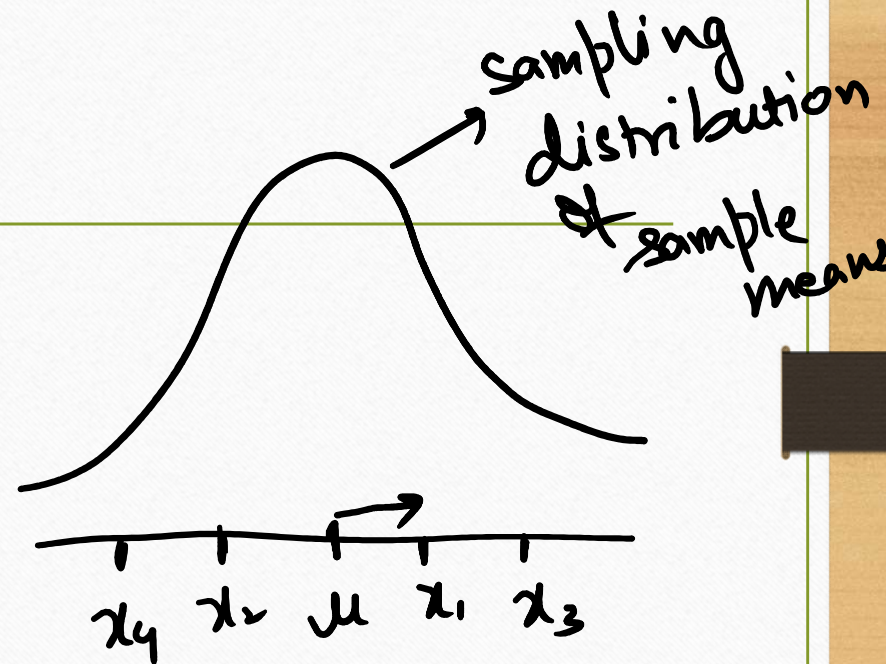


"Central Limit Theorem"

CLT: "given a sufficiently large no. of samples, sampling distribution of sample means will approximate normal distribution irrespective of variable distribution in population".

* $n > 30$ (condition to apply CLT)





$$\text{Std error} = \frac{\sigma}{\sqrt{n}}$$