arrangement ← permutation

XYZ → XY, YX, ZX, XZ, ZY, YZ

combinations

↓

selection

XYZ → XY or YX, YZ or ZY, ZX or XZ

Distributions

Fundamental of Counting

FP of Multiplication

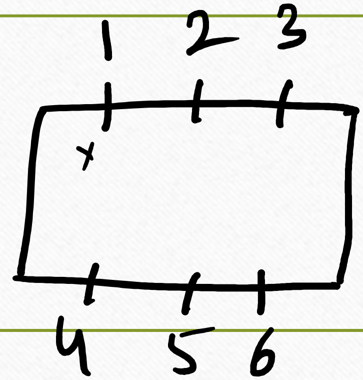
→ and ⇒ ×

→ both job occurs simultaneously

FP of addition

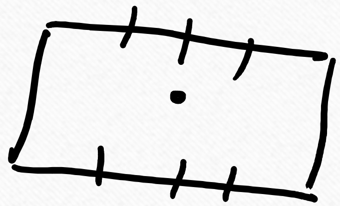
→ OR ⇒ +

→ either of job can occur



$\Rightarrow 6 \times 5 = 30$ ways.

you can enter through any door but can't exit through same door. In how many ways you can enter/exit the room.



You can enter & exit through door. In how ways can you do that?

$$6 \times 6 = 36 \text{ ways}$$

Q Class → 10 boys & 8 girls. You are the mentor of the class. You have to select one boy & one girl as class monitors. In how many ways can you do it?

Sol. $10 \times 8 \Rightarrow 80$ ways

Q Class → 10 boys & 8 girls. You have to choose one boy or one girl to be the class monitor. In how many ways can you do it?

Sol. $10 + 8 = 18$ ways

Q ROSE ① Repitition isn't allowed.

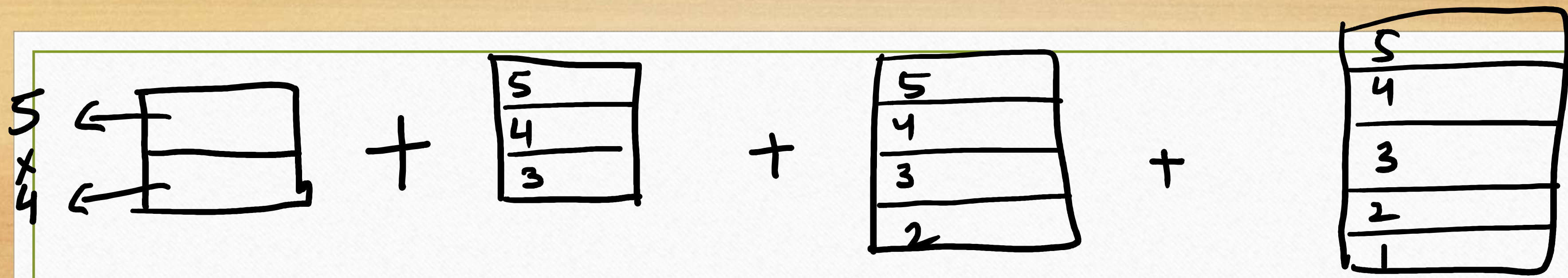
$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1} \Rightarrow 4 \times 3 \times 2 \times 1 = 24$

Q if repitition is allowed!

ROSE

$\underline{4}\ \underline{4}\ \underline{4}\ \underline{4} = 4 \times 4 \times 4 \times 4 = 256$

Q 5 different colours, find different no. of signals that can be generated by using atleast 2 diff. colours in vertical order. Repitition is not allowed!



Ans $\Rightarrow$ $5 \times 4 + 5 \times 4 \times 3 + 5 \times 4 \times 3 \times 2 + 5 \times 4 \times 3 \times 2 \times 1$

$\Rightarrow$ $20 + 60 + 120 + 120 = 320$ ways factorial ⇒ product of first n natural number.

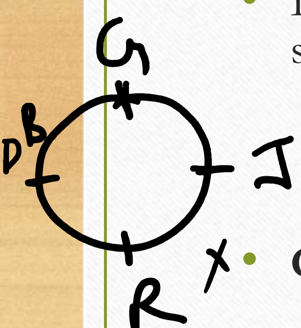
$\lfloor 5$ or $5!$ ⇒ $5 \times 4 \times 3 \times 2 \times 1$ ⇒ 120. $n! = n \times - - - \times 3 \times 2 \times 1$

write code for factorial!

Permutations

- Permutation: A permutation is the number of ways in which you can choose objects from a pool, and where the order in which you choose them counts.
- **The number of ways** in which you can **order n separate objects** is **n!(n factorial)**
- **The number of ways** in which you can **order n separate objects in a circle** is **(n-1)!**
- If you want to arrange **n** objects where **j of one type** are alike, **k of another type** are alike, so are **m of another type** and so on, **the number of arrangements** is given by,

$$\frac{n!}{j!k!m!\ldots} \qquad \frac{n!}{j!\,k!\,m!}$$

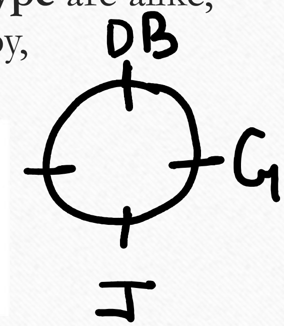


- **General Formula for Permutation**

This is the total number of objects

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

This is the number of positions we want to fill



def function( ):
    function( )

recursion ⇒ calling function within that f^n.

Mathematical Induction

$n=4$

ROOT

$\underbrace{OO}_{O}$

No. of arrangements $= \dfrac{4!}{2!\,1!\,1!}$

$$= \frac{4!}{2!} = \frac{4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}}$$

$$= 12$$

1	2	3	4		r	$r+1$		n
$\square$	$\square$	$\square$	$\square$	$- - -$	$\square$	$\square$	$- - - -$	$\square$
n	$n-1$	$n-2$	$n-3$		$[n-(r-1)]$	$(n-r+\cancel{1}-\cancel{1})$ $=(n-r)$		1

$$\frac{\left[n(n-1)(n-2)\cdots(n-r+1)\times(n-r)(n-r-1)\cdots 1\right]}{(n-r)(n-r-1)\cdots 1}$$

$$\boxed{\frac{n!}{(n-r)!} = {}^{n}P_{r}}$$

Q How many no. lying b/w 100 & 1000 can be formed using digits 0, 1, 2, 3, 4, 5, if repitition isn't allowed?

Sol.

$${}^6P_3 \Rightarrow \frac{6!}{3!} = \frac{6\times5\times4\times\cancel{3!}}{\cancel{3!}} = 120 \quad \times$$

$$\underline{0}\quad \underbrace{\underline{\ 1\ }\quad \underline{\ 2\ }}_{{}^5P_2} \qquad\qquad {}^5P_2$$

$$\text{Total no.s} = {}^6P_3 - {}^5P_2 = 120 - 5\times4 = 100 \text{ ways.}$$

Q INDEPENDENCE. In how many of the arrangements.

a) do words start with P.

$$\frac{n!}{j!\,k!\,m!}$$

I → 1

N → 3

D → 2

E → 4

P → 1

C → 1

P _ _ _ _ _ _ _ _ _ _ _

$\underbrace{\qquad\qquad\qquad\qquad}_{11}$

$1! = 1$

$0! = 1$

$$\frac{11!}{3!\,2!\,4!} \Rightarrow \frac{11!}{3!\,2!\,4!} \Rightarrow \underline{138600}$$

b) all vowel occur together

$\boxed{EEEEI} + 7$

$$\frac{8!}{3!\,2!} \times \frac{5!}{4!} \Rightarrow \frac{8 \times 7 \times 6 \times 5 \times \cancel{4}^{2}}{\cancel{2}} \times 5$$

$$\Rightarrow 420 \times 40$$

$$= 16800$$

c) do vowels never occur together?

Total no. of words $= \dfrac{12!}{4!\,3!\,2!} = 1663200$

No. of arrangements where vowel occur together $= 16800$

Vowels are never together $= 1663200 - 16800 = 1646400$

$$ {}^nC_r \Rightarrow \frac{n!}{r!(n-r)!} = \frac{{}^nP_r}{r!} $$

XYZ

XY or YX

YZ or ZY

XZ or ZX

Combinations

- Combinations: A combination is the number of ways in which you can choose objects from a pool, without caring about the exact order in which you choose them.
- The number of combinations is **the number of ways of choosing r objects from n**, without needing to know the exact order of the objects. The **number of combinations** is written **n Cr** , where

This is the total number of objects.

$$ {}^nC_r = \frac{n!}{r!\,(n-r)!} $$

This bit is calculated in the same way as a permutation.

This is the number of positions we want to fill.

You divide by an extra r! if it's a combination.

$$ {}^nP_r = r! \times {}^nC_r $$

$$\frac{12\times11\times\cancel{10}^{2}\times\cancel{9}^{3}\times\cancel{8}\times\cancel{7}}{\cancel{8}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1}$$

$\underline{792}$

$${}^{12}C_5 = \frac{12!}{(12-5)!\,(5)!}$$

Question to Play!

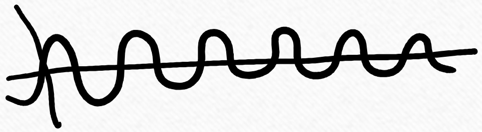
- The Statsville All Stars are due to play a basketball match. There are 12 players in the roster, and 5 are allowed on the court at any one time.
  - How many different arrangements are there for choosing who's on the court at the same time?
  - The coach classes 3 of the players as expert shooters. What's the probability that all 3 of these players will be on the court at the same time, if they're chosen at random?

$$\frac{n(E)}{n(S)} = \frac{{}^{9}C_2}{{}^{12}C_5}$$

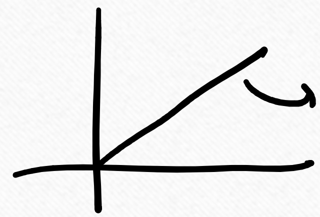
$$\frac{10\times 9\times \cancel{8}^{4}\times 7\times 6}{3\times \cancel{2}\times \cancel{2}} = 2520$$

Question to Play!

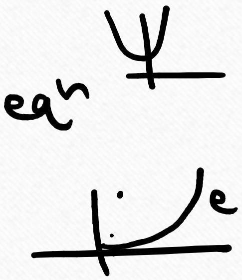
- The Statsville Derby have decided to experiment with their races. They've decided to hold a race between 3 horses, 2 zebras and 5 camels, where all the animals are equally likely to finish the race first.
  - How many ways are there of finishing the race if we're interested in individual animals? $10!$
  - How many ways are there of finishing the race if we're just interested in the species of animal in each position? $10!/3!\,2!\,5!$
  - HW What's the probability that all 5 camels finish the race consecutively if each animal has an equal chance of winning? (Assume we're interested in the species in each position, not the individual animals themselves.). (1+5)!/(3!*2!) = 60→ 60/answer of second question



sine wave



mean



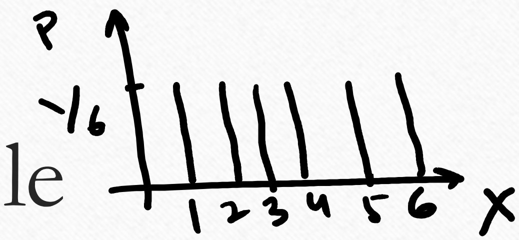
Distributions

- The distribution of a statistical dataset is **the spread of the data which shows all possible values or intervals of the data and how they occur**.
- The distribution provides a parameterized mathematical function that can be used to calculate the probability for any individual observation from the sample space.
- **Density functions** are functions that describe how the proportion of data or likelihood of the proportion of observations change over the range of the distribution.
  - **PDF** → calculate the likelihood of a given observation in a distribution **or** used to summarize the likelihood of observations across the distribution's sample space
  - **CDF** → the CDF calculates the cumulative likelihood for the observation and all prior observations in the sample space. It allows you to quickly understand and comment on how much of the distribution lies before and after a given value.

X	1	2	3	4	5	6
P	1/6	1/6	1/6	1/6	1/6	1/6

discrete

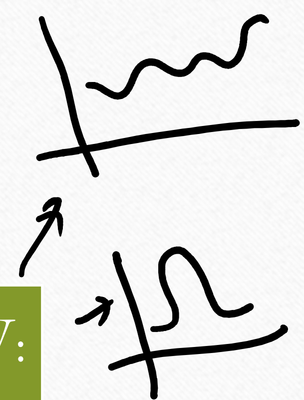
Random Variable



- A variable that can take multiple values in random manner Or take any outcome of the random experiment.

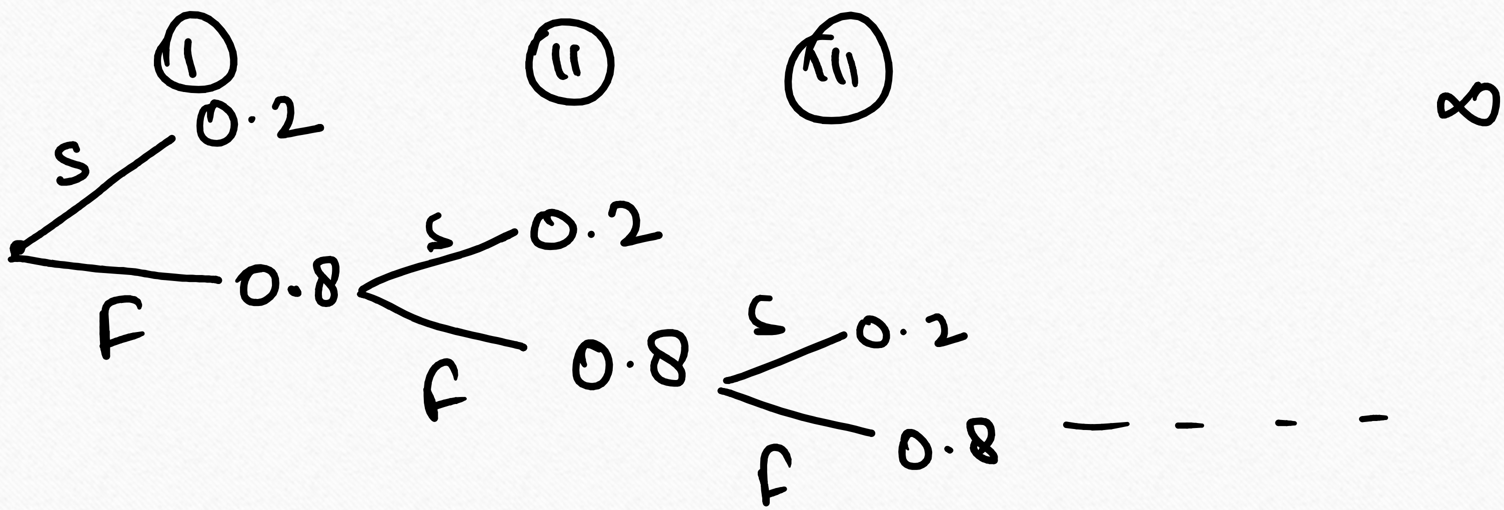
Random Variable

- **Discrete RV:** takes on discrete values like dice roll
- **Continuous RV:** takes on any real value in set of values



$P(F) = 0.8$

$P(S) = 0.2$



Q. What are the chances that I'll succeed at 2^{nd} trial?

$0.8 \times 0.2 = 0.16$

Q What are the chances that you will succeed in I^{st} trial or II^{nd} trial ?

Sol.

$P(S) = 0.2$ $P(F) = 0.8$

$P(X=1) = 0.2$ $P(X=2) = 0.16$

$P(X \leq 2) = 0.2 + 0.16 = 0.36$

Q Prob. of success in 5 trials? $P(S) = 0.2$

$P(F) = 0.8$

$P(X=1) = 0.2$

$P(X=2) = 0.8 \times 0.2 = 0.16$

$P(X=3) = 0.8 \times 0.8 \times 0.2 = 0.128$

$P(X=4) = 0.8 \times 0.8 \times 0.8 \times 0.2 = 0.1024$

$P(X=5) = 0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.2 = 0.08192$

$P(X \leq 5) = 0.672$

$P(X=1) = 0.2 \qquad = 0.8^0 \times 0.2$

$P(X=2) = 0.8 \times 0.2 \qquad = 0.8^1 \times 0.2$

$P(X=3) = 0.8 \times 0.8 \times 0.2 = 0.8^2 \times 0.2$

$P(X=4) = 0.8 \times 0.8 \times 0.8 \times 0.2 = 0.8^3 \times 0.2$

$\vdots$

$P(X=r) = 0.8^{r-1} \times 0.2$

p = prob of success

q = " " failure

$$\boxed{P(X=r) = p\,q^{r-1}}$$

Geometric distribution

$$\boxed{P(X>r) = q^r}$$ $\rightarrow$ prob. of taking more than r trials to win

$P(X \leq r) + P(X>r)$

$r+1$

$p(r)$

1

$$P(X \leq r) + P(X>r) = 1$$

$$\boxed{P(X \leq r) = 1 - P(X>r) = 1 - q^r}$$

Geometric Distribution

- If the probability of success in a trial is represented by p and the probability of failure is 1 - p, or q, we can work out any probability of this nature by using:

RV event/trial

$$P(X = r) = p\, q^{r-1}$$

- The geometric distribution covers situations where:
  - You run a series of independent trials.
  - There can be either a success or failure for each trial, and the probability of success is the same for each trial.
  - The main thing you're interested in is how many trials are needed in order to get the first successful outcome.

Expectations: $p+q=1$

$$E(x) = \sum_{x=1}^{\infty} x\, p(x)$$

$$= \sum_{x=1}^{\infty} x \left[q^{x-1} p\right]$$

$$= \left[1q^{1-1}p + 2q^{2-1}p + 3q^{3-1}p + \cdots\right]$$

$$= \left[p + 2qp + 3q^2 p + \cdots\right]$$

$$= p\left[1 + 2q + 3q^2 + \cdots\right]$$

Applying Binomial theorem,

$$= p(1-q)^{-2}$$

$$= p(p)^{-2} = p^{-1} = \frac{1}{p}$$

$$\boxed{E(x) = \frac{1}{p}}$$

Variance : $Var(X) = E(X^2) - [E(X)]^2 = \frac{\Sigma x^2}{n} - \mu^2$

$\Rightarrow E(X^2 + X - X) - [E(X)]^2$

$\Rightarrow E(X^2 - X + X) - [E(X)]^2$

$\Rightarrow \underbrace{E(X^2 - X)} + E(X) - \frac{1}{p^2} \Rightarrow E(X(X-1)) + \frac{1}{p} - \frac{1}{p^2}$

$\Rightarrow \sum_{X=1}^{\infty} (X-1) q^{X-1} p + \frac{1}{p} - \frac{1}{p^2}$

$$\Rightarrow \left[2qp + 6q^2p + \cdots\right] + \frac{1}{p} - \frac{1}{p^2}$$

$$\Rightarrow 2pq\left[1 + 3q + \cdots\right] + \frac{1}{p} - \frac{1}{p^2}$$

$$\Rightarrow 2pq(1-q)^{-2} + \frac{1}{p} - \frac{1}{p^2} = \frac{q}{p^2}$$

Q Suppose there is a soldier who shoots a target in independent fashion. If prob. of hitting target is 0.7. What is the avg of shots needed by him to target?

Sol. $E(x) = \frac{1}{p} = \frac{1}{0.7} = 1.42$

2 shots.

Geometric Distribution

- The geometric distribution has a distinctive shape. P(X = r) is at its highest when r = 1, and it gets lower and lower as r increases. Notice that the probability of getting a success is highest for the first trial. This means that the mode of any geometric distribution is always 1.
- Expectation E(x) = 1/P
- Variance V(x) = q/P