Module 3:Fuzzy Logic & Defuzzification

- Fuzzy sets – properties, operations on fuzzy set.
- Fuzzy membership functions,
- Methods of membership value assignments
  - intuition,
  - inference,
  - Rank Ordering.
- Fuzzy relations– operations on fuzzy relation.
- Fuzzy Propositions.
- Fuzzy implications.
- Defuzzification– Lamda cuts,
- Defuzzification methods.

Fuzzy Logic

Fuzzy Logic

- Fuzzy- Things which are not clear or vauge
- Fuzzy Logic-Computing based on the “degree of truthness” rather than the usual true or false.
- It is a superset of conventional logic that has been extended to handle the concept of partial truth
- Partial truth means truth values between completely true or completely false
- Fuzzy set infer some knowledge about the membership values
- Returns closed set [0,1].values in between 0 and 1 represent Fuzziness.
- Fuzziness describes the ambiguity of an event.

Fuzzy Logic

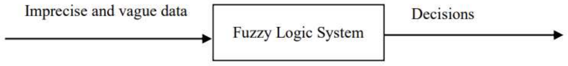


Figure : A fuzzy logic system accepting imprecise data and providing a decision

- In 1965 Lotfi Zadeh, published his famous paper “Fuzzy sets”.
- This new logic for representing and manipulating fuzzy terms was called fuzzy logic, and Zadeh became the Master/Father of fuzzy logic.
- Fuzzy logic is the logic underlying approximate, rather than exact, modes of reasoning.
- It operates on the concept of membership.
- The membership was extended to possess various "degrees of membership" on the real continuous interval [0, l].

Fuzzy Logic

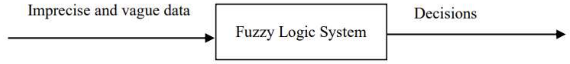


Figure : A fuzzy logic system accepting imprecise data and providing a decision

- In fuzzy systems, values are indicated by a number (called a truth value) ranging from 0 to l, where 0.0 represents absolute falseness and 1.0 represents absolute truth.

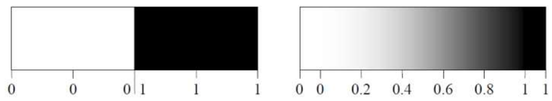
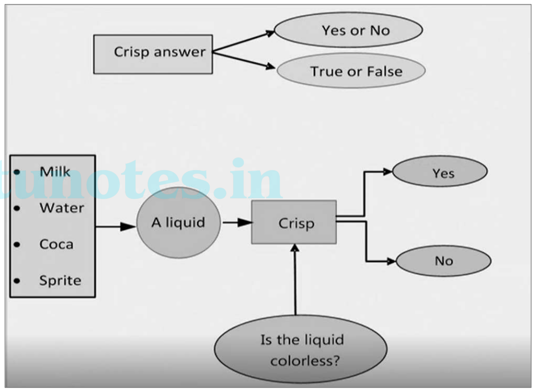


Figure : (a) Boolean Logic (b) Multi-valued Logic

Crisp Set

Crisp Set

- Crisp set has complete membership YES/NO→1/0
- Exact boundaries are there
- No uncertainty about the location of the set boundaries
- A={1,3,5,7,9,11...} is a crisp set/classic set
- 5 € A ? Y→1
- 12 € A ? N→0



Concept of Crisp Set

- To understand the concept of Fuzzy set ,it is better if we clear our idea about Crisp Set.
- Suppose

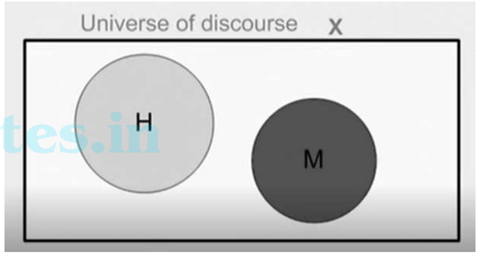
  X=The entire population of India

  M=All Muslim Population

  ={m1,m2,m3,....mk}

  H=All Hindu Population

  ={h1,h2,....hl}



- Here all are the set of finite number of individuals.
- Such a set is called Crisp Set.

Concept of Crisp Set

A *set* is defined as a collection of objects, which share certain characteristics.

A *classical set* is a collection of distinct objects.

Each individual entity in a set is called a *member* or an *element* of the set.

Collection of elements in the universe (U) is called *whole set*.

Number of elements in U is called *cardinal number*.

Collection of elements within a set are called *subsets*.

Classical set is defined as the U is spitted in to two groups: *members and nonmembers*.

No partial membership exists.

Defining a classical Set/Crisp Set

1. The list of all the members of a set may be given.

$$A = \{2, 4, 6, 8, 10\}$$

2. The properties of the set elements may be specified.

$$A = \{x | \ x \ is \ prime \ number < 20 \ \}$$

3. The formula for the definition of a set may be mentioned.

$$A = \{x_i = \frac{x_i + 1}{5}, \ i{=}1 \ to \ 10, \ where \ x_i = 1 \ \}$$

4. The set may be defined on the basis of the results of a logical operation.

$$A = \{x | \ x \ is \ an \ element \ belonging \ to \ P \ AND \ Q \ \}$$

Operations on Classical Set

1. Union

$$A \cup B = \{x | x \in A \ or \ x \in B\}$$

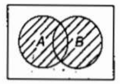


Figure Union of two sets.

2. Intersection

$$A \cap B = \{x | x \in A \ and \ x \in B\}$$

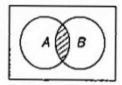


Figure Intersection of two sets.

3. Complement

$$\overline{A} = \{x | x \notin A, \ x \in X\}$$

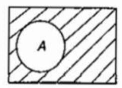
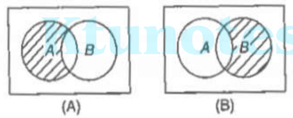


Figure Complement of set A.

Operations on Classical Set

4. Difference(Subtraction)

$$A|B \ or \ (A-B) = \{x | x \in A \ and \ x \notin B\} = A - (A \cap B)$$
$$B|A \ or \ (B-A) = \{x | x \in B \ and \ x \notin A\} = B - (B \cap A)$$



(A) Difference $A|B$ or $(A-B)$; (B) difference $B|A$ or $(B-A)$.

Properties of Classical Set

- Commutativity

$$A \cup B = B \cup A; A \cap B = B \cap A$$

- Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C; A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributivity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C); A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- Idempotency

$$A \cup A = A; A \cap A = A$$

- Transitivity

$$A \subseteq B \subseteq C, \ then \ A \subseteq C$$

Properties of Classical Set

- Identity $A \cup \phi = A; A \cap \phi = \phi$

$$A \cup X = X; A \cap X = A$$

- Involution $\overline{\overline{A}} = A$
- Law of excluded middle

$$A \cup \overline{A} = X$$

- Law of contradiction

$$A \cap \overline{A} = \phi$$

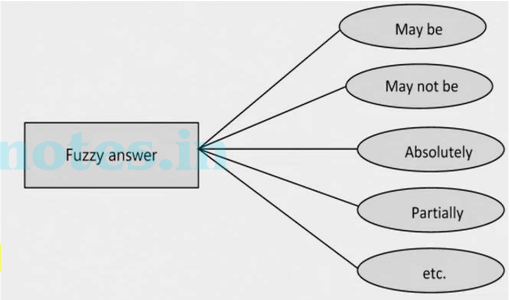
- DeMorgan's law

$$|\overline{A \cap B}| = \overline{A} \cup \overline{B}$$
$$|\overline{A \cup B}| = \overline{A} \cap \overline{B}$$

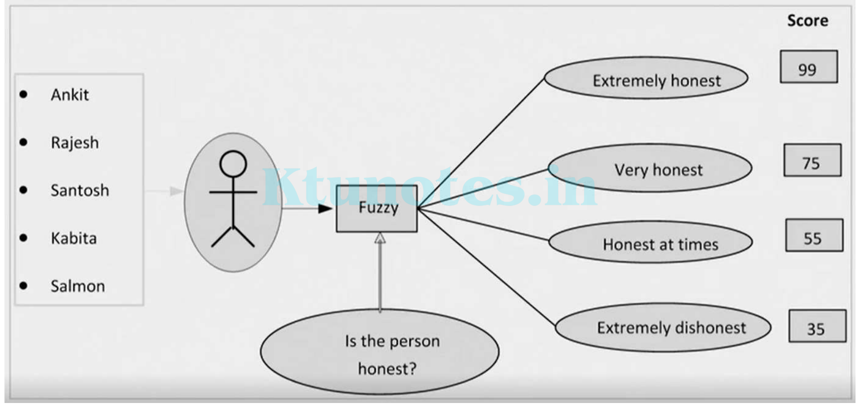
Fuzzy Set

Fuzzy Set

- Fuzzy set has set of partial membership
- Fuzzy set is a set having degrees of membership between 1 and 0.
- A={set of all healthy people}
- B={set of all Young people}
- Fuzzy set can be viewed as a generalization of crisp/classic/conventional set.
- There is uncertainty in the location of set boundaries.



Fuzzy Set Example



Fuzzy Set Example

- Let us discuss about fuzzy set.
- X = All students in SJC
- S = All Good students.
- S = {(s, g(s)) | s ∈ X } and g(s) is a measurement of goodness of the students.
- Example:
- S = {(Raja, 0.8), (Kabita, 0.7) ,(Salman, 0.1) , (Ankit, 0.9)} etc.
- A crisp set is a fuzzy set, but, a fuzzy set is not necessarily a crisp set.
- Example:
- H = {(h1, 1 , (h2, 1) … … … , (hL,1)}
- Person = {(p1, 0 , (p2, 0) … … … , (pN, 0)}
- In crisp set, the elements are with extreme values of degree of membership namely either 1 or 0.

Fuzzy Set Example

- What is the Temperature today??
- How is the Weather condition??
- ANSWER VARIES FROM PERSON TO PERSON..FUZZY ANSWERS
- Other examples
  - Temperature
  - Pressure
  - Taste of Apple
  - Sweetness of Orange
  - Weight of Mango
- Degree of membership values lie in the range [0…1].

Fuzzy Membership function

If X is a universe of discourse and $x \in X$, then a fuzzy set A in X is defined as a set of ordered pairs, that is $A = \{(x, \mu_A(x)) | x \in X\}$ where $\mu_A(x)$ is called the membership function for the fuzzy set A.

- Example:
- X = All cities in India
- A = City of comfort
- A={(New Delhi, 0.7), (Bangalore, 0.9), (Chennai, 0.8), (Hyderabad, 0.6),(Kolkata, 0.3), (Kharagpur, 0)}
- The membership values may be of discrete values
- A fuzzy set is completely characterized by its membership function (sometimes abbreviated as MF and denoted as μ).
- There are other ways of representation of Fuzzy set ,all representations allows partial membership only.

Fuzzy Set

A fuzzy set $\underset{\sim}{A}$ in the universe of discourse U can be defined as

$$\underset{\sim}{A} = \left\{\left(x, \mu_{\underset{\sim}{A}}(x)\right) \mid x \in X\right\}$$

where $\mu_{\underset{\sim}{A}}(x)$ is the degree of membership of x in $\underset{\sim}{A}$ and it indicates the degree that x belongs to $\underset{\sim}{A}$.

In the fuzzy theory, fuzzy set A of universe X is defined by function $\mu_{\underset{\sim}{A}}(x)$ called the membership function of set A.

$\mu_{\underset{\sim}{A}}(x): X \to [0, 1]$, where $\mu_{\underset{\sim}{A}}(x) = 1$ if x is totally in A;

$\mu_{\underset{\sim}{A}}(x) = 0$ if x is not in A;

$0 < \mu_{\underset{\sim}{A}}(x) < 1$ if x is partly in A.

Fuzzy Membership function

There are other ways of representation of fuzzy sets; all representations allow partial membership to be expressed.

When the universe of discourse U is discrete and finite, fuzzy set $\underset{\sim}{A}$ is given as follows:

$$\underset{\sim}{A} = \left\{\frac{\mu_{\underset{\sim}{A}}(x_1)}{x_1} + \frac{\mu_{\underset{\sim}{A}}(x_2)}{x_2} + \frac{\mu_{\underset{\sim}{A}}(x_3)}{x_3} + \cdots\right\} = \left\{\sum_{i=1}^{n} \frac{\mu_{\underset{\sim}{A}}(x_i)}{x_i}\right\}$$

Membership Degree

$$\underset{\sim}{A} = \left\{\frac{0.9}{x_1} + \frac{0.8}{x_2} + \frac{0}{x_3} + \cdots\right\}$$

Elements of A

μ =0 Empty Fuzzy Set

μ =1 ,Universal Fuzzy set

Universe of Discourse is defined as the set X of possible values, that can take for variable x

Crisp set vs Fuzzy set

Crisp set	Fuzzy set
$S = \{s \mid s \in X\}$	$F = (s, \mu(s)) \mid s \in X$ and $\mu(s)$ is the degree of s.
It is a collection of elements.	It is a collection of ordered pairs.
Inclusion of an element $s \in X$ into S is crisp, that is, has strict boundary yes or no.	Inclusion of an element $s \in X$ into F is fuzzy, that is, if present, then with a degree of membership.

Properties of Fuzzy Set

Properties of Fuzzy Set

Fuzzy sets follow the same properties as crisp sets except for the law of excluded middle and law of contradiction. That is, for fuzzy set $\underset{\sim}{A}$

$$\underset{\sim}{A} \cup \bar{\underset{\sim}{A}} = U; \qquad \underset{\sim}{A} \cap \bar{\underset{\sim}{A}} = \emptyset$$

1. **Commutativity** $\quad \underset{\sim}{A} \cup \underset{\sim}{B} = \underset{\sim}{B} \cup \underset{\sim}{A}; \ \underset{\sim}{A} \cap \underset{\sim}{B} = \underset{\sim}{B} \cap \underset{\sim}{A}$

2. **Associativity**

$$\underset{\sim}{A} \cup \left(\underset{\sim}{B} \cup \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) \cup \underset{\sim}{C}$$
$$\underset{\sim}{A} \cap \left(\underset{\sim}{B} \cap \underset{\sim}{C}\right) = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cap \underset{\sim}{C}$$

3. **Distributivity**

$$\underset{\sim}{A} \cup \left(\underset{\sim}{B} \cap \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) \cap \left(\underset{\sim}{A} \cup \underset{\sim}{C}\right)$$
$$\underset{\sim}{A} \cap \left(\underset{\sim}{B} \cup \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cap \underset{\sim}{B}\right) \cup \left(\underset{\sim}{A} \cap \underset{\sim}{C}\right)$$

Properties of Fuzzy Set

4. **Idempotency** $\quad \underset{\sim}{A} \cup \underset{\sim}{A} = \underset{\sim}{A}; \ \underset{\sim}{A} \cap \underset{\sim}{A} = \underset{\sim}{A}$

5. **Transitivity** $\quad If\ \underset{\sim}{A} \subseteq \underset{\sim}{B} \subseteq \underset{\sim}{C}, then\ \underset{\sim}{A} \subseteq \underset{\sim}{C}$

6. **Identity** $\quad \underset{\sim}{A} \cup \emptyset = \underset{\sim}{A}\ \ and\ \underset{\sim}{A} \cup U = U$

$\quad \underset{\sim}{A} \cap \emptyset = \emptyset\ and\ \underset{\sim}{A} \cap U = \underset{\sim}{A}$

7. **Involution (double negation)** $\quad \bar{\bar{\underset{\sim}{A}}} = \underset{\sim}{A}$

8. **DeMorgans law**

$$\left|\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}\right| = \bar{\underset{\sim}{A}} \cup \bar{\underset{\sim}{B}}; \quad \left|\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}\right| = \bar{\underset{\sim}{A}} \cap \bar{\underset{\sim}{B}};$$

Fuzzy Set Operations

Fuzzy Set Operations

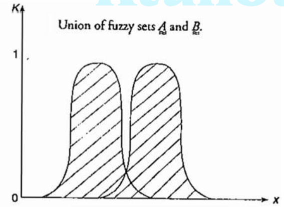
1. Union

The union of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$, denoted by $\underset{\sim}{A} \cup \underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A}\cup\underset{\sim}{B}}(x) = max\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right] = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x)\ for\ all\ x \in U$$

where $\vee$ indicates max operator.

The Venn diagram for union operation of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is shown below figure.



Fuzzy Set Operations

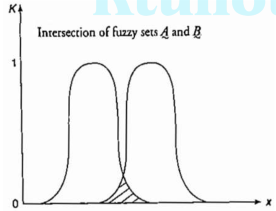
2. Intersection

The union of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$, denoted by $\underset{\sim}{A} \cap \underset{\sim}{B}$, is defined as

$$\mu_{\underset{\sim}{A}\cap\underset{\sim}{B}}(x) = min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\right] = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x)\ for\ all\ x \in U$$

where $\wedge$ indicates min operator.

The Venn diagram for intersection operation of fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is shown below figure.



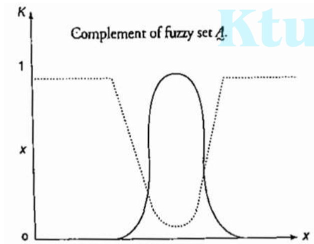
Fuzzy Set Operations

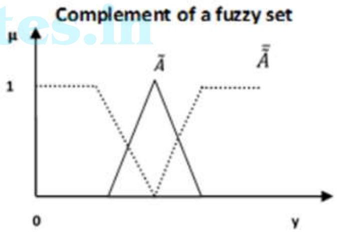
3. Complement

When $\mu_{\underset{\sim}{A}}(x) \in [0,1]$, the complement of $\underset{\sim}{A}$, denoted as $\bar{A}$ is defined by,

$$\mu_{\bar{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)\ for\ all\ x \in U$$

The Venn diagram for complement operation of fuzzy set $\underset{\sim}{A}$ is shown below figure.





More Operations on Fuzzy Sets

a. Algebraic sum

The algebraic sum $\left(\underset{\sim}{A} + \underset{\sim}{B}\right)$ of fuzzy sets, fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A}+\underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x) - \mu_{\underset{\sim}{A}}(x).\mu_{\underset{\sim}{B}}(x)$$

b. Algebraic product

The algebraic product $\left(\underset{\sim}{A}.\underset{\sim}{B}\right)$ of fuzzy sets, fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A}.\underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x).\mu_{\underset{\sim}{B}}(x)$$

c. Bounded sum

The bounded sum $\left(\underset{\sim}{A} \oplus \underset{\sim}{B}\right)$ of fuzzy sets, fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A}\oplus\underset{\sim}{B}}(x) = \min\{1, \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x)\}$$

d. Bounded difference

The bounded difference $\left(\underset{\sim}{A} \odot \underset{\sim}{B}\right)$ of fuzzy sets, fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is defined as

$$\mu_{\underset{\sim}{A}\odot\underset{\sim}{B}}(x) = \max\{0, \mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x)\}$$

Problems of Fuzzy Set

Problems

Consider two given fuzzy sets

$$A = \left\{\frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8}\right\}$$

$$B = \left\{\frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8}\right\}$$

Perform union, intersection, difference and complement over fuzzy sets A and B.

(a) Union $A \cup B = \max\{\mu_A(x), \mu_B(x)\}$

$$= \left\{\frac{1}{2} + \frac{0.4}{4} + \frac{0.5}{6} + \frac{1}{8}\right\}$$

(b) Intersection $A \cap B = \min\{\mu_A(x), \mu_B(x)\}$

$$= \left\{\frac{0.5}{2} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.2}{8}\right\}$$

(c) Complement

$$\bar{A} = 1 - \mu_A(x) = \left\{\frac{0}{2} + \frac{0.7}{4} + \frac{0.5}{6} + \frac{0.8}{8}\right\}$$

$$\bar{B} = 1 - \mu_B(x) = \left\{\frac{0.5}{2} + \frac{0.6}{4} + \frac{0.9}{6} + \frac{0}{8}\right\}$$

(d) Difference $A|B = A \cap \bar{B} = \left\{\frac{0.5}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0}{8}\right\}$

$$B|A = B \cap \bar{A} = \left\{\frac{0}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{0.8}{8}\right\}$$

Problems

Given the two fuzzy sets

$$B_1 = \left\{\frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\right\}$$

$$B_2 = \left\{\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}\right\}$$

find the following:

(a) $B_1 \cup B_2$; (b) $B_1 \cap B_2$; (c) $\bar{B}_1$;

(d) $\bar{B}_2$; (e) $B_1|B_2$; (f) $\overline{B_1 \cup B_2}$;

(g) $\overline{B_1 \cap B_2}$; (h) $B_1 \cap \bar{B}_1$; (i) $B_1 \cup \bar{B}_1$;

(j) $B_2 \cap \bar{B}_2$; (k) $B_2 \cup \bar{B}_2$

(a) $B_1 \cup B_2 = \left\{\frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\right\}$

(b) $B_1 \cap B_2 = \left\{\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}\right\}$

(c) $\bar{B}_1 = \left\{\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}\right\}$

(d) $\bar{B}_2 = \left\{\frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0}\right\}$

(e) $B_1|B_2 = B_1 \cap \bar{B}_2$

$$= \left\{\frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\right\}$$

(f) $\overline{B_1 \cup B_2} = \left\{\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}\right\}$

(g) $\overline{B_1 \cap B_2} = \left\{\frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0}\right\}$

Problems

Given the two fuzzy sets

$$B_1 = \left\{\frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\right\}$$

$$B_2 = \left\{\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}\right\}$$

find the following:

(a) $B_1 \cup B_2$; (b) $B_1 \cap B_2$; (c) $\bar{B}_1$;

(d) $\bar{B}_2$; (e) $B_1|B_2$; (f) $\overline{B_1 \cup B_2}$;

(g) $\overline{B_1 \cap B_2}$; (h) $B_1 \cap \bar{B}_1$; (i) $B_1 \cup \bar{B}_1$;

(j) $B_2 \cap \bar{B}_2$; (k) $B_2 \cup \bar{B}_2$

(h) $B_1 \cap \bar{B}_1 = \left\{\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\right\}$

(i) $B_1 \cup \bar{B}_1 = \left\{\frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}\right\}$

(j) $B_2 \cap \bar{B}_2 = \left\{\frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}\right\}$

(k) $B_2 \cup \bar{B}_2 = \left\{\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0}\right\}$

Problems

It is necessary to compare two sensors based upon their detection levels and gain settings. The table of gain settings and sensor detection levels with a standard item being monitored providing typical membership values to represent the detection levels for each sensor is given in Table 1.

Gain setting	Detection level of sensor 1	Detection level of sensor 2
0	0	0
10	0.2	0.35
20	0.35	0.25
30	0.65	0.8
40	0.85	0.95
50	1	1

Now given the universe of discourse $X = \{0, 10, 20, 30, 40, 50\}$ and the membership functions for the two sensors in discrete form as

$$D_1 = \left\{\frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50}\right\}$$

$$D_2 = \left\{\frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50}\right\}$$

find the following membership functions:

(a) $\mu_{D_1 \cup D_2}(x)$; (b) $\mu_{D_1 \cap D_2}(x)$; (c) $\mu_{\overline{D_1}}(x)$;

(d) $\mu_{\overline{D_2}}(x)$; (e) $\mu_{D_1 \cup \overline{D_1}}(x)$; (f) $\mu_{D_1 \cap \overline{D_1}}(x)$;

(g) $\mu_{D_2 \cup \overline{D_2}}$; (h) $\mu_{D_2 \cap \overline{D_2}}(x)$; (i) $\mu_{D_1 | D_2}(x)$;

(j) $\mu_{D_2 | D_1}(x)$

(a) $\mu_{D_1 \cup D_2}(x)$

$$= \max\{\mu_{D_1}(x), \mu_{D_2}(x)\}$$

$$= \left\{\frac{0}{0} + \frac{0.35}{10} + \frac{0.35}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50}\right\}$$

(b) $\mu_{D_1 \cap D_2}(x)$

$$= \min\{\mu_{D_1}(x), \mu_{D_2}(x)\}$$

$$= \left\{\frac{0}{0} + \frac{0.2}{10} + \frac{0.25}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50}\right\}$$

(c) $\mu_{\overline{D_1}}(x)$

$$= 1 - \mu_{D_1}(x)$$

$$= \left\{\frac{1}{0} + \frac{0.8}{10} + \frac{0.65}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50}\right\}$$

(d) $\mu_{\overline{D_2}}(x)$

$$= 1 - \mu_{D_2}(x)$$

$$= \left\{\frac{1}{0} + \frac{0.65}{10} + \frac{0.75}{20} + \frac{0.2}{30} + \frac{0.05}{40} + \frac{0}{50}\right\}$$

(e) $\mu_{D_1 \cup \overline{D_1}}(x)$

$$= \max\{\mu_{D_1}(x), \mu_{\overline{D_1}}(x)\}$$

$$= \left\{\frac{1}{0} + \frac{0.8}{10} + \frac{0.65}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50}\right\}$$

(f) $\mu_{D_1 \cap \overline{D_1}}(x)$

$$= \min\{\mu_{D_1}(x), \mu_{\overline{D_1}}(x)\}$$

$$= \left\{\frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50}\right\}$$

(g) $\mu_{D_2 \cup \overline{D_2}}(x)$

$$= \max\{\mu_{D_2}(x), \mu_{\overline{D_2}}(x)\}$$

$$= \left\{\frac{1}{0} + \frac{0.65}{10} + \frac{0.75}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50}\right\}$$

(h) $\mu_{D_2 \cap \overline{D_2}}(x)$

$$= \min\{\mu_{D_2}(x), \mu_{\overline{D_2}}(x)\}$$

$$= \left\{\frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.2}{30} + \frac{0.05}{40} + \frac{0}{50}\right\}$$

Problems

Design a computer software to perform image processing to locate objects within a scene. The two fuzzy sets representing a plane and a train image are:

$$\text{Plane} = \left\{\frac{0.2}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.8}{\text{plane}} + \frac{0.1}{\text{house}}\right\}$$

$$\text{Train} = \left\{\frac{1}{\text{train}} + \frac{0.2}{\text{bike}} + \frac{0.4}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.2}{\text{house}}\right\}$$

Find the following:

(a) Plane $\cup$ Train; (b) Plane $\cap$ Train;

(c) $\overline{\text{Plane}}$; (d) $\overline{\text{Train}}$;

(e) Plane|Train; (f) $\overline{\text{Plane} \cup \text{Train}}$;

(g) $\overline{\text{Plane} \cap \text{Train}}$; (h) Plane $\cup$ $\overline{\text{Plane}}$;

(i) Plane $\cap$ $\overline{\text{Plane}}$; (j) Train $\cup$ $\overline{\text{Train}}$;

(k) Train $\cup$ $\overline{\text{Train}}$

Problems

(a) Plane $\cup$ Train

$$= \max\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$

$$= \left\{\frac{1.0}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.4}{\text{boat}} + \frac{0.8}{\text{plane}} + \frac{0.2}{\text{house}}\right\}$$

(b) Plane $\cap$ Train

$$= \min\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$

$$= \left\{\frac{0.2}{\text{train}} + \frac{0.2}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.1}{\text{house}}\right\}$$

(c) $\overline{\text{Plane}} = 1 - \mu_{\text{Plane}}(x)$

$$= \left\{\frac{0.8}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.7}{\text{boat}} + \frac{0.2}{\text{plane}} + \frac{0.9}{\text{house}}\right\}$$

(d) $\overline{\text{Train}} = 1 - \mu_{\text{Train}}(x)$

$$= \left\{\frac{0}{\text{train}} + \frac{0.8}{\text{bike}} + \frac{0.6}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.8}{\text{house}}\right\}$$

(e) Plane|Train

$$= \text{Plane} \cap \overline{\text{Train}}$$

$$= \min\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Train}}}(x)\}$$

$$= \left\{\frac{0}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.1}{\text{house}}\right\}$$

(f) $\overline{\text{Plane} \cup \text{Train}}$

$$= 1 - \max\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$

$$= \left\{\frac{0}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.6}{\text{boat}} + \frac{0.2}{\text{plane}} + \frac{0.8}{\text{house}}\right\}$$

Problems

(g) $\overline{\underset{\sim}{\text{Plane}} \cap \overline{\underset{\sim}{\text{Train}}}}$

$$= 1 - \min\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$

$$= \left\{\frac{0.8}{\text{train}} + \frac{0.8}{\text{bike}} + \frac{0.7}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.9}{\text{house}}\right\}$$

(h) $\underset{\sim}{\text{Plane}} \cup \overline{\underset{\sim}{\text{Plane}}}$

$$= \max\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Plane}}}(x)\}$$

$$= \left\{\frac{0.8}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.7}{\text{boat}} + \frac{0.8}{\text{plane}} + \frac{0.9}{\text{house}}\right\}$$

(i) $\underset{\sim}{\text{Plane}} \cap \overline{\underset{\sim}{\text{Plane}}}$

$$= \min\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Plane}}}(x)\}$$

$$= \left\{\frac{0.2}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.2}{\text{plane}} + \frac{0.1}{\text{house}}\right\}$$

(j) $\underset{\sim}{\text{Train}} \cup \overline{\underset{\sim}{\text{Train}}}$

$$= \max\{\mu_{\text{Train}}(x), \mu_{\overline{\text{Train}}}(x)\}$$

$$= \left\{\frac{1.0}{\text{train}} + \frac{0.8}{\text{bike}} + \frac{0.6}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.8}{\text{house}}\right\}$$

(k) $\underset{\sim}{\text{Train}} \cap \overline{\underset{\sim}{\text{Train}}}$

$$= \min\{\mu_{\text{Train}}(x), \mu_{\overline{\text{Train}}}(x)\}$$

$$= \left\{\frac{0}{\text{train}} + \frac{0.2}{\text{bike}} + \frac{0.4}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.2}{\text{house}}\right\}.$$

Problem-H/W

For aircraft simulator data the determination of certain changes in its operating conditions is made on the basis of hard break points in the mach region. We define two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ representing the condition of "near" a mach number of 0.65 and "in the region" of a mach number of 0.65, respectively, as follows

$\underset{\sim}{A}$ = near mach 0.65

$$= \left\{\frac{0}{0.64} + \frac{0.75}{0.645} + \frac{1}{0.65} + \frac{0.5}{0.655} + \frac{0}{0.66}\right\}$$

$\underset{\sim}{B}$ = in the region of mach 0.65

$$= \left\{\frac{0}{0.64} + \frac{0.25}{0.645} + \frac{0.75}{0.65} + \frac{1}{0.655} + \frac{0.5}{0.66}\right\}$$

For these two sets find the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$; (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$; (c) $\overline{\underset{\sim}{A}}$;

(d) $\overline{\underset{\sim}{B}}$; (e) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$; (f) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$

Mach-It is used as a unit of measurement in stating the Speed of a moving object in relation to the speed of sound.

- *Ex: if the aircraft is travelling at Mach 1 ,it is travelling at exactly the speed of sound.*

Problem

Let U be the universe of military aircraft of interest' as defined below:

$$U = \{a10,\ b52,\ c130,\ f2,\ f9\}$$

Let $\underset{\sim}{A}$ be the fuzzy set of bomber class aircraft:

$$\underset{\sim}{A} = \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{1}{f9}\right\}$$

Let $\underset{\sim}{B}$ be the fuzzy set of fighter class aircraft:

$$\underset{\sim}{B} = \left\{\frac{0.1}{a10} + \frac{0.2}{b52} + \frac{0.8}{c130} + \frac{0.7}{f2} + \frac{0}{f9}\right\}$$

Find the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$; (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$; (c) $\overline{\underset{\sim}{A}}$; (d) $\overline{\underset{\sim}{B}}$;

(e) $\underset{\sim}{A} | \underset{\sim}{B}$; (f) $\underset{\sim}{B} | \underset{\sim}{A}$; (g) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$;

(h) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$; (i) $\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}}$; (j) $\overline{\underset{\sim}{B}} \cup \underset{\sim}{A}$

(a) $\underset{\sim}{A} \cup \underset{\sim}{B} = \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.8}{c130} + \frac{0.7}{f2} + \frac{1}{f9}\right\}$$

(b) $\underset{\sim}{A} \cap \underset{\sim}{B} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.1}{a10} + \frac{0.2}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{0}{f9}\right\}$$

(c) $\overline{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x)$

$$= \left\{\frac{0.7}{a10} + \frac{0.6}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{0}{f9}\right\}$$

(d) $\overline{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x)$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{1}{f9}\right\}$$

(c) $\overline{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x)$

$$= \left\{\frac{0.7}{a10} + \frac{0.6}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{0}{f9}\right\}$$

(d) $\overline{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x)$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{1}{f9}\right\}$$

(e) $\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{1}{f9}\right\}$$

(e) $\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{1}{f9}\right\}$$

(g) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = 1 - \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.7}{a10} + \frac{0.6}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{0}{f9}\right\}$$

(h) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = 1 - \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{1}{f9}\right\}$$

(i) $\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}} = \max\{\mu_{\overline{\underset{\sim}{A}}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{1}{f9}\right\}$$

(j) $\overline{\underset{\sim}{B}} \cup \underset{\sim}{A} = \max\{\mu_{\overline{\underset{\sim}{B}}}(x), \mu_{\underset{\sim}{A}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{1}{f9}\right\}$$

Problem

Consider two fuzzy sets

$$\underset{\sim}{A} = \left\{ \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.5}{4} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.2}{3} + \frac{1}{4} \right\}$$

Find the algebraic sum, algebraic product, bounded sum and bounded difference of the given fuzzy sets.

(a) Algebraic sum

$$\mu_{\underset{\sim}{A}+\underset{\sim}{B}}(x) = [\mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x)] - [\mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)]$$

$$= \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{1.5}{4} \right\} - \left\{ \frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4} \right\}$$

$$= \left\{ \frac{0.28}{1} + \frac{0.44}{2} + \frac{0.52}{3} + \frac{1}{4} \right\}$$

(b) Algebraic product

$$\mu_{\underset{\sim}{A}\cdot\underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$$

$$= \left\{ \frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4} \right\}$$

(c) Bounded sum

$$\mu_{\underset{\sim}{A}\oplus\underset{\sim}{B}}(x)$$

$$= \min[1, \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x)]$$

$$= \min\left\{1, \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{1.5}{4} \right\}\right\}$$

$$= \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{1}{4} \right\}$$

(d) Bounded difference

$$\mu_{\underset{\sim}{A}\odot\underset{\sim}{B}}(x)$$

$$= \max[0, \mu_{\underset{\sim}{A}}(x) - \mu_{\underset{\sim}{B}}(x)]$$

$$= \max\left\{0, \left\{ \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{-0.5}{4} \right\}\right\}$$

$$= \left\{ \frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0}{4} \right\}$$

Problem -H/w

Consider a local area network (LAN) of inter-connected workstations that communicate using Ethernet protocols at a maximum rate of 12 Mbit/s. The two fuzzy sets given below represent the loading of the LAN:

$$\mu_{\underset{\sim}{S}}(x) = \left\{ \frac{1.0}{0} + \frac{1.0}{1} + \frac{0.8}{2} + \frac{0.2}{5} + \frac{0.1}{7} + \frac{0.0}{9} + \frac{0.0}{10} \right\}$$

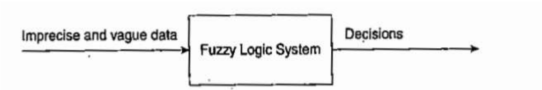
$$\mu_{\underset{\sim}{C}}(x) = \left\{ \frac{0.0}{0} + \frac{0.0}{1} + \frac{0.0}{2} + \frac{0.5}{5} + \frac{0.7}{7} + \frac{0.8}{9} + \frac{1.0}{10} \right\}$$

where $\underset{\sim}{S}$ represents silent and $\underset{\sim}{C}$ represents congestion. Perform algebraic sum, algebraic product, bounded sum and bounded difference over the two fuzzy sets.

Fuzzy inference systems

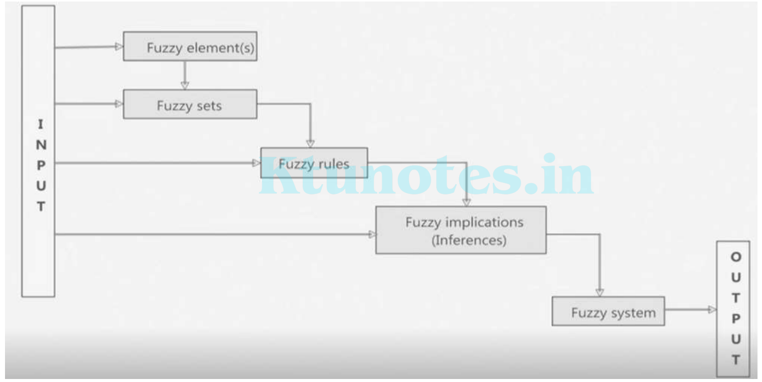
Fuzzy Logic

- Fuzzy logic is a mathematical language ,deals with Fuzzy set or fuzzy algebra,
- Fuzzy logic models uncertainty associated with imprecision and lack of information regarding a problem or a system.
- Fuzziness describes the ambiguity of an event
- The values 0 and 1 describes ‘Not belonging to’ & ‘belongining to’ in conventional set. But the values in between represent “Fuzziness”



A fuzzy logic system accepting imprecise data and providing a decision.

Concept of Fuzzy Logic



Fuzzy Logic

- Developed by Lotfi Zadeh in 1965,Based on a system of non-digital (continuous & fuzzy without crisp boundaries) set theory and rules,
- Its advantage is its ability to deal with vague systems and its use of linguistic variables.
- Fuzzy logic consists of Fuzzy Inference systems also called Rule based system.

  IF-THEN→Rules

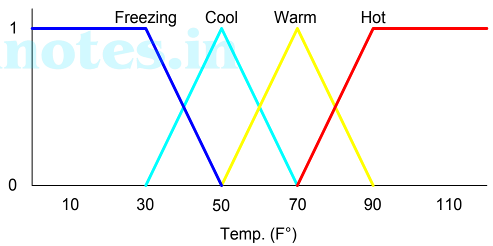
- Examples
- IF speed is TOO SLOW and acceleration is DECELERATING, THEN INCREASE POWER GREATLY
- IF speed is SLOW and acceleration is DECREASING,THEN INCREASE POWER SLIGHTLY
- IF distance is CLOSE,THEN DECREASE POWER SLIGHTLY

Fuzzy Logic

- A way to represent variation or imprecision in logic
- A way to make use of natural language in logic
- Approximate reasoning
- Humans say things like "If it is sunny and warm today, I will drive fast"
- Linguistic variables:
  - Temp: {freezing, cool, warm, hot}
  - Cloud : {overcast, partly cloudy, sunny}
  - Speed: {slow, fast}

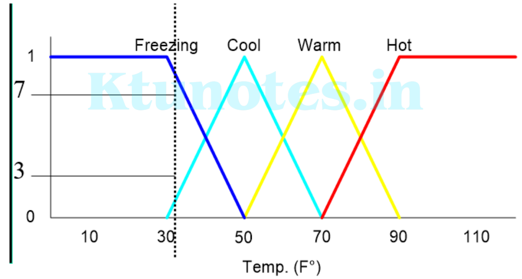
Fuzzy Linguistic variables

- Fuzzy Linguistic Variables are used to represent qualities spanning a particular spectrum
- Temp: {**Freezing**, **Cool**, **Warm**, **Hot**}
- Question: What is the temperature?
- Answer: It is warm.
- Question: How warm is it?
- Degree of Truth or "Membership"



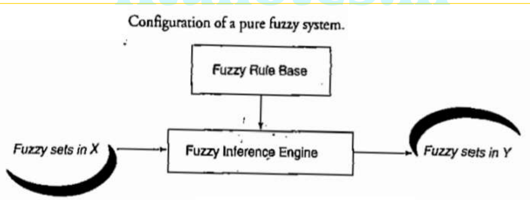
Fuzzy Membership Functions

- How cool is 36 F° ?
- It is 30% Cool and 70% Freezing



Fuzzy IF-THEN Rules

- Fuzzy IF-THEN rules has the general form:
  IF x is A THEN y is B where A&B are fuzzy sets
- Fuzzy system →refers to the s/m's that are governed by fuzzy IF-THEN rules.
- IF part→Antecedent
- THEN part→Consequent
- Fuzzy Inference Engine combines Fuzzy IF-THEN rules into a mapping from the fuzzy sets in the i/p space X to the fuzzy set,in the o/p space Y based on fuzzy logic principles.



Fuzzy Relations

Fuzzy Relation

- Fuzzy relations are important tools that are used in fuzzy modeling, fuzzy diagnosis, and fuzzy control
- Fuzzy relation defines the mapping of variables from one fuzzy set to another
- Fuzzy relations are very important because they can describe interactions between variables.
- A fuzzy relation is the cartesian product of mathematical fuzzy sets.
- Two fuzzy sets are taken as input, the fuzzy relation is then equal to the cross product of the sets which is created by vector multiplication.
- Fuzzy relations also map elements of one universe, say X, to those of another universe, say Y, through the Cartesian product of the two universes.

Crisp Relations

Cardinality of Power Set

Find the power set and cardinality of the given set $X = \{2, 4, 6\}$. *Also find cardinality of power set.*

Set X contains 3 elements, so, $n_X = 3$

The power set of X is,

$$P(X) = \{\phi, \{2\}, \{4\}, \{6\}, \{2,4\}, \{2,6\}, \{4,6\}, \{2,4,6\}\}$$

The cardinality of power set P(X) is, $n_{P(X)} = 2^{n_X} = 2^3 = 8$

- Power set-Set of all subsets of the set A ,including the set itself and null and empty set.

Cardinality of a classical Relation

Consider n elements of the universe X being related to m elements of universe Y. When the cardinality of X= n_X and the cardinality of Y =n_Y , then the cardinality of relation R between the two universe is

$$n_{X\times Y} = n_X \times n_Y$$

The cardinality of the power set P(X × Y) describing the relation is given by

$$n_{P(X\times Y)} = 2^{(n_X n_Y)}$$

Operations on a classical/crisp Relation

Let R and S be two separate relations on the Cartesian universe X ×Y.

The null relation and the complete relation are defined by the relation matrices $\emptyset_R$ and E_R.

An example of a 3 X 3 form of the $\emptyset_R$ and E_R matrices is given below:

$$\emptyset_R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad E_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1. **Union** $R \cup S \rightarrow \chi_{R\cup S}(x,y): \chi_{R\cup S}(x,y) = \max[\chi_R(x,y), \chi_S(x,y)]$
2. **Intersection** $R \cap S \rightarrow \chi_{R\cap S}(x,y): \chi_{R\cap S}(x,y) = \min[\chi_R(x,y), \chi_S(x,y)]$
3. **Complement** $\overline{R} \rightarrow \chi_{\overline{R}}(x,y) : \chi_{\overline{R}}(x,y) = 1 - \chi_{\overline{R}}(x,y)$
4. **Containment** $R \subset S \rightarrow \chi_R(x,y): \chi_R(x,y) \leq \chi_S(x,y)$
5. **Identity** $\emptyset \rightarrow \emptyset_R$ and $X \rightarrow E_R$

Cartesian Product of a Relation

- Consider two universes X *and* Y, their Cartesian product is given by,

$$X \times Y = \{(x,y) | x \in X, y \in Y\}$$

- The *characteristic function*, χ, gives the strength of the relationship between ordered pair of elements in each universe.

$$\chi_{X\times Y}(x,y) = \begin{cases} 1, (x,y) \in X \times Y \\ 0, (x,y) \notin X \times Y \end{cases}$$

- When the sets are finite, then the relation is represented by a matrix called *relation matrix*.

Cartesian Product and Relation Matrix of a Relation

- Consider, $X = \{p, q, r\}$ $\quad Y = \{2, 4, 6\}$

Cartesian product of these two sets, $X \times Y$, is,
$\{(p,2),(p,4),(p,6),(q,2),(q,4),(q,6),(r,2),(r,4),(r,6)\}$

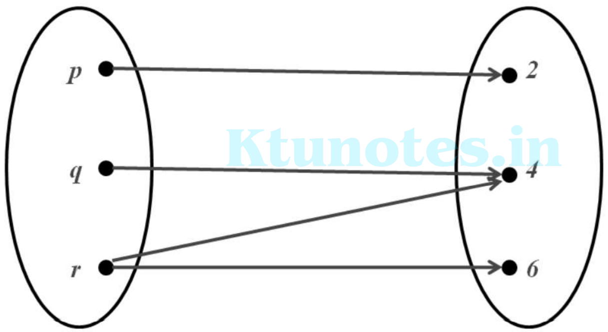
From this set one may select a subset such that,

$$R = \{(p,2),(q,4),(r,4),(r,6)\}$$

Relation matrix is,

$$\begin{array}{c c} & \begin{array}{ccc} 2 & 4 & 6 \end{array} \\ \begin{array}{c} p \\ q \\ r \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

Mapping Representation of a Relation



Crisp Relation-Example

Consider the two crisp sets A and B as given below.
$A = \{1, 2, 3, 4\}$ $B = \{3, 5, 7\}$.

Then, $A \times B = \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7)\}$

define a relation as $R = \{(a,b) | b = a + 1, (a,b) \in A \times B\}$

$$R = \{(2,3), (4,5)\}$$

We can represent the relation R in a matrix form as follows.

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 3 & 5 & 7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Cartesian Product of a crisp Relation

The elements in two sets A and B are given as $A = \{2, 4\}$ and $B = \{a, b, c\}$
Find the various Cartesian products of these two sets.

$A \times B = \{(2, a), (2, b), (2, c), (4, a), (4, b), (4, c)\}$
$B \times A = \{(a, 2), (a, 4), (b, 2), (b, 4), (c, 2), (c, 4)\}$
$A \times A = A^2 = \{(2, 2), (2, 4), (4, 2), (4, 4)\}$
$B \times B = B^2 = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$

Fuzzy Relations

Fuzzy Relation

- *Fuzzy relations* relate elements of one universe to those of another universe through the Cartesian product of the two universes.
- Based on the concept that everything is related to some extent or unrelated.
- A fuzzy relation is a fuzzy set defined on the Cartesian product of classical sets, $\{X_1, X_2, ..., X_n\}$
  where tuples, $(x_1, x_2, ..., x_n)$
  may have varying degrees of membership,
  $\mu_R(x_1, x_2, ..., x_n)$ within the relation.

Cartesian Product of a Fuzzy Relation

Let $\underset{\sim}{A}$ be a fuzzy set on universe X and $\underset{\sim}{B}$ be a fuzzy set on universe Y.

The Cartesian product over $\underset{\sim}{A}$ and $\underset{\sim}{B}$ results in fuzzy relation $\underset{\sim}{R}$ and is contained within the entire (complete) Cartesian space, i.e.,

$\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R}$ where $\underset{\sim}{R} \subset X \times Y$

The membership function of fuzzy relation is given by

$$\boxed{\mu_{\underset{\sim}{R}}(x, y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x, y) = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)\right]}$$

Cartesian Product of a Fuzzy Relation

Consider the following two fuzzy sets:

$$\underset{\sim}{A} = \left\{\frac{0.3}{x_1} + \frac{0.7}{x_2} + \frac{1}{x_3}\right\} \text{ and } \underset{\sim}{B} = \left\{\frac{0.4}{y_1} + \frac{0.9}{y_2}\right\}$$

Perform the Cartesian product over these given fuzzy sets.

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The fuzzy Cartesian product performed over fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ results in fuzzy relation $\underset{\sim}{R}$ given by $\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B}$.

$$\underset{\sim}{R} = \begin{bmatrix} 0.3 & 0.3 \\ 0.4 & 0.7 \\ 0.4 & 0.9 \end{bmatrix}$$

The calculation for $\underset{\sim}{R}$ is as follows:

$$\mu_{\underset{\sim}{R}}(x_1, y_1) = \min[\mu_{\underset{\sim}{A}}(x_1), \mu_{\underset{\sim}{B}}(y_1)] = \min(0.3, 0.4) = 0.3$$

$$\mu_{\underset{\sim}{R}}(x_1, y_2) = \min[\mu_{\underset{\sim}{A}}(x_1), \mu_{\underset{\sim}{B}}(y_2)] = \min(0.3, 0.9) = 0.3$$

$$\mu_{\underset{\sim}{R}}(x_2, y_1) = \min[\mu_{\underset{\sim}{A}}(x_2), \mu_{\underset{\sim}{B}}(y_1)] = \min(0.7, 0.4) = 0.4$$

$$\mu_{\underset{\sim}{R}}(x_2, y_2) = \min[\mu_{\underset{\sim}{A}}(x_2), \mu_{\underset{\sim}{B}}(y_2)] = \min(0.7, 0.9) = 0.7$$

$$\mu_{\underset{\sim}{R}}(x_3, y_1) = \min[\mu_{\underset{\sim}{A}}(x_3), \mu_{\underset{\sim}{B}}(y_1)] = \min(1, 0.4) = 0.4$$

$$\mu_{\underset{\sim}{R}}(x_3, y_2) = \min[\mu_{\underset{\sim}{A}}(x_3), \mu_{\underset{\sim}{B}}(y_2)] = \min(1, 0.9) = 0.9$$

Cartesian Product of a Fuzzy Relation-Home Work

1. The elements in two sets X and Y are given as $X = \{1, 2, 3\}$, $Y = \{p, q, r\}$.
   Find the various Cartesian products of these two sets.
2. For the fuzzy sets given

$$\underset{\sim}{A} = \left\{\frac{0.5}{x_1} + \frac{0.2}{x_2} + \frac{0.9}{x_3}\right\} \quad \underset{\sim}{B} = \left\{\frac{1}{y_1} + \frac{0.5}{y_2} + \frac{1}{y_3}\right\}$$

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find relation $\underset{\sim}{R}$ by performing Cartesian product over the given fuzzy sets.

Graphical representation of a fuzzy relation:

- A relation R also can be graphically represented.
- For example, suppose, a fuzzy relation R in V × W, where V = {1,2,3) and W = {1,2,3,4} has the following definition.
- R = [{{{1,1},1},{{1,2},.2},{{1,3},.7},{{1,4},0},{{2,1},.7},{{2,2},1},{{2,3},.4},{{2,4},.8},{{3,1},0},{{3,2},.6},{{3,3},.3},{{3,4},.5}}]
- This relation can be represented in the form of a graph.

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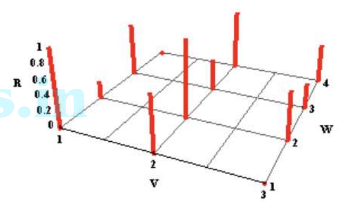
Graphical representation of a relation R



Fuzzy Matrix

- Let, $X = \{x_1, x_2, ..., x_n\}$ and $Y = \{y_1, y_2, ..., y_n\}$

Fuzzy relation $R(x, y)$ can be expressed as an $n \times m$ matrix as:

$$R(x,y) = \begin{bmatrix} \mu_R(x_1,y_1) & \mu_R(x_1,y_2) & ..... & \mu_R(x_1,y_m) \\ \mu_R(x_2,y_1) & \mu_R(x_2,y_2) & ..... & \mu_R(x_2,y_m) \\ \cdots & \cdots & \cdots & \\ \cdots & \cdots & \cdots & \\ \mu_R(x_n,y_1) & \mu_R(x_n,y_2) & ..... & \mu_R(x_n,y_m) \end{bmatrix}$$

- The matrix representing a fuzzy relation is called *Fuzzy matrix*.

Fuzzy Matrix Example

Example:

$X = \{typhoid, viral, cold\}, Y = \{running\ nose, high\ temp, shivering\}$

The fuzzy relation R is defined as

$$R = \begin{array}{c} \\ typhoid \\ viral \\ cold \end{array} \begin{array}{c} \begin{array}{ccc} running\ nose & high\ temperature & shivering \end{array} \\ \begin{bmatrix} 0.1 & 0.9 & 0.8 \\ 0.2 & 0.9 & 0.7 \\ 0.9 & 0.4 & 0.6 \end{bmatrix} \end{array}$$

Fuzzy Graph

- *Fuzzy graph* is a graphical representation of binary fuzzy relation.
- Each element in X and Y corresponds to a node in the fuzzy graph.
- The connection links are established between the nodes by the elements of $X \times Y$ with nonzero membership grades in $R(X, Y)$.
- The links may also be present in the form of arcs.
- Links are labeled with the membership values as $\mu_R(x_i, y_j)$.

Fuzzy Relation Example

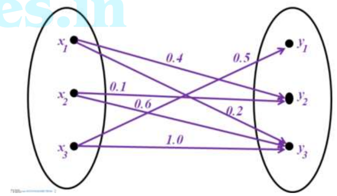
Let, $X = \{x_1, x_2, x_3, x_4\}$ *and* $Y = \{y_1, y_2, y_3, y_4\}$

Let R be a relation from X and Y given by,

$$R = \frac{0.2}{(x_1, y_3)} + \frac{0.4}{(x_1, y_2)} + \frac{0.1}{(x_2, y_2)} + \frac{0.6}{(x_2, y_3)} + \frac{1.0}{(x_3, y_3)} + \frac{0.5}{(x_3, y_1)}$$

Fuzzy matrix for relation R is,

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 0.2 \\ 0 & 0.1 & 0.6 \\ 0.5 & 0 & 1.0 \end{bmatrix} \end{array}$$



Operations on Fuzzy Relation

- <u>Union</u>

$$\mu_{R \cup S}(x, y) = max\{\mu_R(x, y), \mu_S(x, y)\}$$

- <u>Intersection</u>

$$\mu_{R \cap S}(x, y) = min\{\mu_R(x, y), \mu_S(x, y)\}$$

- <u>Complement</u>

$$\mu_{\overline{R}}(x, y) = 1 - \mu_R(x, y)$$

- <u>Containment</u>

$$R \subset S \Longrightarrow \mu_R(x, y) \le \mu_S(x, y)$$

- <u>Inverse</u>

$$R^{-1}(y, x) = R(x, y) \text{ for all pairs } (y, x) \in Y \times X$$

- <u>Projection</u>

$$\mu_{[R \downarrow Y]}(x, y) = max_x \mu_R(x, y)$$

Operations on Fuzzy Relation

Union: $\underline{R} \cup \underline{S} = \{ (a, b), \mu_{\underline{A} \cup \underline{B}}(a, b) \}$

$$\begin{array}{cc} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \bar{R} = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \bar{S} = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.4 & 0.0 & 0.9 & 0.6 \\ 0.9 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0.0 & 0.8 & 0.5 \end{bmatrix} \end{array}$$

$\underline{R} \cup \underline{S} = \{ (a, b), \mu_{\underline{A} \cup \underline{B}}(a, b) \}$

$\mu_{\underline{R} \cup \underline{S}}(a, b) = \max(\mu_{\underline{R}}(a, b), \mu_{\underline{S}}(a, b))$

$\mu_{\underline{R} \cup \underline{S}}(x_1, y_1) = \max(\mu_{\underline{R}}(x_1, y_1), \mu_{\underline{S}}(x_1, y_1))$

$\mu_{\underline{R} \cup \underline{S}}(x_1, y_1) = \max(\mu_{\underline{R}}(x_1, y_1), \mu_{\underline{S}}(x_1, y_1)) = \max(0.8, 0.4) = 0.8$

$\mu_{\underline{R} \cup \underline{S}}(x_1, y_2) = \max(\mu_{\underline{R}}(x_1, y_2), \mu_{\underline{S}}(x_1, y_2)) = \max(0.1, 0.0) = 0.1$

$\mu_{\underline{R} \cup \underline{S}}(x_1, y_3) = \max(\mu_{\underline{R}}(x_1, y_3), \mu_{\underline{S}}(x_1, y_3)) = \max(0.1, 0.9) = 0.9$

$\mu_{\underline{R} \cup \underline{S}}(x_1, y_4) = \max(\mu_{\underline{R}}(x_1, y_4), \mu_{\underline{S}}(x_1, y_4)) = \max(0.7, 0.6) = 0.7$

$\vdots$

$\mu_{\underline{R} \cup \underline{S}}(x_3, y_4) = \max(\mu_{\underline{R}}(x_3, y_4), \mu_{\underline{S}}(x_3, y_4)) = \max(0.8, 0.5) = 0.8$

the final matrix for union operation would be,

$$\begin{array}{cc} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \bar{R} \cup \bar{S} = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.8 & 0.1 & 0.9 & 0.7 \\ 0.9 & 0.8 & 0.5 & 0.7 \\ 0.9 & 1.0 & 0.8 & 0.8 \end{bmatrix} \end{array}$$

Operations on Fuzzy Relation

Intersection: $\underline{R} \cap \underline{S} = \{ (a, b), \mu_{\underline{A} \cap \underline{B}}(a, b) \}$

$\mu_{\underline{R} \cap \underline{S}}(a, b) = \min(\mu_{\underline{R}}(a, b), \mu_{\underline{S}}(a, b))$

$$\begin{array}{cc} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \bar{R} = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \bar{S} = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.4 & 0.0 & 0.9 & 0.6 \\ 0.9 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0.0 & 0.8 & 0.5 \end{bmatrix} \end{array}$$

$\mu_{\underline{R} \cap \underline{S}}(x_1, y_1) = \min(\mu_{\underline{R}}(x_1, y_1), \mu_{\underline{S}}(x_1, y_1)) = \min(0.8, 0.4) = 0.4$

$\mu_{\underline{R} \cap \underline{S}}(x_1, y_2) = \min(\mu_{\underline{R}}(x_1, y_2), \mu_{\underline{S}}(x_1, y_2)) = \min(0.1, 0.0) = 0.0$

$\mu_{\underline{R} \cap \underline{S}}(x_1, y_3) = \min(\mu_{\underline{R}}(x_1, y_3), \mu_{\underline{S}}(x_1, y_3)) = \min(0.1, 0.9) = 0.1$

$\mu_{\underline{R} \cap \underline{S}}(x_1, y_4) = \min(\mu_{\underline{R}}(x_1, y_4), \mu_{\underline{S}}(x_1, y_4)) = \min(0.7, 0.6) = 0.6$

$\vdots$

$$\begin{array}{cc} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \bar{R} \cap \bar{S} = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.4 & 0.0 & 0.1 & 0.6 \\ 0.0 & 0.4 & 0.0 & 0.0 \\ 0.3 & 0.0 & 0.7 & 0.5 \end{bmatrix} \end{array}$$

Intersection of relation

Operations on Fuzzy Relation

Complement: $\underline{R}^c = \{ (a, b), \mu_{\underline{R}}{}^c(a, b) \}$

$\mu_{\underline{R}}{}^c(a, b) = 1 - \mu_{\underline{R}}(a, b)$

$$\begin{array}{cc} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \bar{R} = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

$\mu_{\underline{R}}{}^c(x_1, y_1) = 1 - \mu_{\underline{R}}(x_1, y_1) = 1 - 0.8 = 0.2$

$\mu_{\underline{R}}{}^c(x_1, y_2) = 1 - \mu_{\underline{R}}(x_1, y_2) = 1 - 0.1 = 0.9$

$\mu_{\underline{R}}{}^c(x_1, y_3) = 1 - \mu_{\underline{R}}(x_1, y_3) = 1 - 0.1 = 0.9$

$\vdots$

$$\begin{array}{cc} & \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \bar{R}^c = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.2 & 0.9 & 0.9 & 0.3 \\ 1.0 & 0.2 & 1.0 & 1.0 \\ 0.1 & 0.0 & 0.3 & 0.2 \end{bmatrix} \end{array}$$

Complement of fuzzy relation

Operations on Fuzzy Relation

Projection:

The projection of $\underline{R}$ on X : $\prod_X(x) = \sup(\, \underline{R}(x, y) \mid y \in Y)$

The projection of $\underline{R}$ on Y : $\prod_Y(y) = \sup(\, \underline{R}(x, y) \mid x \in X)$

$$\bar{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

The projection of $\underline{R}$ on X :

$\prod_X(x_1) = 0.8$

$\prod_X(x_2) = 0.8$

$\prod_X(x_3) = 1.0$

The projection of $\underline{R}$ on Y :

$\prod_Y(y_1) = 0.9$

$\prod_Y(y_2) = 1.0$

$\prod_Y(y_3) = 0.7$

$\prod_Y(y_4) = 0.8$

$$\bar{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array}$$

Operations on Fuzzy Relation

Containment

This operation is to test whether a fuzzy relation is a complete subset of other

R contains in S if and only if the following condition holds:

$$R \subset S \equiv \mu_R(x, y) \leq \mu_S(x, y)$$

Properties of Fuzzy Relation

- Commutativity
- Associativity
- Distributivity
- Identity
- Idempotency
- DeMorgan's law

Fuzzy Composition

Let $\underset{\sim}{A}$ be a fuzzy set on universe X and $\underset{\sim}{B}$ be a fuzzy set on universe Y.

The Cartesian product over $\underset{\sim}{A}$ and $\underset{\sim}{B}$ results in fuzzy relation $\underset{\sim}{R}$ and is contained within the entire (complete) Cartesian space, i.e.,

$\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{R}$ where $\underset{\sim}{R} \subset X \times Y$

The membership function of fuzzy relation is given by

$$\boxed{\mu_{\underset{\sim}{R}}(x, y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x, y) = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)\right]}$$

There are two types of fuzzy composition techniques:

1. **Fuzzy max-min composition**
2. **Fuzzy max-product composition**

Composition of fuzzy relation is defined over two fuzzy relations.

Fuzzy Composition

Let $\underset{\sim}{A}$ be a fuzzy set on universe X and $\underset{\sim}{B}$ be a fuzzy set on universe Y.

The Cartesian product over $\underset{\sim}{A}$ and $\underset{\sim}{B}$ results in fuzzy relation $\underset{\sim}{R}$ and is contained within the entire (complete) Cartesian space, i.e.,

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There are two types of fuzzy composition techniques:

1. **Fuzzy max-min composition**
2. **Fuzzy max-product composition**

Composition of fuzzy relation is defined over two fuzzy relations. if x1 related y1 and if y1 is related to z1 then with transitive property we can say x1 is related to z1

Fuzzy max-min Composition and min-max composition

Let $\underset{\sim}{R}$ be fuzzy relation on Cartesian space $X \times Y$, and $\underset{\sim}{S}$ be fuzzy relation on Cartesian space $Y \times Z$.

The max-min composition of R(X,Y) and S(Y,Z), denoted by $R(X, Y) \circ S(Y, Z)$ is defined by T(X,Z) as

$$\mu_{\underset{\sim}{T}}(x, z) = \mu_{\underset{\sim}{R} \circ \underset{\sim}{S}}(x, z) = \max_{y \in Y}\left\{\min\left[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(y, z)\right]\right\}$$

$$= \vee_{y \in Y}\left[\mu_{\underset{\sim}{R}}(x, y) \wedge \mu_{\underset{\sim}{S}}(y, z)\right] \quad \forall x \in X,\ z \in Z$$

The min-max composition of R(X,Y) and S(Y,Z), denoted by $R(X, Y) \circ S(Y, Z)$ is defined by T(X,Z) as

$$\mu_{\underset{\sim}{T}}(x, z) = \mu_{\underset{\sim}{R} \circ \underset{\sim}{S}}(x, z) = \min_{y \in Y}\left\{\max\left[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(y, z)\right]\right\}$$

$$= \wedge_{y \in Y}\left[\mu_{\underset{\sim}{R}}(x, y) \vee \mu_{\underset{\sim}{S}}(y, z)\right] \quad \forall x \in X,\ z \in Z$$

Fuzzy max-product Composition

Let $\underset{\sim}{R}$ be fuzzy relation on Cartesian space $X \times Y$, and $\underset{\sim}{S}$ be fuzzy relation on Cartesian space $Y \times Z$.

Fuzzy max-product composition

The max-product composition of R(X,Y) and S(Y,Z), denoted by $R(X, Y) \circ S(Y, Z)$ is defined by T(X,Z) as

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \mu_{\underset{\sim}{T}}(x, z) = \bigvee_{y \in Y}(\mu_{\underset{\sim}{R}}(x, y) . \mu_{\underset{\sim}{S}}(y, z))$$

$$= max_{y \in Y}\{(\mu_{\underset{\sim}{R}}(x, y) \times \mu_{\underset{\sim}{S}}(y, z))\}$$

Problem

Fuzzy Composition

Q1 *Two fuzzy relations are given by,*

$$R = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array} \text{ and}$$

$$S = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain fuzzy relation T as a composition between the fuzzy relations R and S.

Fuzzy max-min Composition

Two fuzzy relations are given by,

$$R = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array} \text{ and } S = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

max(min((0.6,1),(0.3,0.8)))

max(0.6,0.3)

=0.6

Composition of fuzzy relation is defined over two fuzzy relations. if x1 related y1 and if y1 is related to z1 then with transitive property we can say x1 is related to z1.Here x1 is related to z1 with 0.3 in the case of max–min composition

(a) Max−min composition

$$\begin{aligned}\mu_T(x_1, z_1) &= max\{min[\mu_R(x_1, y_1), \mu_S(y_1, z_1)], min[\mu_R(x_1, y_2), \mu_S(y_2, z_1)]\} \\ &= max\{min[0.6, 1], min[0.3, 0.8]\} \\ &= max\{0.6, 0.3\} = 0.6\end{aligned}$$

$$\begin{aligned}\mu_T(x_1, z_2) &= max\{min[\mu_R(x_1, y_1), \mu_S(y_1, z_2)], min[\mu_R(x_1, y_2), \mu_S(y_2, z_2)]\} \\ &= max\{min[0.6, 0.5], min[0.3, 0.4]\} \\ &= max\{0.5, 0.3\} = 0.5\end{aligned}$$

$$\begin{aligned}\mu_T(x_1, z_3) &= max\{min[\mu_R(x_1, y_1), \mu_S(y_1, z_3)], min[\mu_R(x_1, y_2), \mu_S(y_2, z_3)]\} \\ &= max\{min[0.6, 0.3], min[0.3, 0.7]\} \\ &= max\{0.3, 0.3\} = 0.3\end{aligned}$$

$$T = R \circ S = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ & & \end{bmatrix} \end{array}$$

$$\begin{aligned}\mu_T(x_2, z_1) &= max\{min[\mu_R(x_2, y_1), \mu_S(y_1, z_1)], min[\mu_R(x_2, y_2), \mu_S(y_2, z_1)]\} \\ &= max\{min[0.2, 1], min[0.9, 0.8]\} \\ &= max\{0.2, 0.8\} = 0.8\end{aligned}$$

$$\begin{aligned}\mu_T(x_2, z_2) &= max\{min[\mu_R(x_2, y_1), \mu_S(y_1, z_2)], min[\mu_R(x_2, y_2), \mu_S(y_2, z_2)]\} \\ &= max\{min[0.2, 0.5], min[0.9, 0.4]\} \\ &= max\{0.2, 0.4\} = 0.4\end{aligned}$$

$$\begin{aligned}\mu_T(x_2, z_3) &= max\{min[\mu_R(x_2, y_1), \mu_S(y_1, z_3)], min[\mu_R(x_2, y_2), \mu_S(y_2, z_3)]\} \\ &= max\{min[0.2, 0.3], min[0.9, 0.7]\} \\ &= max\{0.2, 0.7\} = 0.7\end{aligned}$$

$$T = R \circ S = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Fuzzy max-product Composition

(b) $\underline{Max\text{–}product\ composition}$

$$\boxed{\mu_T(x_1, z_1) = max\{[\mu_R(x_1, y_1) \bullet \mu_S(y_1, z_1)], [\mu_R(x_1, y_2) \bullet \mu_S(y_2, z_1)]\}}$$

Two fuzzy relations are given by,

$$R = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array} \quad and \quad S = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

max((0.6 X 1),(0.3 X 0.8))
max(0.6,0.24)
=0.6

(b) $\underline{Max\text{–}product\ composition}$

$$\begin{aligned} \mu_T(x_1, z_1) &= max\{[\mu_R(x_1, y_1) \bullet \mu_S(y_1, z_1)], \\ &\qquad [\mu_R(x_1, y_2) \bullet \mu_S(y_2, z_1)]\} \\ &= max\{[0.6 \times 1], [0.3 \times 0.8]\} \\ &= max\{0.6, 0.24\} = 0.6 \end{aligned}$$

$$\begin{aligned} \mu_T(x_1, z_2) &= max\{[\mu_R(x_1, y_1) \bullet \mu_S(y_1, z_2)], \\ &\qquad [\mu_R(x_1, y_2) \bullet \mu_S(y_2, z_2)]\} \\ &= max\{[0.6 \times 0.5], [0.3 \times 0.4]\} \\ &= max\{0.3, 0.12\} = 0.3 \end{aligned}$$

$$\begin{aligned} \mu_T(x_1, z_3) &= max\{[\mu_R(x_1, y_1) \bullet \mu_S(y_1, z_3)], \\ &\qquad [\mu_R(x_1, y_2) \bullet \mu_S(y_2, z_3)]\} \\ &= max\{[0.6 \times 0.3], [0.3 \times 0.7]\} \\ &= max\{0.18, 0.21\} = 0.21 \end{aligned}$$

$$T = R \circ S$$

$$= \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ \end{array} & \begin{bmatrix} 0.6 & 0.3 & 0.21 \\ & & \end{bmatrix} \end{array}$$

$$\begin{aligned} \mu_T(x_2, z_1) &= max\{[\mu_R(x_2, y_1) \bullet \mu_S(y_1, z_1)], \\ &\qquad [\mu_R(x_2, y_2) \bullet \mu_S(y_2, z_1)]\} \\ &= max\{[0.2 \times 1], [0.9 \times 0.8]\} \\ &= max\{0.2, 0.72\} = 0.72 \end{aligned}$$

$$\begin{aligned} \mu_T(x_2, z_2) &= max\{[\mu_R(x_2, y_1) \bullet \mu_S(y_1, z_2)], \\ &\qquad [\mu_R(x_2, y_2) \bullet \mu_S(y_2, z_2)]\} \\ &= max\{[0.2 \times 0.5], [0.9 \times 0.4]\} \\ &= max\{0.1, 0.36\} = 0.36 \end{aligned}$$

$$\begin{aligned} \mu_T(x_2, z_3) &= max\{[\mu_R(x_2, y_1) \bullet \mu_S(y_1, z_3)], \\ &\qquad [\mu_R(x_2, y_2) \bullet \mu_S(y_2, z_3)]\} \\ &= max\{[0.2 \times 0.3], [0.9 \times 0.7]\} \\ &= max\{0.06, 0.63\} = 0.63 \end{aligned}$$

$$T = R \circ S$$

$$= \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix} \end{array}$$

Problem

Fuzzy max-min Composition

For a speed control of DC motor, the membership functions of series resistance, armature current and speed are given as follows:

$$SR = \{\frac{0.4}{30} + \frac{0.6}{60} + \frac{1.0}{100} + \frac{0.1}{120}\}$$

$$I = \{\frac{0.2}{20} + \frac{0.3}{40} + \frac{0.6}{60} + \frac{0.8}{80} + \frac{1.0}{100} + \frac{0.2}{120}\}$$

$$N = \{\frac{0.35}{500} + \frac{0.67}{1000} + \frac{0.97}{1500} + \frac{0.25}{1800}\}$$

Compute relation T for relating series resistance to motor speed. Perform max−min composition only.

$$SR = \{\frac{0.4}{30} + \frac{0.6}{60} + \frac{1.0}{100} + \frac{0.1}{120}\}$$

$$I = \{\frac{0.2}{20} + \frac{0.3}{40} + \frac{0.6}{60} + \frac{0.8}{80} + \frac{1.0}{100} + \frac{0.2}{120}\}$$

$$N = \{\frac{0.35}{500} + \frac{0.67}{1000} + \frac{0.97}{1500} + \frac{0.25}{1800}\}$$

$$\boxed{\mu_{\underset{\sim}{R}}(x_1, y_1) = \min[\mu_{\underset{\sim}{A}}(x_1), \mu_{\underset{\sim}{B}}(y_1)]}$$

$$SR \times I = \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccccc} 20 & 40 & 60 & 80 & 100 & 120 \end{array} \\ \begin{bmatrix} 0.2 & & & & & \\ 0.2 & & & & & \\ 0.2 & & & & & \\ 0.1 & & & & & \end{bmatrix} \end{array}$$

min(0.4,0.2)=0.2
min(0.6,0.2)=0.2
min(1.0,0.2)=0.2
min(0.1,0.2)=0.1

The membership function of fuzzy relation is given by

$$\mu_{\underset{\sim}{R}}(x, y) = \mu_{\underset{\sim}{A} \times \underset{\sim}{B}}(x, y) = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)\right]$$

$$\boxed{\mu_{\underset{\sim}{R}}(x_1, y_1) = \min[\mu_{\underset{\sim}{A}}(x_1), \mu_{\underset{\sim}{B}}(y_1)]}$$

$$R = SR \times I = \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccccc} 20 & 40 & 60 & 80 & 100 & 120 \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0.4 & 0.4 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.6 & 0.6 & 0.6 & 0.2 \\ 0.2 & 0.3 & 0.6 & 0.8 & 1.0 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

$$S = I \times N = \begin{array}{c} \\ 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 & 0.25 \\ 0.35 & 0.6 & 0.6 & 0.25 \\ 0.35 & 0.67 & 0.8 & 0.25 \\ 0.35 & 0.67 & 0.97 & 0.25 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

max-min composition

$$\mu_{\underset{\sim}{T}}(x, z) = \mu_{\underset{\sim}{R} \circ \underset{\sim}{S}}(x, z) = \max_{y \in Y}\left\{\min\left[\mu_{\underset{\sim}{R}}(x, y), \mu_{\underset{\sim}{S}}(y, z)\right]\right\}$$

$$R = \begin{array}{cc} & \\ x_1 & 30 \\ x_2 & 60 \\ x_3 & 100 \\ x_4 & 120 \end{array} \begin{array}{c} \begin{array}{cccccc} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \\ 20 & 40 & 60 & 80 & 100 & 120 \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0.4 & 0.4 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.6 & 0.6 & 0.6 & 0.2 \\ 0.2 & 0.3 & 0.6 & 0.8 & 1.0 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

$$S = \begin{array}{c} \\ 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 & 0.25 \\ 0.35 & 0.6 & 0.6 & 0.25 \\ 0.35 & 0.67 & 0.8 & 0.25 \\ 0.35 & 0.67 & 0.97 & 0.25 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

$$T = R \circ S = \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} 0.35 & 0.4 & 0.4 & 0.25 \\ 0.35 & 0.6 & 0.6 & 0.25 \\ 0.35 & 0.67 & 0.97 & 0.25 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

- Max(min(0.2,0.2),(0.3,0.3),(04.,0.35),(0.4,0.35),(0.4,0.35),(0.2,0.2))
- =max(0.2,0.3,0.35,0.35,0.35,0.2)
- =0.35

Fuzzy max-min Composition

Consider two fuzzy sets given by

$$\underset{\sim}{A} = \left\{ \frac{1}{\text{low}} + \frac{0.2}{\text{medium}} + \frac{0.5}{\text{high}} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0.9}{\text{positive}} + \frac{0.4}{\text{zero}} + \frac{0.9}{\text{negative}} \right\}$$

(a) Find the fuzzy relation for the Cartesian product of $\underset{\sim}{A}$ and $\underset{\sim}{B}$, i.e., $\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B}$.

(b) Introduce a fuzzy set $\underset{\sim}{C}$ given by

$$\underset{\sim}{C} = \left\{ \frac{0.1}{\text{low}} + \frac{0.2}{\text{medium}} + \frac{0.7}{\text{high}} \right\}$$

Find the relation between $\underset{\sim}{C}$ and $\underset{\sim}{B}$ using Cartesian product, i.e., find $\underset{\sim}{S} = \underset{\sim}{C} \times \underset{\sim}{B}$.

(c) Find $\underset{\sim}{C} \circ \underset{\sim}{R}$ using max-min composition.

(d) Find $\underset{\sim}{C} \circ \underset{\sim}{S}$ using max-min composition.

Solution:

(a) The Cartesian product between $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is obtained as

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(y)]$$

$$= \begin{array}{c} \\ \text{low} \\ \text{medium} \\ \text{high} \end{array} \begin{array}{c} \begin{array}{ccc} \text{positive} & \text{zero} & \text{negative} \end{array} \\ \begin{bmatrix} 0.9 & 0.4 & 0.9 \\ 0.2 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.5 \end{bmatrix} \end{array}$$

The new fuzzy set is

$$\underset{\sim}{C} = \left\{ \frac{0.1}{\text{low}} + \frac{0.2}{\text{medium}} + \frac{0.7}{\text{high}} \right\}$$

The Cartesian product between $\underset{\sim}{C}$ and $\underset{\sim}{B}$ is obtained as

$$\underset{\sim}{S} = \underset{\sim}{C} \times \underset{\sim}{B} = \min[\mu_{\underset{\sim}{C}}(x), \mu_{\underset{\sim}{B}}(y)]$$

$$= \begin{array}{c} \\ \text{low} \\ \text{medium} \\ \text{high} \end{array} \begin{array}{c} \begin{array}{ccc} \text{positive} & \text{zero} & \text{negative} \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.7 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

$$\underset{\sim}{C} \circ \underset{\sim}{R} = [0.1 \;\; 0.2 \;\; 0.7]_{1\times3} \begin{bmatrix} 0.9 & 0.4 & 0.9 \\ 0.2 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.5 \end{bmatrix}_{3\times3}$$

$$= [0.5 \;\; 0.4 \;\; 0.5]$$

For instance,

$$\mu_{\underset{\sim}{C} \circ \underset{\sim}{R}}(x_1, y_1) = \max[\min(0.1, 0.9), \min(0.2, 0.2), \min(0.7, 0.5)]$$
$$= \max(0.1, 0.2, 0.5) = 0.5$$

(d)

$$\underset{\sim}{C} \circ \underset{\sim}{S} = [0.1 \;\; 0.2 \;\; 0.7]_{1\times3} \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.7 & 0.4 & 0.7 \end{bmatrix}_{3\times3}$$

$$= [0.7 \;\; 0.4 \;\; 0.7]$$

Fuzzy max-min Composition

Consider a universe of aircraft speed near the speed of sound as $X = \{0.72, 0.725, 0.75, 0.775, 0.78\}$ and a fuzzy set on this universe for the speed "near mach 0.75" $= \underset{\sim}{M}$ where

$$\underset{\sim}{M} = \left\{ \frac{0}{0.72} + \frac{0.8}{0.725} + \frac{1}{0.75} + \frac{0.8}{0.775} + \frac{0}{0.78} \right\}.$$

Define a universe of altitudes as $Y = \{21, 22, 23, 24, 25, 26, 27\}$ in k-feet and a fuzzy set on this universe for the altitude fuzzy set "approximately 24,000 feet" $= \underset{\sim}{N}$ where

$$\underset{\sim}{N} = \left\{ \frac{0}{21k} + \frac{0.2}{22k} + \frac{0.7}{23k} + \frac{1}{24k} + \frac{0.7}{25k} + \frac{0.2}{26k} + \frac{0}{27k} \right\}$$

(b) For another aircraft speed, say $\underset{\sim}{M}_1$, in the region of mach 0.75 where

$$\underset{\sim}{M}_1 = \left\{ \frac{0}{0.72} + \frac{0.8}{0.725} + \frac{1}{0.75} + \frac{0.6}{0.775} + \frac{0}{0.78} \right\}$$

find relation $\underset{\sim}{S} = \underset{\sim}{M}_1 \circ \underset{\sim}{R}$ using max-min composition.

Solution: The two given fuzzy sets are

$$\underset{\sim}{M} = \left\{ \frac{0}{0.72} + \frac{0.8}{0.725} + \frac{1}{0.75} + \frac{0.8}{0.775} + \frac{0}{0.78} \right\}$$

$$\underset{\sim}{N} = \left\{ \frac{0}{21k} + \frac{0.2}{22k} + \frac{0.7}{23k} + \frac{1}{24k} + \frac{0.7}{25k} + \frac{0.2}{26k} + \frac{0}{27k} \right\}$$

Fuzzy max-min Composition

(a) Relation $\underset{\sim}{R} = \underset{\sim}{M} \times \underset{\sim}{N}$ is obtained by using Cartesian product

$$\underset{\sim}{R} = \min[\mu_{\underset{\sim}{M}}(x), \mu_{\underset{\sim}{N}}(y)]$$

$$= \begin{array}{c} \\ 0.72 \\ 0.725 \\ 0.75 \\ 0.775 \\ 0.78 \end{array} \begin{array}{c} \begin{array}{ccccccc} 21k & 22k & 23k & 24k & 25k & 26k & 27k \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.7 & 0.8 & 0.7 & 0.2 & 0 \\ 0 & 0.2 & 0.7 & 1 & 0.7 & 0.2 & 0 \\ 0 & 0.2 & 0.7 & 0.8 & 0.7 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b) Relation $\underset{\sim}{S} = \underset{\sim}{M}_1 \circ \underset{\sim}{R}$ is found by using max-min composition

$$\underset{\sim}{S} = \max\{\min[\mu_{\underset{\sim}{M}}(x), \mu_{\underset{\sim}{R}}(x, y)]\}$$

$$= [0 \;\; 0.8 \;\; 1 \;\; 0.6 \;\; 0]_{1\times5} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0.7 & 0.8 & 0.7 & 0.2 & 0 \\ 0 & 0.2 & 0.7 & 1 & 0.7 & 0.2 & 0 \\ 0 & 0.2 & 0.7 & 0.8 & 0.7 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5\times7}$$

$$\underset{\sim}{S} = [0 \;\; 0.2 \;\; 0.7 \;\; 1 \;\; 0.7 \;\; 0.2 \;\; 0]_{1\times7}$$

Fuzzy Composition

Consider two relations

$$\underset{\sim}{R} = \begin{array}{c} \\ 9 \\ 18 \\ 27 \\ 36 \end{array} \begin{array}{c} \begin{array}{ccccc} -100 & -50 & 0 & 50 & 100 \end{array} \\ \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1 & 0.8 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

and

$$\underset{\sim}{S} = \begin{array}{c} \\ -100 \\ -50 \\ 0 \\ 50 \\ 100 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 4 & 8 & 16 & 20 \end{array} \\ \begin{bmatrix} 1 & 0.8 & 0.6 & 0.3 & 0.1 \\ 0.7 & 1 & 0.7 & 0.5 & 0.4 \\ 0.5 & 0.6 & 1 & 0.8 & 0.8 \\ 0.3 & 0.4 & 0.6 & 1 & 0.9 \\ 0.9 & 0.3 & 0.5 & 0.7 & 1 \end{bmatrix} \end{array}$$

If $\underset{\sim}{R}$ is a relationship between frequency and temperature and $\underset{\sim}{S}$ represents a relation between temperature and reliability index of a circuit, obtain the relation between frequency and reliability index using (a) max-min composition and (b) max-product composition.

(a) Max-min composition is performed as follows.

$$\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \max\{\min[\mu_{\underset{\sim}{R}}(x,y), \mu_{\underset{\sim}{S}}(x,y)]\}.$$

$$= \begin{array}{c} \\ 9 \\ 18 \\ 27 \\ 36 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 4 & 8 & 16 & 20 \end{array} \\ \begin{bmatrix} 0.9 & 0.6 & 0.7 & 1 & 0.9 \\ 0.8 & 0.6 & 0.7 & 1 & 0.9 \\ 0.6 & 0.6 & 0.8 & 0.9 & 0.9 \\ 0.9 & 1 & 0.8 & 0.8 & 0.8 \end{bmatrix} \end{array}$$

Fuzzy Composition

$$\mu_T(x_1, z_1) = max\{[\mu_R(x_1, y_1) \bullet \mu_S(y_1, z_1)], [\mu_R(x_1, y_2) \bullet \mu_S(y_2, z_1)]\}$$

Consider two relations

$$\underset{\sim}{R} = \begin{array}{c} \\ 9 \\ 18 \\ 27 \\ 36 \end{array} \begin{array}{c} \begin{array}{ccccc} -100 & -50 & 0 & 50 & 100 \end{array} \\ \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1 & 0.8 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

and

$$\underset{\sim}{S} = \begin{array}{c} \\ -100 \\ -50 \\ 0 \\ 50 \\ 100 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 4 & 8 & 16 & 20 \end{array} \\ \begin{bmatrix} 1 & 0.8 & 0.6 & 0.3 & 0.1 \\ 0.7 & 1 & 0.7 & 0.5 & 0.4 \\ 0.5 & 0.6 & 1 & 0.8 & 0.8 \\ 0.3 & 0.4 & 0.6 & 1 & 0.9 \\ 0.9 & 0.3 & 0.5 & 0.7 & 1 \end{bmatrix} \end{array}$$

(b) Max-product composition is performed as follows.

$$= \begin{array}{c} \\ 9 \\ 18 \\ 27 \\ 36 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 4 & 8 & 16 & 20 \end{array} \\ \begin{bmatrix} 0.81 & 0.5 & 0.7 & 1.0 & 0.9 \\ 0.72 & 0.5 & 0.7 & 1.0 & 0.9 \\ 0.42 & 0.6 & 0.8 & 0.9 & 0.81 \\ 0.9 & 1.0 & 0.8 & 0.64 & 0.64 \end{bmatrix} \end{array}$$

Thus the relation between frequency and reliability index has been found using composition techniques.

Fuzzy Composition

Three fuzzy sets are given as follows:

$$\underset{\sim}{P} = \left\{\frac{0.1}{2} + \frac{0.3}{4} + \frac{0.7}{6} + \frac{0.4}{8} + \frac{0.2}{10}\right\}$$

$$\underset{\sim}{Q} = \left\{\frac{0.1}{0.1} + \frac{0.3}{0.2} + \frac{0.3}{0.3} + \frac{0.4}{0.4} + \frac{0.5}{0.5} + \frac{0.2}{0.6}\right\}$$

$$\underset{\sim}{T} = \left\{\frac{0.1}{0} + \frac{0.7}{0.5} + \frac{0.3}{1}\right\}$$

(a) $\boxed{\underset{\sim}{R} = \underset{\sim}{P} \times \underset{\sim}{Q} = \min[\mu_{\underset{\sim}{P}}(x), \mu_{\underset{\sim}{Q}}(y)]}$

$$= \begin{array}{c} \\ 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccccc} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.4 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.4 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

(b) $\boxed{\underset{\sim}{S} = \underset{\sim}{Q} \times \underset{\sim}{T} = \min[\mu_{\underset{\sim}{Q}}(x), \mu_{\underset{\sim}{T}}(y)]}$

$$= \begin{array}{c} \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 0.5 & 1 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.4 & 0.3 \\ 0.1 & 0.5 & 0.3 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

Fuzzy Composition

$\boxed{\underset{\sim}{T} = \underset{\sim}{R} \circ \underset{\sim}{S} = \max\{\min[\mu_{\underset{\sim}{R}}(x,y) \times \mu_{\underset{\sim}{S}}(x,y)]\}}$

$$= \begin{array}{c} \\ 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 0.5 & 1 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 \\ 0.1 & 0.5 & 0.3 \\ 0.1 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

(d) $\boxed{\underset{\sim}{M} = \underset{\sim}{R} \circ \underset{\sim}{S} = \max[\mu_{\underset{\sim}{R}}(x,y) \times \mu_{\underset{\sim}{S}}(x,y)]}$

$$= \begin{array}{c} \\ 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 0.5 & 1 \end{array} \\ \begin{bmatrix} 0.01 & 0.05 & 0.03 \\ 0.03 & 0.05 & 0.09 \\ 0.05 & 0.25 & 0.15 \\ 0.04 & 0.20 & 0.12 \\ 0.02 & 0.0 & 0.06 \end{bmatrix} \end{array}$$

Thus the operations were performed over the given fuzzy sets.

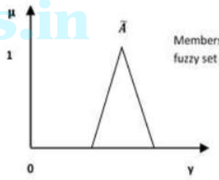
Properties of Fuzzy Composition

1. $R \circ S = S \circ R$
2. $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$
3. $(R \circ S) \circ M = R \circ (S \circ M)$

Fuzzy membership functions,

Fuzzy membership functions,

- fuzzy logic is not ,the logic that is fuzzy
- Fuzzy logic is used to describe fuzziness.
- This fuzziness is best characterized by its membership function.
- The Membership function represents the degree of truthness in fuzzy logic.
- Rules for defining fuzziness are fuzzy too.



Membership Function

Fuzzy membership functions,

- Membership functions were first introduced in 1965 by Lofti A. Zadeh in his first research paper "fuzzy sets".
- Membership functions characterize fuzziness (i.e., all the information in fuzzy set), whether the elements in fuzzy sets are discrete or continuous.
- Membership functions can be defined as a technique to solve practical problems by experience rather than knowledge.
- Membership functions are represented by graphical forms.
- Membership functions represents all the information contained in a fuzzy set.
- A fuzzy set $\tilde{A}$ in the universe of information U can be defined as a set of ordered pairs and it can be represented mathematically as

$$\tilde{A} = \{(x, \mu_A(x)) | x \in X\}$$

Fuzzy membership functions,

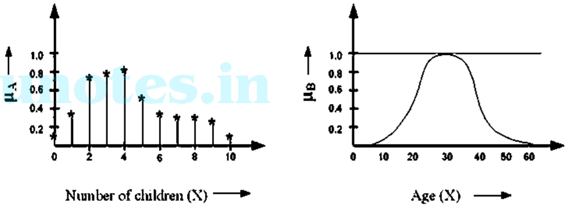
- Membership functions are represented by graphical forms.

A membership function can be on

a) a discrete universe of discourse and

b) a continuous universe of discourse.

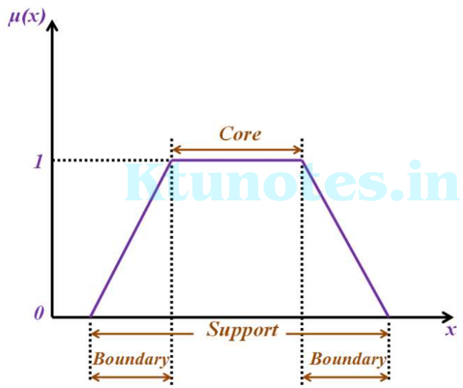
Example:



A = Fuzzy set of "Happy family"   B = "Young age"

Features of Fuzzy membership functions,

Features of Fuzzy membership functions,



Features of Fuzzy membership functions,

Core

- For any fuzzy set A˜, the core of a membership function is that region of the universe that is characterized by full membership in the set.
- Hence, core consists of all those elements x of the universe of information such that,

$$\boxed{\mu_A(x) = 1}$$

- Core of a fuzzy set may be an empty set also.

Features of Fuzzy membership functions,

.Support

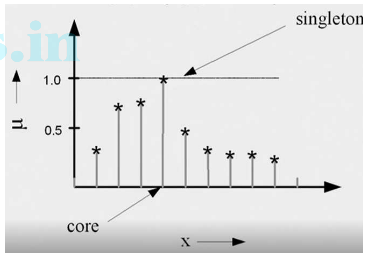
- For any fuzzy set A˜, the support of a membership function is the region of the universe that is characterized by a nonzero membership in the set.
- Hence support consists of all those elements x of the universe of information such that,

$$\mu_A(x) > 0$$

- A fuzzy set whose support is a single element in x with

$$\mu_A(x) = 1$$

is referred to as Fuzzy Singleton.



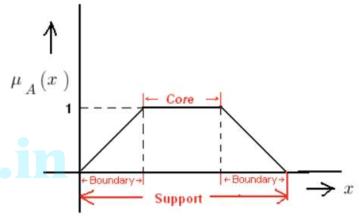
Features of Fuzzy membership functions,

Boundary

- For any fuzzy set A˜, the boundary of a membership function is the universe region characterized by a nonzero but incomplete membership in the set.
- Hence, the Boundary consists of all those elements x of the universe of information such that,
- Boundary (which possess partial membership in the fuzzy set A˜.

$$0 < \mu_A(x) < 1$$

(Non-complete and Non-zero Membership value).

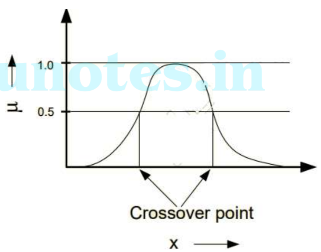


$$\mu_A(x) \in (0,1)$$

Features of Fuzzy membership functions,

- Cross-over Points of a Membership Function:-
- It is defined as the elements of a fuzzy set A whose membership value is equal to 0.5.

$$\mu_A(x) = 0.5$$



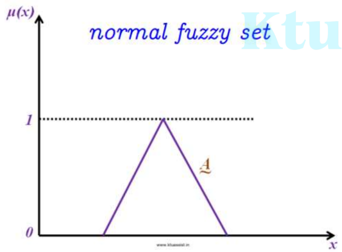
Height of a Membership Function:-

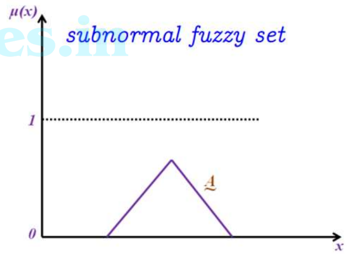
- Height of a membership function is the maximum value of the membership function of a fuzzy set.
- If the height of a fuzzy set is < 1 then it subnormal fuzzy set. Whereas if its height is equal to 1 then it is a normal fuzzy set.

Types of fuzzy set

normal and subnormal fuzzy set

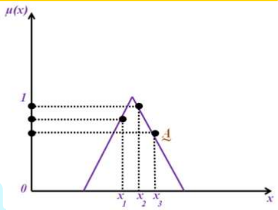
- A fuzzy set with at least one element x in the universe, whose membership value is unity is called a normal fuzzy set.
- A fuzzy set where in, no membership function has its value equal to 1 is called a subnormal fuzzy set.





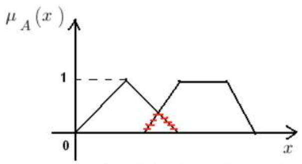
convex fuzzy set

- Convex fuzzy set is described by a membership function whose membership values are strictly Monotonically Increasing or Monotonically Decreasing or Initially Monotonically Increasing then Monotonically Decreasing with the increase in the values of the elements of that particular fuzzy set.
- For elements x1,x2,x3 in a fuzzy set A, if



$$\mu_A(x_2) \geq min[\mu_A(x_1), \mu_A(x_3)]$$

- Then A is said to be a convex fuzzy set.
- The intersection of two convex fuzzy sets is also a convex fuzzy set.
-



Intersection of Two Convex Fuzzy Set

convex fuzzy set

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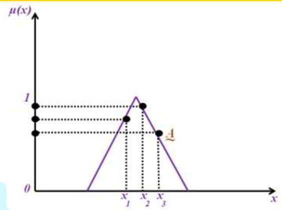

$$\mu_A(x_2) \geq min[\mu_A(x_1), \mu_A(x_3)]$$

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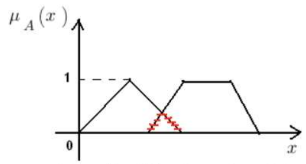
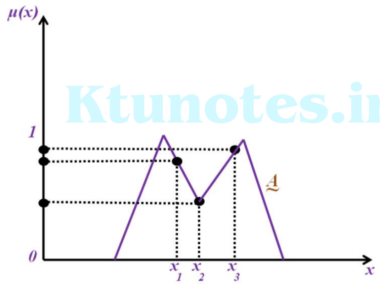


Intersection of Two Convex Fuzzy Set

nonconvex fuzzy set

- A fuzzy set possessing characteristics opposite to that of a convex fuzzy set is called a nonconvex fuzzy set.



FUZZIFICATION

Fuzzification

- **Fuzzification** may be defined as the process of transforming a crisp set to a fuzzy set or a fuzzy set to a fuzzier set
- Fuzzification translates the crisp input data into linguistic variables which are represented by fuzzy sets.
- After that, it applies the membership functions to measure and determine the degree of membership.
- For example, when I say the temperature is 45° Celsius the viewer converts the crisp input value into a linguistic variable like favorable temperature for the human body, hot or cold.

Fuzzification

- Ex..When one is told thst the temperature is 9 degree,the person translates the crisp value to linguistic variable cold/very cold/light warm according to ones knowlege and makes a decision ,about the need to wear a jacket or not
- If one fails to fuzzify ,it is not possible to continue the decision process /error decision may be reached.

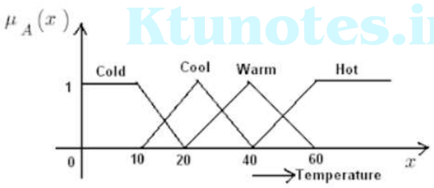
Methods of membership value assignment

methods of assigning membership values

- Different methods of assigning membership values to elements are as follows:
- Intuition
- Inference
- Rank Ordering
- Angular Fuzzy Sets
- Neural Networks
- Genetic Algorithm
- Inductive Reasoning

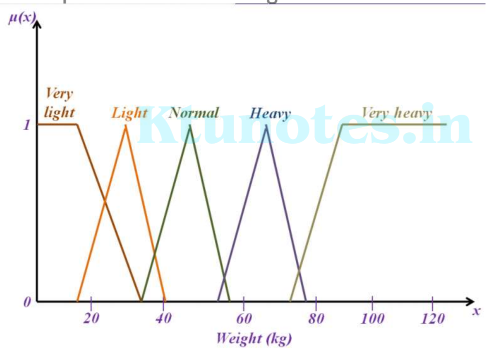
methods of assigning membership values

- **Intuition**
- Intuition is based upon the common intelligence of human
- It is simply derived from the capacity of humans to develop membership functions on the basis of their own intelligence and understanding capability



Membership function for the fuzzy variable weight

- **Intuition**
- Intuition is based upon the common intelligence of human



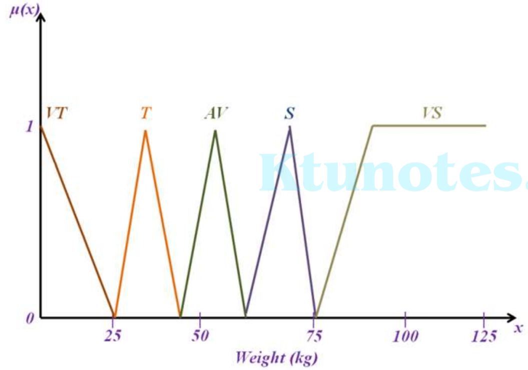
Membership function for the fuzzy variable weight

(1) *Using your own intuition and definitions of the universe of discourse, plot fuzzy membership functions for "weight of people".*

U = *weight of people*
Let the weights be in kilogram.
Let the linguistic variables be the following:

$$\begin{aligned} Very\ thin(VT) &: W \leq 25 \\ Thin(T) &: 25 < W \leq 45 \\ Average(AV) &: 45 < W \leq 60 \\ Stout(S) &: 60 < W \leq 75 \\ Very\ stout(VS) &: W > 75 \end{aligned}$$

Membership function for the fuzzy variable weight



$$\begin{aligned} Very\ thin(VT) &: W \leq 25 \\ Thin(T) &: 25 < W \leq 45 \\ Average(AV) &: 45 < W \leq 60 \\ Stout(S) &: 60 < W \leq 75 \\ Very\ stout(VS) &: W > 75 \end{aligned}$$

Intuition –Problem 2

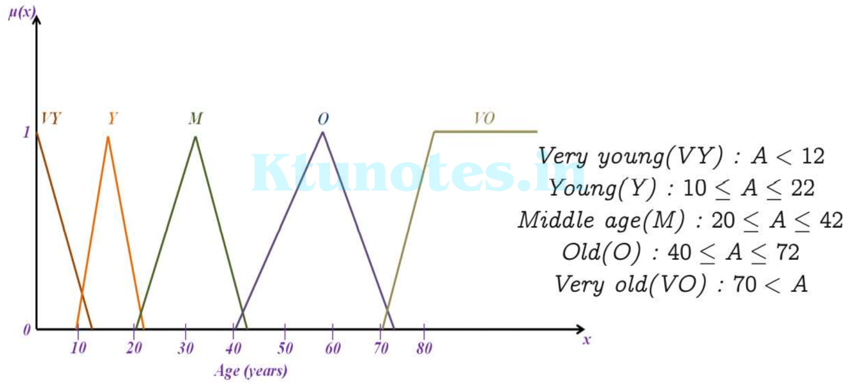
Membership function for the fuzzy variable "age of people"

(2) *Using your own intuition and definitions of the universe of discourse, plot fuzzy membership functions for "age of people".*

U = *age of people*
Let A denote age of people in years.
Let the linguistic variables be the following:

$$\begin{aligned} Very\ young(VY) &: A < 12 \\ Young(Y) &: 10 \leq A \leq 22 \\ Middle\ age(M) &: 20 \leq A \leq 42 \\ Old(O) &: 40 \leq A \leq 72 \\ Very\ old(VO) &: 70 < A \end{aligned}$$

Membership function for the fuzzy variable “age of people”



Membership function for the fuzzy variable “liquid level in the tank”

Using intuition and your own definition of the universe of discourse, plot fuzzy membership functions to the following variables:

(i) Liquid level in the tank

(a) Very small
(b) Small
(c) Empty
(d) Full
(e) Very full

TUTORIAL QUESTION

Interference

methods of assigning membership values

- **Interference**
- In the inference method we use knowledge to perform deductive reasoning.
- To deduce or infer a conclusion, we have to use the facts and knowledge on that particular problem.
- Let us consider the example of Geometric shapes for the identification of a triangle.
- Let A, B, and C be the interior angles of a triangle such that
  $A \geq B \geq C > 0°$ and $A + B + C = 180°$

methods of assigning membership values

Interference

- The knowledge of geometrical shapes and geometry is used for defining membership values.
- The membership functions may be defined using various shapes: *triangular, trapezoidal, bell−shaped, Gaussian etc.*
- The inference method here is via triangular shape.
- Consider a triangle, where X, Y and Z are the angles, such that $X > Y > Z > 0$

  Let U be the universe of triangles, *ie,*

  $$U = \{(X, Y, Z) | X \geq Y \geq Z \geq 0; X + Y + Z = 180\}$$

methods of assigning membership values

Interference

- There are various types of triangles available:
  1. $I = \textit{isosceles triangle}$
  2. $E = \textit{equilateral triangle}$
  3. $R = \textit{right− angle triangle}$
  4. $IR = \textit{isosceles and right− angle triangle}$
  5. $T = \textit{other triangles}$

Inference-Isosceles Triangle(I)

- The membership values of approximate isosceles triangle is obtained using,

$$\boxed{\mu_I(X, Y, Z) = 1 - \frac{1}{60^o} min(X - Y, Y - Z)}$$

where,

$$X \geq Y \geq Z \geq 0 \text{ and } X + Y + Z = 180$$

If $X = Y$ or $Y = Z$, then $\mu_I(X, Y, Z) = 1$

If $X = 120^o$ or $Y = 60^o$ and $Z = 0^o$, then $\mu_I(X, Y, Z) = 0$

Inference-Right Angle Triangle(R)

- The membership value of approximate right angle triangle is given by,

$$\boxed{\mu_R(X, Y, Z) = 1 - \frac{1}{90^o} |X - 90^o|}$$

If $X = 90^o$, then $\mu_R(X, Y, Z) = 1$

If $X = 180^o$, then $\mu_R(X, Y, Z) = 0$

Inference-Isosceles Right Angle Triangle(IR)

- The membership value of approximate isosceles right angle triangle is obtained by,

$$IR = I \cap R$$

and it is given by,

$$\mu_{IR}(X,Y,Z) = min[\mu_I(X,Y,Z), \mu_R(X,Y,Z)]$$

$$\boxed{= 1 - max[\frac{1}{60^o} min(X-Y, Y-Z), \frac{1}{90^o}|X-90^o|]}$$

Inference-Equilateral Triangle(E)

- The membership function for a fuzzy equilateral triangle is given by,

$$\boxed{\mu_E(X,Y,Z) = 1 - \frac{1}{180^o}|X-Z|}$$

Inference-Other Triangle(E)

- The membership function of other triangles is given by,

$$T = \overline{I \cup R \cup E}$$

By using DeMorgan's law,

$$T = \overline{I} \cap \overline{R} \cap \overline{E}$$

The membership value can be obtained using,

$$\boxed{\mu_T(X,Y,Z) = min[1-\mu_I(X,Y,Z), 1-\mu_E(X,Y,Z), 1-\mu_R(X,Y,Z)]}$$

$$= \frac{1}{180^o} \text{Min}[\ 3(X-Y)\ ,\ 3(Y-Z)\ ,\ 2\ |X-90|\ ,\ X-Z]$$

Inference-Problems

(1) *Using the inference approach, find the membership values for the triangular shapes I,R,E,IR and T for a triangle with angles* $45^o, 55^o$ *and* 80^o.

Let the universe of discourse be,

$$U = \{(X, Y, Z) | X = 80^o \geq Y = 55^o \geq Z = 45^o \geq 0, \quad X + Y + Z = 80^o + 55^o + 45^o = 180^o\}$$

Membership value of isosceles triangle, I

$$\mu_I = 1 - \frac{1}{60^o} min(X - Y, Y - Z)$$

Membership value of isosceles triangle, I

$$X = 80^o \geq Y = 55^o \geq Z = 45^o \geq 0$$

$$\mu_I = 1 - \frac{1}{60^o} min(X - Y, Y - Z)$$

$$= 1 - \frac{1}{60^o} min(80^o - 55^o, 55^o - 45^o)$$

$$= 1 - \frac{1}{60^o} min(25^o, 10^o)$$

$$= 1 - \frac{1}{60^o} \times 10^o = 0.833$$

Membership value of right angle triangle, R

$$\mu_R = 1 - \frac{1}{90^o} |X - 90^o|$$

$$= 1 - \frac{1}{90^o} |80^o - 90^o|$$

$$= 1 - \frac{1}{90^o} \times 10^o = 0.889$$

Membership value of equilateral triangle, E

$$\mu_E = 1 - \frac{1}{180^o} (X - Z)$$

$$= 1 - \frac{1}{180^o} (80^o - 45^o)$$

$$= 1 - \frac{1}{180^o} \times 35^o = 0.8056$$

Inference-Problems

Membership value of isosceles and right angle triangle, IR

$$\mu_{IR} = min(\mu_I, \mu_R)$$
$$= min(0.833, 0.889) = 0.833$$

Membership value of other triangles, T

$$\mu_T = min(1-\mu_I, 1-\mu_E, 1-\mu_R)$$
$$= min(0.167, 0.1944, 0.111) = 0.111$$

Inference-Problem2

Inference-Problems

Using the inference approach, obtain the membership values for the triangular shapes $(\underset{\sim}{I}, \underset{\sim}{R}, \underset{\sim}{T})$ for a triangle with angles 40°, 60° and 80°.

Let the universe of discourse be

$$U = \{(X, Y, Z) : X = 80^\circ \geq Y = 60^\circ \geq Z = 40^\circ \text{ and } X + Y + Z = 80^\circ + 60^\circ + 40^\circ = 180^\circ\}$$

Inference-Problems

- Membership value of isosceles triangle, $\underset{\sim}{I}$:

$$\mu_{\underset{\sim}{I}} = 1 - \frac{1}{60^\circ} \min(X - Y, Y - Z)$$
$$= 1 - \frac{1}{60^\circ} \min(80^\circ - 60^\circ, 60^\circ - 40^\circ)$$
$$= 1 - \frac{1}{60^\circ} \min(20^\circ, 20^\circ)$$
$$= 1 - \frac{1}{60^\circ} \times 20^\circ = 0.667$$

Inference-Problems

- Membership value of right-angle triangle, $\underset{\sim}{R}$:

$$\mu_{\underset{\sim}{R}} = 1 - \frac{1}{90^\circ}|X - 90^\circ| = 1 - \frac{1}{90^\circ}|80^\circ - 90^\circ|$$

$$= 1 - \frac{1}{90^\circ} \times 10^\circ = 0.889$$

- Membership value of other triangles, $\underset{\sim}{T}$:

$$\mu_{\underset{\sim}{T}} = \min[1-\mu_{\underset{\sim}{I}}, 1-\mu_{\underset{\sim}{R}}] = \min[1-0.667, 1-0.889]$$

$$= \min[0.333, 0.111] = 0.111$$

Inference-Tutorial

Using inference approach outlined in this chapter, find the membership values for each of the triangular shapes ($\underset{\sim}{I}$, $\underset{\sim}{R}$, $\underset{\sim}{E}$, $\underset{\sim}{IR}$, $\underset{\sim}{T}$) for each of the following (all in degrees):

(a) 20°, 40°, 120°

(b) 90°, 45°, 45°

(c) 35°, 75°, 70°

(d) 10°, 60°, 110°

(e) 50°, 75°, 55°

Rank Ordering

Rank Ordering

- On the basis of the preferences made by an individual, a committee, a poll and other opinion methods.
- Pairwise comparisons enables to determine preferences.
- This results in determining the order of the membership.
- Ex.The formation of Government is based on the Polling concept,to identify the best student ,Ranking may be performed,to Buy a car,one can ask for several opinions and so on,
- Prefernces is made based on the pairwise comparisons and these determines the ordering of the membership.

Rank Ordering

- Suppose 10000 people respond to a questionaire about the pairwise preferences among five colors.

  X={red,orange,yellow,green,blue}

  Define a fuzzy set A on the Universe of Colors "Best Color"

  10 color comparison -->R->O,R->Y,R->G,R->B, O->Y,O->G,O->B,Y->G,Y->B,G->B

Rank Ordering

X={red,orange,yellow,green,blue}

Define a fuzzy set A on the Universe of Colors "Best Color"

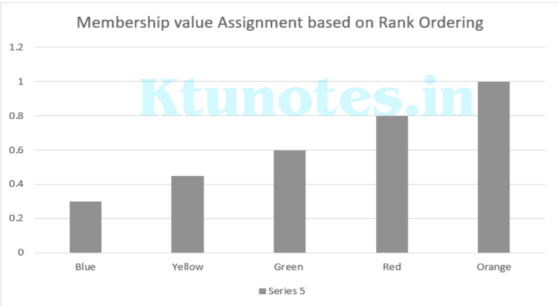
10 color comparison -->R->O,R->Y,R->G,R->B, O->Y,O->G,O->B,Y->G,Y->B,G->B

Color	Red	Orange	Yellow	Green	Blue	Total	Percentage	Rank
Red	-----	517	525	545	661	2248	22.5	2
Orange	483	------	841	477	576	2377	23.8	1
Yellow	475	159	-----	534	614	1782	17.8	4
Green	455	523	466	-----	643	2087	20.9	3
Blue	339	424	386	357	----	1506	15	5

- 1000 persons
- Opinion about
- 10 colors=10000 results

Rank Ordering

- The above is the survey of opinion survey.In the table ,out of 10000 people 517 prefered the color Red,to the color Orange.841 prefered the color orange to the color yellow and so on.



Rank Ordering-problem

Rank Ordering

- Suppose 1,000 people responds to a questionnaire about the pair wise preference among five cars, x– {Palio, Siena, Astra, Easter, Baleno}.
- Define a fuzzy set as A~ on the universe of cars,"Best Car"

Solution: Table shows the rank ordering for performance of cars is a summary of the opinion survey. In Table for example, out of 1000 people, 192 preferred Maruti 800 to the Scorpio, etc. The total number of responses is 10,000 (10 comparisons). On the basis of the preferences, the percentage is calculated. The ordering is then performed. It is found that Octavia is selected as the best car. Figure shows the membership function for this example.

Table

	Number who preferred							
	Maruti 800	Scorpio	Matiz	Santro	Octavia	Total	Percentage	Rank order
Maruti 800	—	192	246	592	621	1651	16.5	5
Scorpio	403	—	621	540	391	1955	19.6	2
Matiz	235	336	—	797	492	1860	18.6	4
Santro	523	364	417	—	608	1912	19.1	3
Octavia	616	534	746	726	—	2622	26.2	1
Total						10000		

Rank Ordering

Table

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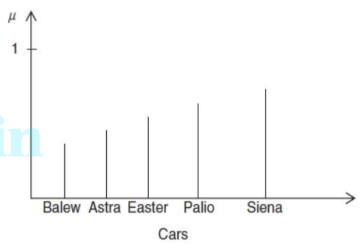
Matiz	235	336	—	797	492	1860	18.6	4
Santro	523	364	417	—	608	1912	19.1	3
Octavia	616	534	746	726	—	2622	26.2	1
Total						10000		



Rank Ordering –tutorial question

- The following data were determined by the pairwise comparison of work preferences of 100 people. When it was compared with Software (S), 69 of persons polled preferred Hardware (H), 45 of them preferred Educational (E), 55 of them preferred Business (B) and 25 preferred Textile (T). When it was compared with hardware (H), the preferences was 58-S, 45-E, 60-B, 30-T. When it was compared with educational, 39-S, 56-H, 34- B, 25-T. When it was compared business, the preferences was 52-S, 49-H, 38-E, 20-T. When it was compared with textile, the preferences was 69-S, 65-H, 44-E, 40-B. Using rank ordering, plot the membership function for the "m o s t p r e f e r r e d w o r k" .

Fuzzy Propositions -introduction

Fuzzy Propositions

- The fundamental difference between classical propositions and fuzzy propositions is in the range of their truth values.
- Each **classical proposition is required to be either true or false**,
- Each proposition has its opposite, which is called as a negation of the proposition. For example the proposition " 3 is a prime number" has negation " 3 is not a prime number", which is false proposition.
- The truth or falsity of fuzzy propositions is a matter of degree.
- Consider another sentence " Mathematics is an easy subject", which is not a statement as it is a perception which may change from person to person
- Assuming that truth and falsity are expressed by values 1 and 0, respectively, the degree of truth of each fuzzy proposition is expressed by a number in the unit interval [0, 1].

Fuzzy Propositions

- Any two propositions can be combined in various way to from new proposition.
- The words such as " and", "or" used to combine propositions are called as connectives.
- The classical two valued logic can be extended into three valued logic in various ways.
- It is common in these logic to denote the truth, falsity and interminacy by 1,0 and 1/2 respectively.
- Also it is possible to generalize three valued logic to n-valued logic.
- Fuzzy logic stems from the mathematical study of multivalued logic.

Reasoning, Proposition, Negation

- *Reasoning* has logic as its basis, whereas *propositions* are text sentences expressed in any languages.
- Proposition is expressed in an canonical form as,

  $$\boxed{Z \text{ is } P}$$

  where,
  Z is the subject
  P is the predicate designing the characteristics of subject.
- Every proposition has its opposite called *negation*.

Truth values and Tables in Fuzzy Logic

- *Truth tables* define logic functions of two propositions.
- Let X and Y be two propositions, either of which can be *True* or *False*.
- The basic logic operations performed over these propositions are:

1. *Conjunction*($\wedge$): X AND Y
2. *Disjunction*($\vee$): X OR Y
3. *Implication or Conditional*($\Rightarrow$): IF X THEN Y
4. *Bidirectional or Equivalence*($\Leftrightarrow$): X IF AND ONLY IF Y

Truth values and Tables in Fuzzy Logic

- *Truth tables* define logic functions of two propositions.
- Let X and Y be two propositions, either of which can be *True* or *False*.
- The basic logic operations performed over these propositions are:

		AND	OR	IF THEN	IFF
X	Y	$X \wedge Y$	$X \vee Y$	$X \Rightarrow Y$	$X \Leftrightarrow Y$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Implication

- Implication / if-then (⟶)
- An implication A⟶B,A⟶B is the proposition “if A, then B”. It is false if A is true and B is false. The rest cases are true.
- The truth table is as follows –

A	B	A⟶B
True	True	True
True	False	False
False	True	True
False	False	True

Equivalence

- If and only if (⇔)
- A⇔B,A⇔B is a bi-conditional logical connective which is true when A and B are same, i.e., both are false or both are true.
- The truth table is as follows –

A	B	A⇔B
True	True	True
True	False	False
False	True	False
False	False	True

Tautology

- On the basis of these operations on propositions, inference rules can be formed.
- These rules produce certain propositions that are always true irrespective of the truth values of propositions.

> A **tautology** is a compound statement which is true for every value of the individual statements.

- Either Mohan will go home or Mohan will not go home.
- He is healthy or he is not healthy
- A number is odd or a number is not odd.

Show that, $(P \Rightarrow Q) \vee (Q \Rightarrow P)$ *is a tautology.*

Tautology

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \vee (Q \Rightarrow P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

$(P \Rightarrow Q) \vee (Q \Rightarrow P)$ *is a tautology.*

Truth Values

- The truth values of propositions are allowed to range over the unit interval $[0, 1]$.
- A truth value in fuzzy logic "*very true*" may be interpreted as a fuzzy set in $[0, 1]$.
- Truth value of proposition " Z *is* A" or truth value of A, $tv(A)$, is defined by a point in $[0, 1]$ called *numerical truth value* or a fuzzy set in $[0, 1]$ called *linguistic truth value*.
- Truth value of a proposition can be obtained from the logic operations of other propositions whose truth values are known.

Operations

- If $tv(X)$ and $tv(Y)$ are numerical truth values of propositions X and Y, then,
  - *Intersection*

    $tv(X \text{ AND } Y) = tv(X) \wedge tv(Y) = min\{tv(X), tv(Y)\}$
  - *Union*

    $tv(X \text{ OR } Y) = tv(X) \vee tv(Y) = max\{tv(X), tv(Y)\}$
  - *Complement*

    $tv(\text{NOT } X) = 1 - tv(X)$
  - *Implication*

$tv(X \Rightarrow Y) = tv(X) \Rightarrow tv(Y) = max\{1 - tv(X), min\{tv(X), tv(Y)\}\}$

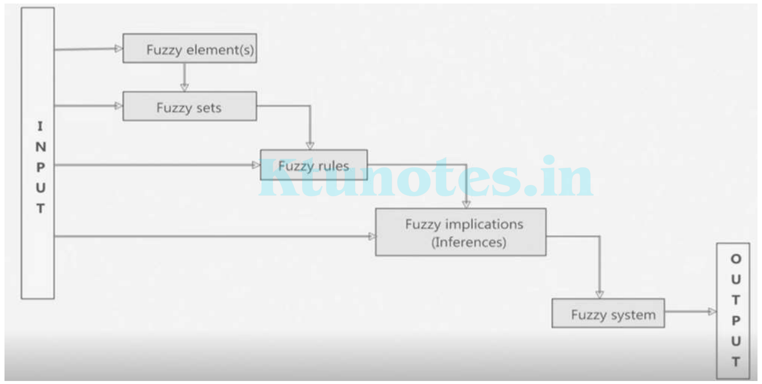
Equivalence

- If and only if (⇔)
- A⇔B,A⇔B is a bi-conditional logical connective which is true when A and B are same, i.e., both are false or both are true.
- The truth table is as follows –

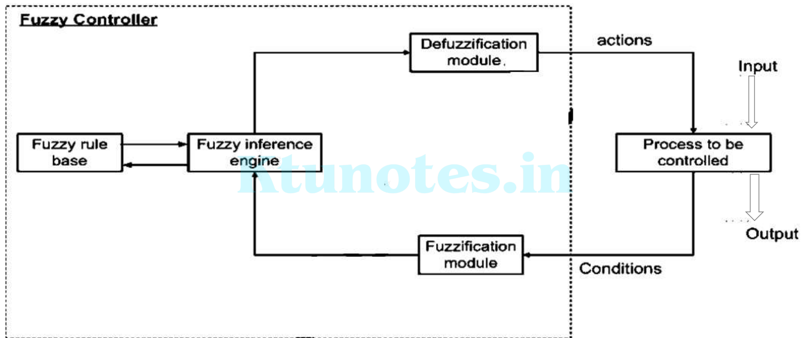
A	B	A⇔B
True	True	True
True	False	False
False	True	False
False	False	True

Fuzzy Propositions

Concept of Fuzzy Logic



Fuzzy Logic Controller



Fuzzy Propositions

Fuzzy Propositions

- A fuzzy proposition is a sentence having a fuzzy truth value. That is the degree of truth of a fuzzy proposition is a number in the unit interval [0,1].
- For the given fuzzy proposition p, T(p) represents the truth value of p.
- A proposition is a collection of declarative statements that have either a truth value of "true" or a truth value of "false".
- A proposition consists of propositional variables and connectives.
- The propositional variables are denoted by capital letters (A, B, etc).
- The connectives connect the propositional variables and values.
- A few examples of classical Propositions are given below –
  - "Ram is a Boy", returns truth value "TRUE"
  - "12 + 9 = 3 – 2", it returns truth value "FALSE"
- The following is not a classical Proposition –
  - **"A is less than 2"** – It is because unless we give a specific value of A, we cannot say whether the statement is true or false.

Fuzzy Propositions

- A fuzzy proposition is a sentence having a fuzzy truth value.
- The degree of truth of a fuzzy proposition is a number in the unit interval [0,1].
- Fuzzy proposition may be quantified by a suitable fuzzy quantifier
- Different types of fuzzy proposition
  - Fuzzy Predicates
  - Fuzzy Predicate Modifiers
  - Fuzzy Quantifiers
  - Fuzzy Qualifiers

Fuzzy Propositions

- (1) *Fuzzy predicates*: In fuzzy logic, the predicates can be fuzzy.

  In this preposition , the predicate in the natural language are fuzzy rather than Crisp.
  Ex: tall, short, quick, hot ,warm ,fast etc

  Subject **Predicate**
  Ex: Peter is Tall

- (2) *Fuzzy predicate modifiers*: In fuzzy logic, there exists a wide range of predicate modifiers that acts as hedges. They are necessary for generating the values of a linguistic variable.

  Ex: Water is slightly hot
  Climate is moderately cool

Fuzzy Propositions

- (3) *Fuzzy quantifiers*: It can be interpreted as a fuzzy proposition, which provides an imprecise characterization of the cardinality of one or more fuzzy or non-fuzzy sets.
  It can be used to represent the meaning of propositions that containing probabilities.

Predicate

Ex: Many People are educated

Subject

Fuzzy Quantifier

Fuzzy Propositions

(4) *Fuzzy qualifiers*:

(a) *Fuzzy truth qualification*

x *is* τ in which τ is a fuzzy truth value.

(b) *Fuzzy probability qualification*

x *is* λ where λ is fuzzy probability. → Likely, very Likely, unlikely

(c) *Fuzzy possibility qualification*

x *is* π where π is a fuzzy possibility. → Possible, Quite Possible, almost Impossible

(d) *Fuzzy usuality qualification*

$$usually(X) = usually(X \text{ is } F)$$

in which the subject X is a variable taking values in a U and the predicate F is a fuzzy subset of U. $U(X) = F$

Fuzzy Propositions

(4) *Fuzzy qualifiers*:

(a) *Fuzzy truth qualification*

x *is* τ in which τ is a fuzzy truth value.

Fuzzy truth values claims the degree of truth of a fuzzy proposition

Ex) (Paul is Young) is NOT VERY TRUE

(Car is Black) is NOT VERY TRUE

Qualified Proposition

Qualifying Fuzzy Truth value

Fuzzy Propositions

(b) *Fuzzy probability qualification*

x *is* λ where λ is fuzzy probability. → Likely, very Likely, unlikely

Fuzzy Probability is expressed in terms of likely,very likely, unlikely, around and so on

Ex: (Paul is Young) is Likely True

Qualified Proposition

Qualifying Fuzzy Probability

Fuzzy Propositions

(c) *Fuzzy possibility qualification*

$x\ is\ \pi$ where π is a fuzzy possibility. → Possible / Quite Possible / almost Impossible

Fuzzy possibility has the following forms:

Possible , Quite Possible , almost Impossible

Ex:(Paul is Young) is Almost Impossible

Qualified Proposition

Qualifying Fuzzy Possibility

Fuzzy Propositions

(d) *Fuzzy usuality qualification*

$$usually(X) = usually(X\ is\ F)$$

in which the subject X is a variable taking values in a U and the predicate F is a fuzzy subset of U.

$$U(X) = F$$

X is a variable taking values in a universe of discourse U and F is a fuzzy subset of U which many be interpreted as a usual value of X. ex)Snow is usually white

The propositions that are usually True or the events that have high probability of occurrence are related by the concept of Usability Qualification

Fuzzy Propositions-Example 1

P: Ram is honest

T(P) = 0.0	: Absolutely false
T(P) = 0.2	: Partially false
T(P) = 0.4	: May be false or not false
T(P) = 0.6	: May be true or not true
T(P) = 0.8	: Partially true
T(P) = 1.0	: Absolutely true.

Fuzzy Propositions-Example 2

P : Mary is efficient ; T(P) = 0.8

Q : Ram is efficient ; T(Q) = 0.6

- **Mary is not efficient.**

  $$T(\neg P) = 1 - T(P) = 0.2$$

- **Mary is efficient and so is Ram**

  $$T(P \wedge Q) = min\{T(P), T(Q)\} = 0.6$$

Fuzzy Propositions-Example 3

P : Mary is efficient ; T(P) = 0.8
Q : Ram is efficient ; T(Q) = 0.6

- **Either Mary or Ram is efficient**

  $$T(P \vee Q) = max\{T(P), T(Q)\} = 0.8$$

- **If Mary is efficient then so is Ram**

  $$T(P \Rightarrow Q) = \max\{1 - T(P), T(Q)\} = 0.6$$

Canonical form of Fuzzy Propositions

- Suppose, X is a universe of discourse of five persons. Intelligent of $x \in X$ is a fuzzy set as defined below.

  Intelligent: $\{(x_1, 0.3), (x_2, 0.4), (x_3, 0.1), (x_4, 0.6), (x_5, 0.9)\}$

- We define a fuzzy proposition as follows:
  $P :$ x is Intelligent
- The canonical form of fuzzy proposition of this type, P is expressed by the sentence $P : v \text{ is } F$.

Fuzzy Rules and Implications

Formation of fuzzy rules

- Fuzzy Systems is also known as Rule Based System
- General form of a rule is :

  $\boxed{IF\ antecedent\ THEN\ consequent}$

- This form is also known as IF-THEN rule based form.
- IF-THEN rule based form is considered as the general way of representing human knowledge by forming natural language expressions.
- Three General forms of Linguistic variables are used in the Rule based System.

Formation of fuzzy rules

- There are three general forms that exist for any linguistic variable:
  1. assignment statements → = statement
  2. conditional statements → If-then statement
  3. unconditional statements → Stop,start etc

Formation of fuzzy rules

Assignment Statements

y = small
color = orange
$a = s$
Climate = autumn
Outside temperature = normal

Unconditional Statements

Goto sum.
Stop.
Divide by a.
Turn the pressure low.

Conditional Statements

IF y is very cool THEN stop.
IF A is high THEN B is low ELSE B is not low.
IF temperature is high THEN climate is hot.

Canonical form of Fuzzy rule based system

Rule 1: IF condition C_1, THEN restriction R_1
Rule 2: IF condition C_2, THEN restriction R_2
.
.
.
Rule n: IF condition C_n, THEN restriction R_n

Fuzzy Implications

- **A fuzzy implication, also known as a fuzzy if-then rule or a fuzzy conditional statement**, takes the form: If x is A then y is B.
  Here, A and B are linguistic variables (defined by the two fuzzy sets A and B) on universes of discourses X and Y respectively.
- 'x is A' is often called the antecedent and 'y is B' is often called the consequence.
- Here are a few examples of fuzzy implications:
- If the *temperature is high*, then the *pressure is high*.
- If the *number is less than or equal to zero*, then the *number is not a natural number*.
- If the *fruit is ripe* then the *fruit is sweet*, else the *fruit is sour*.

Aggregation of FuzzyRules

- *Aggregation of rule* is the process of obtaining the overall consequences from the individual consequences provided by each rule.
- *Methods*
  1. Conjunctive system of rules
  2. Disjunctive system of rules

Conjunctive System of Rules

- For a system of rules to be jointly satisfied.
- Rules are connected by "*and*" connectives.
- Aggregated output, y is determined by,

$$\boxed{\begin{array}{c} y = y_1 \ \ and \ \ y_2 \ \ and \cdots and \ \ y_n \\ or \\ y = y_1 \cap y_2 \cap \cdots \cap y_n \end{array}}$$

- This aggregated output can be defined by the membership function

$$\boxed{\mu_y(y) = min[\mu_{y_1}(y), \mu_{y_2}(y), \cdots, \mu_{y_n}(y)] \ \ for \ y \in Y}$$

Disjunctive System of Rules

- The satisfaction of at least one rule is required.
- Rules are connected by "*or*" connectives.
- Aggregated output, y is determined by,

$$\boxed{\begin{array}{c} y = y_1 \ \ or \ \ y_2 \ \ or \cdots or \ \ y_n \\ or \\ y = y_1 \cup y_2 \cup \cdots \cup y_n \end{array}}$$

- This aggregated output can be defined by the membership function

$$\boxed{\mu_y(y) = max[\mu_{y_1}(y), \mu_{y_2}(y), \cdots, \mu_{y_n}(y)] \ \ for \ y \in Y}$$

DECOMPOSITION OF RULES

- A *compound rule* is a collection of many simple rules combined together.
- Any compound rule structure may be decomposed and reduced to a number of simple canonical rule forms.
- Rules are generally based on natural language representations.

DECOMPOSITION OF RULES

- *Methods*
  1. Multiple conjunctive antecedents
  2. Multiple disjunctive antecedents
  3. Conditional statements
  4. Nested−IF−THEN rules

Multiple Conjunctive Antecedents

$IF\ x\ is\ A_1, A_2, \cdots, A_n\ \ THEN\ y\ is\ B_m$

Assume a new fuzzy subset A_m defined as,

$$A_m = A_1 \cap A_2 \cap \cdots \cap A_n$$

The compound rule may be rewritten as,

$$IF\ x\ is\ A_m\ \ THEN\ y\ is\ B_m$$

Expressed by means of membership function,

$$\mu_{A_m}(x) = min[\mu_{A_1}(x), \mu_{A_2}(x), \cdots, \mu_{A_n}(x)]$$

2. Multiple Disjunctive Antecedents

$IF\ x\ is\ A_1\ \textbf{or}\ x\ is\ A_2\ \textbf{or} \ldots.\ x\ is\ A_n\ \ THEN\ y\ is\ B_m$

Assume a new fuzzy subset A_m defined as,

$$A_m = A_1 \cup A_2 \cup \cdots \cup A_n$$

Expressed by means of membership function,

$$\mu_{A_m}(x) = max[\mu_{A_1}(x), \mu_{A_2}(x), \cdots, \mu_{A_n}(x)]$$

The compound rule may be rewritten as,

$$IF\ x\ is\ A_m\ \ THEN\ y\ is\ B_m$$

3. Conditional Statements(with ELSE and UNLESS)

Statements of the kind

$$IF\ A_1\ \ THEN\ (B_1\ \ ELSE\ B_2)$$

Can be decomposed into two simple canonical rule forms, connected by "OR":

$$IF\ A_1\ \ THEN\ B_1$$
$$OR$$
$$IF\ NOT\ A_1\ \ THEN\ B_2$$

3. Conditional Statements(with ELSE and UNLESS)

$$IF\ A_1\ (THEN\ B_1)\ UNLESS\ A_2$$

Can be decomposed as,

$$IF\ A_1\ THEN\ B_1$$
$$OR$$
$$IF\ A_2\ THEN\ NOT\ B_1$$

4.Nested IF-THEN RULES

The rule

$$IF\ A_1\ THEN\ [IF\ A_2\ THEN\ (B_1)]$$

Can be of the form

$$IF\ A_1\ AND\ A_2\ THEN\ (B_1)$$

Fuzzy Implication Example 1

- If pressure is High then temperature is Low
- If mango is Yellow then mango is Sweet else mango is Sour
- If road is Good then driving is Smooth else traffic is High
- The fuzzy implication is denoted as $R:\ A \rightarrow B$
- In essence, it represents a binary fuzzy relation R on the (Cartesian) product of $A \times B$

Fuzzy Implication Example 2

- Suppose, P and T are two universes of discourses representing pressure and temperature, respectively as follows.

  $P = \{1,2,3,4\}$ and $T = \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$

- Let the linguistic variable High temperature and Low pressure are given as

$$T_{HIGH} = \{(20, 0.2), (25, 0.4), (30, 0.6), (35, 0.6), (40, 0.7), (45, 0.8), (50, 0.8)\}$$

$$P_{LOW} = \{(1, 0.8), (2, 0.8), (3, 0.6), (4, 0.4)\}$$

Fuzzy Implication Example 2

- If pressure is High then temperature is Low

Then the fuzzy implication If temperature is High then pressure is Low can be defined as

$$R : T_{HIGH} \rightarrow P_{LOW}$$

$$R = \begin{array}{c} \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 45 \\ 50 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.4 \\ 0.6 & 0.6 & 0.6 & 0.4 \\ 0.7 & 0.7 & 0.6 & 0.4 \\ 0.8 & 0.8 & 0.6 & 0.4 \\ 0.8 & 0.8 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

Defuzzification

Defuzzification

- *Defuzzification* is a mapping process from a space of fuzzy control actions defined over an output universe of discourse into a space of crisp control actions.
- It has the capability:
  - to reduce a fuzzy set into a crisp set
  - to convert a fuzzy matrix into a crisp matrix
  - to convert a fuzzy number into a crisp number
- Mathematically, the defuzzification process may also be termed as *rounding it off*.

Defuzzification Methods

A number of defuzzification methods are known. Such as

1. **Lambda-cut method**
2. **Weighted average method**
3. **Maxima methods**
4. **Centroid methods**

Defuzzification Methods

- **Fuzzification** converts the crisp input into a fuzzy value.
- Defuzzification **converts the fuzzy output of the fuzzy inference engine into a crisp value** so that it can be fed to the controller.
- The fuzzy results generated can not be used in an application, where a decision has to be taken only on crisp values.
- A controller can only understand the crisp output. So it is necessary to convert the fuzzy output into a crisp value.
- There is no systematic procedure for choosing a good defuzzification strategy.
- The selection of defuzzification procedure depends on the properties of the application

Lambda-cut method converts a fuzzy set (or a fuzzy relation) into a crisp set (or relation).

Lambda-Cuts for Fuzzy Sets

- Consider a fuzzy set $\underset{\sim}{A}$.
- The set $A_\lambda (0 < \lambda < 1)$, called the $\lambda-cut\ or\ \alpha-cut\ set$, is a crisp set of the fuzzy set.
- It is defined as:

$$\boxed{A_\lambda = \{x | \mu_A(x) \geq \lambda\} \quad \text{where, } \lambda \in [0, 1]}$$

Compare
$\mu_A(x) \geq \lambda$
If true x will be added to the crisp set.

- Any particular fuzzy set A can be transformed into an infinite number of $\lambda-$cut sets.

Lambda-Cuts for Fuzzy Sets

- **Weak Alpha Cut-** A Set is called Weak Alpha Cut if It contains all the elements of the fuzzy set ,whose membership functions have values greater than or equal to a specified value.

$$A_\lambda = \{x | \mu_{\underset{\sim}{A}}(x) \geq \lambda\}; \quad \lambda \in [0, 1]$$

- **Strong Alpha Cut** A set is called Strong Alpha Cut if It contains all the elements of the fuzzy set ,whose membership values strictly greater than a specified value.

$$A_\lambda = \{x | \mu_{\underset{\sim}{A}}(x) > \lambda\}; \quad \lambda \in [0, 1]$$

Lambda-Cuts for Fuzzy Sets

Properties of Lambda-Cuts for Fuzzy Sets

1. $(A \cup B)_\lambda = A_\lambda \cup B_\lambda$
2. $(A \cap B)_\lambda = A_\lambda \cap B_\lambda$
3. $(\overline{A})_\lambda \neq (\overline{A_\lambda})$ *except when* $\lambda = 0.5$
4. *For any* $\lambda \leq \beta$, *where* $0 \leq \beta \leq 1$, *it is true that*, $A_\beta \subseteq A_\lambda$, *where* $A_0 = X$.

Features of Membership Functions

1. The *core* of A is the $\lambda = 1$–cut set A_1.
2. The *support* of A is the λ–cut set A_{0^+}, where $\lambda = 0^+$, and it can be defined as, $A_{0^+} = \{x | \mu_A(x) > 0\}$
3. The interval $[A_{0^+}, A_1]$ forms the *boundaries* of the fuzzy set A.

Lambda-Cuts for Fuzzy Sets

Consider the discrete fuzzy set defined on the universe, $X = \{a, b, c, d, e\}$ *as,*

$$A = \{\frac{1}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.3}{d} + \frac{0}{e}\}$$

Find the λ–*cut sets for* $\lambda = 1, 0.9, 0.6, 0.3, 0^+$ *and* 0.

$$\boxed{A_\lambda = \{x | \mu_A(x) \geq \lambda\} \quad \text{where, } \lambda \in [0, 1]}$$

x1=1,x2=0.9,x3=0.6,x4=0.3,x5=0

Compare

$(a)\lambda = 1, A_1 = \{\frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e}\}$ ={x1}

Lambda-Cuts for Fuzzy Sets

$(b)\lambda = 0.9, A_{0.9} = \{\frac{1}{a} + \frac{1}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e}\}$ ={a,b}

$(c)\lambda = 0.6, A_{0.6} = \{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d} + \frac{0}{e}\}$ ={a,b,c}

$(d)\lambda = 0.3, A_{0.3} = \{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{0}{e}\}$ ={a,b,c,d}

$(e)\lambda = 0^+, A_{0^+} = \{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{0}{e}\}$ ={a,b,c,d}

$(f)\lambda = 0, A_0 = \{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e}\}$ ={a,b,c,d,e}

Lambda-Cuts for Fuzzy Sets-problem 2

Consider two fuzzy sets A and B, both defined on X, given as follows:

$\mu(x_i X)$	x_1	x_2	x_3	x_4	x_5
A	0.2	0.3	0.4	0.7	0.1
B	0.4	0.5	0.6	0.8	0.9

Express the following λ– *sets using Zadeh's notation:*

$(a)(\overline{A})_{0.7}$ $(b)(B)_{0.2}$
$(c)(A \cup B)_{0.6}$ $(d)(A \cap B)_{0.5}$
$(e)(A \cup \overline{A})_{0.7}$ $(f)(B \cap \overline{B})_{0.3}$
$(g)(\overline{A \cap B})_{0.6}$ $(h)(\overline{A} \cup \overline{B})_{0.8}$

Lambda-Cuts for Fuzzy Sets-problem 2

The two fuzzy sets given are,

$$A = \{\frac{0.2}{x_1} + \frac{0.3}{x_2} + \frac{0.4}{x_3} + \frac{0.7}{x_4} + \frac{0.1}{x_5}\}$$

$$B = \{\frac{0.4}{x_1} + \frac{0.5}{x_2} + \frac{0.6}{x_3} + \frac{0.8}{x_4} + \frac{0.9}{x_5}\}$$

$$(a)(\overline{A}) = 1 - \mu_A(x) = \{\frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.3}{x_4} + \frac{0.9}{x_5}\}$$

$$(\overline{A})_{0.7} = \{x_1, x_2, x_5\}$$

$$(b)\ (B)_{0.2} = \{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} + \frac{1}{x_5}\}$$

$$(B)_{0.2} = \{x_1, x_2, x_3, x_4, x_5\}$$

Lambda-Cuts for Fuzzy Sets-problem 2

$$(c)(A \cup B) = max\{\mu_A(x), \mu_B(x)\}$$
$$= \{\frac{0.4}{x_1} + \frac{0.5}{x_2} + \frac{0.6}{x_3} + \frac{0.8}{x_4} + \frac{0.9}{x_5}\}$$
$$(A \cup B)_{0.6} = \{x_3, x_4, x_5\}$$

$$(d)(A \cap B) = min\{\mu_A(x), \mu_B(x)\}$$
$$= \{\frac{0.2}{x_1} + \frac{0.3}{x_2} + \frac{0.4}{x_3} + \frac{0.7}{x_4} + \frac{0.1}{x_5}\}$$

$$(A \cap B)_{0.5} = x_4$$

$$(e)(A \cup \overline{A}) = max\{\mu_A(x), \mu_{\overline{A}}(x)\}$$
$$= \{\frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.7}{x_4} + \frac{0.9}{x_5}\}$$
$$(A \cup \overline{A})_{0.7} = \{x_1, x_2, x_4, x_5\}$$

$$(f)(\overline{B}) = 1 - \mu_B(x) = \{\frac{0.6}{x_1} + \frac{0.5}{x_2} + \frac{0.4}{x_3} + \frac{0.2}{x_4} + \frac{0.1}{x_5}\}$$

$$(B \cap \overline{B}) = min\{\mu_B(x), \mu_{\overline{B}}(x)\} = \{\frac{0.4}{x_1} + \frac{0.5}{x_2} + \frac{0.4}{x_3} + \frac{0.2}{x_4} + \frac{0.1}{x_5}\}$$

$$(B \cap \overline{B})_{0.3} = \{x_1, x_2, x_3\}$$

$$(g)(\overline{A \cap B}) = 1 - \mu_{A \cap B}(x) = \{\frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.3}{x_4} + \frac{0.9}{x_5}\}$$
$$(\overline{A \cap B})_{0.6} = \{x_1, x_2, x_3, x_5\}$$

$$(h)(\overline{A} \cup \overline{B}) = max\{\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)\}$$
$$= \{\frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.3}{x_4} + \frac{0.9}{x_5}\}$$
$$(\overline{A} \cup \overline{B})_{0.8} = \{x_1, x_5\}$$

Lambda-Cuts for Fuzzy Sets-Tutorial

Two fuzzy sets P and Q are defined on x as follows.

$\mu(x)$	x_1	x_2	x_3	x_4	x_5
P	0.1	0.2	0.7	0.5	0.4
Q	0.9	0.6	0.3	0.2	0.8

Find the following :

(a) $P_{0.2}$, $Q_{0.3}$

(b) $(P \cup Q)_{0.6}$

(c) $(P \cup \overline{P})_{0.8}$

(d) $(P \cap Q)_{0.4}$

Lambda cut for fuzzy relation

Lambda Cut for Fuzzy Relation

- Consider a fuzzy relation R.
- The relation $R_\lambda (0 < \lambda < 1)$, called the $\lambda-cut$ *or* $\alpha-cut$ *relation*, is a crisp relation of the fuzzy relation.
- It is defined as:

$$R_\lambda = \{(x, y) | \mu_R(x, y) \geq \lambda\}$$

$$\boxed{R_\lambda = \{1 | \mu_{R(x,y)} \geq \lambda; 0 | \mu_{R(x,y)} < \lambda\} \text{ where, } \lambda \in [0, 1]}$$

Properties of Lambda Cut sets

1. $(R \cup S)_\lambda = R_\lambda \cup S_\lambda$
2. $(R \cap S)_\lambda = R_\lambda \cap S_\lambda$
3. $(\overline{R})_\lambda \neq (\overline{R_\lambda})$ *except when* $\lambda = 0.5$
4. *For any* $\lambda \leq \beta$, *where* $0 \leq \beta \leq 1$, *it is true that*, $R_\beta \subseteq R_\lambda$.

Problems of Lambda Cut sets for fuzzy relation

Determine the crisp $\lambda-cut$ *relation when* $\lambda = 0.1, 0^+, 0.3$ *and* 0.9 *for the following relation* R:

$$R = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.3 & 0.7 & 0.1 \\ 0.8 & 0.9 & 1.0 \end{bmatrix}$$

$$\boxed{R_\lambda = \{1 | \mu_{R(x,y)} \geq \lambda; 0 | \mu_{R(x,y)} < \lambda\} \text{ where, } \lambda \in [0, 1]}$$

$(a) \lambda = 0.1$

$$R_{0.1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Problems of Lambda Cut sets for fuzzy relation

$(b)\lambda = 0^+$

$$R_{0^+} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$(c)\lambda = 0.3$

$$R_{0.3} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$(d)\lambda = 0.9$

$$R_{0.9} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Lambda Cut sets for fuzzy relation-problem 2

For the fuzzy relation $\underset{\sim}{R}$,

$$\underset{\sim}{R} = \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1 & 0.8 & 0.6 & 0.4 \end{bmatrix}$$

find the λ-cut relation for $\lambda = 0.2, 0.4, 0.7$ and 0.9

$$R_\lambda = \{1|\mu_{R(x,y)} \geq \lambda; 0|\mu_{R(x,y)} < \lambda\} \text{ where, } \lambda \in [0, 1]$$

Solution: For the given fuzzy relation, the λ-cut relation is given by

$$R_\lambda = \begin{cases} 1, & \mu_{\underset{\sim}{R}(x,y)} \geq \lambda \\ 0, & \mu_{\underset{\sim}{R}(x,y)} < \lambda \end{cases}$$

$$\underset{\sim}{R} = \begin{bmatrix} 0.2 & 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.5 & 0.7 & 1 & 0.8 \\ 0.4 & 0.6 & 0.8 & 0.9 & 0.4 \\ 0.9 & 1 & 0.8 & 0.6 & 0.4 \end{bmatrix}$$

(a) $\lambda = 0.2$,

$$R_{0.2} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(b) $\lambda = 0.4$,

$$R_{0.4} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(c) $\lambda = 0.7$,

$$R_{0.7} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(d) $\lambda = 0.9$,

$$R_{0.9} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Lambda Cut sets for fuzzy relation-problem 3

For a fuzzy relation R

$$R = \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.5 & 0.9 & 0.6 \\ 0.4 & 0.8 & 0.7 \end{bmatrix}$$

find λ-cut relations for the following values of $\lambda = 0, 0.2, 0.9, 0.5$

$$R_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } R_{0.2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and}$$

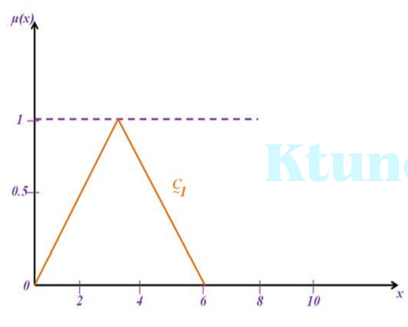
$$R_{0.9} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } R_{0.5} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Output of a Fuzzy process

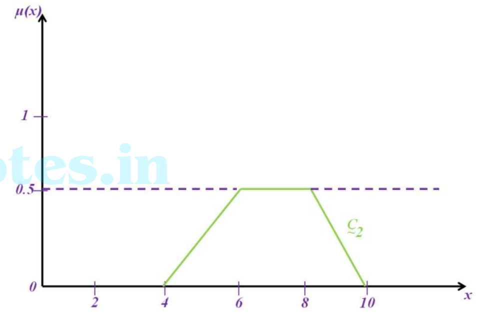
- *Defuzzification* is the process of conversion of a fuzzy quantity into a precise quantity.
- The output of a fuzzy process may be union of two or more fuzzy membership functions defined on the universe of discourse of the output variable.
- A fuzzy output process may involve many output parts, and the membership function representing each part of the output can have any shape.

$$C_n = \cup_{i=1}^{n} C_i = C$$

Output of a Fuzzy process



C_1, a triangular membership shape



C_2, a trapezoidal shape

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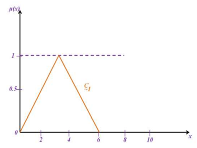
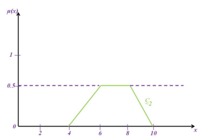
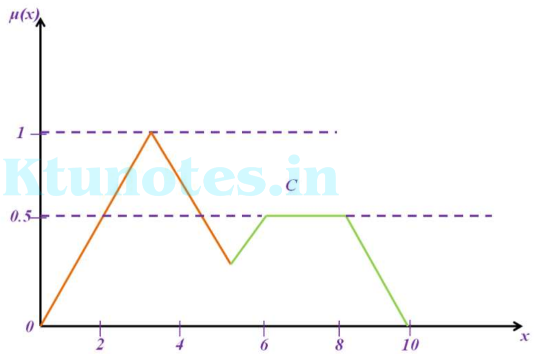
Output of a Fuzzy process



C_1, a triangular membership shape



C_2, a trapezoidal shape



$C = C_1 \cup C_2$, which is the outer envelope of the two shapes

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Other Defuzzification Methods

1. *Max–membership principle*
2. *Centroid method*
3. *Weighted average method*
4. *Mean–max membership*
5. *Center of sums*
6. *Center of largest area*
7. *First of maxima, Last of maxima*

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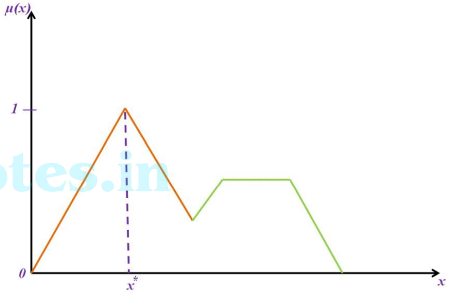
Max-Membership Principle

- Also known as *height method*.
- It is limited to peak output functions.
- This method is given by,

$$\boxed{\mu_C(x^*) \geq \mu_C(x) \text{ for all } x \in X}$$



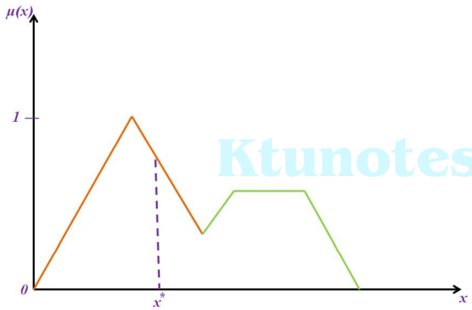
Centroid Method

- Also known as *center of mass*, *center of area* or *Center of gravity* method.
- It is the most commonly used defuzzification method.
- The defuzzified output x^* is defined as,

$$x^* = \frac{\int \mu_C(x) \cdot x dx}{\int \mu_C(x) dx}$$

where the symbol $\int$ denotes algebraic integration.

Centroid Method



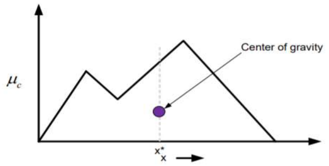
$$\boxed{x^* = \frac{\int \mu_C(x) \cdot x dx}{\int \mu_C(x) dx}}$$

Centroid Method

1. The basic principle in CoG method is to find the point x^* where a vertical line would slice the aggregate into two equal masses.
2. Mathematically, the CoG can be expressed as follows :

$$x^* = \frac{\int x.\mu_C(x)dx}{\int \mu_C(x)dx}$$

3. Graphically,



Centroid Method

Note:

1. x^* is the x-coordinate of center of gravity.
2. $\int \mu_C(x)dx$ denotes the area of the region bounded by the curve μ_C.
3. If μ_C is defined with a discrete membership function, then CoG can be stated as :

   $x^* = \frac{\sum_{i=1}^{n} x_i \cdot \mu_C(x_i)}{\sum_{i=1}^{n} \mu_C(x_i)}$;

4. Here, x_i is a sample element and n represents the number of samples in fuzzy set C.

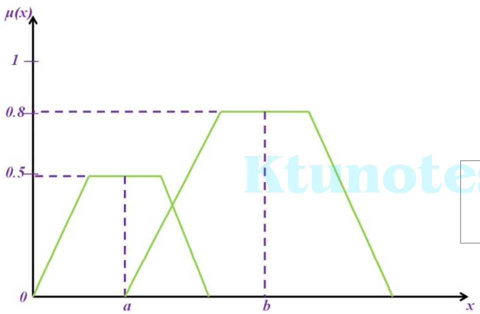
Weighted Average Method

- Each membership function is weighted by its maximum membership value.
- This method is valid for symmetrical output membership functions only.
- The defuzzified output x^* is defined as,

$$\boxed{x^* = \frac{\sum \mu_C(\overline{x}_i) \cdot \overline{x}_i}{\sum \mu_C(\overline{x}_i)}}$$

where $\sum$ denotes algebraic sum and $\overline{x}_i$ is the maximum of the i^{th} membership function.

Weighted Average Method



$$\boxed{x^* = \frac{0.5a + 0.8b}{0.5 + 0.8}}$$

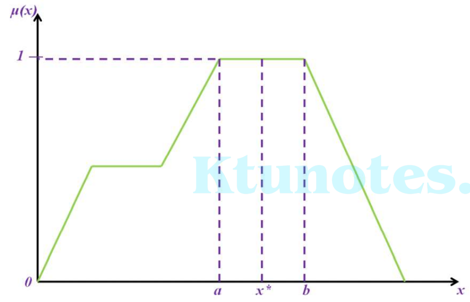
a and b are the means of their respective shapes.

Mean Max Method

- Also known as *middle of the maxima*.
- It is closely related to max−membership method except that the locations of the maximum membership can be non−unique.
- The defuzzified output x^* is defined as,

$$\boxed{x^* = \frac{\sum_{i=1}^{n} \overline{x}_i}{n}}$$

Mean Max Method



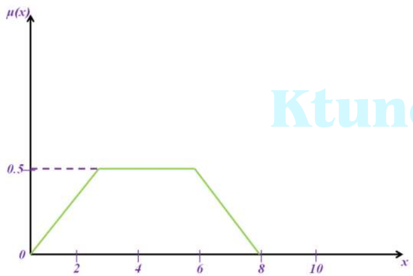
$$x^* = \frac{a+b}{2}$$

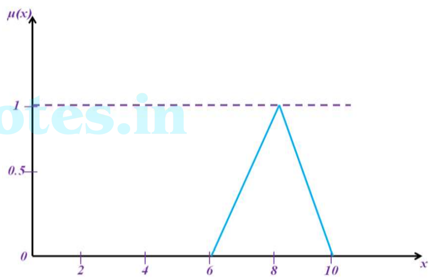
Center of Sums

- This method employs the algebraic sum of the individual fuzzy subsets instead of their union.
- The weights are the areas of the respective membership functions.
- The defuzzified output x^* is defined as,

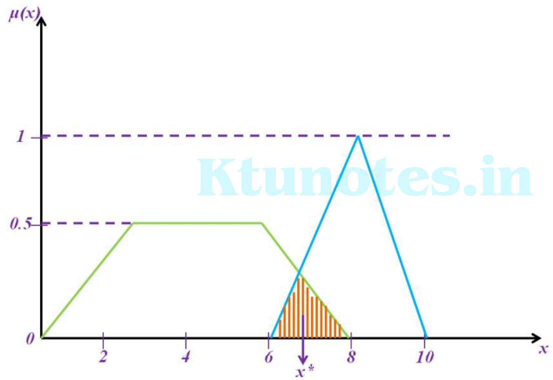
$$\boxed{x^* = \frac{\int_x x \sum_{i=1}^{n} \mu_{C_i}(x)\,dx}{\int_x \sum_{i=1}^{n} \mu_{C_i}(x)\,dx}}$$

Center of Sums





Center of Sums



Center of Largest Area

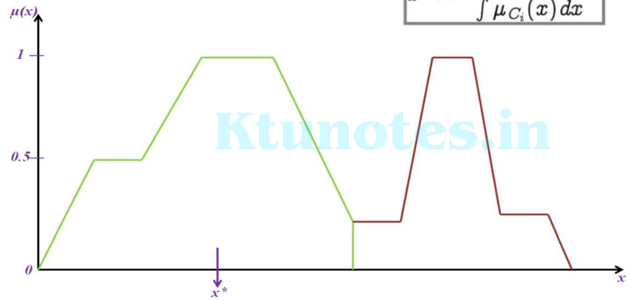
- This method can be adopted when the output consists of at least two convex fuzzy subsets which are not overlapping.
- The output in this case is biased towards a side of one membership function.
- The defuzzified output x^* is defined as,

$$\boxed{x^* = \frac{\int \mu_{C_i}(x) \cdot x dx}{\int \mu_{C_i}(x) dx}}$$

where C_i is the convex subregion that has the largest area.

Center of Largest Area

$$\boxed{x^* = \frac{\int \mu_{C_i}(x) \cdot x dx}{\int \mu_{C_i}(x) dx}}$$



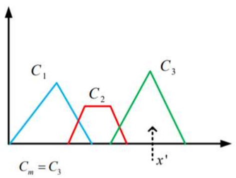
Center of Largest Area

If the fuzzy set has two subregions, then the center of gravity of the subregion with the largest area can be used to calculate the defuzzified value.

Mathematically, $x^* = \frac{\int \mu_{cm}(x).x' dx}{\int \mu_{cm}(x) dx}$;

Here, C_m is the region with largest area, x' is the center of gravity of C_m.

Graphically,



First of Maxima(Last of Maxima)

- This method uses the overall output or union of all individual output fuzzy sets for determining the smaller value of the domain with maximized membership.
- The steps for obtaining x^* are:
  1. Initially, the maximum height in the union is found:

$$\boxed{hgt(C_i) = \sup_{x \in X} \mu_{C_i}(x)}$$

where sup is supremum,ie,the least upper bound.

First of Maxima(Last of Maxima)

2 Then the first of maxima is found:

$$\boxed{x^* = \inf_{x \in X}\{x \in X | \mu_{C_i}(x) = hgt(C_i)\}}$$

where inf is infimum,ie,the greatest lower bound.

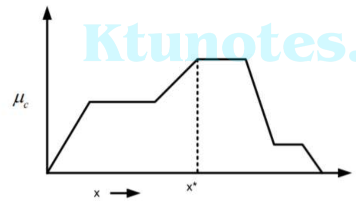
3 After this the last of maxima is found:

$$\boxed{x^* = \sup_{x \in X}\{x \in X | \mu_{C_i}(x) = hgt(C_i)\}}$$

where sup is supremum,ie,the least upper bound.

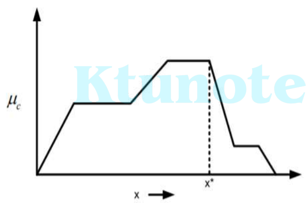
First of Maxima

FoM: First of Maxima : $x^* = min\{x|C(x) = max_w C\{w\}\}$

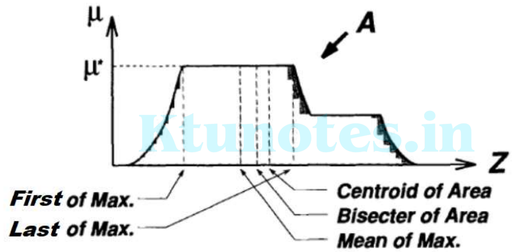


Last of Maxima

LoM : Last of Maxima : $x^* = max\{x|C(x) = max_w C\{w\}\}$



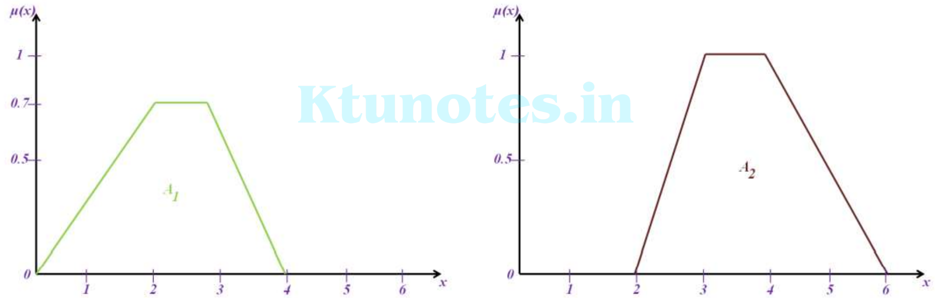
Example

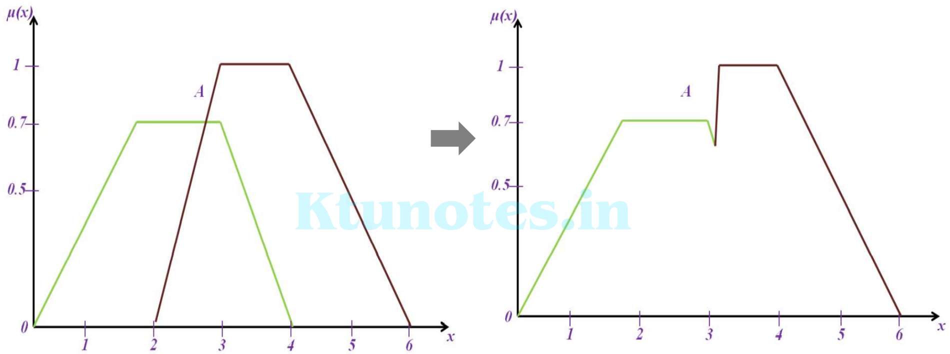


Defuzzification Methods-Summary

- **Maxima Methods**
  1. Height method
  2. First of maxima (FoM)
  3. Last of maxima (LoM)
  4. Mean of maxima(MoM)
- **Centroid methods**
  1. Center of gravity method (CoG)
  2. Center of sum method (CoS)
  3. Center of area method (CoA)
- **Weighted average method**

(1) *For the given membership functions as shown in figure, determine the defuzzified output value by seven methods.*

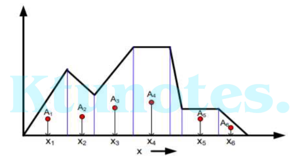




Geometrical Calculation of Centroid Method

Steps:

1. Divide the entire region into a number of small regular regions (e.g. triangles, trapizoid etc.)

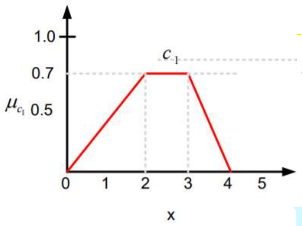


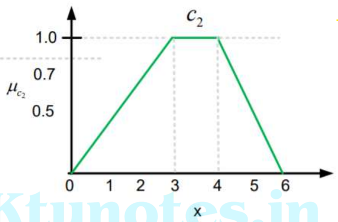
2. Let A_i and x_i denotes the area and c.g. of the i-th portion.
3. Then x^* according to CoG is

$$x^* = \frac{\sum_{i=1}^{n} x_i.(A_i)}{\sum_{i=1}^{n} A_i}$$

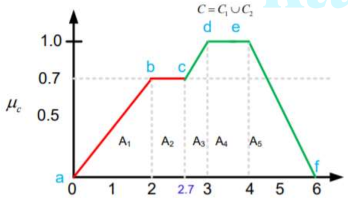
where n is the number of smaller geometrical components.

Geometrical Calculation of Centroid Method





y2-y1/x2-x1=y-y1/x-x1
x1=0,y1=0,x2=2,y2=0.7 for A1
0.7-0/2-0=y-0/x-0
y-0=0.7/2(x-0)



For A_1 : $y - 0 = \frac{0.7}{2}(x-0)$, or $y = 0.35x$

For A_2 : $y = 0.7$

For A_3 : $y - 0 = \frac{1-0}{3-2}(x-2)$, or $y = x - 2$

For, A_4 : $y = 1$

For, A_5 : $y - 1 = \frac{0-1}{6-4}(x-4)$, or $y = -0.5x + 3$

Geometrical Calculation of Centroid Method

For A_1 : $y - 0 = \frac{0.7}{2}(x-0)$, or $y = 0.35x$

For A_2 : $y = 0.7$

For A_3 : $y - 0 = \frac{1-0}{3-2}(x-2)$, or $y = x - 2$

For, A_4 : $y = 1$

For, A_5 : $y - 1 = \frac{0-1}{6-4}(x-4)$, or $y = -0.5x + 3$

$$\mu_c(x) = \begin{cases} 0.35x & 0 \le x < 2 \\ 0.7 & 2 \le x < 2.7 \\ x - 2 & 2.7 \le x < 3 \\ 1 & 3 \le x < 4 \\ (-0.5x + 3) & 4 \le x \le 6 \end{cases}$$

Geometrical Calculation of Centroid Method

$$\boxed{\text{Thus, } x^* = \frac{\int x.\mu_c(x)dx}{\int \mu_c(x)dx} = \frac{N}{D}}$$

$N = \int_0^2 0.35x^2dx + \int_2^{2.7} 0.7x^2dx + \int_{2.7}^3 (x^2 - 2x)dx + \int_3^4 xdx + \int_4^6 (-0.5x^2 + 3x)dx$

= 10.98

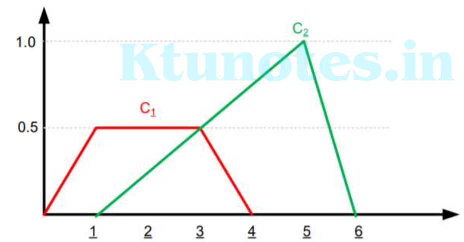
$D = \int_0^2 0.35xdx + \int_2^{2.7} 0.7xdx + \int_{2.7}^3 (x-2)dx + \int_3^4 dx + \int_4^6 (-0.5x+3)dx$

= 3.445

Thus, $x^* = \frac{10.98}{3.445} = 3.187$

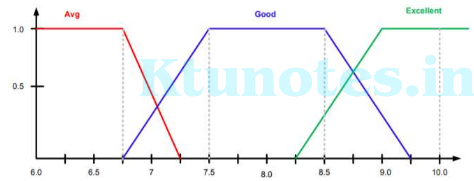
Exercise 1

Find the crisp value of the following using all defuzzified methods.



Exercise 3

- The membership function defining a student as Average, Good, and Excellent denoted by respective membership functions are as shown below.



- Find the crisp value of "Good Student"
- Hint: Use CoG method to the portion "Good" to calculate it.

Summary-What is Defuzzification?

- Defuzzification means the fuzzy to crisp conversion.
- **Example 1:**

  Suppose, T_{HIGH} denotes a fuzzy set representing **temperature is High.**

  T_{HIGH} is given as follows.

  $$T_{HIGH} = \left\{ \frac{0.1}{15} + \frac{0.4}{20} + \frac{0.45}{25} + \frac{0.55}{30} + \frac{0.65}{35} \right\}$$

- What is the crisp value that implies for the high temperature?

Summary-Why Defuzzification?

The fuzzy results generated can not be used in an application, where decision has to be taken only on crisp values.

Example:

If T_{HIGH} then rotate R_{FIRST}.

Here, may be input T_{HIGH} is fuzzy, but action **rotate** should be based on the crisp value of R_{FIRST}.