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ENGINEERSGATEWAY

ENGG. PHYSICS

AS PER NEW SYLLABUS

(BAS-201)

UNIT 1

QUANTUM MECHANICS

TOPICS- PHASE VELOCITY AND GROUP VELOCITY

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SYLLABUS

Quantum Mechanics: Inadequacy of classical mechanics, Planck's theory of black body radiation(qualitative), Compton effect, de-Broglie concept of matter waves, Davisson and Germer Experiment, **Phase velocity and group velocity**, Time-dependent and time-independent Schrodinger wave equations, Physical interpretation of wave function, Particle in a one-Dimensional box.

Wave velocity (Phase velocity) :- When a monochro-matic wave (wave of a single freq. and wavelength) travels through a medium, its velocity of advancement in the medium is called the wave velocity. For example a plane harmonic wave travelling along the +ve x-direction is given by,

$$y = a \sin(\omega t - kx)$$

where $a \rightarrow$ amplitude $\omega = 2n\pi \rightarrow$ angular freq.

$k = \frac{2\pi}{\lambda} \rightarrow$ propagation constant

By defination, ratio of angular freq. (ω) to propogation const. k is given by, $v_p = \frac{\omega}{k}$

$y = A\sin(\omega t - kx)$

$(\omega t - kx)$ is the phase of wave-motion, therefore, the planes of constant phase (wavefronts) are defined as, $\omega t - kx = \text{constant}$

Now diff. this w.r.t 't' then, $\omega - k\frac{dx}{dt} = 0$

Ration velocity $= \frac{dx}{dt} = \frac{\omega}{k} \Rightarrow v_p = \frac{\omega}{k}$

Thus, the wave velocity is the velocity with which plane of constant phase advanced through the medium.

Phase Velocity :- A group consisting of a no. of waves of slightly different fr^{q}'s supperimposed over each other is called a wave packet.

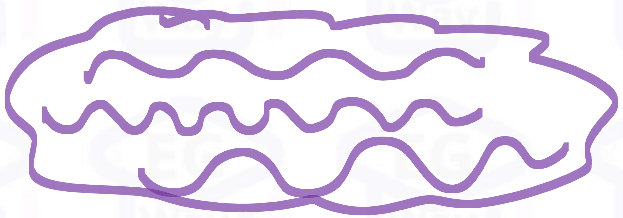
The velocity of each individual wave of a wave packet is known as phase velocity. It is define as,

$$v_p = \frac{\omega}{k}$$

Group Velocity :- The avg. velocity with which the wave packet propogates in the medium is known as group velocity. It is define as

$$v_g = \frac{d\omega}{dk}$$

Wave packet



Expression for Group velocity :- To find an expression for group velocity of a wave packet, consider a wave packet comprising of a group of two waves slightly different in angular frequencies. and phase velocity but of equal amplitude.

If ω_1 and ω_2 & k_1 and k_2 and a- amplitude

$$y_1 = a\sin(\omega_1 t - k_1 x) \quad \text{—①}$$

$$y_2 = a\sin(\omega_2 t - k_2 x) \quad \text{—②}$$

$$Y = y_1 + y_2$$

$$y = a\sin(\omega_1 t - k_1 x) + a\sin(\omega_2 t - k_2 x)$$

$\sin A + \sin B = 2\sin\frac{A+B}{2} \cdot \cos\frac{A-B}{2}$ then we get,

$$y = 2a\sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{k_1+k_2}{2}\right)x\right] \cdot \cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right]$$

$$y = 2a\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t - \left(\frac{k_1-k_2}{2}\right)x\right] \cdot \sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t - \left(\frac{k_1+k_2}{2}\right)x\right]$$

Let $\frac{\omega_1+\omega_2}{2} = \omega$ $\frac{k_1+k_2}{2} = k$ $(\omega_1 - \omega_2) = \Delta\omega$ $k_1 - k_2 = \Delta k$

$$y = 2a\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) \cdot \sin(\omega t - kx)$$

$$y = 2a\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right] \cdot \sin(\omega t - kx) \quad \text{---} \quad (a)$$

The velocity with which this wave packet moves or group velocity v_g of that wave packet is given by,

$$v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\Delta\omega}{\Delta k}$$

$$v_g = \frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

at zero cost