Hypothesis Testing

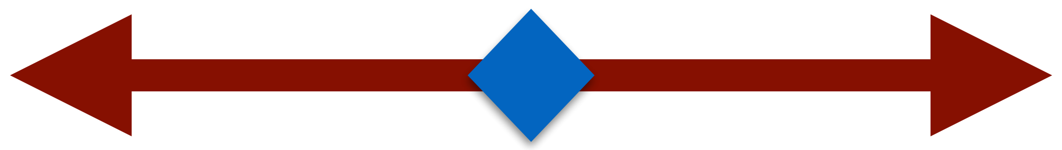
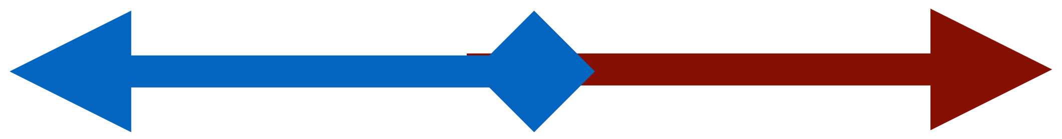
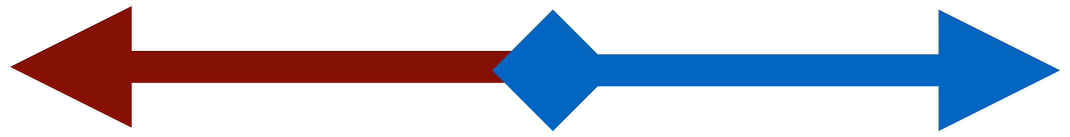
Hypothesis Tests Concerning Variance

1. Chi-Square Test
2. F Test
3. Non-parametric Tests

Chi-Square Test

Used for hypothesis tests concerning the variance of a <u>normally distributed population</u>

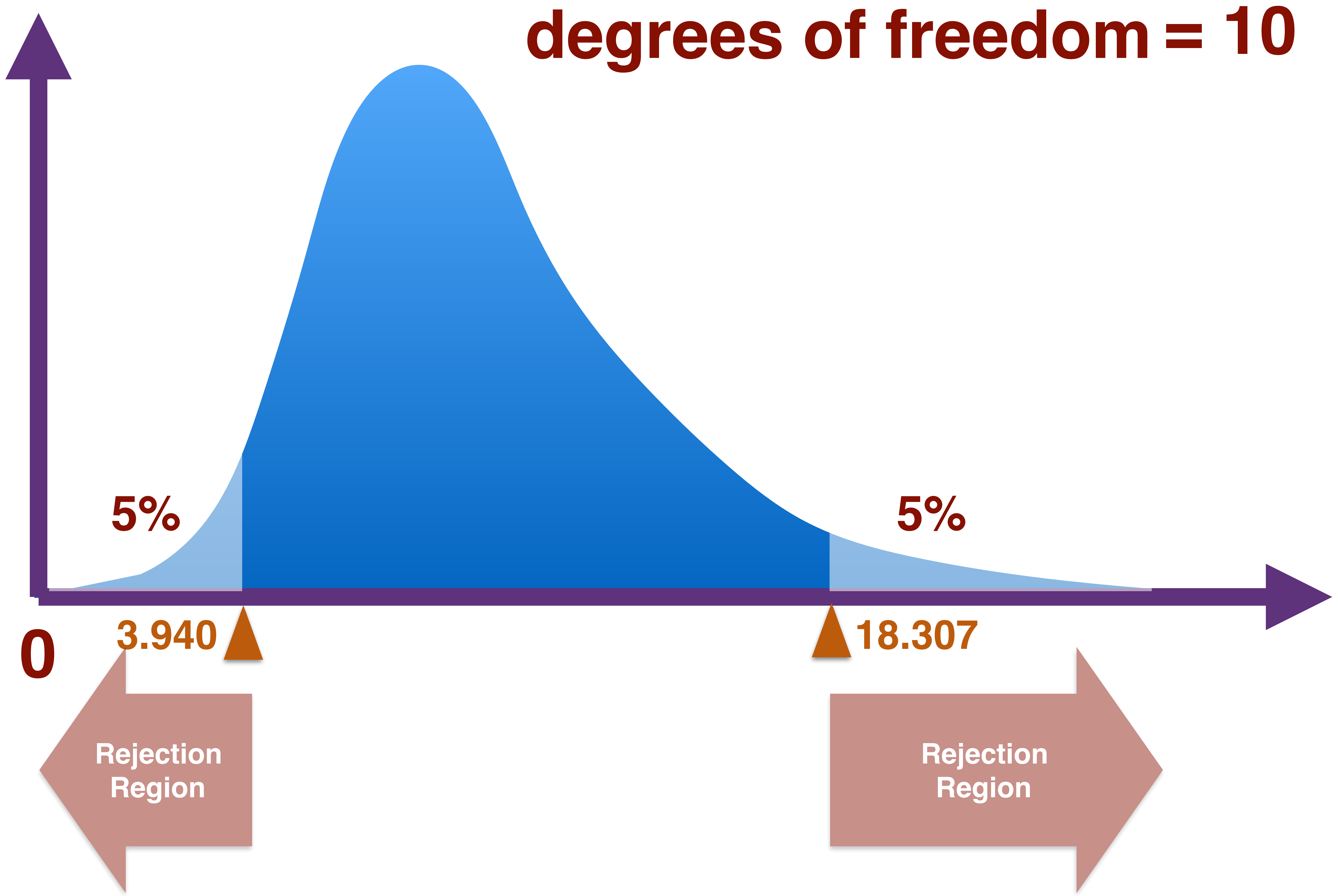
σ_0^2: Hypothesised value of the variance

Null Hypothesis, H_0		Alternative Hypothesis, H_A
$\sigma^2 = \sigma_0^2$		$\sigma^2 \neq \sigma_0^2$
$\sigma^2 \leq \sigma_0^2$		$\sigma^2 > \sigma_0^2$
$\sigma^2 \geq \sigma_0^2$		$\sigma^2 < \sigma_0^2$

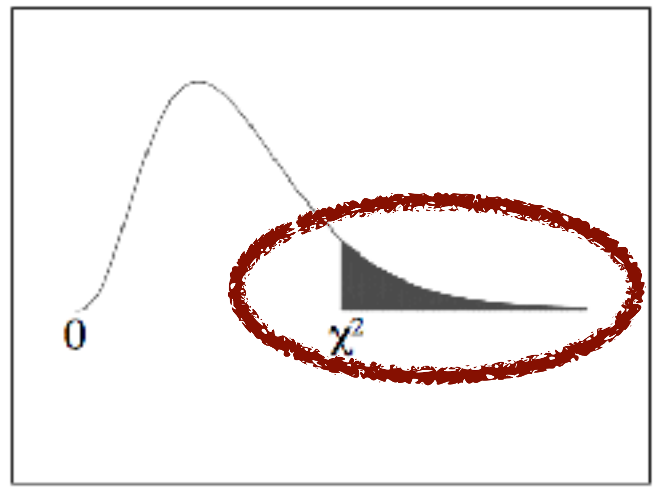
Chi-Square Test

Used for hypothesis tests concerning the variance of a <u>normally distributed population</u>

<u>Two-tailed test, α=10%</u>



Chi-Square Distribution Table



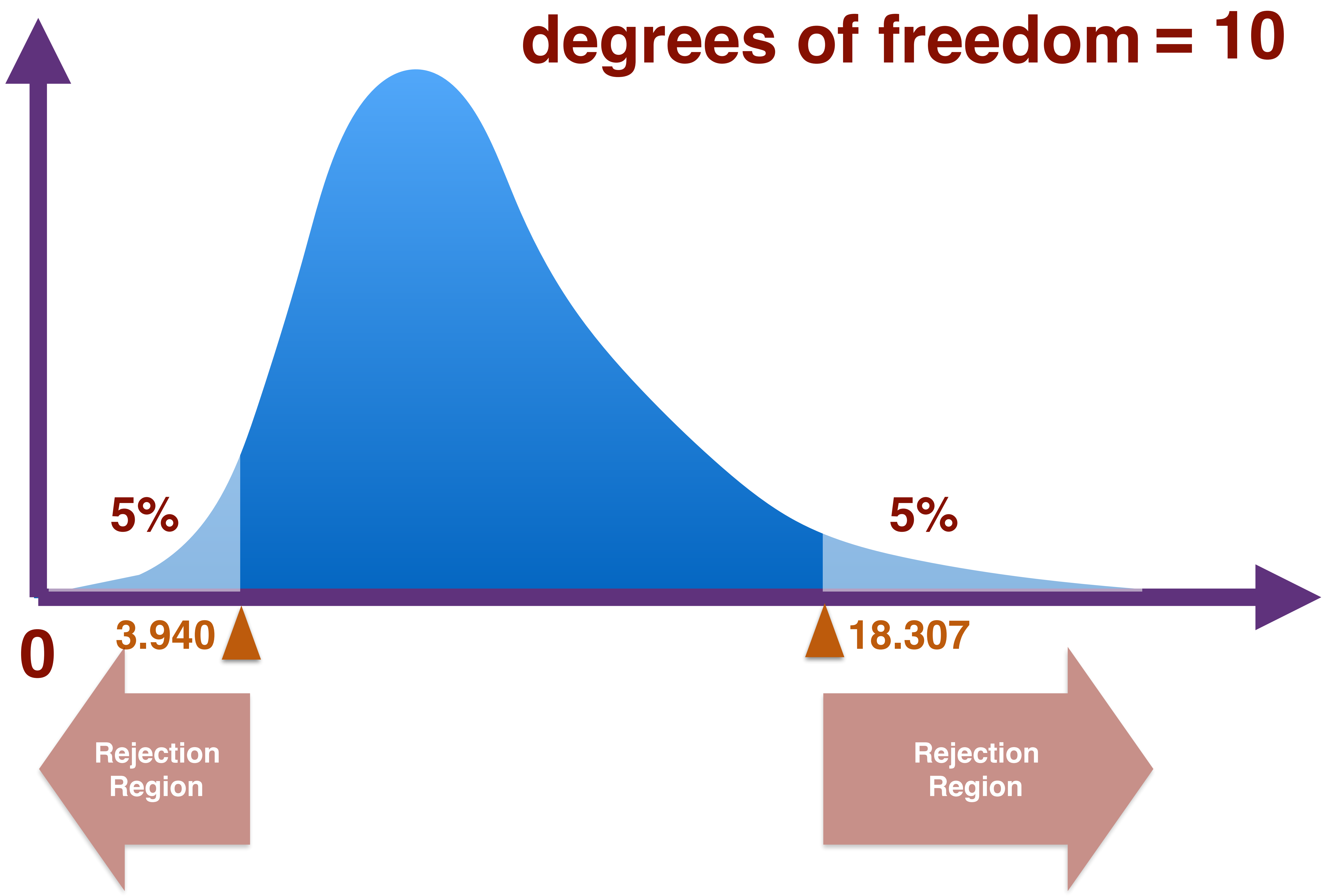
The sha[illegible] area is equal to α for $\chi^2 = \chi^2_\alpha$.

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819

Chi-Square Test

Used for hypothesis tests concerning the variance of a normally distributed population

Two-tailed test, α=10%



Decision Rule

Reject H_0 if $\chi^2 < 3.94$ or $\chi^2 > 18.307$

Test Statistic

$$\chi^2_{n-1} = (n-1)s^2 / \sigma_0^2$$

Suzie Li, CFA would like to investigate if the standard deviation of a fund is under-reported. The monthly standard deviation of the fund is advertised at 6%. Suzie obtained a random sample of 15 monthly standard deviations of the fund and measured a standard deviation of monthly returns of 8.2%. Is this sufficient evidence (at 5% significance level) to prove that the standard deviation of the fund is greater than the advertised rate of 6%?

σ^2: Variance of fund returns

s^2: Variance of sample

1. State the hypothesis

$H_0: \sigma^2 \leq 0.0036$

$H_A: \sigma^2 > 0.0036$

$\sigma_0^2 = 0.06^2 = 0.0036$

2. Select the test statistic

$$\chi^2 = \frac{(n-1)s^2}{0.0036}$$

3. Specify significance level

$\alpha = 5\%$

4. State decision rule

Reject H_0 if $\chi^2 > 23.685$

5. Calculate test statistic

$$\chi^2 = \frac{14 \times 0.082^2}{0.0036} = 26.149$$

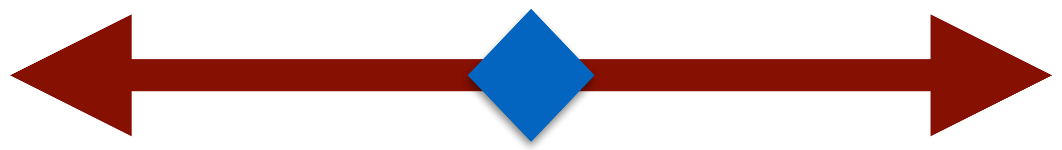
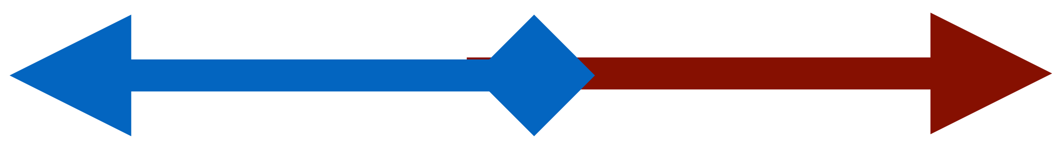
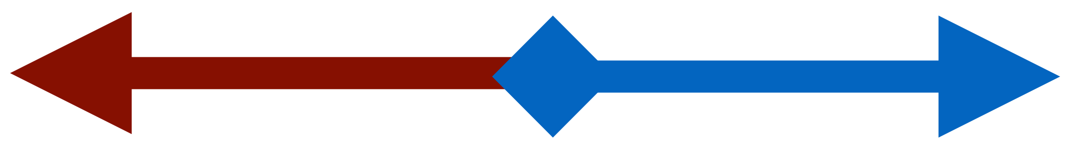
6. Statistical decision

Reject H_0 at 5% significance.

Conclude that the actual standard deviation is higher than the advertised rate of 6%

F-Test

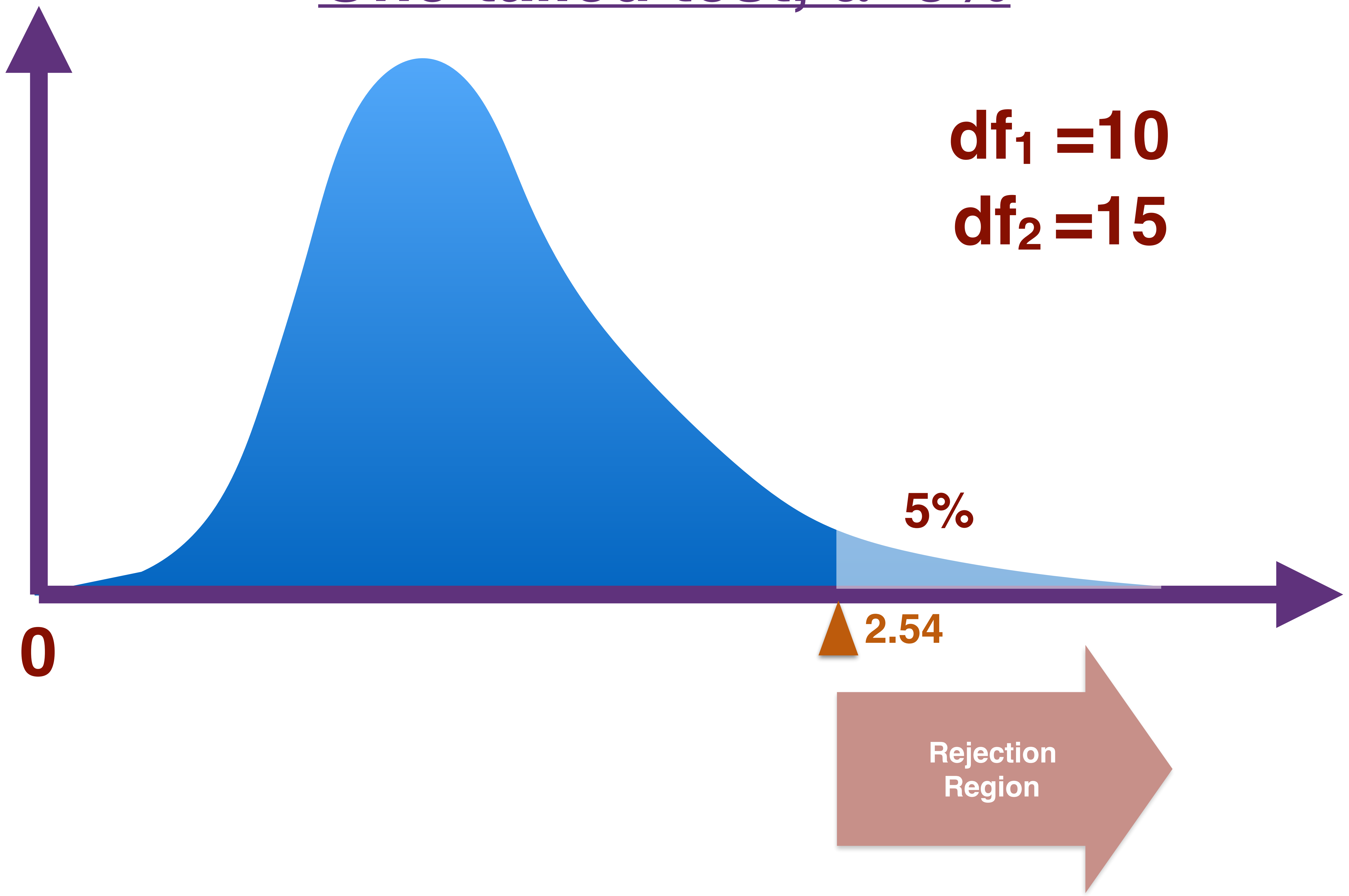
Used for hypothesis tests concerning the equality of variances between two ***normally distributed populations*** *and* ***independent samples***

Null Hypothesis, H_0	Compare t-test for *difference* between means		Alternative Hypothesis, H_A
$\sigma_1^2 = \sigma_2^2$	$\mu_1 - \mu_2 = \theta_0$		$\sigma_1^2 \neq \sigma_2^2$
$\sigma_1^2 \leq \sigma_2^2$	$\mu_1 - \mu_2 \leq \theta_0$		$\sigma_1^2 > \sigma_2^2$
$\sigma_1^2 \geq \sigma_2^2$	$\mu_1 - \mu_2 \geq \theta_0$		$\sigma_1^2 < \sigma_2^2$

F-Test

Used for hypothesis tests concerning the equality of variances between two ***normally distributed populations*** *and* ***independent samples***

One-tailed test, α=5%



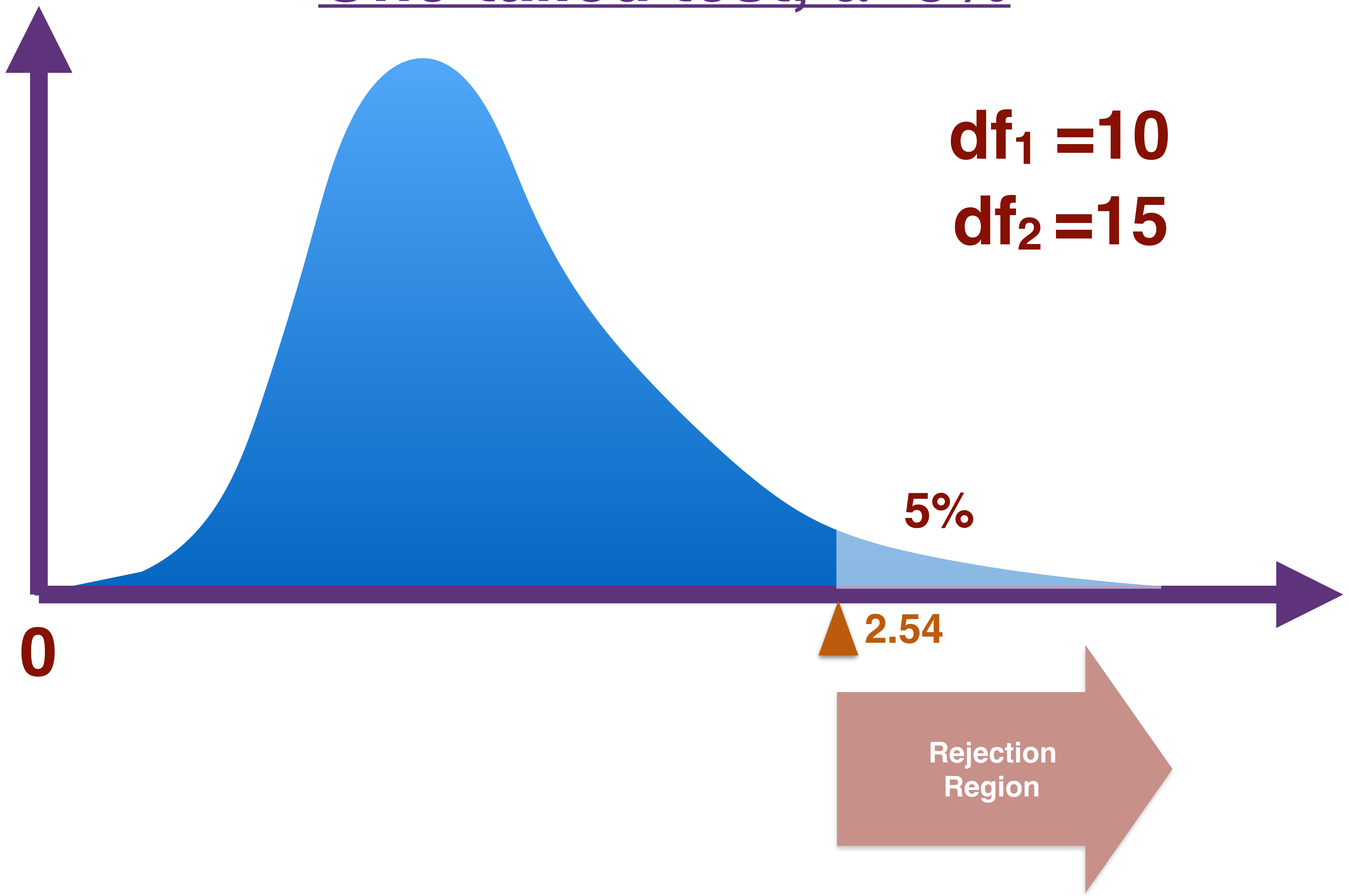
Panel A. Critical values for right-hand tail area equal to 0.05

	df1:1	2	3	4	5	6	7	8	9	10	11	12	1
df2: 1	161	200	216	225	230	234	237	239	241	242	243	244	24
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.7
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.8
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.6
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.9
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.5
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.2
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.0
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.8
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.7
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.6
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.5
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.4
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.4
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.3

F-Test

Used for hypothesis tests concerning the equality of variances between two ***normally distributed populations*** *and* ***independent samples***

One-tailed test, α=5%



Decision Rule

Reject H_0 if $F > 2.54$

Test Statistic

$F = s_1^2 / s_2^2$

Assign numerator with the higher value to force a right-tail test

H_0: $\sigma_1^2 \leq \sigma_2^2$

H_A: $\sigma_1^2 > \sigma_2^2$

Jon suspects that the dispersion of growth stocks are more volatile than value stocks. He obtained 16 randomly selected growth stocks and 20 randomly selected value stocks and derived the following figures from these samples.

	Growth Stocks	Value Stocks
Number of Samples	16	20
Standard deviation	10.9%	6.6%

Determine if Jon's findings are sufficient to conclude that growth stocks are indeed more volatile than value stocks.

1. State the hypothesis

$H_0: \sigma_{growth}^2 \leq \sigma_{value}^2$

$H_A: \sigma_{growth}^2 > \sigma_{value}^2$

2. Select the test statistic

$$F = \frac{s_{growth}^2}{s_{value}^2}$$

3. Specify significance level

$\alpha = 5\%$

4. State decision rule

Reject H_0 if $F > 2.23$

5. Calculate test statistic

$$F = \frac{10.9^2}{6.6^2} = 2.728$$

6. Statistical decision.

Reject H_0 at 5% significance level.

Conclude that the volatility of growth stocks is higher than that of value stocks

Parametric Tests

1. Parameters

e.g. mean, variance

2. Assumptions

e.g. population distribution is normal, independence between sample sets

Non-Parametric Tests

1. Parameters

e.g. mean, variance

Not concerned

2. Assumptions

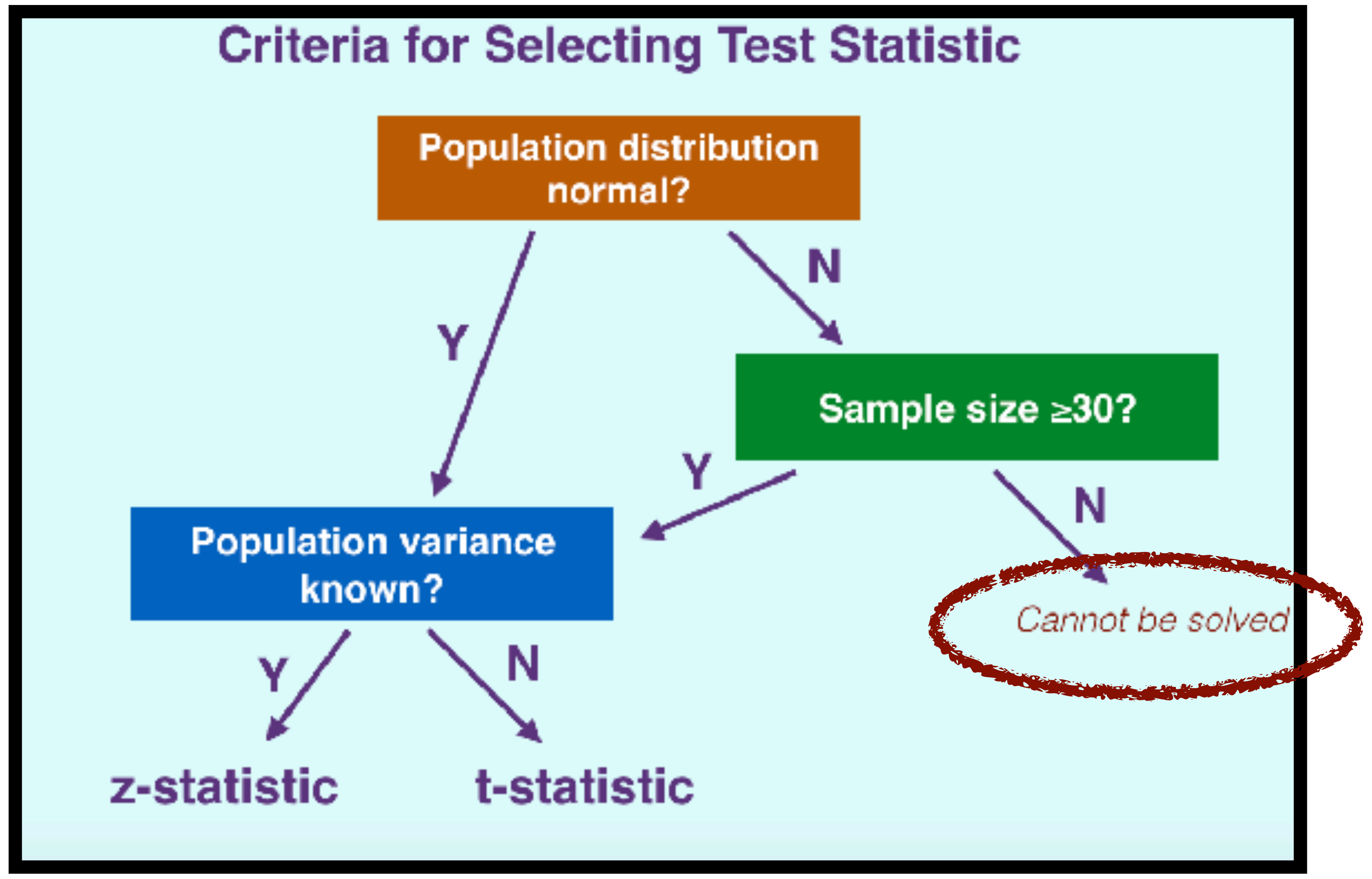
e.g. population distribution is normal, independence between sample sets

Minimal assumptions

Non-Parametric Tests

When is it appropriate?

1 Distributional assumptions not met



2 Ranked data

- *Ranked data is not normally distributed*
- *Parametric tests require stronger scale than rank*

3 Not concerned with parameter

Examples:

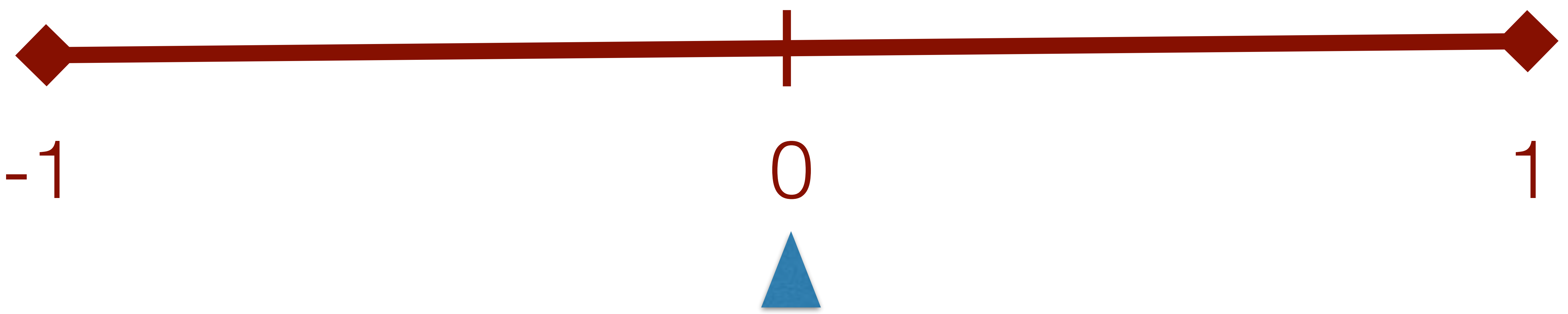
- *Is the sample random?*
- *Is this sample from a population following a normal distribution?*

Non-Parametric Tests

Spearman Rank Correlation Test

X **Strength of linear relationship** Y

Spearman rank correlation coefficient



Perfect inverse straight line relationship

No straight line relationship

Perfect positive straight line relationship