AE 682: Introduction to Thermoacoustics

RIJKE TUBE COMPUTATION PROJECT

M Vishnu Sankar - 18B030013

Course Instructor : Prof. Vineeth Nair

Indian Institute of Technology, Bombay

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1 Theory

1.1 Governing Equations

$$\gamma M \frac{\partial u'}{\partial t} + \frac{\partial P'}{\partial x} = 0$$

$$\frac{\partial P'}{\partial t} + \gamma M \frac{\partial u'}{\partial x} = k\left[\sqrt{\left|1/3 + u'_f(t-\tau)\right|} - \sqrt{\frac{1}{3}}\right]\delta(x - x_f)$$

where u'_f is given by

$$u'_f(t-\tau) = \sum_{i=1}^{\infty} \eta_i(t-\tau)cos(i\pi x_f)$$

Where
τ is the time delay
x_f is the heater location
$\gamma = 1.4$ for air
k is power delivered by the heater in Watts
$\zeta_j = \frac{1}{2\pi}\left(c_1\frac{\omega_j}{\omega_1} + c_2\sqrt{\frac{\omega_1}{\omega_j}}\right)$
$\omega_j = j \times \pi$

Expressing velocity and pressure in duct modes

$$u' = \sum_{j=1}^{\infty} \eta_j cos(j\pi x)$$

$$P' = \sum_{j=1}^{\infty} \frac{\gamma M}{j\pi} \dot{\eta}_j sin(j\pi x)$$

substituting the above equations modal expressions in the above equations we get

$$\frac{d\eta}{dt} = \dot{\eta} = f(t, \eta, \dot{\eta})$$

$$\frac{d\dot{\eta}}{dt} = -2\zeta\dot{\eta} - \omega^2\eta - \frac{2k}{\gamma M} j\pi\left[\sqrt{\left|1/3 + u'_f(t-\tau)\right|} - \sqrt{\frac{1}{3}}\right] sin(j\pi x_f) = g((t, \eta, \dot{\eta})$$

1.2 Solving the Equations using Range-Kutta Fourth Order Scheme

Let's say the initial conditions are given at $t_0 = 0$, $\eta_j(t_0)$ and $\dot{\eta}_j(t_0)$. Choose a time step $\Delta t = h$, $RK(O(4))$ method is given by:

$$k_1 = hf(t_i, \eta_i, \dot{\eta}_i)$$

$$l_1 = hg(t_i, \eta_i, \dot{\eta}_i)$$

$$k_2 = hf(t_i + \frac{h}{2}, \eta_i + \frac{k_1}{2}, \dot{\eta}_i + \frac{l_1}{2})$$

$$l_2 = hg(t_i + \frac{h}{2}, \eta_i + \frac{k_1}{2}, \dot{\eta}_i + \frac{l_1}{2})$$

$$k_3 = hf(t_i + \frac{h}{2}, \eta_i + \frac{k_2}{2}, \dot{\eta}_i + \frac{l_2}{2})$$

$$l_3 = hg(t_i + \frac{h}{2}, \eta_i + \frac{k_2}{2}, \dot{\eta}_i + \frac{l_2}{2})$$

$$k_4 = hf(t_i + h, \eta_i + k_3, \dot{\eta}_i + l_3)$$

$$l_4 = hg(t_i + h, \eta_i + k_3, \dot{\eta}_i + l_3)$$

$$\eta_{i+1} = \eta_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\dot{\eta}_{i+1} = \dot{\eta}_i + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

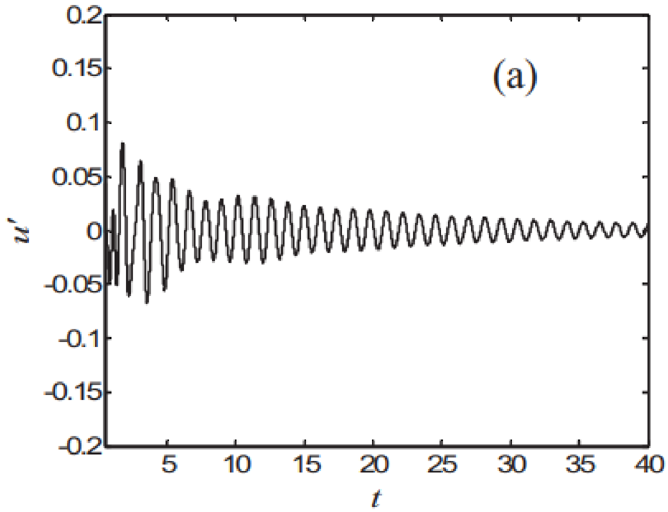
Let the final time be T_{max}, hence h can be given by

$$h = \frac{T_{max}}{N-1}$$

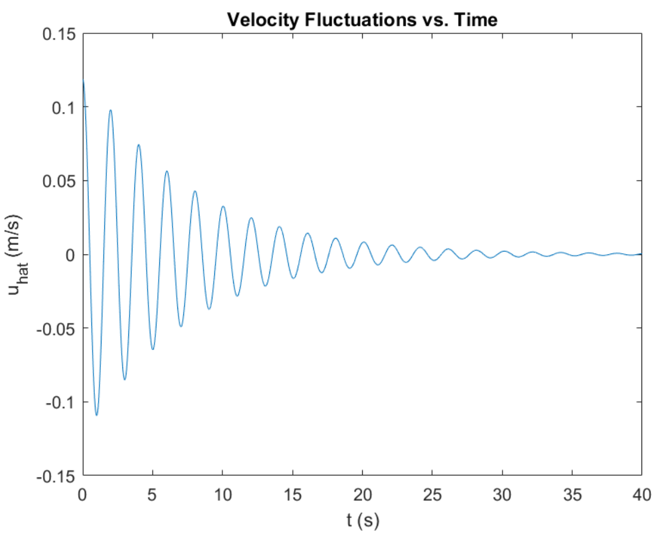
where N is the number of points needed

2 Validation and Results

2.1 Validation



(a) From Source[2]



(b) Code using RK(O(4))

Figure 1: Validating Code

First, the code was bench-marked from the results found in[2]. Here $c_1$and c_2 are taken as zero as per the definition given in [2]. Hence there is no need to define initial conditions of $\dot{\eta}_j$. The only initial condition is $\eta_1(0) = 0.15$ and only one mode is chosen for the problem. Value of k was chosen after experimentation, and it is 0.0002

2.2 Plots and Results

Modes	x_f	c_1	c_2	u_0	a_0	η_j	$\dot{\eta}_j$	τ
10	0.25	0.1	0.06	0.5	340	1.0	0.0	0.45π

The plots of u' and P' were obtained for different values of k to show

2.2.1 Stable System

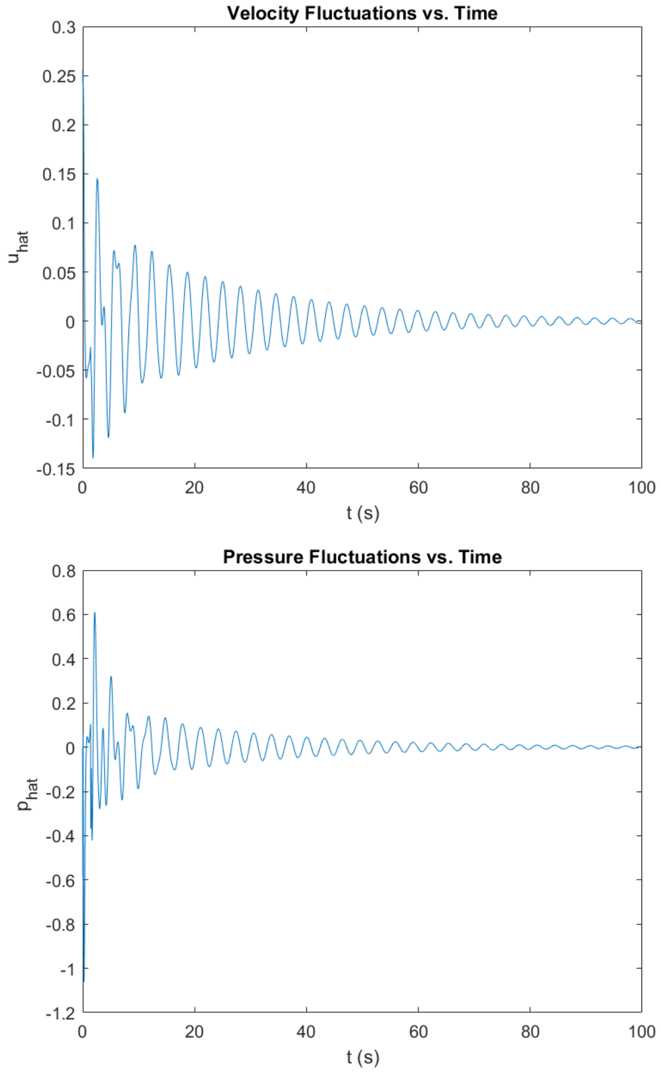


Figure 2: Stable System for $k = 0.005$

2.2.2 Unstable System reaching Limit Cycle

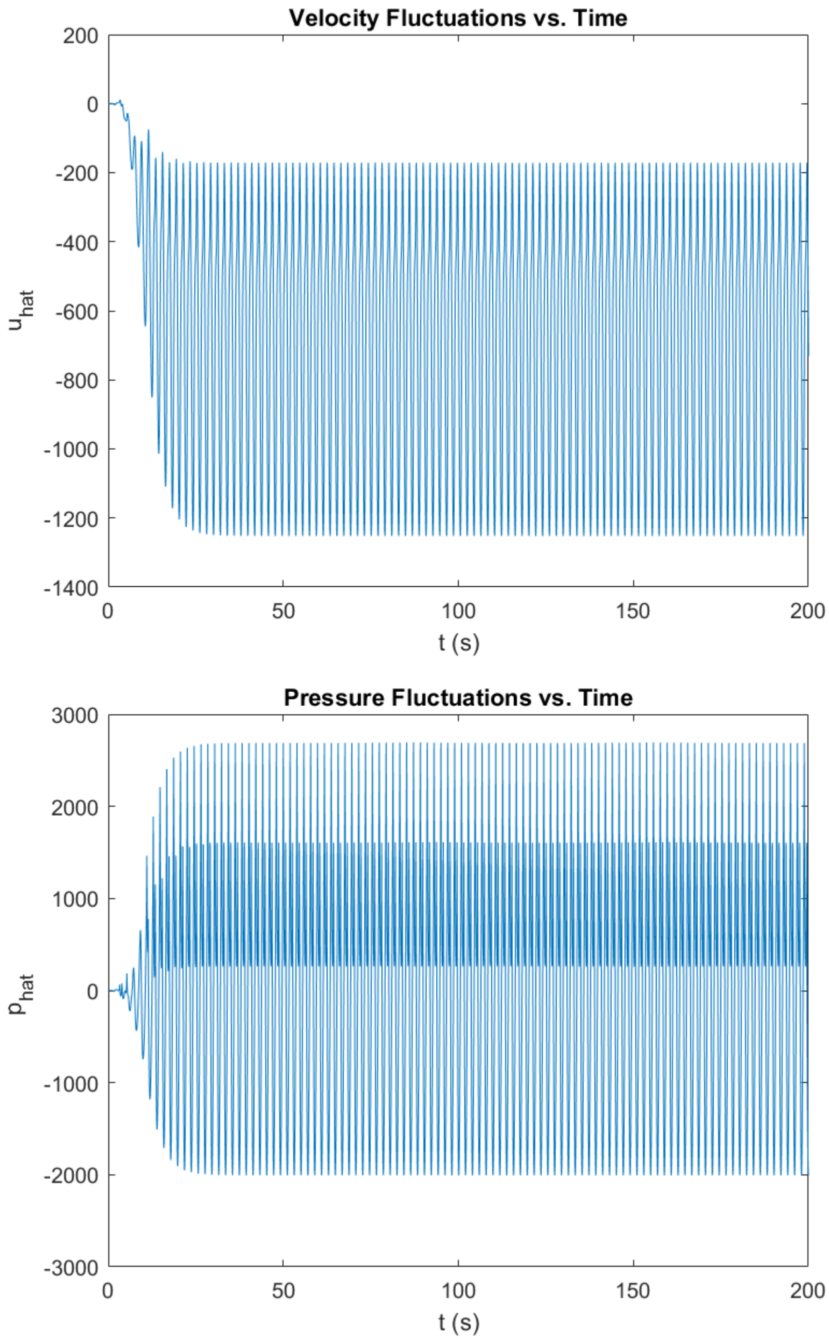


Figure 3: Unstable System, reaching limit cycle for $k = 0.1$

The pressure curve seems to be self intersecting but actually it oscillates so fast that it seems like. In a more shorter time frame we can see:

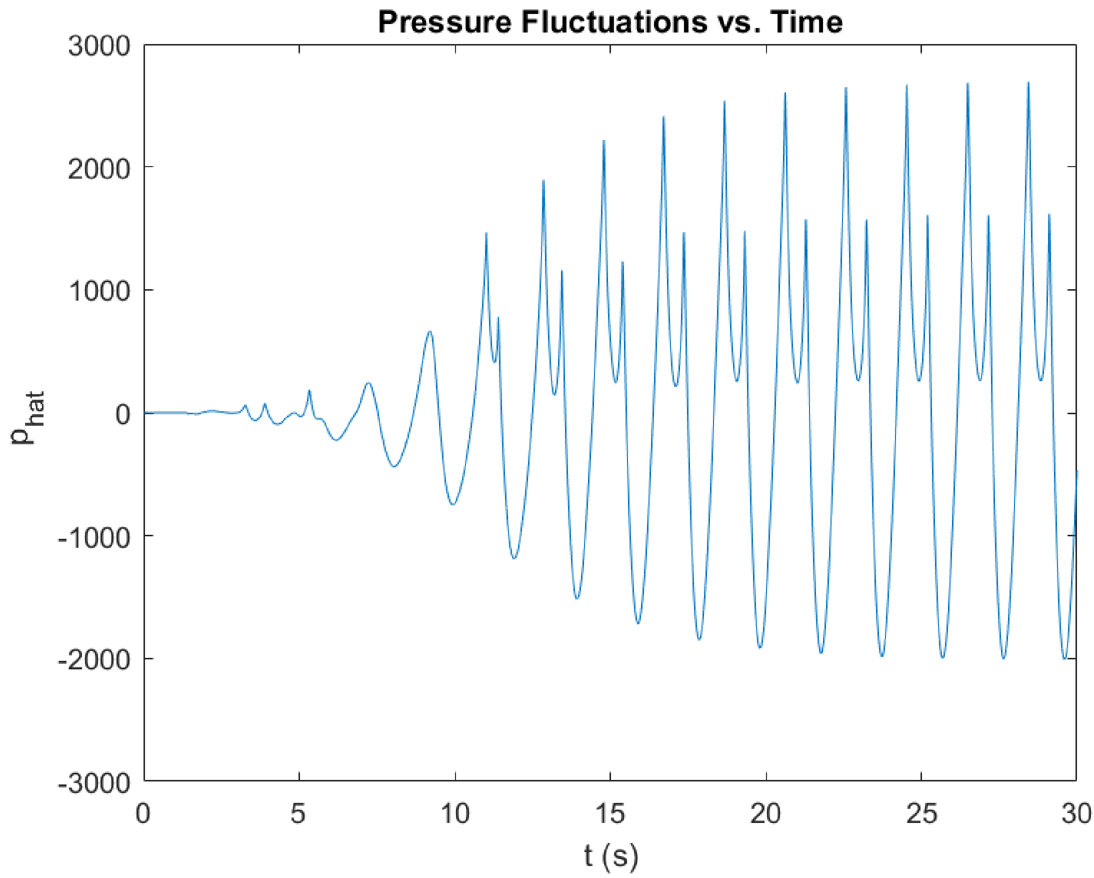


Figure 4: Pressure at short time period $k = 0.1$

2.3 Discussion

1. The stability of the system depends directly on the power delivered by the hot wire. High power destabilizes the system until it reaches a limit cycle. Very small heat release rate results in zero fluctuations at longer time intervals

2. Increasing the damping constants tends to stabilizes the system. Infact an unstable system becomes a stable system at high values of damping constant. Consider figure: 3, by changing the damping constants to $c_1 = 1.5$ and the $c_2 = 1/5$ it completely returns to a stable system as shown in the below images

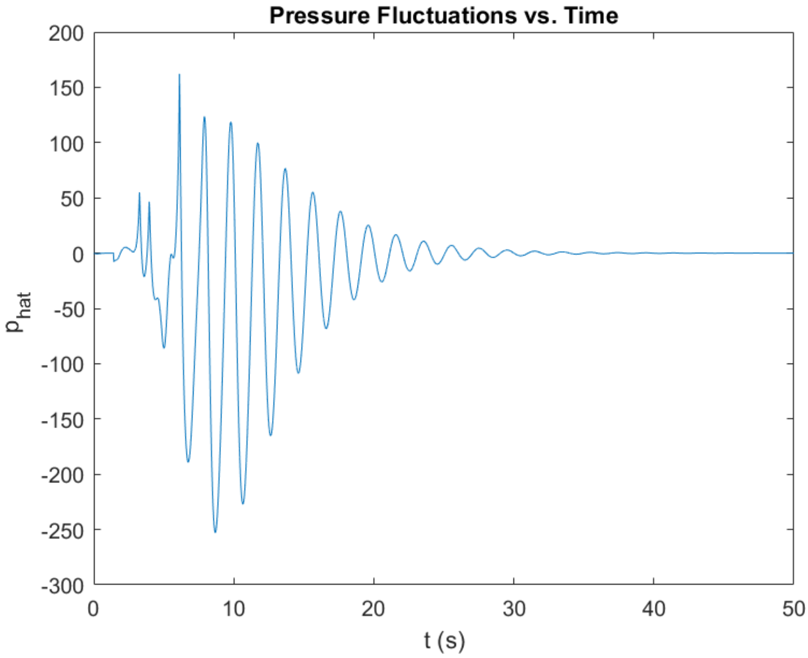


Figure 5: Due to high damping $k = 0.1$, $c_1 = 1.5$, $c_2 = 1.5$

3. A general comment on unstable system: The amplitude of the fluctuations continue to grow in time but eventually gets balanced by the damping mechanisms and hence the unstable system saturates to the limit cycle.

3 REFERENCES

[1] Thesis by GOPALAKRISHNAN E.A., *"BISTABILITY AND NOISE INDUCED TRANSITION IN A HORIZONTAL RIJKE TUBE"*, 2016

[2] Koushik Balasubramanian and R. I. Sujith, *"Thermoacoustic instability in a Rijke tube: Non-normality and nonlinearity"*, Physics of Fluids, 2007