Defuzzification Methods

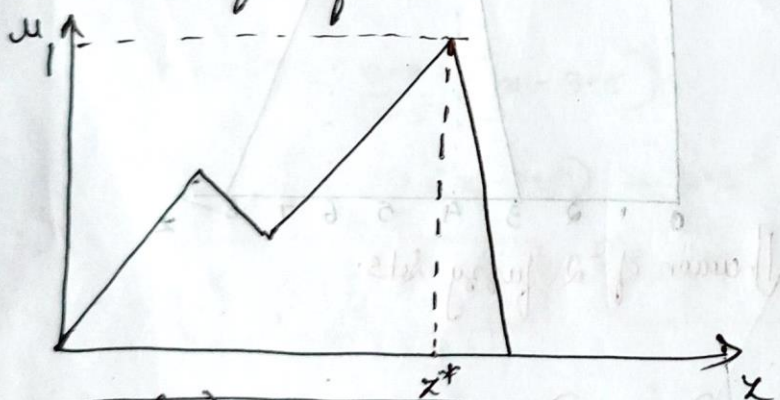
1. Max-membership principle
2. Centroid Method
3. Weighted average method
4. Mean-Max membership
5. Center of Sums
6. Center of largest area
7. First of Maximum, Last of maxima

(1) Max – Membership principle

Height Method

Maximum membership value.

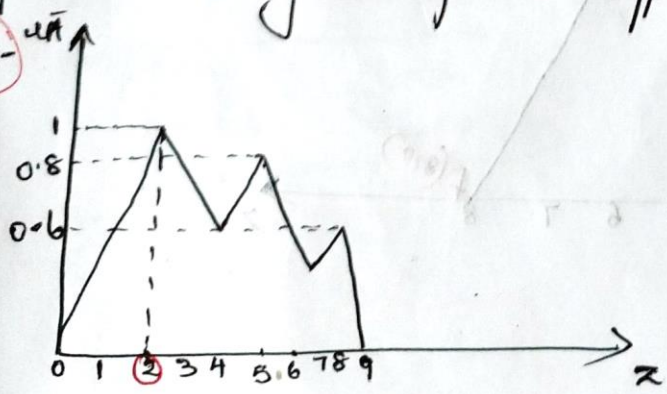
z^* = defazzified value.



$$\mu_A(z^*) \geq \mu_A(z) \quad \text{for all } z \in A$$

→ applicable only to peaked o/p function.

eg:-

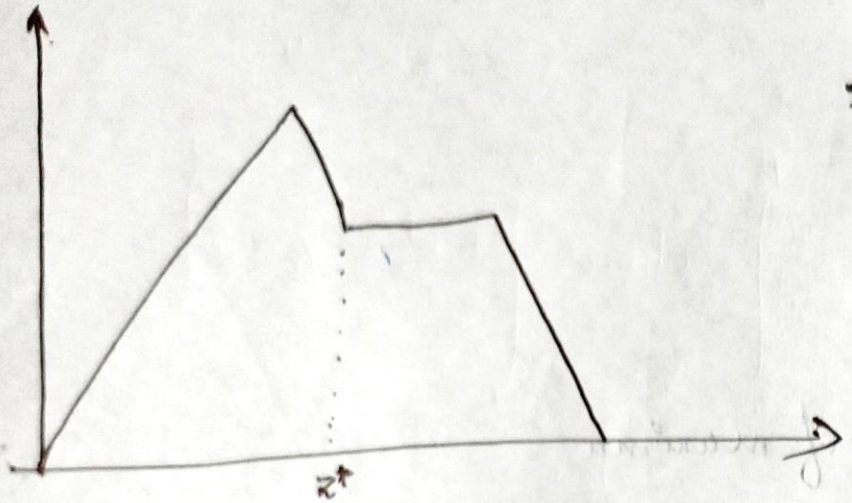


$z^* = 3$

merging of different fuzzy sets

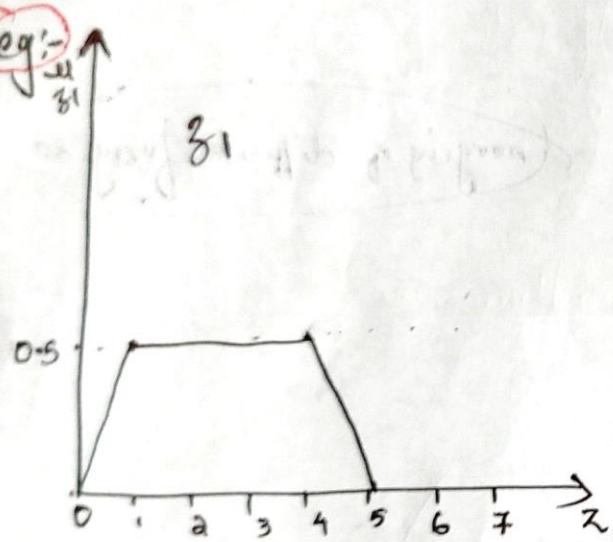
Centroid method

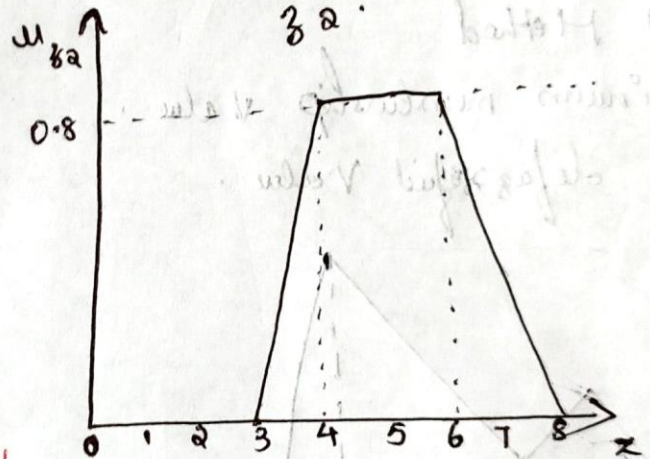
→ Centre of mass, Centre of area or Centre of Gravity.



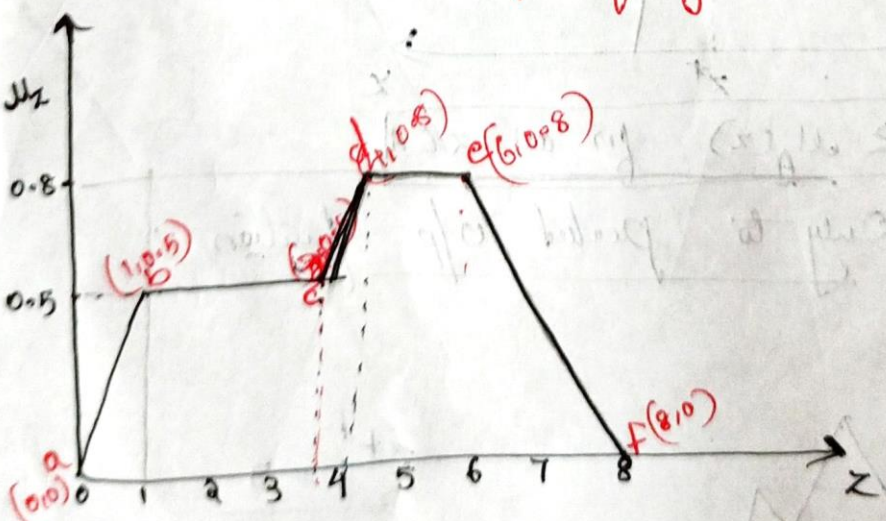
$$z^* = \frac{\int \mu_A \cdot z \, dz}{\int \mu_A \cdot dz}$$

eg:-





↓ union of 2 fuzzy sets.



apply equation of straight line.

ab ⇒ $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$x_1 = 0 \quad y_1 = 0$
$x_2 = 1 \quad y_2 = 0.5$

$\frac{y-0}{x-0} = \frac{0.5-0}{1-0} = 0.5$

$y = 0.5x$ range 0–1

bc ⇒ Straight parallel to x axis

$y = 0.5$ range 1 – 3.5

cd ⇒ $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$x_1 = 3.5 \quad y_1 = 0.5$
$x_2 = 4 \quad y_2 = 0.8$

$\Rightarrow \frac{y-0.5}{x-3.5} = \frac{0.8-0.5}{4-3.5} = \frac{0.3}{0.5}$

$y - 0.5 = \frac{0.3}{0.5}(x-3.5)$

$y = \frac{0.3}{0.5}(x-3.5) + 0.5$

$y = \frac{3}{5}x - \frac{8}{5}$ range 3.5 to 4

de ⇒ parallel to x axis

$y = 0.8$ range 4 – 6

ef ⇒

$x_1 = 8 \quad y_1 = 0$
$x_2 = 6 \quad y_2 = 0.8$

$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{y_2-x_1}$

$y = -0.4x + 3.2$ range 6 – 8

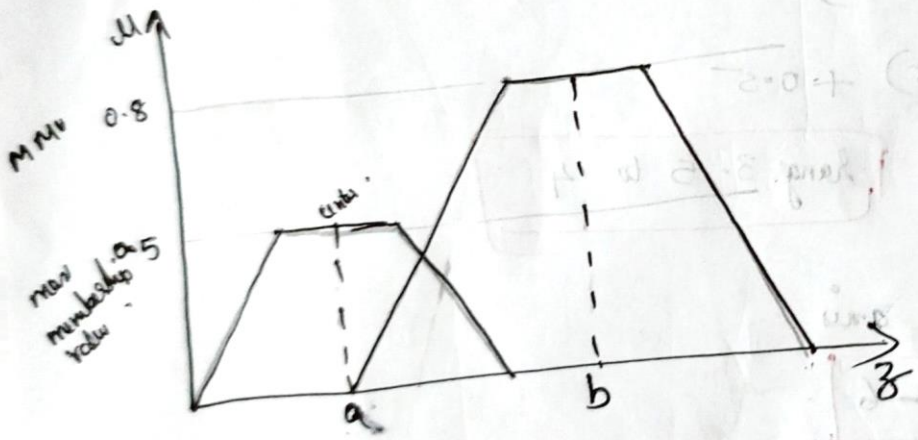
$$z^* = \frac{\int \mu_{\tilde{z}} \cdot z \, dz}{\int \mu_{\tilde{z}} \cdot dz}$$

Putting x as z.

$$= \frac{\int_0^1 0.5z \cdot z \, dz + \int_1^{3.5} 0.5 \cdot z \, dz + \int_{3.5}^{4} \left(\frac{3}{5}z - \frac{8}{5}\right) \cdot z \, dz + \int_4^6 0.8 \cdot z \, dz + \int_6^8 (-0.4z + 3.2) z \, dz}{\int_0^1 0.5z \, dz + \int_1^{3.5} 0.5 \, dz + \int_{3.5}^{4} \left(\frac{3}{5}z - \frac{8}{5}\right) dz + \int_4^6 0.8 \, dz + \int_6^8 (-(0.4z) + 3.2) \, dz}$$

$$= 4.151.$$

(3) weighted average method



$$z^* = \frac{0.5 \times a + 0.8 \times b}{0.5 + 0.8}$$

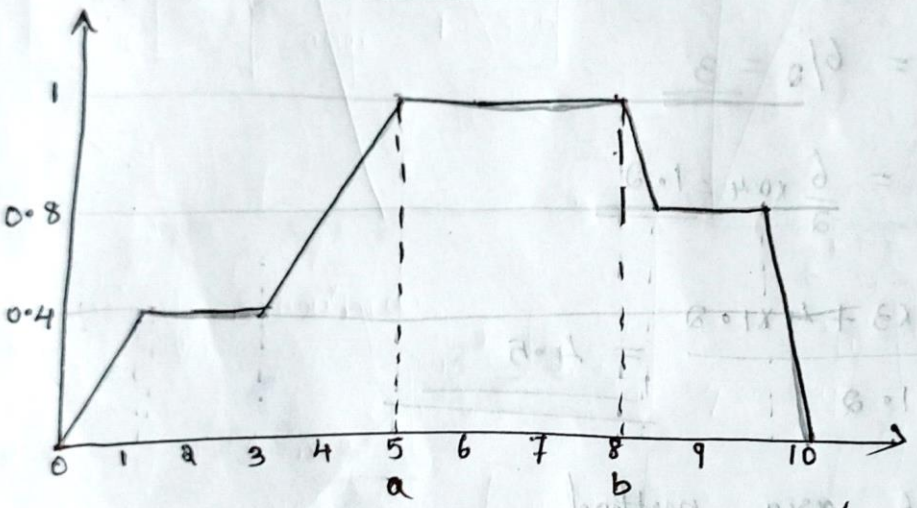
$$z^* = \frac{\sum \mu_A(z) \cdot z}{\sum \mu_A(z)}$$

applicable only for Symmetric functions.

eg:- if $a = 3$
$b = 8$

then $z^* = \frac{0.5 \times 3 + 0.8 \times 8}{0.5 + 0.8} = 6.0769$

(4) Mean - Max membership (Middle of maxima)

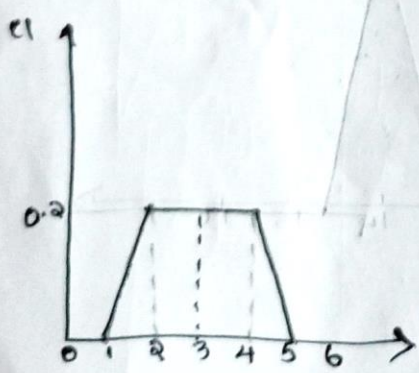


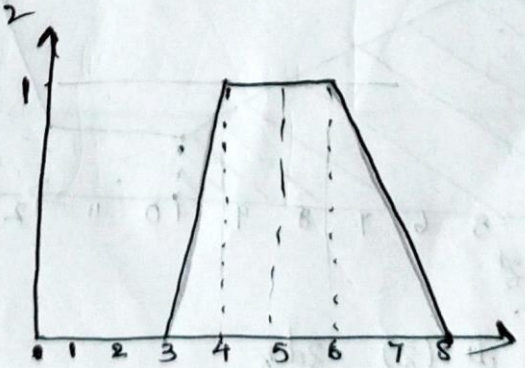
middle value of maximum membership function

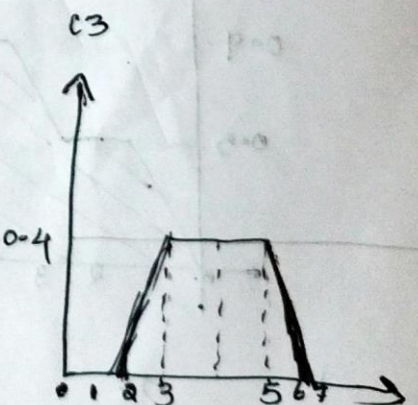
Only applicable to Symmetric function.

$z^* = \frac{a+b}{2} = \frac{5+8}{2} = 13/2 = 6.5$

(5) Centre of Sum Method







need to find Area & geometrical centre.

A_1, A_2, A_3 $z_1 = 3$ $z_2 = 5$ $z_3 = 4$

$$z^* = \frac{\sum_{i=1}^{n} z_i A_{ci}}{\sum_{i=1}^{n} A_{ci}}$$

area of trapezium $= \frac{1}{2} bh$

$= \frac{1}{2}(b_1 + b_2)h$

$A_1 = \frac{1}{2}(2+4)0.2 = \frac{1}{2} \times 6 \times 0.2 = \underline{0.6}$

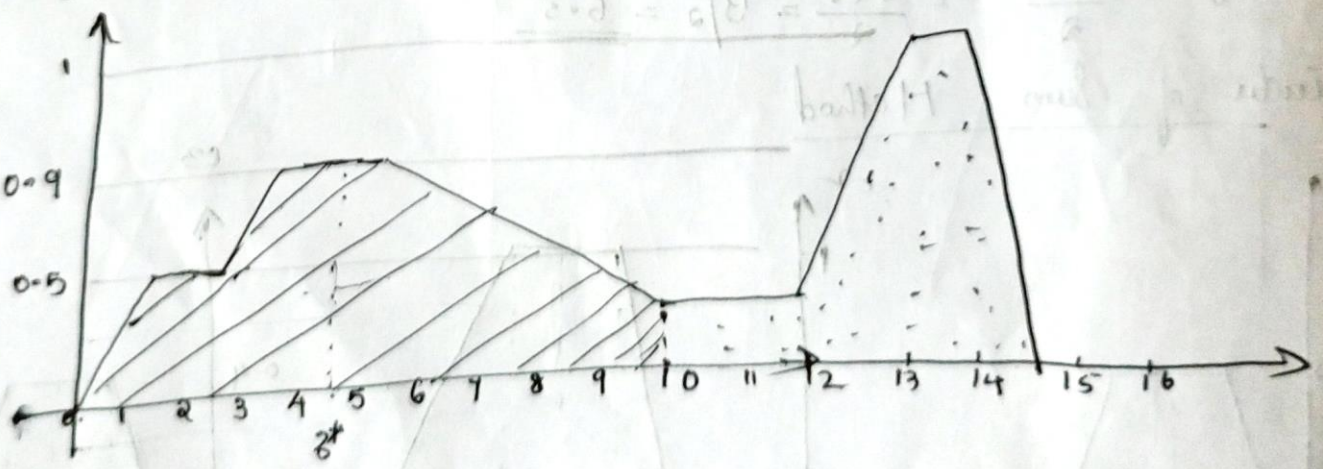
$A_2 = \frac{1}{2}(2+4)1 = 6/2 = \underline{3}$

$A_3 = \frac{1}{2}(2+4)0.4 = \frac{6}{2} \times 0.4 = \underline{1.2}$

$$z^* = \frac{3 \times 0.6 + 5 \times 3 + 4 \times 1.2}{0.6 + 3 + 1.2} = 4.5$$

(6) Center of largest area method

finding the largest area and find the center of that area.

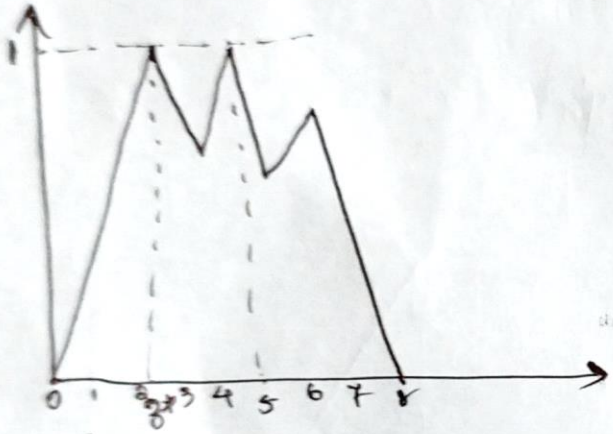


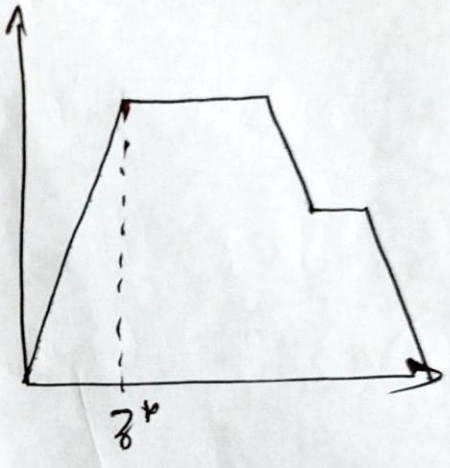
$$z^* = \frac{\int \mu_c(z) \cdot z \, dz}{\int \mu_c(z) \, dz}$$

First of Maxima (last of maxima)

infimum
lowest upper bound

$$z^* = \inf_{z \in I} \left\{ z \in z \mid u_{cv} = \text{highest}(z) \right\}$$





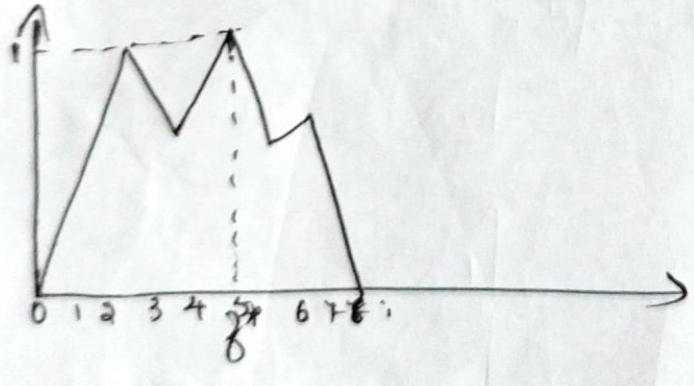
first of maxima

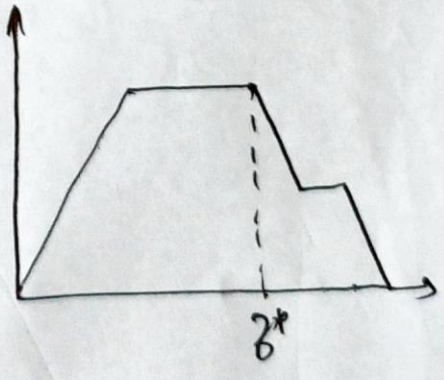
$z^* = 2$.

last of Maxima

$$z^* = \sup_{z \in z} \left\{ z \in z \mid u_c = \text{highest}(C_{cv}) \right\}$$

Supremum
largest upper bound





$z^* = 5$