EE2703: Applied programming Lab
Assignment 4: Fourier Approximations

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1 Defining and Plotting the functions

The function should take a vector or scalar, So numpy module is used while defining the function. The function definition is as follows

```
def exp(x):
    return np.exp(x)
def ccos(x):
    return np.cos(np.cos(x))
```

$cos(cos(x))$ is a periodic function but e^x is not so to evaluate the fourier series it is made 2π periodic. Plotting the functions over the interval $[-2\pi, 4\pi)$ in figure 1 and 2 respectively.

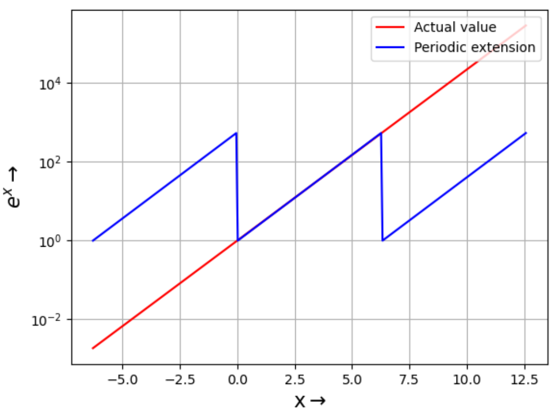

Figure 1: Semilog plot of e^x

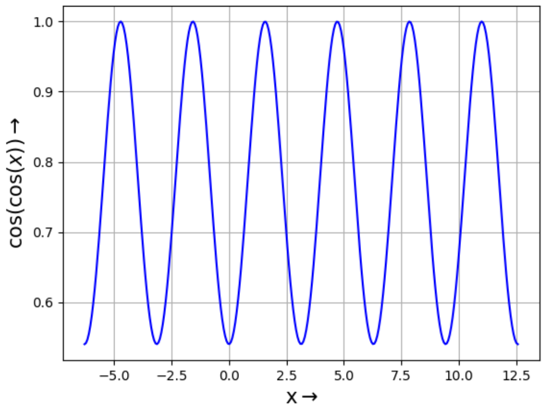

Figure 2: Plot of $cos(cos(x))$

e^x between $[0, 2\pi]$ is taken and repeated to get a 2π periodic function, while $cos(cos(x))$ is a periodic function with period π we plot it as it is. We can expect the approximation for $cos(cos(x))$ will be more accurate than that of e^x.

2 Fourier Series Coefficients

To integrate we use the option in quad to pass the extra arguments to the function being integrated. The following code returns the coefficients taking the number of coefficients and the function name as defined above as arguments.

```
def fourier_coef(n,func):
    coef = np.zeros(n)
    u = lambda x,k: func(x)*np.cos(k*x)
    v = lambda x,k: func(x)*np.sin(k*x)
    coef[0]= quad(func,0,2*np.pi)[0]/(2*np.pi)
    for i in range(1,n,2):
        coef[i] = quad(u,0,2*np.pi,args=((i+1)/2))[0]/np.pi
    for i in range(2,n,2):
        coef[i] = quad(v,0,2*np.pi,args=(i/2))[0]/np.pi
    return coef
```

Obtaining the 51 coefficients using the above code. We can plot the magnitude of coefficients. The coefficient vector is as follows:

$$\begin{bmatrix} a_0 \\ a_1 \\ b_1 \\ \dots \\ a_{25} \\ b_{25} \end{bmatrix}$$

Plots of fourier coefficients of e^x and $cos(cos(x))$ in semilog and loglog scale is as follows

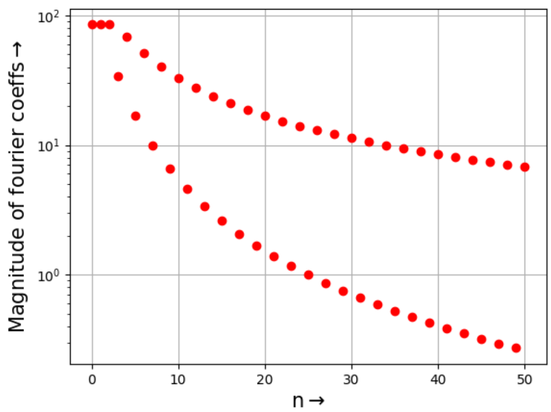

Figure 3: Semilog plot of coefficients of e^x

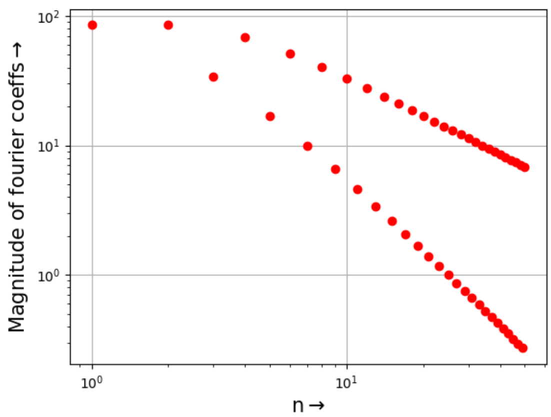

Figure 4: Loglog plot of coefficients of e^x

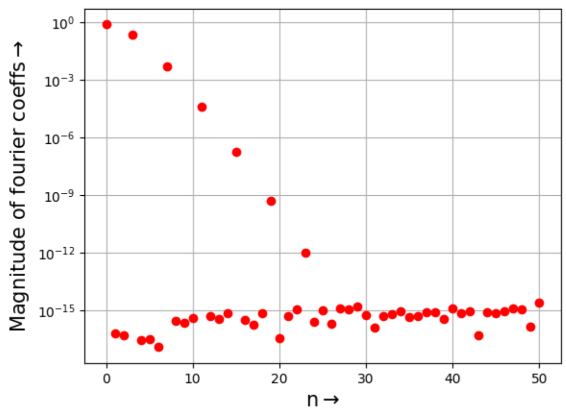

Figure 5: Semilog plot of coefficients of $cos(cos(x))$

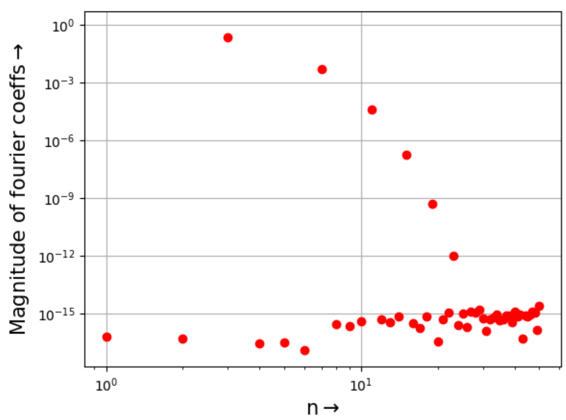

Figure 6: Loglog plot of coefficients of $cos(cos(x))$

b_n coefficients in the second case is nearly zero beacuse $cos(cos(x))$ is an even function so the odd component is zero. It is nearly zero and not exactly due to the limitation in computation capability of π in the fourier formula. In the first case the coefficients do not decay as quickly as the coefficients in second case because e^x has a wide range of frequencies in its fourier series expansion, whereas $cos(cos(x))$ has relatively low frequency of $1/\pi$ so contribution by high frequency sinusoids will be less and the coefficients decay much faster with increasing n.
Magnitude of coefficients of e^x is inversely proportional to n^2 so taking a loglog plot will be approximately linear. whereas the coefficients of $cos(cos(x))$ is approximately exponential with n, So semilog plot will be linear.

3 Least Squares Approach

$$\begin{pmatrix} 1 & \cos(x_1) & \sin(x_1) & .... & \cos(25x_1) & \sin(25x_1) \\ 1 & \cos(x_2) & \sin(x_2) & .... & \cos(25x_2) & \sin(25x_2) \\ ... & ... & ... & .... & ... & ... \\ 1 & \cos(x_{400}) & \sin(x_{400}) & .... & \cos(25x_{400}) & \sin(25x_{400}) \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ b_1 \\ ... \\ a_{25} \\ b_{25} \end{pmatrix} = \begin{pmatrix} f(x_1) \\ f(x_2) \\ ... \\ f(x_{400}) \end{pmatrix}$$

We create the matrix on the left side and call it A . We want to solve $Ac = b$ where c are the fourier coefficients. Using lstsq to find out the best fit numbers that will satisfy c the coefficients obtained are plotted for both functions in semilog and loglog scale.

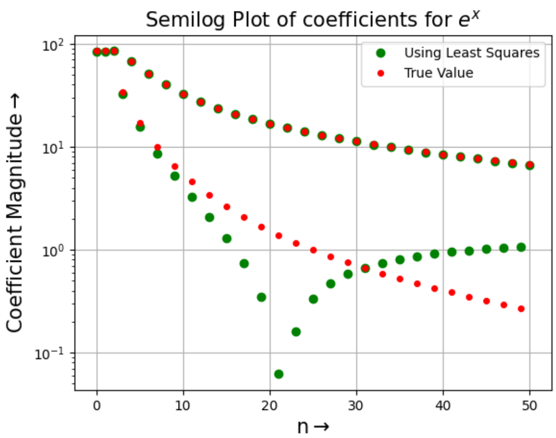

Figure 7: Semilog plot of coefficients of e^x

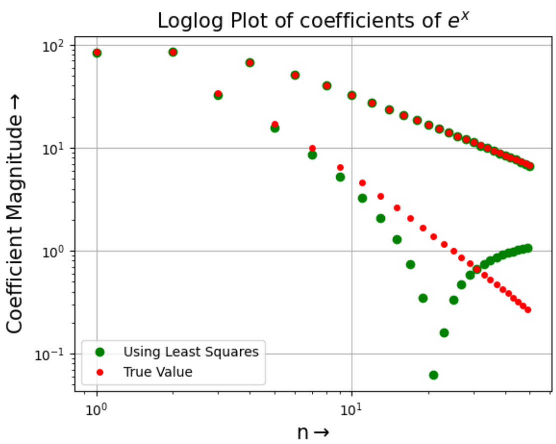

Figure 8: Loglog plot of coefficients of e^x

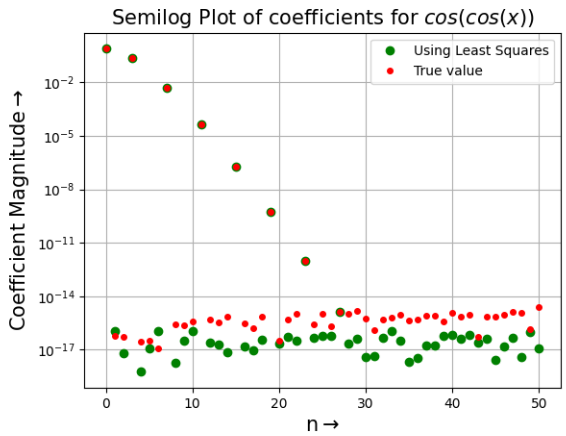

Figure 9: Semilog plot of coefficients of $cos(cos(x))$

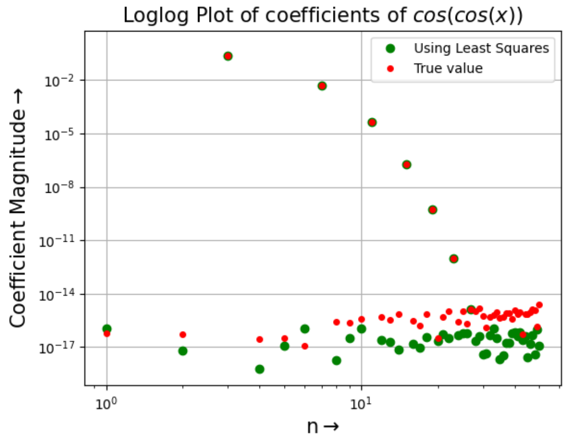

Figure 10: Loglog plot of coefficients of $cos(cos(x))$

Coefficients of e^x deviate much more than that of $cos(cos(x))$ it is as expected because $cos(cos(x))$ is a periodic function whereas e^x is an increasing function which is periodically extended.

4 Deviation in Coefficients

We have two sets of values of magnitude of coefficients obtained from direct integration using fourier formula and the other using least squares approach. The absolute difference between them gives the deviation. The maximum deviation is found from this data.
Maximum Deviation in e^x = 1.3327308703353964

Maximum Deviation in $cos(cos(x))$ = 2.60622479658205859e-15
As expected the deviation in e^x is much higher than the deviation in $cos(cos(x))$.

5 Function Approximation

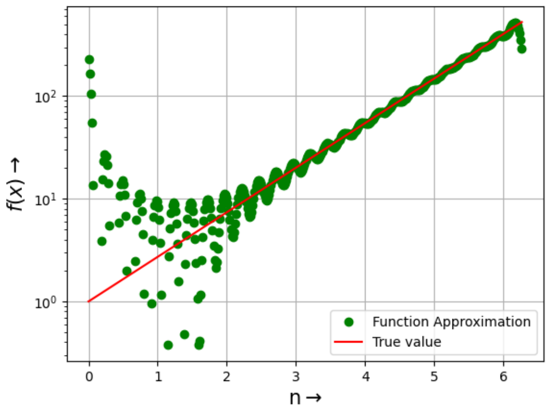

Figure 11: Plot of e^x and its Fourier series approximation

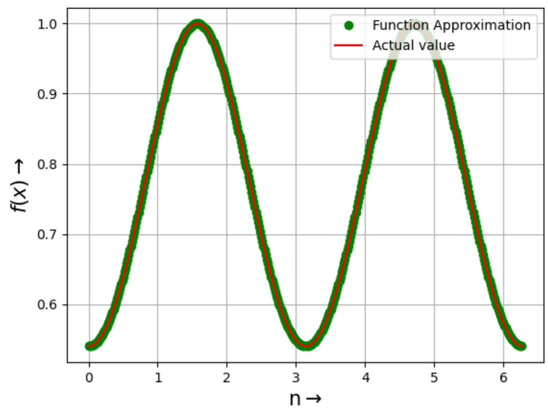

Figure 12: Plot of $cos(cos(x))$ and its Fourier series approximation

It should be noted that e^x is a non periodic function and Fourier series exists only for periodic functions. Hence we have considered a variation of e^x with period 2π that has the actual value of e^x only in the range $[0,2\pi)$. Hence it is acceptable that there is a large discrepancy in the predicted value of e^x at these boundaries

6 Conclusion

We tried to approximate the given functions using their fourier coefficients, we observe that it was quite accurate for $cos(cos(x))$ while deviated largely for e^x because $cos(cos(x))$ is a continuous periodic function whereas the periodic extension of e^x gives a function with discontinuities. We also used direct fourier series formula to calculate the coefficients and another method using Least Squares and computed the deviation least squares method shows from the true value.