EE2703 : Applied Programming Lab
Endsem Assignment Jan-May 2021
Magnetic field due to a current carrying loop

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May 30, 2021

Contents

1 Introduction

In this assignment we try to find the Vector Potential for a given current carrying loop of wire, using this Vector Potential finding the Magnetic field variation. Then fitting this variation in Magnetic field to an exponential and find the exponent and the multiplication factor using least squares method.

2 Pseudo Code

Declare the limits to axes required for this problem
    $0 < x < 2$; $0 < y < 2$; $1 < z < 1000$
Create Meshgrid to get points seperately in all three dimensions
Initialise the variables
    radius = 10cm
    $0 < \phi < 2pi$
Create an array with x and y components
    $x = radius * cos(\phi)$; $y = radius * sin(\phi)$
Find the x and y components of the current elements corresponding to x and y coordinates
Plot the loop and the current elements in the x-y plane
Divide the loop into N = 100 parts
dl is the small division which is a vector perpendicular to the loop
    $dl_x = -k * sin(phi)$; $dl_y = k * cos(phi)$
    $k = 2\pi * radius/N$
Function $calc()$: calculate vector potential for elemental length
    Pass In: elemental section (0 to 99)
    Calculate the distance between space points and the elemental length
    Find the vector potential(A) corresponding to the elemental length
    Pass Out: x and y components of A
EndFunction
Vector potential is divided into x and y component
    $A_x = 0$; $A_y = 0$
for $iteration = 0, 1, \dots 99$ **do**
  call the $calc()$ function
  summate the x component of vector potential obtained for elemental length
  summate the y component of vector potential obtained for elemental length
end for
Calculate B using the vector potential components A_x and A_y
Fit $B(z)$ to cz^b and obtain c and b using least squares method
Plot B vs z True value and the estimate obtained in loglog scale

3 Initialisation

Based on the given information in the problem we initialise some variables. For the given problem loop with radius $10cm$ is placed with centre at origin and lies in the $x - y$ plane and the magnetic field variation is to be analysed on the z axis from $1cm$ to $1000cm$,
For our calculations the loop is to be divided into N (100 in this problem) sections.
Since we have a loop the parameter used in polar coordinates apart from radius is the angle ϕ, so declaring it with $N + 1$ points and removing the last point as 0 and 2π both are the same.
The following code does the initialisation of the variables mentioned earlier:

```
x = np.linspace(0,2,3)
y = np.linspace(0,2,3)
z= np.linspace(1,1000,1000)
X,Y,Z = np.meshgrid(x,y,z)

radius = 10
N = 100
phi = np.linspace(0,2*np.pi,N+1)
phi = phi[:-1]
```

4 Current carrying loop of wire

Now that we have the parameters radius and ϕ we can represent all the points on the loop in polar coordinates. Where $x = rcos\phi$ and $y = rsin\phi$, this is stored in an array ro with first column representing x component and second column the y component. Plotting the loop which is centered at origin with radius $10cm$. The following code does that:

```
ro = np.array([radius*np.cos(phi),radius*np.sin(phi)])
ro = ro.T

plt.figure(0)
plt.plot(ro[:,0],ro[:,1],'ro',label="wire loop")
plt.title("Loop of wire with radius 10cm")
plt.xlabel(r'x$\rightarrow$')
plt.ylabel(r'y$\rightarrow$')
plt.legend()
plt.grid()
plt.show()
```

The plot of the loop of wire is as follows:

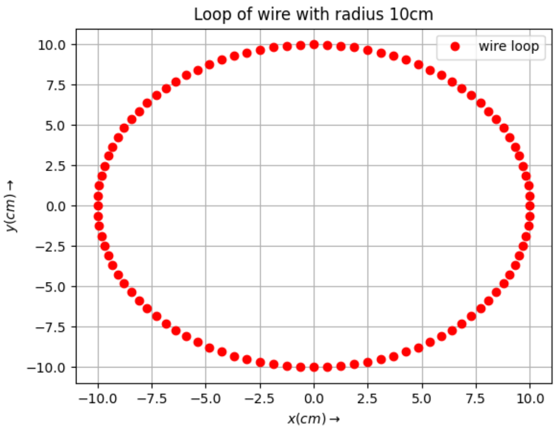

Figure 1: Loop of wire with radius $10cm$

Now let us find the current elements in $x - y$ and plot it using a quiver plot. The Current variation in the loop is given by the following equation:

$$I = \frac{4\pi}{\mu_o}\cos(\phi)exp(j\omega t)$$

The following code finds the current elements corresponding to x and y coordinates and plot it:

```
mu0 = 1.25663706e-6
ix = -(4*np.pi/mu0)*np.cos(phi)*ro[:,1]
iy = (4*np.pi/mu0)*np.cos(phi)*ro[:,0]
plt.figure(1)
plt.quiver(ro[:,0],ro[:,1],ix,iy,label="Current Elements")
plt.title("Quiver plot of current")
plt.xlabel(r'x$\rightarrow$')
plt.ylabel(r'y$\rightarrow$')
plt.legend(loc='upper right')
plt.grid()
plt.show()
```

The plot is as follows:

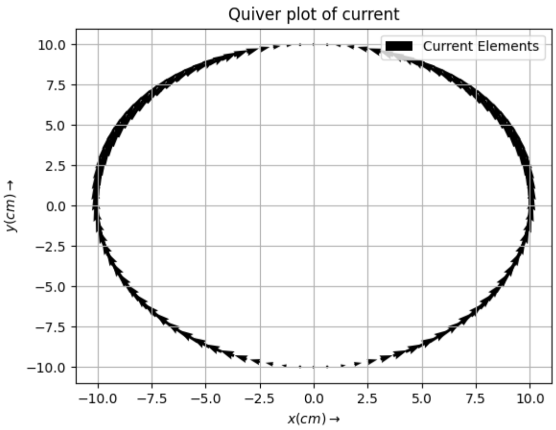

Figure 2: Quiver plot of current

Now let us divide this loop into N sections using these points we can calculate the effect of each element on the vector potential. dl is the elemental length perpendicular to the loop in the x-y plane. First row contains the x component of that element of the loop and second row the y component. The elemental length of the loop is given by the circumference divided by the number of sections the loop is divided. The following code defines dl used further:

```
dl = (2*np.pi*radius/(N))*np.array([-np.sin(phi),np.cos(phi)])
dl = dl.T
```

5 calc() function

This function calc is defined initially to calculate the distance between the point in space and the point on the loop so that its effect on the vector potential can be found.
Then calc() function is extended to generate the terms whose sum will lead to the x and y components of the vector potential.
R is the distance between the point in space (In this assignment we choose to take points in the proximity of the z axis from 1cm to 1000cm) and the points on the loop given by the array ro.
We divide the dl into x and y component, Generating vector potential corresponding to the x and y compenents of dl with other parameters corresponding to the elemental lenght being considered in the iteration of the for loop using the formula to calculate magnetic vector potential.
Returning the vector potentials which are the x and y components of the vector potential corresponding to the elemental length considered.
The following code shows the definition of the calc() function:

```
def calc(l):
    R = np.sqrt((X-ro[l,0])**2 + (Y-ro[l,1])**2 + (Z)**2)
    dl_x =dl[l,0]
    dl_y =dl[l,1]
```

```
    A_1 = np.cos(phi)[l]*np.exp(-0.1*1j*R)*dl_x/R
    A_2 = np.cos(phi)[l]*np.exp(-0.1*1j*R)*dl_y/R
    return A_1,A_2
```

6 Vector Potential

Using all the variables and parameters declared earlier now we find the vector potential divided into x and y components.
The equation for Vector Potential is given by:

$$\vec{A}(r,\phi,z) = \frac{\mu_o}{4\pi}\int \frac{I(\phi)\hat{\phi}e^{-jkR}ad\phi}{R}$$

This equation is reduced to :

$$\vec{A}_{ijk} = \sum_{l=0}^{N-1} \frac{cos(\phi_l')exp(-jkR_{ijkl})\vec{dl}'}{R_{ijkl}}$$

The above equation is used to calculate the vector potential for elemental dl in $calc()$ function. Initialising the x and y components of vector potential with 0, Now we iterate through each element of the loop using the calc() function.
To iterate we have used a for loop the reason for it is : Even though we vectorize this we need to use *np.sum* function to add them on an axis which internally uses a for loop, using both the methods we get almost same efficiency so we are using a for loop to iterate
It takes the element section as input and returns the vector potential corresponding to the elemental length divided into x and y components as output. Now these are the x and y components of the elemental length to get the vector potential for the overall loop we have to add the elemental vector potential in accordance with the principle of superposition.
The following code iterates through all the elements of the loop and adding them we get the overall vector potential divided into x and y components:

```
A_x = 0
A_y = 0
for l in range(N):
    A_1,A_2 = calc(l)
    A_x  += A_1
    A_y  += A_2
```

7 Magnetic Field

Using the vector potential found earlier we can find the Magnetic field. It is found by calculating the curl of vector potential given by:

$$\vec{B} = \nabla \times \vec{A}$$

it can be approximated to the following equation:

$$B_z = \frac{A_y(\Delta x, 0, z) - A_y(-\Delta x, 0, z)}{2\Delta x} - \frac{A_x(0, \Delta y, z) - A_x(0, -\Delta y, z)}{2\Delta y}$$

In the vector potential found upon investigating the first index corresponds to y second index corresponds to x and the third index to z. The above equation in terms of the vectorized vector potential is given by the following code, This gives the Magnetic field variation with respect to z axis from $z = 1cm$ to $1000cm$. Plotting the variation of Magnetic field with z using a loglog plot.

```
B=(A_y[1,2,:]-A_x[2,1,:]-A_y[1,0,:]+A_x[0,1,:])/2
plt.figure(2)
plt.loglog(z,np.abs(B),'g-',label="Magnetic Field variation")
plt.title("Loglog plot of Magnetic field variation along z axis")
plt.xlabel(r'z$\rightarrow$')
plt.ylabel(r'B$\rightarrow$')
plt.legend()
plt.grid()
plt.show()
```

The Maximum and Minimum values of the Magnetic field obtained are as follows:

```
Maximum value of B : 0.06939414690930745
Minimum value of B : 3.1409514094555867e-06
```

The plot of Magnetic field vs z is given as follows:

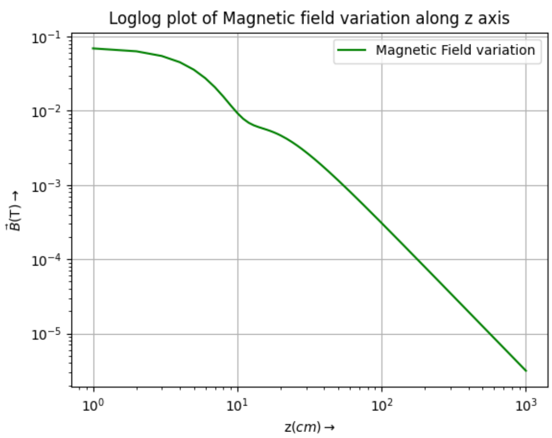

Figure 3: Loglog plot of **B** vs **z**

8 Fitting data

Now let us fit the field variation with z to $B_z = cz^b$, using least squares method. The following code does that and prints c and the exponent b:

```
logB = np.log(abs(B))
param = np.zeros((len(B),2))
param[:,0] = 1
param[:,1] = np.log(z)
logc,b = np.linalg.lstsq(param,np.transpose(logB),rcond=None)[0]
c = np.exp(logc)
print("c :",c)
print("b :",b)
```

The values of c and b obtained are as follows:

c : 1.4048479369071087
b : −1.8698222544166256

The Magnetic field obtained by the parameters found by using the least squares method is the estimated Magnetic field and we already found earlier the True Magnetic field. They are plotted as follows:

```
B_est = c*(z**b)
plt.figure(3)
plt.loglog(z,np.abs(B),'g-',label="True Value")
plt.loglog(z,np.abs(B_est),'r-',label="Estimated Value")
plt.title("Estimated Magnetic field plot based on lstsq")
plt.xlabel(r'z$\rightarrow$')
plt.ylabel(r'B$\rightarrow$')
plt.legend()
plt.grid()
plt.show()
```

The plot is as follows:

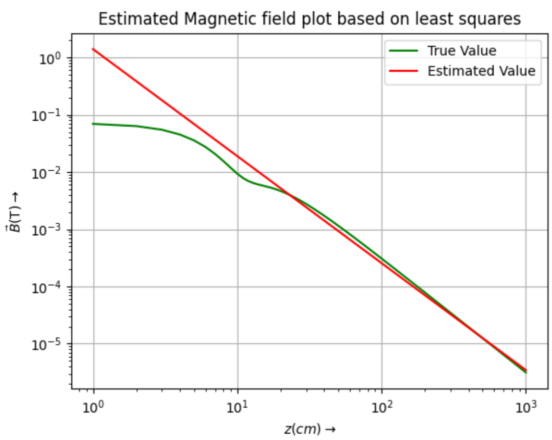

Figure 4: Estimated Magnetic field plot based on lstsq

9 Findings

From the estimation using the least squares method we can notice that the exponent is approximately 2,that implies Magnetic field is varying inversely with z^2. The magnetic field is given by the following equation:

$$\vec{B} = \frac{\mu_o I r^2}{2(z^2 + r^2)^{3/2}} \hat{z}$$

Which is approximated as

$$\vec{B} = \frac{\mu_o I r^2}{2z^3} \hat{z}$$

when $z >> r$

From the equation we expect a decay rate of 3 for a static case, but in this case we have taken a sinusoidally varying current, so Magnetic field varies with a decay rate of 1, using the least squares method the exponent obtained is very close to 2. The slight difference arising is due to the computational accuracy of the CPU, and the fit obtained very much agrees with expectations.

10 Conclusion

We analysed the Magnetic field variation using the vector potential for a current carrying loop. The loop was broken into elements and the effect of which on the vector potential was found adding all such vector potentials along x and y components provided the overall vector potential in terms of their x and y components. Using this we could evaluate the Magnetic Field and analyse the variation in Magnetic Field along the z axis. The Magnetic field obtained was fitted using least squares method and the exponent and the multiplication factor was found. Expected decay rate and the decay rate obtained by fitting was discussed.