Q] Consider 2 fuzzy sets

$$A = \left\{ \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.5}{4} \right\}$$

$$B = \left\{ \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.2}{3} + \frac{0}{4} \right\}$$

Algebraic Sum

$$\mu_{A+B}(x) = \left[\mu_A(x) + \mu_B(x)\right] - \left[\mu_A(x) \cdot \mu_B(x)\right]$$

$$= \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4} \right\} - \left\{ \frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + 0 \right\}$$

$$= \left\{ \frac{0.28}{1} + \frac{0.44}{2} + \frac{0.52}{3} + \frac{0.5}{4} \right\}$$

Algebraic product

$$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$$

$$= \left\{ \frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0}{4} \right\}$$

Bounded Sum

$$\mu_{A \oplus B}(x) = \min \left\{ 1, \mu_A(x) + \mu_B(x) \right\}$$

$$= \min \left\{ 1, \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4} \right\} \right\}$$

$$= \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4} \right\}$$

Bounded difference

$$\mu_{A \ominus B}(x) = \max \left\{ 0, \mu_A(x) - \mu_B(x) \right\}$$

$$= \max \left\{ 0, \left\{ \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4} \right\} \right\}$$

$$= \left\{ \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4} \right\}$$

Q] $\underset{\sim}{A} = \left\{ \frac{0.4}{x_1} + \frac{0.3}{x_2} + \frac{0.2}{x_3} \right\}$

$\underset{\sim}{B} = \left\{ \frac{0.3}{y_1} + \frac{0.5}{y_2} \right\}$

Cartesian product

$\mu_R(x, y_1) = \min\left(\mu_A(x_1), \mu_B(y_1)\right] = \min(0.4, 0.3) = 0.3$

$$\underset{\sim}{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.3 & 0.4 \\ 0.3 & 0.3 \\ 0.2 & 0.2 \end{bmatrix} \end{array}$$

Q] given fuzzy Relation

$$\underset{\sim}{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{bmatrix} y_1 & y_2 \\ 0.6 & 0.5 \\ 0.1 & 1 \\ 0.0 & 0.7 \end{bmatrix}_{3 \times 2} \qquad \underset{\sim}{S} = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.7 & 0.3 & 0.4 \\ 0.9 & 0.1 & 0.6 \end{bmatrix}_{2 \times 3} \end{array}$$

Composition between two of these.

a) Max - Min Composition

$$\underset{\sim}{T} = R \circ S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.6 & 0.5 \\ 0.1 & 1 \\ 0 & 0.7 \end{bmatrix} \end{array} \circ \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.7 & 0.3 & 0.4 \\ 0.9 & 0.1 & 0.6 \end{bmatrix} \end{array}$$

$\mu_{\underset{\sim}{T}}(x_1 z_1) = \max\left\{ \min\left[\mu_R(x_1 y_1), \mu_S(y_1 z_1)\right], \min\left[\mu_R(x_1 y_2), \mu_S(y_2 z_1)\right] \right\}$

$= \max \left\{ \min(0.6, 0.7), \min(0.5, 0.9) \right\}$

$= \max \{ 0.6, 0.5 \}$

$= \underline{\underline{0.6}}$

$$T = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.3 & 0.5 \\ 0.9 & 0.1 & 0.6 \\ 0.7 & 0.1 & 0.6 \end{bmatrix} \end{array}$$

Max - product composition

$\mu_T(x_1 z_1) = \max \left(\{ \mu_R(x_1, y_1) \cdot \mu_S(y_1, z_1), \mu_R(x_1, y_2) \cdot \mu_S(y_2 \cdot z_1) \right)$

$= \max \left(0.6 \times 0.7 , 0.5 \times 0.9 \right)$

$= \max (0.42, 0.45) = 0.45$

$$\underset{\sim}{T} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.45 & 0.18 & 0.3 \\ 0.9 & 0.1 & 0.6 \\ 0.63 & 0.07 & 0.42 \end{bmatrix} \end{array}$$

Q] consider fuzzy Relation

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\ \begin{bmatrix} 1 & 0.8 & 0 & 0.1 & 0.2 \\ 0.8 & 1 & 0.4 & 0 & 0.9 \\ 0 & 0.4 & 1 & 0 & 0 \\ 0.1 & 0 & 0 & 1 & 0.5 \\ 0.2 & 0.9 & 0 & 0.8 & 1 \end{bmatrix} \end{array}$$

Show above Relation is a tolerance Relation or not.

Ans) if both Reflexive & Symmetric property

$$\left.\begin{array}{l} \mu_R(x_i, y_i) = 1 \\ \mu_R(x_1, y_1) = 1 \end{array}\right\} \text{Reflexive.}$$

if upper and lower are same.

$$\mu_R(x_i, y_j) = \mu_R(x_j, y_i)$$

then Symmetric.