EXAMPLE 1.1

Distinguish between the basic and auxiliary functional elements of a measurement system.

Solution : The basic (functional) elements of a generalised measurement system are integral parts of all instruments. They are :

(*i*) *Detector-transducer* that senses the state, value or condition of a system and converts the desired input to a form that can be conveniently handled by the measurement system.

(*ii*) *Signal conditioning* that serves to manipulate and process the output of the transducer in a suitable form.

(*iii*) *Data-presentation* that serves to give the information about the measurand (measured variable) in the quantitative from.

The auxiliary elements refer to those units which may be incorporated in a particular measurement system depending upon the type of measurement and the nature of measurement technique. Some of the auxiliary elements are :

(*i*) Calibration element that serves to provide a built in calibration facility.

(*ii*) External power source that may be needed for the operation of various basic elements comprising the measurement system.

(*iii*) Feed back element needed to control any variation of the physical quantity.

EXAMPLE 1.2

Identify the various functional elements in dial indicator comprising a spindle, gear train, and a pointer associated with scale. The linear motion of the spindle is being transmitted and converted into an angular displacement of the pointer by means of the gear train.

Solution :

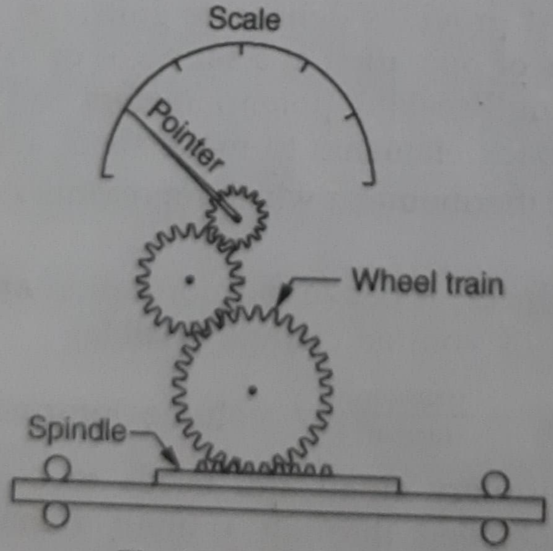


Fig 1.6. Dial indicator

(1) The spindle is sensitive to the linear displacement and acts as the primary sensing element.

(2) The gear train performs different, functions and constitutes the following elements:

(*i*) transmission of input signal from the spindle to the pointer (transmission element)

(*ii*) change in the form of signal from translation to rotation (transducer element)

(iii) multiplication or amplification of the input signal so that a large output displacement occurs (manipulation element)

(3) The pointer and the associated scale comprise the data presentation element.

EXAMPLE 1.3

Refer to the *pressure-actuated* thermometer for the measurement of temperature (Fig 1.7). Identify the various functional elements of the system.

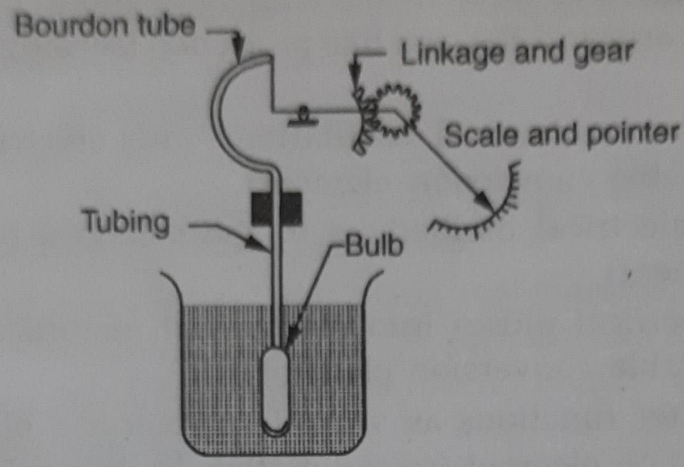


Fig. 1.7. Pressure-actuated thermometer

Solution : The liquid filled bulb acts as the primary sensor and the variable conversion element. It senses the input (temperature), receives the input signal in the form of thermal energy and the constrained thermal expansion of the filling fluid results in a pressure (mechanical energy) build up within the bulb. The pressure tubing is employed to transmit the pressure to the bourdon tube and thus functions as the data transmission element. The bourdon tube converts the fluid pressure into displacement of its tip, and as such acts as the variable conversion element. The displacement is manipulated by the linkage and gearing (manipulation elements) to give a larger pointer motion. The scale and the pointer serve as the data presentation element.

EXAMPLE 1.4

One method of indicating the revolutions of a rotating shaft is shown in Fig. 1.8. Identify the various functional elements of the measurement system.

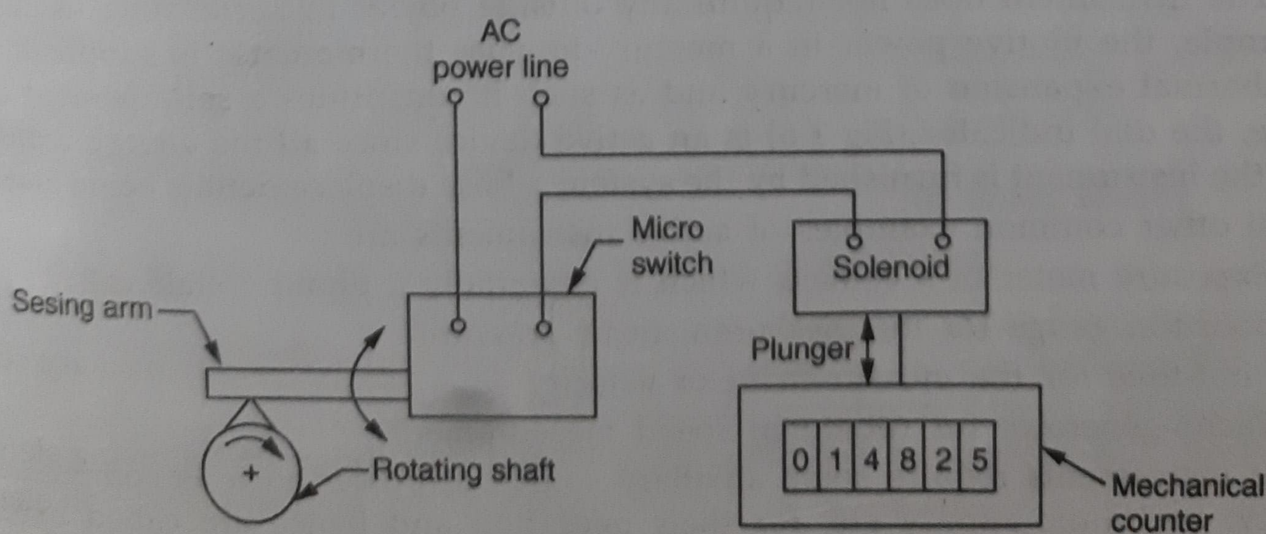


Fig. 1.8. Digital revolution counter

Solution : The sensing arm, operated by the rotating shaft, operates the micro switch and gives a sequence of electrical pulses. These pulses are transmitted over long distances to a solenoid which converts the electrical pulses to mechanical reciprocation of the plunger. Indications of the shaft revolutions are then given on the mechanical counter. The measurement system can be considered to be comprised of the following functional elements:

(1) Sensing of the rotations of the shaft and their subsequent conversion into mechanical oscillations by the arm and the cam like projection (sensing and variable conversion elements)

(2) Conversion of the mechanical oscillations into electrical oscillations by the microswitch (variable conversion element)

(3) Transmission of electrical oscillations to the solenoid by electrical wires (data transmission element)

(4) Conversion of electrical pulses into mechanical reciprocation of the plunger by the solenoid (variable conversion element).

The mechanical counter functions as variable conversion element (reciprocation to rotary motion), manipulation element (rotary motion to decimalised rotary motion) and the data presentation element.

EXAMPLE 2.1

A thermometer reads 73.5°C and the true value of the temperature is 73.15°C. Determine the error and the correction for the given thermometer.

Solution : Error E_s = measured value V_m – true value V_t

$$= 73.5 - 73.15 = 0.35°C$$

$$\text{Correction } C_s = -E_s = -\mathbf{0.35°C}$$

EXAMPLE 2.2

A temperature transducer has a range of 0°C to 100°C and an accuracy of ± 0.5 percent of full scale value. Find the error in a reading of 55°C.

Solution : Error = E_s = ± 0.5°C percent of full scale value

$$= \pm \frac{0.5}{100} \times 100 = \pm\ \mathbf{0.5°C}$$

Thus a nominal reading of 55°C actually indicates a temperature in the range 54.5°C to 55.5°C.

EXAMPLE 2.3

A pressure gauge of range 0-20 bar is stated to have an error of ± 0.25 bar when calibrated by the manufacturer. Calculate the percentage error on the basis of maximum scale value. What would be the possible error as a percentage of the indicated value when a reading of 5 bar is obtained in a test ?

Solution: The percentage error calculated on the basis of the maximum scale value is

$$= \pm \frac{0.25}{20} \times 100 = \pm 1.25\%$$

The percentage error on the basis of indicated value of 5 bar pressure is

$$= \pm \frac{0.25}{5} \times 100 = \pm 5\%$$

Apparently the gauge is more unreliable at the lower end of its range.

EXAMPLE 2.4

A pressure gauge having a range of 1000 kN/m² has a guaranteed accuracy of 1 percent of full deflection.

(i) What would be the possible readings for a true value of 100 kN/m² ?

(ii) Estimate the possible readings if the instrument has an error of 1% of the true value.

Solution : (i) The magnitude of the limiting error of instrument

$$\frac{1}{100} \times 1000 = 10 \text{ kN/m}^2$$

The pressure being measured is between the limits of :

$$100 \pm 10 = 110 \text{ kN/m}^2 \text{ or } 90 \text{ kN/m}^2$$

(ii) Limiting error $= \dfrac{1}{100} \times 100 = 1 \text{ kN/m}^2$

Possible pressure values are

$$100 \pm 1 = 101 \text{ or } 99 \text{ kN/m}^2$$

The example illustrates that an accuracy specified as a percentage of full scale deflection implies a less accurate instrument than one having the same accuracy specified as percentage of true value.

EXAMPLE 2.5

(a) The accuracy of instrument has been specified as "accurate to within ± *x* for the prescribed or full range of the instrument". How do you interpret it ?

(b) A thermometer is quoted as having the following specification :

Range and subdivision °C	Maximum error
− 0.75 to + 37.5 × 0.1	0.25°C

How will you interpret this catalogue ?

Solution : (a) The statement means that the instrument is accurate to within ± *x* at all points on the scale unless specified otherwise. This implies that irrespective of the indicated value, the error remains the same. For example, a given thermometer may be stated to read within ± 0.5°C between 100°C and 230°C. Likewise a scale of length may be read within ± 0.025 cm.

(*b*) The given specification implies that thermometer can be used for temperature measurement between – 0.75°C and + 37.5°C and has a scale which is subdivided into 0.1°C intervals. Further, the error has a temperature within a region bounded by plus or minus 0.25°C of the indicated value. Thus if meniscus of the mercury-in-glass thermometer were read at 28.5°C, the actual temperature would lie between (28.5 ± 0.25)°C.

EXAMPLE 2.6

The pressure at a remote point has been measured by a system comprising a transmitter, a relay and a receiver element.

Transmitter	:	**within 0.2%**
Relay	:	**within 1.1%**
Receiver	:	**within 0.7%**

Estimate the maximum possible error and the root-square accuracy of the measurement system.

Solution : Maximum possible error

$$= \pm (\alpha_1 + \alpha_2 + \alpha_3)$$

$$= \pm (0.2 + 1.1 + 0.7) = \pm 2\%$$

Least square accuracy or root-sum square error

$$= \pm (\alpha_1^2 + \alpha_2^2 + \alpha_3^2)^{1/2}$$

$$= \pm [0.2^2 + 1.1^2 + 0.7^2]^{1/2} = \mathbf{1.319\%}$$

Thus the error is possible as large as 2% but probably not larger than 1.319%.

EXAMPLE 2.7

Following data is taken while calibrating a bourdon gauge with a dead weight gauge teste

Actual pressure (bar)	5	10	15	20	25	30	25	20	15	10	5
Gauge reading (bar)	4.5	9.6	14.2	18.0	22.5	28.0	26.0	21.0	16.2	11.4	7.0

Draw the calibration, the error and the correction curves. Make suitable comments on th results.

Solution :

(*i*) Plot the given data, taking actual pressure along the abscissa and the correspondin gauge indication along the ordinate. The curves for rising and falling pressure do not overla A difference between the two gauge indications at the same actual pressure is due to hysteres including slack motion in bearings, gears, etc., and friction in the bearings.

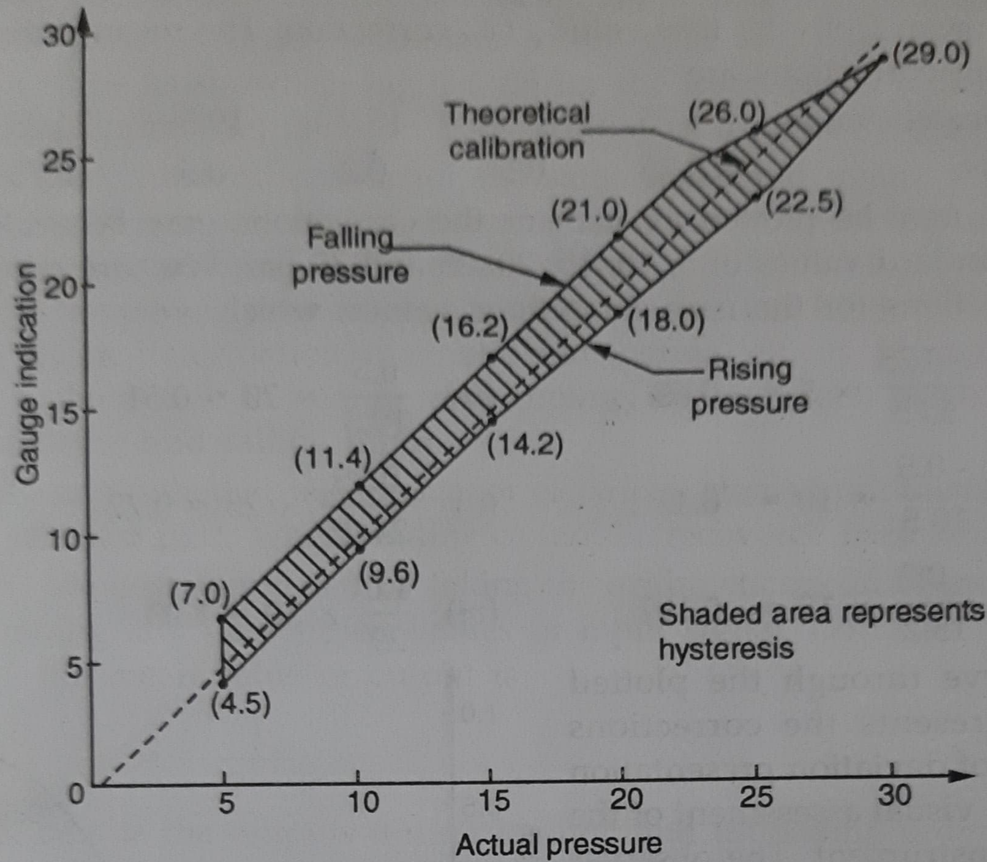


Fig. 2.3. (a) Calibration curve

(ii) Theoretical calibration, shown dotted, refers to a line which is sloped at 45° each to the two ordinates.

(iii) Since, there exists an appreciable dead zone or hysteresis, we take average value of the gauge readings up and down, these average values are :

5.75, 10.5, 15.2, 19.5, 24.25 and 29.0 bar

The corresponding errors are :

$5.75 - 5 = 0.75$; $19.5 - 20 = -0.5$

$10.5 - 10 = 0.5$; $24.25 - 25 = -0.75$

$15.2 - 15 = 0.2$; $29.0 - 30 = -1.0$

The errors are plotted against the actual pressure values and the resulting curve constitutes the error curve.

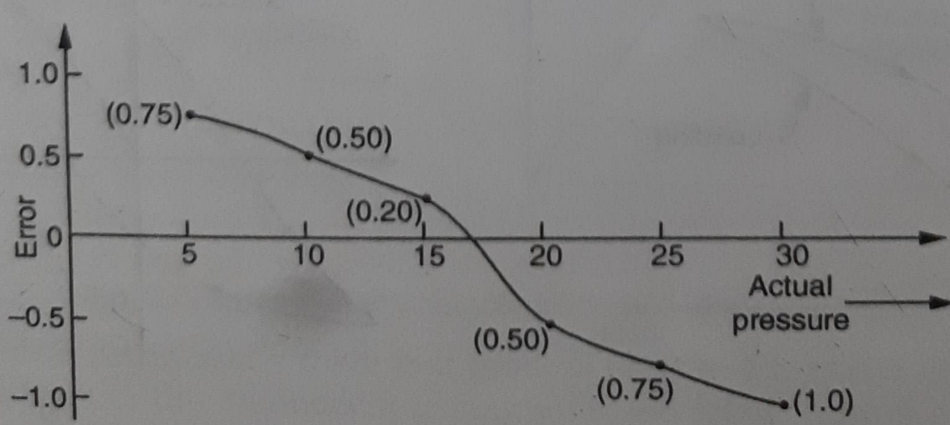


Fig. 2.3. (b) Error curve

(*iv*) Quite often, the indicated values are plotted as abscissa, and the ordinate represents the variation of mean from the true value, *i.e.*, correction. The mean indicated values and the corresponding corrections are :

Mean indicated values	5.75	10.5	15.2	19.5	24.25	29.0
Corrections	– 0.75	– 0.5	– 0.2	+ 0.5	+ 0.75	+ 1.0

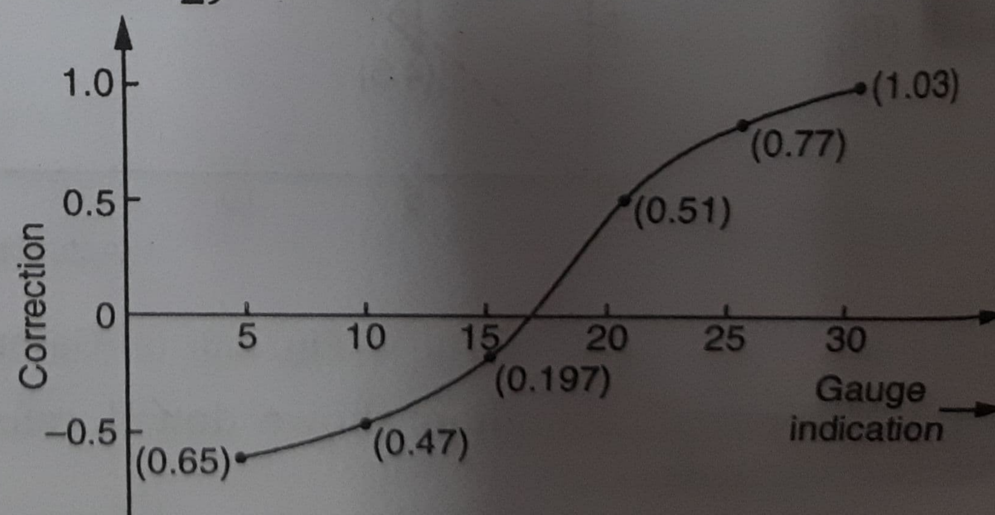
These values may be plotted as such or the corrections may be evaluated for gauge indications of standard values of 5, 10, 15, 20, 25 and 30 bar. We here adopt this approach and make calculations for the new corrections values; which are :

(*i*) $\dfrac{-0.75}{5.75} \times 5 = -0.65$; (*iv*) $\dfrac{0.5}{19.5} \times 20 = 0.51$

(*ii*) $\dfrac{-0.5}{10.5} \times 10 = -0.47$; (*v*) $\dfrac{0.75}{24.25} \times 25 = 0.77$

(*iii*) $\dfrac{-0.2}{15.2} \times 15 = -0.197$; (*vi*) $\dfrac{1.0}{29} \times 30 = 1.03$

A faired curve through the plotted points then represents the corrections curve. This type of deviation presentation facilitates a rapid visual assessment of the accuracy of the instrument. The observer reads the gauge, then refers to the curve, and finds a correction to add to that reading in order to make it more nearly correct.



EXAMPLE 2.8

A spring scale requires a change of 150 N in the applied weight to produce a 2 cm change in the deflection of the spring scale. Determine the static sensitivity,

Solution:

$$k = \frac{\text{change of output signal}}{\text{change of input signal}} = \frac{2}{150} = 0.0133 \text{ cm/N}$$

EXAMPLE 2.9

Explain the following statements :

(i) A galvanometer has sensitivity specified as 15 mm/μA

(ii) An automatic balance has a quoted sensitivity of 1 vernier division/0.1 mg

Solution :

(i) This means that for 1 mA input the display (which is the light spot moving across a scale) shows a movement of an index of 15 mm.

(ii) This means that the index moves through one division when the mass changes by 0.1 mg.

EXAMPLE 2.10

Workout sensitivity in respect of the followings :

(i) A dial indicator gives one complete revolution of pointer on a 25 mm radius scale for 1 mm displacement of plunger.

(ii) The scale of a pressure gauge with range 0-150 bar extends over an arc of 60 mm radius for 240-degree.

Solution :

$$\text{Sensitivity} = \frac{\text{change in output of instrument}}{\text{change in quantity being measured}}$$

$$= \frac{\text{circumference of scale}}{\text{plunger movement}}$$

$$= \frac{2\pi \times 25}{1} = 157 \text{ (dimensionless)}$$

This implies that the pointer moves 157 mm for every 1 mm movement of plunger.

(ii)

$$\text{Sensitivity} = \frac{\text{change in output of instrument}}{\text{change in quantity being measured}}$$

$$= \frac{\text{length of scale arc}}{\text{range of scale}} = \frac{2\pi \times 60 \times 240/360}{150} = 1.675 \ \frac{\text{mm}}{\text{bar}}$$

EXAMPLE 2.11

A measuring system consists of a transducer, an amplifier and a recorder, and their individual sensitivities are stated as follows :

Transducer sensitivity k_1 = 0.25 mV/°C

Amplifier gain k_2 = 2.5 V/mV

Recorder sensitivity k_3 = 4 mm/V

What would be the overall sensitivity of the measuring system ?

Solution : The overall sensitivity of a measuring system, having a number of elements arranged in series, is prescribed by the relation :

$$k = k_1 \, k_2 \, k_3$$

$$= 0.25 \ \frac{\text{mV}}{°\text{C}} \times 2.5 \ \frac{\text{V}}{\text{mV}} \times 4 \ \frac{\text{mm}}{\text{V}} = 25 \text{ mm/°C}$$

EXAMPLE 2.12

A pressure measuring system consists of a piezoelectric transducer, a charge amplifier and a ultra violet charge recorder. The sensitivities of these elements are stated as follows

Piezoelectric transducer k_1 = 8.5 pC/bar

Charge amplifier k_2 = 0.004 V/pc

Ultraviolet charge recorder k_3 = 20 mm/V

What would be the deflection on the chart due to a pressure change of 30 bar ?

Solution : The overall sensitivity of the measuring system with its elements arranged in series is given by :

$$k = k_1\, k_2\, k_3$$

$$= 8.5\ \frac{\text{pC}}{\text{bar}} \times 0.004\ \frac{\text{V}}{\text{pC}} \times 20\ \frac{\text{mm}}{\text{V}} = 0.68\ \frac{\text{mm}}{\text{bar}}$$

Again, $\quad$ sensitivity $= \dfrac{\text{change of output signal}}{\text{change of input signal}}\ ;\ 0.68\ \dfrac{\text{mm}}{\text{bar}} = \dfrac{\text{output}}{30\ \text{bar}}$

$\therefore$ Deflection on the chart $= 0.68\ \dfrac{\text{mm}}{\text{bar}} \times 30\ \text{bar} = \mathbf{20.4\ mm}$

[redacted]

EXAMPLE 2.13

How resolution is reckoned for the analog and digital read out devices ?

A force transducer measures a range of 0-150 N with a resolution of 0.1 percent of full scale. Find the smallest change which can be measured.

Solution : $\quad$ Resolution = 0.1% of full scale value

$$= 0.1/100 \times 150 = 0.15\ \text{N}$$

and this represents the smallest measurable change in force.

EXAMPLE 2.14

Distinguish between threshold and resolution (or discrimination).

The pointer scale of a thermometer has 100 uniform divisions, full scale reading is 200°C and 1/10th of a scale division can be estimated with a fair degree of accuracy. Determine the resolution of the instrument.

Solution : 1 scale division $= 200/100 = 2°C$

Resolution $= 1/10$th of scale division

$= 1/10 \times 2 = 0.2°C$

EXAMPLE 2.15

(a) All the watches and clocks in a watch shop are set at 8.30 but are not working. What you have to say about the accuracy and precision of the indicated readings ?

(b) "Repeatability and reproducibility are synonymous terms" Comment.

Solution : (a) The indicated readings remain at 8.30 and thus have a high degree of precision. However, the clocks are not accurate as they show the same readings irrespective of any time. The indicated readings will be accurate only twice a day when the actual time is 8.30 AM or 8.30 PM.

(b) Though both the terms refer to precision, the closeness of repeated measurements is under different sets of conditions. Repeatability is the closeness between successive measurements of the same input signal (measurand) with the same instrument/method and apparatus, and by the same operator over a short time span (same conditions of instrument use). Reproducibility is the closeness between successive measurements of the same measured quantity where individual measurements are made by different operators, at different locations, with different measuring instruments, over longer time periods and under different conditions of instrument use.

The measurement of 10 mm diameter of ring by an operator A at site B with micrometer C as (10 ± 0.005) mm would indicate repeatability. Consider the same 10 mm ring being manufactured by different workers employed in different factories. Naturally the size of the ring would be measured by different persons and with different instruments. If the size of the rings can be controlled within specified limits, then the rings would be fully interchangeable and all such measurements would be termed reproducible.

EXAMPLE 2.16

The following specifications for a particular linear displacement transducer have been noted from the manufacturer's catalogue :

Range	0-60 mm
Input voltage	10 V
Full scale voltage	9.9 V
Output voltage at zero displacement	0.1 V
Accuracy	Better than 0.1% of full scale output
Repeatability	Better than 0.01% of full scale output
Maximum sensitivity	0.25 V/mm
Resolution	0.1% of full scale displacement
Linearity	Better than 0.15% of full scale output at 80% of displacement with a minimum load of 120 kΩ.

Explain the meaning of each term in this specification in relation to the quoted displacement transducer.

Solution : For proper understanding of the meaning of different terms s given below, reference may be made to the description of linear displacement transducer (Article 4.15.2, chapter 4).

(i) *Range* : The region between the limits within which instrument is designed to operate for measuring, indicating or recording a physical quantity is called the range of instrument. The particular displacement transducer is capable of measuring linear displacements from zero to a maximum of 60 mm. This also implies that travel of the slider from the fully retracted position to the fully extended position is 600 mm. With reference to Fig. 14.34, distance x_m = 60 mm.

(ii) *Input voltage/full scale voltage/output voltage at zero displacement* : An input voltage V = 10 V is recommended to be applied to the input terminals of the transducer. The full scale voltage of 9.9 V refers to the voltage at output terminal (V_0) when the slider has traversed a full length of 60 mm. Likewise, at zero displacement, the voltage at output terminals is stated to be 0.1 V. Ideally, the full scale voltage must be equal to the input supply voltage and the output voltage at zero displacement must be zero. The discrepancies may be attributed to the losses which are incurred at each of the two extremities, and that the sliders of potentiometric transducers are not able to traverse completely the full extent of the resistor.

(*iii*) *Accuracy* : Accuracy of an indicated (measured) value is defined as its conformity with or closeness to the accepted standard value (true value). The accuracy of the transducer has been stated as 0.1 percent of full scale output and accordingly the output voltage reading will be within

$$\pm \frac{0.1}{100} \times 9.9, \text{ that is } 9.9 \times 10^{-3} \text{ V or } 9.9 \text{ mV}$$

(*iv*) *Maximum sensitivity* : Sensitivity of an instrumentation system is the ratio of the magnitude of response (output signal) to the magnitude of quantity being measured (input signal).

$$\text{Sensitivity} = \frac{\text{change of output signal}}{\text{change of input signal}}$$

A sensitivity of 0.25 V/mm implies that for every mm displacement of the slider the output voltage will change by 0.25 V.

(*v*) *Resolution* : When the input signal is increased from non-zero value, one observes that the instrument output does not change until a certain input increment is exceeded. This increment is termed resolution or discrimination. Thus resolution defines the smallest change of input for which there will be a change of output. The smallest displacement that can be detected by the given potentiometric transducer is

0.1% of 60 mm, that is, 0.006 mm

(*vi*) *Linearity* : The working range of most of the instruments provides a linear relationship between the output (reading taken from the scale of instrument) and input (measurand *i.e.*, signal presented to the measuring system). The closeness of calibration curve to a specified straight line relationship is the linearity of the instrument. The maximum deviation from linearity for this particular potentiometric transducer is

$$\frac{0.15}{100} \times 9.9, \text{ that is, } 14.85 \times 10^{-3} \text{ V or } 14.85 \text{ mV}$$

This maximum deviation occurs when the slider moves to 80 percent of 60 mm, which is a displacement of 48 mm. To achieve this linearity, the output voltage measuring instrument must have an input resistance of atleast 120 kΩ.

EXAMPLE 2.17

A voltmeter with internal resistance 150 kΩ is connected across an unknown resistance which has a milliammeter of very small internal resistance connected in series with it. Make calculations for the loading error when (*i*) the voltmeter and the milliammeter read 200 V and 8 mA respectively. (*ii*) the same voltmeter and milliammeter with another resistance read 75 V an 1.5 A respectively. Comment on the results.

Solution : Total resistance of the circuit $R_i = \dfrac{V_t}{I_t}$

$$= \frac{200}{8 \times 10^{-3}} = 25 \times 10^3\ \Omega = 25\ \text{k}\Omega$$

Since the milliammeter is stated to have negligible resistance, the total resistance as calculated above equals the apparent resistance of the resistor

$$R_{app} = 25\ \text{k}\Omega$$

The actual value of resistance of the resistor is obtained from the condition that voltmeter is connected in parallel with the resistor. That is

$$\frac{1}{R_t} = \frac{1}{R_{act}} + \frac{1}{R_v}$$

or
$$R_{act} = \frac{R_t R_v}{R_v - R_t} = \frac{25 \times 150}{150 - 25} = 30\ \text{k}\Omega$$

$$\text{Percentage loading error} = \frac{R_{act} - R_{app}}{R_{act}} \times 100 = \frac{30 - 25}{30} \times 100 = \mathbf{16.67\ \%}$$

(*ii*) Total resistance $R_t = \dfrac{75}{1.5} = 50\ \Omega = 0.05\ \text{k}\Omega$

$$R_{app} = 0.05\ \text{k}\Omega \quad \text{and} \quad R_v = 150\ \text{k}\Omega$$

$$R_{act} = \frac{R_t R_v}{R_v - R_t} = \frac{0.05 \times 150}{150 - 0.05} = 0.050016\ \text{k}\Omega$$

$$\rightarrow \text{Percentage loading error} = \frac{0.050016 - 0.05}{0.050016} \times 100 = \mathbf{0.032\ \%}$$

Apparently we may conclude that :

- a voltmeter with high internal resistance when connected across two points in a high resistance circuit has an appreciable loading effect and therefore gives a misleading voltage reading.

- the same voltmeter has negligible loading error when connected across a low resistance circuit and thus gives more reliable voltage reading.

EXAMPLE 2.18

When a step input of 100 bar is applied to a pressure gauge, the pointer swings to pressure of 102.5 bar and finally comes to rest at 101.3 bar. Determine the overshoot of the gauge reading and express it as a percentage of the final reading. Also calculate the percentage error of the gauge.

Solution : Overshoot = 102.5 – 101.3 = 1.2 bar

$$\text{Percentage overshoot} = \frac{1.2}{101.3} \times 100 = \mathbf{1.18\%}$$

$$\text{Percentage error} = \frac{101.3 - 100}{101.3} \times 100 = \mathbf{1.3\%}$$

EXAMPLE 3.1.

Two students re confronted with the problem of determining the speed of a car by timing its passage over a distance. They are provided with a pistol, stop watch and a measuring tape. Explain how can they proceed to achieve the objective. Present your analysis for the possible sources of various errors.

Solution: The students can mark off a distance on the road and can stand at the two opposite ends. The first observer will fire the pistol at the instant car passes him. The second observer standing at a known distance at the other end would start the stop watch at the instant he receives the firing signal and will stop it at the instant car passes him. Knowing the time interval, speed of the car can be determined by the relation :

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

The uncertainties in the above measurement may arise due to the following :

(1) **Blunders and chaotic errors**

(*i*) Wrong reading of the tape when noing the distance between the marks.

(*ii*) Wrong recording of the distance noted.

(*iii*) Starting the watch beofre or after the given signal.

(*iv*) Stopping the watch a little before or after the car crosses the second mark.

(*v*) Improper signalling.

These errors can be minimized by repeating the measurement.

(2) **Systematic errors**

(*i*) Slow or fast running of the stop watch

(*ii*) Measuring tape may have a certain portion cut off from the measuring end.

(*iii*) Parallax effect in reading the tape and the stop watch.

These errors can be minimized by a careful checking of the apparatus.

(3) ***Random errors***

(*i*) Irregularities on the road and wrong placing of the tape along the road.

(*ii*) Changes in the reaction time of the stop-watch holder during the act of measurement.

EXAMPLE 3.4.

Distinguish between systematic and random errors and give examples of each kind of error.

(b) A spring balance, graduated 0-50 N over a scale length of 100 mm, was calibrated using laboratory masses stated by their manufacturer to be within ± 0.1% of their marked values. The following observations were made :

mass (kg)	0.0	1.0	2.0	3.0	4.0	5.0
force reading (N)	0.0	9.5	19.5	30.0	39.5	49.0

What adjustment needs to be done so that the gauge gives all errros withina band of ± 0.5 N from the standard value ? Corresponding to that, determine the accuracy as percentage of full scale deflection of the balance. Also workout the sensitivity of the balance and spring constant.

Solution : *Systematic errors* are repeated consistently with the repetition of the experiment, and have same magnitude and sign for a given set of conditions. They alter the instrument reading by fixed mgnitude and with same sign from one reading to another. Because of the same algebraic sign, systematic errors tend to accumulate and hence are often called cummulative errors. Instrument bias is another term for systematic errors. These errors are caused by such effects as sensitivity shifts, zero offset and known non-linearity. Systematic errors cannot be determined by direct and repetitive observations of the measurand made each time with same technique. The only way to locate these errors is to have repeated measurements under different methods. Some factors leading to systematic errors are :

(*i*) pointer offset

(*ii*) change in ambient temperature

(*iii*) poor design and construction of instrument

(*iv*) buoyant effect of the wind and the weights of a chemical balance

(*v*) inequality of the arms of a beam balance

(*vi*) change in the original state of the system due to interaction between the instrument and the system.

Random errors are accidental, small and independent, and are mainly due to inconsistent factors such as spring hysteresis, stickness, friction, noise and threshold limitations. Since these errors vary both in magnitude and sign (are positive or negative on the basis of chance alone), they tend to compensate one another and are referred to as chance/accidental/compensating errors. The random errors are detected by lack of consistency in the measured value when the same input is imposed repeatedly on the instrument (measured values are not precise and show a considerable scatter). The magnitude and direction of random errors cannot be predicted from a knowledge of the measurement system; howeveer, these errors are assumed to follow the law of probabilities. Some factors leading to random errors are :

(*i*) stickness and friction
(*ii*) line voltage fluctuations
(*iii*) vibration of instrument supports
(*iv*) large dimensional tolerances between the mating parts
(*v*) spring hysteresis and elastic deformation
(*vi*) incosistencies associated with accurate measurement of small quantities

(*b*) The magnitude of force applied can be calucalated from the mass readings by assuming a standard value of g as 9.80 m/s.

Mass (kg)	0.00	1.00	2.00	3.00	4.00	5.00
Applied force (N)	0.00	9.80	19.60	29.40	39.20	49.00
Force reading (N)	0.00	9.50	19.50	30.00	39.50	49.00
Error ($V_m - V_s$)	0.00	– 0.3	– 0.1	+ 0.6	+ 0.3	0.00

The maximum positive error is 0.6 N, and the maximum negative error is 0.3 N. If the no load reading is increased to 0.1 N, then all errors would lie within a band of ± 0.5 N from the standard value.

Then

Accuracy as % full scale deflection

$$= \pm \frac{0.5}{50} \times 100 = \pm 1\%$$

Evidently the standard against which the balance is calibrated is 10 times as accuracte as the balance.

$$\text{Sensitivity} = \frac{\text{change of output signal}}{\text{change of input signal}}$$

$$= \frac{100 \text{ mm}}{50 \text{ N}} = 2 \text{ mm/N}$$

The spring constant represents the force needed to extend the spring by unit length

$$= \frac{-0.077}{1.226} \times 100 = -6.28\%$$

EXAMPLE 3.5.

To measure the stiffness of a cantilever beam, a gravity force equivalent to 1 kg mass was applied at the free end of the beam and the resulting end deflection was noted by a dial gauge. The plunger of the dial gauge exerts a force $F_d = 0.12\,R + 0.25$ newton where R is the gauge deflection in mm. When the beam is in the initial horizontal position before the application of load, the gauge deflection is 2 mm. Calculate the percentage error in the measurement of the deflection of the free end of the cantilever due to the influence of the dial gauge.

The beam has a length of 250 mm and a rectangular cross-section 20 mm wide and 5 mm deep. The modulus of elasticity of the beam material is 200 GPa.

Solution: The deflection at the free end of cantilever is

$$y = \frac{Wl^3}{3EI}\ ; \quad I = \frac{bd^3}{12}$$

$\therefore$ Deflection per unit force $\left(\frac{W}{y}\right) = \frac{4l^3}{Ebd^3}$

$$= \frac{4 \times (0.25)^3}{200 \times 10^9 \times 0.02 \times (0.005)^3} = 125000 \times 10^{-9} \text{ m/N}$$

$$= 0.125 \text{ mm/N}$$

When spring force due to plunger of the dial gauge is neglected then force on the cantilever is

$$F = F_g = \text{gravitational force due to mass of 1 kg}$$

$$= 1 \times 9.81 = 9.81 \text{ N}$$

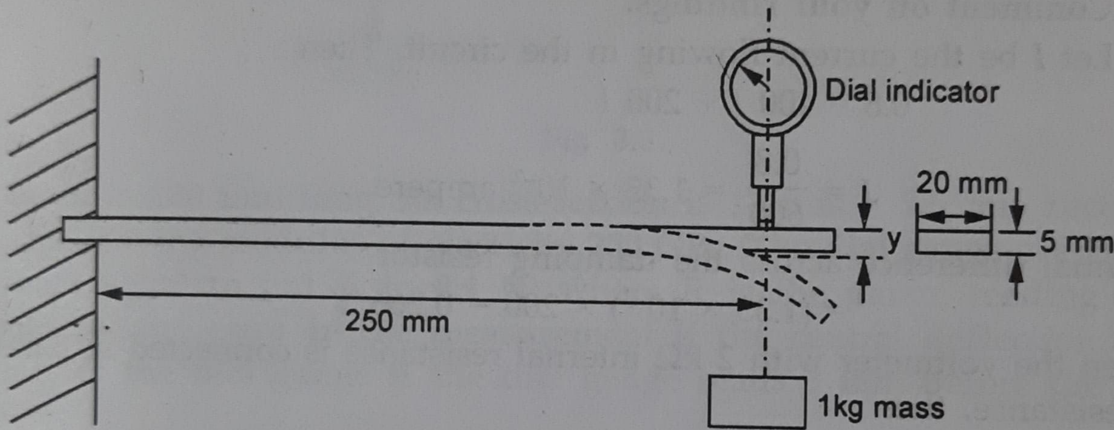


Fig. 3.6.

$\therefore$ True deflection of cantilever

$$= \text{deflection per unit force} \times \text{force}$$

$$= 0.125 \times 9.81 = 1.226 \text{ m}$$

Allowing for spring force,

$$F = F_g + F_d$$

$$= 9.81 + [0.12\,(2 - y_1) + 0.25] = 10.3 - 0.12\,y_1$$

where y_1 is the deflection indicated by the gauge

$\therefore$ Indicated deflection

$$y_1 = 0.125 \times (10.3 - 0.12\,y_1) = 1.2875 - 0.12\,y_1$$

$$y_1 = \frac{1.2875}{1 + 0.12} = 1.149 \text{ mm}$$

Error in measurement = measured value – true value

$$= 1.149 - 1.226 = -0.077 \text{ mm}$$

$$\text{Percentage error} = \frac{-0.077}{1.226} \times 100 = -6.28\%$$

EXAMPLE 3.6. Refer figure given below which shows the matching circuit for a U/V recorder.

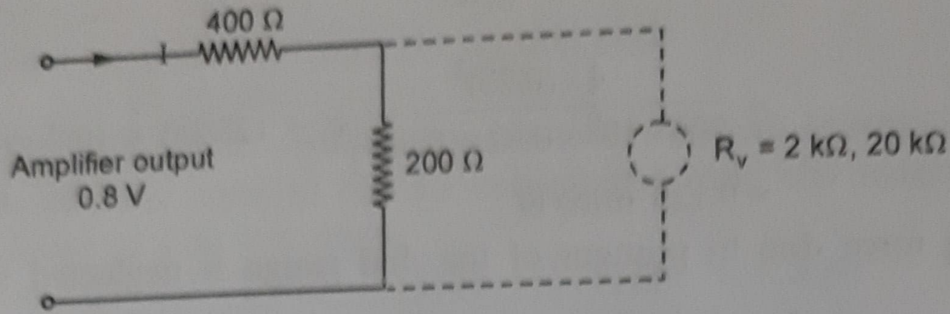


Fig. 3.7.

What will be the interference error in the measurement of potential difference across the 200 W damping register using a voltmeter having an input resistance of (*a*) 2 kΩ and (*b*) 20 kΩ. Comment on your findings.

Solution : Let I be the current flowing in the circuit. Then

$$0.8 = 400\, I + 200\, I$$

or $$I = \frac{0.8}{600} = 1.33 \times 10^{-3} \text{ ampere}$$

∴ Potential difference across the damping resistor

$$= (1.33 \times 10^{-3}) \times 200 = 0.266 \text{ V}$$

(*a*) When the voltmeter with 2 kΩ internal resistance is connected in parallel with the damping resistance, then

$$R_{eq} = \frac{R_d \times R_v}{R_d + R_v} = \frac{200 \times 2000}{200 + 2000} = 181.82\ \Omega$$

Then, the current flowing through the circuit is

$$I = \frac{0.8}{400 + 181.82} = 1.375 \times 10^{-3} \text{ amp}$$

As such the potential difference across the damping resistor is

$$= (1.375 \times 10^{-3}) \times 181.82 = 0.250 \text{ V}$$

$$\text{Percentage error} = \frac{0.250 - 0.266}{0.266} \times 100 = -\ \mathbf{6.015\%}$$

(*b*) When the voltmeter with 20 kW internal resistance is connected in parallel with damping resistance, then

$$R_{eq} = \frac{R_d \times R_v}{R_d + R_v} = \frac{200 \times 20000}{200 + 20000} = 198.02\ \Omega$$

Then the current flowing through the circuit is,

$$I = \frac{0.8}{400 + 198.02} = 1.338 \times 10^{-3} \text{ amp}$$

The corresponding potential difference across the damping resistor is

$$= (1.338 \times 10^{-3}) \times 198.02 = 0.265 \text{ V}$$

$$\text{Percentage error} = \frac{0.265 - 0.266}{0.266} \times 100 = -\mathbf{0.376\%}.$$

Comment : The calculations made above indicate that the interference is reduced if th input resistance of the voltmeter is significantly greater than the damping resistance.

EXAMPLE 3.7.

Refer the figure given below which shows a dial gauge being used to measure the centr deflection of a simply supported beam subjected to a point load of 0.5 kg.

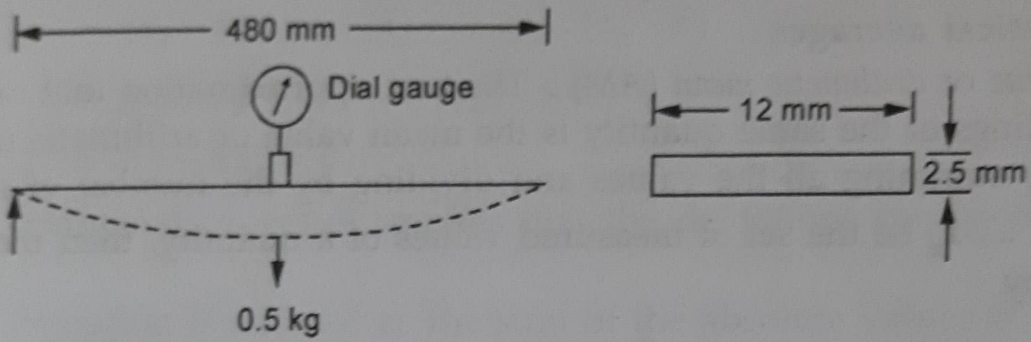


Fig. 3.8.

The beam is 400 mm long, its cross-section is 12 mm × 2.5 mm rectangular and i made of steel having modulus of elasticity 200 GPa. The dial gauge exerts a spring forc on the beam equal to (0.1 R to 0.4 N) where *R* is the gauge reading in mm. Mak calculations for the error in the measurement if the central deflection caused by th interference of the dial gauge if the dial gauge reads 3 mm before the application o load.

Solution : The central deflection of a simply supported beam subjected to a central poin load is given by

$$\delta_t = \frac{Wl^3}{48EI} = \frac{(0.5 \times 9.807)}{48 \times (200 \times 10^9/10^6) \times (12 \times 2.5^3/12)} = 2.092 \text{ mm}$$

This is the theoretical deflection when the dial gauge force is neglected.

When the beam is deflected due to applied mass, the force exerted by the dial gauge is

$$F = [0.1(3 - \delta_a) - 0.4] = 0.7 - 0.1\,\delta_a$$

where δ_a is the actual deflection

The total force applied to the centre of beam is

$$= (0.5 \times 9.807) + (0.7 - 0.1\,\delta_a) = 5.603 - 0.1\,\delta_a$$

$$\therefore \quad \delta_a = (5.603 - 0.1\delta) \times \frac{400^3}{48 \times (200 \times 10^9/10^6) \times (12 \times 2.5^3/12)}$$

$$= (5.603 - 0.1\delta) \times 0.427 = 2.392 - 0.0427\,\delta_a$$

That gives : $\delta_a = \dfrac{2.392}{1.0427} = 2.294$ mm

$\therefore$ Interference error $= 2.294 - 2.092 =$ **0.202 mm**

EXAMPLE 3.8.

The following readings are taken of a certain physical length with the help of a micrometer screw:

1.41, 1.45, 1.53, 1.54, 1.49, 1.51, 1.60, 1.55, 1.47, 1.65, 1.65 mm

Asuming that only random errors are present, calculate the arithmetic mean, the average deviation, standard deviation, variance and the probable error of the reading.

Solution: The statistical values of the given data are computed with the aid of the following table:

Observation number	Length x, mm	Deviation $\delta = x - \bar{x}$	δ^2
1.	1.41	– 0.121	0.0146
2.	1.45	– 0.081	0.00656
3.	1.63	0.099	0.90980
4.	1.55	0.009	0.000081
5.	1.49	– 0.041	0.00168
6.	1.51	– 0.021	0.000441
7.	1.60	0.069	0.00476
8.	1.55	0.019	0.000361
9.	1.47	– 0.061	0.00372
10.	1.66	0.129	0.01664
	$\Sigma x = 15.31$		
$n = 10$	$\bar{x} = \frac{\Sigma x}{n} = \frac{15.31}{10} = 1.531$	$\Sigma\lvert\delta\rvert = 0.65$	$\Sigma\delta^2 = 0.0586$

(*i*) Arithmetic mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{15.31}{10} = 1.531$ mm

(*ii*) Average deviation $D = \frac{\Sigma|\delta|}{n} = \frac{0.65}{10} = 0.65$ mm

(*iii*) Since the number of readings is 10 and it is less than 20, the standard deviation is calculated by using the relation.

$$\sigma = \sqrt{\frac{\Sigma\delta^2}{n-1}} = \sqrt{\frac{0.0586}{10-1}} = 0.0807 \text{ mm}$$

(*iv*) Variance $= \sigma^2 = (0.0807)^2 = 0.00651$ mm^2

(*v*) Probable error $= \pm 0.6745\,\sigma$

$$= \pm 0.6745 \times 0.0807 = \pm 0.0544 \text{ mm}$$

EXAMPLE 3.9.

During a test run, measurements of temperature were made 100 times with variations in apparatus and procedures. After applying corrections for the known systematic errors, the following data were recorded:

Temperature °C	**97**	**98**	**99**	**100**	**101**	**102**	**103**	**104**	**105**
Frequency of occurrence	**2**	**5**	**9**	**19**	**33**	**18**	**8**	**4**	**2**

Make calculations for the arithmetic mean, the average deviation, standard deviation, variance and the probable error of the reading.

Solution: The statictical values of the given data are computed in a tubular form as indicated below:

Temperature °C T	Frequency of occurence f	$T \times f$	Deviation $\delta = x - \bar{x}$	$f \times \delta$	δ^2	$f \times \delta^2$
97	2	194	− 3.94	− 7.88	15.52	31.04
98	5	490	− 2.94	− 14.70	8.64	43.2
99	9	891	− 1.94	− 17.46	3.76	33.84
100	19	1900	− 0.94	− 17.86	0.88	16.72
101	33	3333	+ 0.06	+ 1.98	0.0036	0.1188
102	18	1836	+ 1.06	+ 19.08	1.12	20.16
103	8	824	+ 2.06	+ 16.48	4.24	33.92
104	4	416	+ 3.06	+ 12.24	9.36	37.44
105	2	210	+ 4.06	+ 8.12	16.48	32.96
	$n = 100$ $\bar{x} = \frac{194}{100}$ $= 100.94$	10094		$\Sigma(f \times \delta)$ $= 115.8$	$\Sigma(f \times \delta^2)$ $= 249.39$	

(*i*) Arithmetic mean $= \dfrac{\Sigma T \times f}{n} = \dfrac{10094}{100} = 100.94°C$

(*ii*) Average deviation $= \dfrac{\Sigma(f \times \delta)}{n} = \dfrac{1158}{100} = 1.158°C$

(*iii*) Standard deviation $\sigma = \left[\dfrac{\Sigma(f \times \delta^2)}{n}\right]^{1/2} = \left[\dfrac{249.39}{100}\right]^{1/2} = 1.579°C$

(*iv*) Variance $= \sigma^2 = (1.579)^2 = 2.493°C^2$

(*v*) Probable error $= \pm 0.6745\,\sigma = \pm 0.6745 \times 1.579 = 1.065°C$

MPLE 3.10.

oltmeter is used to measure known voltage of 75 volts. Forty percent of the readings within 0.8 volt of true vlaue. Estimate the standard deviation for the meter and the bability of an error of 1.2 volt.

ution: Considering a normal distribution curve, 20% of the readings have a positive or and 20% have negative error.

From probability tables, it is found that corresponding to a probability of $P(z) = 0.2$,

$$z = 0.5025 \quad \text{or} \quad \frac{x - \bar{x}}{\sigma} = 0.05025$$

$$\therefore \text{ Standard deviation } \sigma = \frac{x - \bar{x}}{0.5025} = 1.592$$

$$\text{(ii) Now, } z = \frac{x - \bar{x}}{\sigma} = \frac{1.2}{1.592} = 0.753$$

Corresponding to $z = 0.753$, the probability as read from the probability tables is

$$P(z) = 0.2743$$

$\therefore$ Probability of an error of $\pm$ 1.2 volt $= 2 \times 0.2743 = 0.5486$

Thus about 55% of the readings are within 1.2 volt of true value.

XAMPLE 3.11.

tachometer has been used to measure the speed of hydraulic turbine model that is eing run at 1000 rpm. For a sample of 20 readings made at this speed, how many of he readings you would expect to fall between 990 and 1010 rpm. Presume that the achometer gives a normal distribution set of deviation with precision index 0.04 at 000 rpm.

olution: Deviation $\delta = x - \bar{x} = \pm 10$ rpm

$$\text{Standard deviation } \sigma = \frac{1}{\sqrt{2}h} = \frac{1}{\sqrt{2} \times 0.04} = 17.68$$

$$\therefore \quad z = \frac{x - \bar{x}}{\sigma} = \frac{10}{17.68} = 0.5656$$

Corresponding to $z = 0.5656$, the probability as read from the probability tables is

$$P(z) = 0.214$$

$\therefore$ Probability of an error of $\pm$ 10 rpm $= 2 \times 0.214 = 0.428$

Thus we expect 42.8% or $4.428 \times 20 = 9$ readings to lie between 990 and 1010 rpm.

EXAMPLE 3.12.

A hook was used to measure the depth of water which had a nominal value of 12 cm. Measurements were taken 28 times and 7 readings were found to lie outside a particular range. Presuming that the measurements conform to normal distribution with precision index 10 cm^{-1}, make calculations for the prescribed range.

Solution: Probability of falling within a prticular range,

$$= \frac{21}{28} = 0.75$$

Half of these measurements have a positive error and half have negative error

$$\therefore \quad P(z) = 0.375$$

From probability tables, it is found that corresponding to a probability $P(z) = 0.375$

$$z = 1.15 \quad \text{or} \quad \frac{x - \bar{x}}{\sigma} = 1.15$$

Now, standard deviation

$$\sigma = \frac{1}{\sqrt{2}h} = \frac{1}{\sqrt{2} \times 10} = 0.0709$$

$$\therefore \quad x - \bar{x} = 1.15 \times 0.0709 = 0.0815 \text{ cm}$$

Thus 75% of depth measurements lie within the range (12 ± 0.0815) cm

EXAMPLE 3.13.

The mean weight of 1000 bearings is 500 gm. The standard deviation is 50.

(*i*) How many bearings are expected to weigh between 400 gm and 575 gm?

(*ii*) How many bearings would weigh over 625 gm?

Solution: (*i*) We first compute the value of z

$$z_1 = \frac{400 - 500}{50} = -2\,; \quad z_2 = \frac{575 - 500}{50} = 1.5$$

From probability tables, it is found that corresponding to $z = 2$ the area is 0.4773. This implies that the probability that a bearings weighs between 400 and 500 gm is 0.4773. Similarly, the probabilities for $z = 1.5$ is 0.4332. Thus we expect that 43.32% of all the bearing would weigh between 500 and 575 gm. Taking two together, we expect $0.4773 + 0.4332 = 0.9105$ *i.e.*, $9105 \times 1000 = 910$ bearings to weigh within the specified range of 400 and 575 gm.

(*iii*) Here we are interested in the area between

$$z = \frac{625 - 500}{50} = 2.5 \text{ and } z = \infty$$

To determine this, we subtract the area between $z = 0$ and $z = 0.25$ from 0.50 which is the area betwen $z = 0$ and $z = \infty$. The requisite area is $0.50 - 0.4938 = 0.0062$. Thus in 1000 bearings, we expect 6 or 7 to weigh more than 625 gm.

EXAMPLE 4.1

The spring of a transducer deflects 0.04 mm when subjected to a force of 8 kN. Find the spring sensitivity and also calculate the input force an for an output displacement of 0.06 m.

Solution : Transducer sensitivity k

$$= \frac{\text{output signal increment}}{\text{input signal increment}} = \frac{0.04 \text{ m}}{8 \text{ kN}} = 0.005 \text{ m/kN}$$

Input force required for 0.06 m displacement,

$$= \frac{0.06}{0.005} = 12 \text{ kN}$$

EXAMPLE 4.2.

An amplifies with amplification factor of 600 has beer used to connect the output of a linear variable differential transformer to a 5V voltameter. When the core moves through a distance of 0.75 mm, an output of 2.25 mV is noted across the terminals of the transformer. The millivoltmeter scale has 100 divisions and it can be read to 1/4 of a division. Make calculation for the sensitivity of the transformer and resolution of the instrument.

Solution. (*a*) The sensitivity of the transformer is given by

$$= \frac{\text{change in output voltage}}{\text{displacsment}} = \frac{2.25}{0.75} = 3 \text{ mV/mm}$$

(*b*) The sensitivity of the instrument would be

$$= \text{amplification factor} \times \text{sensitivity of transformer}$$

$$= 600 \times 3 = 1800 \text{ mV/mm}$$

Now :

$$1 \text{ scale division} = \frac{5 \times 10^3}{100} = 50 \text{ mV}$$

and as such the minium vortage that can be read by the voltmeter

$$= \frac{1}{4} \times 50 = 12.5 \text{ mV}$$

∴ Resolution of the instrument

$$= 12.5 \times \frac{1}{1800} = 0.00694 \text{ mm}$$

EXAMPLE 4.3

A 4 cm long linear resistance potentiometer is uniformly wound with a wire having a resistance 8000 ohm. Under normal conditions, the slider is positioned at the centre of the potentiometer. During operation, the slider moves over the resistance element, and resistance of the potentiometer as measured by a wheat stone bridge is (*i*) 3200 ohm and (*ii*) 6000 ohm.

(*a*) Find the linear displacement and comment on the direction of the two displacements.

(*b*) Find the resolution of the potentiometer if it is feasible to measure a minimum value of 15 Ω resistance with this arrangement.

Solution : (a) At normal position, the slider is at the centre of the potentiometer and accordingly its resistance is equal to 8000/2 = 4000 ohm.

$$\text{Resistance of wire per cm length} = \frac{8000}{4} = 2000 \text{ ohm}$$

(*i*) Change in resistance of potentiometer from its normal position

$$= 4000 - 3200 = 800 \text{ ohm}$$

$$\therefore \quad \text{Displacement} = \frac{8000}{2000} = 0.4 \text{ cm}$$

(*ii*) Change in resistance of potentiometer from its normal position

$$= 6000 - 4000 = 2000 \text{ ohm}$$

$$\therefore \quad \text{Displacement} = \frac{2000}{2000} = 1 \text{ cm}$$

One of the displacements represents a decrease and the other represents an increase in resistance of the potentiometer when compared with the potentiometer resistance under normal conditions. This difference points out that the two displacements are in opposite direction.

$$(b)\ \text{Resolution} = \frac{\text{minimum measurable resistance}}{\text{resistance of potentiometer per unit length}}$$

$$= \frac{15}{2000} = 0.0075\ \text{cm}$$

EXAMPLE 4.4

A transducer is connected to an ammeter which consists of a coil 16 mm square having 160 turns. The coil can rotate about the vertical axis and the suspension stiffness is 2×10^{-3} Nm per degree of rotation. The vertical sides of the coil move in a magnetic field of flux density 0.14 T, the field beings deviated radially relative to the axis of rotation. Calculate the sensitivity of the ammeter in degrees/ampere.

Solution : When a conductor is placed in a magnetic field with its longitudinal axis at right angles to the line of flux and a current is allowed to flow through the conductor, then a mechanical force is generated (Fig. 4.4 b).

For any particular current, the torque is balanced by the spring restoring torque that is

$$2F \times \left(\frac{16}{2} \times 10^{-3}\right) = 2 \times 10^{-3}$$

The factor 2 with force F stems from the fact that force acts on both sides of the coil.

$$\therefore \qquad F = 0.125 \text{ N/degree}$$

The relation between the generated force and the current is given by

$$F = B\,i\,l$$

where B is the flux dustily, i is the conductor current and l is the conductor length. The effective length of coil is

$$l = 160 \times (16 \times 10^{-3}) \quad \text{(one side)}$$

Then the current flowing through the coil is

$$i = \frac{F}{Bl} = \frac{0.125}{0.14 \times \left(160 \times 16 \times 10^{-3}\right)} = \frac{1}{2.867} \text{ ampere/deg}$$

and hence the revsitivity is 2.867 deg/ampere.