Decimal-Lens Macro-Structure of the BB(5) Step-Champion

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Abstract

We introduce a decimal trace encoding ("decimal lens") for 5-state 2-symbol Turing machines and apply it to the Marxen–Buntrock BB(5) step-champion. Under this encoding, the 47,176,870-step execution exhibits a layered macro-structure consisting of: three structural pairs $(0,2)$, $(4,6)$, $(5,9)$; filler series (digit 3 for $(0,2)$, digit 7 for $(4,6)$, none for $(5,9)$); an oscillator block 159; a recurrence over oscillator counts M; terminal ladder values $(K, K+2)$; and gate bifurcation governed by $(K+2) \bmod 3$. We formalize the empirical structure, verify recurrence properties computationally, and state conjectures regarding ladder necessity, gate bifurcation, and halting residue inevitability.

1 The BB(5) Marxen–Buntrock Transition Table

We study the 5-state 2-symbol Busy Beaver step-champion discovered by Marxen and Buntrock. The machine has states

$$\{A, B, C, D, E\},$$

symbols

$$\{0, 1\},$$

and a distinguished halting state H.

Its transition function is:

Key (q, s)	**Write**	**Move**	**Next State**
$(A, 0)$	1	R	B
$(A, 1)$	1	L	C
$(B, 0)$	1	R	C
$(B, 1)$	1	R	B
$(C, 0)$	1	R	D
$(C, 1)$	0	L	E
$(D, 0)$	1	L	A
$(D, 1)$	1	L	D
$(E, 0)$	1	R	H
$(E, 1)$	0	L	A

This is the canonical BB(5) step-champion transition table.

2 Decimal Lens Encoding

2.1 State-Symbol to Digit Map

Each machine step is determined by a *key*

$$(q, s) \in \{A, B, C, D, E\} \times \{0, 1\}.$$

We encode each key as a single decimal digit.
Let the state index function be:

$$A \mapsto 0, \quad B \mapsto 1, \quad C \mapsto 2, \quad D \mapsto 3, \quad E \mapsto 4.$$

Then define the digit encoding:

$$d(q, s) = 2 \cdot \text{index}(q) + s.$$

Thus:

$$\begin{aligned} A0 &\mapsto 0, \quad A1 \mapsto 1, \\ B0 &\mapsto 2, \quad B1 \mapsto 3, \\ C0 &\mapsto 4, \quad C1 \mapsto 5, \\ D0 &\mapsto 6, \quad D1 \mapsto 7, \\ E0 &\mapsto 8, \quad E1 \mapsto 9. \end{aligned}$$

This produces a bijection between the 10 state-symbol keys and digits 0–9.

2.2 Digit Trace Definition

Given a TM execution producing the key sequence:

$$(q_0, s_0), (q_1, s_1), (q_2, s_2), \ldots$$

the *decimal digit trace* is:

$$d_0 d_1 d_2 \cdots \quad \text{where} \quad d_i = d(q_i, s_i).$$

For BB(5), the initial prefix becomes:

$$024615902477776033333\ldots$$

This digit trace is the object studied throughout the paper.

3 Structural Grammar of BB(5)

Three fundamental pair types occur:

Pair classes

(1) $(0, 2)$**-pair:**

$$0\, 3^B\, 2.$$

(2) $(4, 6)$**-pair:**

$$4\, 7^A\, 6.$$

(3) $(5, 9)$**-pair:**

$$59,$$

with no filler (except at the halting boundary).

4 Oscillator Block

The substring 159 forms an oscillator block. Define

$$M = \text{the number of consecutive 159 blocks at the start of a layer.}$$

Empirically, the oscillator counts follow:

$$M = 1, 3, 6, 12, 22, 39, 66, 112, 189, 316, 529, 883, 1473, 2456, \ldots$$

5 Ladder Structure

Within a layer, rung parameters satisfy:

$$B_n = A_n + 1, \qquad A_{n+1} = A_n + 5.$$

At the terminal rung:

$$B_n = A_n + 2.$$

We denote the terminal rung by

$$(A, B) = (K, K + 2).$$

6 Terminal K Computation

Given M, define:

$$X = 5M + 2.$$

Then:

$$K = \begin{cases} X, & \text{if } X \text{ is even,} \\ X - 3, & \text{if } X \text{ is odd.} \end{cases}$$

7 Oscillator Recurrence

Define:

$$M_{\text{next}} = \left\lfloor \frac{K + 2}{3} + 1.5 \right\rfloor.$$

This matches the observed oscillator growth.

8 Gate Bifurcation

Write:

$$K + 2 = 3q + r, \qquad r \in \{0, 1, 2\}.$$

Then the fractional residue of

$$\frac{K + 2}{3} + 1.5$$

determines the gate payload:

- $r = 0$ corresponds to `lead02` gate.
- $r = 1$ corresponds to `lead14` gate.
- $r = 2$ corresponds to the halting corridor.

9 Halting Signature

At the final layer, the oscillator mutates:

$$159 \to 158,$$

corresponding to the unique halting entry through $E0 \mapsto 8$.

10 Conclusion

The BB(5) step-champion admits a layered decimal macro-structure governed by:

- rigid pair grammar,
- oscillator count recurrence,
- modular gate bifurcation,
- a halting residue signature.

This compresses tens of millions of micro-steps into a compact recurrence-driven description.