B.Tech.(Main & COP)
First Semester Examination, 2016-17
Engg. Mathematics-I

Time: 3 Hours **Total Marks: 100**

Note: Attempt all questions. Assume missing data suitably.

1. Attempt any four parts of the following: **(5x4=20)**

(a) If $y^{1/m} + y^{-1/m} = 2x$, prove that

$$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$$

(b) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}) = -\frac{1}{2}\cot u$.

(c) Find the value of $\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z}$ if $u(x,y,z) = \log(\tan x + \tan y + \tan z)$

(d) Trace the curve $y^2(a + x) = x^2(3a - x)$.

(e) If $x+y+z = u$, $y+z = uv$, $z = uvw$ then evaluate the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

(f) Check whether the following functions are functionally dependent. Also find the relation between them $u = x^2+y^2+z^2$, $v=x+y+z$, $w = yz+zx+xy$.

Attempt any two parts of the following: **(10x2=20)**

(a) Divide 120 into three parts so that the sum of their product taken two at a time shall be maximum.

(b) Expand the function $\log[\log(1+x)^{1/x}]$ upto first four terms.

(c) Find approximate value of $[(0.98)^2 + (2.01)^2 + (1.94)^2]^{1/2}$.

Attempt any four parts of the following: **(5x4=20)**

(a) Evaluate the double integral $\iint_R (x+y)\,dxdy$ where R is the region bounded by $y = 0$, $x + y = 2$, $y^2 = x$.

(b) Change the order of integration of $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$ and evaluate.

(c) Find area lying inside the Cardiod $r = a(1 + \cos\theta)$ and outside the circle $r = a$.

(d) Evaluate $\iiint \frac{dx\,dy\,dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$ over the first octant of the sphere $x^2+y^2+z^2=a^2$.

(e) Prove that $\Gamma(m)\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$.

(f) Prove that $\frac{\beta(p,q+1)}{q} = \frac{\beta(p+1,q)}{p} = \frac{\beta(p,q)}{p+q}$.

4. **Attempt any two parts of the following: (10x2=20)**

(a) Find the non-singular matrices P and Q such that the normal form of A is PAQ, where

$$A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$$ Hence, find its rank.

(b) Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

(c) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

5. **Attempt any two parts of the following: (10x2=20)**

(a) Prove the vector identities:-

(i) $\operatorname{div}(\vec{f} \times \vec{g}) = (\vec{g}\cdot \operatorname{curl} \vec{f}) - (\vec{f}\cdot \operatorname{curl} \vec{g})$.

(ii) $\operatorname{grad}(\operatorname{div} \vec{f}) = \operatorname{curl}(\operatorname{curl} \vec{f}) + \nabla^2 \vec{f}$

(b) Verify Green's theorem in xy-plane for $\int_C (xy+y^2)dx + x^2dy$] where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(c) Verify Stoke's theorem for $\vec{F} = xy\hat{i} - 2yz\hat{j} - zx\hat{k}$ where S is open surface of rectangular parallelepiped formed by $x = 0$, $x = 1$, $y = 0$, $y = 2$, $z = 0$, $z = 3$ above xy- plane.