$P(E \cap F) = \frac{n(E \cap F)}{n(S)}$

Q Three coins are tossed simultaneously:

E: three heads or three tails F: atleast two heads

G: at most two heads. $E \cap F = \{HHH\}$

✓ (E, F) $\overset{*HW}{(F, G)}$ & ✓(E, G). Which are independent?

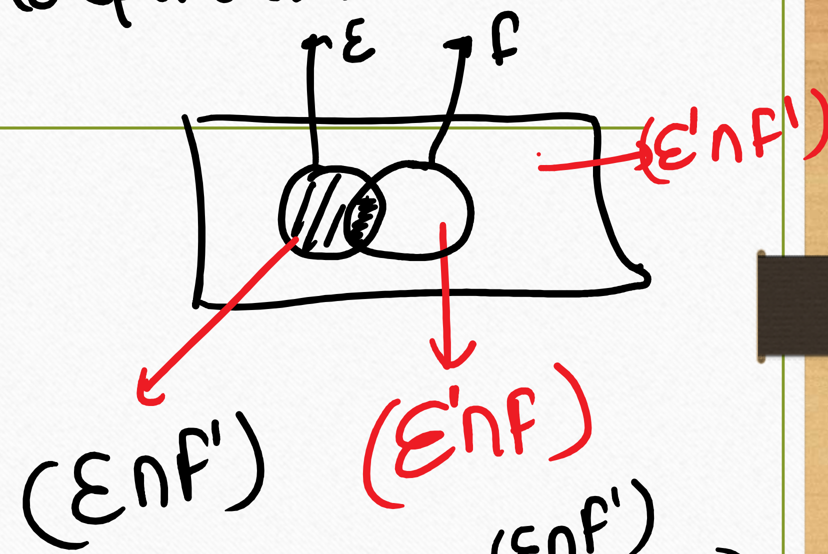
Sol. $S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

$E: \{\underbrace{HHH}, TTT\}$ $F: \{HHT, HTH, THH, \underbrace{HHH}\}$

$\underline{P(E \cap F)} = \frac{1}{8}$ $P(E) = \frac{2}{8} = \frac{1}{4}$ $P(F) = \frac{4}{8} = \frac{1}{2}$

$P(E) \times P(F) = 1/4 \times 1/2 = 1/8$

Q. I know E & F are independent. Can you please check whether E & F' are independent or not?



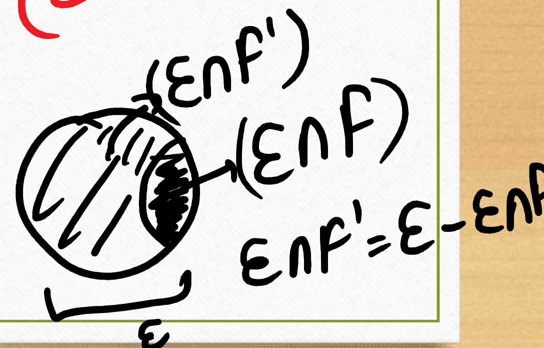
Sol. $P(E\cap F) = P(E)\times P(F)$ — (1)

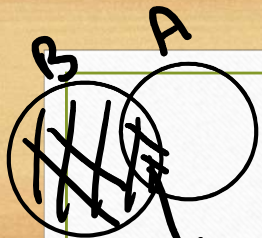
$P(E\cap F') = P(E)\times P(F')$

$P(E\cap F') = \underbrace{P(E) - P(E\cap F)}$

$= P(E) - [P(E)\times P(F)]$

$P(E\cap F') = P(E)[1-P(F)] = P(E)\times P(F')$





$P(B) = P(A \cap B) + P(A' \cap B)$

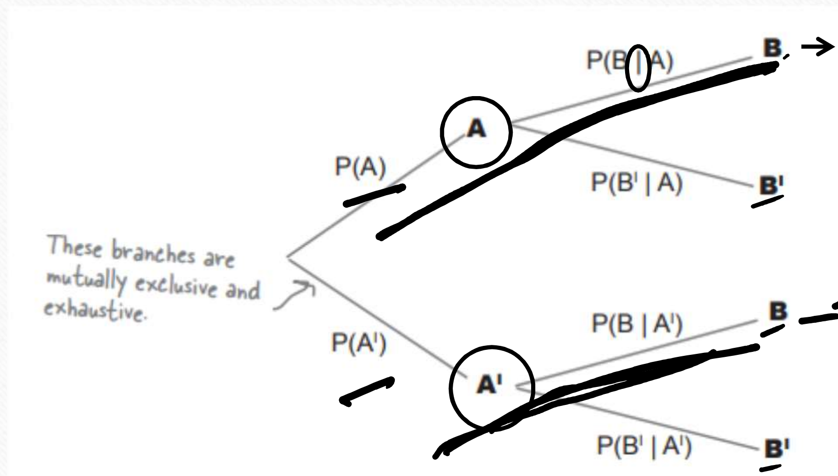
$P(B) = P(A) \times P(B/A) + P(A') \times P(B/A')$

$P(A' \cap B) + P(A \cap B)$

Law of Total Probability

- it gives a way of finding the total probability of a particular event based on conditional probabilities.

$$P(B) = P(A) \times P(B \mid A) + P(A') \times P(B \mid A')$$



$\rightarrow P(A \cap B)$

OR

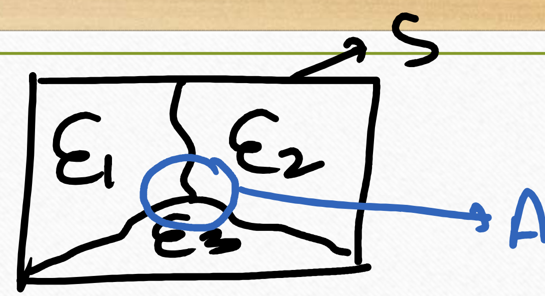
$\rightarrow P(A' \cap B)$

Conditions:

(i) $P(E) > 0$

(ii) Mutually exclusive

(iii) Exhaustive



$$A \cap S = A$$

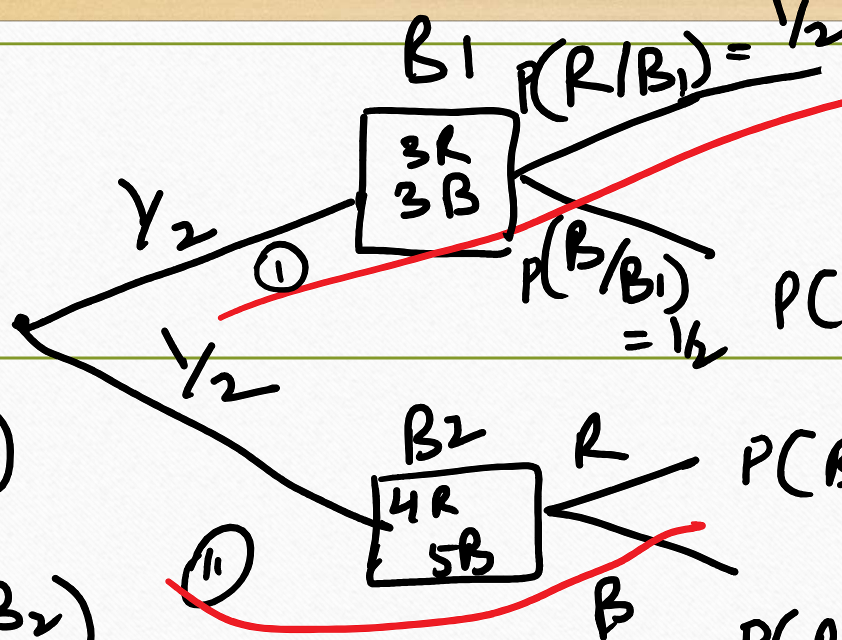
$$P(A) = P[A \cap (E_1 \cup E_2 \cup E_3)] \Rightarrow P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3)$$

$$P(A) = P(E_1)P(E_1/A) + P(E_2)P(E_2/A) + P(E_3)P(E_3/A)$$

$$P(A) = \sum_{i=1}^{n} P(E_i)P(E_i/A)$$

Q =

$P(R)$



$P(R) = P(B_1) \times P(R/B_1) + P(B_2) + P(R/B_2)$

$\rightarrow P(B_1 \cap R) = \frac{1}{4}$

$P(B \cap B_1) = \frac{1}{4}$

$P(B_2 \cap R) = \frac{4}{9} \times \frac{1}{2}$

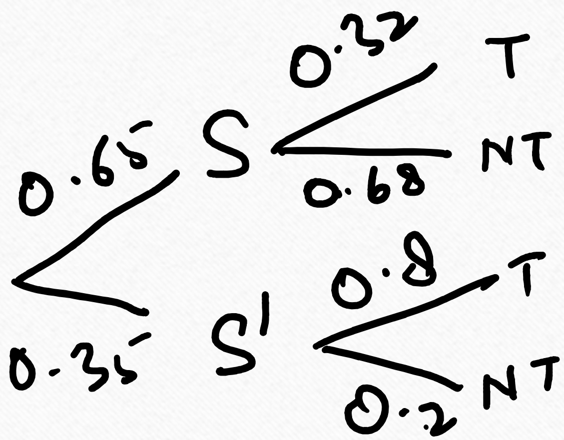
$P(B_2 \cap B) = \frac{5}{9} \times \frac{1}{2}$

$$P(R) = P(B_1 \cap R) + P(B_2 \cap R) = \frac{1}{4} + \frac{4}{9} \times \frac{1}{2}$$

$$\Rightarrow \frac{17}{36}$$

Q Prob. are 0.65 if there is a strike, 0.80 that the job will be done on time if there is no strike. 0.32 that job will be done on time if there is a strike. Calculate prob. of job being completed on time.

Sol.



$P(T) = 0.65 \times 0.32 + 0.35 \times 0.8$

$= 0.488$

$P(NT) = 1 - 0.488 = 0.512$

Bayes' Theorem

Naive Bayes

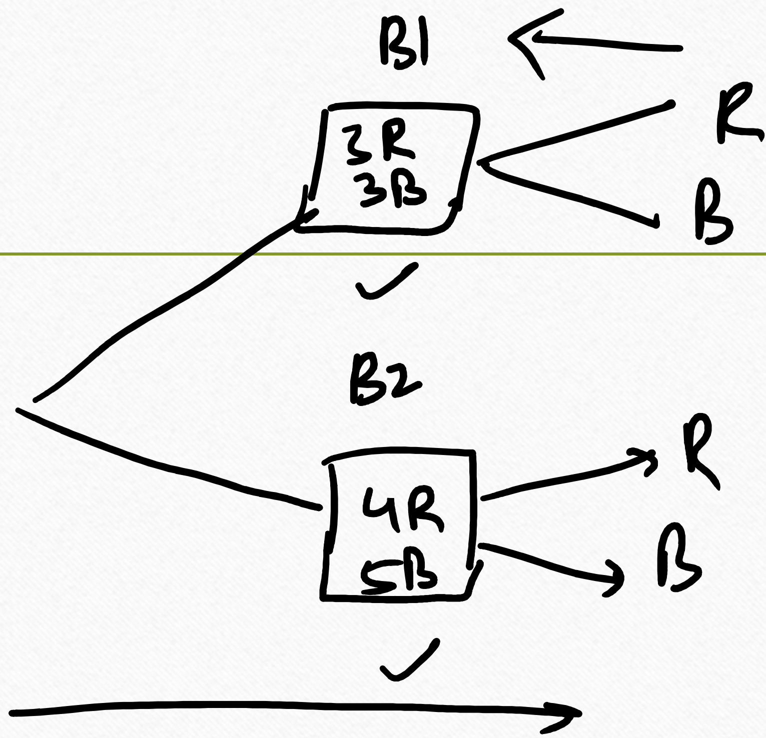
- Best means to find the reverse probability.
- Naïve Bayes ML algorithm is based on Bayes' Theorem.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A) \times P(B \mid A).$$

$$P(B) = P(A) \times P(B \mid A) + P(A') \times P(B \mid A')$$

$$P(A \mid B) = \frac{P(A) \times P(B \mid A)}{P(A) \times P(B \mid A) + P(A') \times P(B \mid A')}$$



$P(B_1)$

$$P(E_i \cap A) = P(A \cap E_i)$$

- $P(E) > 0$
- Mutually Exclusive
- Exhaustive

$E_1 \to A \to P(E_i \cap A)$

$E_2 \to A$

$E_3 \to A$

$$P(E_i / A) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(A \cap E_i)}{P(A)}$$

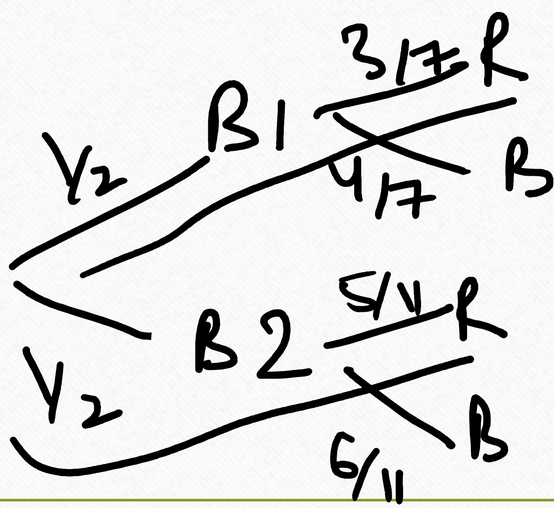
$$P(E_i / A) = \frac{P(E_i) \times P(A/E_i)}{P(A)}$$

$P(A) \to$ LTP

Q. Bag1 → 3R, 4B

Bag2 → 5R, 6B

One ball is drawn at random. We found to be Red. Prob. that it is picked from Bag 2.



$$P(B2/R) = \frac{\frac{1}{2} \times \frac{5}{11}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{5}{11}} = \frac{35}{68}$$

$$= 0.514$$

Q. Box I: 2 Gold coins   Box II: 2 silver coins   Box III: 1 gold, 1 silver.

You have pulled a coin & found it to be gold. Prob that other coin in bag is also gold.

Sol.

$\frac{1}{3}$: $B1 \xrightarrow{1} G$

$\frac{1}{3}$: $B2 \xrightarrow{0} G$

$\frac{1}{3}$: $B3 \xrightarrow{1/2} G$

$$P(B1/G) = \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times 0+\frac{1}{3}\times\frac{1}{2}}$$

$$= 2/3$$