



Statistical Concepts and Market Returns

Measures of Central Tendency

1. Mean
2. Mode
3. Median
4. Harmonic Mean
5. Geometric Mean
6. Quantiles

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Jan	28.6	39.0	-7.7	28.7	2.2	33.9	2.9	0.2	1.8	30.3	-12.9	-8.2	38.0	6.4	-15.1	-4.8	-13.7	-17.3	20.5	-6.1	-4.1	35.3	11.8	19.3	-16.2	-19.3	-9.7
Feb	4.5	30.0	10.0	-13.4	9.8	2.2	28.5	-8.7	20.7	14.2	29.3	2.5	-7.8	-1.6	12.2	-16.4	32.0	-19.0	14.7	-8.9	2.3	32.7	-10.4	35.6	-13.0	30.0	6.5
Mar	20.4	-6.1	-21.4	-19.3	0.6	23.1	33.3	18.5	-1.8	-0.7	18.5	21.1	30.6	34.3	-2.2	21.6	17.6	-19.0	35.6	24.4	27.4	21.0	7.6	-20.2	22.1	6.7	-16.1
Apr	-6.4	16.6	21.0	-16.1	23.3	31.6	-14.0	2.5	12.9	32.8	8.6	4.7	8.9	12.3	-17.8	-9.9	-18.5	-19.0	-14.3	30.0	-20.3	1.7	32.8	-5.0	-15.9	21.2	-11.1
May	30.4	26.8	-8.7	6.4	16.9	36.2	11.9	-1.5	-19.2	38.4	36.8	6.8	32.7	6.8	32.0	-14.0	17.3	-11.9	26.0	27.1	21.6	12.6	1.0	-21.5	6.6	22.2	-1.3
Jun	0.0	32.5	-3.2	36.3	-8.3	1.8	30.1	34.6	27.9	-15.8	38.9	14.7	13.7	25.9	31.5	32.0	-8.0	8.1	-8.4	36.6	2.1	-1.3	31.8	14.7	-14.6	-2.5	-21.1
Jul	0.2	3.7	-6.3	-16.8	-15.1	-11.7	0.9	-20.7	18.6	-19.1	-9.5	-6.9	-20.4	-14.8	29.6	-5.9	34.4	8.3	-13.8	4.7	15.5	35.1	37.2	24.9	9.9	21.8	-19.5
Aug	35.7	36.1	3.0	23.9	14.7	21.1	36.7	37.0	-14.4	2.9	8.6	29.8	-8.8	-5.6	18.2	20.1	-1.1	-0.9	19.3	34.8	8.1	26.8	34.8	15.0	-7.7	-20.2	-7.7
Sep	39.0	27.5	-1.5	-21.0	12.9	25.1	5.5	-14.7	-18.0	37.3	7.2	1.5	-8.0	19.8	-21.8	28.0	29.1	-12.8	6.0	-20.0	8.1	-17.0	32.4	18.0	6.8	-5.4	26.5
Oct	-5.9	-20.3	2.3	-2.1	39.0	3.2	6.2	-5.2	0.7	-14.0	-22.0	11.2	30.8	15.8	11.2	13.6	17.8	21.0	-16.3	34.2	-11.0	5.3	36.1	8.8	25.4	33.1	-14.1
Nov	-1.9	-1.3	-21.7	-20.9	-1.2	-15.2	28.1	28.4	-16.5	-8.5	30.5	1.5	3.6	36.1	-21.9	-18.8	27.0	2.4	26.6	27.2	24.6	-7.3	-11.4	-7.9	28.6	-1.9	-17.6
Dec	34.4	38.5	34.2	-19.9	-12.6	-7.1	37.1	-7.1	21.9	30.9	28.2	28.9	-7.9	-0.3	2.3	-9.3	14.2	3.7	38.6	6.3	3.0	15.4	-3.9	2.2	-23.8	14.9	-17.6

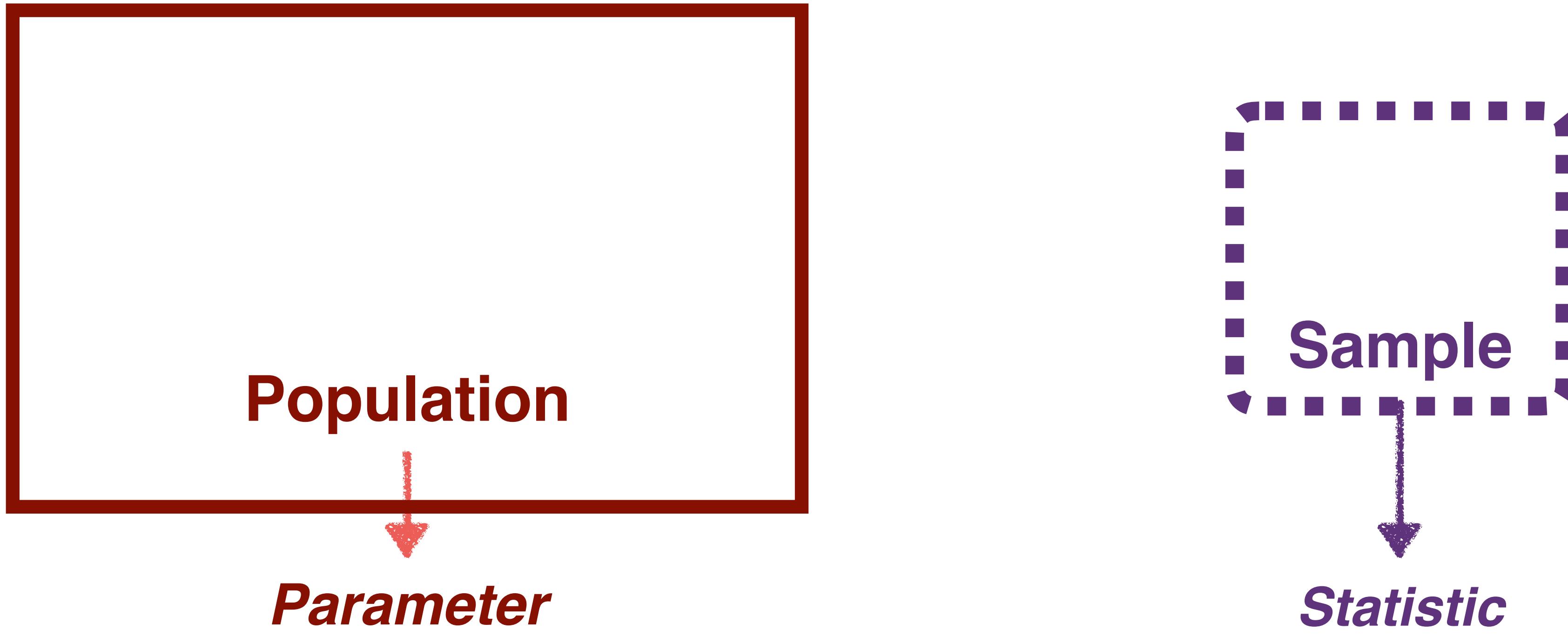
Sample

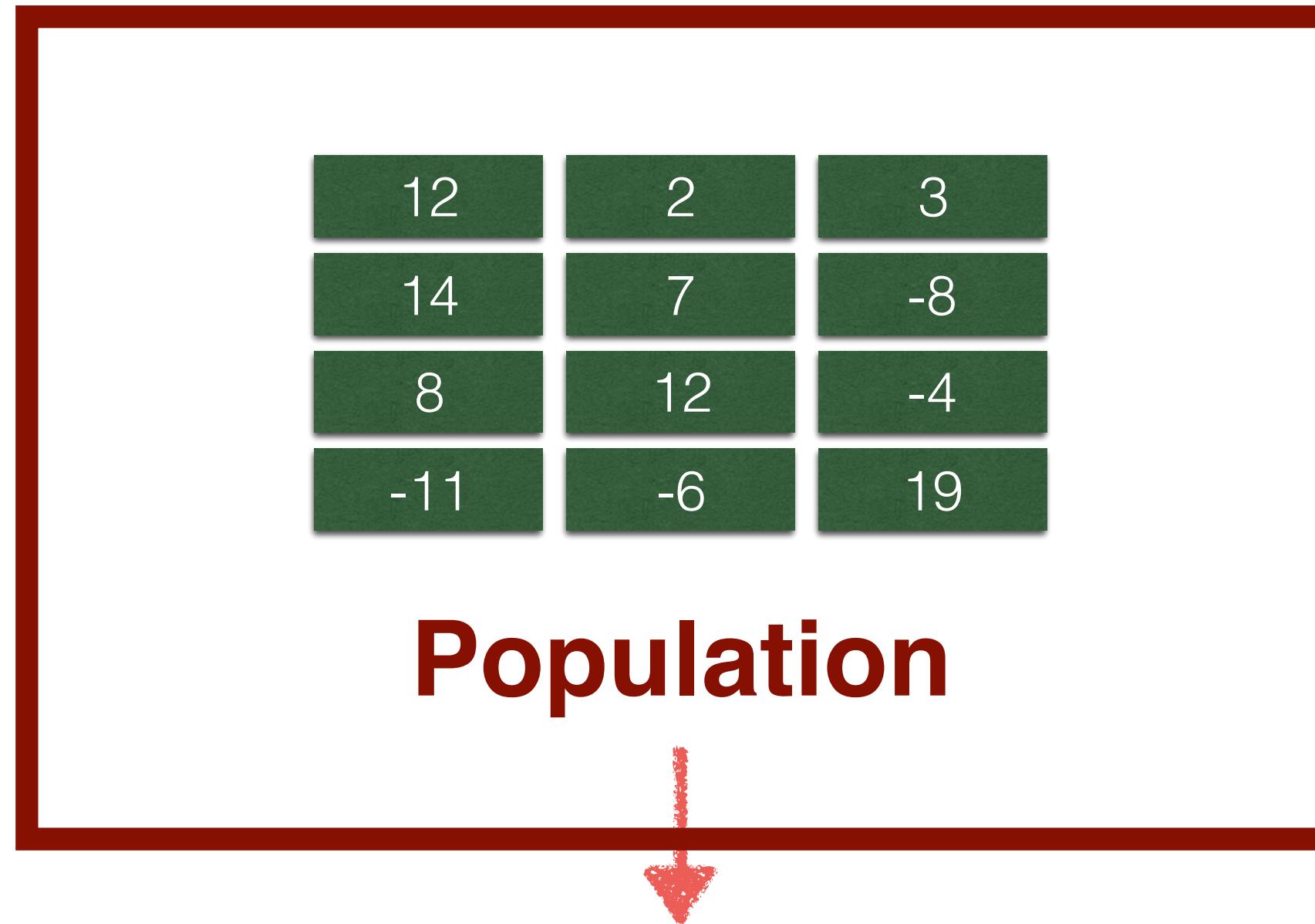
- subset of the population

Statistic

Population - set of ALL possible members of a group of interest

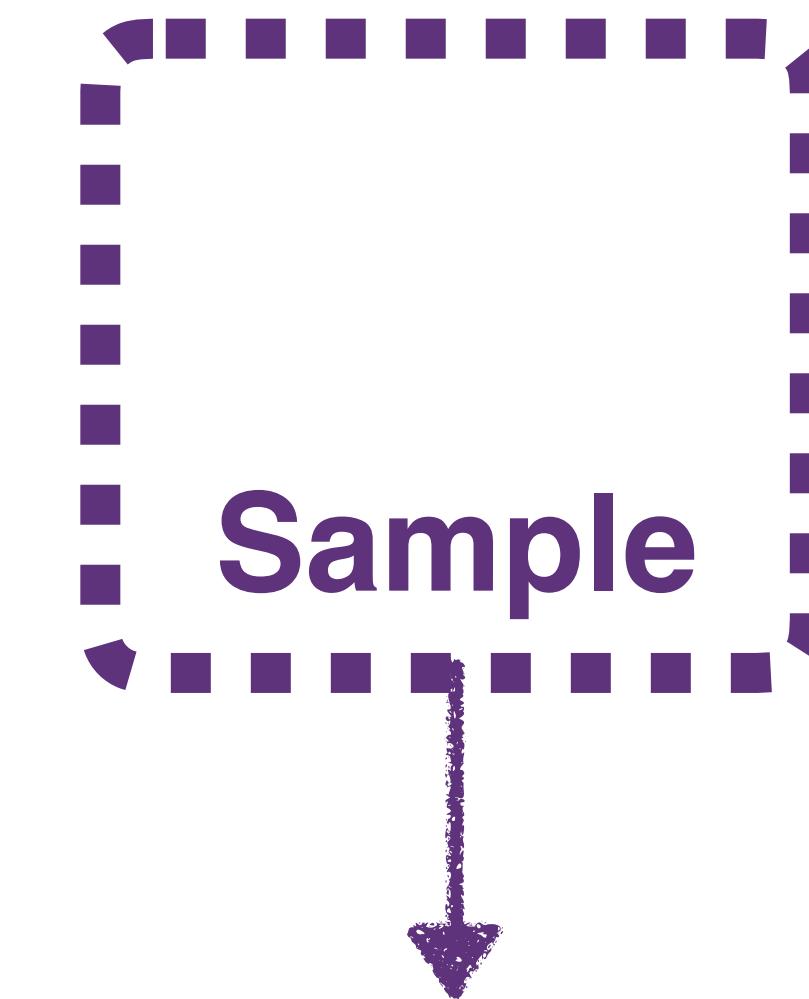
Parameter - measure used to describe a characteristic of the population





Population Mean

$$\mu = \frac{\sum X}{N}$$



Sample Mean

$$\bar{X} = \frac{\sum X}{n}$$

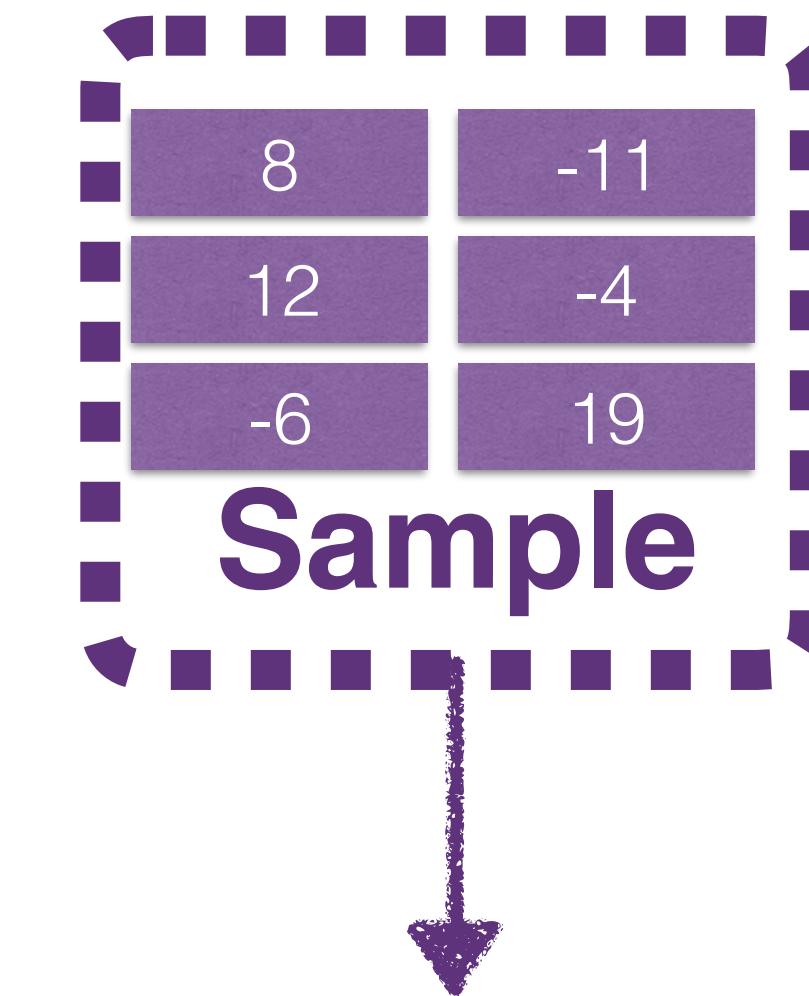
12	2	3
14	7	-8
8	12	-4
-11	-6	19

Population



Population Mean

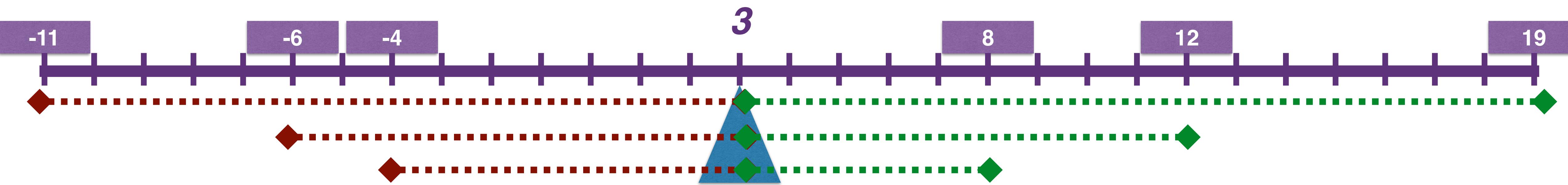
$$\mu = \frac{48}{12} = 4$$



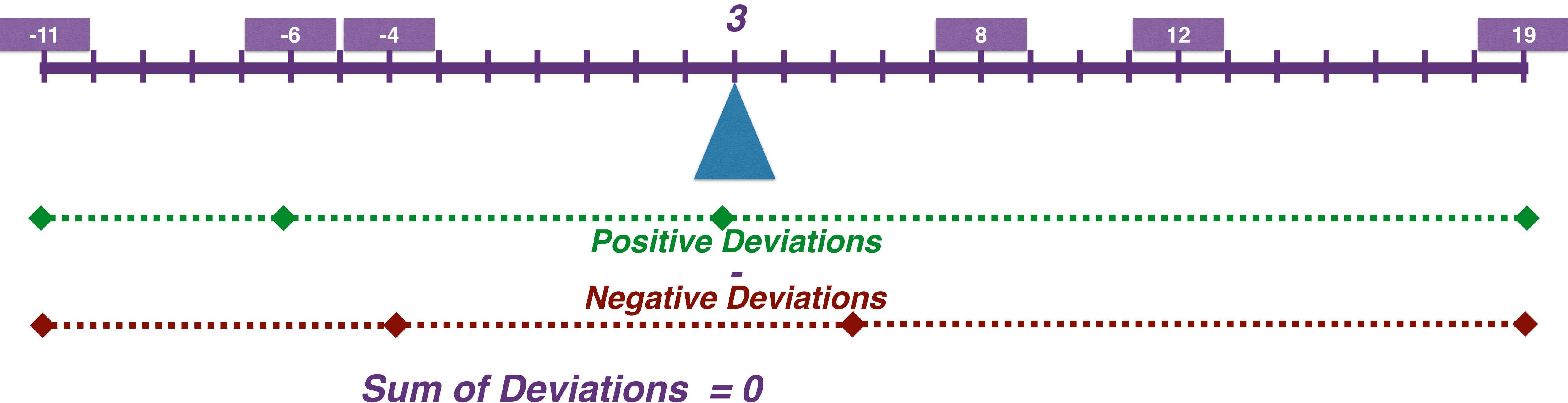
Sample Mean

$$\bar{x} = \frac{18}{6} = 3$$

Arithmetic mean



Arithmetic mean

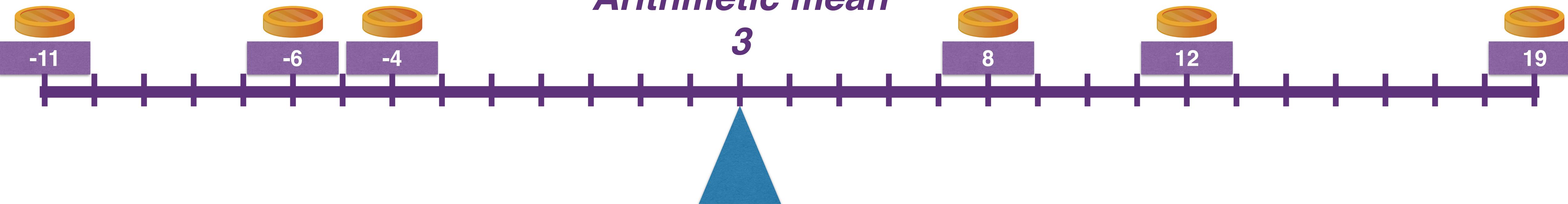


	% allocation	1yr Performance
US Stocks	30	+12%
EU Stocks	10	-6%
Asian Stocks	10	+19%
EM Stocks	20	+8%
Treasuries	20	-4%
HY Bonds	10	-11%

Not even!

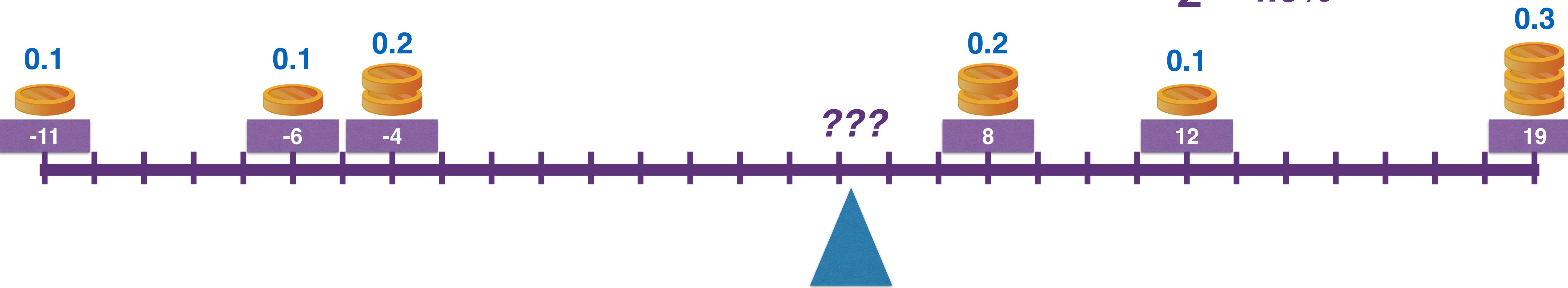
Arithmetic mean

3



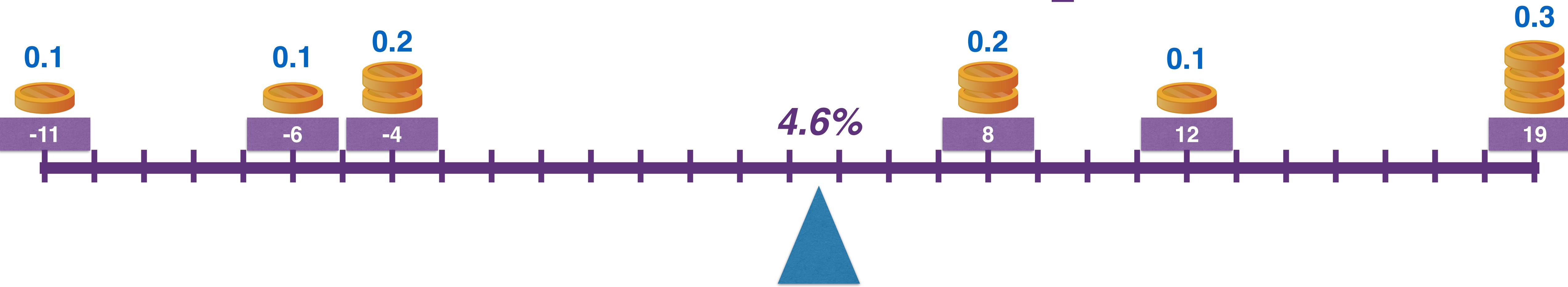
	% allocation	1yr Performance
US Stocks	30	+12%
EU Stocks	10	-6%
Asian Stocks	10	+19%
EM Stocks	20	+8%
Treasuries	20	-4%
HY Bonds	10	-11%

$$\Sigma = 4.6\%$$



	% allocation	1yr Performance	Contribution
US Stocks	30	+12%	+3.6%
EU Stocks	10	-6%	-0.6%
Asian Stocks	10	+19%	+1.9%
EM Stocks	20	+8%	+1.6%
Treasuries	20	-4%	-0.8%
HY Bonds	10	-11%	-1.1%

$$\Sigma = 4.6\%$$



Weighted Mean

Measures of Central Tendency

1. Mean
2. Mode
3. Median
4. Harmonic Mean
5. Geometric Mean
6. Quantiles

Most frequently occurring observation in the series

Mode

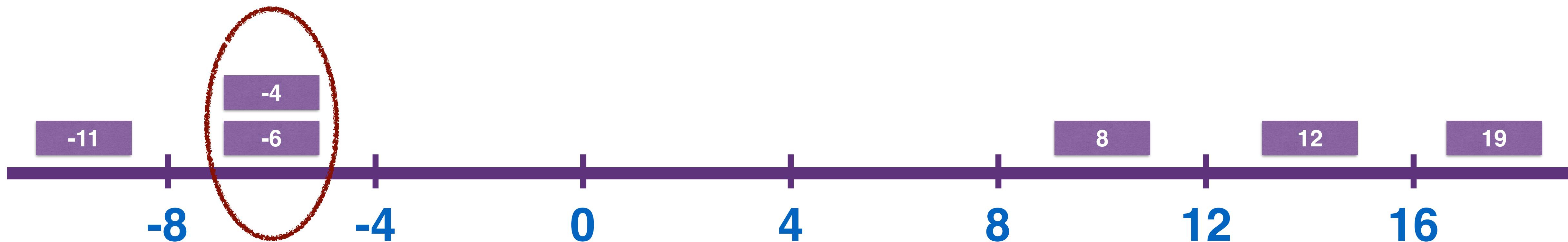
= {-11, -6, -4, 8, 12, 19}
Not very useful!



Modal Interval
 $\{-8 \leq X < -4\}$

Mode

Most frequently occurring observation in the series



Not skewed by outliers!

Middle element in an ordered series

-11

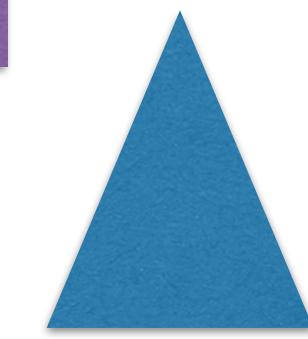
-6

-4

8

12

19



Median

If even number of elements, sum up the 2 middle elements and divide by 2.

$$\frac{8 + (-4)}{2} = 2$$

Harmonic mean

$$\frac{N}{\sum_{i=1}^N \frac{1}{X_i}}$$

Calculate the **average cost** of shares purchased over a period of time

6

$$\frac{1}{-11} + \frac{1}{-6} + \frac{1}{-4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{19}$$

Julian participates in a regular investment plan where he places \$100 every month to buy into a mutual fund. Over the last 3 months, the price paid per unit was \$6, \$8 and \$10.

- (a) What is the total number of units acquired?
- (b) What is the average price paid per unit?

(a) $100/6 + 100/8 + 100/10 = 39.17$ units

(b) $\$300 / 39.17 = \7.66

Shortcut - Harmonic mean

$$\frac{3}{\frac{1}{6} + \frac{1}{8} + \frac{1}{10}} = \$7.66$$

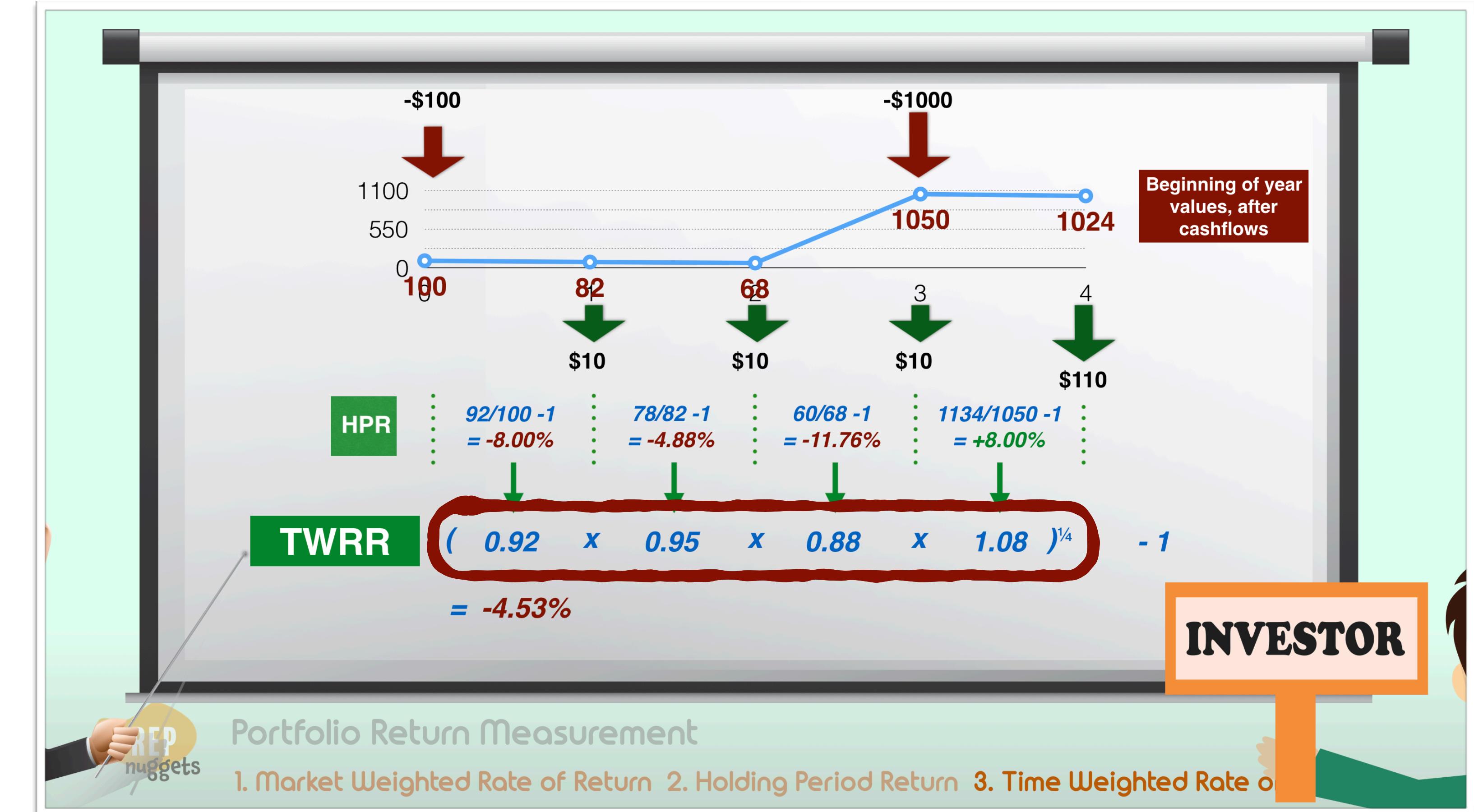
Lower than arithmetic
mean of \$8!!

Dollar cost averaging!

Geometric mean

$$(X_1 \times X_2 \times \dots \times X_n)^{1/n}$$

Calculate **returns** over multiple periods



**Given the following scenarios,
determine the most appropriate
measure for each of them and use it
to solve each problem.**

	Alloc	Return
Cash	20%	+1%
Stocks	50%	+17%
Bonds	20%	-3%
Gold	10%	-23%

Table shows Park's portfolio returns for the past year. What is the total portfolio return for the year?

Weighted mean

	Style	Num
	Value	3
	Value/Growth	5
	Growth	12

Table shows the breakdown of styles of all the funds in a trading house. What is the most representative style of the fund house?

Mode

John bought into a mutual fund when the price per unit was \$4.10, \$4.60 and \$4.75. He placed \$2000 each time. What is the average price paid per unit?

Harmonic mean

The holding period returns of a fund for the past 3 years are 10%, -6% and -9%. Determine an appropriate annualised rate of return for the past 3 years.

Geometric mean

The holding period returns of a stock are -8%, -17%, 102%, 7%, 19%, 7% and -1% for the past 7 years. What is the measure to use if we do not wish to have outlier figures skew the mean?

Median

John bought into a mutual fund when the price per unit was \$4.10, \$4.60 and \$4.75. He placed \$2000 each time. What is the average price paid per unit?

Amount invested must be the same

Harmonic mean

$$\frac{N}{\sum_{i=1}^N \frac{1}{X_i}}$$

$$\frac{3}{1/4.10 + 1/4.60 + 1/4.75} = \$4.47$$

	Alloc	Return
Cash	20%	+1%
Stocks	50%	+17%
Bonds	20%	-3%
Gold	10%	-23%

Table shows Park's portfolio returns for the past year. What is the total portfolio return for the year?

Weighted mean

	Alloc	Weight w	Return r	Contribution w x r
Cash	20%	0.2	+1%	0.2
Stocks	50%	0.5	+17%	8.5
Bonds	20%	0.2	-3%	-0.6
Gold	10%	0.1	-23%	-2.3
Total		1.0		5.8

Sum of weights must equal 1

ANS

Style	Num
Value	3
Value/Growth	5
Growth	12

Table shows the breakdown of styles of all the funds in a trading house. What is the most representative style of the fund house?

Mode

= Growth

Ratio Scale

Interval Scale

Ordinal Scale

Nominal Scale

Mode is the only measure that can be used with nominal data

The holding period returns of a stock are -8%, -17%, 102%, 7%, 19%, 7% and -1% for the past 7 years. What is the measure to use if we do not wish to have outlier figures skew the measurement?

Median

Arranged from low to high, the fourth observation here is the median

-17, -8, -1, **7**, 7, 19, 102

The holding period returns of a fund for the past 3 years are 10%, -6% and -9%. Determine an appropriate annualised rate of return for the past 3 years.

Geometric mean

$$\begin{array}{ccc} \text{1st yr} & \text{2nd yr} & \text{3rd yr} \\ [(1+0.1) \times (1-0.06) \times (1-0.09)]^{1/3} - 1 = 0.98 - 1 \end{array}$$

$$= [1.1 \times 0.94 \times 0.91]^{1/3} - 1$$

$$= 0.98 - 1 = -2.0\%$$

This is TWRR!!!

Weighted mean

Portfolio Returns

	Alloc	Return
Cash	20%	+1%
Stocks	50%	+17%
Bonds	20%	-3%
Gold	10%	-23%

Mode

Ratio Scale

Interval Scale

Ordinal Scale

Nominal Scale

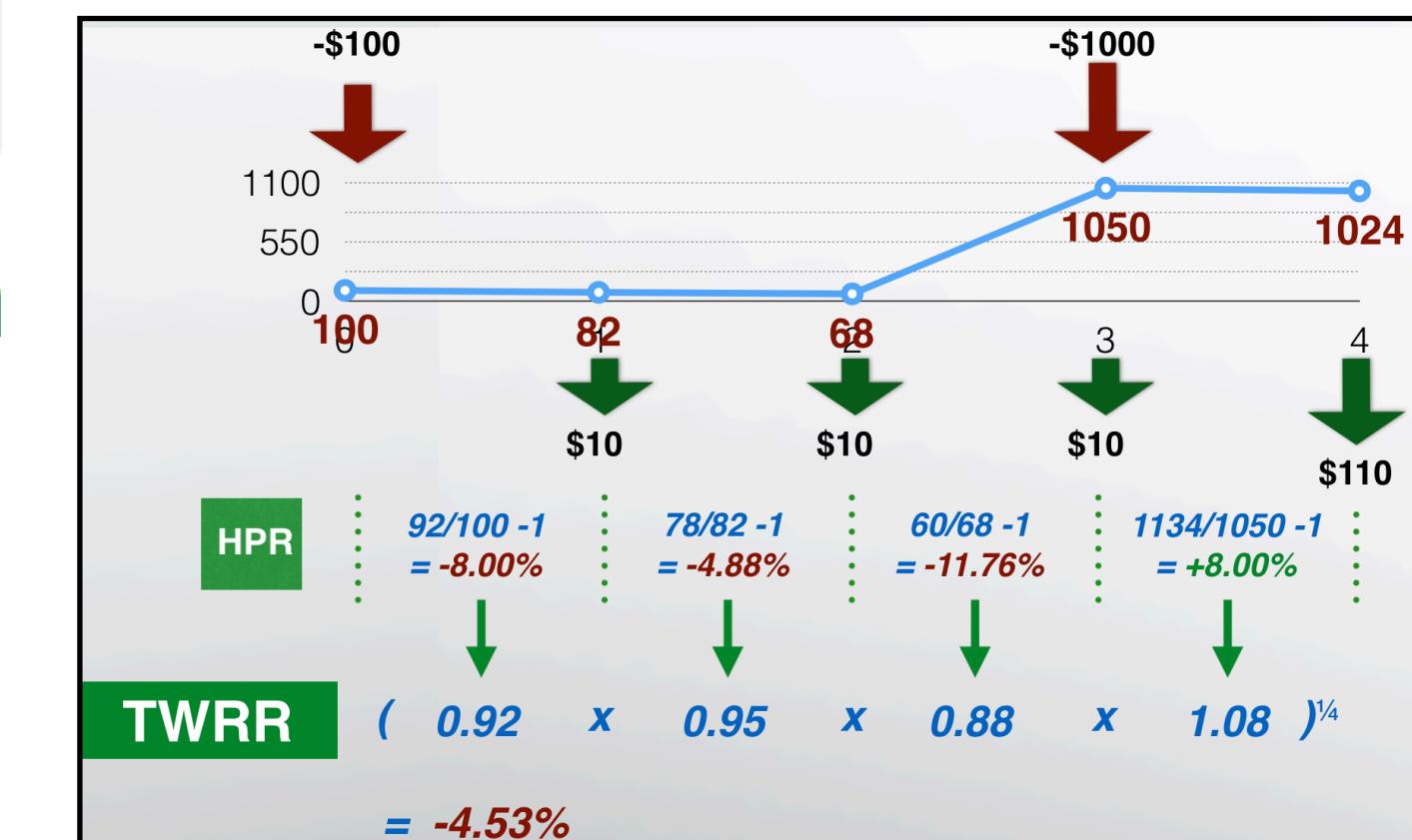
Median

Exclude effect of extreme figures

-17, -8, -1, 7, 7, 19, ~~102~~

Geometric mean

Compute the growth rate of a variable
(e.g. TWRR)



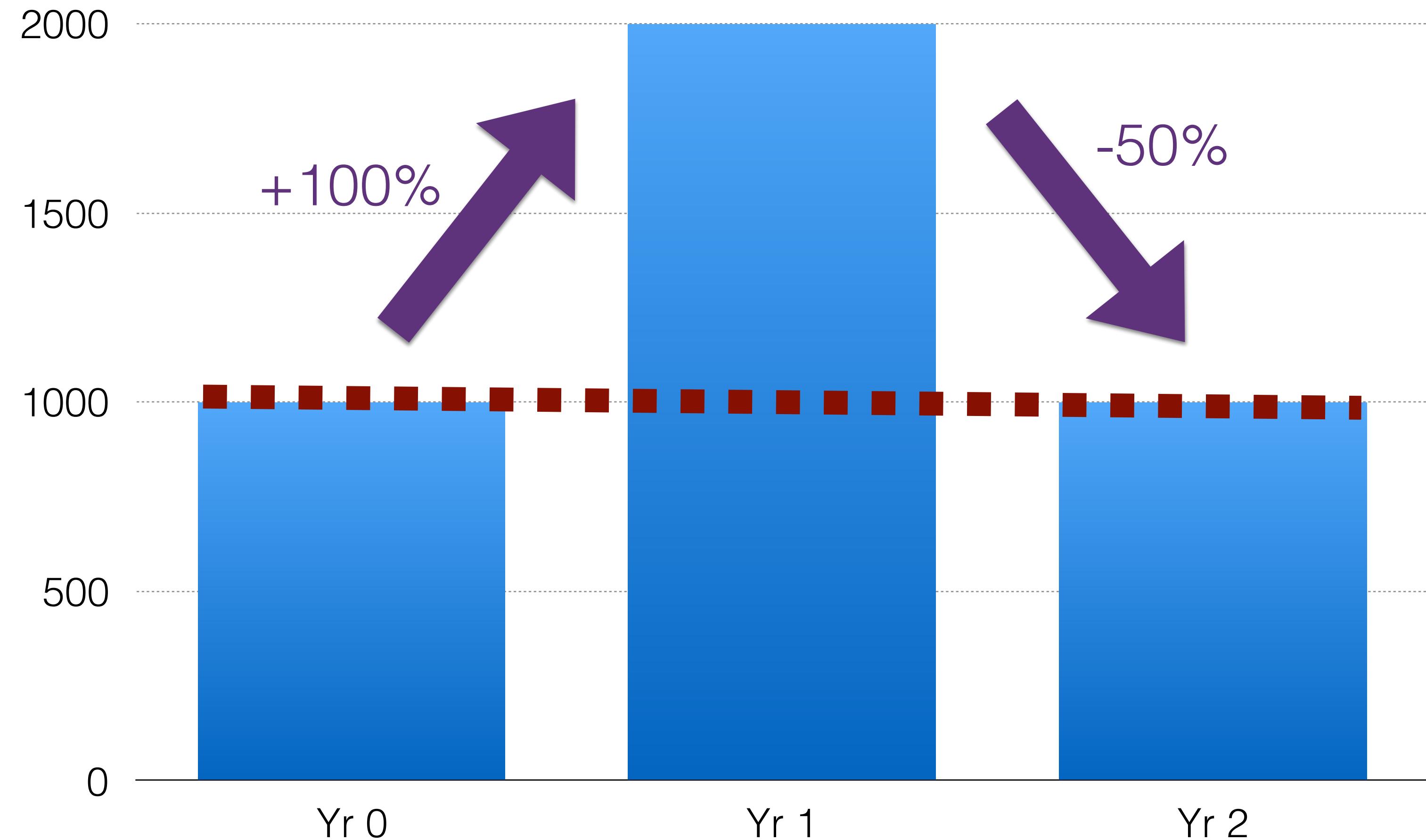
Account Value

Geometric mean

$$= [(1 + 1.0) \times (1 - 0.5)]^{1/2} - 1$$

$$= 0\%$$

Mathematically correct!



Arithmetic mean

$$= (100 - 50)/2$$

$$= 25\%$$

Mathematically incorrect!

Geometric mean

= 0%



I can expect 0%
growth each year in the
future!

Arithmetic mean

= 25%



I can expect
25% growth each
year in the future!

**Which is the better
estimator?**

Geometric mean

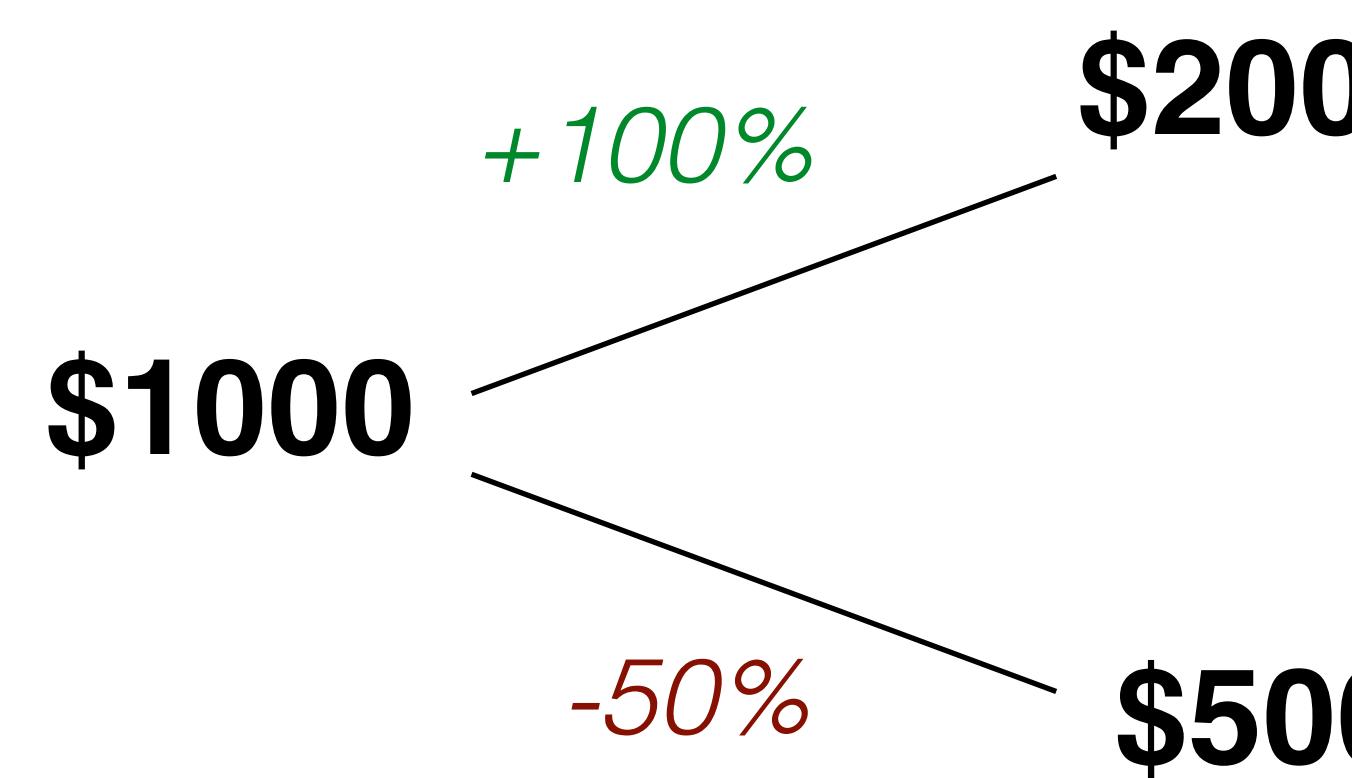
= 0%

Which is the better estimator?

Arithmetic mean

= 25%

Tree Model



Probability

$$\$4000 \times 0.25 = \$1000$$

$$\$1000 \times 0.25 = \$250$$

$$\$1000 \times 0.25 = \$250$$

$$\$250 \times 0.25 = \$62.50$$

$$\$1562.50$$

Measures of Central Tendency

1. Mean
2. Mode
3. Median
4. Harmonic Mean
5. Geometric Mean
6. Quantiles

Geometric mean = 0%

Which is the better estimator?

Arithmetic mean = 25%

Expected future value
 $\$1000 \times 1.0 \times 1.0 = \1000

Expected future value
 $\$1000 \times 1.25 \times 1.25 = \1562.50

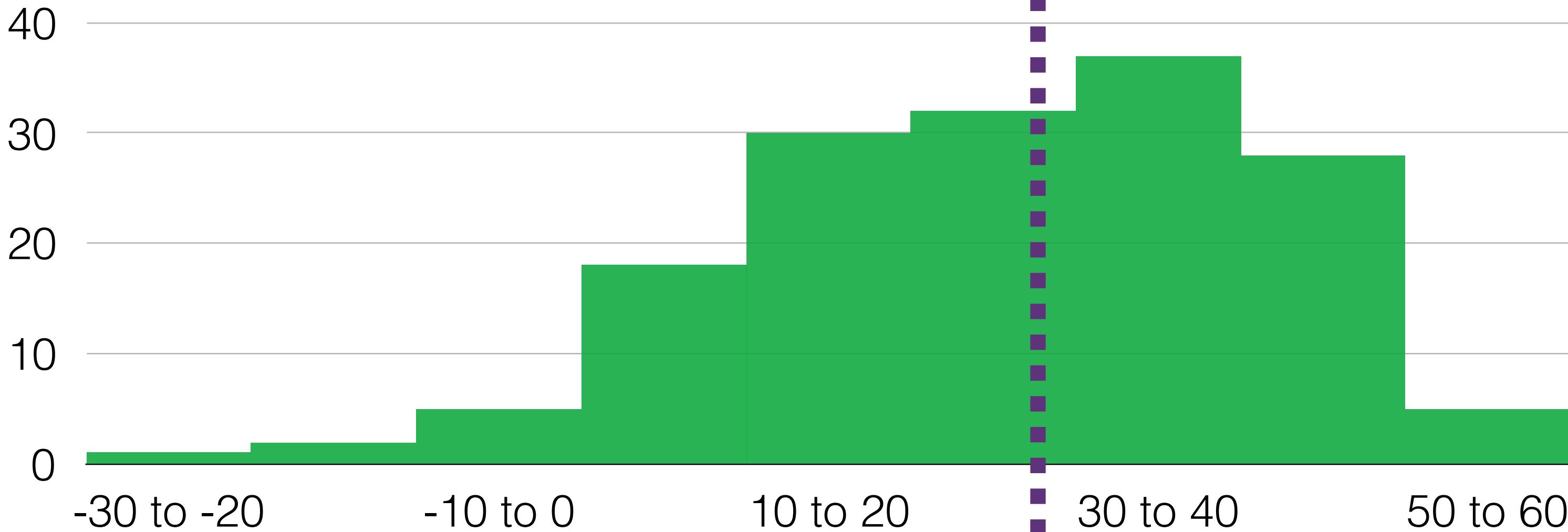
Expected future value
 $\$1562.50$

Geometric mean

**Mathematically correct
calculation of *past* returns**

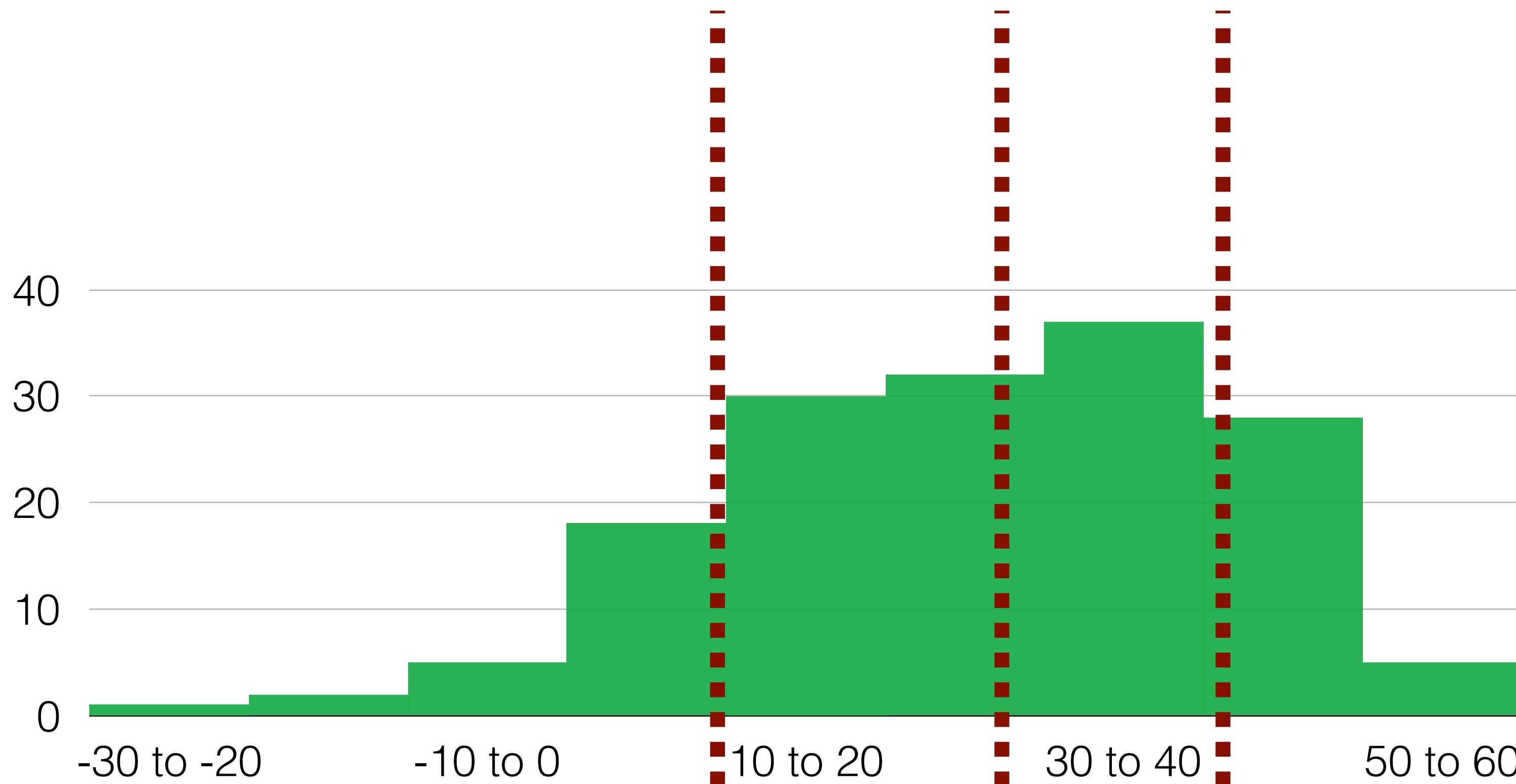
Arithmetic mean

**Better estimator of
future returns**



Median

← 50% → ← 50% →

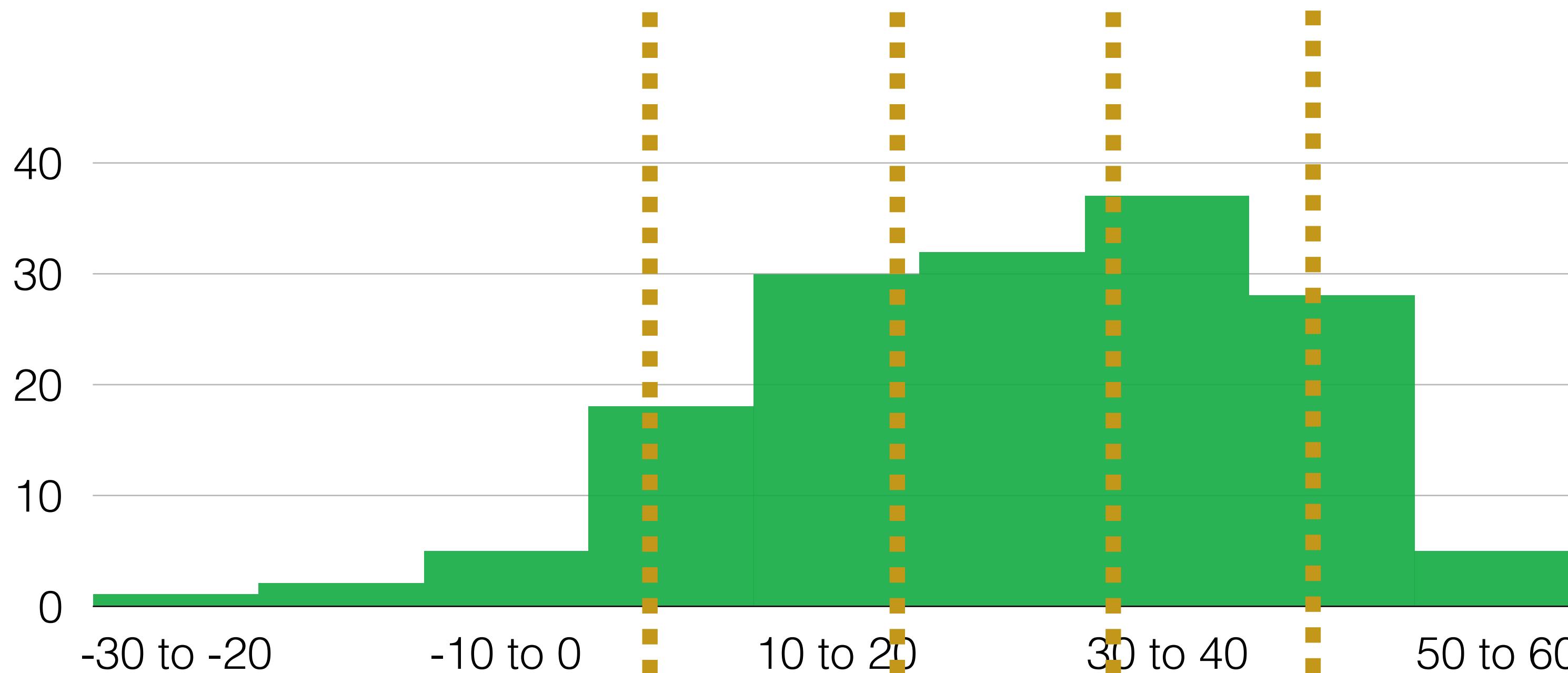


Quartile



Median

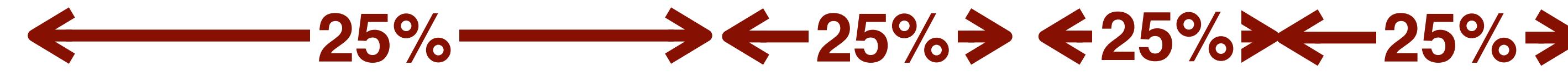




Quintile

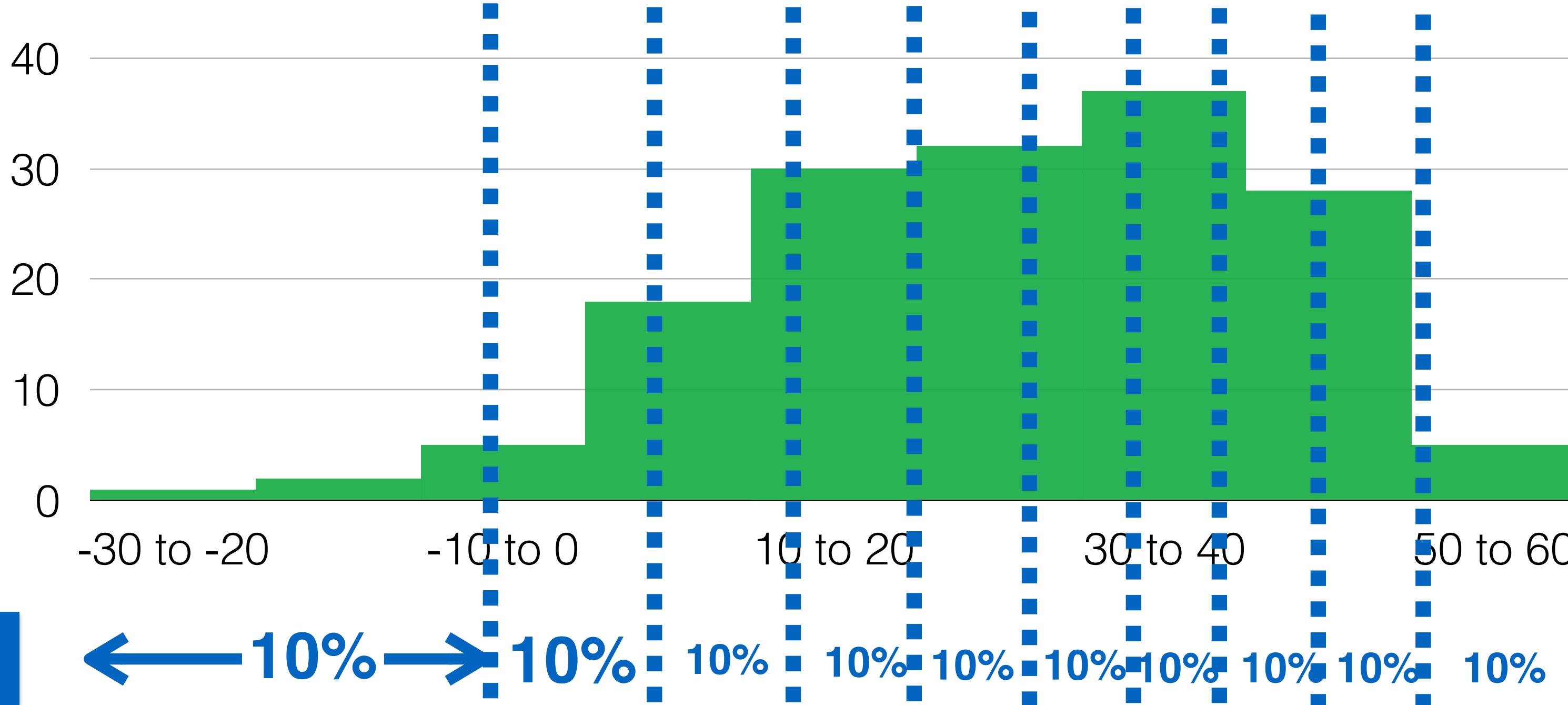


Quartile



Median





Decile

← 10% → 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10% | 10%

Quintile

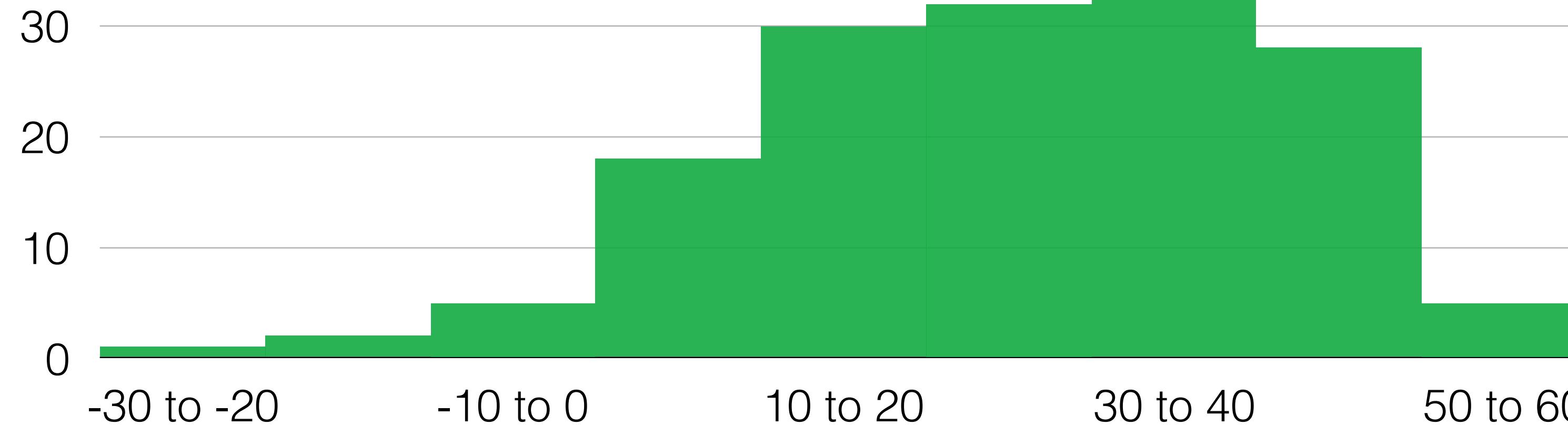
← 20% → < 20% > < 20% > < 20% > < 20% >

Quartile

← 25% → < 25% > < 25% > < 25% >

Median

← 50% → < 50% >



Percentile

... **1%** ...

8th Decile
or
80th Percentile

Decile

10% **10%** **10%** **10%** **10%** **10%** **10%** **10%**

zma Quintile
or
80th Percentile

Quintile

20% **20%** **20%** **20%** **20%**

40th Percentile

Quartile

25% **25%** **25%**

3rd Quartile
or
75th Percentile

Median

50% **50%**

The following are the returns of the 10 stocks listed in Tinyland stock market for the past year. What is the 3rd quartile of the returns?

-12%, -12%, -5%, 0%, 1%, 1%, 8%, 13%, 19%, 54%

$$L_y = (n+1) \times (y/100)$$

L_y : Position

n: number of observations

y: percentile

$$L_y = (10+1) \times (75/100) = 8.25$$

$$\begin{aligned} \text{ANS} &= 13 + (19-13)x0.25 \\ &= 13 + 1.5 \\ &= 14.5\% \end{aligned}$$

The 3rd quartile is 14.5%. This means that 75% of the stock returns were below 14.5%.



prepnuggets.com