(Paper code and roll No. to be filled in your answer book)

Paper code: ECS 302

Roll No. [illegible]

B. Tech.
Third Semester Examination, 2014-15
Discrete Mathematics

Time: 3 Hours **Total Marks: 100**

Note: Attempt all questions. Each question carries equal marks.

1. Attempt any four parts of the following: **(5x4=20)**

(a) In a survey of 100 students; the number of students studying the various languages are found as: English only 18; but not Hindi 23; English and Sanskrit and Hindi 8; English 26; Sanskrit 48 and not languages 24, find

(i) How many students are studying Hindi?

(ii) How many students are studying English and Hindi both?

(b) Using laws of sets prove that:

$(A \cap B) \cup (\bar{A} \cap B) \cup (\bar{A} \cap \bar{B}) = \bar{A} \cup \bar{B}$

(c) If R is an equivalence relation on A, then prove that R-1 is also equivalence relation on A.

(d) What are the different proof methods? Explain the proof by counter with example.

(e) Let X = {1, 2, 3}, Y = {p, q} and Z = {a, b}

Let f: x → y be f = {(1, p), (2, p), (3, q)}

G: Y →Z be g = {(p, b), (q, b)}. Find g of and show it pictorially.

(f) Prove by mathematical induction :

$n^4 - 4n^2$ is divisible by 3 for all $n \geq 2$.

2. Attempt any four parts of the following: **(5x4=20)**

(a) Define group and prove that if every element of a group G is its own inverse then G is its own inverse then G is an abelian group.

(b) State and prove Lagrange's theorem.

(c) Let (G,*) be a group. Prove that (G,*) is abelian, if

$b^{-1} * a^{-1} * b * a = e \ \forall \ a, b \in A$

(d) Prove that Union of two subgroups is not necessarily a subgroup.

(e) If ring R is commutative then Prove $(a+b)^2 = a^2 + 2ab + b^2 \ \forall \ a, b \in R$

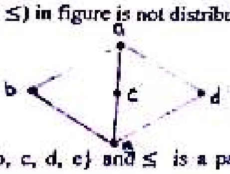
(f) Define the following terms with example :
(1) Subgroup (2) Cyclic group (3) Ring (4) Field

3. Attempt any two parts of the following: **(10x2=20)**

(a) (i) Let D_m denotes the positive divisors of m ordered by divisibility. Draw the Hasse diagram of
(i) D_{12} (ii) D_{15} (iii) D_{16} (iv) D_{17}

(ii) Define the following terms
(i) Properties of Lattices (ii) Sub-lattice (iii) Isomorphic Lattice.

(b) The Lattice (L, ≤) in figure is not distributive lattice. Prove.



Where L = {a, b, c, d, e} and ≤ is a partial ordering relation defined on L.

(c) Use Karnaugh maps to find a minimal form for the following Boolean functions:
(i) f(x, y, z, w) = x'yz w + xy'zw'+ x'y'zw'+xyzw'+ xy'z'w'
(ii) f(x, y, z, w) = xy' + xyz + x'y'z' + x'yzw

4. Attempt any two parts of the following: **(10x2=20)**

(a) Show that the truth values of the following formula are independent of their components
(i) $(p \wedge (p \rightarrow q)) \rightarrow q$ (ii) $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
(iii) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \leftrightarrow r)$ (iv) $(p \vee q) \wedge (\sim p) \wedge (\sim q)$ (v) $(p \leftrightarrow q) \wedge (q \leftrightarrow r) \rightarrow (p \leftrightarrow r)$

(b) (i) The inverse of statement is given. Write the converse and contrapositive of the statement "If a man is not fisherman, then he is not swimmer".

(ii) Write the equivalent formula for $p \wedge (q \rightarrow r) \vee (r \leftrightarrow p)$ which does not contain '↔'.

(c) Find the truth value of each of the following compound statement
(i) if 4+3 = 2, then 5+5=10
(ii) Paris is in England or London is in France.

5. Attempt any two parts of the following: **(10x2=20)**

(a) Describe matrix representation methods for undirected graph and for directed graph with suitable example. Also explain their properties.

(b) (i) Prove that a connected planar graph with n vertices and e edges has e-n+2 regions.
(ii) Discuss two important applications of binary trees.

(c) (i) Solve the recurrence relation given below:
$a_r + 6a_{r-1} + 9a_{r-2} = 3$
Given that $a_0 = 0$, $a_1 = 1$.
(ii) Find the generating function of the following numeric function.
(a) $a_r = 2r + 3$, $r \geq 0$
(b) $a_r = r(3 + 5r)$