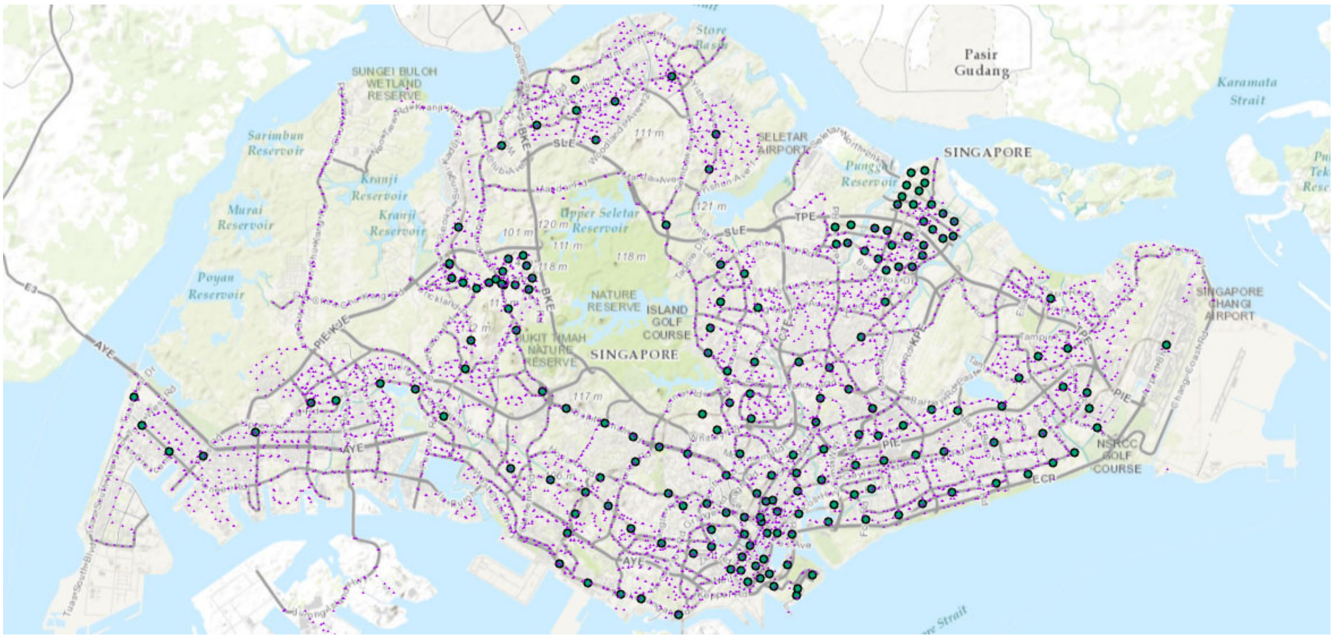


ROBUSTNESS OF SINGAPORE PUBLIC TRANSPORTATION NETWORK

PROJECT REPORT

GROUP 8 : THE DAYLIGHT ANALYSTS	
Chew Tze Hoong	A0123694
Liang Nanxing	A0123702
Vishnu Gowthem Thangaraj	A0134525

EXECUTIVE SUMMARY

There were eleven media-documented cases of train breakdown events in Singapore in the first three months of 2015, triggering waves of commuters' dissatisfaction. The team recognises the need for the Singapore mass public transportation system to be improved, but given limited resources, the need to identify critical stretches of train routes is strong. This report details the team's efforts in mapping the Singapore mass transportation network, and in quantifying impacts brought about by train failures on Singapore as a whole, and on localized residential districts. The team further proposed areas where the developed model may be applied, to generate social-economical impacts.

PROBLEM

There were eleven media-documented mass rapid transit (Includes train networks operated by both SMRT Corporation and SBS Transit) MRT breakdowns in Singapore in the first three months of 2015, sparking off waves of public criticism on the Singapore train network. While the intuitive step to resolve the issue should target at the root problem (i.e. maintenance issues), our team believes that a two-pronged approach should be considered, to include building an overall mass public transportation network that is resilient to failures.

PROJECT OBJECTIVES

The team would like to analyse the current mass public transport network, to understand the robustness of the network, through removing selected edges. This mimics the Singapore train breakdown impacts, i.e. an entire stretch of train station is affected, rather than the mere local site where the train fault is located at. The team would then seek to identify crucial train paths which when removed, will bring about significant impacts to the mass public transport network. The team will also quantify the positive impact brought about by inclusion of new transportation nodes that has been planned, by simulating the benefits brought about by newly-included network edges.

PROJECT SIGNIFICANCE

This project identifies stretches of MRT paths which are critical to the mass public transportation network. Specifically, removing these paths will bring about significant impacts to the public due to a lack of convenient alternative paths. This project also evaluates the efficacy of proposed transportation nodes (new trains stations and bus stops), to understand the benefits on the transportation network brought about by additions of new nodes/edges. The results of this study may be further disseminated to transportation companies so that special attention may be made to reduce chances of breakdown along these paths. The eventual goal would be improved public confidence on the mass public transport network, and better corporate image of train companies.

ASSUMPTIONS

1. Train networks by SMRT Corporation and SBS Transit form the core train network in Singapore.

2. Bus services by SMRT Corporation and SBS Transit form the core bus network in Singapore. All other smaller-scaled bus operators operating on flexible routes (These routes are predominantly in industrial estates, connecting factories to the nearest MRT station) have small-to-negligible impact on the overall transportation network.

3. Bulk of mass public transport commuters will choose to complete their journey via mass public transport, as such, we do not factor impacts of cabs in the study of resilience of mass public transportation network.

4. The time taken by a bus / train to stop at a station for passengers to alight / drop off are not factored into the travelling time as they are not significant.

5. Non-peak hours travelling is not considered.

6. During peak hours, buses travel on an average of 25km/h.

7. Commuters prefer to take the public transport to reach a station if the distance is greater than 400 metres. Research has shown that 400m is the acceptable distance that an average person will walk rather than drive or take the public transport. Short links (less than or equal 400m) between stations (bus, MRT, LRT) are represented by a 4.8km/h travelling speed for walking.

8. The waiting time due to switching mode of transport is estimated at:
    a. change to bus : 8 minutes
    b. change to MRT/LRT : 3 minutes
    c. change MRT/LRT at interchanges : 5 minutes
9. For MRT/LRT stations that are still under construction, we estimate their travelling time to be 2 mins each from station to station.
10. Commuters have perfect knowledge of the public transport system and travel by the shortest path.
11. The traffic conditions are assumed to be optimal and uniform during the peak timings and the travelling and waiting time estimations are accurate throughout the entire network.

RELATED WORK

The analysis of the public transport system's resilience, reliability and robustness (or vulnerability) is a key concern in many countries and has been an important research topic for network analysis. Network vulnerability studies in general, have had contributions from various disciplines and from various countries, evidenced from its available literature. For example, Eduardo et al., 2014[1] outlines measuring the vulnerability and criticality in public transport networks.O Cats and E. Jenelius (2012)[2] considers vulnerability analysis of public transportation in the circumstance of disruption due to non-continuous availability. Murray (2013)[3] also suggested using multiple methodologies for network vulnerability approaches, namely, scenario specific, strategy specific, simulation and mathematical modeling.

Knoop et al., 2012[4] outlined the methodologies for assessing robustness of networks. Scott et al., 2006[5] provides alternative measures of link importance in graphs and Derrible & Kennedy, 2010[6] details how the number of cyclic paths in a metro system seem to directly correlate with the robustness of the network. For Singapore, more specifically, Harold Soh et al., 2010[7] does a network analysis on the public transportation routes in Singapore with certain graph measures and simulation models such as MATSim (Medina et al. 2013)[8] have been developed to stress test the resilience of the Singapore Transport system.

DATA COLLECTION

There were four main sources of data that were required for this project:

Transit Link Bus Routes data

The data were from Transit Link, where users could enter the bus number to obtain bus route information such as, bus direction, bus stop codes, distance between the bus stops and bus stop description. As the webpage was in php, we took a different approach to crawl the bus route information. All bus numbers were stored first and subsequently iterated by appending it to the link (e.g. http://www.transitlink.com.sg/eservice/eguide/service_route.php?service=10. Each of the bus route web pages were then crawled using Python with BeautifulSoup and Selenium (with PhantomJS to "load" the web pages in a browser for scraping) packages.

OpenStreetMap Metro data

The data were from http://metro.teczno.com/#singapore , which stored the osm (OpenStreetMap) XML file for Singapore. Geospatial information for locations such as amenities, transport nodes, hospitals, schools etc were available in the XML file. To identify a MRT or LRT station, nodes with the tag k = 'railway' were scraped. For bus stops, nodes with the tags k='highway' and v='bus_stop' were scraped. Information obtained are the station name / bus stop codes and their, longitude, latitude and description. For the MRT stations, data on stations that were not constructed yet were also available and are included in our study. The information was parsed using Python's BeautifulSoup and only relevantly information were extracted. Note that about 200 bus stop codes obtained from TransitLink could not be mapped to the nodes in the osm file.

Wikipedia (MRT/LRT)

The MRT/LRT station nodes and edges data were obtained from Wikipedia. The travelling time from station to station were obtained from other sources such as MRT mobile applications and SBSTransit. Since the numbers are significantly lesser, we were able to manually copy, paste the information from the internet into a spreadsheet.

District (Sub zone) Planning Boundaries, and Population Density Information

To account for commuter count and their travel start and end destinations, information on how the island is stratified into residential districts, as well as the population count in each district is required. We obtained a geospatial shapefile from data.gov.sg that provided boundary information on planned districts, and the breakdown in population count (by age) for each district from www.singstat.gov.sg.

The codes for obtaining the data are found in Appendix A: Data Retrieval.

DATA TRANSFORMATION

Obtaining Bus Nodes and Edges

After obtaining the bus routes, the next step was to convert the information into nodes and edges. For each bus number and each direction, the bus stops were ordered based on the distance travelled (km) from the beginning terminal in ascending order. For each row, the lag (previous) value for the bus stop code and distance travelled were extracted and merged together with the main data set. The lag value of the bus stop code indicate the start of the edge and the bus stop code indicate the end of the edge. To obtain the distance between the bus stops, we subtract the lag distance travelled from distance travelled.

Building Links for Stations that are 400m apart

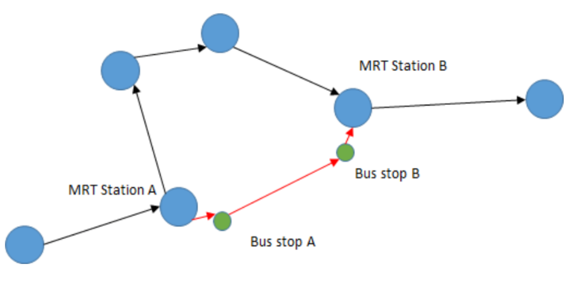
Short travelling distances between stations need to be modeled to consider the walking commute from one station to another nearby station. The assumption made here is that commuters will walk if the distance is less than or equal to 400 metres, and take the public transport if it is more than that. To identify nodes that are of close proximity to each other, we obtained the longitude and latitude of all stations, cross-join on the stations and calculated the Haversine distance between all combinations. There were a total of 4945 stations in our data set, and the number of combinations was approximately 24 million. Since the longitude and latitude obtained had 7 decimals, the precision of the distance was very high. The Haversine distance is one of the more accurate and computationally faster calculation for short distances and is represented by the following formula:

$$2r \arcsin\left(\sqrt{\sin^2\left(\frac{\phi_2-\phi_1}{2}\right)+\cos(\phi_1)\cos(\phi_2)\sin^2\left(\frac{\lambda_2-\lambda_1}{2}\right)}\right)$$

where $\boldsymbol{r}$ is the radius of the sphere (Earth) and approximated to 6378137 metres, $\boldsymbol{\phi}$ is the latitude and λ is the longitude of the coordinates. Note that the decimal degree coordinates need to be converted to radians coordinates.

After the Haversine calculation is completed for all combinations, the next step is to filter out all node to node distances that are less than or equal 400 metres. A walking time using an average walking speed of 4.8km/h is then defined for these edges.

Modelling Waiting Time for Switching Mode of Transport



If we do not penalize the travelling time made by commuters when they switch their mode of transport, there will be scenarios where commuters will switch transport because of the shortest path, but in real-life situations, this will not happen. E.g., the travelling time from MRT station A to station B is 5 minutes and the travelling time from bus stop beside MRT station A to station B is 3 minutes. If waiting time is not penalized, the simulated commuter travelling on an MRT passing by station A and B will proceed to switch to bus transport at station A, reach the bus stop at station B, and subsequently switch back to MRT to continue his travel. In normal circumstances, the commuter will continue his route on the MRT..

To model this, we need to identify edges that are of the following:

a. MRT station to bus stop edge (identified previously by modelling less than 400m edges): 8 minutes waiting time + walking time.

b. Bus stop to MRT station edge (identified previously by modelling less than 400m edges): 3 minutes waiting time + walking time.

c. MRT line switch (identify MRT interchanges): 5 minutes waiting time (walking time factored in).

d. Bus to bus switch at same station (identify stations that have more than one bus route): 8 minutes waiting time.

e. Bus to bus switch at different stations that are connected through a walkable distance (<= 400m): 8 minutes waiting time + walking time. This scenario was realised to be not accounted for during the later stages of modeling. It was discovered after sampling the shortest paths and realising that most of them involved walking from one bus stop to another and taking buses without any waiting time penalty.

Scenarios a to c are relatively easy to identify, but d and e required some transformations.

To achieve scenario d, we split the bus stop nodes into individual bus-stop-route nodes; e.g. bus stop 115 has bus routes 10, 20, 30. We transform bus stop 115 node to create 3 new nodes called 115_10, 115_20, 115_30. We then add waiting time edges to all the combinations of bus transfer than can occur in that single node.

To achieve scenario e, we introduced a walking node for all geospatial nodes. Geospatial nodes are essentially the bus stop codes and the MRT/LRT stations, and each have a longitude and latitude coordinates tied to them.

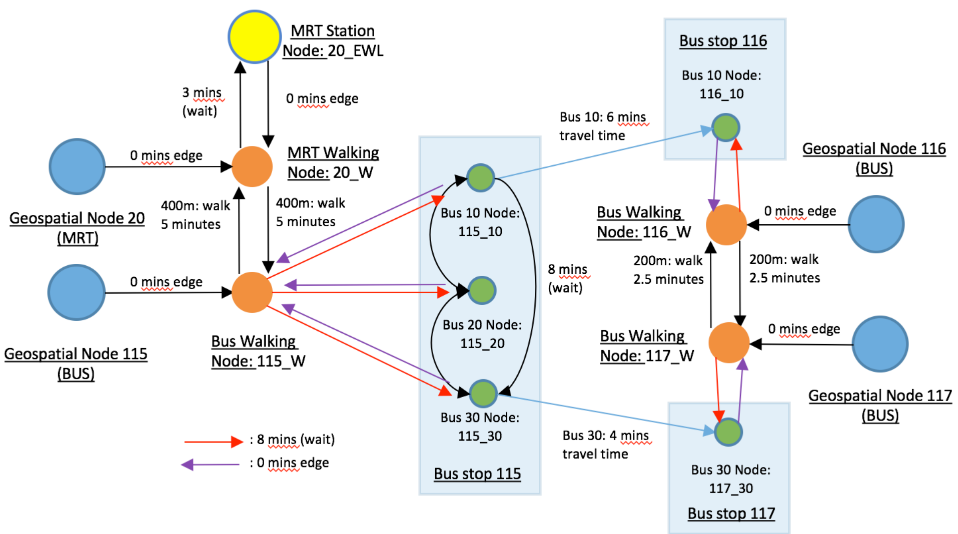
Some of the properties of a walking node are:

a. There will only be one direction from a geospatial node to a walking node with a 0 minutes edge.

b. All edges that end with a walking node will have a 0 minutes edge.

c. All edges that start with a walking node and end with a MRT/LRT/bus node has their respective waiting time as the weight

d. All edges that start and end with a walking node has a weight that equals the time taken to walk from one node to another.



R codes used for the data nodes modelling are found in Appendix B: Data Preparation.

METHODOLOGY

ROBUSTNESS MEASURE

The team believes that an appropriate measure of robustness of the transportation network should possess the following traits:

1. An accurate quantification of the transport network's 'well-being';
2. Ease of translation to the physical domain, so that decision makers can easily relate to the figures produced, and the figures can easily be transformed to actual impacts on breakdown/changes to the transportation network; and
3. Aptness for application to different transportation networks of varying strategies (i.e. spoke-hub network vs point-to-point network), and varying components (e.g. ferry system).

In view of the above mentioned needs, total commute hours is adopted as this study's robustness measure. The team would like to define total commute hours as the sum of travel time for the entire population under study, for a defined set of (a) population, (b) starting and destination location-pair, and (c) travel period (e.g. peak hours between 0830hrs to 0930hrs).

Clearly, usage of total commute hours will allow scalability on the population growth, travel begin and end points evolution, and relevancy across different transport network types. The changes in commute hours itself may also be directly applied to understand productivity impacts brought about by network breakdowns and improvements. Last but not least, the total commute hours may be calculated for specific regions where the population is located, to understand the localized impacts brought about by network breakdown and improvements. A drawback of total commute hours is that the figures will be skewed by population growth/shrink, making direct comparisons difficult. In this case we should adapt the measure to compensate for population figures changes, for example, average commute hours should be used instead. For the purpose of this study, however, we shall use total commute hours as we are studying a snapshot of the population (i.e. constant population), and the total commute hours can thus be translated to productivity impacts.

POPULATION SIMULATION

To account for direction of travel and the magnitude of commuter flow, we have to factor in the population distribution, and the number of commuters within each district. SingStats provided the resident count within age bands for every planned residential district, for which we assumed residents aged between 18 to 65 to be commuters between peak hours 0800-0900hrs. We assume 75% of these commuters have destination within the Central Business District, while the remaining 25% will commute to the rest of the island. We also assume 90% of these commuters take public transport (i.e. bus and MRT) to work. We further assumed that residents in CBD area do not take public transport to work, but walk instead due to close proximity to workplaces.

With these assumptions, commuters between each start and end transportation node pair can be calculated. The shortest path (weighted by travel time) will be calculated for each node pair, and the total commute time can then be subsequently calculated. The team will then apply the following scenarios to understand the impacts on total commute time:

1. No breakdown (baseline robustness measurement);
2. Three actual peak hour MRT breakdown events, with the affected train segment/edges removed from the network; and
3. Inclusion of proposed MR T lines that will be in service in the near future.

DATA PREPARATION

Generation of Commuter Counts Between Node Pairs

All transportation nodes were enriched with information on the residential district (subzone) they are spatially situated in, through a spatial join with subzone boundaries downloaded, executed via existing function on ArcMap/ESRI. With assumption as outlined in Methodology of this report, the exact number of commuter between all possible pairs of nodes (approximately 24 million unique pairs) were generated on R using the following formulas:

$$commuter\ count_{start=subzone\ i,\ \ end=CBD\ node\ j} = 75\% \times \frac{subzone\ i\ working\ population \times 90\%}{subzone\ i\ node\ count \times total\ CBD\ node\ count}$$

$$commuter\ count_{start=subzone\ i,\ \ end=nonCBD\ node\ j} = 75\% \times \frac{subzone\ i\ working\ population \times 90\%}{subzone\ i\ node\ count \times total\ nonCBD\ node\ count}$$

R codes used for this step is included as Appendix B: Data Preparation.

Consolidation of Edge Weights

The main geospatial nodes are the bus stops, MRT and LRT stations, and the intermediate nodes are the walking nodes and bus-stop-route nodes. For the edges, it comprise of the travelling routes of buses/MRT/LRT, the waiting time and the walking distance between the nodes. Detailed factors that contribute to the edge weights include:

a. Buses to Buses: bus travelling time (20 km/h / distance), waiting time: 8 minutes and walking time between nearby stops if buses do not belong to the same stop

b. Buses to MRT/LRT: waiting time: 3 minutes and walking time between nearby stops

c. MRT/LRT to Buses: waiting time: 8 minutes and walking time between nearby stops

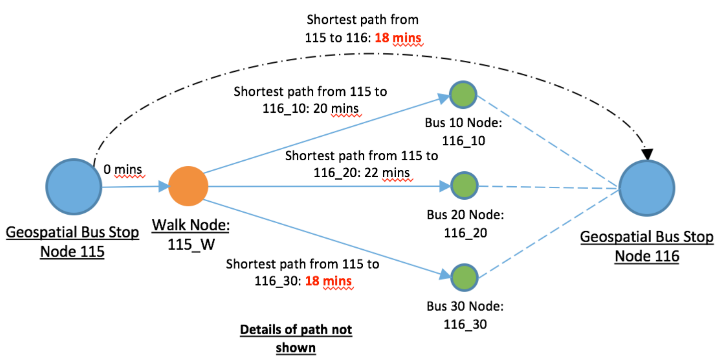
d. MRT/LRT to MRT/LRT: MRT/LRT traveling time: based on standard travelling time and waiting time (Interchange): 5 minutes

After gathering all edge weights for each component that contributes to the commute time, they are aggregated and summed based on the start nodes and end notes. This results in only one edge for each start and end node combination.

ANALYSIS

Shortest Path (Travelling Time) Calculation

After the nodes and edges information were obtained, the next step is the shortest path calculation. The start and end nodes will only include geospatial nodes. However, an important modelling concept to note here is there are no edges from any node to a geospatial node as this will result in "shortcuts" due to the edge weight being 0. A workaround is to first consider all intermediate nodes as end nodes in the shortest path calculation, and subsequently choose the shortest path of all intermediate nodes to determine the shortest path to the geospatial end node. This is illustrated below:



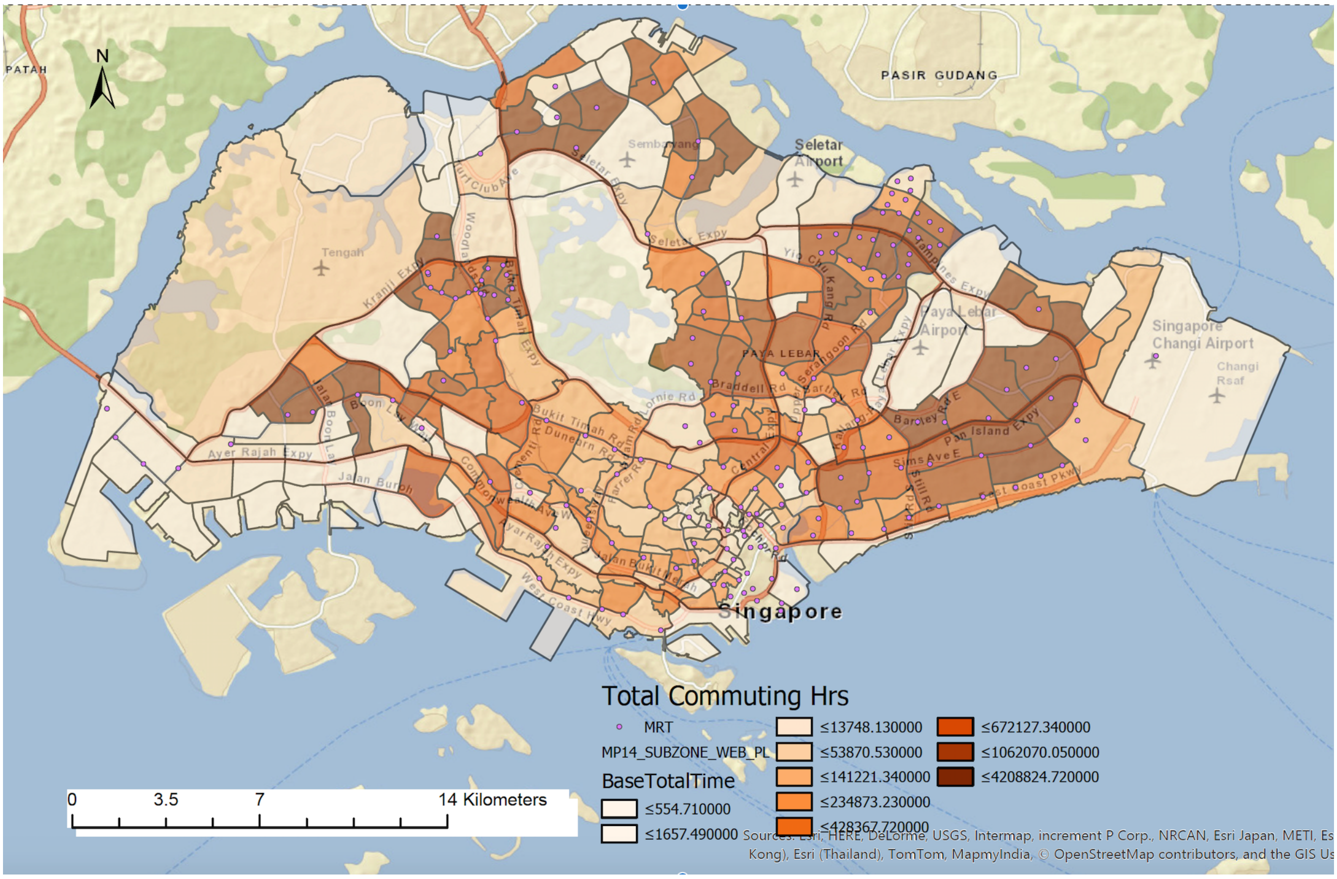
There are a total of 4945 geospatial start nodes and more than 18000 intermediate nodes acting as end nodes. Thus approximately 80 million shortest paths needed to be calculated and it was necessary to utilise an efficient algorithm. After much research, Dijkstra's algorithm was decided upon as it suited uni-directional weighted edges and moreover, for each start node, it also calculates the shortest path (travelling time) for all reachable end nodes in a single run. Python NetworkX package was the main tool for the Dijkstra's algorithm. Python codes used for this step are found in Appendix C: Data Modelling.

Total Commute Time

The last step to obtain the total population commute time is to join the commuter counts data and the shortest travelling time data using the geospatial start and end nodes. The total commute time for each node pair is the product of commuter count and shortest travelling time. Finally, the entire population's commute time is the summation of all the node pairs total commute time. R codes and final results used for this step are found in Appendix C: Data Modelling.

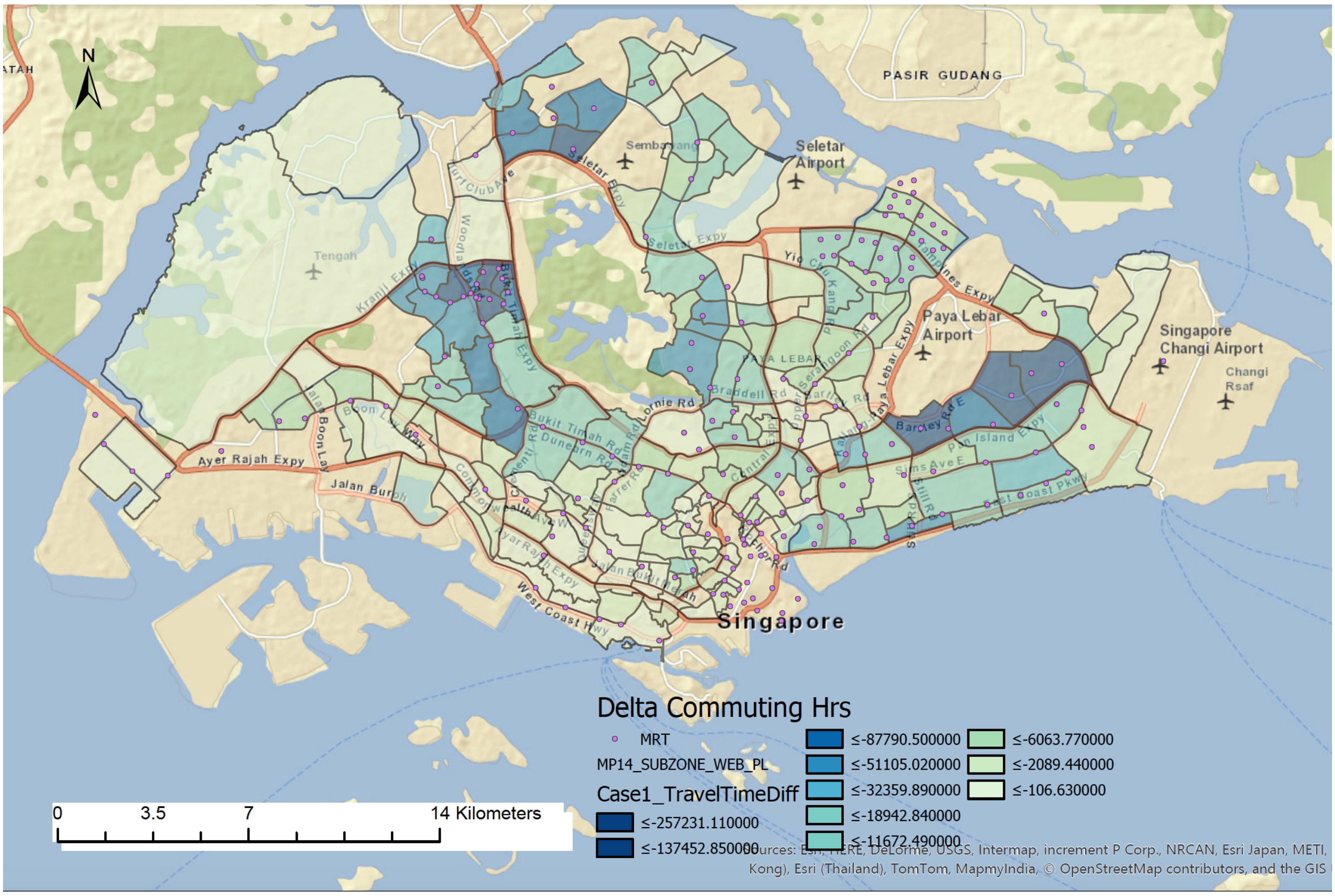
Baseline Scenario

For the baseline robustness measurement, the team considered all the nodes in the Singapore transportation network (without any breakdowns) and calculated the total commute time. The impact on the calculated total commute time from each subzone was visualized using ArcGis and ArcMap, seen in the graph below. As expected, the subzones closer to the CBD area amounted to a lesser total commute time rather than the subzones located further away. The total commute time being a combined measure of both the population and the distance, certain subzones in districts such as Serangoon or Bedok for example, although being relatively closer to the CBD area compared to subzones in districts such as Admiralty had a higher total commute time because of its population. Tampines East, Woodlands East and Tampines West subzones had the highest total commuting time in the network with 4208824.72, 3189378.12 and 2431085.82 hours respectively. The graph also gives us a clear understanding of the heavily commuted areas and least serviced areas, which could be potential zones for future nodes and edges.



Proposed New Nodes

We considered the new MRT lines that are proposed by the Singapore government (ones that are more likely to be built in the future) based on the upcoming LTA plan and measured their impact on the network. Two of the new MRT lines are Thomson East Coast Line and Downtown Line. To measure the robustness and visualize the impact in the network, we used delta commuting time instead of total commuting time. The delta commuting time is a measure of the difference in travel time as a result of the new nodes against the baseline commuting time. The impact on the commuting time for each subzone can be viewed in the ArcGIS map below



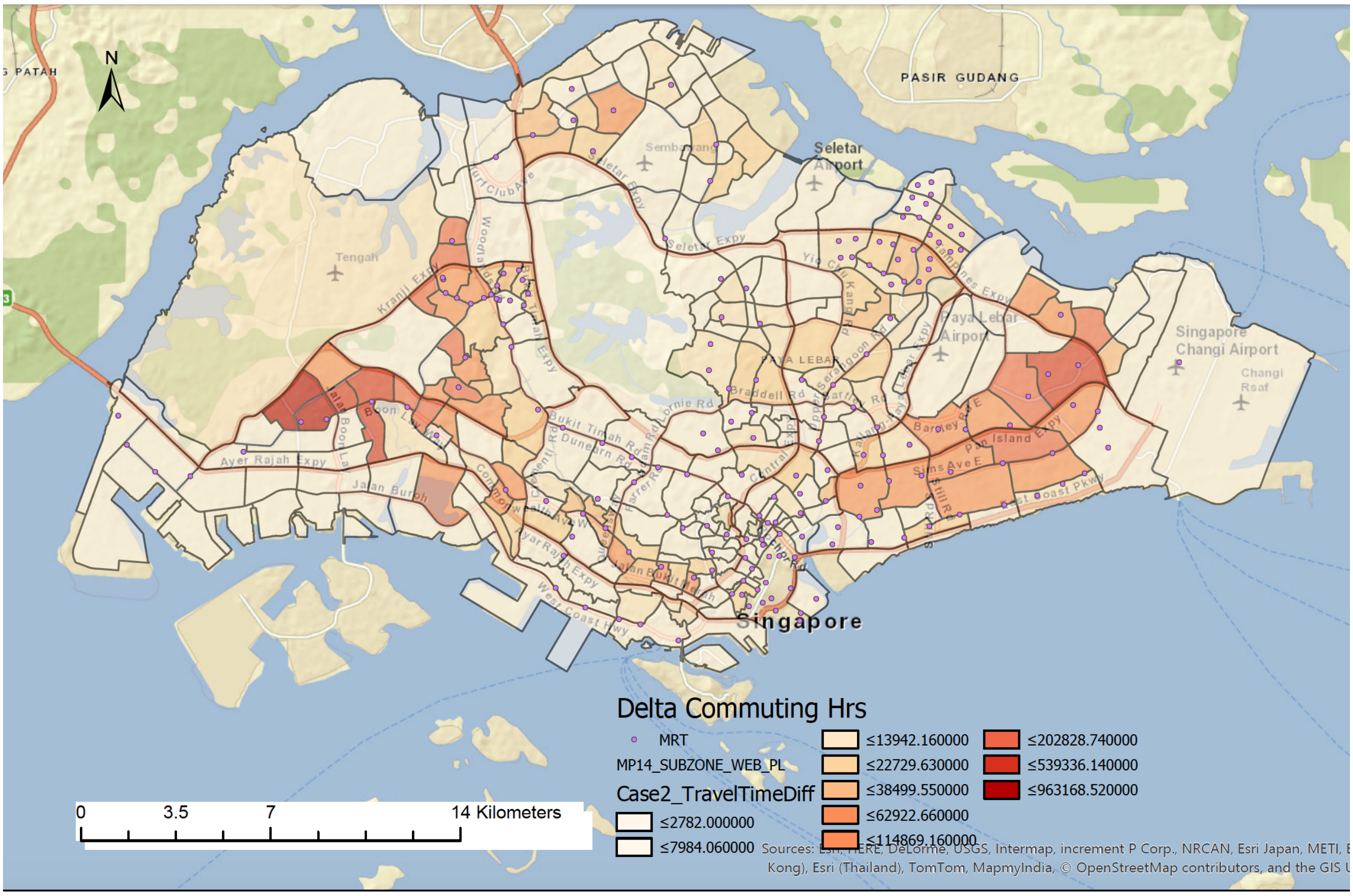
The darker the blue color, the bigger the improvements in terms of total commuting time for that subzone. The top 5 subzones with the biggest impact are Jelebu, Woodlands South, Tampines East, Tampines West and Fajar. It is interesting to note that the new train lines will significantly improve areas where the subzone total commuting time were some of the largest as identified previously in the baseline scenario. This shows that the transport planning authority takes into consideration the population density for planning of new train routes. Jelebu and Fajar belong to the north west portion of the island and with the introduction of the Downtown Line, commuting time to the CBD area will be reduced significantly. Currently, commuters in that area need to travel all the way to Jurong East interchange using the North South Line, and change to the East West Line to travel to the CBD area which was a big detour.

Major Train Breakdowns

When a particular MRT train breakdown occurs, the stations that are connected closely to it will be impacted due to blockage of the route. The main idea here is to identify within the MRT networks, what are the clusters of stations that will be affected if any of the stations suffer a breakdown in the connected route. For example, if a breakdown occurs at Clementi MRT station, the edge between Jurong East, Clementi and Dover will be removed. This is part of the interdiction method of using pre-selected vulnerable links to assess robustness. As mentioned earlier, we considered three real-time major MRT breakdowns that had taken place in the past few years in Singapore to measure their impacts on the model. We also ensured these breakdown events were in different areas of the island.

AFFECTED NODES		DATE OF OCCURENCE (DD/MM/YY)	AFFECTED HOURS	MRT LINE
FROM	TO			
Jurong East	Bugis	03/03/15	1.50	EAST WEST LINE
Woodleigh	Punggol	19/06/13	2.50	NORTH EAST LINE
Bishan	Marina Bay	15/12/11	5.58	NORTH SOUTH LINE

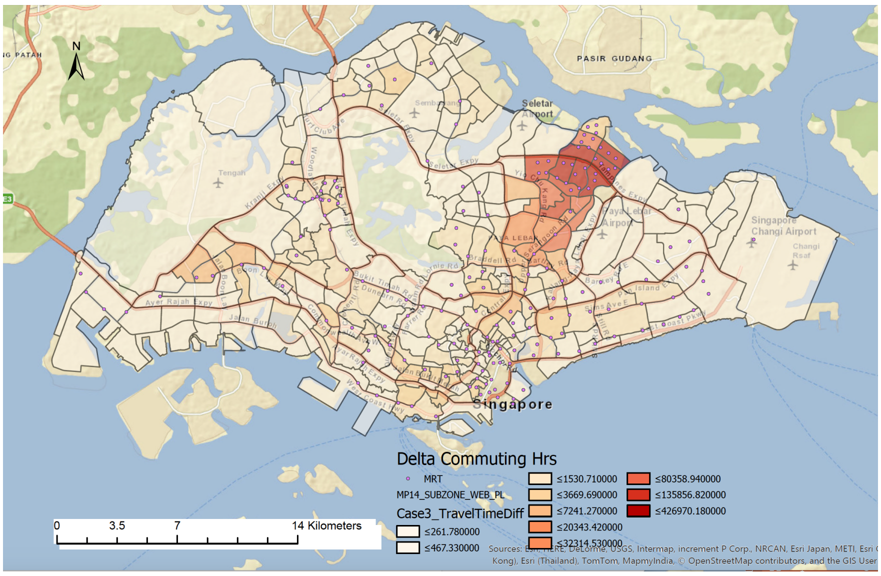
Jurong East to Bugis Breakdown (East West Line)



Jurong East - Bugis Breakdown

We considered one of the most recent breakdowns that occurred in Singapore on March 3, 2015 along the East West MRT Line. The MRT stations from Jurong East to Bugis on the East-West line were affected for around one and half hours as a result of track fault. We factored in the affected mrt stations and the delay in travel time in our model to calculate the delta commuting time in all the subzones. The East West line is a very vital line as it spans several subzones that are not serviced by the other MRT lines and coupled with the fact that the affected MRT stations are closer to the CBD area, the impact of the disruption was felt across the whole island as seen in the above Image. Jurong West Central subzone had the highest impact of about 46% as its total commuting time increased from 2093858.71 to 3057027.22 hours. Yunnan, Hong Kah and Tampines East were the other highly affected subzones.

Woodleigh to Punggol Breakdown (North-East Line)



Woodleigh - Punggol Breakdown

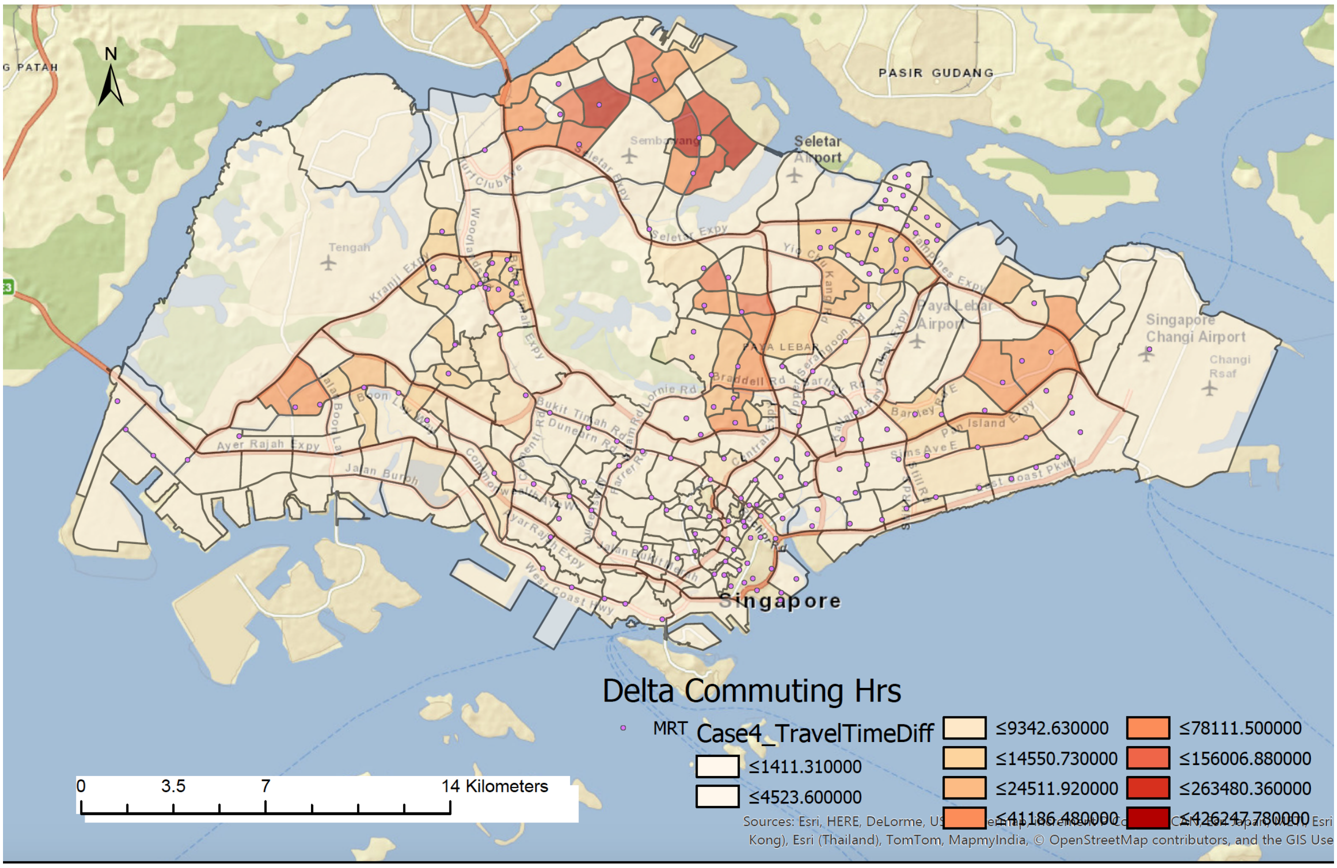
Next, we consider the Woodleigh to Punggol MRT breakdown that happened on June 19, 2013.

Based on the map, we can observe that the most significant increase in commuting hours were the subzones in the North East area, with a 20 to 40% increase in commuting time. The impact is extremely high there as they are only served by a single MRT line and the nearest alternative MRT line which is the North South line are of a certain distance from the area. It is also interesting to note that the North East area consists of a younger working population and the impact there may be greater due to lesser of them owning cars.

Bishan to Marina Bay Breakdown (North South Line)

This case emulates the major breakdown of eleven train stations from North-South line Bishan to Marina Bay stations, which happened in December 2011. The below image visualizes the simulation of increase in commuter hours brought about by the train failure, aggregated by individual subzones. Few trends may be observed from the subzones which are heavily affected: (a) in vicinity of non-CBD train stations between Bishan and Marina Bay, (b) Sembawang and Yishun areas which are north of Bishan, and these areas intuitively would include Bishan to Marina Bay stations as their shortest path to CBD, and (c) Bartley areas, east of Bishan.

While abovementioned (a) and (b) could be intuitively explained as Bishan to Marina Bay stations are along the shortest paths between their affected subzones and the CDB, the same is not immediately apparent to (c). As we analyze deeper into the shortest paths between areas in (c) and the CBD, it was then discovered that commuters in Bartley areas commute to CBD through Circle Line, transiting through Bishan MRT to continue their journey on the North South Line. This explained the extended impact of the train failure to commuters in the east of Bishan.



Bishan - Marina Bay Breakdown

LIMITATIONS

The team's analysis is not without limitations. The following details areas which the team had to knowingly accept their inherent impacts, in order to proceed with the project.

1. This analysis did not factor in 'response reactions (e.g. setting up of temporary bus services to mirror the paths of train segments that ceased)' from the transportation companies in the event of train failures, and may thus overestimate the impacts of train breakdowns. The team, however, feel that the the model is still applicable between point of train failure till 'response reactions' are up, and this time window is non-trivial.

2. Transportation throughput/capacities has not been factored into the model. This may underestimate the impact of train failures, since commuters had to switch to bus networks to proceed with their journeys, and that buses typically have lower capacities than trains.

3. Human behaviours has not been modeled into this analysis. The typical commuter, not knowing how long a train breakdown will last, is likely to stay at the train station to wait for further information on when train services will resume. The lack of factoring such a behaviour in the model will underestimate the impact of train failures.

4. The model assumes all commuters to have full information on the public transportation network, and are able to somehow arrive at the shortest part to their destinations through heuristics or algorithms. Assuming this affects all scenarios to similar extent, we will need to artificially scale up the total commuting hours to compensate.

FUTURE EXTENSIONS

Permute Train Nodes Breakdown

With the framework of evaluating impacts of train breakdowns by removing edges, the team would like to propose the study of impacts of all possible train breakdown scenarios, so as to identify crucial train segments that when down, greatly increases the total commuting hours. This information is useful in identifying areas where train operators can focus on maintaining service levels, as well as for government security agencies to increase attention on. While this is easily doable given luxury of time, every iteration will take the team 1 hour complete, and given the tight timeline, the team in unable to deliver this aspect.

Proposal of New Transportation Nodes/Edges

As demonstrated, the developed model allows the team to assess the impact of newly proposed train stations. This could be used as groundwork to an iterative search model for proposal of new transportation nodes and edges. One possible model is for nodes and edges to be randomly generated in stratas (e.g. 1km by 1km grids) with high population-density-to-existing-transportation-node ratio, and followed by assessing the decrease in total commuting hours brought about by the randomly generated nodes. This identifies areas for transportation planners to look into.

Higher Resolution Starting and Destination Node Pairs

The same developed framework may be applied to the case where start and destinations of each and every commuter is already known. The government may have such information available, as it is reasonable to assume the government to know where her citizens stay, and where they work (for taxation needs). Knowledge at this granularity will improve the reliability of total commuting hours calculated by the model.

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ANNEXE

FUTURE MRT MAP