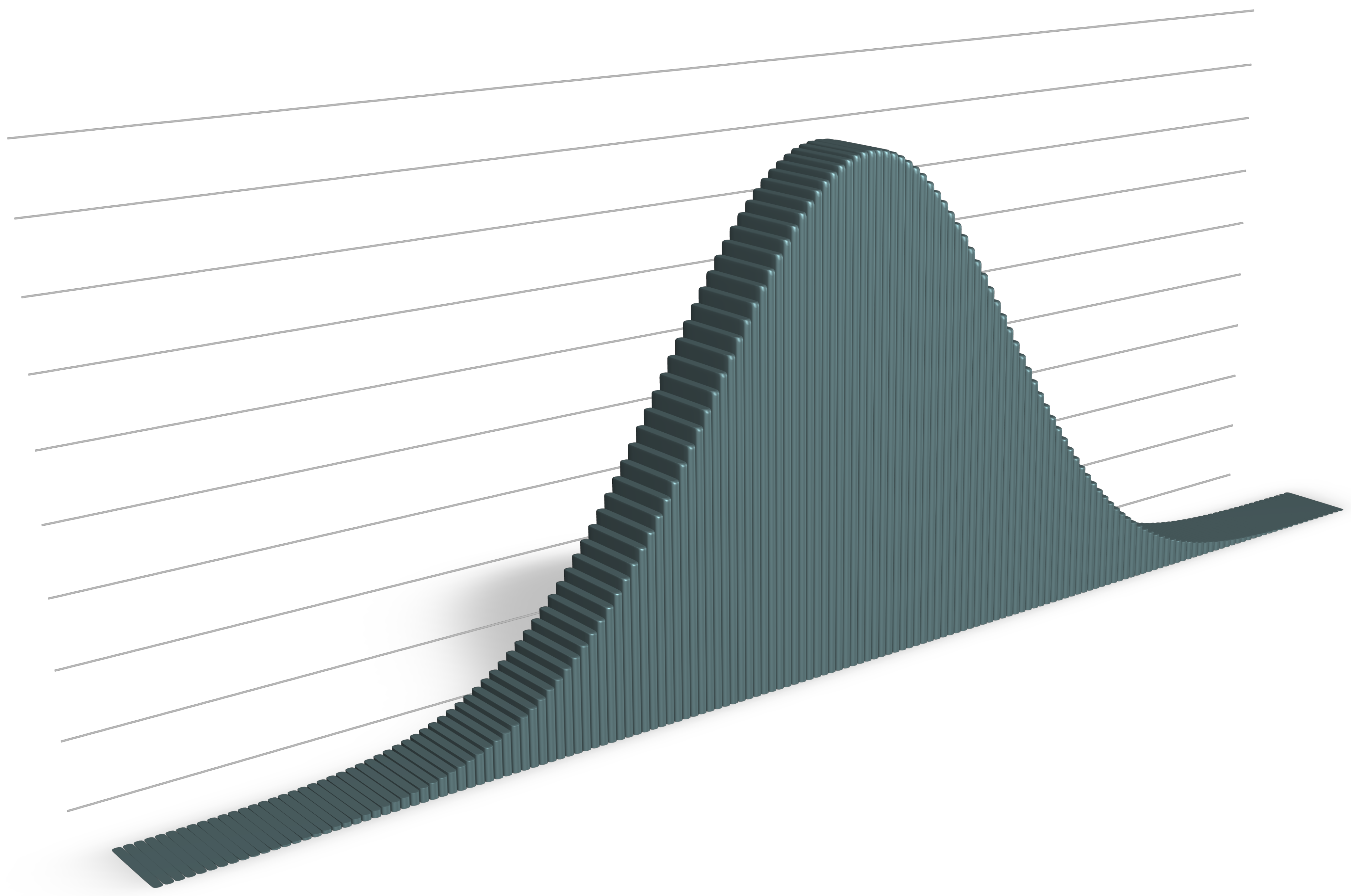


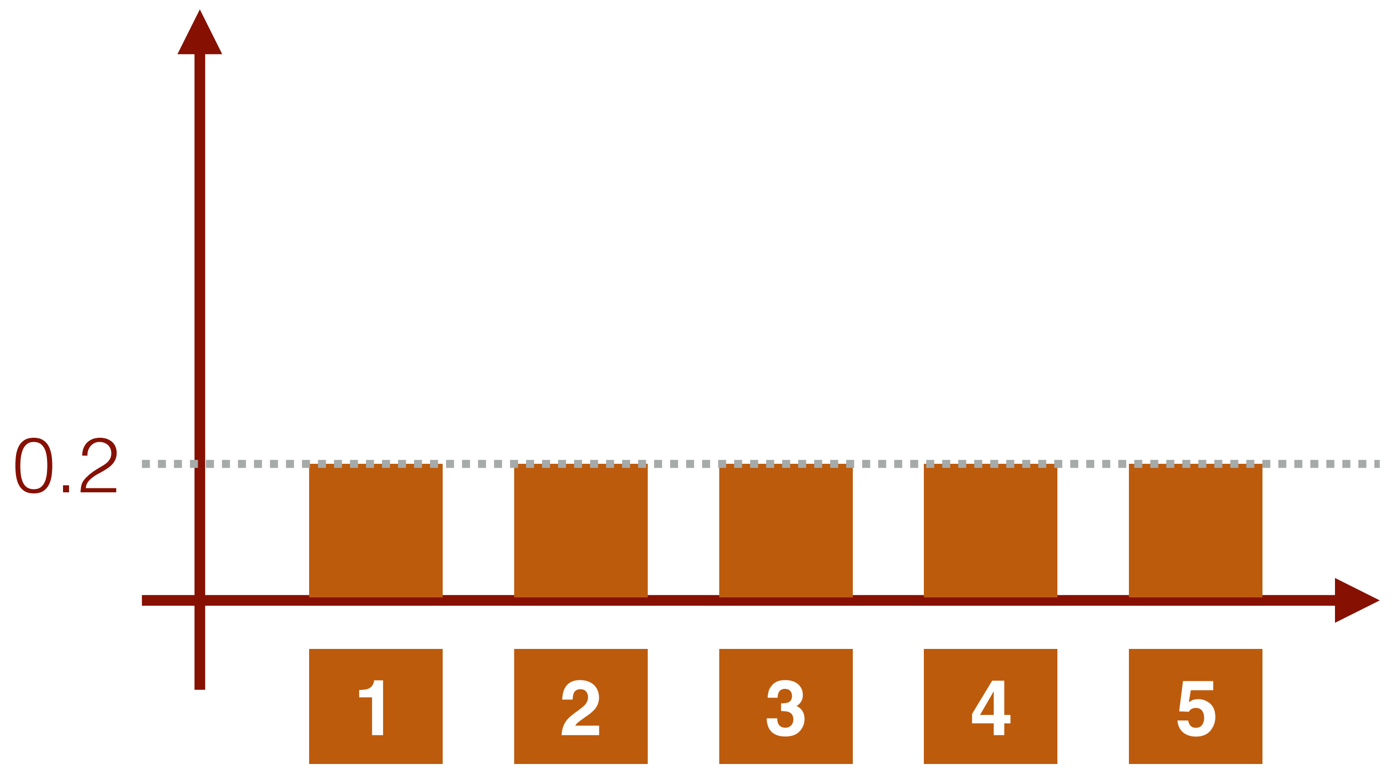
Common Probability Distributions

Continuous Random Variables

1. Continuous Uniform Distributions
2. Normal Distributions
3. Roy's Safety First Ratio
4. Lognormal Distributions

Discrete Random Variable

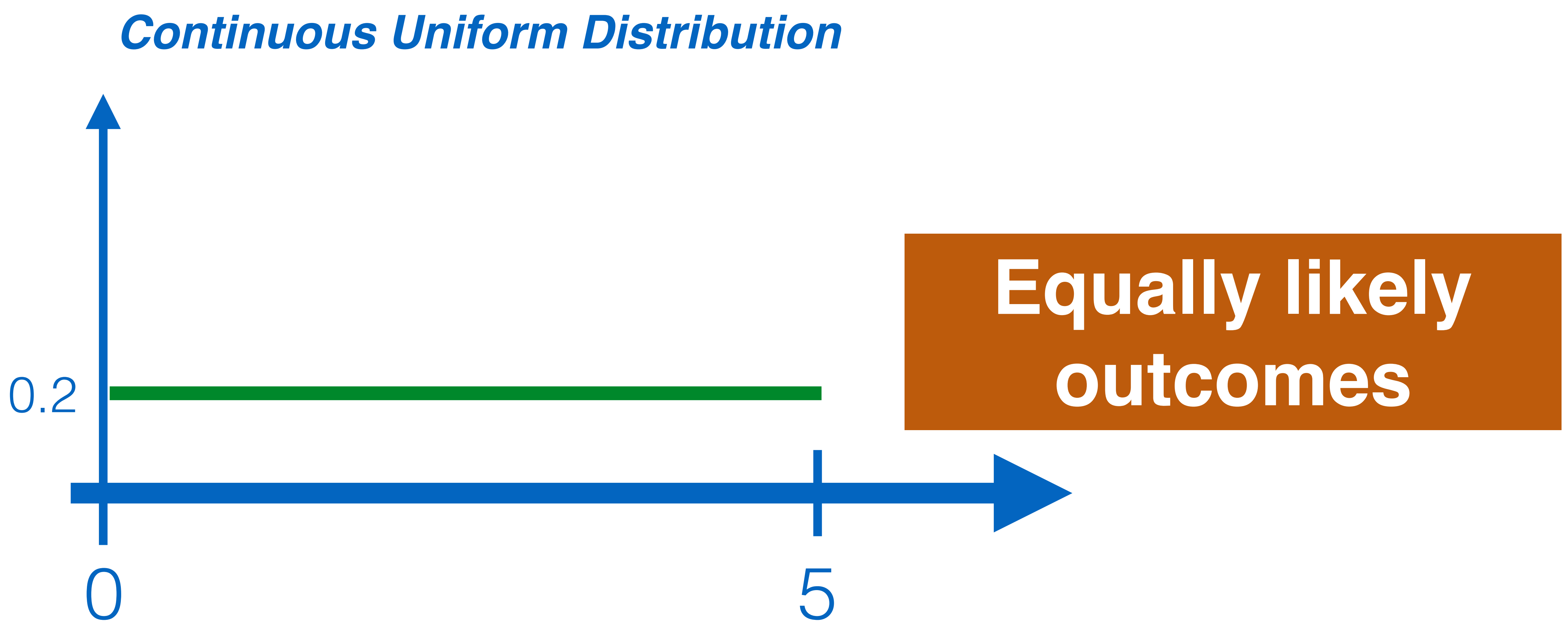
Probability Distribution Function



Continuous Random Variable

Probability Density Function

Continuous Uniform Random Variable

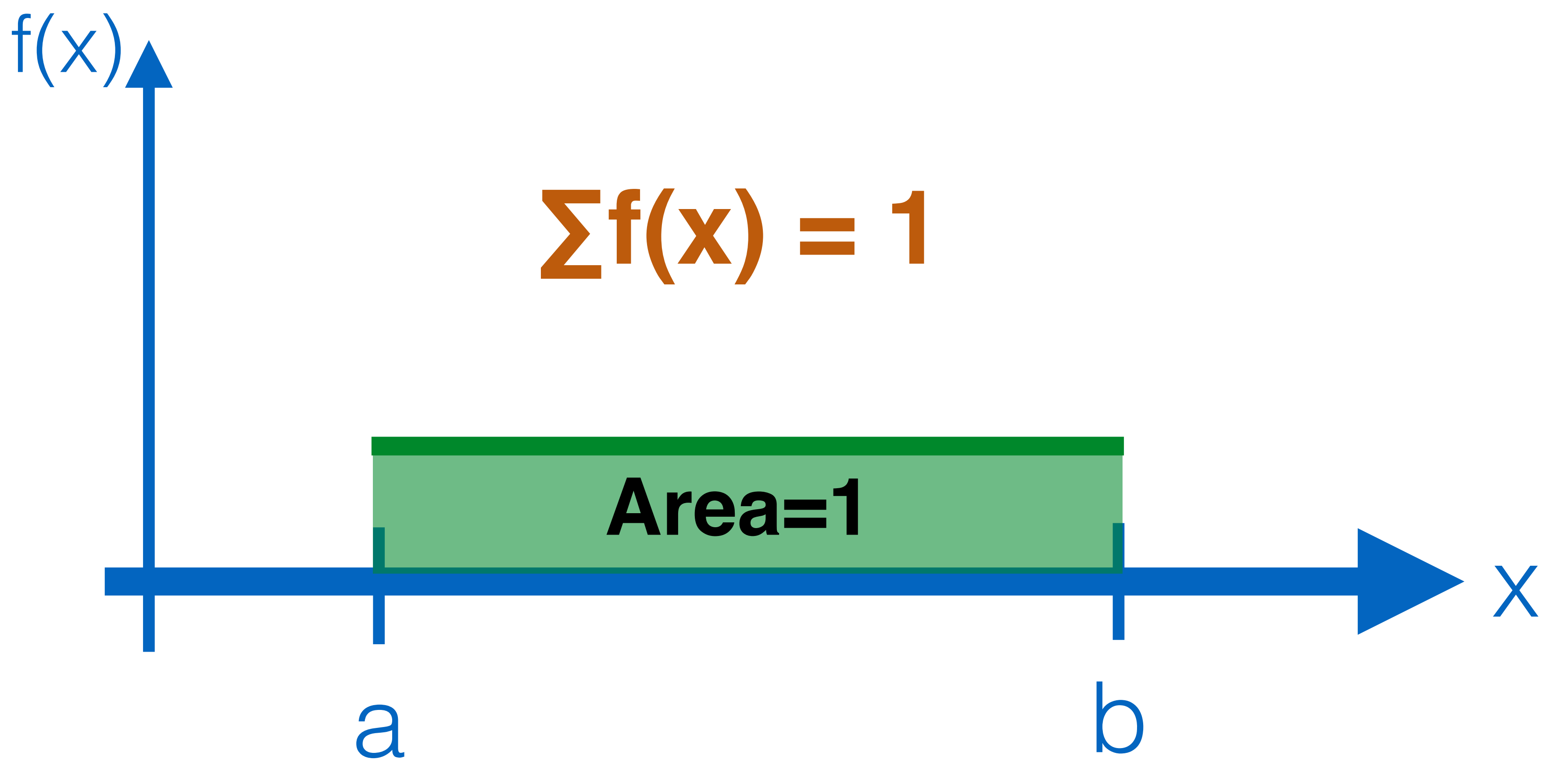


$$f(x) = P(X=x) = 0.2$$

for $0 \leq X \leq 5$

Continuous Uniform Random Variable

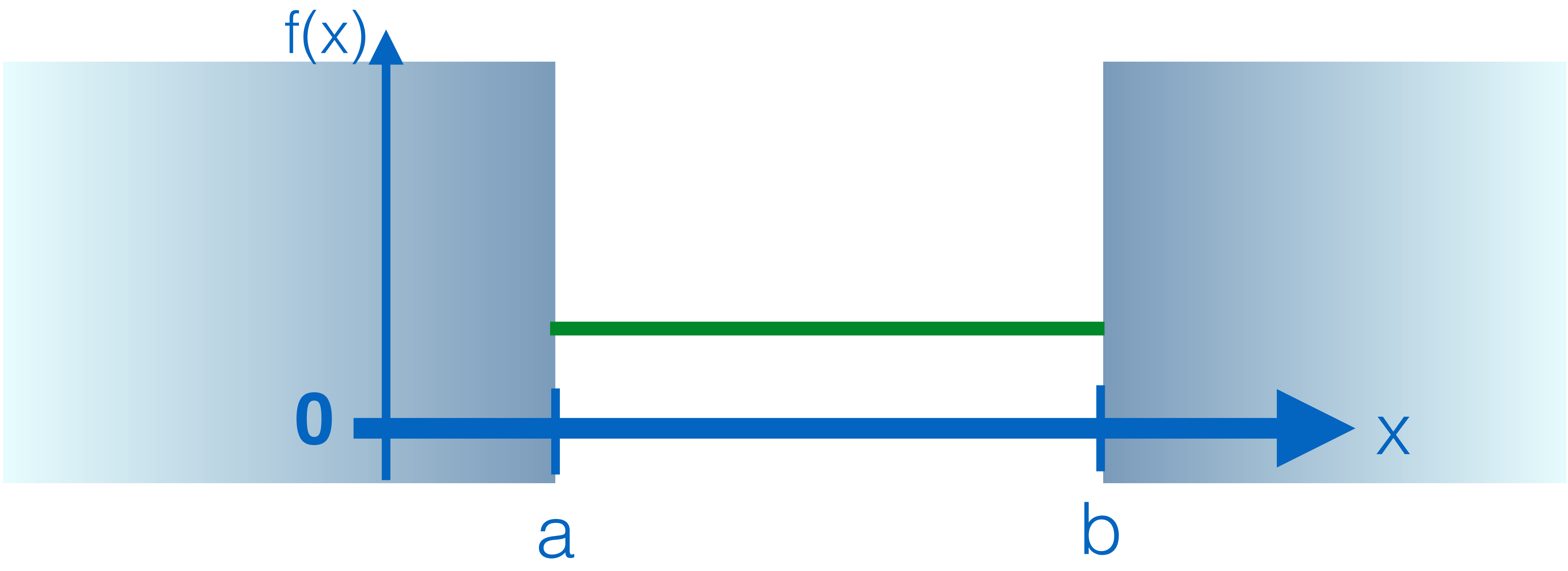
Continuous Uniform Distribution



$$f(x) = \frac{1}{(b-a)} \quad \text{for } a<x<b$$

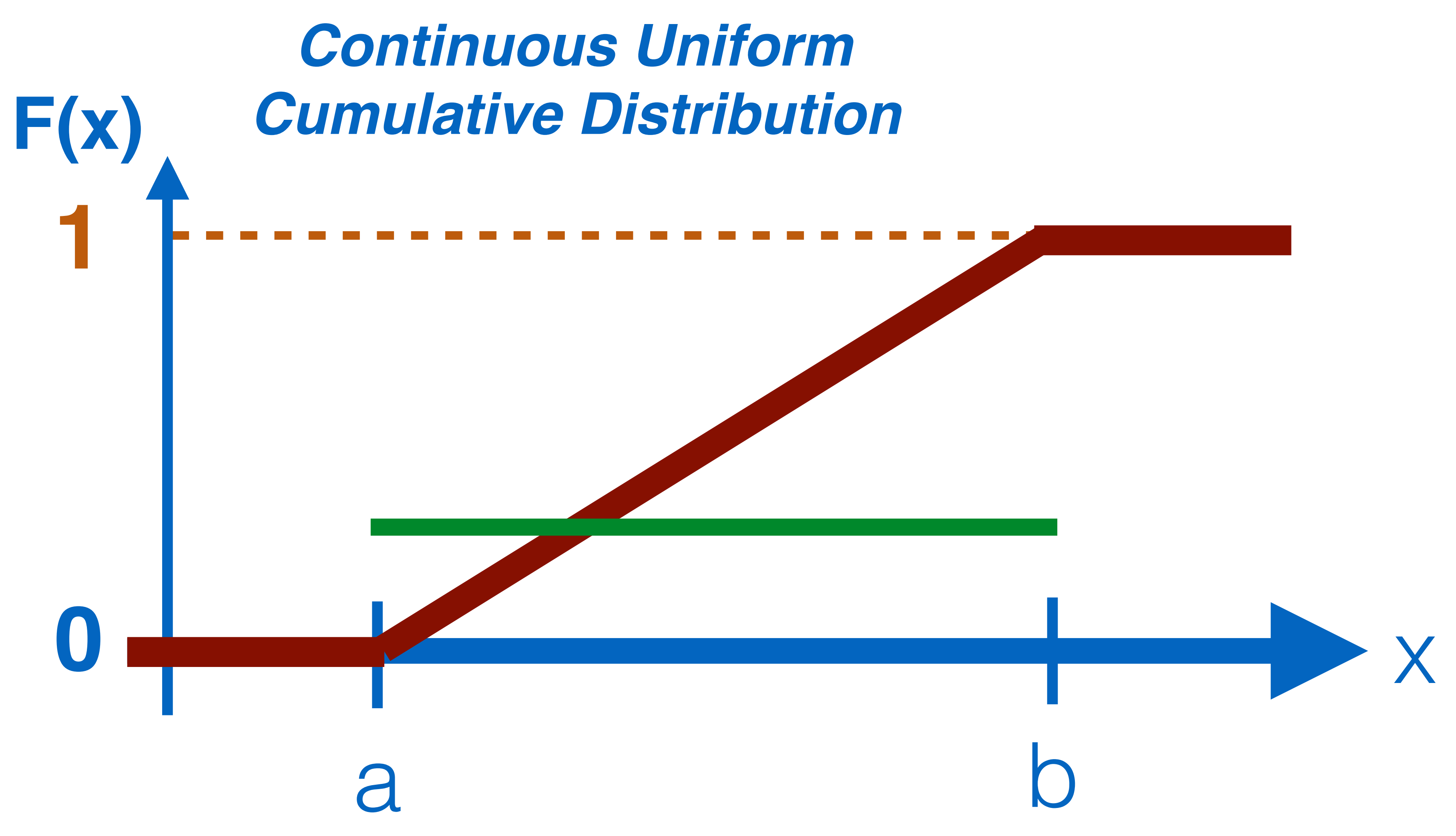
Continuous Uniform Random Variable

Continuous Uniform Distribution



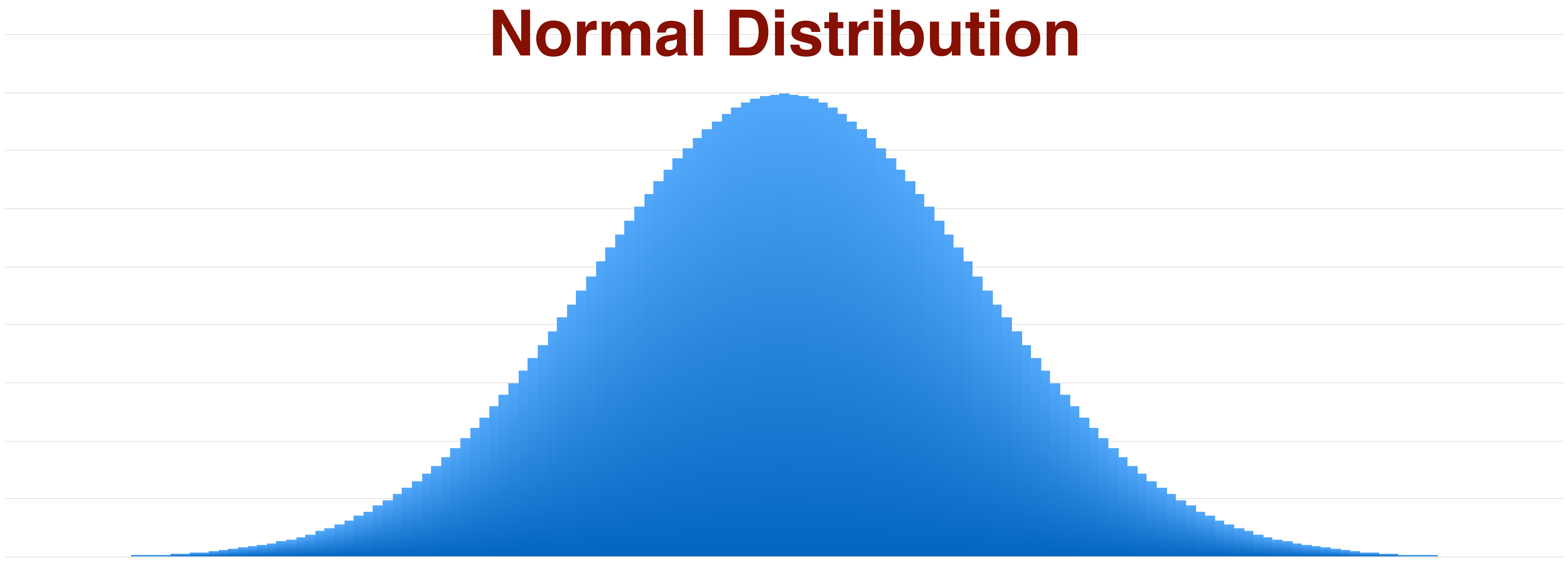
$$\mathbf{f(x)} = \begin{cases} \mathbf{1/(b-a)} & \textit{for } a<x<b \\ \mathbf{0} & \textit{otherwise} \end{cases}$$

Continuous Uniform Random Variable



$$F(x) = \begin{cases} 0 & \textit{for } x<a \\ (x-a)/(b-a) & \textit{for } a<x<b \\ 1 & \textit{for } x>b \end{cases}$$

What makes a distribution “normal”?

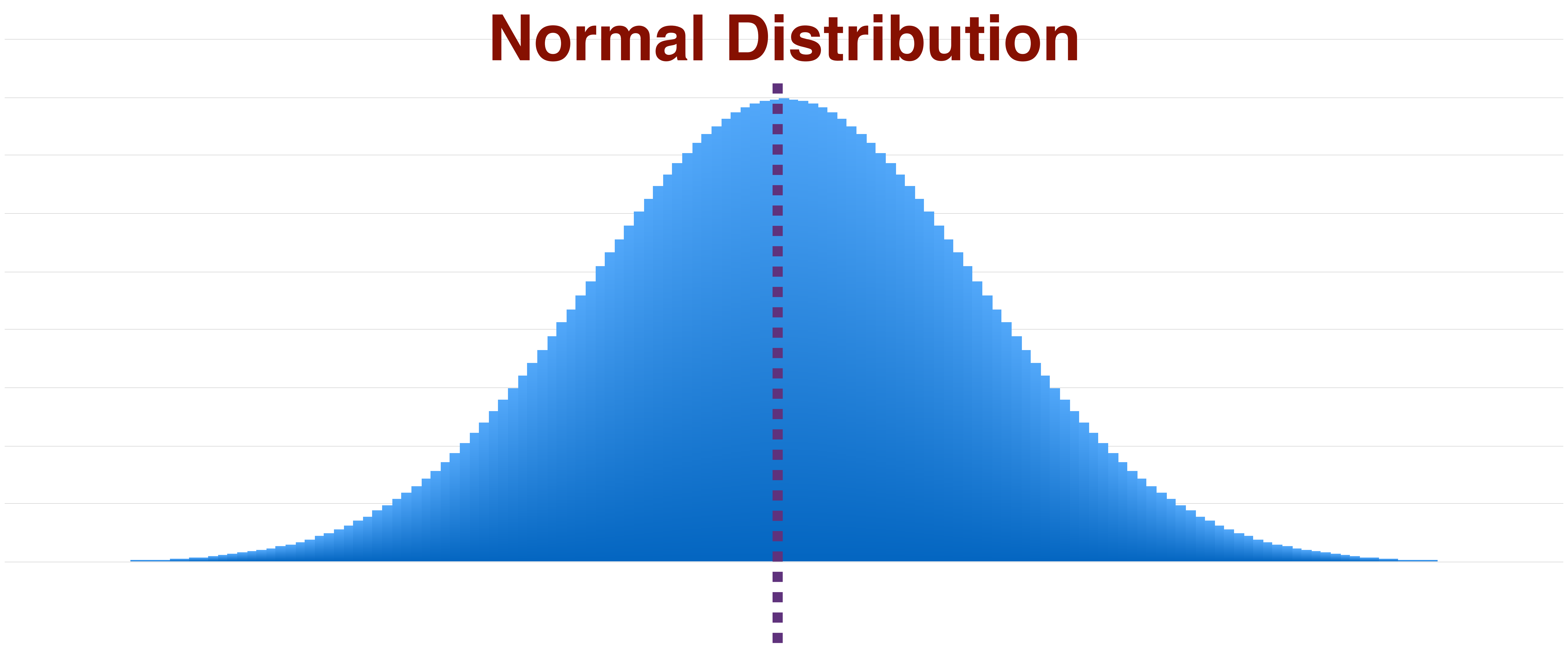


$$X \sim N(\mu, \sigma^2)$$

Mean

Variance

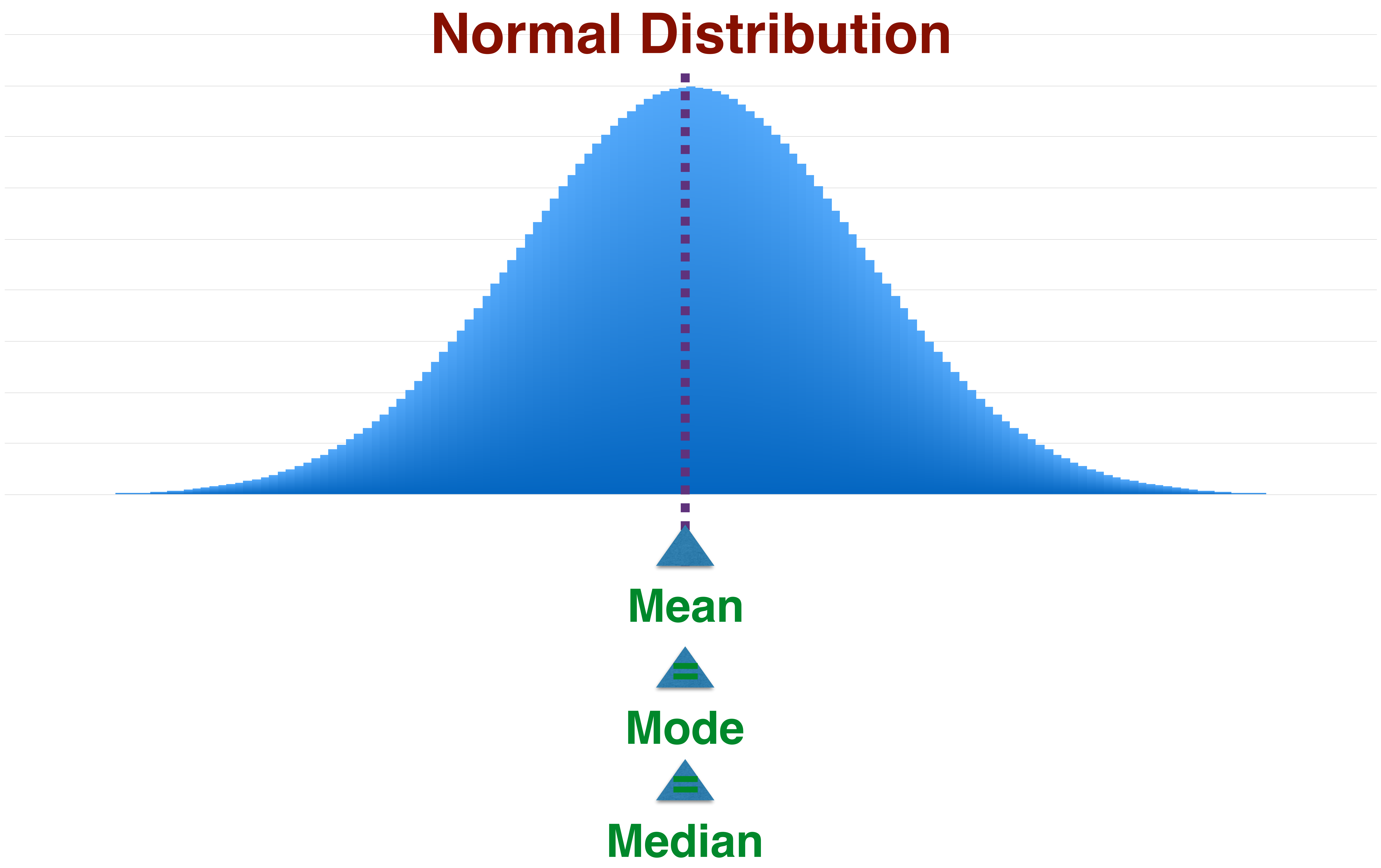
What makes a distribution "normal"?



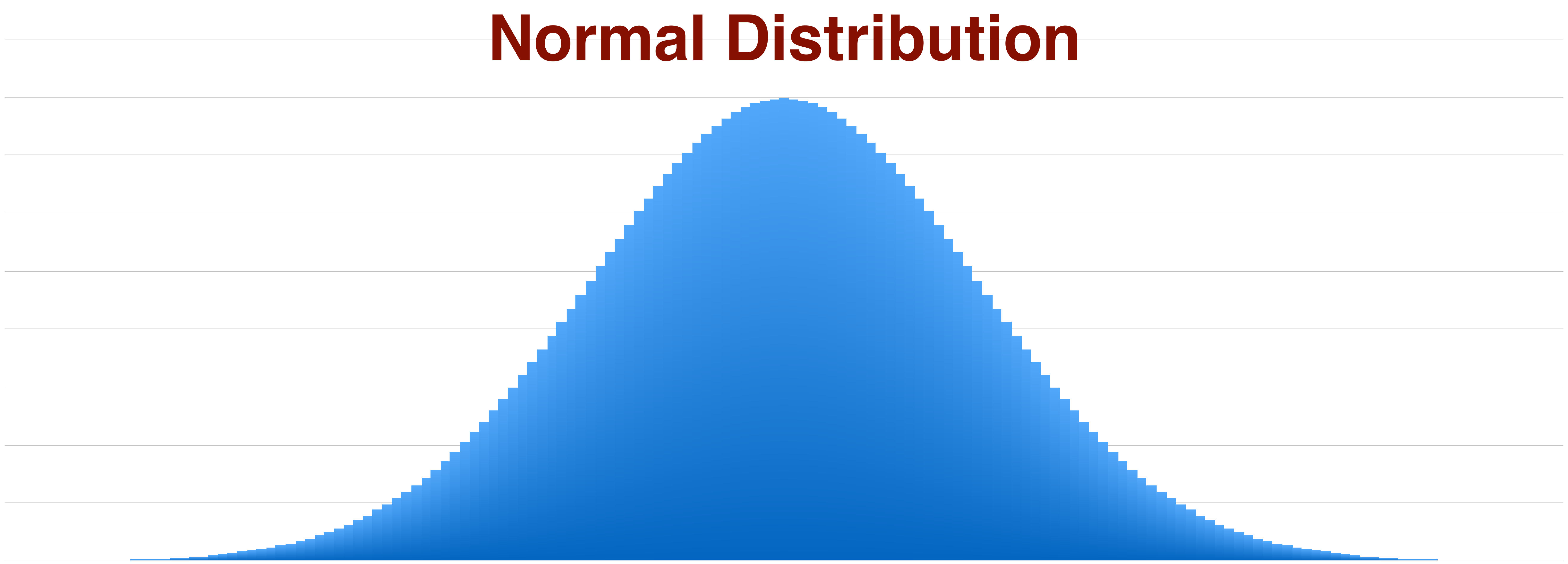
Symmetrical

Skewness = 0

What makes a distribution "normal"?

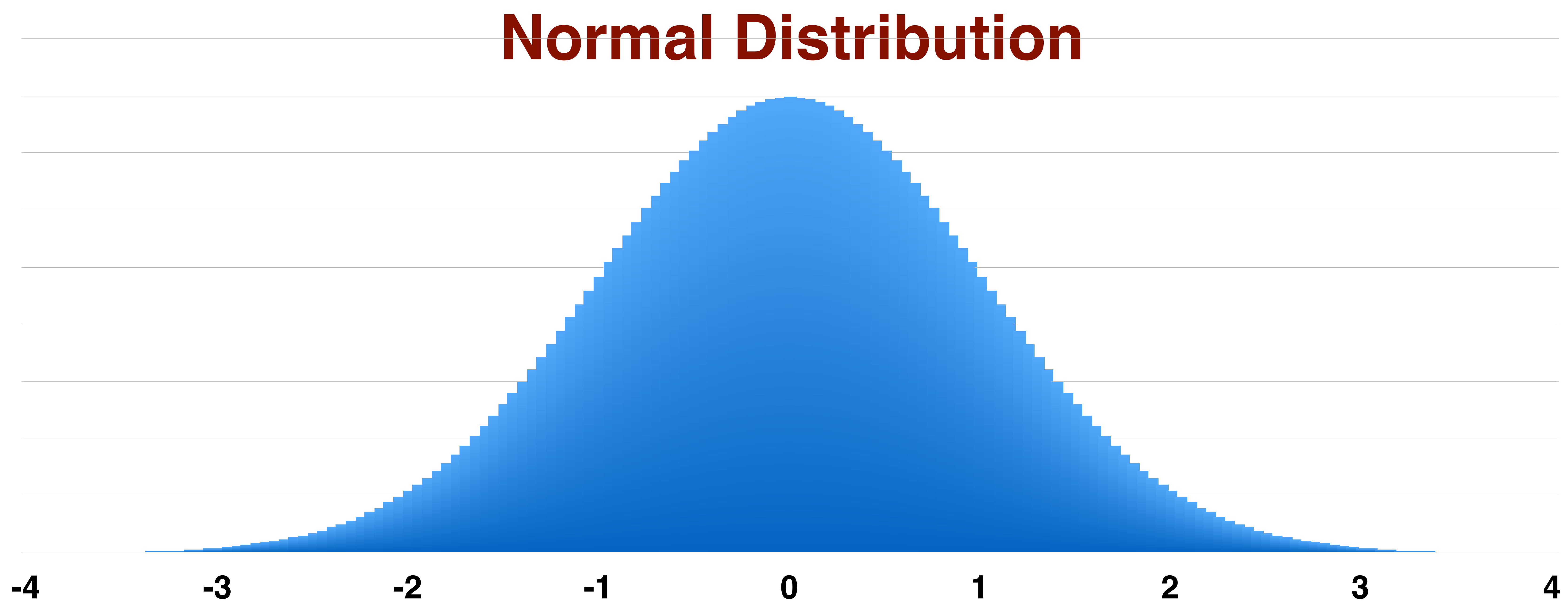


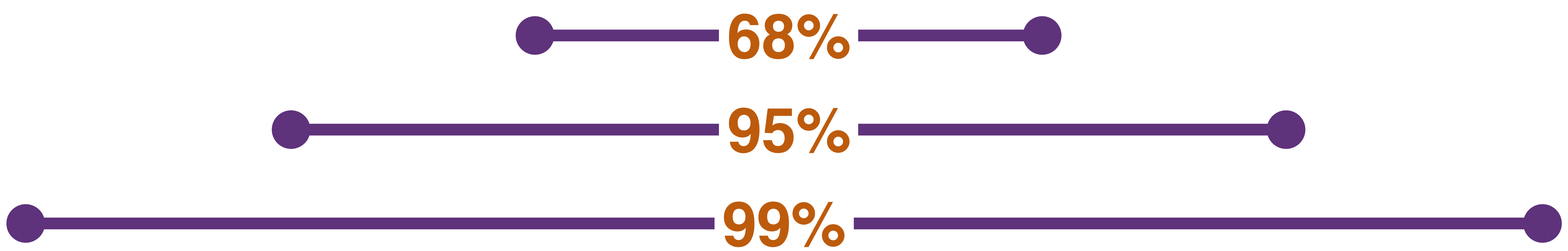
What makes a distribution "normal"?



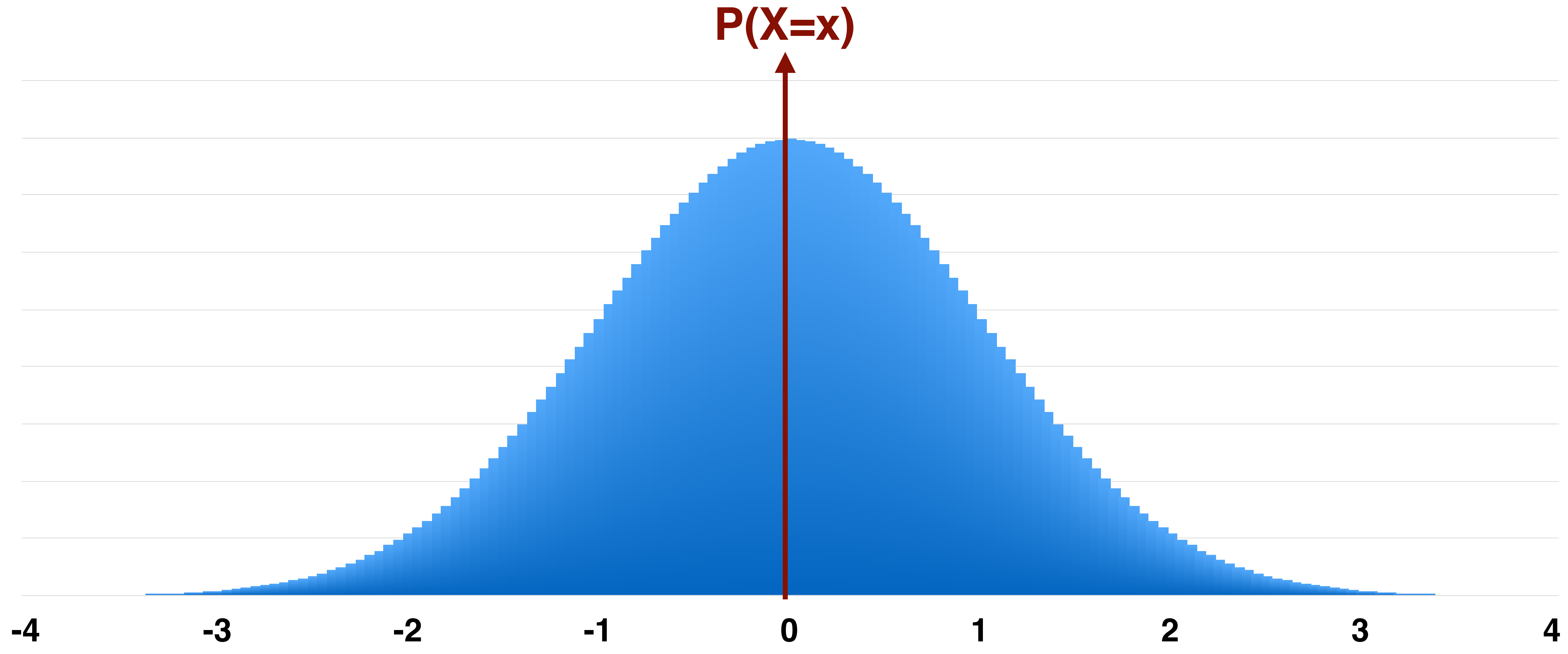
Kurtosis = 3

What makes a distribution “normal”?





Probability Distribution



$\bar{X} - 1.65s < X < \bar{X} + 1.65s$

$\bar{X} - 1.96s < X < \bar{X} + 1.96s$

$\bar{X} - 2.58s < X < \bar{X} + 2.58s$

Confidence Intervals

The average return of a stock index is 8.9% per year and the variance of returns is 256. If the returns are approximately normal, what is the 90% confidence interval for the index return next year?

standard deviation, $s = (256)^{1/2} = 16\%$

90% Confidence Interval

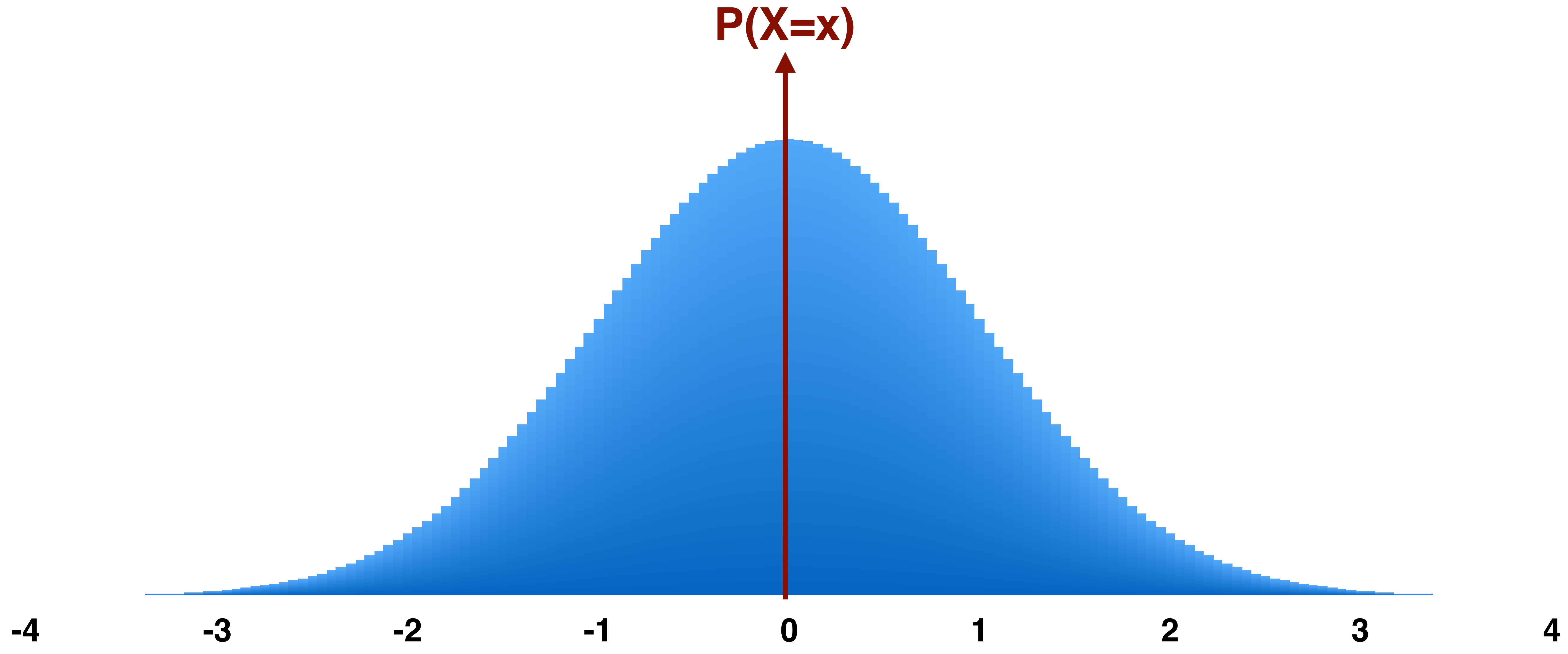
$\bar{X} - 1.65s < X < \bar{X} + 1.65s$

$8.9 - 1.65 \times 16 < X < 8.9 + 1.65 \times 16$

$-17.5 < X < 35.3\%$

We are 90% sure that the returns of the stock index next year will fall within this interval

Probability Distribution



-1.65s — **90%** — +1.65s

-1.96s — **95%** — +1.96s

-2.58s — **99%** — +2.58s

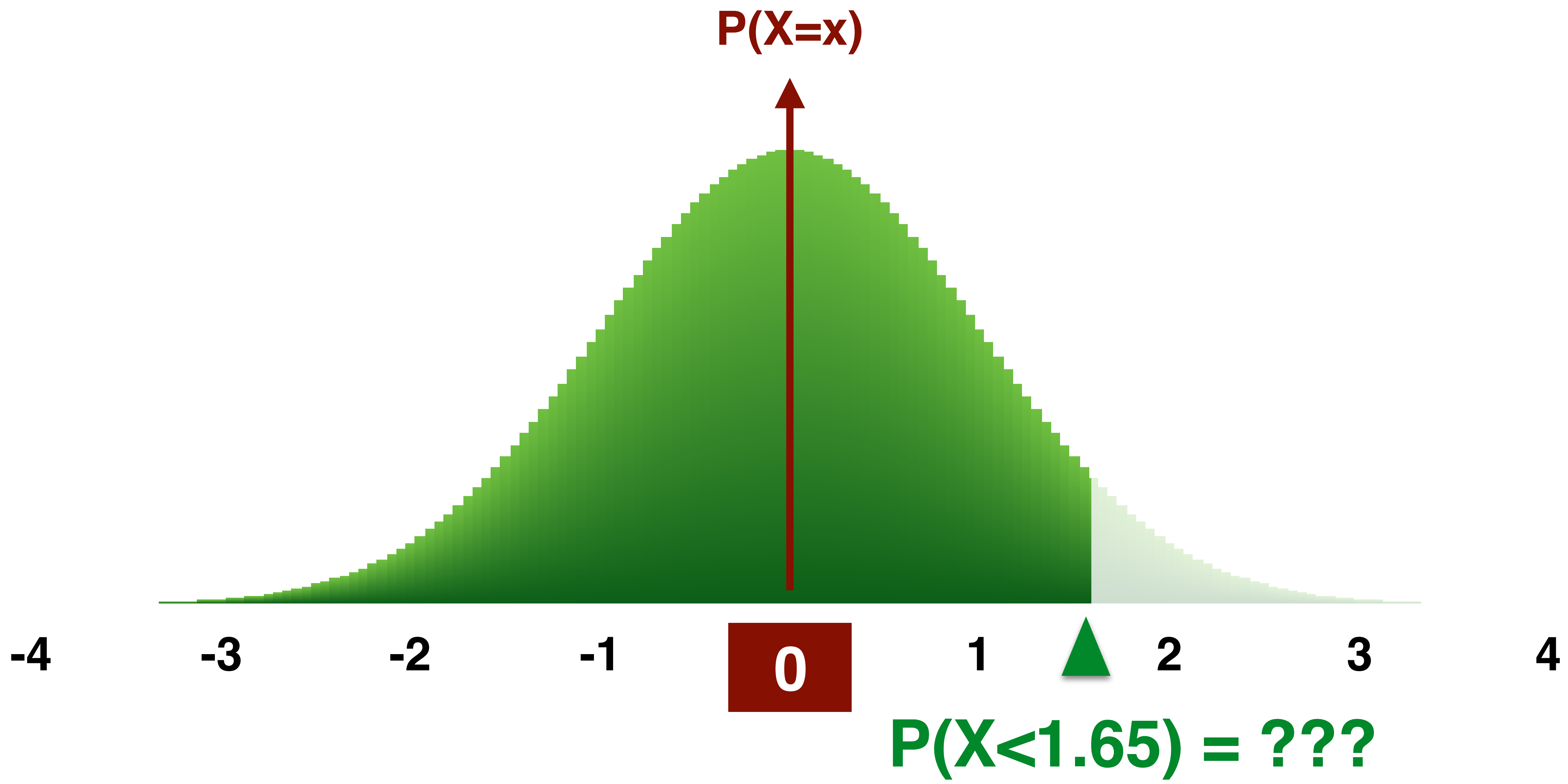
$\bar{X} - 1.65s < X < \bar{X} + 1.65s$

$\bar{X} - 1.96s < X < \bar{X} + 1.96s$

$\bar{X} - 2.58s < X < \bar{X} + 2.58s$

Confidence Intervals

Standard Normal Distribution



Standard Normal Probabilities

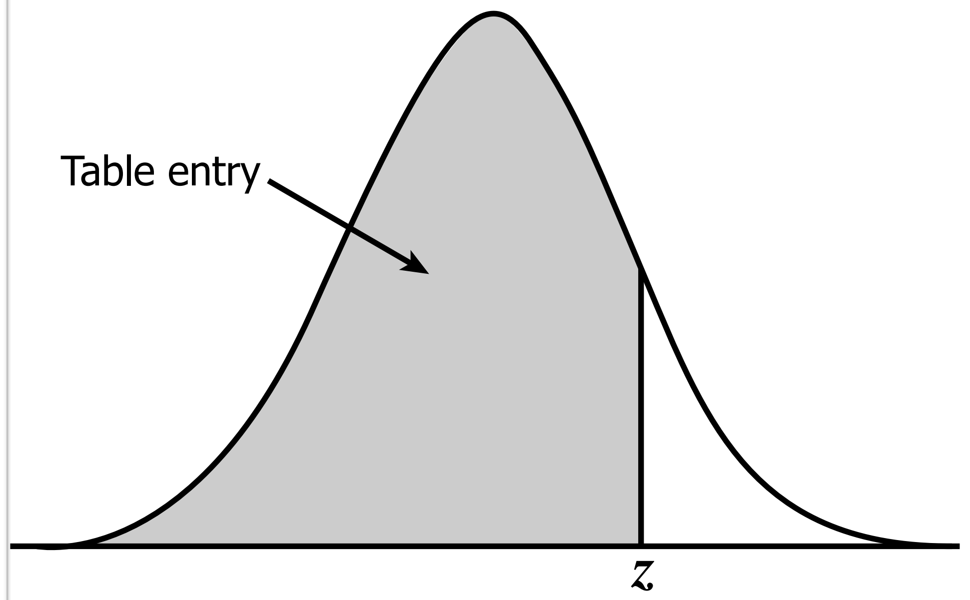
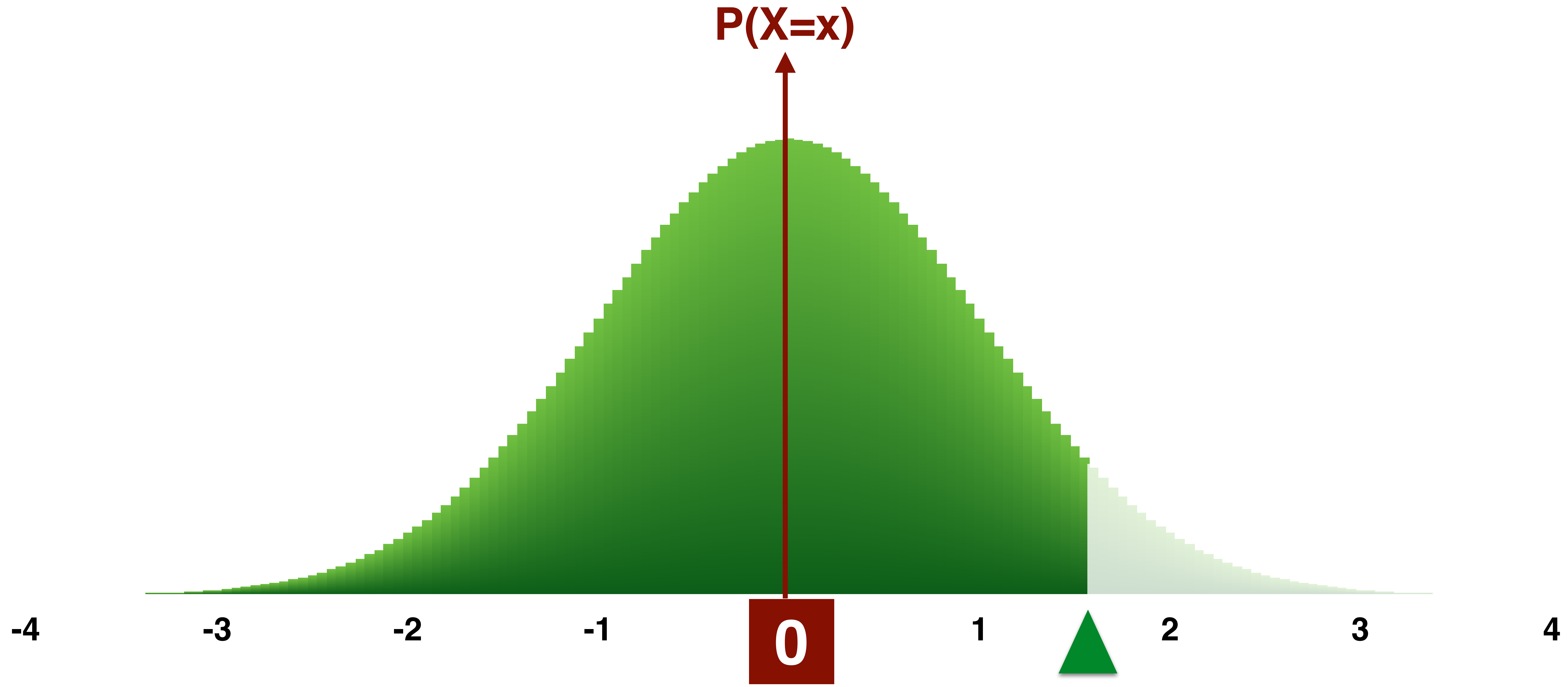


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

Standard Normal Distribution

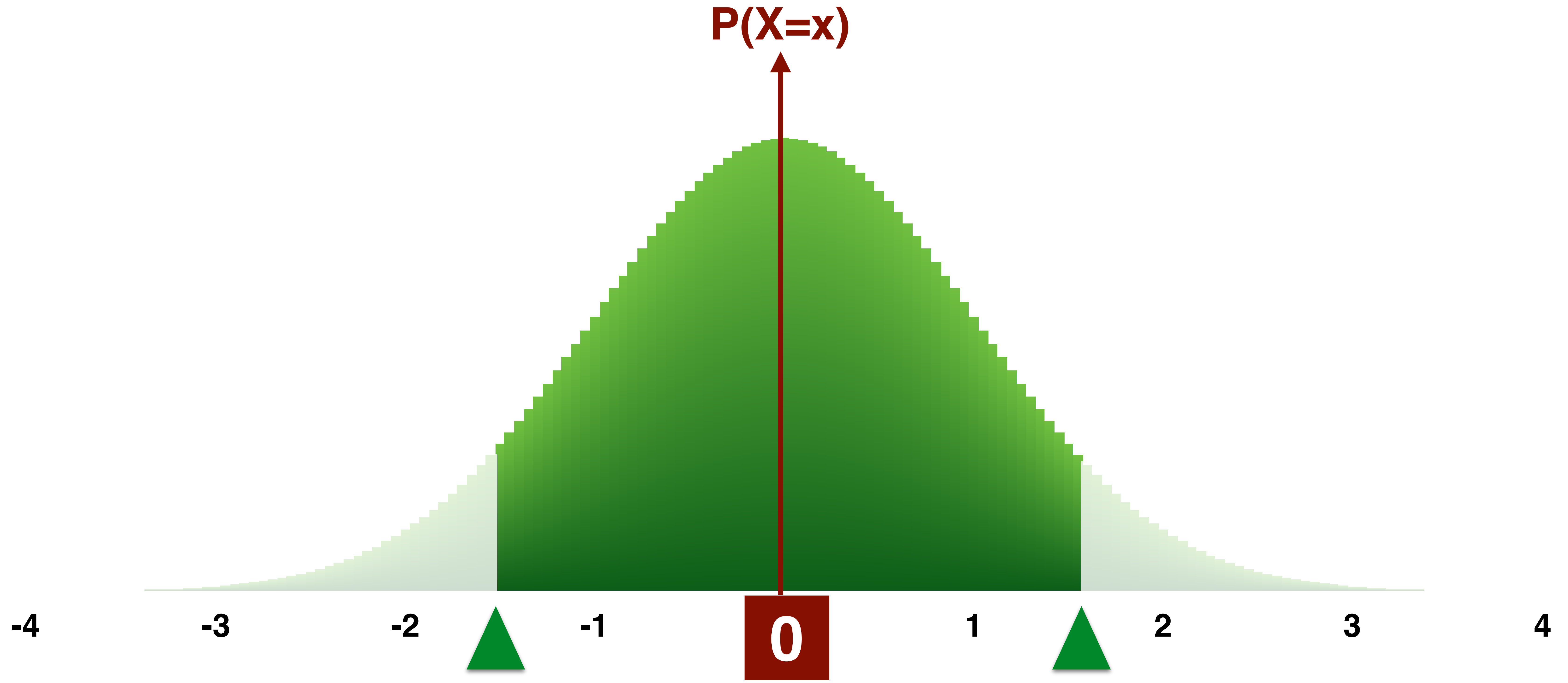


P(X<1.65) = .9505

~95% of the observations lie below 1.65

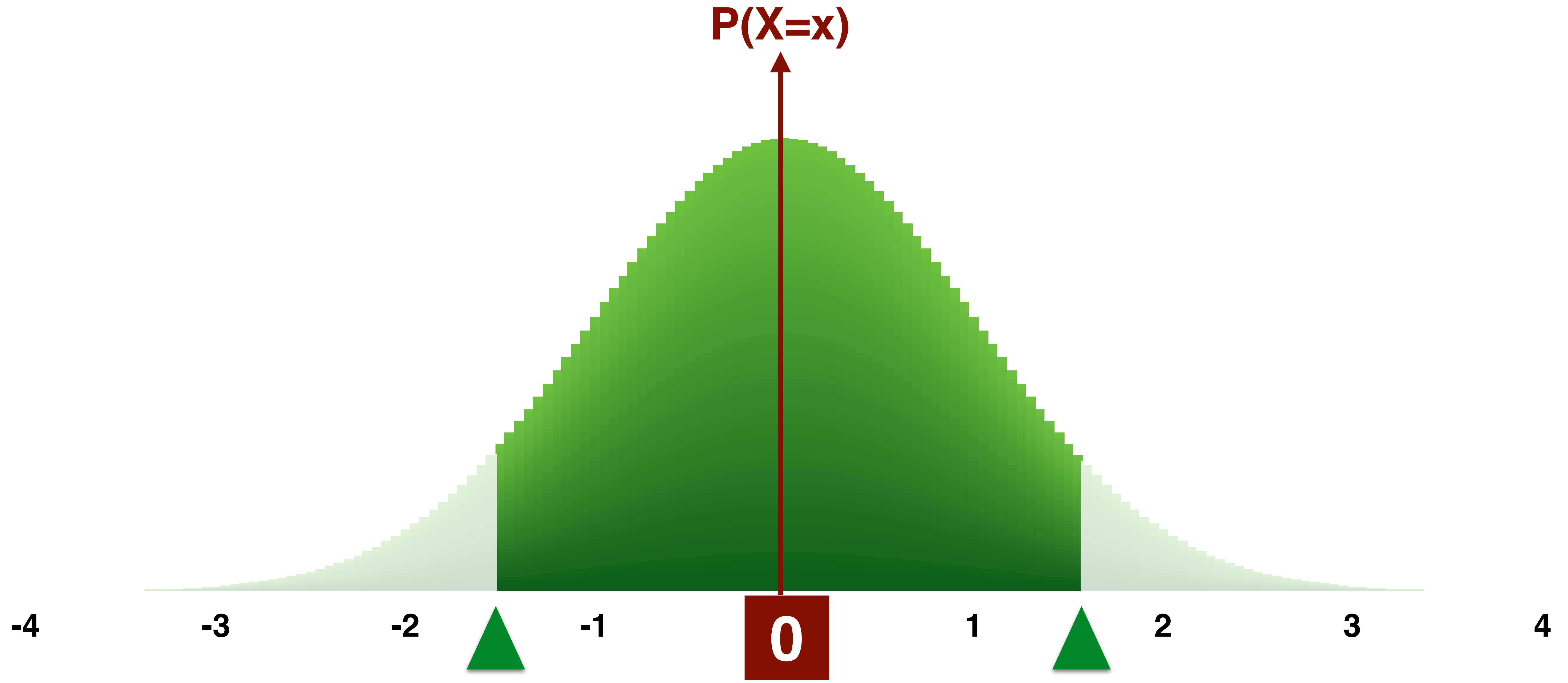
Expect X to be < 1.65 with 95% confidence

Standard Normal Distribution



P(X<-1.65) = 1-0.9505 =.0495 P(X<1.65) = .9505

Standard Normal Distribution

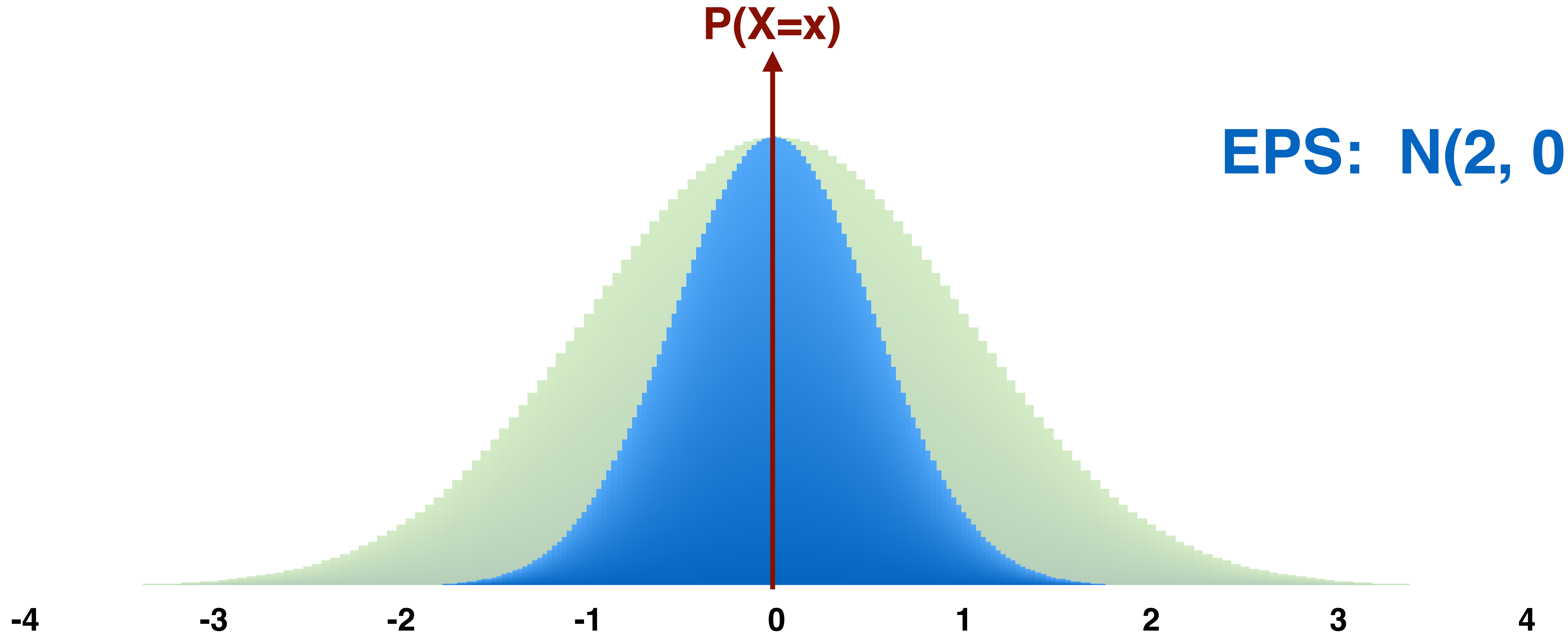


$$P(-1.65<X<1.65) = .9505 - .0495 \approx 0.9$$

-1.65s — 90% — +1.65s

Confidence Interval

Standard Normal Distribution

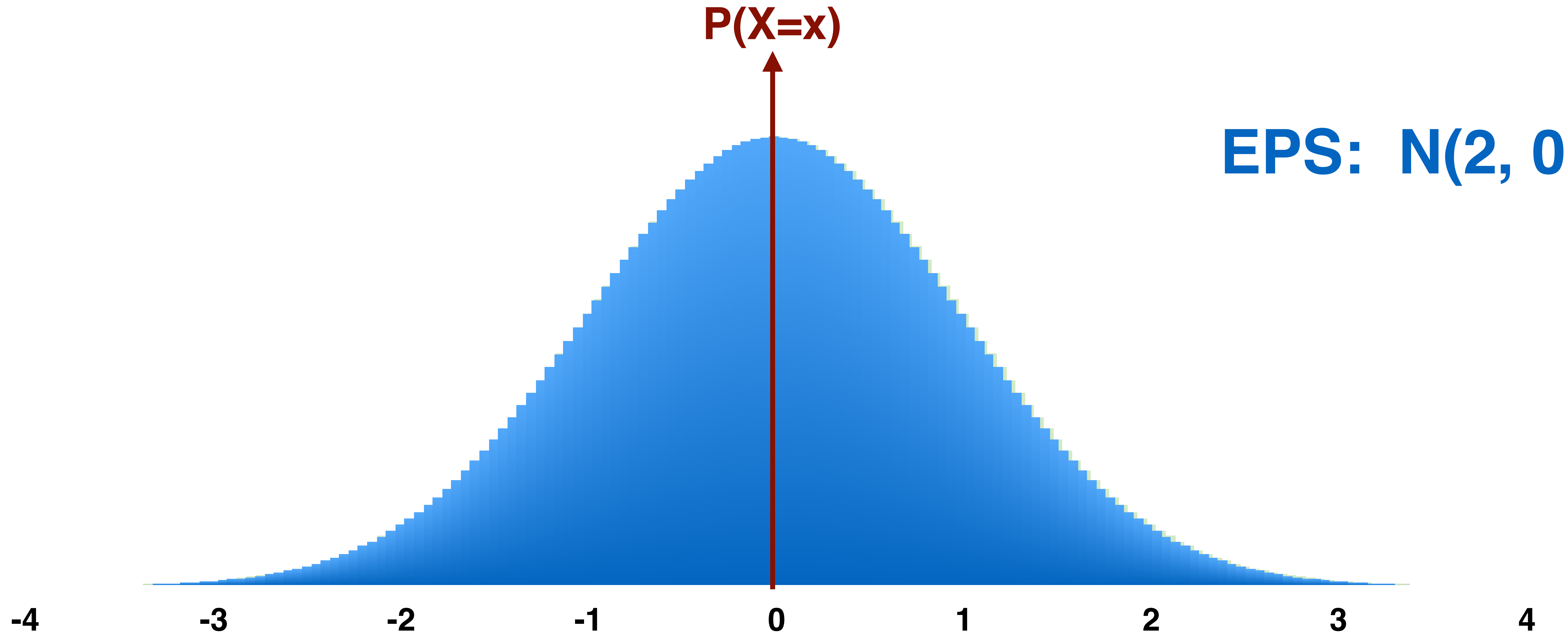


EPS: N(2, 0.5)

align the mean to zero

$$z = x - 2$$

Standard Normal Distribution

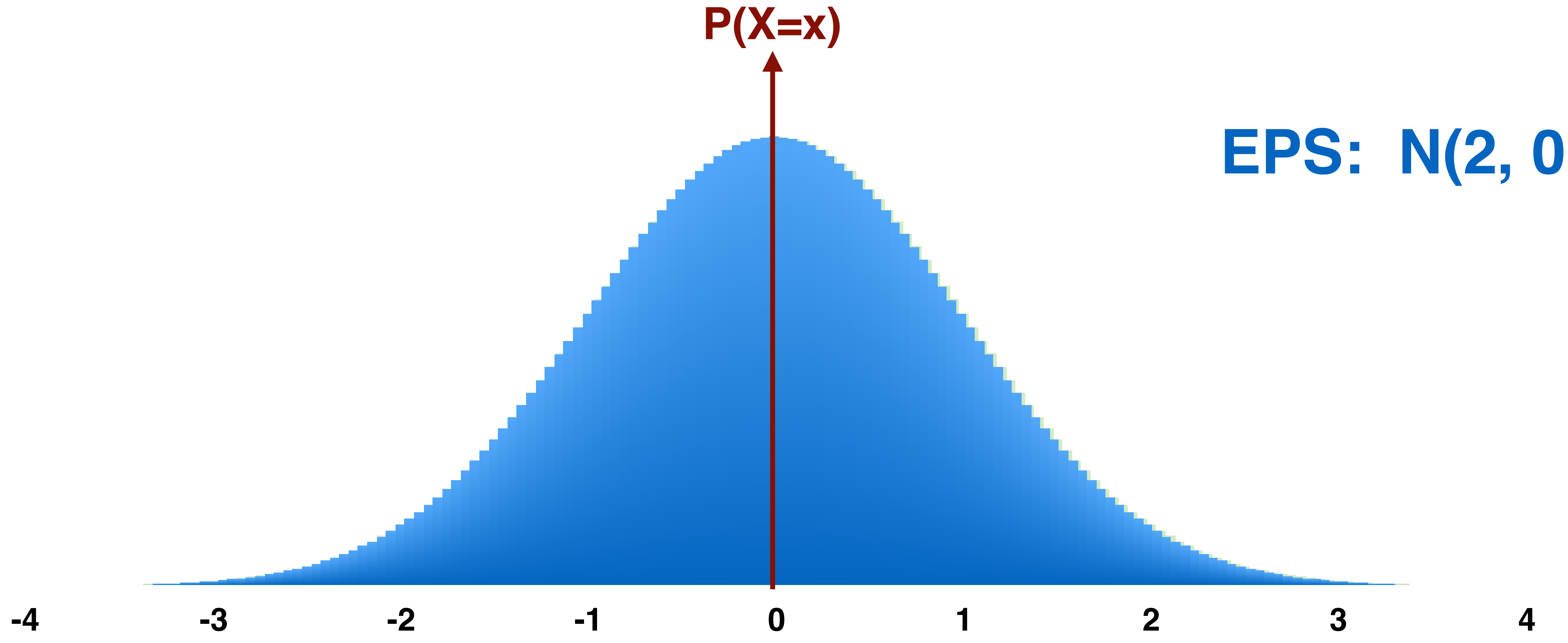


EPS: N(2, 0.5)

scale the the standard deviation to 1

$$z = \frac{x - 2}{0.5}$$

Standard Normal Distribution



EPS: N(2, 0.5)

$$z = \frac{x - \mu}{\sigma}$$

The random variable X denotes the annual earnings per share of stocks in MegaLand. It was found the the mean of X is $2 and the standard deviation is $0.50.

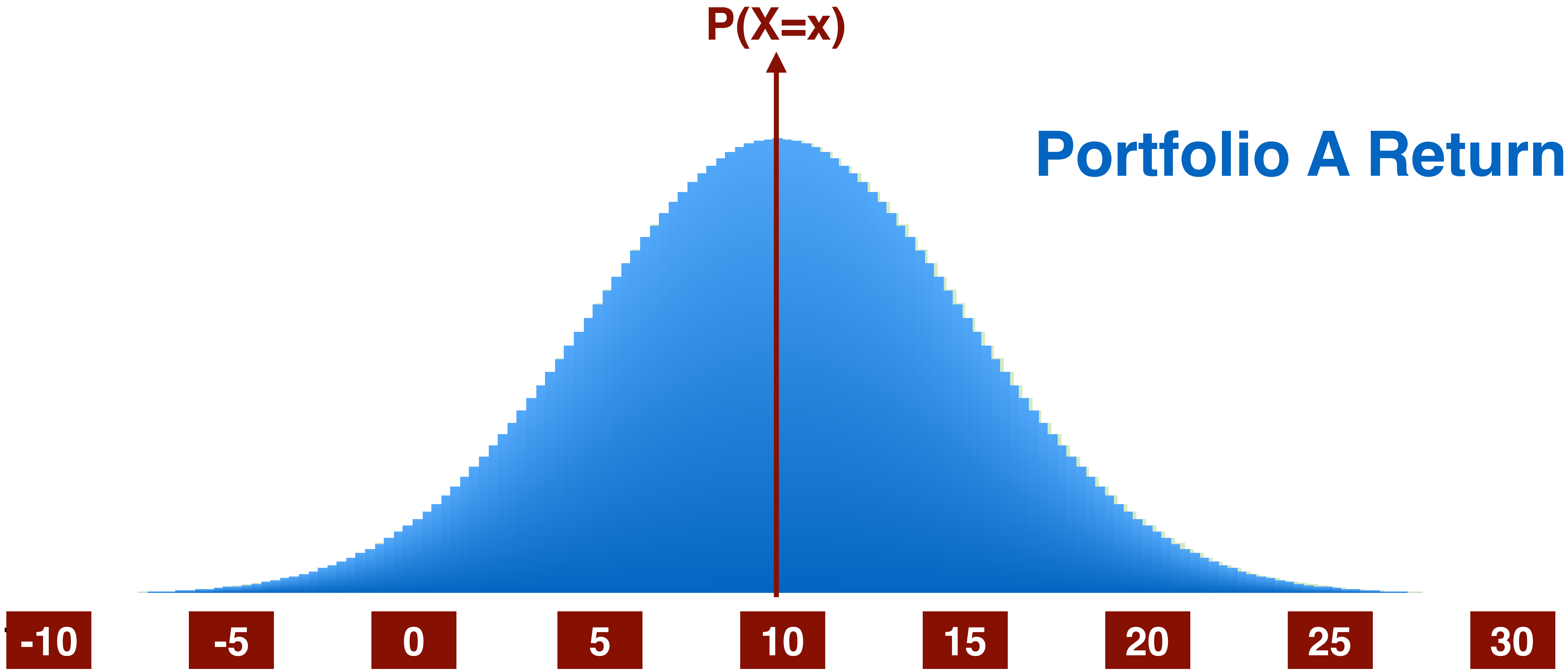
What percentage of the stocks are likely to have an EPS of less than $1?

$$\boxed{z = \frac{x - \mu}{\sigma}}$$

$z = (1-2)/0.5 = -2.0$

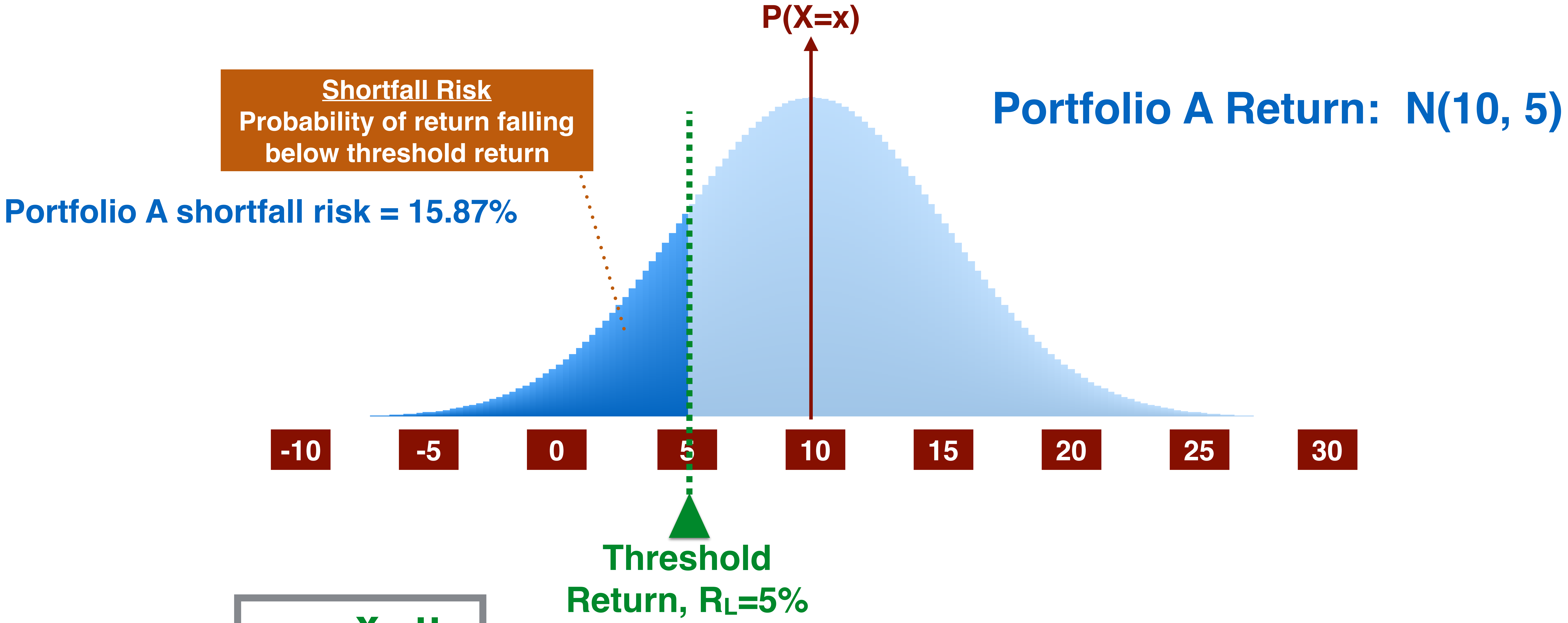
$$F(-2.0) = 1 - F(2.0)$$
$$= 1 - 0.9772$$
$$= 0.0228$$

2.28% of the stocks are likely to have an EPS of less than $1



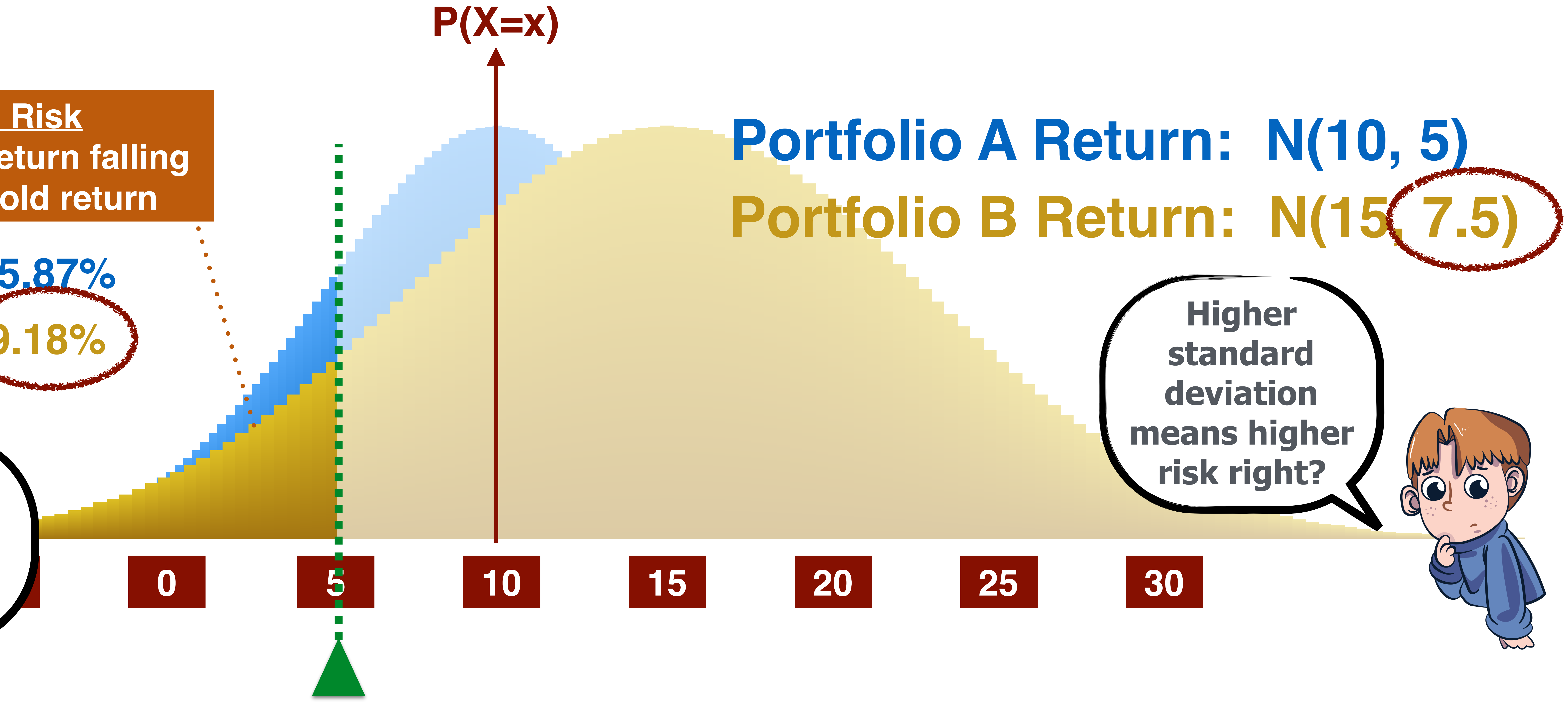
Portfolio A Return: N(10, 5)

$$z = \frac{x - \mu}{\sigma}$$



$$z = \frac{x - \mu}{\sigma}$$

$z_A = (5-10)/5 = -1.0$



<u>Shortfall Risk</u>
Probability of return falling below threshold return

Portfolio A shortfall risk = 15.87%

Portfolio B shortfall risk = 9.18%

Portfolio A Return: N(10, 5)

Portfolio B Return: N(15, 7.5)

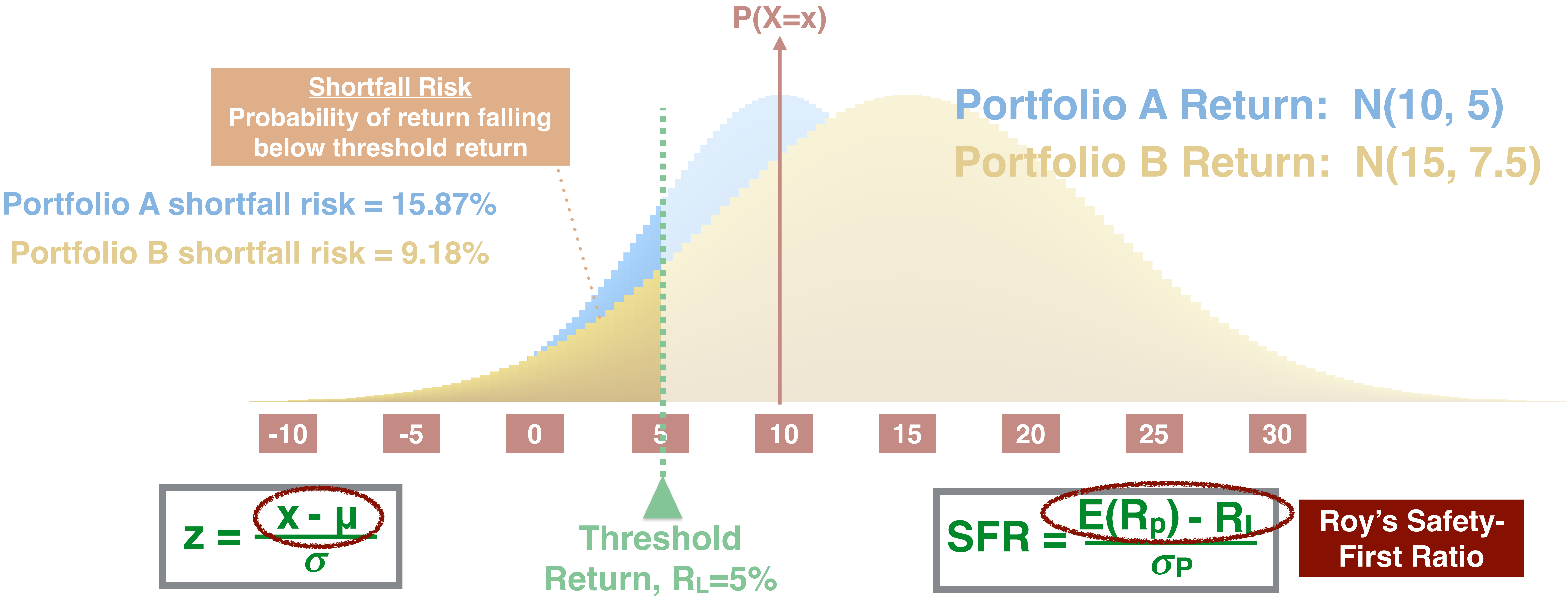
Threshold Return, R_L=5%

$$z = \frac{x - \mu}{\sigma}$$

$z_A = (5-10)/5 = -1.0$

$z_B = (5-15)/7.5 = -1.33$

LOWER Z Value
-> Lower Shortfall Risk



$$z = \frac{x - \mu}{\sigma}$$

$z_A = (5-10)/5 = -1.0$

$z_B = (5-15)/7.5 = -1.33$

LOWER Z Value
-> Lower Shortfall Risk

$$SFR = \frac{E(R_p) - R_L}{\sigma_P}$$

Roy's Safety-First Ratio

$SFR_A = (10-5)/5 = 1.0$

$SFR_B = (15-5)/7.5 = 1.33$

HIGHER SFR Value
-> Lower Shortfall Risk

A client engaged you to invest $100,000. Her requirement is to be able to withdraw at least $3000 per year without the portfolio value falling below the capital invested.

According to the safety-first criterion, which portfolio is more suitable for your client?

$$SFR = \frac{E(R_p) - R_L}{\sigma_P}$$

Portfolio	A	B	C
Expected Return (%)	14	5	8
Standard Deviation (%)	10	2	4
Safety-First Ratio	**1.1**	**1**	**1.25**

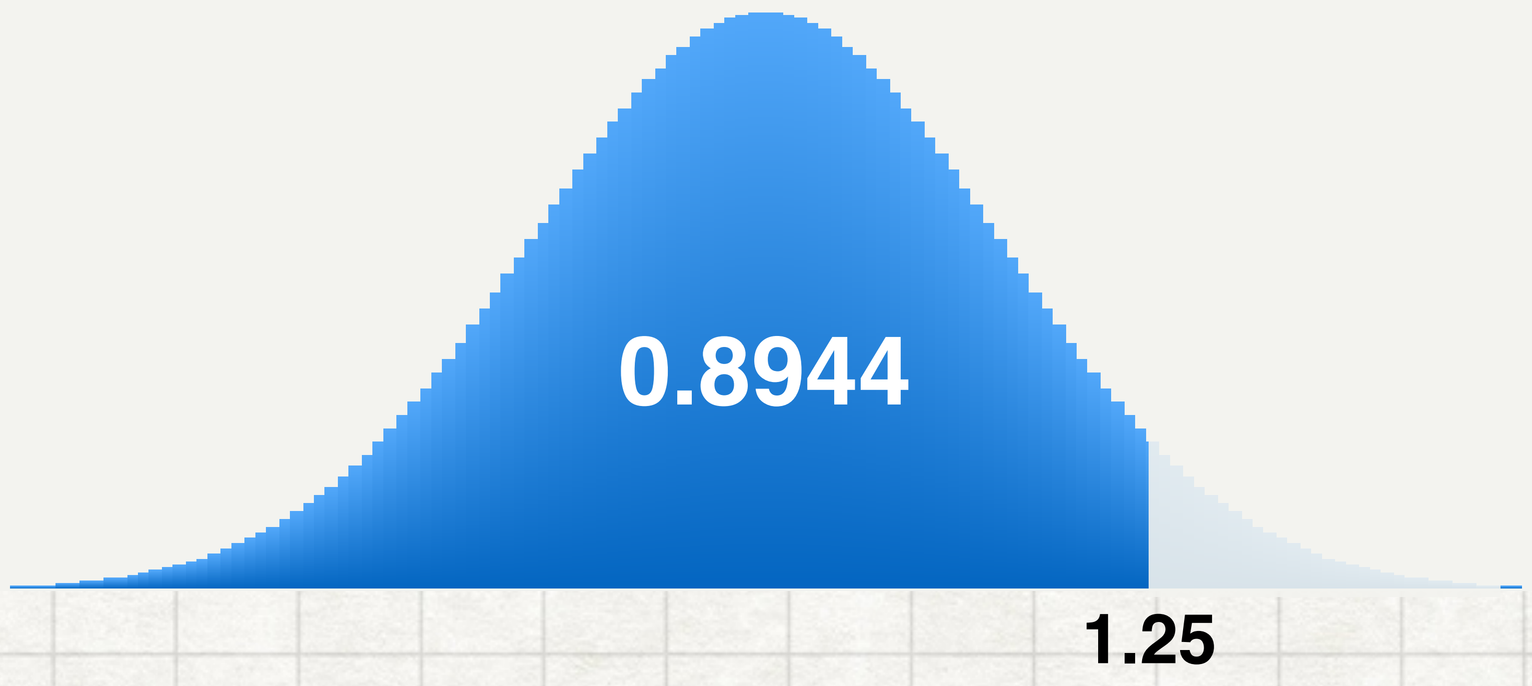
$$R_L = 3000/100000 = 3\%$$

According to the safety-first criterion, portfolio C is the most suitable for the client.

A client engaged you to invest $100,000. Her requirement is to be able to withdraw at least $3000 per year without the portfolio value falling below the capital invested.

What is the shortfall risk for portfolio C?

	A	B	C
Expected Return (%)	14	5	8
Standard Deviation (%)	10	2	4
Safety-First Ratio	**1.1**	**1**	**1.25**

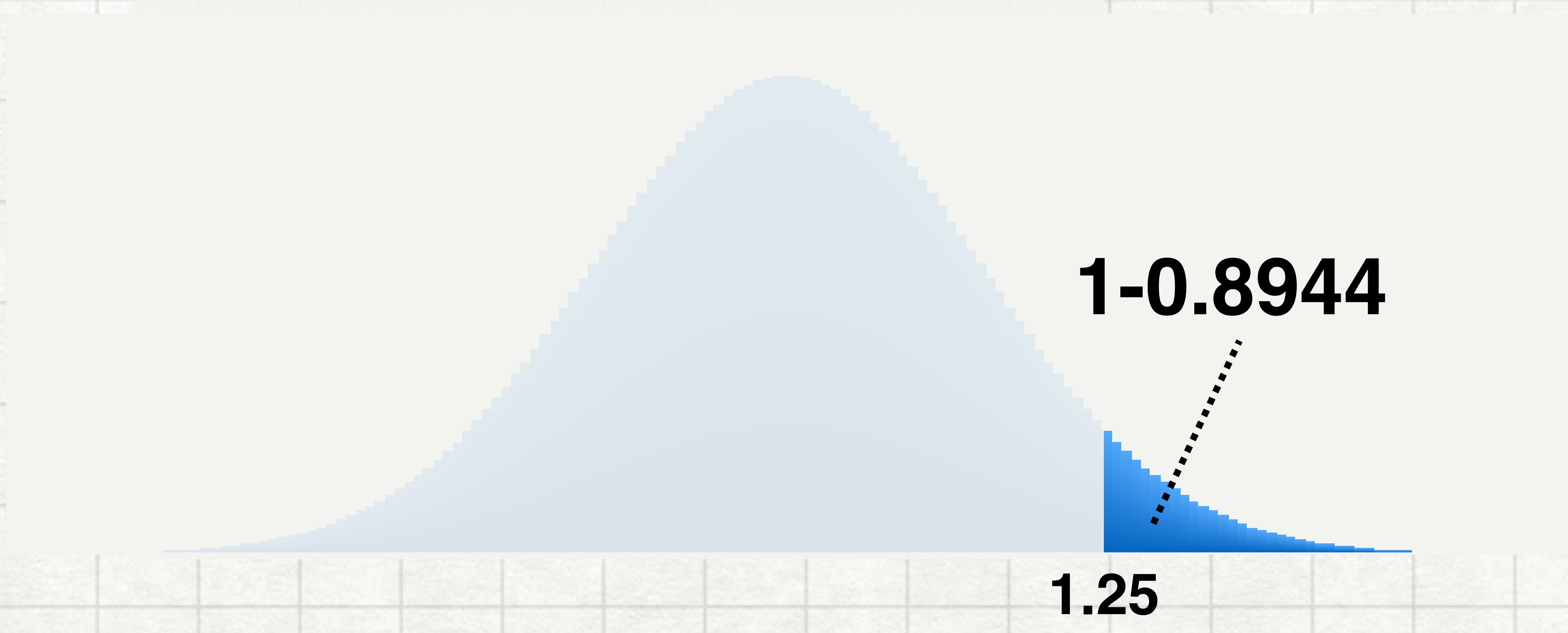


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

A client engaged you to invest $100,000. Her requirement is to be able to withdraw at least $3000 per year without the portfolio value falling below the capital invested.

What is the shortfall risk for portfolio C?

	A	B	C
Expected Return (%)	14	5	8
Standard Deviation (%)	10	2	4
Safety-First Ratio	**1.1**	**1**	**1.25**



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

A client engaged you to invest $100,000. Her requirement is to be able to withdraw at least $3000 per year without the portfolio value falling below the capital invested.

What is the shortfall risk for portfolio C?

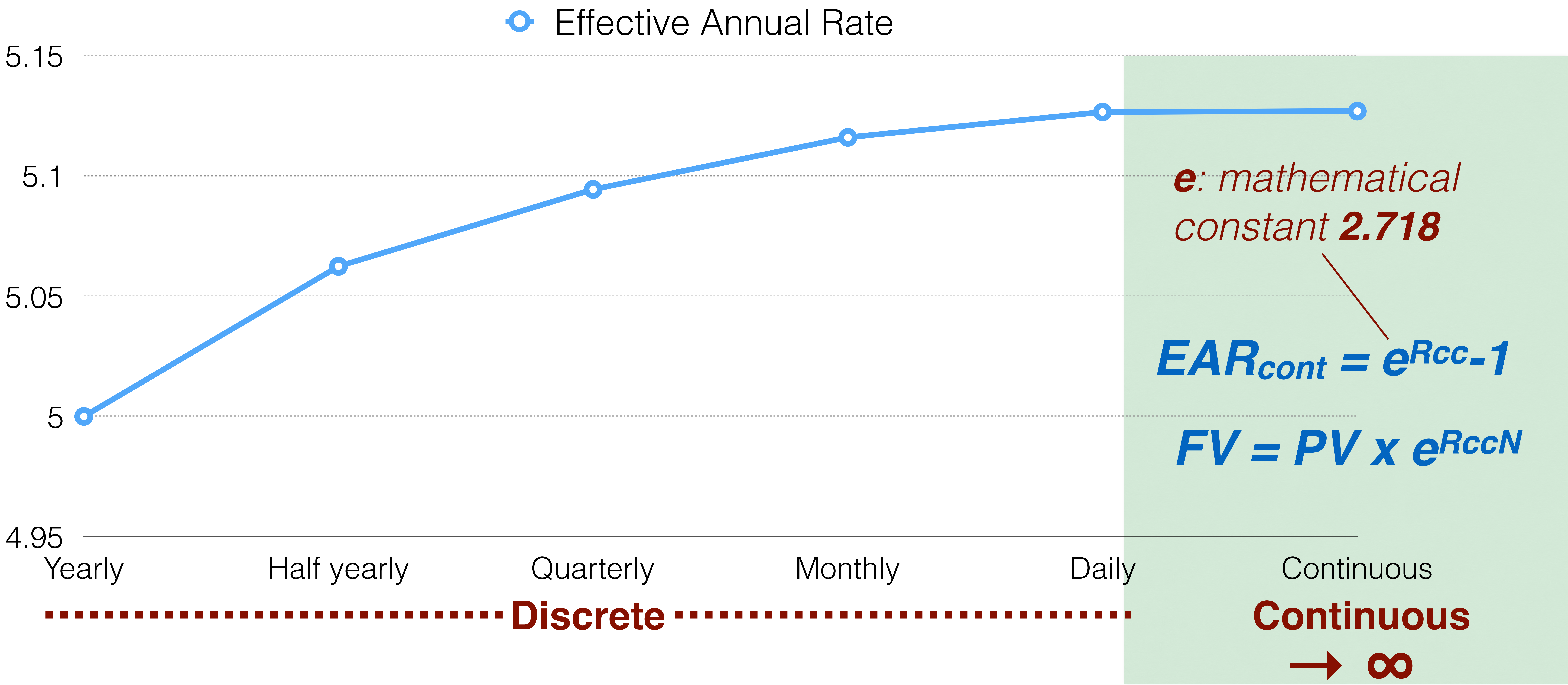
	A	B	C
Expected Return (%)	14	5	8
Standard Deviation (%)	10	2	4
Safety-First Ratio	**1.1**	**1**	**1.25**

Shortfall risk = 1-0.8944

= 10.56%

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

Effective Annual Rate of quoted 5% interest at varying payout frequencies



Price relative or 1+HPR

$$\frac{FV}{PV} = e^{R_{cc}N}$$

$$\ln\left(\frac{FV}{PV}\right) = R_{cc} \times N$$

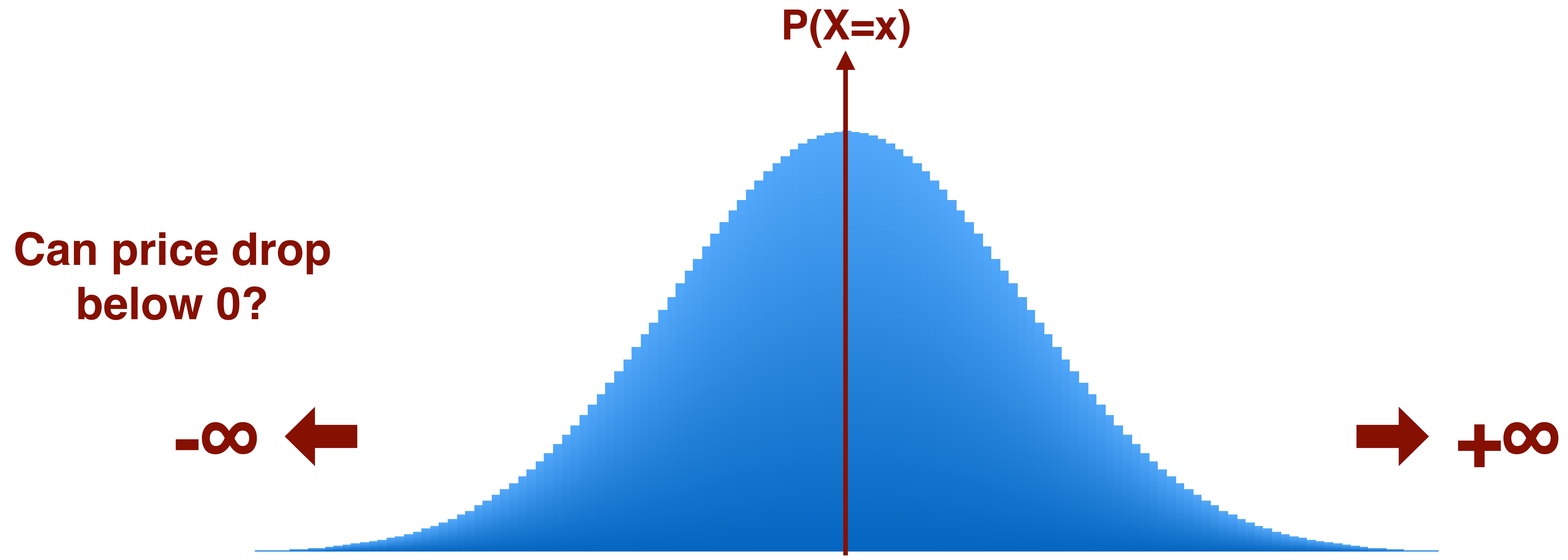
An asset has appreciated by 50% in 3 years. What is the annual rate of return on a continuously compounded basis?

$$R_{cc} \times 3 = \ln(1.5)$$

$$R_{cc} = 13.5\%$$

Price relative or **1+HPR**

$$\frac{FV}{PV} = e^{R_{cc}N}$$



Price relative or 1+HPR

Random Variable

$$\frac{FV}{PV} = e^{R_{cc}N}$$

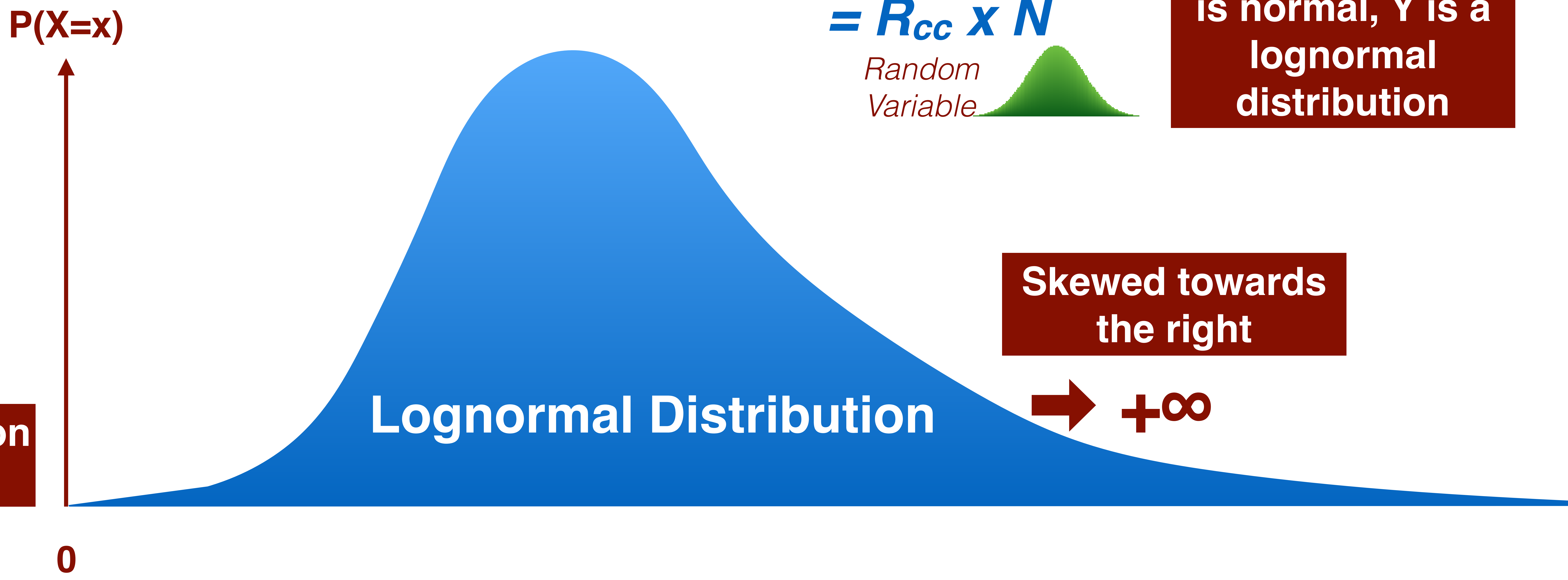
Constant

$$\ln \frac{FV}{PV} = \ln e^{R_{cc}N}$$

$$= R_{cc} \times N$$

Random Variable

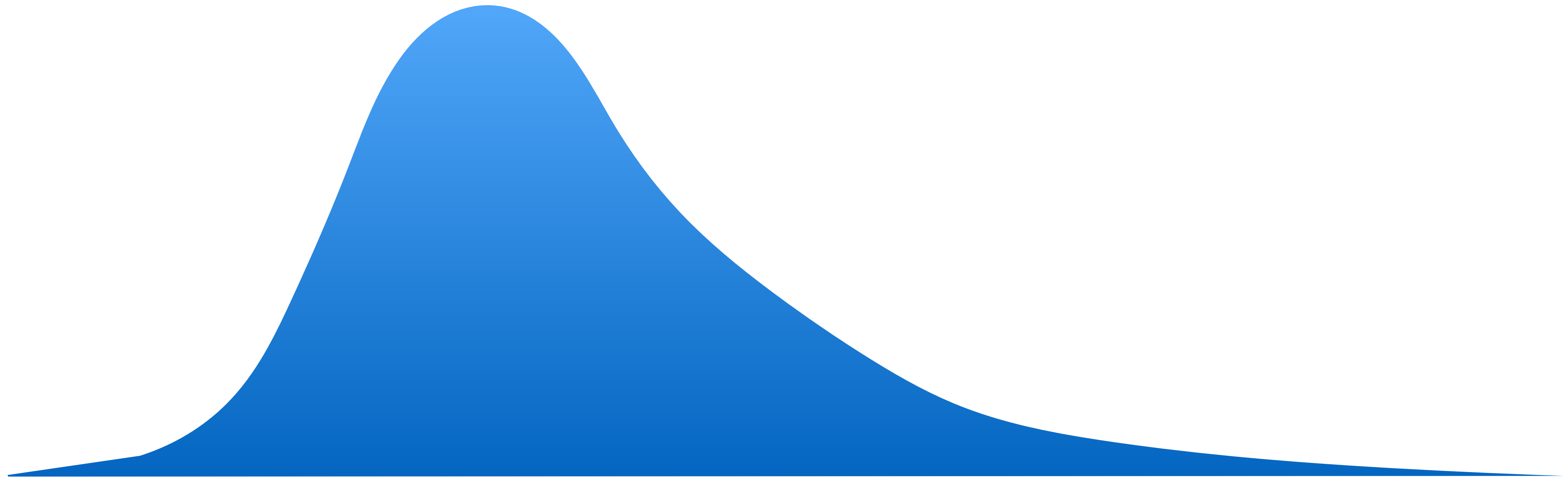
If natural log of Y is normal, Y is a lognormal distribution



Can never take on values below 0

Skewed towards the right

$\rightarrow +\infty$



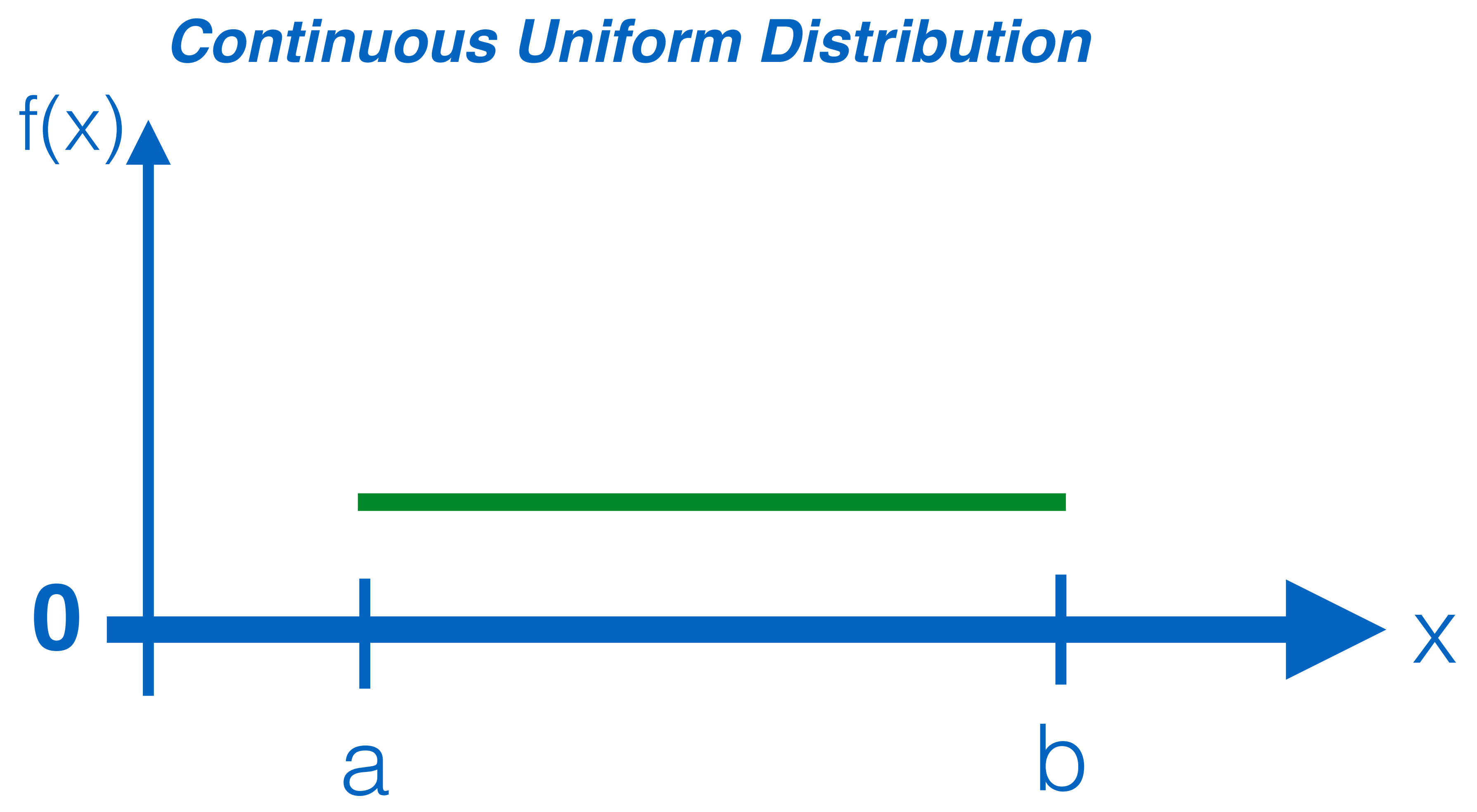
Lognormal Distribution

Modeling Asset Prices

Normal Distribution

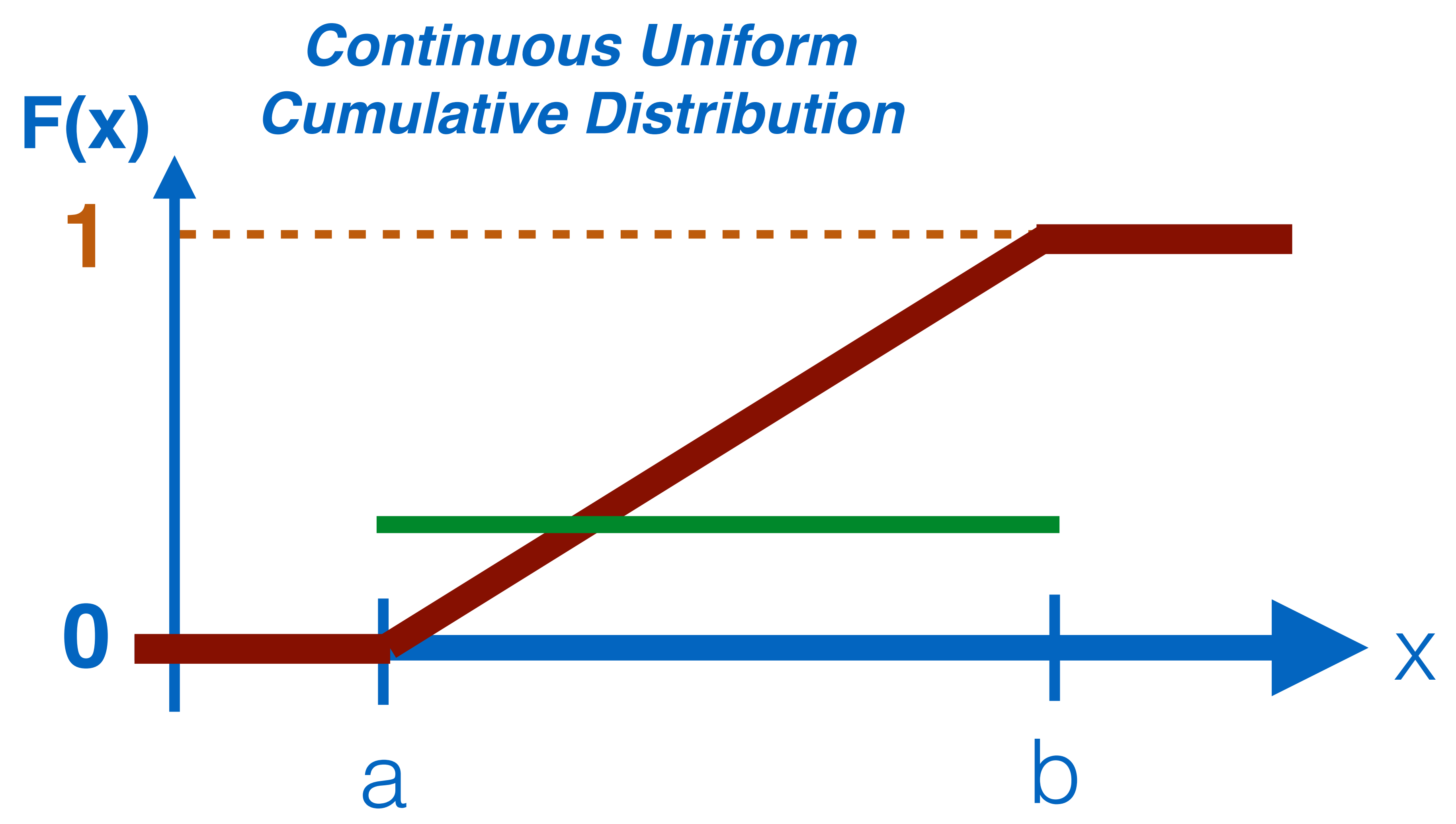
Modeling Asset Returns

Continuous Uniform Random Variable



$$f(x) = \begin{cases} \mathbf{1/(b-a)} & \textit{for } a<x<b \\ \mathbf{0} & \textit{otherwise} \end{cases}$$

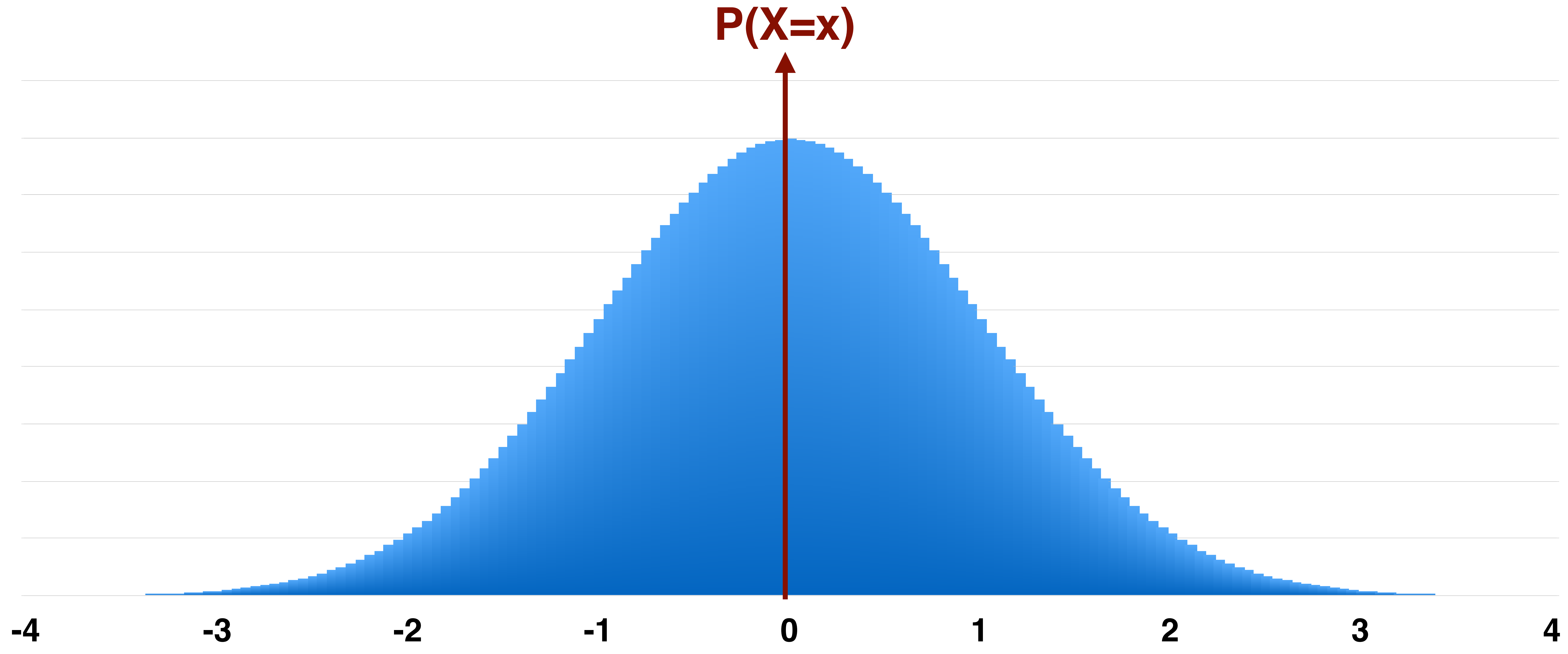
Continuous Uniform Random Variable



$$F(x) = \begin{cases} 0 & \text{for } x<a \\ (x-a)/(b-a) & \text{for } a<x<b \\ 1 & \text{for } x>b \end{cases}$$

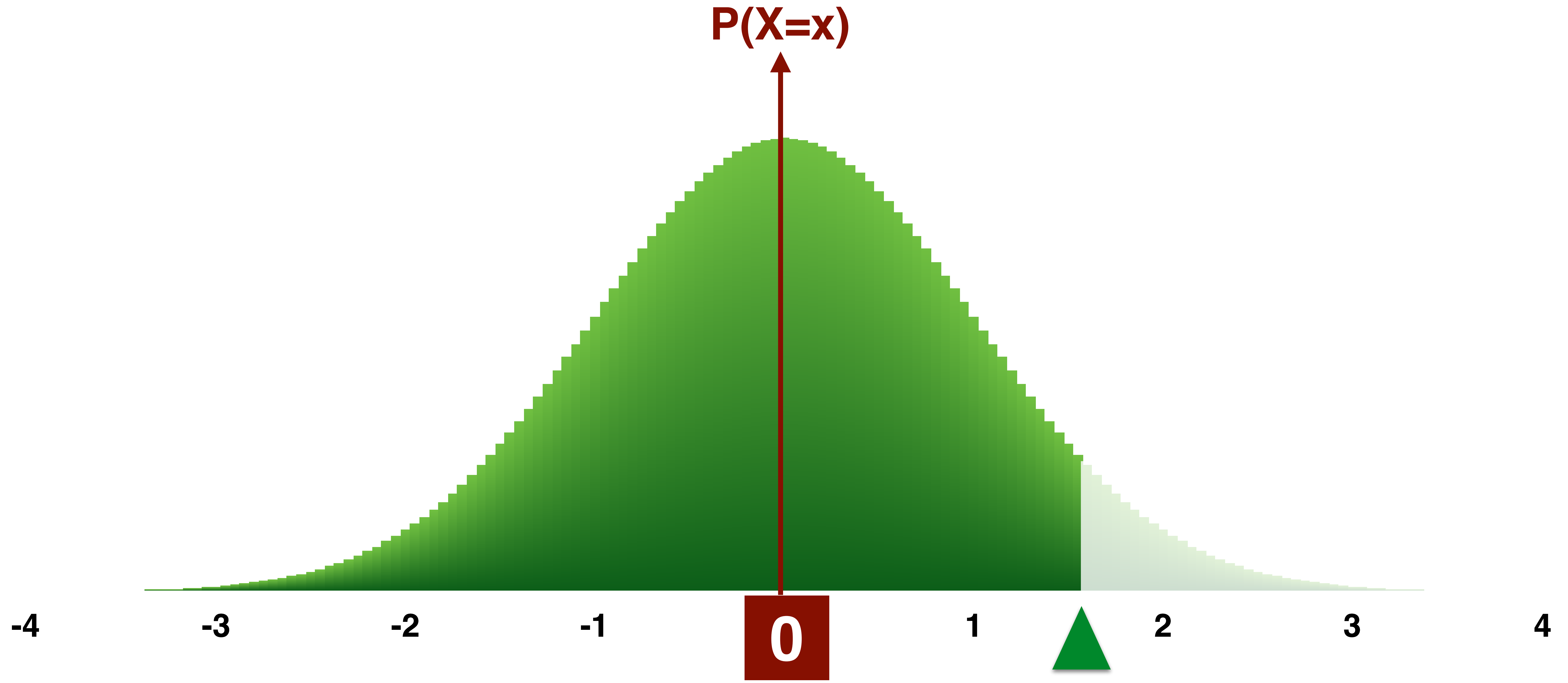
PREP nuggets

Probability Distribution



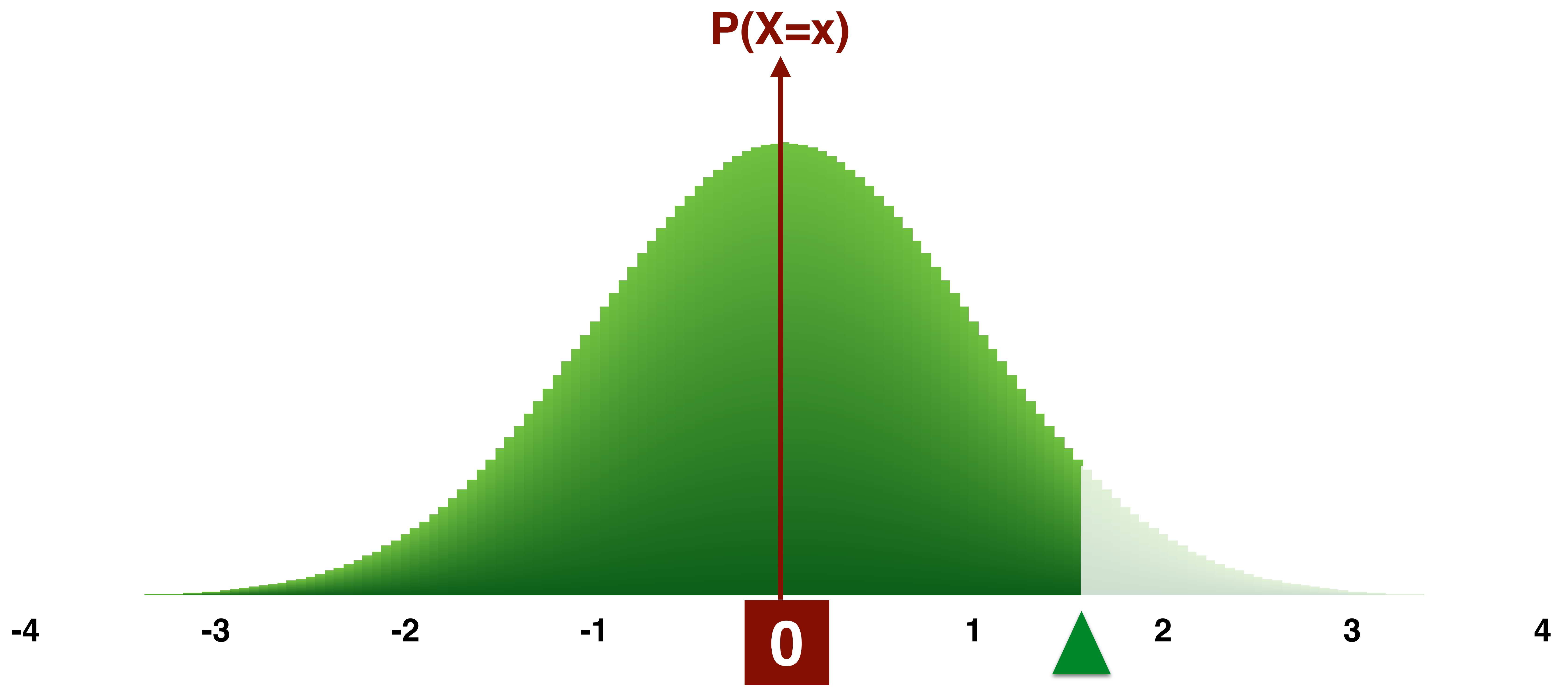
Confidence Intervals

Standard Normal Distribution



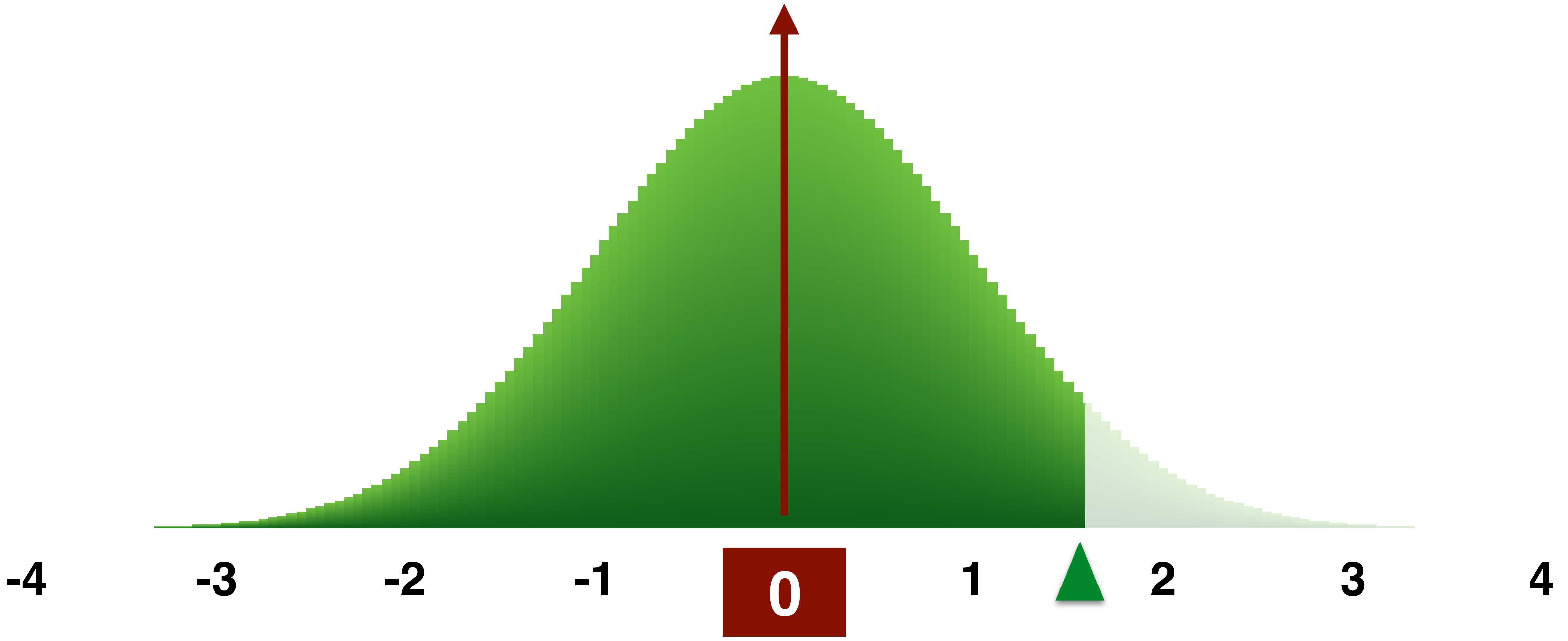
$P(X<1.65) = ???$

Standard Normal Distribution



P(X<1.65) = ???

Standard Normal Distribution



Standard Normal Probabilities

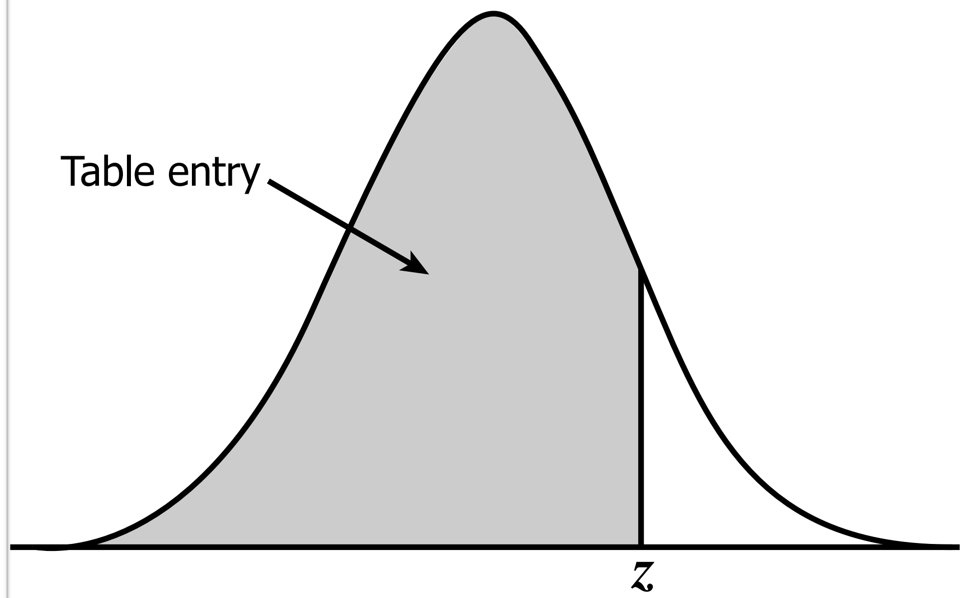
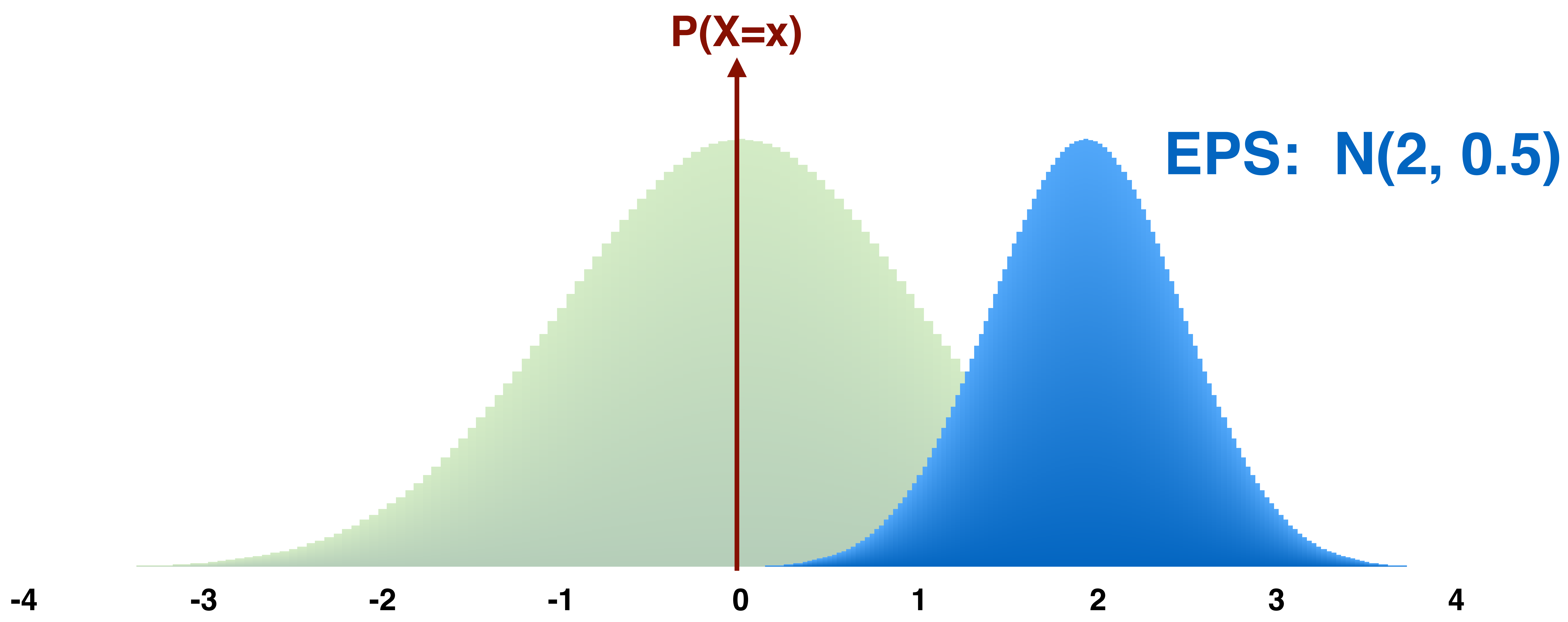


Table entry for z is the area under the standard normal curve to the left of z.

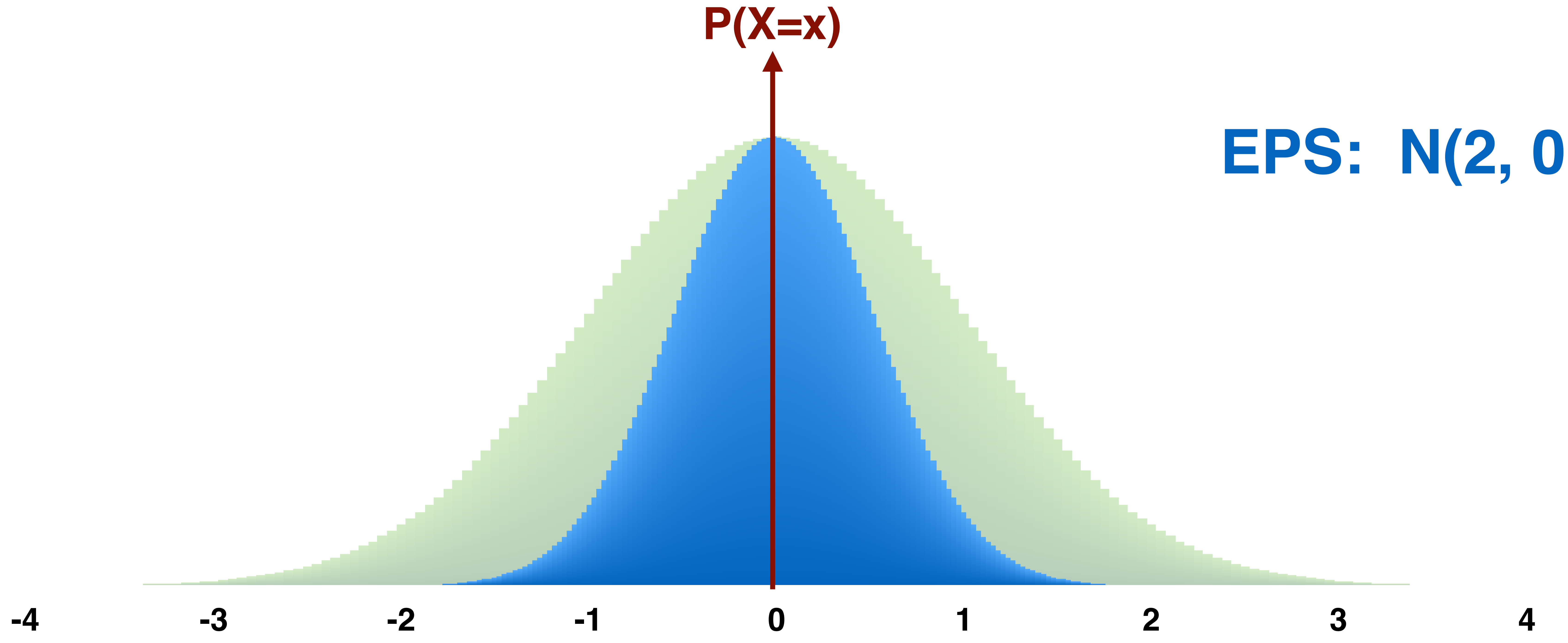
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817

Standard Normal Distribution



$z = x$

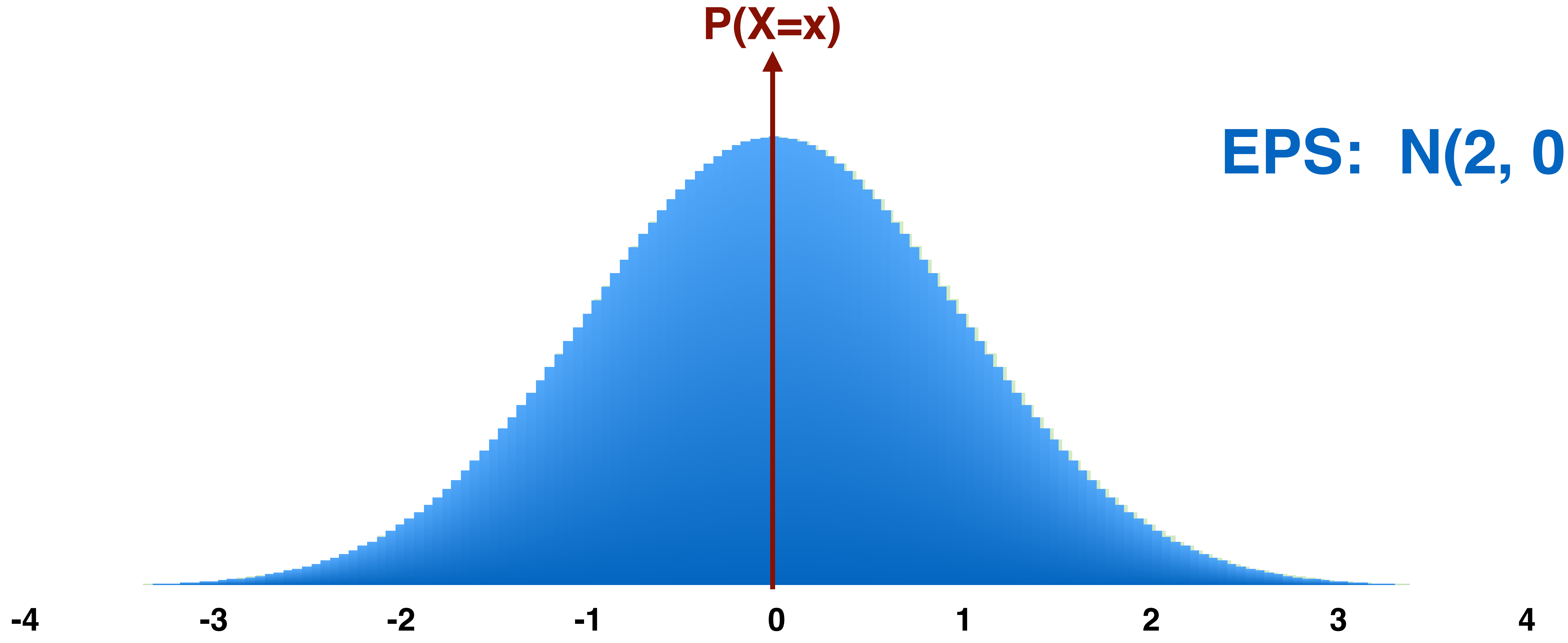
Standard Normal Distribution



EPS: N(2, 0.5)

$$z = x - 2$$

Standard Normal Distribution



EPS: N(2, 0.5)

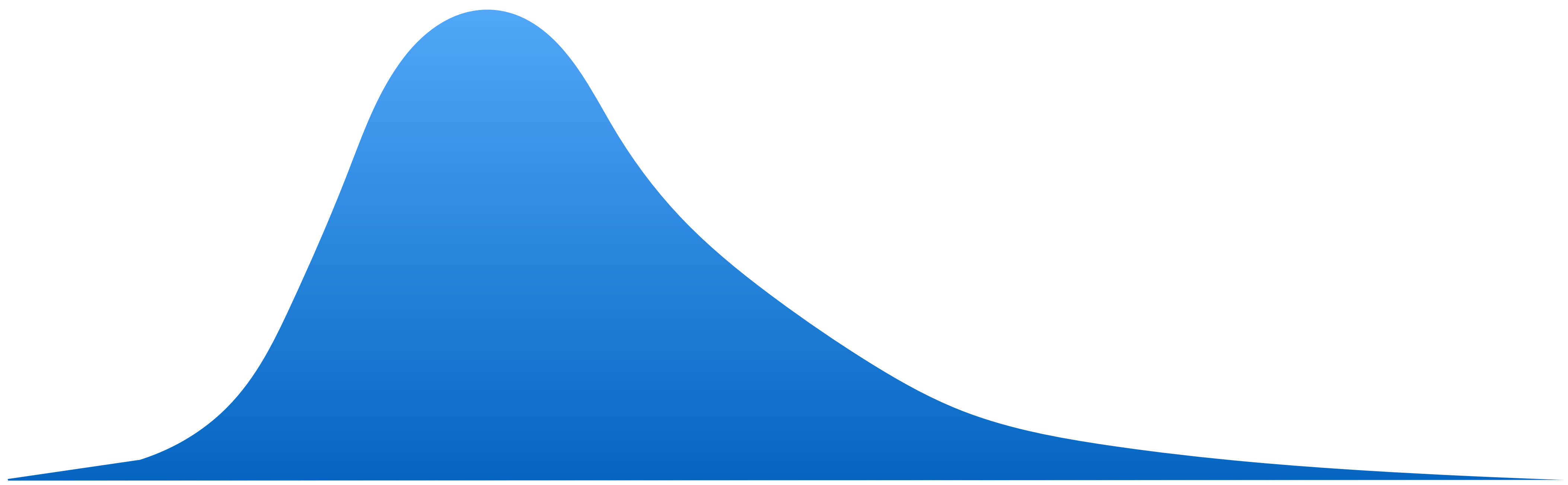
$$z = \frac{x - 2}{0.5}$$

Roy's Safety First Ratio

$$\boxed{SFR = \frac{E(R_p) - R_L}{\sigma_P}}$$

	A	B	C
Expected Return (%)	14	5	8
Standard Deviation (%)	10	2	4
Safety-First Ratio	**1.1**	**1**	**1.25**

PREP nuggets



Lognormal Distribution

Modeling Asset Prices

Normal Distribution

Modeling Asset Returns

PREP nuggets

PREP nuggets

prepnuggets.com