BRIEF EXPLANATION ON THE SLIDES

" As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain they do not refer to reality "

- Albert Einstein

WHAT IS DIMENSION?

To explain the concept of fractal dimension, it is necessary to understand what we mean by dimension in the first place. Obviously, a line has dimension 1, a plane dimension 2, and a cube dimension 3. But why is this? It is interesting to see people struggle to enunciate why these facts are true.

- Dictionary says - a property of space; extension in a given direction
- In *Physics* and *Mathematics*, the dimension of a mathematical space (or object) is informally defined as the minimum number of coordinates needed to specify any point within it.
- In mathematics, the dimension of an object is, roughly speaking, the number of degrees of freedom of a point that moves on this object.
- Science fiction texts often mention the concept of "dimension" when referring to parallel or alternate universes or other imagined planes of existence.

People often say that a line has dimension 1 because there is only 1 way to move on a line. Similarly, the plane has dimension 2 because there are 2 directions in which to move. Of course, there really are 2 directions in a line -- backward and forward -- and infinitely many in the plane. What they are trying to say is there are 2 linearly independent directions in the plane.

Of course, they are right. But the notion of linear independence is quite sophisticated and difficult to articulate. Students often say that the plane is two-dimensional because it has " two dimensions" - length and width. Similarly, a cube is three-dimensional because it has " three dimensions" - length, width, and height. Again, this is a valid notion, though not expressed in particularly rigorous mathematical language.

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WHAT ARE FRACTALS?

Fractals are a paradox. Amazingly simple, yet infinitely complex. New, but older than dirt. What are fractals? Where did they come from? Why should I care?

A fractal is, by definition, a curve whose complexity changes with measurement scale

Fractals appear the same at different levels. Fractals exhibit similar patterns at increasingly small scales called self similarity,

Unconventional 20th century mathematician *Benoit Mandelbrot* created the term fractal from the Latin word *fractus* (meaning irregular or fragmented) in 1975. These irregular and fragmented shapes are all around us. At their most basic, fractals are a visual expression of a *repeating pattern* or formula that starts out simple and gets progressively more complex.

According to Mandelbrot, fractals is "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole"

Mandelbrot found order in this type of branching complex system for which there was previously no geometry, and discovered that the underlying mathematical rules that created these patterns were short and few. In a fractal pattern, roughness can go on forever and ever--infinite roughness--like finding a map of the coastline printed on a single grain of sand.

Fractal geometry is a way to describe the *texture* of a surface. There are four topological dimensions in traditional Euclidean geometry:

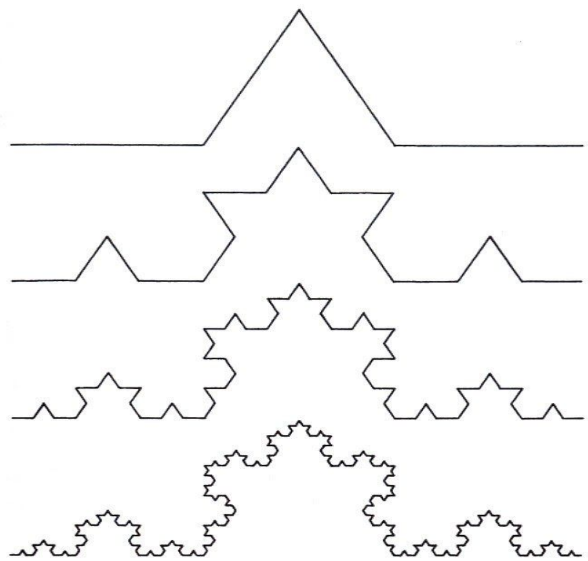
- 0-D for points
- 1-D for straight lines
- 2-D for planes
- 3-D for volumetric objects like cubes and spheres.

An object that is “fractal” has an *intermediate dimensionality*, such as 1.6 for an irregular line or 2.4 for an image “surface”. Generally, the *higher the fractal dimension*, the finer and *“rougher” the texture*.

With these caveats, there are still reasons to be aware of the use of fractal geometry to describe engineering surfaces.

- First, many real surfaces do have a fractal nature, which often extends over many orders of magnitude of dimension.

- Second, there are some indications that the fractal dimension correlates with particular aspects of material properties, processing history and/or performance. In particular this includes the visual appearance of surfaces and the human interpretation of their "roughness".



Many natural objects and surfaces exhibit a statistical sort of fractal geometry (as opposed to the rigorously self-similar geometry of generated features like the snowflake). This geometry produces a pattern of scattered light from such surfaces that is mathematically fractal, and hence human vision seems to have learned to interpret the light patterns in terms of surface roughness. Ranking of surfaces according to their visual appearance generally corresponds very closely to the ranking of fractal dimensions. Hence any applications in which the aesthetic appearance of the surfaces is important are particularly well suited to the use of the fractal technique.

WHAT IS A FRACTAL DIMENSION?

So why is a line one-dimensional and the plane two-dimensional? Note that both of these objects are self-similar. So there is another way we can look at simple dimensions, which brings a mathematical significance to the value of the dimension.

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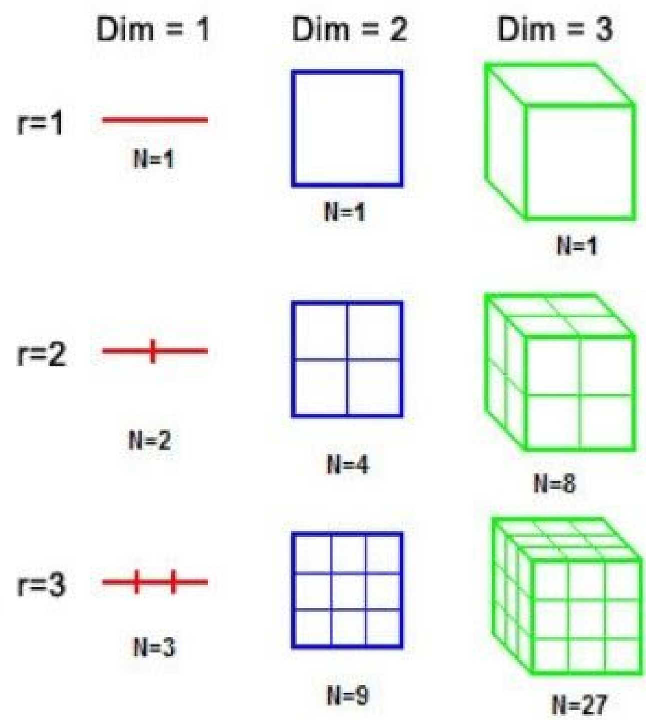
We may break a line segment into 4 self-similar intervals, each with the same length, and each of which can be magnified by a factor of 4 to yield the original segment. We can also break a line segment into 7 self-similar pieces, each with magnification factor 7. In general, we can break a line segment into **N** self-similar pieces, each with magnification factor **N**.



A square is different. We can decompose a square into 4 self-similar sub-squares, and the magnification factor here is 2. Alternatively, we can break the square into 9 self-similar pieces with magnification factor 3. Clearly, the square may be broken into **N^2** self-similar copies of itself, each of which must be magnified by a factor of **N** to yield the original figure.

Similarly, we can decompose a cube into **N^3** self-similar pieces, each of which has magnification factor **N**.

Now, we can describe the relationship between the total number of objects " **N** " the magnification factor " **r** " and the dimension " **D** " with the following equation:

N = rD
D = log(N) / log(r)

This means that the Dimension, **D** equals the log of the number of pieces divided by the log of the magnification factor **r** .

Now we see an alternative way to specify the dimension of a self-similar object: The dimension is simply the exponent of the number of self-similar pieces " **N** " with a magnification factor " **r** " into which the figure may be broken.

Fractal dimension is a measure of how "complicated" a self-similar figure is. A fractal dimension is an index for characterizing fractal patterns or sets by quantifying their complexity as a ratio of the change in detail to the change in scale

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In mathematics, more specifically in fractal geometry, a fractal dimension is a ratio providing a statistical index of complexity comparing how detail in a pattern (strictly speaking, a fractal pattern) changes with the scale at which it is measured. A fractal dimension does not have to be an integer.

COASTLINE PARADOX:

If I ask you "*How long is the coast of India?*", what would be your answer?

The correct answer is that it depends on *how closely you look at it,* or *how long your measuring stick is*. The coastline, technically, gets longer and longer as you measure it more closely, and it *approaches infinity*. This is where the fractal dimension is a very useful concept to describe a coastline.

Since all coastlines are basically infinitely long, we can not describe them using their length. We can, however, describe how fast the measured length of the coastline grows when decreasing the measurement tool length. And this ratio is basically the fractal dimension.

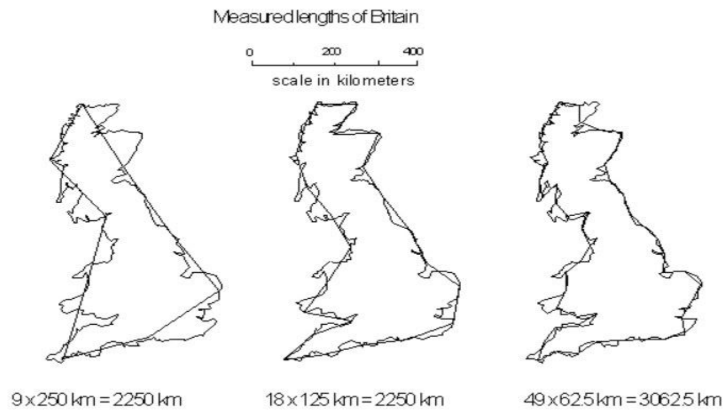
In the year 1950, the English mathematician *Lewis Fry Richardson* was researching the correlation between shared border length and the probability of war among two adjacent countries. Therefore, he wanted to know the length of the border shared between the countries of Portugal and Spain. When he looked up the official length, he noted something strange. The two countries reported two different values for their shared border length.

That would not be unusual, measurement errors are pretty common. The problem was that these values differed by more than 200 kilometers. Such a discrepancy could not have been a mere rounding error. How could two measurements for one and the same object be completely different?

The method that Richardson developed, to solve this issue, is an effective technique for measuring an *object's perimeter fractal dimension*. Although not as precise as the method used for exactly self-similar objects, this procedure enables us to calculate the dimensions of real-world objects, which are not perfectly self-similar.

Let's look closer into Richardson's method for calculating *fractal dimensions* using varying measurement lengths. To calculate a dimension, we first have to establish a logarithmic relationship between the object's overall measured length and the length of the ruler used to measure it.

We do this by graphing various values of $\log p$, where p is the length of the perimeter, against the corresponding values of $\log(1/s)$, where s is the scaling factor (the rulers length), used in measuring the perimeter. The resulting slope of the ($\log p$ / $\log(1/s)$) graph will be the related Fractal dimension of the measured object.



Now there is a very useful reason we take the logarithm of the values before we graph them: It means the points we plot on the graph come out more or less on a straight line.The slope of this line tells us how quickly the perimeter changes versus the magnification factor.

The fractal dimension of the coastline, **D**, is simply the slope of the line in the graph. The *lower* the dimension, the *straighter* and *smoother* the coastline. The *higher* the dimension the more *jagged* and *wiggly* the coastline is.

Fractals have something called "*roughness*," meaning that because the pattern repeats at all scales, the object is difficult to measure. The closer-in you try to measure a coastline, or a lung, the longer it will be.

CONCLUSION:

" Bottomless wonder spring from simple rules….repeated without end "

" Regularity is the contrary to irregularity. Not the other way around. World has always been rough and irregular.roughness has been a part of human nature "

- Mandelbrot

Fractal geometry is one of the mathematical fields whose origins can be found in a natural phenomenon. *Mathematics can be applied to nature, but nature can also be used to create new mathematical areas*. The two share a very special bond in that way.

The fractal dimension is a tool that allows us to characterize patterns and shapes in nature that have previously been outside the reach of mathematics. We can now quantitatively describe a pattern, and that allows us to study it.

Fractal patterns tell a story about the repetitive processes that created them. Examining the fractal dimension can help shed light on the processes. We now have a tool that can allow us to ask powerful questions, and test hypotheses about the underlying nature of... nature.

Ref :
http://math.bu.edu/DYSYS/chaos-game/node6.html
https://en.wikipedia.org/wiki/Dimension
https://www.diygenius.com/fractals-in-nature/
http://www.human-current.com/blog/2015/7/1/fractals-and-measuring-roughness
https://link.springer.com/chapter/10.1007/978-1-4757-3369-3_2
https://www.cs.mcgill.ca/~rwest/wikispeedia/wpcd/wp/f/Fractal.htm
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http://math.bu.edu/DYSYS/chaos-game/node6.html
https://en.wikipedia.org/wiki/Fractal_dimension#:~:text=In%20mathematics%2C%20more%20specifically%20in,at%20which%20it%20is%20measured.
https://en.wikipedia.org/wiki/Hausdorff_dimension
http://fractalfoundation.org/OFC/OFC-10-4.html
http://www.human-current.com/blog/2015/7/1/fractals-and-measuring-roughness
https://www.wahl.org/fe/HTML_version/link/FE4W/c4.htm
https://mathsection.com/from-coastlines-to-fractals/
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