Q1:

If $z=5+t+i\sqrt{25-t^2},(-5\leq t\leq 5)$, then locus of z is a curve which passes through

(A) $5+0i$
(B) $-2+3i$
(C) $2+4i$
(D) $-2-3i$

Q2:

If $|z|\geq 5$, then least value of $\left|z-\frac{1}{z}\right|$ is

(A) 5
(B) $24/5$
(C) 8
(D) $8/3$

Q3:

Let $C_r={}^{15}C_r, 0\leq r\leq 15$. Sum of the series $S=\sum_{r=1}^{15} r\left(\frac{C_r}{C_{r-1}}\right)$ is

(A) 40
(B) 60
(C) 100
(D) 120

Q4:

The least positive integer n such that $1-\frac{2}{3}-\frac{2}{3^2}-\ldots-\frac{2}{3^{n-1}}<\frac{1}{100}$ is

(A) 4
(B) 5
(C) 6
(D) 7

Q5:

If $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$ then $a\theta=$

(A) $x+y-a$
(B) $\sqrt{x^2+y^2-a^2}$
(C) $\sqrt{x^2-y^2+a^2}$
(D) $x-y+a$

Q6:

The system of equations $1z+1-il=\sqrt{2}$ and $|z|=3$ has

(A) no solution
(B) one solution
(C) two solutions
(D) infinite number of solutions

Q7:

If the equations $x^2-ax+b=0$ and $x^2+bx-a=0$ have a common root then

(A) $a=b$ (B) $a+b=-1$
(C) $a-b=-1$ (D) $a-b=1$

Q8:

The number of distinct terms in the expansion of (1 $+3x+3x^2+x^3)^7$ is

(A) 18 (B) 19
(C) 28 (D) 22

Q9:

Suppose $\frac{1}{2}x, \lfloor x+11, \lfloor x-11$ are in A.P., then sum to 10 terms of the A.P. is

(A) 54 (B) 36
(C) 28 (D) none of these

Q10:

The number of solutions of
$\sin\theta+2\sin 2\theta+3\sin 3\theta+4\sin 4\theta=10$
$0<\theta<\pi$ is

(A) 0 (B) 1
(C) 2 (D) 4

Q11:

Suppose $a\in\mathbf{R}$. The set of values of a for which the quadratic equation $x^2-2(a+1)x+a^2-4a+3=0$ has two negative roots is

(A) $(-\infty,-1)$ (B) (1,3)
(C) $(-\infty,1)\cup(3,\infty)$ (D) ϕ

Q12:

The coefficient of x^{60} in $(1+x)^{51}\left(1-x+x^2\right)^{50}$ is

(A) ${}^{50}C_{20}$ (B) $-\left({}^{50}C_{20}\right)$
(C) ${}^{51}C_{20}$ (D) $-\left({}^{51}C_{20}\right)$

Q13:

The sum to 16 terms of the series
$1^3+\frac{1+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\frac{1^3+2^3+3^3+4^3}{1+3+5+7}+\ldots.$ is

(A) 445 (B) 446

(C) 447
(D) 448

Q14:

Suppose $0 < a < b < c$. If the roots α, β of $ax^2 + bx + c = 0$ are imaginary, then

(A) $|\alpha d = \sqrt{\frac{c}{a}}$
(B) $|\beta| = \sqrt{\frac{a}{c}}$
(C) $\alpha + \beta = 0$
(D) $\alpha - \beta = -b/2a$

Q15:

If $x + y = 1$, then value of $\sum_{r=0}^{n}(r)({}^nC_r)x^{n-r}y^r$ is

(A) 1
(B) 0
(C) nx
(D) ny

Q16:

Let z_1, z_2 be two complex numbers such that $z_1 \neq 0$ and z_2/z_1 is purely real, then $\left|\frac{2iz_1+5z_2}{2iz_1-5z_2}\right|$ is equal to

(A) 3
(B) 2
(C) 1
(D) 0

Q17:

The sum to n terms of the series $1^2 + (1)(2) + 3^2 +$ (3) $(4) + 5^2 + (5)$ (6) $+7^2 + \ldots$ upto n terms when n is odd is

(A) $\frac{1}{12}(n+1)\left(4n^2 - n + 3\right)$
(B) $\frac{1}{12}n\left(4n^2 + 3n - 4\right)$
(C) $\frac{1}{6}(n+1)\left(4n^2 - n + 5\right)$
(D) none of these

Q18:

The roots of the equation $|x^2 - x - 6| = x + 2$ are

(A) -2,1,4
(B) 0,2,4
(C) 0,1,4
(D) -2,2,4

Q19:

Value of
$\left({}^{40}C_0\right)\left({}^{40}C_{15}\right) - \left({}^{40}C_1\right)\left({}^{40}C_{16}\right) + \left({}^{40}C_2\right)\left({}^{40}C_{12}\right)\cdots$

$-\left({}^{40}C_{25}\right) \quad \left({}^{40}C_{40}\right)$
equals

(A) $\underline{0}$
(B) ${}^{40}\mathrm{C}_{25}$
(C) ${}^{40}\mathrm{C}_{20}$
(D) -1

Q20:
If $x=\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}+\sin\frac{8\pi}{7}$
and $y=\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}$, then x^2+y^2
is

(A) 1
(B) 2
(C) 3
(D) 4

Q21:
The number of complex numbers z such that
$(1+i)z=i|z|$

Q22:
Let $f(x)=\frac{x^2+4x+1}{x^2+x+1}, x\in\mathbf{R}$.
If $m\le f(x)\le M\forall x\in\mathbf{R}$, then $\frac{1}{10}(M-m)=$

Q23:
If the sixth term in the expansion of
$$\left[3^{\log_3\sqrt{9^{x-1}+7}}+\frac{1}{3^{\log_3\left(3^{x-1}+1\right)}}\right]^7$$
is 84, then sum of the possible values of x is

Q24:
Suppose $\sum_{r=1}^{n} t_r=3^n-1 \quad \forall \quad n\in\mathrm{N}$
then $\sum_{r=1}^{\infty}\frac{1}{t_r}=$ ______.

Q25:
The number of solutions of $2(\cos x+\cos(2x))+$

$\sin(2x)(1 + 2\cos x) = 2\sin x$ lying in the interval $[-\pi, \pi]$ is _______.

Answer Keys

Q1: (C) **Q2:** (B) **Q3:** (D) **Q4:** (D)
Q5: (B) **Q6:** (A) **Q7:** (D) **Q8:** (D)
Q9: (D) **Q10:** (A) **Q11:** (D) **Q12:** (A)
Q13: (B) **Q14:** (C) **Q15:** (D) **Q16:** (C)
Q17: (A) **Q18:** (D) **Q19:** (A) **Q20:** (B)
Q21: (1) **Q22:** (0.4) **Q23:** (3) **Q24:** (0.75)
Q25: (5)

Sol 1:
Let $\quad z=x+iy$ so that
$x=5+t, \quad y=\sqrt{25-t^2}$
$\Rightarrow \quad (x-5)^2+y^2=25$
This clearly passes through $2+4i$

Sol 2:
As $|z|\geq 5, \quad \left|\frac{1}{z}\right|\leq\frac{1}{5}$. Now
$\left|z-\frac{1}{z}\right|\geq\left||z|-\frac{1}{|z|}\right|=|z|-\frac{1}{|z|}\geq 5-\frac{1}{5}=\frac{24}{5}$
The least value is attained when $z=5$.

Sol 3:
$\frac{C_r}{C_{r-1}}=\frac{15!}{r!(15-r)!}\times\frac{(r-1)!(16-r)!}{15!}$
$=\frac{16-r}{r}$
$\Rightarrow S=\sum_{r=1}^{15} r\left(\frac{C_r}{C_{r-1}}\right)=\sum_{r=1}^{15}(16-r)$
$=\frac{1}{2}(15)(16)=120$

Sol 4:
$\frac{99}{100}<\frac{2}{3}+\frac{2}{3^2}+\ldots+\frac{2}{3^{n-1}}$
$\Rightarrow \frac{(2/3)\left[1-(1/3)^{n-1}\right]}{1-1/3}>\frac{99}{100}$
$\Rightarrow \frac{1}{100}>\frac{1}{3^{n-1}}\Rightarrow 3^{n-1}>100$
$\Rightarrow n-1>5\Rightarrow n>6$
Thus, least value of n is 7.

Sol 5:
$x^2+y^2=a^2\left[\cos^2\theta+\sin^2\theta+\theta^2\sin^2\theta+\theta^2\cos^2\theta\right]$
$=a^2\left(1+\theta^2\right)$
$\Rightarrow a\theta=\sqrt{x^2+y^2-a^2}$

Sol 6:
The circle $|z-(-1+i)|=\sqrt{2}$ completely lies inside the circle $|z|=3$

Sol 7:
If α is a common root, then
$\alpha^2-a\alpha+b=0, \alpha^2+b\alpha-a=0$
Subtracting, we get $-(a+b)\alpha+b+a=0$
$\Rightarrow \alpha=1$. Thus, $1-a+b=0\Rightarrow a-b=1$

Sol 8:
$\left(1+3x+3x^2+x^3\right)^7=\left((1+x)^3\right)^7=(1+x)^{21}$

Sol 9:
As $\frac{1}{2}x, \lfloor x+11, \lfloor x-1|$ are in $\mathrm{A.P.}$
$2|x+11=\frac{1}{2}x+|x-1|$ (1)
If $x<-1$, then (1) give us
$-2(x+1)=\frac{1}{2}x-(x-1)$
$\Rightarrow x=-2$
In this case common difference is 2 and sum to 10 terms is
$\frac{10}{2}[2(-1)+(9)(2)]=85$
If $-1\leq x<1$
$2(x+1)=\frac{1}{2}x-(x-1)\Rightarrow x=-2/5$
In this case common difference is $4/5$, and sum to 10 terms is
$\frac{10}{2}\left[2\left(-\frac{1}{5}\right)+(9)\left(\frac{4}{5}\right)\right]=34$
If $x\geq 1$, then (1) becomes
$2x+2=\frac{1}{2}x+x-1$
$\Rightarrow \frac{1}{2}x=-3$
Not possible as $x\geq 1$

Sol 10:
Given equation is possible if
$\sin\theta=\sin 2\theta=\sin 3\theta$

$= \sin 4\theta = 1$ for some $\theta, 0 < \theta < \pi$ which is not possible.

Sol 11:
$D = 4(a+1)^2 - 4\left(a^2 - 4a + 3\right) \geq 0$
$2(a+1) < 0$ and $a^2 - 4a + 3 > 0$
First two imply $a \geq 1/3, a < -1$. Not possible.

Sol 12:
The coefficient of x^{60} in $(1+x)^{51}\left(1 - x + x^2\right)^{50}$
$=$ the coefficient of x^{50} in $(1+x)\left(1 - x^3\right)^{50}$
$= {}^{50}C_{20}$

Sol 13:
$t_r = \frac{1^3+2^3+\ldots+r^3}{1+3+\ldots(2r-1)} = \frac{r^2(r+1)^2}{4r^2} = \frac{(r+1)^2}{4}$
Thus, $\sum_{r=1}^{16} t_r = \frac{1}{4}\left(\sum_{r=1}^{17} r^2 - 1\right)$
$= \frac{1}{4}\left(\frac{1}{6}(17)(18)(35) - 1\right) = 446$

Sol 14:
As the roots are imaginary $b^2 - 4ac < 0$ and
$\alpha, \beta = \frac{1}{2a}\left(-b \pm i\sqrt{4ac - b^2}\right)$
$|\alpha| = \frac{1}{2a}\sqrt{b^2 + 4ac - b^2} = \sqrt{\frac{c}{a}} = |\beta|$

Sol 15:
We have $(a+t)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} t^r$
Differentiating both the sides with respect to t, we get
$n(a+t)^{n-1} = \sum_{r=0}^{n} r\left({}^nC_r\right) a^{n-r} t^{r-1}$
Multiplying both the sides by t and putting $a = x$ and $t = y$, we get

$$n(x+y)^{n-1} y = \sum_{r=0}^{n} r\left({}^nC_r\right) x^{n-r} y'$$

$\Rightarrow \sum_{r=0}^{n} r\left({}^nC_r\right) x^{n-r} y^r = ny \quad [\because \quad x + y = 1]$

Sol 16:
Suppose $z_2/z_1 = \text{a}$, where $a \in \mathbf{R}$.
Now, $\left|\frac{2iz_1+5z_2}{2iz_1-5z_2}\right| = \left|\frac{2i+5(z_2/z_1)}{2i-5(z_2/z_1)}\right| = \left|\frac{2i+5a}{2i-5a}\right| = 1$

Sol 17:
Let $n = 2m - 1$, then

$$\begin{aligned} S_{2m-1} &= 1^2 + (1)(2) + 3^2 + (3)(4) + \ldots \\ &\quad (2m-3)(2m-2) + (2m-1)^2 \\ &= \sum_{r=1}^{2m-1} r^2 + \sum_{r=1}^{m-1}\left[(2r-1)(2r) - (2r)^2\right] \\ &= \frac{1}{6}(2m-1)(2m)(4m-1) - 2\sum_{r=1}^{m-1} r \\ &= \frac{1}{3}m(2m-1)(4m-1) - (m-1)m \\ &= \frac{1}{3}\left(\frac{n+1}{2}\right)n(2n+1) - \left(\frac{n+1}{2}\right)\left(\frac{n-1}{2}\right) \\ &= \frac{1}{12}(n+1)[2n(2n+1) - 3(n-1)] \\ &= \frac{1}{12}(n+1)\left(4n^2 - n + 3\right) \end{aligned}$$

Sol 18:
$x + 2 \geq 0 \Rightarrow x \geq -2$
Now, $1(x+2)(x-3) \mid= x + 2$
$\lfloor x+2 \rfloor \quad \lfloor x - 3 \mid= x + 2$
$\Rightarrow \quad x + 2 = 0$ or $\lfloor x - 3 \mid= 1$
$\Rightarrow \quad x = -2, x = 3 \pm 1$ i.e., $x = -2, 2, 4$

Sol 19:

$$\left({}^{40}C_0\right)\left({}^{40}C_{15}\right) - \left({}^{40}C_1\right)\left({}^{40}C_{16}\right) + \left({}^{40}C_2\right)\left({}^{40}C_{17}\right) + \cdots - \left({}^{40}C_{25}\right)\left({}^{40}C_{40}\right)$$

$= \left({}^{40}C_0\right)\left({}^{40}C_{25}\right) - \left({}^{40}C_1\right)\left({}^{40}C_{24}\right) + \left({}^{40}C_2\right)\left({}^{40}C_{23}\right)$
$+ \cdots - \left({}^{40}C_{25}\right)\left({}^{40}C_0\right)$
$=$ Coefficient of x^{25} in
${}^{40}C_0 - {}^{40}C_1 x + {}^{40}C_2 x^2 \cdots$
$+ {}^{40}C_{40} x^{40}\big] \times \left[{}^{40}C_0 + {}^{40}C_1 x + \cdots + {}^{40}C_{40} x^{40}\right]$
$=$ Coefficient of x^{25} in $(1-x)^{40}(1+x)^{40}$
$=$ Coefficient of x^{25} in $\left(1-x^2\right)^{40} = 0$

Sol 20:
$x^2 + y^2 = 3 + 2\left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right)$
$= 3 + 2(-1/2) = 2$
$(\text{ Ref. Ex } 49,)$

Sol 21:
$|(1+i)z] = |i|z||$
$\Rightarrow \quad \sqrt{2}|z| = |z| \Rightarrow (\sqrt{2}-1)|z| = 0$
$\Rightarrow \quad |z| = 0 \Rightarrow z = 0$

Sol 22:
Let $y = \frac{x^2+4x+1}{x^2+x+1}$
$\Rightarrow \left(x^2+x+1\right)y = x^2 + 4x + 1$
$\Rightarrow (y-1)x^2 + (y-4)x + (y-1) = 0$
As x is real, $(y-4)^2 - 4(y-1)^2 \geq 0$
$\Rightarrow (y-4-2y+2)(y-4+2y-2) \geq 0$
$\Rightarrow -(y+2)(3y-6) \geq 0$
$\Rightarrow (y+2)(y-2) \leq 0$
$\Rightarrow -2 \leq y \leq 2$
Thus $m = -2, M = 2$
and $\frac{1}{10}(M-m) = 0.40$

Sol 23:
$3^{\log_3 \sqrt{9^{1-1}+7}} = \sqrt{9^{x-1}+7} = a$ (say)
and $3^{\log_3 \left(3^{x-1}+1\right)^{1/5}} = \left(3^{x-1}+1\right)^{1/5} = b$ (say)
Now, the sixth term in the expansion of $(a+b)^7$ is
${}^7C_5 a^2/b^5 = 84$
$\Rightarrow \quad 21\left(9^{x-1}+7\right)/\left(3^{x-1}+1\right) = 84$
$\Rightarrow \quad 3^{2x-2} + 7 = 4\left(3^{x-1}+1\right)$
$\Rightarrow \quad \left(3^{x-1}\right)^2 - 4\left(3^{x-1}\right) + 3 = 0$
$\Rightarrow \quad \left(3^{x-1}-1\right)\left(3^{x-1}-3\right) = 0$
$\Rightarrow \quad 3^{x-1} = 1, 3 \Rightarrow x = 1, 2$
$\therefore$ Sum of the values of x is 3.

Sol 24:
$t_n = \left(\sum_{r=1}^{n} t_r\right) - \left(\sum_{r=1}^{n-1} t_{r-1}\right) = (3^n - 1) - \left(3^{n-1} - 1\right)$
$= 2\left(3^{n-1}\right) \quad \forall n \geq 2$
Also, $t_1 = 3^1 - 1 = 2$
$\therefore \quad \sum_{n=1}^{\infty} \frac{1}{t_n} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{3^{n-1}} = \frac{1}{2} \cdot \frac{1}{1-1/3} = 0.75$

Sol 25:
Rewrite the equation as
$2\left(\cos x + 2\cos^2 x - 1\right) + 2\sin x\cos x(1 + 2\cos x)$
$= 2\sin x$
$\Rightarrow 2\left(2\cos^2 x + \cos x - 1\right) + 2\sin x\left(2\cos^2 x +\right.$
$\left.\cos x - 1\right) = 0$
$\Rightarrow 2(1+\sin x)(\cos x + 1)(2\cos x - 1) = 0$
$\Rightarrow \sin x = -1$ or $\cos x = -1$ or $\cos x = 1/2$
As $-\pi \leq x \leq \pi, \sin x = -1 \Rightarrow x = -\pi/2$
$\cos x = -1 \Rightarrow x = -\pi, \pi$
and $\cos x = 1/2 \Rightarrow x = \pi/3, -\pi/3$
Therefore, there are 5 solutions lying in $[-\pi, \pi]$.