EE2703: Applied programming Lab
Assignment 8

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1 Introduction

In this Assignment we analyse circuits using Laplace Transforms using the Symbolic Algebra Capabilities of Python. We deal with two types of filters in this assignment Low Pass Filter and High Pass Filter and determine various aspects of their working.

2 Low Pass Filter

For the given circuit of a Low Pass Filter assigning nodal voltages and following MNA analysis and writing the Matrix equations in Laplace Domain gives us the Nodal voltages at each node in Laplace Domain.

$$\begin{bmatrix} 0 & 0 & 1 & -1/G \\ \frac{-1}{sR_2C_2} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ \frac{-1}{R_1} - \frac{1}{R_2} - s*C_1 & \frac{1}{R_2} & 0 & sC_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_p \\ V_m \\ V_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-V_i(s)}{R_1} \end{bmatrix}$$

The Following code does that:

```
def lowpass(R1,R2,C1,C2,G,Vi):
    s=symbols('s')
    A=Matrix([[0,0,1,-1/G],[-1/(1+s*R2*C2),1,0,0],[0,-G,G,1]
                          ,[-1/R1-1/R2-s*C1,1/R2,0,s*C1]])
    b=Matrix([0,0,0,-Vi/R1])
    V = A.inv()*b
    return (A,b,V)
```

The above code returns the A,b,V Matrices the fourth element of the V matrix gives the output voltage so that is extracted. The parameter values are passed to the above function and the matrices are obtained and fourth element of the V matrix is obtained using the following code:

```
A,b,V=lowpass(10000,10000,1e-9,1e-9,1.586,1)
Vo = V[3]
```

Now the ouptut voltage we have is an expression in Laplace Domain but we need the numerator and denominator coefficients in order to compute using signals toolbox of scipy.
After converting the expression into our required format we shall use *sp.step* function to obtain step response of the Low Pass Filter. Plotting the Step Response of Low Pass Filter for the given parameters yields:

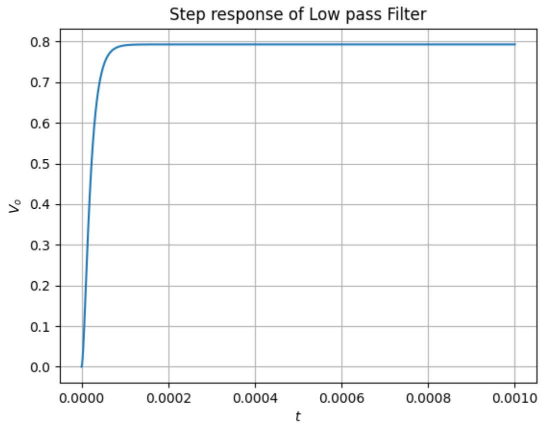

Figure 1: Step response of Low Pass Filter

In the last part we plotted output voltage when input is unit step now we plot the output voltage for the given Mixed frequency input

$$v_i(t) = (sin(2000\pi t) + cos(2 * 10^6 \pi t))u_o(t)Volts$$

We have the Transfer function for this low pass filter now convolving the Transfer function and input voltage yield the output voltage for the given input voltage.
Plotting the input and output voltage:

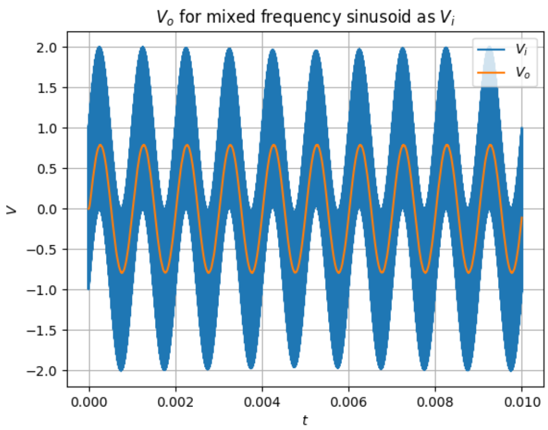

Figure 2: Output for Mixed frequency sinusoidal input

It can be seen that sine term has low frequency whereas the cosine term has high frequency, as this is a low pass filter so the low frequency sine component is passed while the high frequency cosine component is attenuated.

3 High Pass Filter

Next we deal with the High Pass Filter, the same procedure followed for Low Pass Filter by using MNA Analysis and writing the MNA Matrix and obtaining the fourth element of the V matrix which is the output voltage of the High Pass Filter.

$$\begin{bmatrix} 0 & -1 & 0 & 1/G \\ \frac{s*C_2*R_3}{1+s*C_2*R_3} & 0 & -1 & 0 \\ 0 & G & -G & 1 \\ -s*C_2-\frac{1}{R_1}-s*C_1 & 0 & s*C_2 & \frac{1}{R_1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_p \\ V_m \\ V_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -V_i(s)*s*C_1 \end{bmatrix}$$

The Following code does that:

```
def highpass(R1,R3,C1,C2,G,Vi):
    s=symbols('s')
    A=Matrix([[0,0,1,-1/G],[-1/(1+1/(s*R3*C2)),1,0,0],[0,-G,G,1],
                                    [-s*C1-s*C2-1/R1,s*C2,0,1/R1]])
    b=Matrix([0,0,0,-Vi*s*C1])
    #Obtainig the Voltage matrix using A and b
    V = A.inv()*b
    return (A,b,V)
A,b,V = highpass(10000,10000,1e-9,1e-9,1.586,1)
Vo = V[3]
```

Now again converting this symbolic expression into the scipy format we require, we obtain the output voltage.
Now we find the Magniude response for this High Pass Filter using the following code:

```
w=np.logspace(0,8,801)
ss=1j*w
hf=lambdify(s,Vo,'numpy')
v=hf(ss)
plt.loglog(w,abs(v),lw=2)
```

The above code plots the Magniude Response :

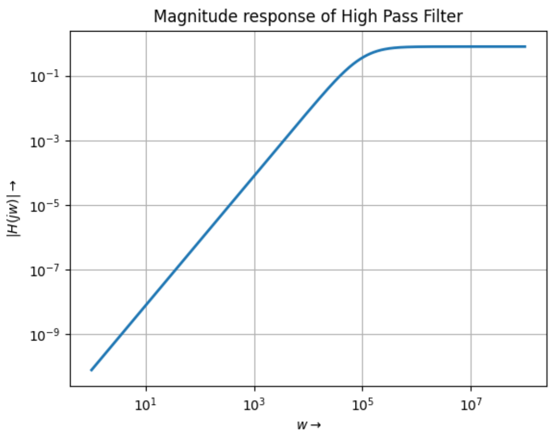

Figure 3: Magnitude Response of High Pass Filter

We can see that as this is a High Pass Filter the Magnitude response passes the high frequency component while attenuating the low frequency component.

4 Response for Damped Sinusoid

Two different input voltages are being considered :

$$v_i = e^{-0.2t} sin(4\pi t)$$

$$v_i = e^{-100t} sin(4\pi * 10^5 t)$$

Since this is a High Pass Filter it is expected to allow High freqency sinusoid whereas attenuate the low frequency sinusoid plotting the output voltages for both the low frequency and high frequency input yields:

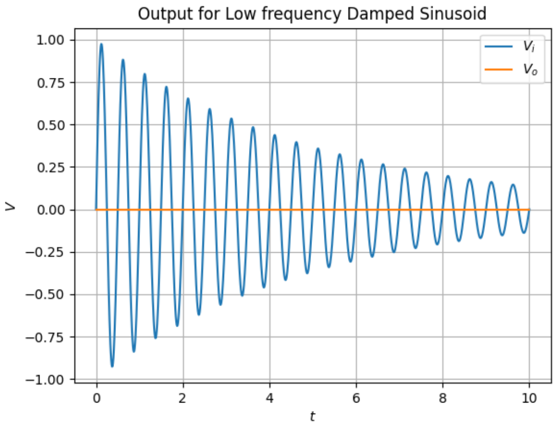

Figure 4: Output Voltage for Low Frequency input

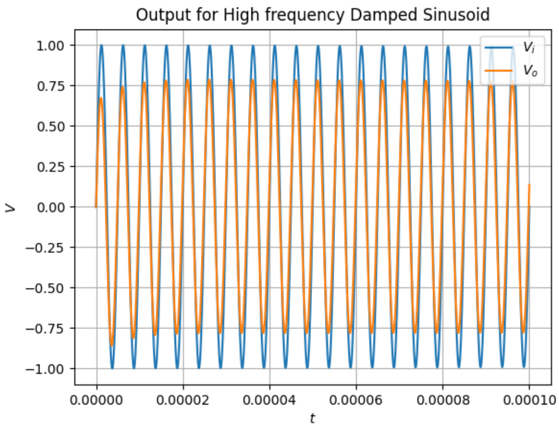

Figure 5: Output Voltage for High Frequency input

Now we plot the step response of the High Pass Filter. The same procedure followed to find the step response of the Low Pass Filter is used to find the step response of the High Pass Filter. Plotting the step response of High Pass Filter yields:

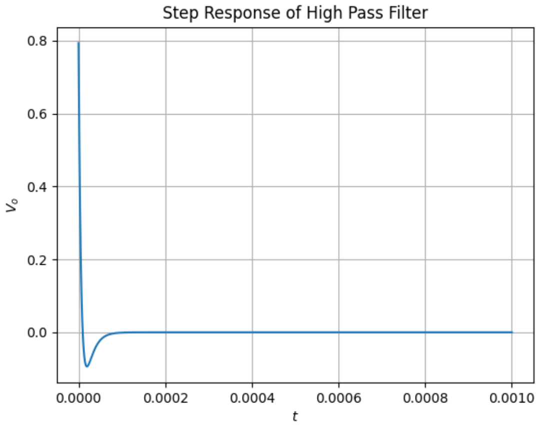

Figure 6: Step Response of High Pass Filter

Initially the capacitors will be shorted, as the gain of the opamp shall be high the output voltage will be high but as the system attains steady state the capacitors become open circuited the output voltage becomes zero. this is clearly depicted in the above plot.

5 Conclusion

Sympy provides an effocient way to deal with circuits in Laplace domain. We have considered the Low Pass Filter and High Pass Filter and Plotted their step response and computed the ouput voltage for different kind of input voltages mixed frequency sinusoid for Low Pass Filter and Damped sinusoid for High Pass Filter and we notice the behaviour of Low Pass Filter to allow low frequency and attenuate high frequency component while the High Pass Filter allows High frequency component and attenuates low frequency component.