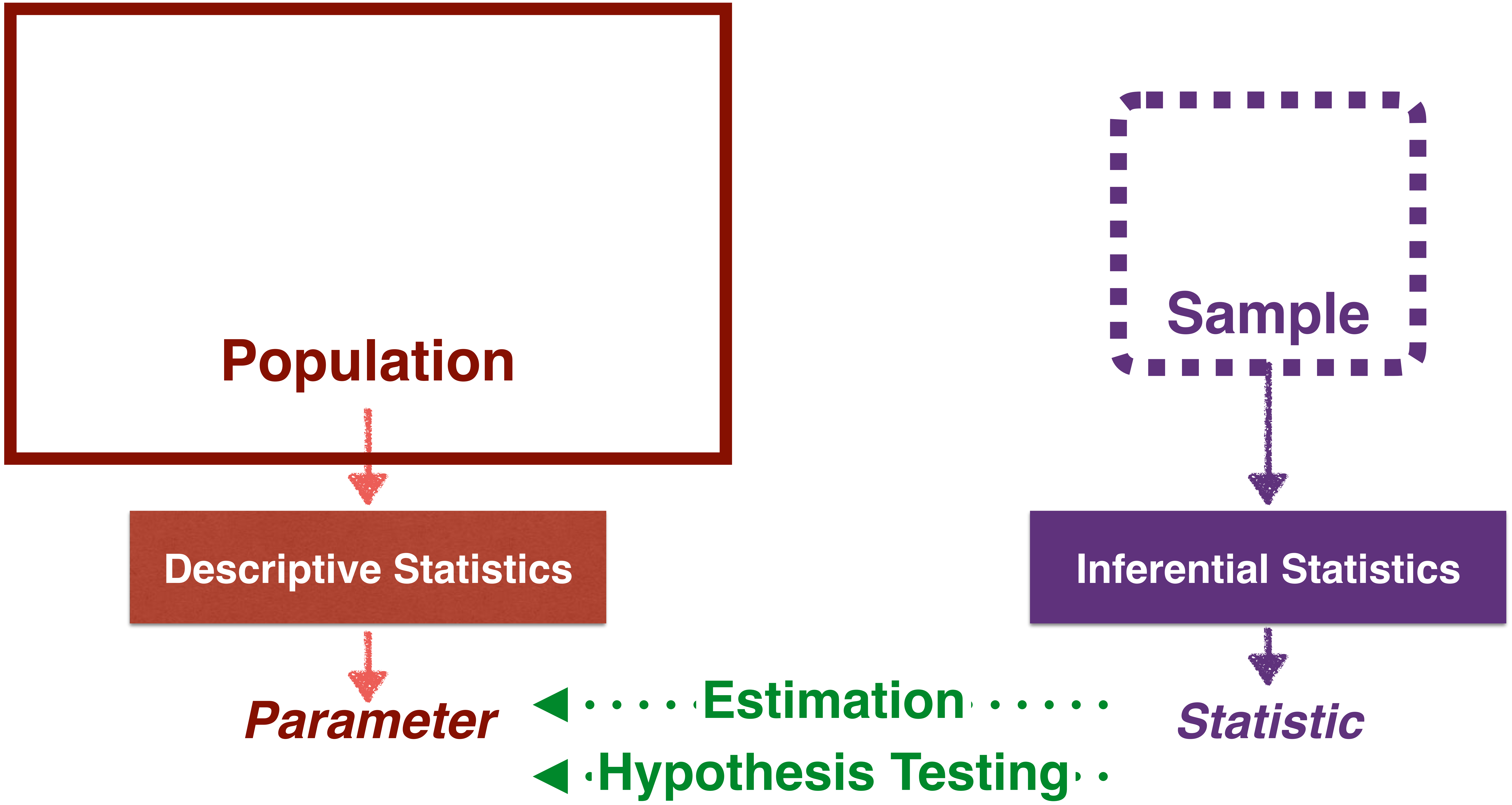
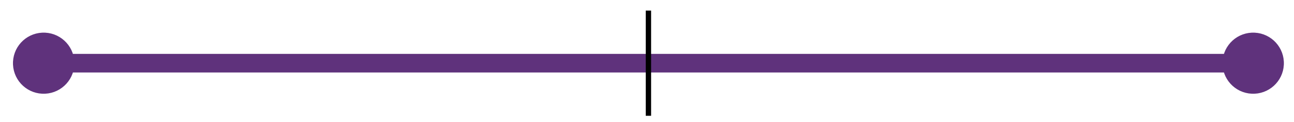
Hypothesis Testing

Hypothesis Testing Procedure


Population
Sample
Descriptive Statistics
Inferential Statistics
Parameter
Estimation
Hypothesis Testing
Statistic

Estimation



Point Estimate

Confidence Interval

Hypothesis Testing

"Is the mean return greater than 0?"

"Is the population standard deviation greater than 10%?"

Null Hypothesis, H_0
hypothesis to be tested

Alternative Hypothesis, H_A
hypothesis accepted when H_0 is rejected

Population parameter, θ

Possible value, θ_0

State the **HYPOTHESIS** | *Identify* **TEST STATISTIC** | *Specify* **SIGNIFICANCE LEVEL** | *State* **DECISION RULE** | **COLLECT & TEST** *Data* | *Make* **STATISTICAL DECISION** | *Make* **ECONOMIC DECISION**

Null Hypothesis, H_0	Alternative Hypothesis, H_A
$\theta = \theta_0$	$\theta \neq \theta_0$
$\theta \leq \theta_0$	$\theta > \theta_0$
$\theta \geq \theta_0$	$\theta < \theta_0$

$$\theta = R_f - R_b$$

H_0: $\theta \leq 0$

H_A: $\theta > 0$

$\theta = R_f - R_b$

$H_0: \theta \leq 0$

$H_A: \theta > 0$

t-statistic $= \dfrac{\bar{X}\% - 0\%}{s_x / \sqrt{30}}$ *(df=29)*

Test Statistic

value that is the basis for rejecting H_0

$$\text{Test statistic} = \frac{\bar{X}\% - 0\%}{s_x / \sqrt{30}}$$ ← CLT

z-distribution
t-distribution
chi-square distribution
F-distribution

Sample

n=30

$\bar{X}$=?%

s_x=?%

Hypothesis Testing Procedure

$\theta = R_f - R_b$

$H_0: \theta \leq 0$

$H_A: \theta > 0$

$$\text{t-statistic} = \frac{\bar{X}\% - 0\%}{s_x / \sqrt{30}} \quad (df=29)$$

Type I Error

rejection of the null hypothesis when it is actually true

Type II Error

failure to reject the null hypothesis when it is actually false

$\theta = R_f - R_b$

$H_0: \theta \leq 0$

$H_A: \theta > 0$

$$\text{t-statistic} = \frac{\bar{X}\% - 0\%}{s_x / \sqrt{30}} \quad (df=29)$$

	True Condition	
Decision	**H_0 is true**	**H_0 is false**
Do not reject H_0	**Correct Decision**	**Type II Error** **P(Type II Error) = β**
Reject H_0	**Type I Error** **P(Type I Error) = α**	**Correct Decision**

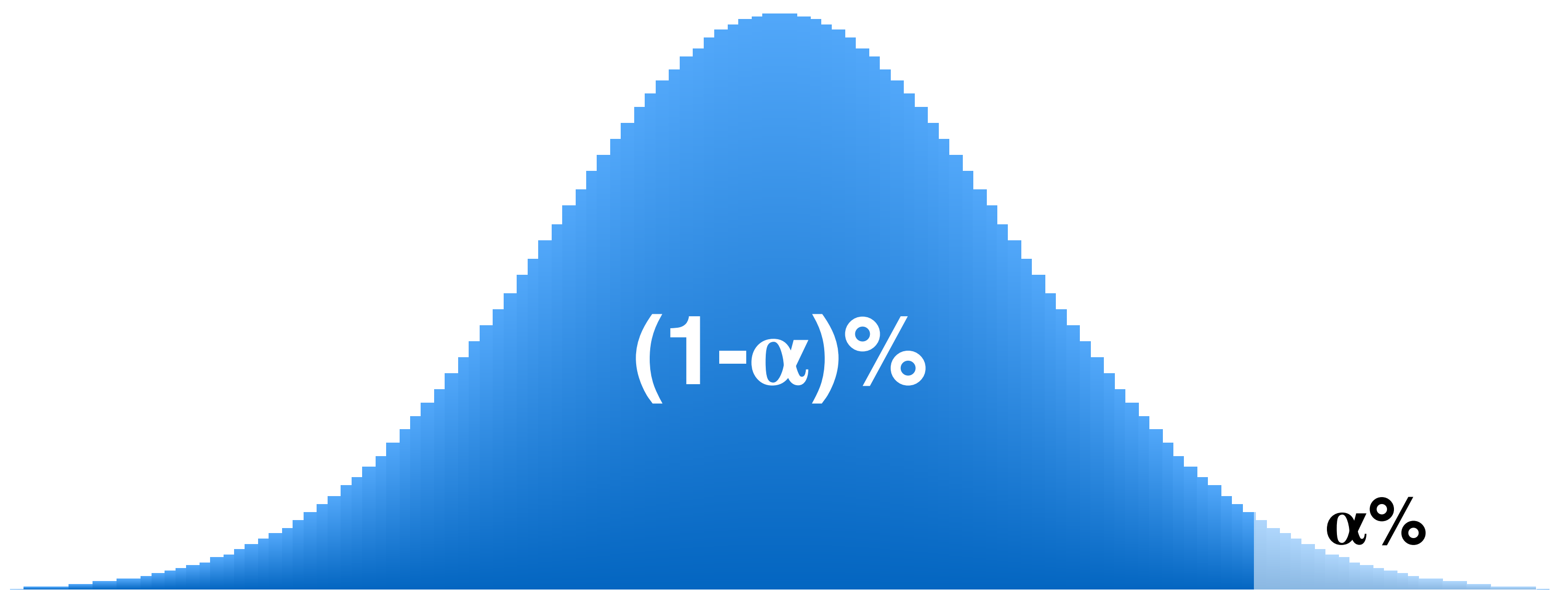
Difficult to quantify

Power of a test

PREP nuggets

Hypothesis Testing Procedure

State the **HYPOTHESIS** | Identify **TEST STATISTIC** | Specify **SIGNIFICANCE LEVEL** | State **DECISION RULE** | **COLLECT & TEST** Data | Make **STATISTICAL DECISION** | Make **ECONOMIC DECISION**



Correct Decision

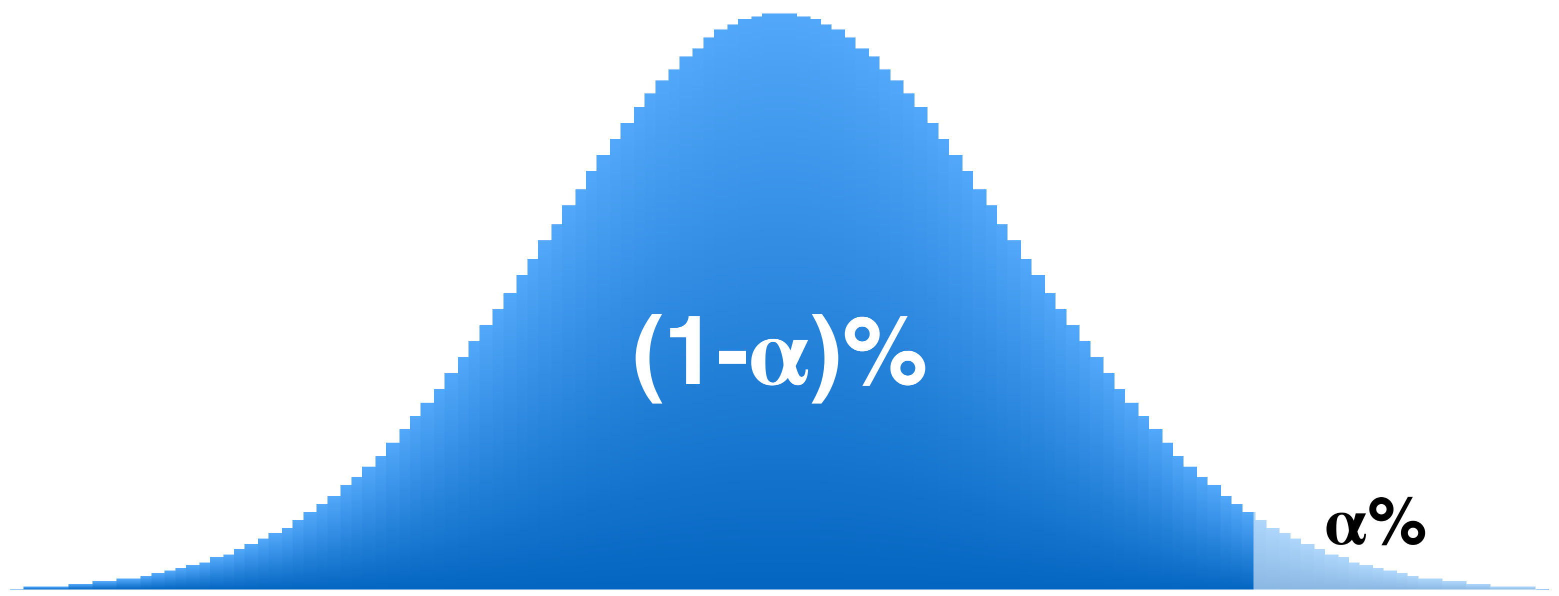
Type I Error
P(Type I Error) = α

Type II Error
P(Type II Error) = β

Correct Decision

Increases as we will reject the null hypothesis less often, including when it is false

Hypothesis Testing Procedure



Correct Decision

Type I Error

P(Type I Error) = α

Type II Error

P(Type II Error) = β

Correct Decision

The only way to decrease both types of errors is to

Sample Size

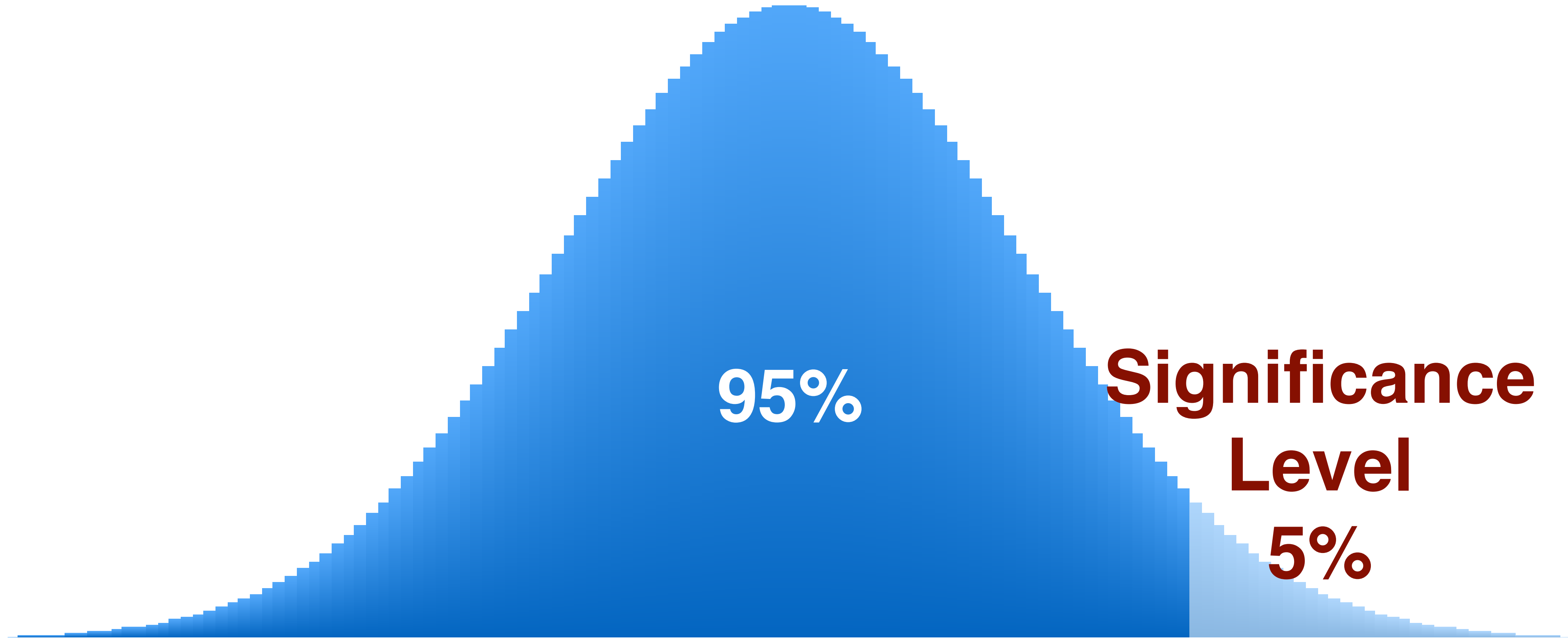
$\theta = R_f - R_b$

$H_0: \theta \leq 0$

$H_A: \theta > 0$

$$\text{t-statistic} = \frac{\bar{X}\% - 0\%}{s_x / \sqrt{30}} \quad (df=29)$$

$\alpha = 5\%$

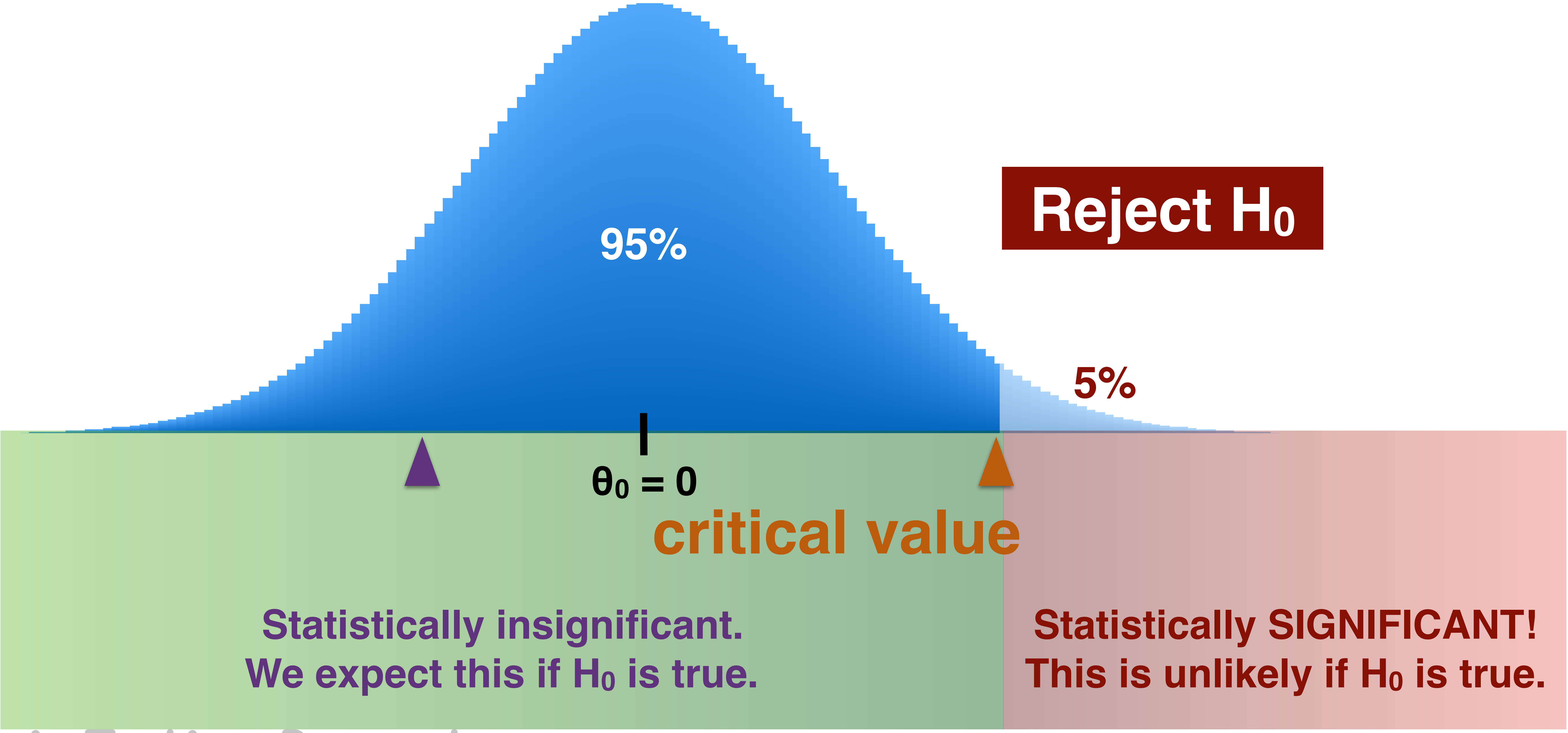


Hypothesis Testing Procedure

One-Tailed Test

H_0: $\theta \leq 0$

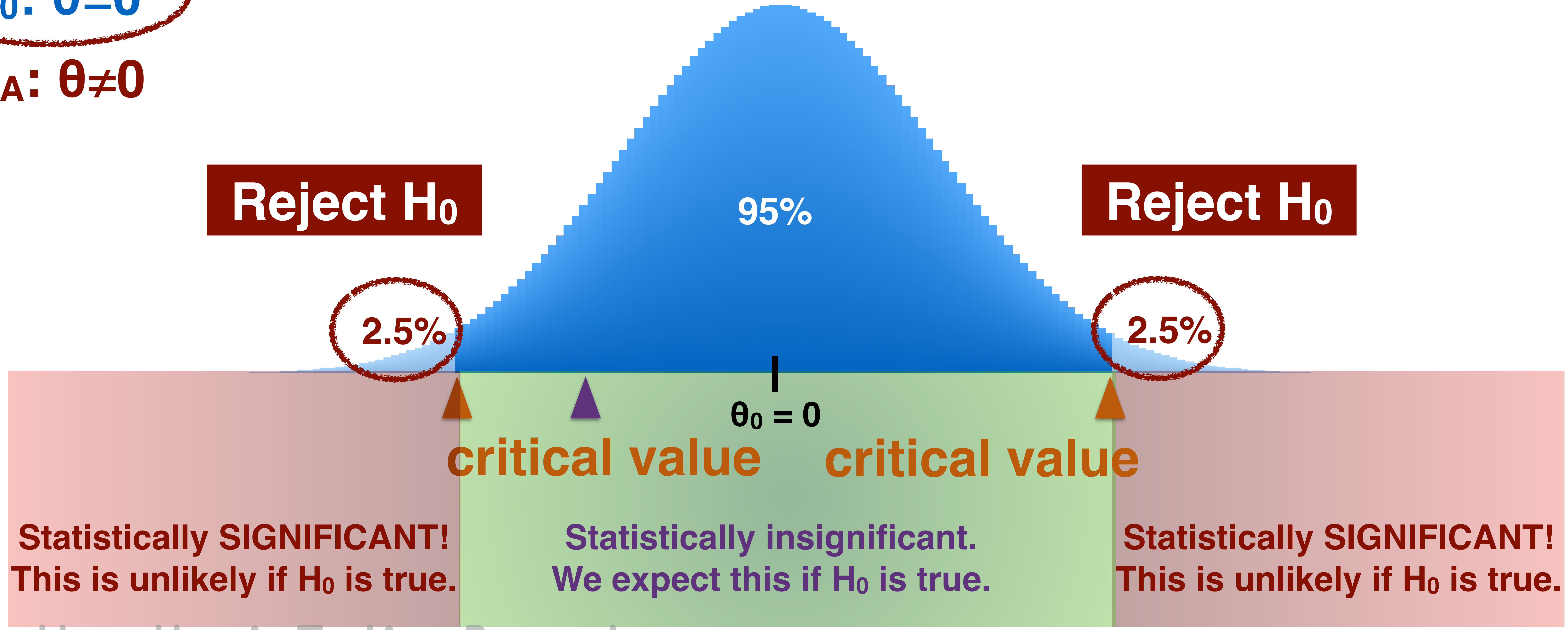
H_A: $\theta > 0$



Two-Tailed Test

$H_0: \theta=0$

$H_A: \theta\neq0$



State the **HYPOTHESIS** | *Identify* **TEST STATISTIC** | *Specify* **SIGNIFICANCE LEVEL** | *State* **DECISION RULE** | **COLLECT & TEST** *Data* | *Make* **STATISTICAL DECISION** | *Make* **ECONOMIC DECISION**

$$\theta = R_f - R_b$$

$$H_0: \theta \leq 0$$

$$H_A: \theta > 0$$

$$\text{t-statistic} = \frac{\bar{X}\% - 0\%}{s_x / \sqrt{30}} \quad (df=29)$$

$$\alpha = 5\%$$

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551

Hypothesis Testing Procedure

PREP nuggets

$\theta = R_f - R_b$

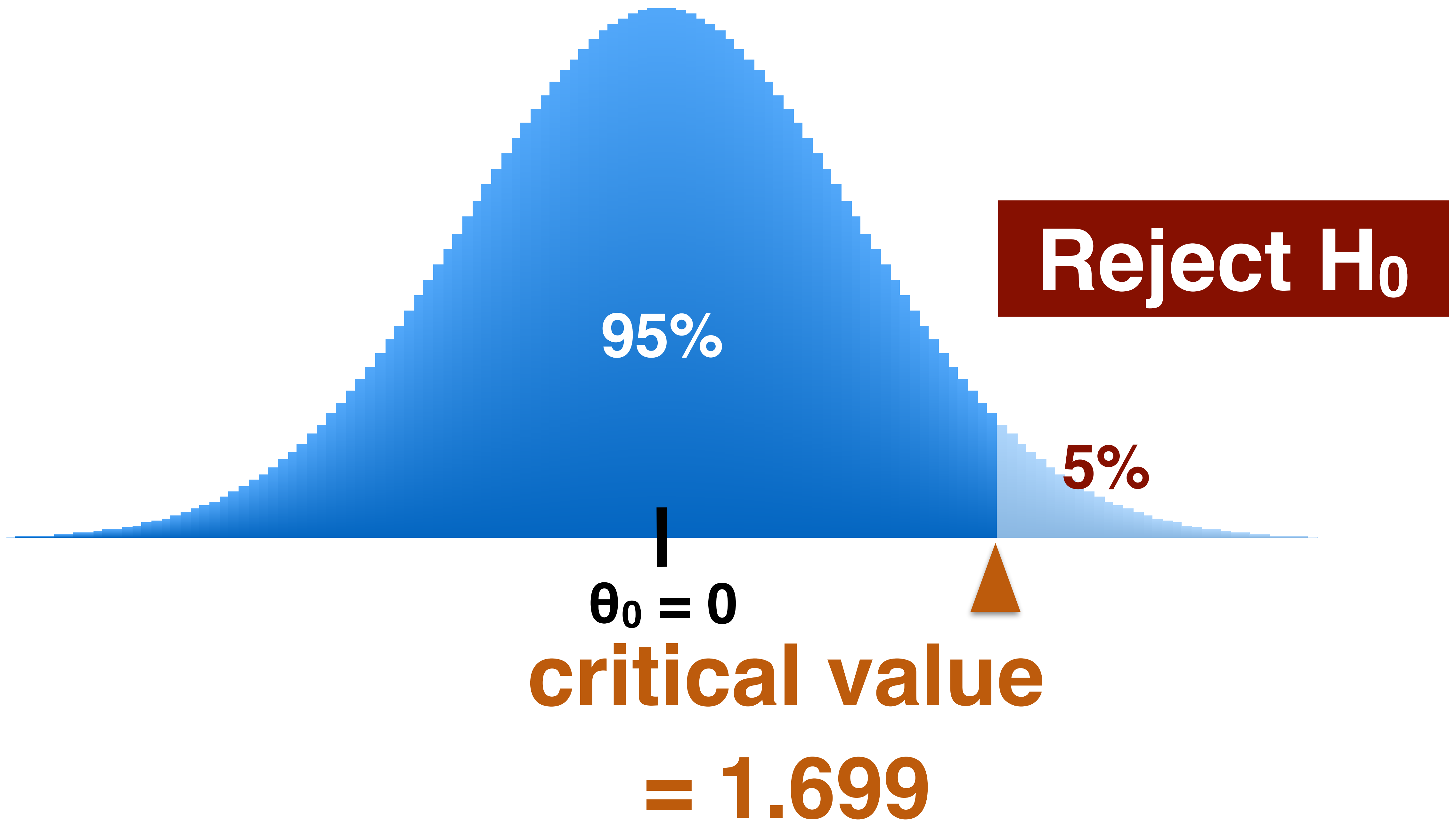
$H_0: \theta \leq 0$

$H_A: \theta > 0$

$$\text{t-statistic} = \frac{\bar{X}\% - 0\%}{s_x / \sqrt{30}} \quad (df=29)$$

$\alpha = 5\%$

Reject H_0 if $t > 1.699$



Hypothesis Testing Procedure

State the **HYPOTHESIS**	*Identify* **TEST STATISTIC**	*Specify* **SIGNIFICANCE LEVEL**	*State* **DECISION RULE**	**COLLECT & TEST** *Data*	*Make* **STATISTICAL DECISION**	*Make* **ECONOMIC DECISION**

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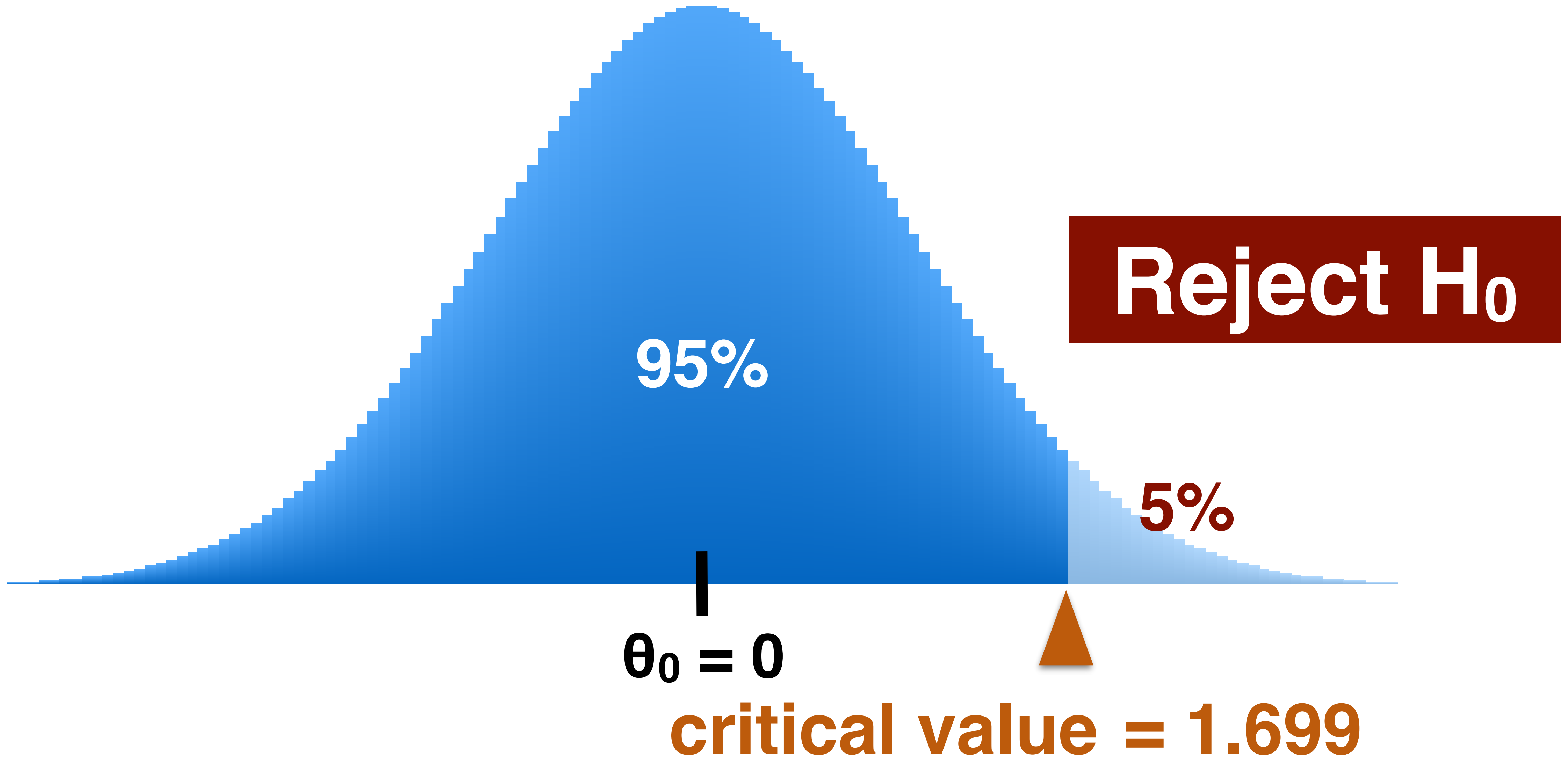

Ensuring the Quality of the Sample

- Check for measurement errors
- Avoid sample selection bias
  - Especially survivorship bias
- Be aware of time period bias

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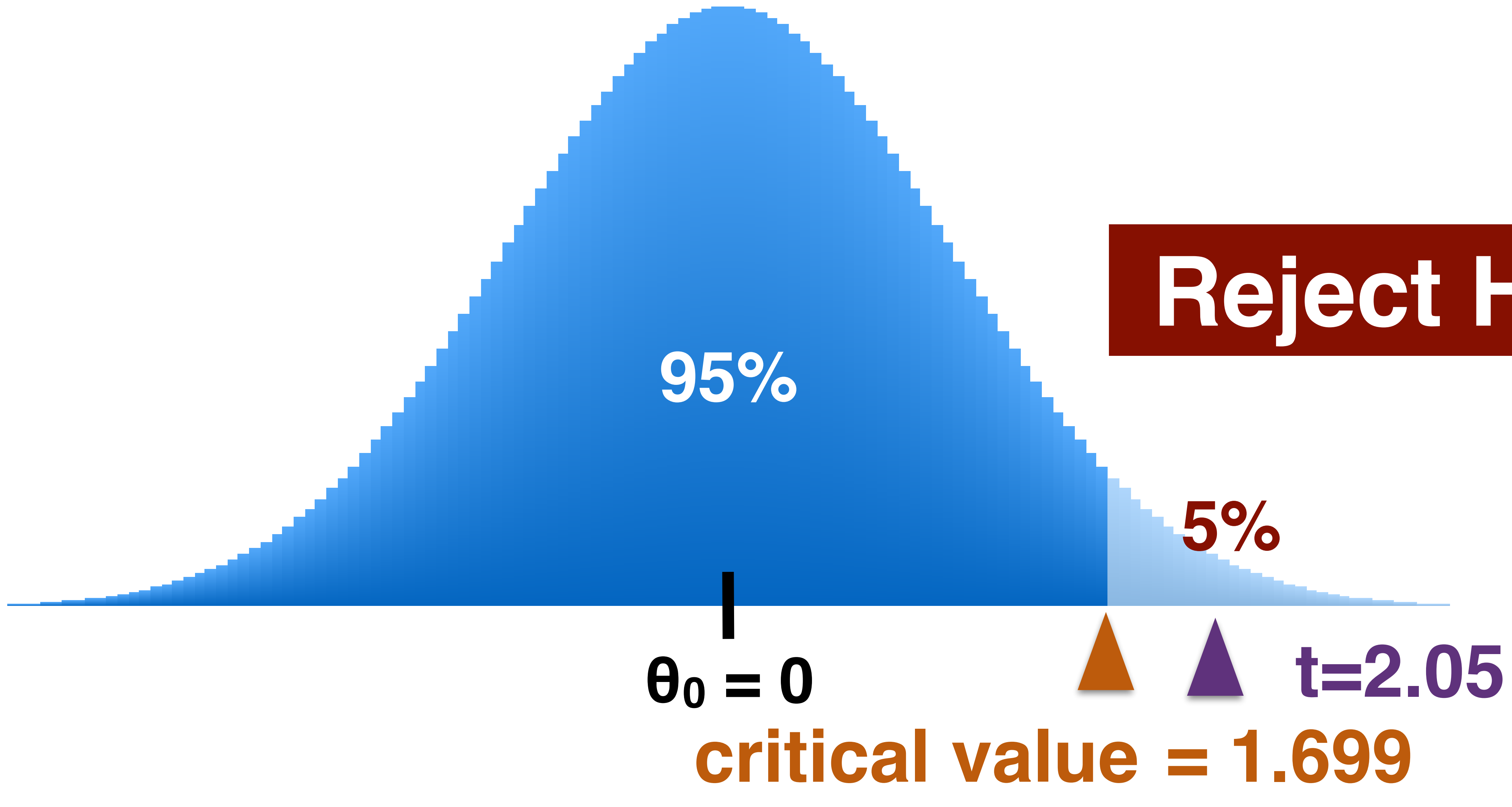

Sample

$n=30$

$\bar{X}=4.5\%$

$s_x= 12\%$

$t=4.5/(12/\sqrt{30})$

$=2.05$

Conclusion: The mutual fund outperforms the benchmark at 5% significance level.

PREP nuggets

Hypothesis Testing Procedure

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Reject H_0 if $t > 1.699$

Statistical significance

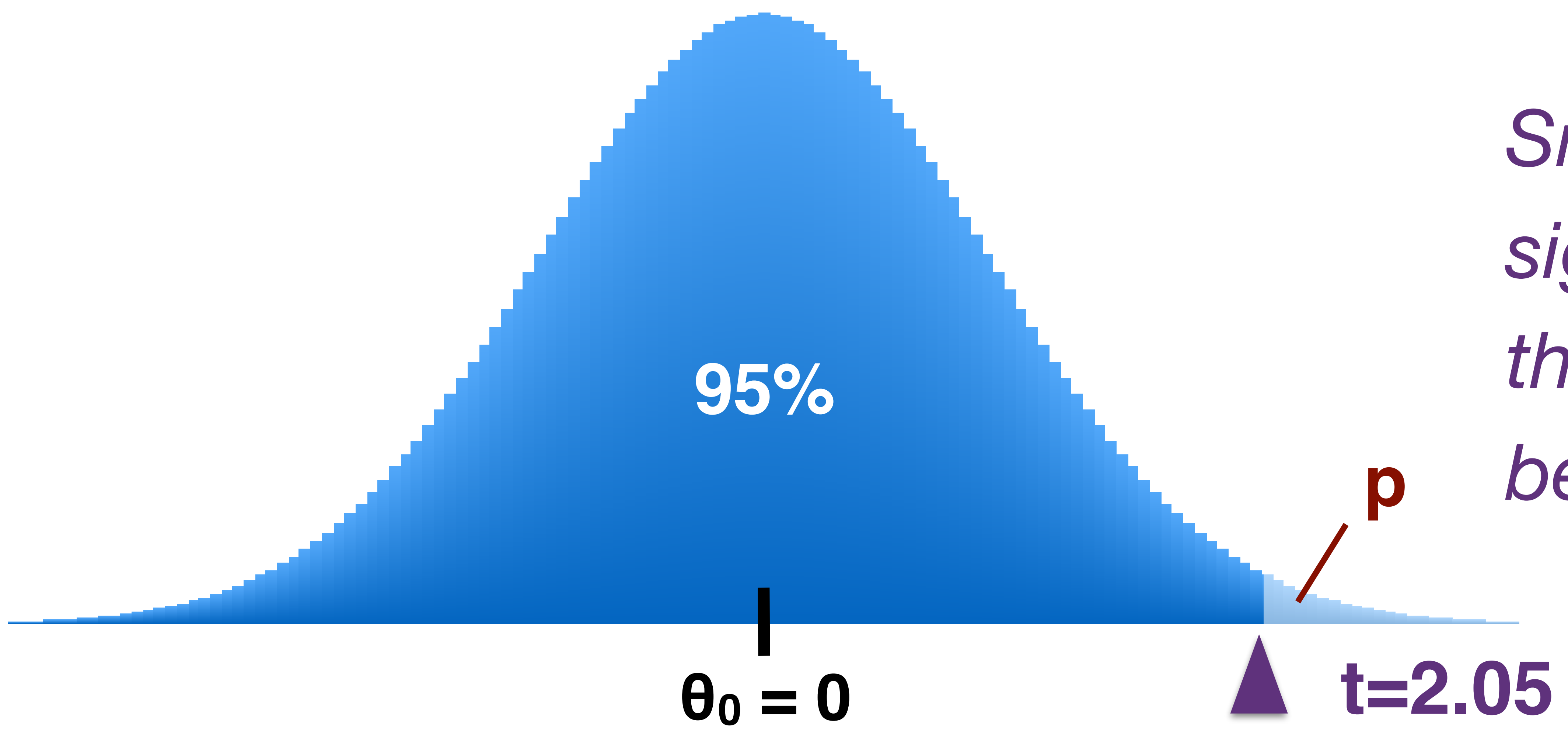
Conclusion: The mutual fund outperforms the benchmark at 5% significance level.

Economic significance

Consider:

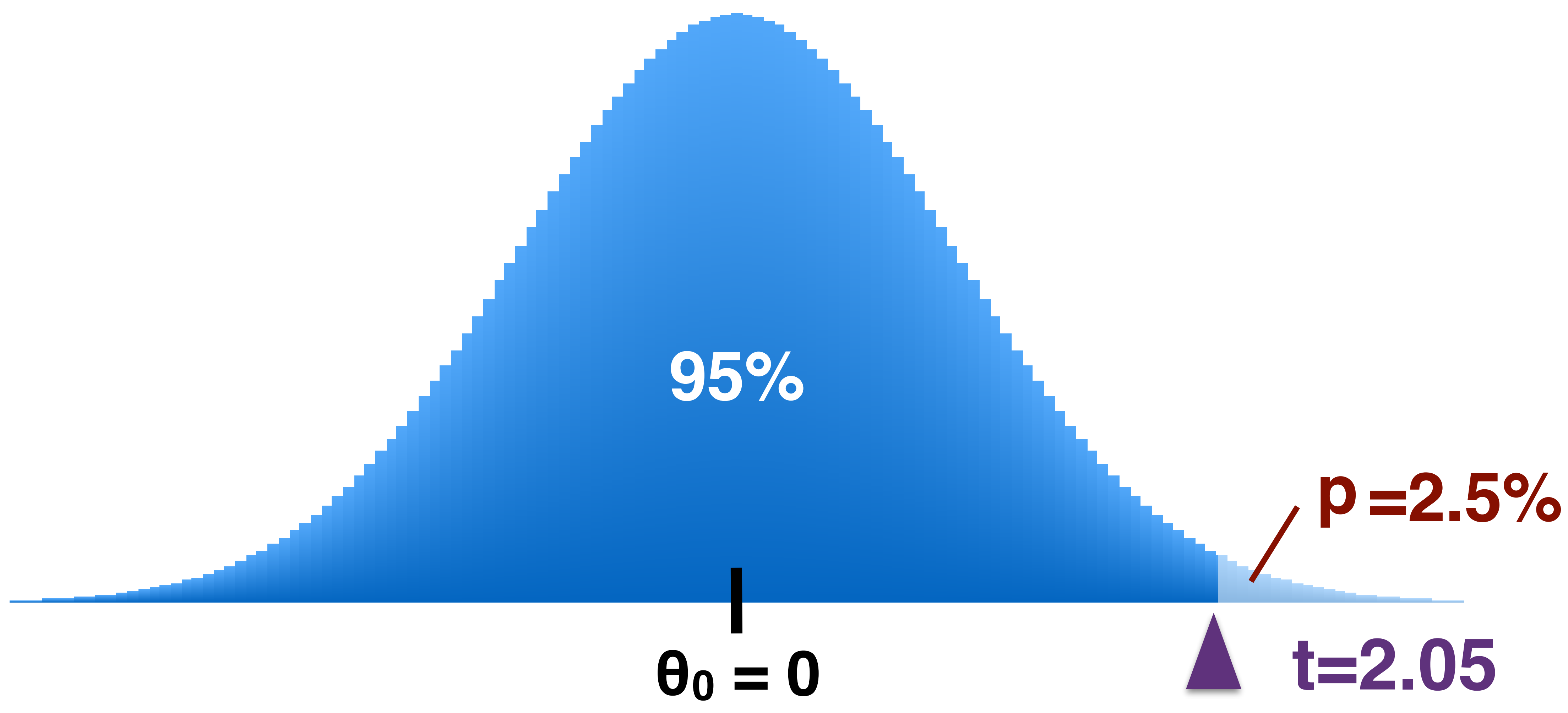
COST	TAXES	RISK
Manager fees *Advisory fees* *Transaction fees* *Brokerage fees*	*Additional taxes?*	*Is the risk level appropriate for the investor?*

p-value Approach



Smallest level of significance for which the null hypothesis can be rejected

p-value Approach



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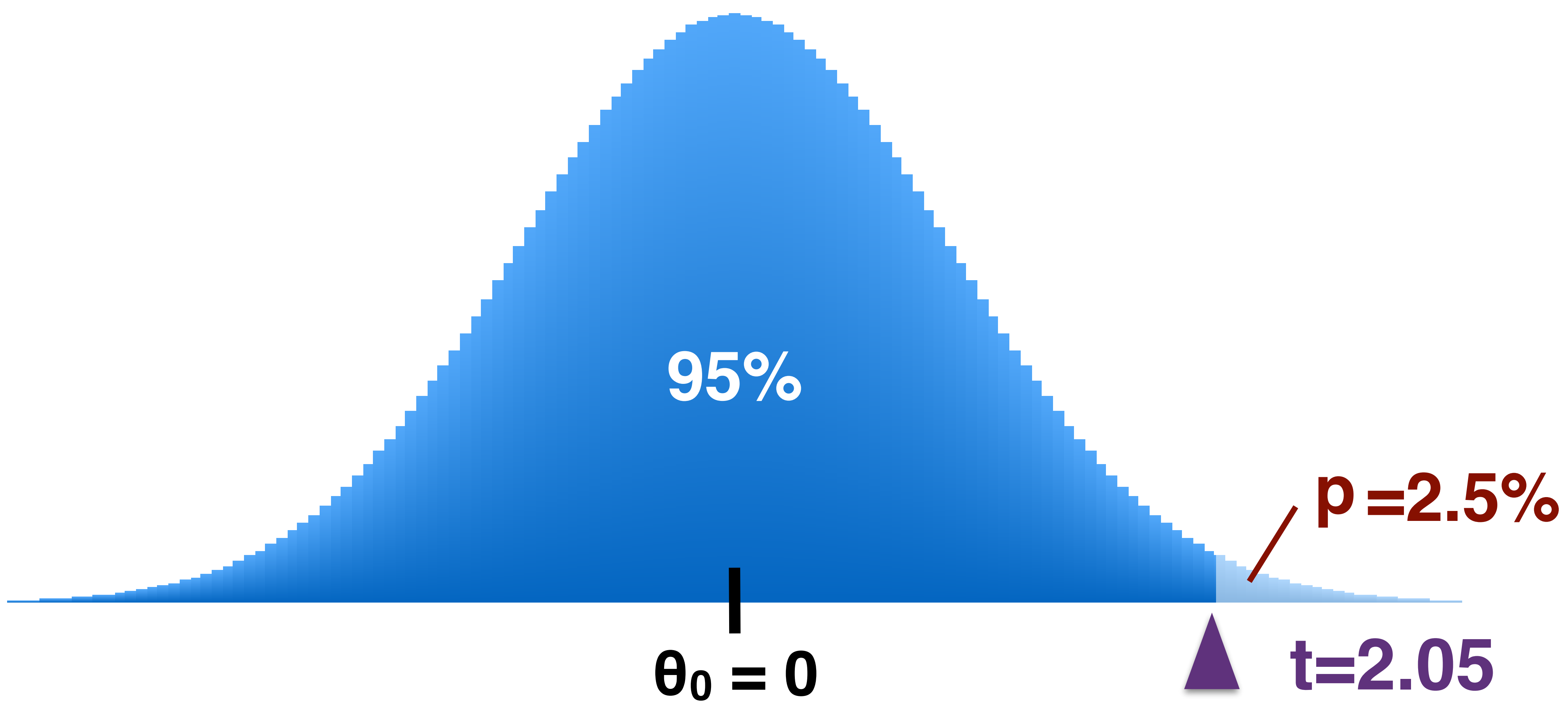
p-value Approach



Just the p-value is reported without selecting a significance level

Allows reader to make own conclusions on the significance of the results