Name: Vishal Shashikant Salvi.

UID: 2019230069

Class: SE Comps

Batch: C

Experiment No 3

Aim: Sum of subset problem using backtracking.

Theory:

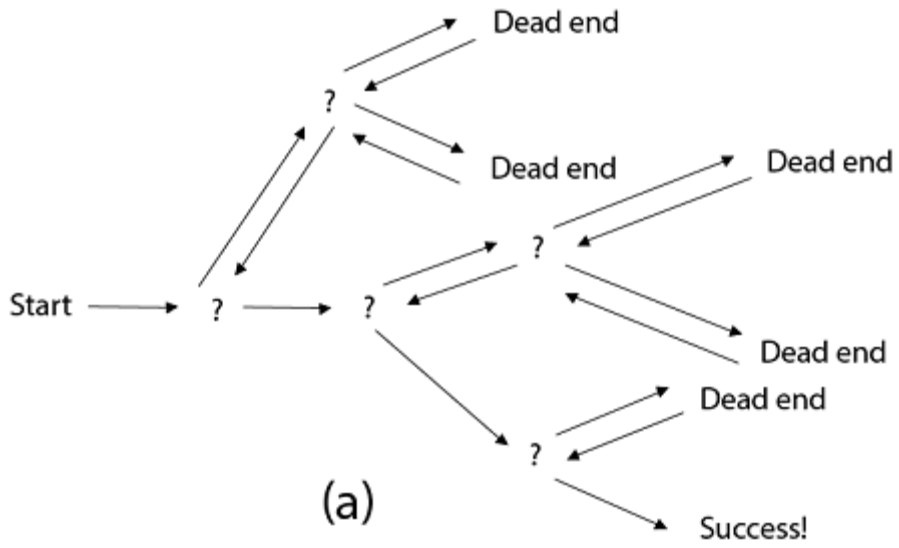
Introduction of Backtracking

The Backtracking is an algorithmic-method to solve a problem with an additional way. It uses a recursive approach to explain the problems. We can say that the backtracking is needed to find all possible combination to solve an optimization problem.

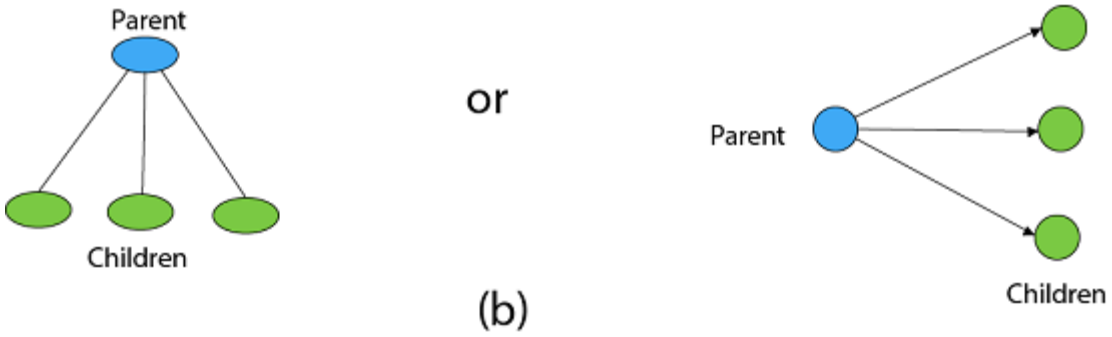
Backtracking is a systematic way of trying out different sequences of decisions until we find one that "works."

In the following Figure:

- Each non-leaf node in a tree is a parent of one or more other nodes (its children)
- Each node in the tree, other than the root, has exactly one parent

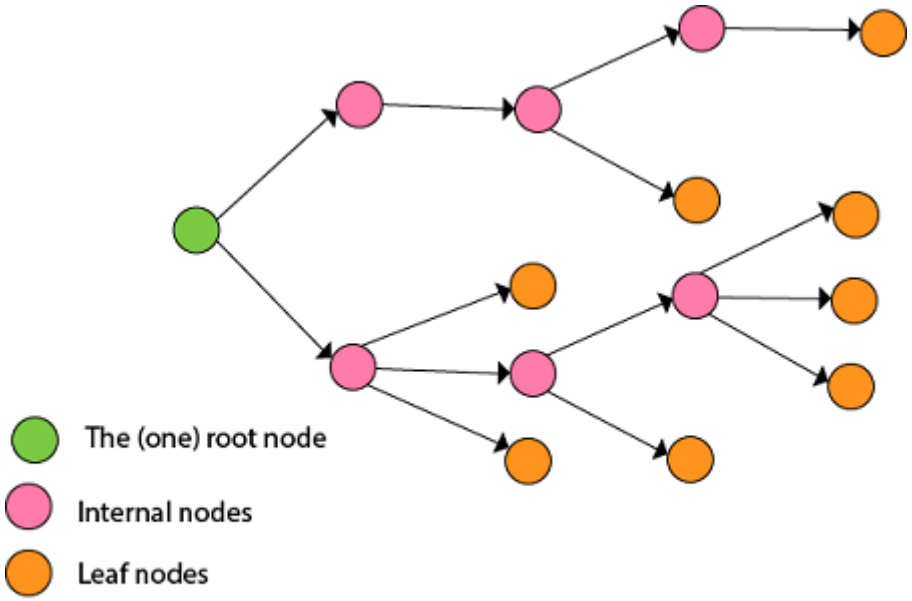


Generally, however, we draw our trees downward, with the root at the top.



(b)

A tree is composed of nodes.



(c)

Backtracking can understand of as searching a tree for a particular "goal" leaf node.

Backtracking is undoubtedly quite simple - we "explore" each node, as follows:

To "explore" node N:

1. If N is a goal node, return "success"
2. If N is a leaf node, return "failure"
3. For each child C of N,
   Explore C
   If C was successful, return "success"
4. Return "failure"

Backtracking algorithm determines the solution by systematically searching the solution space for the given problem. Backtracking is a **depth-first search** with any bounding function. All solution using backtracking is needed to satisfy a complex set of constraints. The constraints may be explicit or implicit.

Explicit Constraint is ruled, which restrict each vector element to be chosen from the given set.

Implicit Constraint is ruled, which determine which each of the tuples in the solution space, actually satisfy the criterion function.

Algorithm:

```
Algorithm Backtrack()
//This is recursive backtracking algorithm
//a[k] is a solution vector. On entering (k–1) remaining next
//values can be computed.
//T(a1,a2,…ak) be the set of all values for a(i+1), such that
//(a1,a2,…ai+1) is a path to problem state.
// Bi+1 is a bounding function such that if Bi+1(a1,a2,…ai+1) is false
//for a path (a1,a2,…ai+1) from root node to a problem state then
//path can not be extended to reach an answer node
{
for ( each ak that belongs to T((a1,a2,…ak-1) do
{
 if(Bk(a1,a2,…ak) = true) then // feasible sequence
 {
    if ((a1,a2,…ak) is a path to answer node then
      print(a[1],a[2],…a[k]);
    if (k<n) then
      Backtrack(k+1); //find the next set.
 }
 }
}
```

Sum of subset Problem

The Subset-Sum Problem is to find a subset's' of the given set S = (S_1 S_2 S_3...S_n) where the elements of the set S are n positive integers in such a manner that s'∈S and sum of the elements of subset's' is equal to some positive integer 'X.'

The Subset-Sum Problem can be solved by using the backtracking approach. In this implicit tree is a binary tree. The root of the tree is selected in such a way that represents that no decision is yet taken on any input. We assume that the elements of the given set are arranged in increasing order:

$S_1 \leq S_2 \leq S_3 ... \leq S_n$

The left child of the root node indicated that we have to include 'S_1' from the set 'S' and the right child of the root indicates that we have to execute 'S_1'. Each node stores the total of the partial solution elements. If at any stage the sum equals to 'X' then the search is successful and terminates.

The dead end in the tree appears only when either of the two inequalities exists:

- The sum of s' is too large i.e.

$s' + S_i + 1 > X$

- The sum of s' is too small i.e.

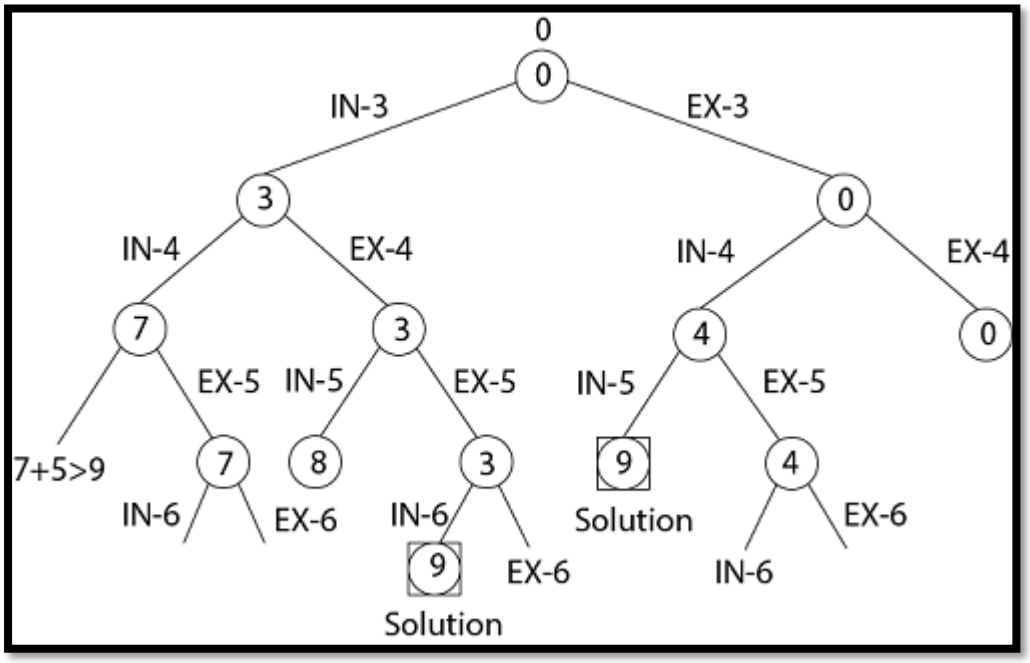
$$s' + \sum_{j=i+1}^{n} S_j < X$$

Example: Given a set S = (3, 4, 5, 6) and X =9. Obtain the subset sum using Backtracking approach.

Solution:

1. Initially S = (3, 4, 5, 6) and X =9.
2. S'= (∅)

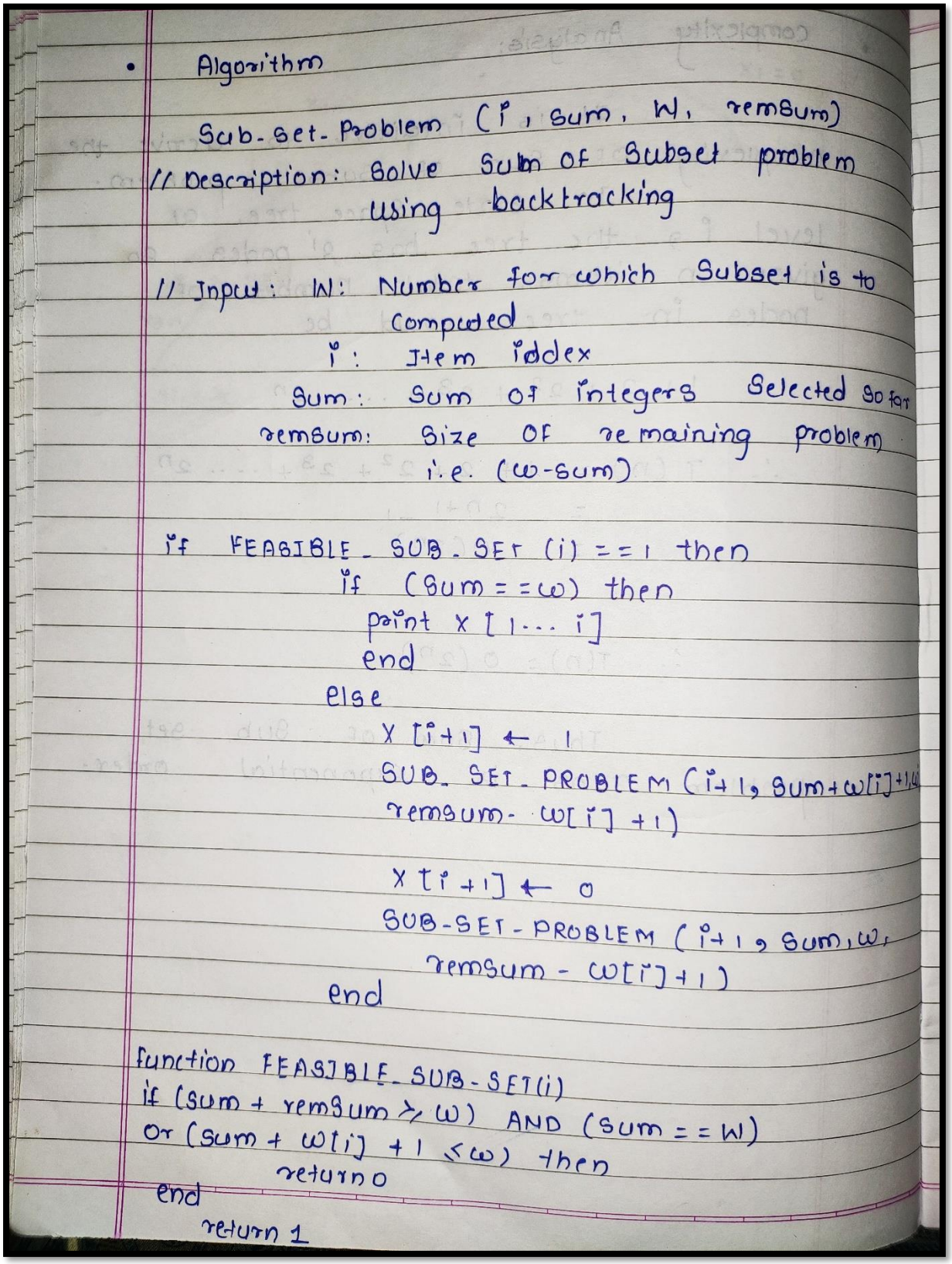
The implicit binary tree for the subset sum problem is shown as fig:



The number inside a node is the sum of the partial solution elements at a particular level.

Thus, if our partial solution elements sum is equal to the positive integer 'X' then at that time search will terminate, or it continues if all the possible solution needs to be obtained.

Algorithm:



- Algorithm

Sub-set-Problem (i, sum, W, remsum)
// Description: Solve Sum of Subset problem using backtracking

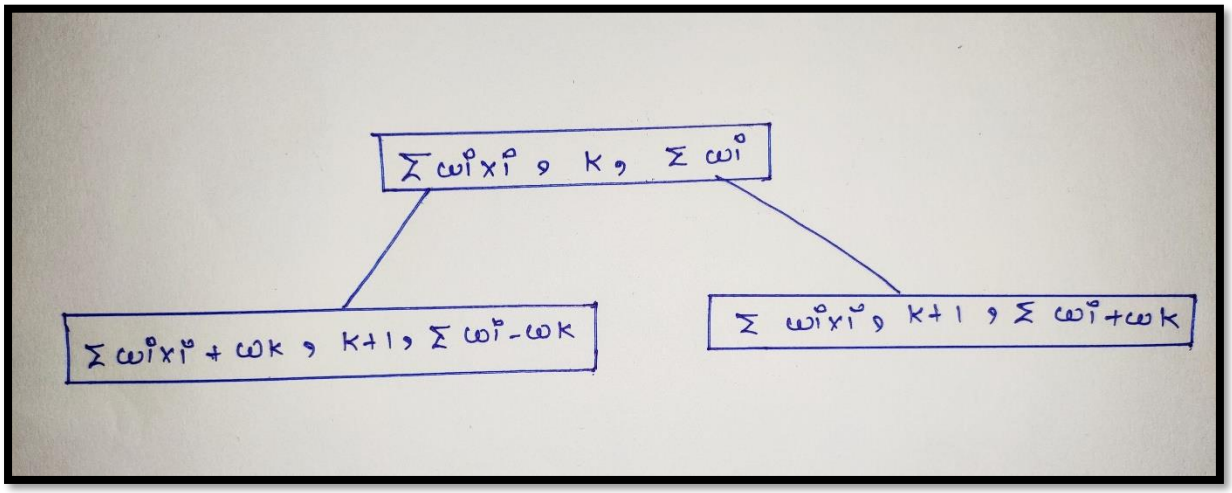
// Input: W: Number for which Subset is to computed
i: Item iddex
Sum: Sum of integers Selected so far
remsum: Size of remaining problem i.e. (w-sum)

if FEASIBLE_SUB_SET (i) == 1 then
if (Sum == w) then
print x [1 ... i]
end
else
x [i+1] ← 1
SUB_SET_PROBLEM (i+1, Sum+w[i]+1,
remsum - w[i] + 1)

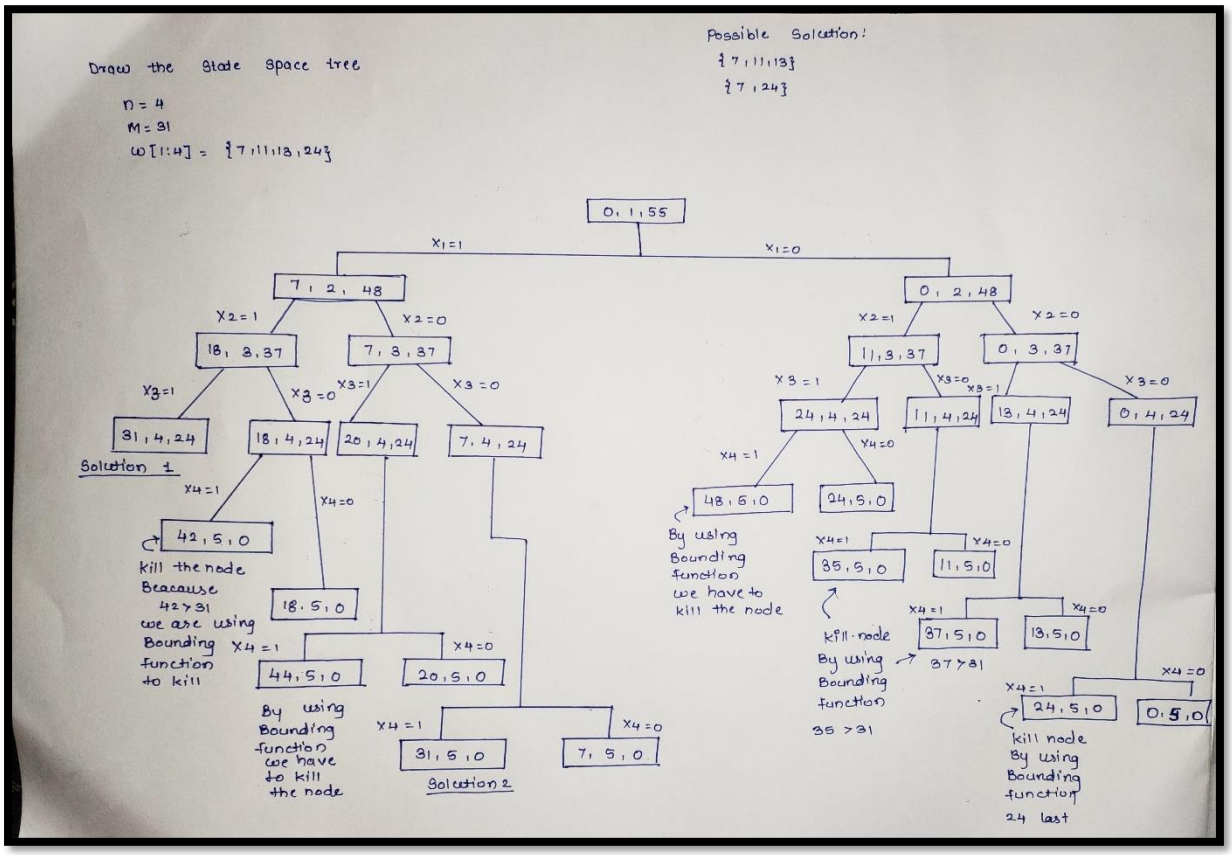
x [i+1] ← 0
SUB-SET-PROBLEM (i+1, Sum, w,
remsum - w[i]+1)
end

function FEASIBLE_SUB-SET(i)
if (sum + remsum ≥ w) AND (sum == W)
or (sum + w[i] + 1 ≤ w) then
return 0
end
return 1

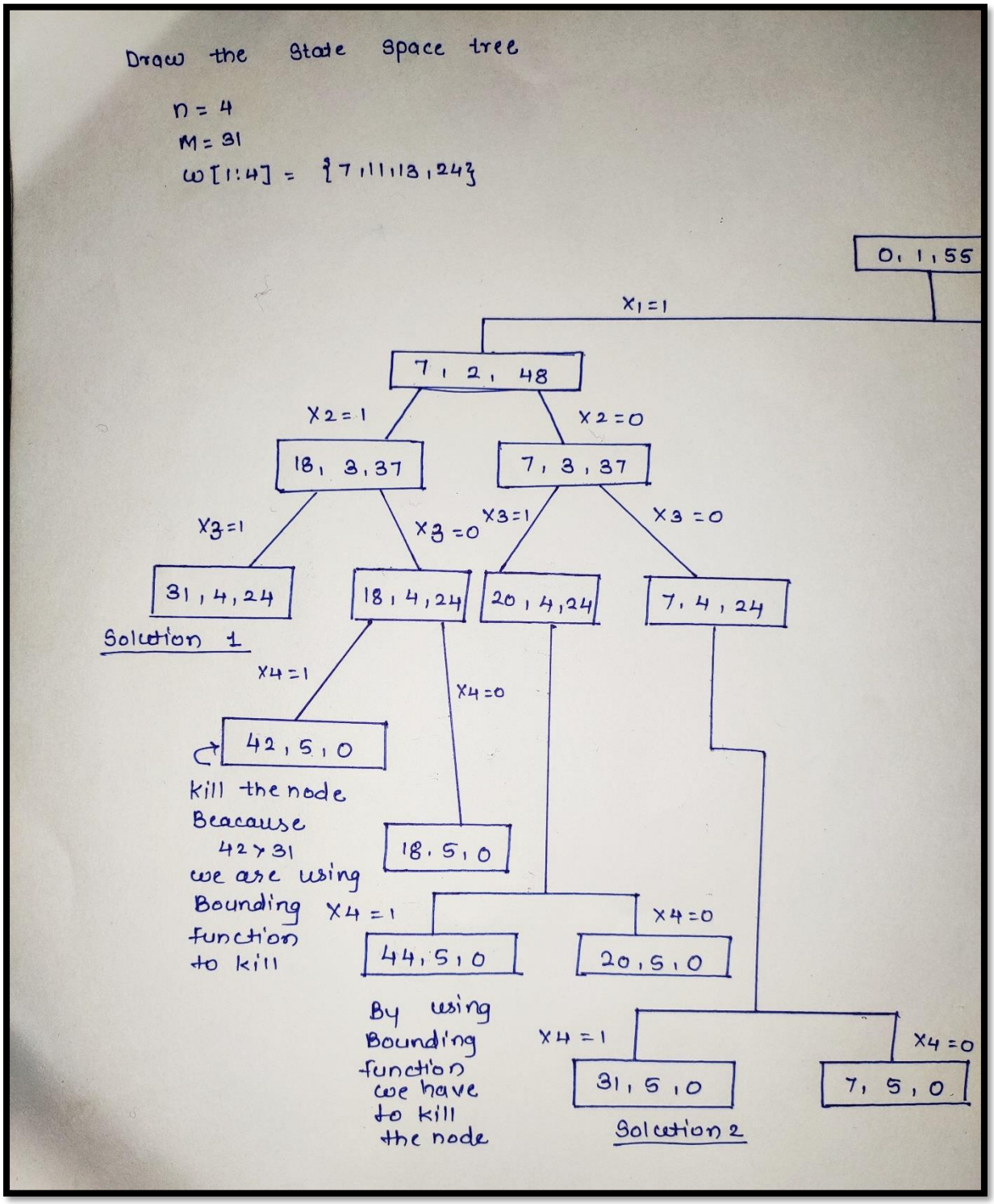
Mathematical Derivation:



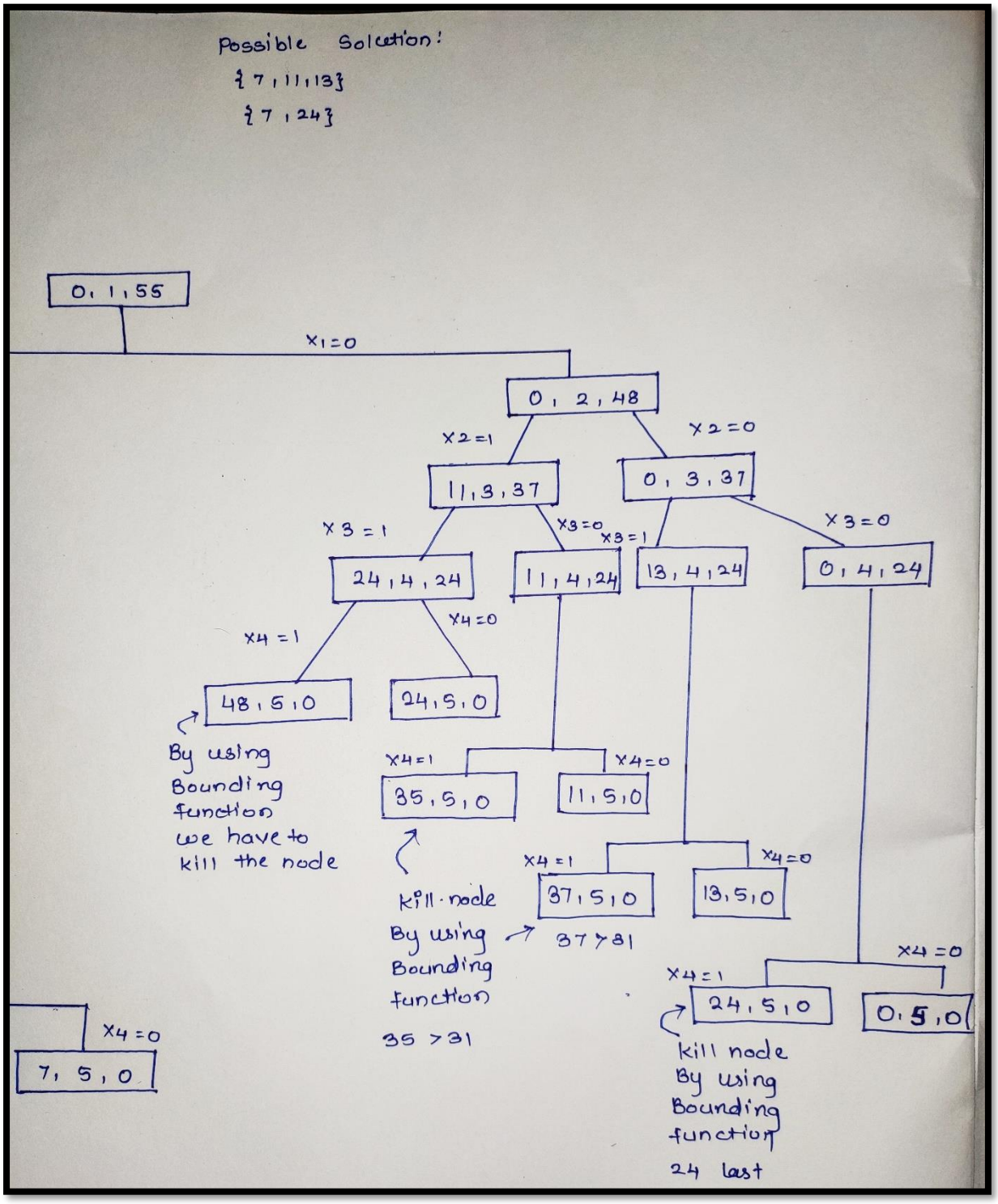
Problem State space tree:



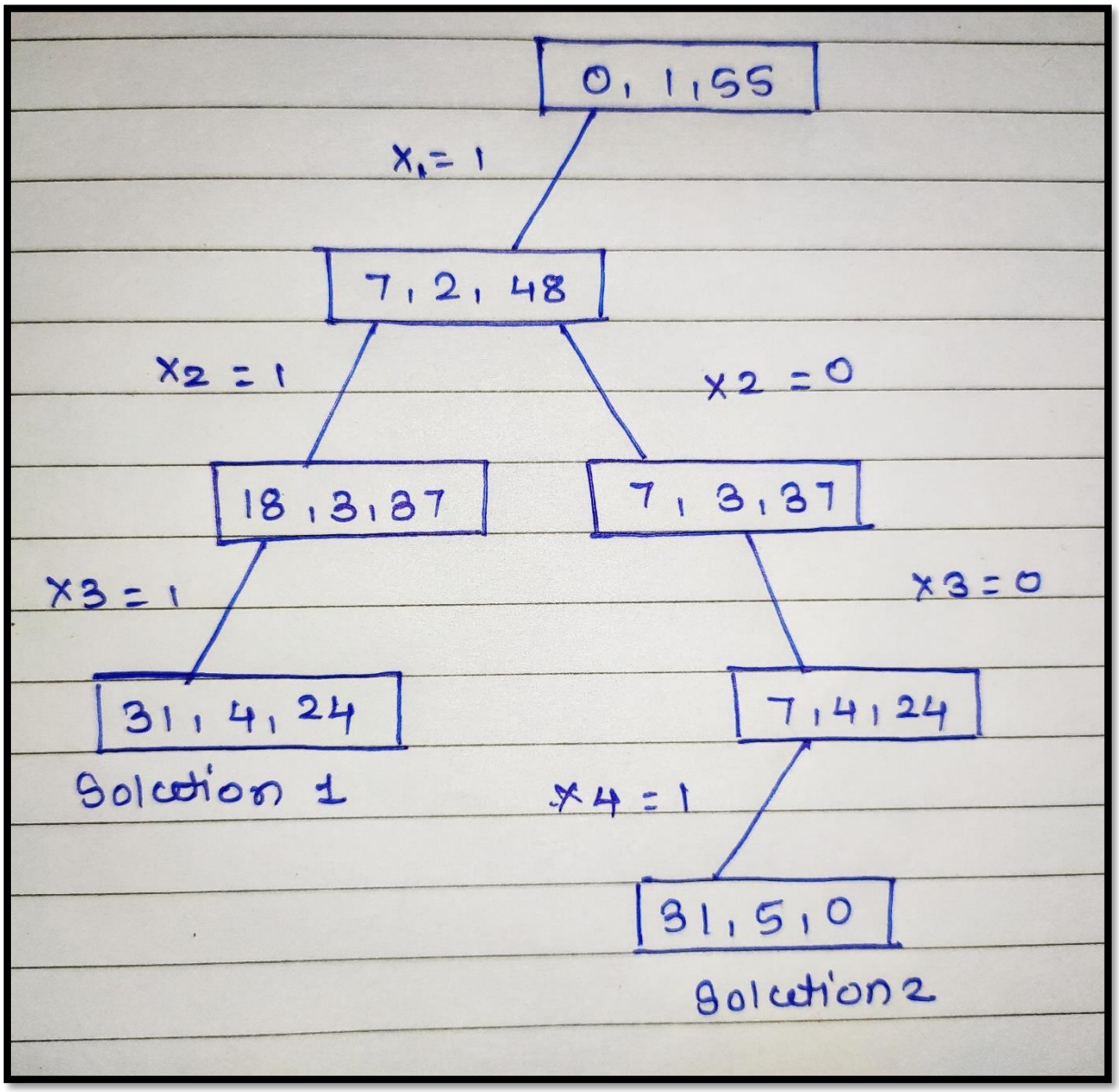
Right State space tree:



Left State space tree:

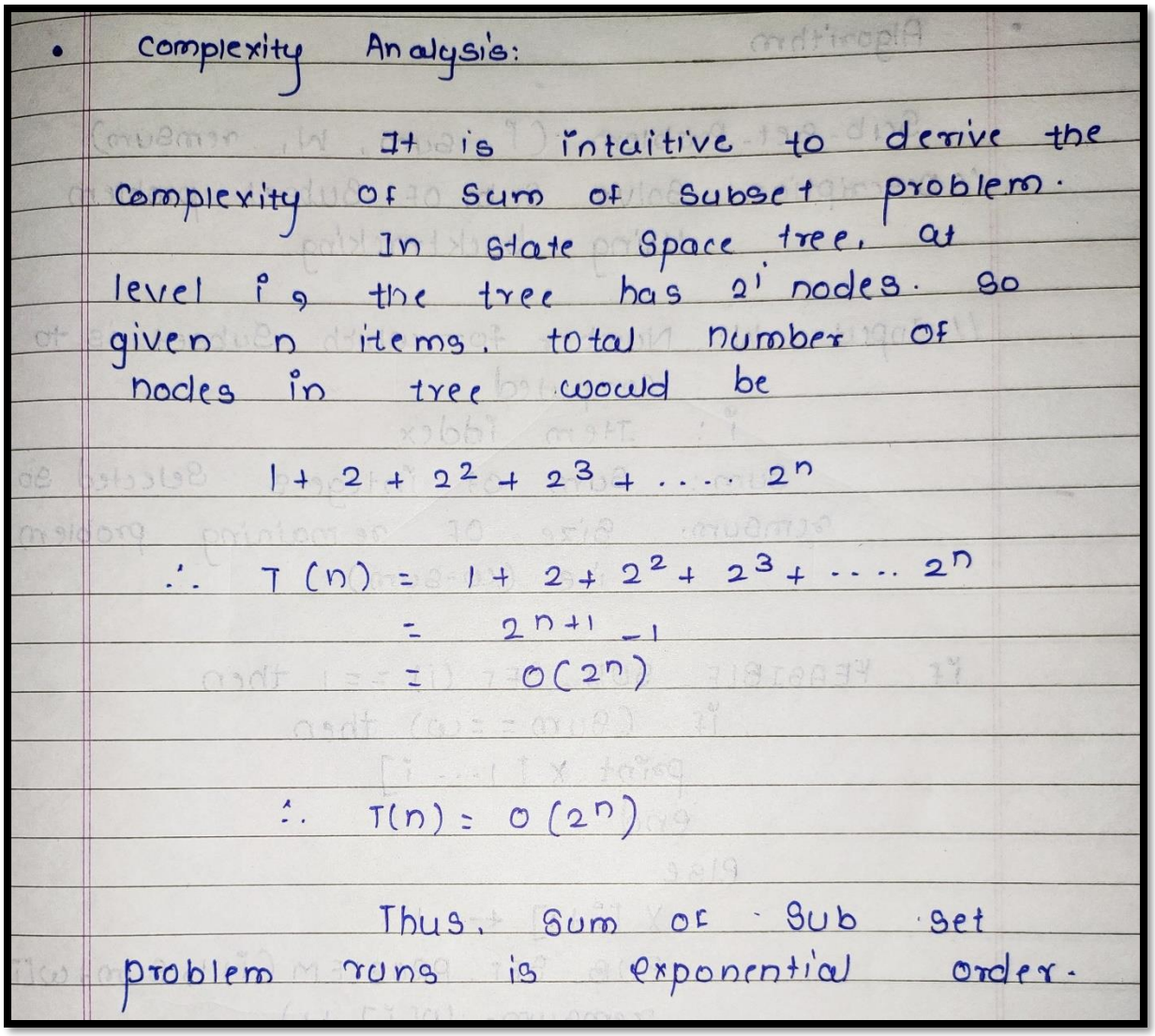


Corresponding tree:



Complexity Analysis:

The time complexity of Sum of Subset problem is $O(2^n)$.



- Complexity Analysis:

It is intuitive to derive the complexity of sum of subset problem. In state space tree, at level i, the tree has 2^i nodes. So given n items, total number of nodes in tree would be

$$1 + 2 + 2^2 + 2^3 + \dots + 2^n$$

$$\therefore T(n) = 1 + 2 + 2^2 + 2^3 + \dots + 2^n$$
$$= 2^{n+1} - 1$$
$$= O(2^n)$$

$$\therefore T(n) = O(2^n)$$

Thus, Sum of Sub set problem runs is exponential order.

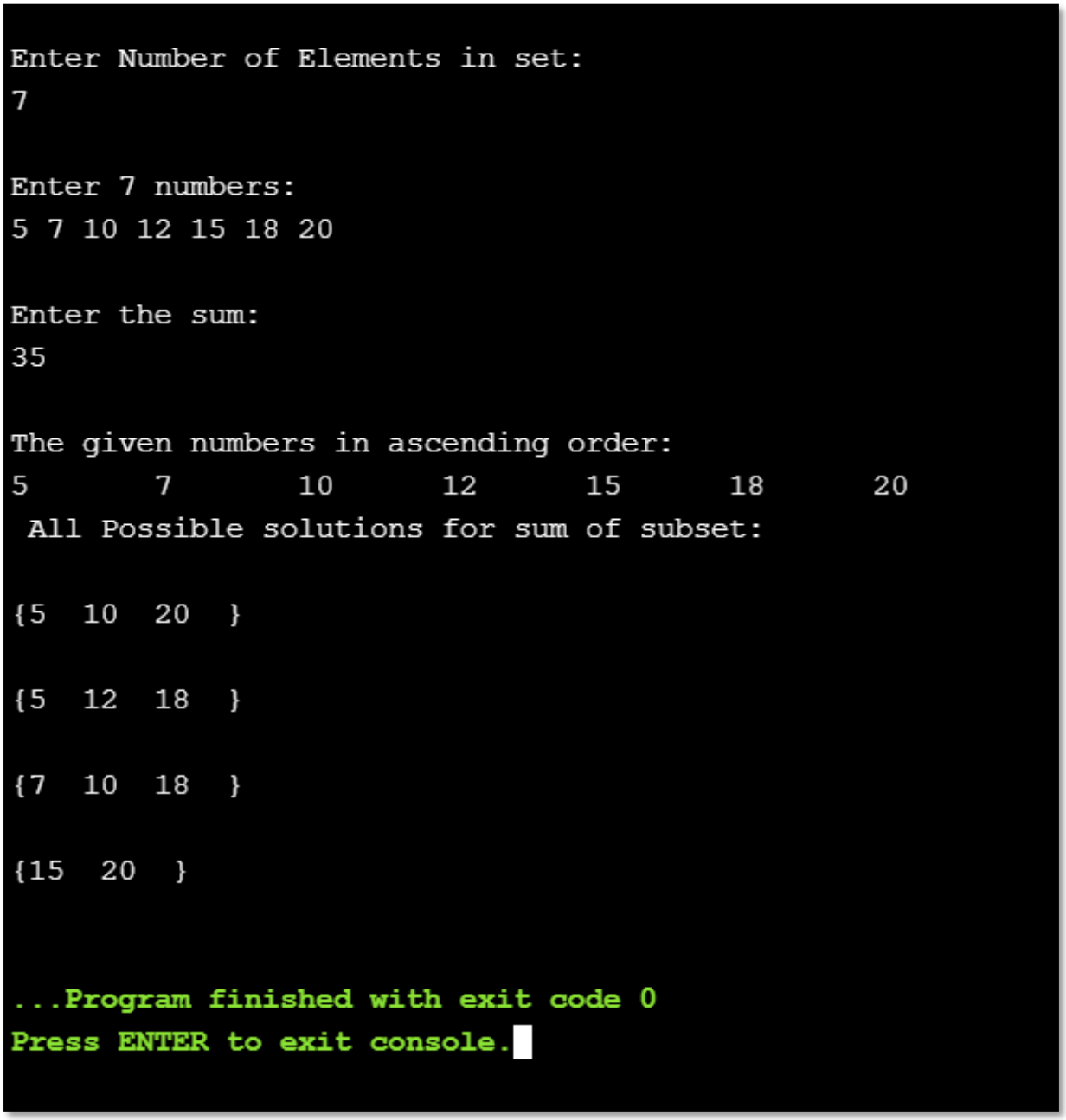
Code:

```
#include<stdio.h>
   #define TRUE 1
   #define FALSE 0
   int inc[100],w[100],sum,n;
   int feasible_sub_set(int i,int wt,int total) {
         return(((wt+total)>=sum)&&((wt==sum)||(wt+w[i+1]<=sum)));
   }

   void subsetproblem(int i,int wt,int total) {
         int j;
         if(feasible_sub_set(i,wt,total)) {
               if(wt==sum) {
                     printf("\n{");
                     for (j=0;j<=i;j++)
                        if(inc[j])
                         printf("%d  ",w[j]);
                     printf("}\n");
               } else {
                     inc[i+1]=TRUE;
                     subsetproblem(i+1,wt+w[i+1],total-w[i+1]);
                     inc[i+1]=FALSE;
                     subsetproblem(i+1,wt,total-w[i+1]);
               }
         }
   }

   int main() {
         int i,j,n,temp,total=0;
         printf("\nEnter Number of Elements in set:\n");
         scanf("%d",&n);
```

```
        printf("\nEnter %d numbers:\n",n);
        for (i=0;i<n;i++) {
                scanf("%d",&w[i]);
                total+=w[i];
        }
        printf("\nEnter the sum:\n");
        scanf("%d",&sum);
        for (i=0;i<=n;i++)
          for (j=0;j<n-1;j++)
           if(w[j]>w[j+1]) {
                temp=w[j];
                w[j]=w[j+1];
                w[j+1]=temp;
        }
        printf("\nThe given numbers in ascending order:\n");
        for (i=0;i<n;i++)
          printf("%d \t",w[i]);
        if((total<sum))
          printf("\nSubset are not possible");
        else {
                for (i=0;i<n;i++)
                   inc[i]=0;
                printf("\n All Possible solutions for sum of subset:\n");
                subsetproblem(-1,0,total);
        }
    return 0;
}
```

Output:

```
Enter Number of Elements in set:
4

Enter 4 numbers:
7 11 13 24

Enter the sum:
31

The given numbers in ascending order:
7       11      13      24
 All Possible solutions for sum of subset:

{7  11  13  }

{7  24  }


...Program finished with exit code 0
Press ENTER to exit console.
```



Conclusion:

Backtracking is an algorithm for capturing some or all solutions to given computational issues, especially for constraint satisfaction issues. The algorithm can only be used for problems which can accept the concept of a "partial candidate solution" and allows a quick test to see if the candidate solution can be a complete solution. Backtracking is considered an important technique to solve constraint satisfaction issues and puzzles. It is also considered a great technique for parsing and also forms the basis of many logic programming languages. We solve sum of subset by using backtracking algorithm.