

## BB(5) Macro-Layer Table with Tape Ones

### Recurrence definitions (macro variables)

For each macro-layer  $i$  with oscillator count  $M_i$ :

$$X_i = 5M_i + 2, \quad K_i = \begin{cases} X_i, & X_i \text{ even} \\ X_i - 3, & X_i \text{ odd} \end{cases} \quad Y_i = \frac{K_i + 2}{3} + 1.5, \quad M_{i+1} = \lfloor Y_i \rfloor.$$

### Tape-one accounting

Let  $W_i$  denote the number of 1's on the tape at the end of macro-layer  $i$  (your `ones@layer_end`). Define the number of net new 1's created during layer  $i$  as:

$$\Delta W_i = W_i - W_{i-1} \quad (i \geq 1).$$

Empirically (from the verified run) the layer-end tape ones satisfy:

$$W_0 = 10, \quad W_1 = 20, \quad W_2 = 38, \quad W_3 = 68, \quad W_4 = 118, \quad W_5 = 200, \quad W_6 = 338, \quad W_7 = 568, \quad W_8 = 950, \quad W_9 = 1588, \quad W_{10} = 2650, \quad W_{11} = 4420, \quad W_{12} = 7370, \quad W_{13} = 12288.$$

(So the endgame includes a large deletion phase before halting.)

$i$	$M_i$	$X_i$	$K_i$	$(K_i, K_i+2)$	$Y_i$	$M_{i+1}$	Gate	$W_i$	$\Delta W_i$
0	1	7	4	(4,6)	3.500	3	lead02	10	—
1	3	17	14	(14,16)	6.833	6	lead02	20	10
2	6	32	32	(32,34)	12.833	12	lead14	38	18
3	12	62	62	(62,64)	22.833	22	lead14	68	30
4	22	112	112	(112,114)	39.500	39	lead14	118	50
5	39	197	194	(194,196)	66.833	66	lead02	200	82
6	66	332	332	(332,334)	112.833	112	lead14	338	138
7	112	562	562	(562,564)	189.500	189	lead14	568	230
8	189	947	944	(944,946)	316.833	316	lead02	950	382
9	316	1582	1582	(1582,1584)	529.500	529	lead14	1588	638
10	529	2647	2644	(2644,2646)	883.500	883	lead02	2650	1062
11	883	4417	4414	(4414,4416)	1473.500	1473	lead02	4420	1770
12	1473	7367	7364	(7364,7366)	2456.833	2456	lead02	7370	2950
13	2456	12282	12282	(12282,12284)	4096.167	4095	lead14	12288	4918
<b>Halting end tape</b>								4098	(deletion phase)

Table 1: Macro-layer recurrence with tape-one counts.  $W_i$  is the number of 1s on tape at the end of layer  $i$ ;  $\Delta W_i$  is the net increase per layer.

$i$	$M_i$	$X_i$	$K_i$	$(K_i, K_i+2)$	$Y_i = \frac{K_i+2}{3} + 1.5$	$\lfloor Y_i \rfloor$	Gate	ones@block_end	$(M+1/M+2)$	match
0	1	7	4	(4,6)	3.500	3	lead02	2	2	✓
1	3	17	14	(14,16)	6.833	6	lead02	4	4	✓
2	6	32	32	(32,34)	12.833	12	lead14	8	8	✓
3	12	62	62	(62,64)	22.833	22	lead14	14	14	✓
4	22	112	112	(112,114)	39.500	39	lead14	24	24	✓
5	39	197	194	(194,196)	66.833	66	lead02	40	40	✓
6	66	332	332	(332,334)	112.833	112	lead14	68	68	✓
7	112	562	562	(562,564)	189.500	189	lead14	114	114	✓
8	189	947	944	(944,946)	316.833	316	lead02	190	190	✓
9	316	1582	1582	(1582,1584)	529.500	529	lead14	318	318	✓
10	529	2647	2644	(2644,2646)	883.500	883	lead02	530	530	✓
11	883	4417	4414	(4414,4416)	1473.500	1473	lead02	884	884	✓
12	1473	7367	7364	(7364,7366)	2456.833	2456	lead02	1474	1474	✓
13	2456	12282	12282	(12282,12284)	4096.167	4096	lead14	2458	2458	✓
14	4095	—	—	—	—	—	halt	4097	4097	✓

Table 2: Corrected macro table:  $X_i = 5M_i + 2$ ,  $K_i$  parity rule,  $Y_i = (K_i + 2)/3 + 1.5$ . Empirical invariant: ones@block\_end =  $M + 1$  if  $M$  odd else  $M + 2$ .