(b) Using Laplace Transform solve the following differential equation $d^2y/dt^2 + 2\,dy/dt + 5y = e^{-t}\sin t$, $y(0) = 0$, $y'(0) = 1$

(c) Find the Laplace Transform of $L[t^2 e^t \sin t]$

(d) Draw the graph of periodic function

$$f(t) = \begin{cases} t & 0 < t < \pi \\ (\pi - t) & \pi < t < 2\pi \end{cases}$$

and find its Laplace Transform.

(e) Using convolution theorem find $L^{-1}[1/s^3(s^2+1)]$

(f) Using Laplace Transform evaluate $\int_0^\infty \frac{e^{-at}\sin^2 t}{t}\,dt$

4. **Attempt any two parts of the following:** **(10x2=20)**

(a) Obtain the Fourier series expansion of the following function in the interval $0 < x < 2\pi$

$$f(x) = 1/12\,(3x^2 - 6\pi x + 2\pi^2)$$

(b) Find the fourier series of the function

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$$

and hence show that $1/1^2 + 1/3^2 + 1/5^2 + \ldots\ldots = \pi^2/8$

c) Solve the linear partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$$

Attempt any two parts of the following: **(10x2=20)**

(a) Use the method of separation of variables to solve the equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u.$$

(b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3(\pi x/l)$. If it is released from rest from this position. Find the displacement $y(x, t)$

(c) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(x, 0) = 3\sin n\pi x$, $u(0, t) = 0$, $u(l, t) = 0$, where $0 < x < l$.

B.Tech. (Main & COP)

Second Semester Theory Examination 2016-17

Mathematics-II

Time: 3 Hours **Total Marks: 100**

Note Attempt all questions.

1. Attempt any four parts of the following: **(5x4=20)**

(a) Solve the differential equation

$$(D^2+3D+2)y = 2\cos(2x+3) + 2e^x + x^2$$

(b) Solve the differential equation by variation of parameters

$$(D^2+1)y = \operatorname{cosec} x \,.\, \cot x. \text{ where } D \equiv (d/dx)$$

(c) Solve the set of simultaneous differential equation

$$d^2x/dt^2 - 3x - 3y = 0, \quad d^2y/dt^2 + x + y = 0$$

(d) Solve by changing the independent variable

$$(1+x^2)^2\, d^2y/dx^2 + 2x(1+x^2)\, dy/dx + 4y = 0$$

(e) Transform the equation $xy'' + y' + 1 = 0$ into a linear equation with constant coefficients and hence solve it.

(f) In an L-C-R circuit the charge q on a plate of condenser is given by $L\, d^2q/dt^2 + R\, dq/dt + q/C = E \sin pt$. The circuit is turned to resonance so that $p^2 = (1/LC)$, if initially the current i and the charge q be zero, show that for small values of R/L, the current i in the circuit at time t is given by $Et/2L \sin pt$.

2. Attempt any two parts of the following: **(10x2=20)**

(a) Find the series solution of the differential equation $3x\, d^2y/dx^2 + 2\, dy/dx + y = 0$

(b) (i) Prove that $\displaystyle\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx = \left(\frac{2n}{4n^2-1}\right)$

(ii) Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre's polynomials.

(c) Show that for Bessel's function

$$\sqrt{\frac{\pi x}{2}}\, J_{3/2}(x) = \frac{1}{x}\sin x - \cos x$$

3. Attempt any four parts of the following: **(5x4=20)**

(a) Using Laplace Transform evaluate $\displaystyle\int_0^\infty \frac{\cos at - \cos bt}{t}\, dt$