Agents with Multiple Monies: Can the Monetary Policy Trilemma be solved?

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ABSTRACT

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An Agent-based Model (ABM) of an economy and/or economies was implemented to investigate whether the Monetary Policy Trilemma can be solved. Money-demanding agents with heterogeneous preferences over interest rates and exchange rates can freely use multiple monies (with each money corresponding to money-supplying agents with heterogeneous preferences). There were no restrictions on capital movement so the hypothesis tests primarily focused on simultaneous monetary autonomy and fixed exchange rates. The ABM uses an extended version of the interest-parity condition. Adaptive expectations w.r.t exchange/interest rates were used by money-demanding agents (using perfectly rational expectations would have been too computationally intensive) and decisions were made according to the money-suppliers' and money-demanders' rationality regarding the impact upon their heterogeneous utility functions. A key finding was that the Trilemma can be 'solved': monetary autonomy and fixed exchange rates are simultaneously possible in various circumstances. The success rate typically increases over time steps and settles/fluctuates around a certain level. Success generally sharply decreases when 'Helicopter Money' is implemented (a more 'extreme' monetary autonomy test). The monies with fixed exchange rates often also correspond with money-supply expansions (even when Helicopter Money is implemented). Ceteris paribus, more monies in the system decreases the success rate but increases economic growth.

Higher economic growth is correlated with less inequality. Ceteris paribus, the number of money-demanders does not have a discernible impact upon economic growth. An alternative interpretation of all these findings is that the Trilemma should not even theoretically exist or be a problem if there are no capital restrictions – contemporary institutional restrictions are briefly discussed. However, the model is demand-driven and the trading mechanisms, protocol, rationality behind money-creation etc. could be more sophisticated still. Nevertheless, the results are still insightful; they offer food for thought in contemporary debates on macroeconomic and monetary policy. The ABM itself is also a rudimentary formalisation of the theory that "money is an instrument of expectations-management". The ABM is generalisable – it could apply, for example, to cities, conurbations, regions, countries, multiple countries, industries, continents, the world, transnational communities etc. It was implemented using Python and MESA (ABM framework).

Keywords: Computational Economics; Agent-based Modelling; Computational Macroeconomics; International Macroeconomics; Monetary Policy; Mundell-Fleming Trilemma; Multi-Agent Systems; Artificial Societies; Complex Systems; Helicopter Money; Adaptive Expectations; Heterogeneous Preferences

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DECLARATION

I hereby certify that this dissertation constitutes my own product, that where the language of others is set forth, quotation marks so indicate, and that appropriate credit is given where I have used the language, ideas, expressions or writings of another.

I declare that the dissertation describes original work that has not previously been presented for the award of any other degree of any institution.

Signed,

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BACKGROUND AND INTRODUCTION

The ‘Monetary Policy Trilemma’ (known variously as ‘the Trilemma’, the ‘Mundell-Fleming trilemma’, the ‘Unholy Trinity’ and ‘the Impossible Trinity’) is a fundamental problem in (and central thesis of) international macroeconomic theory and policy. It is based upon the theories of Mundell (1963) and Fleming (1962) – the Mundell-Fleming model – and was empirically tested by the Obstfeld and Taylor (1997) as well as Obstfeld, Shambaugh and Taylor (2004). More recently, some scholars have sought to argue that the Trilemma can be practically reduced to a ‘dilemma’ (Rey 2015; Farhi and Werning 2014) but this is not widely accepted (Georgiadis and Mehl 2015; Klein and Shambaugh 2013). Nevertheless, the idea of reducing the Trilemma to a ‘dilemma’ for practical purposes bears relevance for this paper but in a distinctly different manner to that which has been previously proposed (this will be clearer within the ‘Design’ and ‘Realisation’ sections). The Trilemma itself states that a country and its policy makers cannot simultaneously pursue (and achieve) these three desirable policy objectives and that one must be sacrificed for the other two to be possible, namely:

1. Free movement of capital (no capital controls).

2. A fixed foreign exchange rate.

3. An ‘independent’ monetary policy (also referred to as a ‘sovereign’ monetary policy or ‘monetary autonomy’).

This project seeks to program and present an Agent-based model (ABM) that will function as a theoretical simulation to see if the Trilemma can be solved when agents have open access to multiple monies and can freely use and trade them. The rationale behind having multiple monies (and, therefore, the multiple exchange rates and multiple interest rates that they entail) is that various agents will have different preferences across interest rates and exchange rates (more precisely, they have heterogeneous expectations-management preferences). Two rather direct examples: exporting communities may prefer monies with relatively low exchange rates (to export more) whilst importing communities may prefer monies with relatively high exchange rates (to constrain inflation). Another two direct examples: savers prefer higher interest rates whilst borrowers prefer lower interest rates. Some subtle, arguably indirect, examples: agents may prefer certain 'monetary rules' to others – certain interest rates and exchange rates for particular monies may be proxies for the soundness of that money, the solvency of and political risks faced by a government, commodity-backed monies versus fiat monies, monies being widely used as a standard medium of exchange in certain industries, indications for the inflation rate and/or growth rates that the monetary policy makers are targeting for their users/key stakeholders etc.

A key assumption made, therefore, is that agents have heterogeneous preferences over interest rates and exchange rates. The reason why an ABM is necessary is because, both historically and contemporarily, the data for instances of multiple monies being freely and openly used is scarce and/or unreliable for the purposes of informing the debate on monetary policy reform. Although there is and has been a literature on 'Free Banking' and 'Free Market Money' (Rothbard 1963;

Hayek 1976; Friedman 1982; Yeager 1983; Selgin 1988; Horwitz 1992; White 1992; Dowd and Timberlake 1998; Dowd 2002; Ögren 2006; Foldvary 2011) as well as continuing, significant questions about systemic monetary reform more broadly, the proposals for 'Free Banking' and 'Free Market Money' are generally taken less seriously than many mainstream macroeconomists' conclusions (Friedman and Schwartz 1986; Svensson 1997; Bernanke and Mishkin 1997; Mishkin 2000; Gonçalves and Salles 2008; McCallum 2015; Garin, Lester and Sims 2016) because the recommendations and arguments are often derived from historical case studies (which are argued to be less applicable to a modern context) and/or relatively abstract theories (that are difficult to formalise). However, a major reason for this is that there is a paucity of reliable, contemporary data because the dominant monetary system(s) globally are monetary monopolies imposed especially via central banking regimes.

More recently, the author of this project wrote papers and articles of his own (Wilde 2015, 2016) for an economics think tank called *The Cobden Centre* which is based around the Austrian School of Economics (as well as for other think tanks and media outlets including the *Adam Smith Institute* and the *Center for a Stateless Society (C4SS)*) surrounding the potential for significantly beneficial monetary reform through agents having access to and using multiple monies – indeed, the modelling of preferences in general is a very rudimentary representation of this author's personal theory of "money, goods and services being instruments of expectations-management" (Wilde 2015).

For the purposes of advancing arguments for Monetary Freedom, Free Market Money and Free Banking, the desirability of a more rigorous, theoretical argument through computational simulations was recognised; these simulations would illustrate and present the potential benefits of agents being able to freely choose, use and trade with multiple monies (they could also yield new insights into economies more broadly). Since ABMs allow for some flexible, illustrative modelling and because they entail a 'sufficiency theorem' (Axtell 2000), they seemed to be an ideal computational method for these purposes. More recent iterations of ABM software/packages (Masad and Kazil 2015) have made the framework even more accessible and, thus, allow for deeper theoretical explorations of policy proposals in social-scientific settings. Indeed, given that policy makers are increasingly sailing uncharted waters for macroeconomic policy more broadly (and monetary policy particularly), theory will necessarily be at the forefront of future policy reforms and responses – the ABM framework offers potent means by which to test and explore these theories.

Relating to the 'fixed exchange rate' objective more precisely, Obstfeld, Shambaugh and Taylor (2004) note that "[in] practice, "fixed" exchange rates are fixed only up to a possibly narrow fluctuation band…" and they state that "their methodology for selecting *de facto* pegs has allowed for this." They go on to say that they "experimented with simulations of an extension of Krugman's (1991) target zone model, using Svensson's (1991) term-structure model to derive interest rates for non-infinitesimal maturities when the fluctuation band is quite narrow (±1%)." Thus, an ABM investigation that seeks to present a potential 'solution' to the Trilemma through must use this very same fluctuation band (±1%) as the benchmark for a successful test. Furthermore, the potential for ‘independent’/’sovereign’ monetary policy can be tested throughout the model run (and, therefore, implicitly) but also very explicitly and in an ‘extreme’ manner through enacting an adaptation of ‘Helicopter Money’ for these purposes (a term which has come to signify many things since Friedman’s (1969) original thought experiment) whilst still retaining a sense of generalisability. Indeed, a key strength of the ABM from this project has been its generalisability to various domains and its extensibility.

Regarding the effectiveness of the solution itself, it was always going to be debateable for macroeconomists – this is due more broadly to social-scientific knowledge being especially contentious due to the value-laden nature of these endeavours – but the results were insightful, many successes were encountered and, furthermore, emergent properties were encountered (emergence being an oft-encountered property of complex systems modelling and agent-based models; see, for example, Axtell 2007). More details of the results/solution are available in the ‘Results’ and ‘Conclusion’ sections in particular.

Project Requirements: An Agent-based Model of a (complex) economy with multiple monies and heterogeneous agents. The interest rates and exchange rates must be determined through an extension/articulation of the Mundell-Fleming model for these purposes so that the hypothesis of potentially ‘solving’ the Trilemma can be validly tested. Software required includes Python 3, MESA (the ABM framework), SciPy, NumPy, pandas and matplotlib.

DESIGN

This section will give high-level details of the original project specification and design with precise details and differences being detailed in the 'Realisation' section to prevent unnecessary replication.

It was proposed that money-supplying and money-demanding agents' randomly-generated heterogeneous preferences over interest rates and exchange rates would be modelled via skew normal and gamma distributions respectively.[1] The skew normal distributions were needed over interest rates to allow for the possibility of negative (as well as positive) interest rates due to the nature of the adapted, extended interest-parity condition. The details of the adapted and extended interest-parity condition will be laid out more explicitly in the 'Realisation' section. The exchange rate over which the agents have preferences over is the multilateral, weighted-average exchange rate.[2] The preferences over interest rates are also preferences over multilateral, weighted-average interest rates. In both cases,[3] the original plan was to have them weighted according to the volume of each money in circulation respectively and then averaged by the number of monies minus one (to exclude itself in the averaging process) but this was found to be problematic in the Realisation.

The bond market is exogenous to this ABM[4] and, therefore, interest payments for bond-holders and bond-issuers are not modelled (which also means the

[1] Appendix 1, p.3-6.
[2] Appendix 1, p.5.
[3] Appendix 1, p.9-10.
[4] Appendix 1, p.11.

extended/revised interest-parity condition still holds within the ABM as proposed). The goods and services markets are not explicitly modelled but, since this ABM models a revised version of the Mundell-Fleming trilemma, the behaviour/responsiveness of aggregate output in the economy/economies can be extrapolated from the behaviour of the model. Indeed, it was necessary for the model to be an extension of the original Mundell-Fleming model and the interest-parity condition because, otherwise, any successful findings would be disputed by macroeconomists even more so than they normally would be. More details will follow in the 'Realisation' section.

In the original design, inflation is not accounted for (assumed to be zero) and, therefore, real and nominal interest rates are assumed to be equal. This was changed during the implementation with inflation and other factors being implicitly/potentially accounted for. A Limit Order Book was proposed in the original specification[5] for determining bilateral exchange rates after each agent's action and/or step but the actual details were not precisely specified. The trade protocol also lacked specific details but was going to be dependent upon the agents' optimisation problems.

Monetary autonomy via 'Helicopter Money' was proposed as a means by which the sovereign/independent monetary policy aspect of the trilemma is tested[6] when there are multiple monies. The term Helicopter Money has its origins in a thought

[5] Appendix 1, p.10.
[6] Appendix 1, p.10-11.

experiment by Friedman (1969) and, although it has come to mean many different things since then, it is still a sufficiently general test of monetary autonomy.

The proposed scheduling regime was either going to be 'synchronous or simultaneous' activation[7] (where all agents act simultaneously) or 'random' activation (where each agent is activated in every step of the model but the order in which they are activated is randomised for each step of the model). It was tentatively proposed that perhaps both variants could be tested to see how results differ in each case.

The money-demanding agents use 'adaptive' expectations rather than 'rational' expectations for simplicity of modelling (rational expectations being much more complex to solve). Adaptive expectations[8] for the money-demanding agents entails their belief that, when they buy monies, they expect the exchange and interest rates to be the same as it was when they bought them (i.e, they do not rationally account for the impact of money-supply expansions by the money-suppliers and the impact upon interest rates and exchange rates). The money-suppliers, however, rationally choose to expand their respective money-supplies on-demand upon predictions formed from the revised interest-parity condition.

More broadly, the hyper-parameters of the ABM include:

1. Number of money-suppliers (and, therefore, number of monies).

[7] Appendix 1, p.8-9.
[8] Appendix 1, p.9-10.

2. Number of money-demanders.

3. Absolute sum of the initial total volume of money supplies in the system (to be equally distributed across monies and money-demanding agents in the initial step).

4. Proportion of money-demanding agents receiving Helicopter Money.

5. Proportion of money-supplying agents participating in Helicopter Money expansions.

6. Quantity of Helicopter Money expressed as a percentage of the absolute sum of all monies in the system.

The criteria for a ‘successful’ test case is elaborated upon in the following Realisation section.

REALISATION

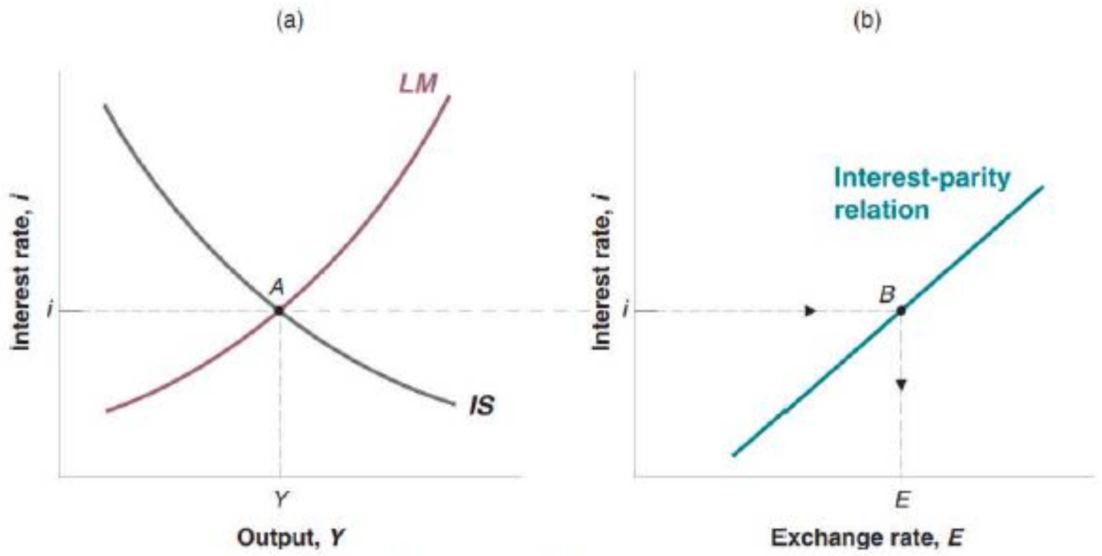
As was specified in the Design, the money-demanding and money-supplying agents' preferences over weighted, multilateral interest rates and exchange rates are represented by skew normal and gamma distributions respectively. Whenever a money-supplying agent or money-demanding agent is created, the value of the shape and scale of each of the distributions is randomly chosen with uniform probability from a certain range. The shape and scale of the gamma distributions, k and θ respectively, are randomly generated with uniform probability such that $k \in [0.01, 10)$ and $\theta \in [0.01, 2)$ since $k, \theta > 0$ for gamma distributions.[9] The shape and scale of the skew normal distributions, α and ω respectively, are randomly sampled with uniform probability such that $\alpha \in [-5, 5)$ and $\omega \in [0.01, 2)$ since $\alpha \in \mathbb{R}$ and $\omega \in \mathbb{R}_{++}$ for skew normal distributions.[10] All agents' distributions' parameter values are randomly chosen with uniform probability from those same ranges so that there is still some degree of validity when making inter-agent utility comparisons but this aspect of the ABM can easily be relaxed in an alternative realisation.

An extended interest-parity condition: weighted, multilateral interest rates and exchange rates

Regarding the weighted, multilateral interest rates and exchange rates, they are based on an extension of the original interest-parity condition and the Mundell-Fleming (1962, 1963) model. The following graph is lifted directly from Blanchard's

[9] Code for this found in Appendix 4: lines 16-50, 185-214.
[10] Code for this found in same code specified in previous footnote.

(2011) standard, undergraduate macroeconomics textbook and it represents the Mundell-Fleming (1962, 1963) model:



Source: Blanchard (2011)

However, this does not suit our purposes since this deals with a domestic currency in relation to a single (world) foreign exchange rate/interest rate because, in our case, we have multiple monies with multiple exchange rate and interest rate equilibria. Thus, it should be extended for our purposes.

Suppose there are Z monies where $M_1^S, M_2^S, \ldots, M_{Z-1}^S, M_Z^S$ denotes the amount/supply of each money currently within the model (across all agents – that is, held by both money-suppliers and money-demanders). The sum of the volume of each of the monies can be denoted as $sum(monies) = M_1^S + M_2^S + \cdots + M_{Z-1}^S + M_Z^S$ and then we denote the proportion that each money $i \in \{1, \ldots, Z\}$ makes up of the sum of the volume of all monies as $prop(M_i^S) = \frac{M_i^S}{sum(monies)}$ and these are our weights for the multilateral weighted exchange rates and interest rates.

The original proposal[11] in the specification was to have each interest rate (except the one being measured) multiplied by the total volume of each of their respective monies and divided by the number of monies minus one; this technically allowed the system to function but the interest/exchange rates fluctuated too wildly (after even modest money-supply expansions) and the division by the $Z - 1$ quantity was found to depress the interest rates to miniscule quantities. The $Z - 1$ division was also originally intended for the multilateral, weighted exchange rates but it would have depressed the value of the exchange rates so significantly that the exchange rates' percentage changes would have been so infinitesimal that it would have been a trivial, unrealistic and unhelpful solution to the Trilemma. Thus, a more realistic extension was found to be necessary over the course of testing.

To begin with, the original interest parity condition can be represented as follows:

$$E_t = \frac{1 + i_t}{1 + i_t^*} E^e$$

In this form, the condition essentially considers the relationship between two monies (or between one money and the 'world' interest rate) – more precisely, it only considers the relationship between a domestic currency's exchange rate E_t as well as the domestic interest rate i_t and the foreign interest rate i_t^* whilst E^e is the expected exchange rate. As such, the condition in this form implies a positive relationship between the domestic interest rate and the exchange rate as well as the expected exchange rate and current exchange rate – conversely, if the foreign

[11] Appendix 1, p.10.

interest rate increases, the exchange rate decreases. For our extension, we assume adaptive expectations (i.e that $E^e = E_t$). When the expected exchange rate for the next moment/period and current exchange rate are equal under the assumption of adaptive expectations, the condition solves to yield $i_t = i_t^*$ but there are no unique solutions in this scenario and it still does not account for the multiple monies and their respective money supplies. The first extension involves accounting for the respective money supplies of the multiple monies. To determine the interest rate i_t^j of a money j in period t, the equation for a model with Z monies under adaptive expectations would be:

$$i_t^j = prop(M_1^S)i_t^1 + prop(M_2^S)i_t^2 \ldots + prop\left(M_{j-1}^S\right)i_t^{j-1} + prop\left(M_{j+1}^S\right)i_t^{j+1} + \cdots$$
$$+ prop(M_{Z-1}^S)i_t^{Z-1} + prop(M_Z^S)i_t^Z$$

Recall that we previously defined $sum(monies) = M_1^S + M_2^S + \cdots + M_{Z-1}^S + M_Z^S$ as well as $prop\left(M_i^S\right) = \frac{M_i^S}{sum(monies)}$ for a money i and thus, within the revised identity, this is how the interest rates are weighted and each of the respective money-supplies are accounted for. By inspection, we can see that the essence of the interest-parity condition is maintained within this extension. The discerning reader may question why the money's own money supply is not included but, on a closer inspection, it is included implicitly because, ceteris paribus, if the money-supply of money j increases: $\Delta M_j^S > 0$, then $\Delta sum(monies) > 0$ and $\Delta prop\left(M_i^S\right) > 0, \forall i \neq j$ and, thus, its i_t^j will fall (which is in line with the conventional prediction that, ceteris paribus, an increase in the money-supply will decrease the 'domestic' interest rate). However, since this is a system of simultaneous equations and, thus, nothing can ever occur in a ‘ceteris paribus’ manner, the interest rate will only decrease so long as any increasing effect exerted upon the *other* interest rates (due to the expansion of the *own* money-supply) does not offset the *own* interest rate-decreasing effect due to $\Delta M_j^S > 0$. Furthermore, these simultaneous equations – in their current form – do not yield a unique solution at all (this was discovered in the process of formulating the system of equations for programming and solving). One more ingredient is necessary (which also sufficiently preserve generalisability); namely, a vector of constants (array of length Z) with ‘delta/error values’ with each element corresponding to each of the Z simultaneous equations respectively: $\Delta_1, \Delta_2, \ldots, \Delta_{Z-1}, \Delta_Z$ where $\Delta_j \in [-5,5)$[12] such that an interest rate i_t^j for money j in period t is given by:

$$i_t^j = prop(M_1^S)i_t^1 + prop(M_2^S)i_t^2 \ldots + prop\left(M_{j-1}^S\right)i_t^{j-1} + prop\left(M_{j+1}^S\right)i_t^{j+1} + \cdots + prop(M_{Z-1}^S)i_t^{Z-1} + prop(M_Z^S)i_t^Z + \Delta_j$$

This extended interest parity condition yields a unique solution (note that the delta values remain constant in each period – this is a non-debilitating limitation of the current ABM).[13] The randomly generated delta values can represent a variety of extraneous/confounding factors such as inflation, geopolitical risks, trust in the money-supplier vs others etc.; this preserves both generalisability and extensibility.

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[12] See Appendix 4, lines 535-541, this implementation also sorts the deltas in ascending order for usage so that the positive relationship between interest rates and exchange rates is still generally adhered to even in the generation of delta/error values.

[13] See Appendix 4, lines 523-558 (547-558 in particular) for how they are initially determined since this is similar to how they are updated throughout each step (see lines 634-671 and 652-671 in particular).

An important point is that the skew normal distributions of the money-suppliers that represents their relative preferences towards interest rates is only concerned with the interest rate of the money they issue and, as such, this is the equation that money-supplying agents' decisions on the rationality of expanding money-supply.[14] For money-demanding agents, it is the interest rates of all the monies they hold in their portfolio that determines the utility they have.[15]

Now we turn to the multilateral, weighted exchange rate that money-supplying and money-demanding agents have preferences over. To begin with, the multilateral weighted average exchange rate does not have a system of simultaneous equations in the same way that the interest rates did. Instead, it is based upon the bilateral exchange rates between each money; these are randomly generated (with uniform probability) and stored in the square matrix $A \in \mathbb{R}_{++}^{Z \times Z}$ since all bilateral exchange rates are positive. In this matrix A, there is necessarily a diagonal of 1s (since each money can, by definition, only buy one of itself) whilst the bottom-left half are randomly generated with uniform probability from the interval $[0.2, 40)$ and the top-right half are the inverse values of the bottom-left (since if one unit of money i can buy x amount of money j, one unit of money j must only be able to buy $1/x$ amount of money i.[16] It is notable that this leads to the weighted exchange rates being naturally sorted such that there is a positive relation between them and the interest rates. However, A remains static across all time periods – which is a major limitation – and, from inspection, if the bilateral exchange rates are randomly generated with

[14] See Appendix 4, lines 16-43 and 65-76 to understand the utility calculation for money-suppliers.
[15] See Appendix 4, lines 185-214 and 216-238 to understand the utility calculation for money-demanders.
[16] See Appendix 4, lines 565-580 for how bilateral exchange rates are determined.

uniform probability, there will most likely be arbitrage opportunities. The original design included a proposal that the bilateral exchange rates would have been determined via modelling a 'Limit Order Book' of sorts.[17] However, this was found to be deeply problematic because to have the bilateral exchange rates alter over time would require assumptions about the elasticities of each money's exchange rates (i.e the bilateral rates' responsiveness to money-demand and money-supply). These would be such specific assumptions that it would not be sufficiently generalisable and a reader would rightly dispute any successful hypothesis tests as being trivial on this basis (in the sense that the system and its tests were 'rigged' from the start). Bearing this in mind, the weighted, multilateral exchange rate $\overline{E_t^J}$ for a money j in period t in a system with Z monies is given by the following equation (where $E^{j,i}$ is the bilateral exchange rate between money j and money i – that is, the amount one unit of money j can buy of one unit of money i):[18]

$$\overline{E_t^J} = prop(M_1^S)E^{j,1} + \cdots + prop\left(M_{j-1}^S\right)E^{j,j-1} + prop\left(M_{j+1}^S\right)E^{j,j+1} + \cdots + prop(M_Z^S)E^{j,Z}$$

Since there are Z monies, there are Z such weighted multilateral exchange rates. Again, like the interest-parity condition, a unilateral increase in *own* money money supply will, ceteris paribus, decrease the exchange rate since the proportions that each of the other monies make up in the system will decrease. On the one hand, the original Trilemma is based upon the empirical behaviour of bilateral exchange rates but, for the reasons mentioned before, precisely predetermining the behaviour

[17] Appendix 1, p.10.
[18] See Appendix 4, lines 582-596 to see how weighted multilateral exchange rates are initially determined – a similar procedure is used to update them in lines 634-650.

of bilateral exchange rates may reduce the ABM to yielding trivial results/solutions. This may seem like a distinctly different criterion to test but it could also be viewed as a stronger test criterion the Trilemma since the volatility of the exchange rate averaged out across exchange rates may be more useful for all users of that money versus just one bilateral exchange rate. Another possibility was to test all the different multilateral weighted exchange rates (all the ones between the bilateral rate and up to $Z - 1$) for all the monies. Although this would predictably increase the success rate, these would be weaker tests.

The Test for a 'Fixed Exchange Rate'

From the preceding details, we have the multilateral weighted exchange rates that we will be testing to see if they remain fixed from one time period to another. Essentially, if the following condition is met then it constitutes a successfully fixed exchange rate for money j:

$$0.99 \leq \overline{E_t^J} \div \overline{E_{t-1}^J} \leq 1.01$$

Since there are Z monies (and, therefore, an equivalent number of multilateral weighted exchange rates), the number of successful tests can be expressed as a percentage of the number of exchange rates/monies.[19]

[19] See Appendix 4, lines 920-929 for the specific short code listing used for this test.

Furthermore, although the trade protocols and money-expansions will be specified later, it is worth noting that the tests that accompany this primary hypothesis test will examine whether the fixed exchange rates correspond to those monies whose money supply expands and whether there are still fixed exchange rates after Helicopter Money is introduced into the system (Helicopter Money was implemented as it was originally specified[20]).

Calculating Money-Demanding and Money-Supplying Agents' Utility

The utility functions were calculated as was proposed in the original project specification and design.[21] For money-demanding agents, the utility of the portfolio they hold is calculated by taking the interest rate and the exchange rate of each of the monies they hold and seeing what pdf value it corresponds to when input as the support of the skew normal and gamma distributions respectively. By adding the pdf value corresponding to the interest rate (from the skew normal distribution) and the pdf value corresponding to the exchange rate (from the gamma distribution), we obtain the utility that the agent gains from holding one of unit of that money. After determining the utility of holding the other monies in a similar fashion, we then obtain a vector of utility values for each money (each composed of two utility values – corresponding to the interest rate and exchange rate of that money respectively). With the vector of utilities, we multiply the agent's utility corresponding to that money with the quantity of that money held (and obtain the sum of this for each money held) to obtain the money-demanding agent's total utility.[22]

[20] Appendix 1, p.10-11.
[21] Appendix 1, p.6-7.
[22] Appendix 4, lines 216-238.

For money-supplying agents the difference is that, since each money-supplier corresponds to a money (i.e. if there are Z money suppliers, then that means there are Z monies and money supplier 1 corresponds to money 1, supplier 2 to money 2, all the way up to supplier Z and money Z), their utilities are determined by the prevailing weighted, multilateral interest rate (according to the interest-parity condition and money supplies) and weighted, multilateral exchange rate (according to the matrix of bilateral exchange rates and money supplies) of their *own* money. Thus, determining a money supplier's utility is more straightforward: they simply need to input the value of the interest rate and exchange rate of their money into their own skew normal and gamma distributions respectively, find the pdf values that correspond to them and then sum the two pdf values to obtain their current utility.[23]

The Money-Demanding and Money-Supplying Agents' Optimisation Problems

There are three key optimisation problems currently included in this model – namely, the money-suppliers' optimisation problems for determining the optimum quantity of each of their monies,[24] the money-demanders' optimisation problems for determining their optimum/desired portfolios[25] and the money-demanders' optimisation problems for making payments in a way that minimises utility loss.[26] We turn to each of these in turn.

[23] Appendix 4, lines 63-76 and also lines 36-48 to see how variables are predefined.
[24] See Appendix 4, lines 101-176 for this.
[25] See Appendix 4, lines 268-355 for this.
[26] See Appendix 4, lines 402-439 and possibly lines 357-400 for completeness.

Originally, the money-suppliers' optimisation problem was not specified in complete detail.[27] Nevertheless, the original essence was maintained but, rather than formulating it as a linear program or a nonlinear optimisation problem, numerical approximation of the optimum solution was employed. Essentially, for the money-suppliers to determine what the optimum quantity of their money would be (given the interest parity condition and predicting how exchange rates may change), they begin by testing/predicting whether 0.1% of their money in circulation would give them the desired interest rate and exchange rate, followed by 0.2%, then 0.3%, all the way up to 99.9% in increments of 0.1% (since the amount of money will necessarily be greater than 0% and less than 100%).[28] Based on what seems to be the optimum proportion for them (given the interest parity condition and bilateral exchange rates, holding the absolute volume of the other monies in the system constant),[29] the money-suppliers will know whether it is rational to create more money and subsequently sell it to the money-demander(s) that want it.

A simplex algorithm is used for the money-demanders' optimisation problem for determining the optimum portfolio of monies.[30] Given their utility functions, this requires recourse to the definition of the bilateral exchange rates matrix. We defined the bilateral exchange rates as being stored in the square matrix $A \in \mathbb{R}_{++}^{Z \times Z}$ and, from this, we can define our constraints. To begin with, the variables are the amount of each money to be held in the portfolio and, thus, the coefficients we are seeking to minimise via the simplex algorithm is the negative of the utility coefficients for each of

[27] Appendix 1, p.6.
[28] See Appendix 4, lines 101-176, also lines 33 and 778.
[29] See Appendix 4, lines 132-169.
[30] See Appendix 4, lines 268-355 and particularly line 323.

the monies (equivalent to maximising the agent's utility with respect to the various quantities of monies held). An obvious set of constraints here is that the optimum quantity of each money held in a money-demanding agent's portfolio must be lesser than or equal to the total volume of each money in the system; this is all represented by an identity matrix (for the upper-bound coefficient matrix) corresponding to bounds which equate to the total volume of each money in the system. Furthermore, the other part of the upper bound matrix includes A^T coupled with bounds that are the sums of the volume of each money present within the money-demanding agent's portfolio multiplied each of the respective elements in the corresponding row of A^T. Each of these inequalities represent 'all-or-nothing' bounds which basically represents the maximum of each money that the money-demanding agent can purchase, given their current portfolio and the purchasing power of their portfolio with respect to each money (translated from the transpose of the bilateral exchange rates matrix).[31]

Regarding the optimisation problem for determining optimal payments (to minimise utility loss from making those payments), unlike the optimisation problem for determining the optimum portfolio, there is no need to minimise the negative of the utility coefficients of the monies because we seek to minimise utility loss itself. The optimal payments algorithm is with respect to a quantity of one particular money[32] and, as such, the equality constraint in this optimisation problem (programmed as two upper bound constraints of the positive and negative of the

[31] See Appendix 4, lines 285-319 for precise details of how the matrix-formulation and its corresponding bounds are coded.
[32] See Appendix 4, lines 402-439.

equality constraint)[33] is the bilateral exchange rates with respect to that money (coefficients to each of the monies used to pay) will equal the quantity of the money being purchased. The remaining upper bound matrix and bound values ($Ax = b$ in simplex standard form) basically limits the monies being paid by the quantities of monies held in the agent's current portfolio[34] (i.e they cannot make the payments with monies they do not have).

The Trading Protocol

The 'Trading Protocol'[35] is initiated by the money-demanding agents (and, as such, it is a demand-driven model). To begin with, the money-demanders determine their optimal portfolios (given their budget constraints)[36] and then, based on which monies they require more of (i.e when the difference in the quantity of a money in the optimum portfolio between that same money in the current portfolio is greater than zero), they approach the money-suppliers directly.[37]

To begin with, the money-demanders try to approach their purchases in a way that gets them approximately close to their optimum portfolio. To do so, they sort the quantity of monies that they need more of in their current portfolio in descending order (so that the money and its quantity that yields the most utility is sought first and the rest are sought in utility-descending order until termination). In each case, the money-demanding agent goes directly to the money-supplying agent that is

[33] See Appendix 4, lines 417-428.
[34] See Appendix 4, lines 430-433.
[35] See Appendix 4, lines 454-513.
[36] See Appendix 4, lines 449-451.
[37] See Appendix 4, lines 454-513.

responsible for issuing the money. If the money-supplying agent already holds the quantity of money desired[38], it sells it to the money-demanding agent and receives the money-demanding agent's optimal payment (such that the money-demanding agent minimises any utility loss from making a payment, as per the previously described optimisation problems)[39].

In the scenario where the money-supplying agent does not have the quantity desired by the money-demander, then the supplier faces a choice of creating money to satisfy that demand.[40] After solving the optimisation problem for money-suppliers described previously, the money-supplier knows whether it is rational to increase the money supply of its own money and sell to the money-demander or not (based on what it will do to the interest rate and exchange rate according to the interest parity condition).[41] Thus, although it is a demand-driven model, the decision to create more money is based not only upon the excess money-demand but also the rationality of money-creation according to the money-suppliers' utility functions. Throughout this process, the exchange rates and interest rates may adjust, the volumes of each money will have to be updated etc. Of course, the money-demanders may not reach their optimum portfolio precisely but they will get approximately close to it.

[38] Appendix 4, lines 460-469.
[39] Appendix 4, line 465.
[40] Appendix 4, lines 471-503.
[41] Appendix 4, lines 473-477 and lines 482-503.

Rationale and Benefit of incorporating 'Market' as an Agent

One aspect of the implementation that may strike one as being peculiar is that the market institution itself is an Agent.[42] This could be an interesting way to maintain and extend the model (from an institutional political economy perspective) but also because it could become the foundations of a computational interpretation/implementation of a model of Soros' (2003) understanding/exposition of 'Reflexivity' in the social sciences. Thus, it would help the extensibility of this ABM.

Testing of Code

The code was tested routinely whilst programming. I constantly ran and re-ran the program when I added new code (thereby debugging whilst programming). This involved many print statements and it would be impossible to recover all this testing. Nevertheless, it is worth noting that this procedure was followed. No IDE was used.

[42] See Appendix 4, lines 515-518.

ETHICAL USE OF DATA

No human data was used in this project. The data generated, analysed and presented in this project was artificial data from the agent-based model. Artificial data can still be unethically manipulated but the initial conditions of the datasets were generated through random distributions (within certain ranges) and random values in each of its runs and, therefore, it is less susceptible to manipulation on this front. I have also presented results that are both ‘good’ and ‘bad’ to be completely honest about the efficacy of the model since there is a worrying trend of irreproducibility of results in the scientific community.

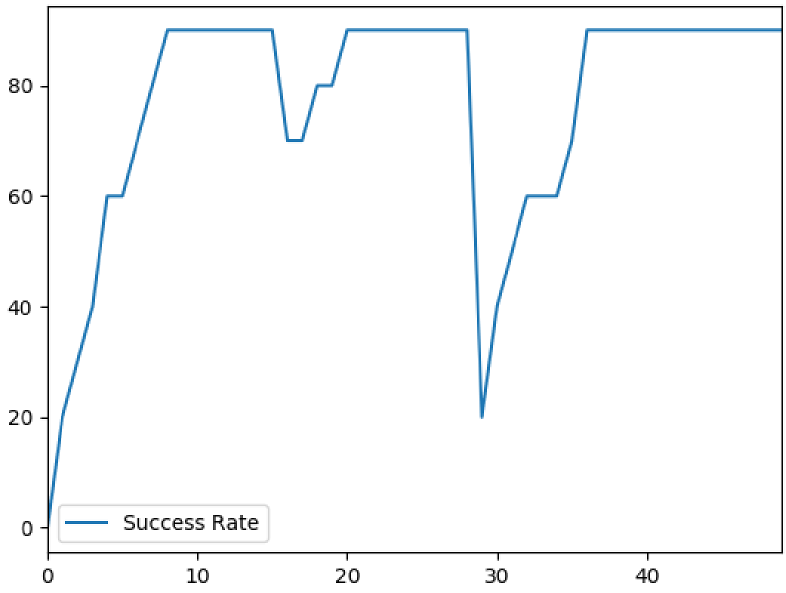
RESULTS

Before proceeding, it should be noted that the following seven graphs are from seven different, individual runs (each consisting of 50 time steps – steps on the x-axis) but they have been chosen as illustrative examples. In each case, the runs are obviously stochastic (with randomly generated attributes) so there will be variations seen across runs but, from my observations, they are relatively representative of general trends.

In each run, there are 10 money-demanding agents, 10 money-supplying agents (and therefore, 10 monies in the system), the sum of the total money the system begins with is 1000 (which means the initial quantity of each money in existence is 100 and each money-demanding agent begins with a portfolio that has 10 of each money – i.e. perfect equality is the initial state), the proportion of Helicopter Money released into the system (step 29) is 10% of the total volume across the system at the time, 50% of money-demanding agents receive the Helicopter Money (chosen randomly, split equally amongst them) and 10% of money-suppliers are randomly chosen to expand their money-supply for Helicopter Money. The six scatter plots that follow the seven graphs are much more varied and are produced through the ‘Batch Runner’ class in MESA; their purposes and nature will be described individually. The initial conditions in all runs are such that the money-demanding agents begin with equal portfolios (equal amounts of each money and, therefore, equal shares of the initial monies in the system).

Graph 1: Success rate over 50 time steps: example run 1

Graph 1: Illustrative example of success rate (percentage, y-axis) over 50 time steps (x-axis) of an individual model run (Helicopter Money at step 29).[43]



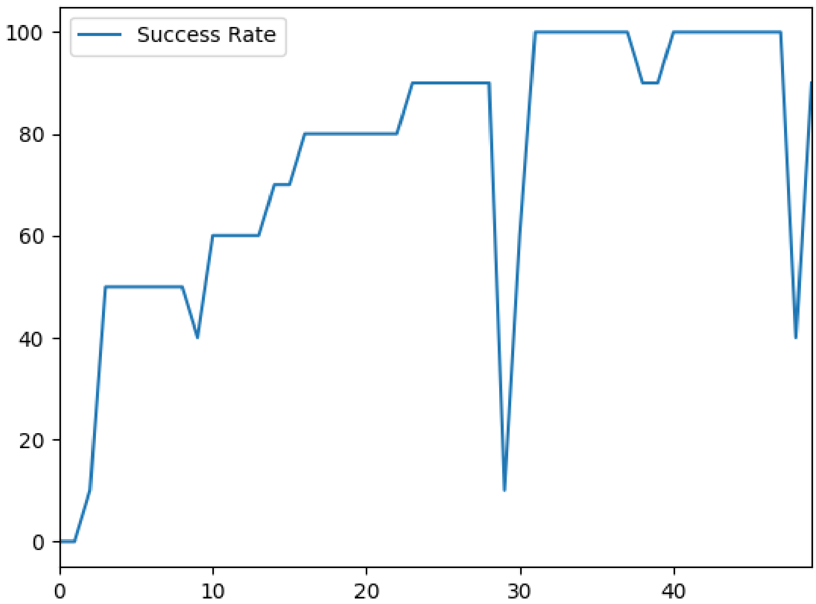
In the above case, the success rate of the model is low (0%) to begin with but peaks at 90% and fluctuates around that state (whilst converging towards it). The sharp drop from 90% (to as low as 20%) around time step 29 corresponds to the step that Helicopter Money is introduced into the system (where 50% of money-demanding agents receive equal portions and one money-supplier expands) and, thus, it would be expected that exchange rates would not remain fixed during this step since it is such an *extreme* test of monetary autonomy (although it does recover afterwards). Importantly, monetary autonomy is also implicitly tested during each model step because money-suppliers expand their money supplies insofar as it is rational for them to do so when approached by money-demanders. Indeed, monetary

[43] Example run from original source code in Appendix 4.

autonomy is only rational and normatively desirable in so far as it improves agents' utility (whether they be suppliers or demanders).

Graph 2: Success rate over 50 time steps: example run 2

Graph 2: Another illustrative example of success rate (percentage, y-axis) over 50 time steps (x-axis) of an individual model run (Helicopter Money at step 29).[44]

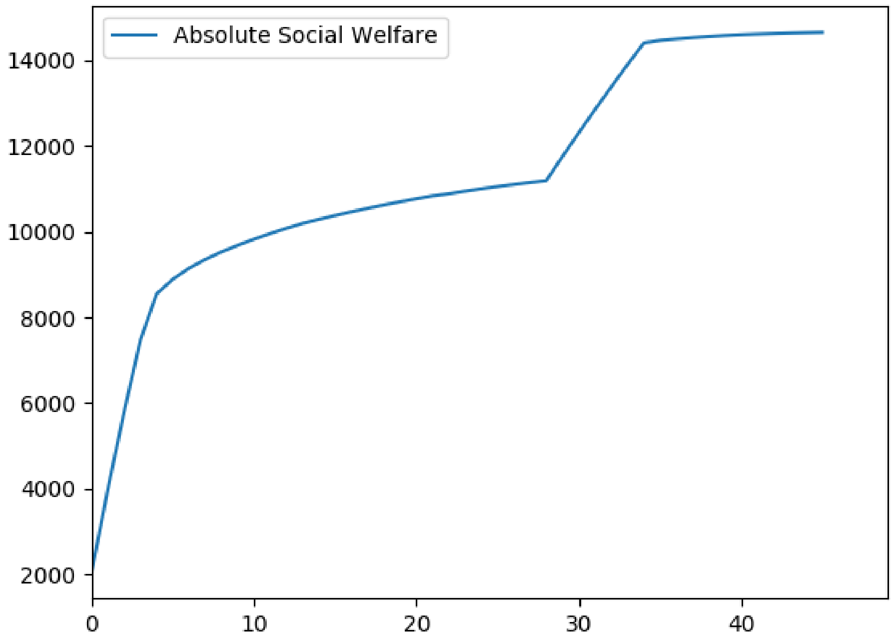


In the second example run, a similar situation is observed but the success rate peaks at 100% instead and, interestingly, in the 49th step there is a sharp drop in the success rate – this is realistic when we consider that economies generally fluctuate around equilibrium conditions (both near and far from them; see Soros 2003) which shows that stochastic, systemic shocks are still possible and it is not a deterministic ABM by any means.

[44] Example run from original source code in Appendix 4.

Graph 3: Social Welfare absolute growth over 50 time steps: example run 3

Graph 3: Absolute social welfare (sum of money-demanders and money-suppliers' utility, y-axis) in a single model run of 50 time steps (x-axis) with Helicopter Money at step 29.[45]



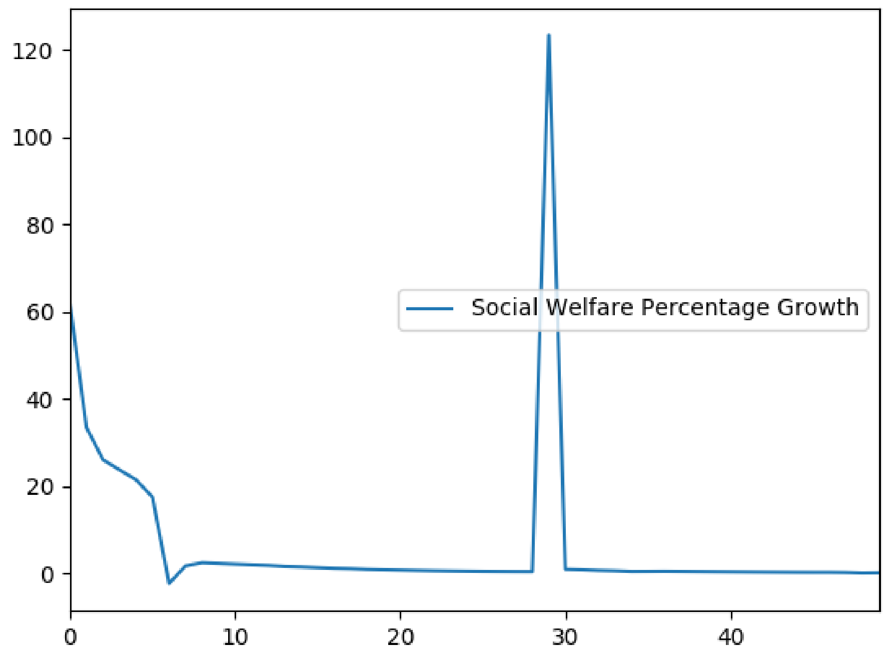
Social welfare is defined as the sum of all agents' utilities (both money-demanders and money-suppliers) although, in this model, social welfare is more heavily weighted towards money-demanders because their utility coefficients are multiplied by the quantities of monies they hold whilst the utility the money-suppliers derive is not linked to the quantity of their money in the system but only with the interest rate and exchange rate of their money in equilibrium. In this example run, the social welfare seems to peak near 11,000 in 'equilibrium' before sharply increasing

[45] Produced from code in Appendix 5.

again from step 29 before settling at 14,000 (consistent with the idea that Helicopter Money would improve agents' utility).

Graph 4: Social Welfare percentage growth over 50 time steps: example run 4

Graph 4: Social Welfare growth rate (percentage, y-axis) in a single model run of 50 time steps (x-axis) with Helicopter Money at step 29.[46]



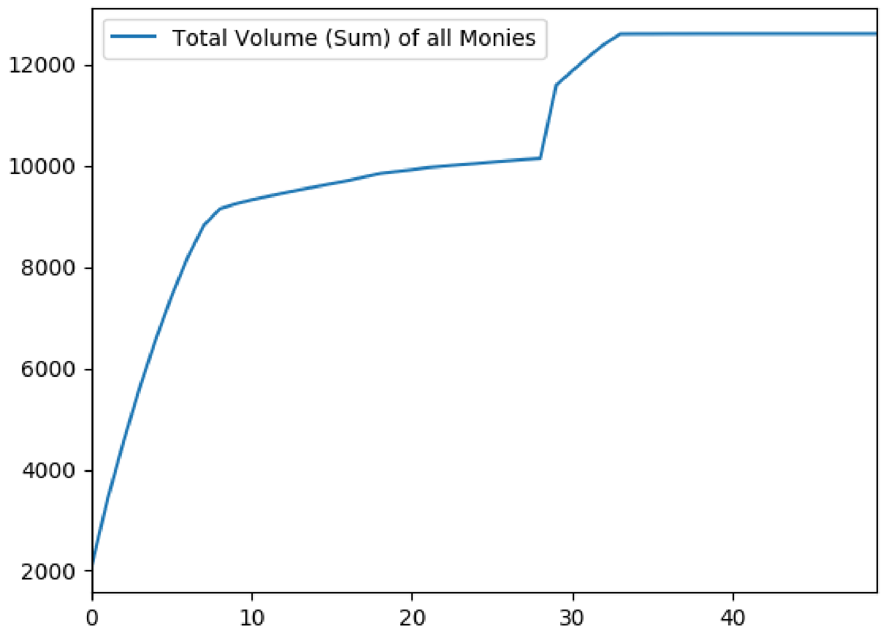
The discerning reader will note that this is obviously from a different run to the previous graphs (each of the graphs are from different runs) and, in this case, social welfare percentage growth experiences an over 120% increase. Obviously, social welfare growth will vary according to a variety of factors (e.g. the money/monies being used as Helicopter Money, the number of money-demanders receiving, the

[46] Produced from code in Appendix 6.

amount of Helicopter Money released into the system and its effect on interest rates and exchange rates). There is also the possibility for negative growth (e.g. step 6).

Graph 5: Growth of total volume (sum) of all monies growth over 50 time steps: example run 5

Graph 5: Total volume (sum) of all monies (y-axis) over 50 time steps (x-axis) with Helicopter Money at step 29.[47]



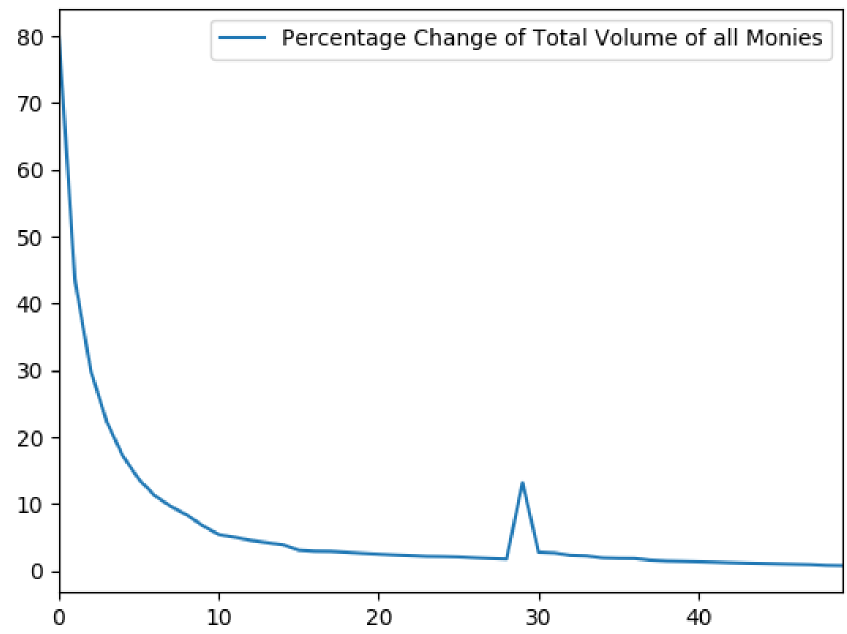
Notably, the success rates (from example run 1 and 2, graphs 1 and 2) increase as the total volume of monies in the system increases at a decreasing rate and, as such, it could be argued that these results are trivial. However, this does not account for the fact that the volume of monies in the system only increases according to whether it is rational for money-suppliers to expand and money-

[47] Produced from code in Appendix 7.

demanders to demand (i.e. it is dependent on their utility functions). They are still, therefore, arguably valid tests for fixed exchange rates whilst also implicitly testing monetary autonomy. As expected, the total volume of monies in the system settles at a higher aggregate quantity post-Helicopter Money.

Graph 6: Total volume (sum) of all monies percentage change over 50 time steps: example run 6

Graph 6: Chart to show percentage change of total volume of all monies (y-axis) over 50 time steps (x-axis) of a single model run with Helicopter Money at step 29.[48]

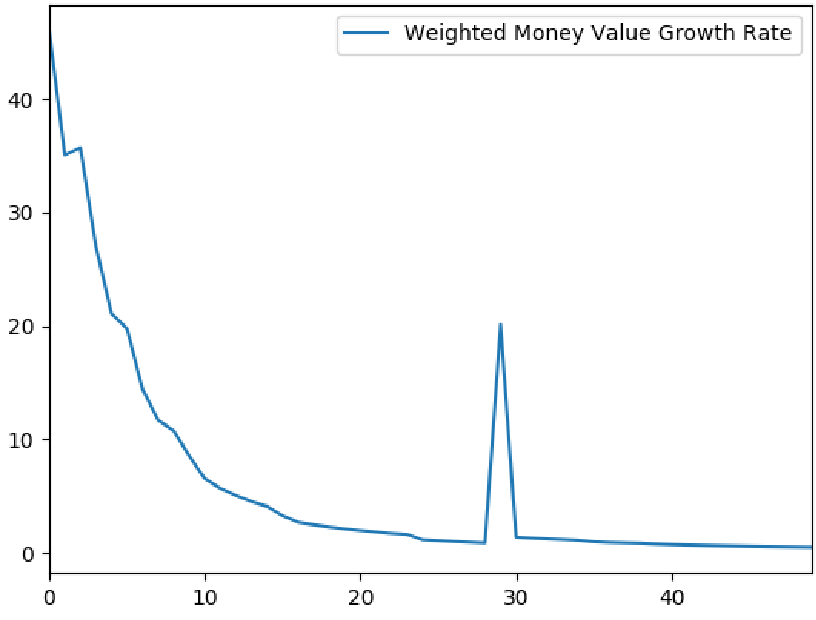


[48] Produced from code in Appendix 8.

This example run re-affirms the typical behaviour of the aggregate quantity of monies increasing at a decreasing rate as time progresses (and spiking during Helicopter Money).

Graph 7: Purchasing power growth rate over 50 time steps: example run 7

Graph 7: Chart to show the percentage growth of the exchange-value weighted percentage growth (y-axis) over 50 time steps (x-axis). This can be extrapolated as being proportional to output/GDP growth rate in the economy/those economies.[49]



This graph shows the exchange-weighted money value (i.e. the aggregate purchasing power) growth rate. Growth is highest at the start and it spikes again around the Helicopter Money step before settling relatively quickly. This implies that

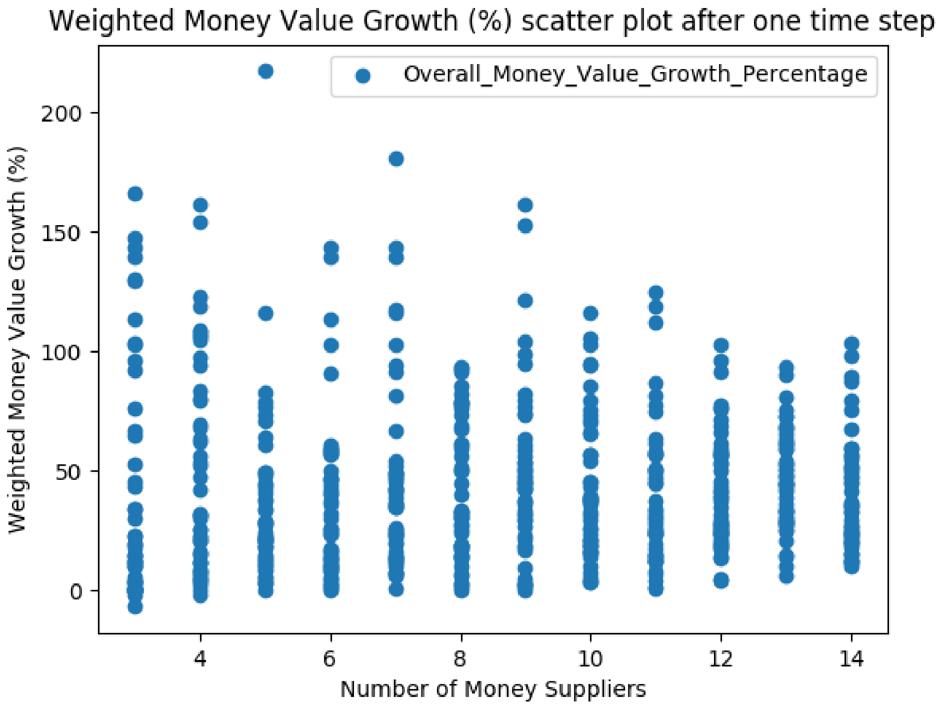
[49] Produced from code in Appendix 9.

the economic growth benefits associated with Helicopter Money will be felt most strongly in the short-term. The growth rate can be extrapolated to GDP/output in the economy/economies because the model is an extension of the Mundell-Fleming (1962, 1963) model.

The above graph is also consistent with recent findings in modern macroeconomics that are gaining increasing prominence – namely, that inequality constrains growth and lesser inequality is associated with higher economic growth (Piketty 2014; Ravallion 2014; Biswas, Chakraborty and Hai 2017). This is a valid observation because, although inequality was not formally measured after each time step, the initial state is such that every money-demanding agent is perfectly equal (each with equal amounts of each money in their respective portfolios) and thus, as they trade over time, they are most likely to become more unequal (when compared to the initial state). A scatter plot would now be worth presenting which shows how economic growth rates immediately after equality (i.e. the first step) vary with the number of monies in the system (thereby further investigating the relationship between multiple monies and economic growth when there is no inequality – this is obviously an unrealistic scenario but, nevertheless, it is interesting to explore).

Scatter Plot 1: Weighted Money Value Growth (%) after equality (first time step) as number of money-suppliers increases

Scatter Plot 1: Batch run (50 iterations each) of influence of number of monies/money-suppliers (x-axis) on final growth rate (percentage, y-axis) [50]



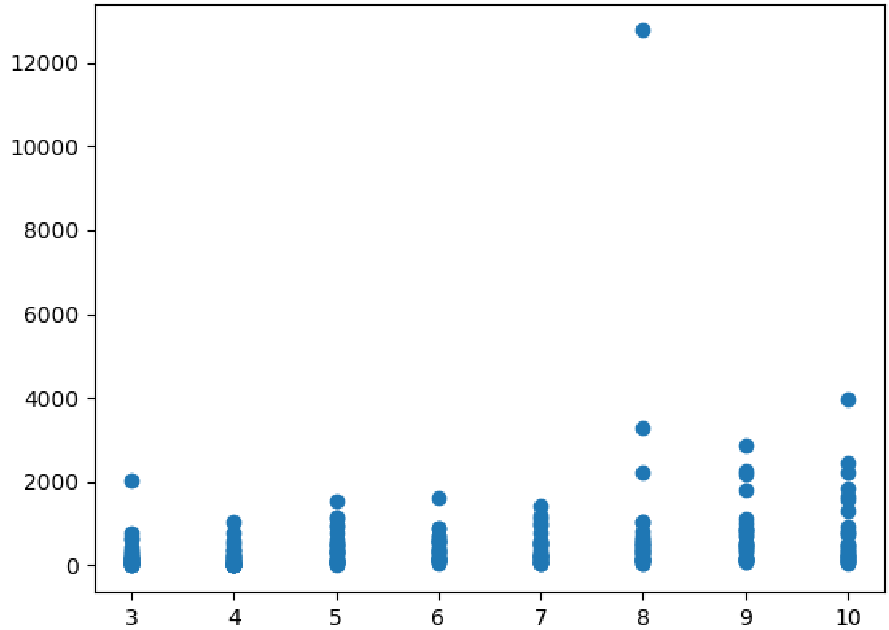
The problem with this plot is it is somewhat difficult to discern relationships because more iterations may have been useful (currently, it is 50 at each money). However, a tentative qualitative examination would suggest that the variability/variance in growth rates across the economy/economies trends lower as the number of monies increases (i.e. growth becomes more stable) and a gentle, positive trend is arguably discernible (some values are clearly negative for fewer monies whilst more monies tend to be associated with a higher minimum growth rate). Readings were taken after the first step to save time when capturing results

[50] Results produced from code in Appendix 10.

(and indeed, in general, this is valid because the largest increases in growth tend to be at the start of the model run – see Graph 7 – but it highlights how more monies can improve growth rates).

Scatter Plot 2: Overall Weighted Money Value Growth after 50 time steps as number of money-suppliers increases

Scatter Plot 2: Batch run (30 iterations each) for economic growth rate (percentage, y-axis) after 50 time steps as number of money-suppliers increases (no. of money suppliers, x-axis)[51]



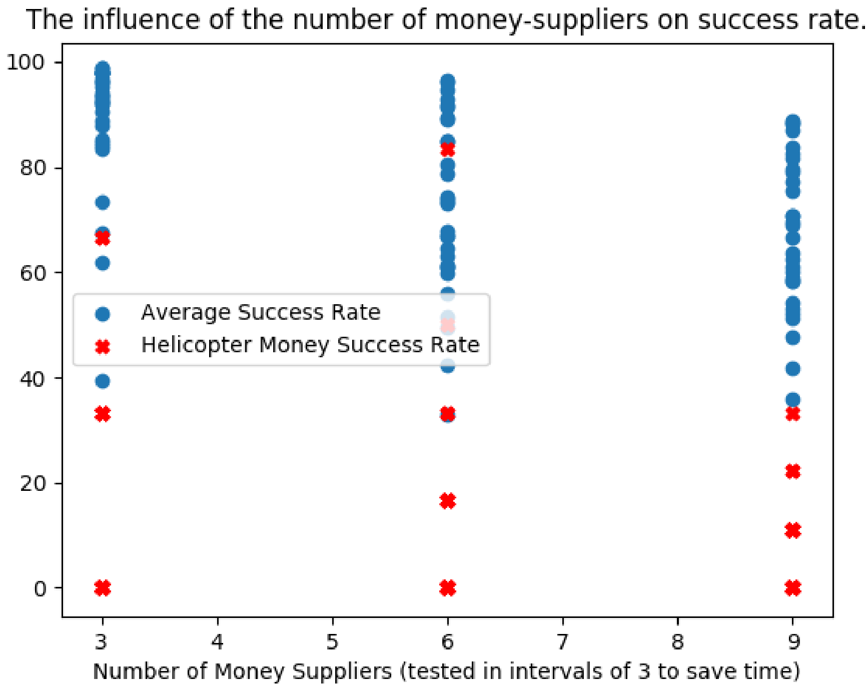
[51] Results produced from code in Appendix 11. A re-run was not conducted even though the outlier made it more difficult to discern a relationship by significantly expanding the scale since this plot took over 30 hours to produce. As an additional caveat, the axes were not labelled within the plot produced by the simulations because this was forgotten and to interrupt it would have been unnecessarily arduous.

This graph includes 240 data points with 30 iterations for each number of monies. This graph measures the final, total exchange-weighted value of all the monies in the system (i.e. the aggregate purchasing power) and compares it to the initial value at the start of the ABM and determines the percentage growth from the start of the model run to the end of the model run (each run being a total of 50 time steps where all the money-demanding agents are activated in a random order). Since the ABM corresponds to an extended version of the Mundell-Fleming model, this can also correspond to the growth rates in output of each of the economies (which could be urban, regional, national, multinational or even global economies that have multiple monies and truly free capital movement) over the 50 steps of the model.

Since the ABM generally converges to/fluctuates around a system equilibrium by step 50 in each model run, this is suitable for trying to determine the correlation between multiple monies and the growth rates in that economy/those economies. In this instance, the variables that are kept constant are as follows: 10 money-demanding agents, 1000 initial money, 50% of demanders receive equal shares of the Helicopter Money, 10% of suppliers expand their supplies in the Helicopter Money step and the Helicopter Money is equivalent to 10% of the total money in the system when it is introduced. Nevertheless, any trend from the above plot would have to be imaginatively extrapolated (or discerned with great difficulty) since the outlier for 8 monies alters the scale completely. Scatter plot 1 was more useful for this purpose.

Scatter Plot 3: The influence of the number of money-suppliers on Average Success Rate and Helicopter Money Success Rate

Scatter Plot 3: Batch run (30 iterations each) of average success rate and Helicopter Money success rate (both percentages, y-axis) vs number of monies (x-axis)[52]



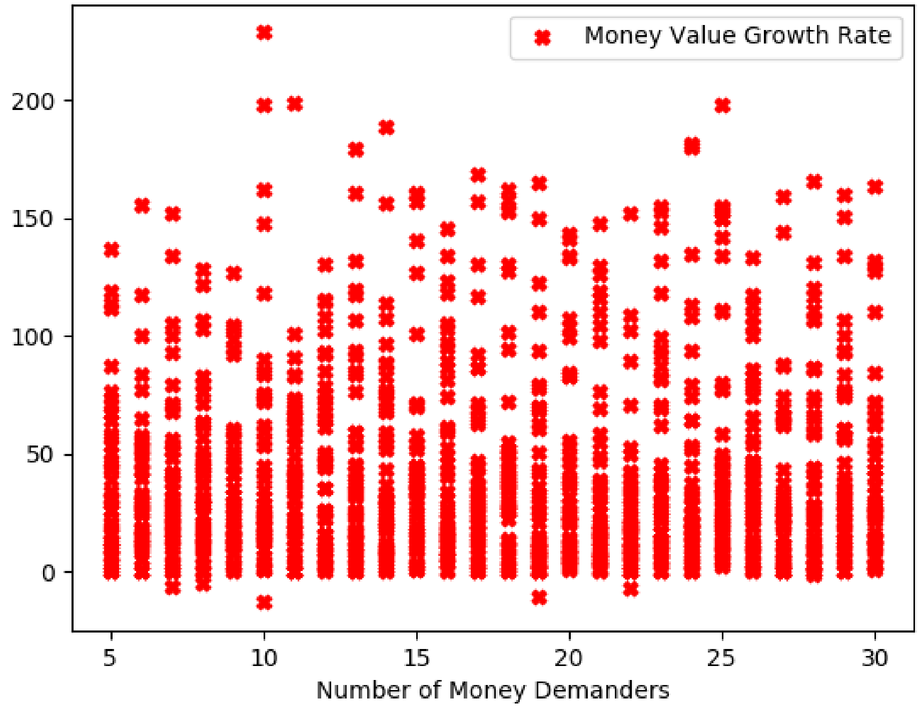
Scatter Plot 3 included 90 data points (30 iterations each for 3, 6 and 9 money-suppliers) and this was measured across all 50 time steps for the runs. Clearly, the average success rate from each model run (averaged across the 50 time steps) seems to decrease as the number of monies increases and, furthermore, the likelihood of Helicopter Money succeeding also decreases. This is consistent with real-world observations since governments have historically wanted to have a single money (or, at the very least, fewer monies) to have greater control over these policy

[52] Produced from code in Appendix 12.

objectives. Nevertheless, the Trilemma itself can still be solved under multiple monies (albeit with decreasing frequency as the number of monies increases).

Scatter Plot 4: The influence of the number of money-demanders on the exchange-weighted money value growth rate (aggregate purchasing power)

Scatter plot 4: Batch run (50 iterations each) of influence of number of money-demanders (x-axis) on economic growth (aggregate purchasing power, %, y-axis) [53]



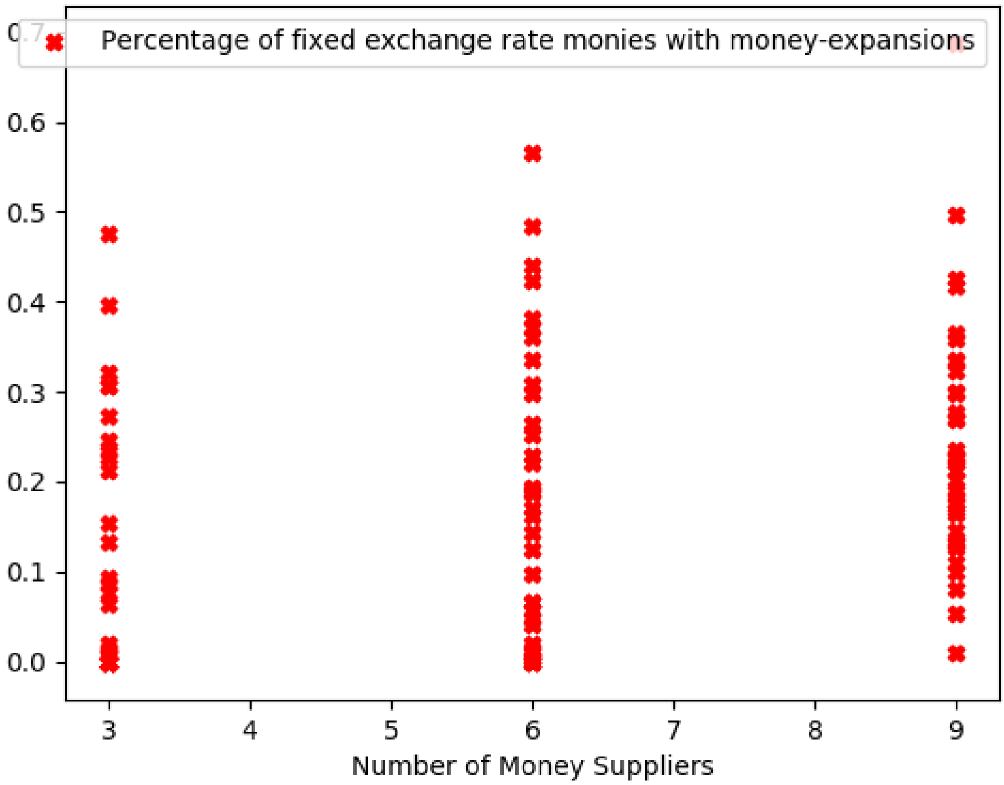
The above plot seems to have no discernible relationship between the number of money-demanders and the exchange-weighted money value (aggregate purchasing power) growth rate which could mean many things but it is worth noting

[53] Produced from code in Appendix 13.

that all these tests (50 iterations each for 5 money-demanders up to 30) were run in an economy with only five monies. Unexpectedly, the number of money-demanders does not seem to discernibly influence economic growth when they share the same initial ‘pie’; this problematically contradicts the intuitively reasonable theory that many Keynesian macroeconomists advocate (i.e. that poorer agents have higher average and marginal propensities to consume and, therefore, extra income for them will stimulate the economy most effectively) but this inconsistency can be rationalised in the sense that the goods, services and labour markets were not explicitly modelled and, thus, the economic growth dynamics resulting from industrial production and consumption are not truly accounted for.

Scatter Plot 5: Investigating whether a money's fixed exchange rate corresponds to an expansion of its supply

Scatter Plot 5: Batch run (35 iterations each) to see how number of monies/money-suppliers (x-axis) influences percentage of monies with fixed exchange rates that also experienced a money-supply expansion (percentage, y-axis – 100x value depicted below)[54]



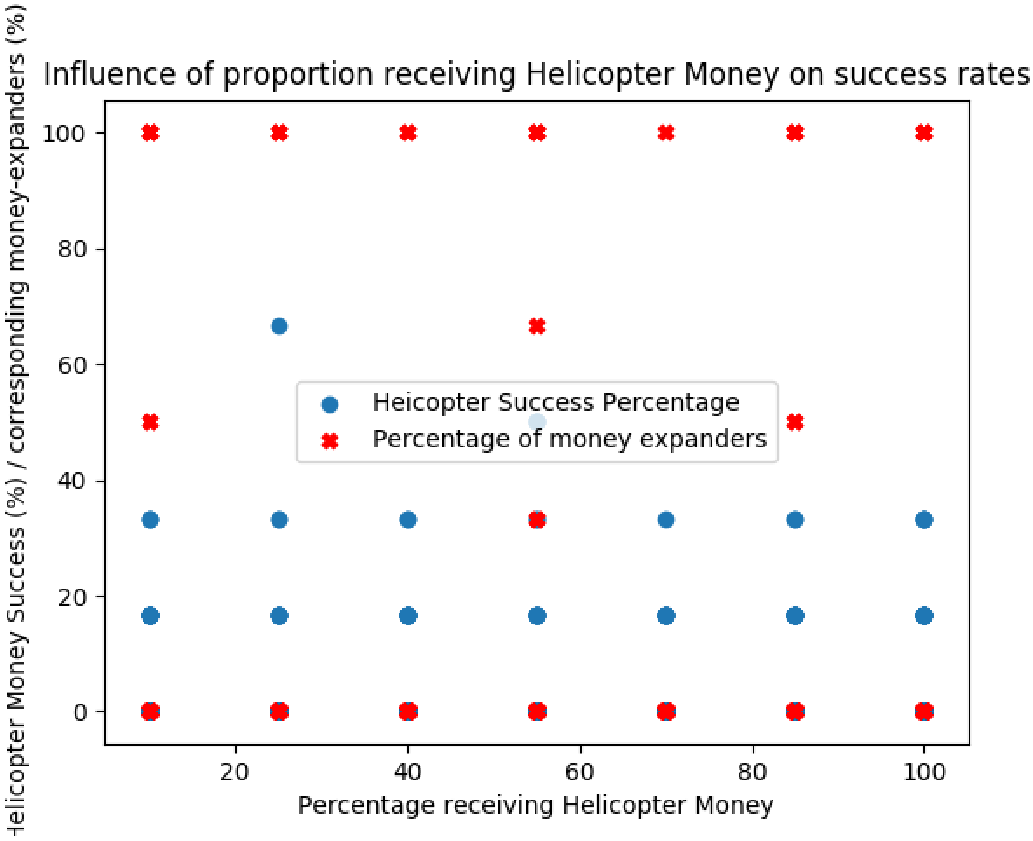
This was run with 35 iterations each for 3, 6 and 9 monies. The percentage on the y-axis is supposed to be 100x the number depicted but it shows that increasing the number of monies does not have a detrimental impact on whether the fixed exchange rates correspond to monies where there have been money-supply

[54] Code that produced this plot is found in Appendix 14.

expansions. Although the percentage of fixed exchange rate monies that also expand their supplies are all less than 70% in these iterations, it still indicates that monetary autonomy and fixed exchange rates are simultaneously possible and that the Trilemma can be solved – thus, they are not ‘trivially’ fixed. We now turn to an even stronger test.

Scatter Plot 6: Investigating influence of percentage of money-demanders that receive Helicopter Money on Helicopter Money success rates

Scatter plot 6: Batch run (30 iterations each) to see if agents receiving Helicopter Money (percentage, x-axis) influences the Helicopter Money success rate and whether the successes correspond to money-expansions (percentages, y-axis) with six monies[55]



This scatter plot has 210 data points, 30 iterations for each of the x-axis percentages tested, it runs for 30 steps and all runs have 6 monies. Clearly Helicopter Money seems to be quite ‘hit-and-miss’ in the sense that, even when a percentage of exchange rates remain successfully fixed, the fixed exchange rates may still not correspond to monies that experience an increase in money-supply (and

[55] Code that produced this is in Appendix 15.

sometimes, conversely, they all do). Most importantly, the proportion of agents receiving Helicopter Money (whether that be 10% or 100%) seems to have no obvious influence on whether exchange rates still remain relatively fixed during this extreme exercise of monetary autonomy (or on whether these fixed exchange rates correspond to money-supply expansions).

EVALUATION

Strengths

A major strength of this project is that the ABM builds upon the well-known Mundell-Fleming (1962, 1963) model and, thus, the results may be considered relatively valid for the proposition being tested. An especially advantageous feature is that since it is an extension of the Mundell-Fleming model, it means that the behaviour/responsiveness of macroeconomic aggregates (GDP/output) can be inferred to some extent from the simulation.

By placing no restrictions on capital movement, only the possibility for simultaneous monetary autonomy and fixed exchange rates needed to be tested. Monetary autonomy was implicitly exercised throughout the model (in so far as it is rational for money-supplying agents to expand their money supplies) and a very extreme conception of monetary autonomy – Helicopter Money – was also tested. The hypothesis tests remain relatively straightforward to understand even when compared to the original criteria used by Obstfeld, Shambaugh and Taylor (2004) as well as Obstfeld and Taylor (1997) for their empirical investigations and, indeed, they potentially have further theoretical advantages (namely, the extent to which the exchange rate remains fixed versus a weighted basket of the remaining monies).

Although each agent’s gamma and skew normal distributions are randomly generated, interpersonal utility comparisons/aggregations are still valid for measuring social welfare (since they are still from the same family of distributions). Thus, this

ABM simultaneously acknowledges the limitations of interpersonal utility comparisons as well as the possibility of making interpersonal utility comparisons.

Weaknesses

The most obvious weakness is that agents use ‘adaptive expectations’ which means that they can (or indeed do) make errors systematically rather than randomly and other possible forms of expectations were not modelled or possible for agents to use; thus, the expectations-formation mechanism was homogeneous and could have been far more sophisticated.

Another weakness pertains to the interest-parity condition and the multilateral weighted interest rates and exchange rates; the weighting between the volume of each money supply is equal when realistically they could vary (and probably do) but this is worth exploring in future iterations (since an unweighted version with respect to the type of monies – but not unweighted with respect to the absolute money supplies themselves – is worth exploring in this first instance). Additionally, it is noteworthy that other potentially stochastic shocks/disturbances to the system were not incorporated. Furthermore, a mechanism by which the bilateral exchange rates adjust in each period (and cease to have arbitrage opportunities or, at the very least, have negligible ones) was not implemented since it was difficult to conceive of a sufficiently general means by which to envision it.

Crucially, the model was demand-driven which meant that money-demanding agents would ultimately only buy money supplied directly from money-supplying agents (rather than trying to engage with other money-demanding agents). However, in the real world, money-suppliers may also try to manipulate the market more actively. However, an earlier implementation included money-suppliers printing all the money they could (and holding it in their own portfolios) to adjust the interest rates and exchange rates optimally – this led to a ridiculous, exponential-trend ballooning of the various money-supplies and it was completely unrealistic. This ABM behaved far more reasonably in this regard. Regarding visualisation of the money usage, demand and production, a spatial element was not incorporated and, therefore, it is still difficult to present how multiple monies could be practically implemented.

A key limitation of this trade protocol is that money-demanders do not approach other money-demanders who may hold their desired money (nor do they approach other money-suppliers) and, as such, it can be seen as a rather rudimentary market structure but, given that the ABM is still based on the interest-parity condition and bilateral exchange rates remain static (due to the previously mentioned impracticability of modelling a Limit Order Book for the purposes of determining them in each time period), the trading protocol is still sufficient and valid for the purpose of this simulation.

Finally, the nature of the solution merely shows that multiple monies are sufficient rather than necessary for the Trilemma to be solved. It does not shed sufficient insight upon the exact conditions, the monetary policy tools that should be used and *how* they should be used to 'solve' the Trilemma but this can be investigated more thoroughly in ABMs. The model also runs very slowly as we add more money-suppliers (and, therefore, monies) which makes it more difficult to thoroughly investigate the implications of the system. However, it does run in polynomial time (from observation and because that is the nature of the simplex algorithms employed in the agents' optimisation functions). Most obviously, a more powerful machine is desirable to execute several 1000s of these model runs very quickly to produce more data and, therefore, more robust, precise and reliable results.

LEARNING POINTS

- Learnt of many of the benefits and challenges of using MESA (an agent-based modelling framework in Python) and will likely use this in future research that makes use of Agent-based modelling; this was a great introduction to the ABM paradigm for scientific modelling and computation.

- Became more familiar with the SciPy and NumPy Python packages for scientific and numerical computing – this will be helpful in future research.

- Learnt numerical approximation methods for non-linear equations (that are relatively non-complex but which cannot be solved through standard linear programming methods or analytically) instead of analytically formulating exact mathematical formulae – I previously thought this to be a significant obstacle to the project's completion. Thus, it is beneficial to understand computational capabilities for scientific computing.

- Regularly consulting my primary supervisor after I had exhausted my own attempts at solving modelling problems was crucial for a successful project and, therefore, I benefitted significantly from his expertise in mathematical modelling and computational solutions.

- In future, I should be careful about over-promising things in the design/specification (such as the Limit Order Book – see Appendix 1) especially since I realised that including this would not only be complex but may also limit the validity of the results. However, further research on the nature of Limit Order Books could inform a future iteration of this ABM for hypothesis-testing in alternative contexts.

- I learnt that I need to significantly improve my management of algorithmic methods so that the time complexity of my software is improved (for the purposes of efficient testing of hyper-parameters in future).

- Overall, the ABM method is profoundly powerful for various purposes and this project has been an invaluably insightful introduction to it. I will continue to employ it in future (policy-relevant), social-scientific research that I conduct.

PROFESSIONAL ISSUES

There are four key aspects of the British Computer Society's Code of Conduct that this project can or does relate to; namely, the 'Public Interest', 'Professional Competence and Integrity', 'Duty to Relevant Authority' and 'Duty to Profession'. It is worth examining each of these aspects in turn.

The 'Public Interest' aspect is most relevant here because the entire project was conceived for improving economic prosperity. Solving the Trilemma and/or making it irrelevant through modelling agents using multiple monies freely would overcome a fundamental problem in macroeconomic policy and could significantly improve peoples' welfare across the world if it successfully influences the direction of macroeconomic policy. This is one of the crucial macroeconomic reforms that is needed for inclusive economic growth and robust financial stability. The way in which agents were modelled does not discriminate on any grounds – all sectors of society could benefit from this.

In terms of 'Professional Competence and Integrity', I have written for various think tanks and media outlets on economic policy and particularly for The Cobden Centre (an economics think tank based around the Austrian School of Economics) on monetary economics, monetary policy issues and monetary reform. I developed my technical knowledge and awareness of agent-based modelling software (through the MESA package) and throughout my course. I have been respectful of various viewpoints and have sought both my supervisors' honest criticisms of my work and have accepted those criticisms and worked on them.

Regarding the ‘Duty to Relevant Authority’ aspect, the University of Liverpool, its Department of Computer Science and my supervisors are probably the institutions that constitute a ‘Relevant Authority’. With respect to both my supervisors, I have never knowingly misrepresented nor withheld information regarding the performance of my ABM, its features or its shortcomings.

Regarding ‘Duty to the Profession’, I’ve collaborated with fellow Computer Scientists to aid in their professional development as well as my own. I do not have any criminal convictions that could bring the BCS and its members to disrepute.

CONCLUSIONS

Summary and main findings

Most importantly, fixed exchange rates and monetary autonomy can co-exist when there are no restrictions on capital movement (both when monetary autonomy was tested implicitly throughout the model run and when it is tested extremely through Helicopter Money). The success rate generally increased over time (as the ABM settles and/or fluctuates around a quasi-equilibrium; Graphs 1 and 2). Typically, social welfare increases at a decreasing rate (Graphs 3 and 4), the total volume of monies in circulation increases at a decreasing rate (Graphs 5 and 6) and economic growth behaves in a similar manner (Graph 7 measured in terms of aggregate purchasing power via the exchange-weighted value of all monies) – indeed, growth can be inferred to some degree because an extension of the Mundell-Fleming model was used (even though the goods and labour markets were not explicitly incorporated).

Due to the ABM's generality, economic growth could correspond to single economy or several economies; this could include urban economies, conurbations, (semi-)rural economies, regions, industries, nations, transnational communities, etc. Economic growth is likely to be higher when there is less inequality and more monies (Graph 7, Scatter plot 1) but, strangely, the number of money-demanders did not have a discernible impact (Scatter plot 4). The latter could be because the goods and labour markets were not explicitly incorporated.

Monies with fixed exchange rates also often simultaneously correspond with money-supply expansions (Scatter plot 5) – this happens both routinely and during the Helicopter Money step (Scatter plots 3 and 6). Interestingly, although Helicopter Money tends to reduce the success rate, the proportion of money-demanding agents receiving/sharing it does not seem to have a discernible impact upon its success (Scatter plot 6). Furthermore, the money of the Helicopter Money phase could also have its exchange rate remain fixed (contingent upon other agents' actions, obviously – Scatter plot 6).

Increasing the number of monies reduces the success rate (Scatter plot 3) which is consistent with the historical observation of governments' imposing a single (or few) currencies. However, there were still numerous successes. An alternative interpretation of the findings here is that the empirically observed Trilemma may not actually theoretically exist in the form we consider and this could be because we do not live in a world of free capital movement. One significant, obviously institutional capital control that comes to mind is the fact that entities can only practically pay taxes in their national money; as such, this artificially increases demand for that money and correspondingly constrains/prevents the free use of multiple monies. Interestingly, the percentage of fixed exchange rates that corresponded to money-expansions seemed to increase as the number of monies increased (Scatter Plot 5) so it is possible that limiting the number of monies in the system (like governments historically have) does not actually stand the test of reason to meet those desirable policy objectives.

Potential directions for future research

Most obviously, the model could use rational expectations to decide agents' actions in each step but this would undoubtedly be more complex (probably requiring the calculation of subgame perfect equilibria in games of potentially incomplete information, for example) and the computational time-complexity would rapidly increase.

Another, very promising avenue is to model the monetary policy preferences of money-suppliers and the monetary rule preferences of money-demanders more precisely (rather than merely representing their heterogeneous preferences by gamma distributions and skew normal distributions) to allow for some more diverse modelling and alternative results. In the current ABM, the distributions could also be proxies for other preferences but the proxies themselves could be explicitly included in future iterations (e.g. preferences for commodity-backed monies, for monies backed by particularly credible entities, for monies being used as a medium of exchange in certain areas and for certain purposes, and so on).

Regarding a spatial element to the model, we can see how the system behaves differently if different monies are used predominantly in different geographies (thereby modelling the impact that multiple monies have depending on the geographic distribution of users). We could also include demographic variables.

The credit markets could also be more explicitly modelled (the bond market was exogenously given to this model, for example) as could the goods and services markets; this is entirely possible since the model is derived from the Mundell-Fleming (1962, 1963) model.

References

1. Axtell, R. L. (2000). “Why Agents? On the Varied Motivations for Agent Computing in the Social Sciences.” *Center on Social and Economic Dynamics Working Paper No. 17.* The Brookings Institution: Washington, DC.

2. Axtell, R. L. (2007). “What economic agents do: How cognition and interaction lead to emergence and complexity.” *The Review of Austrian Economics,* 20(2-3), pp.105-122.

3. Bernanke, B. S., and Mishkin, F. S. (1997). “Inflation Targeting: A New Framework for Monetary Policy?” *NBER Working Paper No. 5893,* National Bureau of Economic Research: Cambridge, MA.

4. Biswas, S., Chakraborty, I., and Hai, R. (2017). “Income Inequality, Tax Policy, and Economic Growth.” *The Economic Journal,* 127(101), pp.688-727.

5. Blanchard, O. (2011). *Macroeconomics.* Pearson.

6. Dowd, K., and Timberlake, R. H. (eds., 1998). *Money and the Nation State: The Financial Revolution, Government and the World Monetary System.* Transaction Publishers: New Brunswick, USA and London, UK.

7. Dowd, K. (ed., 2002). *The experience of free banking.* Routledge: London and New York.

8. Farhi, E., and Werning, I. (2014). “Dilemma not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows.” *IMF Economic Review,* 62(4), pp.569-605.

9. Fleming, J. M. (1962). “Domestic Financial Policies under Fixed and under Floating Exchange Rates.” *IMF Staff Papers,* 9(3), pp.369-380. International Monetary Fund: Palgrave Macmillan.

10. Foldvary, F. E. (2011). “Free Banking Beats Central Banking.” *Foundation for Economic Education,* Available at: https://fee.org/articles/free-banking-beats-central-banking/

11. Friedman, D. D. (1982). “Gold, Paper, or… Is There a Better Money?” *Cato Institute Policy Analysis No. 17.*

12. Friedman, M. (1969). *The Optimum Quantity of Money.* Transaction Publishers: New Brunswick, USA and London, UK.

13. Friedman, M. and Schwartz, A. J. (1986). “Has government any role in money?” *Journal of Monetary Economics,* 17(1), pp.37-62.

14. Garin, J., Lester, R., and Sims, E. (2016). “On the desirability of nominal GDP targeting.” *Journal of Economic Dynamics and Control,* 69, pp.21-44.

15. Georgiadis, G. and Mehl, A. (2015). “Trilemma, Not Dilemma: Financial Globalisation and Monetary Policy Effectiveness.” *Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute,* Working Paper No. 222.

16. Gonçalves, C. E. S., and Salles, J. M. (2008). “Inflation targeting in emerging economies: What do the data say?” *Journal of Development Economics,* 85(1-2), pp.312-318.

17. Hayek, F. A. (1976). *The Denationalisation of Money.* Hobart Papers 70, Institute of Economic Affairs: London, UK.

18. Horwitz, S. (1992). *Monetary Evolution, Free Banking, and Economic Order.* Westview Press.

19. Klein, M., and Shambaugh, J. (2013). “Is there a dilemma with the Trilemma?” *Vox, CEPR’s Policy Portal,* available at: http://voxeu.org/article/dilemma-financial-trilemma.

20. Krugman, P. R. (1991). “Target Zones and Exchange Rate Dynamics.” *Quarterly Journal of Economics,* 106, pp.669-682.

21. Masad, D. and Kazil, J. (2015). “Mesa: An Agent-Based Modeling Framework.” *Proceedings of the 14th Python in Science Conference (SCIPY 2015),* pp.53-60.

22. McCallum, B. T. (2015). “Nominal GDP Targeting: Policy rule or discretionary splurge?” *Journal of Financial Stability,* 17, pp.76-80.

23. Mishkin, F. S. (2000). “Inflation Targeting in Emerging Market Countries.” *NBER Working Paper No. 7618,* National Bureau of Economic Research: Cambridge, MA.

24. Mundell, R. A. (1963). “Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates.” *The Canadian Journal of Economics and Political Science,* 29(4), pp.475-485.

25. Obstfeld, M., Shambaugh, J. C., and Taylor, A. M. (2004). “The Trilemma in History: Tradeoffs among Exchange Rates, Monetary Policies, and Capital Mobility.” *NBER Working Paper No. 10396.* National Bureau of Economic Research: Cambridge, MA.

26. Obstfeld, M., and Taylor, A. M. (1997). “The Great Depression as a Watershed: International Capital Mobility over the Long Run.” *NBER Working Paper No. 5960.* National Bureau of Economic Research: Cambridge, MA.

27. Ögren, A. (2006). “Free or central banking? Liquidity and financial deepening in Sweden, 1834-1913.” *Explorations in Economic History,* 43(1), pp.64-93.

28. Piketty, T. (2014). *Capital in the Twenty-First Century.* Cambridge, MA: Harvard University Press.

29. Ravallion, M. (2014). “Income inequality in the developing world.” *Science,* 344(6186), pp.851-855.

30. Rey, H. (2015). “Dilemma not Trilemma: The global Financial Cycle and Monetary Policy Independence.” *NBER Working Paper No. 21162.* National Bureau of Economic Research: Cambridge, MA.

31. Rothbard, M. N. (1963). *What Has Government Done to Our Money?* Ludwig von Mises Institute: USA.

32. Selgin, G. A. (1988). *The Theory of Free Banking: Money Supply under Competitive Note Issue.* Rowman & Littlefield: Lanham, MD.

33. Soros, G. (2003). *The Alchemy of Finance.* John Wiley & Sons.

34. Svensson, L. E. O. (1991). “The Term Structure of Interest Rate Differentials in a Target Zone: Theory and Swedish Data.” *Journal of Monetary Economics,* 28, pp.87-116.

35. Svensson, L. E. O. (1997). “Inflation forecast targeting: Implementing and monitoring inflation targets.” *European Economic Review,* 41(6), pp.1111-

1146.

36. White, L. H. (1992). *Competition and Currency: Essays on Free Banking and Money.* New York, NY: New York University Press.

37. Wilde, V. (2015). "A Unifying Theory: Money, Goods and Services are Instruments of Expectations-Management." *The Cobden Centre,* Available at: http://www.cobdencentre.org/2015/10/a-unifying-theory-money-goods-and-services-are-instruments-of-expectations-management/ .

38. Wilde, V. (2016). "The Monetary Policy Trilemma in Light of a Choice of Monies." *The Cobden Centre,* Available at: http://www.cobdencentre.org/2016/07/the-monetary-policy-trilemma-in-light-of-a-choice-of-monies/ .

39. Yeager, L. B. (1983). "Stable Money and Free-Market Currencies." *Cato Journal,* 3(1).

APPENDICES

Appendix 1 – Full Documentation of Original Project Specification and Proposed Design

This has been supplied as a separate PDF document within the folder with the file name “Appendix 1 Specification and Proposed Design Vishal Wilde.pdf”.

Appendix 2 – User guide to installation and usage of software

- This project used Python 3.6.1.

- This project used MESA (an agent-based modelling framework in Python) which was developed by David Masad and Jacqueline Kazil (2015) from George Mason University – its latest iteration is Mesa-0.8.1.

- This project also used pandas (an open source Python package for data analysis).

- The software was executed from the terminal.

Appendix 3 – Project Log

Abbreviations:

- RS – Rahul Savani (primary supervisor)
- JF – John Fearnley (secondary supervisor)
- ABM – Agent-based model
- MDAs – Money-demanding agents
- MSAs – Money-supplying agents
- OP – Optimisation problem

Key dates/milestones:*

- June 6th 2017: 1st meeting with RS.
- June 7th – 11th 2017: Decided MESA and pandas would be good packages for the purposes of this project.
- June 29th 2017: Finished and submitted MSc Project Specification and Design document as well as presentation slides.
- 3rd July 2017: Design presentation delivered to RS and JF.
- 6th July – 22nd July 2017: In California, visiting family.

- 2nd August 2017: Met RS to speak about a problem with formulating the extended interest-parity condition's simultaneous equations.

- 16th August 2017: The OP of the MDAs completed.

- 24th August 2017: Met RS to speak about MSAs OPs.

- 26th August 2017: The OP of the MSAs completed.

- 28th August 2017: ABM yields valid and interesting results.

- 29th August 2017: Final presentation slides and report submitted.

- 31st August 2017: Final presentation delivered to RS and JF.

*E-mail correspondence with RS was made throughout the project timeline.

Appendix 4 – Original Source Code

Included as a separate file within the submitted folder of documents and it is entitled "Appendix_4_ABMMultipleMonies_demand_driven.py". A run of this will print many of the individual variables associated with each step but also uses the 'Data Collector' class to collect the success rate at the end of each model step before charting it at the end of the model run. There are variations each time it is run since the model is essentially stochastic and the probability distributions that represent the agents' preferences over interest rates and exchange rates are generated randomly from certain ranges. Two separate runs of this original source code was used to obtain Graphs 1 and 2 (both of success rates from example runs) found within the 'Results' section of the main body.

Appendices 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and 15 – Slight Variations of Original Source Code

These appendices are essentially variations of the original source code to obtain different sets of results (e.g adding new variables, collecting different data, implementing Batch Runner or Data Collector to discern different trends etc.) whilst the core mechanics of the model remain fundamentally unchanged from Appendix 4 (the original source code). The print commands have changed from time to time the commenting will also be somewhat sloppier but, again, these files are just included for the sake of completeness so that the exact code that was executed to get every figure included in this dissertation is available to the reader – of course, the problem is that the exact same results will not occur as they did for the particular run I conduct but, in principle, similar sorts of results should emerge much of the time (with some variation due to the random variations between models).

Appendix 5 – Code used to produce Graph 3

Code included within the submitted folder within file entitled "Appendix_5_ABMMultipleMonies_demand_driven_total_social_welfare_plot.py"

Appendix 6 – Code used to produce Graph 4

Code included within the submitted folder within file entitled "Appendix_6_ABMMultipleMonies_demand_driven_social_welfare_plot.py"

Appendix 7 – Code used to produce Graph 5

Code included within the submitted folder within file entitled “Appendix_7_ABMMultipleMonies_demand_driven_total_money_volume_plot.py”

Appendix 8 – Code used to produce Graph 6

Code included within the submitted folder within file entitled “Appendix_8_ABMMultipleMonies_demand_driven_total_money_volume_percentage_change_plot.py”

Appendix 9 – Code used to produce Graph 7

Code included within the submitted folder within file entitled “Appendix_9_ABMMultipleMonies_demand_driven_money_value_growth_plot.py”

Appendix 10 – Code used to produce Scatter Plot 1

Code included within the submitted folder within file entitled “Appendix_10_ABMMultipleMonies_demand_driven_inequality_final_growth_percentage_plot.py”

Appendix 11 – Code used to produce Scatter Plot 2

Code included within the submitted folder within file entitled “Appendix_11_ABMMultipleMonies_demand_driven_final_growth_percentage_plot.py”

Appendix 12 – Code used to produce Scatter Plot 3

Code included within the submitted folder within file entitled “Appendix_12_ABMMultipleMonies_demand_driven_average_success_plot.py”

Appendix 13 – Code used to produce Scatter Plot 4

Code included within the submitted folder within file entitled “Appendix_13_ABMMultipleMonies_demand_driven_money_demander_growth_plot.py”

Appendix 14 – Code used to produce Scatter Plot 5

Code included within the submitted folder within file entitled “Appendix_14_ABMMultipleMonies_demand_driven_success_rate_money_expander_plot.py”

Appendix 15 – Code used to produce Scatter Plot 6

Code included within the submitted folder within file entitled "Appendix_15_ABMMultipleMonies_demand_driven_helicopter_success_receiver_plot.py"