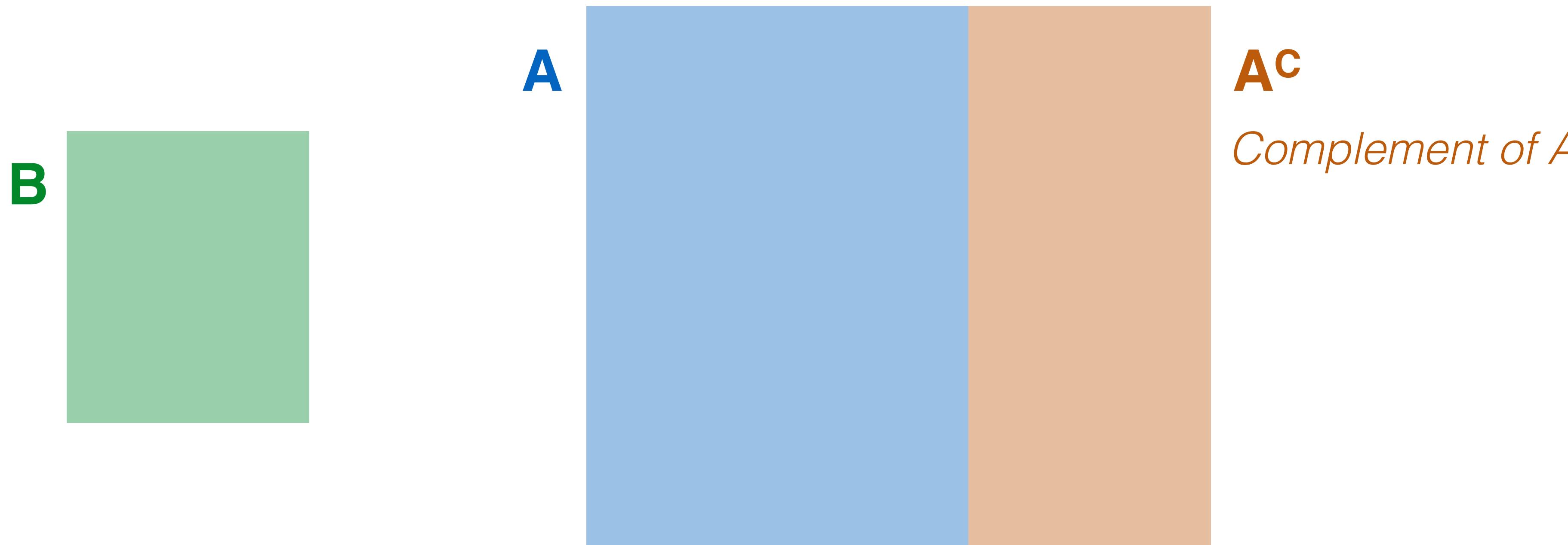


Probability Concepts

Bayes' Formula

A and A^c

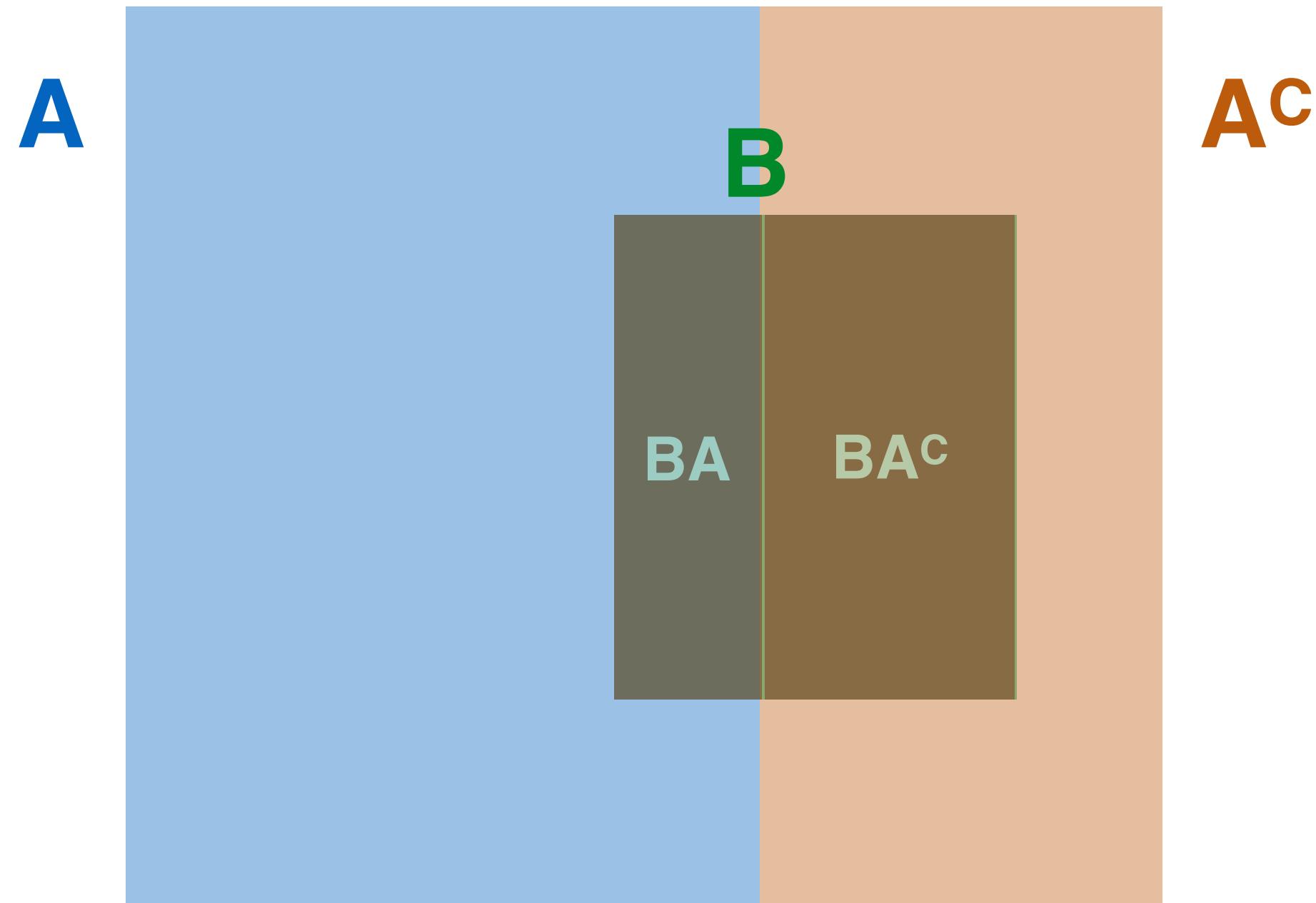
Mutually exclusive and Exhaustive Events



Total Probability Rule

Applying
Multiplication rule

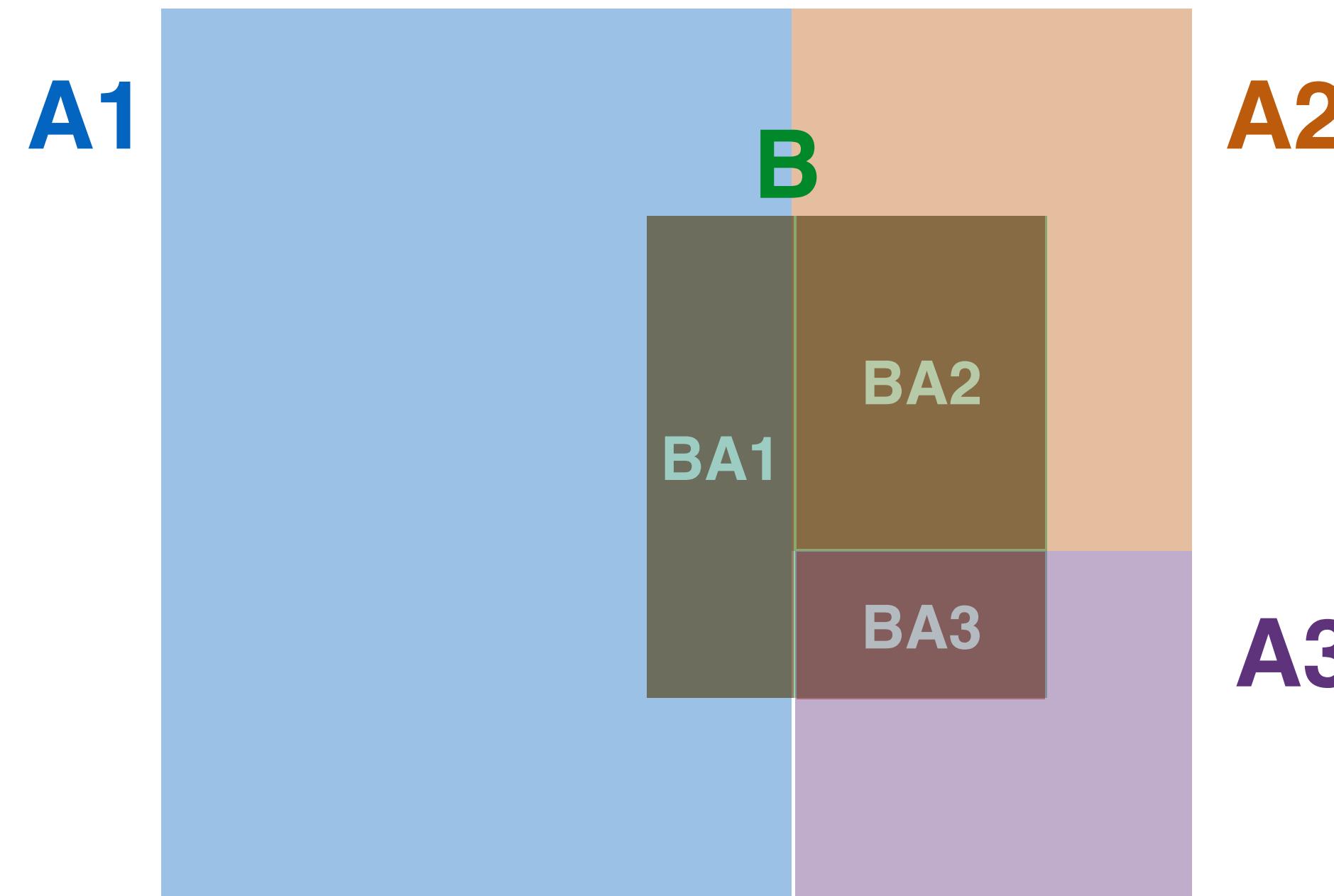
$$\begin{aligned}P(B) &= P(BA) + P(BA^c) \\P(B) &= P(B | A) \times P(A) + P(B | A^c) \times P(A^c)\end{aligned}$$



Total Probability Rule

$$P(B) = P(BA1) + P(BA2) + P(BA3)$$

$$P(B) = P(B | A1) \times P(A1) + P(B | A2) \times P(A2) + P(B | A3) \times P(A3)$$

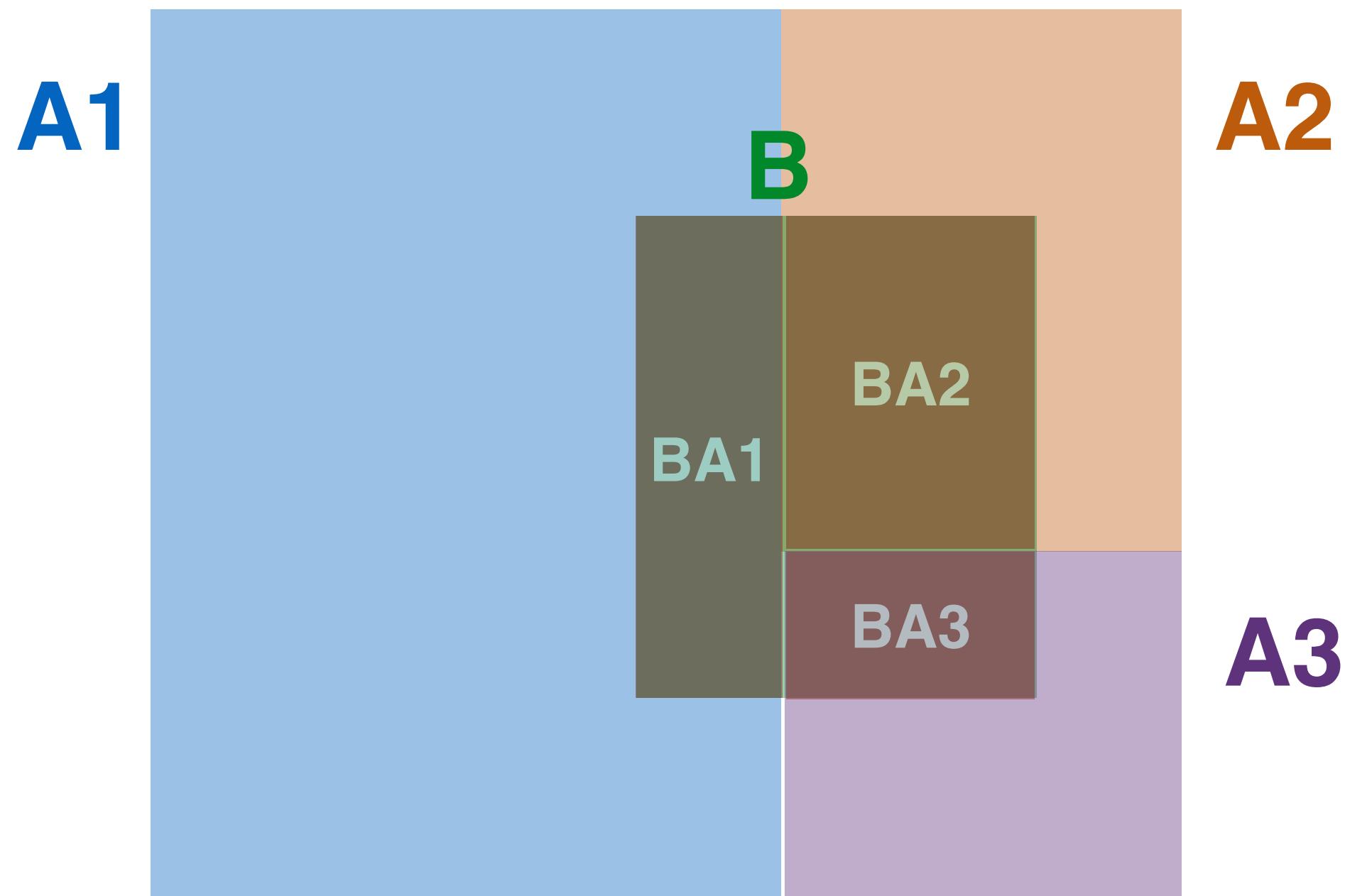


*A₁, A₂ and A₃ are **mutually exclusive** and **exhaustive***
Bayes' Formula

Total Probability Rule

$$\begin{aligned} P(B) &= P(B | A_1) \times P(A_1) + P(B | A_2) \times P(A_2) + P(B | A_3) \times P(A_3) \\ &= 0.55 \times 0.9 + 0.8 \times 0.05 + 0.3 \times 0.05 \\ &= 0.55 \end{aligned}$$

Probability of DJIA going up in any given month is 0.55



$P(A_1)=0.9$ Interest rate unchanged

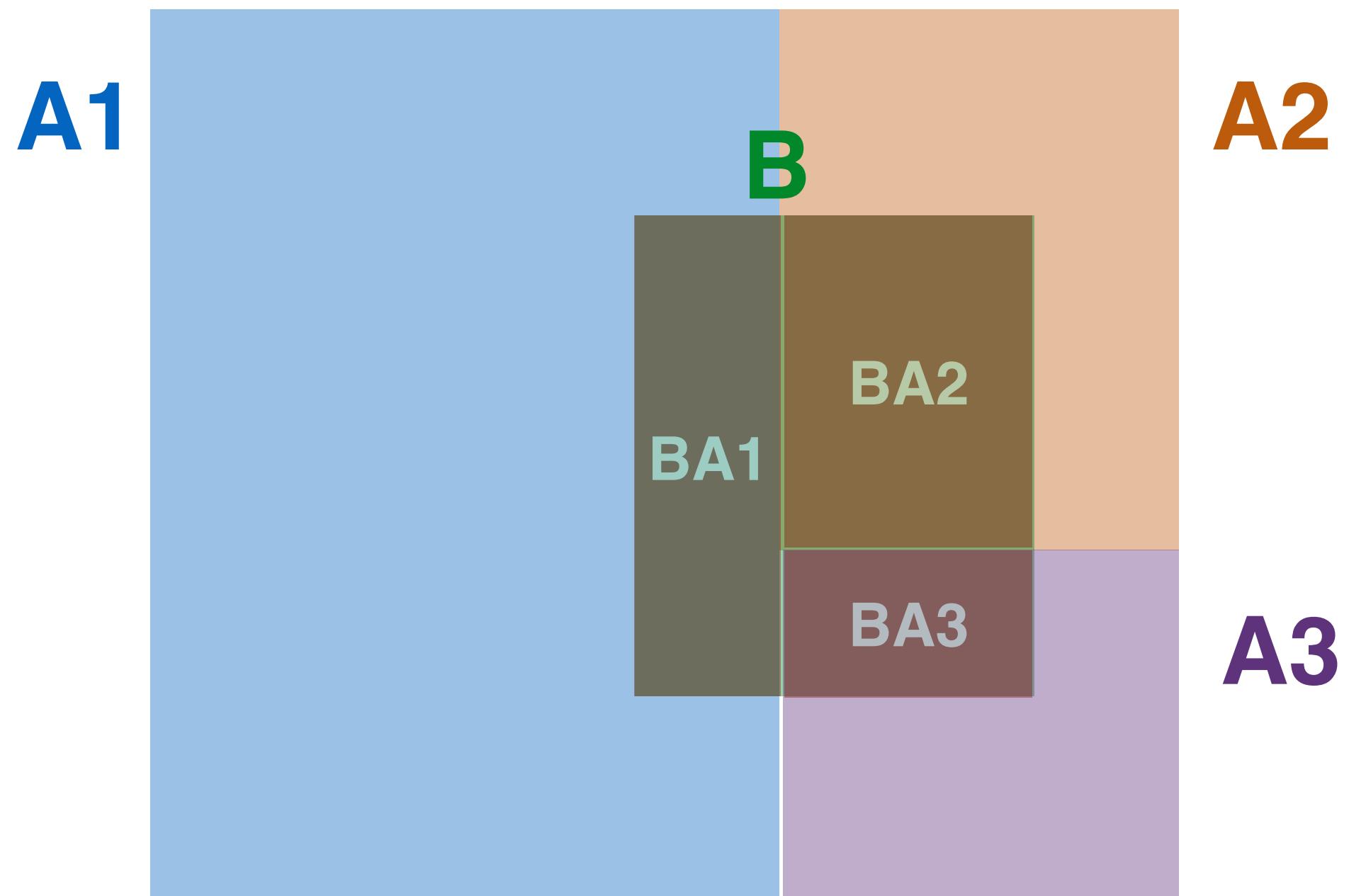
$P(A_2)=0.05$ Interest rate cut

$P(A_3)=0.05$ Interest rate increase

$P(B | A_1)=0.55$ DJIA up given A1

$P(B | A_2)=0.8$ DJIA up given A2

$P(B | A_3)=0.3$ DJIA up given A3



$P(A_1)=0.9$ Interest rate unchanged

$P(A_2)=0.05$ Interest rate cut

$P(A_3)=0.05$ Interest rate increase

$P(B | A_1)=0.55$ DJIA up given A1

$P(B | A_2)=0.8$ DJIA up given A2

$P(B | A_3)=0.3$ DJIA up given A3

This Month

**B**

Likelihood of DJIA going up
 $P(B) = 0.55$

By studying movement of DJIA this month

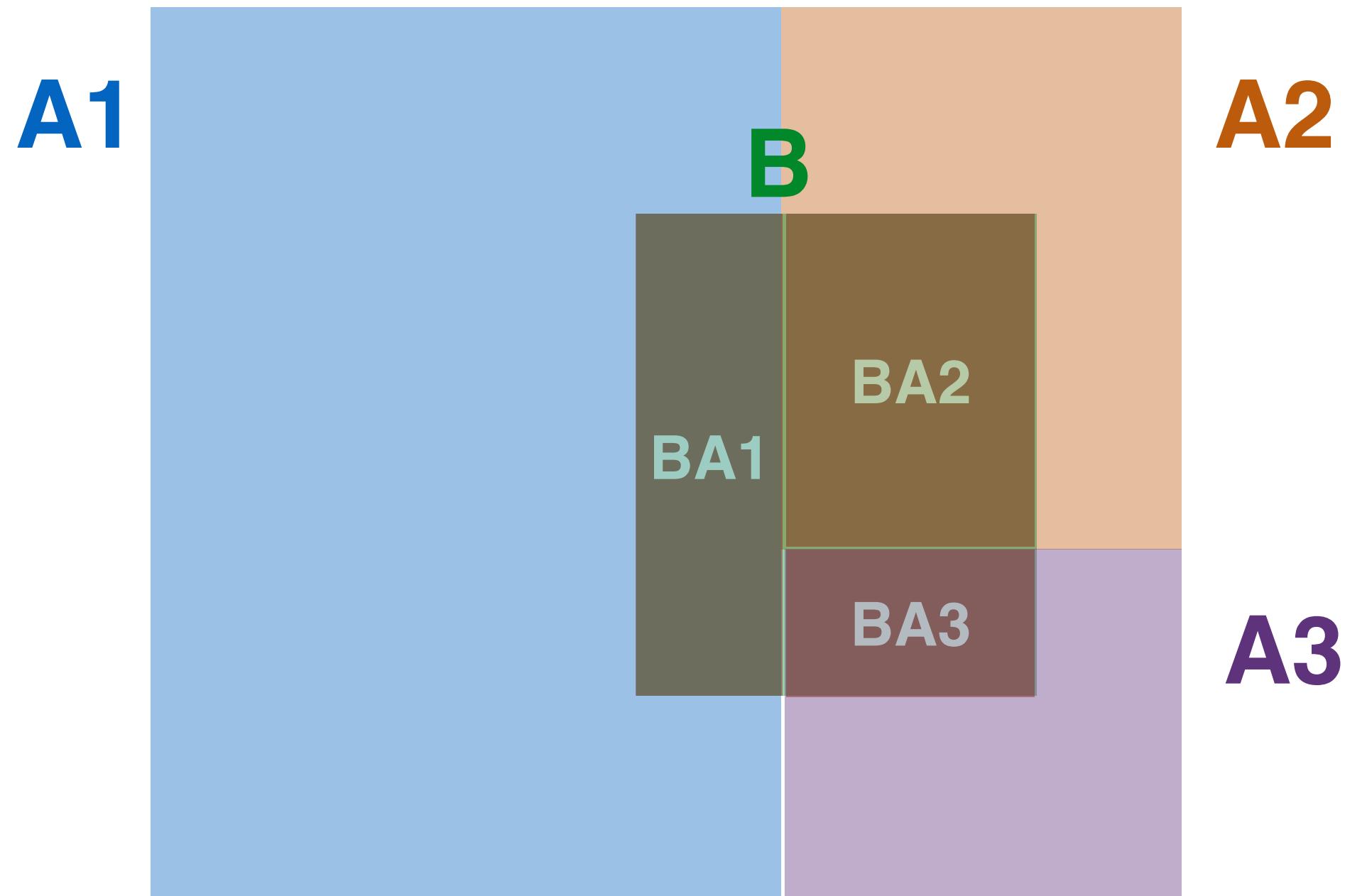
Estimate

Next Month

A

What are the likelihood of changes in interest rates?

Probability of a particular interest rate decision next month



P(A1)=0.9 Interest rate unchanged

P(A2)=0.05 Interest rate cut

P(A3)=0.05 Interest rate increase

P(B | A1)=0.55 DJIA up given A1

P(B | A2)=0.8 DJIA up given A2

P(B | A3)=0.3 DJIA up given A3

This Month

**B**

DJIA has gone UP! 
 $P(B) = 0.55$

By studying movement of DJIA this month

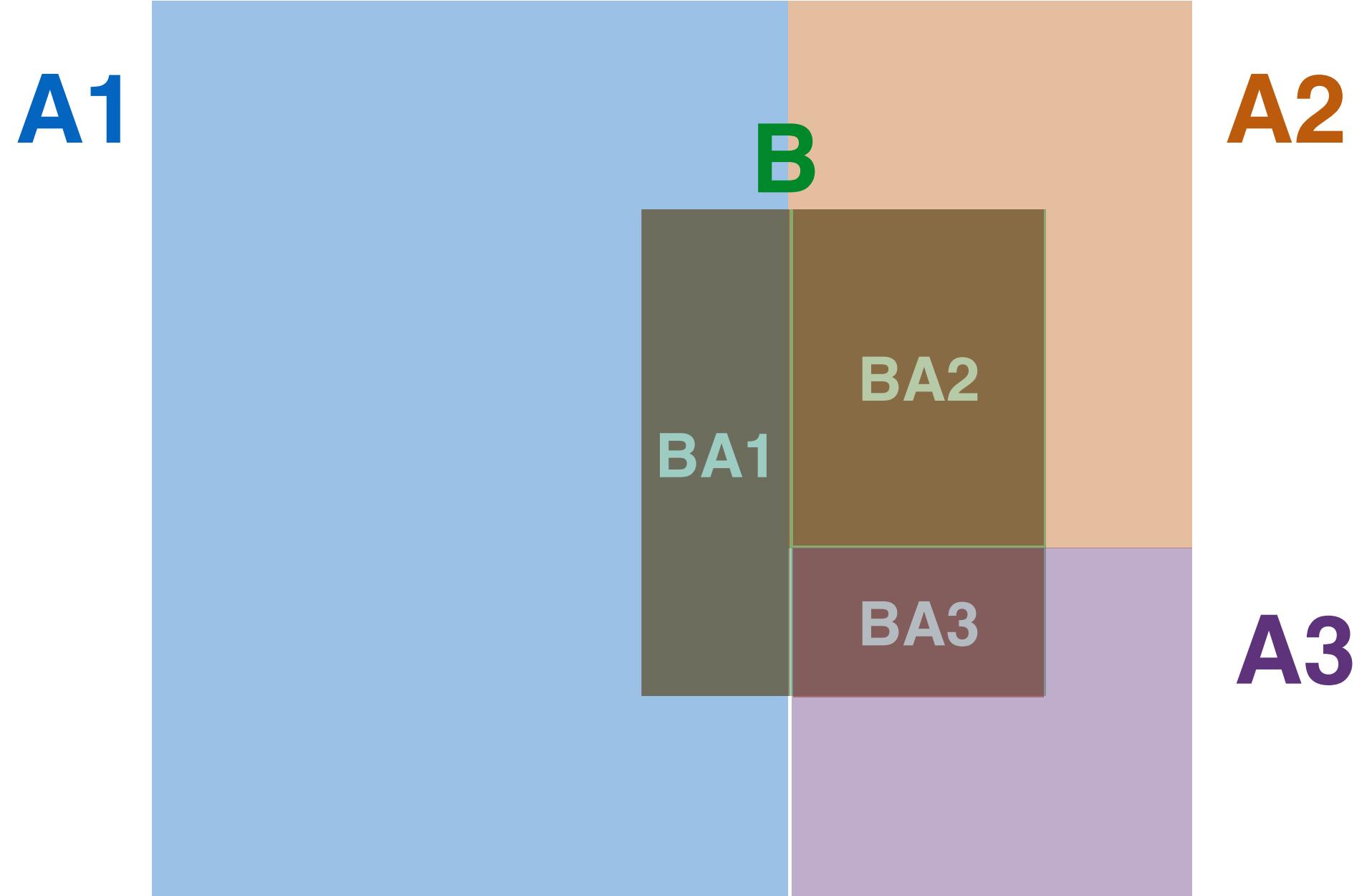
Estimate

Next Month

What is the likelihood of a rate cut?

A2

Probability of a particular interest rate decision next month



P(A1)=0.9 Interest rate unchanged

P(A2)=0.05 Interest rate cut

P(A3)=0.05 Interest rate increase

P(B | A1)=0.55 DJIA up given A1

P(B | A2)=0.8 DJIA up given A2

P(B | A3)=0.3 DJIA up given A3

This Month



B

DJIA has gone UP!
 $P(B) = 0.55$

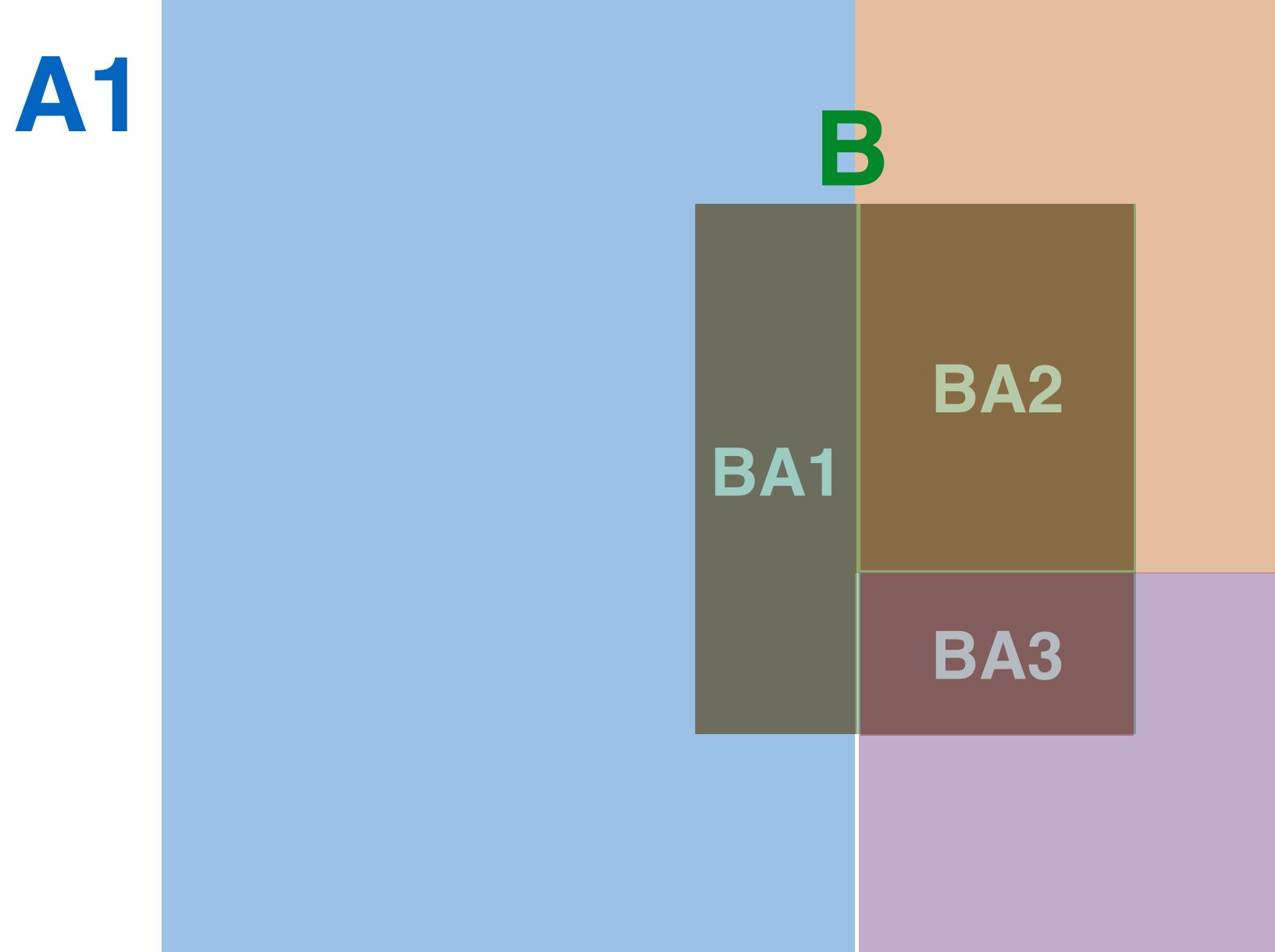
Estimate

Next Month

A2

What is the likelihood of a
rate cut?

New information to update probability



Prior probabilities

- | | | |
|---|---|--------------------------------------|
| <p>A1</p> <p>B</p> <p>BA1</p> <p>BA2</p> <p>BA3</p> <p>A3</p> | <p>A2</p> <p>P(A1)=0.9 Interest rate unchanged</p> <p>P(A2)=0.05 Interest rate cut</p> <p>P(A3)=0.05 Interest rate increase</p> | <p>In the following month</p> |
|---|---|--------------------------------------|

- | | |
|---|--------------------------------------|
| <p>P(B A1)=0.55 DJIA up given A1</p> <p>P(B A2)=0.8 DJIA up given A2</p> <p>P(B A3)=0.3 DJIA up given A3</p> | <p>In the following month</p> |
|---|--------------------------------------|

This Month



B

DJIA has gone UP! ↑
 $P(B) = 0.55$

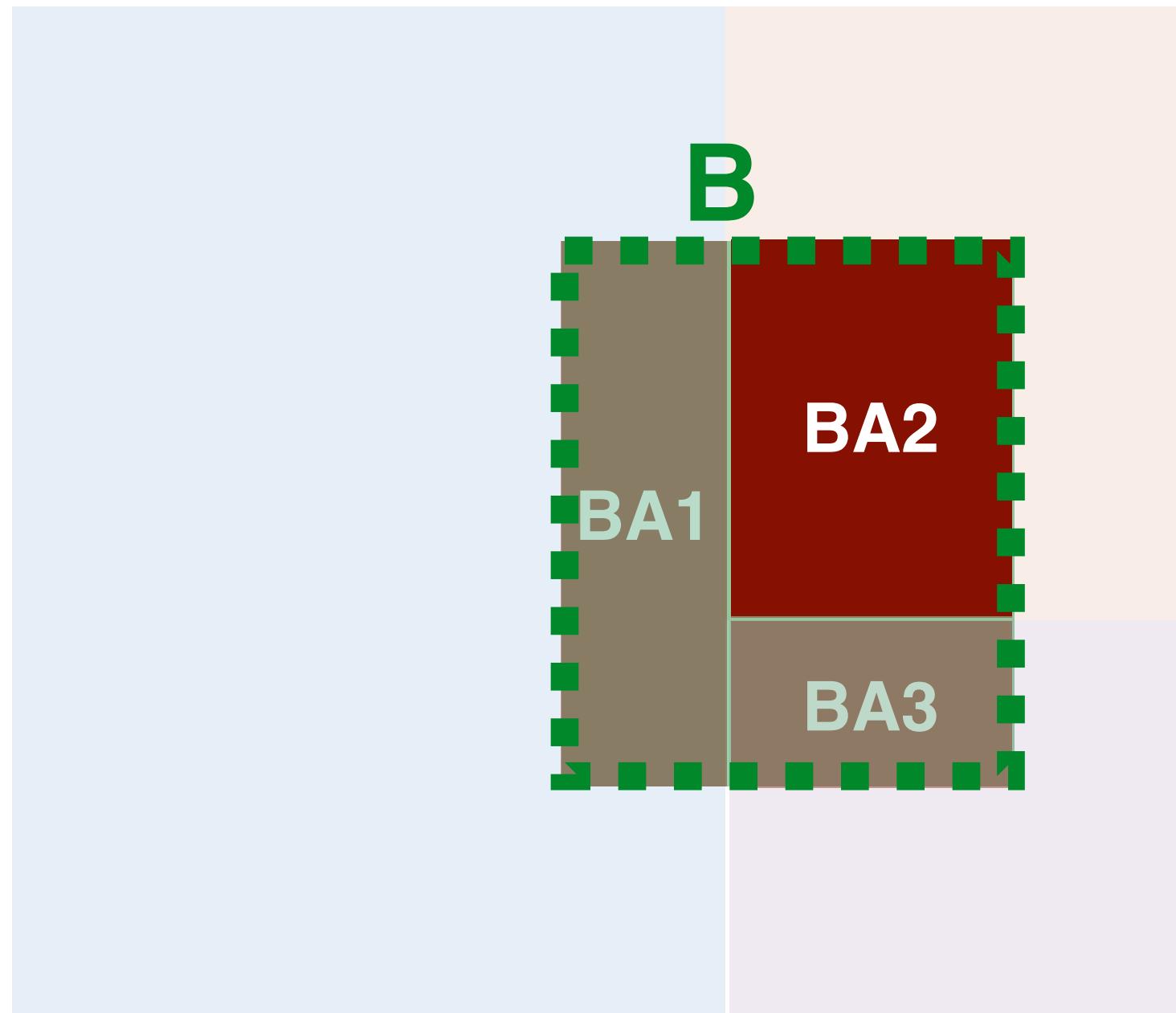
Estimate

Next Month

A2

What is the likelihood of a
rate cut?

A1



A2

A3

Multiplication rule

$P(A2B)$

$$= P(A2 | B) \times P(B)$$

Prior probabilities

$P(A1)=0.9$ Interest rate unchanged

$P(A2)=0.05$ Interest rate cut

$P(A3)=0.05$ Interest rate increase

$P(B | A1)=0.55$ DJIA up given A1

$P(B | A2)=0.8$ DJIA up given A2

$P(B | A3)=0.3$ DJIA up given A3

This Month



B

DJIA has gone UP!
 $P(B) = 0.55$

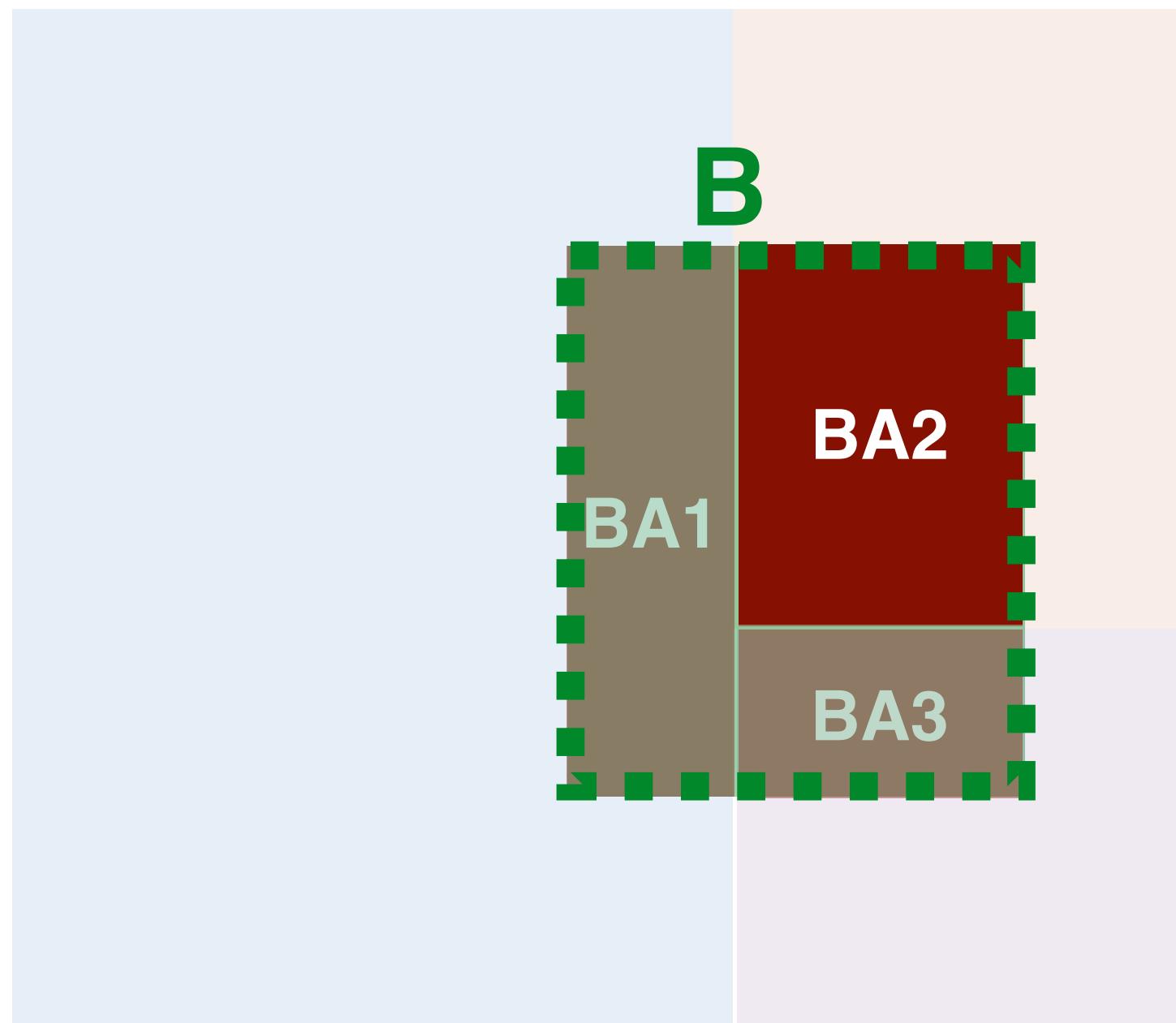
Estimate

Next Month

A2

What is the likelihood of a
rate cut?

A1



Multiplication rule

A2

$$P(A2 | B) = \frac{P(B | A2) \times P(A2)}{P(B)}$$
$$= (0.8 \times 0.05) / 0.55$$
$$= 0.07$$

Prior probabilities

$P(A1)=0.9$ Interest rate unchanged

$P(A2)=0.05$ Interest rate cut

$P(A3)=0.05$ Interest rate increase

$P(B | A1)=0.55$ DJIA up given A1

$P(B | A2)=0.8$ DJIA up given A2

$P(B | A3)=0.3$ DJIA up given A3

This Month



B

DJIA has gone UP! ↑
 $P(B) = 0.55$

Estimate

Next Month

A2

What is the likelihood of a
rate cut?

$$P(A2 | B) = \frac{P(B | A2) \times P(A2)}{P(B)}$$

DJIA has gone up!

$P(A2)=0.05$

Probability of rate cut

New Info
B is TRUE

$P(A2 | B) = 0.07$

Probability of rate cut given that
we know DJIA has gone up!

This Month



B

DJIA has gone UP! ↑
 $P(B) = 0.55$

Estimate

Next Month

A2

What is the likelihood of a
rate cut?

$$P(A2 | B) = \frac{P(B | A2) \times P(A2)}{P(B)}$$

Bayes'
Formula

$$P(Event | Info) = \frac{P(Info | Event) \times P(Event)}{P(Info)}$$

Event: I/R increase
Info: DJIA up

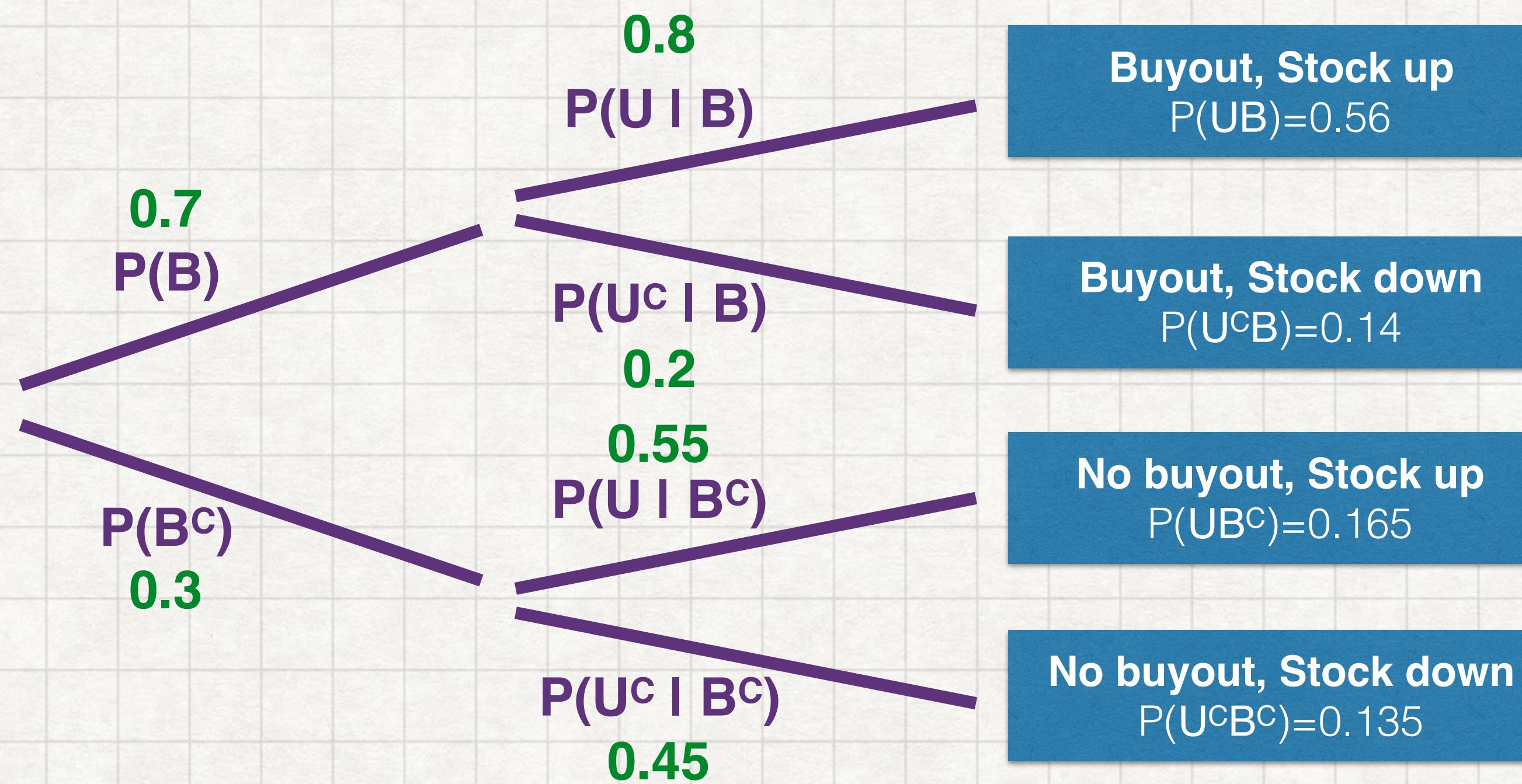
$P(A2) = 0.05$

New Info
B is TRUE

$P(A2 | B) = 0.07$

Robert Holmes, CFA estimates that the probability of a buyout (B) of Pinnacle Inc is 70%. If it does, there is an 80% chance that the stock of Pinnacle Inc will go up (U). If there is no buyout, the chances that the stock will go up is 55%.

Given that the stock of Pinnacle went up, what is the updated probability of a buyout?



Robert Holmes, CFA estimates that the probability of a buyout (B) of Pinnacle Inc is 70%. If it does, there is an 80% chance that the stock of Pinnacle Inc will go up (U). If there is no buyout, the chances that the stock will go up is 55%.

Given that the stock of Pinnacle went up, what is the updated probability of a buyout?

Bayes'
Formula

$$P(\text{Event} \mid \text{Info}) = \frac{P(\text{Info} \mid \text{Event}) \times P(\text{Event})}{P(\text{Info})}$$

Event: Buyout (B)

Info: Stock went up (U)

Robert Holmes, CFA estimates that the probability of a buyout (B) of Pinnacle Inc is 70%. If it does, there is an 80% chance that the stock of Pinnacle Inc will go up (U). If there is no buyout, the chances that the stock will go up is 55%.

Given that the stock of Pinnacle went up, what is the updated probability of a buyout?

Bayes'
Formula

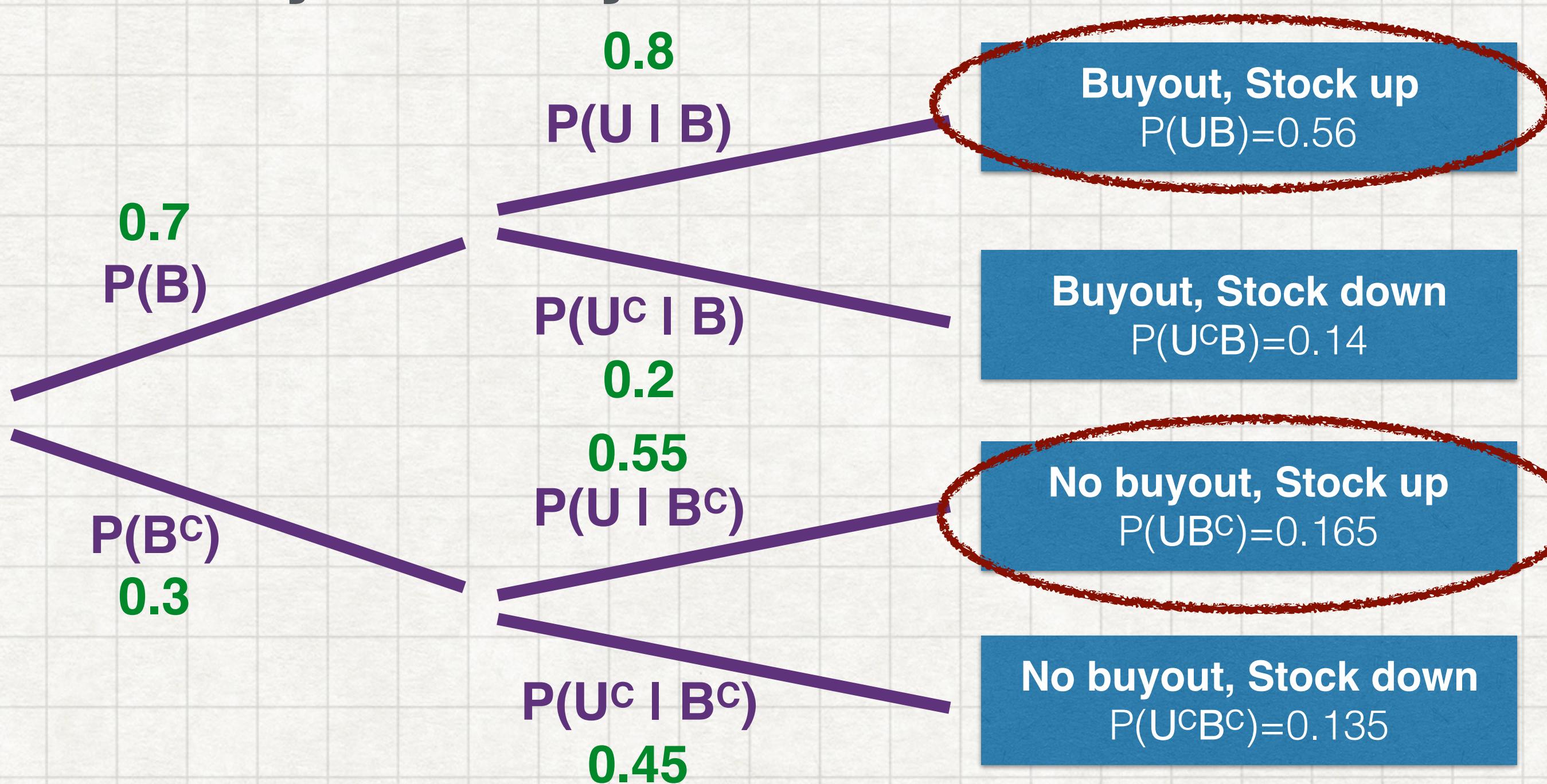
$$P(B | U) = \frac{P(U | B) \times P(B)}{P(U)}$$

Event: Buyout (B)

Info: Stock went up (U)

Robert Holmes, CFA estimates that the probability of a buyout (B) of Pinnacle Inc is 70%. If it does, there is an 80% chance that the stock of Pinnacle Inc will go up (U). If there is no buyout, the chances that the stock will go up is 55%.

Given that the stock of Pinnacle went up, what is the updated probability of a buyout?



$$\begin{aligned}
 P(B|U) &= \frac{P(U|B) \times P(B)}{P(U)} \\
 &= \frac{P(U|B) \times P(B)}{P(UB) + P(U^cB)} \\
 &\quad \text{Total probability rule} \\
 &= \frac{0.8 \times 0.7}{0.56 + 0.165} \\
 &= 0.77
 \end{aligned}$$



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