Session 3

Textures, feature extraction from varying textures

Session delivered by:

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Session Outcomes

At the end of this session, student will be able to:

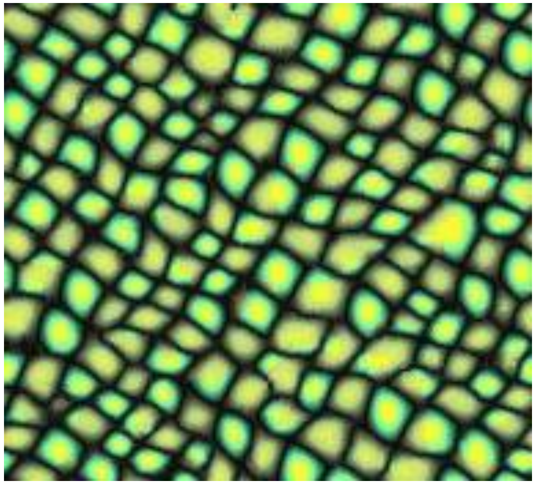
- Discuss Texture
- Analyse Image Texture
- Analyse Textures Features
- Discuss features extraction
- Discuss features extraction from varying textures
- Discuss Texture based matching
- Discuss Texture segmentation
  - Representing texture
- Discuss Texture synthesis
  - useful; also gives some insight into quality of representation
- Discuss Shape from texture

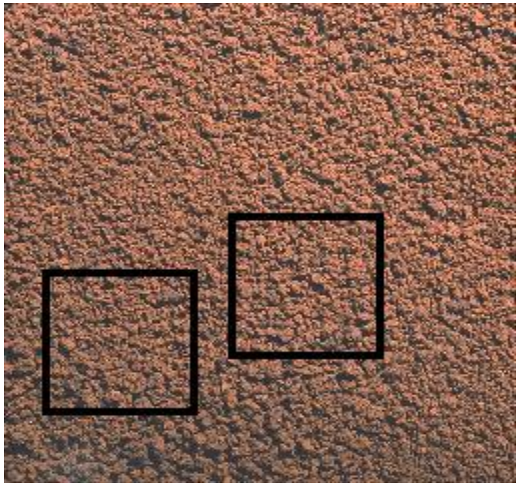
Session Topics

- Texture
- Image Texture
- Textures Features
- Texture features extraction
- Feature extraction from varying textures
- Texture based matching
- Texture segmentation

What is Texture?

- Texture is a feature that can help to segment images into regions of interest and to classify those regions.
- Image texture gives us information about the spatial arrangement of color or intensities in an image or selected region of an image.
- Images containing repeating patterns
- Local & stationary



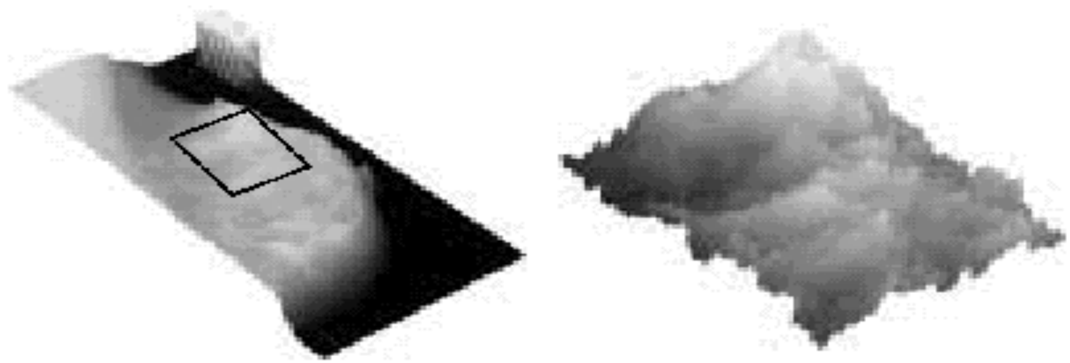


Cont..

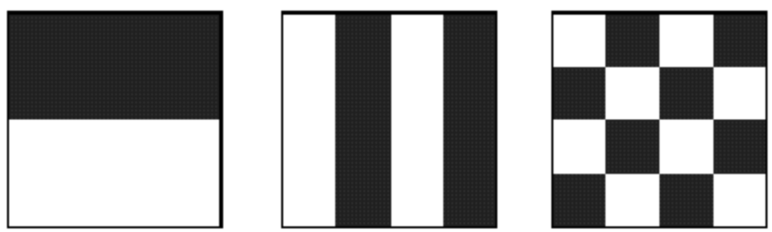
- A feature is used to partition images into regions of interest and to classify those regions
- provides information in the spatial arrangement of colours or intensities in an image
- characterized by the spatial distribution of intensity levels in a neighbourhood
- repeating pattern of local variations in image intensity
- cannot be defined for a point

Cont..

- Texture is a repeating pattern of local variations in image intensity:



- For example, an image has a 50% black and 50% white distribution of pixels.



- 3 different images with the same intensity distribution, but with different textures.

Cont..

- Texture consists of texture primitives or texture elements, sometimes called ***texels***.
  - Texture can be described as fine, coarse, grained, smooth, etc.
  - Such features are found in the tone and structure of a texture.
  - Tone is based on pixel intensity properties in the ***texel***, while structure represents the spatial relationship between ***texels***.
  - If ***texels*** are **small** and tonal differences between texels are large a fine texture results.
  - If ***texels*** are **large** and consist of several pixels, a coarse texture results.

Cont..

- The surface of any visible object is textured at certain scale.
- In general texture refers to surface characteristics and appearance of an object given by the size, shape, density, arrangement, proportion of its elementary parts .
- A texture is usually described as smooth or rough, soft or hard, coarse of fine, matt or glossy, and etc.
- Textures might be divided into 2 categories, ***tactile*** and ***visual textures***.
- **Tactile textures** refer to the immediate tangible feel of a surface. **Visual textures** refer to the visual impression that textures produce to human observer, which are related to local spatial variations of simple stimuli like colour, orientation and intensity in an image.

Cont..

- Def: The regular repetition of an element or pattern on a surface.
- Figures 1 and 2 show a few natural and man-made textures, respectively, which could be met in daily life.

Figure 1: Examples of natural textures

Figure 2: Examples of artificial regular textures

Definition

- An image texture is a set of metrics calculated in image processing designed to quantify the perceived texture of an image
- Image Texture gives us information about the spatial arrangement of color or intensities in an image or selected region of an image.
- Texture can be defined as an entity consisting of mutually related pixels and group of pixels.

Texture Analysis

- Because texture has so many different dimensions no single method of texture representation that is adequate for a variety of textures.

- **Texture analysis** refers to the characterization of regions in an **image** by their **texture** content. **It** attempts to quantify intuitive qualities described by terms such as rough, smooth, silky, or bumpy as a function of the spatial variation in pixel intensities.

- Purpose of texture analysis is:
  - To identify different textured and non-textured regions in an image.
  - To classify/segment different texture regions in an image.
  - To extract boundaries between major texture regions.
  - To describe the texel unit.
  - 3-D shape from texture

Cont..

- Two primary issues in texture analysis:

  - ***texture classification***
  - ***texture segmentation***

- ***Texture classification*** is concerned with identifying a given textured region from a given set of texture classes.

  Each of these regions has unique texture characteristics.

  Statistical methods are extensively used.

  (*e.g. GLCM, contrast, entropy, homogeneity*)

- ***Texture segmentation*** is concerned with automatically determining the boundaries between various texture regions in an image.

Texture classification

- In texture classification the goal is to assign an unknown sample image to one of a set of known texture classes.
- Texture classification is one of the 4 problem domains in the field of texture analysis. The other 3 are
  - **texture segmentation** (partitioning of an image into regions which have homogeneous properties with respect to texture; supervised texture segmentation with a priori knowledge of textures to be separated simplifies to texture classification),
  - **texture synthesis** (the goal is to build a model of image texture, which can then be used for generating the texture) and
  - **shape from texture** (a 2D image is considered to be a projection of a 3D scene and apparent texture distortions in the 2D image are used to estimate surface orientations in the 3D scene).

Cont..

- Texture classification process involves 2 phases: 1. learning phase & 2. recognition phase.
- In the learning phase, the target is to build a model for the texture content of each texture class present in the training data, which generally comprises of images with known class labels.
- The texture content of the training images is captured with chosen texture analysis method, which yields a set of textural features for each image. These features can be scalar numbers or discrete histograms or empirical distributions, characterize given textural properties of the images, such as spatial structure, contrast, roughness, orientation, etc.
- In the recognition phase, texture content of the unknown sample is 1st described with the same texture analysis method. Then the textural features of the sample are compared to those of the training images with a classification algorithm, and the sample is assigned to the category with the best match.

Definition

- An image texture is a set of metrics calculated in image processing designed to quantify the perceived texture of an image

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Why we use texture ?

- Image textures can be artificially created or found in natural scenes captured in an image

- Used to help in segmentation
- Classification of image

Analyze texture in CG
- Structure approach
- Statistical approach

Defining Texture

- There are ***<u>3 approaches</u>*** to defining exactly what texture is:

  – ***Structural*** : texture is a set of primitive texels in some regular or repeated relationship.

  – ***Statistical*** : texture is a quantitative measure of the arrangement of intensities in a region.
  This set of measurements is called a *feature vector*.

  – ***Modeling*** : texture modeling techniques involve constructing models to specify textures.

Cont..

- Statistical methods are particularly useful when the texture primitives are small, resulting in ***micro-textures***.

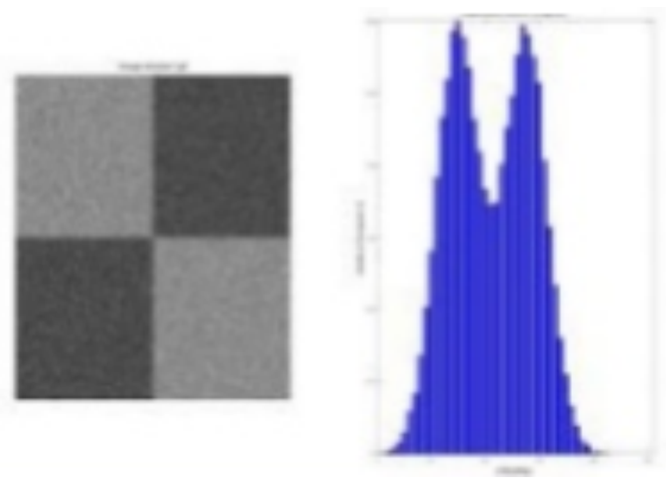
- When the size of the texture primitive is large, first determine the shape and properties of the basic primitive and the rules which govern the placement of these primitives, forming ***macro-textures***.

Structured Approach

- Structural approach: a set of texels in some regular or repeated pattern. Human beings have the ability to perceive the structural characteristics of some textures.

Statistical approach

- Texture is a spatial property.
- A simple 1-D Histogram is not useful in characterizing texture.



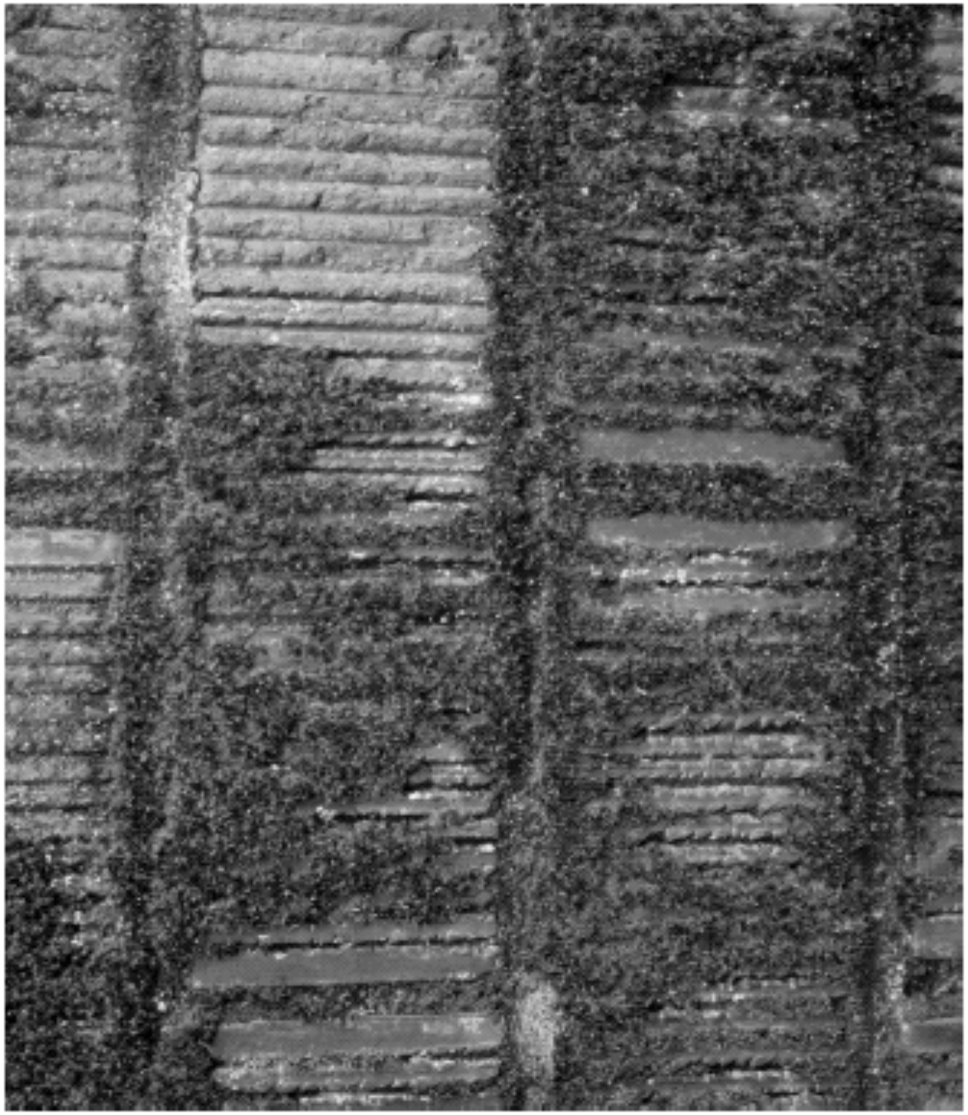
Example: an image in which pixel alternate from black to white in a checkerboard fashion will have the same histogram as an image in which the top half is black and the bottom half is black and the bottom half is white).

Statistical approach

- There are many ways in which intensity might vary, but if the variation does not have sufficient uniformity, the texture may not be characterized sufficiently close to permit recognition or segmentation.

- Thus, the degrees of randomness and of regularity will have to be measured and compared when charactering a texture.

- Often, textures are derived from tiny objects or components that are themselves similar, but that are placed together in ways ranging from purely random to purely regular, such as bricks in a wall, or grains of sand, etc.



Representing textures

- Textures are made up of quite stylized sub-elements, repeated in meaningful ways
- Representation:
  - find the sub-elements, and represent their statistics
- But what are the sub-elements, and how do we find them?
  - recall normalized correlation
  - find sub-elements by applying filters, looking at the magnitude of the response
- What filters?
  - experience suggests spots and oriented bars at a variety of different scales
  - details probably don't matter
- What statistics?
  - within reason, the more the merrier.
  - At least, mean and standard deviation
  - better, various conditional histograms.

❖ **Texture segmentation:**

- Unlike texture classification, texture segmentation is concerned with automatically determining the boundaries between various textured regions in an image.

- Both reign-based methods and boundary-based methods have been attempted to segments texture images.

❖ Shape recovery from texture:

- Image plane variation in the texture properties, such as density, size and orientation of texture primitives, are the cues exploited by shape –from-texture algorithms.

- Quantifying the changes in the shape of texture elements is also useful to determine surface orientation.

TECHNIQUES FOR TEXTURE EXTRACTION

- There are various techniques for texture extraction. Texture feature extraction algorithms can be grouped as follows:

- **Statistical**
- **Geometrical**
- **Model based**
- **Signal Processing**

Statistical method

A. Local features

i. Grey level of central pixels,
ii. Average of grey levels in window,
iii. Median,
iv. Standard deviation of grey levels,
v. Difference of maximum and minimum grey levels,
vi. Difference between average grey level in small and large windows,
vii. Kirsch feature,
viii. Combine features

Cont..

B. Galloway

i. run length matrix

C. Haralick

i. co-occurrence matrix

Geometrical method

- First threshold images into binary images of n grey levels.
- Then calculate statistical features of connected areas.

Model Based method

These involve building mathematical models to describe textures:

- Markov random fields
- Fractals 1
- Fractals 2

Signal processing method

These methods involve transforming original images using filters and calculating the energy of the transformed images.

- Law’s masks
- Laines – Daubechies wavelets
- Fourier transform
- Gabor filters

Texture Measures

- GLCM (Gray level Co-occurrence Matrices)
- Law's Texture Energy Measures
- Wavelets
- Steerable Pyramids

GLCMs

– The statistical measures described so far are easy to calculate, but do not provide any information about the repeating nature of texture.

– A gray level co-occurrence matrix (**GLCM**) contains information about the positions of pixels having similar gray level values.

Cont..

- A co-occurrence matrix is a two-dimensional array, *P*, in which both the rows and the columns represent a set of possible image values.

- A GLCM P_d [*i, j*] is defined by first specifying a displacement vector *d*=(d_x, d_y) and counting all pairs of pixels separated by *d* having gray levels *i* and *j*.

- 2D histogram of image intensities

- *P(i, j, d,* θ *)* : Count of occurrence of gray level *i* with j at distance d and in direction θ

Cont..

- The **GLCM** is defined by:

$$P_d[i, j] = n_{ij}$$

– where n_{ij} is the number of occurrences of the pixel values *(i, j)* lying at distance *d* in the image.

– The co-occurrence matrix P_d has dimension *n × n*, where n is the number of gray levels in the image.

Cont..

50	51	52	50
53	51	51	52
51	50	51	52
52	53	53	52

Intensity image patch

	50	51	52	53
50	0	2	0	0
51	1	1	3	0
52	1	0	0	1
53	0	1	1	1

$P(d,\theta)$, $d=1$, $\theta=0^{o}$

Framework for the GLCM:

e.g. for glcm matrix for an image

image

0	**0**	**1**	**1**
0	0	1	1
0	2	2	2
2	2	3	3

glcm matrix

Spatial relationship between two pixels

GLCM texture considers the relation between two pixels at a time, called the **reference** and the **neighbor** pixel.

neighbor pixel value -> ref pixel value:	0	1	2	3
0	0,0	0,1	0,2	0,3
1	1,0	1,1	1,2	1,3
2	2,0	2,1	2,2	2,3
3	3,0	3,1	3,2	3,3

matrix framework

Cont..

- In the above illustration, the neighbor pixel is chosen to be the one to the east (right) of each reference pixel. This can also be expressed as a (1,0) relation: 1 pixel in the x direction, 0 pixels in the y direction.

Cont..

- Each pixel within the window becomes the reference pixel in turn, starting in the upper left corner and proceeding to the lower right. Pixels along the right edge have no right hand neighbor, so they are not used for this count.

How to read the matrix framework

The top left cell will be filled with the number of times the combination 0,0 occurs, i.e. how many times within the image area a pixel with grey level 0 (neighbor pixel) falls to the right of another pixel with grey level 0 (reference pixel).

How to read the east matrix

Twice in the test image the reference pixel is 0 and its eastern neighbor is also 0. Twice the reference pixel is 0 and its eastern neighbor is 1. Three times the reference pixel is 2 and its neighbor is also 2.

2	**2**	**1**	**0**
0	2	0	0
0	0	3	1
0	0	0	1

east matrix framework

UNDERSTANDING GLCM

- GLCM represents the distance and angular spatial relationship over an image sub-region of specific region of specific size.

- The GLCM calculates, how often a pixel with gray-level (grayscale intensity or Tone) value i occurs either horizontally, vertically, or diagonally to adjacent pixels with the value j.

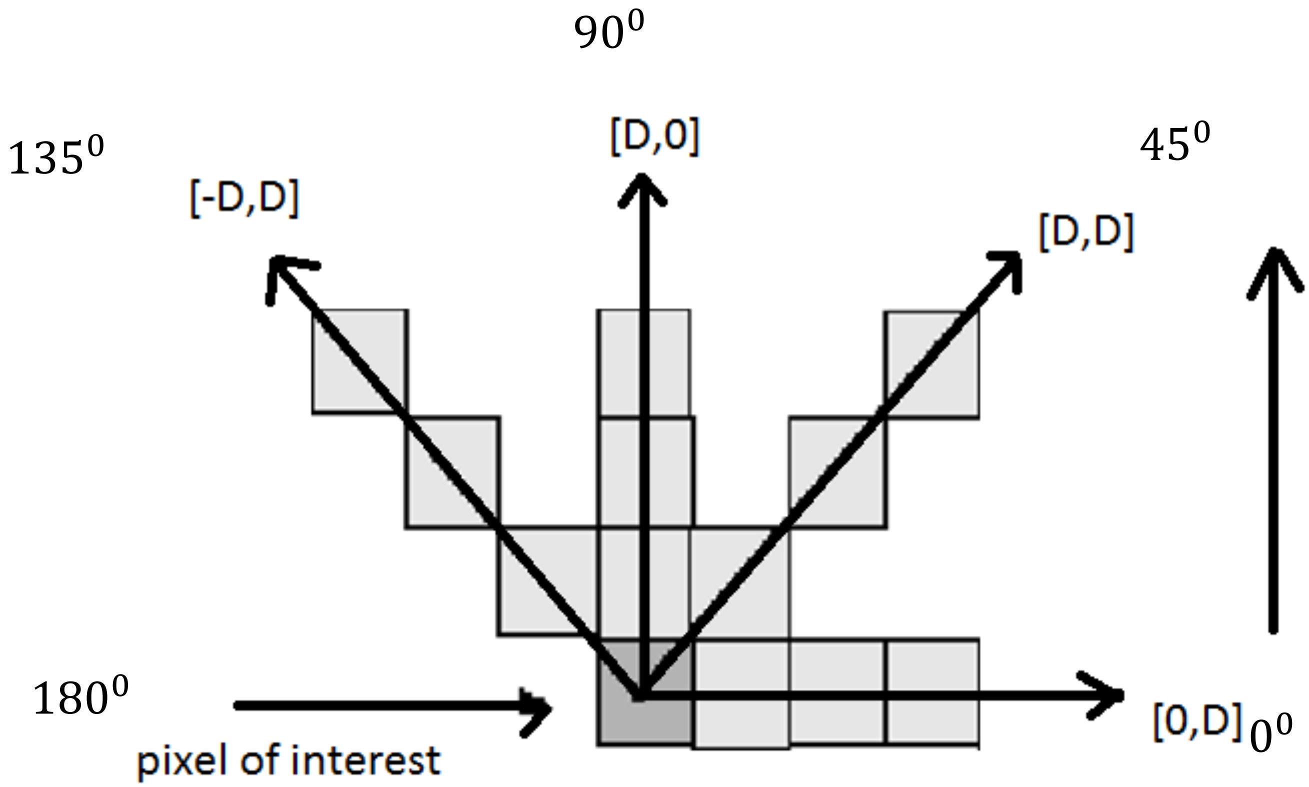
GLCM directions of Analysis

- *Horizontal (0^o)*
- *Vertical (90^o)*
- *Diagonal:*
- *Bottom left to top right (-45^o)*
- *Top left to bottom right (-135^o)*

Denoted P_0, P_{45}, P_{90}, & P_{135} Respectively.

Ex. P_0 (i, j)

GLCM direction analysis:



Cont..

- GLCM of an image is computed using a displacement vector d, defined by its **radius δ** and **orientation θ**.

- Consider a 4x4 image represented by figure with four gray-tone values 0 through 3. A generalized GLCM for that image is shown in figure 1b where #(i,j) stands for number of times i and j have been neighbors satisfying the condition stated by displacement vector d.

Cont..

0	0	1	1
0	0	1	1
0	2	2	2
2	2	3	3

1a. Test image

Gray tone	0	1	2	3
0	#(0,0)	#(0,1)	#(0,2)	#(0,3)
1	#(1,0)	#(1,1)	#(1,2)	#(1,3)
2	#(2,0)	#(2,1)	#(2,2)	#(2,3)
3	#(3,0)	#(3,1)	#(3,2)	#(3,3)

1b. General form of GLCM

Cont..

- **The four GLCM for angles equal to 0°, 45°, 90° and 135° and radius equal to 1 are shown in figure 2 a-d**.

Cont..

4	**2**	**1**	**0**
2	4	0	0
1	0	6	1
0	0	1	2

GLCM for δ*=1 &* θ=0^0

6	**0**	**2**	**0**
0	4	2	0
2	2	2	2
0	0	2	0

GLCM for δ*=1 &* θ=90^0

Cont..

4	**1**	**0**	**0**
1	2	2	0
0	2	4	1
0	0	1	0

GLCM for δ*=1 &* θ=45^0

2	**1**	**3**	**0**
1	2	1	0
3	1	0	2
0	0	2	0

GLCM for δ*=1 &* θ=135^0

Choice of radius δ

- δ values ranging should be ranging from 1, 2 to 10, but the best result is for δ = 1 and 2.

- Applying large displacement value to a fine texture would yield a GLCM that does not capture detailed textural information.

- As a pixel is more likely to be correlated to other closely located pixel than the one located far away, the above consideration is correct.

Choice of angle θ

- Every pixel has eight neighboring pixels allowing eight choices for θ, which are 0°, 45°, 90°, 135°, 180°, 225°, 270° or 315°.

- According to the definition of GLCM, the co-occurring pairs obtained by choosing θ equal to 0° would be similar to those obtained by choosing θ equal to 180°. This concept extends to 0°,45°, 90°and 135° as well. Hence, one has four choices to select the value of θ.

GLCMs

- Too many parameters
- Computationally Expensive
- Not suitable for coarse texture
- Susceptible to noise

Common Statistics Derived From Co-occurrence Probabilities

ENERGY

- Also called **Uniformity or Angular second moment.**
- Measures **the textural uniformity that is pixel pair repetitions**.
- Detects disorders in textures.
- Energy reaches a maximum value equal to one.

$$Energy = \sum_i \sum_j {p_{ij}}^2$$

Entropy

- **Measures the disorder or complexity of an image**.
- The entropy is large when the image is not texturally uniform.
- Complex textures tend to have high entropy.
- Entropy is strongly, but inversely correlated to energy.
- $Entropy(ent) = -\sum_i \sum_j p_{ij} \log_2 p_{ij}$

Contrast

- **Measures the spatial frequency of an image and is difference moment of GLCM**.
- It is the difference between the highest and the lowest values of a contiguous set of pixels.
- It measures the amount of local variations present in the image.
- Contrast(con)=$\sum_i \sum_j (i-j)^2 p_{ij}$

Homogeneity

- Also called as **Inverse Difference Moment**.

- **Measures image homogeneity as it assumes larger values for smaller gray tone differences in pair elements**.

- It is more sensitive to the presence of near diagonal elements in the GLCM.

- It has maximum value when all elements in the image are same.
- Homogeneity decreases **if** contrast increases while energy is kept constant.

- Homogeneity(hom) = $\sum_i \sum_j \frac{1}{1+(i-j)^2} p_{ij}$

Variance

- **This statistic is a measure of heterogeneity and is strongly correlated to first order statistical variable such as standard deviation**.

- Variance increases when the gray level values differ from their mean.

- Variance(var)= $\sum_i \sum_j (i-\mu)^2 \, p_{ij}$

  where μ is the mean of p_{ij}

Cont..

Difference Variance

$=$variance of p_{x-y}

Difference Entropy

$$=\sum_{i=0}^{N-1} p_{x-y}(i)\log\{p_{x-y}(i)\}$$

Law's Texture Energy Features

- Signal-processing-based algorithms use texture filters applied to the image to create filtered images from which texture features are computed.
- The Laws Algorithm
  - Filter the input image using texture filters.
  - Compute texture energy by summing the absolute value of filtering results in local neighborhoods around each pixel.
  - Combine features to achieve rotational invariance.

Law's Texture Energy Measures

- Another approach to generate texture features is to use local masks to detect various types of texture.
- Laws developed a texture-energy approach that measures the amount of variation within a fixed-size window.
- A set of nine *5 x 5* convolution masks is used to compute texture energy.
- Feature Extraction scheme based on gradient operators
- The masks are computed from the following vectors.

Cont..

- 25 Masks by convolution of 5 1-D vectors
  - Level L5 = [1 4 6 4 1]
  - Edge E5 = [-1 -2 0 2 1]
  - Spot S5 = [-1 0 2 0 -1]
  - Wave W5= [-1 2 0 -2 1]
  - Ripple R5= [1 -4 6 -4 1]
- L5(Gaussian) gives a center weighted local average
- E5(Gradient) responds to row or col step edges
- S5(LOG) detects spots
- R5(Gabbor) detects ripples

Cont..

- The names of the vectors describe their purposes.
- The L5 vector gives a center-weighted local average.
- The E5 vector detects edges, the S5 vector detects spots, and the R5 vector detects ripples.
- The 2D convolution masks are obtained by computing outer products of pairs of vectors.
- For example, the mask E5,L5 is computed as the product of E5 and L5 as follows

Law’s Texture Energy Measures

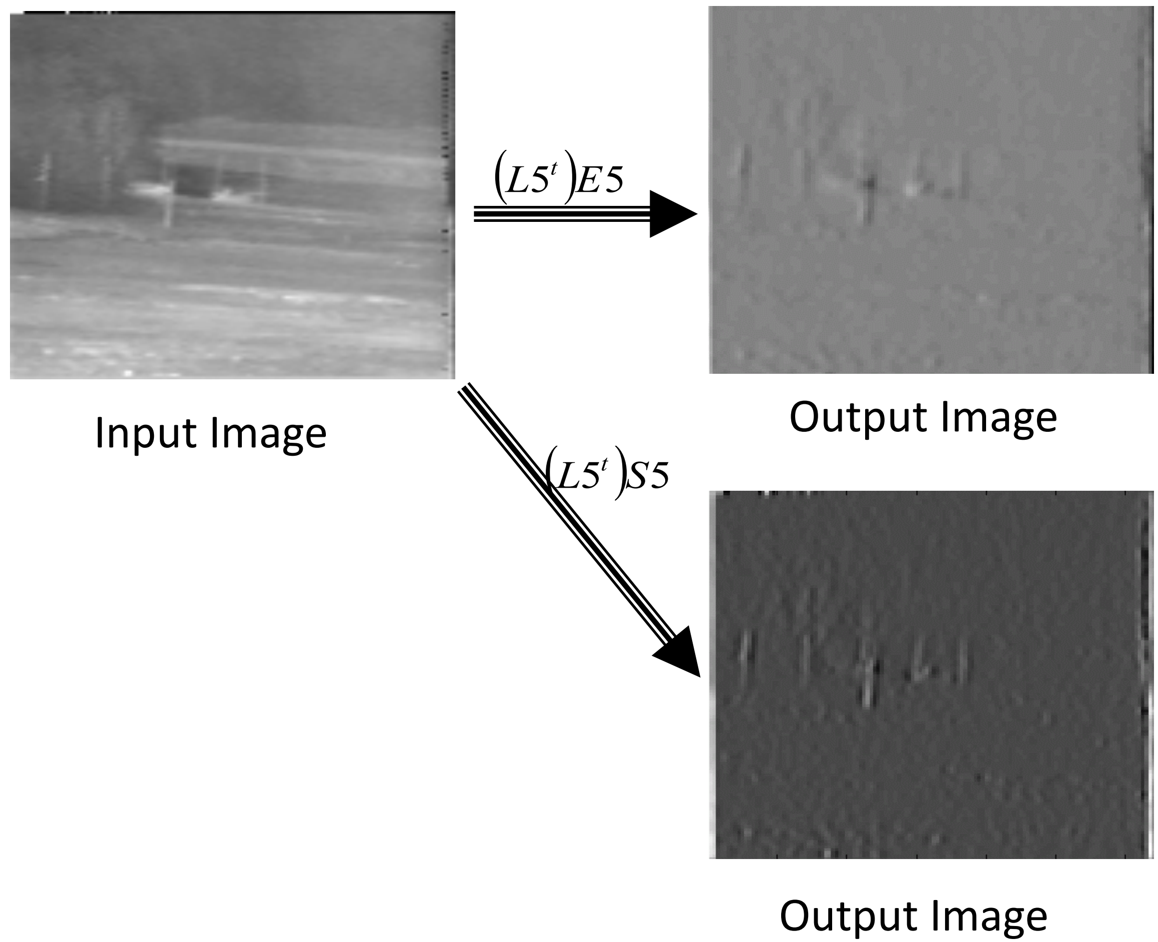
$$(L5^t)E5 = \begin{matrix} 1 & 2 & 0 & 2 & 1 \\ 4 & 8 & 0 & 8 & 4 \\ 6 & 12 & 0 & 12 & 6 \\ 4 & 8 & 0 & 8 & 4 \\ 1 & 2 & 0 & 2 & 1 \end{matrix}$$

$$L5 = [1 \quad 4 \quad 6 \quad 4 \quad 1]$$

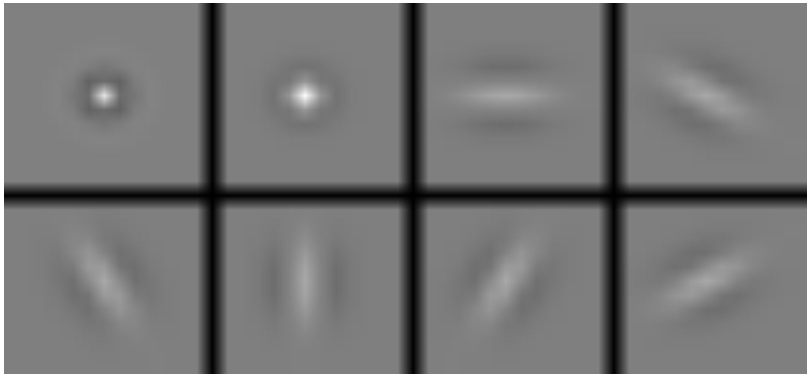
$$E5 = [\ 1 \quad 2 \quad 0 \quad 2 \quad 1]$$

$$(L5^t)S5 = \begin{matrix} 1 & 0 & 2 & 0 & 1 \\ 4 & 0 & 8 & 0 & 4 \\ 6 & 0 & 12 & 0 & 6 \\ 4 & 0 & 8 & 0 & 4 \\ 1 & 0 & 2 & 0 & 1 \end{matrix}$$

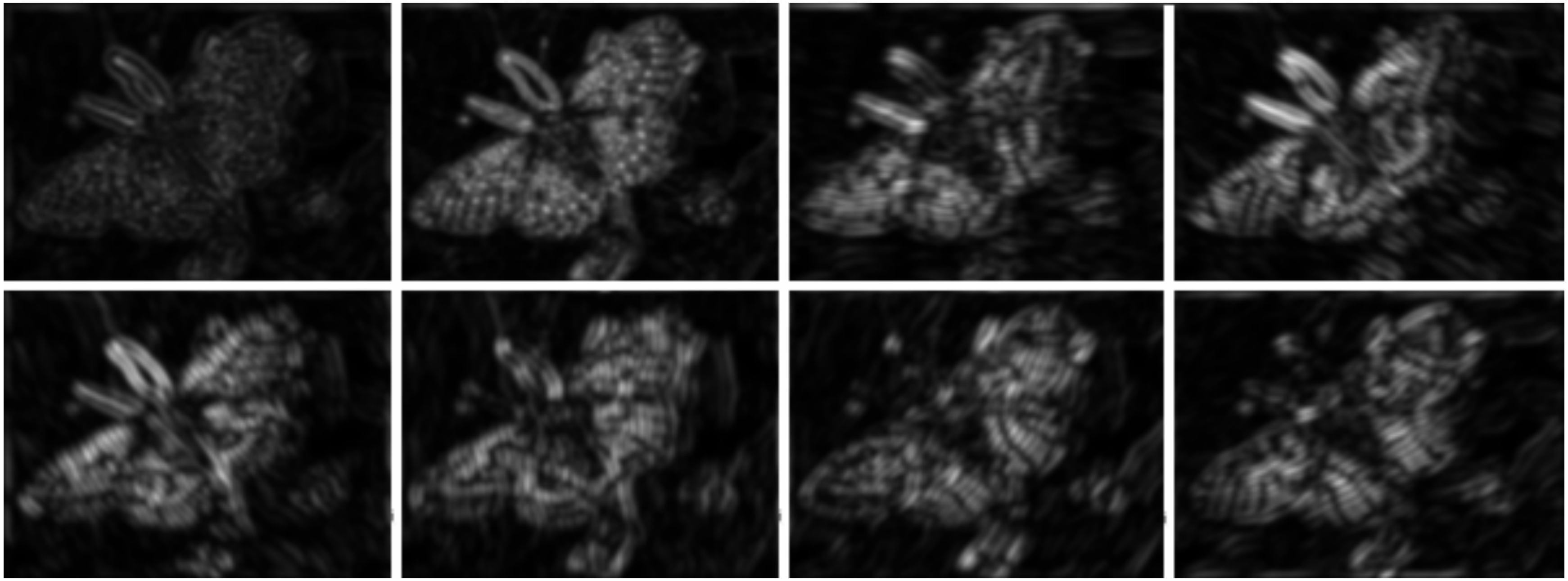
$$S5 = [\ 1 \quad 0 \quad 2 \quad 0 \quad 1]$$



Gabor Filters of different orientations



Spots and bars at a fine scale for the butterfly; images show squared response for corresponding filter



Wavelet Analysis

- Tool for multi-resolution analysis
- Provides localization in both spatial and frequency domain
- Every decomposition contains information of a specified scale and orientation

Overview of wavelet

- What does Wavelet mean?
  - Oxford Dictionary: A wavelet is a small wave.
  - Wikipedia: A wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components.
- A Wavelet Transform is the representation of a function by wavelets.

Historical Development

- 1909 : Alfred Haar –The 1st wavelet related theory .
- 1910 : Alfred Haar : Development of a set of rectangular basis functions.
- 1930s : Paul Levy investigated “The Brownian Motion”. Littlewood and Paley worked on localizing the contributing energies of a function.
- 1946 : Dennis Gabor : Used Short Time Fourier Transform .
- 1975 : George Zweig : The 1st Continuous Wavelet Transform CWT.
- 1985 : Yves Meyer : Construction of orthogonal wavelet basis functions with very good time and frequency localization.
- 1986 : Stephane Mallat : Developing the Idea of Multiresolution Analysis “MRA” for “DWT”.
- 1988 : The Modern Wavelet Theory with Daubechies and Mallat.
- 1992 : Albert Cohen, Jean fauveaux and Daubechies constructed the compactly supported biorthogonal wavelets.

- Wavelet transform decomposes $f(x)$ onto a basis of wavelet functions:

$$(W_a f)(b) = \quad f(x) \quad {}_{a,b}(x)dx$$

$$where$$

$${}_{a,b}(x) = \frac{1}{\sqrt{a}} \qquad \frac{x \quad b}{a}$$

Wavelet Transform

- Fourier transform is an analysis of global frequency content in the signal.
- Some applications in IP require analysis to be localized in the spatial domain (spatial frequency).
- The classical way of doing this is through what is called Windowed Fourier Transform.
- The windowed transform of *f(x)* in Short Time Fourier Transform (STFT) is given as:

$$F(\omega, \alpha) = \int_{-\infty}^{+\infty} f(x) g(x-\alpha) \exp^{-j\omega x} \, dx \qquad (1)$$

Cont..

- where ω: frequency and α: position of the window.
- The Gaussian is well-localized around the time $x = \alpha$. Thus, Eq. 1 transforms the signal $f(x)$ in a small window around α.
- The STFT conveys the localized frequency component present in the signal during the short window of time. The same concept may be extended to a 2-D spatial image where the localized frequency components may be determined from the windowed transform.
- This is one of the basis of the conceptual understanding of wavelet transforms.

Cont..

- Family of functions is constructed by superimposing scaled versions of a given basis function. An arbitrary signal is analyzed in terms of scaling and translation of a single mother wavelet function (basis).
- Mathematically a “wave” is expressed as a sinusoidal (or oscillating) function of time or space.
- Fourier analysis expands an arbitrary signal in terms of an infinite number of sinusoidal functions of its harmonics and has been well studied by the signal processing community for decades.

Cont..

- Fourier representation of signals is effective in analysis of time-invariant (stationary) periodic signals. In contrast to a sinusoidal function, a wavelet is a small wave whose energy is concentrated in time.

- Wavelets allow both time and frequency analysis of signals simultaneously because of the fact that the energy of wavelets is concentrated in time and still possesses the wave-like (periodic) characteristics.

- Wavelet representation provides a tool to analyze transient, time-variant (non-stationary) signals that are not statistically predictable at the region of discontinuities.

Cont..

- Wavelets are functions generated from one single function (basis function) called the *prototype or mother wavelet by dilations (scalings) and translations* (shifts) in time (frequency) domain.

- If the mother wavelet is denoted by (t), the other wavelets $_{a,b}(t)$ *is represented as:*

$$_{a,b}(t) = \frac{1}{\sqrt{|a|}} \quad \frac{t \quad b}{a} \qquad (2)$$

Cont..

- where *a and b are two arbitrary real numbers.*
- *The variables a and b represent* the parameters for *dilations and translations respectively in the time* axis.
- From Eq. 2 , it is obvious that the mother wavelet can be essentially represented as

$$(t) = {}_{1,0}(t) \qquad (3)$$

- For any arbitrary *a # 1 and b = 0, we can derive that*

$${}_{a,0}(t) = \frac{1}{\sqrt{|a|}} \quad \frac{t}{a} \qquad (4)$$

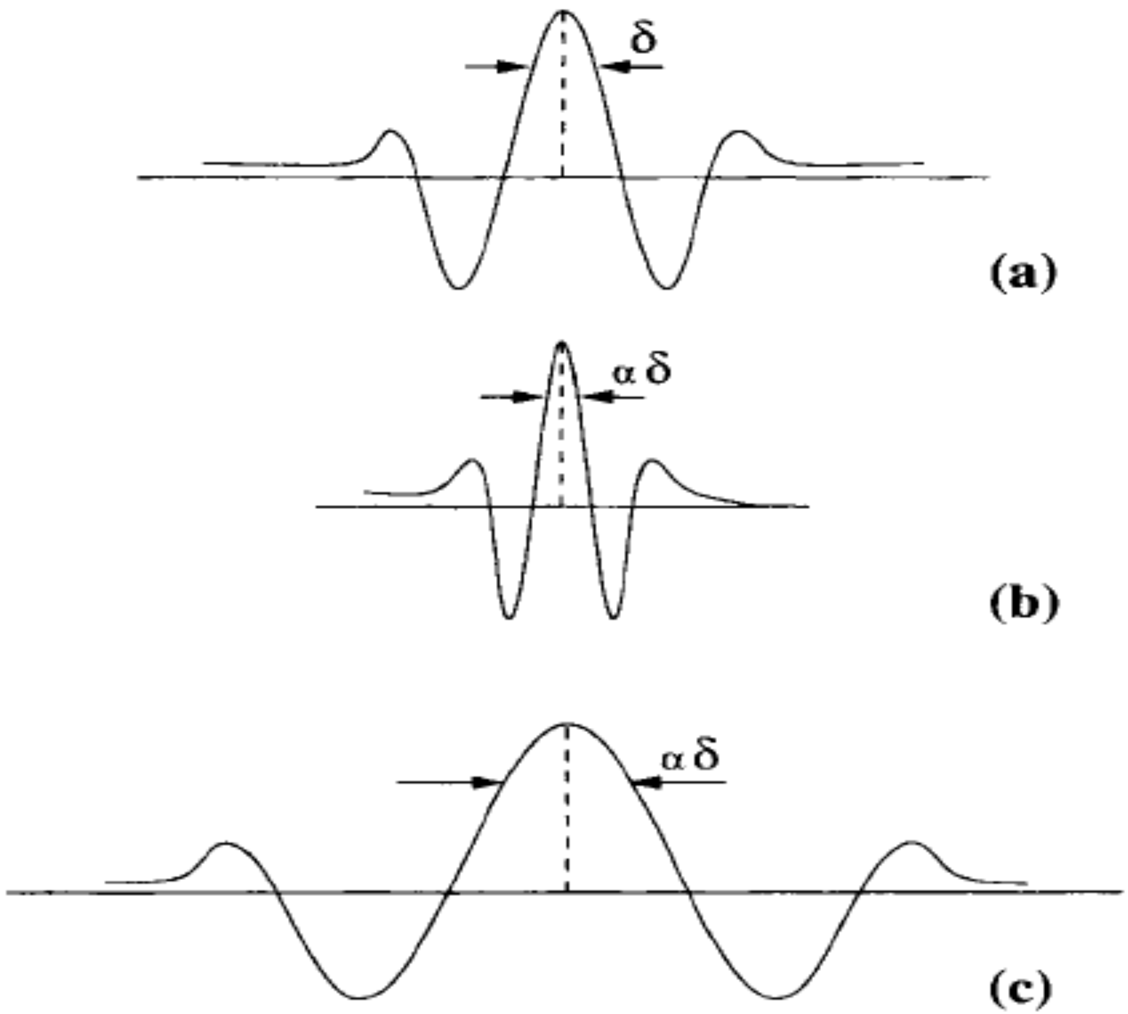
Cont..

- Eq. 4 is a time-scaled (by *a) and* amplitude-scaled (by $\sqrt{a}$) version of the mother wavelet function (t) *in* Eq. *3 .*
- *The parameter a causes*
  - *contraction of* (t) *in the time axis when a < 1 and*
  - *expansion or stretching when a > 1. That's why the parameter*
- Parameter a is called the *dilation (scaling) parameter.*
- *For a < 0, the function* $_{a,b}(t)$ results in time reversal with dilation

Cont..

- Mathematically, we can substitute *t in Eq. 4* $_{a,0}(t)$ *by t - b to cause a translation* or shift in the time axis resulting in the wavelet function *as shown in* Eq. 2.

- Function $_{a,b}(t)$ *is a*
  - *shift* $_{a,b}(t)$ *in right along the time axis* by *b when b > 0*
  - *shift in left along the time axis by b when b < 0.*
- *Variable b represents the translation* in time *(shift in frequency) domain*



(a) A mother wavelet (t) , *(b)* $\frac{t}{a}$ *: 0 <a< 1, (c)* $\frac{t}{a}$ *: a > 1.*

Cont..

- In Figure we see mother wavelet and its dilations in the time domain with the dilation parameter *$a = \alpha$.*
- *Figure (a) is the* mother wavelet (t).
- *A contraction of the signal in* the time axis when *$\alpha < 1$ is shown in Figure (b) and expansion of the* signal in the time axis when *$\alpha > 1$ is shown in Figure (c).*
- *Wavelet transform (WT) of a function (signal) f(t)* is mathematically represented by

$$W(a,b) = \int_{-\infty}^{+\infty} \psi_{a,b}(t) f(t)\, dt. \qquad (5)$$

Continuous wavelet transform

- If *a and b are two continuous (non-discrete) variables and f(t) is also a continuous* function, *W(a,b) is called the continuous wavelet transform (CWT).*

- Hence the CWT maps a one-dimensional function *f(t) to a function W(a,b)* of two continuous real variables *a (dilation) and b (translation).*

Cont..

- *DWT is a versatile signal processing* tool provide multi-resolution representation of signals based on wavelet decomposition.

- The advantage of DWT over FT is that it performs multi-resolution analysis of signals with localization both in time and frequency, popularly known as time-frequency localization.

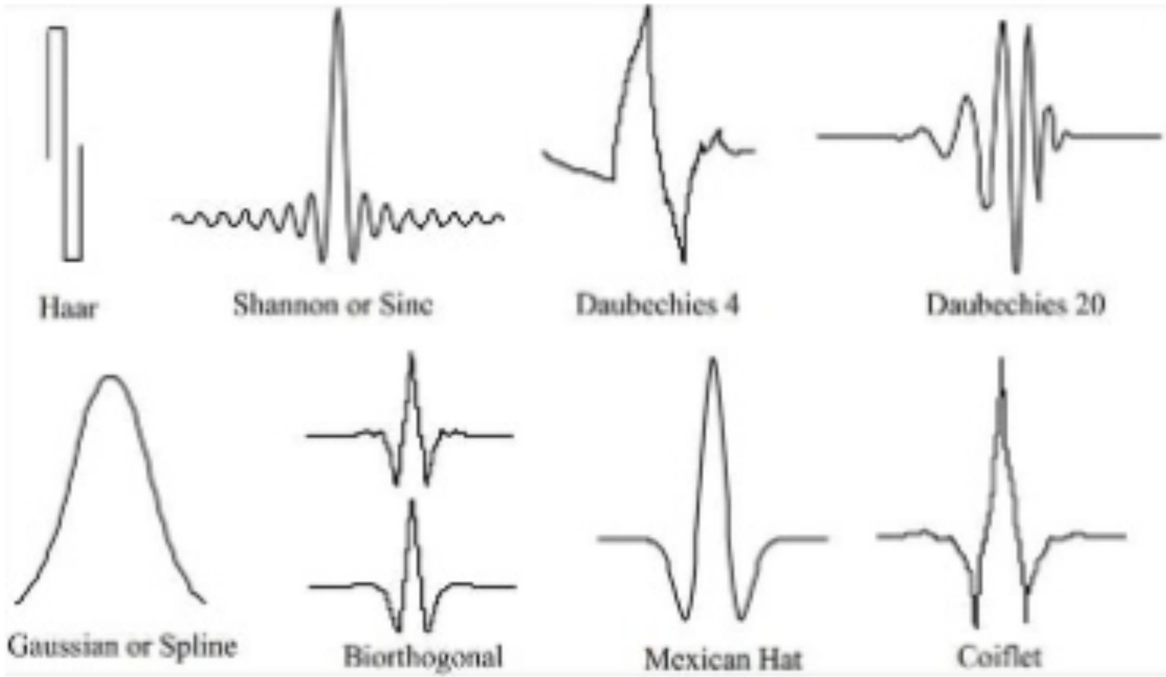
- DWT decomposes a digital signal into different sub-bands so that the LF sub-bands have finer frequency resolution and coarser time resolution compared to the HF sub-bands.

Wavelet Analysis

- Desirable Properties
  - Symmetry
  - Compactness
  - Small Support
  - Shift Invariance

Several families of wavelets

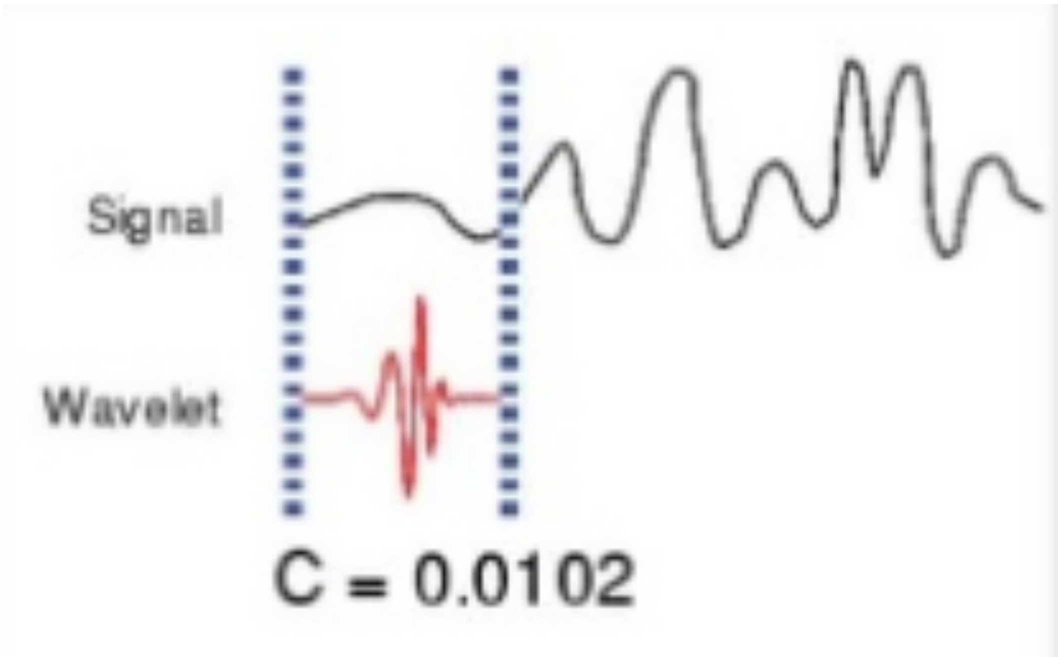
- Haar
- Daubechies
- Biorthogonal
- Coiflets
- Symlets
- Morlet
- Mexican Hat
- Meyer
- Other Real Wavelets
- Complex Wavelets



- Figure shows some of the most popular mother wavelets.

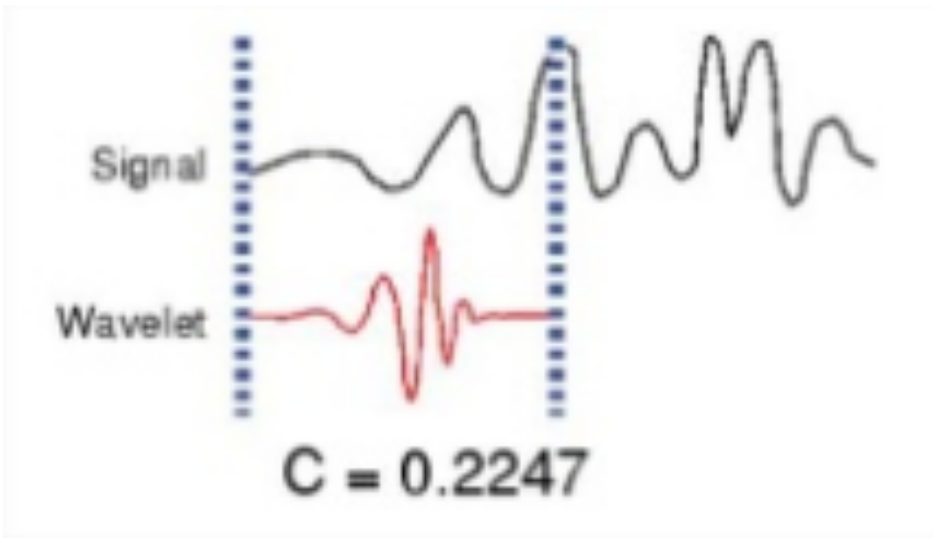
Steps to compute CWT of a given signal

1. Each Mother Wavelet has its own equation
2. Take a wavelet and compare it to section at the start of the original signal, and calculate a correlation coefficient C.



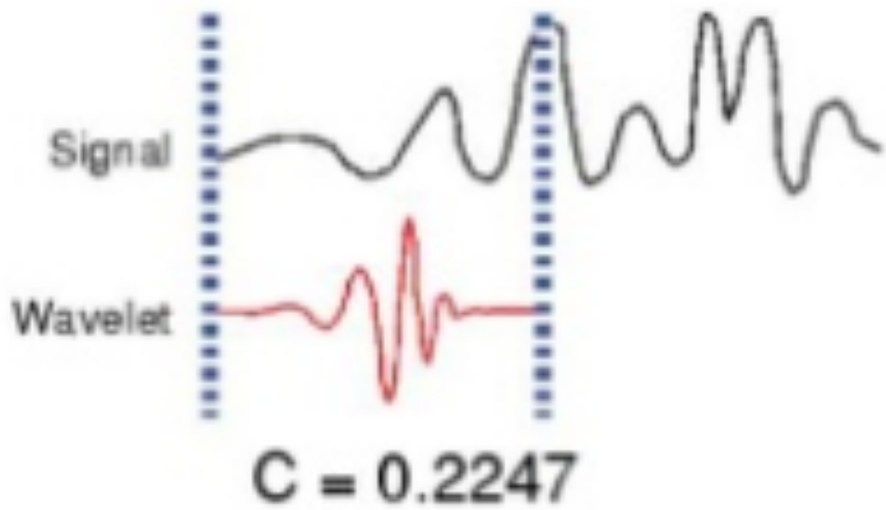
Contd..

3. Shift the wavelet to the right and repeat step 1 until the whole signal is covered.



Cont..

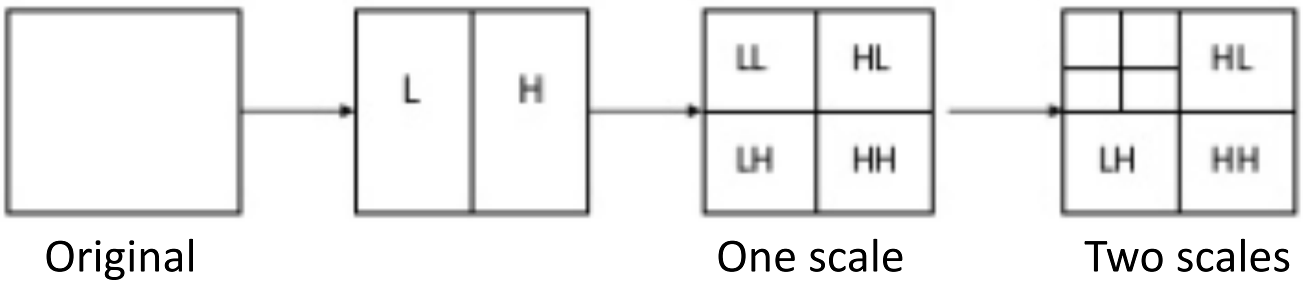
4. Scale (stretch) the wavelet and repeat steps 1 through 2.



5. Repeat steps 1 through 3 for all scales.

2-D Discrete Wavelet Transform

- A 2-D DWT can be done as follows:
  - Step 1: Replace each row with its 1-D DWT;
  - Step 2: Replace each column with its 1-D DWT;
  - Step 3: repeat steps (1) and (2) on the lowest subband for the next scale
  - Step 4: repeat steps (3) until as many scales as desired have been completed



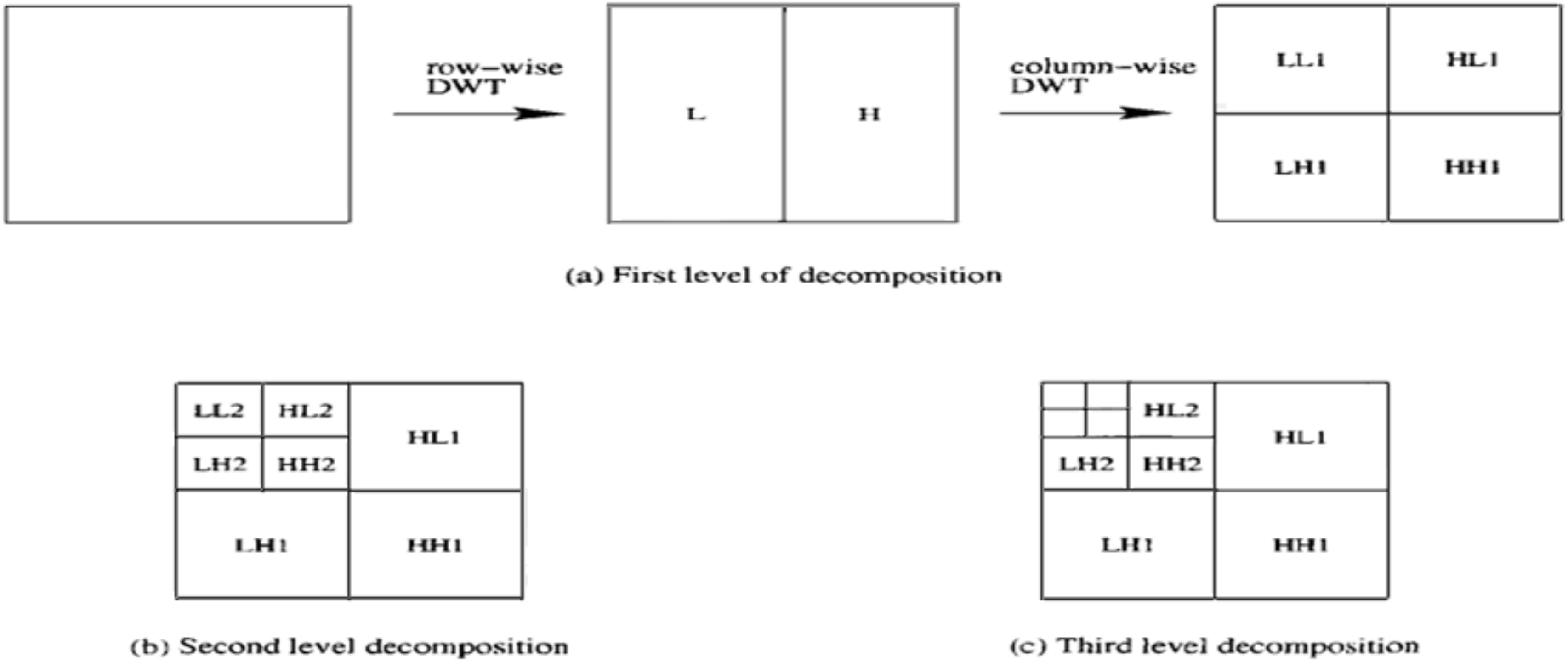
Cont..

- Apply the 1D transform in each row, to produce 2 sub-bands.
- When the low-freq sub-bands of all the rows (L) are put together, it looks like a thin version (of size $M \cdot \frac{N}{2}$) of the input signal as shown in Figure.
- Similarly put together the high-frequency sub-bands of all the rows to produce the H sub-band of size $M \cdot \frac{N}{2}$, which contains the high-frequency

  information around discontinuities (edges in an image) in the input signal.
- Apply 1D DWT column-wise on these L and H sub-bands (intermediate result), we produce 4 sub-bands LL, LH, HL, and HH of size $\frac{M}{2} \cdot \frac{N}{2}$ as in Figure.
- LL is a coarser version of the original input signal. LH, HL, and HH are the high-frequency sub-band containing the detail information.

Cont..

- Apply 1D DWT column-wise 1st and then row-wise to get the same result

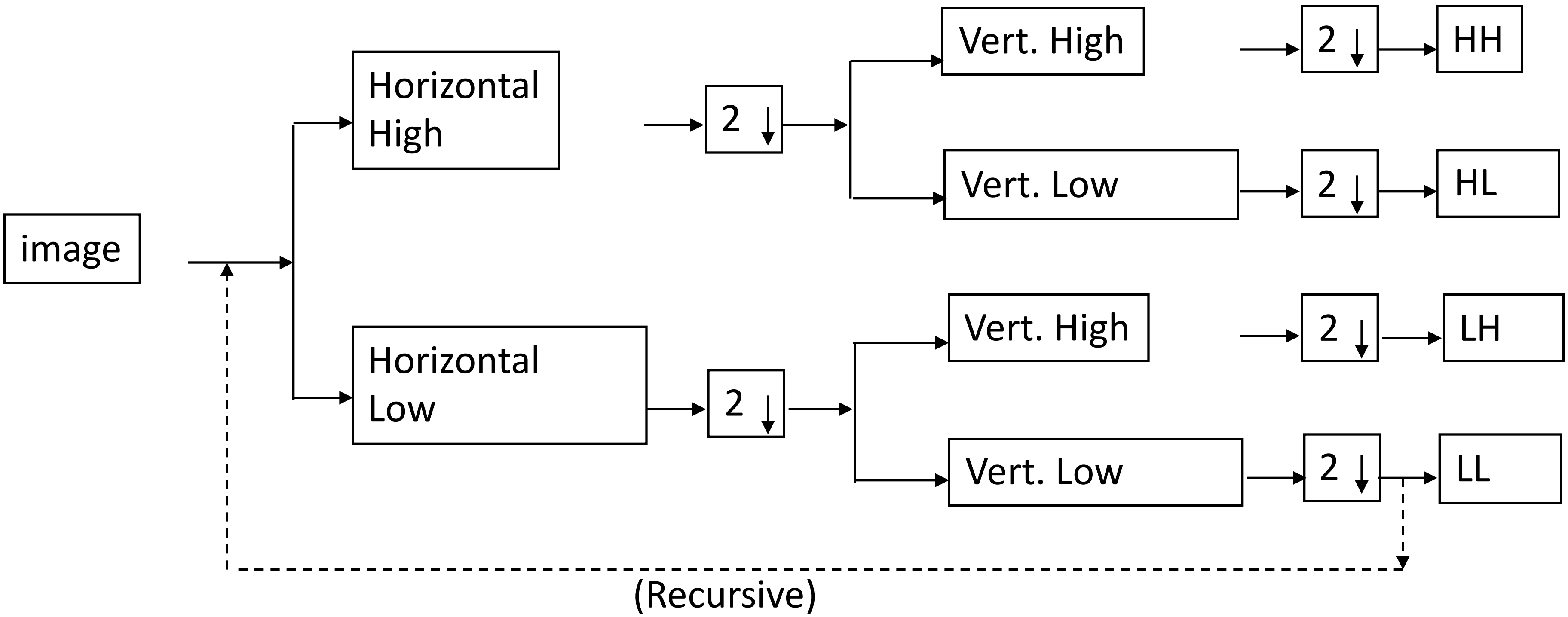


Row-column computation of two-dimensional DWT.

- In Figure (b), the LL1 sub-band is further decomposed into 4 sub-bands LL2, HL2, LH2, and HH2 based on the principle of multiresolution analysis.
- The same computation can continue to further decompose LL2 into higher levels.

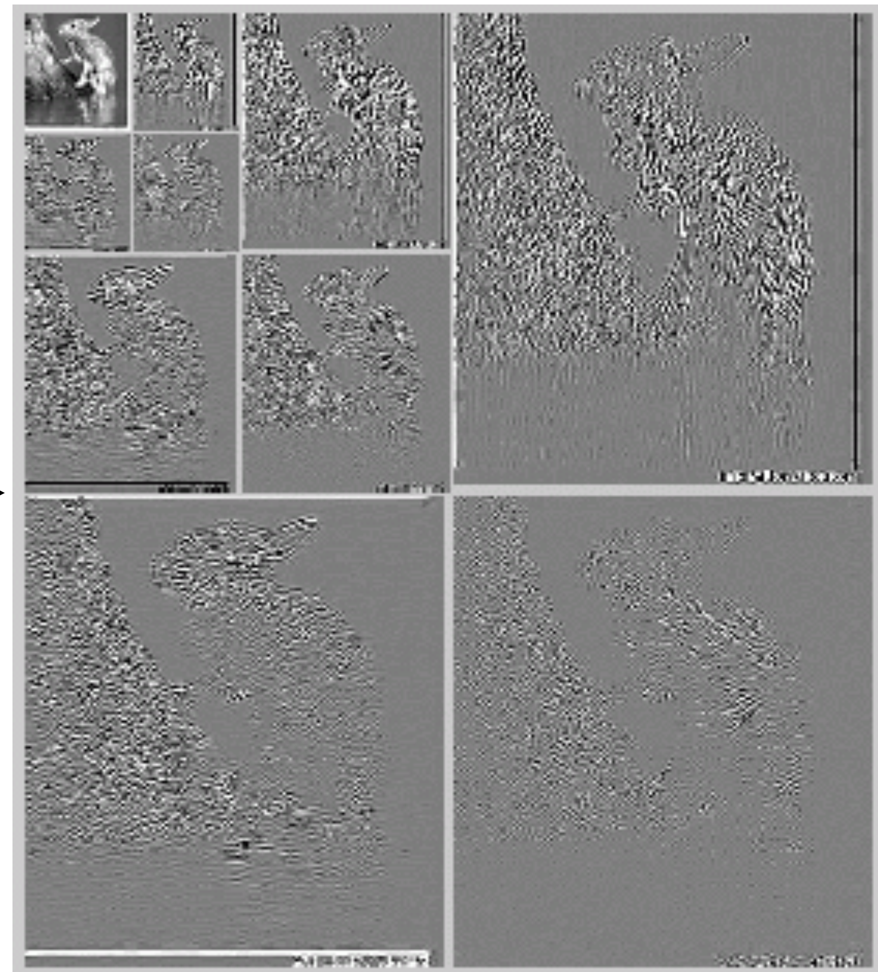
Wavelet Analysis

- 2-D Wavelet decomposition is obtained by separable filter bank.



Wavelet Analysis

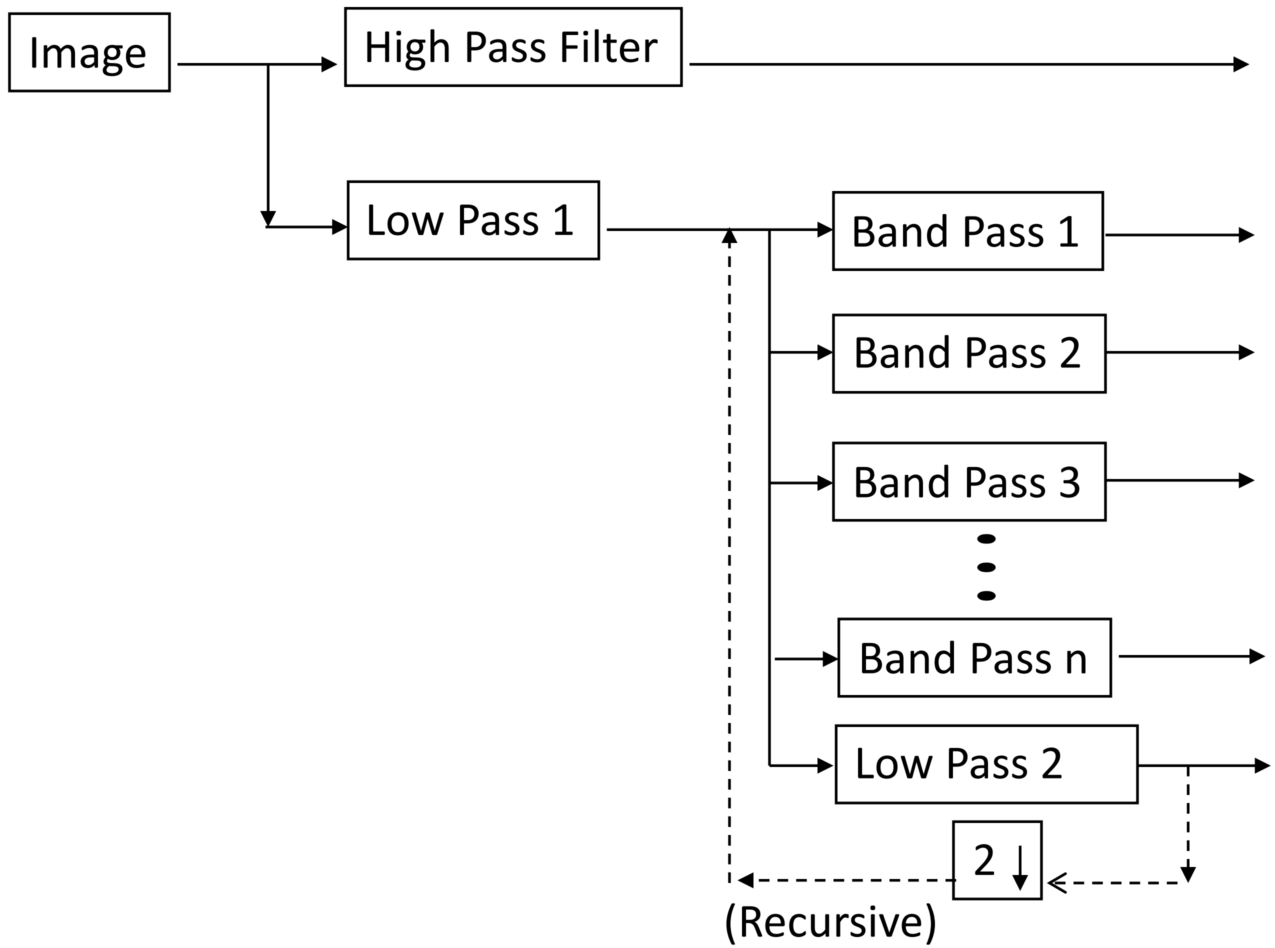
Eric Miller / Reuters



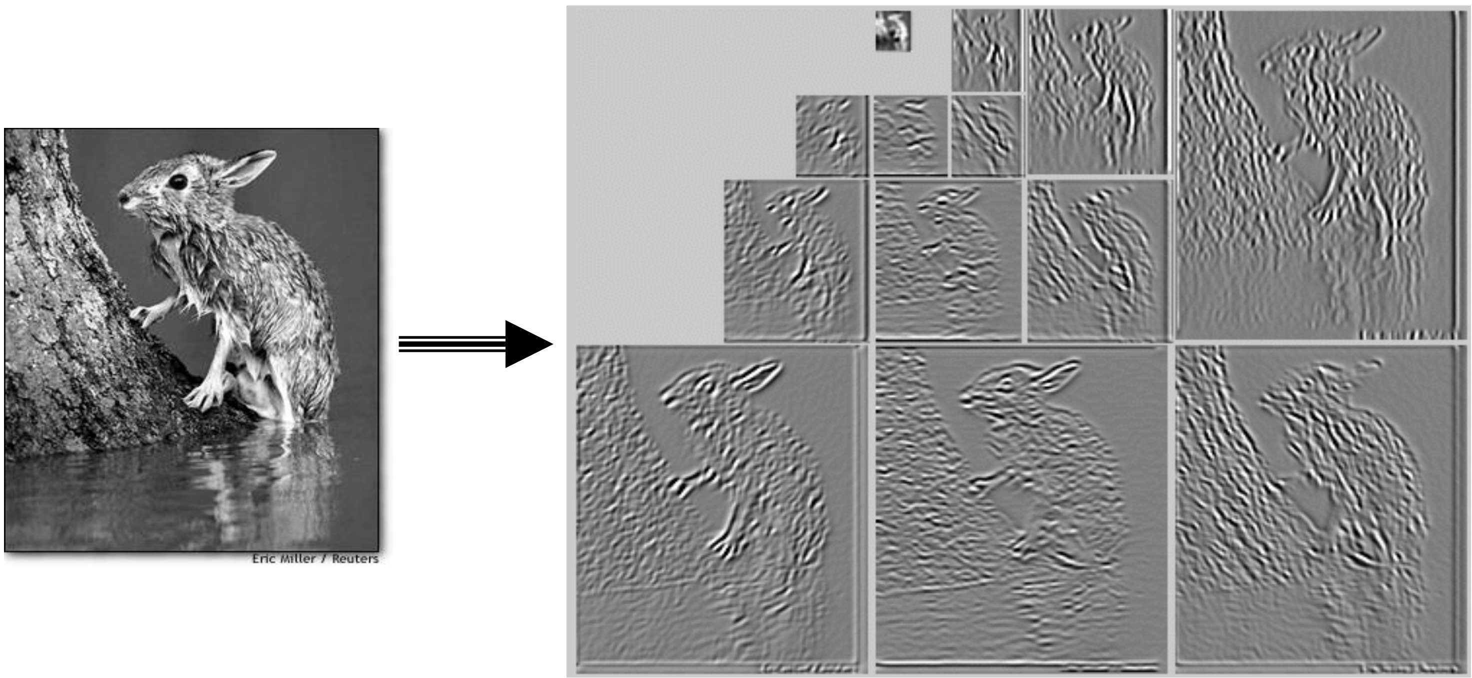
Steerable Pyramids

- Linear multi-scale, multi-orientation image decomposition
- Basis functions are directional derivative operators in different sizes and orientations
- Type of over-complete wavelet transform
- Steerable orientation decomposition

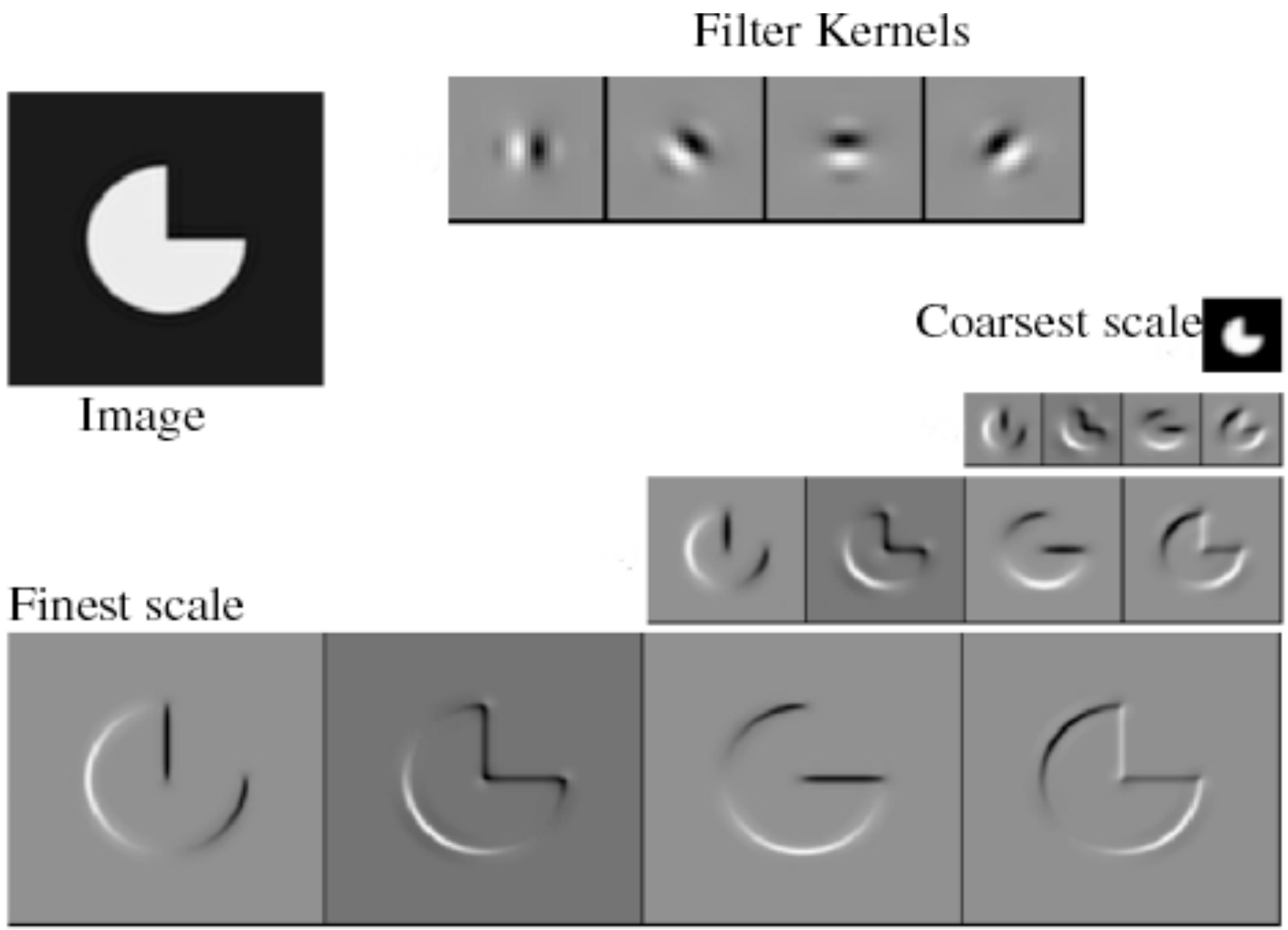
Steerable Pyramids



Steerable Pyramids



Shift-able Multi-Scale Transforms



"Shiftable MultiScale Transforms," by Simoncelli et al., IEEE Transactions on Information Theory, 1992, copyright 1992, IEEE

Steerable Pyramids vs. Wavelets

- Translation invariance
- Rotation invariance
- Aliasing
- Orthogonality
- Completeness
- Computational Efficiency

Features

- Energy

$$e_i = \frac{1}{M \cdot N} \sum_{x=1}^{M} \sum_{y=1}^{N} I_i^2(x, y)$$

- Entropy

$$Entropy_i = \frac{1}{M \cdot N} \sum_{x=1}^{M} \sum_{y=1}^{N} I_i(x, y) \log I_i(x, y)$$

Cont..

- Kurtosis

$$k = \frac{\mathop{}\limits_{x=1}^{M}\ \mathop{}\limits_{y=1}^{N} \left[I_i(x, y) \qquad \right]^4}{M \cdot N \cdot \qquad ^4} \qquad 3$$

- Skew

$$Skew = \frac{\mathop{}\limits_{x=1}^{M}\ \mathop{}\limits_{y=1}^{N} \left[I_i(x, y) \qquad \right]^3}{M \cdot N \cdot \qquad ^3}$$

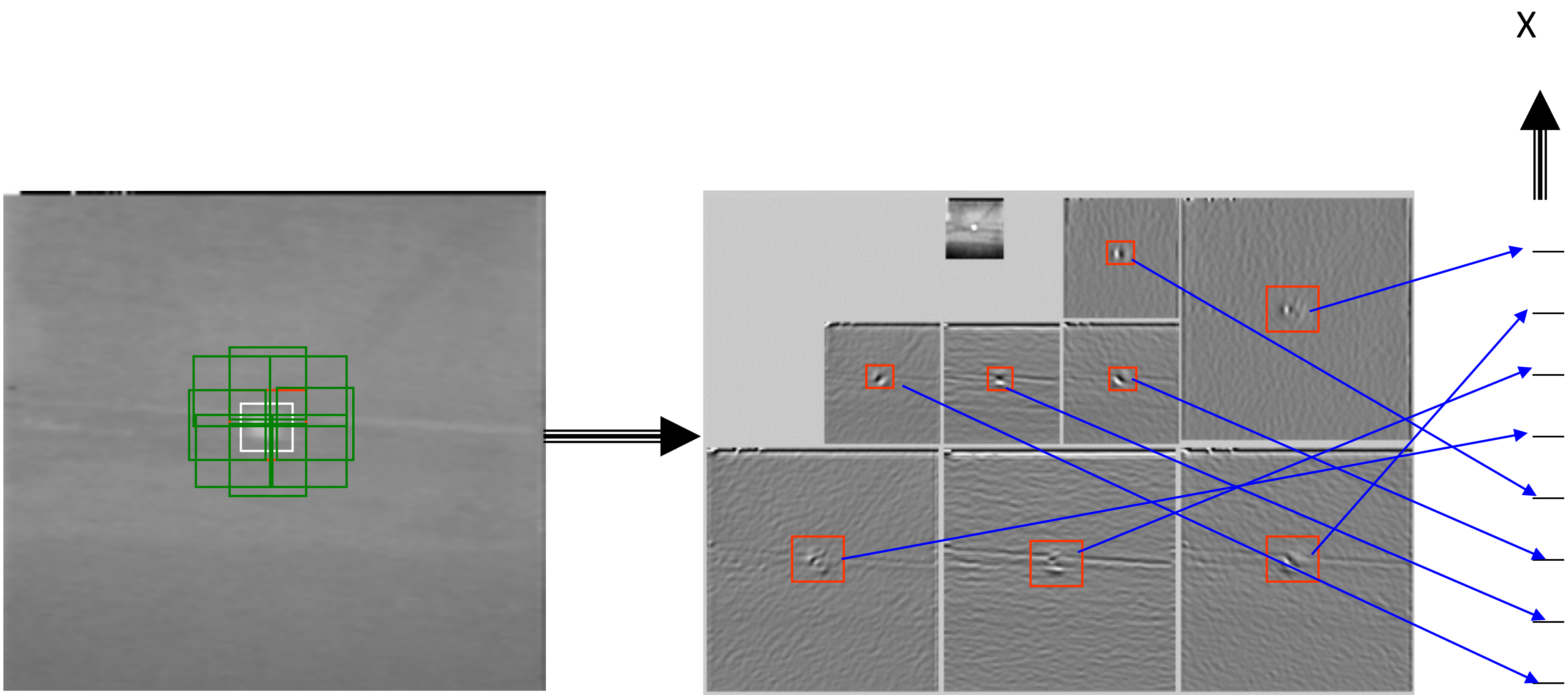
- Variance

$$^2 = \frac{\mathop{}\limits_{x=1}^{M}\ \mathop{}\limits_{y=1}^{N} \left[I_i(x, y) \qquad \right]^2}{M \cdot N}$$

Final texture representation

- Form an oriented pyramid (or equivalent set of responses to filters at different scales and orientations).
- Square the output
- Take statistics of responses
  - e.g. mean of each filter output (are there lots of spots)
  - std of each filter output
  - mean of one scale conditioned on other scale having a particular range of values (e.g. are the spots in straight rows?)

An Application



$X^{(1)}$ $X^{(2)}$ $X^{(3)}$ $X^{(4)}$ $X^{(5)}$ $X^{(6)}$ $X^{(7)}$ $X^{(8)}$

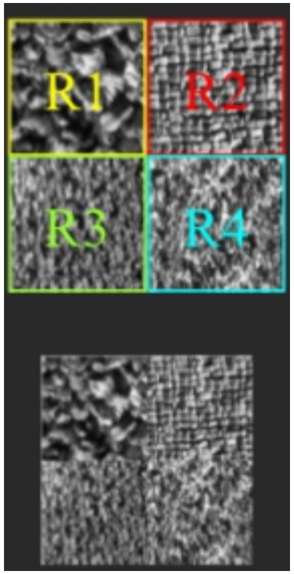
Region Based Texture Segmentation

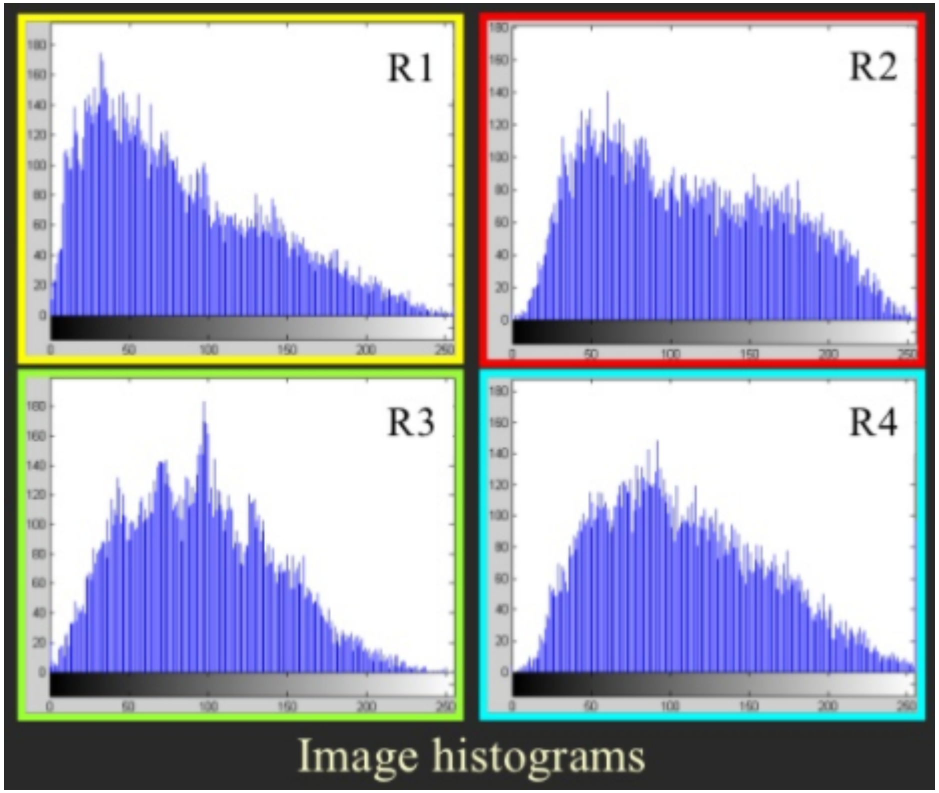
- In some images, it can be the defining characteristic of regions and critical in obtaining a correct analysis. The image of Figure 7.1 has three very distinct textures: the texture of the tiger, the texture of the jungle, and the texture of the water. These textures can be quantified and used to identify the object classes they represent.

Cont..

- We usually operate on **digital** (**discrete**) images:
  - **Sample** the 2D space on a regular grid
  - **Quantize** each sample (round to nearest integer)
- If our samples are Δ apart, we can write this as:

  $f[i\ ,j]$ = Quantize{ $f(i\ \Delta, j\ \Delta)$ }
- The image can now be represented as a matrix of integer values





Session Summary

- Texture is a repeating pattern of local variations in image intensity .
- Texture can be defined as an entity consisting of mutually related pixels and group of pixels.
- Texture consists of texture primitives or texture elements, sometimes called texels.
- Textures might be divided into 2 categories, tactile and visual textures.
- Texture analysis refers to the characterization of regions in an image by their texture content. It attempts to quantify intuitive qualities described by terms such as rough, smooth, silky, or bumpy as a function of the spatial variation in pixel intensities.