



# Time Value of Money

## Annuities



# Time Value of Money

## Annuities

1. Ordinary Annuities
2. Annuity Due
3. Perpetuities

# ***Annuity***

***A Finite set of Level Sequential cash flows.***

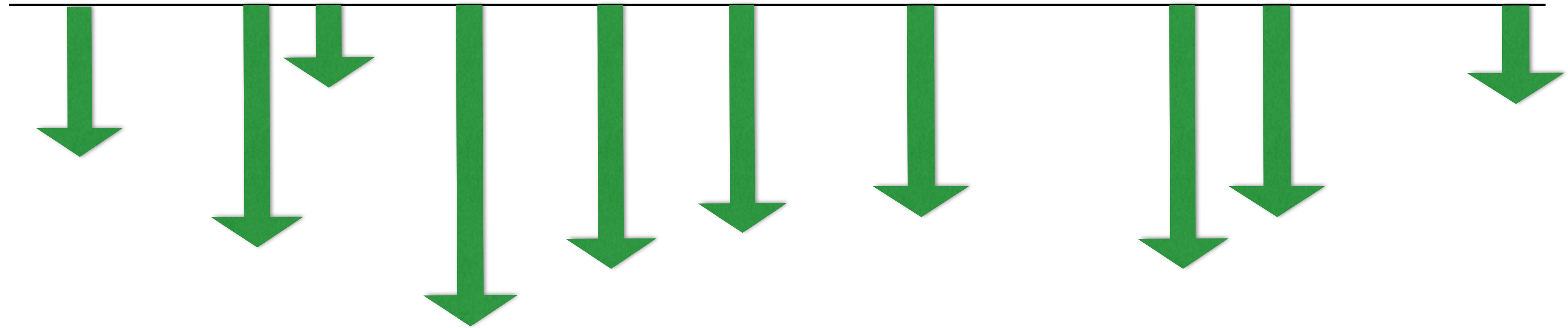


## Annuities

1. Ordinary Annuities

2. Annuity Due

3. Perpetuities



***Finite***

***Level***

***Sequential***

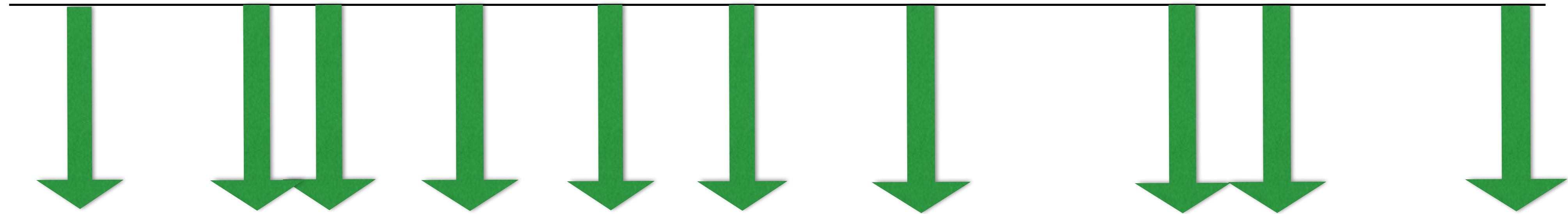


Annuities

1. Ordinary Annuities

2. Annuity Due

3. Perpetuities



*Finite*

*Level*

*Sequential*

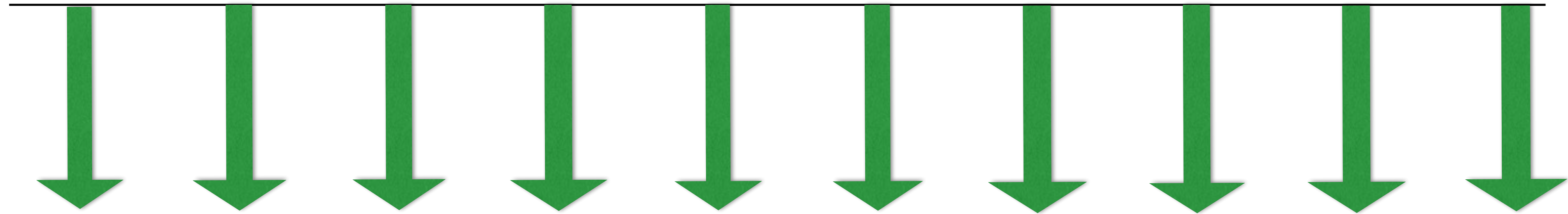


Annuities

1. Ordinary Annuities

2. Annuity Due

3. Perpetuities



***Finite***

***Level***

***Sequential***



Annuities

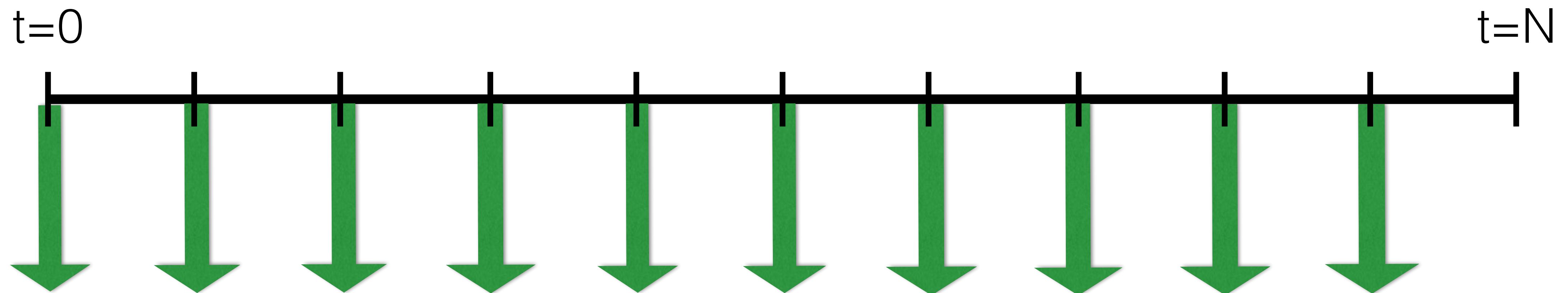
1. Ordinary Annuities

2. Annuity Due

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***Annuity  
Due***

***Ordinary  
Annuity***

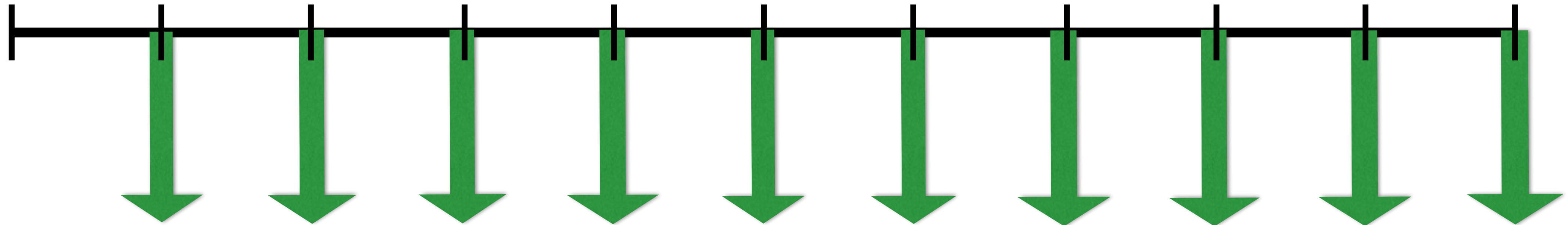


*Annuity  
Due*

*Ordinary  
Annuity*

$t=0$

$t=N$





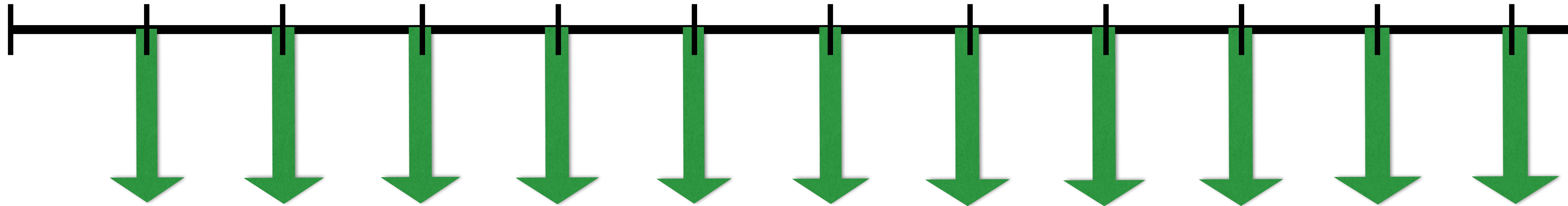
*Annuity  
Due*

*Ordinary  
Annuity*

*Perpetuity*

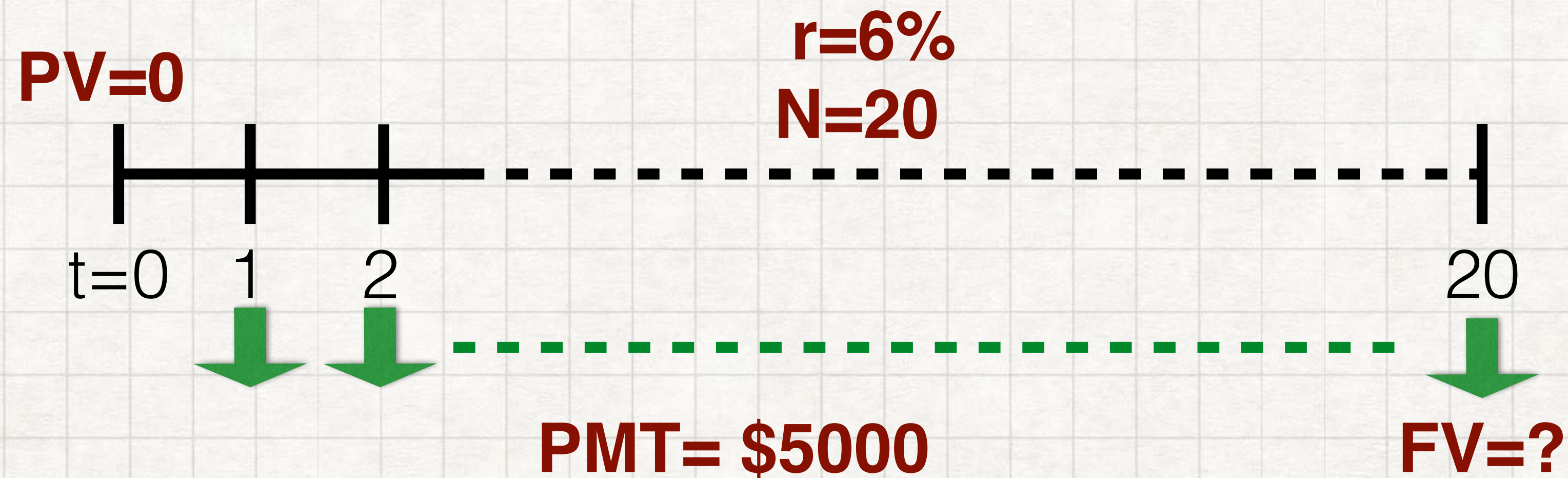
$t=0$

$t \rightarrow \infty$





What is the future value of an **ordinary annuity** that pays \$5000 per year for each of the next 20 years? Assume a discount rate of 6% per year.

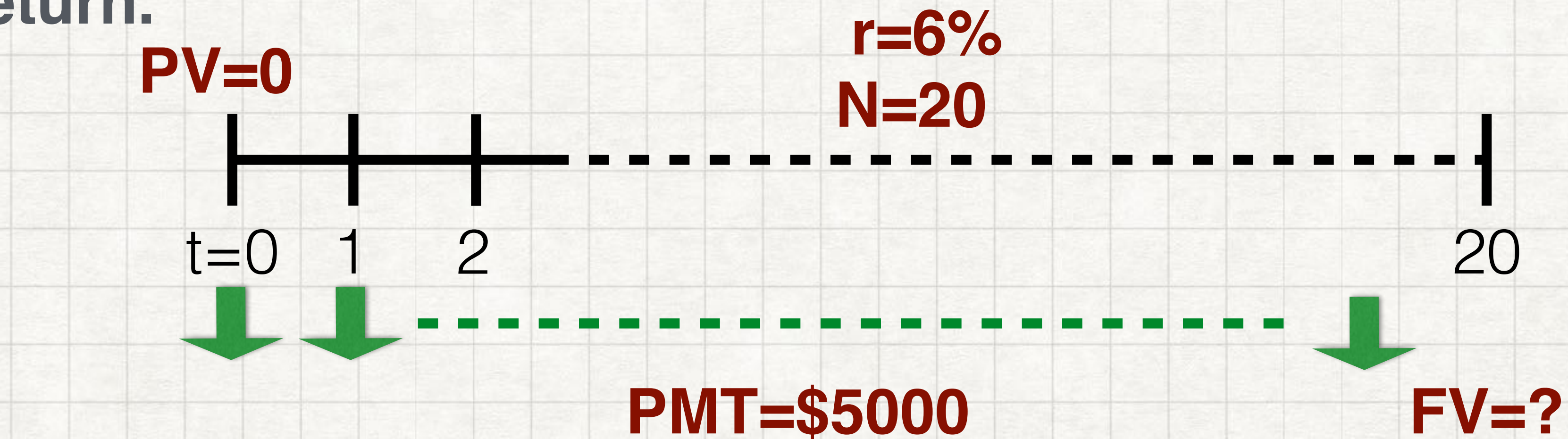


**CALC**  
END Mode

**$FV = \$183,928$**



What is the future value of an **annuity due** that pays \$5000 per year for each of the next 20 years? The investment is expected to earn a 6% rate of return.



2ND BGN → 2ND SET

CALC  
BGN Mode

$FV=\$194,964$

OR

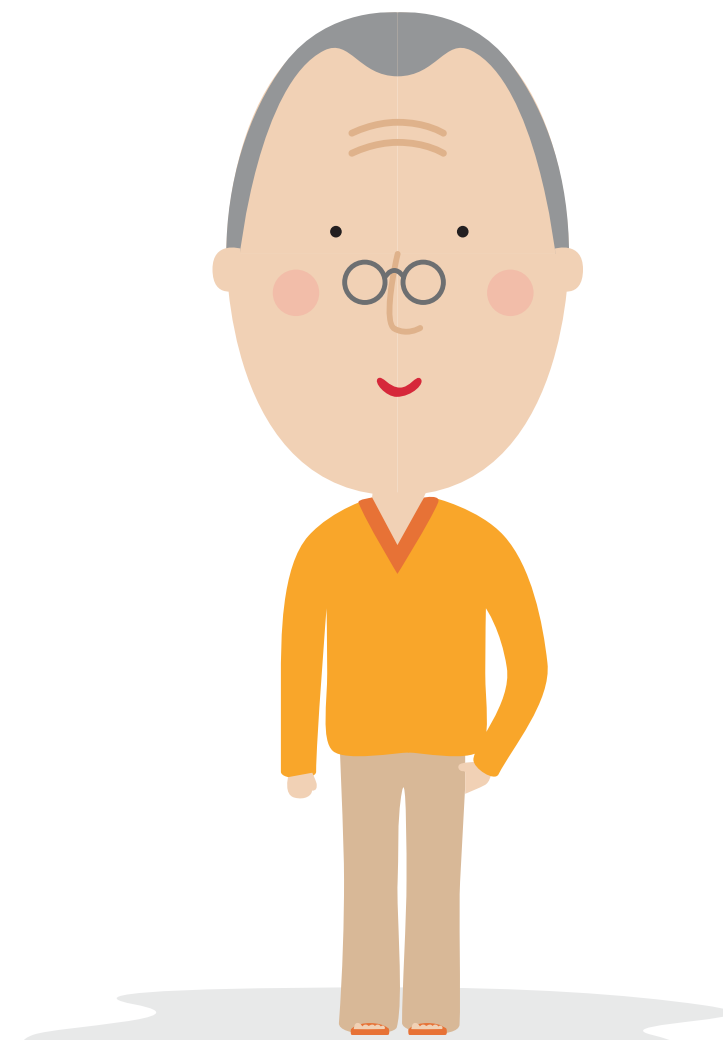
CALC  
END Mode

$FV=\$183,928 \times 1.06$   
 $=\$194,964$

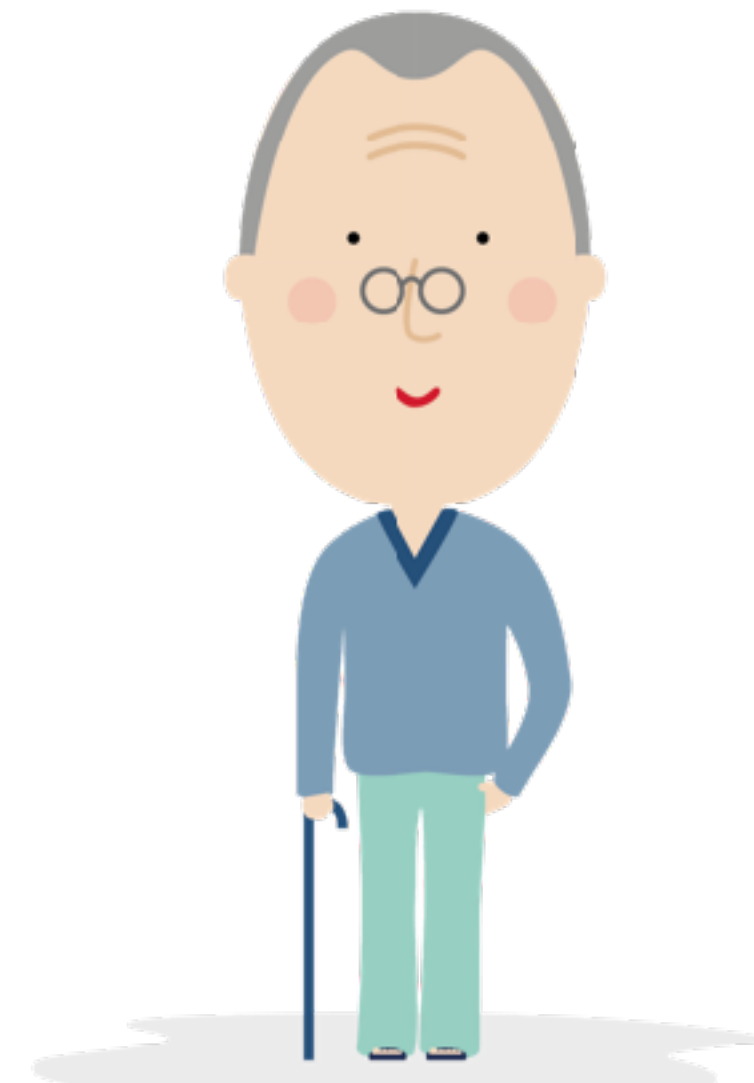




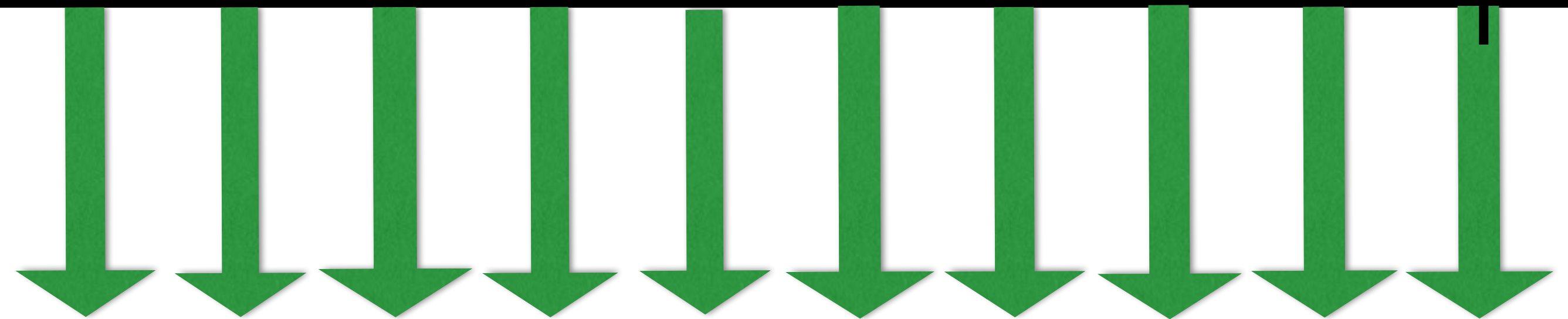
Age 60



Age 65



Age 75



Ordinary Annuity  
\$100,000 / yr

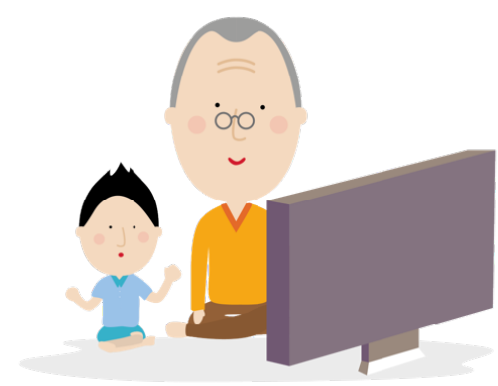


## Annuities

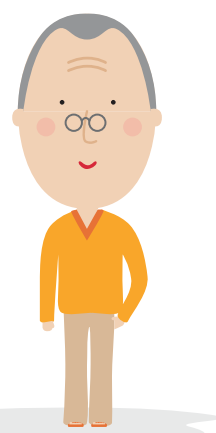
1. Ordinary Annuities

2. Annuity Due

3. Perpetuities



Age 60



Age 65

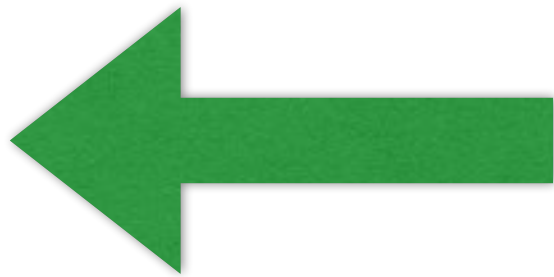


Age 75

Ordinary Annuity  
\$100,000 / yr

4% discount rate

$PV_{60}$



$PV_{65}$

$N=10$

$I/Y=4\%$

$PMT=\$100,000$

$FV=\$0$

**CALC**

END Mode

**$PV=\$811,090$**

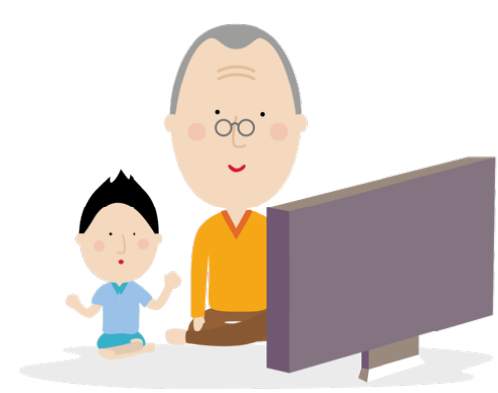


## Annuities

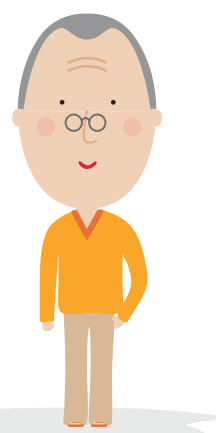
1. Ordinary Annuities

2. Annuity Due

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Age 60



Age 65



Age 75

Ordinary Annuity  
\$100,000 / yr

$PV_{60}$

$PV_{65}$

4% discount rate

← ..... \$811,090

$$\begin{aligned} PV &= FV / (1+r)^N \\ &= 811090 / 1.04^5 \\ &= \$666,657 \end{aligned}$$



## Annuities

1. Ordinary Annuities

2. Annuity Due

3. Perpetuities

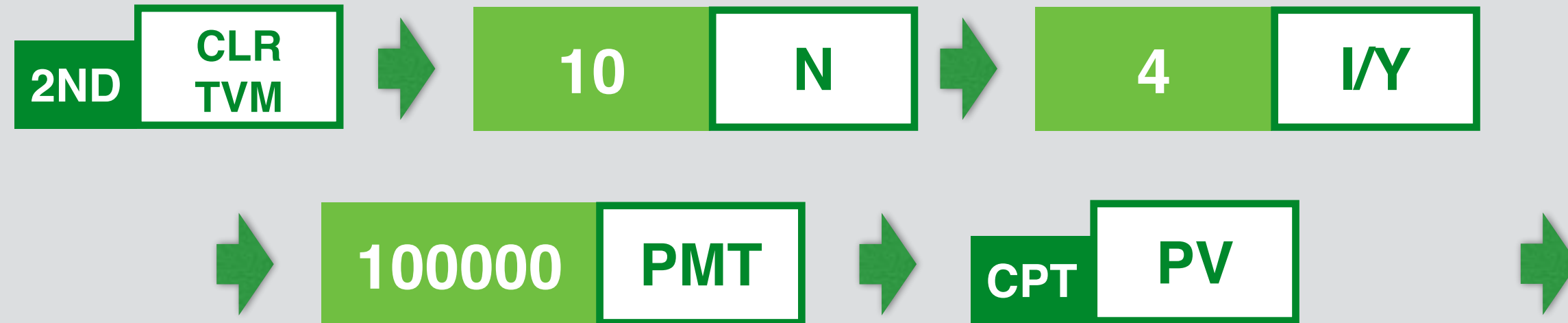


# Calculator Practise

$N=10$   
 $I/Y=4\%$   
 $PMT=\$100,000$   
 $FV=\$0$

$PV_{65}=\$811,090$

## STEP 1 - Calculate $PV_{65}$



## Shortcut



*\*In AOS mode only. Check out video “Setting up your calculator” to find out more.*

## STEP 2 - Store to Memory

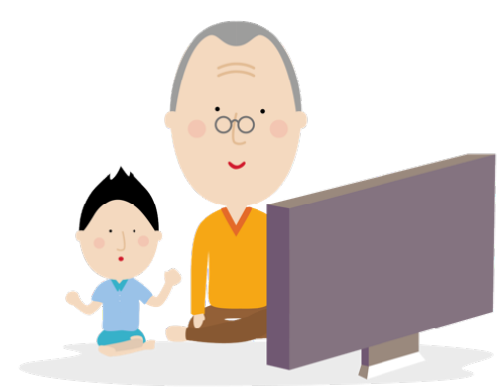


## STEP 3 - Calculate Discount Factor



## STEP 4 - Multiply by stored $PV_{65}$





Age 60



Age 65



Age 75

Ordinary Annuity  
\$100,000 / yr

$PV_{60}$

$PV_{65}$

4% discount rate



..... \$811,090

$$\begin{aligned} PV &= FV / (1+r)^N \\ &= 811090 / 1.04^5 \\ &= \$666,657 \end{aligned}$$



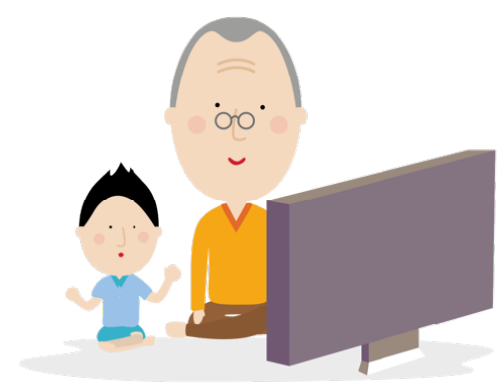
## Annuities

1. Ordinary Annuities

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Age 60



Age 65

Perpetuity  
\$100,000 / yr

→ ∞

$$PV_{\text{perp}} = PMT / r$$

$$PV_{65} = \$100,000 / 0.04 \\ = \$2.5M$$

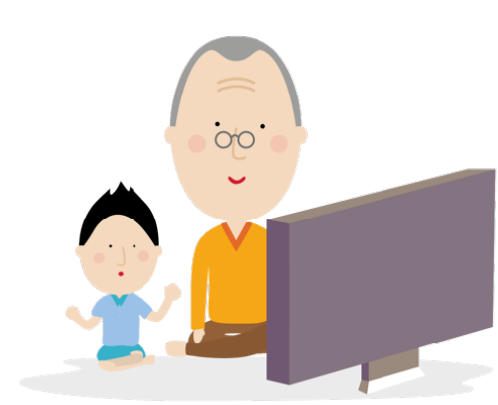


## Annuities

1. Ordinary Annuities

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Age 60



Age 65

Perpetuity  
\$100,000 / yr

→ ∞

$PV_{60}$

$PV_{65}$

4% discount rate

← ..... \$2.5M

$$\begin{aligned} PV &= FV / (1+r)^N \\ &= 2500000 / 1.04^5 \\ &= \$2.055M \end{aligned}$$



Annuities

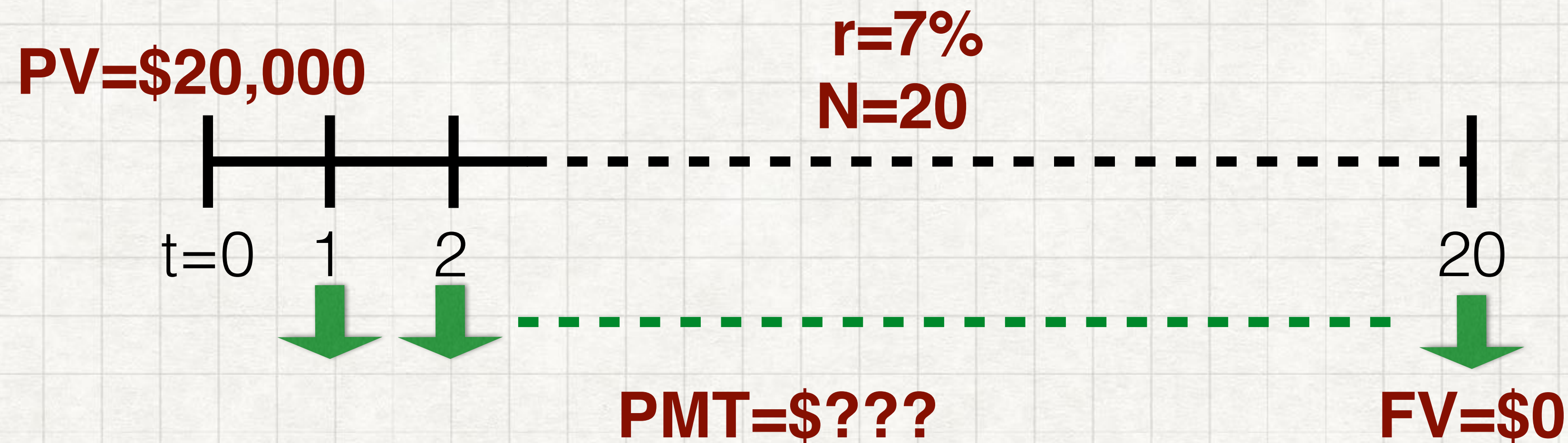
1. Ordinary Annuities

2. Annuity Due

3. Perpetuities



Casey has an ordinary annuity that is worth \$20,000 today. The expected annual return for this annuity is 7% per annum. He is expected to live for another 20 years. How much can he withdraw at the end of each year such that the annuity can last him for the full 20 years?



**CALC**  
END Mode

**$PMT = \$1888$**



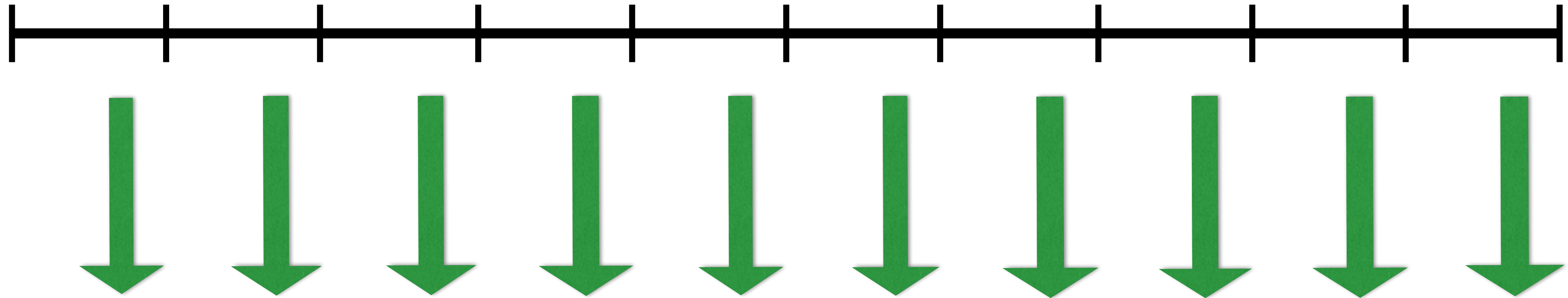
***Annuity  
Due***

***Ordinary  
Annuity***

***Perpetuity***

$t=0$

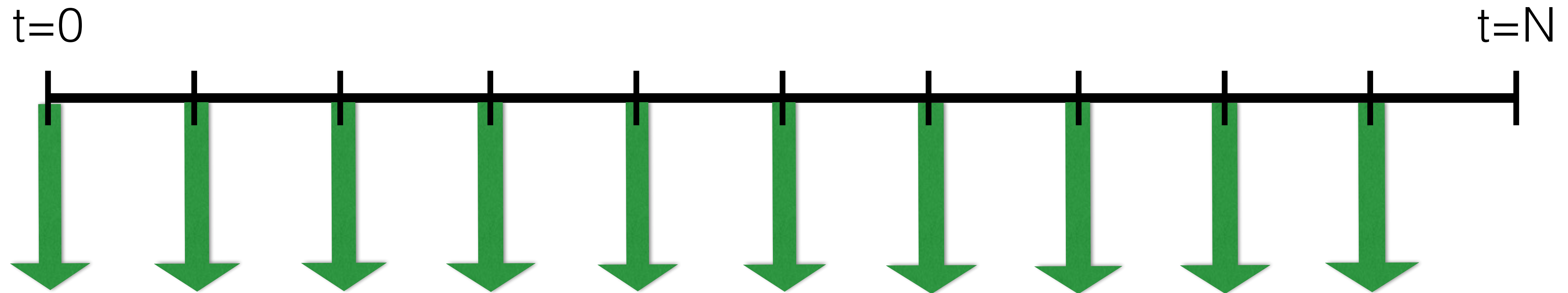
$t=N$



*Annuity  
Due*

*Ordinary  
Annuity*

*Perpetuity*



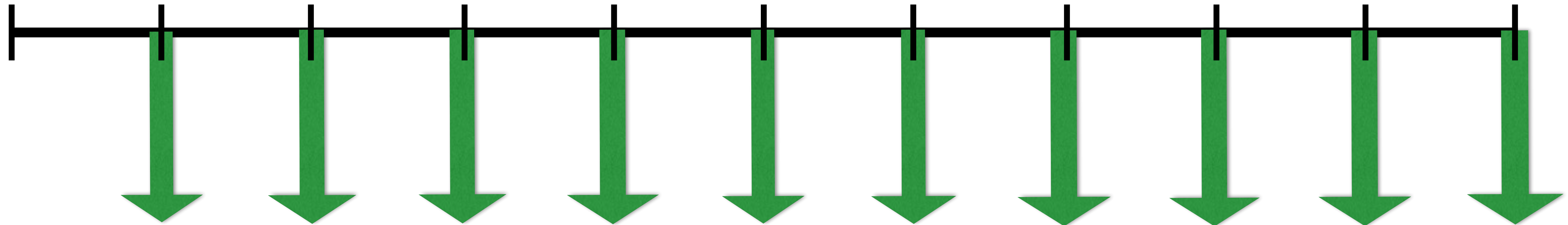
*Annuity  
Due*

*Ordinary  
Annuity*

*Perpetuity*

$t=0$

$t=N$



Annuities

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3. Perpetuities

*Annuity  
Due*

*Ordinary  
Annuity*

*Perpetuity*

$$PV_{\text{perp}} = PMT / r$$

$t=0$

$t \rightarrow \infty$

