```python
import numpy as np
import gym
from collections import deque
import random

# Ornstein-Ulhenbeck Process
# Taken from #https://github.com/vitchyr/rlkit/blob/master/rlkit/exploration_strategies/ou_strategy.py
class OUNoise(object):
    def __init__(self, action_space, mu=0.0, theta=0.15, max_sigma=0.3, min_sigma=0.3, decay_period=100000):
        self.mu           = mu
        self.theta        = theta
        self.sigma        = max_sigma
        self.max_sigma    = max_sigma
        self.min_sigma    = min_sigma
        self.decay_period = decay_period
        self.action_dim   = action_space.shape[0]
        self.low          = action_space.low
        self.high         = action_space.high
        self.reset()

    def reset(self):
        self.state = np.ones(self.action_dim) * self.mu

    def evolve_state(self):
        x  = self.state
        dx = self.theta * (self.mu - x) + self.sigma * np.random.randn(self.action_dim)
        self.state = x + dx
        return self.state

    def get_action(self, action, t=0):
        ou_state = self.evolve_state()
        self.sigma = self.max_sigma - (self.max_sigma - self.min_sigma) * min(1.0, t / self.decay_period)
        return np.clip(action + ou_state, self.low, self.high)


# https://github.com/openai/gym/blob/master/gym/core.py
class NormalizedEnv(gym.ActionWrapper):
    """ Wrap action """

    def action(self, action):
        act_k = (self.action_space.high - self.action_space.low)/ 2.
        act_b = (self.action_space.high + self.action_space.low)/ 2.
        return act_k * action + act_b



class Memory:
    def __init__(self, max_size):
        self.max_size = max_size
        self.buffer = deque(maxlen=max_size)

    def push(self, state, action, reward, next_state, done):
        experience = (state, action, np.array([reward]), next_state, done)
        self.buffer.append(experience)

    def sample(self, batch_size):
        state_batch = []
        action_batch = []
        reward_batch = []
        next_state_batch = []
        done_batch = []

        batch = random.sample(self.buffer, batch_size)

        for experience in batch:
            state, action, reward, next_state, done = experience
            state_batch.append(state)
            action_batch.append(action)
            reward_batch.append(reward)
            next_state_batch.append(next_state)
            done_batch.append(done)

        return state_batch, action_batch, reward_batch, next_state_batch, done_batch

    def __len__(self):
        return len(self.buffer)
```

DDPG uses four neural networks: a Q network, a deterministic policy network, a target Q network, and a target policy network.

$$\text{Parameters:}$$

$$\theta^{Q} : \text{Q network}$$

$$\theta^{\mu} : \text{Deterministic policy function}$$

$$\theta^{Q'} : \text{target Q network}$$

$$\theta^{\mu'} : \text{target policy network}$$

The Q network and policy network is very much like simple Advantage Actor-Critic, but in DDPG, the Actor directly maps states to actions (the output of the network directly the output) instead of outputting the probability distribution across a discrete action space.

The target networks are time-delayed copies of their original networks that slowly track the learned networks. Using these target value networks greatly improve stability in learning.

```
import torch
import torch.nn as nn
import torch.nn.functional as F
import torch.autograd
from torch.autograd import Variable

class Critic(nn.Module):
    def __init__(self, input_size, hidden_size, output_size):
        super(Critic, self).__init__()
        self.linear1 = nn.Linear(input_size, hidden_size)
        self.linear2 = nn.Linear(hidden_size, hidden_size)
        self.linear3 = nn.Linear(hidden_size, output_size)

    def forward(self, state, action):
        """
        Params state and actions are torch tensors
        """
        x = torch.cat([state, action], 1)
        x = F.relu(self.linear1(x))
        x = F.relu(self.linear2(x))
        x = self.linear3(x)

        return x

class Actor(nn.Module):
    def __init__(self, input_size, hidden_size, output_size, learning_rate = 3e-4):
        super(Actor, self).__init__()
        self.linear1 = nn.Linear(input_size, hidden_size)
        self.linear2 = nn.Linear(hidden_size, hidden_size)
        self.linear3 = nn.Linear(hidden_size, output_size)

    def forward(self, state):
        """
        Param state is a torch tensor
        """
        x = F.relu(self.linear1(state))
        x = F.relu(self.linear2(x))
        x = torch.tanh(self.linear3(x))

        return x
```

Now, let's create the DDPG agent. The agent class has two main functions: "get_action" and "update":

- **get_action()**: This function runs a forward pass through the actor network to select a determinisitic action. In the DDPG paper, the authors use Ornstein-Uhlenbeck Process to add noise to the action output (Uhlenbeck & Ornstein, 1930), thereby resulting in exploration in the environment. Class OUNoise (in cell 1) implements this.

$$\mu'(s_t) = \mu(s_t|\theta_t^{\mu}) + \mathcal{N}$$

- **update()**: This function is used for updating the actor and critic networks, and forms the core of the DDPG algorithm. The replay buffer is first sampled to get a batch of experiences of the form **<states, actions, rewards, next_states>**.

The value network is updated similarly as is done in Q-learning. The updated Q value is obtained by the Bellman equation. However, in DDPG, the next-state Q values are calculated with the target value network and target policy network. Then, we minimize the mean-squared loss between the updated Q value and the original Q value:

$$y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$$

$$Loss = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$$

For the policy function, our objective is to maximize the expected return. To calculate the policy loss, we take the derivative of the objective function with respect to the policy parameter. Keep in mind that the actor (policy) function is differentiable, so we have to apply the chain rule.

But since we are updating the policy in an off-policy way with batches of experience, we take the mean of the sum of gradients calculated from the mini-batch:

$$\nabla_{\theta^\mu} J(\theta) \approx \frac{1}{N}\sum_i [\nabla_a Q(s,a|\theta^Q)|_{s=s_i,a=\mu(s_i)} \nabla_{\theta^\mu}\mu(s|\theta^\mu)|_{s=s_i}]$$

We make a copy of the target network parameters and have them slowly track those of the learned networks via “soft updates,” as illustrated below:

$$\theta^{Q'} \leftarrow \tau\theta^{Q} + (1-\tau)\theta^{Q'}$$

$$\theta^{\mu'} \leftarrow \tau\theta^{\mu} + (1-\tau)\theta^{\mu'}$$

$$\text{where} \quad \tau \ll 1$$

```
def soft_update(target, source, tau):
    for target_param, param in zip(target.parameters(), source.parameters()):
        target_param.data.copy_(
            target_param.data * (1.0 - tau) + param.data * tau
        )

def to_tensor(ndarray, volatile=False, requires_grad=False, dtype=torch.FloatTensor):
    return Variable(
        torch.from_numpy(ndarray), volatile=volatile, requires_grad=requires_grad
    ).type(dtype)


import torch
import torch.autograd
import torch.optim as optim
import torch.nn as nn
# from model import *
# from utils import *

class DDPGagent:
    def __init__(self, env, hidden_size=256, actor_learning_rate=1e-4, critic_learning_rate=1e-3, gamma=0.99, tau=1e-2, max_memory_size=50000):
        # Params
        self.num_states = env.observation_space.shape[0]
        self.num_actions = env.action_space.shape[0]
        self.gamma = gamma
        self.tau = tau

        # Networks
        self.actor = Actor(self.num_states, hidden_size, self.num_actions)
        self.actor_target = Actor(self.num_states, hidden_size, self.num_actions)
        self.critic = Critic(self.num_states + self.num_actions, hidden_size, self.num_actions)
        self.critic_target = Critic(self.num_states + self.num_actions, hidden_size, self.num_actions)


        for target_param, param in zip(self.actor_target.parameters(), self.actor.parameters()):
            target_param.data.copy_(param.data)

        for target_param, param in zip(self.critic_target.parameters(), self.critic.parameters()):
            target_param.data.copy_(param.data)

        # Training
        self.memory = Memory(max_memory_size)
        self.critic_criterion  = nn.MSELoss()
        self.actor_optimizer  = optim.Adam(self.actor.parameters(), lr=actor_learning_rate)
        self.critic_optimizer = optim.Adam(self.critic.parameters(), lr=critic_learning_rate)

    def get_action(self, state):
        state = Variable(torch.from_numpy(state).float().unsqueeze(0))
        action = self.actor.forward(state)
        action = action.detach().numpy()[0,0]
        return action

    def update(self, batch_size):
        states, actions, rewards, next_states, _ = self.memory.sample(batch_size)
        # states = torch.FloatTensor(states)
        # actions = torch.FloatTensor(actions)
        # rewards = torch.FloatTensor(rewards)
        # next_states = torch.FloatTensor(next_states)
        states = np.array(states)
        actions = np.array(actions)
        rewards = np.array(rewards)
        next_states = np.array(next_states)

        # Implement actor and critic loss
        next_q_values = self.critic_target(
            to_tensor(next_states, volatile = True),
            self.actor_target(to_tensor(next_states, volatile = True))
        )
        next_q_values.volatile = False
        target_q_batch = to_tensor(rewards) + \
            self.gamma*next_q_values

        # Update actor and critic networks
        self.critic.zero_grad()
        q_batch = self.critic(to_tensor(states), to_tensor(actions))

        value_loss = self.critic_criterion(q_batch, target_q_batch)
        value_loss.backward()
        self.critic_optimizer.step()

        self.actor.zero_grad()

        policy_loss = -self.critic(
            to_tensor(states),
            self.actor(to_tensor(states))
        )

        policy_loss = policy_loss.mean()
        policy_loss.backward()
        self.actor_optimizer.step()
```

```
        # update target networks
        soft_update(self.actor_target, self.actor, self.tau)
        soft_update(self.critic_target, self.critic, self.tau)
```

Putting it all together: DDPG in action.

The main function below runs 100 episodes of DDPG on the "Pendulum-v0" environment of OpenAI gym. This is the inverted pendulum swingup problem, a classic problem in the control literature. In this version of the problem, the pendulum starts in a random position, and the goal is to swing it up so it stays upright.

Each episode is for a maximum of 500 timesteps. At each step, the agent chooses an action, updates its parameters according to the DDPG algorithm and moves to the next state, repeating this process till the end of the episode.

The DDPG algorithm is as follows:

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

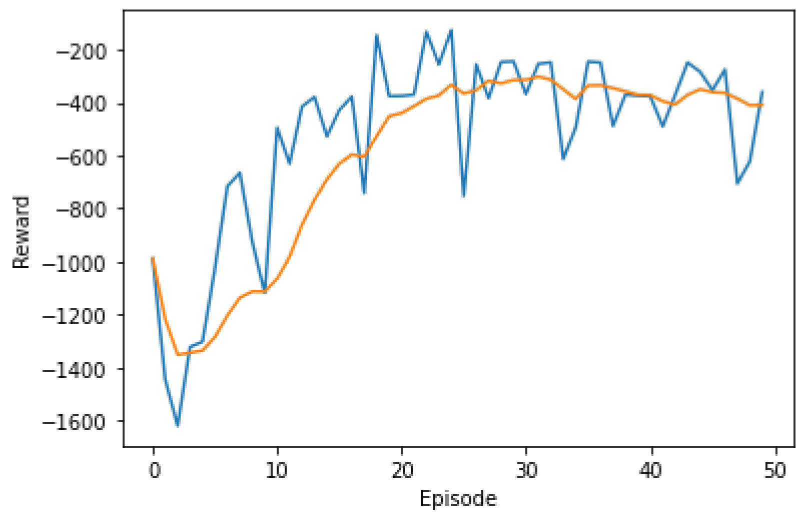
    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

```
import sys
import gym
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

env = NormalizedEnv(gym.make("Pendulum-v0"))

agent = DDPGagent(env)
noise = OUNoise(env.action_space)
batch_size = 128
rewards = []
avg_rewards = []

for episode in range(50):
    state = env.reset()
    noise.reset()
    episode_reward = 0

    for step in range(500):
        action = agent.get_action(state)
        #Add noise to action

        action = noise.get_action(action)
        new_state, reward, done, _ = env.step(action)
        agent.memory.push(state, action, reward, new_state, done)

        if len(agent.memory) > batch_size:
            agent.update(batch_size)

        state = new_state
        episode_reward += reward

        if done:
            sys.stdout.write("episode: {}, reward: {}, average _reward: {} \n".format(episode, np.round(episode_reward, decimals=2), np.mean(rewards[-10:])))
            break

    rewards.append(episode_reward)
    avg_rewards.append(np.mean(rewards[-10:]))

plt.plot(rewards)
plt.plot(avg_rewards)
plt.plot()
plt.xlabel('Episode')
plt.ylabel('Reward')
plt.show()
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:9: UserWarning: volatile was removed and now has no effect. Use `with torch.no_grad():` instead.
  if __name__ == '__main__':
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:57: UserWarning: volatile was removed and now has no effect. Use `with torch.no_grad():` instead.
/usr/local/lib/python3.7/dist-packages/numpy/core/fromnumeric.py:3441: RuntimeWarning: Mean of empty slice.
  out=out, **kwargs)
/usr/local/lib/python3.7/dist-packages/numpy/core/_methods.py:189: RuntimeWarning: invalid value encountered in double_scalars
  ret = ret.dtype.type(ret / rcount)
episode: 0, reward: -989.84, average _reward: nan
episode: 1, reward: -1445.29, average _reward: -989.8404954953085
episode: 2, reward: -1620.63, average _reward: -1217.564403112102
episode: 3, reward: -1322.21, average _reward: -1351.919742255108
episode: 4, reward: -1302.14, average _reward: -1344.4921091180768
episode: 5, reward: -1024.82, average _reward: -1336.0215293915032
episode: 6, reward: -716.75, average _reward: -1284.1540891492966
episode: 7, reward: -665.35, average _reward: -1203.0963010788703
episode: 8, reward: -925.88, average _reward: -1135.8783078268023
episode: 9, reward: -1118.33, average _reward: -1112.5446884682995
episode: 10, reward: -495.68, average _reward: -1113.1235602265642
episode: 11, reward: -630.8, average _reward: -1063.7070895311786
episode: 12, reward: -414.78, average _reward: -982.2586632730145
episode: 13, reward: -378.71, average _reward: -861.6738801020383
episode: 14, reward: -527.63, average _reward: -767.3242926747951
episode: 15, reward: -427.79, average _reward: -689.8736318127528
episode: 16, reward: -377.19, average _reward: -630.1708645435447
episode: 17, reward: -742.04, average _reward: -596.2152814640866
episode: 18, reward: -145.55, average _reward: -603.8841233198704
episode: 19, reward: -375.63, average _reward: -525.8519217462451
episode: 20, reward: -374.73, average _reward: -451.581862123416
episode: 21, reward: -370.01, average _reward: -439.4868208446047
episode: 22, reward: -131.93, average _reward: -413.4075499251368
episode: 23, reward: -255.79, average _reward: -385.1218825402383
episode: 24, reward: -125.58, average _reward: -372.8299941493089
episode: 25, reward: -752.73, average _reward: -332.6245828421931
episode: 26, reward: -255.82, average _reward: -365.11903787142126
episode: 27, reward: -383.43, average _reward: -352.9812473839303
episode: 28, reward: -246.74, average _reward: -317.12005453500007
episode: 29, reward: -243.92, average _reward: -327.2382355858064
episode: 30, reward: -367.67, average _reward: -314.066997238137
episode: 31, reward: -253.33, average _reward: -313.36130792850213
episode: 32, reward: -248.0, average _reward: -301.69337980813714
episode: 33, reward: -613.38, average _reward: -313.3012619094284
episode: 34, reward: -494.5, average _reward: -349.05991601320795
episode: 44, reward: -283.47, average _reward: -370.59645349913507
episode: 45, reward: -353.16, average _reward: -349.49383373284564
episode: 46, reward: -275.19, average _reward: -360.3059425553421
episode: 47, reward: -706.07, average _reward: -362.9614875170043
episode: 48, reward: -622.82, average _reward: -384.7234792466428
episode: 49, reward: -360.31, average _reward: -410.03055555164264
```



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