EE2703: Applied programming Lab
Assignment 7: Laplace Transform

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1 Introduction

In this assignment we shall analyse Linear Time Invariant Systems using Python's Signals Toolbox.

2 Spring System

We find the time response of a spring system given by the following equation:

$$\ddot{x} + 2.25x = f(t)$$

$$f(t) = cos(1.5t)e^{-0.5t}u(t)$$

The Laplace domain equation corresponding to the above time domain equation is given by:

$$X(s) = \frac{F(s)}{(s^2 + 2.25)}$$

Substituting the Laplace expression for $F(s)$ gives:

$$X(s) = \frac{s + 0.5}{(s^2 + 2.25)((s + 0.5)^2 + 2.25)}$$

Initial conditions are given as zero, using them and plotting $x(t)$ for $0 < t < 50s$ yields the following plot:

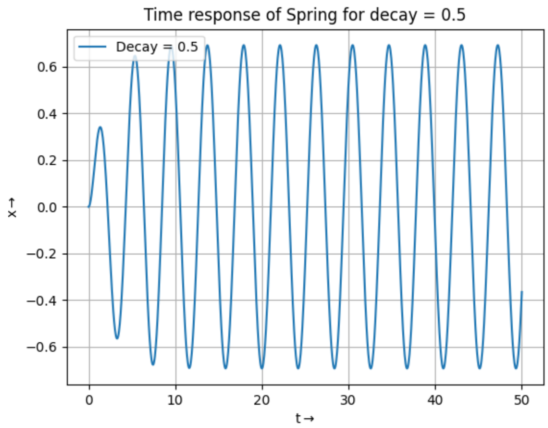

Figure 1: Time response of a spring

For the same system taking a smaller decay and plotting $x(t)$ for $0 < t < 50s$:

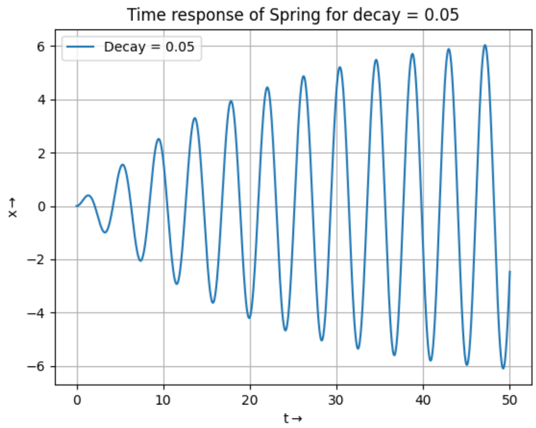

Figure 2: Time response of a spring with smaller decay

3 Response for varying frequency

Considering the problem to be an LTI System with input $f(t)$ and the frequency varying from 1.4 to 1.6 in steps of 0.05 keeping the exponent as $e^{(-0.05t)}$, plotting the responses gives:

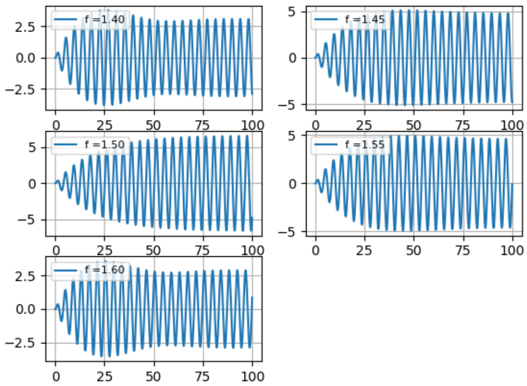

Figure 3: Responses for varying frequency from 1.4 to 1.6

The natural frequency of the given system is 1.5 rad/s and when the input frequency is the same as natural frequency we observe the maximum amplitude of oscillation which signifies the phenomenon of resonance, and on either sides of this frequency we observe similar plots.

4 Coupled Spring

The given coupled equations are:

$$\ddot{x} + (x - y) = 0$$

and

$$\ddot{y} + 2(y - x) = 0$$

with the initial conditions: $\dot{x}(0) = 0, \dot{y}(0) = 0, x(0) = 1, y(0) = 0.$ Taking Laplace Transform and solving for $X(s)$ and $Y(s)$, We get:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s}$$

$$Y(s) = \frac{2}{s^3 + 3s}$$

Solving the Laplace Transform of the above equations gives $x(t)$ and $y(t)$ which have the same frequency but different amplitude and out of phase :

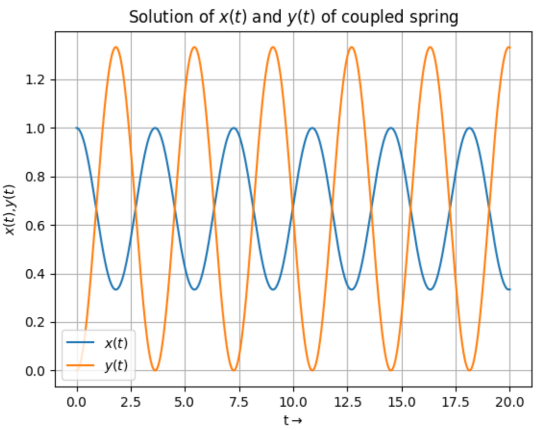

Figure 4: Plot of $x(t)$ and $y(t)$

5 Two Port Network

Transfer function of the given circuit is given by:

$$H(s) = \frac{10^{12}}{s^2 + 10^8 s + 10^{12}}$$

The magnitude and phase response of the circuit are given as follows:

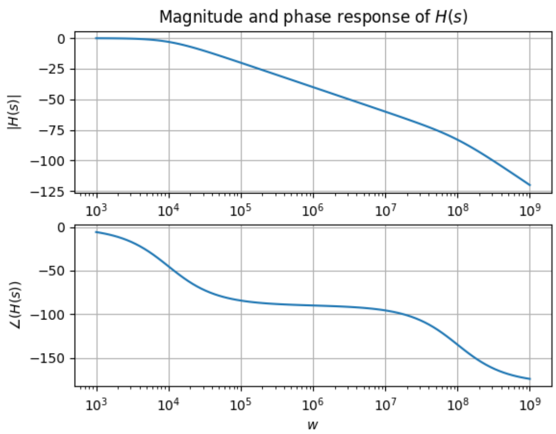

Figure 5: Magnitude and Phase plot of $H(s)$

The system is now fed with input signal $v_i(t)$ given by:

$$v_i(t) = cos(10^3 t)u(t) - cos(10^6 t)u(t)$$

The output can be found out using the below Laplace equation:

$$V_o(s) = V_i(s)H(s)$$

solving the above equation using sp.lsim for $0 < t < 30\mu s$ and plotting yields the following plot:

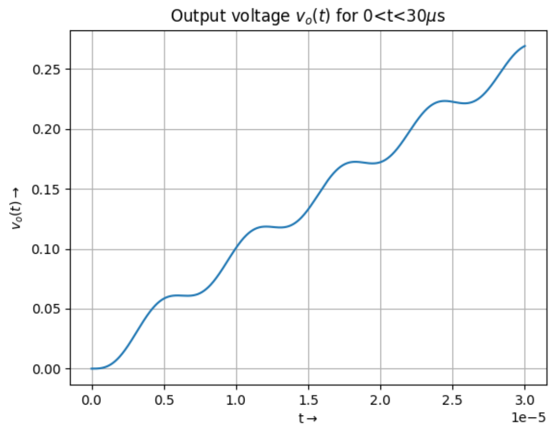

Figure 6: Output voltage $v_o(t)$ for $0 < t < 30\mu s$

Plotting for $0 < t < 10ms$

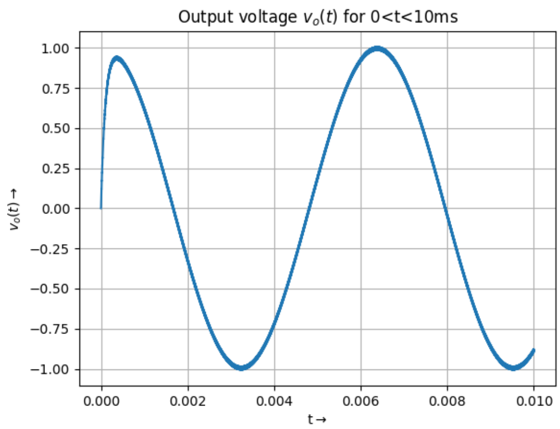

Figure 7: Output voltage $v_o(t)$ for $0 < t < 10ms$

Observing the Bode plot of $H(s)$ we notice that the system provides unity gain for low frequency, thus preserving the low frequency component. The system starts damping for high frequency, this is evident from the fact that it is a low pass filter.

6 Conclusion

In this assignment we analysed the Linear Time Invariant systems using Python's Signals Toolbox. We analysed the forced response of a spring system obtained over various frequencies of the applied force, the amplitude was highest at resonant frequency. Using $sp.impulse$ a coupled spring problem was solved both the sinusoids were obtained and plotted. The two port Network used as a low pass filter was analysed plotting its bode magnitude and phase plot and when $v_i(t)$ was given as input the output was obtained and plotted.