

Image Filtering

Dr. Tushar Sandhan

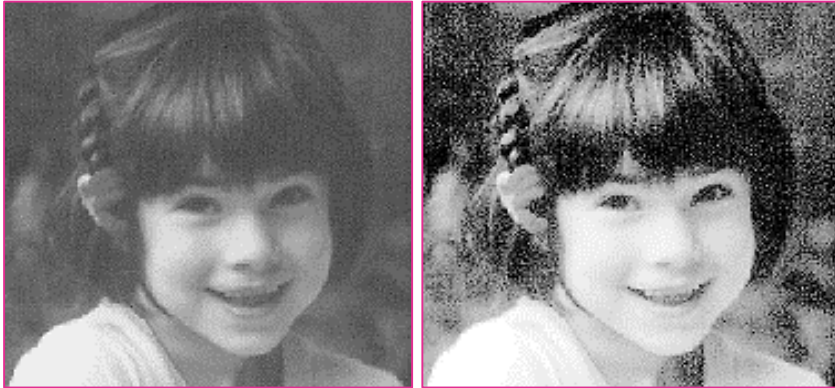
Introduction

Input



Introduction

Input



Introduction

Input



Histeq



Introduction

Input



Histeq



Introduction

Input



Histeq



Noise



Introduction

Input



Histeq



Noise



Introduction

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Histeq



Noise



Filter-1



Introduction

Input



Histeq



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Histeq



Noise



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Filter-2



Introduction

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Filter-2



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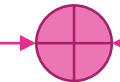
Filter-1



Filter-2



Filter-3



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Noise



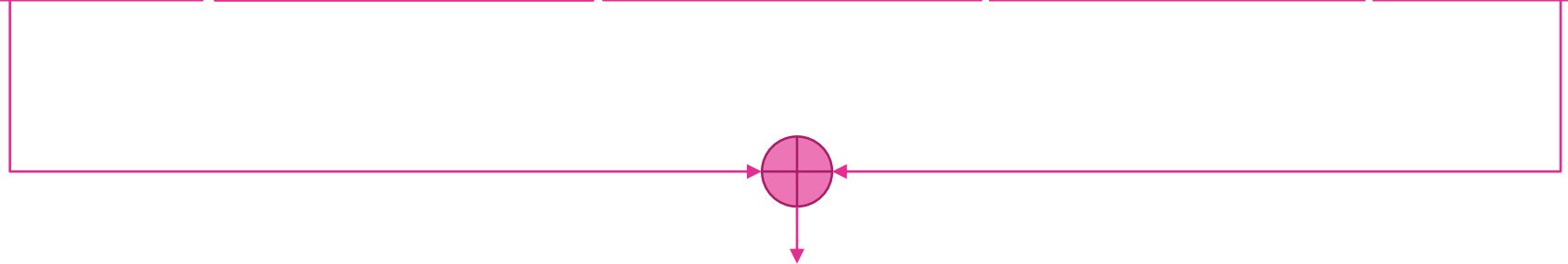
Filter-1



Filter-2



Filter-3



New filter

Introduction

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Histeq



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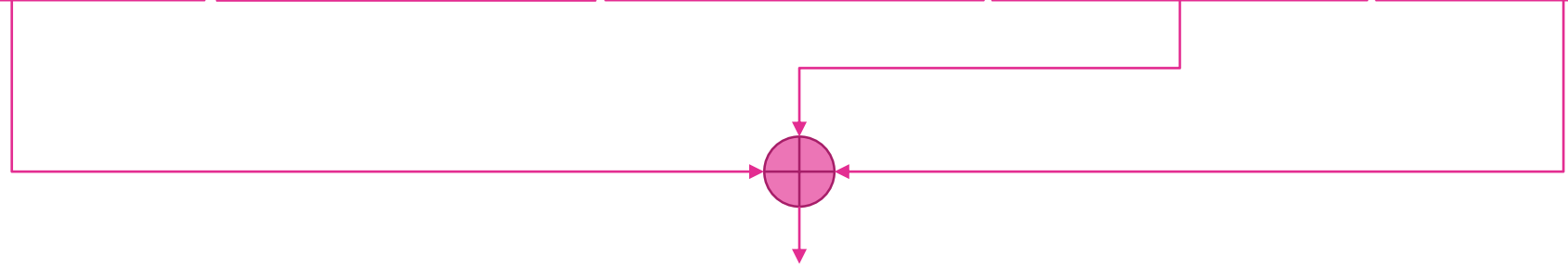
Filter-1



Filter-2



Filter-3



New filter

Linearity

■ Operations

○ linear

- additivity

$$\mathcal{T}[a \cdot f_1(x, y) + b \cdot f_2(x, y)] = \mathcal{T}[a \cdot f_1(x, y)] + \mathcal{T}[b \cdot f_2(x, y)]$$

- homogeneity

$$\mathcal{T}[a \cdot f_1(x, y)] = a \cdot \mathcal{T}[f_1(x, y)]$$

○ non-linear

- not satisfying above

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■ Examples

○ linear

- negatives

○ non-linear

- gammas

Correlation & Convolution

- Correlation
 - measures similarity between the two signals
 - windowed signal (kernel) is not reversed
 - sliding vectors dot product
 - orthogonal signals are uncorrelated

Correlation & Convolution

- Correlation

- measures similarity between the two signals
- windowed signal (kernel) is not reversed
- sliding vectors dot product
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- Convolution

- measure the effect of one signal on the another
- windowed signal (kernel) is reversed
 - for symmetric kernels convolution = correlation

Diagram illustrating the convolution operation $f \star g$. The functions f (blue step function) and g (red ramp function) are shown on the left. The resulting function $f \star g$ (black curve) is shown on the right, with its value at a point x determined by the integral of the product of f and a shifted g over the interval $[x, x+1]$. The diagram shows the shifting of g relative to f and the resulting area under the product curve.

$$R(x) = \int_{-\infty}^{\infty} f(z)g(x+z)dz$$

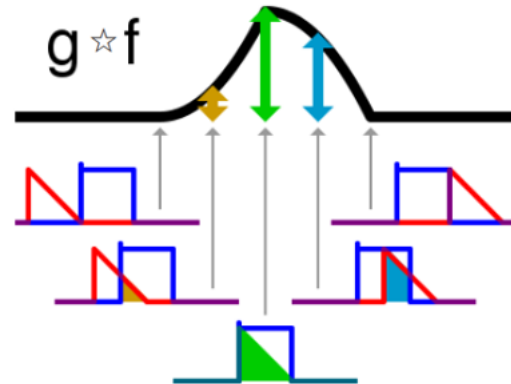
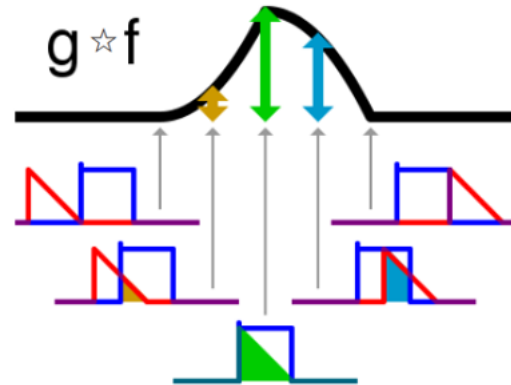


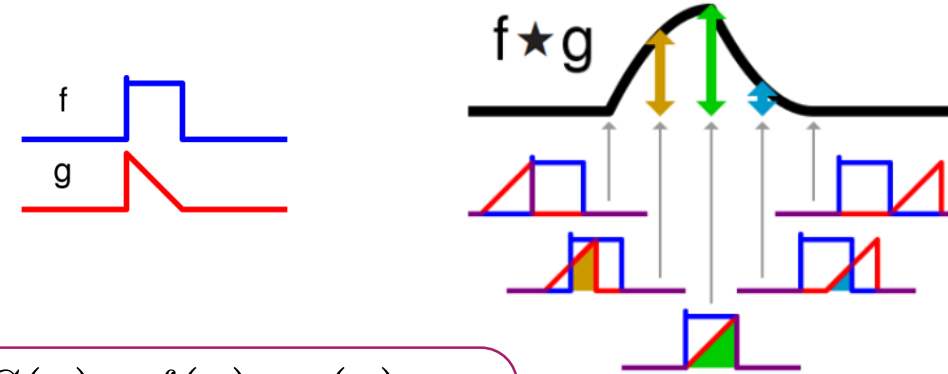
Diagram illustrating the convolution operation $f \star g$. The functions f (blue step function) and g (red trapezoidal function) are shown on the left. The resulting function $f \star g$ (black curve) is shown on the right, with its value at a point x determined by the integral of the product of f and a shifted g over the interval $[x, x+1]$. The diagram shows the shifting of g relative to f and the resulting area under the product curve.

$$R(x) = \int_{-\infty}^{\infty} f(z)g(x+z)dz$$

$$G(x) = \int_{-\infty}^{\infty} f(z)g(x-z)dz$$

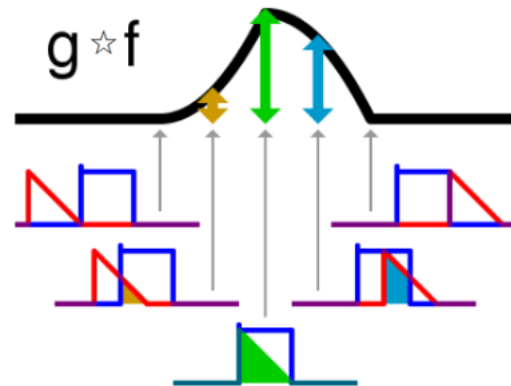


- Convolution

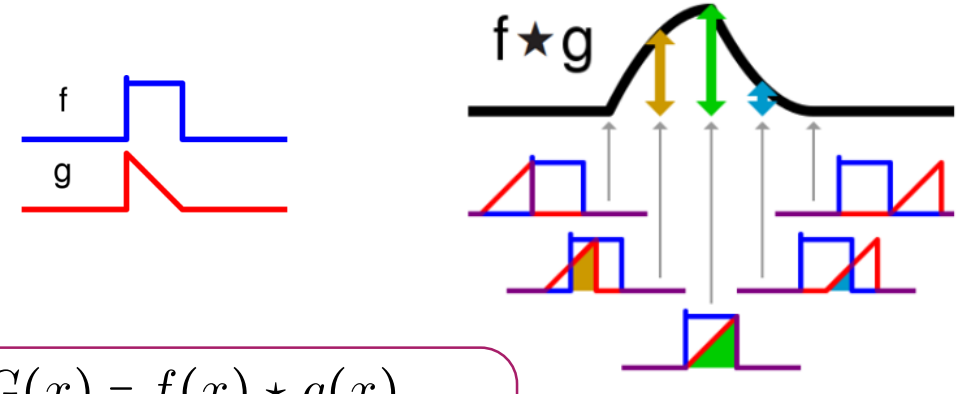


$$G(x) = f(x) \star g(x)$$

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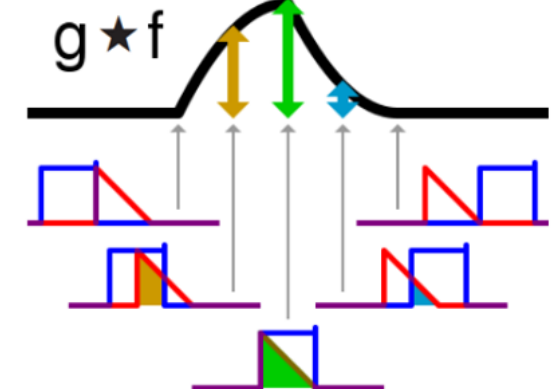


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Correlation & Convolution

- 2D correlation
 - cross-correlation
 - filtering algos internally use it
 - w need to be appropriately reflected before filtering

$$(w \star f)(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

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■ 2D convolution

- $w \rightarrow m \times n$
- $a = \frac{m-1}{2}, b = \frac{n-1}{2}$
 - a, b are assumed to be odd integers
 - note the kernels do not depend on (x, y)

$$(w \star f)(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x - s, y - t)$$

Correlation & Convolution

$$(w \star f)(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

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Property	Correlation	Convolution
Commutative	—	$f \star g = g \star f$
Associative	—	$f \star (g \star h) = (f \star g) \star h$
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$

Filtering

- Image filtering
 - spatial filtering
 - convolving a kernel with an image
 - filtering: $g(x, y) = (w \star f)(x, y)$

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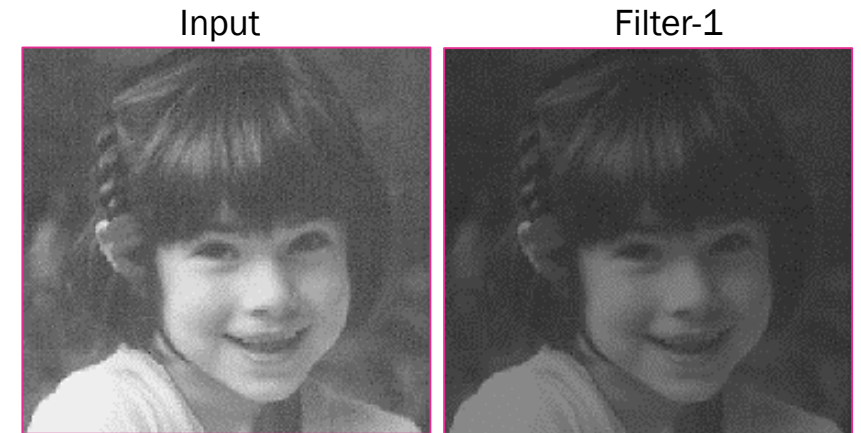
Input



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 - filtering the filtered
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Filter-1

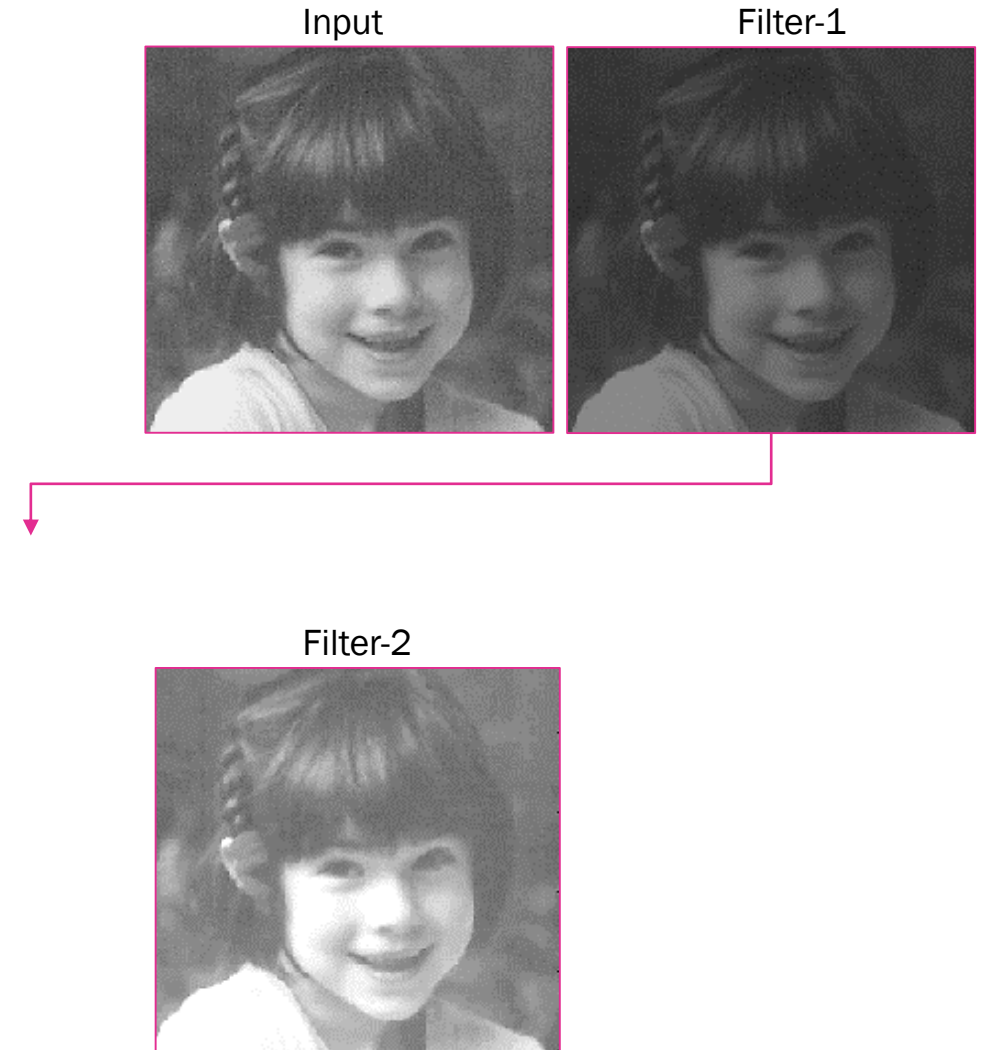


Filter-2



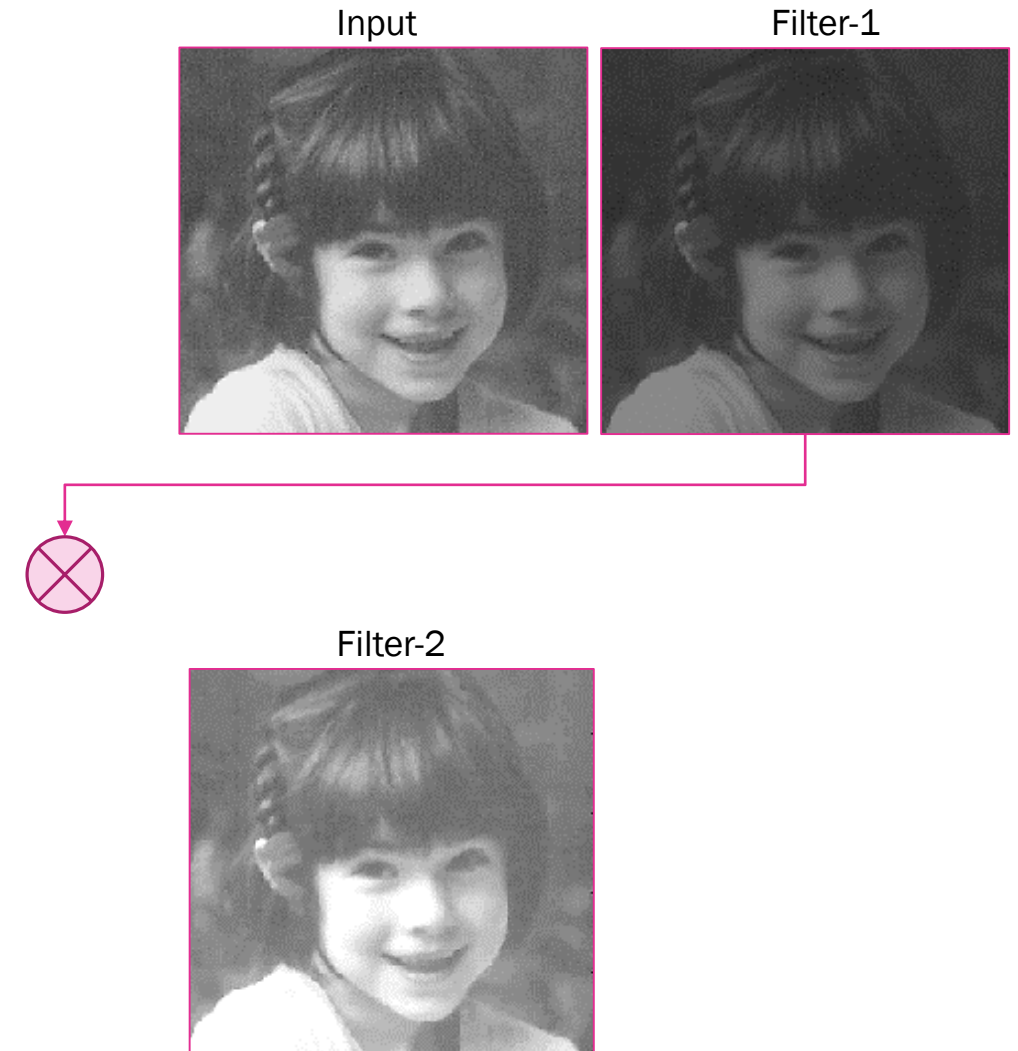
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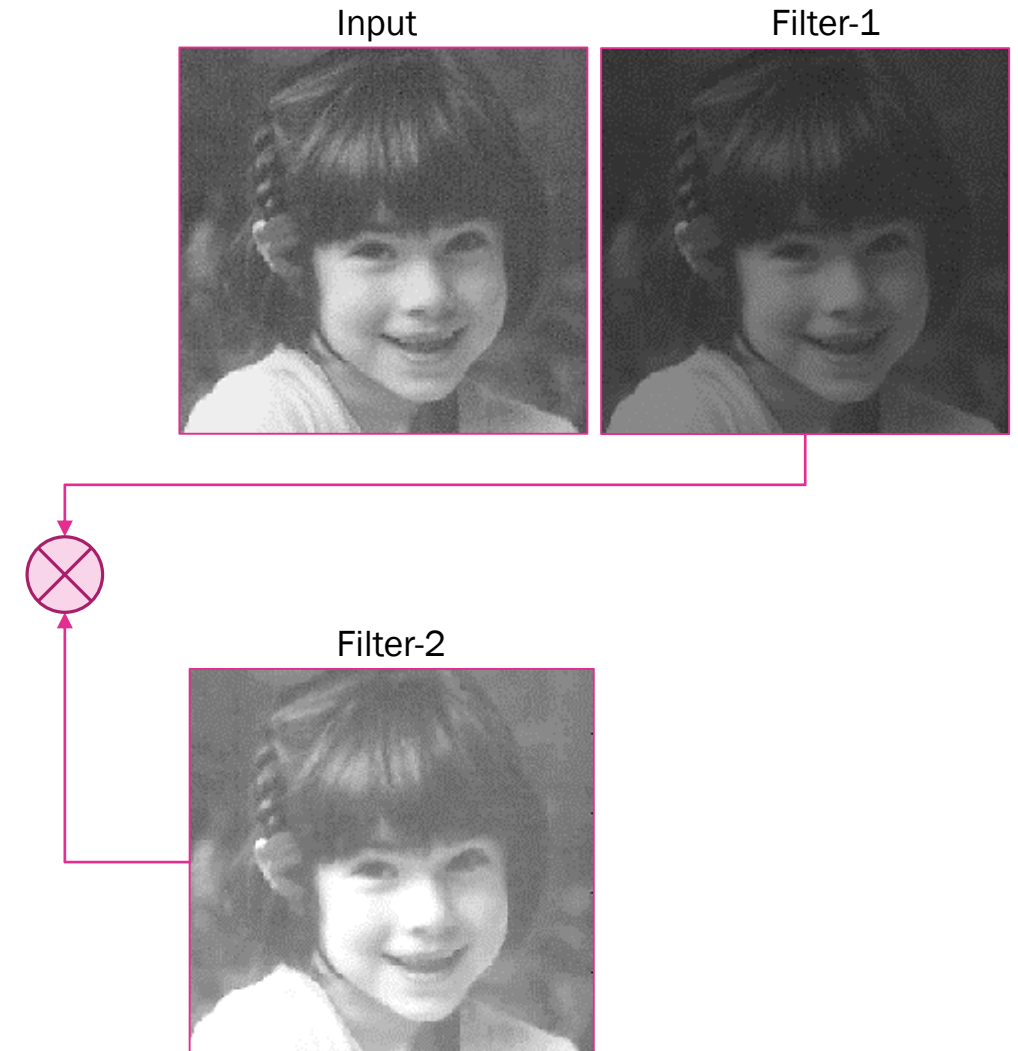
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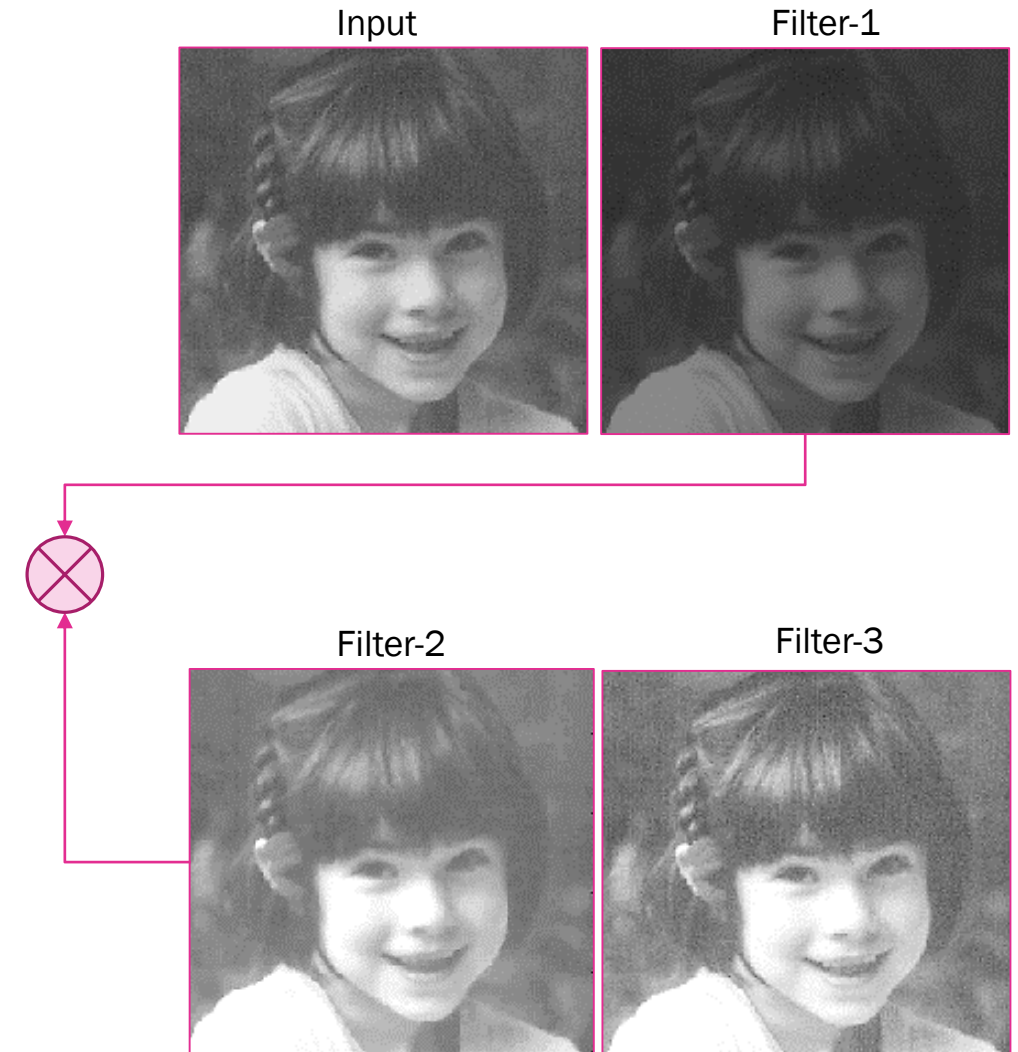
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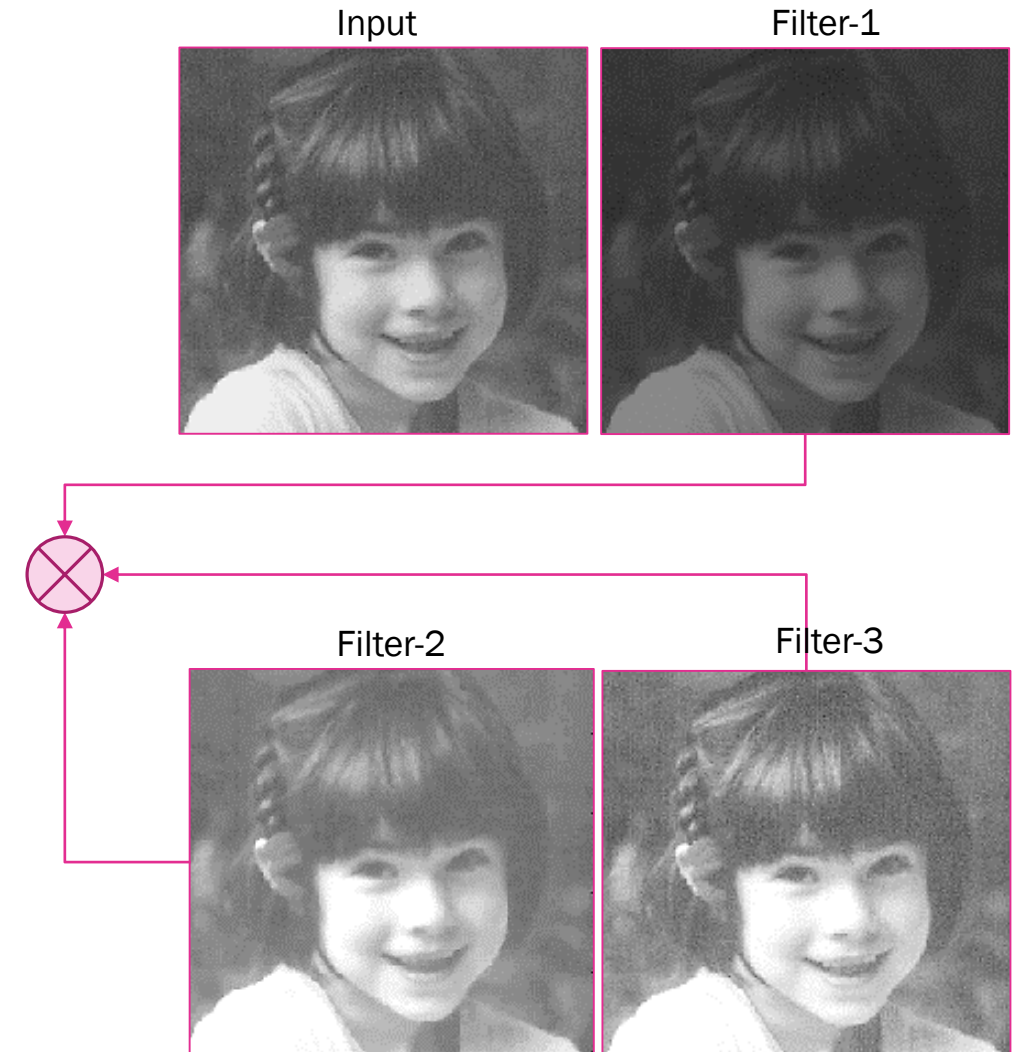
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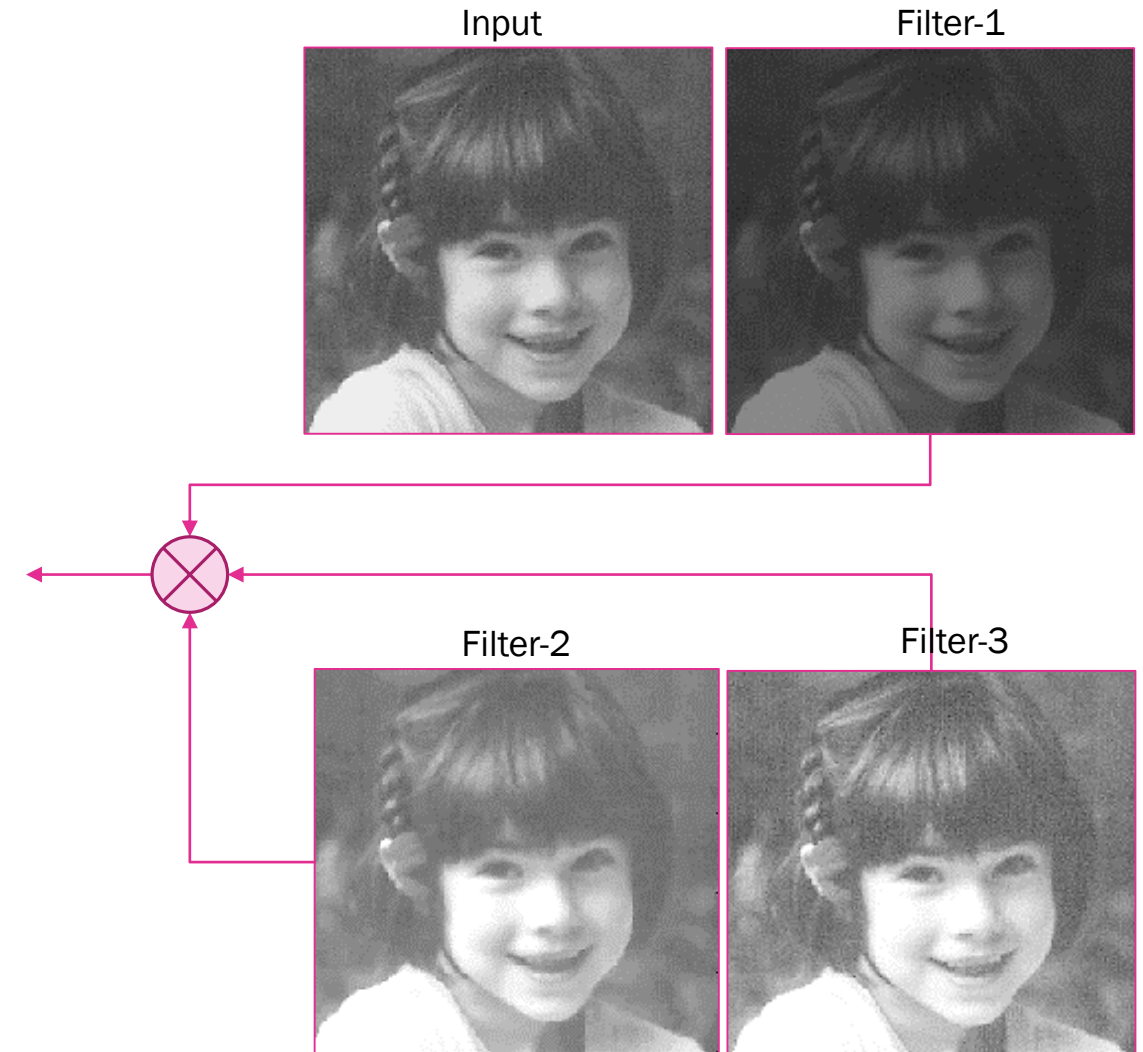
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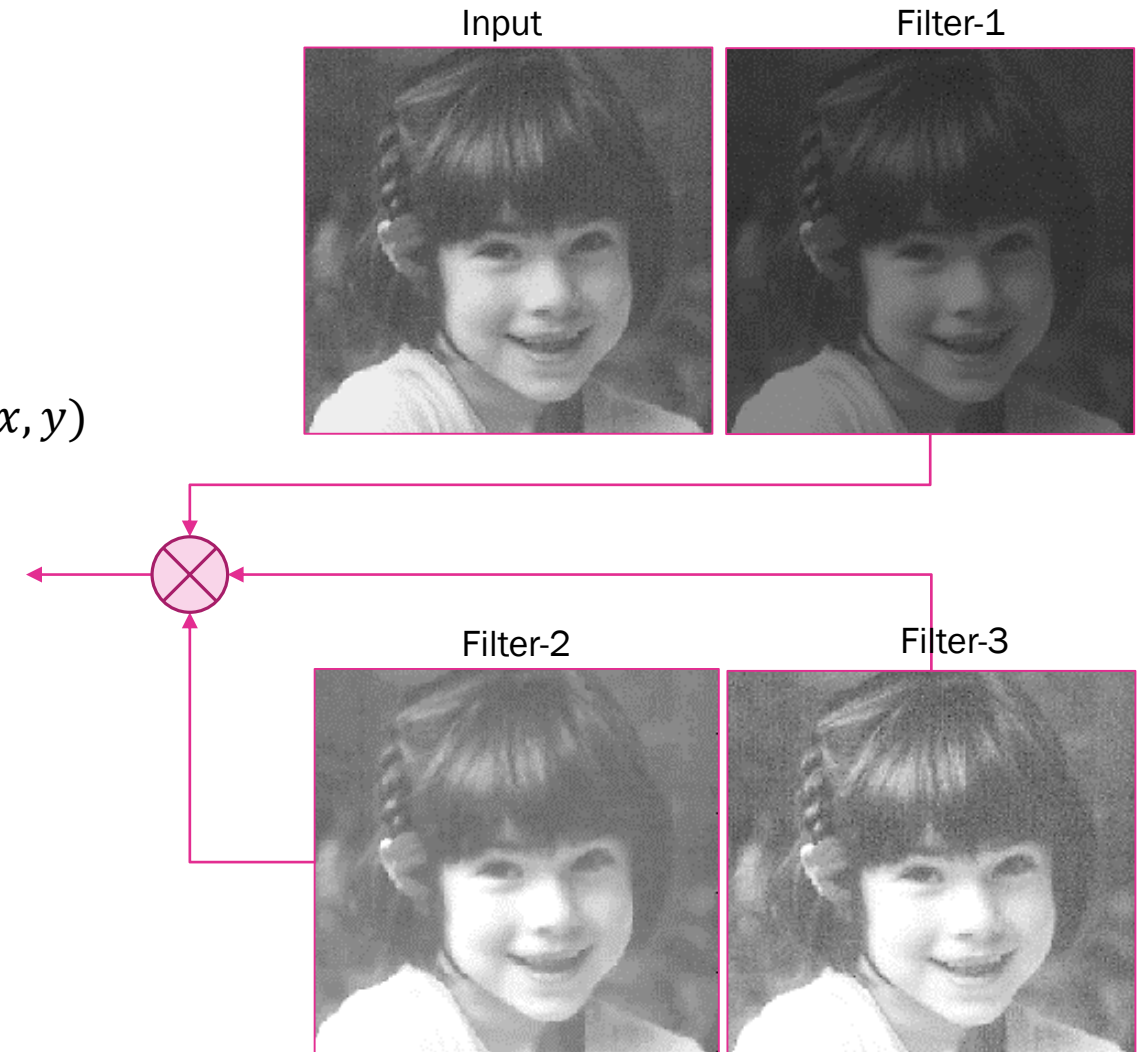


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 - filtering: $g(x, y) = (w \star f)(x, y)$

$$g(x, y) = w_3 \star w_2 \star w_1 \star f(x, y)$$

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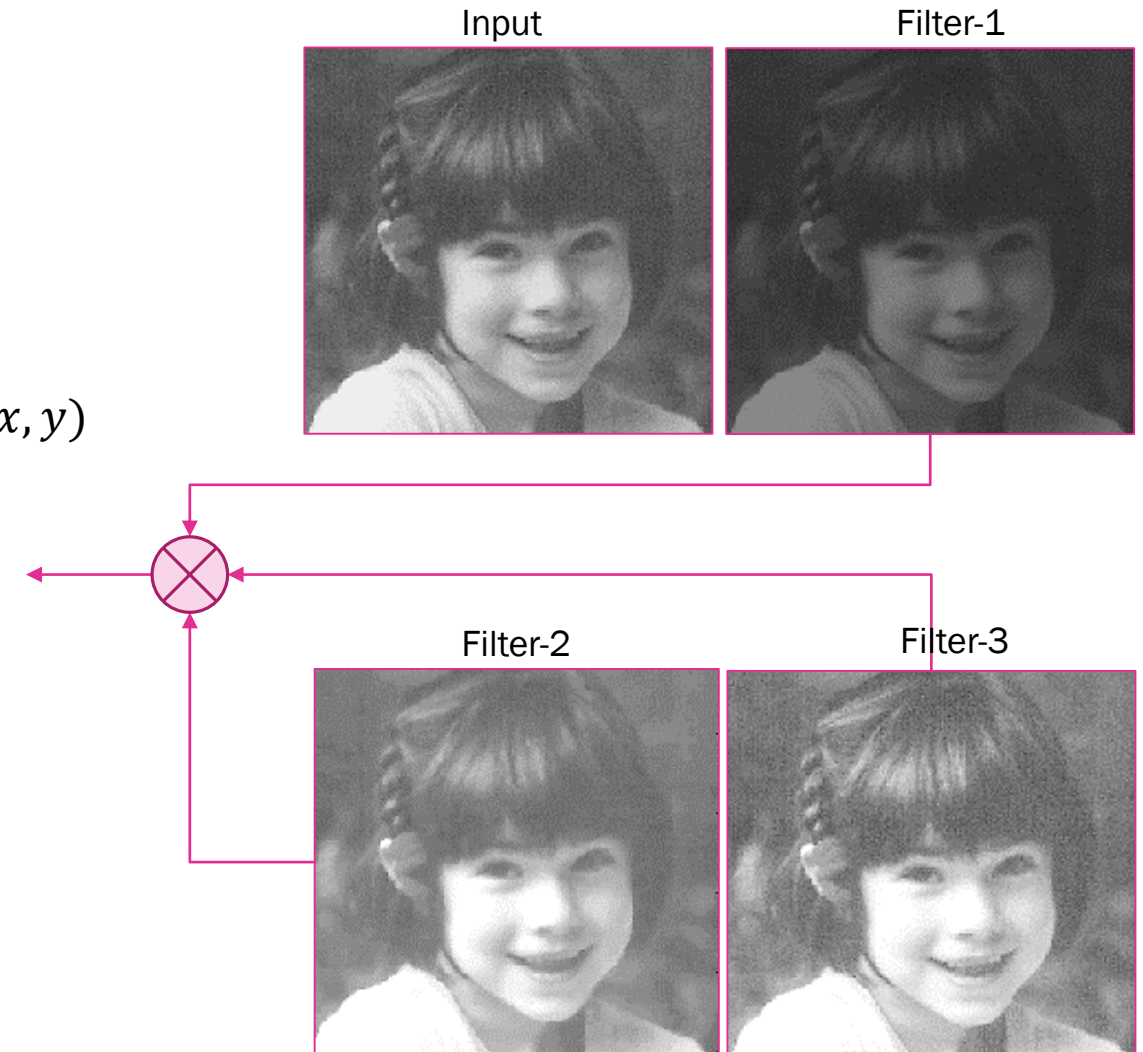
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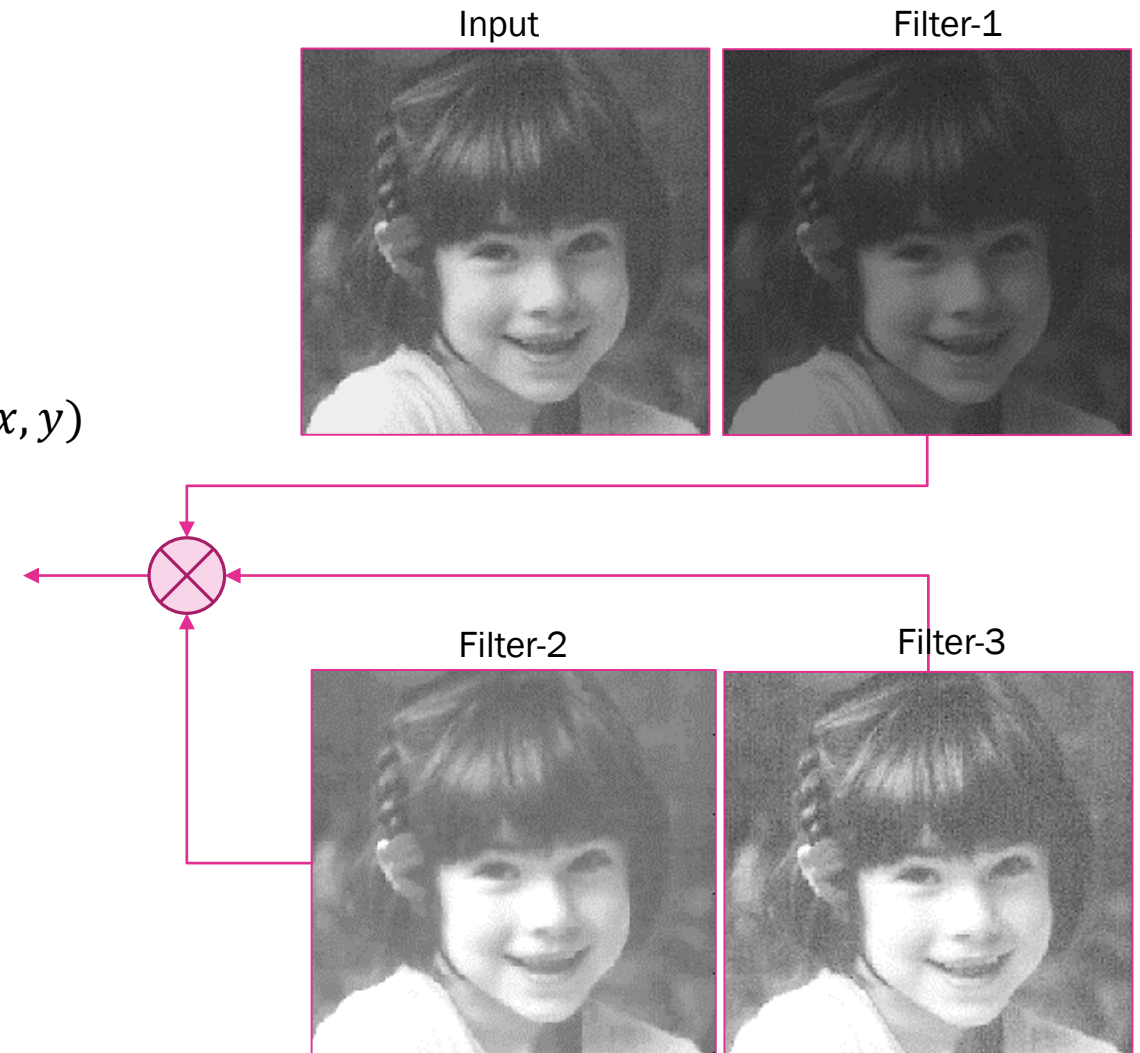
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$$(w_1 \star w_2 \star w_3) \star f(x, y)$$



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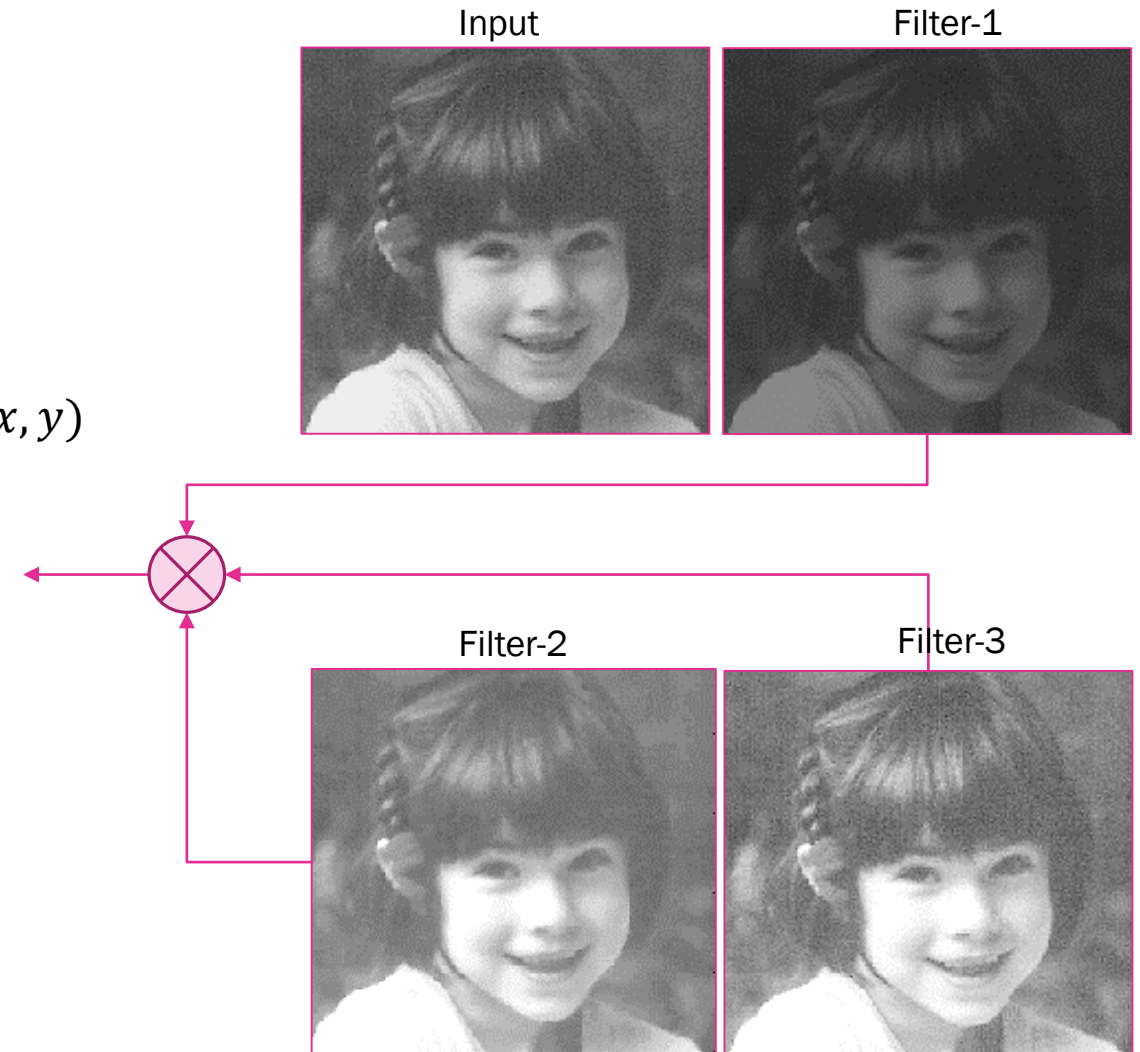
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$$(w_3 \star w_2 \star w_1) \star f(x, y)$$

$$(w_1 \star w_2 \star w_3) \star f(x, y)$$

$$g(x, y) = w \star f(x, y)$$

$$w = w_1 \star w_2 \star w_3$$



Filtering

- Filter

- kernel, mask, window, template
- $w(i,j)$ or $k(i,j) \quad \forall i,j \in N_K$, K - kernel size
 - K : determine neighbourhood of operation
 - $w(i,j)$: filter coefficients – determine nature of the filter

Filtering

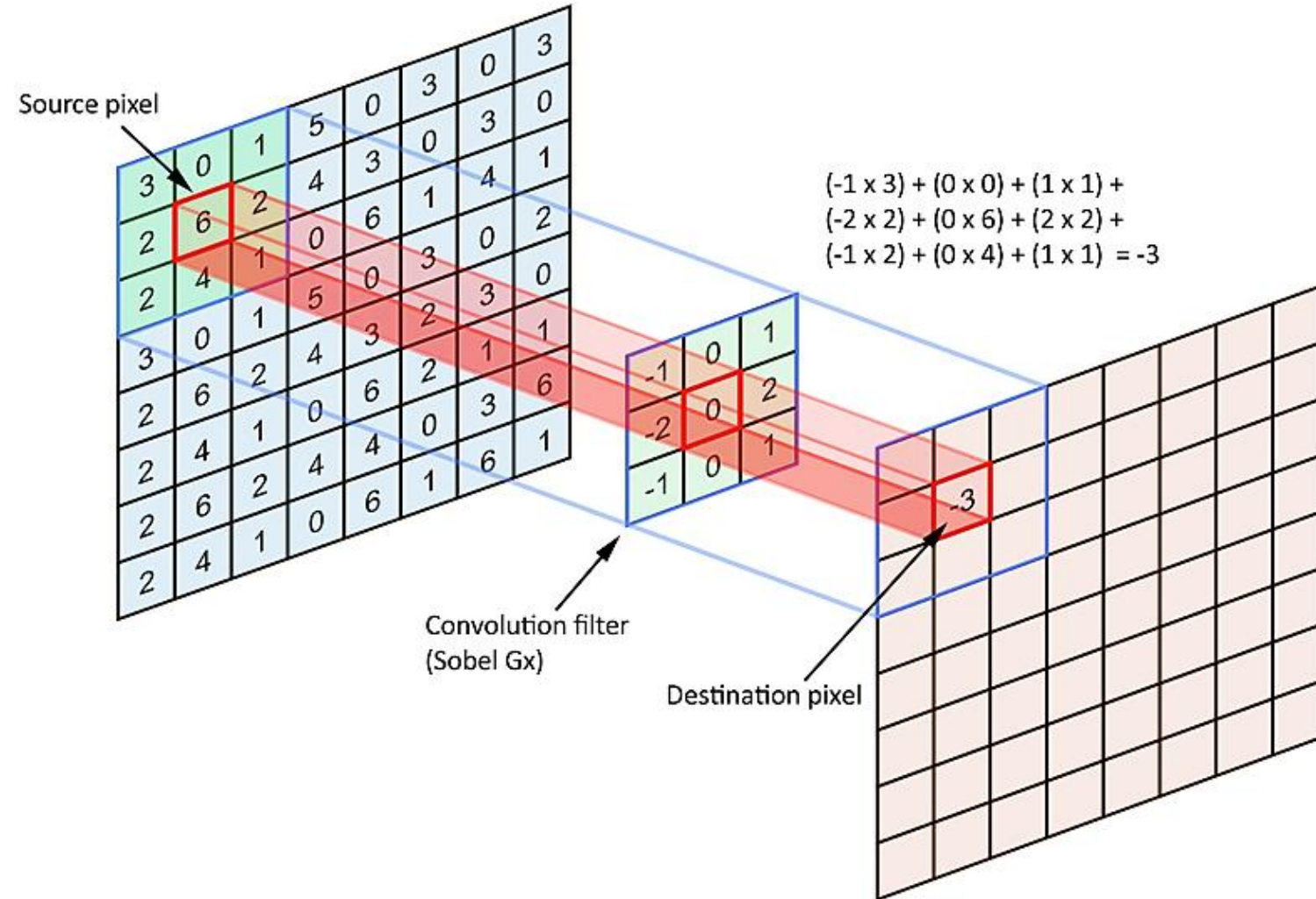
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■ Nature of a filter

- neighbour interactions
 - filter coefficients define severity of interaction
- smoothing
- sharpening
- noise handling capacity

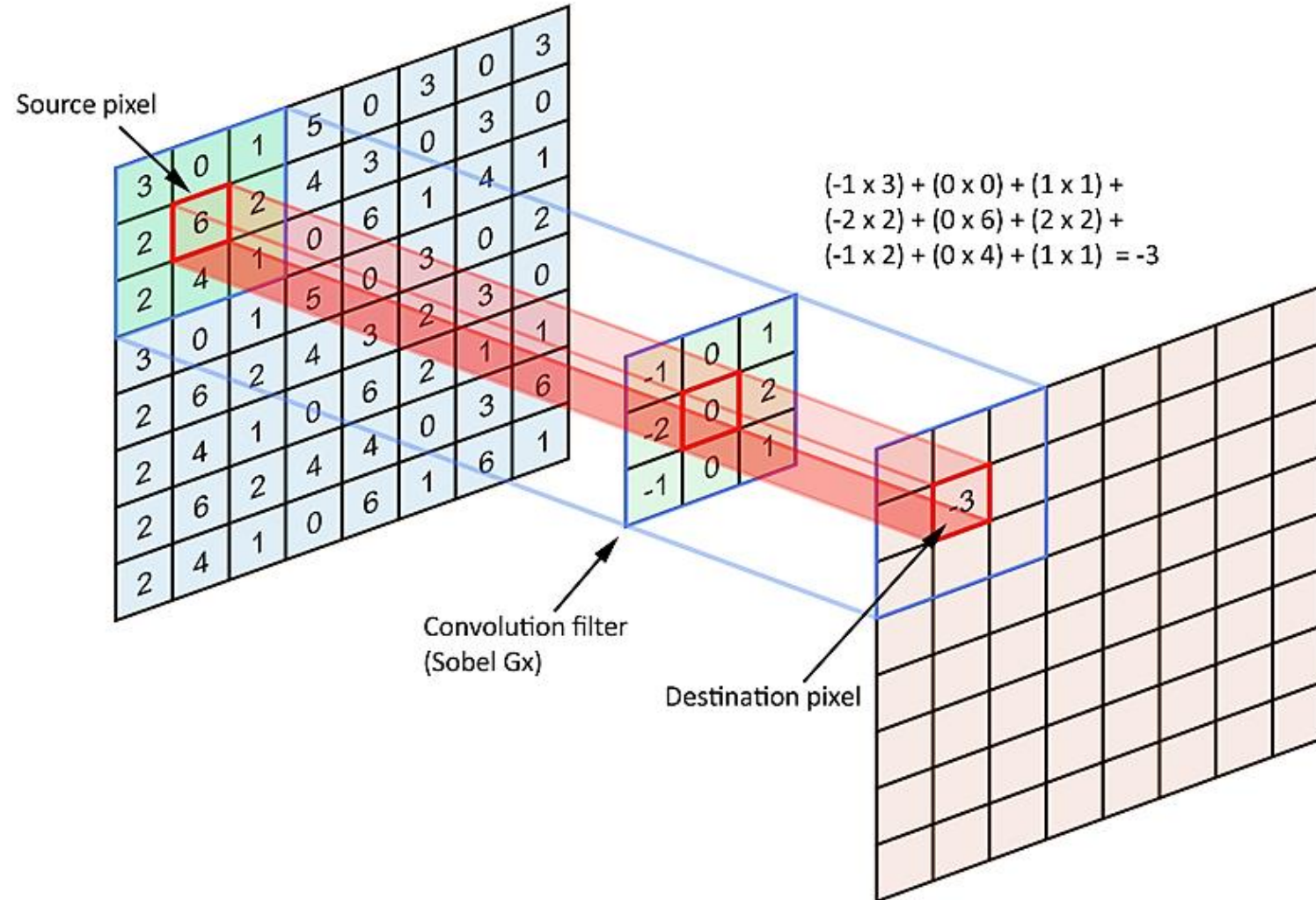
Filtering



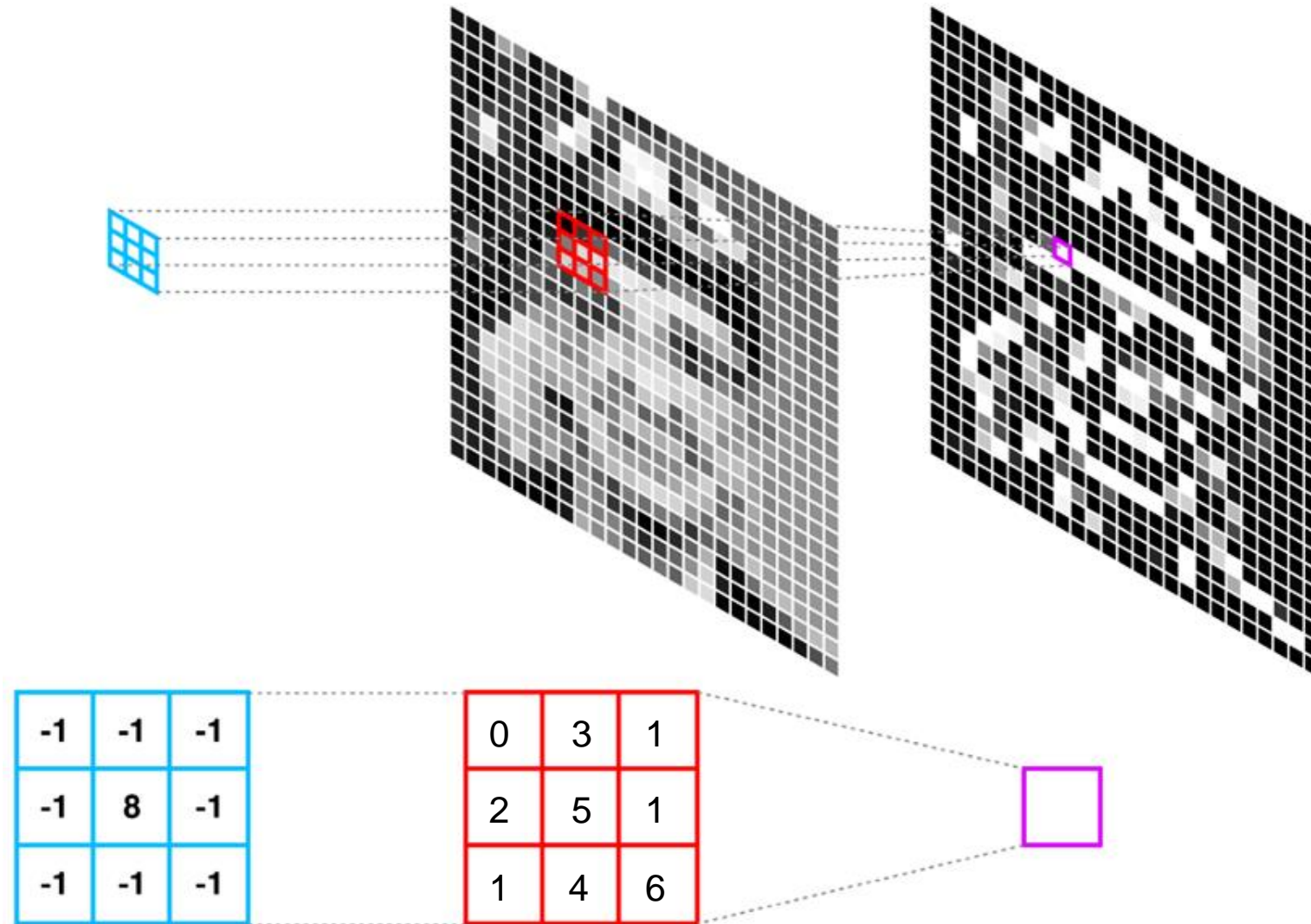
Filtering

■ Paddings

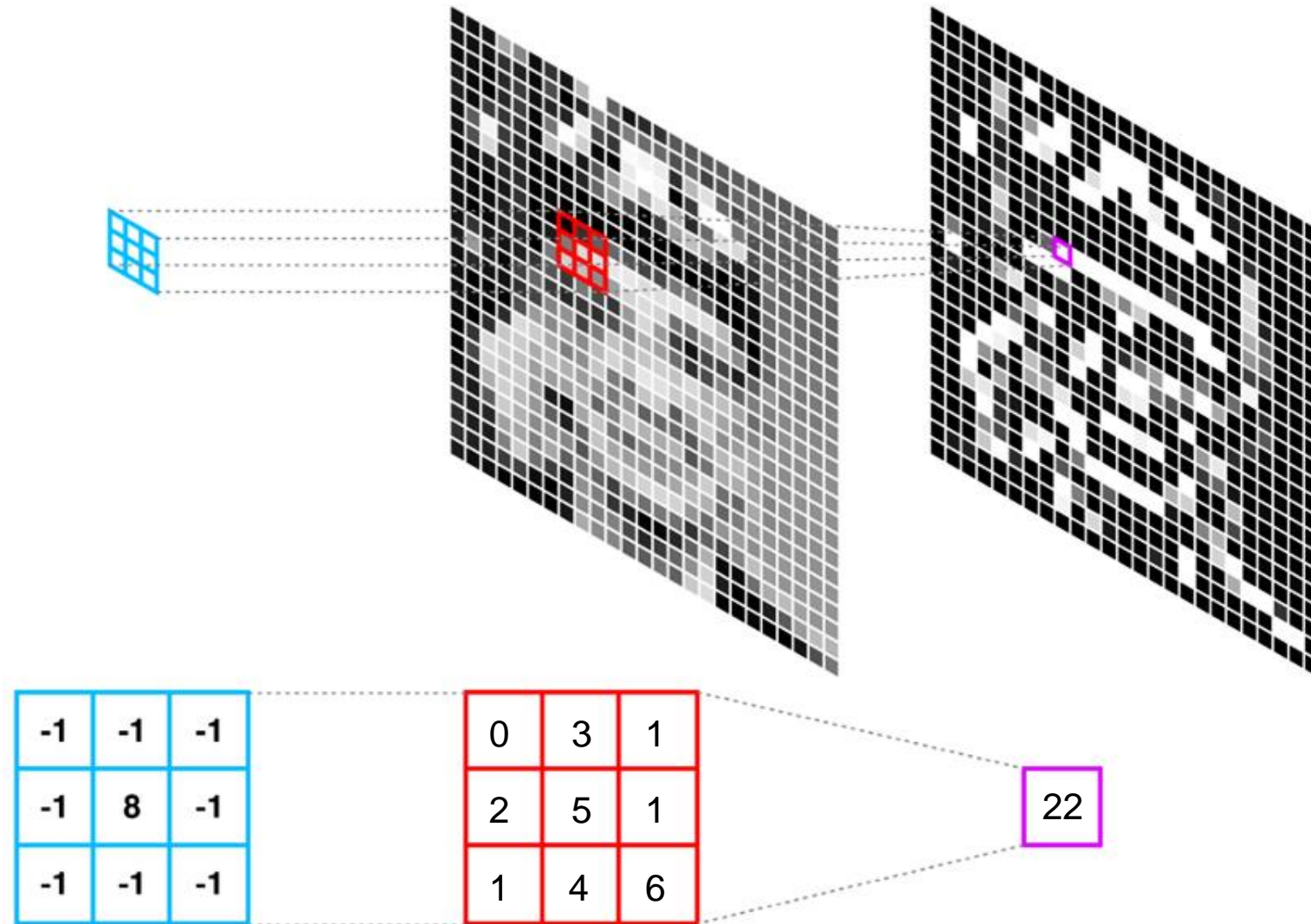
- zero
- mirror
- replicate



Filtering



Filtering



Filtering

- Separable kernels

- a kernel in a matrix form can be represented as outer product of two vectors
- $w = uv^T$
 - $u \in m \times 1$
 - $v \in n \times 1$
 - sq. kernels $w = uu^T$, $w \in m \times m$

$$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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- Advantage: separable kernels

- computationally fast
 - outer product of vectors is same as their 2D conv

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$$\mathbf{c} \mathbf{r}^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = w$$

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■ Advantage: separable kernels

- computationally fast
 - outer product of vectors is same as their 2D conv
 - image: $M \times N$
 - advantage factor = $\frac{mn}{m+n}$

Filtering

- Box filter
 - smoothing filter
 - lowpass filter
 - averaging filter

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

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- Use cases

- random noise reduction
 - reducing sharp transitions in intensity
 - favours blurring along perpendicular directions
- reduce aliasing
 - smoothing prior to resampling
- reduce quantization noise
 - reduce false contours of intensities
- essential in composite filtering
 - multistage filters

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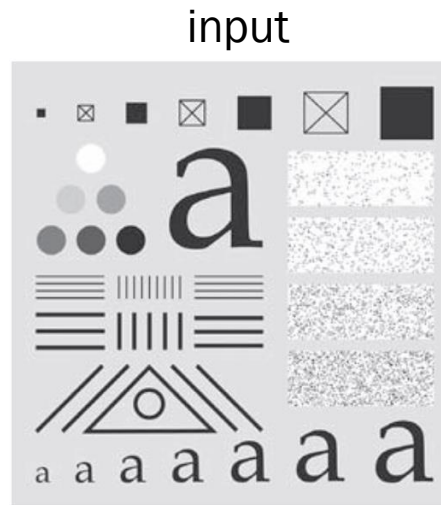
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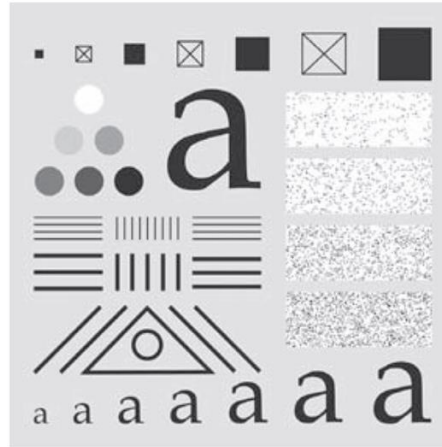
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$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

m=3

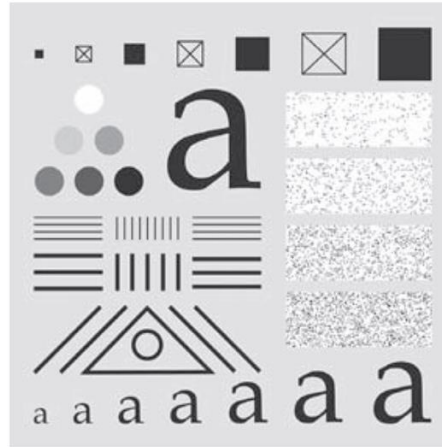


Filtering

- Box filter

- smoothing filter
- lowpass filter
- averaging filter

input

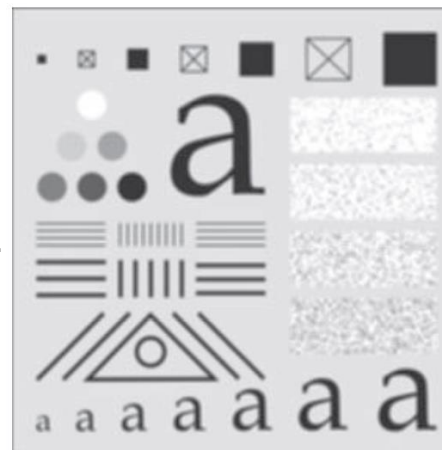


$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

m=3



m=11

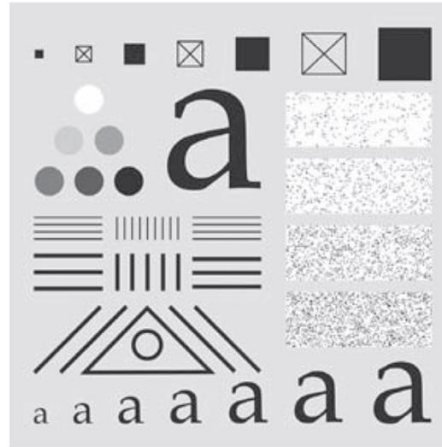


Filtering

- Box filter

- smoothing filter
- lowpass filter
- averaging filter

input



$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

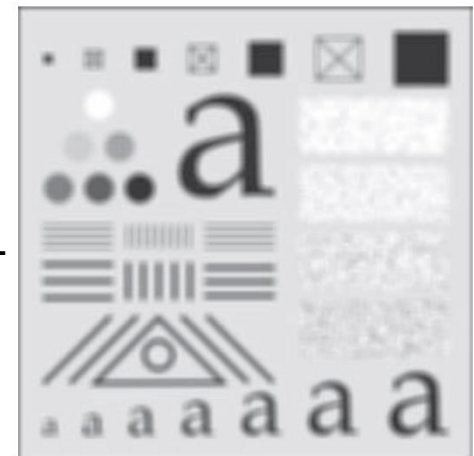
m=3



m=11



m=21



Filtering

- Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

$$w(s,t) = G(s,t) = Ke^{-\frac{s^2 + t^2}{2\sigma^2}}$$

$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|} \hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

Filtering

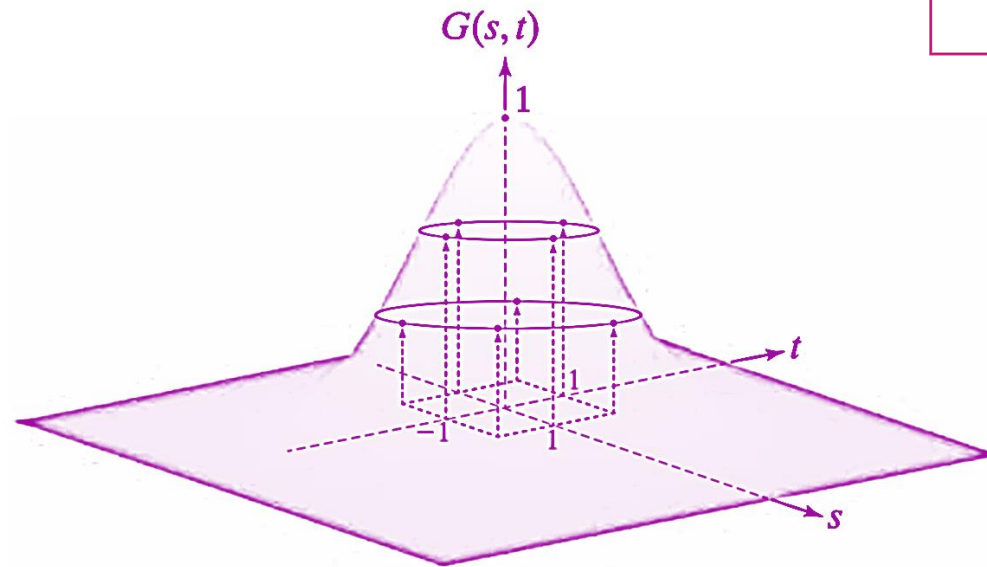
■ Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

$$w(s, t) = G(s, t) = Ke^{-\frac{s^2 + t^2}{2\sigma^2}}$$

$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679



Filtering

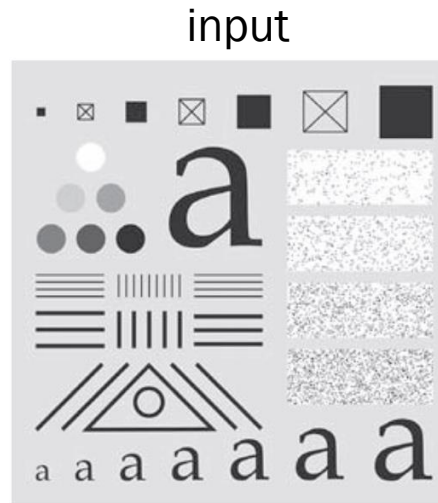
- Gaussian filter
 - smoothing filter
 - defocused lens approximators
 - isotropic
 - response is independent of orientation
 - circularly symmetric

$$\frac{1}{4.8976} \times \begin{array}{|c|c|c|} \hline 0.3679 & 0.6065 & 0.3679 \\ \hline 0.6065 & 1.0000 & 0.6065 \\ \hline 0.3679 & 0.6065 & 0.3679 \\ \hline \end{array}$$

Filtering

- Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric


$$\frac{1}{4.8976} \times$$

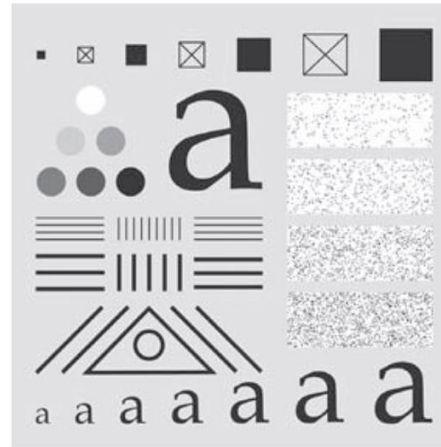
0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

Filtering

- Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

input



$$\frac{1}{4.8976} \times \begin{bmatrix} 0.3679 & 0.6065 & 0.3679 \\ 0.6065 & 1.0000 & 0.6065 \\ 0.3679 & 0.6065 & 0.3679 \end{bmatrix}$$

m=21 σ =3.5 Gauss

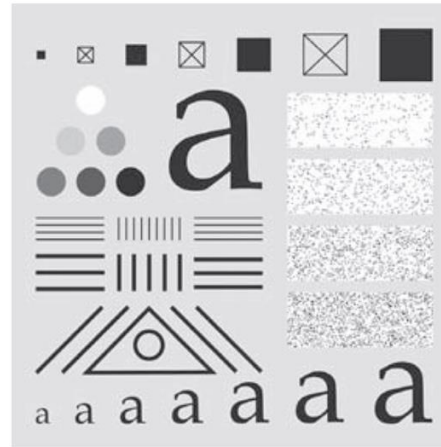


Filtering

■ Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

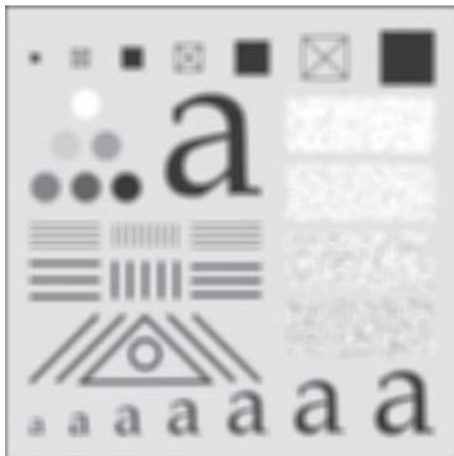
input



$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

m=21 box



m=21 $\sigma=3.5$ Gauss

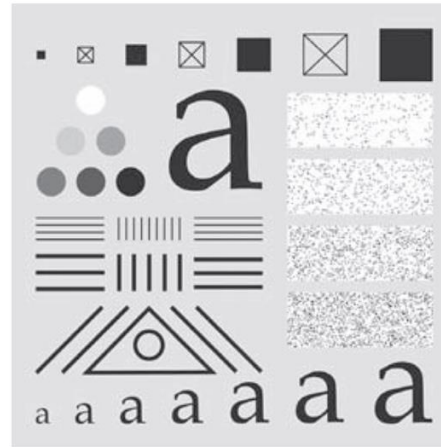


Filtering

■ Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

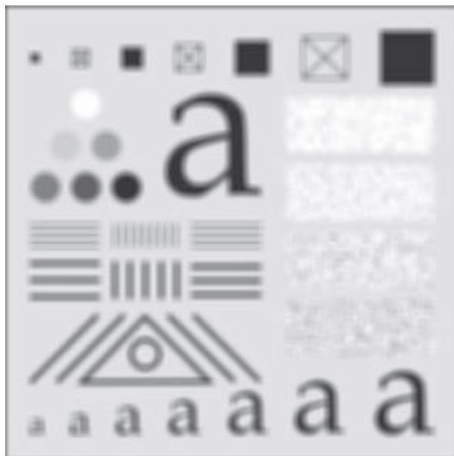
input



$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

m=21 box



m=21 $\sigma=3.5$ Gauss



m=43 $\sigma=7$ Gauss

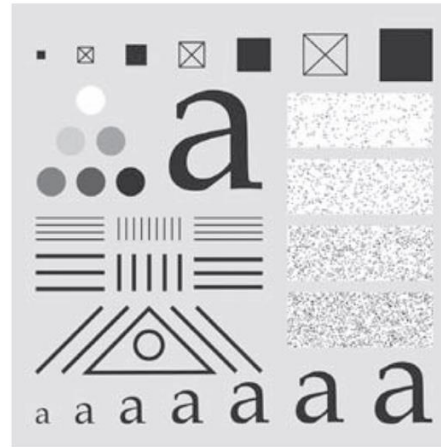


Filtering

■ Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

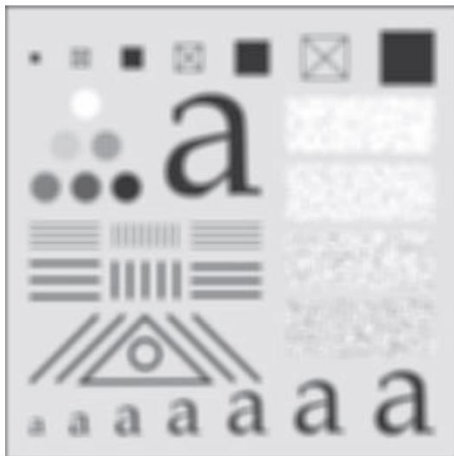
input



$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

m=21 box



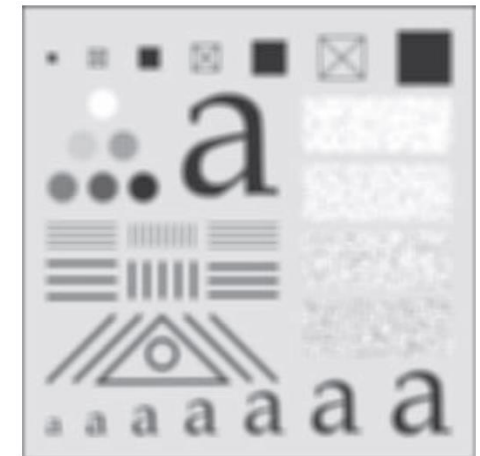
m=21 $\sigma=3.5$ Gauss



m=43 $\sigma=7$ Gauss



m=21 box



Filtering

- Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

input



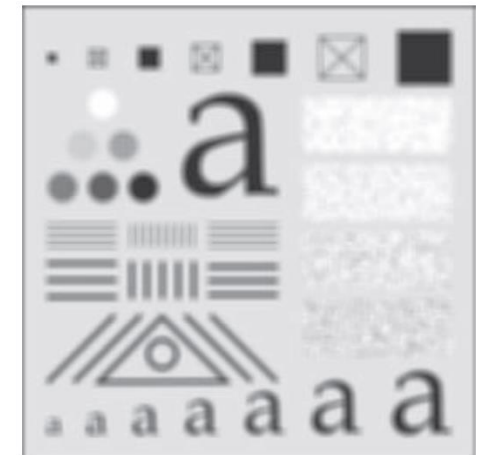
$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

m=43 $\sigma=7$ Gauss



m=21 box

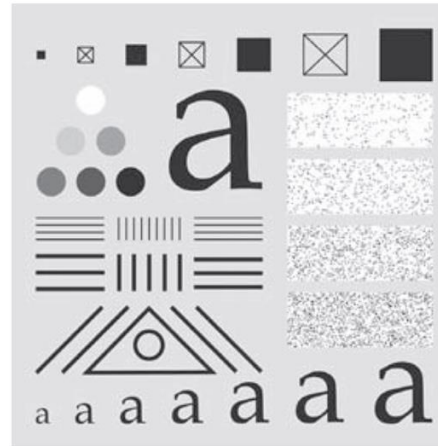


Filtering

■ Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

input



$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

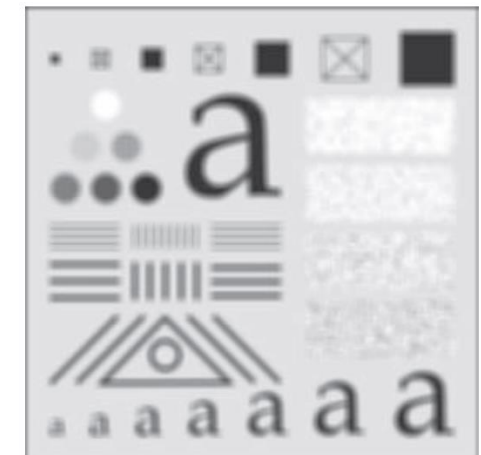
m=85 $\sigma=7$ Gauss



m=43 $\sigma=7$ Gauss



m=21 box

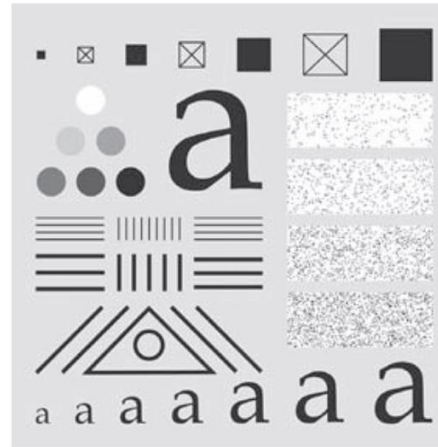


Filtering

■ Gaussian filter

- smoothing filter
- defocused lens approximators
- isotropic
 - response is independent of orientation
 - circularly symmetric

input



$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

difference $m_{85} - m_{43}$



$m=85 \sigma=7$ Gauss



$m=43 \sigma=7$ Gauss

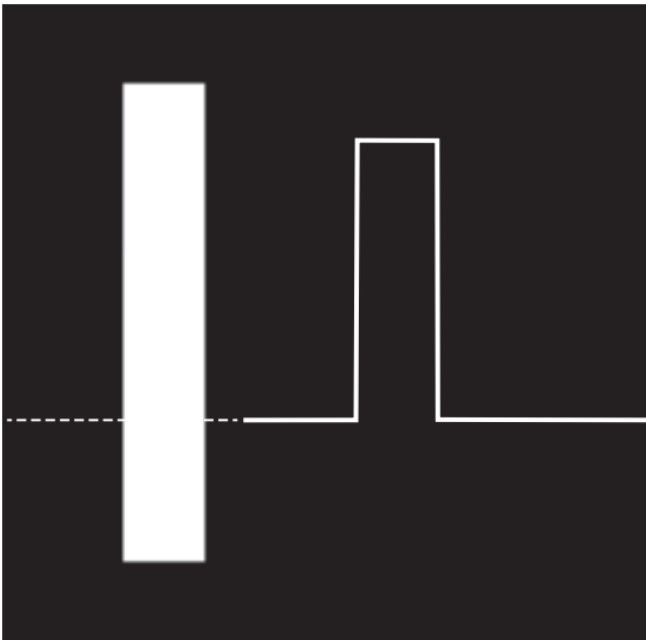


$m=21$ box



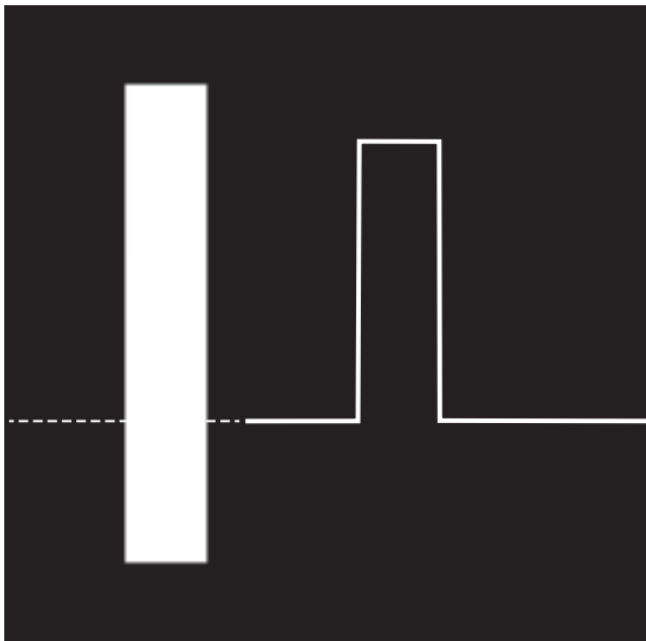
Filtering

- Box vs Gaussian
 - blur profile
 - blurred rects having same shape

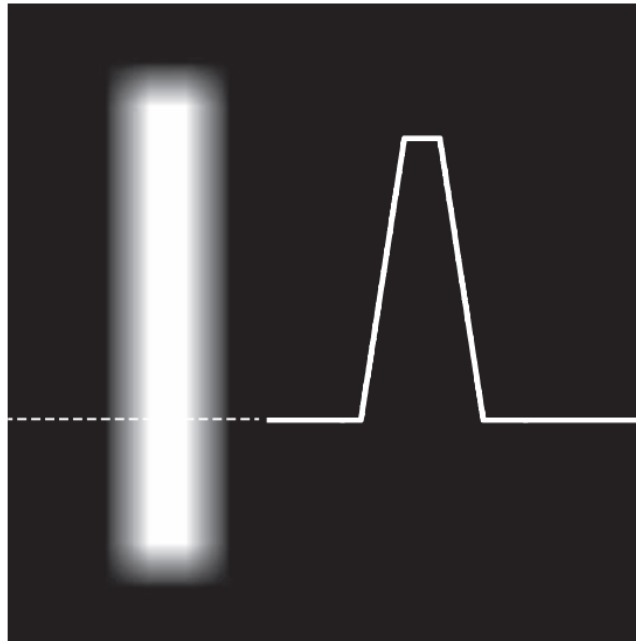


Filtering

- Box vs Gaussian
 - blur profile
 - blurred rects having same shape

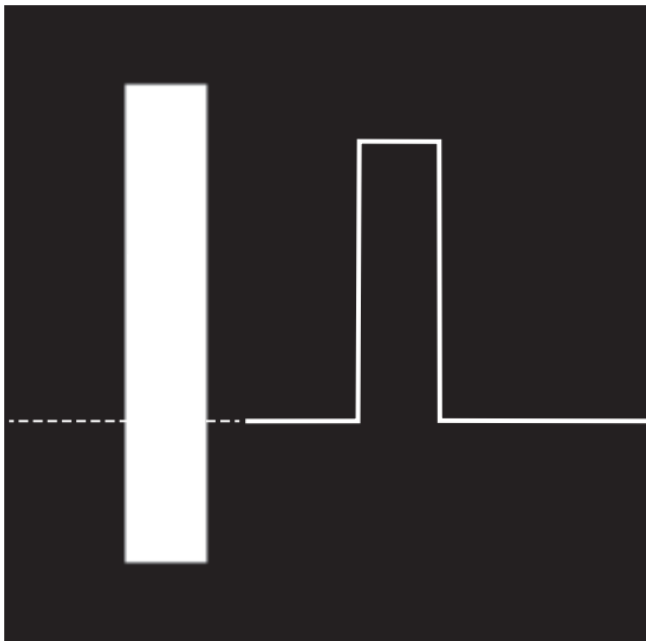


m=71 box

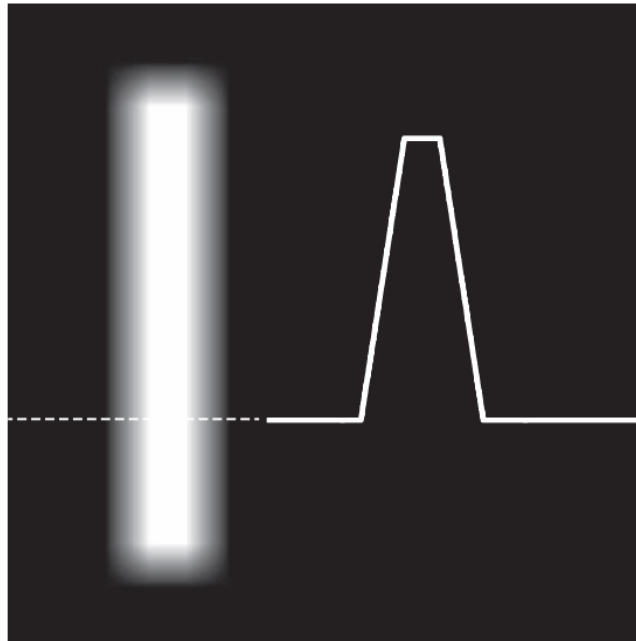


Filtering

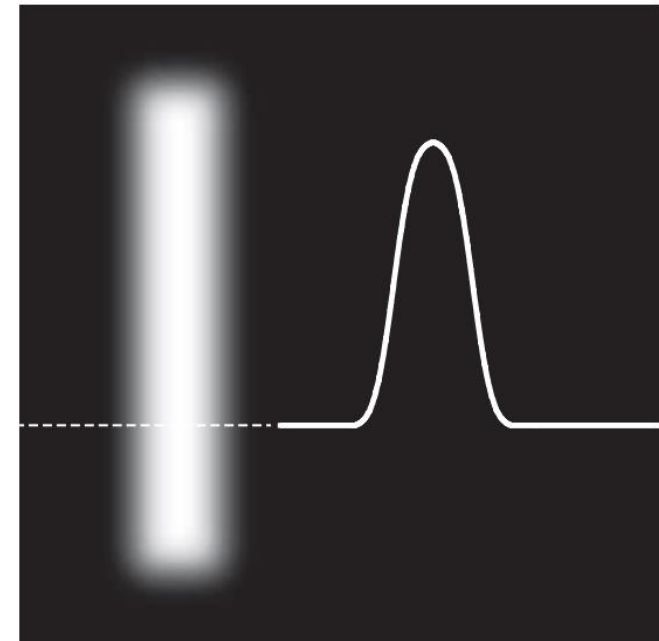
- Box vs Gaussian
 - blur profile
 - blurred rects having same shape



m=71 box



m=151 $\sigma=25$ Gauss

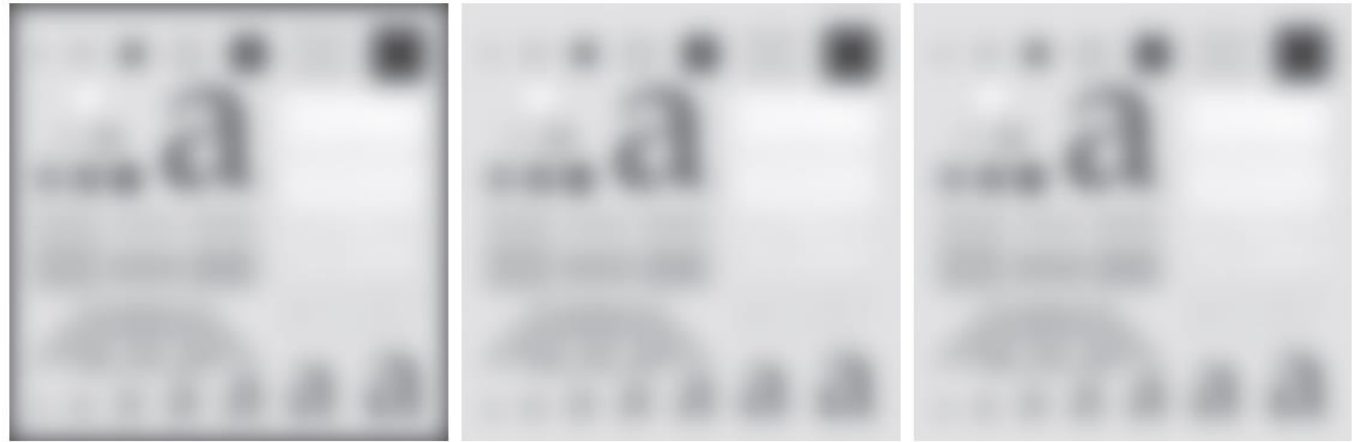


Filtering

- Padding effects

$m=187$ $\sigma=31$ Gauss

image 1024x1024



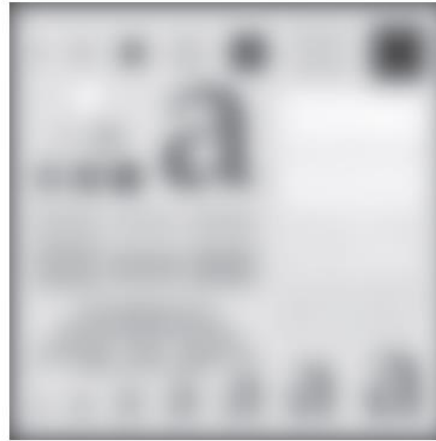
Filtering

- Padding effects

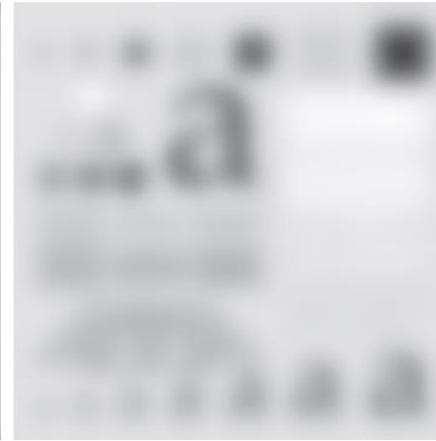
$m=187$ $\sigma=31$ Gauss

image 1024x1024

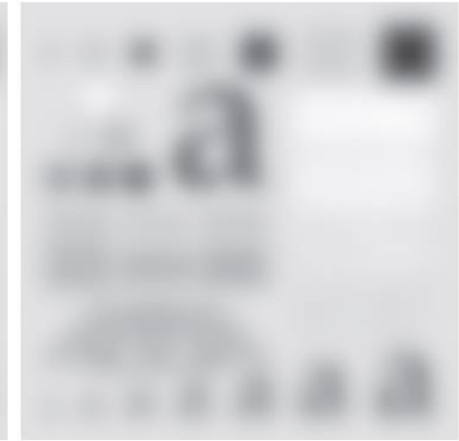
zero



mirror



replicate



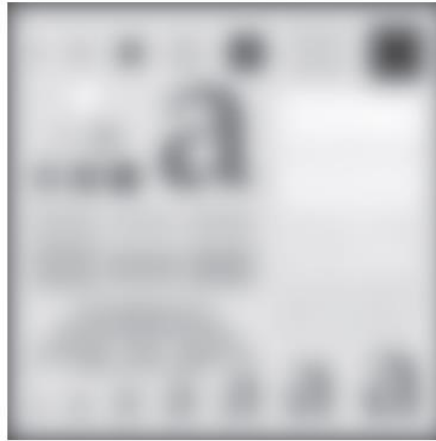
Filtering

- Padding effects

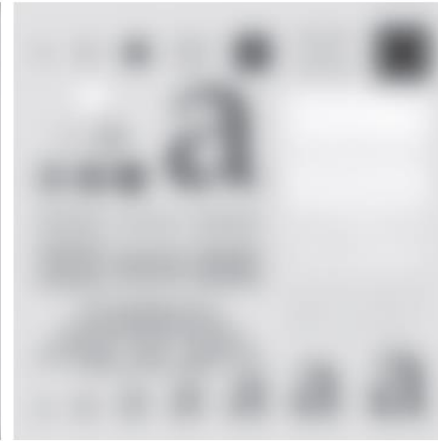
$m=187$ $\sigma=31$ Gauss

image 1024x1024

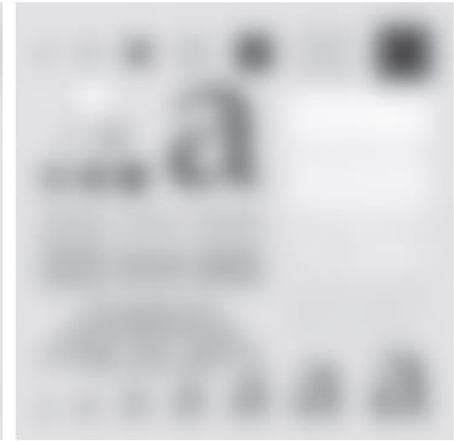
zero



mirror



replicate



- Relative size effect

$m=187$ $\sigma=31$ Gauss

image 4096x4096

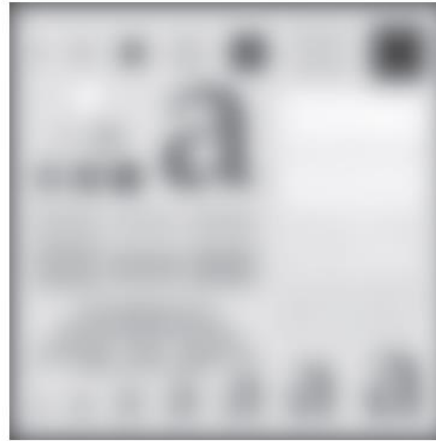
Filtering

- Padding effects

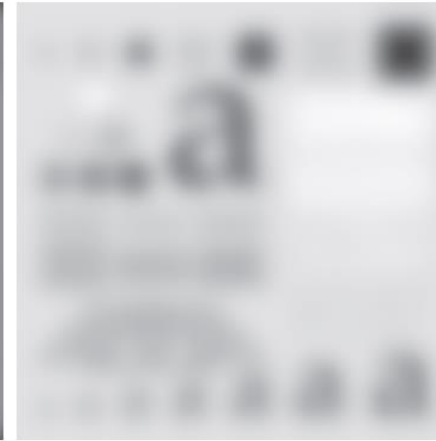
$m=187$ $\sigma=31$ Gauss

image 1024x1024

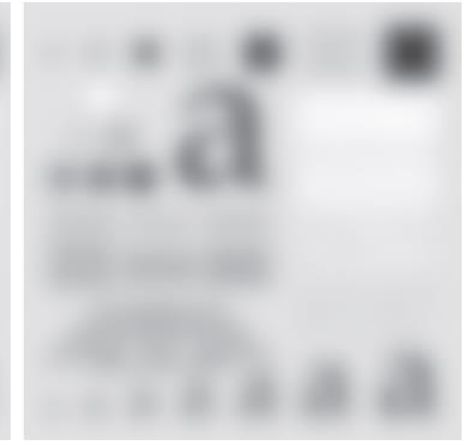
zero



mirror



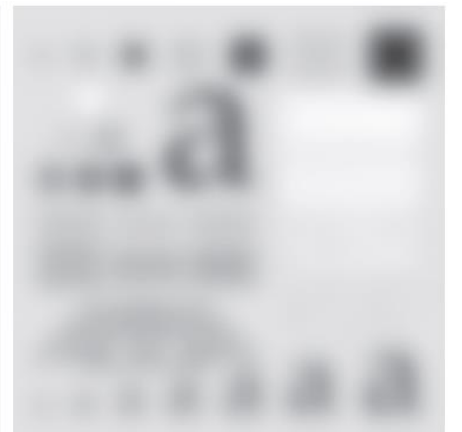
replicate



- Relative size effect

$m=187$ $\sigma=31$ Gauss

image 4096x4096



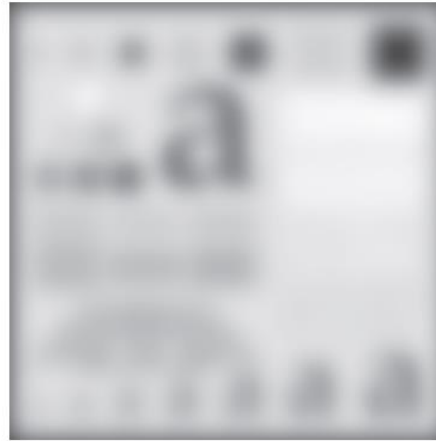
Filtering

- Padding effects

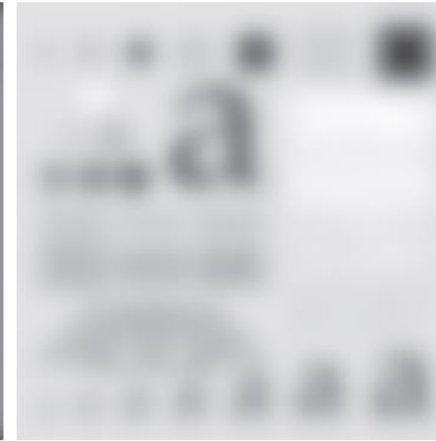
$m=187$ $\sigma=31$ Gauss

image 1024x1024

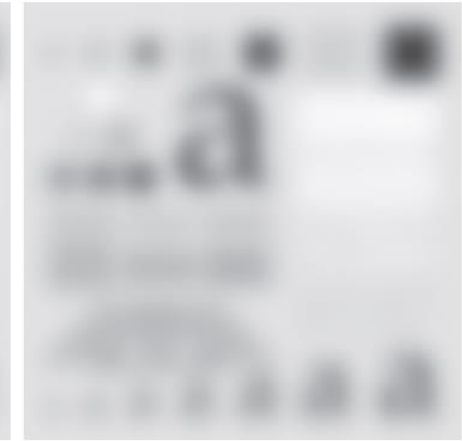
zero



mirror



replicate

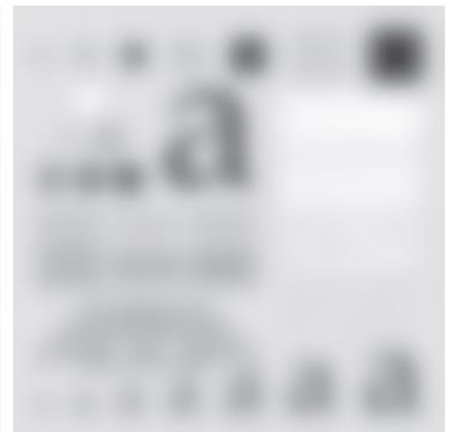


- Relative size effect

$m=187$ $\sigma=31$ Gauss

image 4096x4096

$m=745$ $\sigma=124$ Gauss



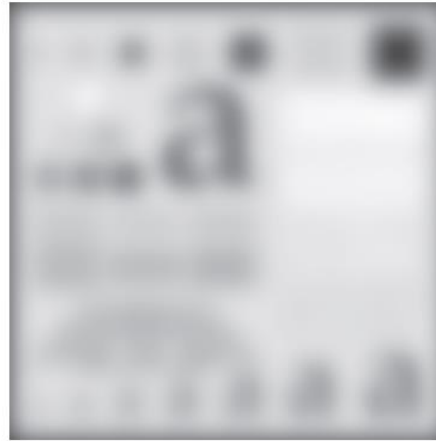
Filtering

- Padding effects

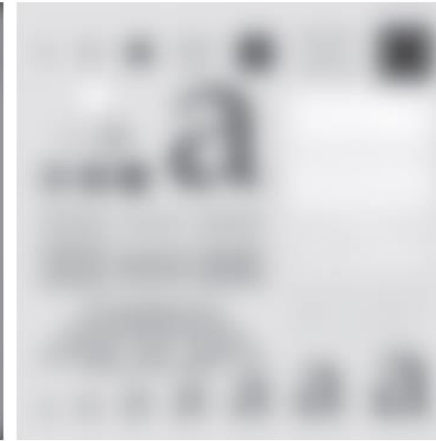
$m=187$ $\sigma=31$ Gauss

image 1024x1024

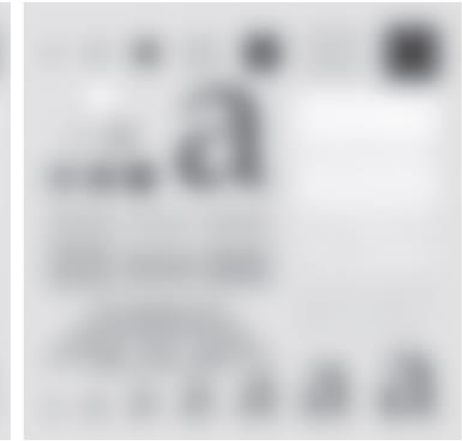
zero



mirror



replicate



- Relative size effect

$m=187$ $\sigma=31$ Gauss

image 4096x4096

$m=745$ $\sigma=124$ Gauss

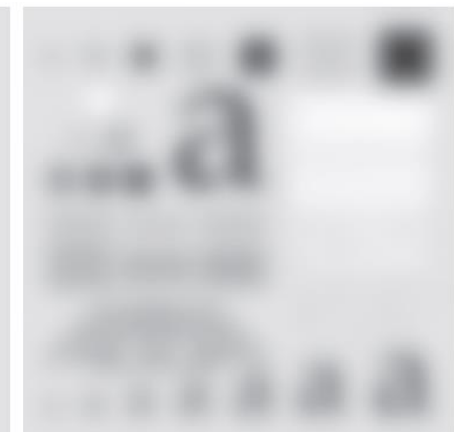
input



$m=187$ $\sigma=31$



$m=745$ $\sigma=124$



Filtering

- Relevant region extraction



Filtering

- Relevant region extraction



Filtering

- Relevant region extraction



Filtering

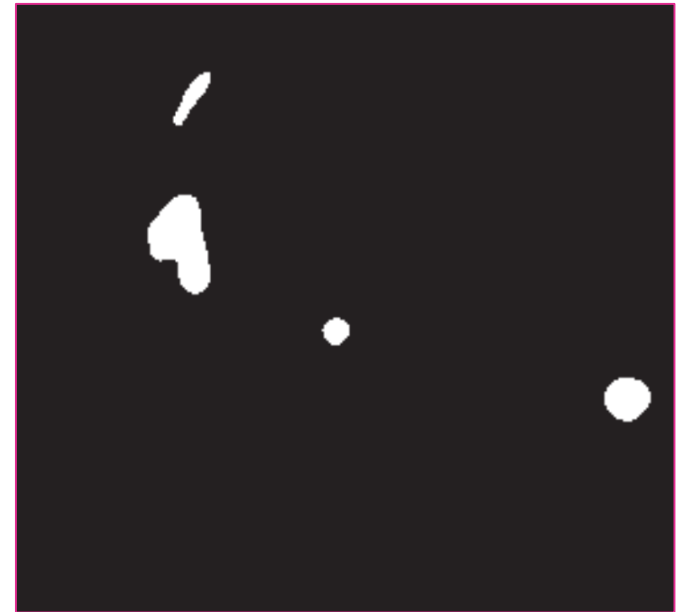
- Relevant region extraction



filtering

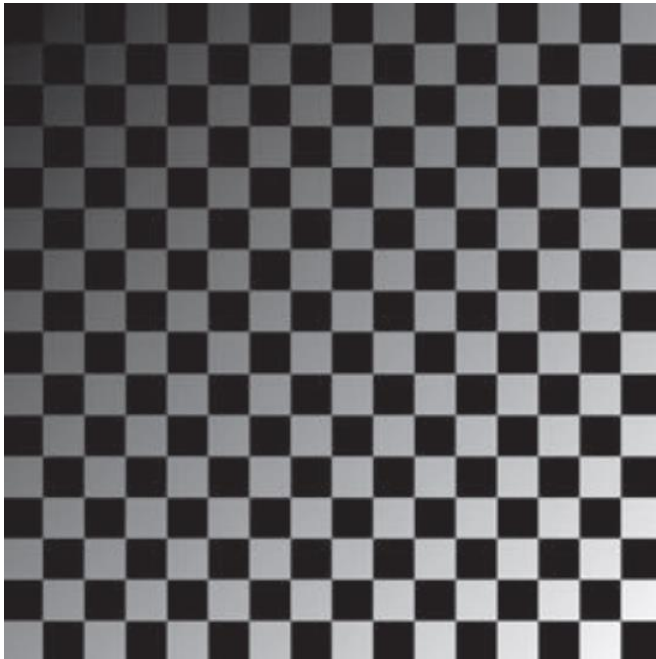


thresholding



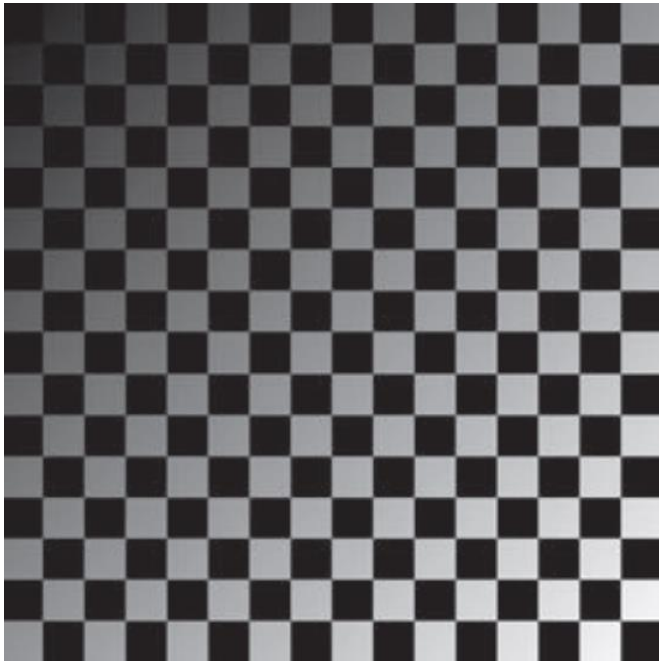
Filtering

- Shading correction



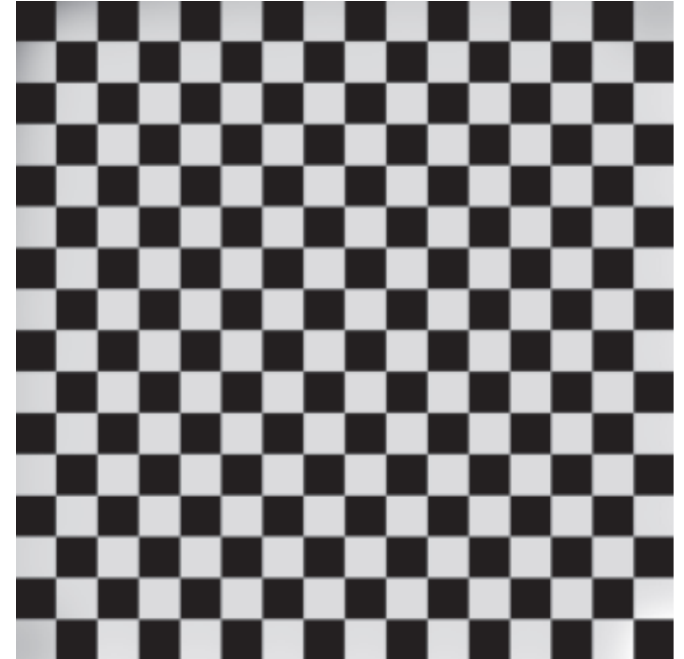
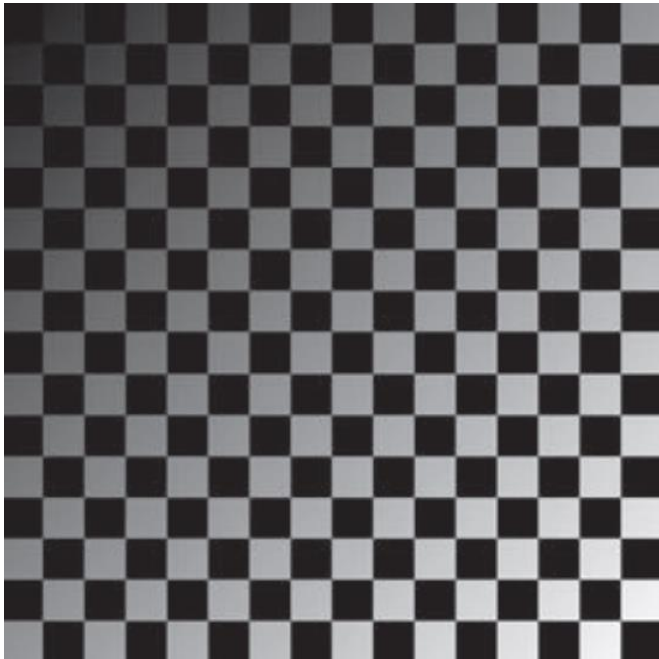
Filtering

- Shading correction



Filtering

- Shading correction

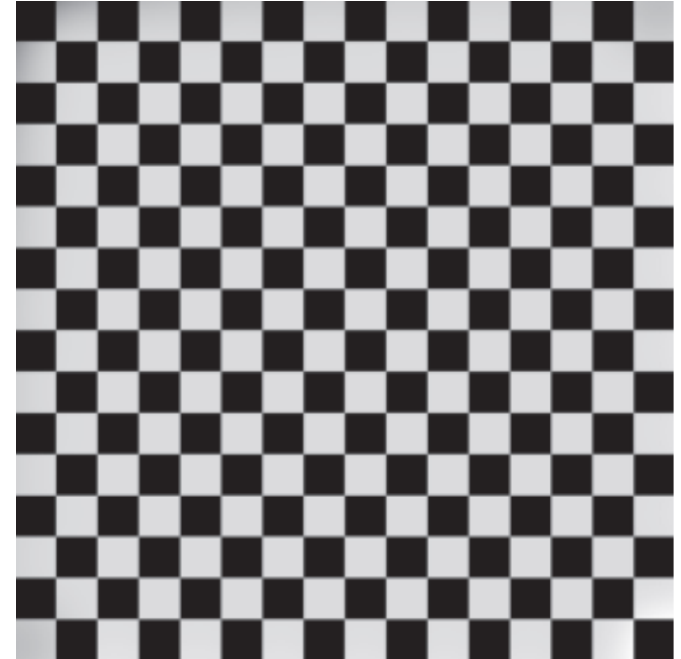
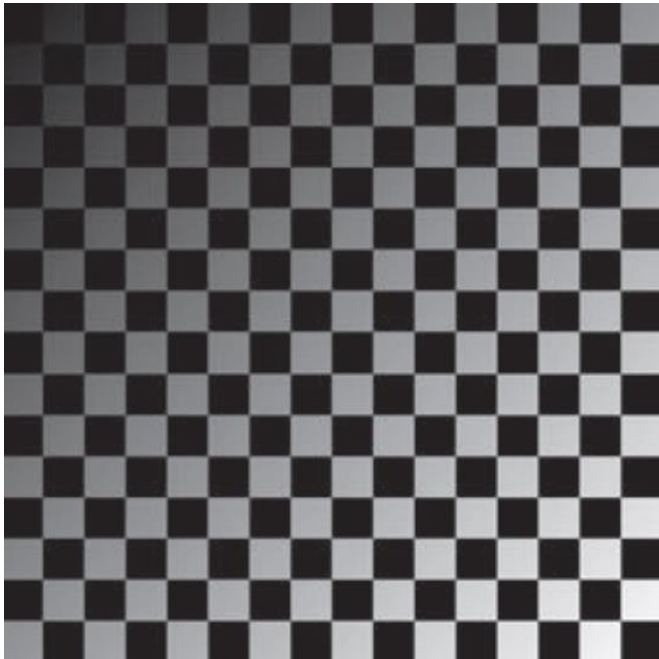


Filtering

- Shading correction

filtering

scaling





Filtering

Conclusion

- Filtering
 - Separable kernels
 - Correlation Vs Convolution
 - Filter properties
 - Smoothing filters

