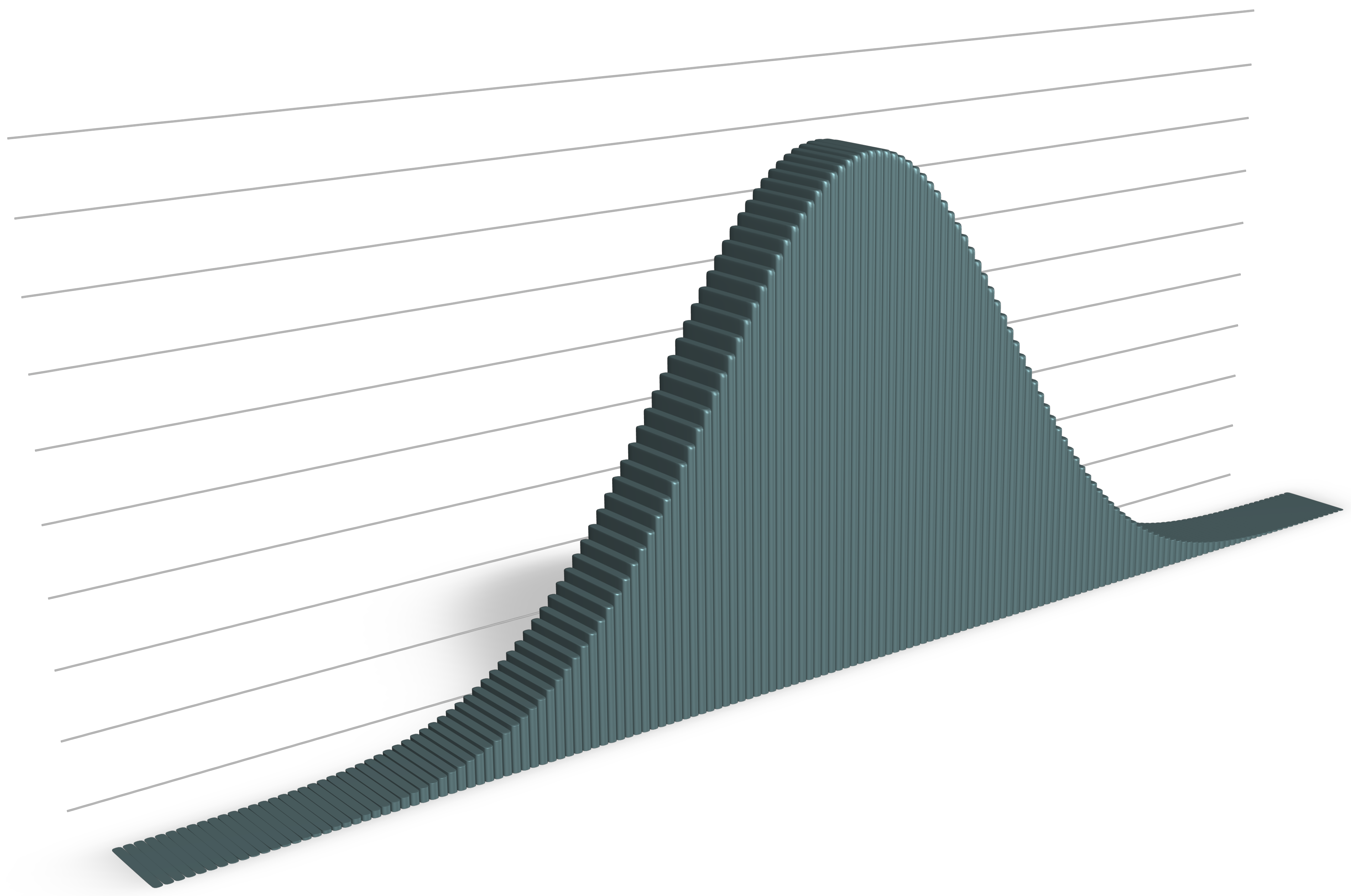


Common Probability Distributions

Discrete Random Variables

1. Discrete vs Continuous
2. Discrete Uniform Distributions
3. Binomial Distributions

Random Variable

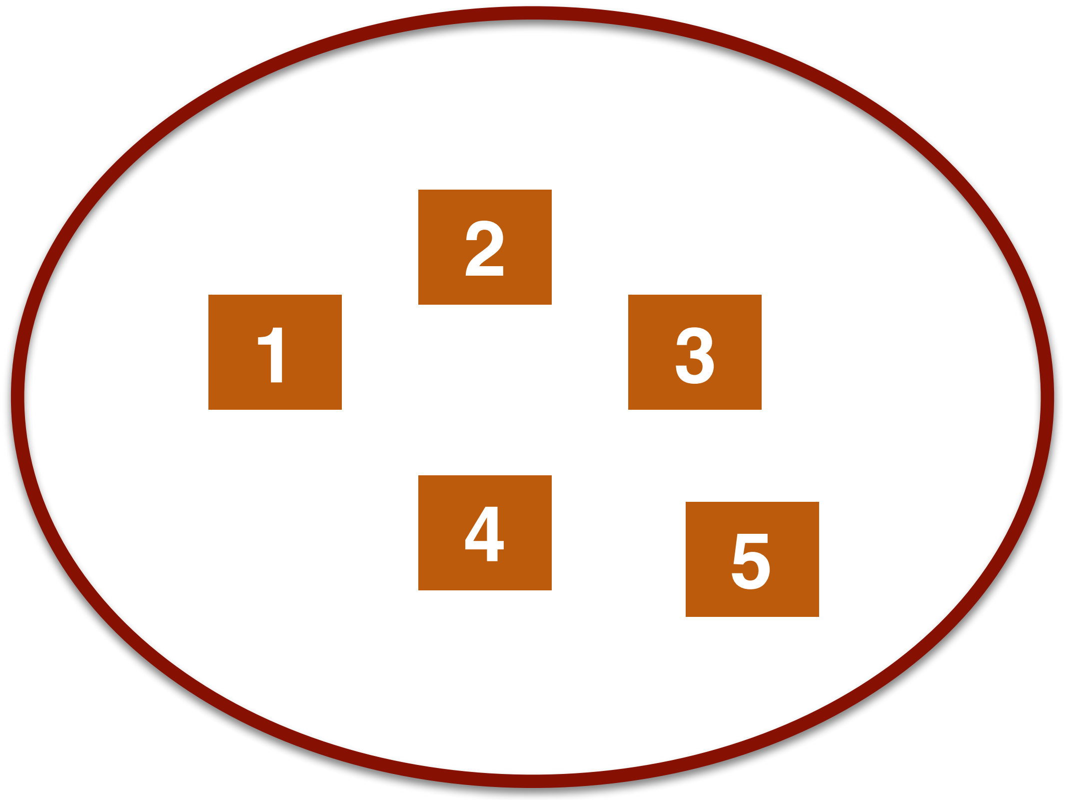
Uncertain quantity or number

Discrete

Continuous

Discrete Random Variable

*number of possible outcomes can be **counted***



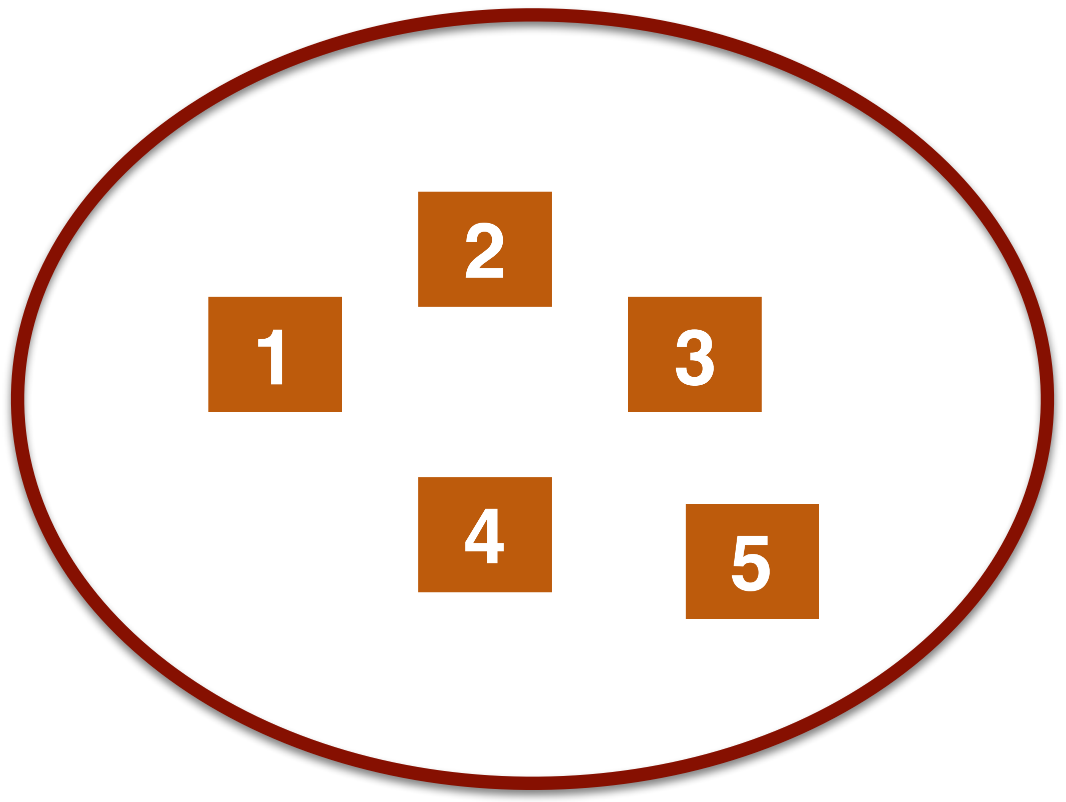
e.g. num of up days for a stock

Continuous Random Variable

*number of possible outcomes is **infinite***

Discrete Random Variable

number of possible outcomes can be ***counted***



e.g. num of up days for a stock

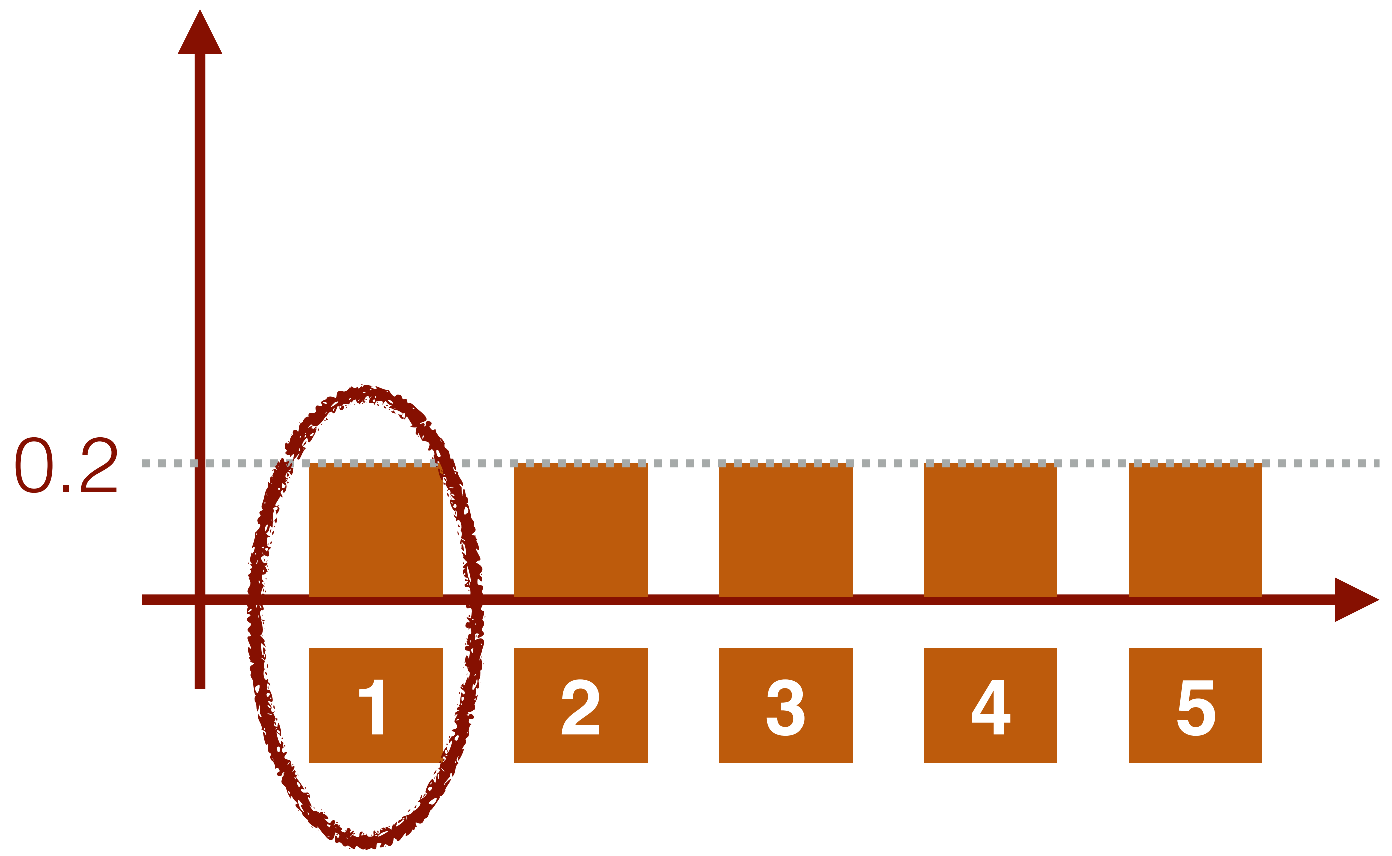
Continuous Random Variable

number of possible outcomes is ***infinite***

e.g. time for stock to move +/- 1%

Discrete Random Variable

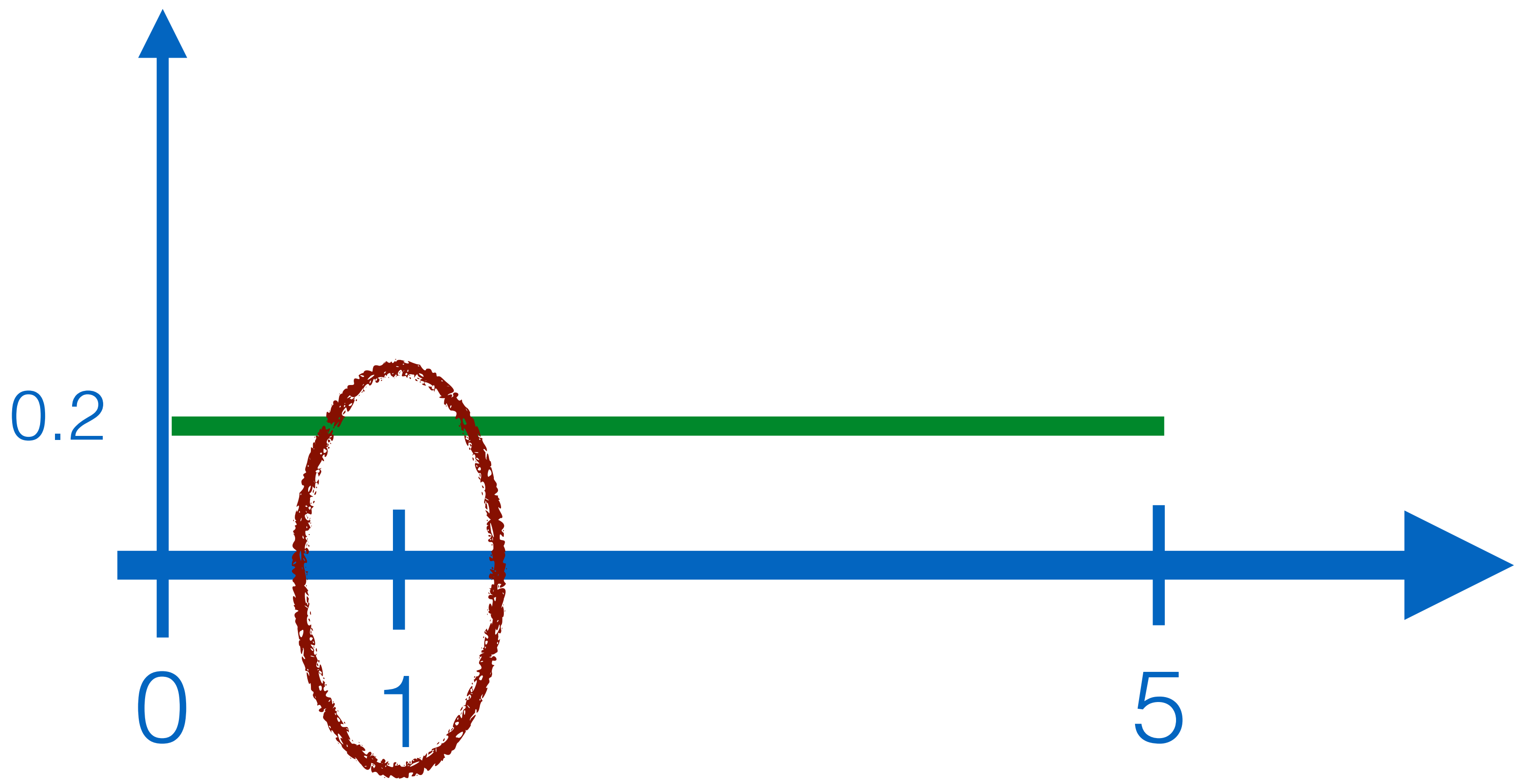
Probability Distribution Function



$$p(1) = P(X=1) = 0.2$$

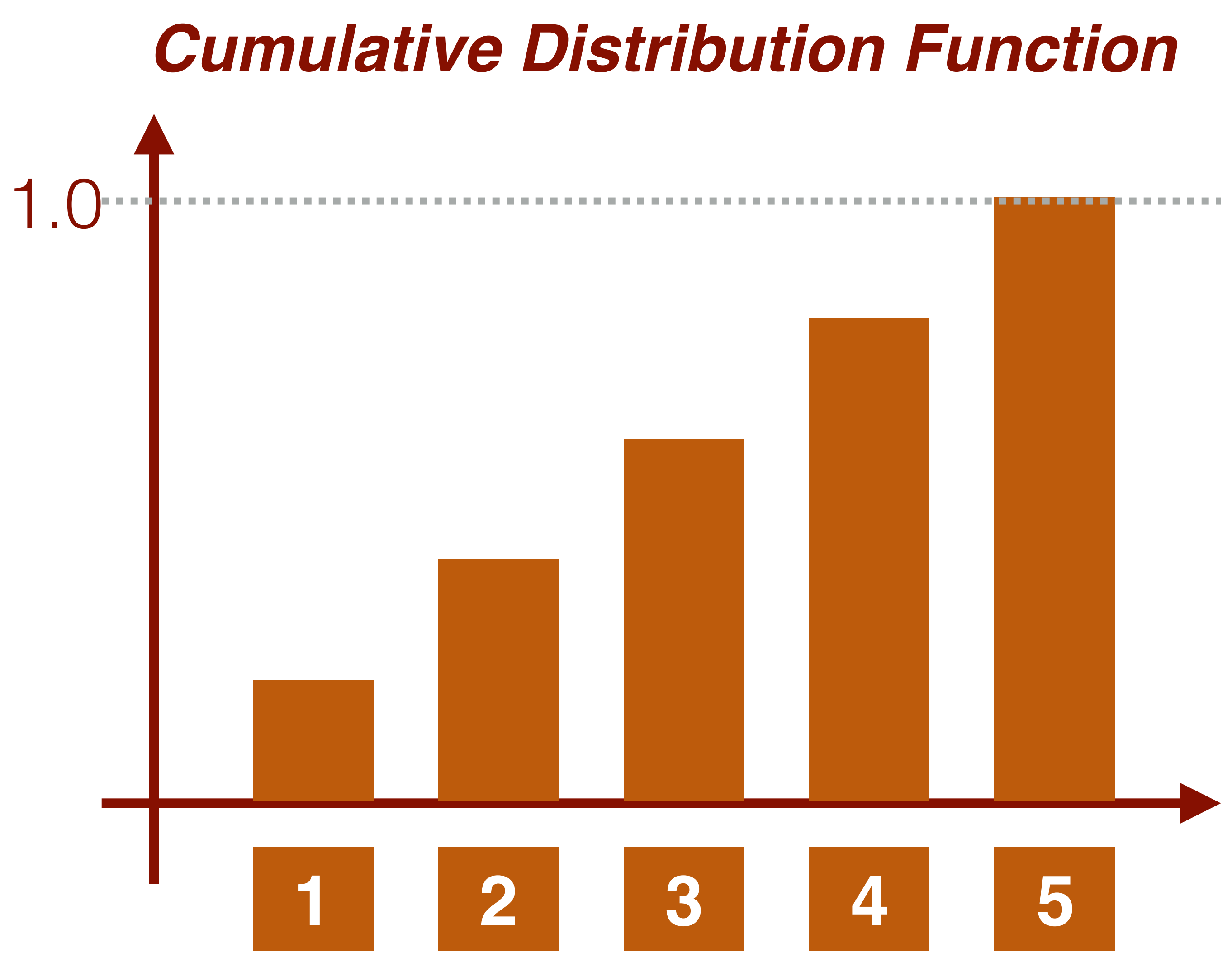
Continuous Random Variable

Probability Density Function



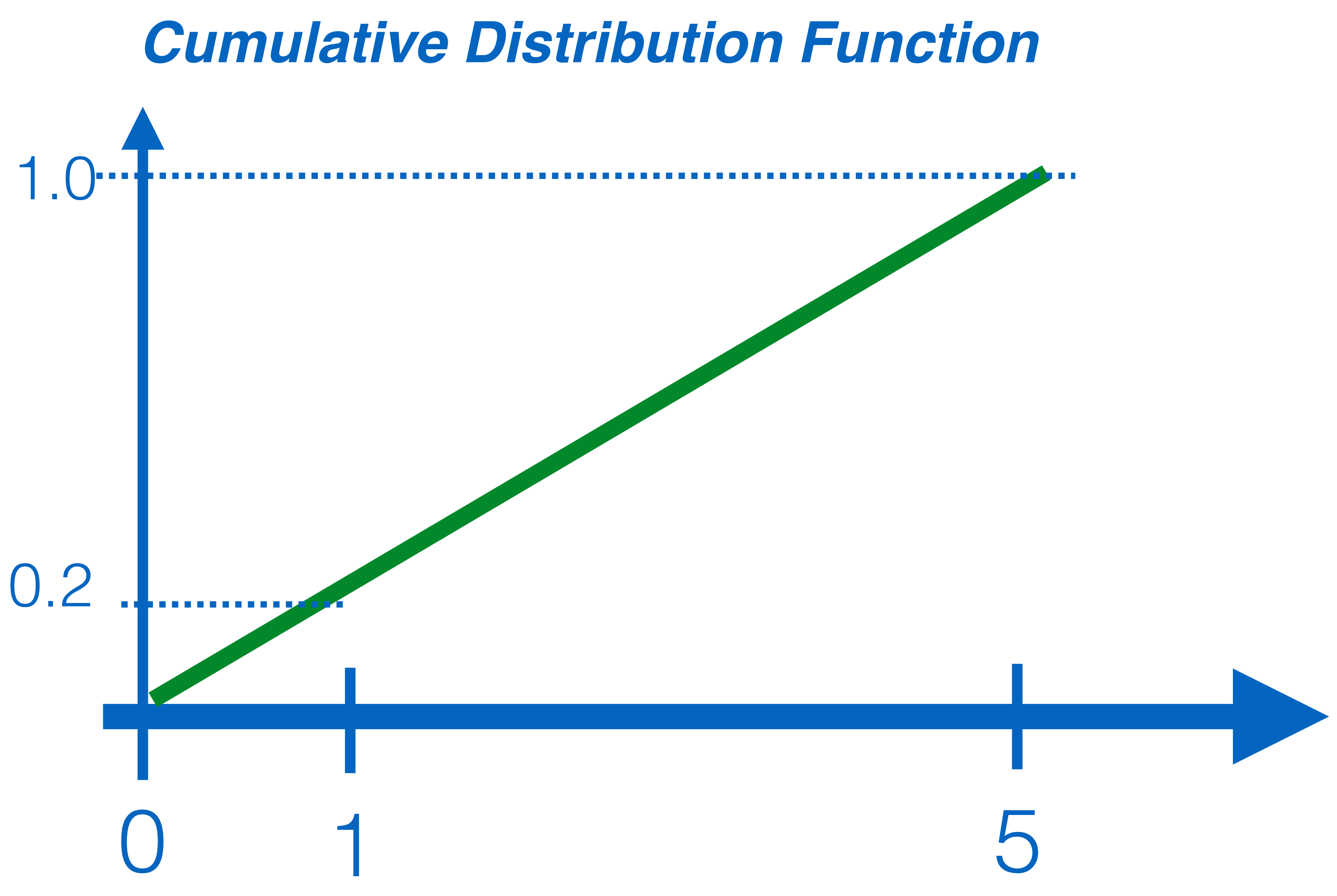
$$f(1) = P(X=1) = 0.2$$

Discrete Random Variable



F(1) = P(X≤1) = 0.2

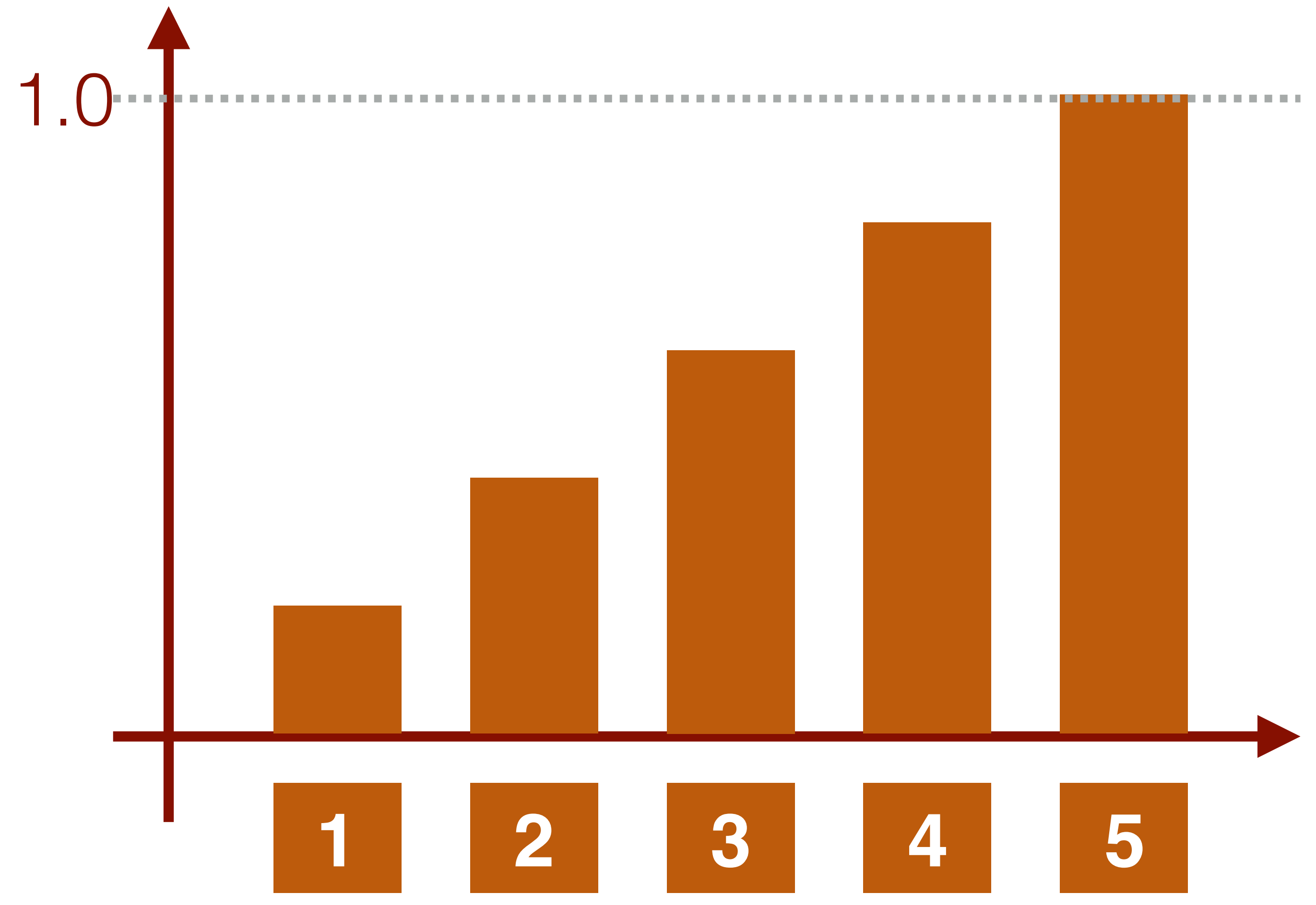
Continuous Random Variable



F(1) = P(X≤1) = 0.2

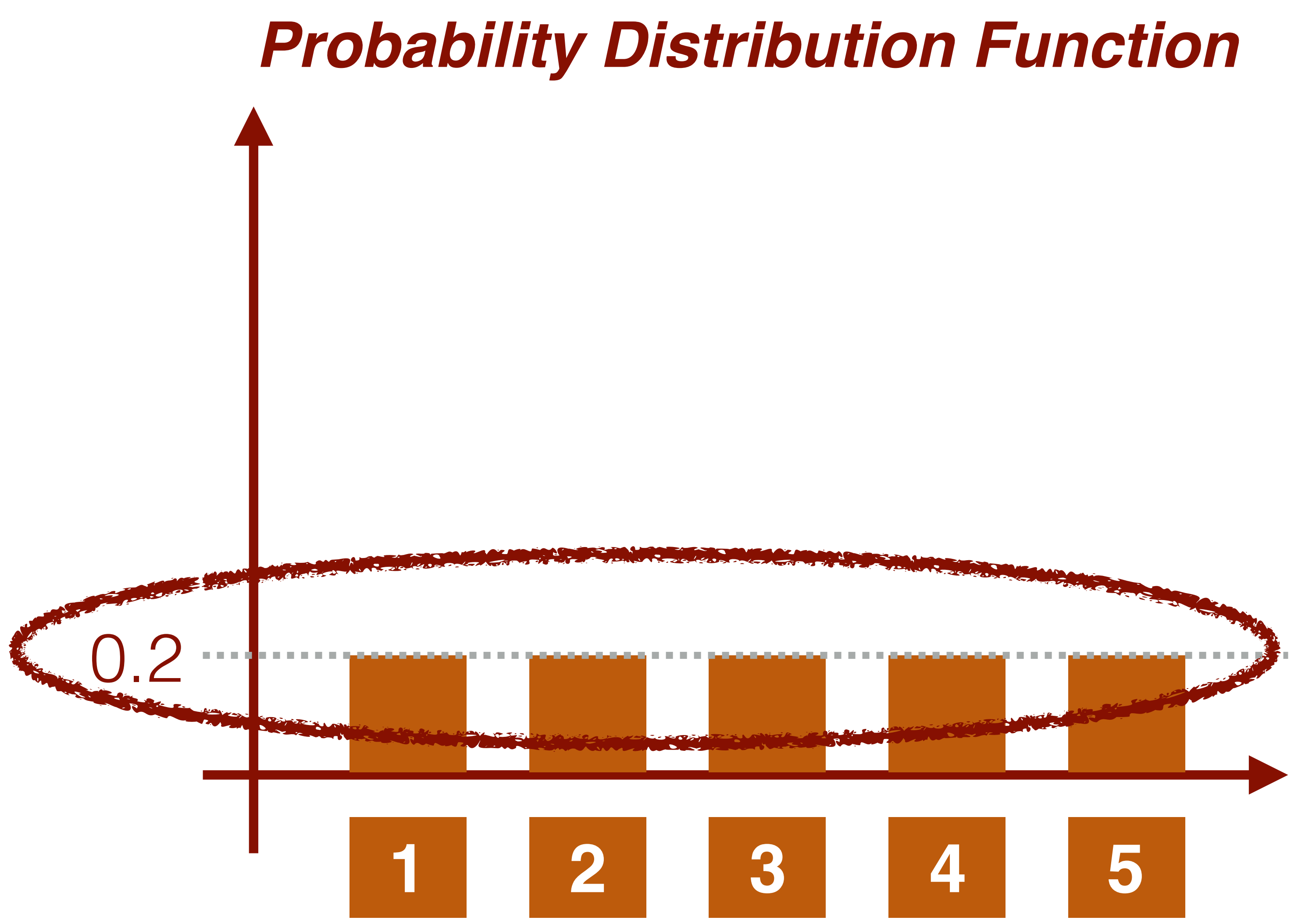
Discrete Random Variable

Cumulative Distribution Function



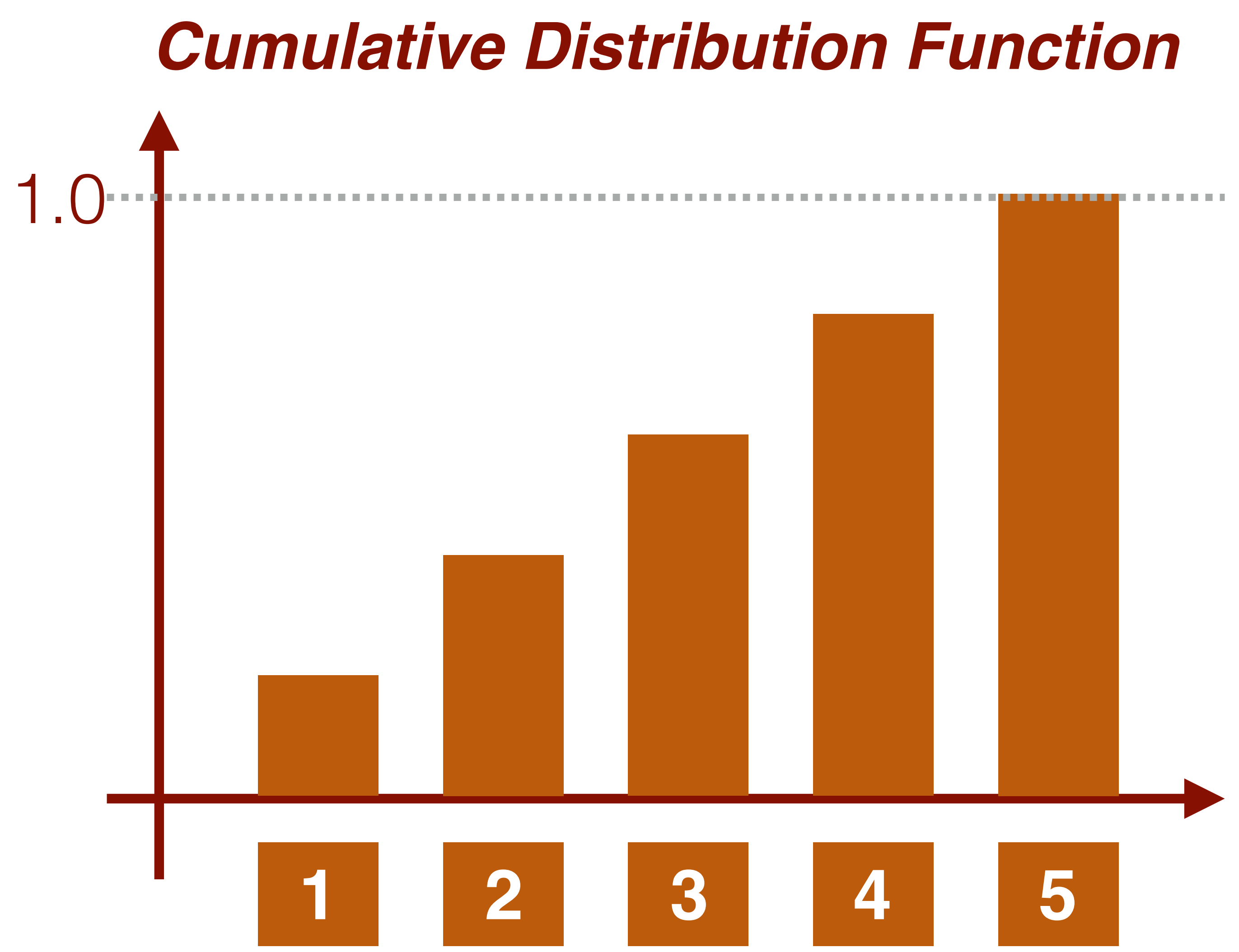
$F(1) = P(X \leq 1) = 0.2$

Discrete Uniform Random Variable



All outcomes equally likely

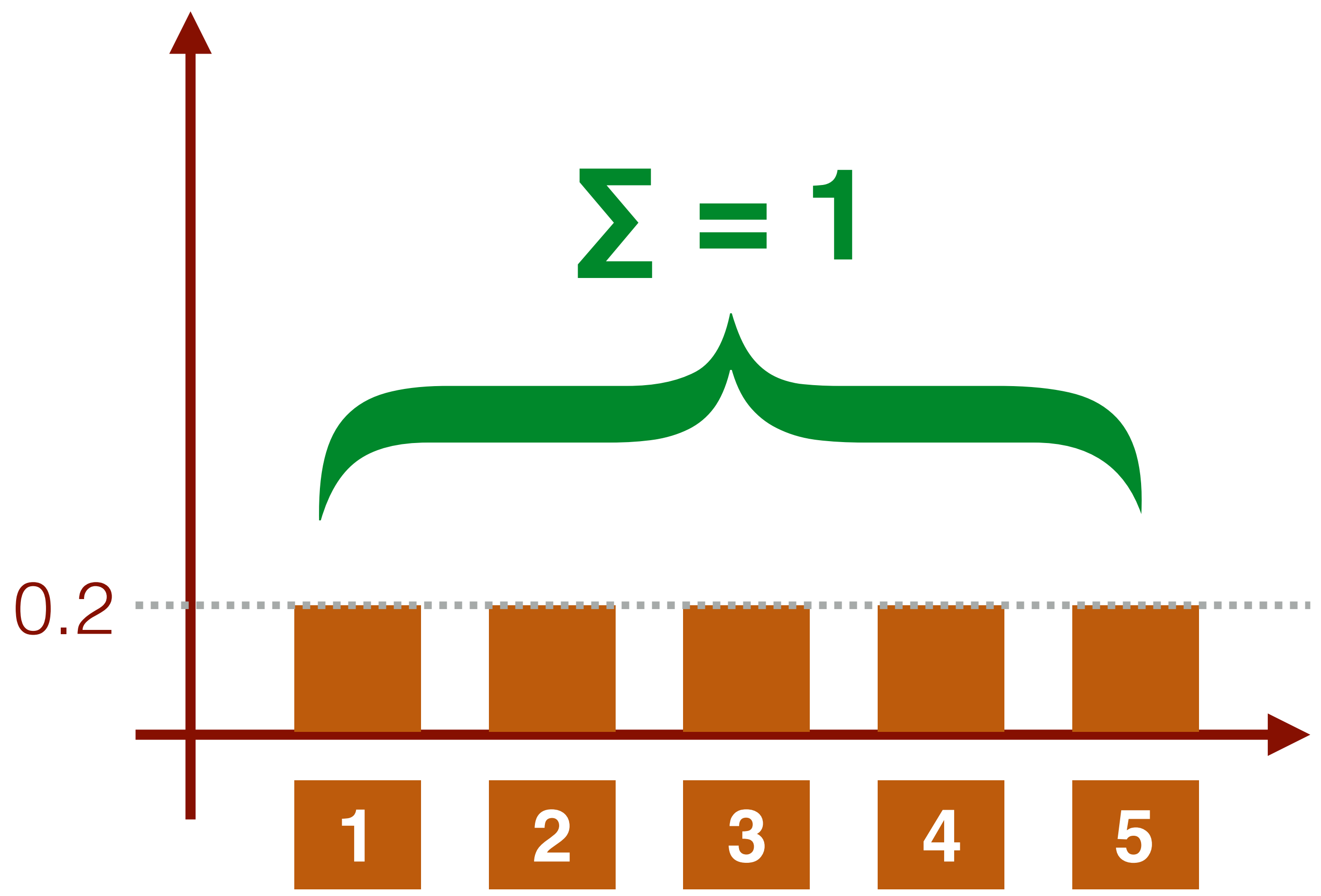
$p(1) = P(X=1) = 0.2$



$F(1) = P(X\leq 1) = 0.2$

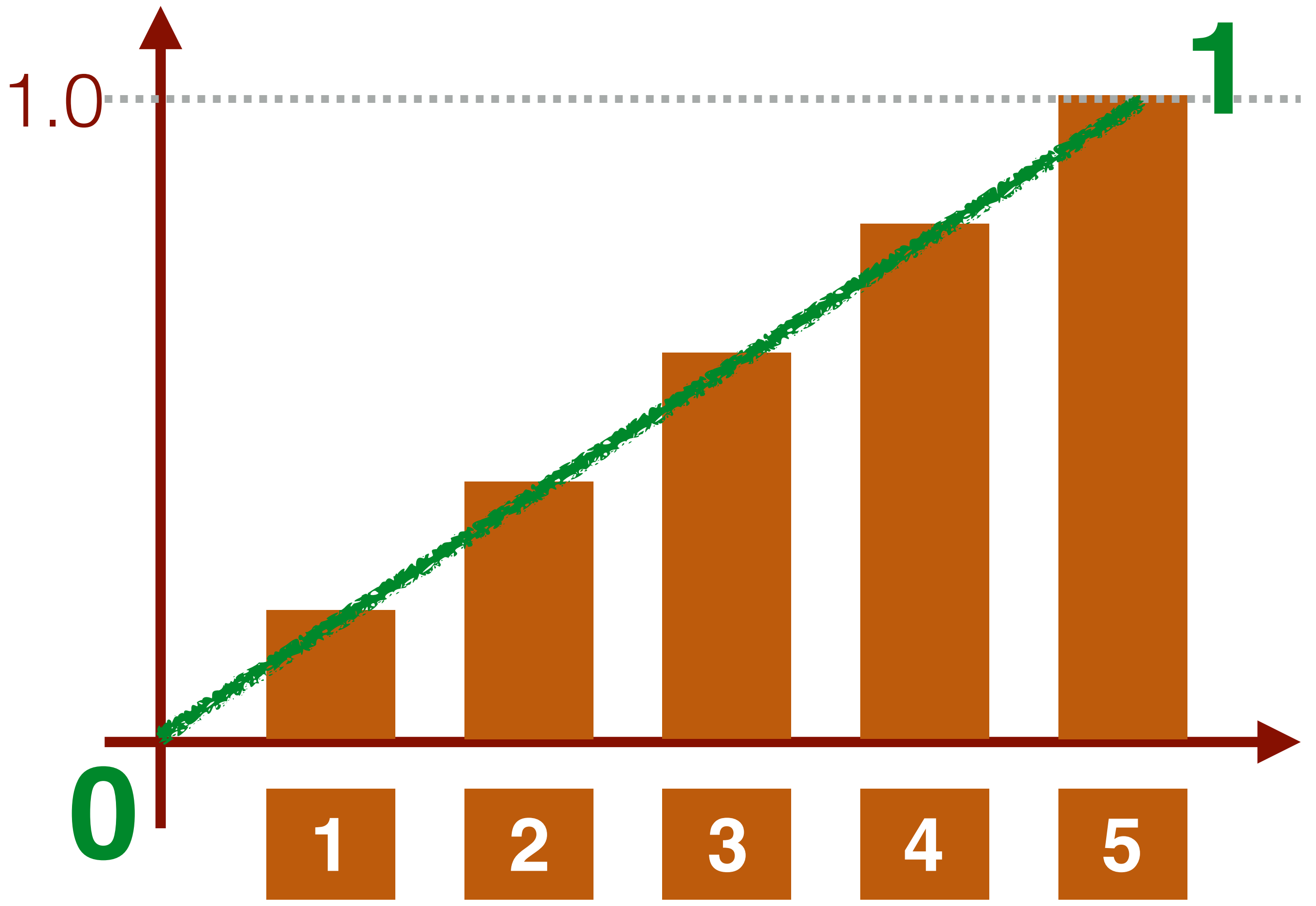
Discrete Uniform Random Variable

Probability Distribution Function



$p(1) = P(X=1) = 0.2$

Cumulative Distribution Function



$F(1) = P(X \leq 1) = 0.2$

Determine the following for the result of one throw of a fair 6-sided dice.

(a) $p(2)$

(b) $F(2)$

(c) $P(X>2)$

(a) $p(2) = P(X=2)$

$= 1/6$

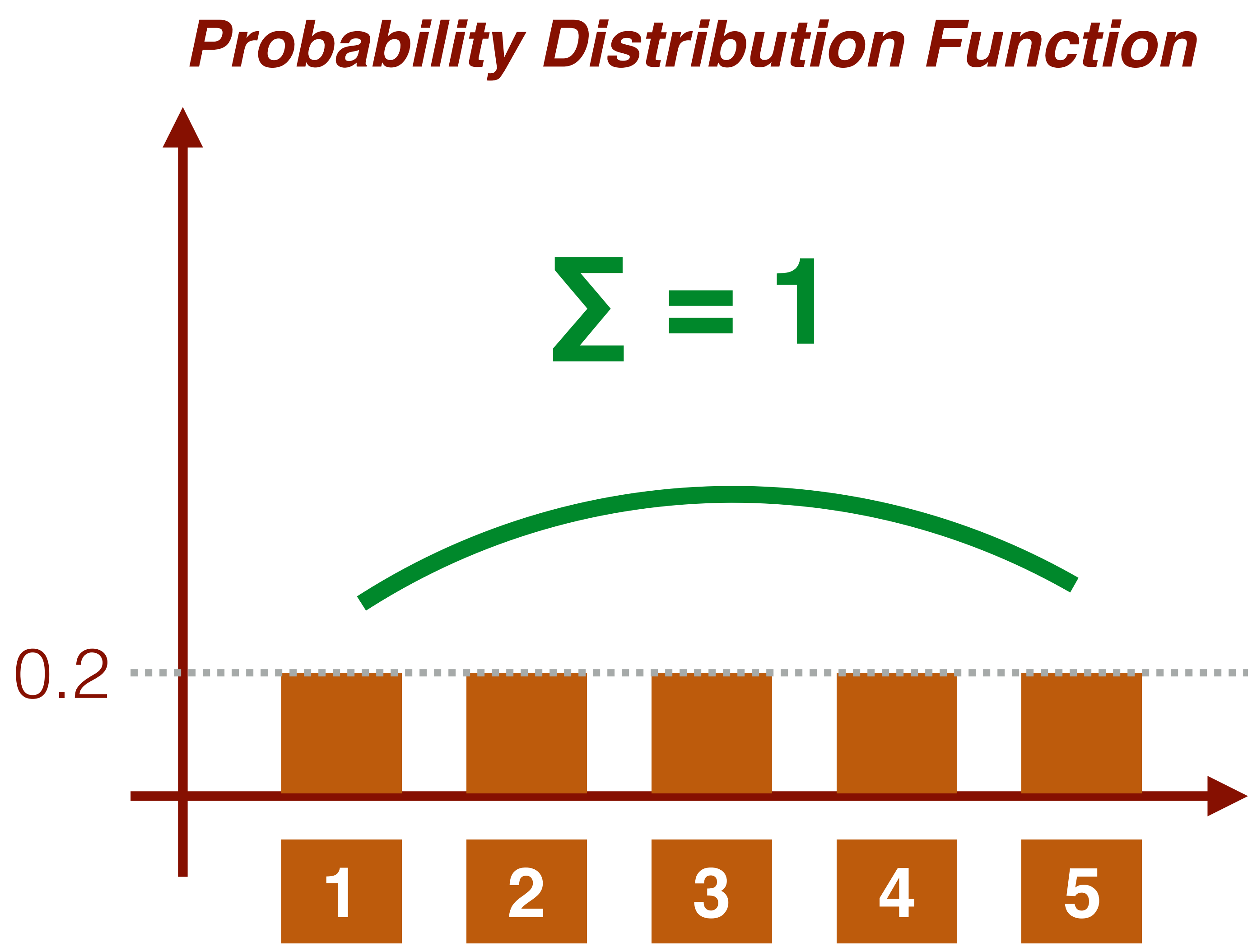
(b) $F(2) = p(1) + p(2)$

$= 1/6 + 1/6 = 1/3$

(c) $P(X>2) = 1 - F(2)$

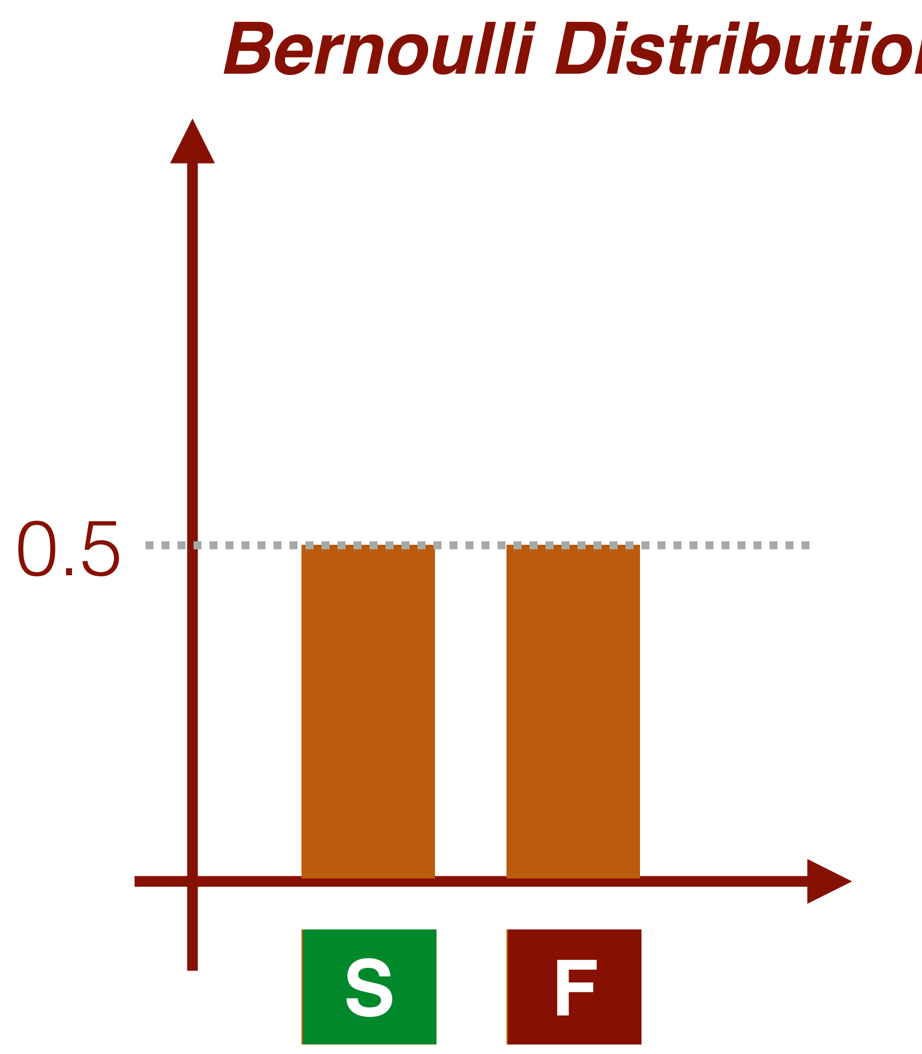
$= 1 - 1/3 = 2/3$

Discrete Uniform Random Variable

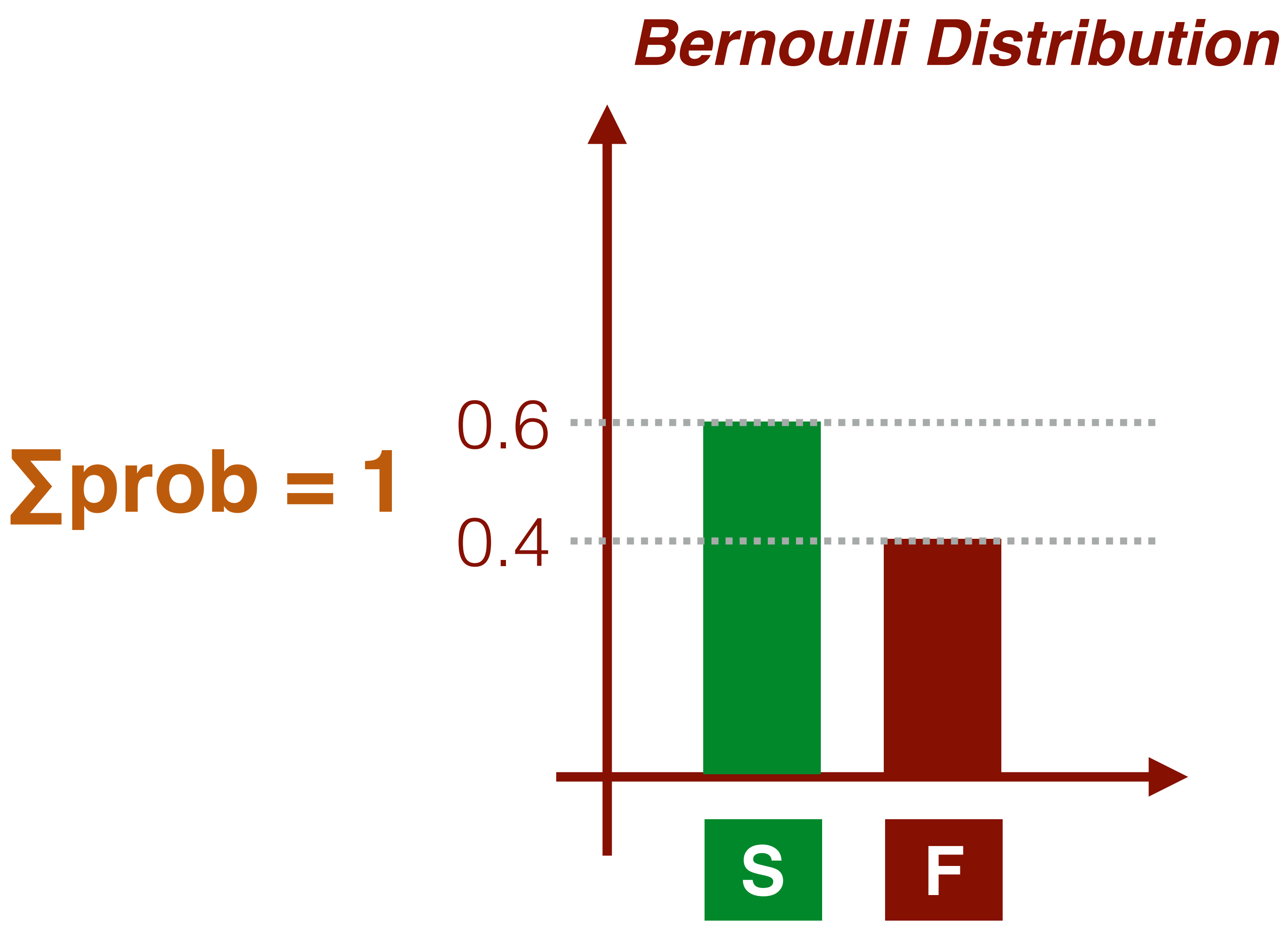


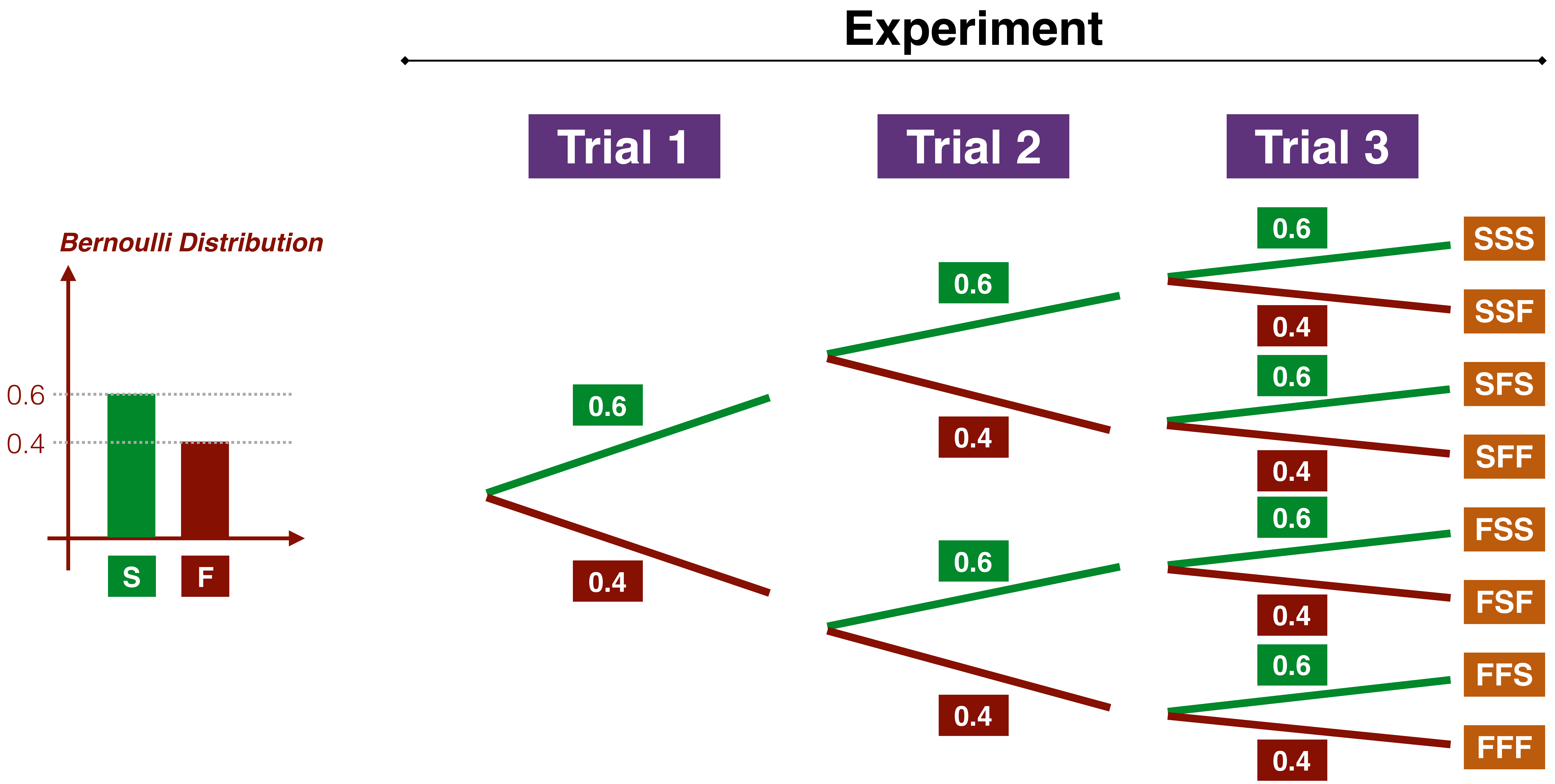
p(1) = P(X=1) = 0.2

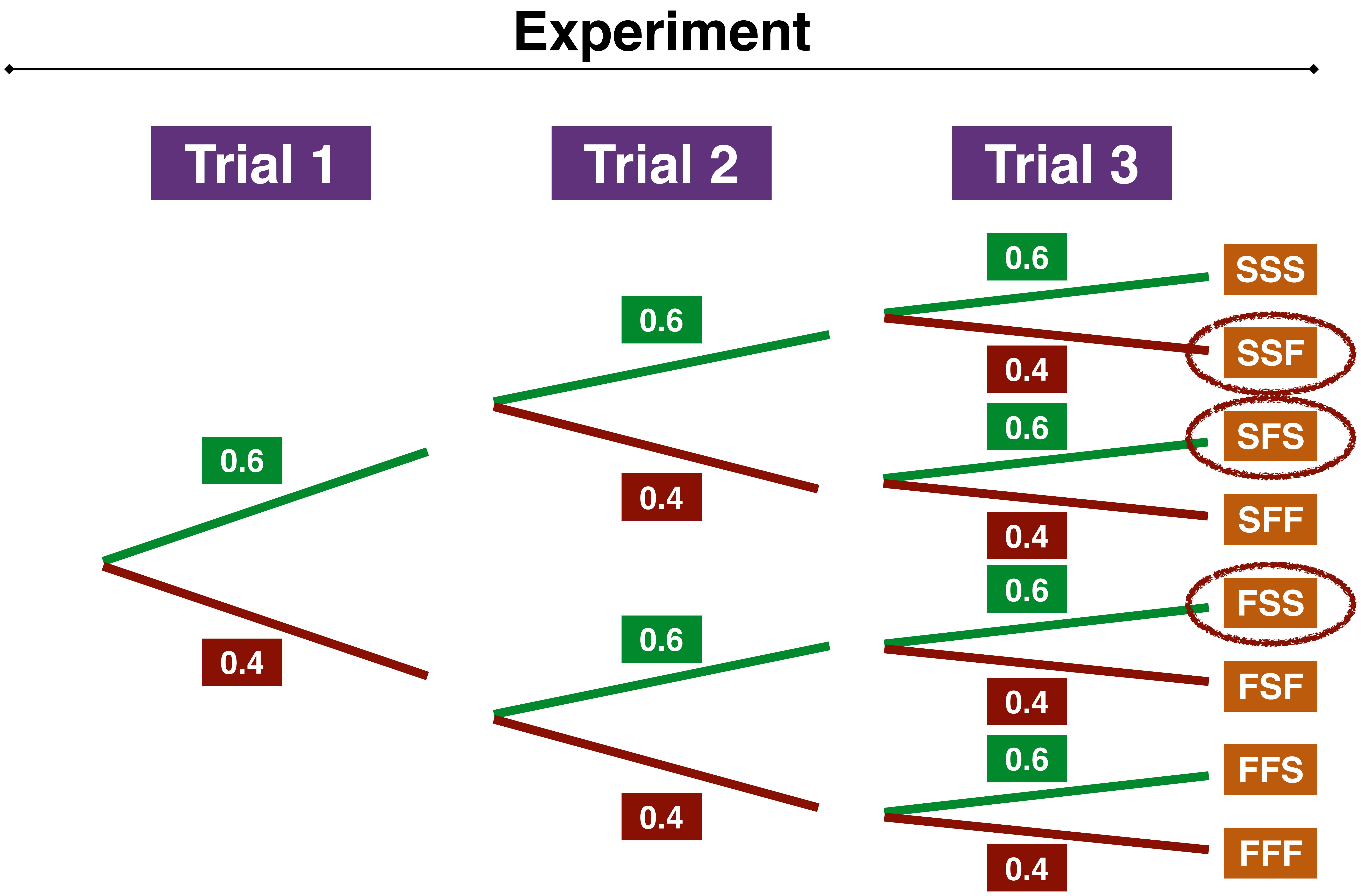
Bernoulli Random Variable



Bernoulli Random Variable




Experiment
Trial 1
Trial 2
Trial 3
Bernoulli Distribution
0.6
0.4
S
F
0.6
0.4
0.6
0.4
0.6
0.4
0.6
0.4
0.6
0.4
0.6
0.4
0.6
0.4
SSS
SSF
SFS
SFF
FSS
FSF
FFS
FFF

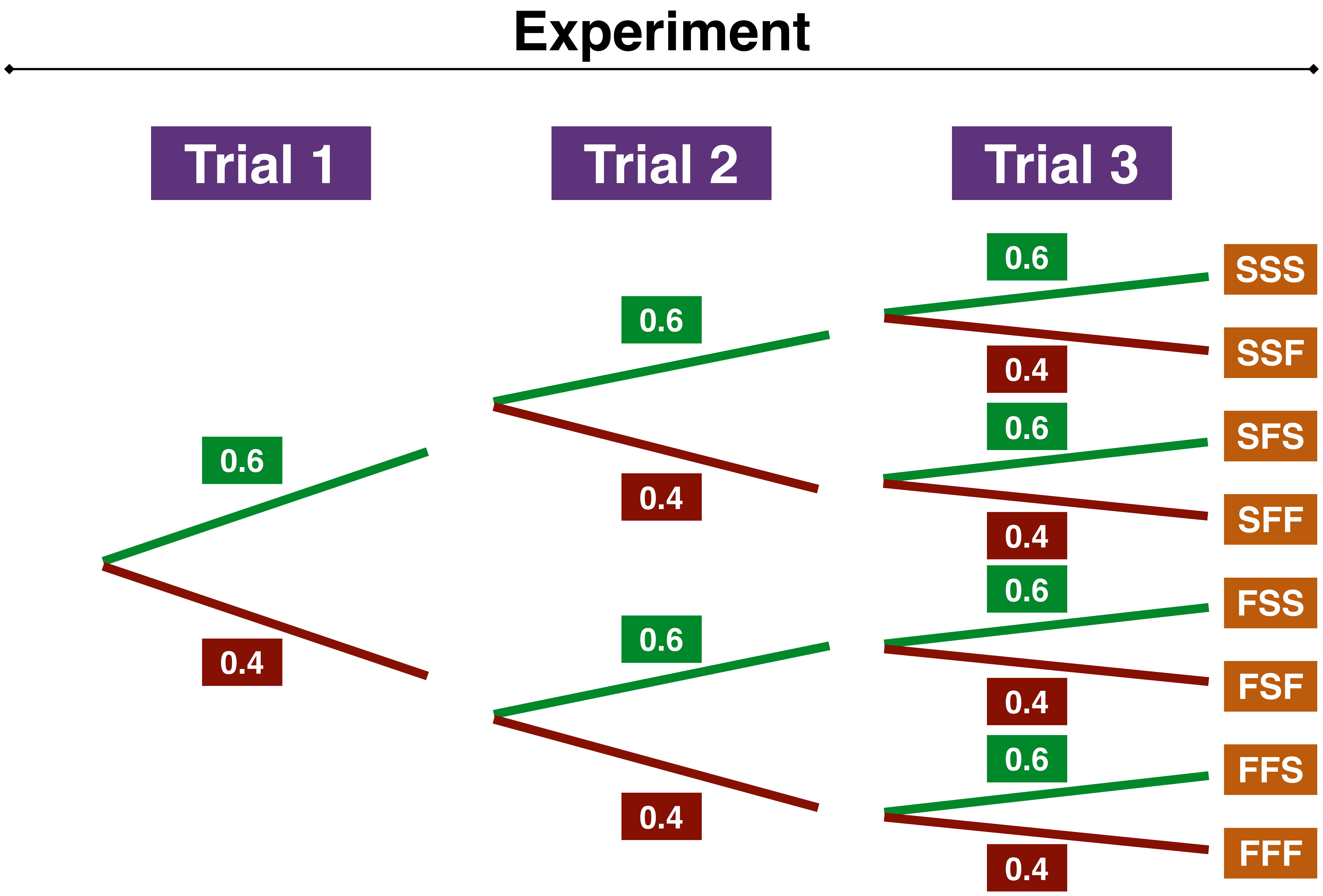


Binomial Distribution

X: number of successes
n: number of trials
p: probability of success

$P(X=2)$

$= 3 \times 0.6^2 \times 0.4$



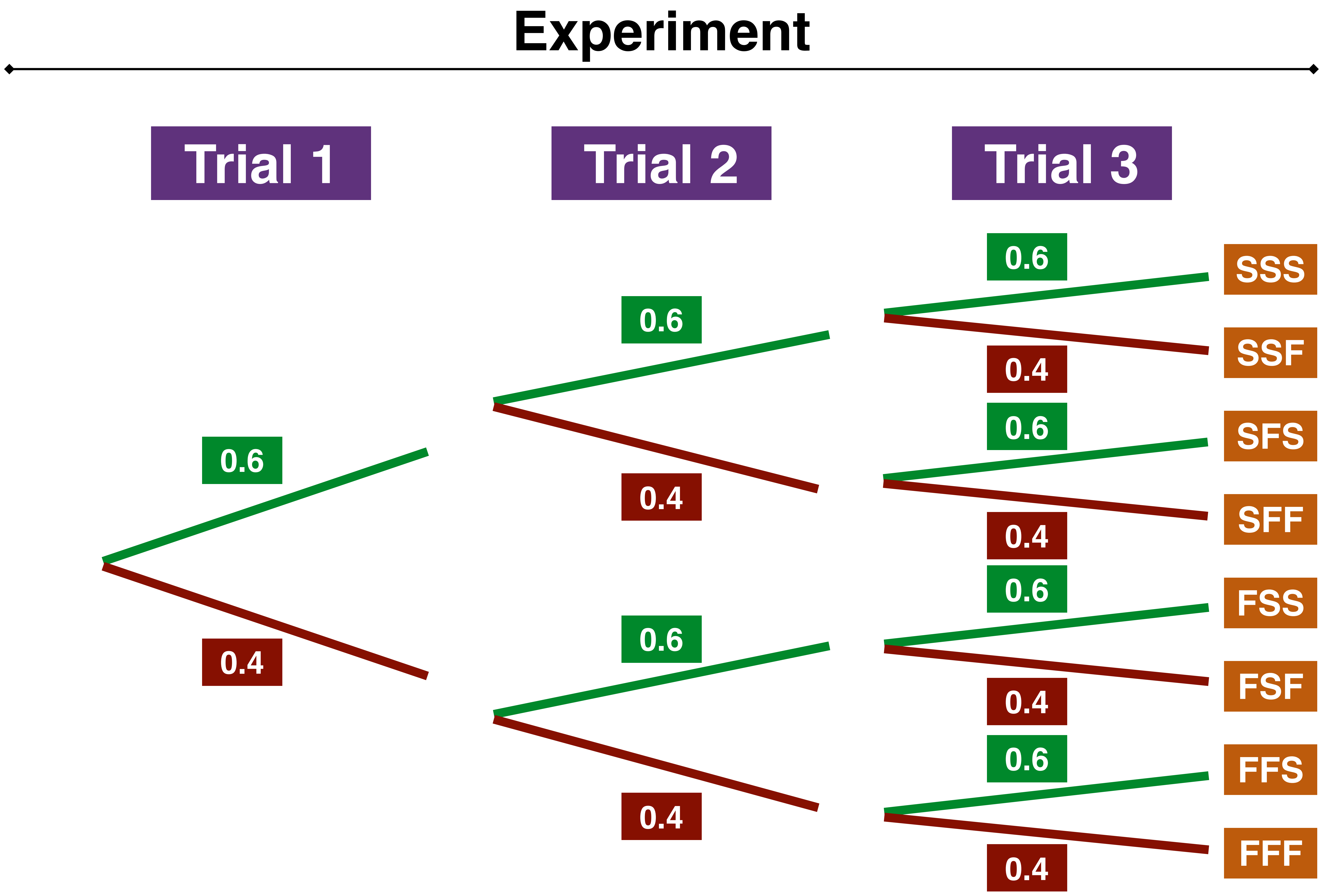
Binomial Distribution

X: number of successes

n: number of trials

p: probability of success

$$p(x) = P(\,X=x\,) = {}_nC_x\, p^x\, (1-p)^{n-x}$$



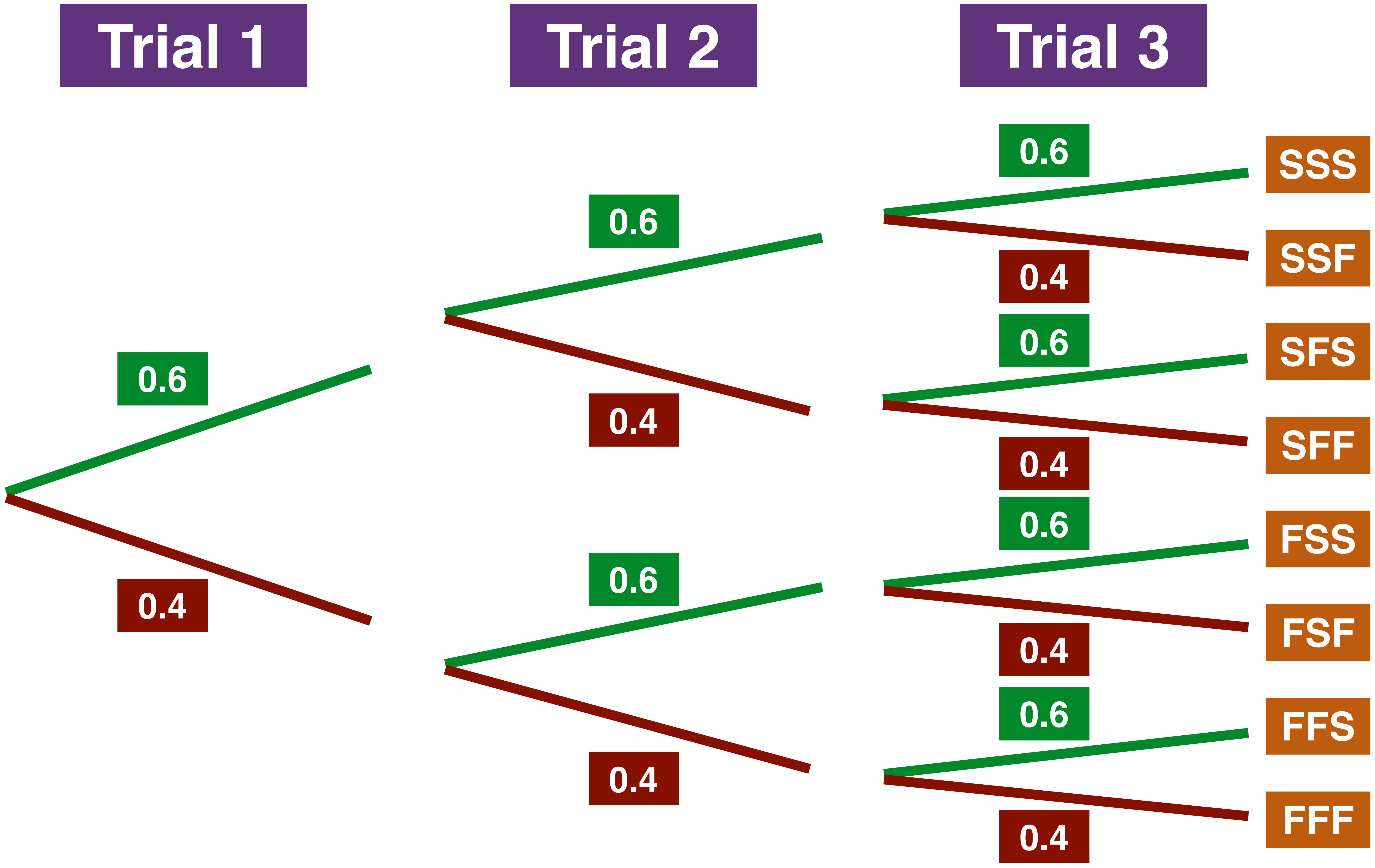
Binomial Distribution

X: number of successes
n: number of trials
p: probability of success

$$p(x) = P(X=x)$$
$$= {}_{n}C_{x}\, p^{x} (1-p)^{n-x}$$

$$E(X) = np$$
$$= 3\text{x}0.6$$
$$= 1.8$$

Experiment



Binomial Distribution

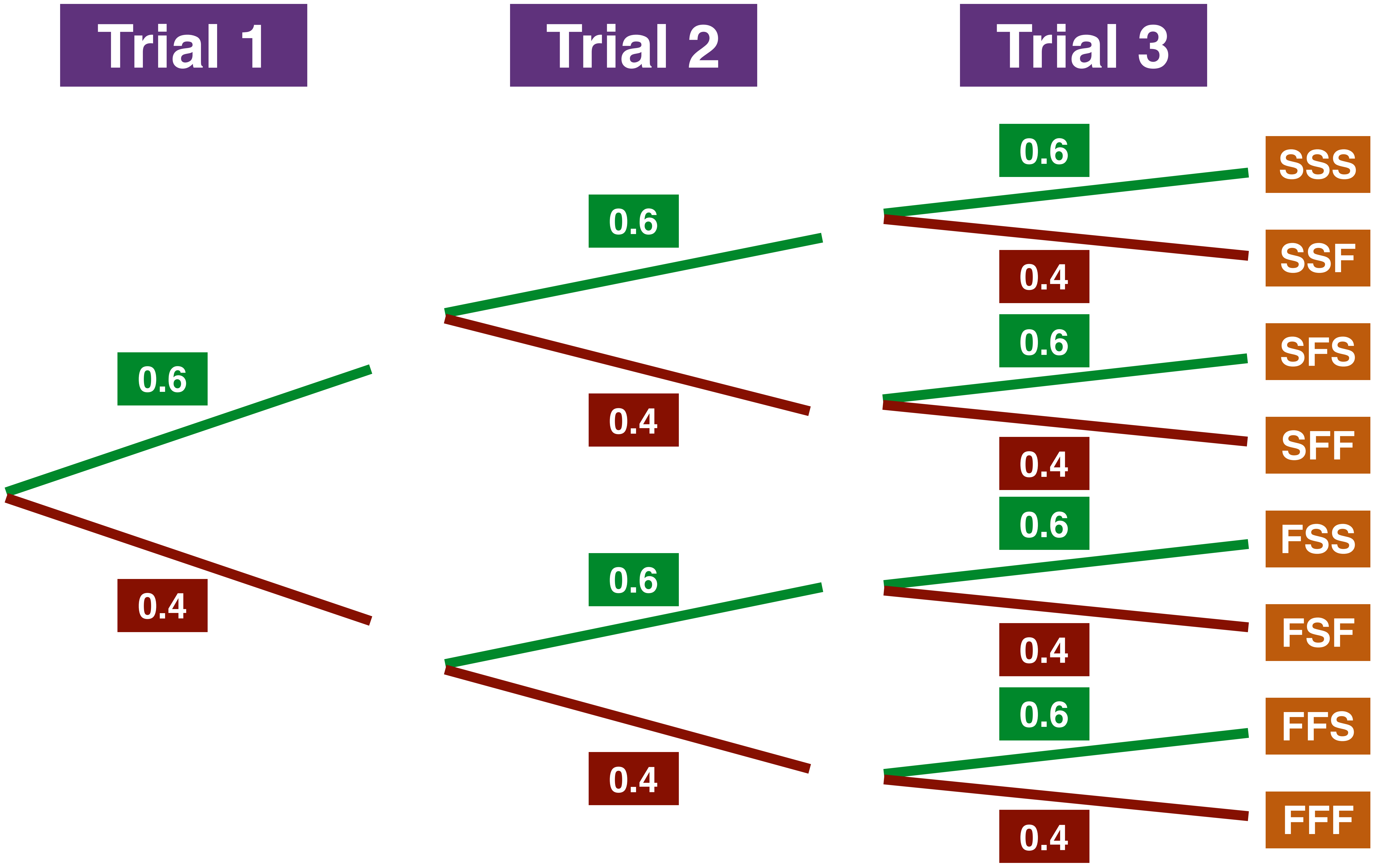
X: number of successes
n: number of trials
p: probability of success

$$p(x) = P(X=x) = {}_{n}C_{x}\, p^{x} (1-p)^{n-x}$$

$$E(X) = np$$

$$Var(X) = np(1-p) = 3 \times 0.6 \times 0.4 = 0.72$$

Experiment



Binomial Distribution

X: number of successes

n: number of trials

p: probability of success

$$p(x) = P(X=x) = {}_nC_x\, p^x (1-p)^{n-x}$$

$$E(X) = np$$

$$Var(X) = np(1-p)$$

A fund manager claims that its US equity fund has a 0.8 probability of keeping to less than +/-1% tracking error from the S&P500 return in any quarter. **Calculate, out of 4 successive quarters, what is the probability of there being at least 3 quarters of success in keeping to this tracking error of +/-1%.**

Tracking Error

Total return of portfolio - Total return of index

	Fund Return (A)	S&P500 Return (B)	Tracking Error (A) - (B)
Q1	+7.2%	+7.7%	-0.5%
Q2	-1.2%	-1.2%	0%
Q3	+8.1%	+8.0%	+0.1%
Q4	-0.7%	+0.7%	-1.4%

Within +/- 1%

X=3

A fund manager claims that its US equity fund has a 0.8 probability of keeping to less than +/-1% tracking error from the S&P500 return in any quarter. **Calculate, out of 4 successive quarters, what is the probability of there being at least 3 quarters of success in keeping to this tracking error of +/-1%.**

$$p(x) = P(X=x)$$
$$= {}_nC_x\, p^x (1-p)^{n-x}$$

Binomial distribution

$n=4 \quad p=0.8$

$$P(X \geq 3) = p(3) + p(4)$$

$$= {}_4C_3(0.8)^3(0.2) + {}_4C_4(0.8)^4$$

$$= 0.41 + 0.41$$

$$= 0.82$$

A fund manager claims that its US equity fund has a 0.8 probability of keeping to less than +/-1% tracking error from the S&P500 return in any quarter. **Out of 4 quarters, what is the expected number of quarters that will NOT meet the tracking error of +/- 1%? What is the standard deviation of this number?**

$$p(x) = P(X=x) = {}_nC_x\, p^x (1-p)^{n-x}$$

Binomial distribution

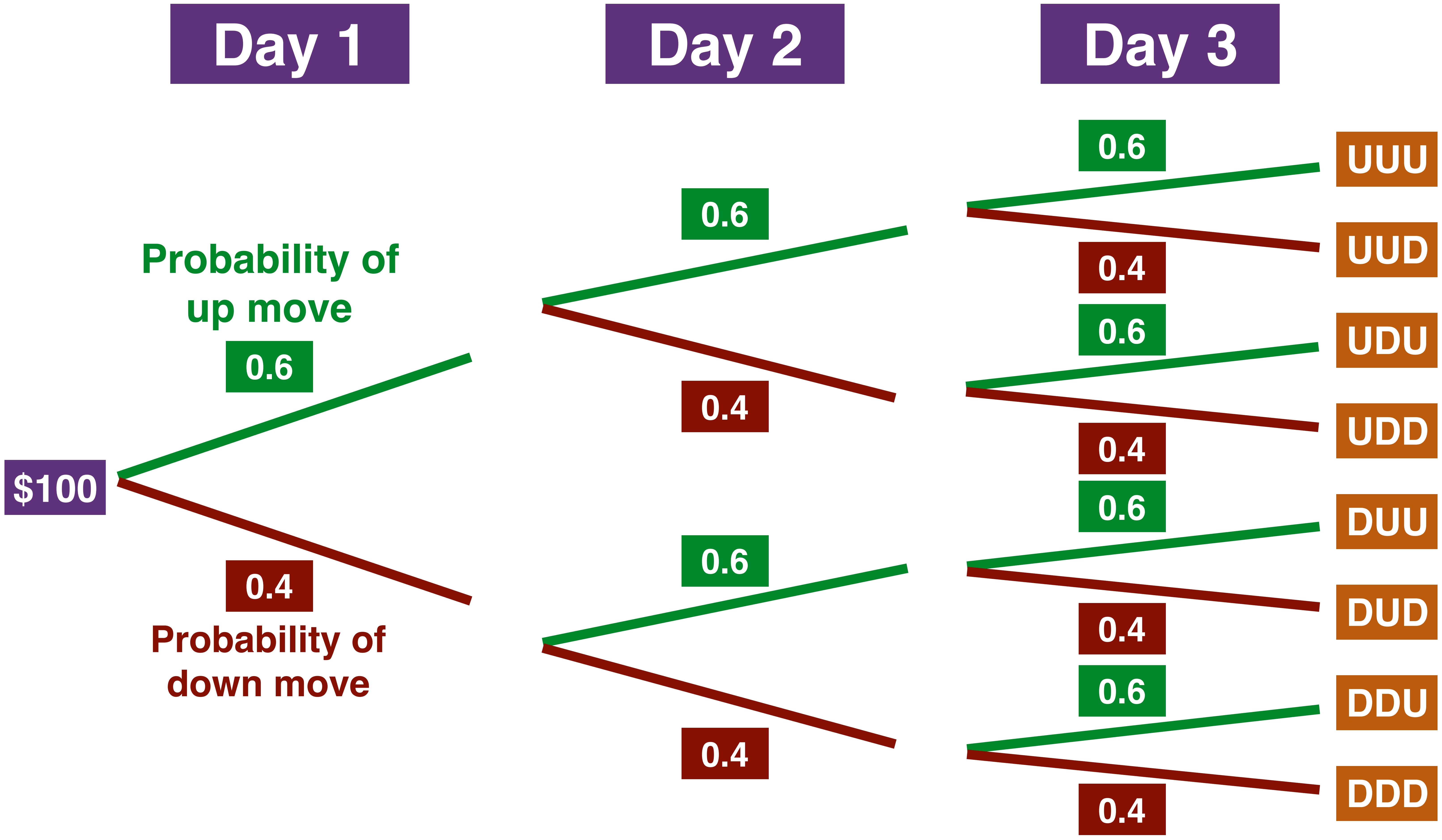
$n=4 \quad p=0.2$

$$E(X) = np = 4\text{x}0.2 = 0.8$$

$$\text{Variance} = np(1-p) = 4\text{x}0.2\text{x}0.8 = 0.64$$

$$\text{Std Dev} = (0.64)^{½} = 0.8$$

Stock Price Movement



Binomial Distribution

X: number of up moves

n: number of days

p: probability of up move

▲ *1% in up move*

U = 1.01

D = 1/1.01

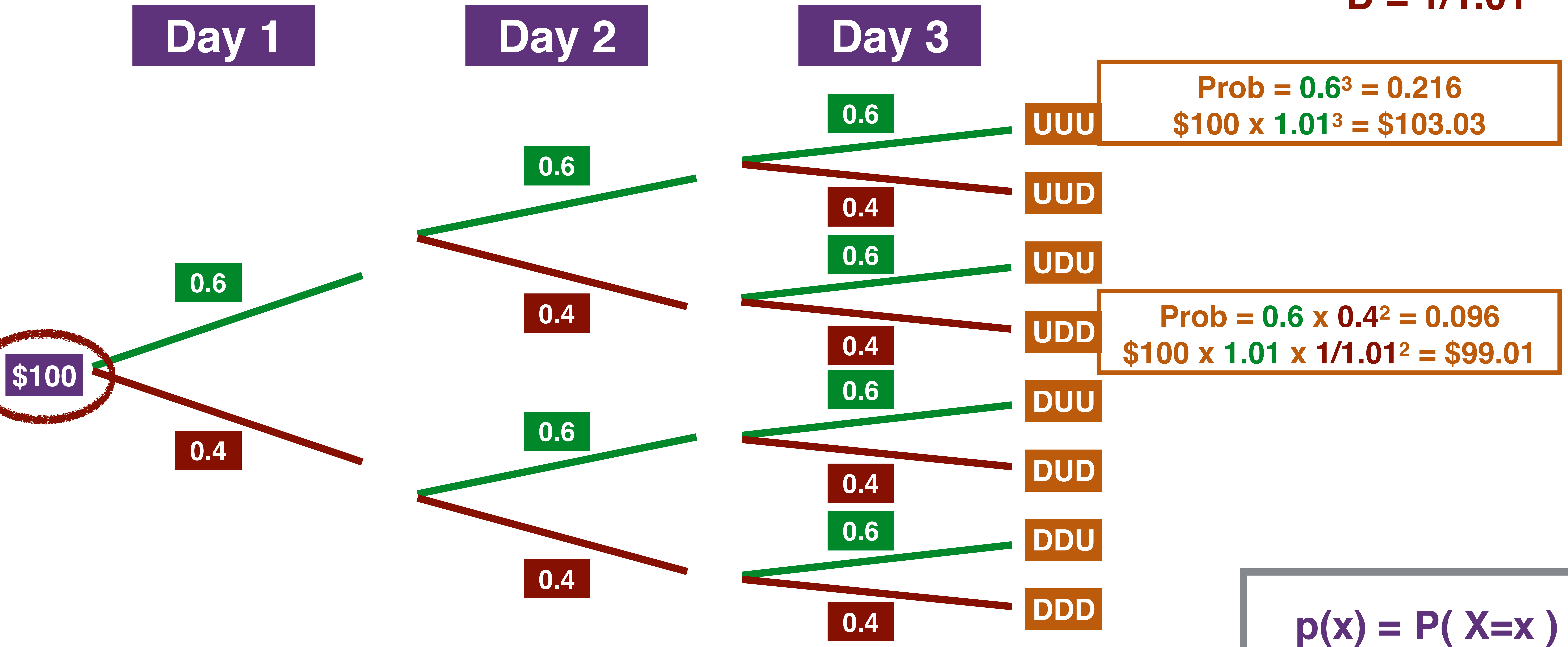
▼ *1% in down move*

$$p(x) = P(X=x)$$
$$= {}_{n}C_{x}\, p^{x} (1-p)^{n-x}$$

Stock Price Movement

U = 1.01

D = 1/1.01



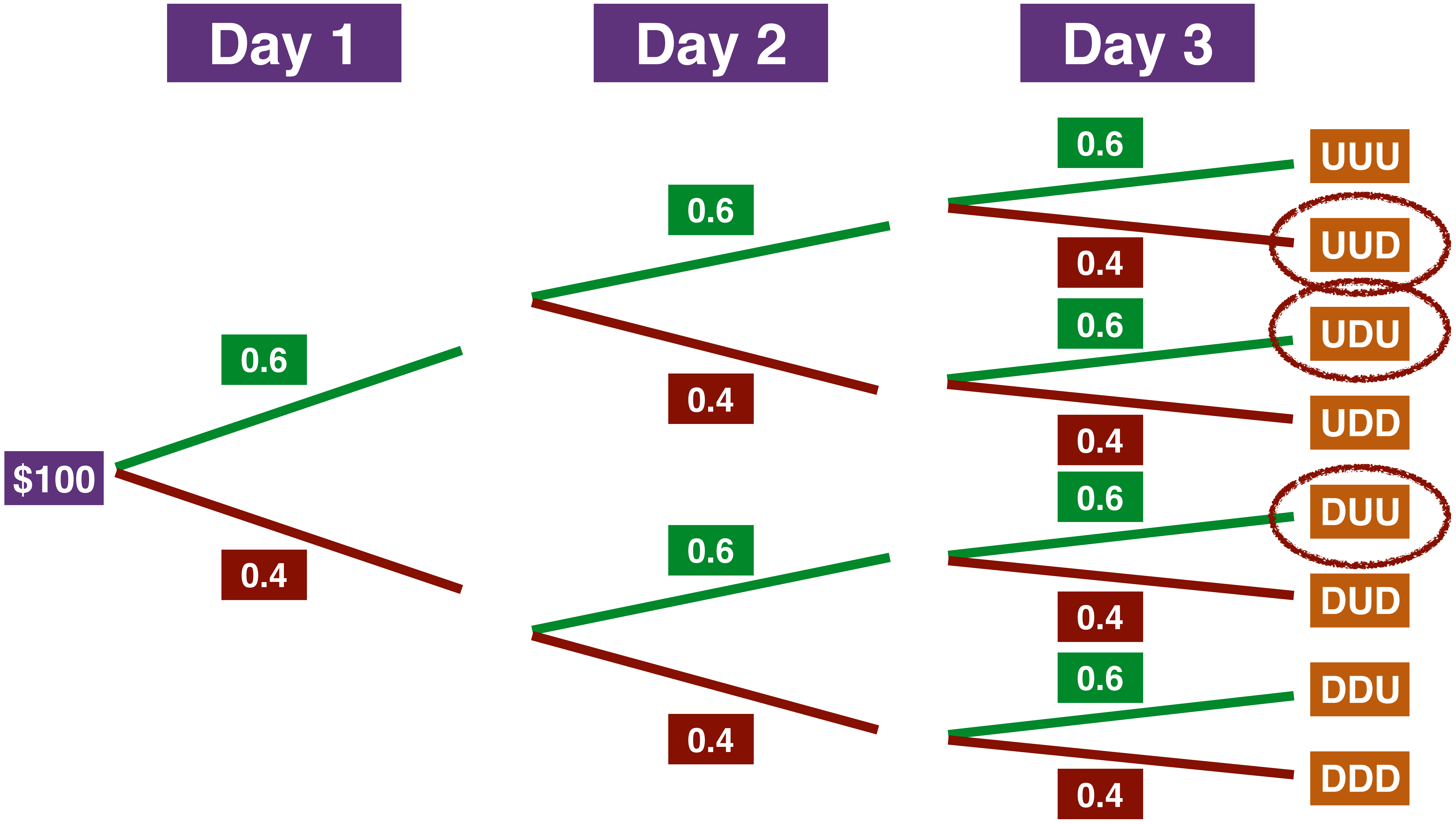
$$p(x) = P(X=x)$$
$$= {}_nC_x\, p^x (1-p)^{n-x}$$

PREP nuggets

Discrete Random Variables

1. Discrete vs Continuous   2. Discrete Uniform Distributions   3. Binomial Distributions

Stock Price Movement



$U = 1.01$

$D = 1/1.01$

What is the probability of having 2 up days and 1 down day? What is the expected price in this case?

$$p(2) = {}_3C_2\,(0.6)^2\,(0.4)$$

$$= 0.432$$

Expected price

$$= \$100 \times 1.01^2 \times 1/1.01$$

$$= \$101$$

$$p(x) = P(X=x)$$
$$= {}_nC_x\, p^x\, (1-p)^{n-x}$$

PREP nuggets

prepnuggets.com