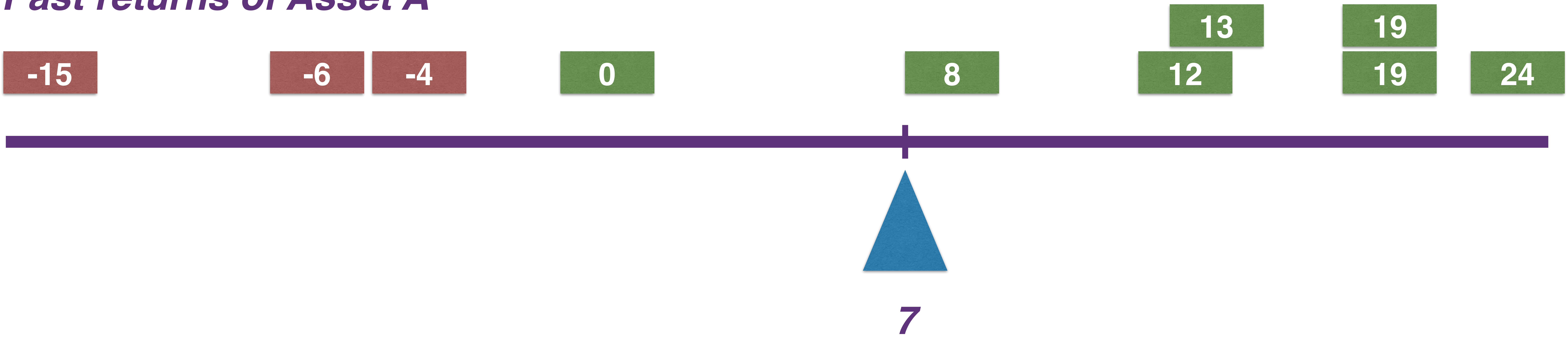
Statistical Concepts and Market Returns

Measures of Dispersion

1. Range and MAD
2. Variance and Std Dev
3. Chebyshev's Inequality
4. COV and Sharpe Ratio

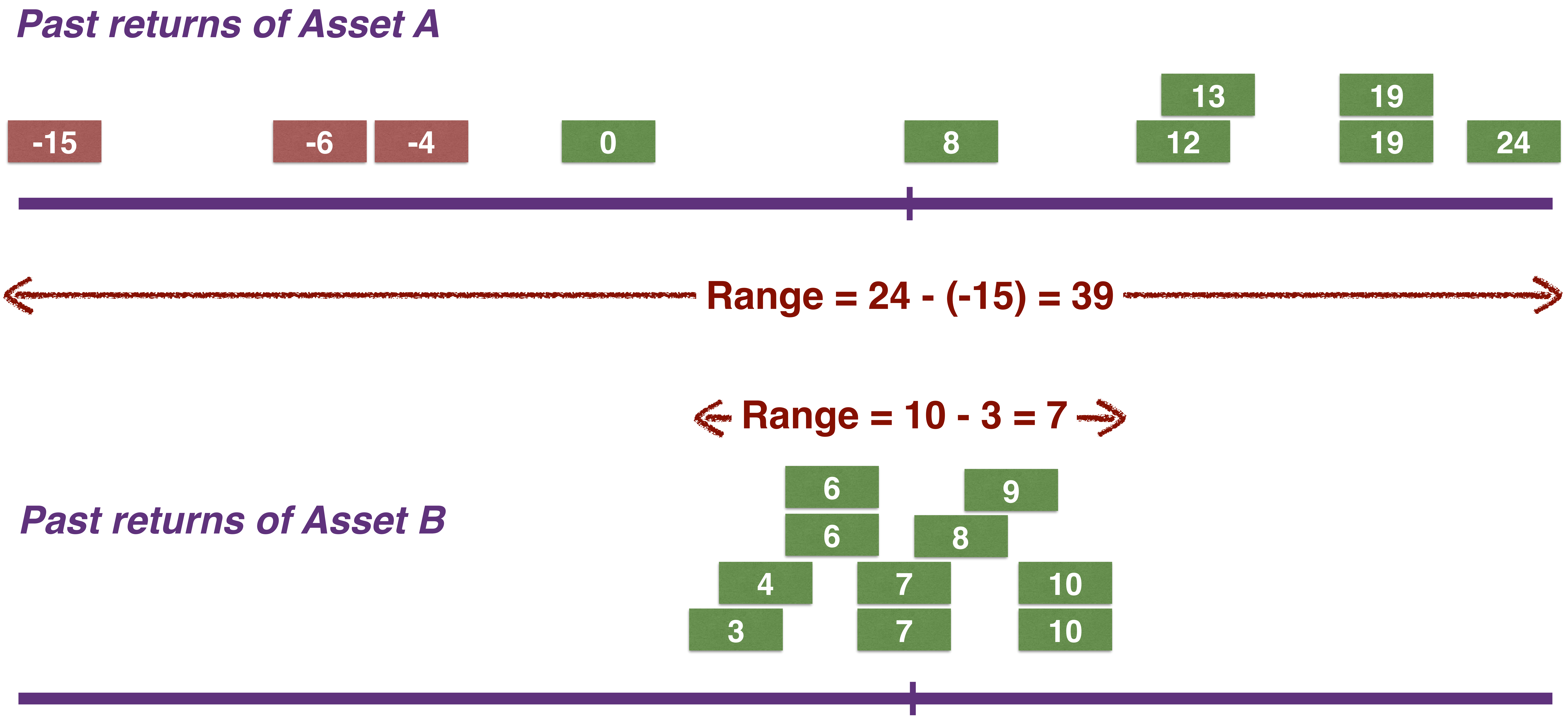
PREP nuggets

Past returns of Asset A



Measures of Dispersion

1. Range and MAD 2. Variance and Std Dev 3. Chebyshev's Inequality 4. COV and Sharpe Ratio

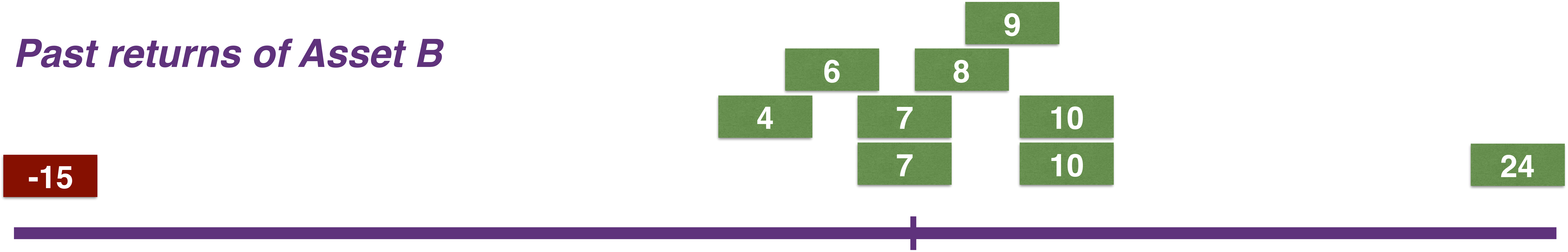

Past returns of Asset A
13
19
-15
-6
-4
0
8
12
19
24
Range = 24 - (-15) = 39
Range = 10 - 3 = 7
6
9
6
8
Past returns of Asset B
4
7
10
3
7
10

Past returns of Asset A

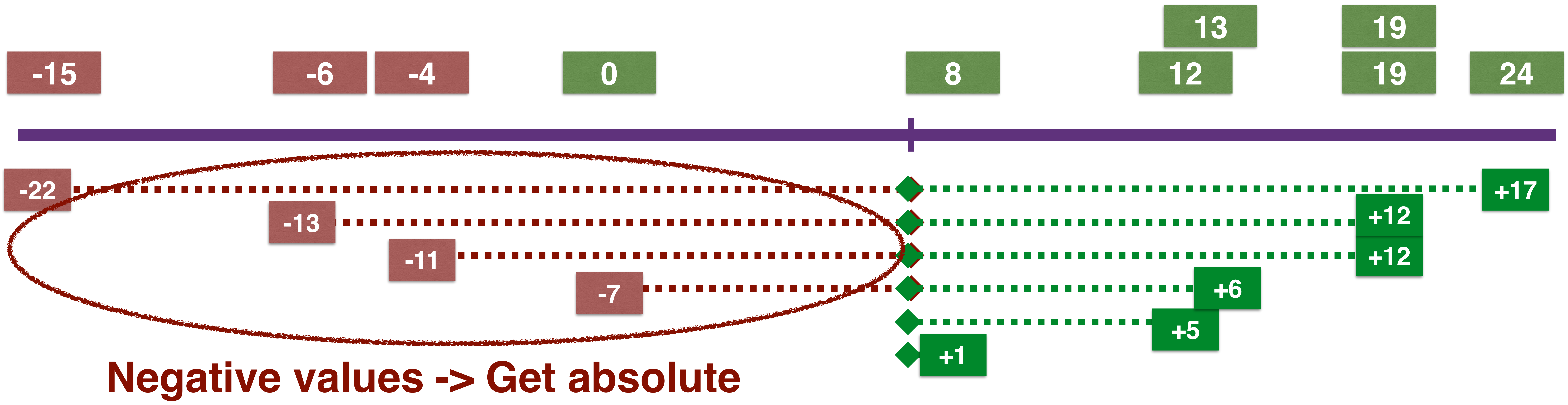
Range = 24 - (-15) = 39

Range = 24 - (-15) = 39

Past returns of Asset B

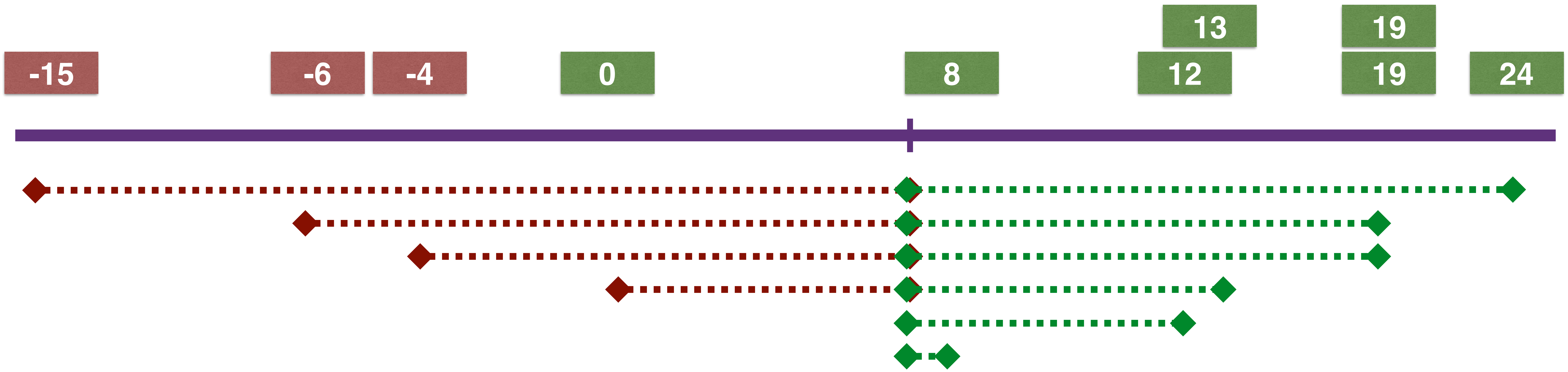


Mean Absolute Deviation

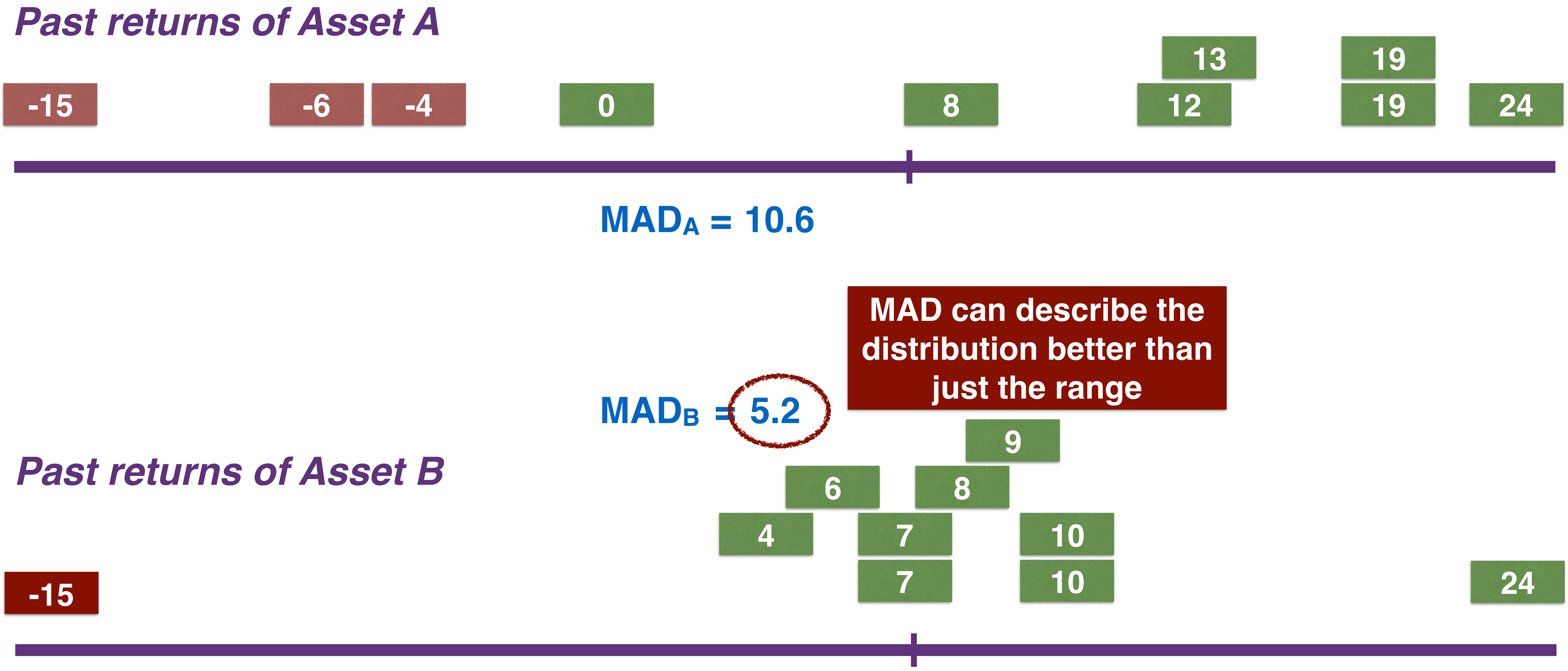


$$MAD = \frac{\sum |X-\bar{X}|}{N}$$

Mean Absolute Deviation

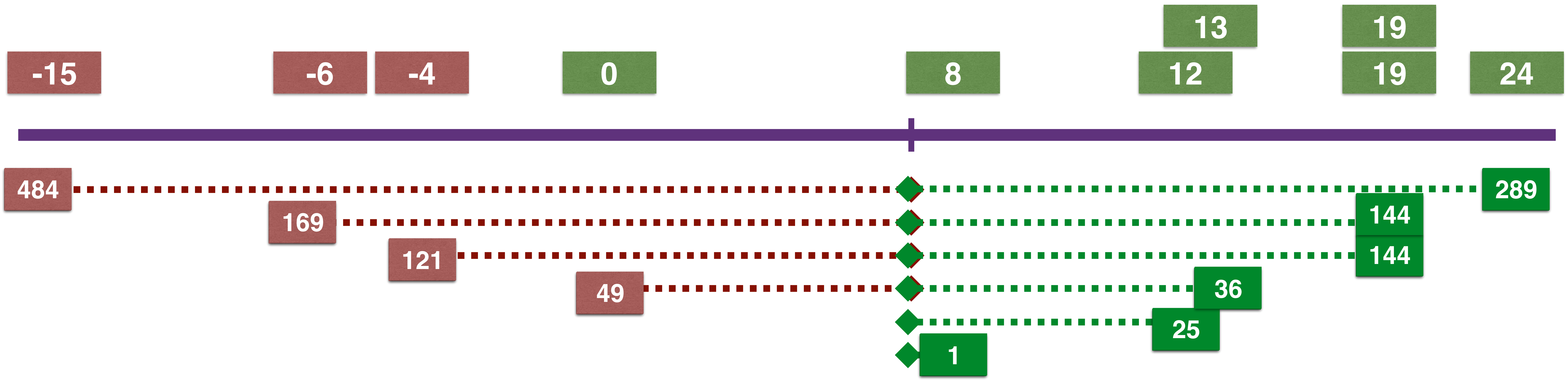


$$\text{MAD} = \frac{\sum |X-\bar{X}|}{N} = \frac{106}{10} = 10.6$$

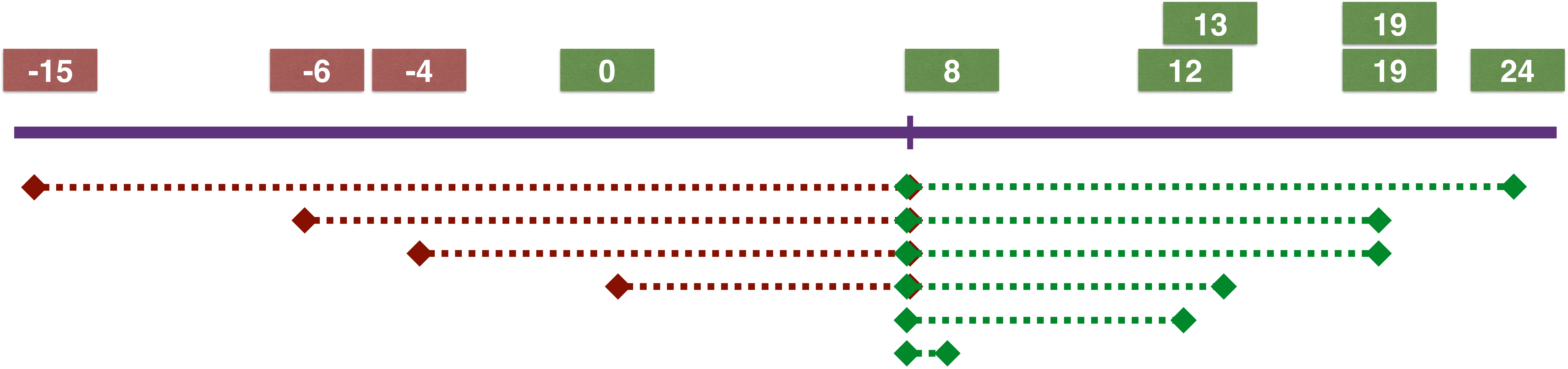


Past returns of Asset A
13
19
-15
-6
-4
0
8
12
19
24
MADA = 10.6
MAD can describe the distribution better than just the range
MADB = 5.2
9
Past returns of Asset B
6
8
4
7
10
-15
7
10
24

$$\text{Variance } \sigma^2 = \frac{\sum(| X-\mu |)^2}{N}$$



$$\text{Variance } \sigma^2 = \frac{\sum(X-\mu)^2}{N}$$



$$\text{Variance } \sigma^2 = \frac{\sum (X-\mu)^2}{N} = \frac{1462}{10} = 146.2$$

$$\text{Variance } \sigma^2 = \sqrt{\frac{\sum(X-\mu)^2}{N}} = 146.2$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum(X-\mu)^2}{N}} = 12.09$$

vs MAD = 10.6

Std Dev* ≥ *MAD

More weight to larger deviations

What if the population is too big or if we don't have all the data?

Population

↓

Descriptive Statistics

↓

Parameter

$$\text{Population Variance } \sigma^2 = \frac{\sum(X-\mu)^2}{N}$$

$$\text{Population Standard deviation } \sigma = \sqrt{\frac{\sum(X-\mu)^2}{N}}$$

What if the population is too big or if we don't have all the data?

Population

Descriptive Statistics

Parameter

Sample

Inferential Statistics

Statistic

Unbiased Estimate

Population Variance $\sigma^2 = \dfrac{\sum(X-\mu)^2}{N}$

Population Standard deviation $\sigma = \sqrt{\dfrac{\sum(X-\mu)^2}{N}}$

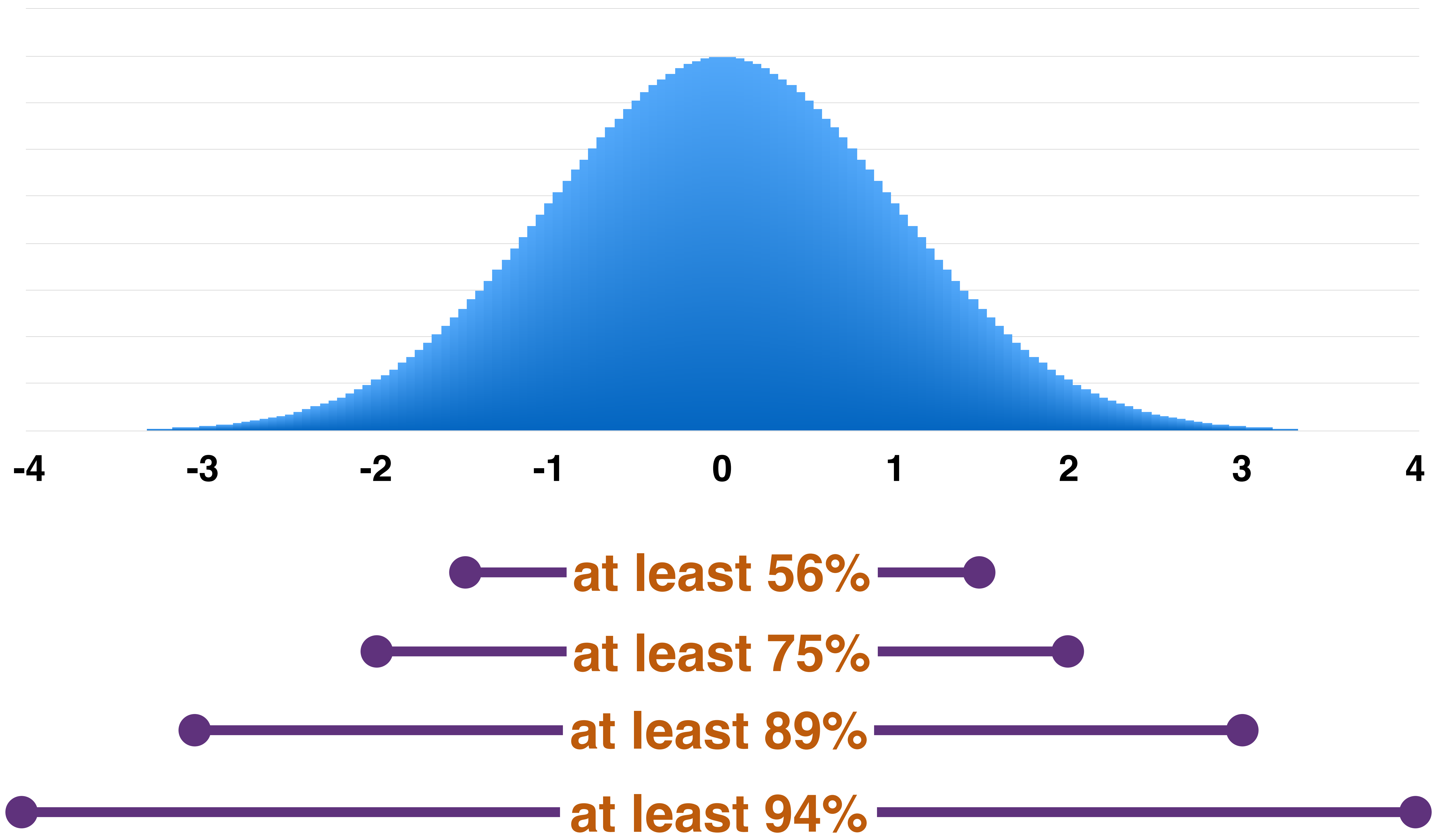
Sample Variance $s^2 = \dfrac{\sum(X-\bar{X})^2}{n-1}$

Sample Standard deviation $s = \sqrt{\dfrac{\sum(X-\bar{X})^2}{n-1}}$

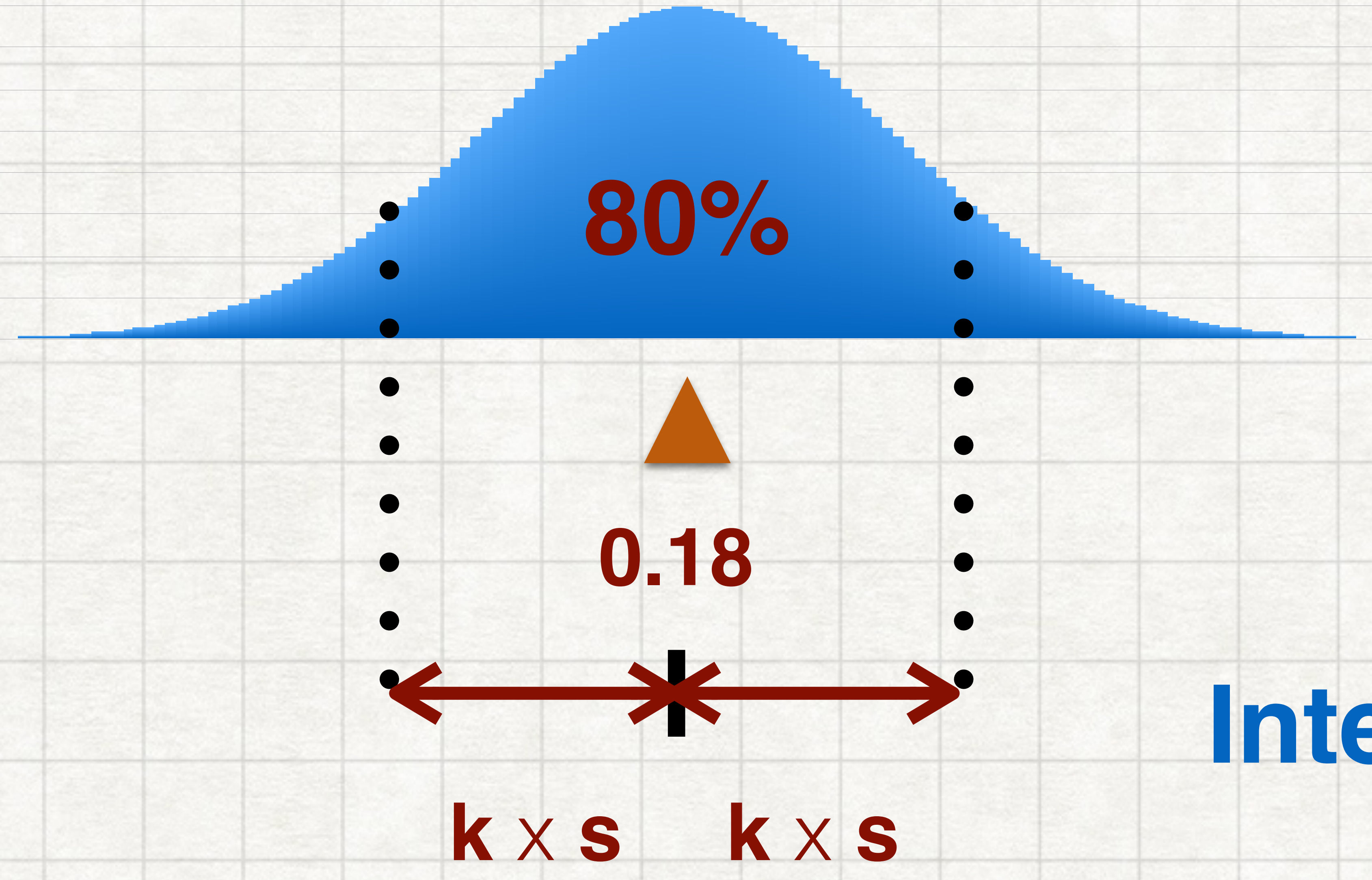
Chebyshev's Inequality

The proportion of observations within k standard deviations is at least $1-1/k^2$

No. Std Devs	$1 - 1/k^2$
1.5	0.56
2.0	0.75
3.0	0.89
4.0	**0.94**



The daily returns of Bubble Inc. stock for the past 1000 trading days have a mean of 0.18% and a standard deviation of 1.25%.
Calculate the endpoints of the interval that contains at least 800 of the observations.



$$0.8 = 1 - 1/k^2$$

$$k^2 = 5$$

$$k = 2.24$$

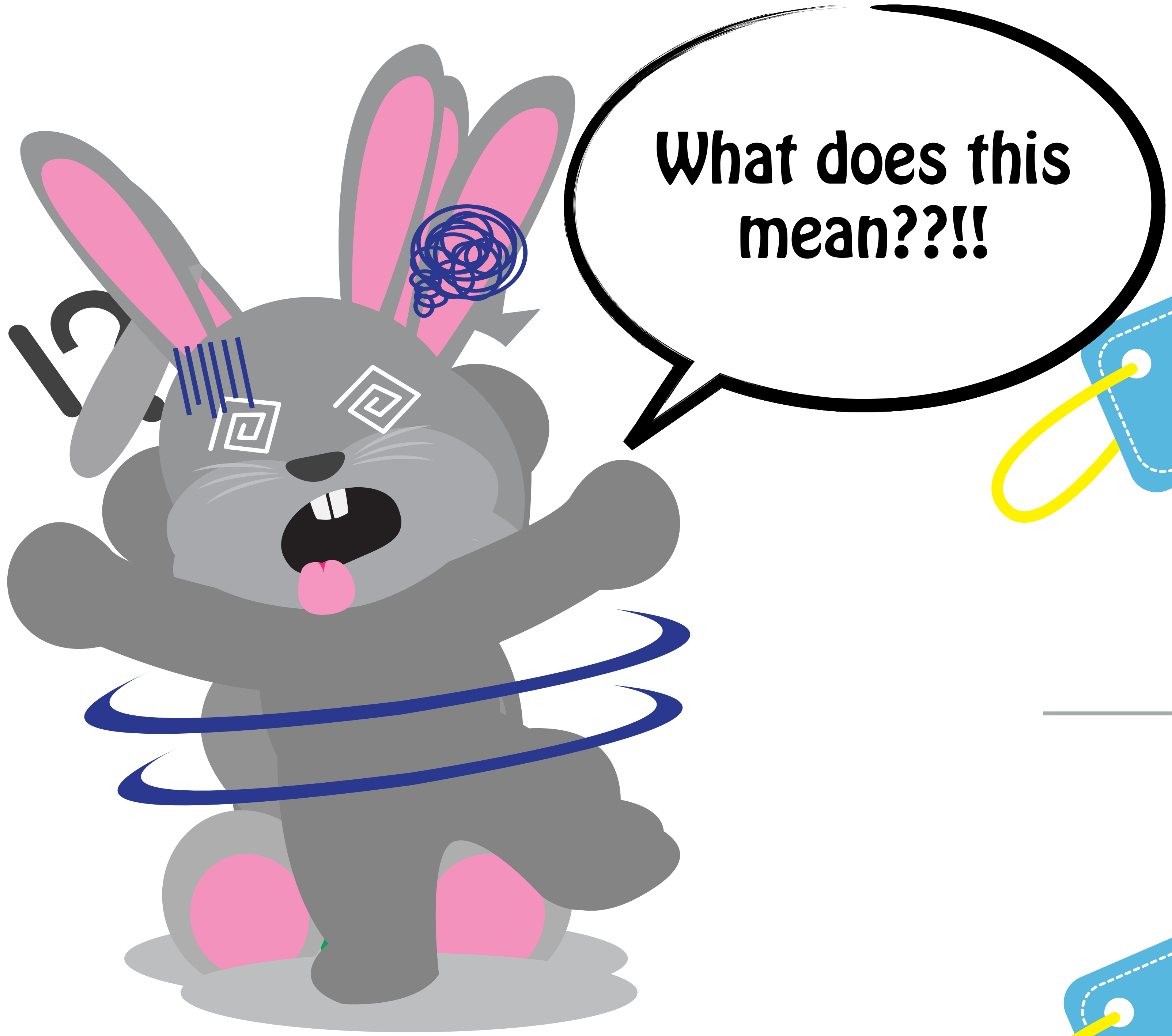
Interval:

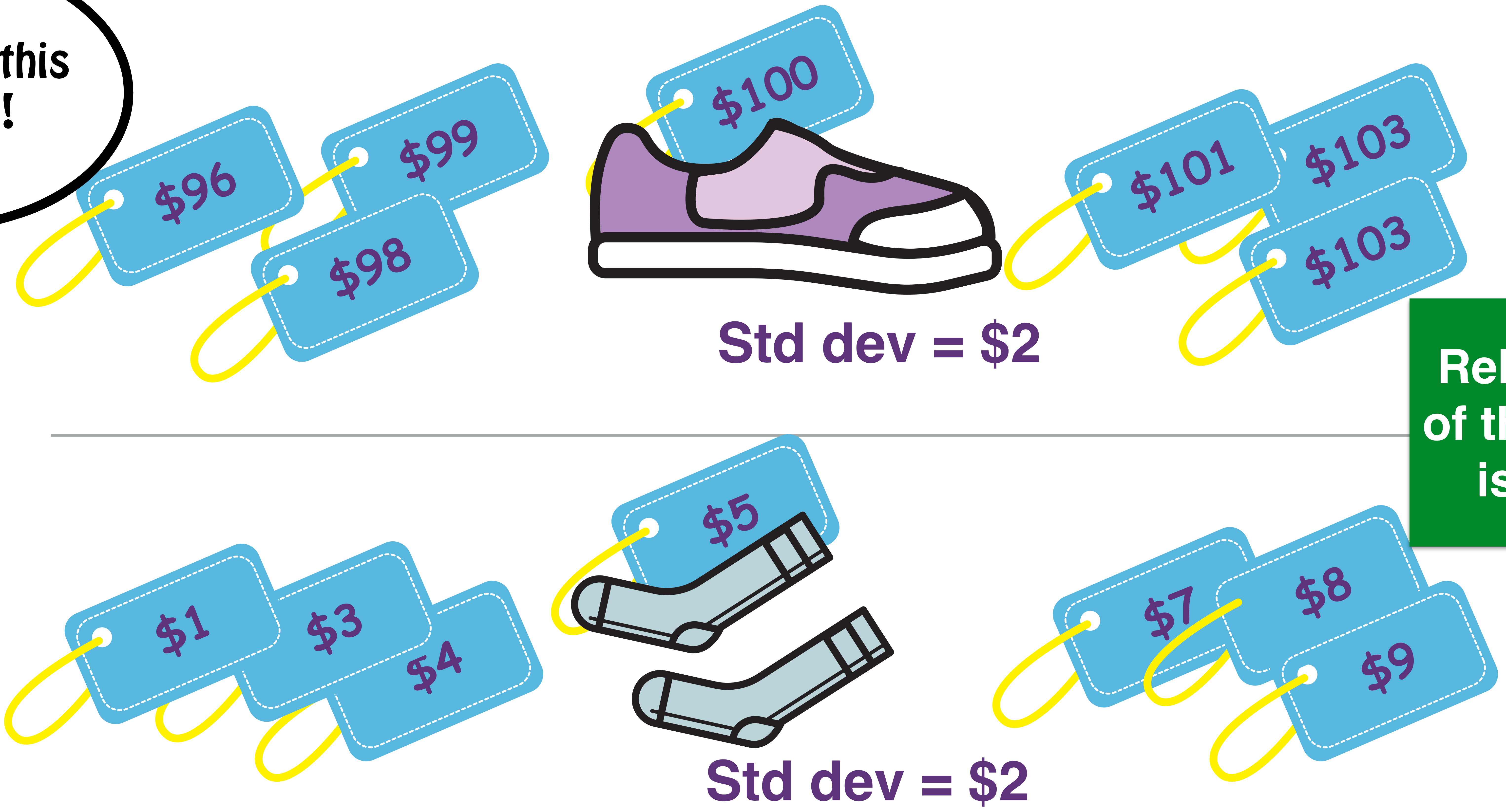
$$0.18 - 2.24 \times 1.25 < X < 0.18 + 2.24 \times 1.25$$

$$\boxed{-2.62\% < X < 2.98\%}$$

Coefficient of Variation

To address the issue of relative degree of variability of different data sets.





Relative effect of the deviation is different

Coefficient of Variation = $\dfrac{\text{Std Dev}}{\text{Avg Return}}$

Risk

Return

Std dev = $2

COV = 2/100 = 0.02

Less risk!

Std dev = $2

COV = 2/5 = 0.4

More risk!

$$\text{Coefficient of Variation} = \frac{\text{Std Dev}}{\text{Avg Return}}$$

Risk

Return

Avg return = +100%

STOCK A

COV = 2/100 = 0.02

Less risk!

Std dev = 2%

Avg return = +5%

STOCK B

COV = 2/5 = 0.4

More risk!

Std dev = 2%

$$\text{Coefficient of Variation} = \frac{\text{Std Dev}}{\text{Avg Return}}$$

Risk

Return

LOWER values desirable

Subtract by risk-free rate
to find the excess return

$$\text{Sharpe Ratio} = \frac{r - r_f}{\text{Std Dev}}$$

Return

Risk

HIGHER values desirable

As an analyst, you are presented with the mean and standard deviation of the monthly returns of T-bills and the S&P 500 for the past 10 years. Calculate the COV and Sharpe ratio for both of them. You may use the T-bill rate to represent the risk-free rate.

	T-Bills	S&P 500
Mean	0.25% r_f	1.22%
Std Dev	0.33%	6.81%

T-Bills

COV = 0.33/0.25 = 1.32 **Less risk**

Sharpe ratio = (0.25-0.25)/0.33 = 0

S&P 500

COV = 6.81/1.22 = 5.58

Sharpe ratio = (1.22-0.25)/6.81 = 0.14

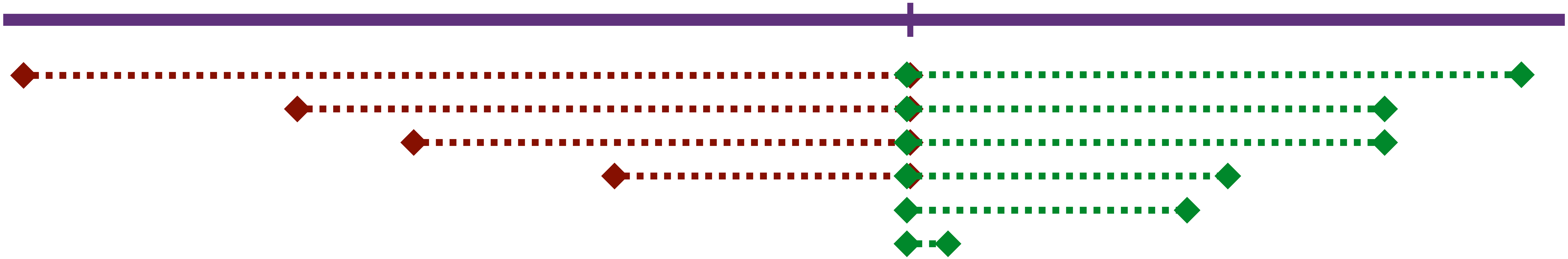
Higher risk-adjusted returns

Range

$$X_{max} - X_{min}$$

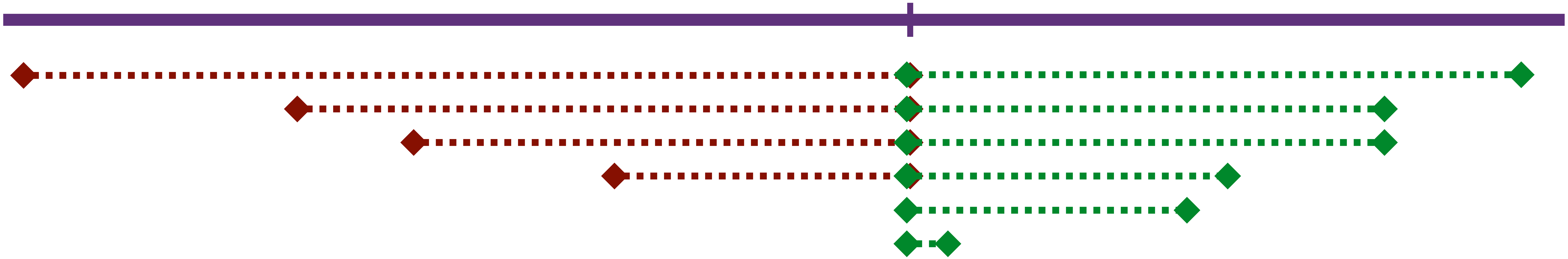
Mean Absolute Deviation

$$\frac{\sum |X-\bar{X}|}{N}$$



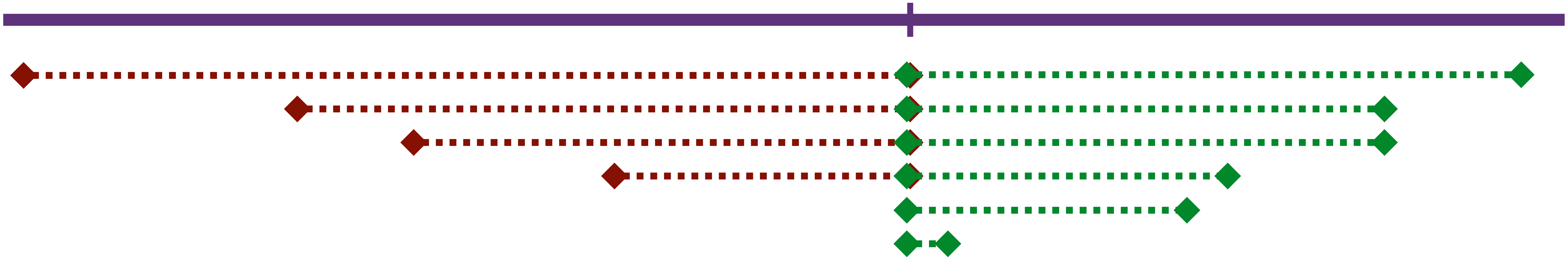
Variance

$$\frac{\sum(| X-\mu |)^2}{N}$$



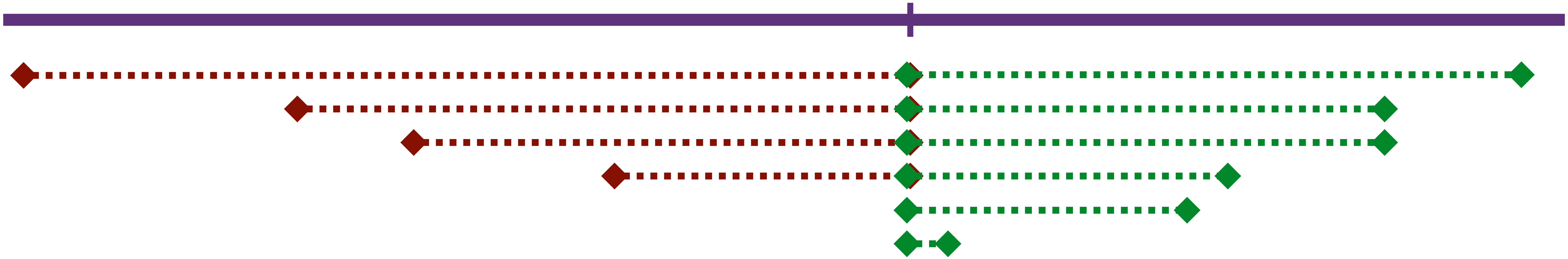
Standard Deviation

$$\sqrt{\frac{\sum (X-\mu)^2}{N}}$$



Sample Standard Deviation

$$\sqrt{\frac{\sum(X-\bar{X})^2}{n-1}}$$



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Chebyshev's Inequality

$$1 - 1/k^2$$

Coefficient of Variation

$$\frac{\text{standard deviation}}{\text{mean}}$$

Sharpe Ratio

$$\frac{r_p - r_f}{\sigma_p}$$

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