

**B. Tech**  
**Third Semester Examination, 2014-15**  
**Mathematics-III**

**Time: 3 Hours**

**Total Marks: 100**

**Note: Attempt all the questions. Each question carries equal marks.**

**1. Attempt any four parts of the following: (5x4=20)**

(a) If  $\phi$  and  $\psi$  are functions satisfying Laplace's equation, show that  $(s + it)$  is analytic, where

$$s = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \quad \text{and} \quad t = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}$$

(b) Find the value of  $\int_c \frac{2z^3 + z}{z^2 - 1} dz$ , where  $c$  is the circle of unit radius with centre at  $z=1$ .

(c) Using Taylor's theorem for an analytic function,

$$\log z = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} - \dots, \quad \text{where } |z-1| < 1$$

(d) Find the value of the integral  $\int_0^{\infty} (x^2 - y^2 - a^2) dx$  along real axis from  $x=0$  to  $x=1$  and then along a line parallel to imaginary axis from  $z=1$  to  $z=1+i$ .

(e) Find the residue of

$$f(z) = \frac{z^3}{(z-1)^2(z-2)(z-3)}$$
 at its pole.

(f) Using residue theorem, evaluate  $\int_0^{2\pi} \frac{d\theta}{4 + 3 \sin \theta}$

**2. Attempt any two parts of the following: (10x2=20)**

(a) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

(b) Using Fourier transform solve  $\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial t^2}$ ,  $-\infty < x < \infty$ ,  $t > 0$

and  $V(x, 0) = f(x)$

(c) (i) Find the z-transform of  $\cos h(n\pi/2 + 0)$

(ii) Solve by z-transform

$$y_{n+2} - 4y_n = 0 \text{ given that } y_0 = 0, y_1 = 2$$

3. Attempt any two parts of the following: (10x2=20)

(a) Find the mean of the binomial distribution. Also find the moment generating function of the binomial distribution about its mean.

(b) (i) If  $\theta$  is the acute angle between the two regression lines in case of two variables  $x$  and  $y$ , show that  $\tan \theta = \frac{1-r^2}{r} \cdot \frac{\sigma_y \sigma_x}{\sigma_x^2 + \sigma_y^2}$

where  $r$  is correlation coefficient between  $x$  and  $y$ ,  $\sigma_x$  and  $\sigma_y$  are their standard deviations. Also, explain the significance of the formula when  $r = 0$  and  $r = \pm 1$ .

(ii) Two lines of regression are given by  $x+2y-5=0$ ,  $2x+3y-8=0$  and  $\sigma_x^2=12$ . Calculate the mean values of  $x$  and  $y$ , the coefficient of correlation between  $x$  and  $y$ .

(c) Calculate the correlation coefficient between  $x$  and  $y$  for the following data:

x	21	23	30	54	57	58	72	78	87	90
y	60	71	72	83	110	84	100	92	113	135

4. Attempt any four parts of the following: (5x4=20)

(a) Find a positive value of  $(17)^{1/3}$  correct to four decimal places by Newton-Raphson method.

(b) Find the rate of convergence for Regula-Falsi method.

(c) Using Newton's divided difference formula, find a polynomial function satisfying the following data:

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1135

(d) Prove that the  $n^{\text{th}}$  difference of a polynomial function of  $n^{\text{th}}$  degree is constant when the values of the independent variable are at equal intervals.

(e) Using method of least squares, fit a straight line from the following data:

x	2	4	6	8	10	12
y	10	5	6	4	8	10

(f) Obtain the least squares fit of the form  $f(t) = a e^{bt}$ , for the data

5. Attempt any two parts of the following:

(10x2=20)

(a) Apply Gauss-Seidel iteration method to solve the following equations (three iterations only)

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

(b) A rod is rotating in a plane. The following table gives the angle  $\theta$  (radians) through which the rod has turned for various values of the time 't' seconds:

t	0	0.2	0.4	0.6	0.8	1.0	1.2
$\theta$	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity of the rod when  $t = 0.6$

(c) Find  $y(2)$  if  $y(x)$  is the solution of  $dy/dx = \frac{1}{2}(x+y)$  using Runge-Kutta method, in two steps taking  $h=1.0$ . Given  $y(0)=2.0$ .