TOPICS IN APPLIED STATISTICS – SPRINT 2019
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Mini Project 2 – Part 2

Topic: Bootstrap Coverage Comparison

Per class materials, the percentile bootstrap "only has much theoretical support when the sampling distribution of the estimator is unbiased and symmetric." With that in mind, we expect the residual bootstrap to have better coverage for skewed distributions and biased estimators.

We provide 2 examples to confirm our expectations. The first, explores the impact of skewness in the performance of the unbiased estimators. The second compares performance of the bootstraps when using biased estimators.

All results are reproducible because we used the R set.seed function. Different seed values tested in the project yielded similar results to those reported herein, much more often than not.

Example 1

To explore the impact of skewness in the performance of the bootstraps, we used a chi-square distribution with df=3. We use the bootstraps to estimate the population mean (mean = 3) from sample means, which are unbiased estimators. We increase the skewness of the chi-square distributions (df=3) by increasing the size of the sample from 10, to 100, to 1,000, as seen in Table 1.

Sample Size N	Skewness
10	0.7008
100	1.476
1,000	1.6187

Table 1. Skewness of Chi-squared distribution with df=3, for sample sizes 10, 100, and 1,000.

To results of the simulation for each sample size are presented in Table 2. They show that for n=10 and n=100 the percentile bootstrap has better coverage than the residual bootstrap. However, for n=1000, the skewness of the chi-square (df=3) presumably will cause the residual bootstrap to outperform the percentile bootstrap. The fourth column in the table, "Meaningful?" identifies when the performance of the residual bootstrap is ½ a percentile closer to the nominal level of coverage than the percentile bootstrap, as required for our project.

Sample Size N	Percentile	Residual	Meaningful?
10	86.1%	84.1%	N/A
100	95.2%	93.9%	N/A
1,000	94.1%	95.0%	Yes

Table 2. Estimation of Population Mean:
Coverage of Percentile vs. Residual (Basic) bootstrap for sample sizes 10, 100, and 1,000.

As stated before, similar meaningful results were frequent with other seed values.

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Example 2

For the second example we explore coverage of the bootstraps using a symmetric distribution. We simulate estimation of standard deviation of normal distributions of the form N(0,1) using first an unbiased estimator, and then a biased estimator. We compare coverage of the bootstraps in each case, for sample sizes 10, 100, and 1,000, as in example 1. Since normal distributions are symmetric by nature, skewness should not be a factor in our analysis, as confirmed by Table 3.

Sample Size N	Skewness
10	0.0075
100	-0.0004
1,000	-0.0072

Table 3. Skewness of normal distributions N(0,1) for sample sizes 10, 100, and 1,000.

Table 4 shows performance of bootstraps when we used the unbiased sample standard deviation as the estimator of the population standard deviation, with "n-1" in the denominator.

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

Sample Size N	Percentile	Residual	Meaningful?
10	76.8%	83.1%	Yes
100	93.2%	93.3%	No
1,000	94.5%	94.5%	N/A

Table 4. Estimation of Population STDev from Unbiased Sample Estimate:
Coverage of Percentile vs. Residual (Basic) bootstrap for sample sizes 10, 100, and 1,000.

Table 5 shows performance of bootstraps when we used the biased sample standard deviation formula as the estimator of the population standard deviation, with "n" in the denominator,

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$$

Sample Size N	Percentile	Residual	Meaningful?
10	71.1%	81.8%	Yes
100	92.3%	93.2%	Yes
1,000	94.5%	94.7%	No

Table 5. Estimation of Population STDev from Biased Sample Estimate:
Coverage of Percentile vs. Residual (Basic) bootstrap for sample sizes 10, 100, and 1,000.

We notice now that the residual bootstrap outperforms the perceptual bootstrap for all sample sizes, and "meaningfully" so for small and medium size samples (N = 10 and 100). As stated before, similar meaningful results were frequent with other seed values.

Note that we should not be surprised that the coverage of the two bootstraps converge for N = 1,000, since the difference between the biased and unbiased standard deviation drops quickly to zero for larger values of N, as seen in Figure 1.

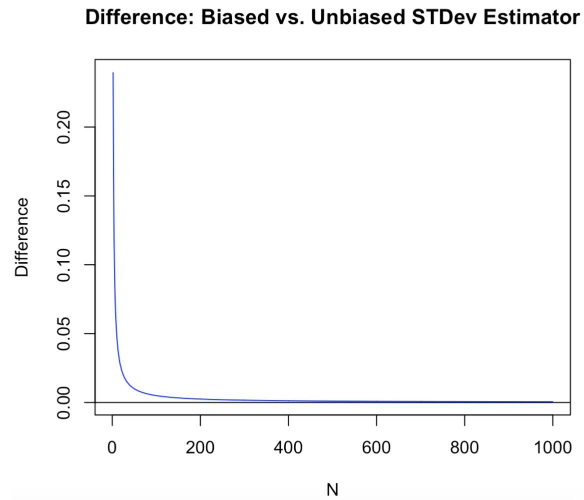

Figure 1. Difference: Biased vs. Unbiased Standard Deviation Estimators

Conclusion

In conclusion, we showed through two examples that the residual bootstrap is likely to have better coverage than the perceptual bootstrap on skewed distributions, and for biased estimators, as suggested in the class materials.