

B.Tech. (Main & COP)
Second Semester Examination, 2015-16
Mathematics-II

Time: 3 Hours

Total Marks: 100

Note: Attempt all questions. Assume missing data if any.

1. Attempt any four parts of the following:

(5x4=10)

(a) Solve the following differential equation

$$\frac{d^4 y}{dx^4} - k^4 y = 0$$

(b) Solve $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{3x}$

(c) Solve the following simultaneous differential equations

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0$$

(d) A body executes damped forced vibrations given by the equation

$$\frac{d^2 x}{dt^2} + 2k \frac{dx}{dt} + b^2 x = e^{-kt} \sin nt$$

Solve the equation when $n^2 = b^2 - k^2$

(e) Use variation of parameters method to solve

$$y'' + y = \sec x$$

(f) Solve the equation

$$(y \log y) dx + (x - \log x) dy = 0$$

2. Attempt any two parts of the following :-

(10x2=20)

(a) Solve the following equation in power series

$$2x(1-x) \frac{d^2 y}{dx^2} + (1-x) \frac{dy}{dx} + 3y = 0$$

(b) Prove

$$\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$$

Where $J_n(x)$ is a Bessel function of the first kind of order n .

Further, show that

$$\int x J_0^2(x) dx = \frac{1}{2} x^2 [J_0^2(x) + J_1^2(x)]$$

(c) Show that

$$(i) \quad n P_n(x) = x P_n'(x) - P_{n-1}'(x)$$

$$(ii) \quad \int_{-1}^1 (1-x^2) P_m'(x) P_n'(x) dx = 0, \quad m \neq n$$

3. Attempt any four parts of the following :-

(5x4=20)

(a) If $L\{f(t)\} = f(s)$ and

$$g(t) = \begin{cases} f(t-a) & , \quad t > a \\ 0 & , \quad 0 < t < a \end{cases}$$

then prove that $L\{g(t)\} = e^{-as} f(s)$

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(b) If $L\{\cos^2(t)\} = \frac{(s^2+2)}{s(s^2+4)}$, find $L\{\cos^2(at)\}$

(c) State convolution theorem and hence evaluate

$$L^{-1} \left\{ \frac{s^2}{(s^2+a^2)(s^2+b^2)} \right\}$$

(d) Using Laplace transform, find the solution of the initial value problem

$$\frac{d^2 y}{dt^2} + 9y = 6 \cos 3t$$

$$\text{where } y(0) = 2, \quad y'(0) = 0$$

(e) Solve the simultaneous differential equation by Laplace transform

$$\frac{dx}{dt} - y = e^t$$

$$\frac{dy}{dt} + x = \sin t$$

(f) Find the Laplace transform of the saw-tooth wave given below

$$F(t) = \frac{K}{T}t, \quad 0 < t < T$$

and $F(t + T) = F(t)$

4. Attempt any two parts of the following :- (10x2=20)

(a) Find the Fourier half range cosine series of the function

$$f(t) = \begin{cases} 2t, & 0 < t < 1 \\ 2(2-t), & 1 < t < 2 \end{cases}$$

(b) Solve the PDE :-

$$(D^2 - 5DD' + 4D'^2)z = \sin(4x + y)$$

(c) (i) $(y^2 + z^2)p - xyq + xz = 0$

(ii) $r + (a + b)s + abt = xy$

where r, s and t have their usual meaning.

5. Attempt any two parts of the following :- (10x2=20)

(a) Using method of separation of variables, solve the following PDE

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$$

$$\text{with } u(0, y) = 0, \quad \frac{\partial u}{\partial x}(0, y) = 1 + e^{-3y}$$

(b) A rod of length l with insulated sides is initially at a uniform temperature u_0 . Their ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature function (x, t) .

(c) Obtain the general solution of one dimensional wave equation by separation of variables.