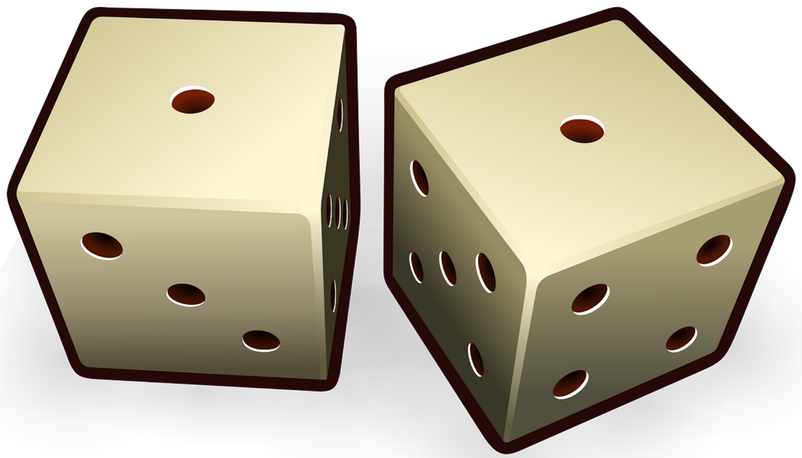


Probability Concepts

Joint Probability and Total Probability Rule

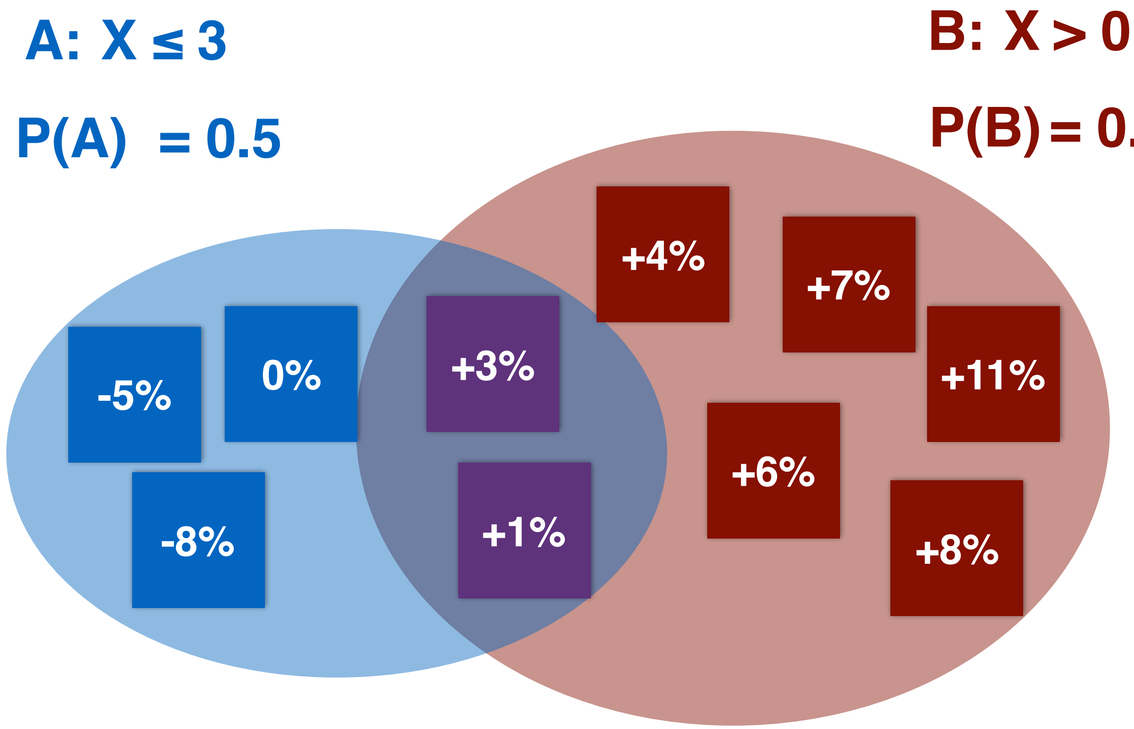
1. Conditional Probability
2. Joint Probability
3. Total Probability Rule

PREP nuggets

Unconditional Probability

"Marginal Probability"

probability of an event regardless of past or future occurrences of other events



X: *monthly return of a stock*

Conditional Probability

P(A | B)

What is the probability that X≤3 if X>0?

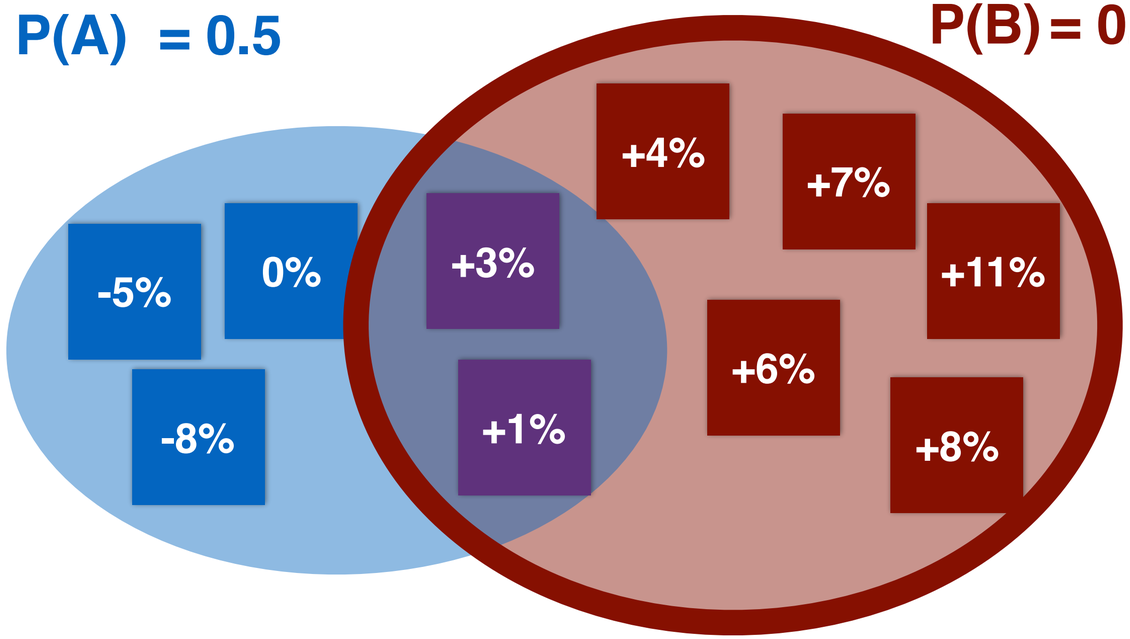
A: X ≤ 3

P(A) = 0.5

B: X > 0

P(B) = 0.7

Limit to this condition



Conditional Probability

$$P(A \mid B) = 2/7 = 0.29$$

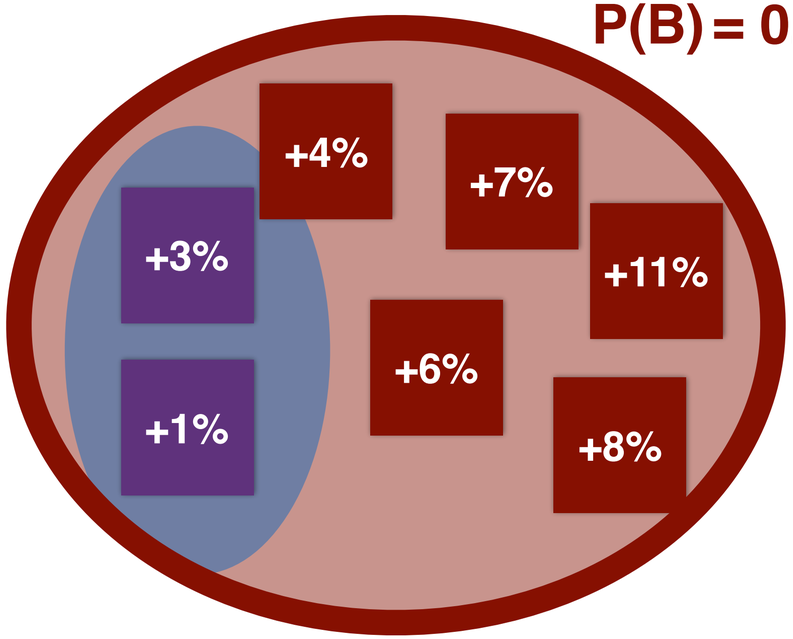
A: $X \leq 3$

$P(A) = 0.5$

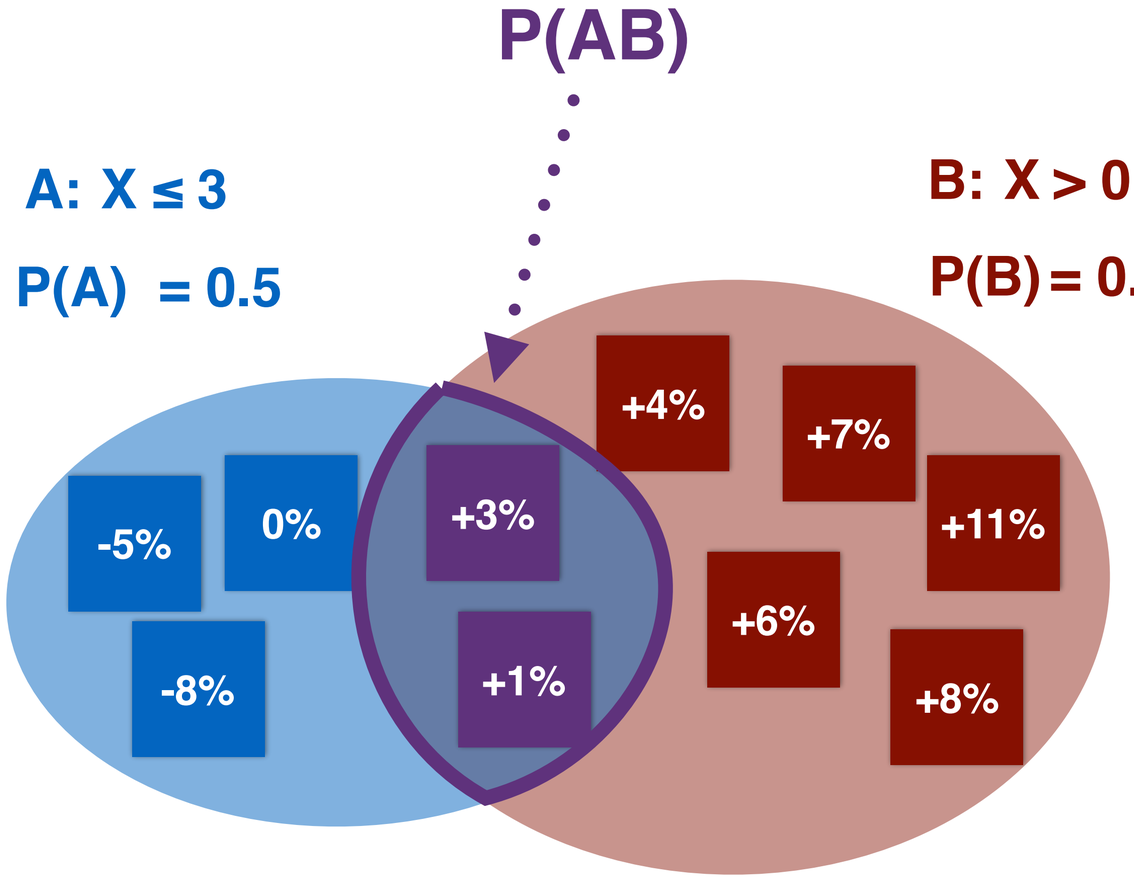
B: $X > 0$

$P(B) = 0.7$

Limit to this condition



Joint Probability



$P(A \mid B) = 0.29$

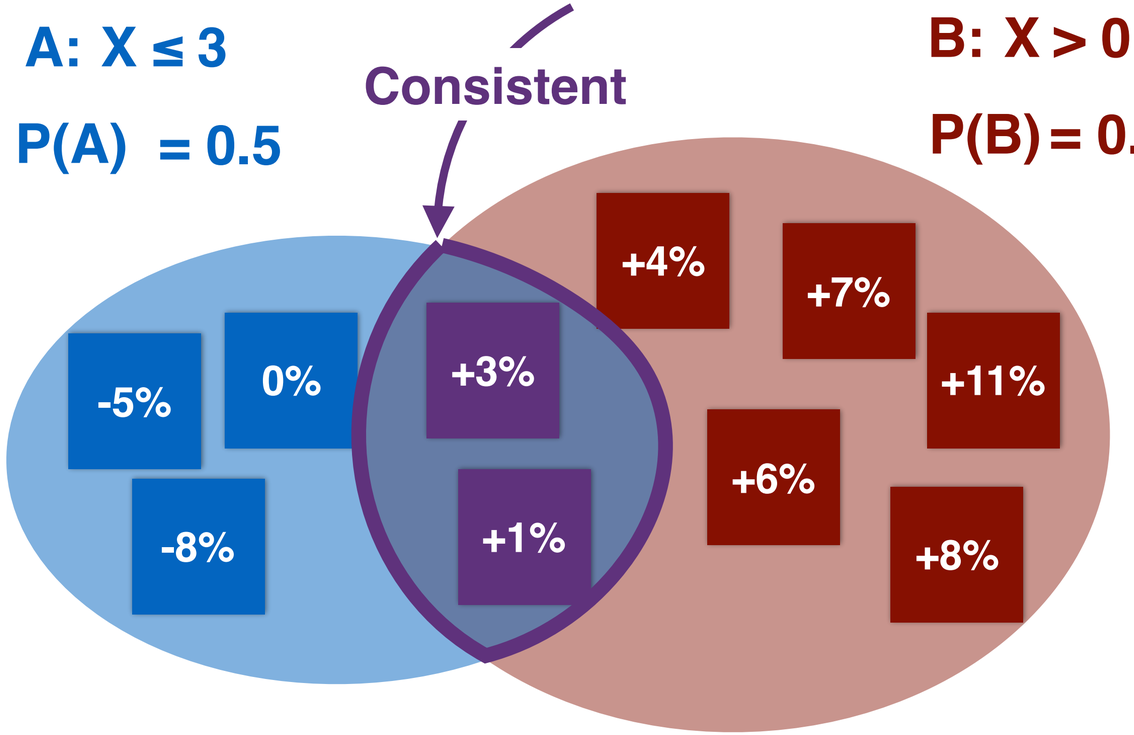
Joint Probability

$$P(AB) = P(A \mid B) \times P(B)$$

$$= 0.29 \times 0.7$$

$$= 0.2$$

Multiplication rule



Joint Probability

$$P(AB) = P(A \mid B) \times P(B)$$

Multiplication rule

** This formula can be simplified if A and B are* ***INDEPENDENT***

events which the occurrence of one has no influence on the occurrence of the other

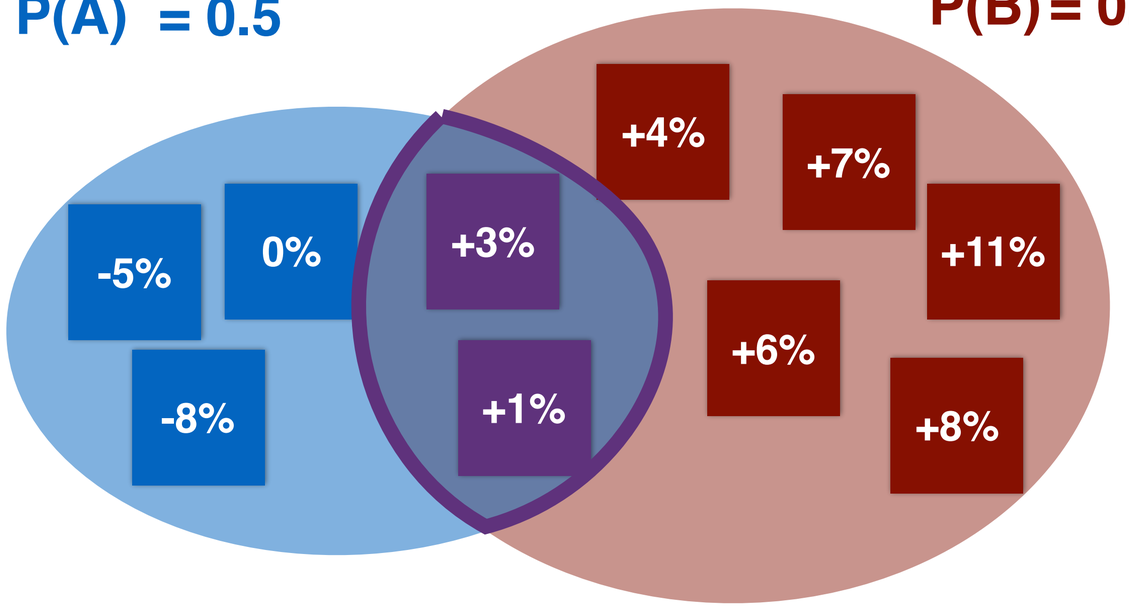
A: X ≤ 3

P(A) = 0.5

A and B are obviously NOT independent

B: X > 0

P(B) = 0.7



PREP nuggets

Joint Probability and Total Probability Rule

1. Conditional Probability 2. Joint Probability 3. Total Probability Rule

Joint Probability

Multiplication rule

$$P(AB) = P(A \mid B) \times P(B)$$

Simplified multiplication rule for independent events

$$P(AE) = P(A) \times P(E)$$

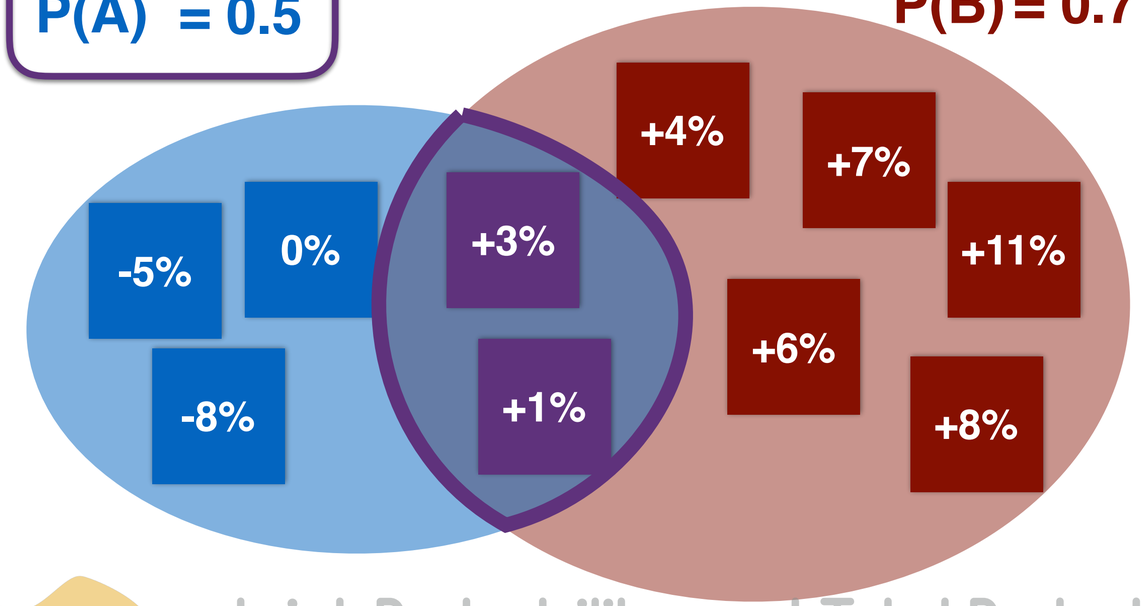
A and E are likely independent ⇒ $P(A \mid E) = P(A)$

A: $X \leq 3$

$P(A) = 0.5$

B: $X > 0$

$P(B) = 0.7$



E: Investor gets hospitalised

$P(E) = 0.01$

PREP nuggets

Joint Probability and Total Probability Rule

1. Conditional Probability 2. Joint Probability 3. Total Probability Rule

Lily observed that ABC stock has 3 up days for every 5 trading days for the past 8 years. She has also determined that the up and down days of this stock are independent. Using the empirical method, compute the probability of the stock closing higher for 4 consecutive days.

$$P(\text{Up}) = 3/5 = 0.6$$

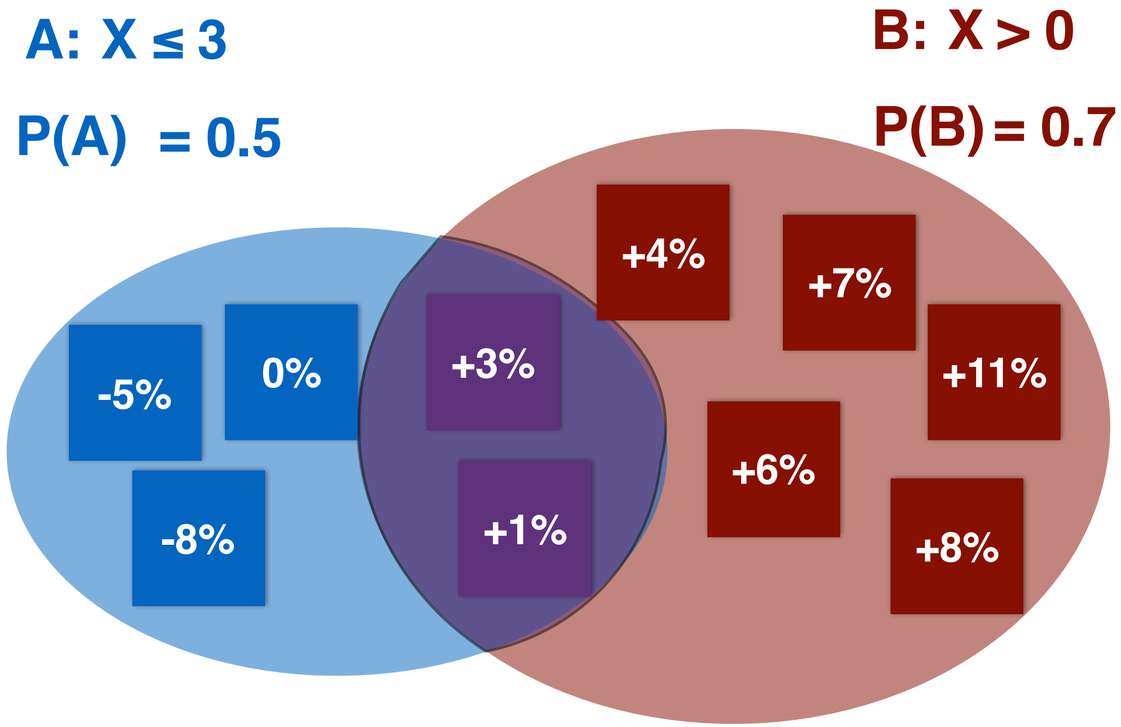
$$P(\text{4 consecutive Up}) = 0.6 \times 0.6 \times 0.6 \times 0.6$$

$$= 0.13$$

Simplified Multiplication rule

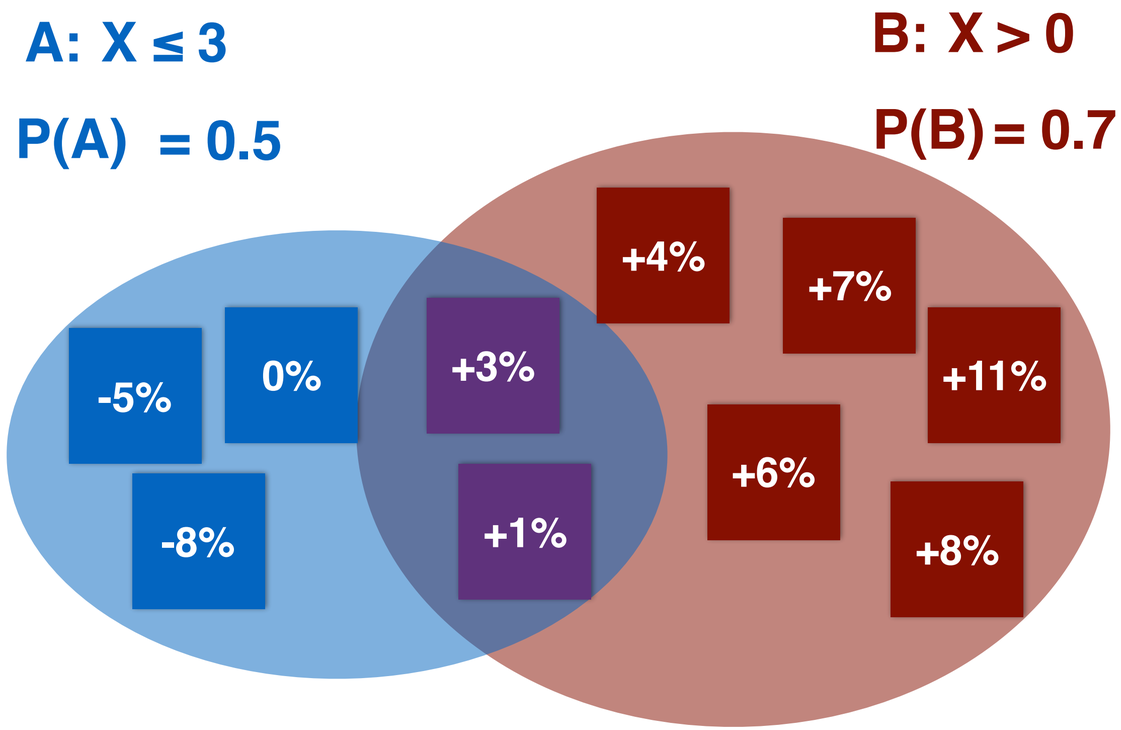
Joint Probability

$A \cap B$



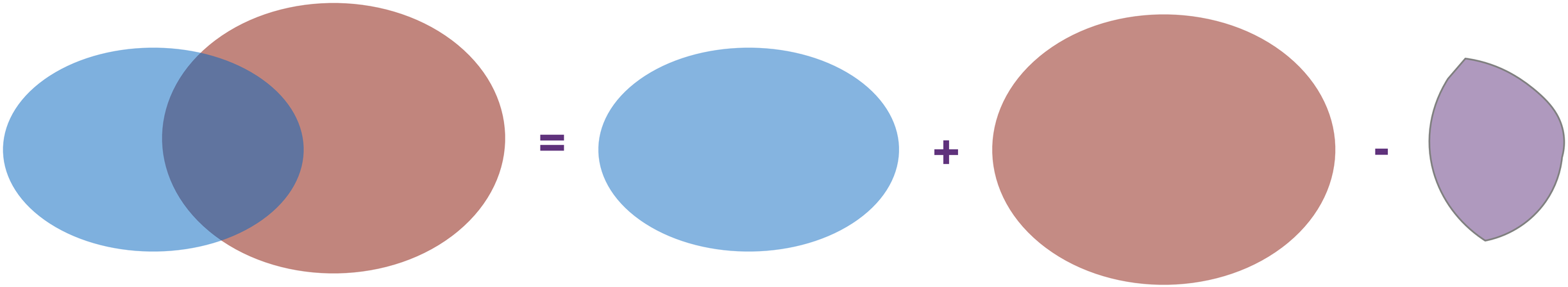
At Least One Event Will Occur

$A \cup B$



At Least One Event Will Occur

$$A \cup B$$

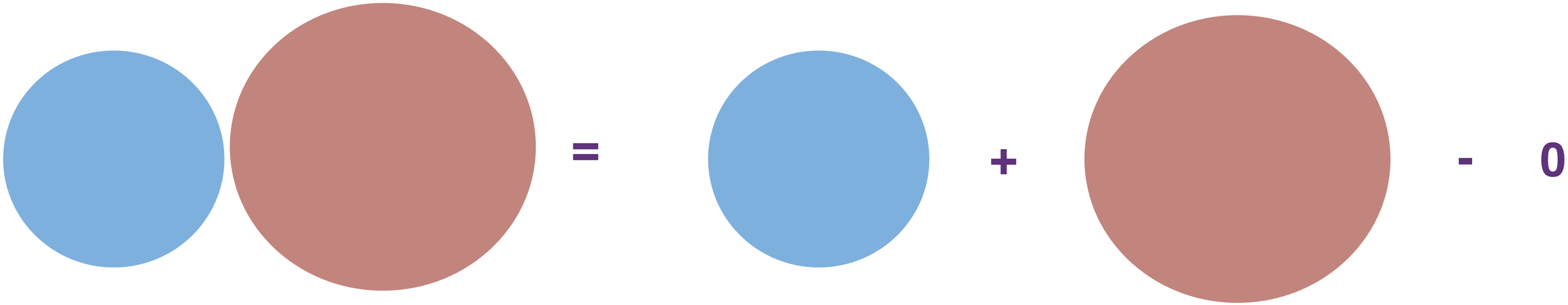


$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Addition Rule

At Least One Event Will Occur

$$A \cup B$$



$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

Simplified **Addition Rule** ***for mutually exclusive events***

As an analyst, David has collated the following information:

- P(C) = 0.05, the probability of a cut in interest rate in any given month
- P(U | C) = 0.8, the probability of a rise in DJIA for the month (U) given a cut in interest rate

(a) **P(C) is derived based on David's interpretation of the language used by the central bank on the rate outlook. What kind of probability is P(C)?**

Personal Judgement → **Subjective probability**

As an analyst, David has collated the following information:

- P(C) = 0.05, the probability of a cut in interest rate in any given month
- P(U | C) = 0.8, the probability of a rise in DJIA for the month (U) given a cut in interest rate

(b) **What is the joint probability of a cut in interest rate (C) and a rise in DJIA (U) in the same month?**

$$P(UC) = P(U \mid C) \times P(C)$$ **Multiplication rule**

$$= 0.8 \times 0.05$$

$$= 0.04$$

As an analyst, David has collated the following information:

- P(C) = 0.05, the probability of a cut in interest rate in any given month
- P(U | C) = 0.8, the probability of a rise in DJIA for the month (U) given a cut in interest rate

(c) **Given that the unconditional probability of a rise in DJIA in any given month is 0.65, what is the probability that there is a cut in interest rate or a rise in DJIA in any given month?**

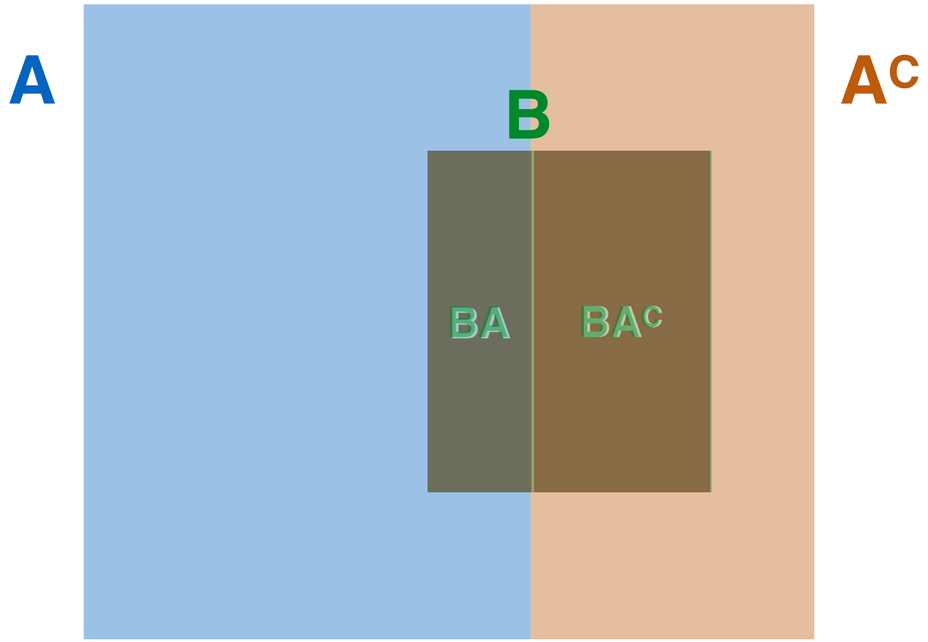
$$P(U \text{ or } C) = P(U) + P(C) - P(UC)$$

$$= 0.65 + 0.05 - 0.04$$

$$= 0.66$$

A and A^C

Mutually exclusive and Exhaustive Events



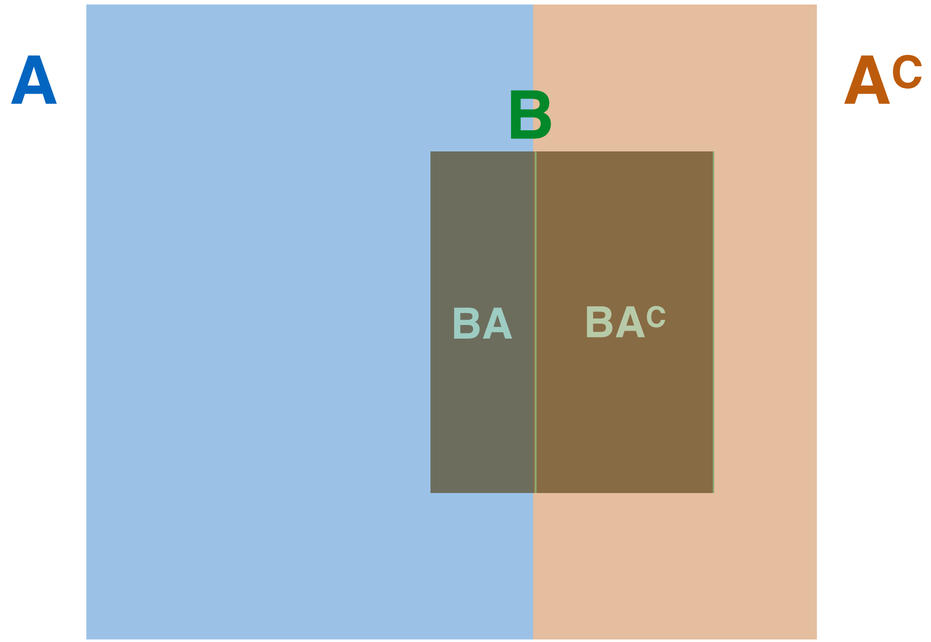
B has to occur with either A OR A^C

Total Probability Rule

$$P(B) = P(BA) + P(BA^C)$$

Applying Multiplication rule

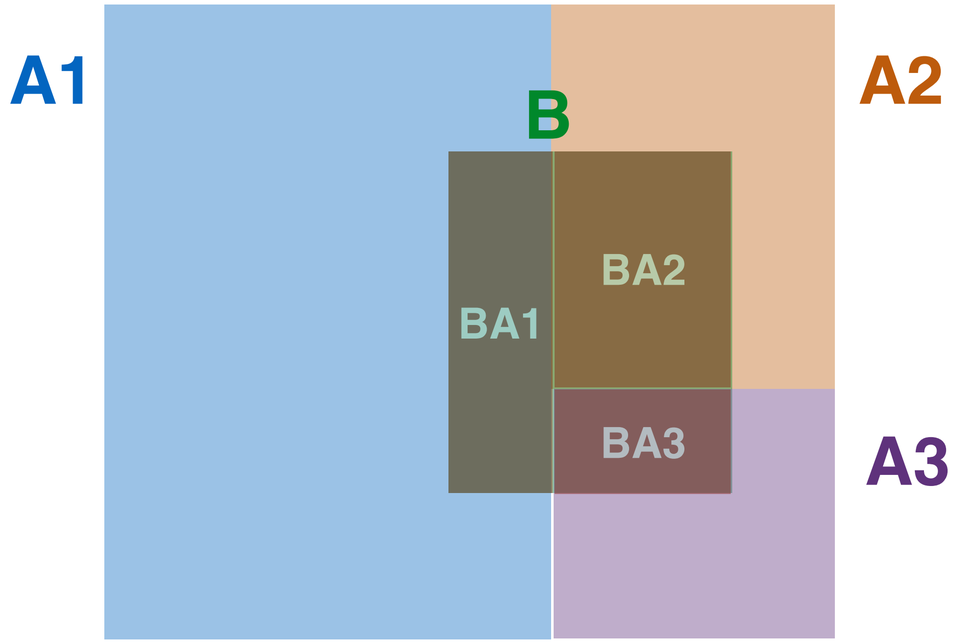
$$P(B) = P(B \mid A) \times P(A) + P(B \mid A^C) \times P(A^C)$$



Total Probability Rule

$$P(B) = P(BA1) + P(BA2) + P(BA3)$$

$$P(B) = P(B \mid A1) \times P(A1) + P(B \mid A2) \times P(A2) + P(B \mid A3) \times P(A3)$$



*A1, A2 and A3 are **mutually exclusive** and **exhaustive***

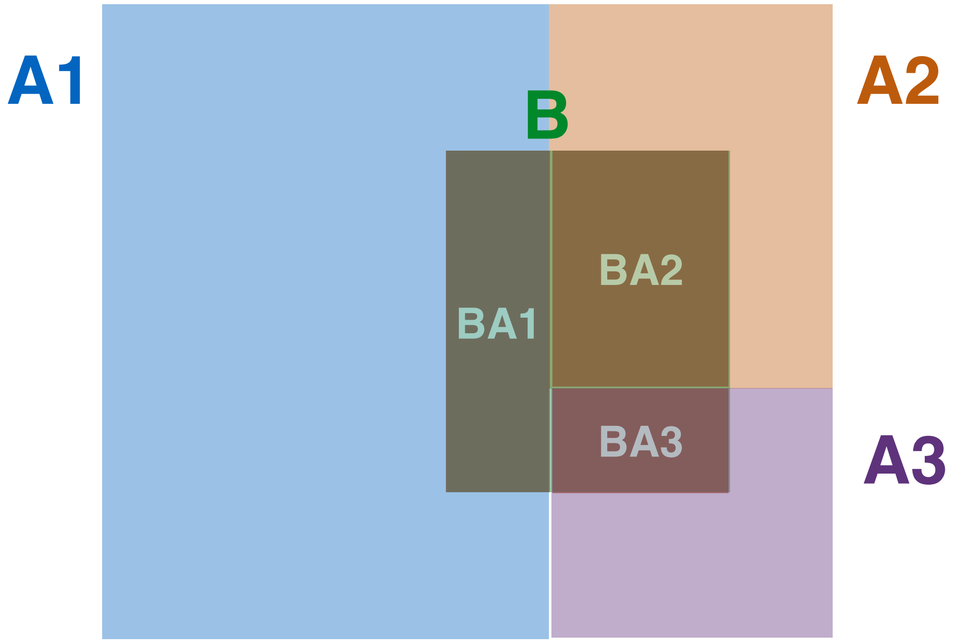
Total Probability Rule

$$P(B) = P(B \mid A1) \times P(A1) + P(B \mid A2) \times P(A2) + P(B \mid A3) \times P(A3)$$

$$= 0.55 \times 0.9 + 0.8 \times 0.05 + 0.3 \times 0.05$$

$$= 0.55$$

Probability of DJIA going up in any given month is 0.55

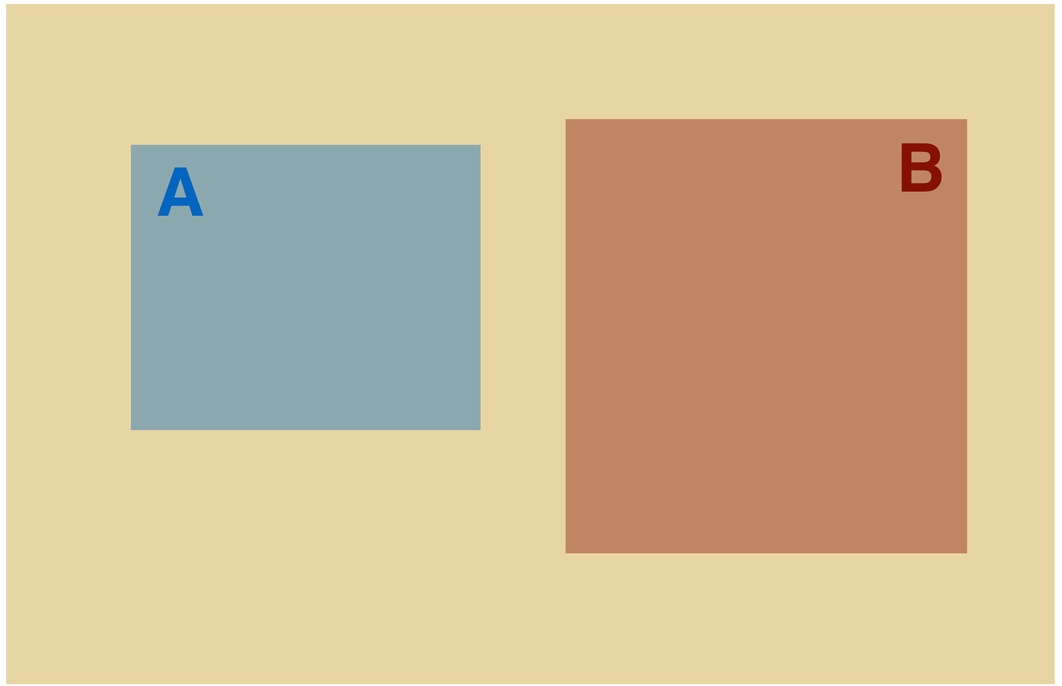


P(A1)=0.9 *Interest rate unchanged*
P(A2)=0.05 *Interest rate cut*
P(A3)=0.05 *Interest rate increase*

P(B | A1)=0.55 *DJIA up given A1*
P(B | A2)=0.8 *DJIA up given A2*
P(B | A3)=0.3 *DJIA up given A3*

Mutually Exclusive

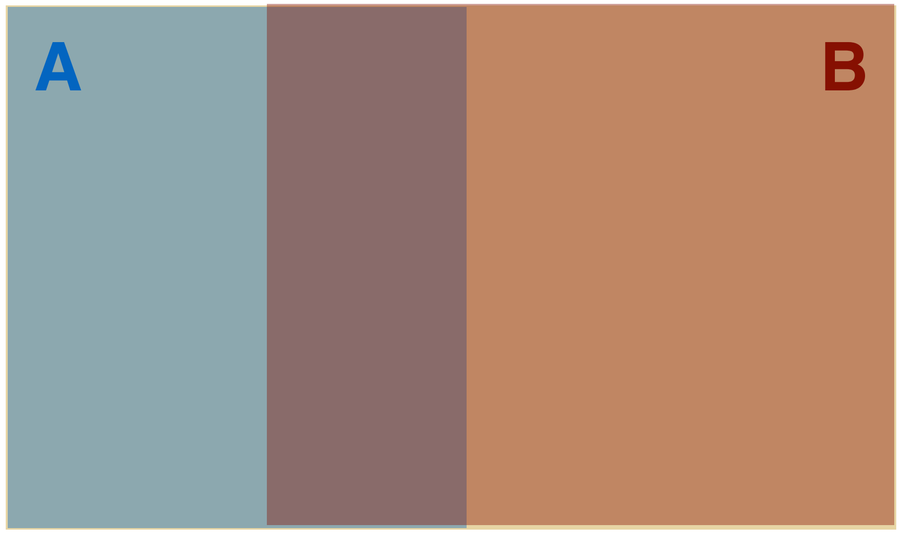
cannot happen at the same time



PREP nuggets

Exhaustive

covers all possible outcomes



All possible outcomes

PREP nuggets

Mutually Exclusive and Exhaustive

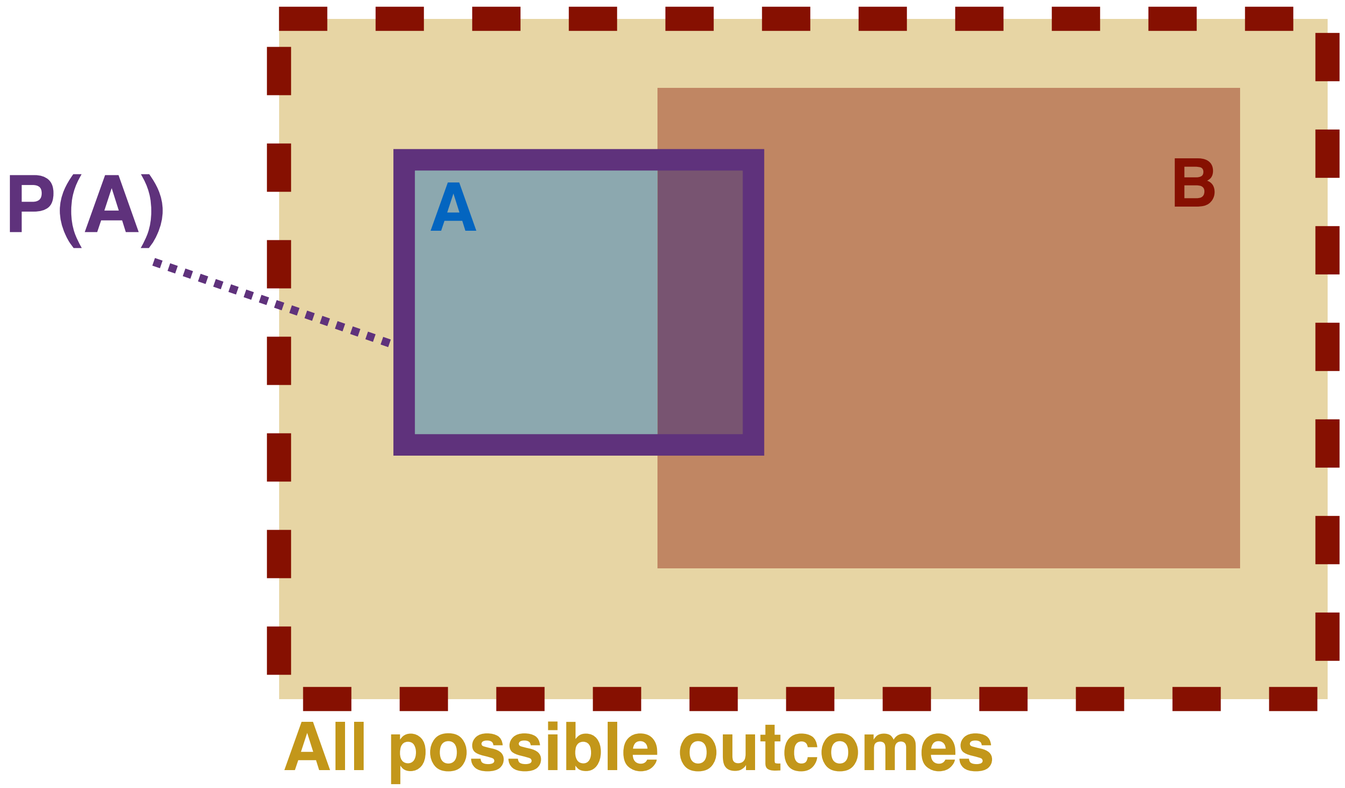
$P(A) + P(B) = 1$

All possible outcomes

PREP nuggets

Unconditional Probability

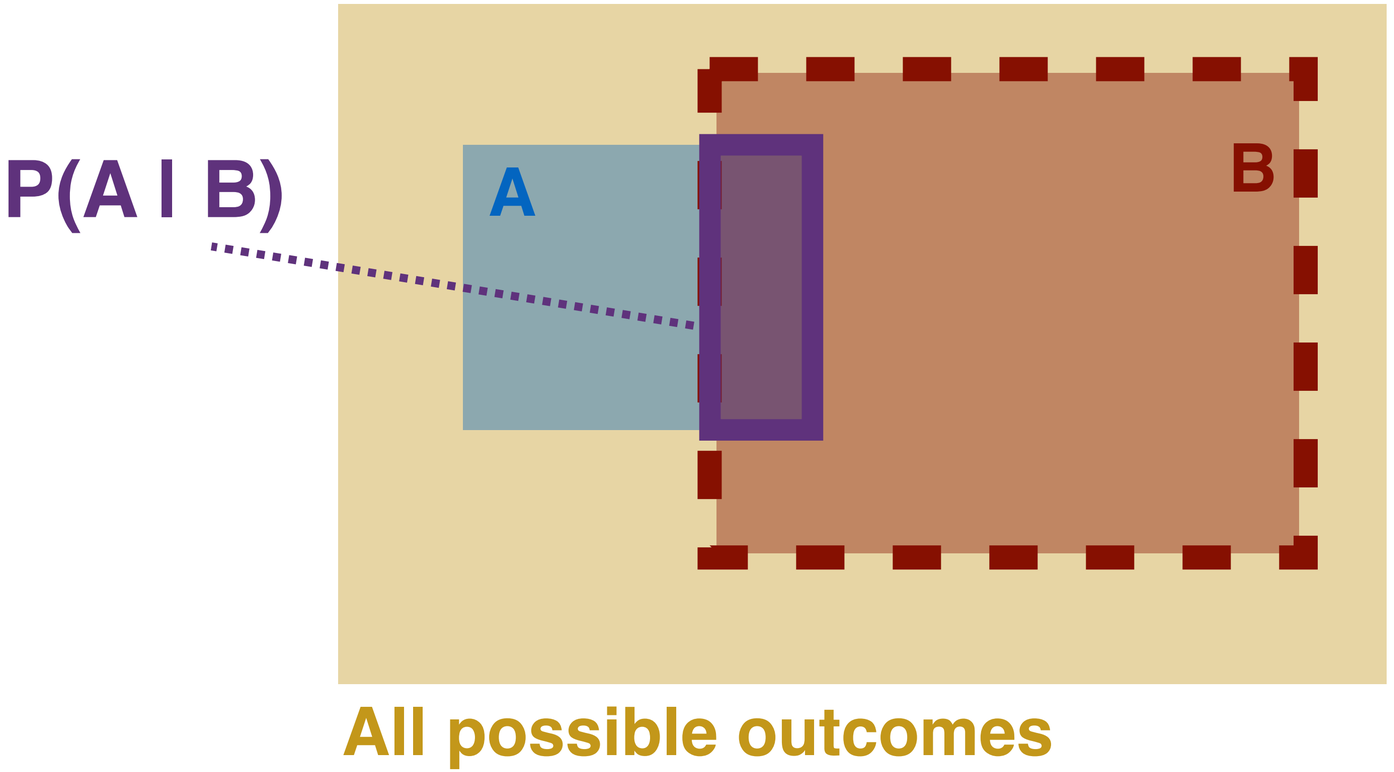
probability of an event regardless of past or future occurrences of other events



PREP nuggets

Conditional Probability

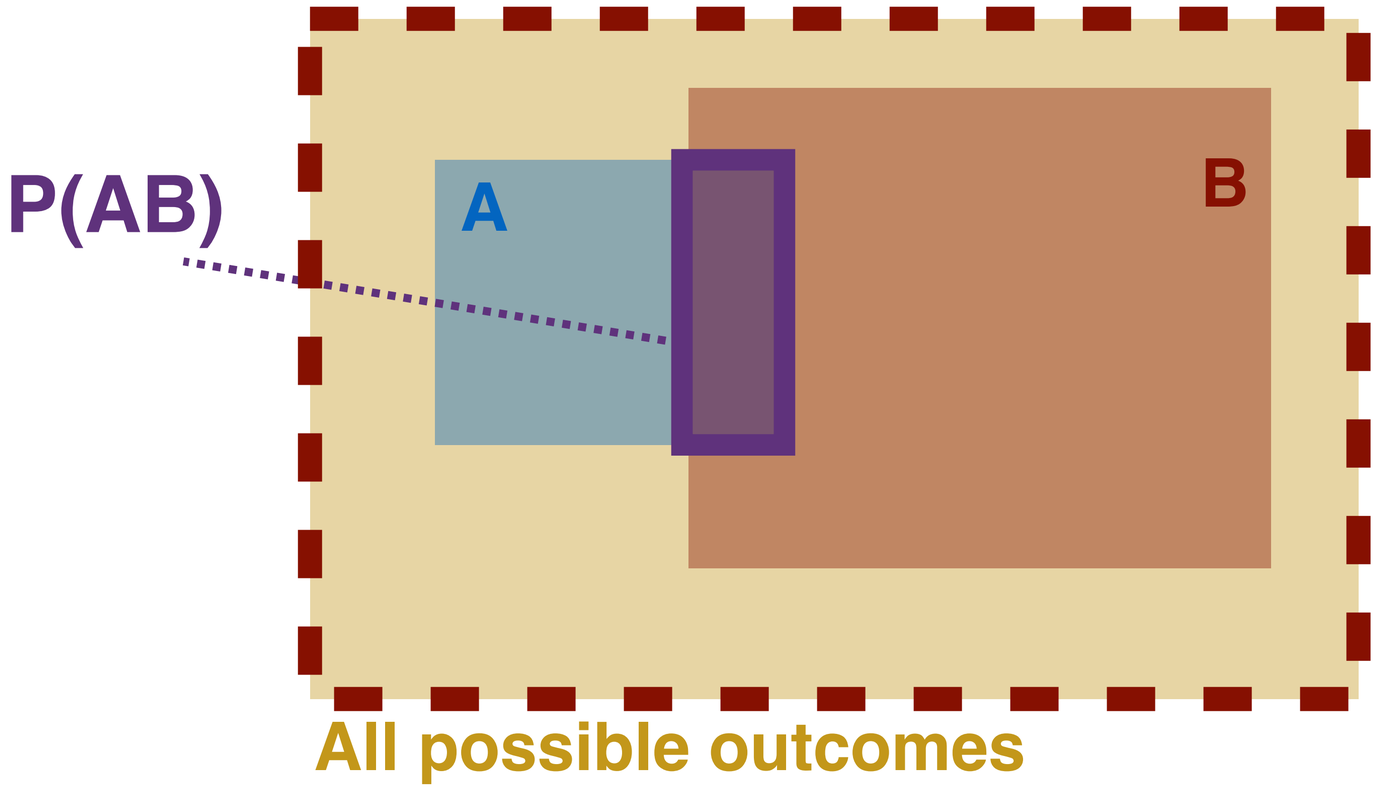
probability of an event in the context of another event



PREP nuggets

Joint Probability

probability of both events happening



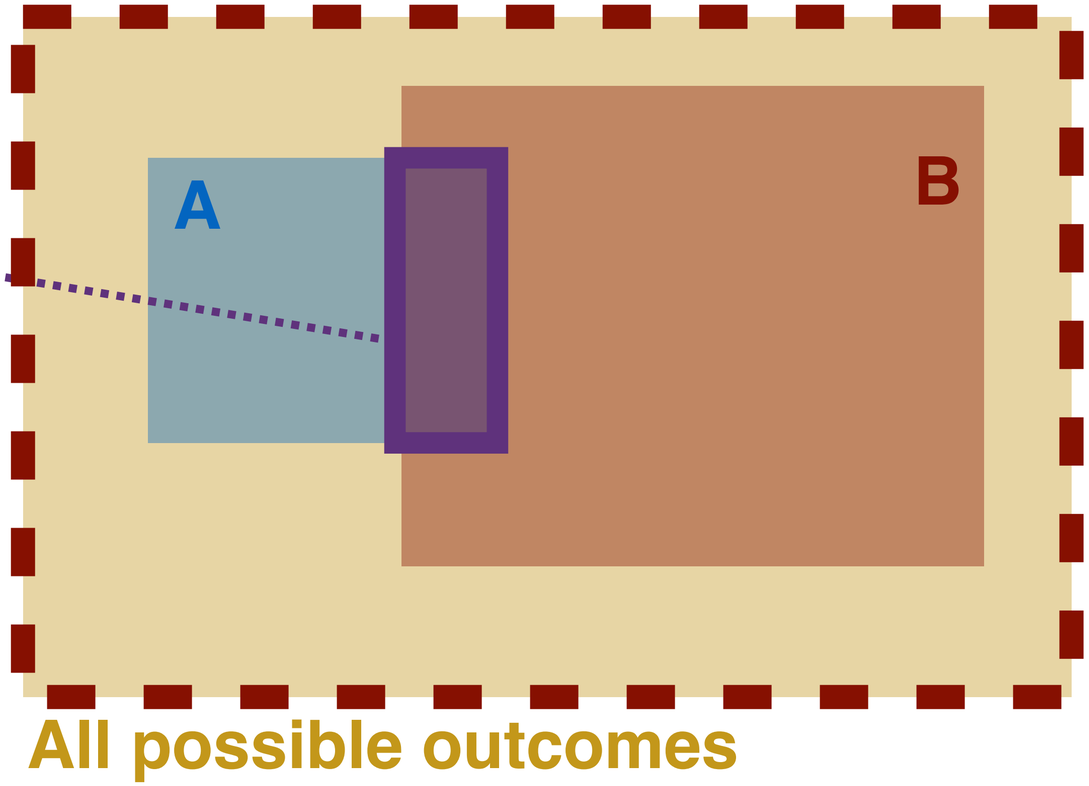
PREP nuggets

Joint Probability

probability of both events happening

P(AB) = P(A | B) x P(B)

Multiplication Rule



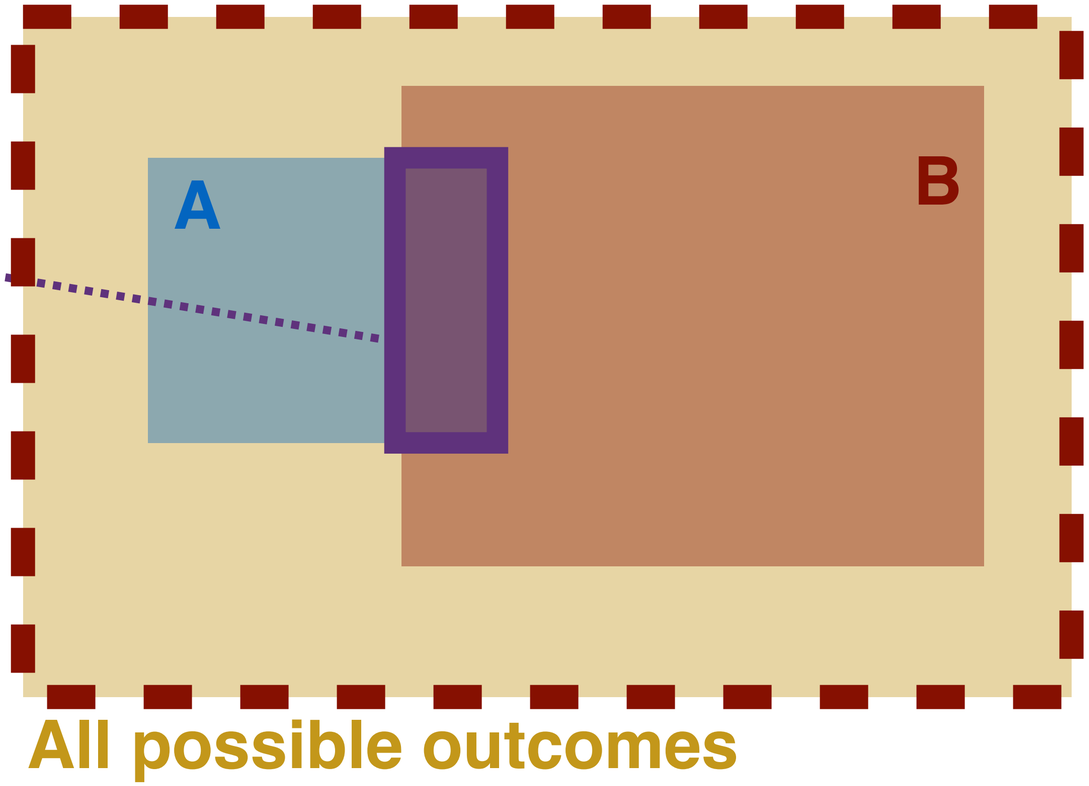
PREP nuggets

Joint Probability

probability of both events happening

P(AB) = P(A | B) x P(B)

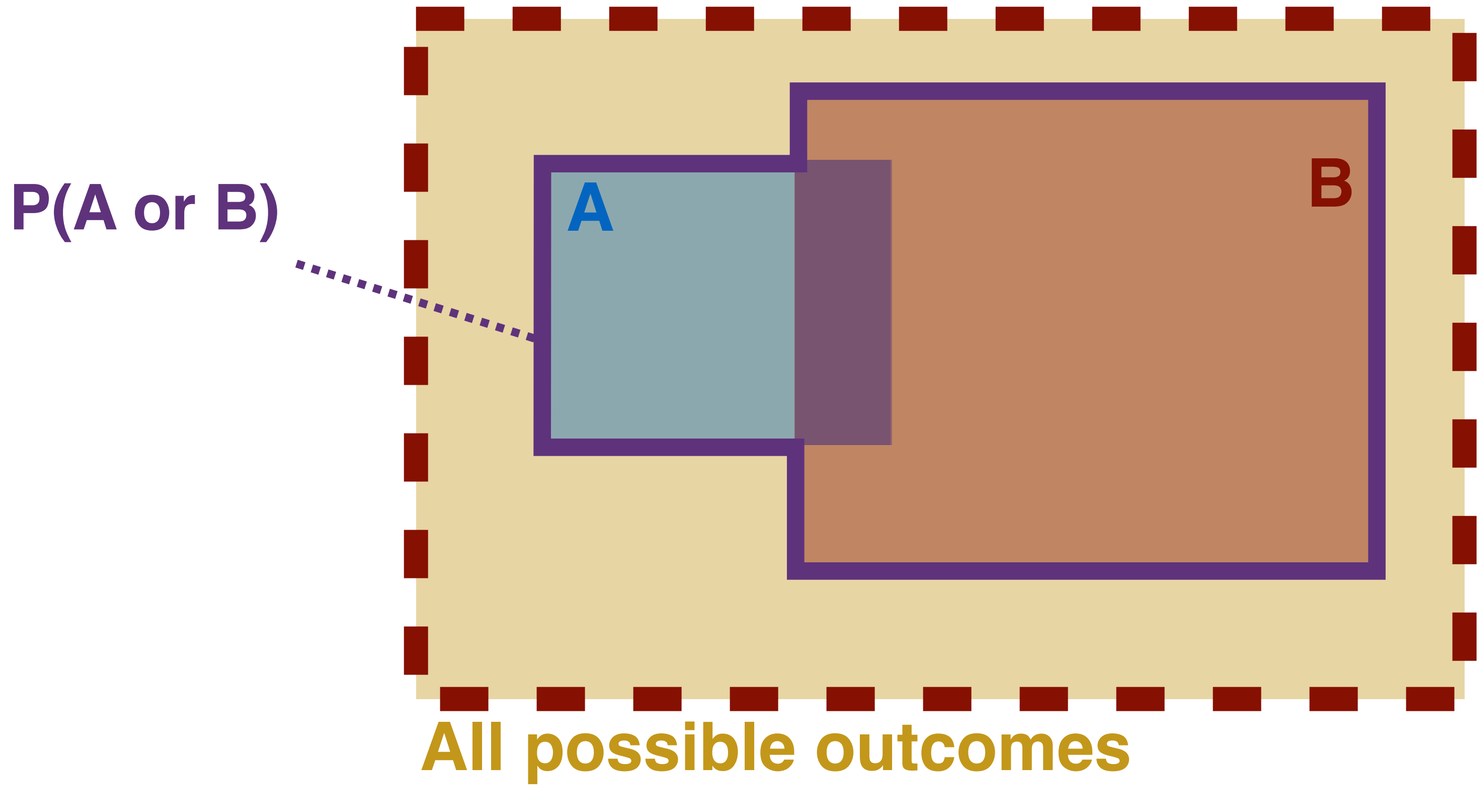
Multiplication Rule



PREP nuggets

Union

probability of either events happening



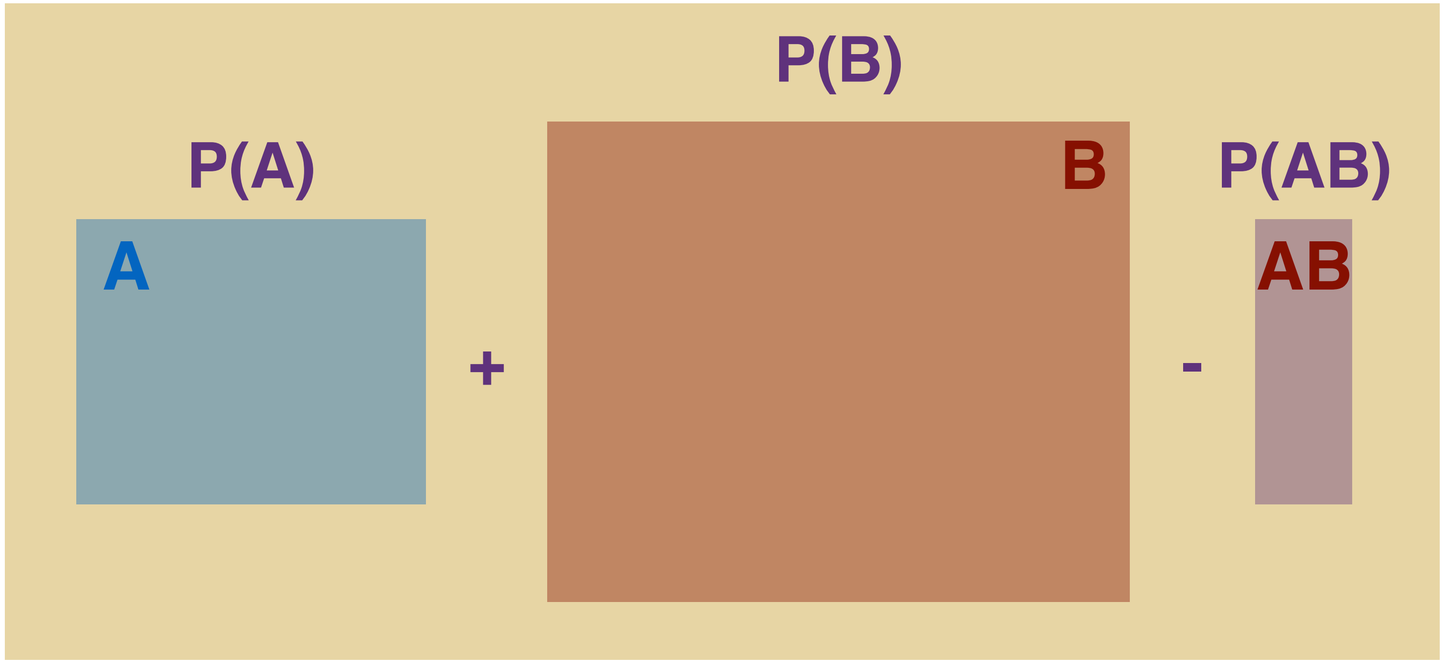
PREP nuggets

Union

probability of either events happening

Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

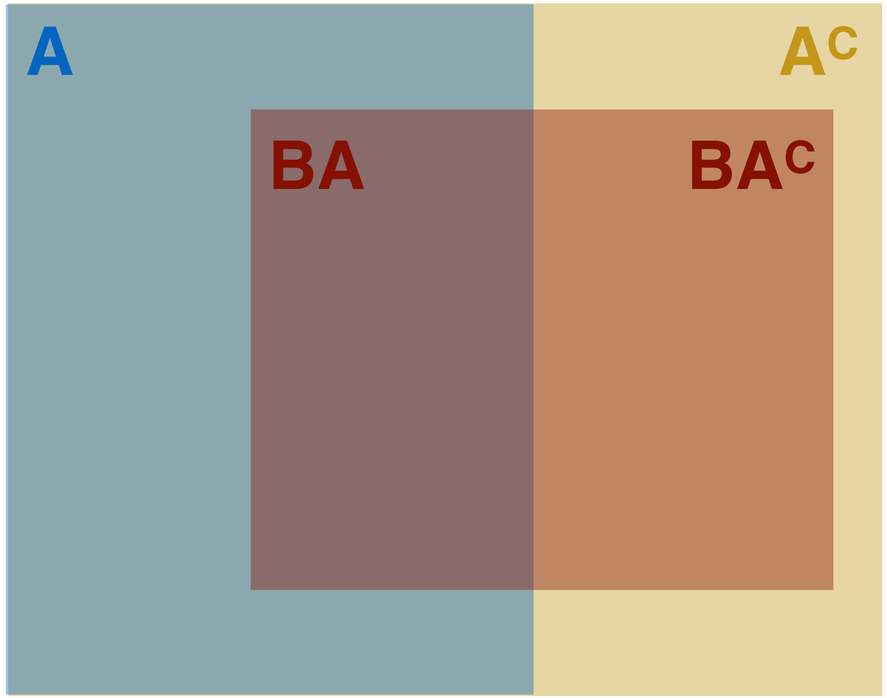


PREP nuggets

Total Probabilty Rule

$$P(B) = P(BA) + P(BA^C)$$

$$= P(B \mid A) \times P(A) + P(B \mid A^C) \times P(A^C)$$

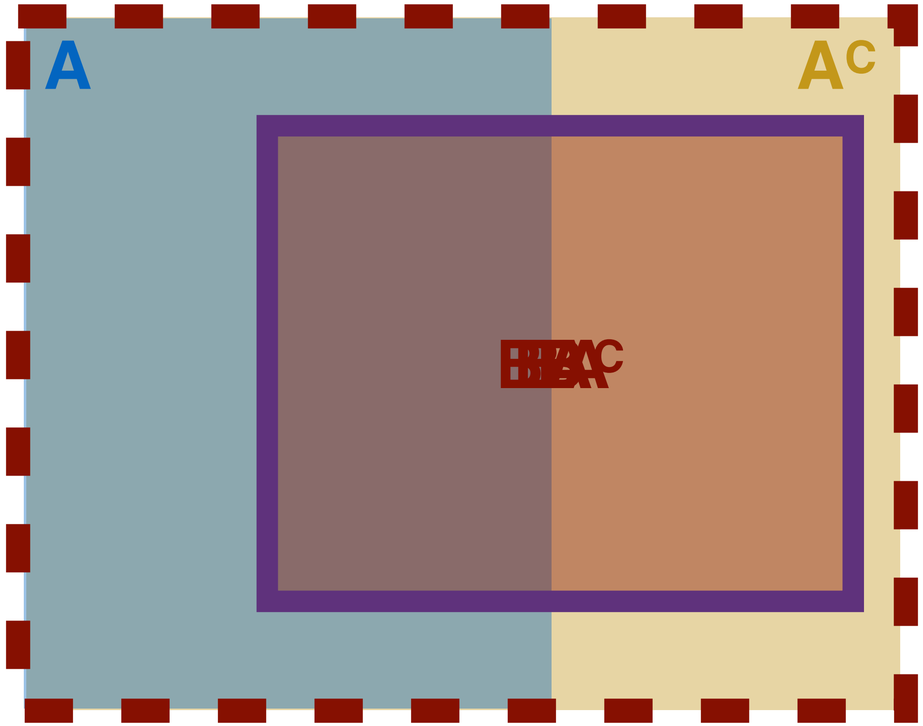


PREP nuggets

Total Probabilty Rule

$$P(B) = P(BA) + P(BA^C)$$

$$= P(B \mid A) \times P(A) + P(B \mid A^C) \times P(A^C)$$



PREP nuggets

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