(Paper code and roll No. to be filled in your answer book) KNIT Sultanpur

Paper code: KAS-103 Roll No. 1 4 6 4 9

B. Tech
First Semester Examination, 2014-15
Mathematics - I

Time: 3 Hours **Total Marks: 100**

Note: Attempt all the questions. Each question carries equal marks.

1. Attempt any four parts of the following: **(5x4=20)**

(a) If $y = x \log\frac{x-1}{x+1}$, show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$

(b) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

(c) If $u = f(x-y, y-z, z-x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

(d) Trace the curve $x^3+y^3=3axy$.

(e) If $u = \sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$

(f) If $y=(x^2-1)^n$, use Leibnitz theorem to show that
$(1-x^2)y_{n+2} - 2xy_{n+1} + n(n+1)y_n = 0$

2. Attempt any two parts of the following: **(10x2=20)**

(a) If $u = xyz$, $v = x^2+y^2+z^2$ and $w = x+y+z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

(b) Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

(c) Expand $\tan^{-1}(y/x)$ in the neighbourhood of (1, 1) up to and inclusive of second degree terms. Hence compute f (1.1, 0.9) approximately.

3. Attempts any two parts of the following: **(10x2=20)**

(a) Find the eigen values and eigen vectors of the matrix:

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

square speed and the most probable speed of the molecules of an ideal gas.

(b) Using Bose-Einstein statistics derive Planck's radiation law of black body.

(c) Discuss energy distribution of free electrons in metals. Derive an expression for Fermi energy.

4. Attempt any four parts from the following: (3x4=12)

(a) What do you mean by spontaneous and stimulated emission of radiations?

(b) What is the concept of three and four level Lasers?

(c) Explain the working of He-Ne lasers

(d) Distinguish the single mode and multimode fibers.

(e) Calculate the numerical aperture, acceptance angle and the critical angle of the fiber if the refractive indices of the core and cladding are 1.50 and 1.45 respectively.

(f) Explain the principle of holography and discuss its characteristics.

(b) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Hence compute A^{-1}.

(b) Reduce the matrix A to its normal form when

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix},$$

Hence find the rank of A.

4. Attempt any four parts of the following: **(5x4=20)**

(a) Evaluate $\int_0^2 \int_1^{e^x} dy\,dx$ by changing the order of integration.

(b) Determine the area of region bounded by the curves $xy=2$, $4y=x^2$ and $y = 4$.

(c) A triangular prism is formed by planes whose equations are $a.y = b.x$, $y=0$ and $x=a$. Find the volume of the prism between the planes $z=0$ and surface $z=c+xy$.

(d) Define Beta and Gamma functions. Prove that $\beta(m,n) = \frac{\Gamma_m \Gamma_n}{\Gamma_{(m+n)}}$.

(e) Evaluate the following integral by changing to spherical polar coordinates: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$

(f) Evaluate $I = \iiint_v x^{m-1} y^{n-1} z^{r-1} dx\,dy\,dz$, where v is the region in first octant bounded by sphere $x^2+y^2+z^2=1$ and the coordinate planes.

5. Answer any four of the following: **(5x4=20)**

(a) Prove that angular velocity at any point is equal to half the Curl of linear velocity at that point of the body.

(b) Define scalar and vector fields. Find the directional derivative of the function $f=x^2-y^2+2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4).

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Paper code: KAS-101 Roll No. 1 4 6 4 9

B. Tech
First Semester Examination, 2014-15
Engg. Physics-I

Time: 2 Hours **Total Marks: 50**

Note: Attempt all four questions. Take standard values wherever required if not provided.

1. Attempt any four parts from the following: (3 ½ x4=14)

(a) What was the aim of Michelson-Morley experiment? Explain its negative results.

(b) Explain why Galilean relativity failed to explain the actual results of Michelson Morley Experiment and hence obtained Lorentz transformations.

(c) Show that no signal can travel faster than light.

(d) Deduce Einstein's mass-energy relation, $E=mc^2$ and discuss it.

(e) Calculate the length of a meter rod in a frame of reference which is moving with a velocity equal to 0.6c, in a direction making an angle of 30° with the rod.

(f) Find the speed of a 0.1 MeV electrons according to classical and relativistic mechanics.

2. Attempt any two parts from the following: (6x2=12)

(a) Explain the wave-particle duality. Derive the de-Broglie wavelength of a particle as a function of temperature.

(b) State and explain Heisenberg's uncertainty principle? Using this principle show that electron cannot reside in an atomic nucleus.

(c) Derive time independent Schrodinger wave equation. Find the probabilities of finding a particle trapped in a box of length L in the region from 0.45L to 0.55L for the ground state and the first excited state.

3. Attempt any two parts from the following: (6x2=12)

(a) Using Maxwell-Boltzmann law of distribution of speeds, derive the expressions for the average speed, root mean