(Paper code and roll No. to be filled in your answer book)		KNIT Sultanpur
Paper code: KAS-103	Roll No.	1 4 6 4 9

B. Tech
First Semester Examination, 2014-15
Mathematics - I

Time: 3 Hours **Total Marks: 100**

Note: Attempt all the questions. Each question carries equal marks.

1. Attempt any four parts of the following: **(5x4=20)**

(a) If $y = x\log\frac{x-1}{x+1}$, show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$

(b) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, show that
$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

(c) If u = f(x-y, y-z, z-x), show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

(d) Trace the curve $x^3+y^3=3axy$.

(e) If $u = \sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$

(f) If $y = (x^2-1)^n$, use Leibnitz theorem to show that
$(1-x^2)y_{n+2} - 2xy_{n+1} + n(n+1)y_n = 0$

2. Attempt any two parts of the following: **(10x2=20)**

(a) If $u = xyz$, $v = x^2+y^2+z^2$ and $w = x+y+z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

(b) Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

(c) Expand $\tan^{-1}(y/x)$ in the neighbourhood of (1, 1) up to and inclusive of second degree terms. Hence compute f (1.1, 0.9) approximately.

3. Attempts any two parts of the following: **(10x2=20)**

(a) Find the eigen values and eigen vectors of the matrix:

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(a) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Hence compute A^{-1}.

(b) Reduce the matrix A to its normal form when

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix},$$

Hence find the rank of A.

4. Attempt any four parts of the following: **(5x4=20)**

(a) Evaluate $\int_0^2 \int_1^{x} dy\,dx$ by changing the order of integration.

(b) Determine the area of region bounded by the curves $xy=2$, $4y=x^2$ and $y=4$.

(c) A triangular prism is formed by planes whose equations are $a.y = b.x$, $y=0$ and ~~$x=0$.~~ $x=a$ Find the volume of the prism between the planes $z=0$ and surface $z=c+xy$.

(d) Define Beta and Gamma functions. Prove that $\beta(m,n) = \frac{\Gamma_m \Gamma_n}{\Gamma_{(m+n)}}$.

(e) Evaluate the following integral by changing to spherical polar coordinates: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$

(f) Evaluate $I = \iiint_v x^{\alpha-1} y^{\beta-1} z^{\gamma-1}\,dx\,dy\,dz$,where v is the region in first octant bounded by sphere $x^2+y^2+z^2=1$ and the coordinate planes.

5. Answer any four of the following: **(5x4=20)**

(a) Prove that angular velocity at any point is equal to half the Curl of linear velocity at that point of the body.

(b) Define scalar and vector fields. Find the directional derivative of the function $f=x^2-y^2+2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4).

(c)Find the value of Curl ($\hat{k}\times$grad $1/r$) + grad ($\hat{k}$ grad $1/r$), where r is the magnitude of position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\hat{k}$ is a unit vector in the direction of oz.

(d) If $\vec{V} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$, find the value of div $\vec{V}$ and curl $\vec{V}$.

(e) Find $\iint_s \vec{F}\cdot\hat{n}\,ds$, where $\vec{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$ and s is the surface of sphere having centre (3, -1, 2) and radius 3.

(f) Evaluate $\oint_c \vec{F}\cdot d\vec{r}$ by Stoke's theorem, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and c is the boundary of triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0).