Revealing Preferences: Why Gerrymanders are Hard to Prove, and What to Do about It

Micah Altman, Massachusetts Institute of Technology Brian Amos, University of Florida Michael P. McDonald, University of Florida Daniel A. Smith, University of Florida

Abstract

Gerrymandering requires illicit intent. We classify six proposed methods to infer the intent of a redistricting authority using a formal framework for causal inferences that encompasses the redistricting process from the release of census data to the adoption of a final plan. We argue all proposed techniques to detect gerrymandering can be classified within this formal framework. Courts have, at one time or another, weighed evidence using one or more of these methods to assess racial or partisan gerrymandering claims. We describe the assumptions underlying each method, raising some heretofore unarticulated critiques revealed by laying bare their assumptions. We then review how these methods were employed in the 2014 Florida district court ruling that the state legislature violated a state constitutional prohibition on partisan gerrymandering, and propose standards that advocacy groups and courts can impose upon redistricting authorities to ensure they are held accountable if they adopt a partisan gerrymander.

1. Introduction

Countries with legislative bodies typically redraw districts' boundaries through a process known as redistricting. Ostensibly, expert judgement is used to equalize districts' populations to achieve generally uniform representation, while balancing against other goals, such as affording communities effective representation. The U.S. stands out as one of the few countries where legislative bodies or politicized commissions are responsible for drawing boundaries (Handley & Grofman 2008; McDonald 2004; Levitt 2011). This creates a conflict of interest so storied in American politics that a word has been coined for it, gerrymandering, named after a salamander-shaped 1812 Massachusetts legislative district signed into law by Governor Elbridge Gerry, designed to strategically group voters to enhance his party's chances of winning elections.

In the 1986 case Davis v. Bandemer, the U.S. Supreme Court ruled partisan gerrymandering is justicable, however, federal courts have never invalidated a redistricting plan on partisan gerrymandering grounds. In 2006, Justice Anthony Kennedy laid down a gauntlet to the social science and legal communities to devise a "reliable standard" to "identify unconstitutional acts of partisan gerrymandering." There has been no lack of trying to satisfy Justice Kennedy. The fractured dissenting opinions in Vieth v. Jubelirer offered various standards devised by notable social scientists, none of which Justice Kennedy found palatable. Subsequently, federal courts in Illinois

¹ LULAC v Perry, 548 U.S. 399, 418 (2006).

and Texas denied plaintiffs' claims of partisan gerrymandering because they failed to provide a "judicially discernible" standard of excessive partisan gerrymandering.² While there has been some success at limiting partisan gerrymandering at the state level,³ a federal standard remains elusive.

Gerrymandering requires illicit intent, when a redistricting authority (any entity that adopts a redistricting plan for implementation, henceforth "Authority") purposefully allocates voters to districts in a manner to further their political goals. Many have proposed methods to detect impermissible gerrymandering in racial (e.g., King 1999; Grofman, Handley, & Lublin 2001) and political contexts (e.g., Stokes 1993, Grofman & King 2007). Sometimes, the evidence of intent is difficult to distinguish from other apolitical goals when traditional redistricting principles and the Authority's intent are intertwined, either intentionally motivated by illicit intent (Parker 1990) or unintentionally as a consequence of faithful adherence to legal requirements (Altman 1998; Chen & Rodden 2013).

We classify six methods to infer the intent of an Authority, employing a formal framework for causal inference that follows the redistricting process from the release of census data to the adoption of a final plan. We argue all proposed techniques to detect gerrymandering can be classified within this formal framework, and courts have weighed evidence using the six methods we describe to assess racial or partisan gerrymandering claims. We

² Committee for a Fair and Balanced Map v. Ill. State Board of Elections, No. 1:11-cv-05065 (N.D. Ill.) (24) and Perez v. Texas, No. 5:11-cv-00360 (W.D. Tex.), Order on Rule 12 motions, Sept. 2, 2011.

³ Romo v. Detzner, Nos. 2012-CA-00412 & 2012-CA-00490 (Fla. 2d Jud. Cir. Ct.).

define the assumptions underlying each method, raising some heretofore unarticulated critiques revealed by laying bare their assumptions. We then review how these methods were employed in the 2014 Florida district court ruling that found the state legislature violated a state constitutional prohibition on partisan gerrymandering, and propose standards that advocacy groups and courts can impose upon Authorities to hold them accountable if they gerrymander.

2. The Redistricting Task

We begin our analysis with describing the redistricting task. Redistricting involves assigning a jurisdiction's geographic units to districts. For most applications in the U.S., these geographic units are called census blocks, small geographies typically the size of a city block, but include fine-grained geographies, such as road medians and streams, or larger areas in sparsely-populated rural areas.

Redistricting is abstractly a task well-known to mathematicians as a partitioning problem - a problem of dividing some set (here, blocks) into mutually exclusive and exhaustive subsets. Mathematically, one can create many representations of the problem - all of which have the same optimal set of solutions. A common approach is to describe redistricting as a graph, a technique often encountered in social network applications. A graph is defined by its nodes (i.e., blocks) and vertices (i.e., edges shared by blocks). The redistricting task is to assign (or partition) all blocks (the graph) to one district, given constraints.

We do not dwell in graph terminology, but it is important to understand mathematicians have studied its application to redistricting (e.g., Grilli di Cortona, et al. 1987). Redistricting belongs to a class of graph partitioning problems known to be NP-Complete (Altman 1997). A characteristic of NP-Complete problems is their tremendously large number of solutions as the number of units increases, such that enumeration of all solutions is practically limited to very small jurisdictions (Rossiter & Johnston 1981). A jurisdiction with as few as 50 blocks has over 10³¹ potential districts. Enumeration of states with hundreds of thousands of blocks likely remains beyond reach forever, given current theoretical computation limits (Altman 1997). The number of solutions is far more than any human can examine in detail. Thus, a proposed method to detect gerrymandering cannot be based on what is possible, but rather what is knowable to an Authority.

3. A Causal Model of Redistricting Decision-making

Causal inference has the purpose of predicting counterfactuals — of answering "why" questions (Pearl 2000). Why did the defendant harm the plaintiff? Why did the defendant create a gerrymander? Scientists state counterfactual questions in terms of a hypothesis statement, seeking to answer the question by comparing it to what would be observed if no nefarious intent was present. Any proposed method to detect gerrymandering makes a causal claim — and thus must be analyzed in these terms to ensure valid causal inference.

Counterfactual causal analysis has been used previously to assess gerrymandering claims. For example, Kousser (1991) applies counterfactual analysis using ranges of hypotheticals to examine gerrymandering in Los Angeles, Gronke and Wilson (1999) apply the approach to North Carolina's 1990's congressional redistricting, and Altman (2002) outlines a more general Bayesian counterfactual framework for evaluating gerrymandering claims based on observations of disproportional electoral outcomes or predicted outcomes from simulated plans. However, no one has previously provided a comprehensive rigorous framework of evaluating the causal evidence for gerrymanders.

Pearl's (2000) innovative method for causal analysis enables such an approach. He enumerates all potential causal relationships using sets (ordered triples) of endogenous variables, exogenous variables, and structural equations, and then applies methods for reasoning over these sets to deduce necessary and sufficient conditions. His "causal diagram" is a visualization of the relationship among these variables (defined as a mathematical graph). Eschewing the background technical details, Pearl's causal diagrams are formal analytic tools that expose underlying general causal assumptions for any causal assertion based on empirical evidence.

We present in Figure 1 all relevant causal pathways affecting how an Authority's intent to create a redistricting plan leads to its adoption. The diagram represents the process from an Authority's perspective since whether or not an Authority created an impermissible gerrymander is the central litigation question. By identifying pathways that relate to proposed gerrymandering detection methods, we clarify underlying assumptions present

for a method to provide valid inference that an Authority intended to gerrymander.

In Figure 1, we identify the 'information' (formally, variables) related to redistricting decisions as circles. Arrows show how this information is connected together, flowing from one circle to the next. Formally, an arrow from A to B exists where A could have a direct causal effect on B. Conversely, the absence of arrow from A to B reflects a basic modeling assumption that A does not directly causally affect B. In Figure 1, the information externally observed is identified in the darker shaded circles. We might think of this information as evidence introduced in courtrooms. Private information (unobservable variables) is identified by light colored circles.

Reliably estimating causal effects from observed data requires correct assumption about which causal relationships can be excluded, observed evidence, and a correct assumptions about about the structural relationships along each step of the causal pathway.⁴ Thus, each arrow in the graph represents not only a causal relationship, but also a set of modeling assumptions that reflect one's understanding about how the causal factor could be plausibly structurally related to the outcome — for example, whether the causal effect is linear, continuous, etc. In the discussion of inference methods below, we reflect on the types of structural assumptions required

⁴ These assumptions might not be necessary in a purely experimental lab setting, using randomized controlled interventions — however they are necessary for observational analysis.

for each method. However, since the main focus of our analysis is to identify the conditions necessary for correct inferences, we need not examine the particular structural models involved in each.

Finally, if one adopts a Bayesian view of inferences, the prior probabilities distribution (or "prior") is needed for estimation. In this case, one's prior can be thought as one's beliefs about the general likelihood an Authority has bad intent, before seeing any evidence. In many legal analyses, priors are excluded; however, the totality of the circumstances method discussed below may provide systematic evidence affecting one's prior beliefs.

We start the causal chain with the dark shaded circle in the upper right corner which represents the geographic, demographic, and voting data used for redistricting.⁵ A causal arrow flows from the redistricting data to the universe of possible plans, which we conceive here as plans meeting minimal legal requirements of contiguity, completeness (all blocks assigned to a single district), and rough population equality, but no other legal constraints.

Blocks can be assigned to a district in a practically infinite number of ways for most jurisdictions' legislative bodies, thus it is impossible

^{9.0}

⁵ See McDonald (2014) for a description of how these data are compiled. Although we indicate redistricting data is public knowledge, this is not strictly true (Altman & McDonald 2014). Technically, one needs to distinguish between the common set of redistricting data collected and known to the actors and the characteristics (variables) that the data measure, which could be independently measured and result in different data (with a common cause). Here, we refer to the former, the data collected by the actor, since this comprises the information available to the actors.

for any Authority to conceive of and evaluate every possible plan. We draw three arrows emanating from the set of possible plans:

- 1. Plans that are privately *knowable* by an Authority in that it might have been able to discover such plans while deliberating redistricting.
- 2. Plans that are publicly introduced during the redistricting process that an Authority could reasonably be aware of prior to adopting a plan.
- 3. Post-hoc plans produced after an Authority adopts a plan. These plans may be introduced by plaintiffs in litigation or drawn by scholars assessing the redistricting.

An Authority sets about the task of drawing a plan to achieve its goals. Some aspects of the redistricting process are observable, which we identify in the dark circle in the middle right of Figure 1. Some observable processes are fixed at the start of redistricting, while an Authority has discretion over others. In the latter case, an Authority's intent might influence their structure, which is represented by drawing a line from intent to observable process. These observables include, but are not limited to, how an Authority conducts itself in terms of voting rules and coalitions that lead to plan adoption (McDonald 2004); whether or not to hold public hearings and solicit public input (Altman, et al. 2005); the constitutional and statutory criteria

⁶ Strictly speaking, we should divide observable processes into two circles: one for those that are fixed prior to the redistricting and one for those that are mutable by an Authority during redistricting. We might also contemplate the existence of private, unobservable, processes that similarly guide redistricting. For the sake of exposition, and since only observable processes can be entered into evidence, we draw only one circle for observable processes.

that an Authority must respect; and additional criteria that an Authority develops as part of its process (e.g., definitions of communities of interest that emerge from hearings).

An Authority does not contemplate all feasible plans, as it is constrained by the process, either as a consequence of limited resources to explore the space of redistricting plans or by the process governing the redistricting. Among the many plans that are potentially knowable to an Authority are a subset of known plans created by an Authority, the latter which reflects how mappers iteratively draw plans until a "best" plan emerges or time runs out and a plan must be adopted. Among the plans privately known to an Authority are publicly known plans by drawn by others that may present configurations of districts worthy of consideration or be dismissed as incompatible with an Authority's goals.

An Authority's intent guides plan development, as does the observable process, which is why we draw arrows from the observable process and an Authority's intent to the privately knowable and known plans. It is this critical causal chain from intent to plan development and selection by which all methods to detect gerrymandering infer intent, save one. Intent may also be inferred from the political environment an Authority operates in. To represent this, we draw an arrow from the totality of the circumstances (defined below) to intent.

⁷ Mapping is tedious, especially drawing plans with *de minimis* population deviations across districts. Promising configurations may not prove viable upon deeper inspection. An Authority may explore options that are incompatible with their goals to explore what may happen in negotiations, if needed, or lawsuits, if they occur.

Eventually, an Authority adopts a plan when deadlines imposed explicitly by constitutions and statutes or implicitly by candidate filing deadlines are reached (or if not, courts take up redistricting). An Authority is an agent that has reached a state, where motivated by its intent, and subject to the process, it selects a plan from among those that are privately known to it. From this state, an Authority selects an adopted plan and makes public statements concerning it, often lauding how the adopted plan achieves constitutionally-mandated requirements.

4. Methods to Detect Gerrymandering

The causal diagram in Figure 1 allows us to evaluate the assumptions that underlie proposed methods to infer an Authority's intent, or more plainly, to detect gerrymandering. As we shall show, Figure 1 suggests there are six methods to detect gerrymandering. It is always possible a new method may be devised, however, we are confident that the six methods are exhaustive of gerrymandering detection methods. Our strategy is to review the six proposed methods in turn and to identify how they fit within the Figure 1 causal framework that leads an Authority to adopt a redistricting plan. We explore how these methods were applied by a Florida district court in finding an unconstitutional partisan gerrymander. The case study and our redistricting experience elsewhere lead us to recommendations as to how advocacy reform efforts and discerning courts may strengthen these methods. The six methods are:

- 1. **Method of Stated Intent:** examination of an Authority's public statements revealing purposive intent.
- 2. **Method of Totality of the Circumstances:** past behavior by an Authority and its past and present political environment that illuminate current intentions with regards to redistricting.
- 3. **Method of Evaluation of Process:** examination of the observable process to reveal intent.
- 4. **Method of Inspection:** examination of characteristics of a plan, visually or aided by descriptive statistics, to provide evidence of unconstitutional behavior.
- 5. **Method of Post-hoc Comparison:** comparing an adopted plan against alternative plans drawn post-hoc to the redistricting process to provide evidence of unconstitutional behavior.
- 6. **Method of Revealed Preferences:** comparing an adopted plan against alternative plans known to an Authority to provide evidence of unconstitutional behavior.

All of these methods have strengths and weaknesses, and in a given situation, one or more methods may be employed by plaintiffs and weighed by courts when adjudicating claims of politically or racially motivated redistricting. Indeed, courts may require plaintiffs present more than one method in support of their claims. For example, in allegations of discriminatory racial effect brought under Section 2 of the Voting Rights Act, plaintiffs are required to demonstrate how a plan has discriminatory effect, evidence of the totality of the circumstances, and provide a post-

hoc remedial plan. Rarely, if ever, does a single method provide sufficient evidence of illicit intent. Thus, we do not claim one method is strictly superior to the others, as their value is conditioned on available evidence and all may simultaneously have a litigation role.

3.1 Method of Stated Intent. Courts have long weighed public statements when assessing legislative intent. In Figure 2, we present the causal diagram relating an Authority's intent from their public statements, which are indicators of purposive intent. We exclude from this diagram all the non-relevant causal pathways in Figure 1, a technique we subsequently repeat to make clear the causal chain embodied in each method. Some statements by an Authority may arise from deliberation that took place during plan development, such as the rationale for a specific boundary. Others may be general statements that courts may infer improper use of race or partisanship in crafting a plan. For these reasons (respectively) there is one causal arrow leading from intent through agent state to public statements, which embodies the confluence of intent and choices available to an Authority when statements are made about plans, and a second causal directly from intent to public statements, which are statements that might generally reveal an Authority's intent.

By analyzing the causal diagram in Figure 2 we can deduce several requirements for inferences when one relies on stated intent about a plan: First, when relying on statements about plans one must make an inference about the agent's internal state, since there is no causal pathway from statements to intent that does not include agent state. Second to do this

requires making assumptions about the structural relationship between statements and agent state - in this case it assumes reliable understanding of human behavior. Third, the agent state is affected by an Authority's privately known plans. Since we do not observe these plans directly, we must infer this set from the available evidence, which means we must consider the geographic and demographic data, the observable process, and publicly available plans. For example, an Authority may laud certain aspects of a plan for how well they achieve apolitical goals, but an Authority may lie by omission when aware of plans that perform as well or better on these aspects but do not achieve its political goals as well.

A potentially less demanding causal chain is the direct link between an Authority's intent and their revealing public statements. This pathway still requires assumptions about the reliable understanding of human behavior, i.e., that an Authority sincerely states its motives. Courts have on occasion used such public statements as an indicator of illicit intent. A federal court found purposeful discrimination in a 1980's Georgia congressional plan in part due to racist statements by the chair of the state house committee responsible for redistricting (but not about specific plans).8

In contemporary times, plaintiffs' ability to demonstrate discriminatory purposive intent through analysis of the public record is difficult since it requires an Authority to candidly speak the breadth of

⁸ Busbee v. Smith 549 F.Supp. 494 (1982).

its motives. Authorities are aware their actions are being monitored and craft their language accordingly (Kousser 1991). They may shield themselves from public scrutiny by closing the doors to their deliberations, even when mandated by state law to hold open meetings, as Ohio's Apportionment Board did. While what is said in these meetings may be unknown, their existence has been held as evidence of suspect behavior. Authorities that hide their actions run the risk that a court will provide plaintiffs access to plans, reports, emails, and other information shielded from public view. Even here, however, plaintiffs and courts will only gather as much evidence as was archived, which usually does not include voice communications.

When political actors do not censor their language, there may be little legal consequence, particularly with regards to partisan gerrymandering. Candidates and spokespeople for political parties routinely disparage the other party as they attempt to win elections and shape policy. As Texas Republican Party Chair Susan Weddington stated concerning the state's redistricting, "We weren't overly sensitive to protecting anyone in particular, and particularly not Democrats. We make no bones about that. We're the Republican Party." Statements such as these did not sway Justice

⁹ Wilson v. Kasich, No. 2012-0019 (Ohio Sup. Ct.).

¹⁰ Romo v. Detzner, pp. 24-25. In his decision, Judge Terry P. Lewis cited a memorandum written by Republican operative, Marc Reichelderfer, following a December 2010 meeting of fellow Republican operatives and lawmakers: "There is nothing necessarily sinister about such meetings. Most of the attendees were friends or professional colleagues and perhaps it could be considered a courtesy extended. But it doesn't look good if you are promoting openness, transparency and neutrality in the redistricting process."

¹¹ Jay Root. "State GOP Rolls Out Redistricting Plan," Fort Worth Star Telegram, June 14, 2001, Metro, p.8.

Kennedy when he ruled against Democrats' allegations of partisan gerrymandering during the Texas re-redistricting in the 2000s. 12

3.2 Method of Totality of the Circumstances. Direct evidence of intent from public statements may be difficult to discover when politicians carefully guard their language. Another method to reveal intent is to scrutinize an Authority's past actions and the general political environment it operates in for evidence of discriminatory behavior, with the counterfactual question again posed that a different plan would have resulted absent such illicit intent. The rather simple causal diagram is presented in Figure 3. Courts broadly use totality of the circumstances to divine intent, for example, in sexual harassment cases (Kreiger & Fox 2013) or, more narrowly relevant to redistricting, in determining legislative intent (Law & Zaring 2010).

In the racial redistricting context, a specific set of "the totality of the circumstances" (ToC) is defined, and it is for this reason we adopt this phrase. (This should not be confused with the totality of the evidence, which may also include post-hoc plans, and other evidence described in the other methods that may be required by courts to find illicit intent). Congress amended Section 2 of the VRA in 1982 to require plaintiffs who argue a redistricting plan has discriminatory effect, without providing evidence of discriminatory purpose, show official and unofficial discrimination in the

 $^{^{12}}$ LULAC v Perry (2006).

democratic process and other state policies (Grofman, Migalski, & Noviello 1985: 200).13

The Senate Committee on the Judiciary's report accompanying the 1982 legislation amending Section 2 of the voting rights suggests seven factors, but notes that these factors are neither exclusive nor comprehensive. ¹⁴ Most of the factors listed in the report rely on analysis of historical evidence of prior actions of an Authority; the prior outcomes from redistricting plans and other electoral choice; and prior patterns of demography and voting. The analysis may also incorporate evidence from current electoral processes (e.g., exclusion of minority groups from candidate slating), or statements by others during the electoral process (e.g., use of racial appeals in campaigns). Although not strictly identical, each relies on a chain of inference analogous to one of the described in methods to detect gerrymandering and is thus subject to analogous inferential limitations, even if we do not explicitly model them.

The historical evidence included under ToC analysis may influence causal inference in two additional ways. First, if we use historical evidence to make inferences about the past intent of previous actors, this may influence our (Bayesian) priors — we may believe that those who have acted badly in the past are more likely to do so in the future. Second, where we are able to witness historical evidence that relates past political choices

 $^{^{13}}$ This amendment was designed to reinstate an effects-based test articulated by the Supreme Court in White v. Regester 412 U.S. 755 (1973) but later found unconstitutional in Mobile v Bolden 446 U.S. 55 (1980). See also Thornburg v. Gingles, 478 U.S. 30 (1986).

¹⁴ S.Rep. No. 97-417, 97th Cong., 2d Sess. (1982), pages 28-29.

and outcomes, this changes our understanding of the structural relationships along each causal path — for example, if we observe that partisan registration predicts partisan voting in the past, this strengthens the assumption that we can use the partisan registration in a proposed plan to predict the likely political outcome of that plan, and, based on this outcome, the intent of the actor.

The use of ToC is limited in modern redistricting litigation. In striking down the Section 4 coverage formula of the VRA, Chief Justice Roberts averred, "voting discrimination still exists; no one doubts that." 15 Yet, evidence of discrimination is difficult to find as the U.S. is far removed from the civil rights battles of a half-century ago. Furthermore, congruent application in racial gerrymandering claims to partisan gerrymandering claims is unclear. In the partisan gerrymandering context, defendants have not been held liable for partisan public statements regarding adopted redistricting plans, so one might reasonably infer courts will give little weight to the totality of the circumstances, especially given current highly charged partisan rhetoric. Still, as we shall see, there are opportunities to apply ToC to partisan gerrymandering claims when unconstitutional partisanship is found in other contexts, perhaps a legal finding that an Authority exhibited impermissible excessive partisanship in the crafting of a redistricting plan for another legislative body or a different election law.

¹⁵ Shelby County v Holder, 133 U.S. 2612 (2013) at 2.

3.3 Method of Evaluation of Process. Process evaluation or process tracing typically refers to the connection between the implementation of a process by an entity to how well the process results in outcomes consistent with the entity's goals (George and Bennett 2005). We borrow this language to describe process evaluation in the narrow context of the Figure 1 causal diagram, focusing solely on the connection between observable processes and intent, as depicted in Figure 4, while reserving analyses that examine outcomes, which may include observable processes, to other methods. We use the terminology "method of evaluation of process", rather than "process evaluation" (or "process tracing") to signify this method is not comprehensive process evaluation.

Courts may consider adoption of certain processes as indicative of an Authority's intent. Consider the following examples: an Authority solicits input only from one party or race, thereby favoring their interests; or an Authority mandated to hold public hearings to ascertain communities of interest intentionally chooses to hold hearings only in certain communities, thereby favoring these communities over others. As another example – which we discuss below – an Authority that chooses to intentionally hide its actions through opaque processes may be indicative of illicit intent. 16

3.4 Method of Inspection. Those who wish to remove politics from redistricting frequently claim that drawing districts blind to politics produces neutral results. The Arizona, California, and Iowa - states with

 $^{^{16}\ \}text{Romo}\ v.\ \text{Detzner}$ (2014). The details of this application are described below.

processes widely considered to be exemplars for redistricting reform - are formally prohibited from evaluating political data at some stage of their process. These commissions must first adhere to traditional redistricting principles, which generally include contiguity, compactness, and respect for existing political boundaries and communities of interest. Some courts have approvingly lauded these principles as a means towards "the prevention of gerrymandering." 18

The method of inspection entails examining a plan, either visually or with the aid of summary statistics, to ascertain if a constitutional violation has occurred. Why, the reasoning goes, would an Authority violate traditional redistricting principles except to gerrymander? Examples of violations of traditional redistricting principles that are indicative of political intent range from Elbridge Gerry's celebrated gerrymander to North Carolina's "bizarre in shape" Twelfth Congressional District, at some points no wider than a highway median, subject of the 1993 case Shaw v Reno.

In Figure 5, we present the causal graph underlying the method of inspection to detect improper intent by an Authority. The graph allows us to deduce several requirements for inferences when one relies on plan inspection. First, as in the method of stated intent, making inferences from inspection requires making inference about the agent state; that state is causally affected both by the agent's intent, and by the plans privately

 17 Arizona Constitution, Article IV, Part 2(1)(15); California Constitution, Article XXI, § 3.3(d)(4)-(5); and Iowa Code § 42.4(5).

¹⁸ State ex rel. Herbert v. Bricker, 139 Ohio St. 499, 509 (1942).

known to it. Second, since we do not observe privately known plans directly, we must again infer them from the observable geographic, demographic, and voting data; the observable process, and any public plans. Correct inference requires making assumptions about the structural relationship between the proposed plan and the agent state. To infer an Authority's intent requires counterfactual analysis of what the attributes of the plan would have been, if an Authority had various different intents (represented by the dotted line connecting the data and the proposed plan).

These assumptions are usually implicit when using descriptive statistics to inspect plans. Only two pieces of public information are used to compute a descriptive statistic: the adopted plan and the redistricting data from which it is was created. A court detects a gerrymander when statistics provided in evidence exceed some critical value, or a court otherwise is persuaded of a violation through visual inspection of the plan. For example, plaintiffs may claim the number of times a plan's districts split local political boundaries and the plan's lack of compactness are excessive, or plaintiffs may display district maps in courtrooms and allow judges to form opinions of violations without statistics.

As is clear from Figure 5, we do not know from the method of inspection the characteristics of alternative plans. Even if we believe that a better plan than the adopted plan exists, the method of inspection may still be challenged on several accounts. First, one runs the risk of mistaking correlation for causation, which is a common confusion (see Rosenbaum 2002 for a discussion). A lack of compactness may be an indicator of improper

political intent when odd fingers carve an incumbent's home or community of partisans from a district. However, a lack of compactness may result from adhering to other valid criteria that arise from the observable process. For example, many local political boundaries have irregular shapes as a consequence of local annexation battles. A district line that follows a local political boundary may thus lack compactness. A single descriptive statistic cannot alone provide an assessment of how well a redistricting plan comports with all legally valid goals. Indeed, in *Shaw v Reno*, the Supreme Court weighed in addition to compactness violations of other traditional redistricting principles such as contiguity, respect of geographic boundaries, and respect of political subdivisions.

Second, the detection measure may be highly correlated with the gerrymander one wishes to detect. Justice White writing in Gaffney v. Cummings noted by following traditional redistricting principles, "this politically mindless approach may produce, whether intended or not, the most grossly gerrymandered results." Parker (1990) describes how adhering to seemingly-apolitical criteria to equalize road lengths across districts effectively resulted in a racial gerrymander when county Supervisorial districts divided an urban African-American community. Districts drawn to a purportedly neutral principle may in fact be a gerrymander in sheep's clothing, what Parker refers to as second order bias.

If we cannot determine if a redistricting plan is a gerrymander by looking at it, perhaps we can more directly measure a plan's political character to assess political intent. A special class of descriptive

statistics, predictive inference measures, summarize predicted political effects of a redistricting plan. Courts widely use such methods to evaluate minority voting rights claims and to determine the effective racial composition of districts to enable a minority community an opportunity to elect a candidate of their choice (Goodman 1953; King 1997).

Redistricting commissions and courts have used similar statistical methods to evaluate compliance with state prohibitions against favoring partisan interests or proactively requiring competitive districts (Scarrow 1982; Stokes 1993; McDonald 2006). 19 Chief Justice Earl Warren, writing the majority opinion in Reynolds v Sims, articulated a standard for detecting partisan gerrymanders invoking the approach academics take by comparing the congruence of vote and seat shares: "Logically, in a society ostensibly grounded on representative government, it would seem reasonable that a majority of the people of a State could elect a majority of that State's legislators." Partisan bias occurs when a party fails to win a majority of seats with a majority of the vote; the magnitude of the bias is the deviation from the fifty-fifty vote to seat shares. A related measure is responsiveness, or how consistently a change in vote shares translates to a change in seat shares.

¹⁹ Examples of expert witnesses who have used social science approaches include Keith Gaddie in *LULAC* v. *Perry*, 126 S. Ct. 2594 (2006); Gary King in *Voinovich* v. *Quilter*, 507 U.S. 146 (1993); Jonathan Katz in *O'Lear* v. *Miller* No. 222 F. Supp. 2d 850 - 2002; Allan Lichtman in *Vieth* v. *Commonwealth of Pennsylvania*, 188 F. Supp. 2d 532 (MD Pa. 2002); and Michael P. McDonald in *In Re 2001 Redistricting Cases* (Case No. S-10504).

Scholars have studied bias and responsiveness of electoral systems through the lens of votes to seats relationships since as early as the late nineteenth century (Edgeworth 1898; Kendall & Stuart 1950; Tufte 1973). The application of the votes to seats relationship to measure and minimize partisan bias was first formally used in redistricting by a Connecticut commission after the 1970 census (Scarrow 1982) and a decade later by the New Jersey Apportionment Board (Stokes 1993). More sophisticated statistical approaches to measure bias and responsiveness have since been proposed (Gelman & King 1994).20 A recently proposed metric transforms the votes to seats relationship into terms of "wasted votes," defined as votes "cast for a losing candidate or for a winning candidate but in excess of what she needed to prevail" (Stephanopoulos & McGhee 2015: 2). However, this measure, and all similar measures, are fundamentally grounded in seats to votes relationships. Despite their long-standing usage and wide acceptance by scholars, and their use by state commissions and courts, the U.S. Supreme Court has been reluctant to endorse these methods to detect partisan gerrymandering.

The use of predictive inference statistics to detect gerrymandering suffers similar problems as descriptive statistics. Measuring how much a redistricting plan deviates from an ideal plan that treats both parties

 $^{^{20}}$ Scholars who analyze the democratic performance of electoral systems recognize that single-member district systems tend to disproportionately translate vote shares to seat shares (e.g., Rae 1967). Grofman & King (2007) thus propose partisan symmetry, that the votes to seats curve be symmetric around 50% so that either major party wins the same number of seats for a given vote share.

fairly does not provide evidence an ideal plan exists, or that it was privately known to an Authority. One must assume an ideal plan exists and was known to an Authority to make valid causal inference, as well as assume that the ideal plan conforms to contiguity, compactness, voting rights compliance, and all other legal components of the observable process.

Predictions may be in error. Two notable alleged Republican gerrymanders that were the subject of U.S. Supreme Court cases - the 1980 Indiana redistricting and 2000 Pennsylvania congressional redistricting - later in the decade yielded Democratic-majority delegations. Redistricting is just one important input into the complicated election process. Candidates, issues, and even the districts' populations may change over the course of a decade. Results should thus not be confused with intent. The latter can be best assessed through evaluation of the information available at the time redistricting takes place rather than post-redistricting election outcomes.

Together, descriptive and predictive inferential statistics describe the geographical, demographic and political characteristics of a redistricting plan. A final caution about descriptive statistics is that they can only provide statistics for redistricting goals that are measurable. For example, a common redistricting requirement is respecting communities of interest, of which there are no widely-accepted standards for identifying. If an Authority fails to geographically identify communities of interest, how much a plan respects such communities devolves into expert judgment as to the location of relevant communities' boundaries. Related, there may be

more than one way to measure a concept. For example, there are at least thirty proposed measures of compactness (Neimi, Grofman, Carlucci, & Hofeller 1990), enabling cherry-picking of the measure that best conforms to one's argument that gerrymandering occurred. Thus, to fully employ the method of inspection, one must be able to consistently measure all legally-relevant redistricting goals.

3.5 Method of Post-hoc Comparison. The primary weaknesses of the method of inspection is the lack of a counterfactual plan to test the adopted plan against. Without an alternative plan, one must assume the existence of a "better" plan, which may or may not exist. An obvious solution is to evaluate a plan against one or more alternatives plans.

Here, we assess the class of gerrymandering detection methods that compare an adopted plan to plans that are generated post-hoc to the redistricting process. The causal diagram is sketched in Figure 6, and the counterfactual is represented by the dotted line connecting the post-hoc plans and the adopted plan. This approach requires comparison of plans on relevant descriptive statistics described in the method of inspection, so in this sense post-hoc comparison (and the described below method of revealed preferences) embodies the same causal assumptions of the method of inspection. An important consideration is that to make proper inference of intent, a post-hoc plan must incorporate the relevant information that led to the agent state - the redistricting data, the observable process, and public plans - minus the illicit intent.

Post-hoc plans are often drawn by plaintiffs in voting rights cases. The U.S. Supreme Court requires plaintiffs to provide a demonstrative plan that rectifies alleged constitutional defects in an adopted plan, while meeting other constitutionally mandated and permissible requirements considered by an Authority, thereby satisfying the causal demands of the method of post-hoc comparison.²¹ The plan proffered by plaintiffs thus demonstrates that a remedy exists for the alleged violations, while satisfying legal requirements.

A gerrymandering detection method of increasing popularity among scholars is to implement a computer algorithm to generate in a random fashion a large number of post-hoc redistricting plans (or districts) and compare these plans to the adopted plan using statistical methods (Shepherd and Jenkins 1972; Cirrincone, Darling, and O'Rourke 2000; Chen and Rodden 2013; Mattingly and Vaughn 2014). Vickrey (1961) was the first to propose to program a computer to prevent gerrymandering by drawing districts according to a set of traditional redistricting principles (see also Weaver & Hess 1963). Nagel (1965) was the first to implement an automated redistricting algorithm and propose that automation could reveal alternatives, and Altman & McDonald (2011) were the first to implement an open-source algorithm employing modern optimization techniques.

²¹ See *Bartlett v. Strickland* 556 U.S. 1 (2009) (finding plaintiffs failed to demonstrate the existence of a majority-minority district that did not violate North Carolina's whole county requirement); *Easley v. Cromartie*, 532 U.S. 234, 258 (2001) (finding plaintiffs did not demonstrate race was favored over politics in the drawing of North Carolina's Twelfth Congressional District).

The automated redistricting approach compares an adopted plan to a large set of "randomly" generated post-hoc plans by appealing to a basic precept of statistics that we can determine how likely an event occurs by comparing it against the distribution frequency of all possible events. For example, if a fair coin has a 50% probability of being a head or tail we can compute the probability of observing 3 heads or fewer out of 10 flips by adding together the probabilities of observing 0 heads, 1 head, 2 heads, and 3 heads. Likewise, if the distribution of plans in a jurisdiction with ten districts is known, we can compare how often a plan with three Democratic-majority or fewer districts would occur, say compared to "fair" plans with five such districts. If plans with three Democratic-majority districts occurs with very low frequency compared to a fair plan, and the adopted plan is such a plan, we may identify the adopted plan a partisan gerrymander.

There are three fundamental problems with this approach. First, one must assume an Authority could know of the automatically generated plans. The number of plans is staggeringly large for any practical application for a moderate-sized state, well beyond the capabilities of an Authority to generate, much less analyze. One might counter an Authority could know these plans if it made an effort to find them, but certain classes of plans were uninvestigated (i.e., fair plans). However, if there are a very large number of minor variations of a fair (or unfair) plan, say an exchange of a single block between districts, computing statistics from the distribution of all plans may not inform us about the choice between a fair and unfair plan.

Second, there may be a good reason why an Authority did not explore fair options: because they do not conform to all legally required criteria. Some automated approaches attempt to draw only contiguous, compact, and equal population districts (Cirrincone, Darling, & O'Rourke 2000; Chen and Rodden 2013; Mattingly & Vaughn 2014). One can only draw inferences how an adopted plan deviates from plans drawn in accordance to these criteria. In survey research this issue is described as an incongruence of the sample frame to the population of interest, a situation that can produce biased estimates. Automated algorithms that do not generate districts in accordance with all legally required criteria present in the observable process, minus the alleged illicit intent, pose the wrong counterfactual question and cannot be used to make inferences of intent.²²

Third, and most problematic since it is often hidden from casual inspection, the implementation of the automated algorithm may be biased. All automated redistricting approaches solve the intractability of enumeration by appealing to the notion that the algorithm randomly samples from the space of feasible redistricting plans. This approach is intuitively appealing from a mathematical standpoint since statistical tests require that observed data are sampled randomly without bias from the population. However, if an algorithm is biased, well-known statistical tests cannot be applied. When researchers develop a new method, they often run their algorithm on a test

_

²² Chen and Rodden, in their expert report for *Romo v. Detzner* attempt to address the sample frame issue by fixing in place voting rights districts and preserving whole counties as they appear in the adopted plan and then applying their sampling algorithm.

problem with a known solution so they can assess its properties, like unbiasedness (Altman, Gill, & McDonald 2003). This has not been the case with proposed automated redistricting algorithms, until now.

To begin this critique, we first describe how automated algorithms work. Unlike the familiar linear regression, where the optimal solution is analytically and quickly computed, redistricting is a discrete optimization problem without a closed-form analytical solution. Redistricting algorithms construct plans out of blocks. Solving discrete optimization problems and using automation to sample requires iteration to comprehensively explore the space of possible solutions, typically by assigning blocks to or trading blocks between districts. Like enumeration, the search across all possible solutions to NP-Complete graph partitioning problems of the size typically encountered in redistricting cannot be completed in finite time, except by reshaping the problem in ways that do not produce legal redistricting plans.²³ All redistricting optimization searches and related sampling methods are thus algorithms, not analytic solutions, that may embody inherent limitations as a consequence of failing to search across the space of all feasible plans.

To illustrate these points, consider a locality composed of 48 blocks, depicted in Figure 7(a). For simplicity, assume each block has equal population. When one district is assigned 24 blocks, a second block is created out of the remainder, both of equal population. The locality is divided into

 $^{^{23}}$ Clark (2004) and Olson (2010) implement algorithms to draw contiguous and equal population plans optimized with a specific compactness algorithm, but ignore voting rights issues and other legal requirements.

four quadrants along a regular grid, creating two separate plans Figures 7(b) and 7(c), each with two equal population districts that are equally compact. A clear difference between the two plans is that 7(b) bisects the locality vertically and 7(c) bisects the locality horizontally. We arrange blocks within the quadrants slightly off a regular grid so that when we implement algorithms there are no ties in distances between a block and those adjoining it. In practice, blocks are not arrayed on grids, so our abstraction reflects reality in this regard.

We implement an algorithm proposed by various scholars to automatically generate districts (Gudgin & Taylor 1979; Cirrincone, Darling, & O'Rourke 2000). The algorithm creates districts by randomly selecting a single block and randomly adding adjacent blocks until the district reaches equal population. We run this algorithm one million times and throw out just over 850,000 instances where one district creates a non-contiguous second district. This seemingly-technical point reveals the proper unit of analysis for gerrymandering detection is the plan, which assigns all blocks to districts, and not individual districts, since geography assigned to one district is conditional on geography assigned to other districts.

We create over 138,000 unique partitions of the locality into two contiguous and equally-populated districts. We would likely discover much more if the algorithm continued to run: we find about 17 new plans for every 20 non-rejections at the one millionth iteration. This concretely illustrates our assertion that enumeration is practically infeasible even for this small locality. Since the number of feasible plans becomes exponentially larger as

the number of blocks increases, the number of feasible plans is unlikely to be appreciably reduced by introducing legal constraints beyond equal population and contiguity.

Any algorithm that purports to sample plans with compact districts in an unbiased manner must find the vertical and horizontal plans in Figures 7(b) and 7(c) with equal probability. In a million iterations, when we utilize the method proposed by Cirrincone, Darling, & O'Rourke (2000), we produce the vertical plan 595 times - three times more often than the second most common plan - but fail to produce the horizontal plan even once. Similarly, when we utilize a method proposed by Chen and Rodden (2013), their algorithm produces the vertical plan 60% of the time and the horizontal plan just 15% of the time out of one million iterations. Further problematic is that both these algorithms produce non-compact districts, albeit with low frequency. If these algorithms are biased in this simple hypothetical example, increasing the size and complexity of the locality is unlikely to resolve this bias.

As this simulation demonstrates, automated approaches, even if they incorporate randomized starting values or randomized algorithms do not yield

One might counter that duplicates can be rejected, yielding an equal probability that all plans are considered. Our response is that this is still obviously not true since some plans are selected from the space of feasible plans with higher probability than others. Furthermore, our example is specifically designed to exploit a potential weakness of automated algorithms. Note the two groupings of smaller-sized blocks in Figure 7(a), which might be analogous to more densely-populated urban areas. Both algorithms favor the vertical configuration in 7(b), that keeps these two urban areas contained within a single district, while they disfavor the horizontal configuration in 7(c) that splits these urban areas. In scaling to larger problems it is unlikely that these algorithms sample uniformly since they favor consolidating urban areas into districts over splitting them.

a random sample. Generally, one cannot rely on randomized methods to produce random samples unless they are explicitly designed to do so (see Knuth 1997). 25 No proposed redistricting plan sampling algorithm is of this character. 26

Moreover, any algorithm capable of sampling must be capable of producing all feasible redistricting plans, which although possible, is provably computationally infeasible (Altman 1997). Barring unanticipated advances in computational theory, the prospect of feasibly sampling redistricting plans randomly remains elusive. Thus, unless authors can prove otherwise, proposed automated redistricting sampling algorithms are biased and no unbiased statistical test can be applied to infer redistricting intent from them.

Two well-known standard methods of random sampling readily apply to graph partitions. The first produces random partitions, ignoring graph structure and constraints (Nijienhus & Wilf 1975), and rejects any solution that is infeasible (aka, applying rejection sampling). This approach yield a uniform random sample of feasible partitions, but unfortunately, generating even one feasible plan by this method is computationally intractable (i.e., takes practically forever) for any real redistricting problem. A second method, the standard method for sampling from graphs (Lovász 1993), requires conducting a random walk across a graph of solutions to the problem being posed: no published automated redistricting algorithms constructs a graph of solutions, related by a deterministic neighborhood function.

The sole exception, to our knowledge, is a draft paper by (Fifield, Higgins, and Imai 2014) which proposes a method of sampling from contiguous, equipopulous partitions using an MCMC method. This method has not yet been peer reviewed and a number of gaps, such as: whether the assumption that the proposed transition function yields the entire space of solutions can be justified; whether the degree of mixing is adequate in practice; and whether the method can be extended to include other common legal constraints. If this method proves theoretically justified, practical, and generalizable it would allow for unbiased sampling of plan. However, since there are exponential number of partitions to explore, MCMC and similar approaches cannot fully explore the sample space, and depend critically on identifying all high density regions of solutions. There is currently no way of proving these have been identified - so the success can only be gauged ex-post, using complex diagnostics, and provides no formal guarantees unbiased sampling.

3.6 Method of Revealed Preferences. The method of revealed preferences, presented in Figure 8, also addresses the lack of an explicit counterfactual present in the method of inspection. In contrast with the generation of post-hoc redistricting plans, the method of revealed preferences compares plans made public by any source through the redistricting process, again represented by a dotted line, such that an Authority was aware of these alternatives when it adopted a plan. A primary virtue of this approach is that we no longer need to assume the distribution of plans knowable to an Authority, as these public plans are among the Authority's privately known plans. While we still do not know all the plans privately knowable (or known) to an Authority, counterfactual public plans may exist that directly reveal an Authority's intent.

The method - like any method used to infer preferences from a rational actor - is built on a fundamental axiom in economics, the Weak Axiom of Revealed Preference or WARP (Samuelson 1948; see also Varian 2006). WARP states that if plan p_1 is chosen from a set of plans $\mathbf{P} = \{p_1...p_n\}$ then the plan must be preferred over all other plans in the set. Comparing plans allows one to map out an Authority's preference.²⁷ Unlike the previously discussed

Formally, plan characteristics measured through descriptive statistics provide metrics to evaluate choices between plans. Consider a Authority's adopted plan plan p_A , being the product of some j considerations, c_j . We might write the values of the descriptive statistics, v, for each of the considerations as $v_j = p_A(c_j)$. For example, if c_1 is minority representation, the value v_1 might be the number of districts that are able to elect a minority candidate of choice, which we might say is 3 for this plan. If c_2 is partisan advantage, the value v_2 might be the share of districts that lean to a party, which we might say is 70% for this plan. If there exists another plan, $p_1(c_j)$, that we can compare against that has $v_1 = 4$ and $v_2 = 70$ % (and is identical on all other considerations $j \neq 1$ or 2) then we can say that the Authority did not prefer to increase minority representation. More generally,

method of analyzing post-hoc plans, the method of revealed preferences does not require the distributional assumptions that limit statistical tests and sampling heuristics to reject competing hypotheses. WARP is deterministic: the probability that an alternative plan known to an Authority was chosen given that an adopted plan exists is zero.

The method of revealed preferences is not novel, although we are the first to rigorously formulate its application to redistricting. Courts and litigants informally use the method of revealed preferences when examining characteristics of plans that were rejected to illuminate why a particular plan was accepted. For example, an Arizona court found probative value in an Arizona commission's failure to adopt a proposed plan with more competitive districts over one with fewer (such districts are a constitutional requirement in the state). Scholars have also used the method to evaluate intent among competing redistricting plans (Kousser 1991; Gronke & Wilson 1999).

The causal chain in Figure 8 shows the method of revealed preferences requires that an Authority is aware of the alternative choices available to it. Computers have enabled redistricting authorities to consider more options, more quickly, and have enabled greater public participation in redistricting to further expand an Authority's menu of plans (Altman &

we might evaluate the adopted plan against all public plans, p_i , where i=1, 2, ..., n. If plans $\{p_1(c_j), p_2(c_j), ..., p_n(c_j)\}$ have identical values on all considerations $j \neq 1$ or 2, the boundaries of a Authority's willingness to trade minority representation for partisan advantage may be mapped out from the observed plans.

 $^{^{28}}$ Minority Coalition v Arizona Independent Redistricting Commission CV 2002-004380.

McDonald 2014). This public mapping revolution also enables courts to apply more robustly the standard of revealed preferences to determine if partisan gerrymandering has occurred, as the Pennsylvania Supreme Court did when overturning the state legislative plan based on the comparison to a plan drawn by a piano teacher that better conformed to state constitutional requirements.²⁹

The method of revealed preferences is not fool-proof. A set of public plans may fail to reject any of the competing hypotheses of an Authority's intent. This is analogous to statistical tests where data may not contain enough information to provide useful estimates. A common solution is to "obtain more data," which in this context means finding more plans that could reasonably have been knowable to an Authority. Some of these plans may be privately known plans internally deliberated by an Authority during the redistricting process, and made public by court order, as happened in Florida and Ohio. Others may be post-hoc plans drawn by plaintiffs' experts (by human or computer) in litigation, with the key assumption that these plans could have been knowable to an Authority.

Because an infinite number of plans exist, we are sensitive to a *laches* argument, that a legal claim against a redistricting plan must be made without unreasonable delay else it will interfere with the conduct of upcoming

²⁹ Patriot News Editorial Board, "Amanda Holt is Pennsylvania's Citizen Activist of the year," *Patriot News*, December 31, 2012.

http://www.pennlive.com/opinion/index.ssf/2012/12/amanda_holt_is_pennsylvania s citizen activist of the year.html

 $[\]overline{^{30}}$ Wilson v. Kasich, No. 2012-0019 (Ohio Sup. Ct.); Romo v. Detzner (2014).

elections.³¹ Since there is always practically an infinite number of alternative plans that have not been evaluated, there is always the possibility that a post-hoc plan exists that better satisfies legal requirements. At some point an Authority or a court must complete its work so that timely elections can be held. If plans are overturned simply because a better plan is found, litigation would likely be endless as new plans emerge from the nearly infinite possibilities. Still, we cannot dismiss that an Authority may consciously choose to ignore a potentially knowable plan in favor of a constitutionally suspect plan.

5. Case Study: Congressional Redistricting in Florida

To illustrate the application of the various methods used to detect gerrymanders, we investigate a real-world example of a redistricting process: the creation of Florida's congressional district plan following the 2010 census. This case is significant as it was the first time a court found an Authority's plan unconstitutional on partisan fairness grounds and required the state to produce a remedy. Plaintiffs provided evidence utilizing all six gerrymandering detection methods, and the Florida District Court Judge Lewis found some, but not all, of this as persuasive evidence of partisan gerrymandering in his ruling.

Florida uses the typical institutional arrangement of placing congressional redistricting in the hands of the state legislature, subject to a governor veto. During the map-drawing process, Republicans held two-

36

 $^{^{31}\,\}mbox{See}$ Veasey v Perry, No. 14-41127 (TX 5th Circuit).

thirds majorities in both chambers of the legislature as well as the governor's seat, which strongly suggests (applying the method of evaluation of process) that the party would pursue a partisan gerrymander strategy if possible. In previous decades, the appearance of partisan bias was primarily a political problem for Florida legislators. The problem became a legal one in 2010 after voters approved two constitutional amendments placing new standards on statewide redistricting plans (Eagleton & Smith 2015). These standards are arranged in two tiers, and standards within the same tier are considered to be on equal footing. The first tier mandates districts should be contiguous, should not abridge minority voting rights, and should not be drawn with the intent to favor a party or incumbent. The second tier, to be followed when not in conflict with the first, mandates equal population, compactness, and the following of existing political and geographic boundaries where possible (Colburn & MacManus 2015).

Members of the House and Senate redistricting committees held public hearings throughout the state in the summer of 2012 to garner input as to what attendees wanted from redistricting (MacManus, et al. 2015). Both chambers provided web-based software allowing individuals to draw and submit maps into the public record. To receive consideration, public submissions were due to the Florida Senate or House redistricting committees by November

_

³² In 2002, following the explicitly partisan gerrymander adopted by the Republican-controlled Florida legislature, the U.S. District Court for the Southern District of Florida observed, "This raw exercise of majority legislative power does not seem to be the best way of conducting a critical task like redistricting, but it does seem to be an unfortunate fact of political life around the country." See Martinez v. Bush, 234 F. Supp. 2d 1275, 1297 (S.D. Fla. 2002).

1, 2011. The state constitution required the legislature to submit a final congressional plan to the Governor for approval by early March, 2012. The Senate, however, did not produce a single map for public examination and comment until late November, 2011; the House, in turn, made public a series of maps in early December. Over the next two months, the Senate and House redistricting committees produced two waves of amendments on these initial plans; the Senate focused on a single plan throughout the process, whereas the House narrowed down an initial seven maps to three, then to a single map. Ultimately, the House's final plan was adopted by the Senate and signed into law by the governor.

Changes to plans made during the committee process are potentially helpful in ascertaining preferences held by the Republicans who controlled redistricting in Florida. However, these plans represent the end of the causal pathway as we have laid out above; it is reasonable to believe that maps introduced by the House and Senate do not represent the full range of possibilities known to legislators, and that preferences had already been exercised through the selection of plans to be made public. Fortuitously, in litigation surrounding the congressional district plan following its passage, the court ordered several draft plans prepared by Republican staff and operatives, dating from early November to the end of the redistricting process, be released to the plaintiffs.33

 $^{^{33}\,\}mbox{Romo}$ v. Detzner (2014), pp. 12-13.

Table 1 presents a selection of the complete or nearly complete publicly available maps that were drafted by Republicans. These can be broadly grouped into four waves: first, maps submitted to the legislature through the Senate or House websites by members of the public, second, private draft plans that preceded the committee process, third, the public plans that were introduced and amended during the committee process, and finally, private plans that were considered in the lead up to the introduction of what would become the enacted map, H000C9047 (we list this plan first due to its importance). Unless otherwise noted, each plan has a maximum population deviation of 1% and two black majority-minority districts in southeastern Florida. For each plan we provide the number of Republican-leaning districts, the number of competitive districts, a compactness score reported by the legislature and favored by the court, counts of city (census place) and county splits, and the black voting age population (VAP) of the Fifth Congressional District represented by Rep. Corrine Brown, which was a point of contention in the litigation.

To measure the number of Republican-leaning districts, we compute in each district the average of the two-party vote shares received by Democrat Barack Obama for president in 2008 and Democrat Alex Sink for governor in 2010. Both elections were competitive, high-profile, statewide races, and the statistics for both were available to lawmakers and legislative staff during the redistricting process. The Florida House publicly released the census block-level election results dataset used in their redistricting application available for download, which we utilize for our analysis.

Given the close margins in both elections - Barack Obama received 51.4% of the two-party vote and Alex Sink received 49.4% - it is not surprising that possible "fair" plans exist in which 13 or 14 of the 27 congressional districts are Republican leaning. Indeed, several early maps found among the court-ordered release are such plans. What is notable, though, is that beyond these early drafts, Republicans drew almost no maps with fewer than 15 Republican districts. The exception is one of the initial plans introduced in the House redistricting committee (H000C9001) with 14 GOP seats, but was not considered further. Moreover, Republicans tended to move away from drawing competitive districts, which is an aspect of incumbent protection and therefore also a potential violation of the Florida constitution. Defining a competitive district as falling between 48% and 52% Democratic performance, early plans and H000C9001 managed to produce nine such districts; the adopted plan contained just six.

Since contiguity was not a point of contention in the process, the other tier-one consideration of note was the ban on diminishing minorities' ability to elect a candidate of their choosing. In the litigation that resulted from the enacted plan, the composition of predominantly African-American districts became especially important, and in particular, the Fifth Congressional district that linked black communities from Jacksonville to Orlando, represented both before and after 2012 by U.S. Representative Corrine Brown. Federal courts had upheld the 2002 version of the district in which 46.2% of the voting-age population was black, but in court testimony, Republican lawmakers and legislative staff claimed that they thought a

majority-black district was a necessary prophylactic to avoid Section 2 voting rights litigation. Since blacks tend to vote for Democrats in overwhelming proportions, packing of minority voters may be equivalent to a Republican gerrymander, and can create tension among the two tier-one requirements. In July 2014, the Florida district court judge ruled that the legislature had drawn the majority-black district with the intent of favoring Republicans, and ultimately accepted a revised version that was only 48.1% black VAP.

That said, over the course of several months of mapmaking by Republican legislators, the staff of the House and Senate redistricting committees, and GOP operatives working behind the scenes, the various plans they produced varied only slightly with respect to the black VAP of the Fifth District, ranging from 47.5% to 50.1%. A number of publicly-submitted maps, on the other hand, did not attempt to create a minority district stretching from Jacksonville to Orlando. Many maps submitted by the public did not include a single black-majority district, even in southeastern Florida, where the legislature consistently managed to create two. Only four complete plans submitted by the public drew Representative Brown's district with a majority black population. Two of these maps, however, were later revealed to be drawn by a Republican consultant and submitted under a false name (HPUBC0132 and HPUBC0133). Also noteworthy was that very early in the process, Republicans had drafted a majority-black Fifth District plan that did not look so much like a Republican gerrymander, as it had just 14 districts that would likely be won by Republicans (Congress 11072011(1) A2).

Reiterating, the second tier requirements are only to be followed inasmuch as they do not conflict with the first tier. Neither the Florida Constitution nor the U.S. Constitution requires exact population equality in a congressional plan, but the courts have shown a low tolerance for population deviations, especially when a rationale for the differences is not present. For this reason, all plans introduced in the legislature, as well as post-hoc alternative plans put forward by litigation plaintiffs, had a population variation of just one person among all districts. However, it makes sense to not expect perfect population equality from draft plans, and almost none display perfect equality; zeroing out is a tedious last step of mapping. As such, it seems unlikely that plans were chosen throughout the process on this basis, with the understanding that a plan that performed well on other metrics could be zeroed out at any time with only marginal shifts in those metrics.

The parties in the lawsuit both chose to measure the tier-two requirement to follow existing political boundaries by counting the number of counties and municipalities split across two or more districts. Moving to the final requirement, there are dozens of ways to measure compactness; here we use the average Reock (1961) score across districts - one of three methods preferred by the legislature and Judge Lewis - calculated by taking the ratio of the area of a district to the area of the smallest possible circle that could contain the district (penalizing highly dispersed districts).

In moving from the second to the third wave of plans, Republicans chose not to introduce a plan with 13 districts likely to be won by Republicans; on the other extreme, they also did not introduce one with 17 Republican

districts.³⁴ Also of note is the makeup of the Fifth District: from the range of draft possibilities, the House chose to pursue a plan that had the black VAP for the Fifth District pegged at roughly 48% (which ultimately was what the court would accept when the legislature provided its remedy plan), while the Senate stuck with a black VAP for the district that was just shy of 50%.³⁵ Moving through iterations of plans introduced and then adopted by the Florida House during the legislative process -- from H000C9011 to H000C9043 to H000C9047 -- Republican lawmakers consistently chose the plan offering the greatest number of Republican seats. While the number of split counties and cities decreased over this chain of adopted plans, this is a tier-two requirement that is secondary to the tier-one requirement of not favoring a particular party or incumbent. Furthermore, the compactness measures suffered in the enacted plan compared to the prior two iterations, a metric which the Florida Constitution places on equal ground as reducing local jurisdiction splits.

During the legislature's penultimate step of amending H000C9043 to become H000C9047, the Senate convinced the House that the Fifth District needed to be majority-minority.³⁶ Due to discovery that took place during the litigation, the legislature's privately known draft maps were made public;

 34 The "Cong 132" series of draft plans, of which we've included just the final of 13 iterations, were a series of edits to HPUBC0132, one of the plans submitted by a Republican consultant under a false name.

³⁵Considering that the Senate racial demographic reports rounded figures to one decimal place, legislative staff and Republican lawmakers may have believed they were introducing a plan with a majority-minority Fifth District.

³⁶ Romo v. Detzner (2014), pp. 31-32.

as such, it is possible to view plans that were drafted by the legislature both before and after the legislature's decision to adopt H000C9047. Although all of the legislature's privately known plans retained 16 Republican districts, H000C9047 25Cities Hollywood 2Counties Hillsborough had fewer local political boundaries splits, was more compact than the enacted plan, and had a smaller uniform partisan swing needed to achieve a Democratic majority. Even limiting ourselves majority-minority plans, to H000C9047 27Cities Hollywood CD5over50 was more compact, had equal local boundary splits, and was marginally less biased against Democrats than H000C9047. Though difficult to quantify across all maps, one major difference between the legislature's January draft maps and the enacted plan was the adoption of a non-compact "appendage" to District 10 that reached into Orlando and was present in the Senate's plans, but not the House's plans. This appendage was ultimately ruled to have been added to benefit Republican incumbent Rep. Daniel Webster, and Judge Lewis struck it down in his July 2014 decision.³⁷

Judge Lewis' found constitutional violations in just two congressional districts: the Fifth and Tenth Districts. Within his ruling, Judge Lewis considered all six methods of inference in finding improper partisan intent. To begin, Judge Lewis echoes our concerns regarding the method of stated intent, "In this inquiry it its extremely unlikely that the bill's sponsor

³⁷ Ibid., pp. 31-34. In email on November 27, 2011, from Republican operative Marc Reichelderfer to Kirk Pepper, Deputy Chief of Staff to then Speaker of the House, Dean Cannon, noted that Representative Webster's district in the Senate's first plan was "a bit messed up." Pepper responded, "performance or geography?" Ibid., p. 29.

would stand up on the floor of the House or Senate and advise his or her colleagues that the intent of the legislation is to favor the Republican Party."³⁸ However, along with ordering draft maps to be made public, Lewis also ordered the release of private email communications between Republican legislators and Republican operatives. Contrasting evasive statements they made during the trial to the private emails between Republican lawmakers and operatives, Lewis applied the method of evaluation of process to conclude, "[T]hey managed to taint to redistricting process and the resulting map with improper partisan intent."³⁹

The methods of totality of the circumstances and post-hoc comparison were also considered by Judge Lewis in his decision. The legislature's amendments to the penultimate plan, H000C9043, which became the adopted plan, H000C9047, were crucial to Judge Lewis' decision to strike down District 5 and District 10. These last-minute changes were largely driven by the Senate's redistricting members and legislative staff. Plaintiff's used the fact that the Florida Supreme Court found improper intent in its 2012 decision striking down the legislature's Senate map⁴⁰ as circumstantial evidence by plaintiffs that improper intent could also be found in the congressional map.⁴¹ Although analyses using automated redistricting methods were presented by the plaintiffs to Judge Lewis through an expert report, he did not reference them

³⁸ *Ibid.*, p. 15.

³⁹ *Ibid.*, p. 22.

 $^{^{40}}$ Re: Senate Joint Resolution of Legislative Apportionment 1176 (2012), 83 So. 3d 597.

⁴¹ Romo v. Detzner (2014), p. 31.

in his decision; he does, however, reference post-hoc plans drawn manually by the plaintiffs. 42

Judge Lewis leads with reasoning based on the method of inspection to reject the two congressional districts. He references simple visual tests—"there is an odd-shaped appendage...Such appendages render a district not compact pursuant to tier—two standards"43—as well as descriptive statistics with regard to the Fifth District particularly its compactness scores, black VAP, and local jurisdiction splits. Despite not explicitly using the method of revealed preferences to make a judgement on the partisan intent of the map as a whole, Judge Lewis notes that House's plans did not contain the odd-shaped appendages that he found particularly offensive, and that in particular districts, the appendages created by the Senate allowed for more favorable conditions for Republican incumbents. Because the legislature ultimately chose a plan influenced by the Senate containing dubious appendages, Judge Lewis found a tier—one requirement violation.

Plaintiffs appealed Judge Lewis's ruling to the Florida Supreme Court, and at this time a decision is pending. As we demonstrate, applying the method of revealed preferences, there is cause for a broader ruling regarding the intent of the whole plan. The characteristics the privately and publicly known sequential plans considered by the legislature reveal Republican lawmakers cared about pushing the bounds of partisan advantage and incumbent protection, as evidenced by the existence of more equitable partisan plans

_

⁴² *Ibid.*, p. 18; 33.

⁴³ *Ibid.*, pp. 32-33.

that would also have satisfied first tier requirements of contiguity and minority voting rights, as well as second tier requirements.

6. Discussion and Recommendations

Courts routinely weigh all evidence brought before them. Our Florida case study demonstrates that courts may consider, at one time or another, all six of the gerrymandering detection methods we discuss. In the future it is likely that new metrics will be invented and old ones refashioned to divine an Authority's intent. At their heart, though, all must fundamentally be classified by one of these methods, in accordance with the causal chain presented in Figure 1. We believe that understanding the strengths and weaknesses of these six methods enables plaintiffs to make more cogent arguments and provides opportunities for courts and reformers to further buttress strengths and shore up weaknesses by demanding more transparency from redistricting authorities, and to judge them unfavorably when they act in an opaque manner. We thus make the following recommendations.

First, to avoid post-hoc justification for a gerrymander, an Authority should operate with a well-defined set of criteria at the outset of the redistricting task. These criteria may be found in constitutions or statutes, or an Authority may develop these criteria before redistricting begins. If a goal is to reduce gerrymandering, these criteria should explicitly include minimizing the type of gerrymandering one wishes to avoid, as has been done in Arizona and Washington with respect to promoting competitiveness, and Connecticut and New Jersey with respect to minimizing partisan bias. The Authority should be clear how criteria are measured, especially in the case

of measurement of ambiguous measures such as compactness. If an Authority will respect communities of interest, these should be geographically mapped prior to the redistricting to avoid post-hoc rationalizations. An Authority should justify their adopted plan, especially where tradeoffs exist and there is no clear ordering of competing criteria in a state constitution or statutes.

Second, an Authority should make all data and software tools necessary to draw and evaluate plans open-source. This sunshine avoids contestation over measurement of criteria that might occur if an Authority uses alternative data or software than persons independent of the Authority. This further allows the public to participate in the redistricting process through independent evaluation and mapping utilizing recent technological innovations, which have produced a marked increase in legal redistricting plans drawn by the public (Altman & McDonald 2014). A technical addition in our experience is that plans must made publicly available in a machine-readable format, not as pictures, as the latter deters accurate representation of alternative plans.

Related to our second recommendation, open-source data and software, combined with clear criteria, permit experts to generate post-hoc plans using the same process as an Authority, be they generated by human or computer. Building on Nagel (1965), Altman and McDonald (2007) suggest that while automation is limited in statistical tests due to potential unknown biases, automation may generate plans potentially knowable to an Authority and the method of revealed preferences can then be applied to these post-hoc plans.

Third, an Authority should operate in sunlight such that deliberations over all competing plans are discussed in open sessions. This may reveal an Authority's intent through their public statements, but its primary benefit is to prevent an Authority from hiding their intent by conducting itself differently in private than in public. When an Authority develops and deliberates plans in private, the public and courts cannot assess the motivation that may have been present leading to the release of their public plans. If the truth is to be known, courts should not grant legislative immunity to Authorities and should be skeptical when an Authority engages in extraordinary measures to hide plan development, such as hiring a law firm to draw plans and invoking attorney-client privilege when facing litigation.

Fourth, an Authority should allow ample public commenting periods so that the public can evaluate plans and make recommendations. Encouraging public submissions is important to establish intent through the method of revealed preferences, as plans knowable to an Authority that perform better than an Authority's plan on its stated goals are strong evidence of constitutionally impermissible intent. An Authority cannot be relied upon to fully explore their options, even if some plans are easily discoverable. When an Authority abandons a promising line of plans in pursuit of constitutionally suspect goals, comparison of an adopted plans to post-hoc plans drawn by humans or computers, using the promising plan as a starting point, may be employed to answer the question – using the method of revealed preferences – what might have happened if the authority had continued virtuously.

Fifth, change the process to reduce the incentives for purposeful discrimination by adopting a citizen commission removed from the legislature (McDonald 2007; Alman & McDonald 2012, 2014). In theory, a citizen commission is one whose members are selected through a vetting process that weeds out political actors with conflict of interests between the districts they create and other goals. Of course, it is unlikely that political motives can be fully removed from the process, or as Madison stated in Federalist 51, "If men were angels, no government would be necessary." Still, breaking strong ties between an Authority and politics potentially reduces discriminatory intent, reducing the need to detect such intent.

Aside from our fifth recommendation, these standards can be applied to any Authority, be it a legislature or a commission. When an Authority fails to achieve these standards, courts should consider such failure as indicative of intent to gerrymander since it limits the ability for courts to establish intent. There are additional benefits to our recommendations beyond detecting gerrymandering. Transparency and public participation can provide better representation when the public expresses their representational needs to an Authority and these wishes are incorporated into an adopted plan (Altman & McDonald 2014). The public can have greater confidence that an Authority acted without improper intent when its actions are open to public scrutiny. Preventing gerrymandering through improved detection methods is thus merely a means to an end of fostering better representation and confidence in democratic governance.

References

Allen, Liz, Amy Brand, Jo Scott, Micah Altman, and Marjorie Hlava. 2014. "Credit Where Credit is Due," Nature 508: 312-13.

Altman, Micah, 1997. "Is Automation the Answer: The Computational Complexity of Automated Redistricting," Rutgers Computer and Law Technology Journal 23 (1), 81-142.

Altman, Micah. 1999. "Modeling the Effect of Mandatory District Compactness on Partisan Gerrymanders," Political Geography 17: 989-1012.

Altman, Micah. 2002. "A Bayesian Approach to Detecting Electoral Manipulation," Political Geography 21: 39-48.

Altman, Micah, and Michael P. McDonald. 2007. "The Limitations of Quantitative Methods for Analyzing Gerrymanders: Indicia, Algorithms, Statistics and Revealed Preference." Paper presented at the 2007 Midwest Political Science Association Meeting, Chicago, IL.

Altman, Micah, and Michael P. McDonald. 2015. "Paradoxes of Political Reform: Congressional Redistricting in Florida," in Seth C. McKee, ed., *Jigsaw Puzzle Politics in the Sunshine State*. Gainesville: University of Florida Press, pp. 163-84.

Altman, Micah, Karin Mac Donald, and Michael P. McDonald. 2005. "Pushbutton Gerrymanders? How Computing Has Changed Redistricting," in Bruce Cain and Thomas Mann, eds., Party Lines: Competition, Partisanship and Congressional Redistricting. Brookings Press, Washington, DC.

Altman, Micah, and Michael P. McDonald. 2012. "Redistricting Principles for the Twenty-First Century," Case-Western Law Review 62: 1-26.

Altman Micah, and Michael P. McDonald. 2014. "How Independent Commissions Could Use the Internet and Open Software to Maximize Transparency and Public Engagement in Redistricting." Brief, Scholars Strategy Network.

Chen, Wei, and Jonathan Rodden. 2013. "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures," Quarterly Journal of Political Science 8: 239-69.

Cirincione, C., T.A. Darling, and T.G. O'Rourke. 2000. "Assessing South Carolina's 1990's Congressional Districting," *Political Geography* 19: 189-211.

Clark, George L. 2004. Stealing Our Votes: How Politicians Conspire to Control Elections and How to Stop Them. Pittsburgh: Dorrance Publishing Company.

Colburn, David C. and Susan A. MacManus. 2015. "Forward," in Seth C. McKee, ed., *Jigsaw Puzzle Politics in the Sunshine State*. Gainesville: University of Florida Press, pp. xii-xix.

Eagleton, Joseph P., and Daniel A. Smith. 2015. "Drawing the Line: Public Support for Amendments 5 and 6," in Seth C. McKee, ed., *Jigsaw Puzzle Politics in the Sunshine State*. Gainesville: University of Florida Press, pp. 109-25.

Edgeworth, Frances Y. 1898. "Miscellaneous Applications of the Calculus of Probabilities." Journal of the Royal Statistical Society 51: 534-542. Fifield, B., M. Higgins, K. Imai, and A. Tarr. 2014. "A New Automated Redistricting Simulator Using Markov Chain Monte Carlo." Draft Working Paper. (retrieved Dec 12, 2014)

Gelman, Andrew, and Gary King. 1994. "Enhancing Democracy Through Legislative Redistricting," American Political Science Review 88: 541-59.

George, Alexander L. and Andrew Bennett. 2005. Case Studies and Theory Development. Cambridge Massachusetts: MIT Press.

Goodman, Leo. 1953. "Ecological regression and the behavior of individuals," American Sociological Review 18: 663-64.

Grilli di Cortona, Pietro, Cecilia Manzi, Aline Pennisi, Fredrica Ricca, and Bruno Simeone. 1987. Evaluation and Optimization of Electoral Systems. Cambridge: Society for Industrial and Applied Mathematics.

Grofman, Bernard, and Lisa Handley, eds. 2008. Redistricting in Comparative Perspective. New York: Oxford University Press.

Grofman, Bernard, Lisa Handley, and David Lublin. 2001. "What Minority Populations and Sufficient to Afford Minorities a Realistic Chance to Elect a Candidate of Their Choice? Drawing Effective Minority Districts: A Conceptual Framework and Some Evidence," North Carolina Law Review 79: 1383-430.

Grofman, Bernard, and Gary King. 2007. "The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering after LULAC v. Perry," Election Law Journal 6: 2-35.

Grofman, Bernard, Michael Migalski, and Nicholas Noviella. 1985. "The 'Totality of the Circumstances Test' in Section 2 of the 1982 Extension of the Voting Rights Act: A Social Science Perspective," Law and Policy 7: 199-223.

Gronke, Paul, and J. Matthew Wilson. 1999. "Competing Redistricting Plans as Evidence of Political Motives: The North Carolina Case," American Politics Research 27: 147-76.

Gudgin, Graham, and Peter J. Taylor. 1979. Seats, Votes and the Spatial Organization of Elections. London: Pion Limited.

Kendall, M. G. and A. Stuart. 1950. "The Law of Cubic Proportions in Electoral Results," British Journal of Sociology 1(1): 183-197.

King, Gary. 1997. A Solution to the Ecological Inference Problem: Reconstructing Individual Behavior from Aggregate Data. Princeton University Press, Princeton.

Kousser. Morgan. 1991. "How to Determine Intent: Lessons from L.A.," Journal of Law & Politics 7: 591-732.

Kreiger, Linda J., and Cindi Fox. 2013. "Evidentiary Issues in Sexual Harassment Litigation," Berkeley Journal of Gender, Law & Justice 1: 115-39.

Law, David S. and David T. Zaring. 2010. "Law Versus Ideology: The Supreme Court and the Use of Legislative History," William and Mary Law Review 51: 1653-1743.

Levitt, Justin. 2011. "Essay: Weighing the Potential of Citizen Redistricting," Loyola of Los Angeles Law School Review 44: 513-43.

Lovász, László. "Random walks on graphs: A survey." Combinatorics, Paul erdos is eighty 2, no. 1 (1993): 1-46.

MacManus, Susan A., et al. 2015. "Redistricting in Florida: Loud Voices from the Grassroots," in Seth C. McKee, ed., *Jigsaw Puzzle Politics in the Sunshine State*. Gainesville: University of Florida Press, pp. 126-162.

Mattingly, Jonathan C., and Christy Vaughn. 2014. "Redistricting and the Will of the People." Available: http://www.math.duke.edu/~jonm/PaperArchive/14/gerrymandering.pdf.

McDonald, Michael P. 2006. "Re-Drawing the Line on District Competition," PS: Political Science and Politics 39: 99-102.

McDonald, Michael P. 2014. "Calculating Presidential Vote in Legislative Districts," State Politics and Policy Quarterly 14: 196-204.

McDonald, Michael D., and Richard C. Engstrom. 1990. "Detecting Gerrymandering," in B. Grofman, ed., *Political Gerrymandering and the Courts*. Agathon: New York.

Nagel, Stuart. 1965. "Simplified Bipartisan Computer Redistricting," Stanford. Law Review 17: 863-99.

Neimi, Richard, Bernard Grofman, Carl Carlucci, and Thomas Hofeller. 1990. "Measuring Compactness and the Role of a Compactness Standard in a Test for Partisan and Racial Gerrymandering," Journal of Politics 52: 1155-81.

Nijenhuis, Albert, and Herbert S. Wilf. "A method and two algorithms on the theory of partitions." Journal of Combinatorial Theory, Series A 18, no. 2 (1975): 219-222.

Olson, Brian. 2010. Redistricter: A Non-Gerrymandered Impartial Redistricting Program. Available: http://code.google.com/p/redistricter/(last visited Feb. 26, 2015)

Parker, Frank R. 1990. *Black Votes Count*. Chapel Hill: University of North Carolina Press.

Pearl, Judea. 2000. Causality: Models, Reasoning, and Inference. New York: Cambridge University Press.

Rae, Douglas. 1967. The Political Consequences of Electoral Laws. New Haven: Yale University Press.

Reock, Ernest C., Jr. 1961. "A Note: Measuring Compactness as a Matter of Legislative Apportionment," Midwest Journal of Political Science 5: 70-4.

Rosenbaum, Paul R. 2002. Observational Studies, 2nd ed. New York: Springer.

Rossiter, D.J. and R.J. Johnston. 1981. "Program GROUP: the identification of all possible solutions to a constituency-delimitation problem," *Environment and Planning* 13: 231-8.

Scarrow, Howard A. 1982. "Partisan Gerrymandering-Invidious or Benevolent? Gaffney v. Cummings and Its Aftermath," The Journal of Politics 44: 810-21.

Samuelson, Paul. 1948. "Consumption Theory in Terms of Revealed Preference," Economica 60: 243-53.

Shepherd, J.W., and M.A. Jenkins. 1972. "Decentralizing High School Administration in Detroit: A Computer Evaluation of Alternative Strategies of Political Control," *Economic Geography* 48: 95-106.

Stephanopoulos, Nicholas O., and Eric M. McGhee. 2015. "Partisan Gerrymandering and the Efficiency Gap," *University of Chicago Law Review* 82 (forthcoming).

Stokes, Donald. 1993. Legislative Redistricting by the New Jersey Plan. New Brunswick, NJ: Fund for New Jersey.

Tufte, Edward R. 1973. "The Relationship between Seats and Votes in Two-Party Systems." The American Political Science Review 67(2): 540-554.

Varian, Hal R. 2006. "Revealed Preference," in Michael Szenberg, ed., Samuelsonian Economics and the 21st Century. Oxford: Oxford University Press.

Vickrey, William. 1961. "On the Prevention of Gerrymandering," Political Science Quarterly 76: 105-10.

Weaver, James B. and Sidney W. Hess. 1963. "A Procedure for Nonpartisan Districting: Development of Computer Techniques," The Yale Law Journal 72: 288-308.

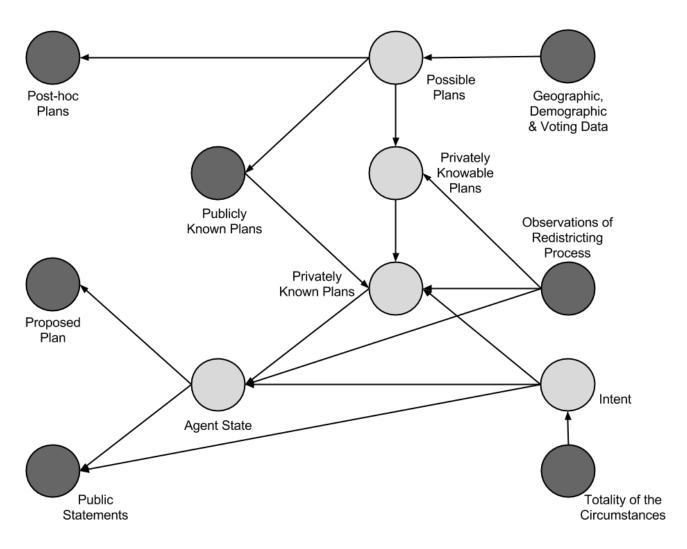


Figure 1. An Authority's Causal Process to Adopt a Plan

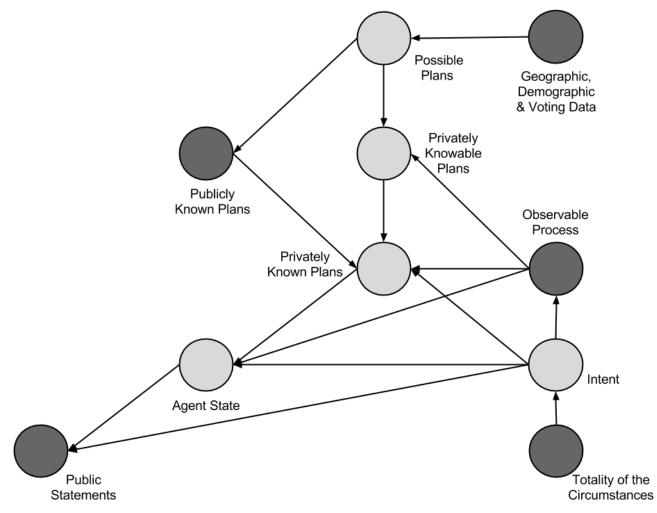


Figure 2. Causal Diagram for Method of Stated Intent

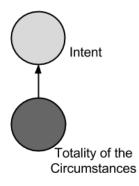


Figure 3. Causal Diagram for Method of Totality of the Circumstances

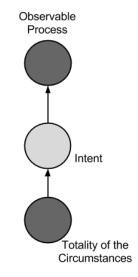


Figure 4. Causal Diagram for Method of Evaluation of Process

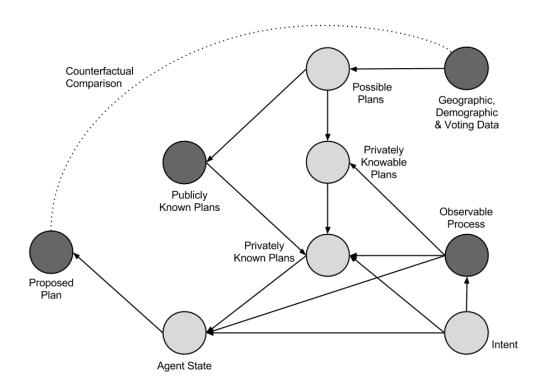


Figure 5. Causal Diagram for Method of Inspection

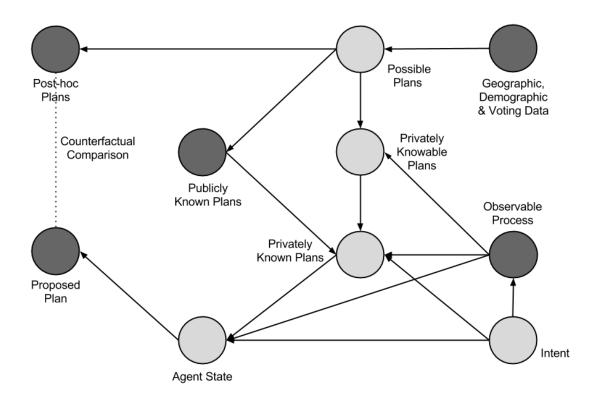


Figure 6. Causal Diagram for Method of Post-Hoc Plans

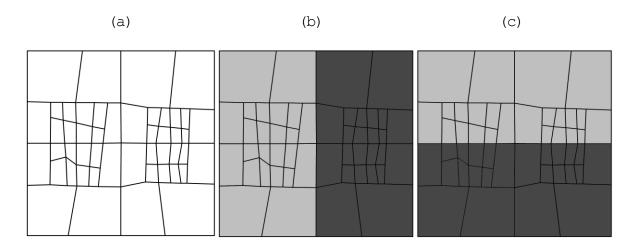


Figure 7. A Hypothetical Locality (a) Composed of 48 Equal Population Blocks Partitioned into Two Equally Compact and Equal Population Districts (b) & (c)

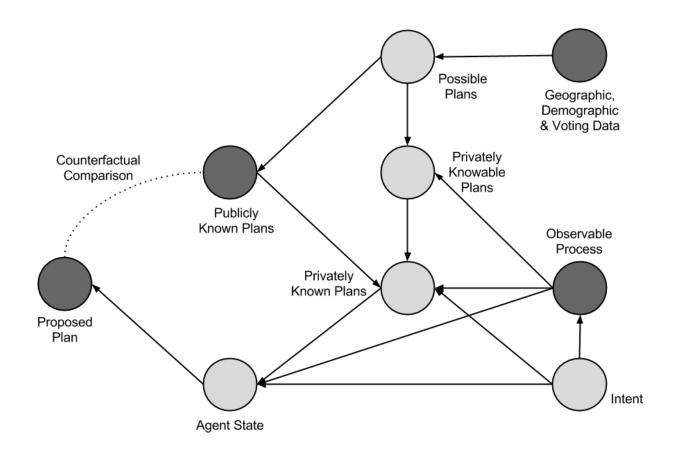


Figure 8. Causal Diagram for Revealed Preferences

	Enacted Map (Jan	uary 25 2012\				
	Republican	Competitive	Compactness	Split	Split	% Black VAP,
Map	Districts	Districts	(Reock)	Cities	Counties	District 5
H000C9047	16	6	0.39	27	21	50.06%
Maps Submitt	ed by Public (May	27, 2011 - Novem	nber 1, 2011)			
	Republican	Competitive	Compactness	Split	Split	% Black VAP,
Map HPUBC0001	Districts	Districts 3	(Reock)	Cities	Counties	District 5
	16		0.43	84	33	50.26% *
SPUBC0068 (no black majority districts or District 5 analogue		5	0.43	85	36	
HPUBC0132 (6 districts with population deviation)	17	3	0.37	91	24	50.13%
HPUBC0139	15	3	0.40	81	28	50.17%
SPUBC0154	15	6	0.32	115	40	48.00%
Pre-Committee D	Draft Maps (Novem Republican	iber 7, 2011 - Dec Competitive	cember 1, 2011) Compactness	Split	Split	% Black VAP,
Мар	Districts	Districts	(Reock)	Cities	Counties	District 5
Congress_11072011(1)/Congressional 1	13	9	0.38	51	24	47.54%
Congress_11072011(1)_A2	14	6	0.41	73	25	50.24%
Congress_11072011(1)_A4	15	5	0.38	77	27	50.24%
Congress_11072011(1)_A5	15	7	0.38	74	27	50.24%
Cong 132 rev13	17	6	0.36	93	27	50.59%
Congressional 2	15	6	0.40	59	26	48.27%
Congressional 2revised6	16	6	0.38	69	26	50.02%
Congressional 2revised7	16	6	0.39	72	26	50.02%
Congressional 3.kmz	15	8	0.39	53	22	48.10%
		6			24	
Congressional 4.kmz	15	7	0.40	58		48.27%
Congressional 5.kmz	15		0.40	55	26	48.10%
Congressional 6.kmz	16	6	0.42	48	22	48.09%
Congressional 7.kmz	15	7	0.40	59	24	48.10%
Committee Introduc	D					
Committee-Introduc	Republican	Competitive	Compactness	Split	Split	% Black VAP,
Мар	Districts	Districts	(Reock)	Cities	Counties	District 5
First Roui	nd (November 28, 2	011 – December	6, 2011)			
H000C9001	14	9	0.20	F-1	24	47.53%
1100000000			0.38	51		
HUUUC9UU3	16	6	0.38	57	26	48.27%
H000C9005	16 15				26 22	
H000C9003 H000C9005 H000C9007		6	0.41	57		48.27%
H000C9005	15	6 8	0.41 0.39	57 50	22	48.27% 48.10%
H000C9005 H000C9007	15 16	6 8 6	0.41 0.39 0.40	57 50 55	22 24	48.27% 48.10% 48.27%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041)	15 16 15	6 8 6 7	0.41 0.39 0.40 0.45	57 50 55 52	22 24 26	48.27% 48.10% 48.27% 48.10%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045)	15 16 15 16	6 8 6 7 6	0.41 0.39 0.40 0.45 0.42	57 50 55 52 48 58	22 24 26 22 23	48.27% 48.10% 48.27% 48.10% 48.09% 48.10%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006)	15 16 15 16 15	6 8 6 7 6 7 6	0.41 0.39 0.40 0.45 0.42 0.40 0.39	57 50 55 52 48	22 24 26 22	48.27% 48.10% 48.27% 48.10% 48.09%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006)	15 16 15 16 15 15 15 ound (December 30)	6 8 6 7 6 7 6	0.41 0.39 0.40 0.45 0.42 0.40 0.39	57 50 55 52 48 58 63	22 24 26 22 23 23	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Re	15 16 15 16 15 15 25 20und (December 30)	6 8 6 7 6 7 6 2, 2011 – January	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012)	57 50 55 52 48 58 63	22 24 26 22 23 23	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047)	15 16 15 16 15 15 15 15 16	6 8 6 7 6 7 6 7, 2011 – January 7	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42	57 50 55 52 48 58 63 44 39	22 24 26 22 23 23 26 22	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045	15 16 15 16 15 15 15 25 20 21 15 16 16 15	6 8 6 7 6 7 6 7, 2011 – January 7 6	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40	57 50 55 52 48 58 63 44 39 48	22 24 26 22 23 23 26 22 23	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045	15 16 15 16 15 15 15 0und (December 30) 15 16 15	6 8 6 7 6 7 6 0, 2011 – January 7 6 7	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42	57 50 55 52 48 58 63 44 39	22 24 26 22 23 23 26 22	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014)	15 16 15 16 15 15 0und (December 30 15 16 15 15	6 8 6 7 6 7 6 2, 2011 – January 7 6 7 6	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40 0.38	57 50 55 52 48 58 63 44 39 48 45	22 24 26 22 23 23 26 22 23 24	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05% 48.05% 49.96%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014)	15 16 15 16 15 15 15 0und (December 30) 15 16 15	6 8 6 7 6 7 6 0, 2011 – January 7 6 7	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40	57 50 55 52 48 58 63 44 39 48	22 24 26 22 23 23 26 22 23	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014)	15 16 15 16 15 15 20und (December 30) 15 16 15 15 Third Round (January)	6 8 6 7 6 7 6 0, 2011 – January 7 6 7 6 20ary 12, 2012)	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40 0.38	57 50 55 52 48 58 63 44 39 48 45	22 24 26 22 23 23 26 22 23 24	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05% 49.96%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014)	15 16 15 16 15 15 20 ound (December 30) 15 16 15 15 Third Round (Janual)	6 8 6 7 6 7 6 7, 2011 – January 7 6 7 6 2017 12, 2012) 6	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40 0.38	57 50 55 52 48 58 63 44 39 48 45	22 24 26 22 23 23 26 22 23 24	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05% 49.96%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014)	15 16 15 16 15 15 20und (December 30) 15 16 15 15 Third Round (January)	6 8 6 7 6 7 6 0, 2011 – January 7 6 7 6 20ary 12, 2012)	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40 0.38	57 50 55 52 48 58 63 44 39 48 45	22 24 26 22 23 23 26 22 23 24	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05% 49.96%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014) S004C9014 Pre-H000C9047	15 16 15 16 15 15 15 10 15 15 16 15 16 15 16 15 17 17 18 19 19 19 19 19 19 19 19 19 19 19 19 19	6 8 6 7 6 7 6 7, 2011 – January 7 6 7 6 2017 12, 2012) 6	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40 0.38 0.38 uary 24, 2012) Compactness	57 50 55 52 48 58 63 44 39 48 45 46	22 24 26 22 23 23 26 22 23 24 24	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05% 49.96% 49.96%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014) S004C9014 Pre-H000C9047 Map	15 16 15 16 15 15 15 00und (December 30) 15 16 15 15 Third Round (January 15) Praft Maps (January 15) Republican Districts	6 8 6 7 6 7 6 7, 2011 – January 7 6 7 6 2017 (2012) 6	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40 0.38 0.38 uary 24, 2012) Compactness (Reock)	57 50 55 52 48 58 63 44 39 48 45 46 Split Cities	22 24 26 22 23 23 26 22 23 24 24 Split Counties	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05% 49.96% % Black VAP, District 5
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014) S004C9014 Pre-H000C9047 Map AlexJPAmendment_2to9043	15 16 15 16 15 15 15 10 15 15 16 15 16 15 16 15 17 17 18 18 19 19 19 19 19 19 19 19 19 19 19 19 19	6 8 6 7 6 7 6 7, 2011 – January 7 6 7 6 2017 12, 2012) 6 ary 10, 2012 - January Competitive Districts	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40 0.38 0.38 uary 24, 2012) Compactness (Reock) 0.41	57 50 55 52 48 58 63 44 39 48 45 46 Split Cities 31	22 24 26 22 23 23 26 22 23 24 24 Split Counties 21	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05% 49.96% % Black VAP, District 5 48.03%
H000C9005 H000C9007 H000C9009 (predecessor to H000C9041) H000C9011 (predecessor to H000C9043) H000C9013 (predecessor to H000C9045) S000C9002 (predecessor to S000C9006) Second Rel H000C9041 H000C9043 (predecessor to H000C9047) H000C9045 S000C9006 (predecessor to S000C9014) S004C9014 Pre-H000C9047 Map AlexiPAmendment_2to9043 H000C9047_24Cities_Hollywood_2Counties	15 16 15 16 15 15 15 15 16 15 15 16 15 16 15 17 17 18 19 19 19 19 19 19 19 19 19 19 19 19 19	6 8 6 7 6 7 6 7, 2011 – January 7 6 7 6 2017 12, 2012) 6 ary 10, 2012 - January Competitive Districts 7	0.41 0.39 0.40 0.45 0.42 0.40 0.39 4, 2012) 0.40 0.42 0.40 0.38 0.38 uary 24, 2012) Compactness (Reock) 0.41 0.40	57 50 55 52 48 58 63 44 39 48 45 46 Split Cities 31 24	22 24 26 22 23 23 26 22 23 24 24 Split Counties 21 20	48.27% 48.10% 48.27% 48.10% 48.09% 48.10% 49.96% 48.05% 48.05% 49.96% % Black VAP, District 5 48.03% 48.01%

H000C9047- in progress

0.39

50.04%