GANITA PRAKASH

Textbook of Mathematics for Grade 6

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NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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Textbook of Mathematics for Grade 6

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Foreword

The National Education Policy (NEP) 2020 envisages a system of education in the country that is rooted in Indian ethos and its civilisational accomplishments in all domains of human endeavour and knowledge, while at the same time preparing the students to constructively engage with the prospects and challenges of the twenty-first century. The basis for this aspirational vision has been well laid out by the National Curriculum Framework for School Education (NCFSE) 2023 across curricular areas at all stages. Nurturing the students' inherent abilities touching upon all the five planes of human existence, the *pañchakośhas*, in the Foundational and the Preparatory Stages has paved the way for the progression of their learning further at the Middle Stage. The Middle Stage acts as a bridge between the Preparatory and the Secondary Stages, spanning three years from Grade 6 to 8.

This framework, at the Middle Stage, aims to equip students with the skills that are needed to grow as they advance in their lives. It endeavours to enhance their analytical, descriptive, and narrative capabilities, and to prepare them for the challenges and opportunities that await them. A diverse curriculum, covering nine subjects ranging from three languages—including at least two languages native to India—to Science, Mathematics, Social Sciences, Art Education, Physical Education and Well-being, and Vocational Education promotes their holistic development.

Such a transformative learning culture requires certain essential conditions. One of them is to have appropriate textbooks in different curricular areas, as these textbooks will play a central role in mediating between content and pedagogy—a role that will strike a judicious balance between direct instruction and opportunities for exploration and inquiry. Among other conditions, classroom arrangement and teacher preparation are crucial to establish conceptual connections both within and across curricular areas.

The National Council of Educational Research and Training (NCERT), in its part, is committed to provide students with such high-quality textbooks. Various Curricular Area Groups, which have been constituted for this purpose, comprising notable subject-experts,

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pedagogues, and practising teachers as their members, have made all possible efforts to develop such textbooks. *Ganita Prakash*, the textbook of Mathematics for Grade 6, is one of these. The textbook is a captivating journey through the world of mathematics designed for Grade 6 students. The book begins with encouraging the students to observe and explore the patterns around them and discover mathematical concepts on their own. The book further delves into the realm of numbers, where young learners are introduced to the magic of numbers and shapes. Through colourful illustrations and interactive exercises, children develop a strong foundation in arithmetic, paving the way for more complex mathematical concepts. Throughout the book, stories, conversations and anecdotes have been incorporated to make abstract mathematical concepts more relatable and accessible to young learners. Content has been evolved using puzzles and innovative problems that will not only engage the students in thoughtfully relating the mathematical concepts to the world around them and help them in deepening their understanding of mathematics, but also prepare them to understand the concepts of the emerging field of Computational Thinking. Indian rootedness and relation to Indian Knowledge Systems (IKS) has been embedded in the content of the textbook.

However, in addition to this textbook, students at this stage should also be encouraged to explore various other learning resources. School libraries play a crucial role in making such resources available. Besides, the role of parents and teachers will also be invaluable in guiding and encouraging students to do so.

With this, I express my gratitude to all those who have been involved in the development of this textbook and hope that it will meet the expectations of all stakeholders. At the same time, I also invite suggestions and feedback from all its users for further improvement in the coming years.

DINESH PRASAD SAKLANI
Director
National Council of Educational
Research and Training

New Delhi
July, 2024

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About the Book

Mathematics helps students develop not only basic arithmetic skills, but also the crucial capacities of logical reasoning, creative problem solving, and clear and precise communication (both oral and written). Mathematical knowledge also plays a crucial role in understanding concepts in other school subjects, such as Science and Social Science, and even Art, Physical Education, and Vocational Education. Learning Mathematics can also contribute to the development of capacities for making informed choices and decisions. Understanding numbers and quantitative arguments is necessary for effective and meaningful democratic and economic participation. Mathematics thus has an important role to play in achieving the overall Aims of School Education.

Mathematics at the Middle Stage is a major challenge and has to perform the dual role of being both close to the experience and environment of the child and being abstract. It must perform the dual role of developing intuition while also maintaining and emphasizing rigour. It must perform the dual role of enhancing critical and logical thinking while also developing artistry and creativity and a sense of elegance and aesthetics. Finally, Mathematics must perform the dual role of providing students plenty of opportunities for exploration and discovery of concepts on their own while also teaching best-known methods in the global repertoire of mathematics.

The present textbook has made an attempt to address the above-mentioned goals and challenges of learning mathematics. The writers of this book have aimed to strike a judicious balance between informal and formal definitions and methods to develop in students both intuition and rigour. The book also provides numerous opportunities for student–student and student–teacher interaction in the classroom to promote active and experiential learning. A number of questions, puzzles, and interactive exercises are posed throughout the book to encourage constant exploration. Many of the questions are open-ended to stimulate in-class discussion. Finally, some famous unsolved problems have also been included so that students can appreciate that Mathematics is still a very active subject, with much that is already known and discovered, but also many exciting frontiers that remain unknown and unseen. Such unknown realms and unresolved questions will require new ideas and a new generation of adventurers to explore and understand, and to thereby solve these exciting problems.

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Among the world's greatest problem solvers and most creative minds of the current generation is world-renowned mathematician Manjul Bhargava. He has resolved decades-old, and in some cases centuries-old, problems of a fundamental nature across Mathematics, particularly in the areas of number theory, algebra, representation theory, and arithmetic geometry. For his pioneering breakthroughs in Mathematics, in 2014 he became the first person of Indian origin to receive the Fields Medal, the highest honour given to mathematicians, awarded every four years and known as the 'Nobel Prize of Mathematics'.

We are thrilled and honoured that the beautiful Chapter 1 of this book, 'Patterns in Mathematics', has been kindly composed and contributed by Professor Bhargava. In this chapter, in the section 'What is Mathematics?', Bhargava eloquently speaks of mathematics as a creative art—as a search for beautiful patterns, and the explanations of those patterns. In later sections of the chapter, he describes a sampling of some of the most basic patterns in mathematics—sequences of numbers and sequences of shapes—and their remarkable and often-surprising interrelations. These patterns are regularly revisited in later chapters of this book, to emphasise the unity of mathematics, and will also be revisited in future years. We hope that this exploratory chapter will help in inspiring a new generation to explore and pursue mathematics.

Building on the idea of exploring patterns in mathematics, the book then turns to a journey across different areas of mathematics. Chapter 2, 'Lines and Angles', introduces the building blocks of geometry—points, line segments, rays, lines, angles, and how to measure angles. Chapter 3, 'Number Play', is an exploratory adventure through some instructive but fun games and puzzles in mathematics—some of which are still unsolved! Chapter 4, 'Data Handling', is an introduction to the art of collecting and presenting data, including both its analytic and aesthetic aspects. Chapter 5, 'Prime Time', is a playful adventure through prime numbers—the building blocks of the universe of whole numbers—and factorization. Chapter 6, 'Perimeter and Area', is a revision of these fundamental notions, with a variety of challenging puzzles to keep children on their toes and enhance understanding. Chapter 7, 'Fractions', will be many students' first encounter with this important concept; the chapter aims to build intuition about fractions gradually, starting with fractional units like 1/10 as the foundation, and gradually building up to working with general fractions, including

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their comparison, addition, and subtraction. Chapter 8, 'Playing with Constructions', is a hands-on experience of drawing shapes, including using a compass and a ruler, to enhance students' geometric intuition and comprehension. Chapter 9, 'Symmetry', is an artistic and hands-on exploration of this most important and ubiquitous concept in Mathematics and beyond. Finally, Chapter 10, 'The Other Side of Zero', aims for students to gain intuition for negative numbers by visiting Bela's Building of Fun, and gradually working up to understanding the laws of addition and subtraction of all integers as laid down by Brahmagupta.

In all chapters, an attempt has been made to emphasise connections with other subjects including Art, History, and Science. Many pictures and drawings have been included to illustrate patterns, numbers, constructions, symmetry, games, puzzles, etc., to thereby develop visual and artistic imagination and intuition for mathematical objects and principles. The history of various mathematical concepts has been described, including Brahmagupta's world-changing discoveries in the year 628 C.E. of the laws for addition and subtraction of fractions and of zero and negative numbers. Other discoveries from around the world, of unit fractions, searching for primes, Collatz Conjecture, Kaprekar numbers, etc., have also been described with their history to help students appreciate and humanize the joy and process of discovery. Examples from Science (e.g., the use of negative numbers to measure temperature or heights above or below sea level) also abound to illustrate the importance of the use of mathematical concepts in Science.

By weaving together storytelling and hands-on activities, we hope that an immersive learning experience will be created that ignites curiosity and fosters a love for mathematics. It is hoped that teachers would give children the opportunity to discuss, play, engage with each other, provide logical arguments for different ideas, and find loopholes in arguments presented. This is necessary for the learners to eventually develop the ability to understand what it means to prove something and also become confident about underlying concepts. The mathematics classroom should not expect a blind application of algorithms but should rather encourage children to find many different ways to solve problems.

As per the NEP 2020, Computational Thinking has also been gently introduced through puzzles, games, and interactive exercises that encourage such thinking. Indian rootedness has also been kept in

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mind while giving contexts for different concepts. The contributions of Indian mathematicians have been given as part of a problem-solving approach to make students aware of India's rich mathematical heritage and its global contributions to mathematics.

The concepts and problems are related to daily life situations. An attempt has been made to use contexts and materials with which the students are familiar. Learning material sheets have been given at the back of the book that may be photocopied and used. At many places, exercises or activities are given to encourage peer group efforts and discussions. The textbook intends to address the learning needs of a diverse group of students in the classroom.

We have tried to link concepts learnt in initial chapters with ideas in subsequent chapters to show the connectedness and unity of mathematics. We hope that teachers will use this as an opportunity to revise these concepts in a spiralling way so that children are able to appreciate the entire conceptual structure of mathematics. We hope that teachers may give more time to the ideas of fractions, negative numbers and other notions that are new to students. Many of these are the basis for further learning in mathematics.

Finally, this book aims to be more than just a textbook—it's a passport to a world of mathematical discovery and exploration. Whether used in the classroom or at home, we hope that it may inspire students to embark on their own mathematical adventures, empowering them to see the beauty and relevance of mathematics in everything around them. With its engaging approach and comprehensive coverage of Grade 6 mathematics concepts, this book hopes and aims to captivate young minds and set them on a lifelong journey of mathematical discovery.

I thank again all the writers of and contributors to this textbook for this important and valuable contribution and service to the nation's mathematics teachers, learners and enthusiasts.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching and learning, to be included in future editions.

Ashutosh Wazalwar
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Note to the Teacher

We hope that this book, Ganita Prakash, will serve as a strong support and guide to you in achieving the exciting task that you have before you: that of passing on the joy of learning the beautiful subject of mathematics to the next generation.

This task calls for providing a fertile environment that allows for the flowering of mathematical thinking in the minds of students.

Classrooms, where students just listen and write down whatever is being told to them or written on the board, are deficient in the conditions required for learning mathematics. Instead, classrooms need to be places where students are engaged in playing with mathematical concepts, finding and discussing patterns, and developing creative strategies together to solve problems. Students should also be posing problems to each other and discussing possible solutions with each other. In fact, these are the very conditions that have led to the development of the entire field of mathematics so far, and so one cannot expect students to pick up mathematical thinking and understanding without these conditions.

Fortunately, it is not difficult to create such conditions in the classroom. It just requires an interesting question, problem, pattern, or challenge to be thrown open to the students on a regular basis, and sufficient time to be given to them to play with, discuss, and work on it as a class or in pairs or groups.

Along with it, an environment that accepts mistakes and acknowledges their importance in learning needs to be nurtured.

While creating the spark for initiating mathematical thinking in classrooms is not difficult, sustaining it may be challenging and may involve efforts from your side. Nevertheless, even if just the first part of throwing open a question, problem, pattern, or challenge is done at least once or twice a week, accompanied by sufficient waiting time from your side for students to play, discuss, and work on it, it can have a great positive impact on how the students view and approach mathematics.

It should be noted that this positive impact will not happen overnight. That takes time and depends on various factors such as the number of opportunities you give for problem solving, your patience, and the encouragement you give to the students.

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To support you in posing problems, all the problems or questions in this book are marked using the icon . This icon is an indicator of a potential opportunity to start off a process of problem solving and exploration in the classroom. You will find some of the problems labelled “Math Talk”. Such questions can especially be made as topics for classroom discussion.

To develop students’ mathematical thinking and understanding of concepts, a sufficient number of problems are given. Trying to “cover” all of them must not happen at the cost of students not getting to spend quality time on playing with and discussing them.

It is important to understand that the exploratory problems are not only for promoting problem solving skills; they also serve in strengthening procedural fluency when children start engaging in exploration.

Efforts must be made in making students independent learners. One essential aspect required for this is an ability to read and understand mathematical text. To promote this skill, students should be encouraged to read the book by themselves and in groups. Give opportunities to them to interpret what they read and express it to others. This will also address the big problem that students face in speaking mathematics and interpreting word problems.

This book contains a number of open-ended problems. It also contains new treatments of certain concepts. If you are not able to solve them or follow some of them immediately, it is perfectly okay! Not everyone knows everything. Along with trying to understand and reflect upon such content, it will be very useful to take it to the classroom and open it up for discussion. After the discussion, things that are clear and those that are not yet clear can be clearly summarised. This process itself can throw a lot of light on the content. In these discussions, you can participate as a fellow seeker, and when students see a teacher seek and think to understand something, it sets a wonderful example for them.

It is hoped that you and your students will have a great and fruitful time using this book!

Summary of Key points

Time for Exploration

1. It is important to routinely pose new problems, questions, patterns, or challenges to the students and give them sufficient time to play with, discuss, and work on them, individually and in groups.

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2. During this time, an environment that accepts mistakes and acknowledges their importance in learning needs to be nurtured.
3. There should be a culture where students pose problems to each other and discuss with each other various ways to approach the problems.

About the Problems in the Book

1. The exploratory problems in the book not only promote problem solving; they also aim to strengthen procedural fluency when children start engaging in exploration.
2. Trying to "cover" all the problems in the book must not happen at the cost of students not getting to spend quality time on playing with, discussing, and solving them.

Reading

1. Encourage students to read the book by themselves and in groups.
2. Give opportunities to them to interpret what they read and to express it to others.

Right of Not Knowing!

1. It is perfectly okay if some of the content is not understood immediately. Along with trying to understand and reflect upon such content, it can also be taken to the classroom and opened up for discussion. After the discussion, things that are clear and those that are not yet clear can be clearly summarised. In these discussions, you can participate as a fellow seeker, and when students see a teacher seek and think to understand something, it sets a wonderful example for them!
2. Learning is a continual process. Indeed, there is so much in mathematics that is still not known and requires further exploration!

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A Note to Students!

To be able to appreciate the art of mathematics, it is not enough to just be a passive spectator. You need to immerse yourself in its process like a detective getting into action to solve a mystery.

This is especially required when you see a new question or when a question arises from your own sense of wonder, or when you come across a new beautiful pattern. When you encounter these, pause your reading, and use your creativity to work out the question or understand and appreciate the pattern.

You will find that some questions are accompanied by their answers. Even if this is the case, it is worthwhile to work on the problems by yourself or in a group before you see the answer. This will enrich your experience of going through the book!

Whenever there are questions coming up, you will see this icon: . This indicates that it is time for figuring things out! Sometimes you will find many questions collected together in a single place under the title '**Figure It Out**'.

Some questions are marked Math Talk .These questions are meant to be discussed and worked out with your friends.

Finally, there are questions marked Try This . These questions demand more creativity to be answered, and therefore will also often be more fun to answer as a result!

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THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

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1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

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1 PATTERNS IN MATHEMATICS

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1.1 What is Mathematics?

Mathematics is, in large part, the search for patterns, and for the explanations as to why those patterns exist.

Such patterns indeed exist all around us—in nature, in our homes and schools, and in the motion of the sun, moon, and stars. They occur in everything that we do and see, from shopping and cooking, to throwing a ball and playing games, to understanding weather patterns and using technology.

The search for patterns and their explanations can be a fun and creative endeavour. It is for this reason that mathematicians think of mathematics both as an art and as a science. This year, we hope that you will get a chance to see the creativity and artistry involved in discovering and understanding mathematical patterns.

It is important to keep in mind that mathematics aims to not just find out what patterns exist, but also the explanations for why they exist. Such explanations can often then be used in applications well beyond the context in which they were discovered, which can then help to propel humanity forward.

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For example, the understanding of patterns in the motion of stars, planets, and their satellites led humankind to develop the theory of gravitation, allowing us to launch our own satellites and send rockets to the Moon and to Mars; similarly, understanding patterns in genomes has helped in diagnosing and curing diseases—among thousands of other such examples.

Figure it Out

Math Talk

1. Can you think of other examples where mathematics helps us in our everyday lives?
2. How has mathematics helped propel humanity forward? (You might think of examples involving: carrying out scientific experiments; running our economy and democracy; building bridges, houses or other complex structures; making TVs, mobile phones, computers, bicycles, trains, cars, planes, calendars, clocks, etc.)

1.2 Patterns in Numbers

Among the most basic patterns that occur in mathematics are **patterns of numbers**, particularly patterns of whole numbers:

0, 1, 2, 3, 4, ...

The branch of Mathematics that studies patterns in whole numbers is called **number theory**.

Number sequences are the most basic and among the most fascinating types of patterns that mathematicians study.

Table 1 shows some key number sequences that are studied in Mathematics.

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Table 1 Examples of number sequences

1, 1, 1, 1, 1, 1, 1, ...	(All 1’s)
1, 2, 3, 4, 5, 6, 7, ...	(Counting numbers)
1, 3, 5, 7, 9, 11, 13, ...	(Odd numbers)
2, 4, 6, 8, 10, 12, 14, ...	(Even numbers)
1, 3, 6, 10, 15, 21, 28, ...	(Triangular numbers)
1, 4, 9, 16, 25, 36, 49, ...	(Squares)
1, 8, 27, 64, 125, 216, ...	(Cubes)
1, 2, 3, 5, 8, 13, 21, ...	(Virahānka numbers)
1, 2, 4, 8, 16, 32, 64, ...	(Powers of 2)
1, 3, 9, 27, 81, 243, 729, ...	(Powers of 3)

Figure it Out

1. Can you recognize the pattern in each of the sequences in Table 1?
2. Rewrite each sequence of Table 1 in your notebook, along with the next three numbers in each sequence! After each sequence, write in your own words what is the rule for forming the numbers in the sequence.

Math Talk

1.3 Visualising Number Sequences

Many number sequences can be visualised using pictures. Visualising mathematical objects through pictures or diagrams can be a very fruitful way to understand mathematical patterns and concepts.

Let us represent the first seven sequences in Table 1 using the following pictures.

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Table 2 Pictorial representation of some number sequences

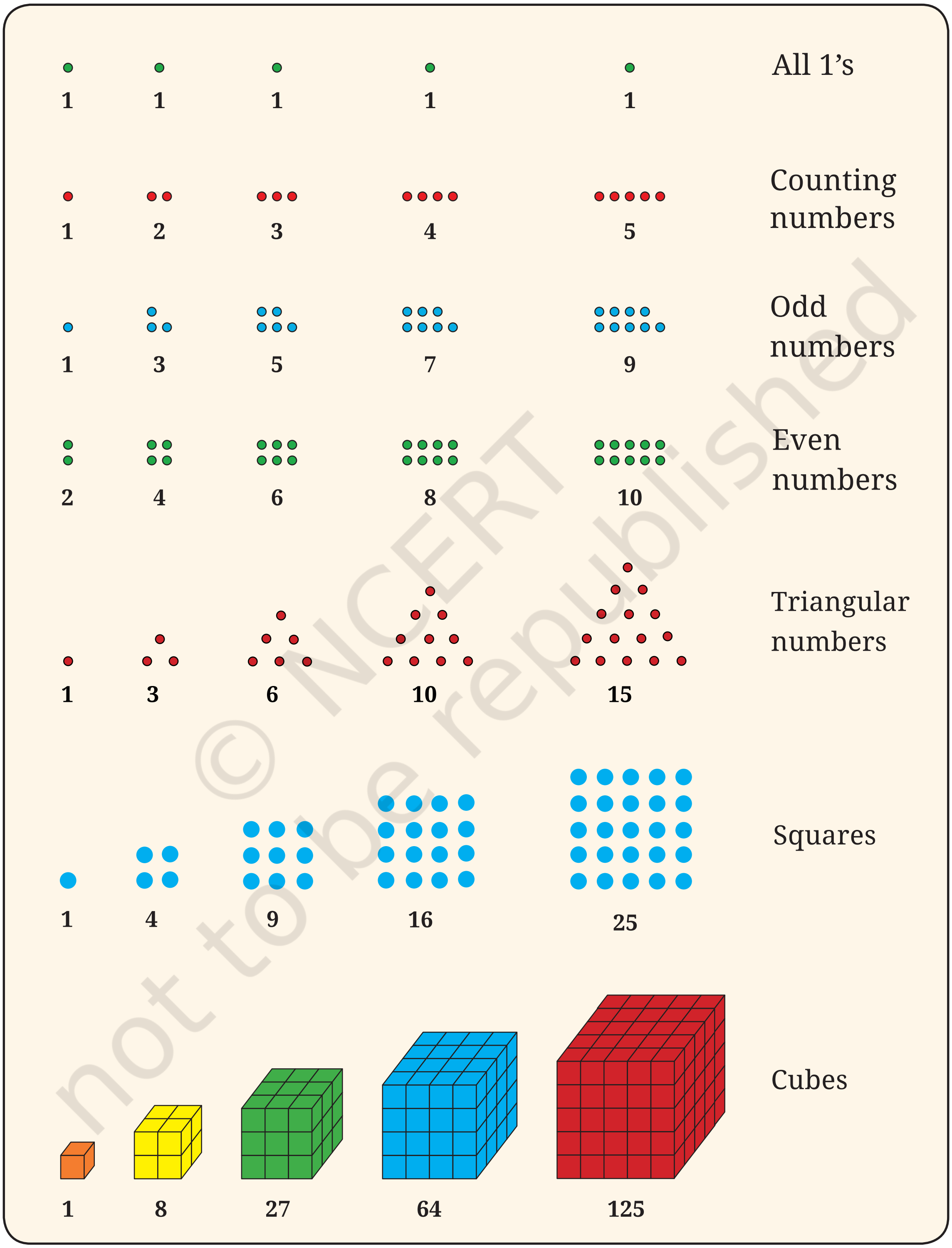

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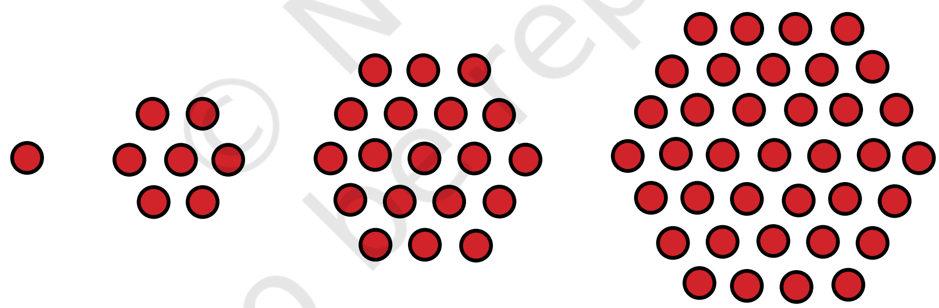
Figure it Out

Math Talk

1. Copy the pictorial representations of the number sequences in Table 2 in your notebook, and draw the next picture for each sequence!
2. Why are 1, 3, 6, 10, 15, … called **triangular numbers**? Why are 1, 4, 9, 16, 25, … called **square numbers** or **squares**? Why are 1, 8, 27, 64, 125, … called **cubes**?
3. You will have noticed that 36 is both a triangular number and a square number! That is, 36 dots can be arranged perfectly both in a triangle and in a square. Make pictures in your notebook illustrating this!

   This shows that the same number can be represented differently, and play different roles, depending on the context. Try representing some other numbers pictorially in different ways!
4. What would you call the following sequence of numbers?



1 **7** **19** **37**

   That's right, they are called **hexagonal numbers**! Draw these in your notebook. What is the next number in the sequence?
5. Can you think of pictorial ways to visualise the sequence of Powers of 2? Powers of 3?

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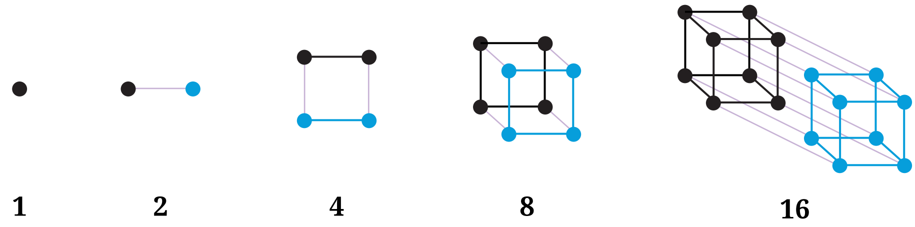
Here is one possible way of thinking about Powers of 2:



1.4 Relations among Number Sequences

Sometimes, number sequences can be related to each other in surprising ways.

Example: What happens when we start adding up odd numbers?

$$1 = 1$$
$$1 + 3 = 4$$
$$1 + 3 + 5 = 9$$
$$1 + 3 + 5 + 7 = 16$$
$$1 + 3 + 5 + 7 + 9 = 25$$
$$1 + 3 + 5 + 7 + 9 + 11 = 36$$
$$\vdots$$

This is a really beautiful pattern!

Why does this happen? Do you think it will happen forever?

The answer is that the pattern does happen forever. But why? As mentioned earlier, the reason why the pattern happens is just as important and exciting as the pattern itself.

A picture can explain it

Visualising with a picture can help explain the phenomenon. Recall that square numbers are made by counting the number of dots in a square grid.

How can we partition the dots in a square grid into odd numbers of dots: 1, 3, 5, 7,... ?

Think about it for a moment before reading further!

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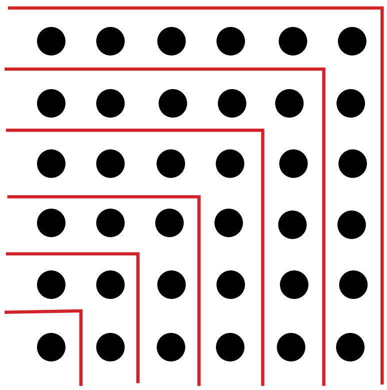
Here is how it can be done:



This picture now makes it evident that

$$1 + 3 + 5 + 7 + 9 + 11 = 36.$$

Because such a picture can be made for a square of any size, this explains why adding up odd numbers gives square numbers.

- By drawing a similar picture, can you say what is the sum of the first 10 odd numbers?
- Now by imagining a similar picture, or by drawing it partially, as needed, can you say what is the sum of the first 100 odd numbers?

Another example of such a relation between sequences: Adding up and down

Let us look at the following pattern:

$$1 = 1$$
$$1 + 2 + 1 = 4$$
$$1 + 2 + 3 + 2 + 1 = 9$$
$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$$
$$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25$$
$$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36$$
$$\vdots$$

This seems to be giving yet another way of getting the square numbers—by adding the counting numbers up and then down!

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Can you find a similar pictorial explanation?

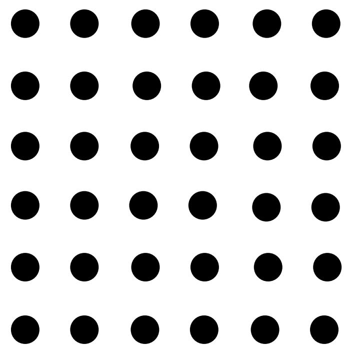

Figure it Out

1. Can you find a similar pictorial explanation for why adding counting numbers up and down, i.e., 1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1, ..., gives square numbers?
2. By imagining a large version of your picture, or drawing it partially, as needed, can you see what will be tha value of 1 + 2 + 3 + ··· + 99 + 100 + 99 + ··· + 3 + 2 + 1?
3. Which sequence do you get when you start to add the All 1's sequence up? What sequence do you get when you add the All 1's sequence up and down?
4. Which sequence do you get when you start to add the Counting numbers up? Can you give a smaller pictorial explanation?
5. What happens when you add up pairs of consecutive triangular numbers? That is, take 1 + 3, 3 + 6, 6 + 10, 10 + 15, ... ? Which sequence do you get? Why? Can you explain it with a picture?
6. What happens when you start to add up powers of 2 starting with 1, i.e., take 1, 1 + 2, 1 + 2 + 4, 1 + 2 + 4 + 8, ... ? Now add 1 to each of these numbers—what numbers do you get? Why does this happen?

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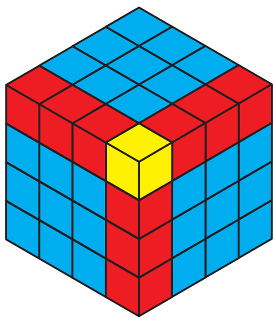
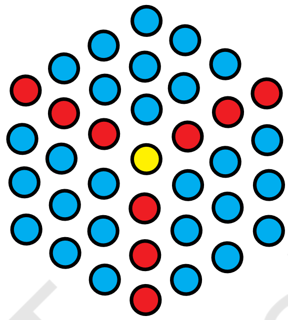
7. What happens when you multiply the triangular numbers by 6 and add 1? Which sequence do you get? Can you explain it with a picture?
8. What happens when you start to add up hexagonal numbers, i.e., take 1, 1 + 7, 1 + 7 + 19, 1 + 7 + 19 + 37, ... ? Which sequence do you get? Can you explain it using a picture of a cube?





9. Find your own patterns or relations in and among the sequences in Table 1. Can you explain why they happen with a picture or otherwise?

1.5 Patterns in Shapes

Other important and basic patterns that occur in Mathematics are **patterns of shapes**. These shapes may be in one, two, or three dimensions (1D, 2D, or 3D)—or in even more dimensions. The branch of Mathematics that studies patterns in shapes is called **geometry**.

Shape sequences are one important type of shape pattern that mathematicians study. Table 3 shows a few key shape sequences that are studied in Mathematics.

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Table 3 Examples of shape sequences

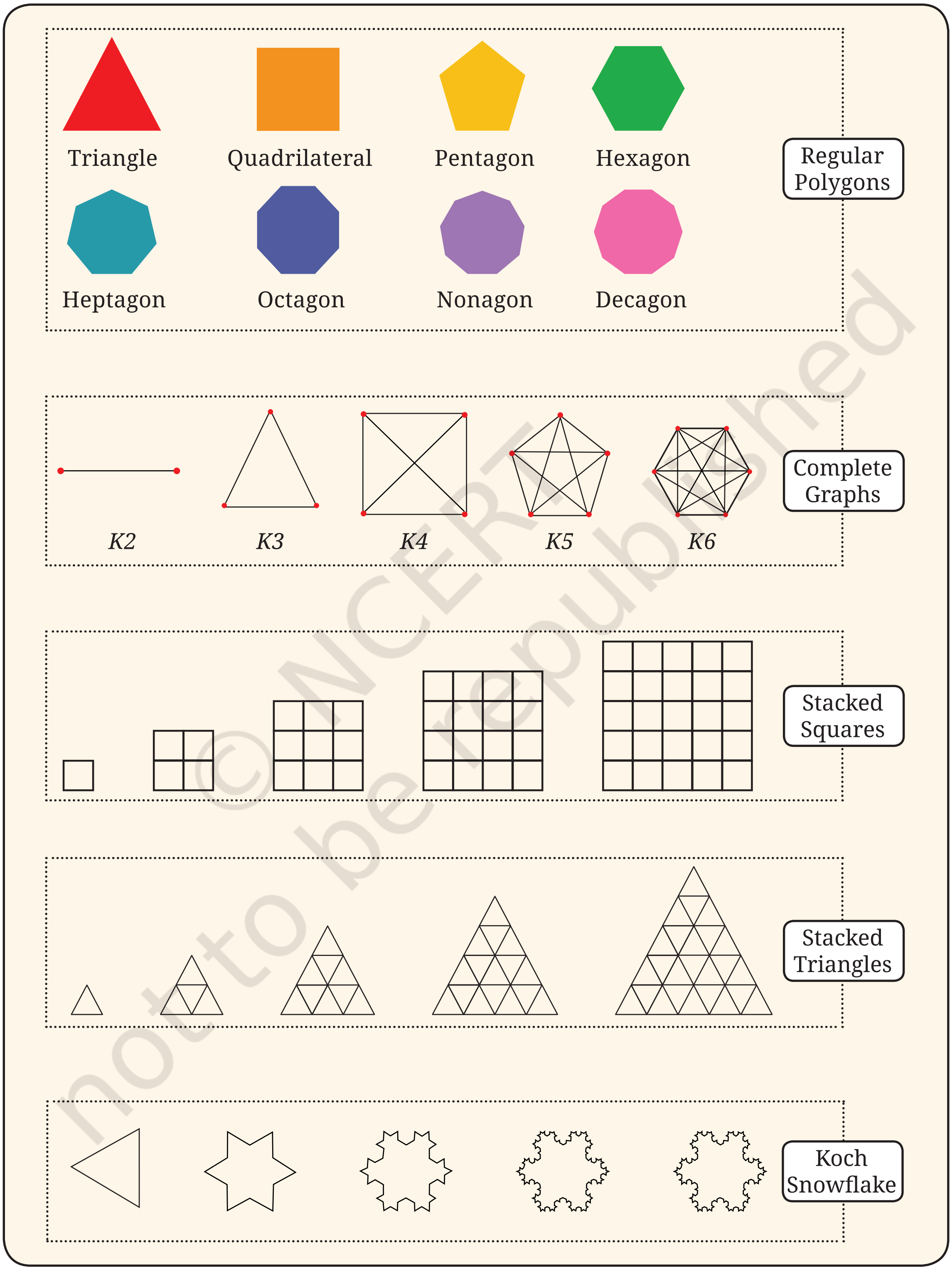

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Figure it Out

Math Talk

1. Can you recognise the pattern in each of the sequences in Table 3?
2. Try and redraw each sequence in Table 3 in your notebook. Can you draw the next shape in each sequence? Why or why not? After each sequence, describe in your own words what is the rule or pattern for forming the shapes in the sequence.

1.6 Relation to Number Sequences

Often, shape sequences are related to number sequences in surprising ways. Such relationships can be helpful in studying and understanding both the shape sequence and the related number sequence.

Example: The number of sides in the shape sequence of Regular Polygons is given by the counting numbers starting at 3, i.e., 3, 4, 5, 6, 7, 8, 9, 10, ... That is why these shapes are called, respectively, **regular triangle**, **quadrilateral** (i.e., **square**), **pentagon**, **hexagon**, **heptagon**, **octagon**, **nonagon**, **decagon**, etc., respectively.

The word ‘regular’ refers to the fact that these shapes have equal-length sides and also equal ‘angles’ (i.e., the sides look the same and the corners also look the same). We will discuss angles in more depth in the next chapter.

The other shape sequences in Table 3 also have beautiful relationships with number sequences.

Figure it Out

Try This

1. Count the number of sides in each shape in the sequence of Regular Polygons. Which number sequence do you get? What about the number of corners in each shape in the sequence of Regular Polygons? Do you get the same number sequence? Can you explain why this happens?
2. Count the number of lines in each shape in the sequence of Complete Graphs. Which number sequence do you get? Can you explain why?

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3. How many little squares are there in each shape of the sequence of Stacked Squares? Which number sequence does this give? Can you explain why?
4. How many little triangles are there in each shape of the sequence of Stacked Triangles? Which number sequence does this give? Can you explain why? (*Hint*: In each shape in the sequence, how many triangles are there in each row?)
5. To get from one shape to the next shape in the Koch Snowflake sequence, one replaces each line segment '—' by a 'speed bump' . As one does this more and more times, the changes become tinier and tinier with very very small line segments. How many total line segments are there in each shape of the Koch Snowflake? What is the corresponding number sequence? (The answer is 3, 12, 48, ..., i.e. 3 times Powers of 4; this sequence is not shown in Table 1)

Try This

SUMMARY

- Mathematics may be viewed as the search for patterns and for the explanations as to why those patterns exist.
- Among the most basic patterns that occur in mathematics are **number sequences.**
- Some important examples of number sequences include the counting numbers, odd numbers, even numbers, square numbers, triangular numbers, cube numbers, Virahānka numbers, and powers of 2.
- Sometimes number sequences can be related to each other in beautiful and remarkable ways. For example, adding up the sequence of odd numbers starting with 1 gives square numbers.
- Visualizing number sequences using pictures can help to understand sequences and the relationships between them.
- **Shape sequences** are another basic type of pattern in mathematics. Some important examples of shape sequences include regular polygons, complete graphs, stacked triangles and squares, and Koch snowflake iterations. Shape sequences also exhibit many interesting relationships with number sequences.

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2

LINES AND ANGLES

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In this chapter, we will explore some of the most basic ideas of geometry including points, lines, rays, line segments and angles. These ideas form the building blocks of 'plane geometry', and will help us in understanding more advanced topics in geometry such as the construction and analysis of different shapes.

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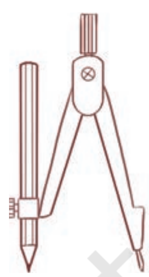
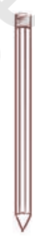
2.1 Point

Mark a dot on the paper with a sharp tip of a pencil. The sharper the tip, the thinner will be the dot. This tiny dot will give you an idea of a point. A **point** determines a precise location, but it has no length, breadth or height. Some models for a point are given below.



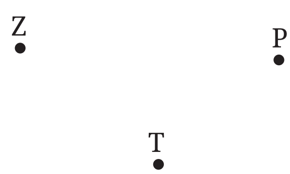
The tip of a compass



The sharpened end of a pencil

The pointed end of a needle

If you mark three points on a piece of paper, you may be required to distinguish these three points. For this purpose, each of the three points may be denoted by a single capital letter such as



Z, P and T. These points are read as 'Point Z', 'Point P' and 'Point T'. Of course, the dots represent precise locations and must be imagined to be invisibly thin.

2.2 Line Segment

Fold a piece of paper and unfold it. Do you see a crease? This gives the idea of a line segment. It has two end points, A and B.

Mark any two points A and B on a sheet of paper. Try to connect A to B by various routes (Fig. 2.1).

What is the shortest route from A to B? This shortest path from Point A to Point B (including A and B) as shown here is called the **line segment** from A to B. It is denoted by either $\overline{AB}$ or $\overline{BA}$. The points A and B are called the **end points** of the line segment $\overline{AB}$.

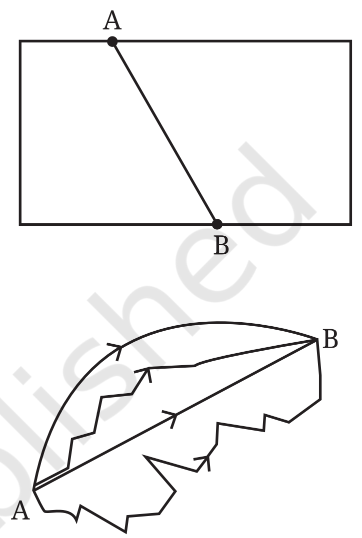


Fig. 2.1

2.3 Line

Imagine that the line segment from A to B (i.e., $\overline{AB}$) is extended beyond A in one direction and beyond B in the other direction without any end (see Fig 2.2). This is a model for a **line.** Do you think you can draw a complete picture of a line? No. Why?

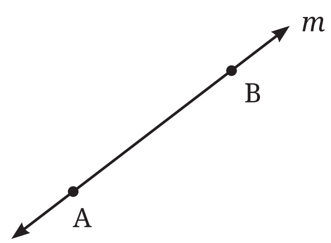


Fig. 2.2

A line through two points A and B is written as $\overleftrightarrow{AB}$. It extends forever in both directions. Sometimes a line is denoted by a letter like *l* or *m*.

Observe that any two points determine a unique line that passes through both of them.

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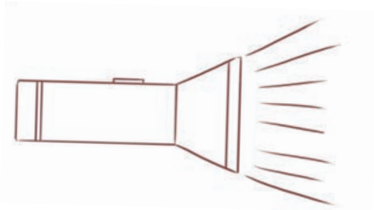
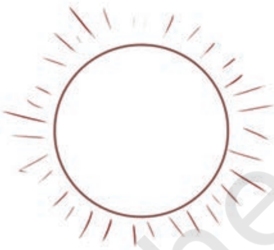
2.4 Ray

A **ray** is a portion of a line that starts at one point (called the **starting point** or **initial point** of the ray) and goes on endlessly in a direction. The following are some models for a ray:

Beam of light from a lighthouse



Ray of light from a torch



Sun rays

Look at the diagram (Fig. 2.3) of a ray. Two points are marked on it. One is the starting point A and the other is a point P on the path of the ray. We then denote the ray by $\overrightarrow{AP}$.

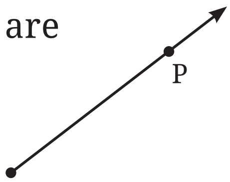


Fig. 2.3

Figure it Out

1. Rihan marked a point on a piece of paper. How many lines can he draw that pass through the point?

   Sheetal marked two points on a piece of paper. How many different lines can she draw that pass through both of the points?

   Can you help Rihan and Sheetal find their answers?

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2. Name the line segments in Fig. 2.4. Which of the five marked points are on exactly one of the line segments? Which are on two of the line segments?

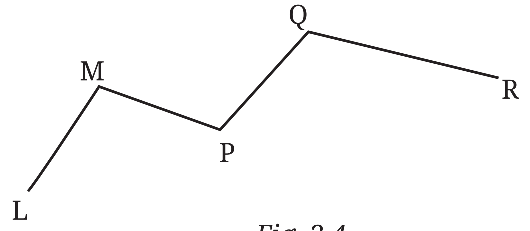

Fig. 2.4

3. Name the rays shown in Fig. 2.5. Is T the starting point of each of these rays?

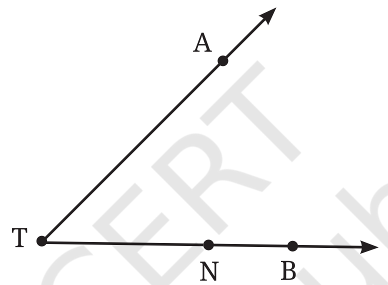

Fig. 2.5

4. Draw a rough figure and write labels appropriately to illustrate each of the following:

   a. $\overleftrightarrow{OP}$ and $\overleftrightarrow{OQ}$ meet at O.

   b. $\overrightarrow{XY}$ and $\overleftrightarrow{PQ}$ intersect at point M.

   c. Line l contains points E and F but not point D.

   d. Point P lies on AB.

5. In Fig. 2.6, name:

   a. Five points

   b. A line

   c. Four rays

   d. Five line segments

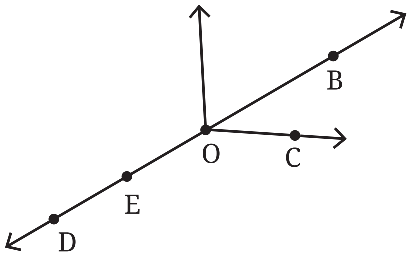

Fig. 2.6

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6. Here is a ray $\overrightarrow{OA}$ (Fig. 2.7). It starts at O and passes through the point A. It also passes through the point B.
   a. Can you also name it as $\overrightarrow{OB}$? Why?
   b. Can we write $\overrightarrow{OA}$ as $\overrightarrow{AO}$? Why or why not?

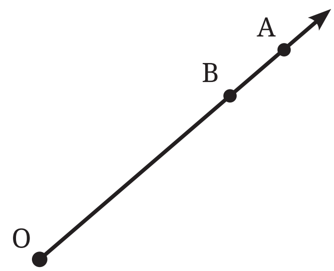


Fig. 2.7

2.5 Angle

An **angle** is formed by two rays having a common starting point. Here is an angle formed by rays $\overrightarrow{BD}$ and $\overrightarrow{BE}$ where B is the common starting point (Fig. 2.8).

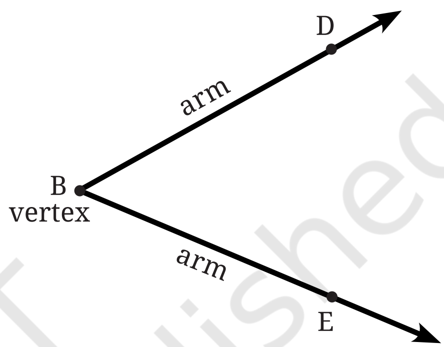
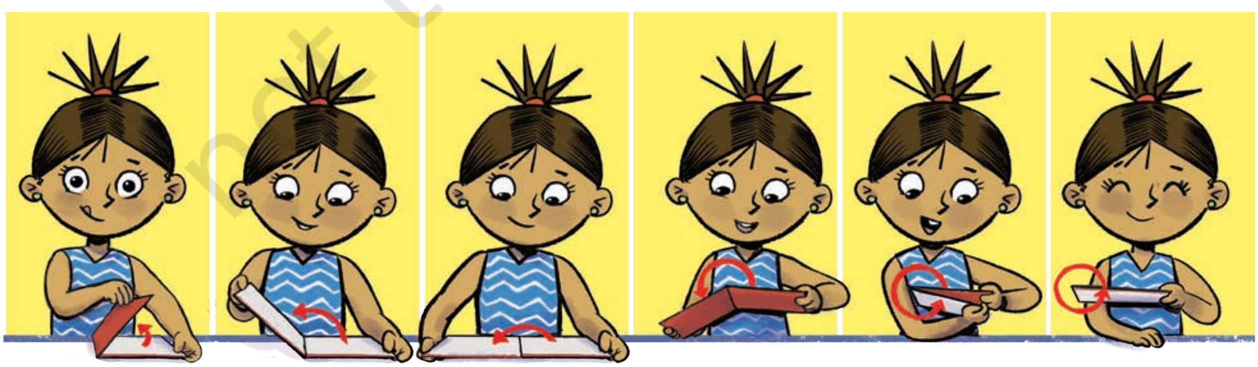


Fig. 2.8

The point B is called the **vertex** of the angle, and the rays $\overrightarrow{BD}$ and $\overrightarrow{BE}$ are called the **arms** of the angle. How can we name this angle? We can simply use the vertex and say that it is the Angle B. To be clearer, we use a point on each of the arms together with the vertex to name the angle. In this case, we name the angle as Angle DBE or Angle EBD. The word angle can be replaced by the symbol '$\angle$', i.e., $\angle$DBE or $\angle$EBD. Note that in specifying the angle, the vertex is always written as the middle letter.

To indicate an angle, we use a small curve at the vertex (refer to Fig. 2.9).

Vidya has just opened her book. Let us observe her opening the cover of the book in different scenarios.



Case 1 Case 2 Case 3 Case 4 Case 5 Case 6

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Do you see angles being made in each of these cases? Can you mark their arms and vertex?

Which angle is greater—the angle in Case 1 or the angle in Case 2?

Just as we talk about the size of a line based on its length, we also talk about the size of an angle based on its amount of rotation.

So, the angle in Case 2 is greater as in this case she needs to rotate the cover more. Similarly, the angle in Case 3 is even larger than that of Case 2, because there is even more rotation, and Cases 4, 5, and 6 are successively larger angles with more and more rotation.

The size of an angle is the amount of rotation or turn that is needed about the vertex to move the first ray to the second ray.

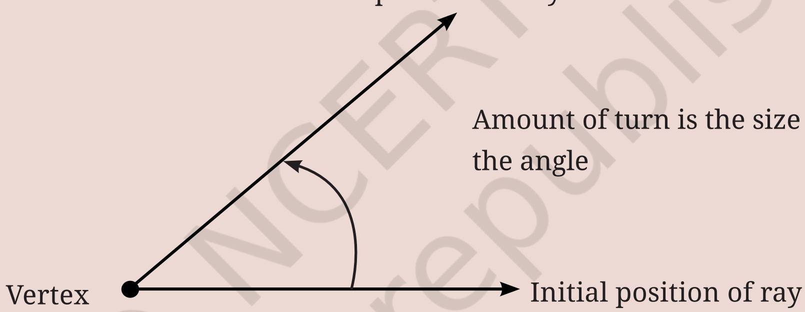

Fig. 2.9

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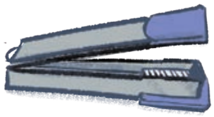
Let's look at some other examples where angles arise in real life by rotation or turn:

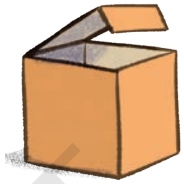
- In a compass or divider, we turn the arms to form an angle. The vertex is the point where the two arms are joined. Identify the arms and vertex of the angle.

- A pair of scissors has two blades. When we open them (or 'turn them') to cut something, the blades form an angle. Identify the arms and the vertex of the angle.

- Look at the pictures of spectacles, wallet and other common objects. Identify the angles in them by marking out their arms and vertices.







Do you see how these angles are formed by turning one arm with respect to the other?

Teacher's Note

Teacher needs to organise various activities with the students to recognise the size of an angle as a measure of rotation.

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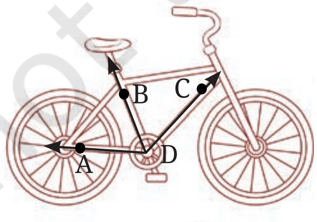
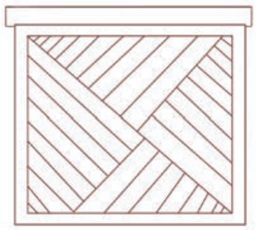
Figure it Out

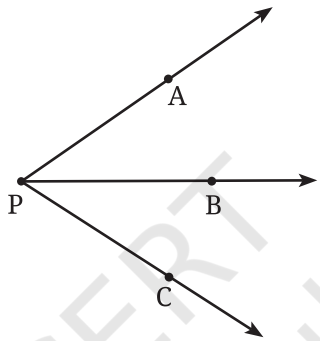
1. Can you find the angles in the given pictures? Draw the rays forming any one of the angles and name the vertex of the angle.

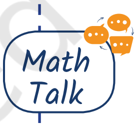




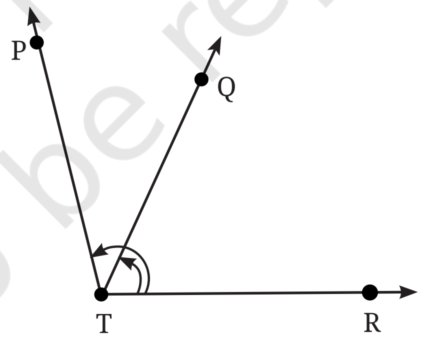
2. Draw and label an angle with arms ST and SR.

3. Explain why $\angle APC$ cannot be labelled as $\angle P$.





4. Name the angles marked in the given figure.



5. Mark any three points on your paper that are not on one line. Label them A, B, C. Draw all possible lines going through pairs of these points. How many lines do you get? Name them. How many angles can you name using A, B, C? Write them down, and mark each of them with a curve as in Fig. 2.9.

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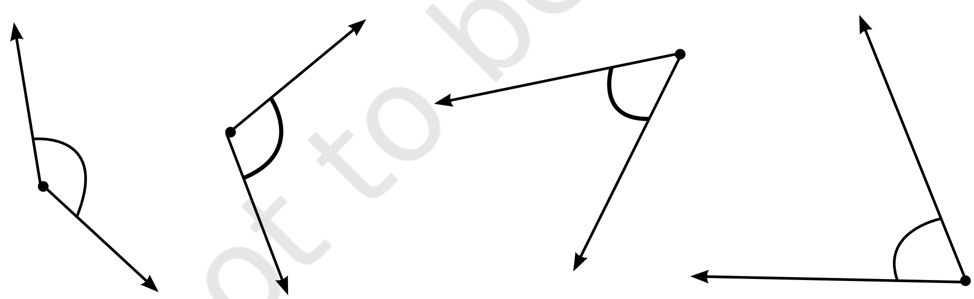
6. Now mark any four points on your paper so that no three of them are on one line. Label them A, B, C, D. Draw all possible lines going through pairs of these points. How many lines do you get? Name them. How many angles can you name using A, B, C, D? Write them all down, and mark each of them with a curve as in Fig. 2.9.

2.6 Comparing Angles

Look at these animals opening their mouths. Do you see any angles here? If yes, mark the arms and vertex of each one. Some mouths are open wider than others; the more the turning of the jaws, the larger the angle! Can you arrange the angles in this picture from smallest to largest?

Is it always easy to compare two angles?



Math Talk

Here are some angles. Label each of the angles. How will you compare them?

Draw a few more angles; label them and compare.

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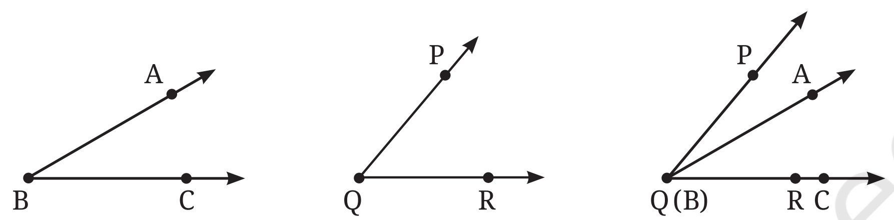
Comparing angles by superimposition

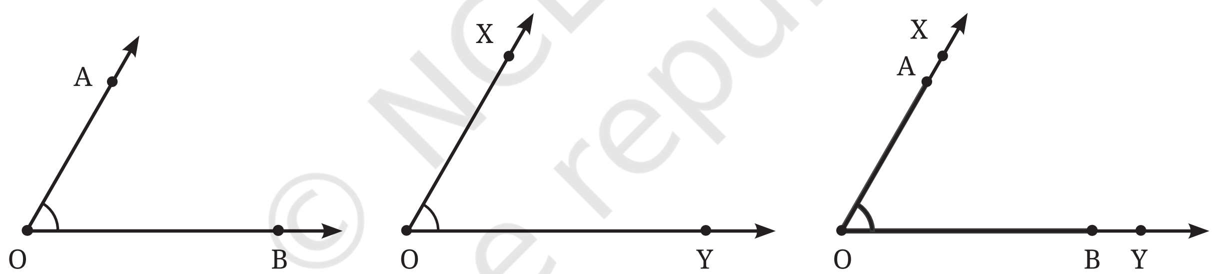
Any two angles can be compared by placing them one over the other, i.e., by superimposition. While superimposing, the vertices of the angles must overlap.

After superimposition, it becomes clear which angle is smaller and which is larger.



The picture shows the two angles superimposed. It is now clear that ∠PQR is larger than ∠ABC.

Equal angles. Now consider ∠AOB and ∠XOY in the figure. Which is greater?



The corners of both of these angles match and the arms overlap with each other, i.e., OA ↔ OX and OB ↔ OY. So, the angles are **equal** in size.

The reason these angles are considered to be equal in size is because when we visualise each of these angles as being formed out of rotation, we can see that there is an equal amount of rotation needed to move $\overrightarrow{OB}$ to $\overrightarrow{OA}$ and $\overrightarrow{OY}$ to $\overrightarrow{OX}$.

From the point of view of superimposition, when two angles are superimposed, and the common vertex and the two rays of both angles lie on top of each other, then the sizes of the angles are equal.

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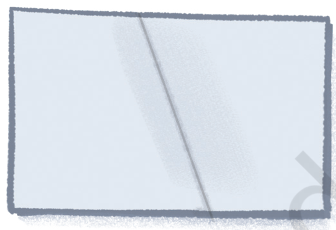
- Where else do we use superimposition to compare?

Math Talk

Figure it Out

1. Fold a rectangular sheet of paper, then draw a line along the fold created. Name and compare the angles formed between the fold and the sides of the paper. Make different angles by folding a rectangular sheet of paper and compare the angles. Which is the largest and smallest angle you made?



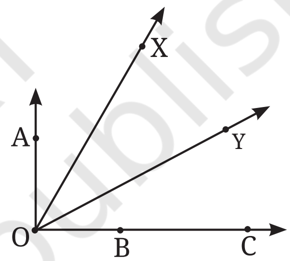
2. In each case, determine which angle is greater and why.

   a. ∠AOB or ∠XOY

   b. ∠AOB or ∠XOB

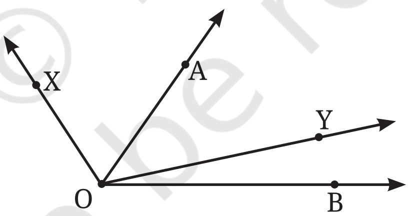
   c. ∠XOB or ∠XOC

   Discuss with your friends on how you decided which one is greater.



Math Talk

3. Which angle is greater: ∠XOY or ∠AOB? Give reasons.



Comparing angles without superimposition

Two cranes are arguing about who can open their mouth wider, i.e., who is making a bigger angle.

Let us first draw their angles. How do we know which one is bigger? As seen

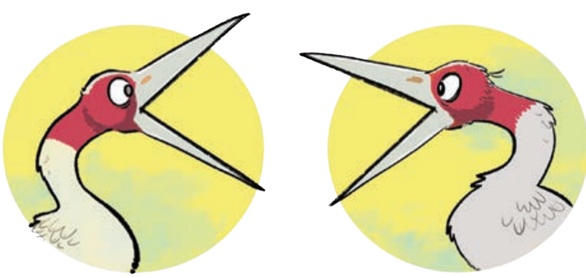


Fig. 2.10

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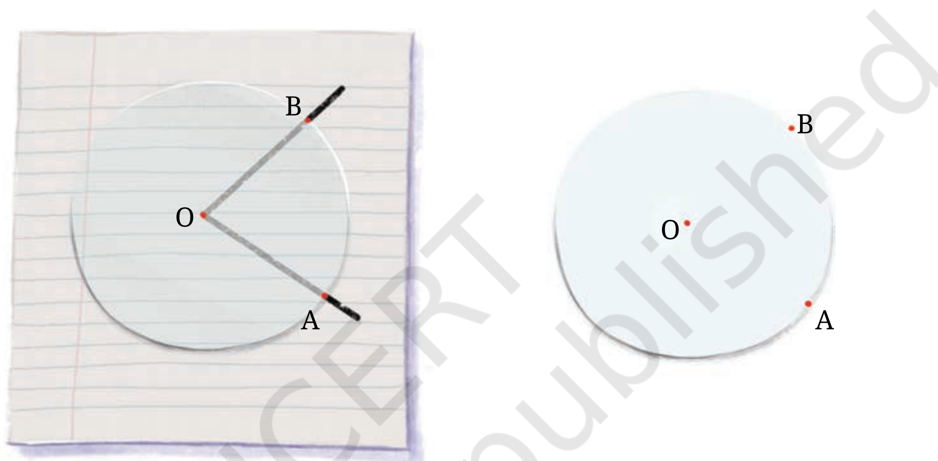
before, one could trace these angles, superimpose them and then check. But can we do it without superimposition?

Suppose we have a transparent circle which can be moved and placed on figures. Can we use this for comparison?

Let us place the circular paper on the angle made by the first crane. The circle is placed in such a way that its centre is on the vertex of the angle. Let us mark the points A and B on the edge circle at the points where the arms of the angle pass through the circle.



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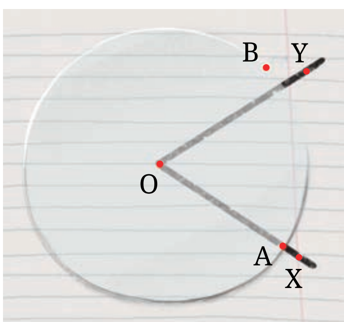
Can we use this to find out if this angle is greater than, or equal to or smaller than the angle made by the second crane?

Let us place it on the angle made by the second crane so that the vertex coincides with the centre of the circle and one of the arms passes through OA.



Can you now tell which angle is bigger?

Which crane was making the bigger angle?

If you can make a circular piece of transparent paper, try this method to compare the angles in Fig. 2.10 with each other.

Teacher's Note

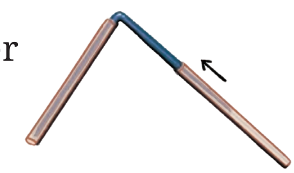
A teacher needs to check the understanding of the students around the notion of an angle. Sometimes students might think that increasing the length of the arms of the angle increases the angle. For this, various situations should be posed to the students to check their understanding on the same.

2.7 Making Rotating Arms

Let us make 'rotating arms' using two paper straws and a paper clip by following these steps:

1. Take two paper straws and a paper clip.

2. Insert the straws into the arms of the paper clip.



3. Your rotating arm is ready!

Make several 'rotating arms' with different angles between the arms. Arrange the angles you have made from smallest to largest by comparing and using superimposition.

Passing through a slit: Collect a number of rotating arms with different angles; do not rotate any of the rotating arms during this activity.

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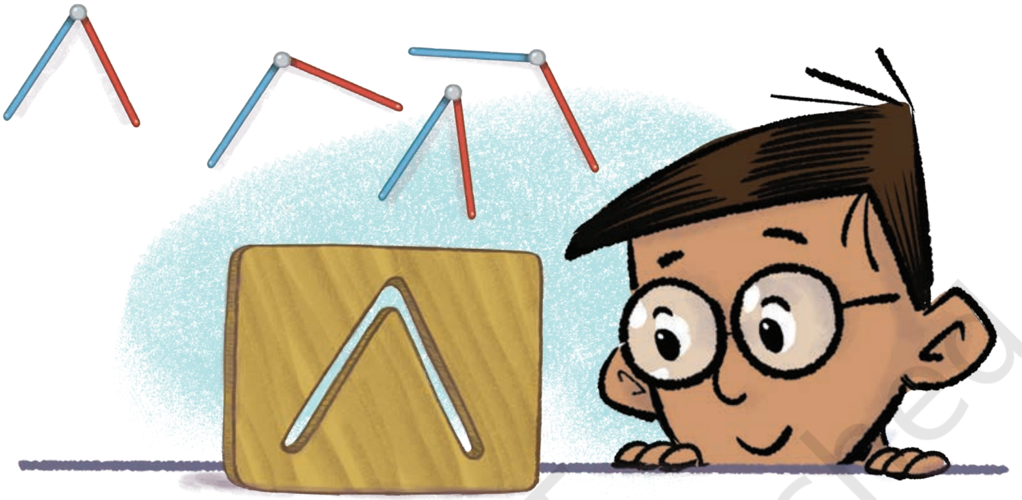
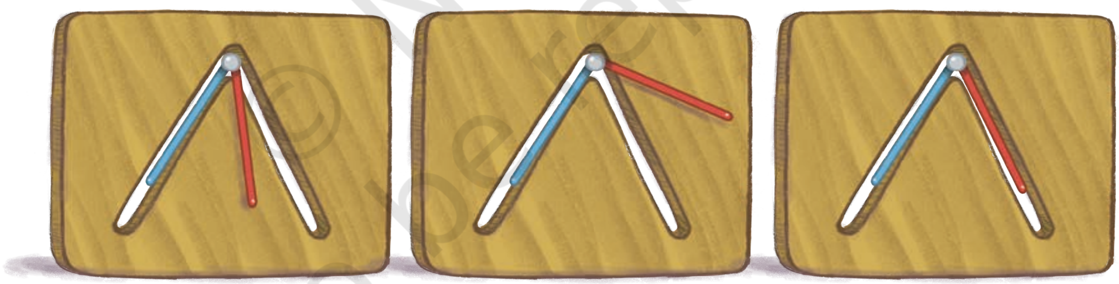
Take a cardboard and make an angle-shaped slit as shown below by tracing and cutting out the shape of one of the rotating arms.



Now, shuffle and mix up all the rotating arms. Can you identify which of the rotating arms will pass through the slit?

The correct one can be found by placing each of the rotating arms over the slit. Let us do this for some of the rotating arms:

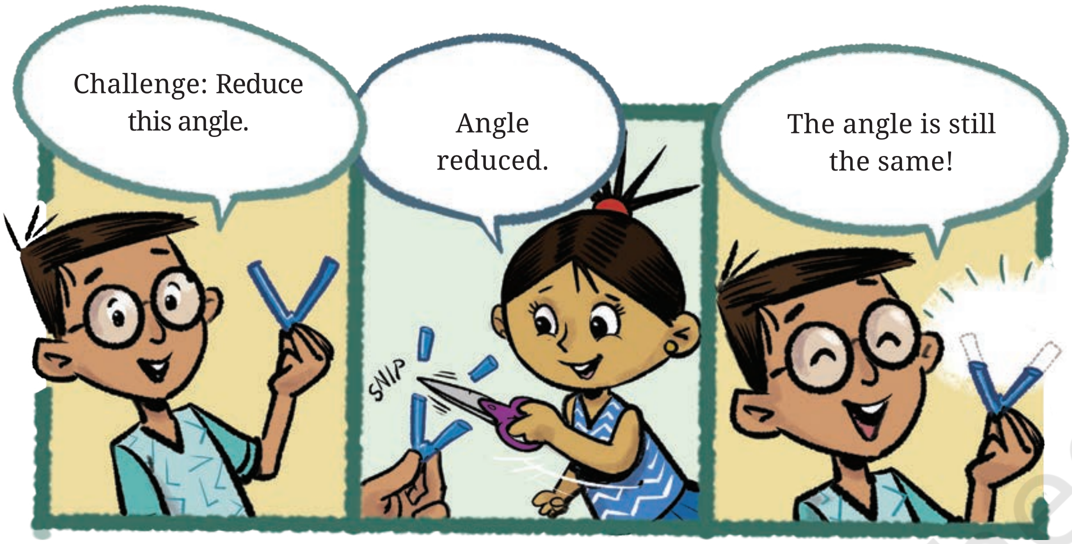


Slit angle is greater than the arms' angle. The arms will not go through the slit.

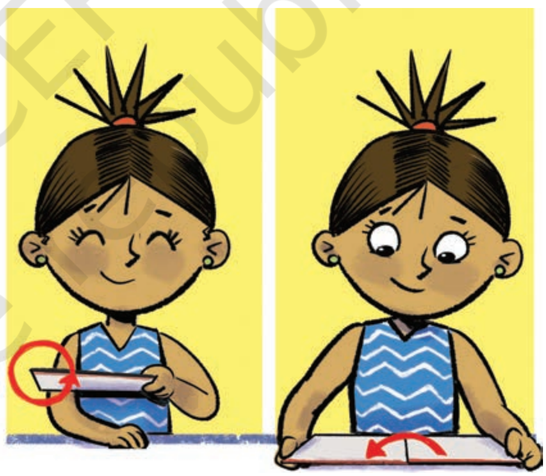
Slit angle is less than the arms' angle. The arms will not go through the slit.

Slit angle is equal to the arms' angle. The arms will go through the slit.

Only the pair of rotating arms where the angle is equal to that of the slit passes through the slit. Note that the possibility of passing through the slit depends only on the angle between the rotating arms and not on their lengths (as long as they are shorter than the length of the slit).



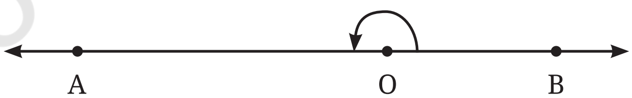
2.8 Special Types of Angles



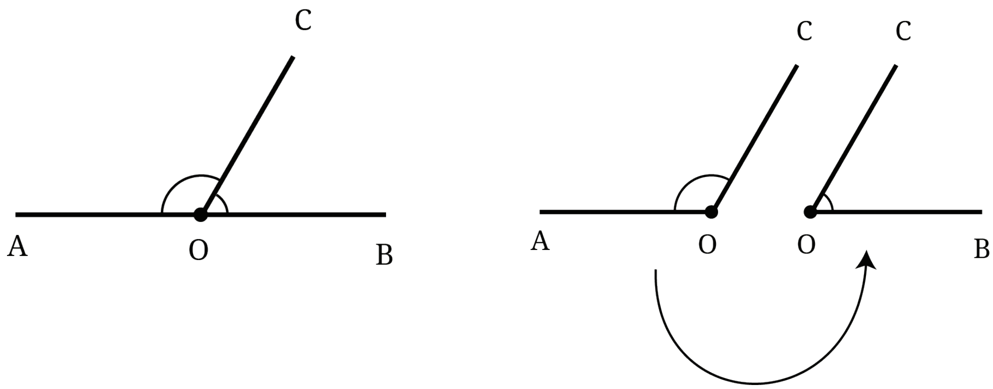
Let us go back to Vidya's notebook and observe her opening the cover of the book in different scenarios.

She makes a full turn of the cover when she has to write while holding the book in her hand.

She makes a half turn of the cover when she has to open it on her table. In this case, observe the arms of the angle formed. They lie in a straight line. Such an angle is called a **straight angle**.



Let us consider a straight angle $\angle AOB$. Observe that any ray $\overrightarrow{OC}$ divides it into two angles, $\angle AOC$ and $\angle COB$.



- Is it possible to draw $\overrightarrow{OC}$ such that the two angles are equal to each other in size?

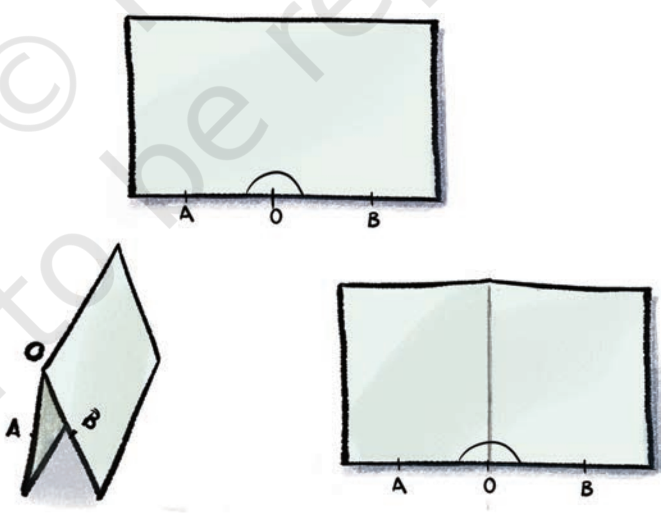
Math Talk

Let's Explore

We can try to solve this problem using a piece of paper. Recall that when a fold is made, it creates a crease which is straight.

Take a rectangular piece of paper and on one of its sides, mark the straight angle AOB. By folding, try to get a line (crease) passing through O that divides ∠AOB into two equal angles.

How can it be done?



Fold the paper such that OB overlaps with OA. Observe the crease and the two angles formed.

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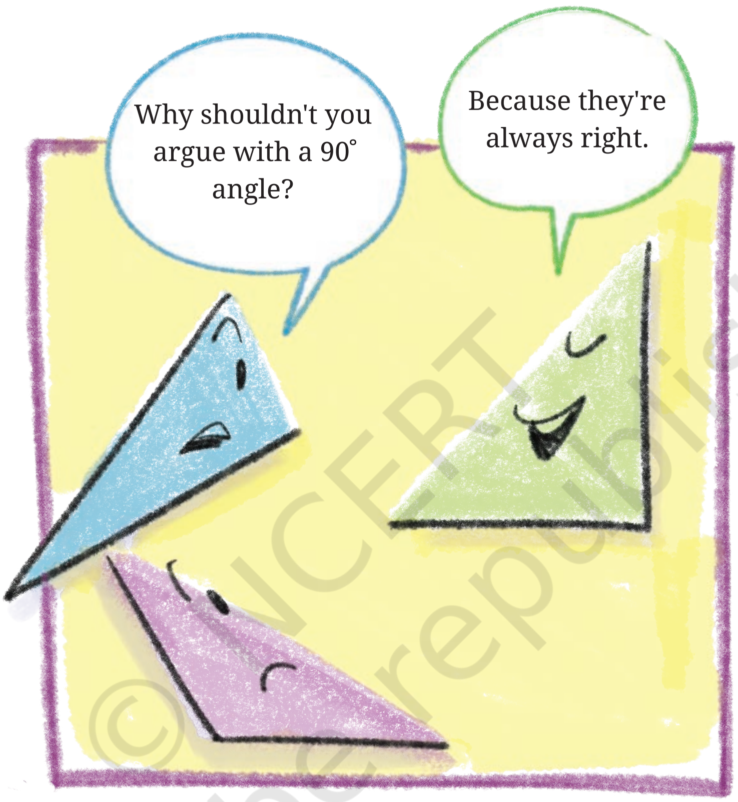
Justify why the two angles are equal. Is there a way to superimpose and check? Can this superimposition be done by folding?

Each of these equal angles formed are called right angles. So, a straight angle contains two **right angles**.



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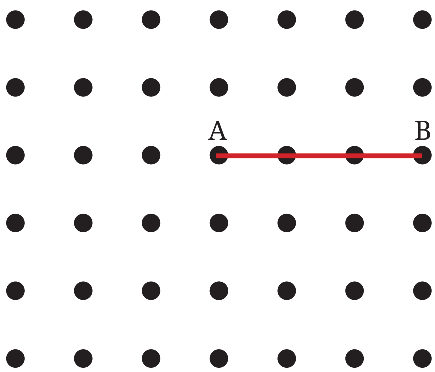
If a straight angle is formed by half of a full turn, how much of a full turn will form a right angle?

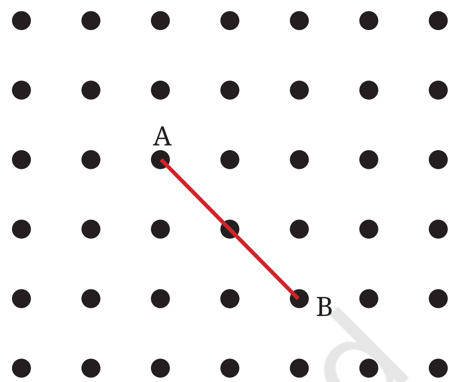
Observe that a right angle resembles the shape of an 'L'. An angle is a right angle only if it is exactly half of a straight angle. Two lines that meet at right angles are called **perpendicular** lines.

Figure it Out

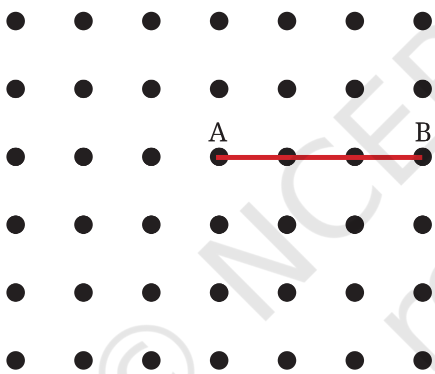
1. How many right angles do the windows of your classroom contain? Do you see other right angles in your classroom?

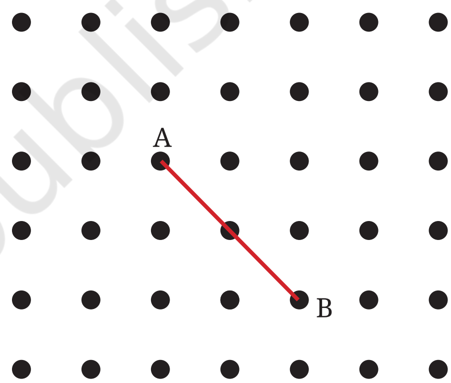
2. Join A to other grid points in the figure by a straight line to get a straight angle. What are all the different ways of doing it?



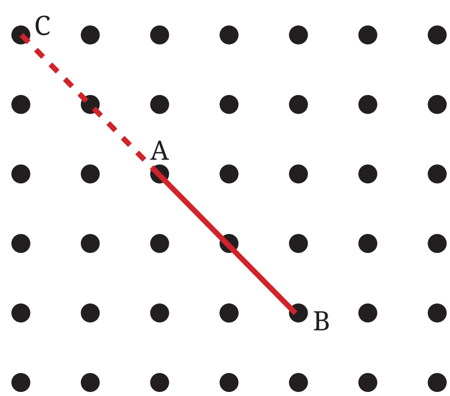


3. Now join A to other grid points in the figure by a straight line to get a right angle. What are all the different ways of doing it?





Hint: Extend the line further as shown in the figure below. To get a right angle at A, we need to draw a line through it that divides the straight angle CAB into two equal parts.



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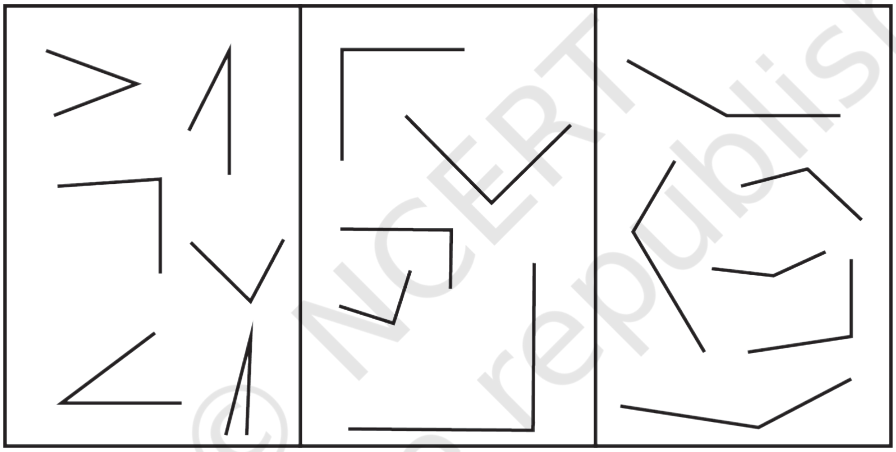
4. Get a slanting crease on the paper. Now, try to get another crease that is perpendicular to the slanting crease.
   a. How many right angles do you have now? Justify why the angles are exact right angles.
   b. Describe how you folded the paper so that any other person who doesn't know the process can simply follow your description to get the right angle.

Classifying Angles

Angles are classified in three groups as shown below. Right angles are shown in the second group. What could be the common feature of the other two groups?



In the first group, all angles are less than a right angle or in other words, less than a quarter turn. Such angles are called **acute angles**.

In the third group, all angles are greater than a right angle but less than a straight angle. The turning is more than a quarter turn and less than a half turn. Such angles are called **obtuse angles**.

Figure it Out

1. Identify acute, right, obtuse and straight angles in the previous figures.
2. Make a few acute angles and a few obtuse angles. Draw them in different orientations.

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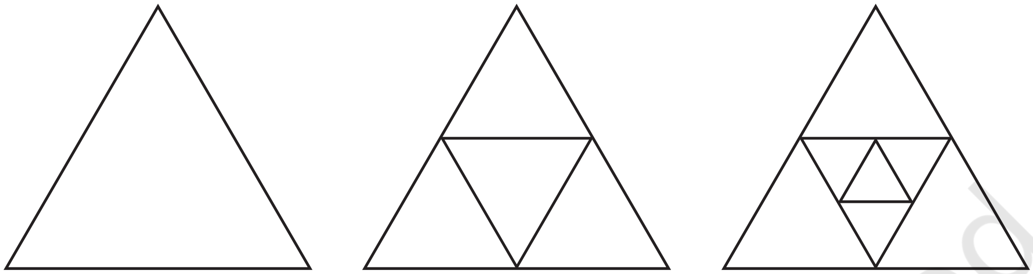
3. Do you know what the words acute and obtuse mean? Acute means sharp and obtuse means blunt. Why do you think these words have been chosen?
4. Find out the number of acute angles in each of the figures below.



What will be the next figure and how many acute angles will it have? Do you notice any pattern in the numbers?

2.9 Measuring Angles

We have seen how to compare two angles. But can we actually quantify how big an angle is using a number without having to compare it to another angle?

We saw how various angles can be compared using a circle. Perhaps a circle could be used to assign measures for angles?

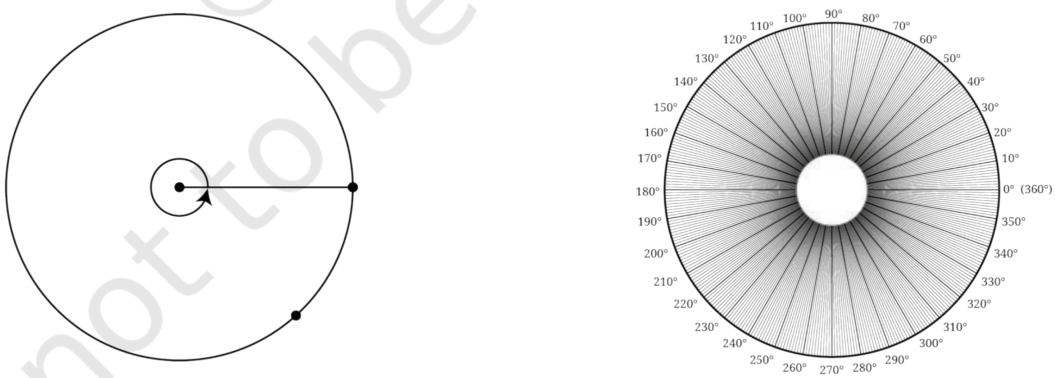


Fig. 2.12

To assign precise measures to angles, mathematicians came up with an idea. They divided the angle in the centre of the circle into

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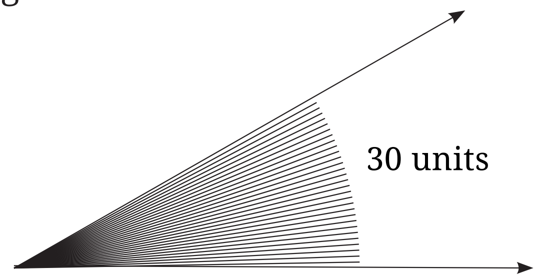
360 equal angles or parts. The angle measure of each of these unit parts is 1 degree, which is written as 1°.

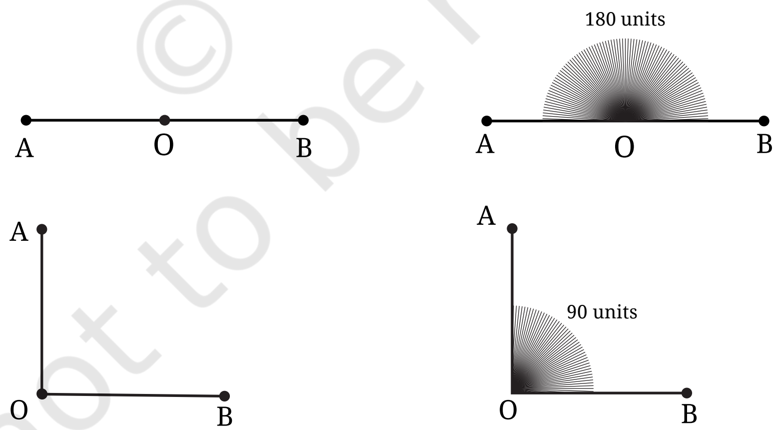
This unit part is used to assign measure to any angle: the measure of an angle is the number of 1° unit parts it contains inside it. For example, see this figure:



It contains 30 units of 1° angle and so we say that its angle measure is 30°.

Measures of different angles: What is the measure of a full turn in degrees? As we have taken it to contain 360 degrees, its measure is 360°.

What is the measure of a straight angle in degrees? A straight angle is half of a full turn. As a full-turn is 360°, a half turn is 180°.
What is the measure of a right angle in degrees? Two right angles together form a straight angle. As a straight angle measures 180°, a right angle measures 90°.



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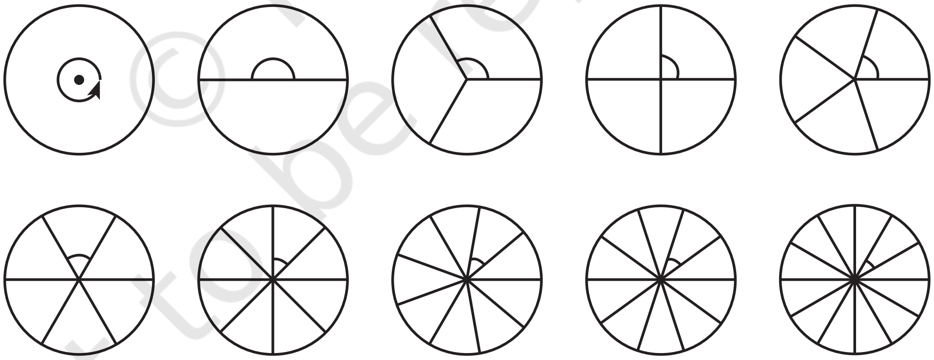
A pinch of history

A full turn has been divided into 360°. Why 360? The reason why we use 360° today is not fully known. The division of a circle into 360 parts goes back to ancient times. The *Rigveda*, one of the very oldest texts of humanity going back thousands of years, speaks of a wheel with 360 spokes (Verse 1.164.48). Many ancient calendars, also going back over 3000 years—such as calendars of India, Persia, Babylonia and Egypt—were based on having 360 days in a year. In addition, Babylonian mathematicians frequently used divisions of 60 and 360 due to their use of sexagesimal numbers and counting by 60s.

Perhaps the most important and practical answer for why mathematicians over the years have liked and continued to use 360 degrees is that 360 is the smallest number that can be evenly divided by all numbers up to 10, aside from 7. Thus, one can break up the circle into 1, 2, 3, 4, 5, 6, 8, 9 or 10 equal parts, and still have a whole number of degrees in each part! Note that 360 is also evenly divisible by 12, the number of months in a year, and by 24, the number of hours in a day. These facts all make the number 360 very useful.

The circle has been divided into 1, 2, 3, 4, 5, 6, 8, 9 10 and 12 parts below. What are the degree measures of the resulting angles? Write the degree measures down near the indicated angles.



Degree measures of different angles

How can we measure other angles in degrees? It is for this purpose that we have a tool called a **protractor** that is either a circle divided into 360 equal parts as shown in Fig. 2.12 (on page 32), or a half circle divided into 180 equal parts.

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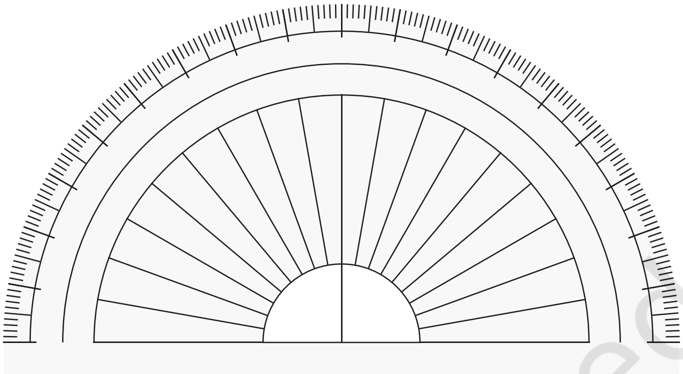
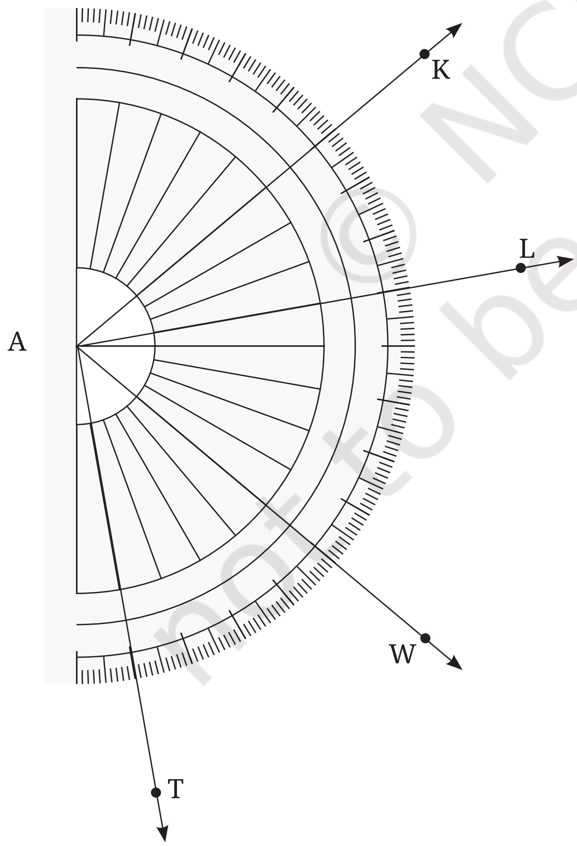
Unlabelled protractor

Here is a protractor. Do you see the straight angle at the center divided into 180 units of 1 degree? Only part of the lines dividing the straight angle are visible, though!



Starting from the marking on the rightmost point of the base, there is a long mark for every 10°. From every such long mark, there is a medium sized mark after 5°.

Figure it out



1. Write the measures of the following angles:

   a. ∠ KAL

   Notice that the vertex of this angle coincides with the centre of the protractor. So the number of units of 1 degree angle between KA and AL gives the measure of ∠KAL. By counting, we get

   $$\angle KAL = 30°.$$

   Making use of the medium sized and large sized marks, is it possible to count the number of units in 5s or 10s?

   b. ∠WAL

   c. ∠TAK

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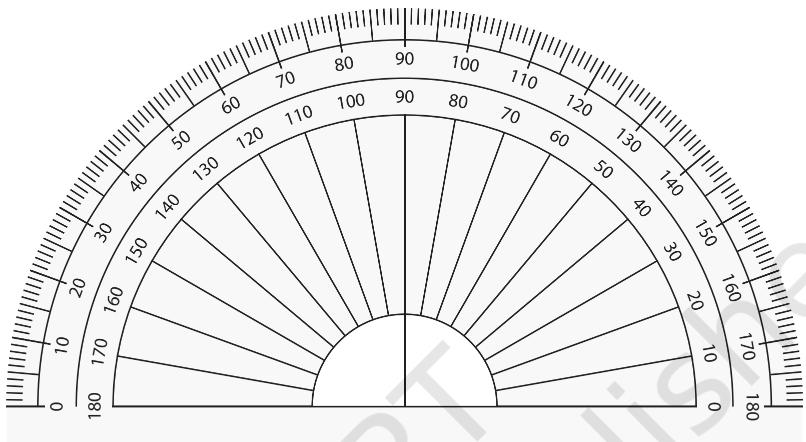
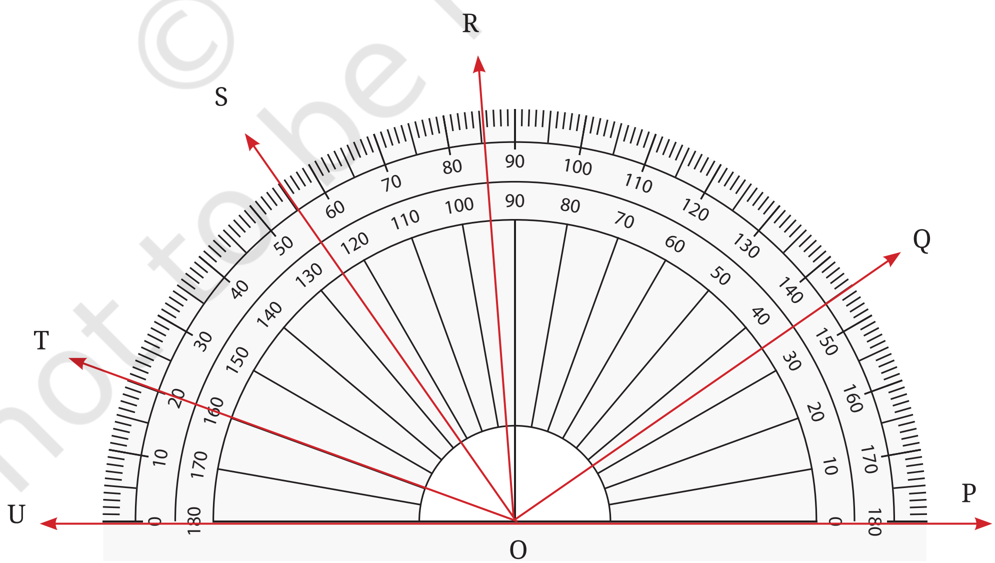
Labelled protractor

This is a protractor that you find in your geometry box. It would appear similar to the protractor above except that there are numbers written on it. Will these make it easier to read the angles?



There are two sets of numbers on the protractor: one increasing from right to left and the other increasing from left to right. Why does it include two sets of numbers?

- Name the different angles in the figure and write their measures.



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Did you include angles such as ∠TOQ?

Which set of markings did you use - inner or outer?

What is the measure of ∠TOS?

Can you use the numbers marked to find the angle without counting the number of markings?

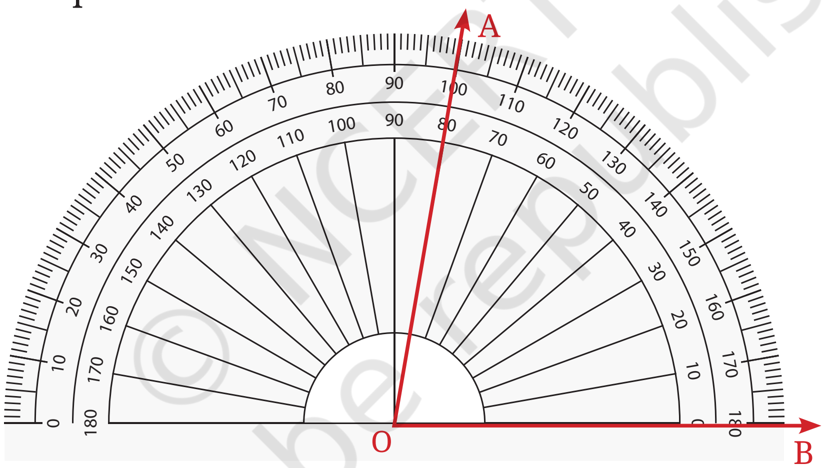
Here, OT and OS pass through the numbers 20 and 55 on the outer scale. How many units of 1 degree are contained between these two arms?

Can subtraction be used here?

How can we measure angles directly without having to subtract?

Place the protractor so the center is on the vertex of the angle.

Align the protractor so that one the arms passes through the 0° mark as in the picture below.



What is the degree measure of ∠AOB?

Make your own Protractor!

You may have wondered how the different equally spaced markings are made on a protractor. We will now see how we can make some of them!

1. Draw a circle of a convenient radius on a sheet of paper. Cut out the circle (Fig. 2.13). A circle or one full turn is 360°.
2. Fold the circle to get two equal halves and cut it through the crease to get a semicircle. Write '0°' in the bottom right corner of the semi-circle.

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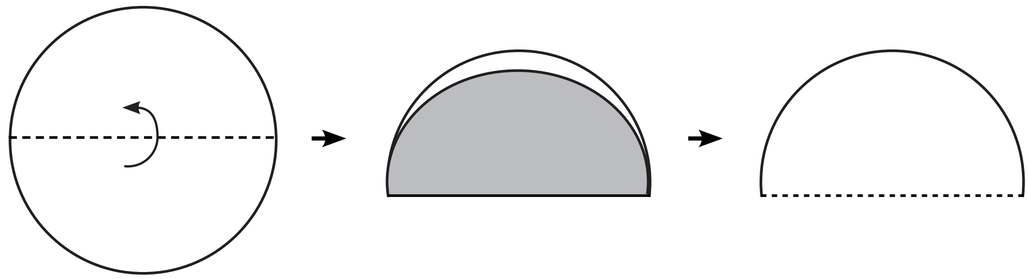
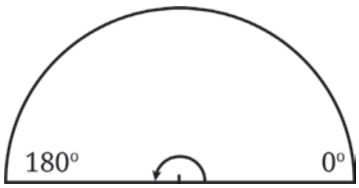
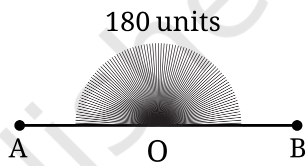
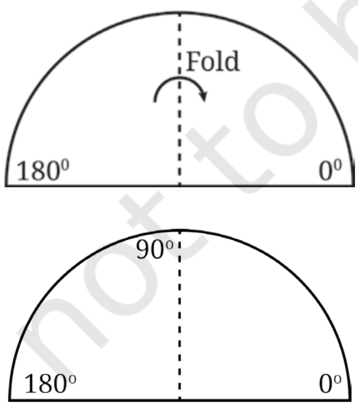
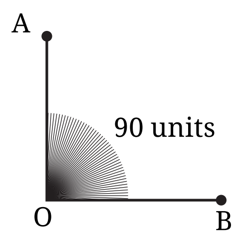


Fig. 2.13

 *Fig. 2.14*	The measure of half a circle is $\frac{1}{2}$ of a full turn. (Fig. 2.14) So, the measure of half a turn = $\frac{1}{2}$ of ____ = 180°. Thus, write 180° in the left bottom corner of the semicircle.	

3. Fold the semi-circular sheet in half as shown in Fig. 2.15 to form a quarter circle.

 *Fig. 2.15*	The measure of a quarter circle is $\frac{1}{4}$ of a full turn. The measure of a $\frac{1}{4}$ turn = $\frac{1}{4}$ of 360° = ________. Or, the measure of a $\frac{1}{4}$ turn = $\frac{1}{2}$ of a half turn = $\frac{1}{2}$ of 180° = ______. Thus, mark 90° at the top of the semicircle.	

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4. Fold the sheet again as shown in Figs. 2.16 and 2.17:

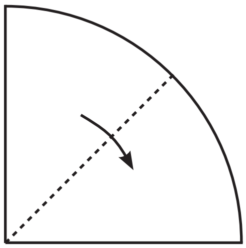

Fig. 2.16

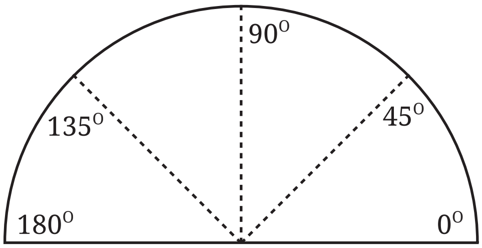

Fig. 2.17

When folded, this is $\frac{1}{8}$ of the circle, or $\frac{1}{8}$ of a turn, or $\frac{1}{8}$ of 360°, or $\frac{1}{4}$ of 180° or $\frac{1}{2}$of 90° = ________________________.

The new creases formed give us measures of 45° and 180°– 45° = 135° as shown. Write 45° and 135° at the correct places on the new creases along the edge of the semicircle.

5. Continuing with another half fold as shown in Fig. 2.18, we get an angle of measure ________________________.

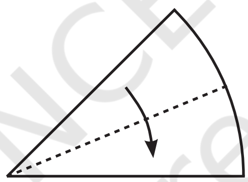

Fig. 2.18

6. Unfold and mark the creases as OB, OC, ..., etc., as shown in Fig. 2.19 and Fig. 2.20.

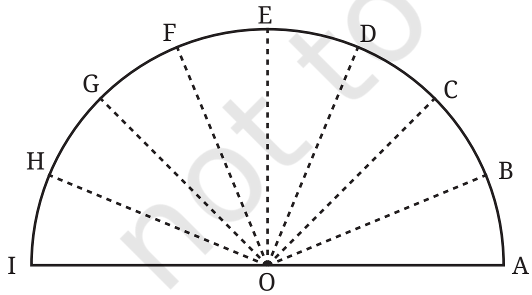

Fig. 2.19

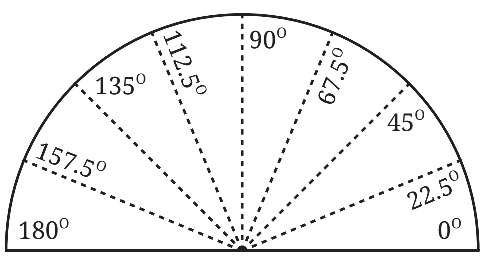

Fig. 2.20

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Think !

In Fig. 2.20, we have ∠AOB = ∠BOC = ∠COD = ∠DOE = ∠EOF = ∠FOG = ∠GOH = ∠HOI=_____. Why?

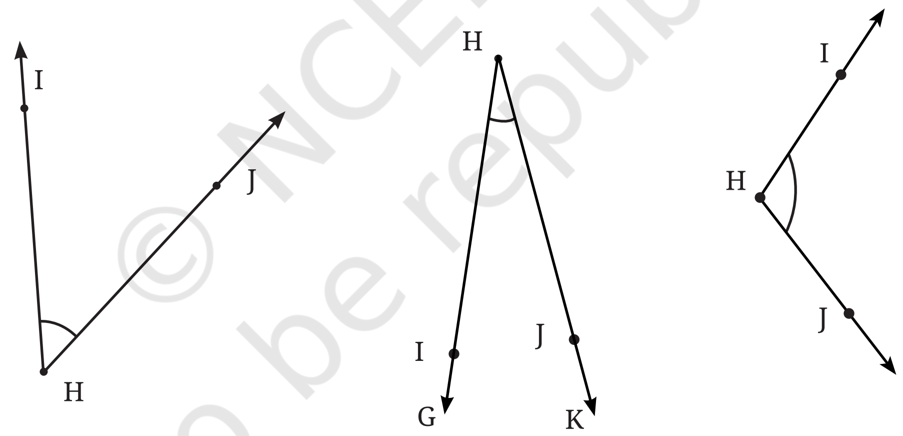
Angle Bisector

At each step, we folded in halves. This process of getting half of a given angle is called **bisecting the angle**. The line that bisects a given angle is called the **angle bisector** of the angle.

Identify the angle bisectors in your handmade protractor. Try to make different angles using the concept of angle bisector through paper folding.

Figure it Out

1. Find the degree measures of the following angles using your protractor.



2. Find the degree measures of different angles in your classroom using your protractor.

Teacher's Note

It is important that students make their own protractor and use it to measure different angles before using the standard protractor so that they know the concept behind the marking of the standard protractor.

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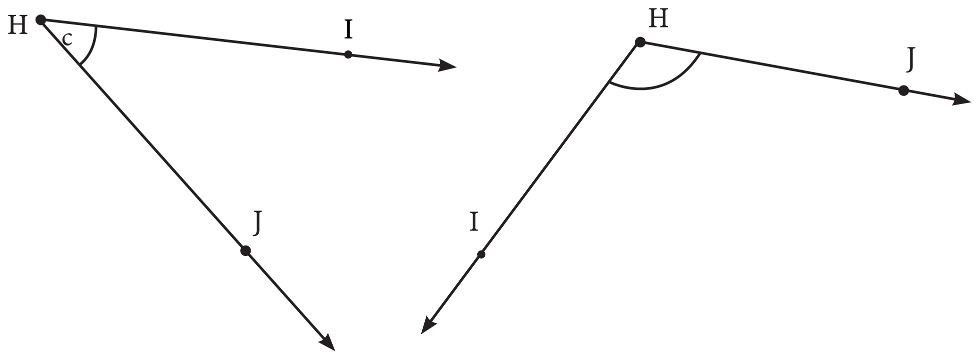
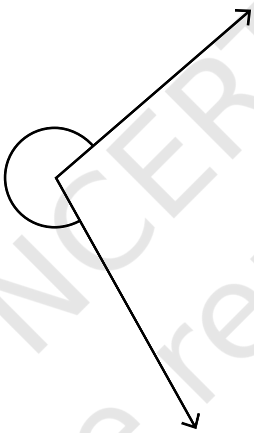
3. Find the degree measures for the angles given below. Check if your paper protractor can be used here!

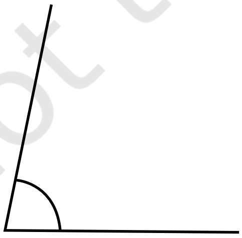


4. How can you find the degree measure of the angle given below using a protractor?

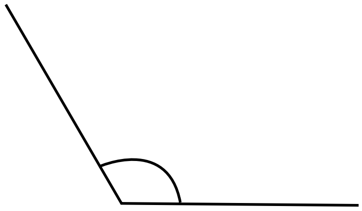


5. Measure and write the degree measures for each of the following angles:

a.



b.



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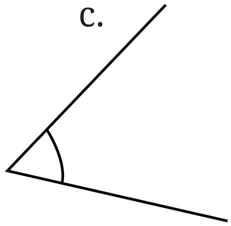
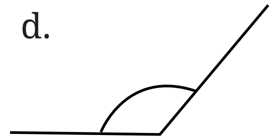
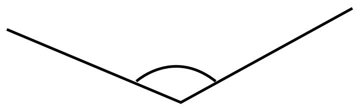
c.



d.

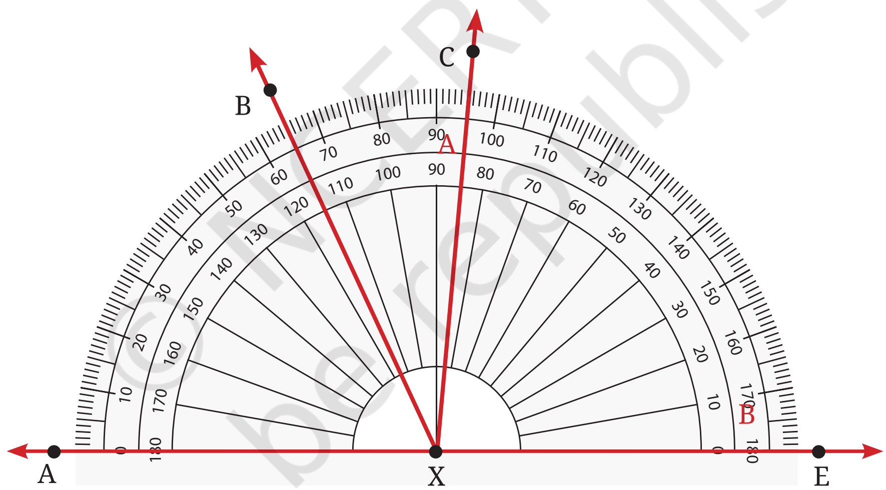


e.

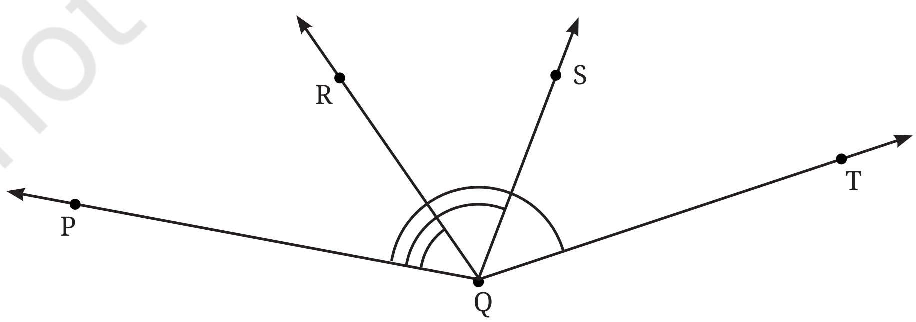


f.

6. Find the degree measures of ∠BXE, ∠CXE, ∠AXB and ∠BXC.



7. Find the degree measures of ∠PQR, ∠PQS and ∠PQT.



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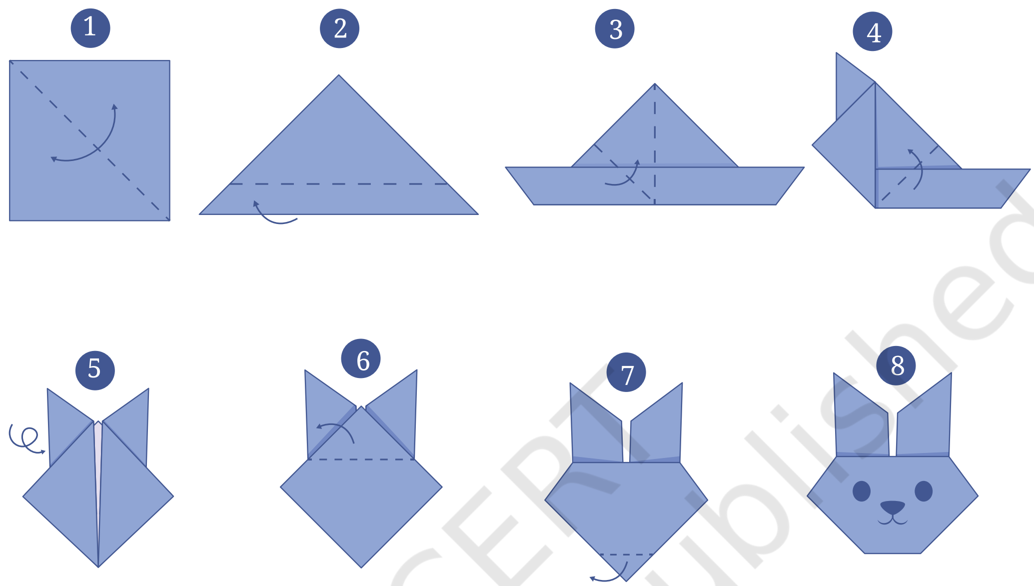
8. Make the paper craft as per the given instructions. Then, unfold and open the paper fully. Draw lines on the creases made and measure the angles formed.



9. Measure all three angles of the triangle shown in Fig. 2.21 (a), and write the measures down near the respective angles. Now add up the three measures. What do you get? Do the same for the triangles in Fig. 2.21 (b) and (c). Try it for other triangles as well, and then make a conjecture for what happens in general! We will come back to why this happens in a later year.

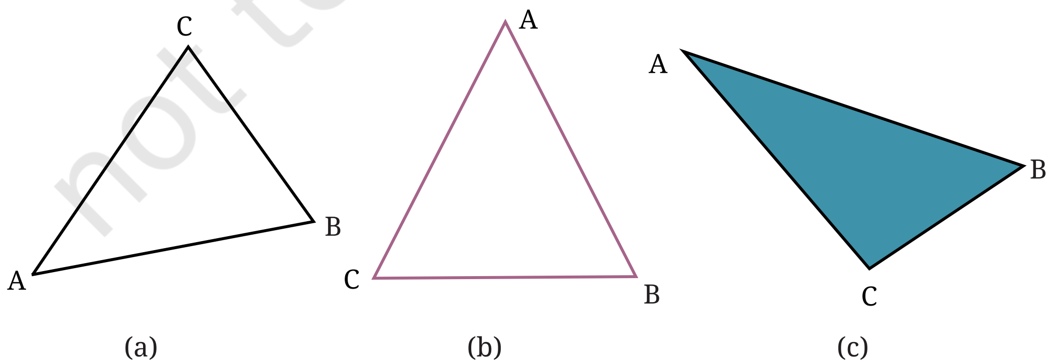


Fig. 2.21

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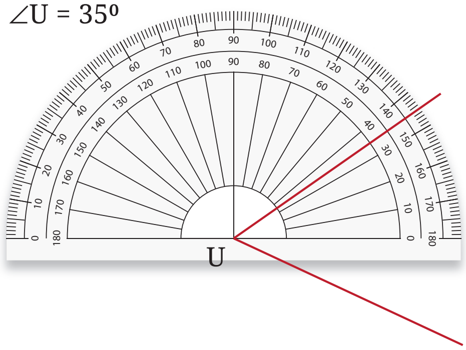
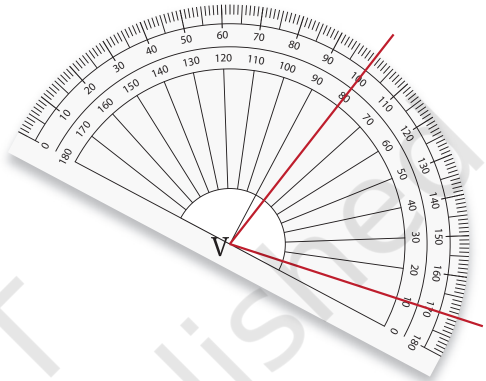
Mind the Mistake, Mend the Mistake!

A student used a protractor to measure the angles as shown below. In each figure, identify the incorrect usage(s) of the protractor and discuss how the reading could have been made and think how it can be corrected.

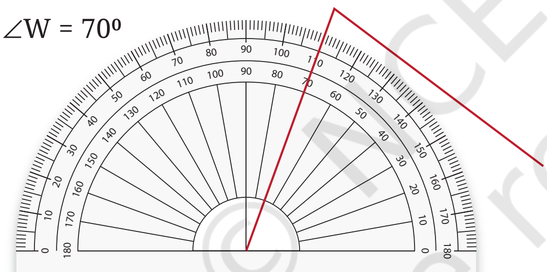
∠U = 35⁰



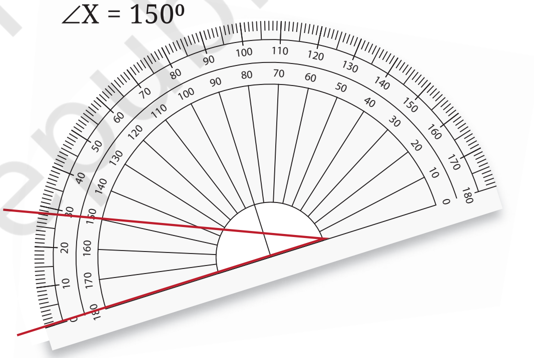
∠V = 80⁰



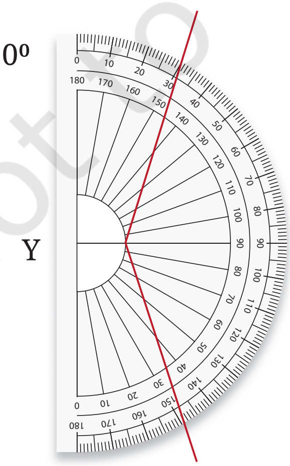
∠W = 70⁰



∠X = 150⁰



∠Y = 120⁰



∠Z = 85⁰

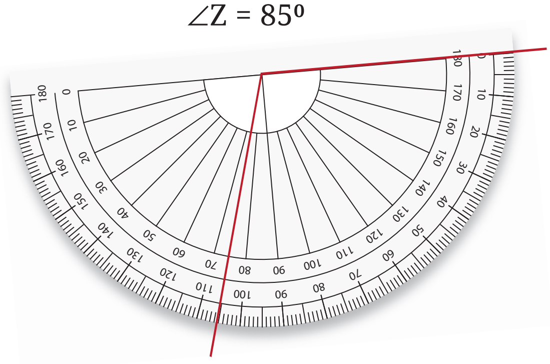


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Figure it Out

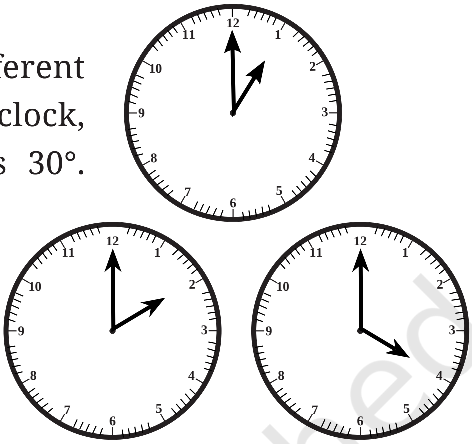
Where are the angles?

1. Angles in a clock:

    a. The hands of a clock make different angles at different times. At 1 o'clock, the angle between the hands is 30°. Why?

    b. What will be the angle at 2 o'clock? And at 4 o'clock? 6 o'clock?

    c. Explore other angles made by the hands of a clock.



2. The angle of a door:

    Is it possible to express the amount by which a door is opened using an angle? What will be the vertex of the angle and what will be the arms of the angle?

3. Vidya is enjoying her time on the swing. She notices that the greater the angle with which she starts the swinging, the greater is the speed she achieves on her swing. But where is the angle? Are you able to see any angle?

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4. Here is a toy with slanting slabs attached to its sides; the greater the angles or slopes of the slabs, the faster the balls roll. Can angles be used to describe the slopes of the slabs? What are the arms of each angle? Which arm is visible and which is not?

5. Observe the images below where there is an insect and its rotated version. Can angles be used to describe the amount of rotation? How? What will be the arms of the angle and the vertex?

   Hint: Observe the horizontal line touching the insects.

Teacher's Note

It is important that students see the application of each mathematical concept in their daily lives. Teacher can organise some activities where students can appreciate the practical applications of angles in real-life situations, e.g., clocks, doors, swings, concepts of uphill and downhill, location of the sun, the giving of directions, etc.

2.10 Drawing Angles

Vidya wants to draw a 30° angle and name it ∠TIN using a protractor.

In ∠TIN, I will be the vertex, IT and IN will be the arms of the angle. Keeping one arm, say IN, as the reference (base), the other arm IT should take a turn of 30°.

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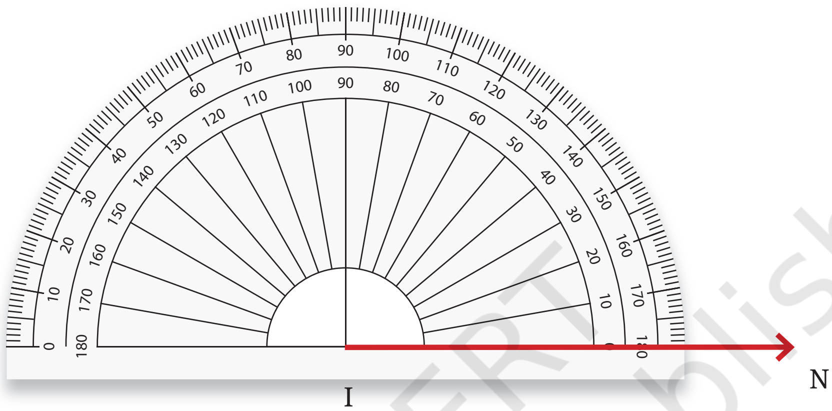
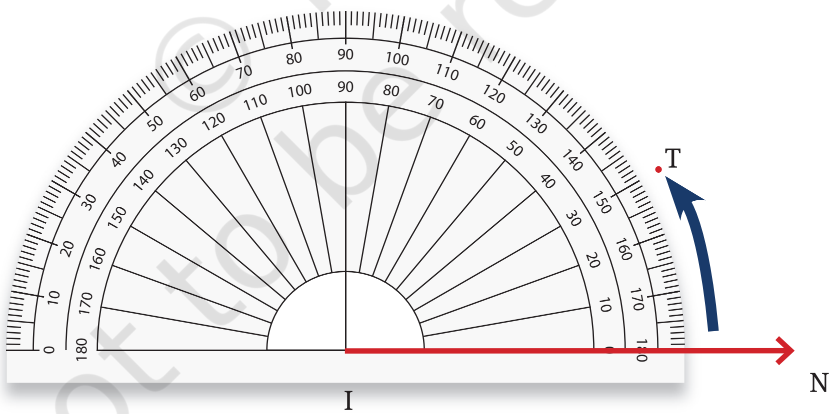
Step 1: We begin with the base and draw $\overrightarrow{IN}$:

Step 2: We will place the centre point of the protractor on I and align IN to the 0 line.



Step 3: Now, starting from 0, count your degrees (0, 10, 30) up to 30 on the protractor. Mark point T at the label 30°.



Step 4: Using a ruler join the point I and T.

∠TIN = 30° is the required angle.

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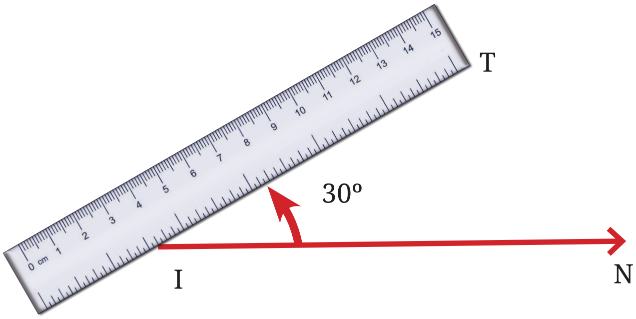

Let's Play a Game #1

This is an angle guessing game! Play this game with your classmates by making two teams, Team 1 and Team 2. Here are the instructions and rules for the game:

- **Team 1** secretly choose an angle measure, for example, 49° and makes an angle with that measure using a protractor without Team 2 being able to see it.
- **Team 2** now gets to look at the angle. They have to quickly discuss and guess the number of degrees in the angle (without using a protractor!).
- **Team 1** now demonstrates the true measure of the angle with a protractor.
- **Team 2** scores the number of points that is the absolute difference in degrees between their guess and the correct measure. For example, if Team 2 guesses 39°, then they score 10 points (49°–39°).
- Each team gets five turns. The winner is the team with the lowest score!

Let's Play a Game #2

We now change the rules of the game a bit. Play this game with your classmates by again making two teams, Team 1 and Team 2. Here are the instructions and rules:

- **Team 1** announces to all, an angle measure, e.g., 34°.
- A player from **Team 2** must draw that angle on the board without using a protractor. Other members of **Team 2** can help the player by speaking words like 'Make it bigger!' or 'Make it smaller!'.
- A player from **Team 1** measures the angle with a protractor for all to see.
- **Team 2** scores the number of points that is the absolute difference in degrees between **Team 2**'s angle size and the intended angle size. For example, if player's angle from **Team 2** is measured to be 25°, then **Team 2** scores 9 points (34°–25°).
- Each team gets five turns. The winner is again the team with the lowest score.

Teacher's Note

These games are important to play to build intuition about angles and their measures. Return to this game at least once or twice on different days to build practice in estimating angles. Note that these games can also be played between pairs of students.

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Figure it Out

1. In Fig. 2.23, list all the angles possible. Did you find them all? Now, guess the measures of all the angles. Then, measure the angles with a protractor. Record all your numbers in a table. See how close your guesses are to the actual measures.

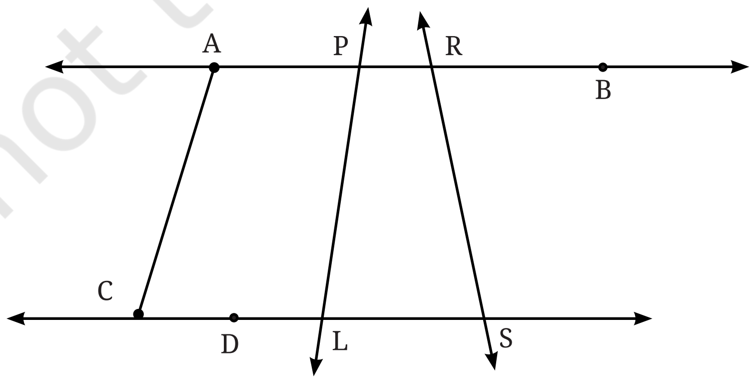
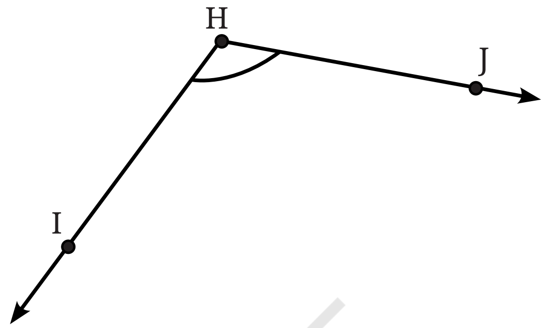


Fig. 2.23

2. Use a protractor to draw angles having the following degree measures:

   a. 110° b. 40° c. 75° d. 112° e. 134°

3. Draw an angle whose degree measure is the same as the angle given below:



Also, write down the steps you followed to draw the angle.

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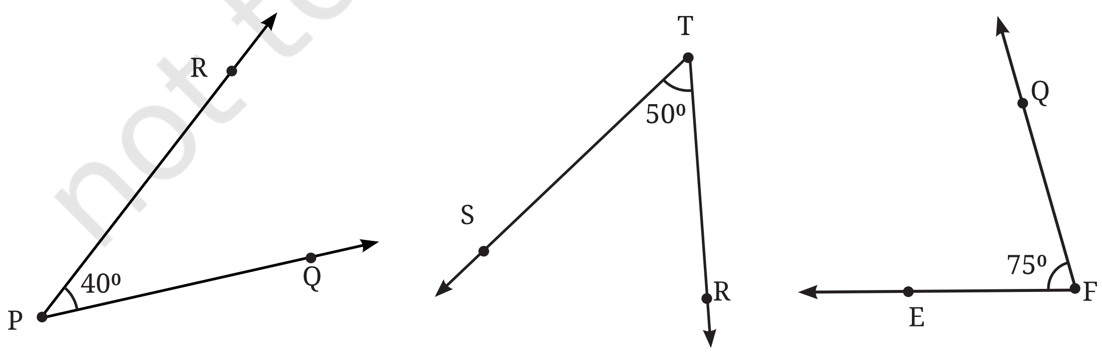
2.11 Types of Angles and their Measures

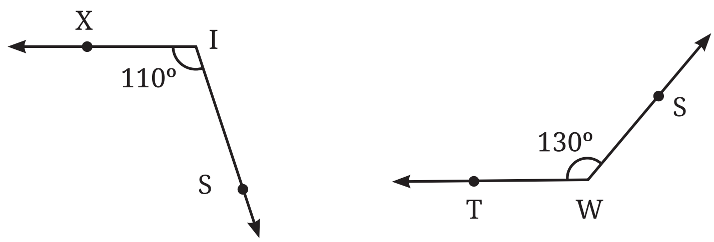
We have read about different types of angles in this chapter. We have seen that a straight angle is 180° and a right angle is 90°. How can other types of angles—acute and obtuse—be described in terms of their degree measures?

Acute Angle: Angles that are smaller than the right angle, i.e., less than 90° and are greater than 0°, are called **acute** angles.



Examples of acute angles

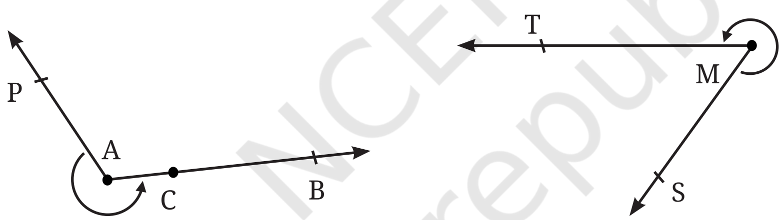
Obtuse Angle: Angles that are greater than the right angle and less than the straight angle, i.e., greater than 90° and less than 180°, are called **obtuse** angles.



Examples of obtuse angles

Have we covered all the possible measures that an angle can take? Here is another type of angle.

Reflex angle: Angles that are greater than the straight angle and less than the whole angle, i.e., greater than 180° and less than 360°, are called **reflex** angles.



Examples of reflex angles

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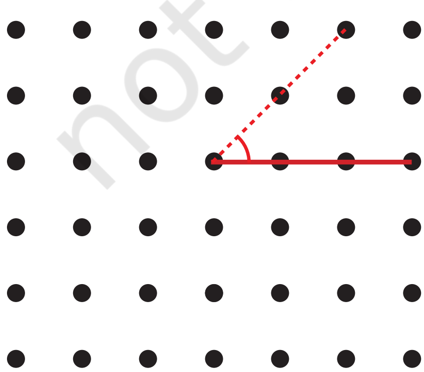
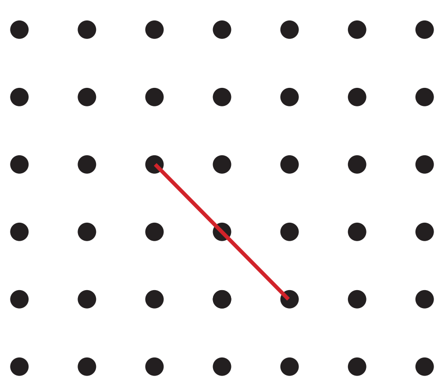
Figure it Out

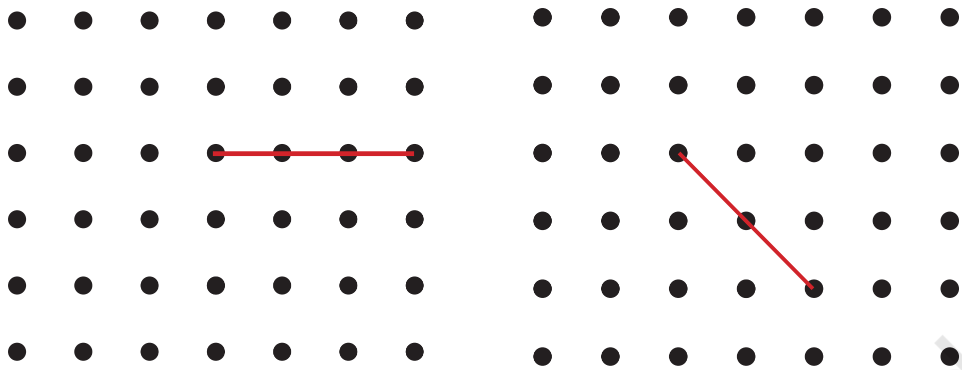
1. In each of the below grids, join A to other grid points in the figure by a straight line to get:

   a. An acute angle

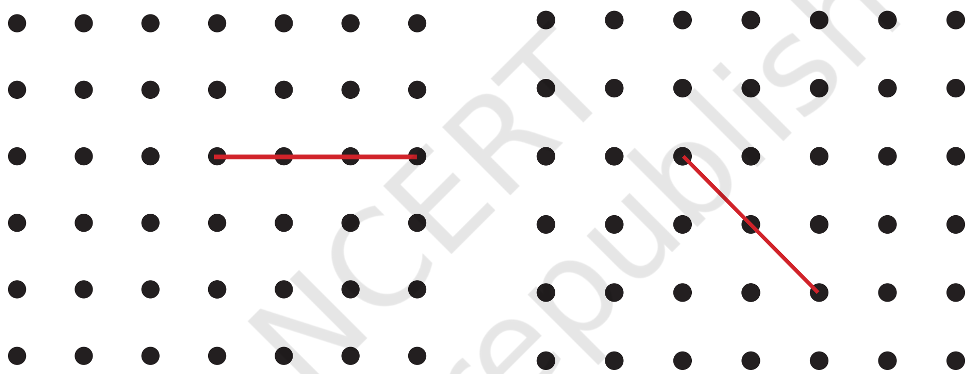




b. An obtuse angle



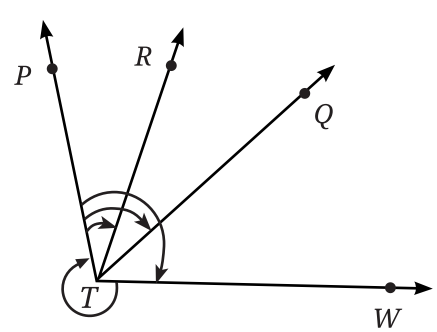
c. A reflex angle



Mark the intended angles with curves to specify the angles. One has been done for you.

2. Use a protractor to find the measure of each angle. Then classify each angle as acute, obtuse, right, or reflex.

a. ∠PTR b. ∠PTQ c. ∠PTW d. ∠WTP



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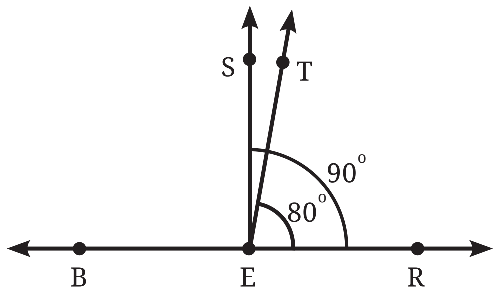
Let's Explore:

In this figure, ∠TER = 80°. What is the measure of ∠BET? What is the measure of ∠SET?



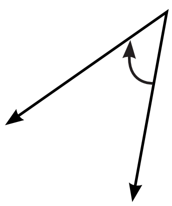
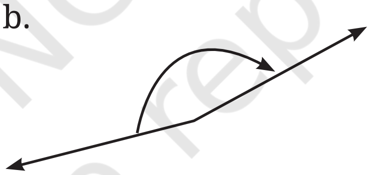
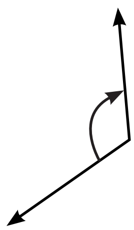
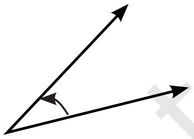
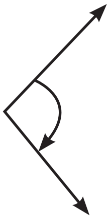
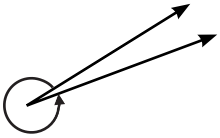
Hint: Observe that ∠REB is a straight angle. Hence the degree measure of ∠REB = 180° of which 80° is covered by ∠TER. A similar argument can be applied to find the measure of ∠SET.

Figure it Out

1. Draw angles with the following degree measures:

   a. 140° b. 82° c. 195° d. 70° e. 35°

2. Estimate the size of each angle and then measure it with a protractor:

   a.
   
   b.
   
   c.
   
   d.
   
   e.
   
   f.
   

   Classify these angles as acute, right, obtuse or reflex angles.

3. Make any figure with three acute angles, one right angle and two obtuse angles.
4. Draw the letter 'M' such that the angles on the sides are 40° each and the angle in the middle is 60°.
5. Draw the letter 'Y' such that the three angles formed are 150°, 60° and 150°.

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6. The Ashoka Chakra has 24 spokes. What is the degree measure of the angle between two spokes next to each other? What is the largest acute angle formed between two spokes?

7. **Puzzle:** I am an acute angle. If you double my measure, you get an acute angle. If you triple my measure, you will get an acute angle again. If you quadruple (four times) my measure, you will get an acute angle yet again! But if you multiply my measure by 5, you will get an obtuse angle measure. What are the possibilities for my measure?

SUMMARY

- A **point** determines a location. It is denoted by a capital letter.
- A **line segment** corresponds to the shortest distance between two points. The line segment joining points S and T is denoted by $\overline{ST}$.
- A **line** is obtained when a line segment like $\overline{ST}$ is extended on both sides indefinitely; it is denoted by $\overleftrightarrow{ST}$ or sometimes by a single small letter like *m*.
- A **ray** is a portion of a line starting at a point D and going in one direction indefinitely. It is denoted by $\overrightarrow{DP}$ where P is another point on the ray.
- An angle can be visualised as two rays starting from a common starting point. Two rays $\overrightarrow{OP}$ and $\overrightarrow{OM}$ form the angle ∠POM (also called ∠MOP); here, O is called the **vertex** of the angle, and the rays $\overrightarrow{OP}$ and $\overrightarrow{OM}$ are called the **arms** of the angle.
- The size of an angle is the amount of rotation or turn needed about the vertex to rotate one ray of the angle onto the other ray of the angle.
- The sizes of angles can be measured in **degrees**. One full rotation or turn is considered as 360 degrees and denoted as 360°.
- Degree measures of angles can be measured using a **protractor**.
- Angles can be **straight** (180°), **right** (90°), **acute** (more than 0° and less than 90°), **obtuse** (more than 90° and less than 180°), and **reflex** (more than 180° and less than 360°).

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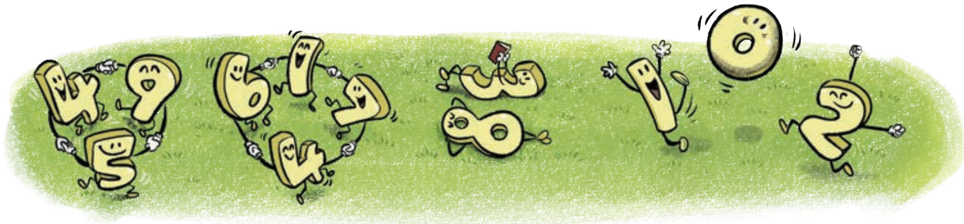

3 NUMBER PLAY

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Numbers are used in different contexts and in many different ways to organise our lives. We have used numbers to count, and have applied the basic operations of addition, subtraction, multiplication and division on them, to solve problems related to our daily lives.

In this chapter, we will continue this journey, by playing with numbers, seeing numbers around us, noticing patterns, and learning to use numbers and operations in new ways.

Math Talk

- Think about various situations where we use numbers. List five different situations in which numbers are used. See what your classmates have listed, share, and discuss.

3.1 Numbers can Tell us Things

What are these numbers telling us?

Some children in a park are standing in a line. Each one says a number.

- What do you think these numbers mean?

The children now rearrange themselves, and again each one says a number based on the arrangement.

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Did you figure out what these numbers represent?
Hint: Could their heights be playing a role?

A child says '1' if there is only one taller child standing next to them. A child says '2' if both the children standing next to them are taller. A child says '0', if neither of the children standing next to them are taller. That is each person says the number of taller neighbours they have.

Try answering the questions below and share your reasoning:

Math Talk

1. Can the children rearrange themselves so that the children standing at the ends say '2'?
2. Can we arrange the children in a line so that all would say only 0s?
3. Can two children standing next to each other say the same number?
4. There are 5 children in a group, all of different heights. Can they stand such that four of them say '1' and the last one says '0'? Why or why not?
5. For this group of 5 children, is the sequence 1, 1, 1, 1, 1 possible?
6. Is the sequence 0, 1, 2, 1, 0 possible? Why or why not?
7. How would you rearrange the five children so that the maximum number of children say '2'?

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3.2 Supercells

Observe the numbers written in the table below. Why are some numbers coloured? Discuss.

43	79	75	63	10	29	28	34

200	577	626	345	790	694	109	198

A cell is coloured if the number in it is larger than its adjacent cells. 626 is coloured as it is larger than 577 and 345 whereas 200 is not coloured as it is smaller than 577. The number 198 is coloured as it has only one adjacent cell with 109 in it, and 198 is larger than 109.

Figure it Out

1. Colour or mark the supercells in the table below.

6828	670	9435	3780	3708	7308	8000	5583	52

2. Fill the table below with only 4-digit numbers such that the supercells are exactly the coloured cells.

5346			1258				9635	

3. Fill the table below such that we get as many supercells as possible. Use numbers between 100 and 1000 without repetitions.

4. Out of the 9 numbers, how many supercells are there in the table above? ________

5. Find out how many supercells are possible for different numbers of cells.

   Math Talk

   Do you notice any pattern? What is the method to fill a given table to get the maximum number of supercells? Explore and share your strategy.

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6. Can you fill a supercell table without repeating numbers such that there are no supercells? Why or why not?
7. Will the cell having the largest number in a table always be a supercell? Can the cell having the smallest number in a table be a supercell? Why or why not?
8. Fill a table such that the cell having the second largest number is not a supercell.
9. Fill a table such that the cell having the second largest number is not a supercell but the second smallest number is a supercell. Is it possible?
10. Make other variations of this puzzle and challenge your classmates.

Try This

Let's do the supercells activity with more rows.

Here the neighbouring cells are those that are immediately to the left, right, top and bottom.

The rule remains the same: a cell becomes a supercell if the number in it is greater than all the numbers in its neighbouring cells. In Table 1, 8632 is greater than all its neighbours 4580, 8280, 4795 and 1944.

Table 1

2430	7500	7350	9870
3115	4795	9124	9230
4580	8632	8280	3446
5785	1944	5805	6034

Complete Table 2 with 5-digit numbers whose digits are '1', '0', '6', '3', and '9' in some order. Only a coloured cell should have a number greater than all its neighbours.

The biggest number in the table is ___________ .

Table 2

	96,301	36,109	
	13,609	60,319	19,306
		60,193	
	10,963		

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The smallest even number in the table is ___________.

The smallest number greater than 50,000 in the table is ___________.

Once you have filled the table above, put commas appropriately after the thousands digit.

3.3 Patterns of Numbers on the Number Line

We are quite familiar with number lines now. Let's see if we can place some numbers in their appropriate positions on the number line. Here are the numbers: 2180, 2754, 1500, 3600, 9950, 9590, 1050, 3050, 5030, 5300 and 8400.

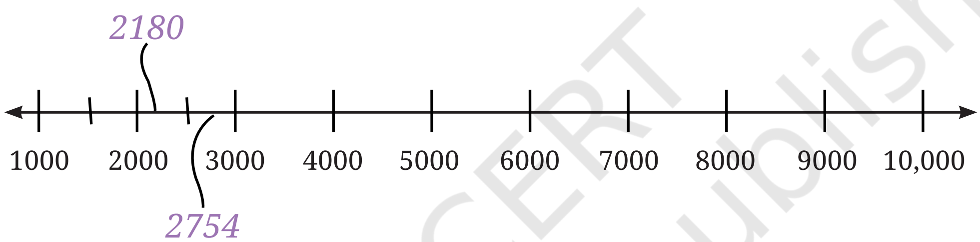
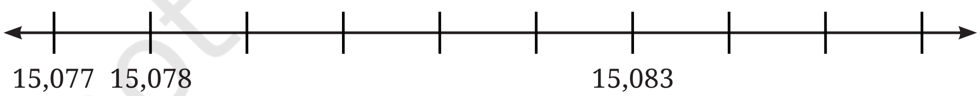


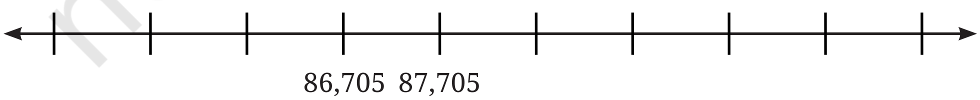
Figure it Out

Identify the numbers marked on the number lines below, and label the remaining positions.

a. 

b. 

c. 

d. 

Put a circle around the smallest number and a box around the largest number in each of the sequences above.

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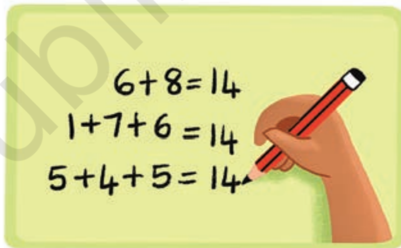
3.4 Playing with Digits

We start writing numbers from 1, 2, 3 ... and so on. There are nine 1-digit numbers.

Find out how many numbers have two digits, three digits, four digits, and five digits:

1-digit numbers From 1–9	2-digit numbers	3-digit numbers	4-digit numbers	5-digit numbers
9				

Digit Sums of Numbers



Komal observes that when she adds up digits of certain numbers the sum is the same.

For example, adding the digits of the number 68 will be same as adding the digits of 176 or 545.

Figure it Out

Math Talk

1. **Digit sum 14**
   a. Write other numbers whose digits add up to 14.
   b. What is the smallest number whose digit sum is 14?
   c. What is the largest 5-digit whose digit sum is 14?
   d. How big a number can you form having the digit sum 14? Can you make an even bigger number?
2. Find out the digit sums of all the numbers from 40 to 70. Share your observations with the class.
3. Calculate the digit sums of 3-digit numbers whose digits are consecutive (for example, 345). Do you see a pattern? Will this pattern continue?

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Digit Detectives

After writing numbers from 1 to 100, Dinesh wondered how many times he would have written the digit '7'!

Among the numbers 1–100, how many times will the digit '7' occur? Among the numbers 1–1000, how many times will the digit '7' occur?

3.5 Pretty Palindromic Patterns

What pattern do you see in these numbers: 66, 848, 575, 797, 1111? These numbers read the same from left to right and from right to left. Try and see. Such numbers are called **palindromes** or **palindromic numbers**.

All palindromes using 1, 2, 3

The numbers 121, 313, 222 are some examples of palindromes using the digits '1', '2', 3'.

Write all possible 3-digit palindromes using these digits.

Reverse-and-add palindromes

Now look at these additions. Try to figure out what is happening.

Steps to follow: Start with a 2-digit number. Add this number to its reverse. Stop if you get a palindrome; else repeat the steps of reversing the digits and adding.

34	29	48	76
43	92	84	67
77	121	132	143
		231	341
		363	484

Try the same procedure for some other numbers, and perform the same steps. Stop if

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you get a palindrome. There are numbers for which you have to repeat this a large number of times.

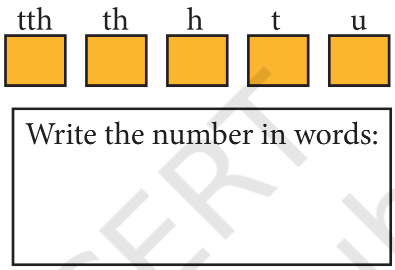
Are there numbers for which you do not reach a palindrome at all?

Explore

Will reversing and adding numbers repeatedly, starting with a 2-digit number, always give a palindrome? Explore and find out.*

Math Talk

Puzzle time



I am a 5-digit palindrome.
I am an odd number.
My ‘t’ digit is double of my ‘u’ digit.
My ‘h’ digit is double of my ‘t’ digit.
Who am I? ______________

3.6 The Magic Number of Kaprekar

D.R. Kaprekar was a mathematics teacher in a government school in Devlali, Maharashtra. He liked playing with numbers very much and found many beautiful patterns in numbers that were previously unknown.

In 1949, he discovered a fascinating and magical phenomenon when playing with 4-digit numbers.

**The answer is yes! For 3-digit numbers the answer is unknown. It is suspected that starting with 196 never yields a palindrome!*

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Follow these steps and experience the magic for yourselves! Pick any 4-digit number, say 6382.

Take a 4-digit number.

↓

Make the largest number from these digits. Call it A.

↓

Make the smallest number from these digits. Call it B.

↓

Subtract B from A. Call it C.
C = A – B

Use digits of C

What happens if we continue doing this?

A = 8632
B = 2368
C = 8632–2368
= 6264

A = 6642
B = 2466
C = 6642–2466
= 4176

A = 7641
B = 1467
C = 7641–1467
= 6174

A =
B =
C =

Explore

Take different 4-digit numbers and try carrying out these steps. Find out what happens. Check with your friends what they got.

You will always reach the magic number '6174'! The number '6174' is now called the 'Kaprekar constant'.

Carry out these same steps with a few 3-digit numbers. What number will start repeating?

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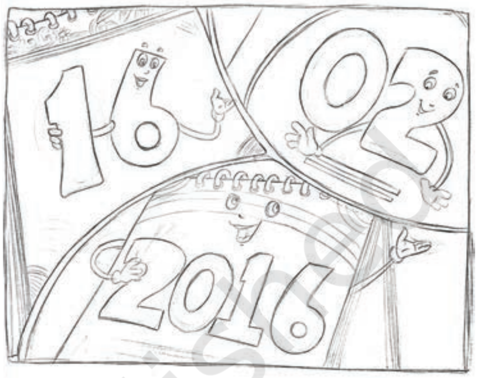
3.7 Clock and Calendar Numbers

On the usual 12-hour clock, there are timings with different patterns. For example, 4:44, 10:10, 12:21.

- Try and find out all possible times on a 12-hour clock of each of these types.

Manish has his birthday on 20/12/2012 where the digits '2', '0', '1', and '2' repeat in that order.



- Find some other dates of this form from the past.

His sister Meghana has her birthday on 11/02/2011 where the digits read the same from left to right and from right to left.

- Find all possible dates of this form from the past.

Jeevan was looking at this year's calendar. He started wondering, "Why should we change the calendar every year! Can we not reuse a calendar?" What do you think?

You might have noticed that last year's calendar was different from this year's. Also, next year's calendar is also different from the previous years.

- But, will any year's calendar repeat again after some years? Will all dates and days in a year match exactly with that of another year?

Try This

Figure it Out

1. Pratibha uses the digits '4', '7', '3' and '2', and makes the smallest and largest 4-digit numbers with them: 2347 and 7432. The difference between these two numbers is 7432 – 2347 = 5085. The sum of these two numbers is 9779. Choose 4–digits to make:
   a. the difference between the largest and smallest numbers greater than 5085.

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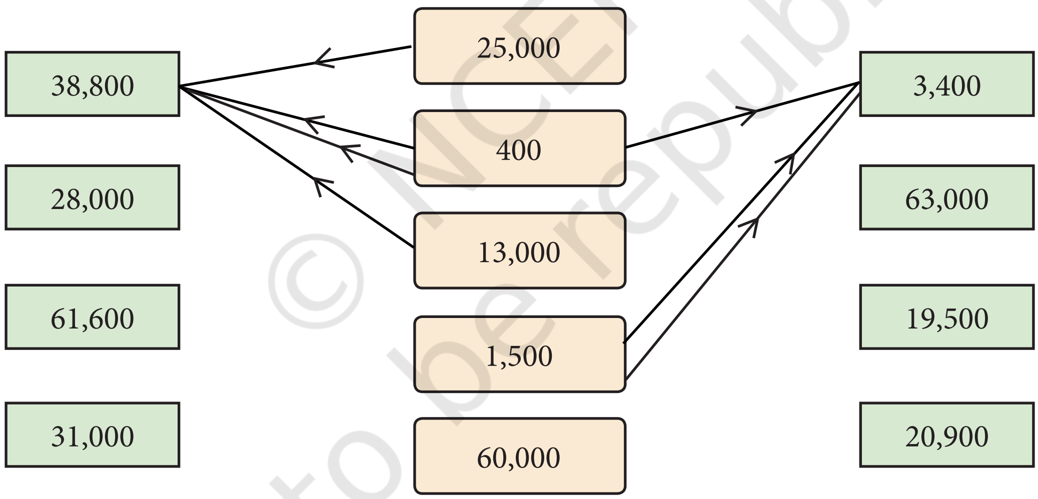
b. the difference between the largest and smallest numbers less than 5085.
c. the sum of the largest and smallest numbers greater than 9779.
d. the sum of the largest and smallest numbers less than 9779.

2. What is the sum of the smallest and largest 5-digit palindrome? What is their difference?
3. The time now is 10:01. How many minutes until the clock shows the next palindromic time? What about the one after that?
4. How many rounds does the number 5683 take to reach the Kaprekar constant?

3.8 Mental Math

Observe the figure below. What can you say about the numbers and the lines drawn?



Numbers in the middle column are added in different ways to get the numbers on the sides (1500 + 1500 + 400 = 3400). The numbers in the middle can be used as many times as needed to get the desired sum. Draw arrows from the middle to the numbers on the sides to obtain the desired sums.

Two examples are given. It is simpler to do it mentally!

38,800 = 25,000 + 400 × 2 + 13,000
3400 = 1500 + 1500 + 400

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Can we make 1,000 using the numbers in the middle? Why not? What about 14,000, 15,000 and 16,000? Yes, it is possible. Explore how. What thousands cannot be made?

Math Talk

Adding and Subtracting

Here, using the numbers in the boxes, we are allowed to use both addition and subtraction to get the required number. An example is shown.

40,000	7,000
300	1,500
12,000	800

39,800 = 40,000 – 800 + 300 + 300

45,000 =

5,900 =

17,500 =

21,400 =

Digits and Operations

An example of adding two 5-digit numbers to get another 5-digit number is 12,350 + 24,545 = 36,895.

An example of subtracting two 5-digit numbers to get another 5-digit number is 48,952 – 24,547 = 24,405.

Figure it Out

1. Write an example for each of the below scenarios whenever possible.

5-digit + 5-digit to give a 5-digit sum more than 90,250	5-digit + 3-digit to give a 6-digit sum	4-digit + 4-digit to give a 6-digit sum	5-digit + 5-digit to give a 6-digit sum	5-digit + 5-digit to give 18,500
5-digit – 5-digit to give a difference less than 56,503	5-digit – 3-digit to give a 4-digit difference	5-digit – 4-digit to give a 4-digit difference	5-digit – 5-digit to give a 3-digit difference	5-digit – 5-digit to give 91,500

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Could you find examples for all the cases? If not, think and discuss what could be the reason. Make other such questions and challenge your classmates.

Math Talk

2. *Always, Sometimes, Never?*

   Below are some statements. Think, explore and find out if each of the statement is 'Always true', 'Only sometimes true' or 'Never true'. Why do you think so? Write your reasoning; discuss this with the class.

   a. 5-digit number + 5-digit number gives a 5-digit number

   b. 4-digit number + 2-digit number gives a 4-digit number

   c. 4-digit number + 2-digit number gives a 6-digit number

   d. 5-digit number – 5-digit number gives a 5-digit number

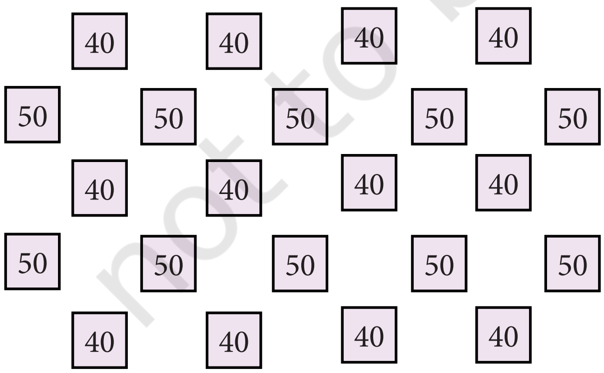
   e. 5-digit number – 2-digit number gives a 3-digit number

3.9 Playing with Number Patterns

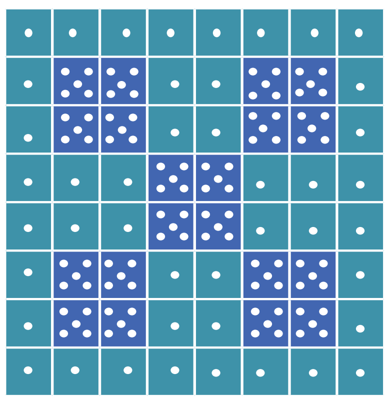
Here are some numbers arranged in some patterns. Find out the sum of the numbers in each of the below figures. Should we add them one by one or can we use a quicker way?

Share and discuss in class the different methods each of you used to solve these questions.

a.



b.



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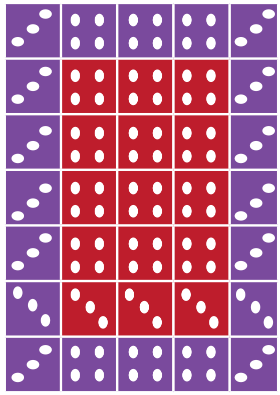
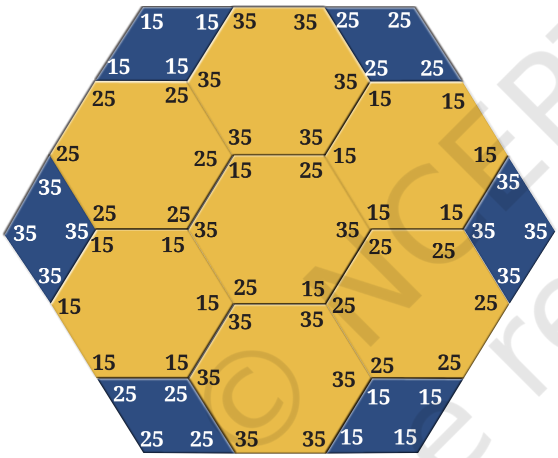
c.

32	32	32	32	32	32	32	32
32	32	32	32	32	32	32	32
32	32	32	32	32	32	32	32
32	32	32	32	32	32	32	32
64	64	64					64
64	64	64					64
64	64	64					64
64	64	64					64

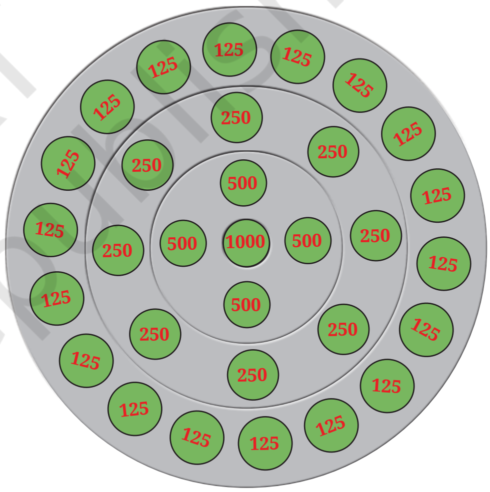
d.



e.



f.



3.10 An Unsolved Mystery - the Collatz Conjecture!

Look at the sequences below—the same rule is applied in all the sequences:

a. 12, 6, 3, 10, 5, 16, 8, 4, 2, 1

b. 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

c. 21, 64, 32, 16, 8, 4, 2, 1

d. 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

Do you see how these sequences were formed?

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The rule is: one starts with any number; if the number is even, take half of it; if the number is odd, multiply it by 3 and add 1; repeat.

Notice that all four sequences above eventually reached the number 1. In 1937, the German mathematician Lothar Collatz conjectured that the sequence will *always* reach 1, regardless of the whole number you start with. Even today—despite many mathematicians working on it — it remains an unsolved problem as to whether Collatz's conjecture is true! Collatz's conjecture is one of the most famous unsolved problems in mathematics.

Make some more Collatz sequences like those above, starting with your favourite whole numbers. Do you always reach 1?

Do you believe the conjecture of Collatz that all such sequences will eventually reach 1? Why or why not?

3.11 Simple Estimation

At times, we may not know or need an exact count of things and an estimate is sufficient for the purpose at hand. For example, your school headmaster might know the exact number of students enrolled in your school, but you may only know an estimated count. How many students are in your school? About 150? 400? A thousand?

Paromita's class section has 32 children. The other 2 sections of her class have 29 and 35 children. So, she estimated the number of children in her class to be about 100. Along with Class 6, her school also has Classes 7–10 and each class has 3 sections each. She assumed a similar number in each class and estimated the number of students in her school to be around 500.

Figure it Out

We shall do some simple estimates. It is a fun exercise, and you may find it amusing to know the various numbers around us. Remember,

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we are not interested in the exact numbers for the following questions. Share your methods of estimation with the class.

1. Steps you would take to walk:
   a. From the place you are sitting to the classroom door
   b. Across the school ground from start to end
   c. From your classroom door to the school gate
   d. From your school to your home
2. Number of times you blink your eyes or number of breaths you take:
   a. In a minute
   b. In an hour
   c. In a day
3. Name some objects around you that are:
   a. a few thousand in number
   b. more than ten thousand in number

Estimate the answer

Try to guess within 30 seconds. Check your guess with your friends.

1. Number of words in your maths textbook:
   a. More than 5000
   b. Less than 5000
2. Number of students in your school who travel to school by bus:
   a. More than 200
   b. Less than 200
3. Roshan wants to buy milk and 3 types of fruit to make fruit custard for 5 people. He estimates the cost to be ₹ 100. Do you agree with him? Why or why not?
4. Estimate the distance between Gandhinagar (in Gujarat) to Kohima (in Nagaland).
   [*Hint*: Look at the map of India to locate these cities.]

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5. Sheetal is in Grade 6 and says she has spent around 13,000 hours in school till date. Do you agree with her? Why or why not?
6. Earlier, people used to walk long distances as they had no other means of transport. Suppose you walk at your normal pace. Approximately how long would it take you to go from:
   a. Your current location to one of your favourite places nearby.
   b. Your current location to any neighbouring state's capital city.
   c. The southernmost point in India to the northernmost point in India.
7. Make some estimation questions and challenge your classmates!

3.12 Games and Winning Strategies

Numbers can also be used to play games and develop winning strategies.

Here is a famous game called 21. Play it with a classmate. Then try it at home with your family!

Rules for Game #1: The first player says 1, 2 or 3. Then the two players take turns adding 1, 2, or 3 to the previous number said. The first player to reach 21 wins!

Play this game several times with your classmate. Are you starting to see the winning strategy?

Which player can always win if they play correctly? What is the pattern of numbers that the winning player should say?

There are many variations of this game. Here is another common variation:

Rules for Game #2: The first player says a number between 1 and 10. Then the two players take turns adding a number between 1 and 10 to the previous number said. The first player to reach 99 wins!

Play this game several times with your classmate. See if you can figure out the corresponding winning strategy in this case! Which

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player can always win? What is the pattern of numbers that the winning player should say this time?

Make your own variations of this game — decide how much one can add at each turn, and what number is the winning number. Then play your game several times, and figure out the winning strategy and which player can always win!

Figure it Out

1. There is only one supercell (number greater than all its neighbours) in this grid. If you exchange two digits of one of the numbers, there will be 4 supercells. Figure out which digits to swap.

16,200	39,344	29,765
23,609	62,871	45,306
19,381	50,319	38,408

Try This

2. How many rounds does your year of birth take to reach the Kaprekar constant?
3. We are the group of 5-digit numbers between 35,000 and 75,000 such that all of our digits are odd. Who is the largest number in our group? Who is the smallest number in our group? Who among us is the closest to 50,000?
4. Estimate the number of holidays you get in a year including weekends, festivals and vacation. Then try to get an exact number and see how close your estimate is.
5. Estimate the number of liters a mug, a bucket and an overhead tank can hold.
6. Write one 5-digit number and two 3-digit numbers such that their sum is 18,670.
7. Choose a number between 210 and 390. Create a number pattern similar to those shown in Section 3.9 that will sum up to this number.

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8. Recall the sequence of Powers of 2 from Chapter 1, Table 1. Why is the Collatz conjecture correct for all the starting numbers in this sequence?
9. Check if the Collatz Conjecture holds for the starting number 100.
10. Starting with 0, players alternate adding numbers between 1 and 3. The first person to reach 22 wins. What is the winning strategy now?

SUMMARY

- Numbers can be used for many different purposes, including to convey information, make and discover patterns, estimate magnitudes, pose and solve puzzles, and play and win games.
- Thinking about and formulating set procedures to use numbers for these purposes is a useful skill and capacity (called "computational thinking").
- Many problems about numbers can be very easy to pose, but very difficult to solve. Indeed, numerous such problems are still unsolved (e.g., Collatz's Conjecture).

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4 DATA HANDLING AND PRESENTATION

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If you ask your classmates about their favourite colours, you will get a list of colours. This list is an example of data. Similarly, if you measure the weight of each student in your class, you would get a collection of measures of weight—again data.

Any collection of facts, numbers, measures, observations, or other descriptions of things that convey *information* about those things is called **data**.

We live in an age of information. We constantly see large amounts of data presented to us in new and interesting ways. In this chapter, we will explore some of the ways that data is presented, and how we can use some of those ways to correctly display, interpret and make inferences from such data!

4.1 Collecting and Organising Data

Navya and Naresh are discussing their favourite games.

Cricket is my favourite game!

I play cricket sometimes but hockey is the game I like the most.

I think cricket is the most popular game in our class.

I am not sure. How can we find the most popular game in our class?

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To figure out the most popular game in their class, what should Navya and Naresh do? Can you help them?

Naresh and Navya decided to go to each student in the class and ask what their favourite game is. Then they prepared a list.

Navya is showing the list:

Mehnoor – *Kabaddi*	Pushkal – *Satoliya* (*Pittu*)	Anaya – *Kabaddi*
Jubimon – Hockey	Densy – Badminton	Jivisha – *Satoliya* (*Pittu*)
Simran – *Kabaddi*	Jivika – *Satoliya* (*Pittu*)	Rajesh – Football
Nand – *Satoliya* (*Pittu*)	Leela – Hockey	Thara – Football
Ankita – *Kabaddi*	Afshan – Hockey	Soumya – Cricket
Imon – Hockey	Keerat – Cricket	Navjot – Hockey
Yuvraj – Cricket	Gurpreet – Hockey	Hemal – *Satoliya* (*Pittu*)
Rehana – Hockey	Arsh – *Kabaddi*	Debabrata – Football
Aarna – Badminton	Bhavya – Cricket	Ananya – Hockey
Kompal – Football	Sarah – *Kabaddi*	Hardik – Cricket
Tahira – Cricket		

She says (happily) , "I have collected the data. I can figure out the most popular game now!"

A few other children are looking at the list and wondering, "We can't yet see the most popular game. How can we get it from this list?"

Figure it Out

1. What would you do to find the most popular game among Naresh's and Navya's classmates?
2. What is the most popular game in their class?
3. Try to find out the most popular game among your classmates.
4. Pari wants to respond to the questions given below. Put a tick (✓) for the questions where she needs to carry out data collection and

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put a cross (×) for the questions where she doesn't need to collect data. Discuss your answers in the classroom.

a. What is the most popular TV show among her classmates? ☐

b. When did India get independence? ☐

c. How much water is getting wasted in her locality? ☐

d. What is the capital of India? ☐

Shri Nilesh is a teacher. He decided to bring sweets to the class to celebrate the new year. The sweets shop nearby has jalebi, gulab jamun, gujiya, barfi, and rasgulla. He wanted to know the choices of the children. He wrote the names of the sweets on the board and asked each child to tell him their preference. He put a tally mark '|' for each student and when the count reached 5, he put a line through the previous four and marked it as 𝍸.

Sweets	Tally Marks	No. of Students
Jalebi	𝍸 |	6
Gulab Jamun	𝍸 ||||	9
Gujiya	𝍸 𝍸 |||	______
Barfi	|||	______
Rasgulla	𝍸 ||	______

Figure it Out

1. Complete the table to help Shri Nilesh to purchase the correct numbers of sweets:
   - How many students chose jalebi? ☐
   - Barfi was chosen by ☐ students?
   - How many students chose gujiya? ☐
   - Rasgulla was chosen by ☐ students?
   - How many students chose gulab jamun? ☐

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Shri Nilesh requested one of the staff members to bring the sweets as given in the table. The above table helped him to purchase the correct numbers of sweets.

2. Is the above table sufficient to distribute each type of sweet to the correct student? Explain. If it is not sufficient, what is the alternative?

To organise the data, we can write the name of each sweet in one column and using tally signs, note the number of students who prefer that sweet. The numbers 6, 9, ... are the **frequencies** of the sweet preferences for *jalebi, gulab jamun* ... respectively.

Sushri Sandhya asked her students about the sizes of the shoes they wear. She noted the data on the board—

4	5	3	4	3	4	5	5	4
5	5	4	5	6	4	3	5	6
4	6	4	5	7	5	6	4	5

She then arranged the shoe sizes of the students in ascending order—

3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7

Figure it Out

1. Help her to figure out the following –
   - The largest shoe size in the class is _________
   - The smallest shoe size in the class is _________
   - There are _________ students who wear shoe size 5.
   - There are _________ students who wear shoe sizes larger than 4.
2. How did arranging the data in ascending order help to answer these questions?

Math Talk

3. Are there other ways to arrange the data?

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4. Write the names of a few trees you see around you. When you observe a tree on the way from your home to school (or while walking from one place to another place), record the data and fill in the following table—

Tree	No. of Trees
Peepal	
Neem	
...	
....	

a. Which tree was found in the greatest number?

b. Which tree was found in the smallest number?

c. Were there any two trees found in the same numbers?

5. Take a blank piece of paper and paste any small news item from a newspaper. Each student may use a different article. Now, prepare a table on the piece of paper as given below. Count the number of each of the letters 'c', 'e', 'i', 'r', and 'x' in the words of the news article, and fill in the table.

Letter	c	e	i	r	x	Any other letter of your choice:
Number of times found in the news item						

a. The letter found the most number of times is ________

b. The letter found the least number of times is ________

c. List the five letters 'c', 'e', 'i', 'r', 'x' in ascending order of frequency. Now, compare the order of your list with that of your classmates. Is your order the same or nearly the same as theirs? (Almost everyone is likely to get the order 'x, c, r, i, e'.) Why do you think this is the case?

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d. Write the process you followed to complete this task.

e. Discuss with your friends the processes they followed.

f. If you do this task with another news item, what process would you follow?

Teacher's Note

Provide more opportunities to collect and organise data. Ask students to guess what is the most popular colour, game, toy, school subject, etc., amongst the students in their classroom, and then collect the data for it. It can be a fun activity in which they also learn about their classmates. Discuss how they can organise the data in different ways, each way having its own advantages and limitations. For all these tasks, and the tasks under 'Figure it Out', discuss the tasks with the children and let them understand the tasks, and then let them plan and present their research processes and conclusions in the class.

4.2 Pictographs

Pictographs are one visual and suggestive way to represent data without writing any numbers. Look at this picture—you may be familiar with it from previous classes.

Modes of Travelling	Number of Students ☺ = 1 Student
Private car	☺ ☺ ☺ ☺
Public bus	☺ ☺ ☺ ☺ ☺
School bus	☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺ ☺
Cycle	☺ ☺ ☺
Walking	☺ ☺ ☺ ☺ ☺ ☺ ☺

This picture helps you understand at a glance the different modes of travel used by students. Based on this picture, answer the following questions:

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- Which mode of travel is used by the most number of students?
- Which mode of travel is used by the least number of students?

A pictograph represents data through pictures of objects. It helps answer questions about data with just a quick glance.

In the above pictograph, one unit or symbol (☺) is used to represent one student. There are also other pictographs where one unit or symbol stands for many people or objects.

Example: Nand Kishor collected responses from the children of his middle school in Berasia regarding how often they slept at least 9 hours during the night. He prepared a pictograph from the data:

Response	Number of Children (▲ = 10 Children)
Always	
Sometimes	
Never	

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Answer the following questions using the pictograph:

a. What is the number of children who always slept at least 9 hours at night?

b. How many children sometimes slept at least 9 hours at night?

c. How many children always slept less than 9 hours each night? Explain how you got your answer.

Solutions

a. In the table, there are 5 pictures ▲ for 'Always'. Each picture ▲ represents 10 children. Therefore, 5 pictures indicate 5 × 10 = 50 children.

b. There are 2 complete pictures ▲ (2 × 10 = 20) and a half picture ◢ (half of 10 = 5). Therefore, the number of children who sleep at least 9 hours only sometimes is 20 + 5 = 25.

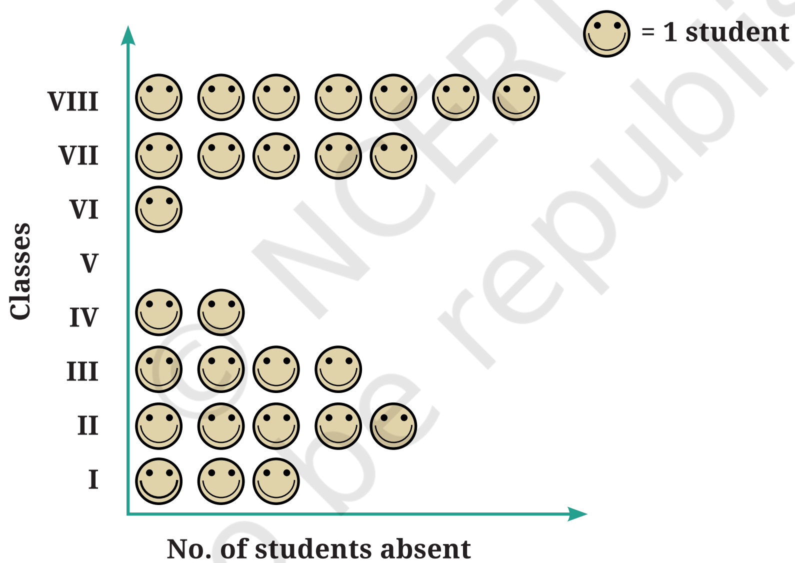
c. There are 4 complete pictures for 'Never'. Hence 4 × 10 = 40 children never sleep at least 9 hours in a night, i.e., they always sleep less than 9 hours.

Drawing a Pictograph

One day, Lakhanpal collected data on how many students were absent in each class—

Class	1	2	3	4	5	6	7	8
No. of students	3	5	4	2	0	1	5	7

He created a pictograph to present this data and decided to show 1 student as ☺ in the pictograph—



Meanwhile, his friends Jarina and Sangita collected data on how many students were present in each class—

Class	1	2	3	4	5	6	7	8
No. of students	30	35	20	25	30	25	30	20

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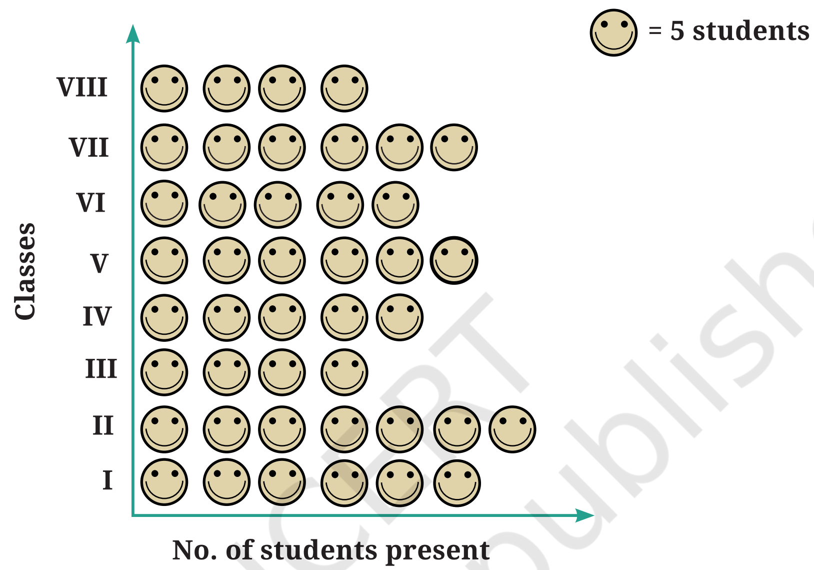
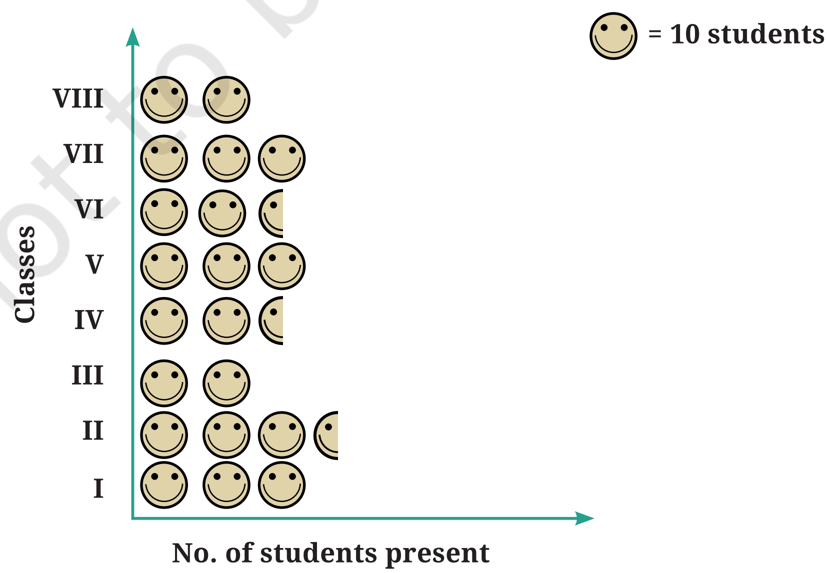
If they want to show their data through a pictograph, where they also use one symbol for each student, as Lakhanpal did, what are the challenges they might face?

Jarina made a plan to address this—since there were many students, she decided to use to represent 5 students. She figured that would save time and space too.



Sangita decided to use one to represent 10 students instead.

Since she used one to show 10 students, she had a problem in showing 25 students and 35 students in the pictograph. Then, she realised she could use to show 5 students.



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What could be the problems faced in preparing such a pictograph, if the total number of students present in a class is 33 or 27?

Math Talk

- **Pictographs** are a nice visual and suggestive way to represent data. They represent data through pictures of objects.
- Pictographs can help answer questions and make inferences about data with just a quick glance (in the examples above— about favorite games, favorite colours, most common modes of conveyance, number of students absent etc.).
- By reading a pictograph, we can quickly understand the frequencies of the different categories (for example, cricket, hockey, etc.), and the comparisons of these frequencies.
- In a pictograph, the categories can be arranged horizontlly or vertically. For each category, simple pictures and symbols are then drawn in the designated columns or rows according to the frequency of that category.
- A **scale** or **key** (for example, ☺ : 1 student or ☺ : 5 students) is added to show what each symbol or picture represents. Each symbol or picture can represent one unit or multiple units.
- It can be more challenging to prepare a pictograph when the amount of data is large or when the frequencies are not exact multiples of the scale or key.

Figure it Out

1. The following pictograph shows the number of books borrowed by students, in a week, from the library of Middle School, Ginnori—

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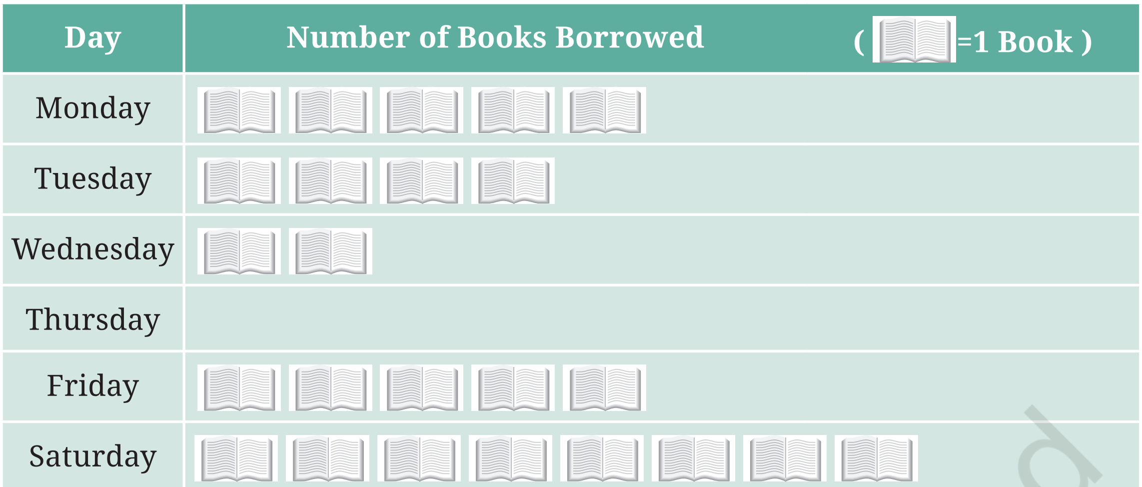

a. On which day were the minimum number of books borrowed?

b. What was the total number of books borrowed during the week?

c. On which day were the maximum number of books borrowed? What may be the possible reason?

2. Magan Bhai sells kites at Jamnagar. Six shopkeepers from nearby villages come to purchase kites from him. The number of kites he sold to these six shopkeepers are given below—

Shopkeeper	Number of Kites sold
Chaman	250
Rani	300
Rukhsana	100
Jasmeet	450
Jetha Lal	250
Poonam Ben	700

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Prepare a pictograph using the symbol to represent 100 kites.

Answer the following questions:

a. How many symbols represent the kites that Rani purchased?

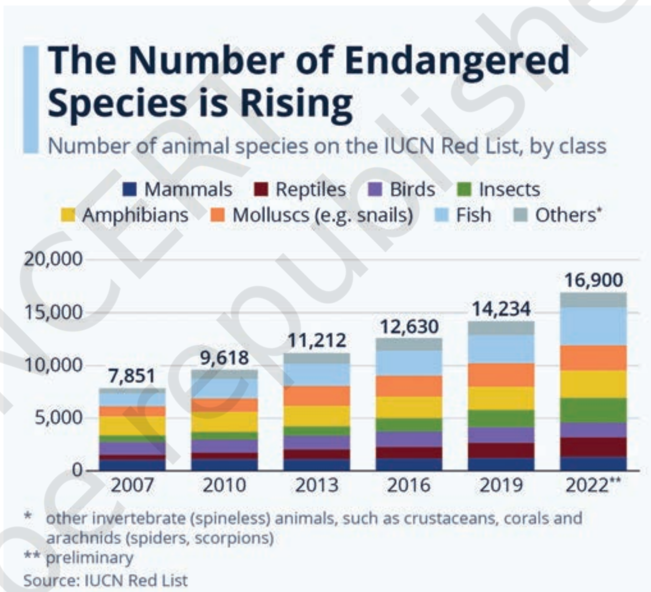
b. Who purchased the maximum number of kites?

c. Who purchased more kites, Jasmeet or Chaman?

d. Rukhsana says Poonam Ben purchased more than double the number of kites that Rani purchased. Is she correct? Why?

4.3 Bar Graphs

Have you seen graphs like this on TV or in a newspaper?



https://www.statista.com/chart/17122/number-of-threatened-species-red-list/

Like pictographs, such **bar graphs** can help us to quickly understand and interpret information, such as the highest value, the comparison of values of different categories, etc. However, when the amount of data is large, presenting it by a pictograph is not only time consuming but at times difficult too. Let us see how data can be presented instead using a bar graph.

Let's take the data collected by Lakhanpal earlier, regarding the number of students absent on one day in each class—

Class	1	2	3	4	5	6	7	8
No. of students	3	5	4	2	0	1	5	7

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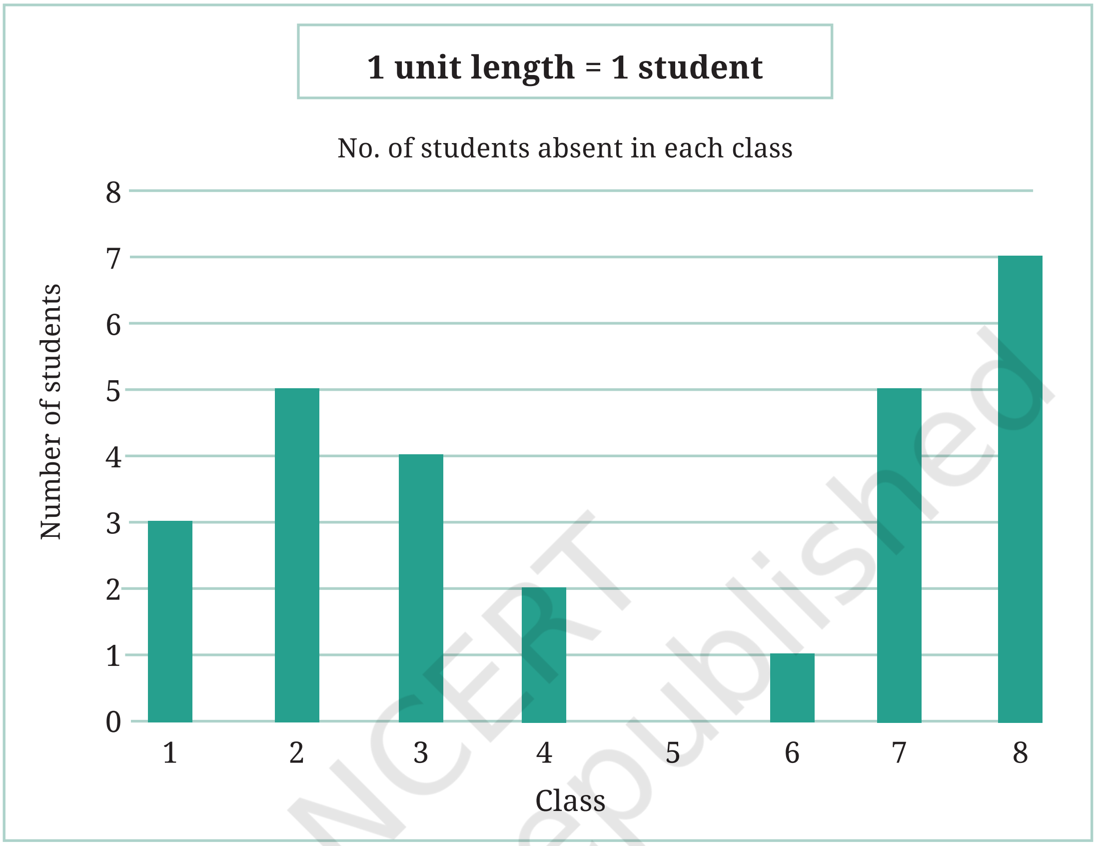
He presented the same data using a bar graph—



Teacher's Note

If the students have not noticed, please point out the equally spaced horizontal lines. Explain that this means that each pair of consecutive numbers on the left has the same gap.

Answer the following questions using the bar graph:

1. In Class 2, ___________ students were absent that day.
2. In which class were the maximum number of students absent? ___________
3. Which class had full attendance that day? ___________

When making bar graphs, bars of uniform width can be drawn horizontally or vertically with equal spacing between them; then the

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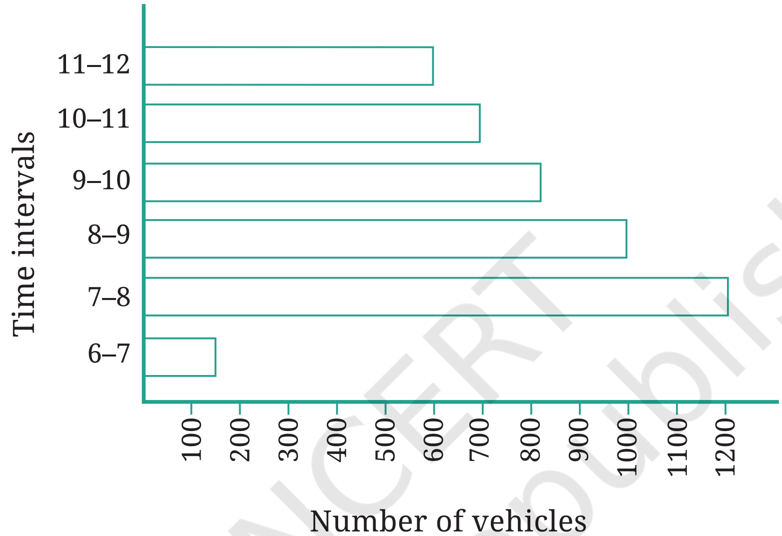
length or height of each bar represents the given number. As we saw in pictographs, we can use a scale or key when the frequencies are larger.

Let us look at an example of vehicular traffic at a busy road crossing in Delhi, which was studied by the traffic police on a particular day. The number of vehicles passing through the crossing each hour from 6 am to 12:00 noon is shown in the bar graph. One unit of length stands for 100 vehicles.



We can see that the maximum traffic at the crossing is shown by the longest bar, i.e., for the time interval 7-8 a.m. The bar graph shows that 1200 vehicles passed through the crossing at that time. The second longest bar is for 8-9 a.m. During that time, 1000 vehicles passed through the crossing. Similarly, the minimum traffic is shown by the smallest bar, i.e., the bar for the time interval 6-7 a.m. During that time, only about 150 vehicles passed through the crossing. The second smallest bar is that for the time interval 11 a.m.-12 noon, when about 600 vehicles passed through the crossing.

The total number of cars passing through the crossing during the two-hour interval 8.00-10.00 am as shown by the bar graph is about $1000 + 800 = 1800$ vehicles.

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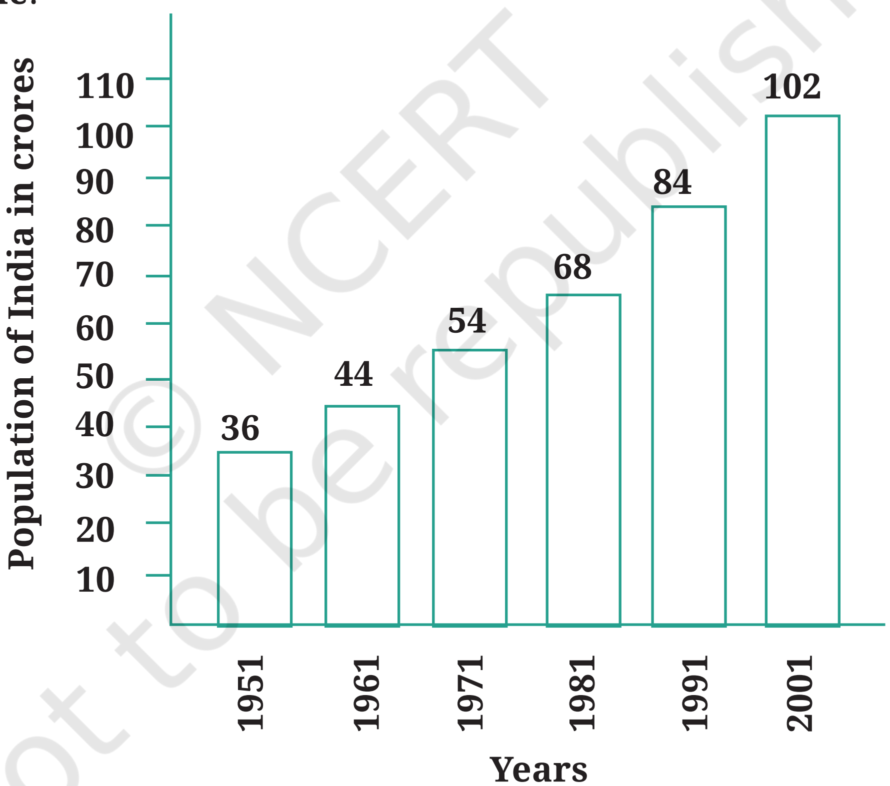
Figure it Out

1. How many total cars passed through the crossing between 6 am and noon?
2. Why do you think so little traffic occurred during the hour of 6–7 am, as compared to the other hours from 7 am-noon?
3. Why do you think the traffic was the heaviest between 7 am and 8 am?
4. Why do you think the traffic was lesser and lesser each hour after 8am all the way until noon?

Example:



Population of India in crores

This bar graph shows the population of India in each decade over a period of 50 years. The numbers are expressed in crores. If you were to take 1 unit length to represent one person, drawing the bars will be difficult! Therefore, we choose the scale so that 1 unit represents 10 crores. The bar graph for this choice is shown in the figure. So a bar of length 5 units represents 50 crores and of 8 units represents 80 crores.

- On the basis of this bar graph, what may be a few questions you may ask your friends?
- How much did the population of India increase over 50 years? How much did the population increase in each decade?

4.4 Drawing a Bar Graph

In a previous example, Shri Nilesh prepared a frequency table representing the sweet preferences of the students in his class. Let`s try to prepare a bar graph to present his data—

Sweet	No. of Students
Jalebi	6
Gulab Jamun	9
Gujiya	13
Barfi	3
Rasgulla	7

1. First, we draw a horizontal line and a vertical line. On the horizontal line, we will write the name of each of the sweets, equally spaced, from which the bars will rise in accordance with their frequencies; and on the vertical line we will write the frequencies representing the number of students.
2. We must choose a scale. That means we must decide how many students will be represented by a unit length of a bar so that it fits nicely on our paper. Here, we will take 1 unit length to represent 1 student.
3. For Jalebi, we therefore need to draw a bar having a height of 6 units (which is the frequency of the sweet Jalebi), and similarly for the other sweets we have to draw bars as high as their frequencies.

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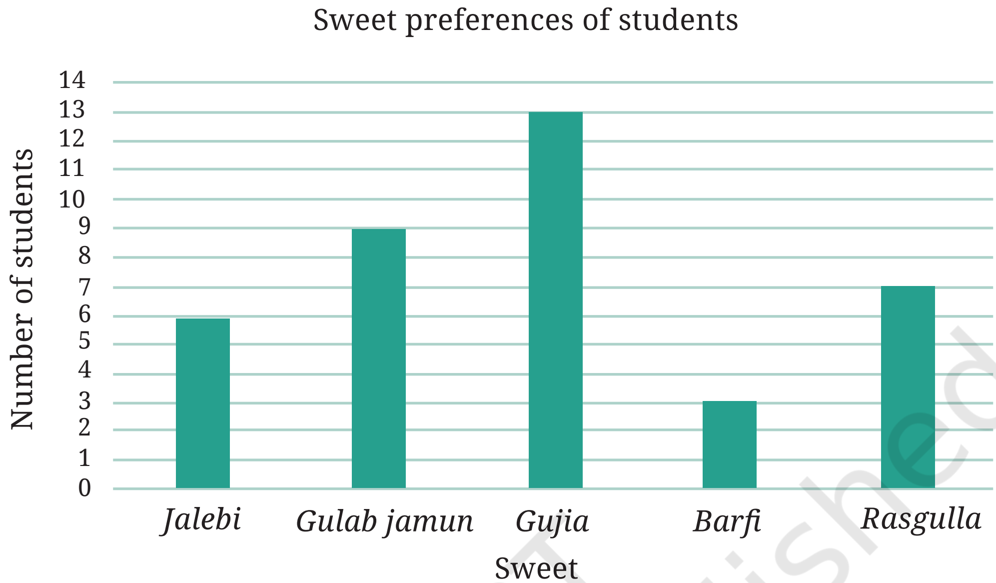
4. We therefore get a bar graph as shown below—



When the frequencies are larger and we cannot use the scale of 1 unit length = 1 number (frequency), we need to choose a different scale like we did in the case of pictographs.

Example: The number of runs scored by Smriti in each of 8 matches are given in the table below:

Match	Match 1	Match 2	Match 3	Match 4	Match 5	Match 6	Match 7	Match 8
Runs	80	50	10	100	90	0	90	50

In this example, the minimum score is 0 and the maximum score is 100. Using a scale of 1 unit length = 1 run would mean that we have to go all the way from 0 to 100 runs in steps of 1. This would be unnecessarily tedious. Instead, let us use a scale where 1 unit length = 10 runs. We mark this scale on the vertical line and draw the bars according to the scores in each match. We get the following bar graph representing the above data.

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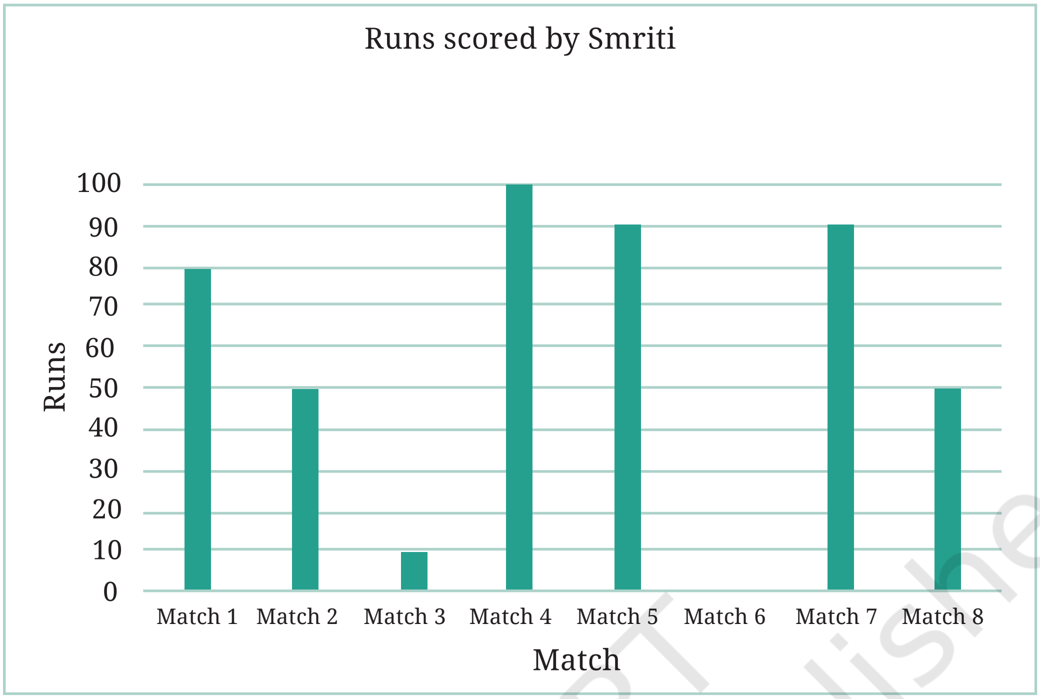

Example: The following table shows the monthly expenditure of Imran's family on various items:

Items	Expenditure (in ₹)
House rent	3000
Food	3400
Education	800
Electricity	400
Transport	600
Miscellaneous	1200

To represent this data in the form of a bar graph, here are the steps—

- Draw two perpendicular lines, one horizontal and one vertical.
- Along the horizontal line, mark the 'Items' with equal spacing between them, and along the vertical line, mark the corresponding expenditures.

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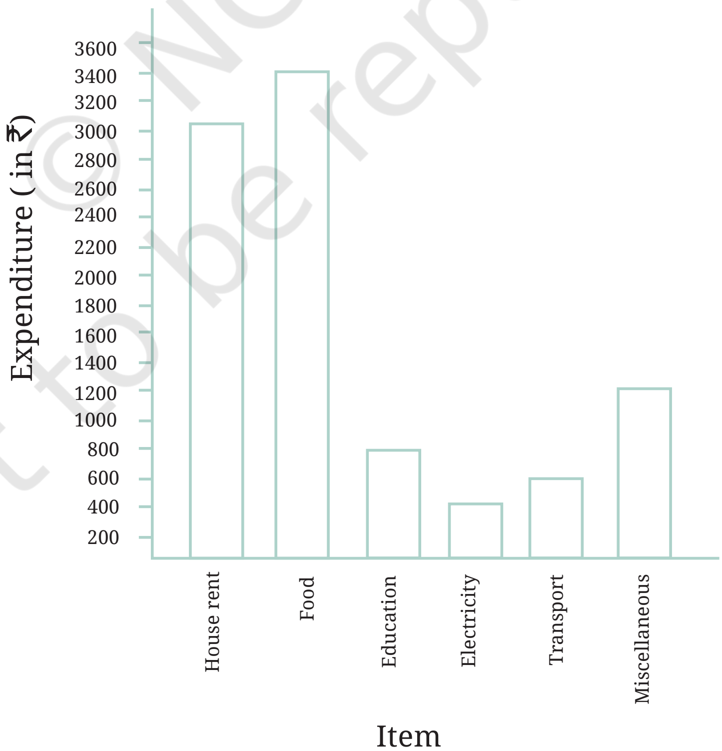
- Take bars of the same width, keeping a uniform gap between them.
- Choose a suitable scale along the vertical line. Let 1 unit length = ₹ 200, and then mark and write the corresponding values (₹ 200, ₹ 400, etc.) representing each unit length.

Finally, calculate the heights of the bars for various items as shown below—

House rent	3000 ÷ 200	15 units
Food	3400 ÷ 200	17 units
Education	800 ÷ 200	4 units
Electricity	400 ÷ 200	2 units
Transport	600 ÷ 200	3 units
Miscellaneous	1200 ÷ 200	6 units

Here is the bar graph that we obtain based on the above steps:



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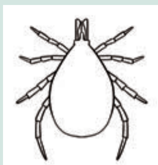
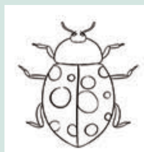
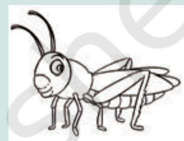
- Use the bar graph to answer the following questions:
  1. On which item does Imran's family spend the most and the second most?
  2. Is the cost of electricity about one-half the cost of education?
  3. Is the cost of education less than one-fourth the cost of food?

Figure it Out

1. Samantha visited a tea garden and collected data of the insects and critters she saw there. Here is the data she collected—

				
Mites	Caterpillars	Beetles	Butterflies	Grasshoppers
6	10	5	3	2

Help her prepare a bar graph representing this data.

2. Pooja collected data on the number of tickets sold at the Bhopal railway station for a few different cities of Madhya Pradesh over a 2-hour period.

City	Vidisha	Jabalpur	Seoni	Indore	Sagar
Number of tickets	24	20	16	28	16

She used this data and prepared a bar graph on the board to discuss the data with her students, but someone erased a portion of the graph.

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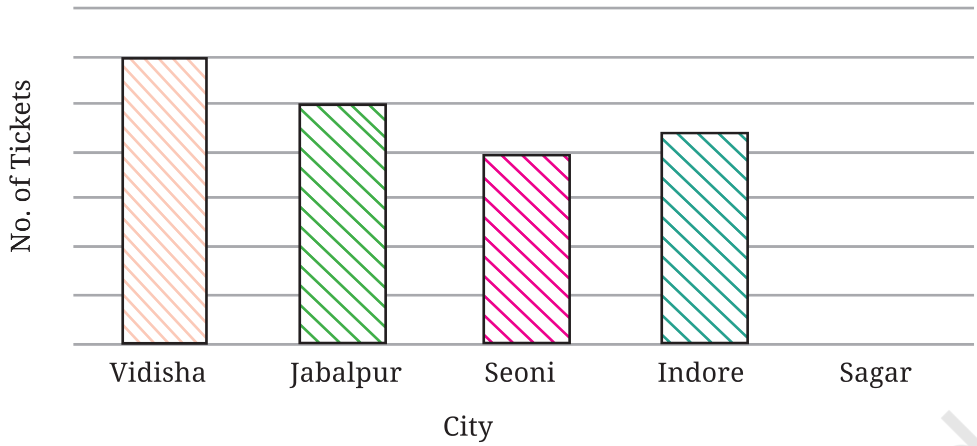

a. Write the number of tickets sold for Vidisha above the bar.

b. Write the number of tickets sold for Jabalpur above the bar.

c. The bar for Vidisha is 6 unit lengths and the bar for Jabalpur is 5 unit lengths. What is the scale for this graph?

d. Draw the correct bar for Sagar.

e. Add the scale of the bar graph placing the correct numbers on the vertical axis.

f. Are the bars for Seoni and Indore correct in this graph? If not, draw the correct bar(s).

3. Chinu listed the various means of transport that passed across the road in front of his house from 9 AM to 10 AM:

bike	car	bike	bus	bike	bike
bike	auto	bicycle	bullock cart	bicycle	auto
car	scooter	car	auto	bicycle	bike
car	auto	bike	scooter	bike	car
bicycle	scooter	bicycle	scooter	bike	bus
auto	auto	bike	bicycle	bus	bike
bicycle	scooter	bus	scooter	auto	bike
scooter	bicycle	bike	bullock cart	auto	scooter
car	scooter				

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a. Prepare a frequency distribution table for the data.

b. Which means of transport was used the most?

c. If you were there to collect this data, how could you do it? Write the steps or process.

4. Roll a die 30 times and record the number you obtain each time. Prepare a frequency distribution table using tally marks. Find the number that appeared:

a. The minimum number of times.

b. The maximum number of times.

c. Find numbers that appeared an equal number of times.

5. Faiz prepared a frequency distribution table of data on the number of wickets taken by Jaspreet Bumrah in his last 30 matches:

Wickets Taken	Number of Matches
0	2
1	4
2	6
3	8
4	3
5	5
6	1
7	1

a. What information is this table giving?

b. What may be the title of this table?

c. What caught your attention in this table?

d. In how many matches has Bumrah taken 4 wickets?

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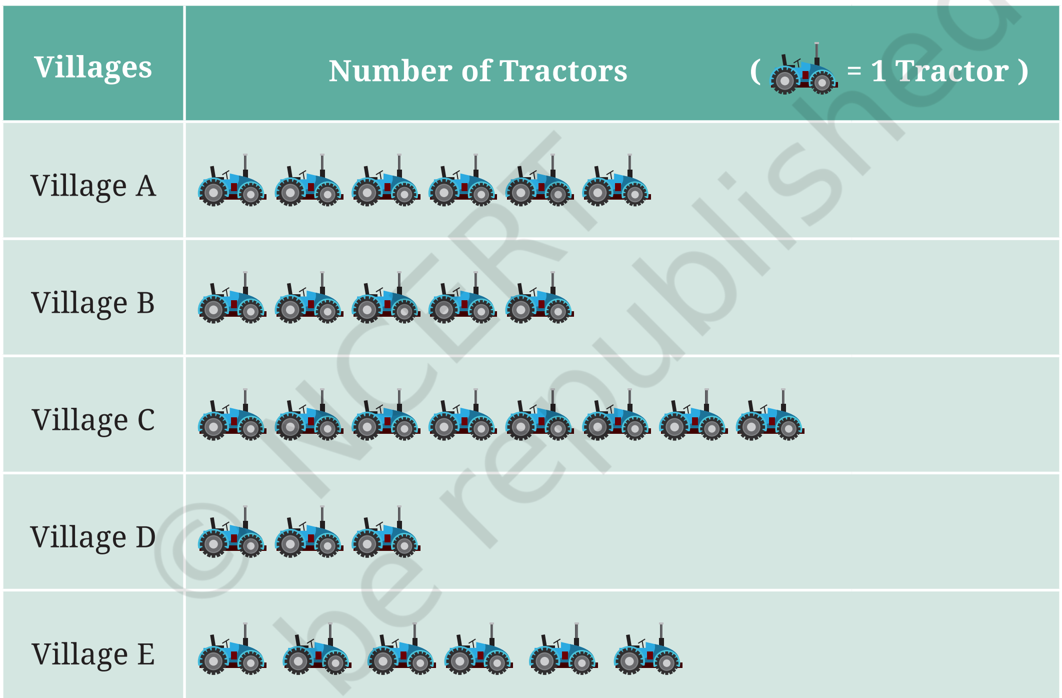
e. Mayank says "If we want to know the total number of wickets he has taken in his last 30 matches, we have to add the numbers 0, 1, 2, 3 ..., up to 7." Can Mayank get the total number of wickets taken in this way? Why?

f. How would you correctly figure out the total number of wickets taken by Bumrah in his last 30 matches, using this table?

6. The following pictograph shows the number of tractors in five different villages.



Observe the pictograph and answer the following questions—

a. Which village has the smallest number of tractors?

b. Which village has the most tractors?

c. How many more tractors does Village C have than Village B?

d. Komal says, "Village D has half the number of tractors as Village E." Is she right?

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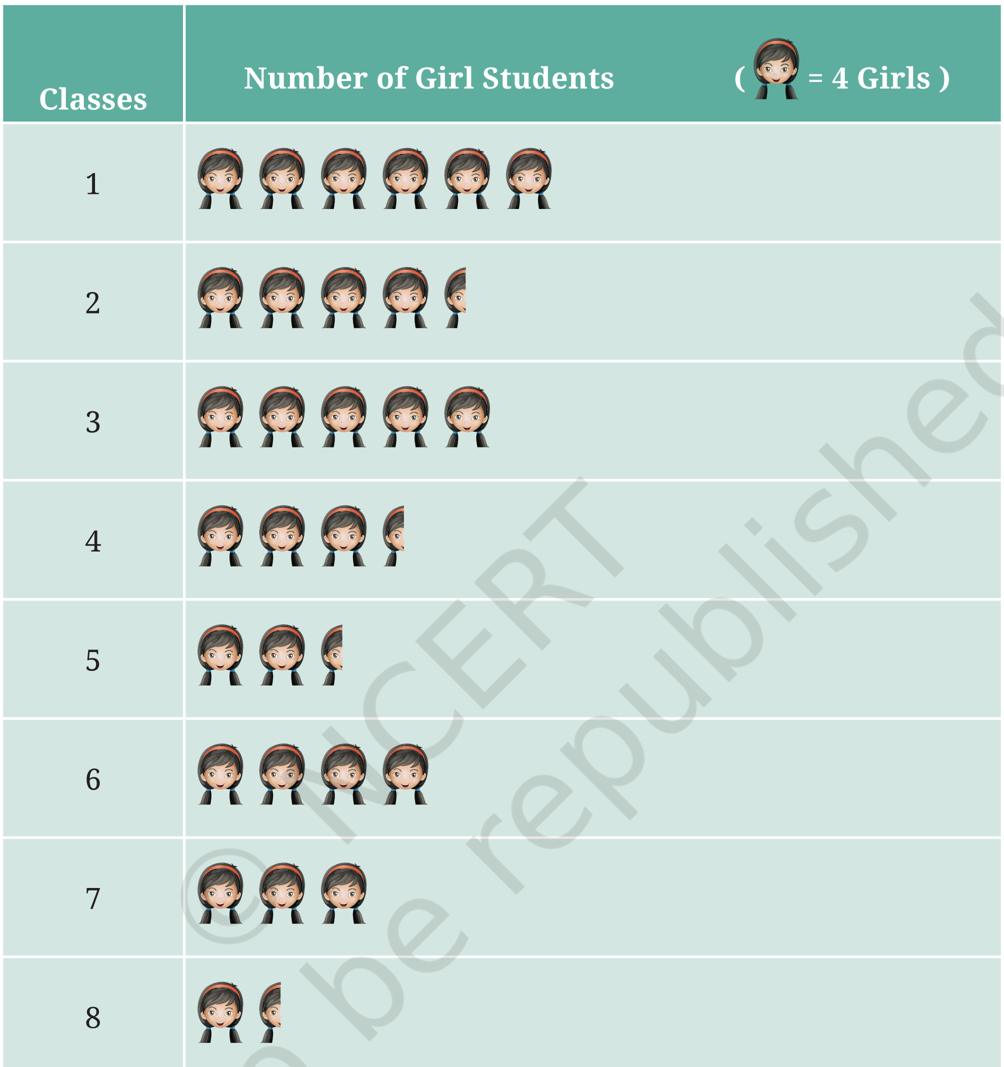
7. The number of girl students in each class of a school is depicted by a pictograph:



Observe this pictograph and answer the following questions:

a. Which class has the least number of girl students?

b. What is the difference between the number of girls in Classs 5 and 6?

c. If 2 more girls were admitted in Class 2, how would the graph change?

d. How many girls are there in Class 7?

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8. Mudhol Hounds (a type of breed of Indian dogs) are largely found in North Karnataka's Bagalkote and Vijaypura districts. The government took an initiative to protect this breed by providing support to those who adopted these dogs. Due to this initiative, the number of these dogs increased. The number of Mudhol dogs in six villages of Karnataka are as follows—

   Village A : 18, Village B : 36, Village C : 12, Village D : 48, Village E : 18, Village F : 24

   Prepare a pictograph and answer the following questions:

   a. What will be a useful scale or key to draw this pictograph?

   b. How many symbols will you use to represent the dogs in Village B?

   c. Kamini said that the number of dogs in Village B and Village D together will be more than the number of dogs in the other 4 villages. Is she right? Give reasons for your response.

9. A survey of 120 school students was conducted to find out which activity they preferred to do in their free time.

Preferred Activity	Number of Students
Playing	45
Reading story books	30
Watching TV	20
Listening to music	10
Painting	15

   Draw a bar graph to illustrate the above data taking the scale of 1 unit length = 5 students. Which activity is preferred by most students other than playing?

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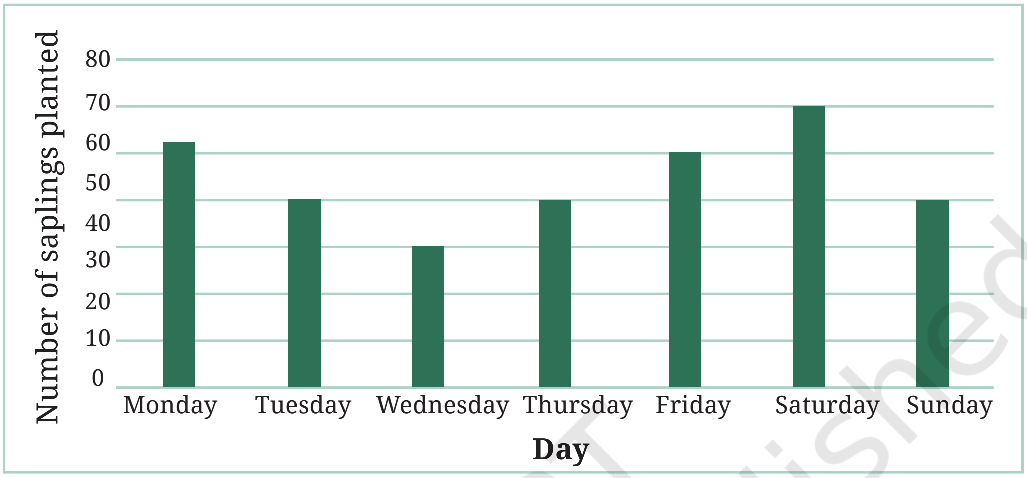
10. Students and teachers of a primary school decided to plant tree saplings in the school campus and in the surrounding village during the first week of July. Details of the saplings they planted are as follows—



a. The total number of saplings planted on Wednesday and Thursday is __________.

b. The total number of saplings planted during the whole week is __________.

c. The greatest number of saplings were planted on __________, and the least number of saplings were planted on __________. Why do you think that is the case? Why were more saplings planted on certain days of the week and less on others? Can you think of possible explanations or reasons? How could you try and figure out whether your explanations are correct?

11. The number of tigers in India went down drastically between 1900 and 1970. Project Tiger was launched in 1973 to track and protect tigers in India. Starting in 2006, the exact number of tigers in India was tracked. Shagufta and Divya looked up information about the number of tigers in India between 2006 and 2022 in 4-year intervals. They prepared a frequency table for this data and a bar graph to present this data, but there are a few mistakes in the graph. Can you find those mistakes and fix them?

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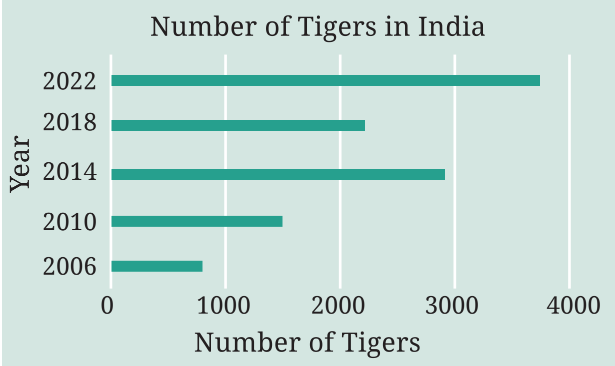
Year	Number of Tigers (approx.)
2006	1400
2010	1700
2014	2200
2018	3000
2022	3700



- Like pictographs, bar graphs give a nice visual way to represent data. They represent data through equally-spaced bars, each of equal width, where the lengths or heights give frequencies of the different categories.
- Each category is represented by a bar where the length or height depicts the corresponding frequency (for example, cost) or quantity (for example, runs).
- The bars have uniform spaces between them to indicate that they are free standing and represent equal categories.
- The bars help in interpreting data much faster than a frequency table. By reading a bar graph, we can compare frequencies of different categories at a glance.
- We must decide the scale (for example, 1 unit length = 1 student or 1 unit length = ₹ 200) for a bar graph on the basis of the data, including the minimum and maximum frequencies, so that the resulting bar graph fits nicely and looks visually appealing on the paper or poster we are preparing. The markings of the unit lengths as per the scale must start from zero.

Teacher's Note

The main focus of this chapter is to learn how to handle data to find answers to specific questions or inquiries, to test hypotheses or to take specific decisions. This should be kept in mind when providing practice opportunities to collect, organise, and analyse data.

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4.5 Artistic and Aesthetic Considerations

In addition to the steps described in previous sections, there are also some other more artistic and aesthetic aspects one can consider when preparing visual presentations of data to make them more interesting and effective. First, when making a visual presentation of data, such as a pictograph or bar graph, it is important to make it fit in the intended space; this can be controlled, e.g., by choosing the scale appropriately, as we have seen earlier. It is also desirable to make the data presentation visually appealing and easy-to-understand, so that the intended audience appreciates the information being conveyed.

Let us consider an example. Here is a table naming the tallest mountain on each continent, along with the height of each mountain in meters.

Continent	Asia	South America	North America	Africa	Europe	Antarctica	Australia
Tallest Mountain	Everest	Aconcagua	Denali	Kiliman-jaro	Elbrus	Vinson Massif	Koscuiszko
Height	8848m	6962m	6194m	5895m	5642m	4892m	2228m

How much taller is Mount Everest than Mount Koscuiszko? Are Mount Denali and Mount Kilimanjaro very different in height? This is not so easy to quickly discern from a large table of numbers.

As we have seen earlier, we can convert the table of numbers into a bar graph, as shown on the right. Here, each value is drawn as a horizontal box. These are longer or shorter depending on the number they represent. This makes it easier to compare the heights of all these mountains at a glance.

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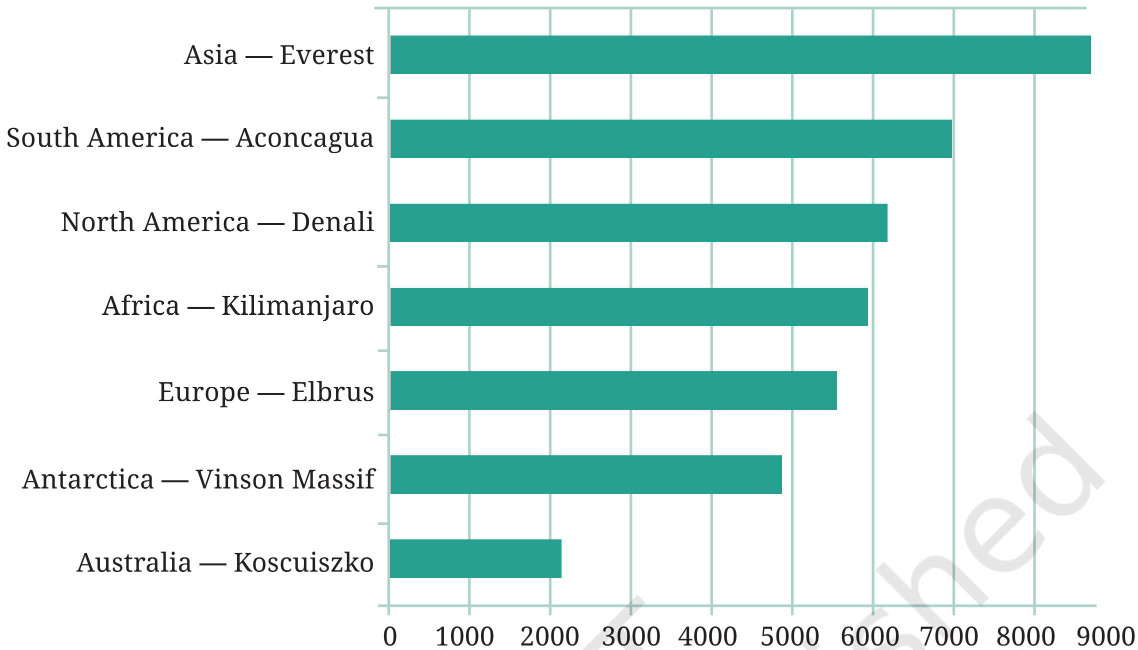
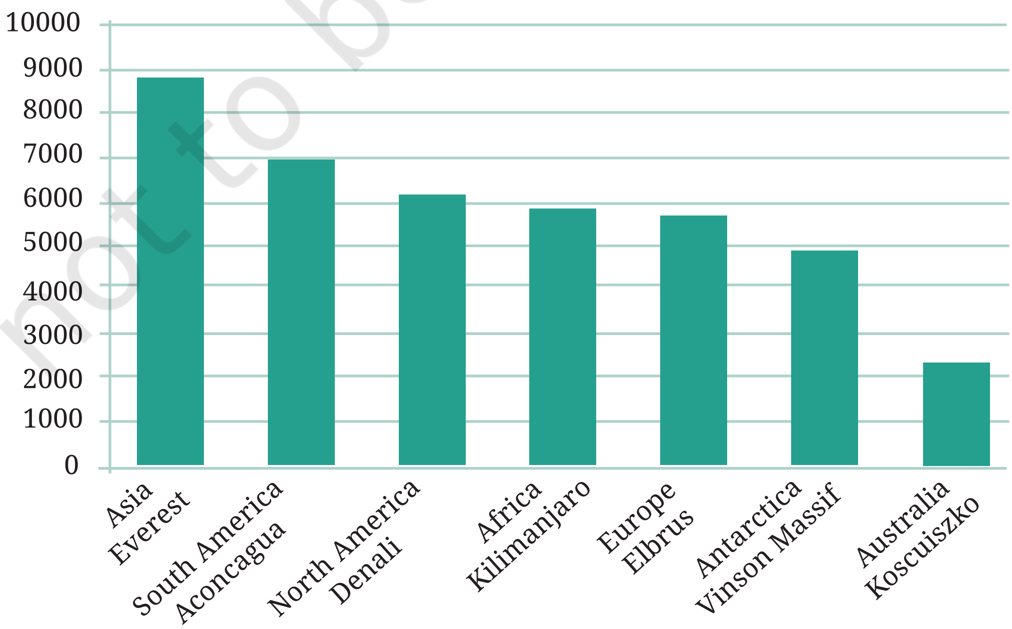


However, since the boxes represent heights, it is better and more visually appealing to rotate the picture, so that the boxes grow upward vertically from the ground like mountains. A bar graph with vertical bars is also called a column graph. Columns are the pillars you find in a building that hold up the roof.

Below is a column graph for our table of tallest mountains. From this column graph, it becomes easier to compare and visualize the heights of the mountains.



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In general, it is more intuitive, suggestive, and visually appealing to represent heights, that are measured upwards from the ground, using bar graphs that have vertical bars or columns. Similarly, lengths that are parallel to the ground (e.g., distances between location on Earth) are usually best represented using bar graphs with horizontal ars.

Figure it Out

1. If you wanted to visually represent the data of the heights of the tallest persons in each class in your school, would you use a graph with vertical bars or horizontal bars? Why?
2. If you were making a table of the longest rivers on each continent and their lengths, would you prefer to use a bar graph with vertical bars or with horizontal bars? Why? Try finding out this information, and then make the corresponding table and bar graph! Which continents have the longest rivers?

Infographics

When data visualisations such as bar graphs are further beautified with more extensive artistic and visual imagery, they are called **information graphics**, or **infographics** for short. The aim of infographics is to make use of attention-attracting and engaging visuals to communicate information even more clearly and quickly, in a visually pleasing way.

As an example of how infographics can be used to communicate data even more suggestively, let us go back to the table above listing the tallest mountain on each continent. We drew a bar graph with vertical bars (columns), rather than horizontal bars, to be more indicative of mountains. But instead of rectangles, we could instead use triangles, which look a bit more like mountains. And we can add a splash of colour as well. Here is the result.

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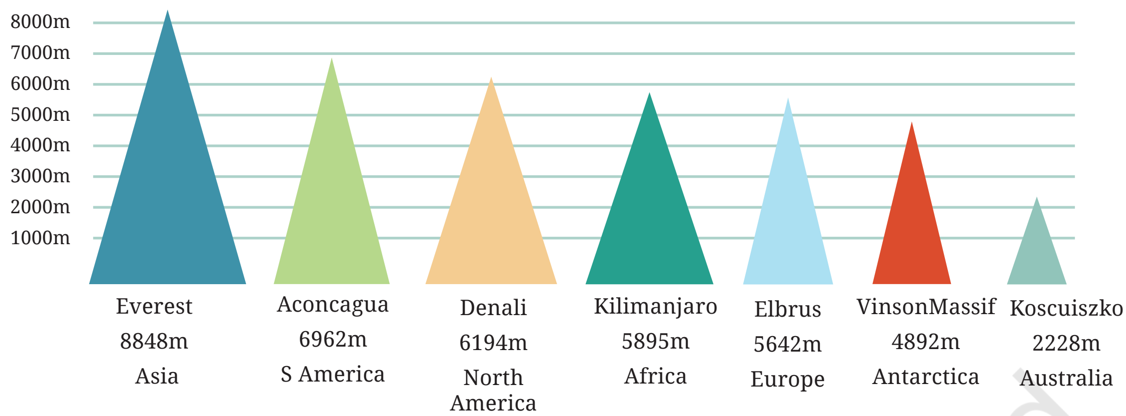

While this infographic might look more appealing and suggestive at first glance, it does have some issues. The goal of our bar graph earlier was to represent the heights of various mountains - using bars of the appropriate heights but the same widths. The purpose of using the same widths was to make it clear that we are only comparing heights. However, in this infographic, the taller triangles are also wider! Are taller mountains always wider? The infographic is implying additional information that may be misleading and may or may not be correct. Sometimes going for more appealing pictures can also accidentally mislead.

Taking this idea further, and to make the picture even more visually stimulating and suggestive, we can further change the shapes of the mountains to make them look even more like mountains, and add other details, while attempting to preserve the heights. For example, we can create an imaginary mountain range that contains all these mountains.

Is the infographic below better than the column graph with rectangular columns of equal width? The mountains look more realistic, but is the picture accurate?

For example, Everest appears to be twice as tall as Elbrus.

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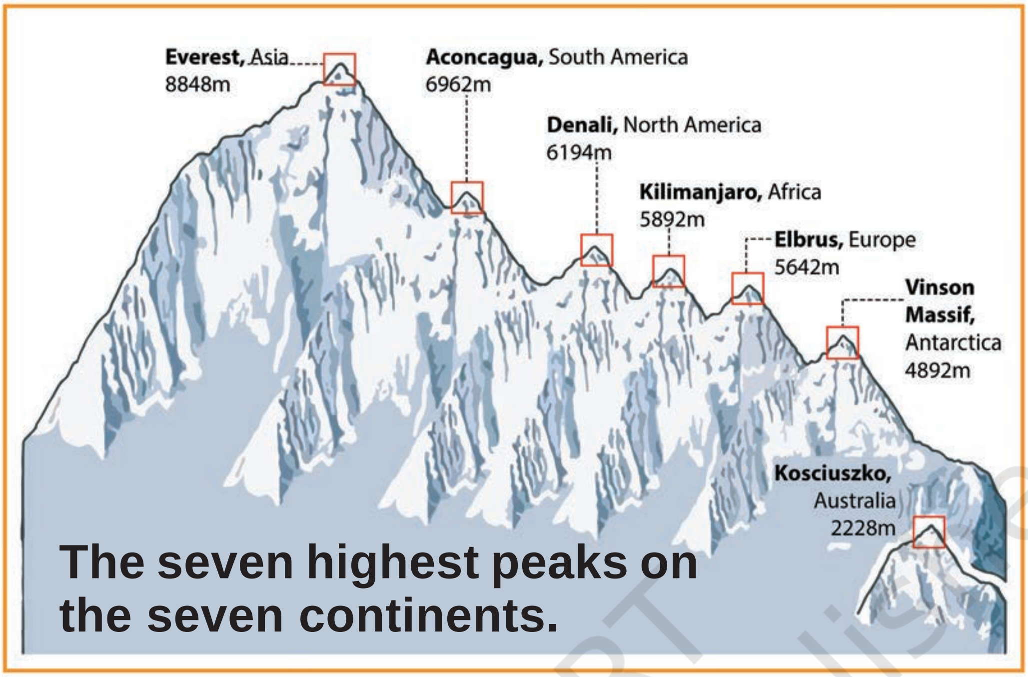

What is 5642 × 2?

While preparing visually-appealing presentations of data, we also need to be careful that the pictures we draw do not mislead us about the facts. In general, it is important to be careful when making or reading infographics, so that we do not mislead our intended audiences and so that we, ourselves, are not misled.

SUMMARY

- Facts, numbers, measures, observations, and other descriptions of things that convey information about those things is called **data**.
- Data can be organised in a tabular form using tally marks for easy analysis and interpretation.
- **Frequencies** are the counts of the occurrences of values, measures, or observations.

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- **Pictographs** represent data in the form of pictures/objects or parts of objects. Each picture represents a frequency which can be 1 or more than 1 – this is called the **scale**, and it must be specified.
- **Bar graphs** have bars of uniform width; the length or height then indicates the total frequency of occurrence. The **scale** that is used to convert length/height to frequency again must again be specified.
- Choosing the appropriate scale for a pictograph or bar graph is important to accurately and effectively convey the desired information/data and to also make it visually appealing.
- Other aspects of a graph also contribute to its effectiveness and visual appeal, such as how colours are used, what accompanying pictures are drawn, and whether the bars are horizontal or vertical. These aspects correspond to the artistic and aesthetic side of data handling and presentation.
- However, making visual representations of data too “fancy” can also sometimes be misleading.
- By reading pictographs and bar graphs accurately, we can quickly understand and make inferences about the data presented.

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5 PRIME TIME

5.1 Common Multiples and Common Factors

Idli-Vada Game

Children sit in a circle and play a game of numbers.

One of the children starts by saying '1'. The second player says '2', and so on. But when it is the turn of 3, 6, 9, ... (multiples of 3), the player should say 'idli' instead of the number. When it is the turn of 5, 10, ... (multiples of 5), the player should say 'vada' instead of the number. When a number is both a multiple of 3 and a multiple of 5, the player should say 'idli-vada'! If a player makes any mistake, they are out.

The game continues in rounds till only one person remains.

For which numbers should the players say 'idli' instead of saying the number? These would be 3, 6, 9, 12, 18, ... and so on.

For which numbers should the players say 'vada'? These would be 5, 10, 20, ... and so on.

Which is the first number for which the players should say, 'idli-vada'? It is 15, which is a multiple of 3, and also a multiple of 5. Find out other such numbers that are multiples of both 3 and 5. These numbers are called ____________________.

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Figure it Out

1. At what number is 'idli-vada' said for the 10th time?
2. If the game is played for the numbers from 1 till 90, find out:
   a. How many times would the children say 'idli' (including the times they say 'idli-vada')?
   b. How many times would the children say 'vada' (including the times they say 'idli-vada')?
   c. How many times would the children say 'idli-vada'?
3. What if the game was played till 900? How would your answers change?
4. Is this figure somehow related to the 'idli-vada' game?

   Hint: Imagine playing the game till 30. Draw the figure if the game is played till 60.

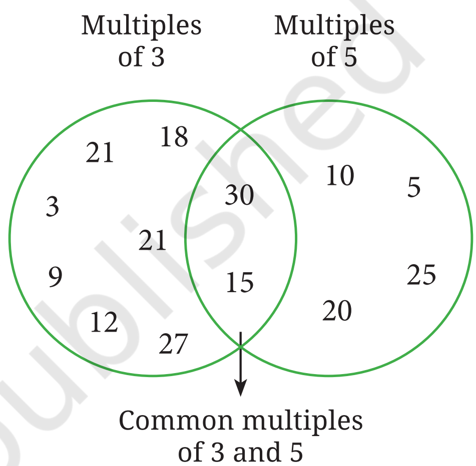
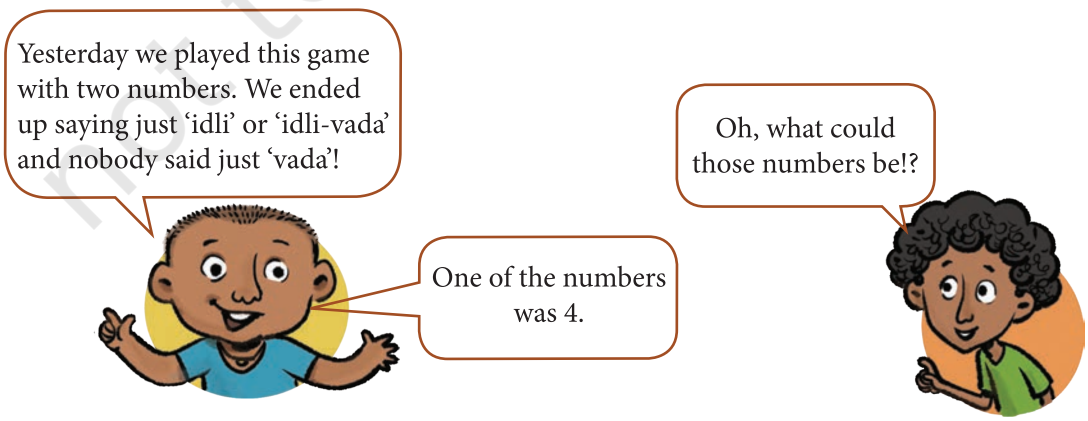


Fig. 5.1

Let us now play the 'idli-vada' game with different pairs of numbers:

a. 2 and 5,
b. 3 and 7,
c. 4 and 6.

We will say 'idli' for multiples of the smaller number, 'vada' for multiples of the larger number and 'idli-vada' for common multiples. Draw a figure similar to Fig. 5.1 if the game is played up to 60.



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Which of the following could be the other number:

2, 3, 5, 8, 10?

Jump Jackpot

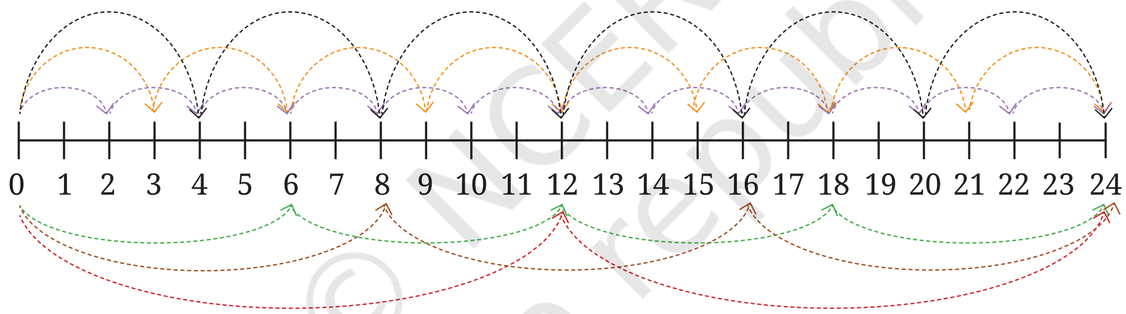
Jumpy and Grumpy play a game.

- Grumpy places a treasure on some number. For example, he may place it on 24.
- Jumpy chooses a jump size. If he chooses 4, then he has to jump only on multiples of 4, starting at 0.
- Jumpy gets the treasure if he lands on the number where Grumpy placed it.

Which jump sizes will get Jumpy to land on 24?

If he chooses 4: Jumpy lands on 4 → 8 → 12 → 16 → 20 → 24 → 28 → ...

Other successful jump sizes are 2, 3, 6, 8 and 12.



What about jump sizes 1 and 24? Yes, they also will land on 24.

The numbers 1, 2, 3, 4, 6, 8, 12, 24 all divide 24 exactly. Recall that such numbers are called **factors** or **divisors** of 24.

Grumpy increases the level of the game. Two treasures are kept on two different numbers. Jumpy has to choose a jump size and stick to it. Jumpy gets the treasures only if he lands on both the numbers with the chosen jump size. As before, Jumpy starts at 0.

Grumpy has kept the treasures on 14 and 36. Jumpy chooses a jump size of 7.

Will Jumpy land on both the treasures? Starting from 0, he jumps to 7 → 14 → 21 → 28 → 35 → 42 ... We see that he landed on 14 but

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did not land on 36, so he does not get the treasure. What jump size should he have chosen?

The factors of 14 are: 1, 2, 7, 14. So these jump sizes will land on 14.

The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36. These jump sizes will land on 36.

So, the jump sizes of 1 or 2 will land on both 14 and 36. Notice that 1 and 2 are the common factors of 14 and 36.

The jump sizes using which both the treasures can be reached are the **common factors** of the two numbers where the treasures are placed.

- What jump size can reach both 15 and 30? There are multiple jump sizes possible. Try to find them all.

- Look at the table below. What do you notice?

31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

In the table,

1. Is there anything common among the shaded numbers?
2. Is there anything common among the circled numbers?
3. Which numbers are both shaded and circled? What are these numbers called?

Math Talk

Figure it Out

1. Find all multiples of 40 that lie between 310 and 410.

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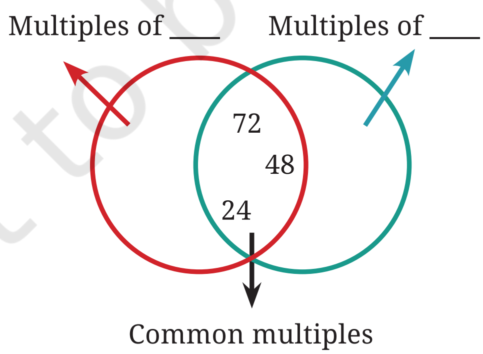
2. Who am I?
   a. I am a number less than 40. One of my factors is 7. The sum of my digits is 8.
   b. I am a number less than 100. Two of my factors are 3 and 5. One of my digits is 1 more than the other.
3. A number for which the sum of all its factors is equal to twice the number is called a **perfect number**. The number 28 is a perfect number. Its factors are 1, 2, 4, 7, 14 and 28. Their sum is 56 which is twice 28. Find a perfect number between 1 and 10.
4. Find the common factors of:
   a. 20 and 28
   b. 35 and 50
   c. 4, 8 and 12
   d. 5, 15 and 25
5. Find any three numbers that are multiples of 25 but not multiples of 50.
6. Anshu and his friends play the 'idli-vada' game with two numbers, which are both smaller than 10. The first time anybody says 'idli-vada' is after the number 50. What could the two numbers be which are assigned 'idli' and 'vada'?
7. In the treasure hunting game, Grumpy has kept treasures on 28 and 70. What jump sizes will land on both the numbers?
8. In the diagram below, Guna has erased all the numbers except the common multiples. Find out what those numbers could be and fill in the missing numbers in the empty regions.

Math Talk



9. Find the smallest number that is a multiple of all the numbers from 1 to 10 except for 7.
10. Find the smallest number that is a multiple of all the numbers from 1 to 10.

Try This

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5.2 Prime Numbers

Guna and Anshu want to pack figs (*anjeer*) that grow in their farm. Guna wants to put 12 figs in each box and Anshu wants to put 7 figs in each box.

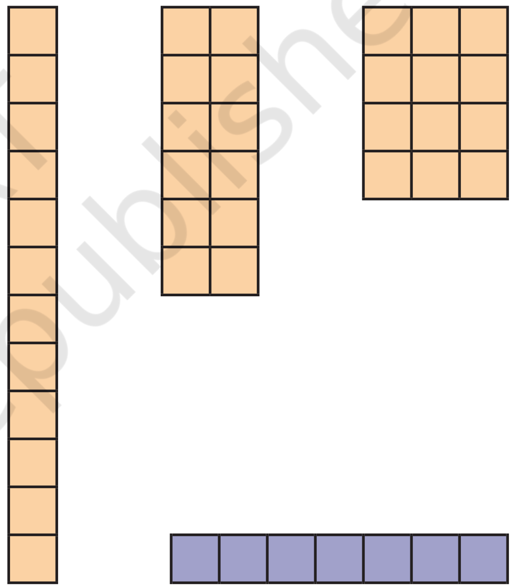
How many arrangements are possible?

Think and find out the different ways how—

i. Guna can arrange 12 figs in a rectangular manner.

ii. Anshu can arrange 7 figs in a rectangular manner.

Guna has listed out these possibilities.



Observe the number of rows and columns in each of the arrangements. How are they related to 12?

In the second arrangement, for example, 12 figs are arranged in two columns of 6 each or $12 = 2 \times 6$.

Anshu could make only one arrangement: 7×1 or 1×7. There are no other rectangular arrangements possible.

In each of Guna's arrangements, multiplying the number of rows by the number of columns gives the number 12. So, the number of rows or columns are factors of 12.

We saw that the number 12 can be arranged in a rectangle in more than one way as 12 has more than two factors. The number 7 can be arranged in only one way, as it has only two factors—1 and 7.

Numbers that have only two factors are called **prime numbers** or *primes*. Here are the first few primes—2, 3, 5, 7, 11, 13, 17, 19. Notice that the factors of a prime number are 1 and the number itself.

What about numbers that have more than two factors? They are called **composite numbers**. The first few composite numbers are—4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

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What about 1, which has only one factor? The number 1 is neither a prime nor a composite number.

How many prime numbers are there from 21 to 30? How many composite numbers are there from 21 to 30?

Can we list all the prime numbers from 1 to 100?

Here is an interesting way to find prime numbers. Just follow the steps given below and see what happens.

Step 1: Cross out 1 because it is neither prime nor composite.

Step 2: Circle 2, and then cross out all multiples of 2 after that, i.e., 4, 6, 8 and so on.

Step 3: You will find that the next uncrossed number is 3. Circle 3 and then cross out all the multiples of 3 after that, i.e., 6, 9, 12 and so on.

Step 4: The next uncrossed number is 5. Circle 5 and then cross out all the multiples of 5 after that, i.e., 10, 15, 20 and so on.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Step 5: Continue this process till all the numbers in the list are either circled or crossed out.

All the circled numbers are prime numbers. All the crossed out numbers, other than 1, are composite numbers. This method is called the Sieve of Eratosthenes.

This procedure can be carried on for numbers greater than 100 also. Eratosthenes was a Greek mathematician who lived around 2200 years ago and developed this method of listing primes.

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Guna and Anshu started wondering how this simple method is able to find prime numbers! Think how this method works. Read the steps given above again and see what happens after each step is carried out.

Figure it Out

1. We see that 2 is a prime and also an even number. Is there any other even prime?
2. Look at the list of primes till 100. What is the smallest difference between two successive primes? What is the largest difference?
3. Are there an equal number of primes occurring in every row in the table on the previous page? Which decades have the least number of primes? Which have the most number of primes?

Primes through the Ages

Prime numbers are the building blocks of all whole numbers. Starting from the time of the Greek civilisation (more than 2000 years ago) to this day, mathematicians are still struggling to uncover their secrets!

Food for thought: is there a largest prime number? Or does the list of prime numbers go on without an end? A mathematician named Euclid found the answer and so will you in a later class!

Fun fact: The largest prime number that anyone has 'written down' is so large that it would take around 6500 pages to write it! So they could only write it on a computer!

4. Which of the following numbers are prime? 23, 51, 37, 26
5. Write three pairs of prime numbers less than 20 whose sum is a multiple of 5.
6. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers up to 100.
7. Find seven consecutive composite numbers between 1 and 100.
8. **Twin primes** are pairs of primes having a difference of 2. For example, 3 and 5 are twin primes. So are 17 and 19. Find the other twin primes between 1 and 100.

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9. Identify whether each statement is true or false. Explain.
   a. There is no prime number whose units digit is 4.
   b. A product of primes can also be prime.
   c. Prime numbers do not have any factors.
   d. All even numbers are composite numbers.
   e. 2 is a prime and so is the next number, 3. For every other prime, the next number is composite.
10. Which of the following numbers is the product of exactly three distinct prime numbers: 45, 60, 91, 105, 330?
11. How many three-digit prime numbers can you make using each of 2, 4 and 5 once?
12. Observe that 3 is a prime number, and $2 \times 3 + 1 = 7$ is also a prime. Are there other primes for which doubling and adding 1 gives another prime? Find at least five such examples.

5.3 Co-prime Numbers for Safekeeping Treasures

Which pairs are safe?

Let us go back to the treasure finding game. This time, treasures are kept on two numbers. Jumpy gets the treasures only if he is able to reach both the numbers with the same jump size. There is also a new rule—a jump size of 1 is not allowed.

Where should Grumpy place the treasures so that Jumpy cannot reach both the treasures?

Will placing the treasure on 12 and 26 work? No! If the jump size is chosen to be 2, then Jumpy will reach both 12 and 26.

What about 4 and 9? Jumpy cannot reach both using any jump size other than 1. So, Grumpy knows that the pair 4 and 9 is safe.

Check if these pairs are safe:

a. 15 and 39 b. 4 and 15
c. 18 and 29 d. 20 and 55

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What is special about safe pairs? They don't have any common factor other than 1. Two numbers are said to be **co-prime** to each other if they have no common factor other than 1.

Example: As 15 and 39 have 3 as a common factor, they are not co-prime. But 4 and 9 are co-prime.

- Which of the following pairs of numbers are co-prime?

  a. 18 and 35   b. 15 and 37   c. 30 and 415

  d. 17 and 69   e. 81 and 18

- While playing the '*idli-vada*' game with different number pairs, Anshu observed something interesting!

  a. Sometimes the first common multiple was the same as the product of the two numbers.

  b. At other times the first common multiple was less than the product of the two numbers.

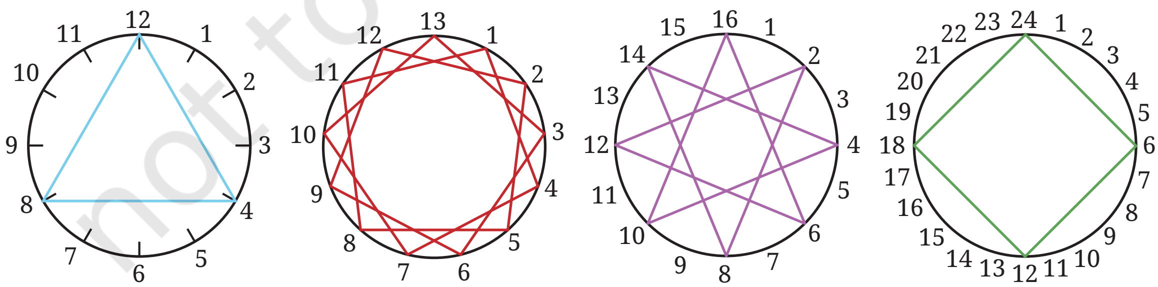
Find examples for each of the above. How is it related to the number pair being co-prime?

Math Talk

Co-prime Art

- Observe the following thread art. The first diagram has 12 pegs and the thread is tied to every fourth peg (we say that the thread-gap is 4). The second diagram has 13 pegs and the thread-gap is 3. What about the other diagrams? Observe these pictures, share and discuss your findings in class.

Math Talk



In some diagrams, the thread is tied to every peg. In some, it is not. Is it related to the two numbers (the number of pegs and the thread-gap) being co-prime?

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Make such pictures for the following:

a. 15 pegs, thread-gap of 10

b. 10 pegs, thread-gap of 7

c. 14 pegs, thread-gap of 6

d. 8 pegs, thread-gap of 3

5.4 Prime Factorisation

Checking if two numbers are co-prime

Teacher: Are 56 and 63 co-prime?

Anshu and Guna: If they have a common factor other than 1, then they are not co-prime. Let us check.

Anshu: I can write $56 = 14 \times 4$ and $63 = 21 \times 3$. So, 14 and 4 are factors, of 56. Further, 21 and 3 are factors of 63. So, there are no common factors. The numbers are co-prime.

Guna: Hold on. I can also write $56 = 7 \times 8$ and $63 = 9 \times 7$. We see that 7 is a factor of both numbers, so, they are not co-prime.

Clearly Guna is right, as 7 is a common factor.

But where did Anshu go wrong?

Writing $56 = 14 \times 4$ tells us that 14 and 4 are both factors of 56, but it does not tell all the factors of 56. The same holds for the factors of 63.

Try another example: 80 and 63. There are many ways to factorise both numbers.

$80 = 40 \times 2 = 20 \times 4 = 10 \times 8 = 16 \times 5 = ???$

$63 = 9 \times 7 = 3 \times 21 = ???$

We have written '???' to say that there may be more ways to factorise these numbers. But if we take any of the given factorisations, for example, $80 = 16 \times 5$ and $63 = 9 \times 7$, then there are no common factors. Can we conclude that 80 and 63 are co-prime? As Anshu's mistake above shows, we cannot conclude that as there may be other ways to factorize the numbers.

What this means is that we need a more systematic approach to check if two numbers are co-prime.

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Prime Factorisation

Take a number such as 56. It is composite, as we saw that it can be written as **56 = 4 × 14** . So, both 4 and 14 are factors of 56. Now take one of these, say 14. It is also composite and can be written as 14 = 2 × 7. Therefore, **56 = 4 × 2 × 7**. Now, 4 is composite and can be written as 4 = 2 × 2. Therefore, **56 = 2 × 2 × 2 × 7**. All the factors appearing here, 2 and 7, are prime numbers. So, we cannot divide them further.

In conclusion, we have written 56 as a product of prime numbers. This is called a **prime factorisation** of 56. The individual factors are called *prime factors*. For example, the prime factors of 56 are 2 and 7.

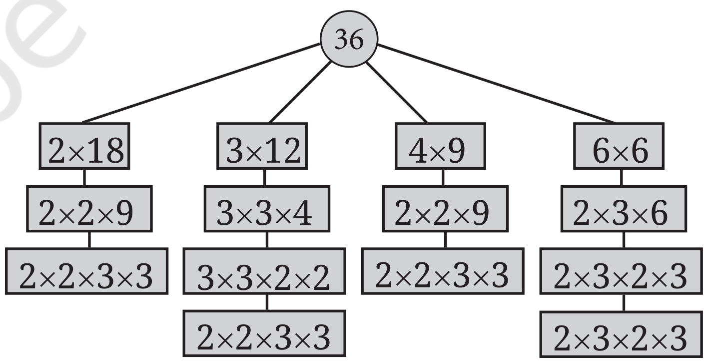
Every number greater than 1 has a prime factorisation. The idea is the same: Keep breaking the composite numbers into factors till only primes are left.

The number 1 does not have any prime factorisation. It is not divisible by any prime number.

What is the prime factorisation of a prime number like 7? It is just 7 (we cannot break it down any further).

Let us see a few more examples.

By going through different ways of breaking down the number, we wrote 63 as 3 × 3 × 7 and as 3 × 7 × 3. Are they different? Not really! The same prime numbers 3 and 7 occur in both cases. Further, 3 appears two times in both and 7 appears once.



Here, you see four different ways to get prime factorisation of 36. Observe that in all four cases, we get two 2s and two 3s.

Multiply back to see that you get 36 in all four cases.

For any number, it is a remarkable fact that there is only one prime factorisation, except that the prime factors may come in different

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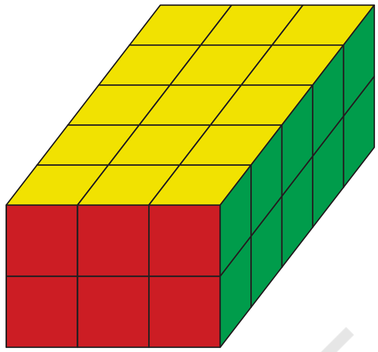
orders. As we explain below, the order is not important. However, as we saw in these examples, there are many ways to arrive at the prime factorisation!

Does the order matter?

Using this diagram,



can you explain why **30 = 2 × 3 × 5,** no matter which way you multiply 2, 3, and 5?

When multiplying numbers, we can do so in any order. The end result is the same. That is why, when two 2s and two 3s are multiplied in any order, we get 36. In a later class, we shall study this under the names of **commutativity and associativity of multiplication**.

Thus, the order does not matter. Usually we write the prime numbers in increasing order. For example, $225 = 3 \times 3 \times 5 \times 5$ or $30 = 2 \times 3 \times 5$.

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Prime factorisation of a product of two numbers

When we find the prime factorisation of a number, we first write it as a product of two factors. For example, $72 = 12 \times 6$. Then, we find the prime factorisation of each of the factors. In the above example, $12 = 2 \times 2 \times 3$ and $6 = 2 \times 3$. Now, can you say what the prime factorisation of 72 is?

The prime factorisation of the original number is obtained by putting these together.

$$72 = 2 \times 2 \times 3 \times 2 \times 3$$

We can also write this as $2 \times 2 \times 2 \times 3 \times 3$. Multiply and check that you get 72 back!

Observe how many times each prime factor occurs in the factorisation of 72.

Compare it with how many times it occurs in the factorisations of 12 and 6 put together.

Figure it Out

1. Find the prime factorisations of the following numbers: 64, 104, 105, 243, 320, 141, 1728, 729, 1024, 1331, 1000.
2. The prime factorisation of a number has one 2, two 3s, and one 11. What is the number?
3. Find three prime numbers, all less than 30, whose product is 1955.
4. Find the prime factorisation of these numbers without multiplying first a. 56×25 b. 108×75 c. 1000×81
5. What is the smallest number whose prime factorisation has:
   a. three different prime numbers?
   b. four different prime numbers?

Prime factorisation is of fundamental importance in the study of numbers. Let us discuss two ways in which it can be useful.

Using prime factorisation to check if two numbers are co-prime

Let us again take the numbers 56 and 63. How can we check if they are co-prime? We can use the prime factorisation of both numbers—

$$56 = 2 \times 2 \times 2 \times 7 \text{ and } 63 = 3 \times 3 \times 7.$$

Now, we see that 7 is a prime factor of 56 as well as 63. Therefore, 56 and 63 are not co-prime.

What about 80 and 63? Their prime factorisations are as follows:

$$80 = 2 \times 2 \times 2 \times 2 \times 5 \text{ and } 63 = 3 \times 3 \times 7.$$

There are no common prime factors. Can we conclude that they are co-prime? Suppose they have a common factor that is composite. Would the prime factors of this composite common factor appear in the prime factorisation of 80 and 63?

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Therefore, we can say that if there are no common prime factors, then the two numbers are co-prime.
Let us see some examples.

Example: Consider 40 and 231. Their prime factorisations are as follows:

$$40 = 2 \times 2 \times 2 \times 5 \text{ and } 231 = 3 \times 7 \times 11$$

We see that there are no common primes that divide both 40 and 231. Indeed, the prime factors of 40 are 2 and 5 while, the prime factors of 231 are 3, 7, and 11. Therefore, 40 and 231 are co-prime!

Example: Consider 242 and 195. Their prime factorisations are as follows:

$$242 = 2 \times 11 \times 11 \text{ and } 195 = 3 \times 5 \times 13.$$

The prime factors of 242 are 2 and 11. The prime factors of 195 are 3, 5, and 13. There are no common prime factors. Therefore, 242 and 195 are co-prime.

Using prime factorisation to check if one number is divisible by another

We can say that if one number is divisible by another, the prime factorisation of the second number is included in the prime factorisation of the first number.

We say that 48 is divisible by 12 because when we divide 48 by 12, the remainder is zero. How can we check if one number is divisible by another without carrying out long division?

Example: Is 168 divisible by 12? Find the prime factorisations of both:

$$168 = 2 \times 2 \times 2 \times 3 \times 7 \text{ and } 12 = 2 \times 2 \times 3.$$

Since we can multiply in any order, now it is clear that,

$$168 = 2 \times 2 \times 3 \times 2 \times 7 = 12 \times 14$$

Therefore, 168 is divisible by 12.

Example: Is 75 divisible by 21? Find the prime factorisations of both:

$$75 = 3 \times 5 \times 5 \text{ and } 21 = 3 \times 7.$$

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As we saw in the discussion above, if 75 was a multiple of 21, then all prime factors of 21 would also be prime factors of 75. However, 7 is a prime factor of 21 but not a prime factor of 75. Therefore, 75 is not divisible by 21.

Example: Is 42 divisible by 12? Find the prime factorisations of both:

$$42 = 2 \times 3 \times 7 \text{ and } 12 = 2 \times 2 \times 3.$$

All prime factors of 12 are also prime factors of 42. But the prime factorisation of 12 is not included in the prime factorisation of 42. This is because 2 occurs twice in the prime factorisation of 12 but only once in the prime factorisation of 42. This means that 42 is not divisible by 12.

We can say that if one number is divisible by another, then the prime factorisation of the second number is included in the prime factorisation of the first number.

Figure it Out

1. Are the following pairs of numbers co-prime? Guess first and then use prime factorisation to verify your answer.
   a. 30 and 45   b. 57 and 85
   c. 121 and 1331   d. 343 and 216
2. Is the first number divisible by the second? Use prime factorisation.
   a. 225 and 27   b. 96 and 24
   c. 343 and 17   d. 999 and 99
3. The first number has prime factorisation $2 \times 3 \times 7$ and the second number has prime factorisation $3 \times 7 \times 11$. Are they co-prime? Does one of them divide the other?
4. Guna says, "Any two prime numbers are co-prime". Is he right?

5.5 Divisibility Tests

So far, we have been finding factors of numbers in different contexts, including to determine if a number is prime or not, or if a given pair of numbers is co-prime or not.

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It is easy to find factors of small numbers. How do we find factors of a large number?

Let us take 8560. Does it have any factors from 2 to 10 (2, 3, 4, 5, ..., 9, 10)?

It is easy to check if some of these numbers are factors or not without doing long division. Can you find them?

Divisibility by 10

Let us take 10. Is 8560 divisible by 10? This is another way of asking if 10 is a factor of 8560.

For this, we can look at the pattern in the multiples of 10.

The first few multiples of 10 are: 10, 20, 30, 40, ... Continue this sequence and observe the pattern.

Is 125 a multiple of 10? Will this number appear in the previous sequence? Why or why not?

Can you now answer if 8560 is divisible by 10?

Consider this statement:

Numbers that are divisible by 10 are those that end with '0'. Do you agree?

Math Talk

Divisibility by 5

The number 5 is another number whose divisibility can easily be checked. How do we do it?

Explore by listing down the multiples: 5, 10, 15, 20, 25, ... What do you observe about these numbers? Do you see a pattern in the last digit?

What is the largest number less than 399 that is divisible by 5? Is 8560 divisible by 5?

Consider this statement:

Numbers that are divisible by 5 are those that end with either a '0' or a '5'. Do you agree?

Math Talk

Divisibility by 2

The first few multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...
What do you observe? Do you see a pattern in the last digit?

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Is 682 divisible by 2? Can we answer this without doing the long division?

Is 8560 divisible by 2? Why or why not?

- Consider this statement:

Numbers that are divisible by 2 are those that end with '0', '2', '4, '6' or '8'. Do you agree?

What are all the multiples of 2 between 399 and 411?

Divisibility by 4

Checking if a number is divisible by 4 can also be done easily!

Look at its multiples: 4, 8, 12, 16, 20, 24, 28, 32, ...

Are you able to observe any patterns that can be used? The multiples of 10, 5 and 2 have a pattern in their last digits which we are able to use to check for divisibility. Similarly, can we check if a number is divisible by 4 by looking at the last digit?

It does not work! Look at 12 and 22. They have the same last digit, but 12 is a multiple of 4 while 22 is not. Similarly 14 and 24 have the same last digit, but 14 is not a multiple of 4 while 24 is. Similarly, 16 and 26 or 18 and 28. What this means is that by looking at the last digit, we cannot tell whether a number is a multiple of 4.

Can we answer the question by looking at more digits? Make a list of multiples of 4 between 1 and 200 and search for a pattern.

- Find numbers between 330 and 340 that are divisible by 4. Also, find numbers between 1730 and 1740, and 2030 and 2040, that are divisible by 4. What do you observe?

- Is 8536 divisible by 4?

- Consider these statements:

  a. Only the last two digits matter when deciding if a given number is divisible by 4.

  b. If the number formed by the last two digits is divisible by 4, then the original number is divisible by 4.

  c. If the original number is divisible by 4, then the number formed by the last two digits is divisible by 4.

  Do you agree? Why or why not?

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Divisibility by 8

Interestingly, even checking for divisibility by 8 can be simplified. Can the last two digits be used for this?

- Find numbers between 120 and 140 that are divisible by 8. Also find numbers between 1120 and 1140, and 3120 and 3140, that are divisible by 8. What do you observe?

- Change the last two digits of 8560 so that the resulting number is a multiple of 8.

- Consider this statement:

  a. Only the last three digits matter when deciding if a given number is divisible by 8.
  b. If the number formed by the last three digits is divisible by 8, then the original number is divisible by 8.
  c. If the original number is divisible by 8, then the number formed by the last three digits is divisible by 8.

  Do you agree? Why or why not?

Math Talk

We have seen that long division is not always needed to check if a number is a factor or not. We have made use of certain observations to come up with simple methods for 10, 5, 2, 4, 8. Do we have such simple methods for other numbers as well? We will discuss simple methods to test divisibility by 3, 6, 7, and 9 in later classes!

Figure it Out

1. 2024 is a leap year (as February has 29 days). Leap years occur in the years that are multiples of 4, except for those years that are evenly divisible by 100 but not 400.
   a. From the year you were born till now, which years were leap years?
   b. From the year 2024 till 2099, how many leap years are there?
2. Find the largest and smallest 4-digit numbers that are divisible by 4 and are also palindromes.
3. Explore and find out if each statement is always true, sometimes true or never true. You can give examples to support your reasoning.

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a. Sum of two even numbers gives a multiple of 4.
b. Sum of two odd numbers gives a multiple of 4.

4. Find the remainders obtained when each of the following numbers are divided by i) 10, ii) 5, iii) 2.

   78, 99, 173, 572, 980, 1111, 2345

5. The teacher asked if 14560 is divisible by all of 2, 4, 5, 8 and 10. Guna checked for divisibility of 14560 by only two of these numbers and then declared that it was also divisible by all of them. What could those two numbers be?
6. Which of the following numbers are divisible by all of 2, 4, 5, 8 and 10: 572, 2352, 5600, 6000, 77622160.
7. Write two numbers whose product is 10000. The two numbers should not have 0 as the units digit.

5.6 Fun with Numbers

Special Numbers

There are four numbers in this box. Which number looks special to you? Why do you say so?

9	16
25	43

Look at the what Guna's classmates have to share:

- Karnawati says, "9 is special because it is a single-digit number whereas all the other numbers are 2-digit numbers.
- Gurupreet says, "9 is special because it is the only number that is a multiple of 3"
- Murugan says, "16 is special because it is the only even number and also the only multiple of 4".
- Gopika says, "25 is special as it is the only multiple of 5".
- Yadnyikee says, "43 is special because it is the only prime number".
- Radha says, "43 is special because it is the only number that is not a square".

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Below are some boxes with four numbers in each box. Within each box try to say how each number is special compared to the rest. Share with your classmates and find out who else gave the same reasons as you did. Did anyone give different reasons that may not have occurred to you?!

Math Talk

5	7
12	35

3	8
11	24

27	3
123	31

17	27
44	65

A Prime Puzzle

The figure on the left shows the puzzle. The figure on the right shows the solution of the puzzle. Think what the rules can be to solve the puzzle.

Math Talk

			75
			42
			102
170	30	63	

5	5	3	75
2	3	7	42
17	2	3	102
170	30	63	

Rules

Fill the grid with prime numbers only so that the product of each row is the number to the right of the row and the product of each column is the number below the column.

			105
			20
			30
28	125	18	

			8
			105
			70
30	70	28	

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			63
			27
			190
45	42	171	

			343
			660
			44
28	154	231	

SUMMARY

- If a number is divisible by another, the second number is called a **factor** of the first. For example, 4 is a factor of 12 because 12 is divisible by 4 ($12 \div 4 = 3$).
- **Prime numbers** are numbers like 2, 3, 5, 7, 11, ... that have only two factors, namely 1 and themselves.
- **Composite numbers** are numbers like 4, 6, 8, 9, ... that have more than 2 factors, i.e., at least one factor other than 1 and themselves. For example, 8 has the factor 4 and 9 has the factor 3, so 8 and 9 are both composite.
- Every number greater than 1 can be written as a product of prime numbers. This is called the number's **prime factorisation**. For example, $84 = 2 \times 2 \times 3 \times 7$.
- There is only one way to factorise a number into primes, except for the ordering of the factors.
- Two numbers that do not have a common factor other than 1 are said to be **co-prime**.
- To check if two numbers are co-prime, we can first find their prime factorisations and check if there is a common prime factor. If there is no common prime factor, they are co-prime, and otherwise they are not.
- A number is a factor of another number if the prime factorisation of the first number is included in the prime factorisation of the second number.

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6 PERIMETER AND AREA

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6.1 Perimeter

Do you remember what the perimeter of a closed plane figure is? Let us refresh our understanding!

The perimeter of any closed plane figure is the distance covered along its boundary when you go around it once. For a **polygon**, i.e., a closed plane figure made up of line segments, the perimeter is simply the sum of the lengths of its all sides, i.e., the total distance along its outer boundary.

The perimeter of a polygon = the sum of the lengths of its all sides.

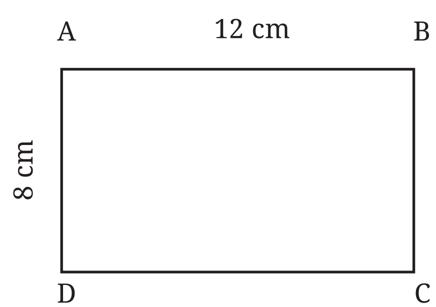
Let us revise the formulas for the perimeter of rectangles, squares, and triangles.

Perimeter of a Rectangle

Consider a rectangle ABCD whose length and breadth are 12 cm and 8 cm, respectively. What is its perimeter?

Perimeter of the rectangle = Sum of the lengths of its four sides

= AB + BC + CD + DA



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= AB + BC + AB + BC

= 2 × AB + 2 × BC

= 2 × (AB + BC)

= 2 × (12 cm + 8 cm)

= 2 × (20 cm)

= 40 cm.

Opposite sides of a rectangle are always equal. So, AB = CD and AD = BC

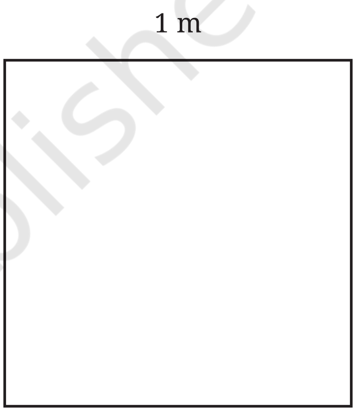
From this example, we see that—

Perimeter of a rectangle = length + breadth + length + breadth.

Perimeter of a rectangle = 2 × (length + breadth).

The perimeter of a rectangle is twice the sum of its length and breadth.

Perimeter of a Square



Debojeet wants to put coloured tape all around a square photo frame of side 1m as shown. What will be the length of the coloured tape he requires? Since Debojeet wants to put the coloured tape all around the square photo frame, he needs to find the perimeter of the photo frame.

Thus, the length of the tape required = perimeter of the square

= sum of the lengths of all four sides of the square

= 1 m + 1 m + 1 m + 1 m = 4 m.

Now, we know that all four sides of a square are equal in length. Therefore, in place of adding the lengths of each side, we can simply multiply the length of one side by 4.

Thus, the length of the tape required = 4 × 1 m = 4 m.

From this example, we see that

Perimeter of a square = 4 × length of a side.

The perimeter of a square is quadruple the length of its side.

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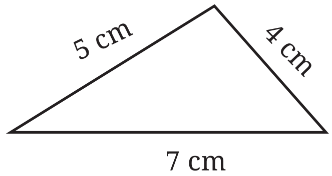
Perimeter of a Triangle



Consider a triangle having three given sides of lengths 4 cm, 5 cm and 7 cm. Find its perimeter.

Perimeter of the triangle = 4 cm + 5 cm + 7 cm

= 16 cm.

Perimeter of a triangle = sum of the lengths of its three sides.

Example: Akshi wants to put lace all around a rectangular tablecloth that is 3 m long and 2 m wide. Find the length of the lace required.

Solution

Length of the rectangular table cover = 3 m.

Breadth of the rectangular table cover = 2 m.

Akshi wants to put lace all around the tablecloth.

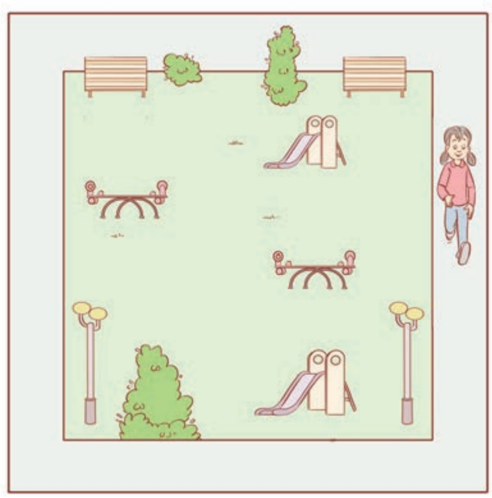
Therefore, the length of the lace required will be the perimeter of the rectangular tablecloth.

Now, the perimeter of the rectangular tablecloth = 2 × (length + breadth)

= 2 × (3 m + 2 m) = 2 × 5 m = 10 m.

Hence, the length of the lace required is 10 m.

Example: Find the distance travelled by Usha if she takes three rounds of a square park of side 75 m.



Solution

Perimeter of the square park = 4 × length of a side = 4 × 75 m = 300 m.

Distance covered by Usha in one round = 300 m.

Therefore, the total distance travelled by Usha in three rounds = 3 × 300 m = 900 m.

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Figure it Out

1. Find the missing terms:
   a. Perimeter of a rectangle = 14 cm; breadth = 2 cm; length = ?.
   b. Perimeter of a square = 20 cm; side of a length = ?.
   c. Perimeter of a rectangle = 12 m; length = 3 m; breadth = ?.
2. A rectangle having sidelengths 5 cm and 3 cm is made using a piece of wire. If the wire is straightened and then bent to form a square, what will be the length of a side of the square?
3. Find the length of the third side of a triangle having a perimeter of 55 cm and having two sides of length 20 cm and 14 cm, respectively.
4. What would be the cost of fencing a rectangular park whose length is 150 m and breadth is 120 m, if the fence costs ₹40 per metre?
5. A piece of string is 36 cm long. What will be the length of each side, if it is used to form:
   a. A square,
   b. A triangle with all sides of equal length, and
   c. A hexagon (a six sided closed figure) with sides of equal length?
6. A farmer has a rectangular field having length 230 m and breadth 160 m. He wants to fence it with 3 rounds of rope as shown. What is the total length of rope needed?

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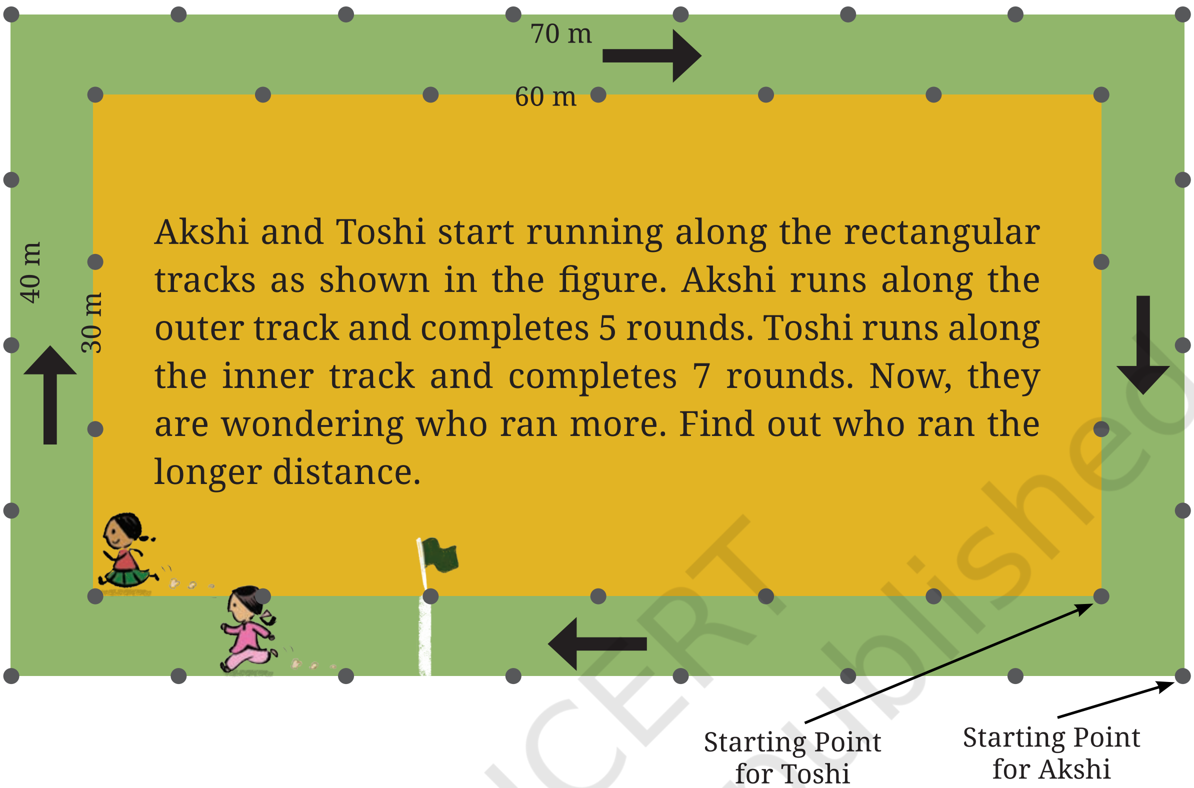
Matha Pachchi!



Each track is a rectangle. Akshi's track has length 70 m and breadth 40 m. Running one complete round on this track would cover 220 m, i.e., 2 × (70 + 40) m = 220 m. This is the distance covered by Akshi in one round.

Figure it Out

1. Find out the total distance Akshi has covered in 5 rounds.
2. Find out the total distance Toshi has covered in 7 rounds. Who ran a longer distance?
3. Think and mark the positions as directed—
   a. Mark 'A' at the point where Akshi will be after she ran 250 m.
   b. Mark 'B' at the point where Akshi will be after she ran 500 m.
   c. Now, Akshi ran 1000 m. How many full rounds has she finished running around her track? Mark her position as 'C'.
   d. Mark 'X' at the point where Toshi will be after she ran 250 m.
   e. Mark 'Y' at the point where Toshi will be after she ran 500 m.

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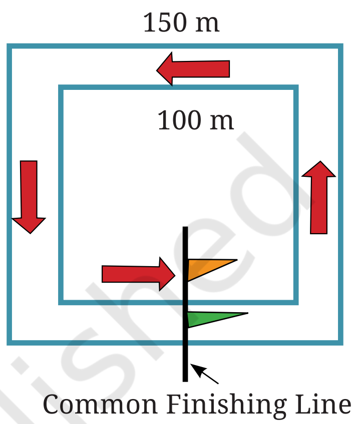
f. Now, Toshi ran 1000 m. How many full rounds has she finished running around her track? Mark her position as 'Z'.

Deep Dive: In races, usually there is a common finish line for all the runners. Here are two square running tracks with the inner track of 100 m each side and outer track of 150 m each side. The common finishing line for both runners is shown by the flags in the figure which are in the center of one of the sides of the tracks.

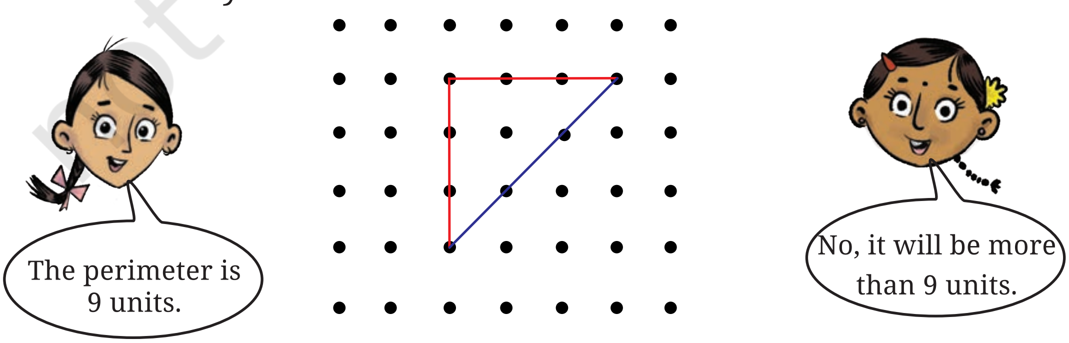


If the total race is of 350 m, then we have to find out where the starting positions of the two runners should be on these two tracks so that they both have a common finishing line after they run for 350 m. Mark the starting points of the runner on the inner track as 'A' and the runner on the outer track as 'B'.

Estimate and Verify

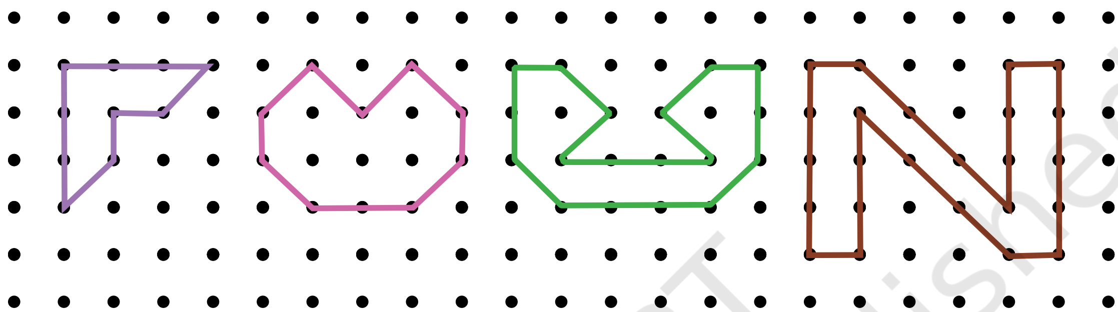
Take a rough sheet of paper or a sheet of newspaper. Make a few random shapes by cutting the paper in different ways. Estimate the total length of the boundaries of each shape then use a scale or measuring tape to measure and verify the perimeter for each shape.

Akshi says that the perimeter of this triangle shape is 9 units. Toshi says it can't be 9 units and the perimeter will be more than 9 units. What do you think?



This figure has lines of two different unit lengths. Measure the lengths of a red line and a blue line; are they same? We will call the red lines–straight lines and the blue lines–diagonal lines. So, the perimeter of this triangle is 6 straight units + 3 diagonal units. We can write this in a short form as: 6s + 3d units.

Write the perimeters of the figures below in terms of straight and diagonal units.



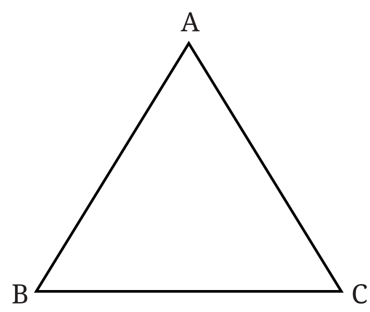
Perimeter of a Regular Polygon

Like squares, closed figures that have all sides and all angles equal are called **regular polygons**. We studied the sequence of regular polygons as 'Shape Sequence' #1 in Chapter 1. Examples of regular polygons are the equilateral triangle (where all three sides and all three angles are equal), regular pentagon (where all five sides and all five angles are equal), etc.

Perimeter of an equilateral triangle

We know that for any triangle its perimeter is sum of all three sides.

Using this understanding, we can find the perimeter of an equilateral triangle.



Perimeter of an equilateral triangle

= AB + BC + AC = AB + AB + AB

= 3 times length of one side.

Perimeter of an equilateral triangle = 3 × length of a side.

What is a similarity between a square and an equilateral triangle?

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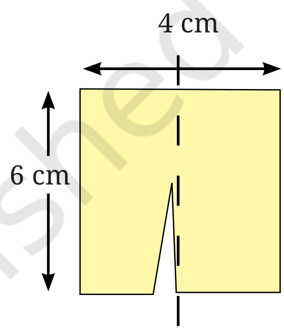
Find various objects from your surroundings that have regular shapes and find their perimeters. Also, generalise your understanding for the perimeter of other regular polygons.

Teacher's Note

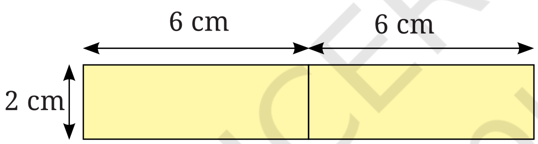
Discuss more about regular polygons and encourage students to come up with a general formula for the perimeter of a regular polygon.

Split and Rejoin

A rectangular paper chit of dimension 6 cm × 4 cm is cut as shown into two equal pieces. These two pieces are joined in different ways.



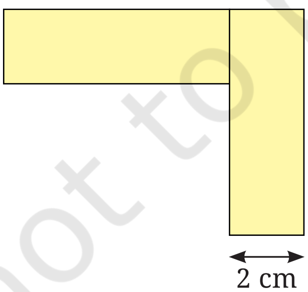
a.



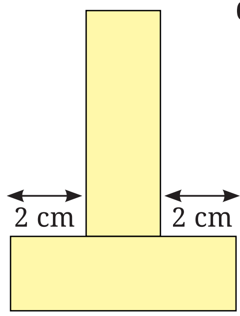
For example, the arrangement a. has a perimeter of 28 cm.

Find out the length of the boundary (i.e., the perimeter) of each of the other arrangements below.

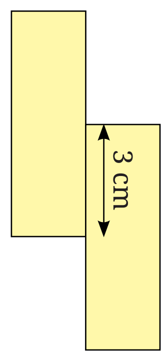
b.



c.



d.



Arrange the two pieces to form a figure with a perimeter of 22 cm.

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6.2 Area

We have studied the areas of closed figures (regular and irregular) in previous grades. Let us recall some key points.

The amount of region enclosed by a closed figure is called its **area**.

In previous grades, we arrived at the formula for the area of a rectangle and a square using square grid paper. Do you remember?

Area of a square = ________

Area of a rectangle = _______

Teacher's Note

Help students in recalling the method of finding the area of a rectangle and a square using grid papers. Provide square grid papers to students and let them come up with the formula.

Let's see some real-life problems related to these ideas.

Example: A floor is 5 m long and 4 m wide. A square carpet of sides 3 m is laid on the floor. Find the area of the floor that is not carpeted.

Solution

Length of the floor = 5 m.

Width of the floor = 4 m.

Area of the floor = length × width = 5 m × 4 m = 20 sq m.

Length of the square carpet = 3 m.

Area of the carpet = length × length = 3 m × 3 m = 9 sq m.

Hence, the area of the floor laid with carpet is 9 sq m.

Therefore, the area of the floor that is not carpeted is: area of the floor minus the area of the floor laid with carpet = 20 sq m – 9 sq m = 11 sq m.

Example: Four square flower beds each of side 4 m are in four corners on a piece of land 12 m long and 10 m wide. Find the area of the remaining part of the land.

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Solution

Length of the land (l) = 12 m.

Width of land (w) = 10 m.

Area of the whole land = $l \times w$ = 12 m × 10 m = 120 sq m.

The sidelength of each of the four square flower beds is (s) = 4 m.

Area of one flower bed = $s \times s$ = 4 m × 4 m = 16 sq m.

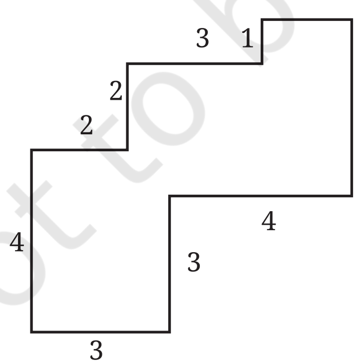
Hence, the area of the four flower beds = 4 × 16 sq m = 64 sq m.

Therefore, the area of the remaining part of the land is: area of the complete land minus the area of all four flower beds = 120 sq m – 64 sq m = 56 sq m.

Figure it Out

1. The area of a rectangular garden 25 m long is 300 sq m. What is the width of the garden?
2. What is the cost of tiling a rectangular plot of land 500 m long and 200 m wide at the rate of ₹8 per hundred sq m?
3. A rectangular coconut grove is 100 m long and 50 m wide. If each coconut tree requires 25 sq m, what is the maximum number of trees that can be planted in this grove?
4. By splitting the following figures into rectangles, find their areas (all measures are given in metres):

a.



b.

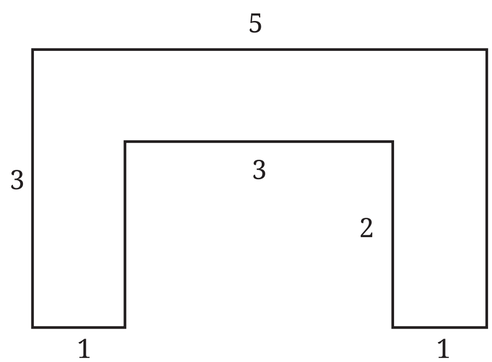


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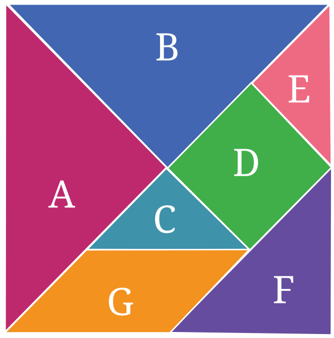
Figure it Out

Cut out the tangram pieces given at the end of your textbook.



1. Explore and figure out how many pieces have the same area.
2. How many times bigger is Shape D as compared to Shape C? What is the relationship between Shapes C, D and E?
3. Which shape has more area: Shape D or F? Give reasons for your answer.
4. Which shape has more area: Shape F or G? Give reasons for your answer.
5. What is the area of Shape A as compared to Shape G? Is it twice as big? Four times as big?

   Hint: In the tangram pieces, by placing the shapes over each other, we can find out that Shapes A and B have the same area, Shapes C and E have the same area. You would have also figured out that Shape D can be exactly covered using Shapes C and E, which means Shape D has twice the area of Shape C or shape E, etc.
6. Can you now figure out the area of the big square formed with all seven pieces in terms of the area of Shape C?
7. Arrange these 7 pieces to form a rectangle. What will be the area of this rectangle in terms of the area of Shape C now? Give reasons for your answer.
8. Are the perimeters of the square and the rectangle formed from these 7 pieces different or the same? Give an explanation for your answer.

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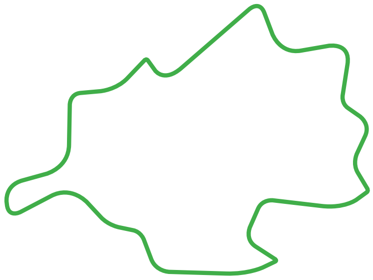
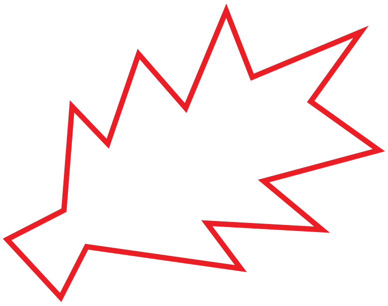
Look at the figures below and guess which one of them has a larger area.



a.



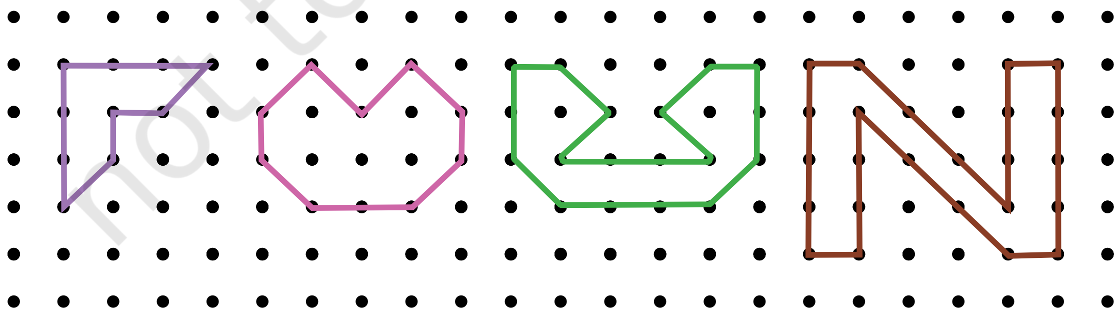
b.

We can estimate the area of any simple closed shape by using a sheet of squared paper or graph paper where every square measures 1 unit × 1 unit or 1 square unit.

To estimate the area, we can trace the shape onto a piece of transparent paper and place the same on a piece of squared or graph paper and then follow the below conventions—

1. The area of one full small square of the squared or graph paper is taken as 1 sq unit.
2. Ignore portions of the area that are less than half a square.
3. If more than half of a square is in a region, just count it as 1 sq unit.
4. If exactly half the square is counted, take its area as $\frac{1}{2}$ sq unit.

Find the area of the following figures.



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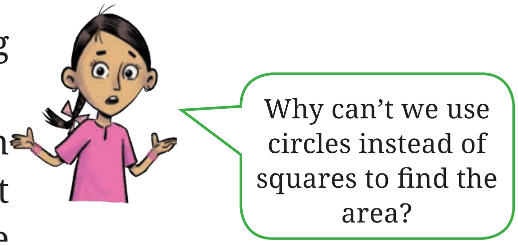
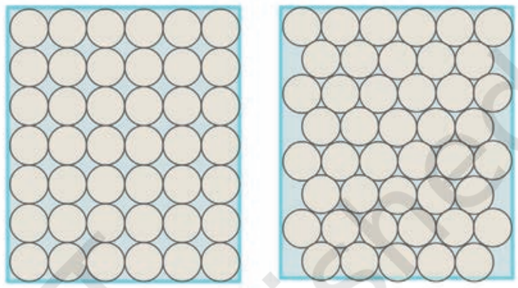
Let's Explore!

Why is area generally measured using squares?



Draw a circle on a graph sheet with diameter (breadth) of length 3. Count the squares and use them to estimate the area of the circular region.



As you can see, circles can't be packed tightly without gaps in between. So, it is difficult to get an accurate measurement of area using circles as units. Here, the same rectangle is packed in two different ways with circles—the first one has 42 circles and the second one has 44 circles.

Try using different shapes (triangle and rectangle) to fill the given space (without overlaps and gaps) and find out the merits associated with using a square shape to find the area rather than another shape. List out the points that make a square the best shape to use to measure area.

1. Find the area (in square metres) of the floor outside of the corridor.
2. Find the area (in square metres) occupied by your school playground.

Let's Explore!

On a squared grid paper (1 square = 1 square unit), make as many rectangles as you can whose lengths and widths are a whole number of units such that the area of the rectangle is 24 square units.

Math Talk

a. Which rectangle has the greatest perimeter?

b. Which rectangle has the least perimeter?

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c. If you take a rectangle of area 32 sq cm, what will your answers be? Given any area, is it possible to predict the shape of the rectangle with the greatest perimeter as well as the least perimeter? Give examples and reasons for your answer.

6.3 Area of a Triangle

Draw a rectangle on a piece of paper and draw one of its diagonals. Cut the rectangle along that diagonal and get two triangles.

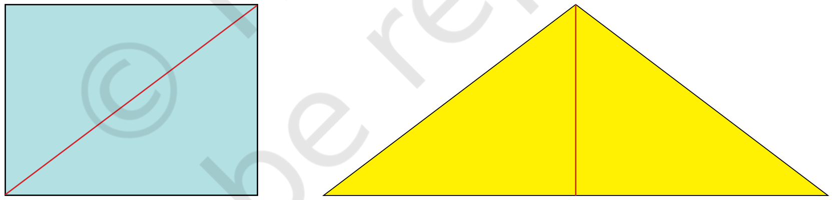
Check! whether the two triangles overlap each other exactly. Do they have the same area?

Try this with more rectangles having different dimensions. You can check this for a square as well.

Can you draw any inferences from this exercise? Please write it here. ______________________________

____

Now, see the figures below. Is the area of the blue rectangle more or less than the area of the yellow triangle? Or is it the same? Why?



Can you see some relationship between the blue rectangle and the yellow triangle and their areas? Write the relationship here.

____

Teacher's Note

Help students in articulating their inferences and in defining the relationships they have observed in their own words, gradually leading to a common statement for whole classroom. Recall the definition of a diagonal in the classroom.

Draw suitable triangles on grid paper to verify your inferences and relationships observed in the above exercises.

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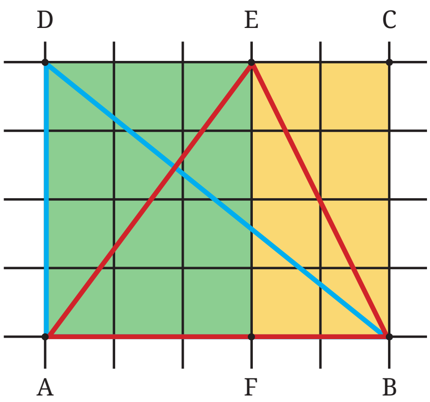
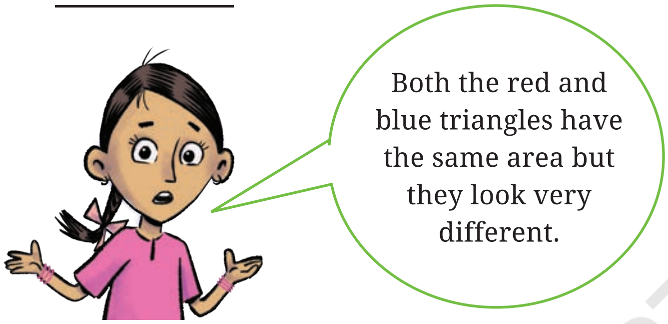
Use your understanding from previous grades to calculate the area of any closed figure using grid paper and—

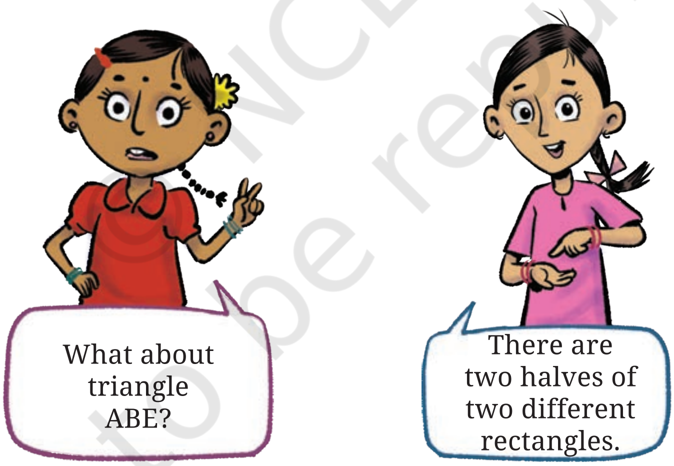


1. Find the area of blue triangle BAD. ________
2. Find the area of red triangle ABE. ________



Area of rectangle ABCD = ______________

So, the area of triangle BAD is half of the area of the rectangle ABCD.



Area of triangle ABE = Area of triangle AEF + Area of triangle BEF.

Here, the area of triangle AEF = half of the area of rectangle AFED.
Similarly, the area of triangle BEF = half of the area of rectangle BFEC.

Thus, the area of triangle ABE = half of the area of rectangle AFED + half of the area of rectangle BFEC

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= half of the sum of the areas of the rectangles AFED and BFEC

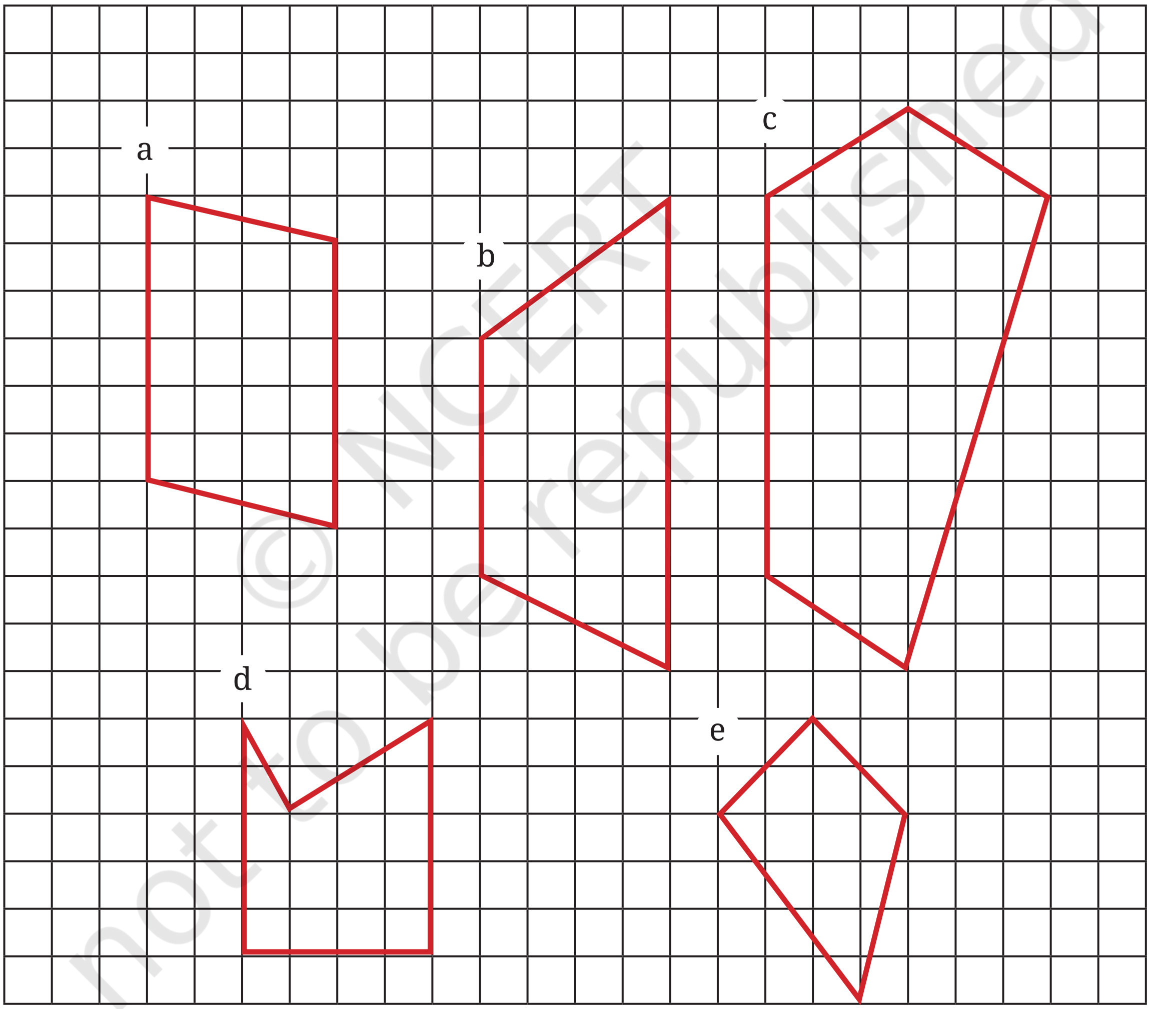
= half of the area of rectangle ABCD.

Conclusion ____

____

Figure it Out

1. Find the areas of the figures below by dividing them into rectangles and triangles.



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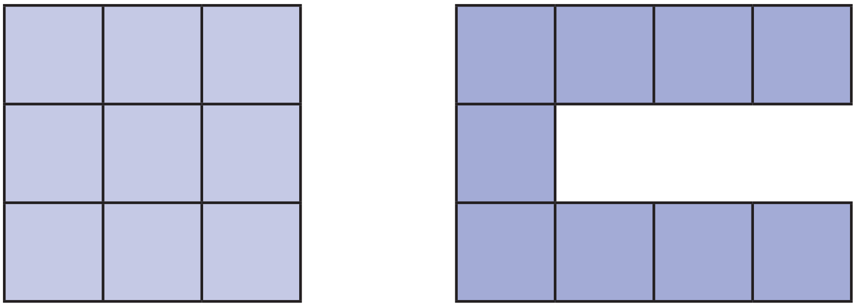
Making it 'More' or 'Less'

Observe these two figures. Is there any similarity or difference between the two?



Using 9 unit squares (having an area of 9 sq units), we have made figures with two different perimeters—the first figure has a perimeter of 12 units and the second has a perimeter of 20 units.

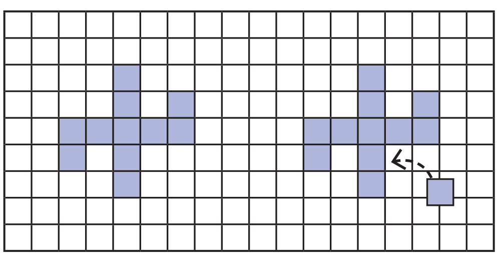
Arrange or draw different figures with 9 sq units to get other perimeters. Each square should align with at least one other square on at least one side completely and together all squares should form a single connected figure with no holes.

Using 9 unit squares, solve the following.

1. What is the smallest perimeter possible?
2. What is the largest perimeter possible?
3. Make a figure with a perimeter of 18 units.
4. Can you make other shaped figures for each of the above three perimeters, or is there only one shape with that perimeter? What is your reasoning?

Let's do something tricky now! We have a figure below having perimeter 24 units.

Without calculating all over again, observe, think and find out what will be the change in the perimeter if a new square is attached as shown on the right.



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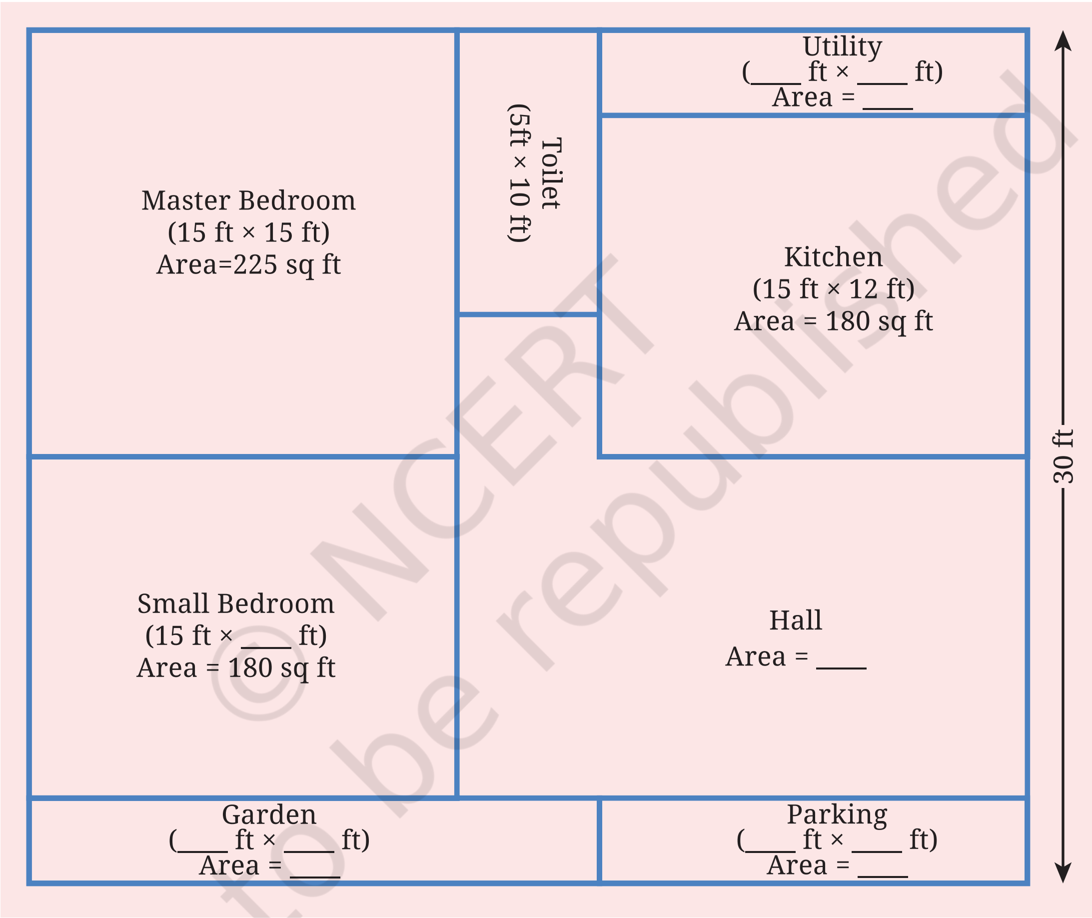
Experiment placing this new square at different places and think what the change in perimeter will be. Can you place the square so that the perimeter: a) increases; b) decreases; c) stays the same?

Below is the house plan of Charan. It is in a rectangular plot. Look at the plan. What do you notice?



Some of the measurements are given.

a. Find the missing measurements.

b. Find out the area of his house.

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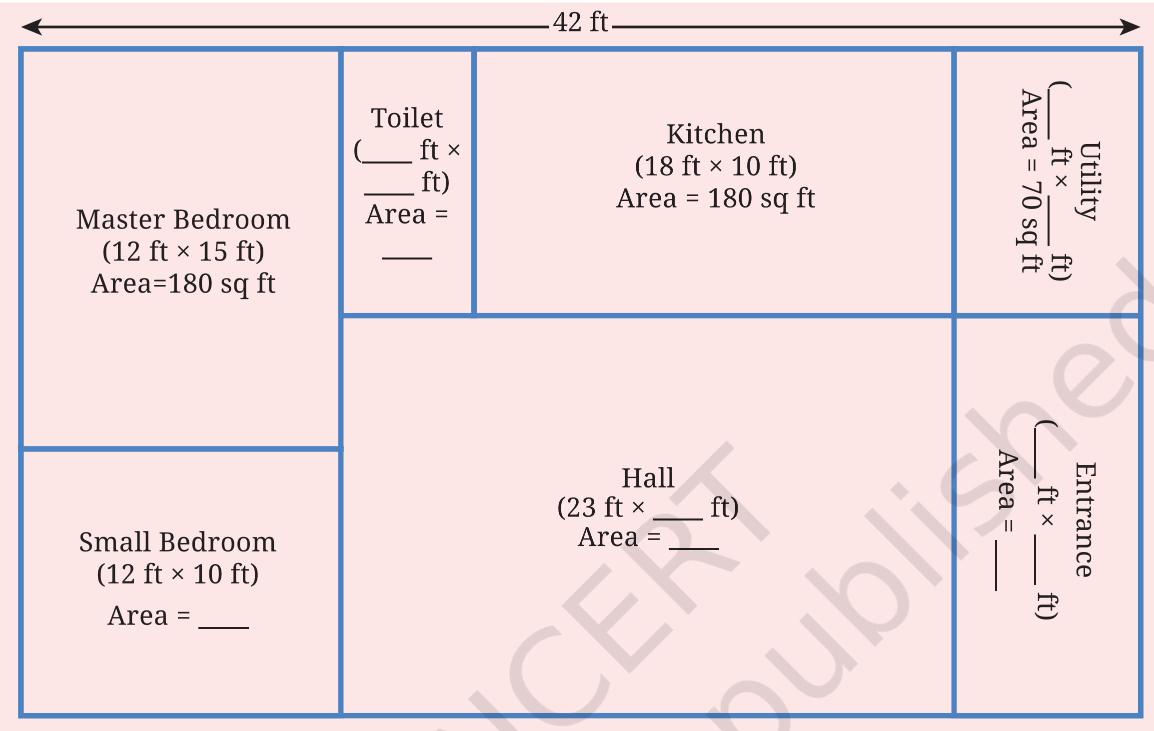
Now, find out the missing dimensions and area of Sharan's home. Below is the plan:



Some of the measurements are given.

a. Find the missing measurements.

b. Find out the area of his house.

What are the dimensions of all the different rooms in Sharan's house? Compare the areas and perimeters of Sharan's house and Charan's house.

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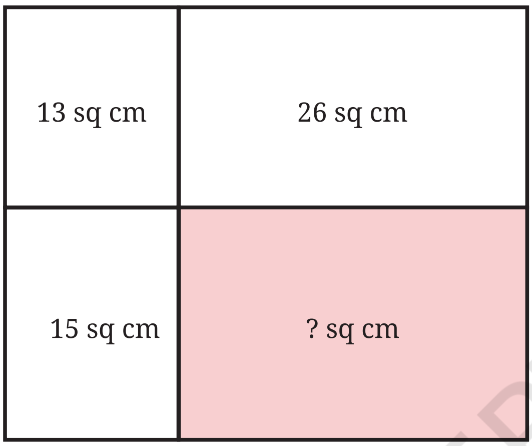
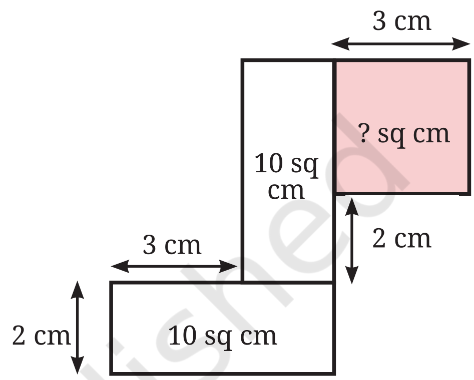
Area Maze Puzzles

In each figure, find the missing value of either the length of a side or the area of a region.

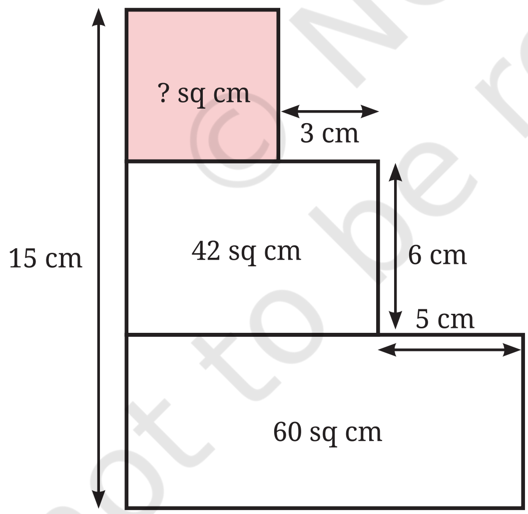
a.



b.



c.



d.

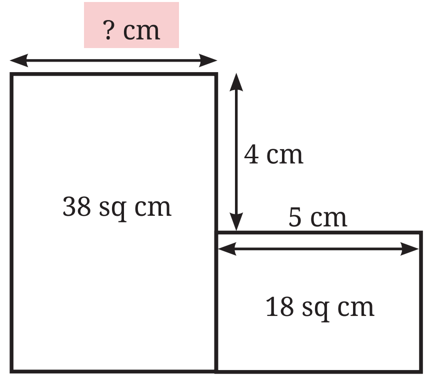


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Figure it Out

1. Give the dimensions of a rectangle whose area is the sum of the areas of these two rectangles having measurements: 5 m × 10 m and 2 m × 7 m.
2. The area of a rectangular garden that is 50 m long is 1000 sq m. Find the width of the garden.
3. The floor of a room is 5 m long and 4 m wide. A square carpet whose sides are 3 m in length is laid on the floor. Find the area that is not carpeted.
4. Four flower beds having sides 2 m long and 1 m wide are dug at the four corners of a garden that is 15 m long and 12 m wide. How much area is now available for laying down a lawn?
5. Shape A has an area of 18 square units and Shape B has an area of 20 square units. Shape A has a longer perimeter than Shape B. Draw two such shapes satisfying the given conditions.
6. On a page in your book, draw a rectangular border that is 1 cm from the top and bottom and 1.5 cm from the left and right sides. What is the perimeter of the border?
7. Draw a rectangle of size 12 units × 8 units. Draw another rectangle inside it, without touching the outer rectangle that occupies exactly half the area.
8. A square piece of paper is folded in half. The square is then cut into two rectangles along the fold. Regardless of the size of the square, one of the following statements is always true. Which statement is true here?

a. The area of each rectangle is larger than the area of the square.

b. The perimeter of the square is greater than the perimeters of both the rectangles added together.

c. The perimeters of both the rectangles added together is always $1\frac{1}{2}$ times the perimeter of the square.

d. The area of the square is always three times as large as the areas of both rectangles added together.

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SUMMARY

- The perimeter of a polygon is the sum of the lengths of all its sides.
  a. The perimeter of a rectangle is twice the sum of its length and width.
  b. The perimeter of a square is four times the length of any one of its sides.
- The area of a closed figure is the measure of the region enclosed by the figure.
- Area is generally measured in square units.
- The area of a rectangle is its length times its width. The area of a square is the length of any one of its sides multiplied by itself.
- Two closed figures can have the same area with different perimeters, or the same perimeter with different areas.
- Areas of regions can be estimated (or even determined exactly) by breaking up such regions into unit squares, or into more general-shaped rectangles and triangles whose areas can be calculated.

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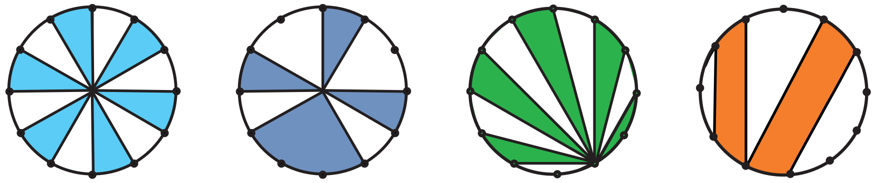
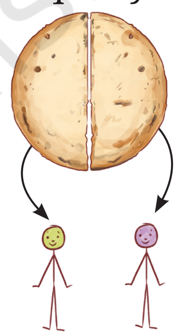


7 FRACTIONS

0674CH07

Recall that when some whole number of things are shared equally among some number of people, fractions tell us how much each share is.

Shabnam: Do you remember, if one *roti* is divided equally between two children, how much *roti* will each child get?



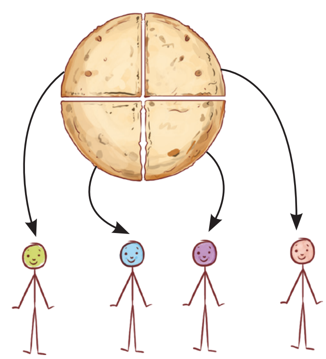
Mukta: Each child will get half a *roti*.

Shabnam: The fraction 'one half' is written as $\frac{1}{2}$. We also sometimes read this as 'one upon two.'

Mukta: If one roti is equally shared among 4 children, how much roti will one child get?

Shabnam: Each child's share is $\frac{1}{4}$ *roti*.

Mukta: And which is more $\frac{1}{2}$ *roti* or $\frac{1}{4}$ *roti*?



Shabnam: When 2 children share 1 *roti* equally, each child gets $\frac{1}{2}$ *roti*. When 4 children share 1 *roti* equally, each child gets $\frac{1}{4}$ *roti*. Since, in the second group more children share the

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same one *roti*, each child gets a smaller share. So, $\frac{1}{2}$ *roti* is more than $\frac{1}{4}$ *roti*.

$\frac{1}{2} > \frac{1}{4}$

7.1 Fractional Units and Equal Shares

Beni: Which fraction is greater? $\frac{1}{5}$ or $\frac{1}{9}$?

Arvin: 9 is bigger than 5. So I would guess that $\frac{1}{9}$ is greater than $\frac{1}{5}$. Am I right?

Beni: No! That is a common mistake. Think of these fractions as shares.

Arvin: If one roti is shared among 5 children, each one gets a share of $\frac{1}{5}$ roti. If one roti is shared among 9 children, each one gets a share of $\frac{1}{9}$ roti?

Beni: Exactly! Now think again - which share is higher?

Arvin: If I share with more people, I will get less. So $\frac{1}{9} < \frac{1}{5}$.

Beni: You got it!

Oh, so $\frac{1}{100}$ is bigger than $\frac{1}{200}$!

When one unit is divided into several equal parts, each part is called a **fractional unit**. These are all fractional units:

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{10}, \ldots, \frac{1}{50}, \ldots, \frac{1}{100},$ etc.

We also sometimes refer to fractional units as '**unit fractions**.'

Figure it Out

Fill in the blanks with fractions.

1. Three guavas together weigh 1 kg. If they are roughly of the same size, each guava will roughly weigh ____kg.
2. A wholesale merchant packed 1 kg of rice in four packets of equal weight. The weight of each packet is ___ kg.
3. Four friends ordered 3 glasses of sugarcane juice and shared it equally among themselves. Each one drank ____ glass of sugarcane juice

Math Talk

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4. The big fish weighs $\frac{1}{2}$ kg. The small one weighs $\frac{1}{4}$ kg. Together they weigh ____ kg.

Math Talk

Knowledge from the past!

Fractions have been used and named in India since ancient times. In the *Rig Veda*, the fraction $\frac{3}{4}$ is referred to as *tri-pada*. This has the same meaning as the words for $\frac{3}{4}$ in many Indian languages today, e.g., '*teen paav*' in colloquial Hindi and '*mukkaal*' in Tamil. Indeed, words for fractions used today in many Indian languages go back to ancient times.

Find out and discuss the words for fractions that are used in the different languages spoken in your home, city, or state. Ask your grandparents, parents, teachers, and classmates what words they use for different fractions, such as for one and a half, three quarters, one and a quarter, half, quarter, and two and a half, and write them here:

__________ __________ __________ __________ __________ __________

5. Arrange these fraction words in order of size from the smallest to the biggest in the empty box below:
   One and a half, three quarters, one and a quarter, half, quarter, two and a half.

Write your answer here.

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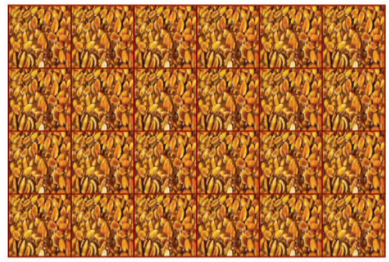
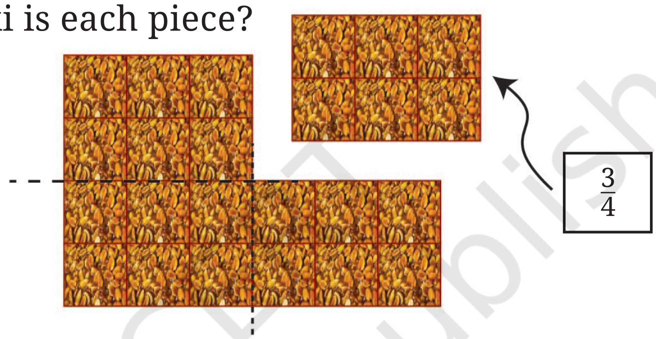
7.2 Fractional Units as Parts of a Whole

The picture shows a whole *chikki.*

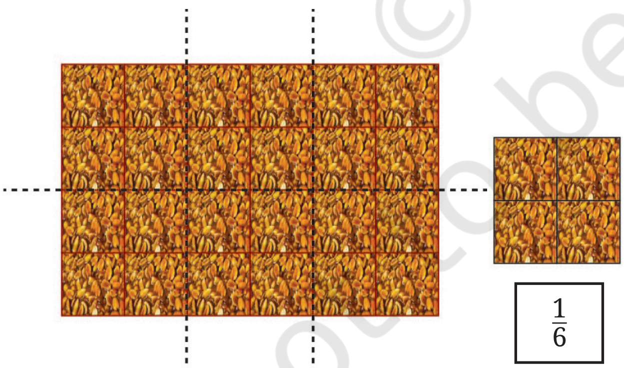


A whole chikki

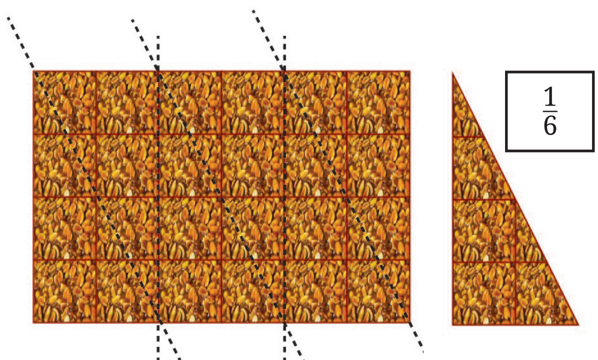
Below, a picture of the chikki broken into 2 pieces is shown. How much of the original chikki is each piece?



We can see that the bigger piece has 3 pieces of $\frac{1}{4}$ chikki in it. So, we can measure the bigger piece using the fractional unit $\frac{1}{4}$. We see that the bigger piece is $\frac{3}{4}$ chikki.



A whole chikki cut into 6 equal pieces.



A whole chikki cut into 6 equal pieces in a different way.

By dividing the whole chikki into 6 equal parts in different ways, we get $\frac{1}{6}$ chikki pieces of different shapes. Are they of the same size?

Math Talk

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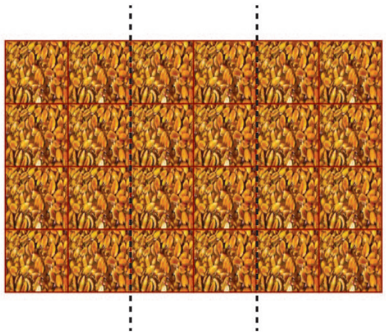
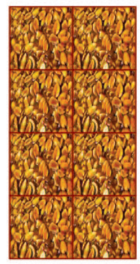
What is the fractional unit of chikki shown below?



A whole chikki



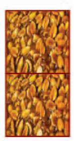
$\boxed{\frac{1}{3}}$

We get this piece by breaking the chikki into 3 equal pieces. So this is $\frac{1}{3}$ chikki.

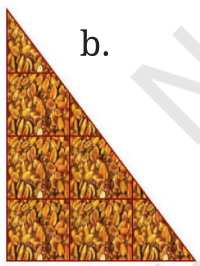
Figure it Out

The figures below show different fractional units of a whole chikki. How much of a whole chikki is each piece?

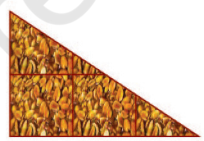
a.



b.



c.



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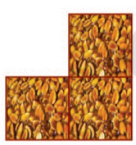
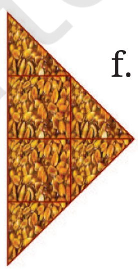


f.



g.

h.

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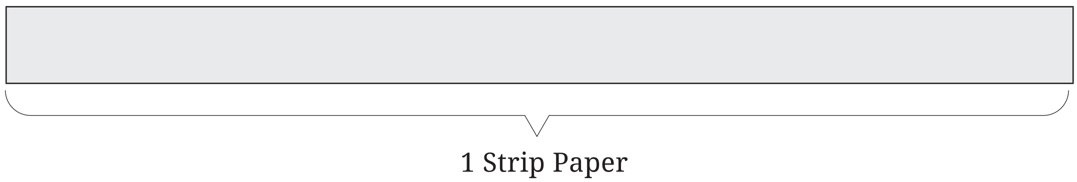
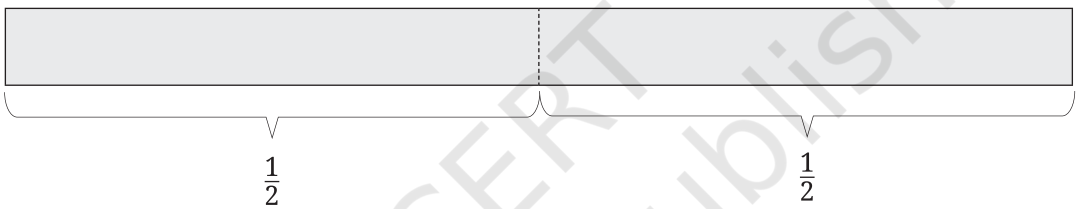
7.3 Measuring Using Fractional Units

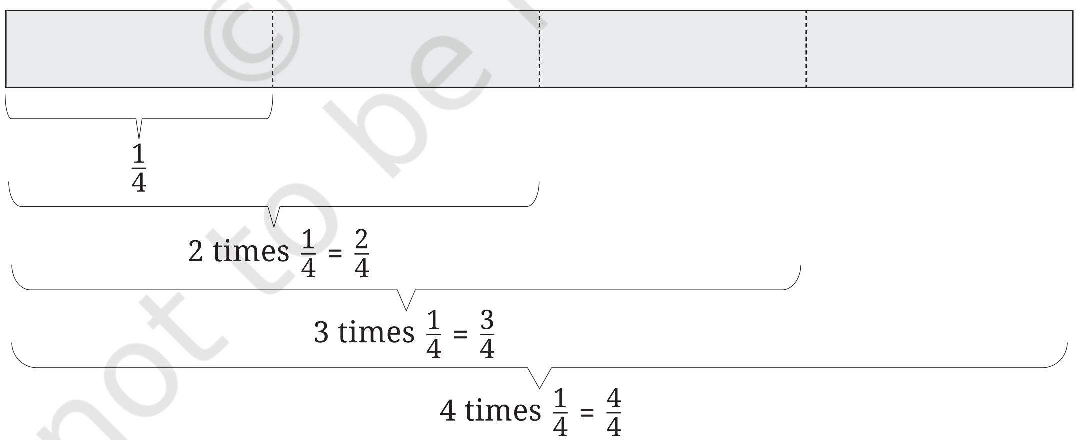
Take a strip of paper. We consider this paper strip to be one unit long.



Fold the strip into two equal parts and then open up the strip again. Taking the strip to be one unit in length, what are the lengths of the two new parts of the strip created by the crease?



What will you get if you fold the previously-folded strip again into two equal parts? You will now get four equal parts.



Do it once more! Fill in the blank boxes.

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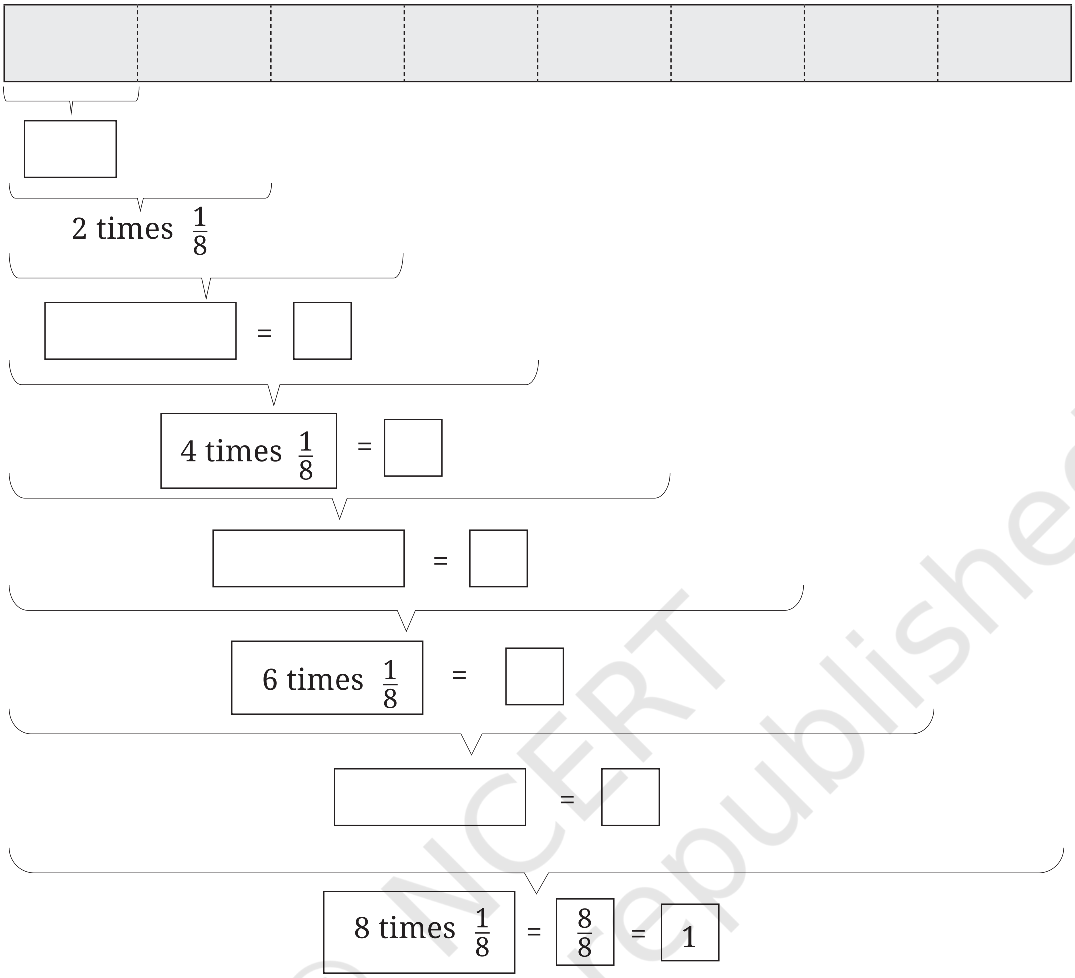
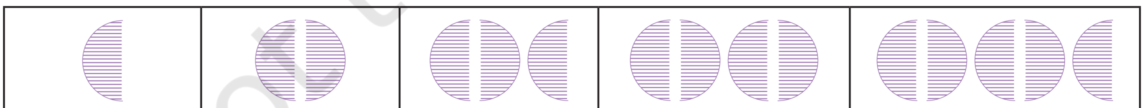


Fractional quantities can be measured using fractional units.

Let us look at another example,


Represents a full *roti* (whole)



$\frac{1}{2}$ = 1 times half	$\frac{1}{2}+\frac{1}{2}$ = 2 times half	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ = 3 times half	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ = 4 times half	$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ = 5 times half

We can describe how much the quantity is by collecting together the fractional units.

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Figure it Out

1. Continue this table of $\frac{1}{2}$ for 2 more steps.
2. Can you create a similar table for $\frac{1}{4}$?
3. Make $\frac{1}{3}$ using a paper strip. Can you use this to also make $\frac{1}{6}$?
4. Draw a picture and write an addition statement as above to show:

   a. 5 times $\frac{1}{4}$ of a *roti* b. 9 times $\frac{1}{4}$ of a *roti*

5. Match each fractional unit with the correct picture:

$\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{8}$ $\frac{1}{6}$

Reading Fractions

We usually read the fraction $\frac{3}{4}$ as 'three quarters' or 'three upon four', but reading it as '3 times $\frac{1}{4}$' helps us to understand the size of the fraction because it clearly shows what the fractional unit is ($\frac{1}{4}$) and how many such fractional units (3) there are.

Recall what we call the top number and the bottom number of fractions. In the fraction $\frac{5}{6}$, 5 is the **numerator** and 6 is the **denominator.**

Teacher's Note

Give several opportunities to the children to explore the idea of fractional units with different shapes like circles, squares, rectangles, triangles, etc.

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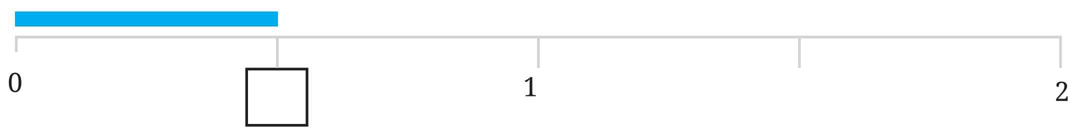
7.4 Marking Fraction Lengths on the Number Line

We have marked lengths equal to 1, 2, 3, ... units on the number line. Now, let us try to mark lengths equal to fractions on the number line.

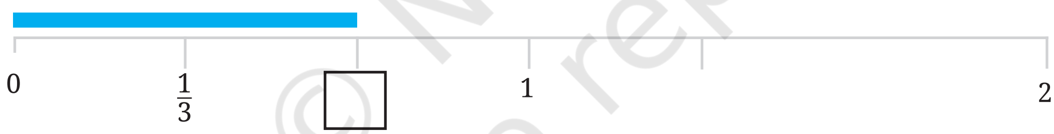
What is the length of the blue line? Write the fraction that gives the length of the blue line in the box?



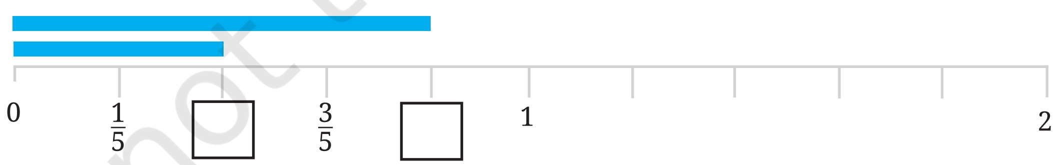
The distance between 0 and 1 is one unit long. It is divided into two equal parts. So, the length of each part is $\frac{1}{2}$ unit. So, this blue line is $\frac{1}{2}$ unit long.

Now, can you find the lengths of the various blue lines shown below? Fill in the boxes as well.

1. Here, the fractional unit is dividing a length of 1 unit into three equal parts. Write the fraction that gives the length of the blue line in the box or in your notebook.



2. Here, a unit is divided into 5 equal parts. Write the fraction that gives the length of the blue lines in the respective boxes or in your notebook.



3. Now, a unit is divided into 8 equal parts. Write the appropriate fractions in your notebook.

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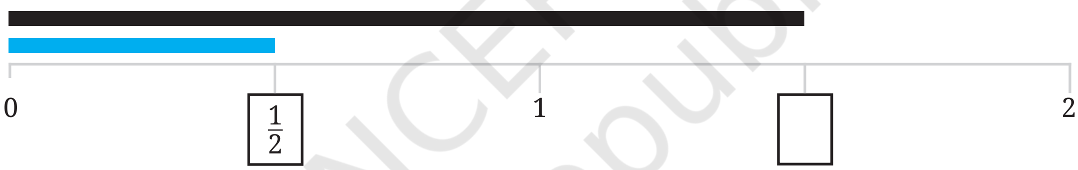
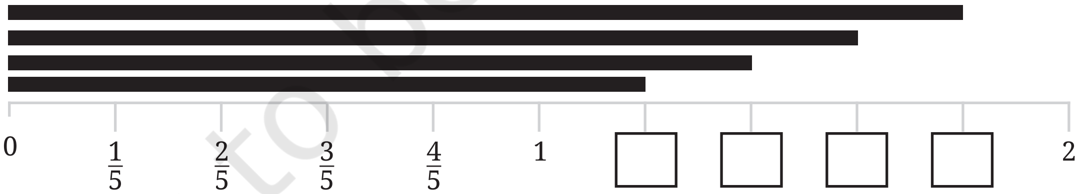
Figure it Out

Math Talk

1. On a number line, draw lines of lengths $\frac{1}{10}$, $\frac{3}{10}$, and $\frac{4}{5}$.
2. Write five more fractions of your choice and mark them on the number line.
3. How many fractions lie between 0 and 1? Think, discuss with your classmates, and write your answer.
4. What is the length of the blue line and black line shown below? The distance between 0 and 1 is 1 unit long, and it is divided into two equal parts. The length of each part is $\frac{1}{2}$. So the blue line is $\frac{1}{2}$ units long. Write the fraction that gives the length of the black line in the box.



5. Write the fraction that gives the lengths of the black lines in the respective boxes.



Teacher's Note

Draw these lines on the board and ask the students to write the answers in their notebooks.

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7.5 Mixed Fractions

Fractions greater than one

You marked some fractions on the number line earlier. Did you notice that the lengths of all the blue lines were less than one and the lengths of all the black lines were more than 1?

Write down all the fractions you marked on the number line earlier.

Now, let us classify these in two groups:

Lengths less than 1 unit	Lengths more than 1 unit

Did you notice something common between the fractions that are greater than 1?

In all the fractions that are less than 1 unit, the numerator is smaller than the denominator, while in the fractions that are more than 1 unit, the numerator is larger than the denominator.

We know that $\frac{3}{2}$, $\frac{5}{2}$ and $\frac{7}{2}$ are all greater than 1 unit. But can we see how many whole units they contain?

$$\frac{3}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1 + \frac{1}{2}$$

$$\frac{5}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2 + \frac{1}{2}$$

I know that $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$. If I add one more $\frac{1}{3}$,

I will get more than 1 unit! So, $\frac{4}{3} > 1$.

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Figure it Out

1. How many whole units are there in $\frac{7}{2}$?

2. How many whole units are there in $\frac{4}{3}$ and in $\frac{7}{3}$?

Math Talk

Writing fractions greater than one as mixed numbers

We saw that: $\frac{3}{2} = 1 + \frac{1}{2}$.

We can write other fractions in a similar way. For example,

$$\frac{4}{3} = \underbrace{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}_{3 \times \frac{1}{3} = 1} + \frac{1}{3} = 1 + \frac{1}{3} .$$

Figure it Out

1. Figure out the number of whole units in each of the following fractions:

   a. $\frac{8}{3}$ b. $\frac{11}{5}$ c. $\frac{9}{4}$

   We saw that

   $$\frac{8}{3} = 2 + \frac{2}{3}$$

   Fraction Mixed number

   This number is thus also called 'two and two thirds'. We also write it as $2\frac{2}{3}$.

2. Can all fractions greater than 1 be written as such mixed numbers?

A **mixed number / mixed fraction** contains a whole number (called the whole part) and a fraction that is less than 1 (called the fractional part).

3. Write the following fractions as mixed fractions (e.g., $\frac{9}{2} = 4\frac{1}{2}$):

   a. $\frac{9}{2}$ b. $\frac{9}{5}$ c. $\frac{21}{19}$ d. $\frac{47}{9}$ e. $\frac{12}{11}$ f. $\frac{19}{6}$

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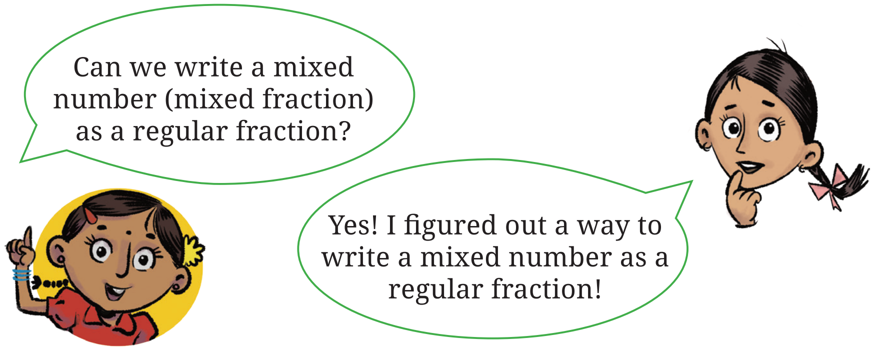

Jaya: When I have $3 + \frac{3}{4}$, this means $1 + 1 + 1 + \frac{3}{4}$. I know

$$1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}.$$

So I get

$$\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right) = \frac{15}{4}.$$

Therefore, $\left(4 \times \frac{1}{4}\right) + \left(4 \times \frac{1}{4}\right) + \left(4 \times \frac{1}{4}\right) + \left(3 \times \frac{1}{4}\right) = \frac{15}{4}$.

Figure it Out

Write the following mixed numbers as fractions:

a. $3\frac{1}{4}$ b. $7\frac{2}{3}$ c. $9\frac{4}{9}$

d. $3\frac{1}{6}$ e. $2\frac{3}{11}$ f. $3\frac{9}{10}$

Math Talk

7.6 Equivalent Fractions

Using a fraction wall to find equal fractional lengths!

In the previous section, you used paper folding to represent various fractions using fractional units. Let us do some more activities with the same paper strips.

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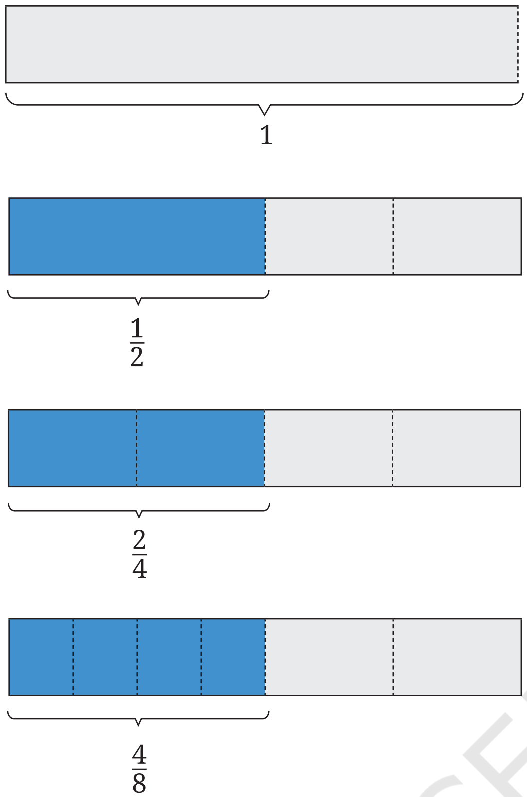


What do you observe?

- Are the lengths $\frac{1}{2}$ and $\frac{2}{4}$ equal?
- Are the lengths $\frac{2}{4}$ and $\frac{4}{8}$ equal?

We can say that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

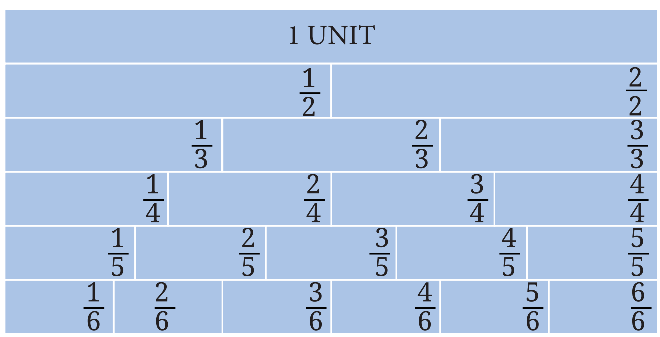
These are 'equivalent fractions' that denote the same length, but they are expressed in terms of different fractional units.

Now, check whether $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent fractions or not, using paper strips.

Make your own fraction wall using such strips as given in the picture below!

Answer the following questions after looking at the fraction wall:

1. Are the lengths $\frac{1}{2}$ and $\frac{3}{6}$ equal?
2. Are $\frac{2}{3}$ and $\frac{4}{6}$ equivalent fractions? Why?
3. How many pieces of length $\frac{1}{6}$ will make a length of $\frac{1}{2}$?



4. How many pieces of length $\frac{1}{6}$ will make a length of $\frac{1}{3}$?

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We can extend this idea to make a fraction wall up to the fractional unit $\frac{1}{10}$. (This fraction wall is given at the end of the book.)

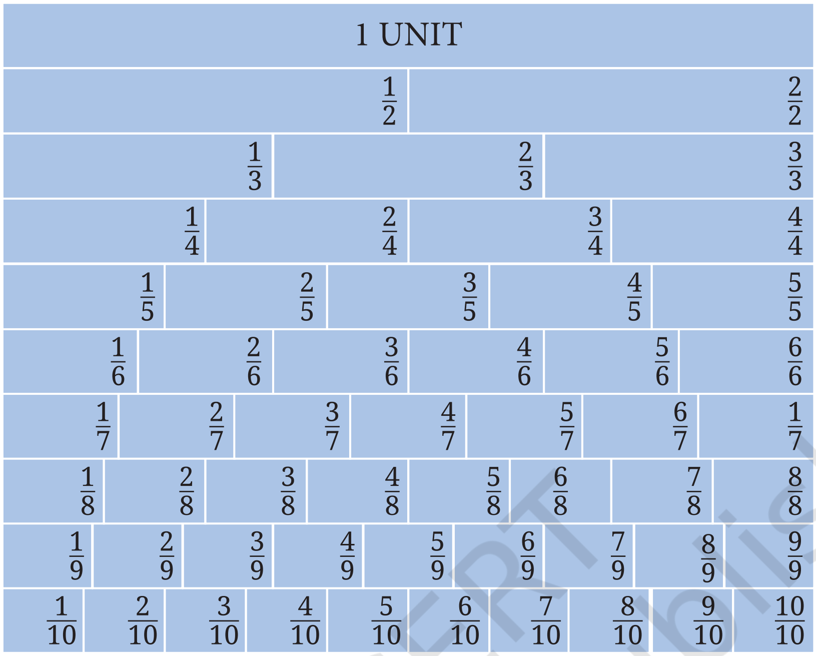


Figure it Out

1. Are $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$ equivalent fractions? Why?

2. Write two equivalent fractions for $\frac{2}{6}$.

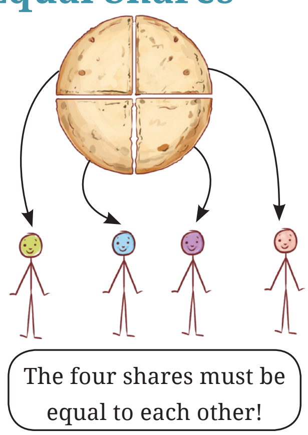
3. $\frac{4}{6} = \boxed{\frac{}{}} = \boxed{\frac{}{}} = \boxed{\frac{}{}} =$ .... (Write as many as you can)

Understanding Equivalent Fractions using Equal Shares

One *roti* was shared equally by four children. What fraction of the whole did each child get?

The adjoining picture shows the division of a *roti* among four children.

Fraction of *roti* each child got is $\frac{1}{4}$.



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You can also express this event through division facts, addition facts, and multiplication facts.

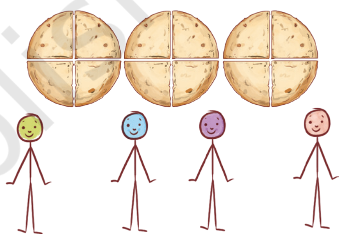
The division fact is $1 \div 4 = \frac{1}{4}$.

The addition fact is $1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.

The multiplication fact is $1 = 4 \times \frac{1}{4}$.

Figure it Out

1. Three *rotis* are shared equally by four children. Show the division in the picture and write a fraction for how much each child gets. Also, write the corresponding division facts, addition facts, and multiplication facts.

   Fraction of *roti* each child gets is ______.

   Division fact:

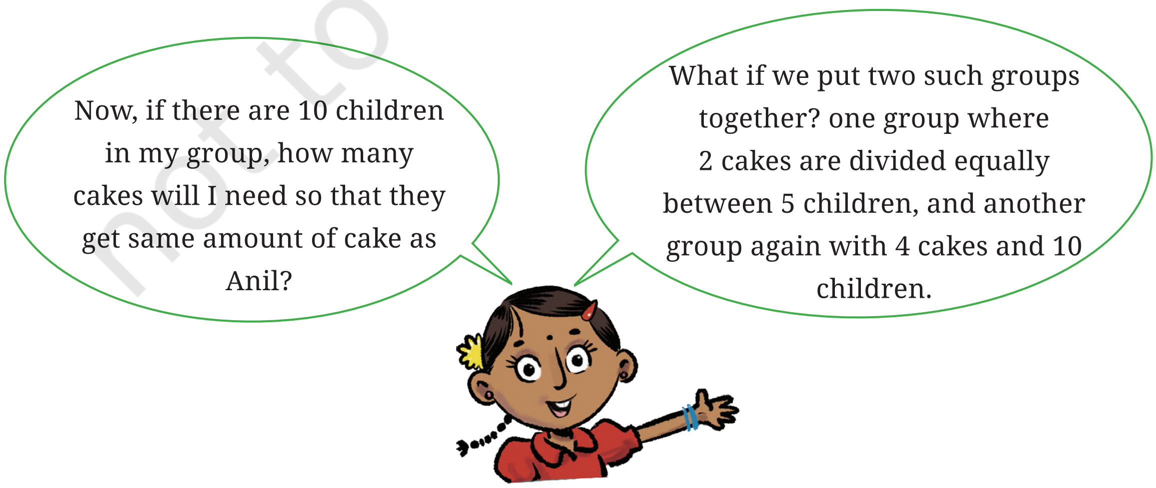
   Addition fact:

   Multiplication fact:

   Compare your picture and answers with your classmates!

2. Draw a picture to show how much each child gets when 2 *rotis* are shared equally by 4 children. Also, write the corresponding division facts, addition facts, and multiplication facts.

3. Anil was in a group where 2 cakes were divided equally among 5 children. How much cake would Anil get?



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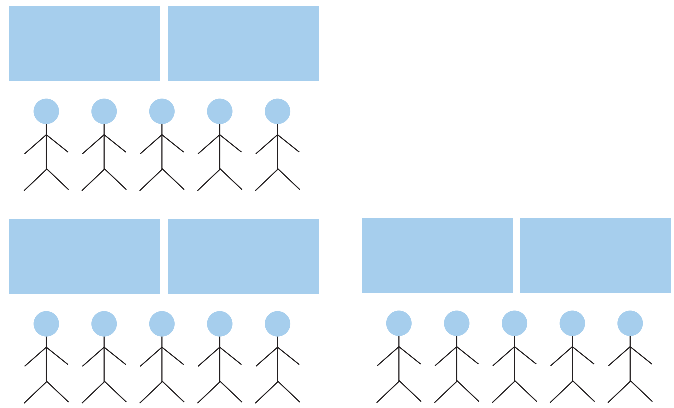
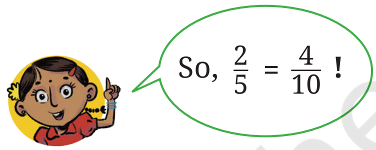
Group 1

Group 2



So, the share of each child is the same in both these situations!

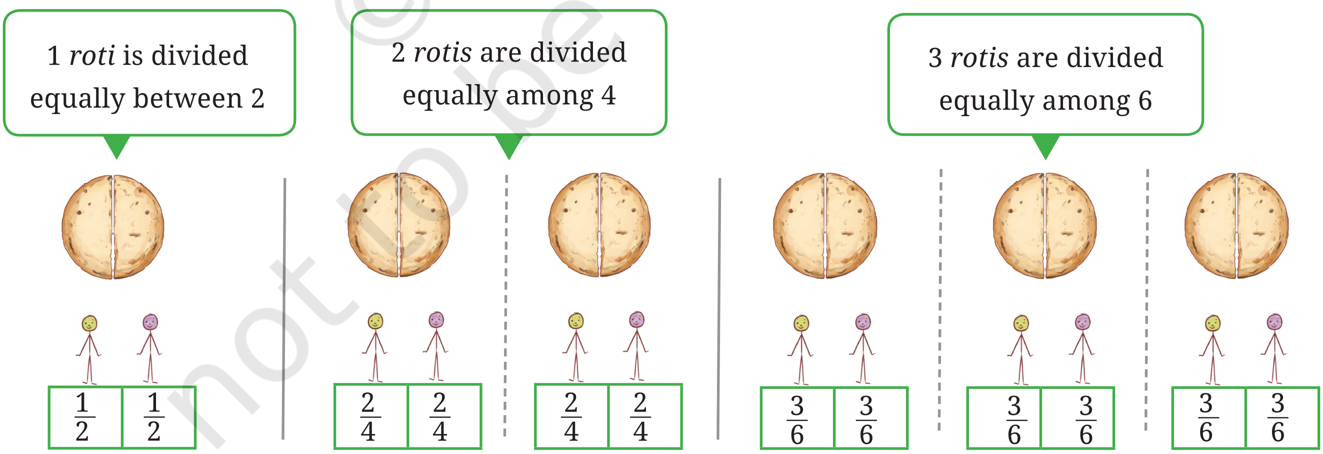


Let us examine the shares of each child in the following situations.

- 1 *roti* is divided equally between 2 children.
- 2 *rotis* are divided equally among 4 children.
- 3 *rotis* are divided equally among 6 children.

Let us draw and share!

Did you notice that in each situation the share of every child is the same? So, we can say that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$.



Fractions where the shares are equal are called 'equivalent fractions'.

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So, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are all **equivalent fractions**.

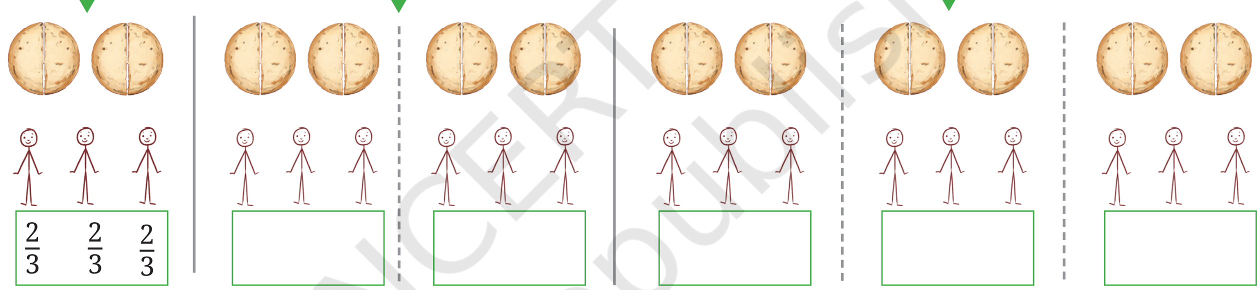
Find some more fractions equivalent to $\frac{1}{2}$. Write them in the boxes here: $\square$ $\square$ $\square$ $\square$

Equally divide the *rotis* in the situations shown below and write down the share of each child. Are the shares in each of these cases the same? Why?

2 *rotis* divided equally among 3 children

4 *rotis* divided equally among 6 children

6 *rotis* divided equally among 9 children



$\frac{2}{3}$ is also called the simplest form of $\frac{4}{6}$. It is also the simplest form of $\frac{6}{9}$ as well.

Do you notice anything about the relationship between the numerator and denominator in each of these fractions?

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Figure it Out

Find the missing numbers:

Math Talk

a. 5 glasses of juice shared equally among 4 friends is the same as ____ glasses of juice shared equally among 8 friends.

So, $\frac{5}{4} = \frac{\square}{8}$.

b. 4 kg of potatoes divided equally in 3 bags is the same as 12 kgs of potatoes divided equally in ___ bags.

So, $\frac{4}{3} = \frac{12}{\square}$

c. 7 *rotis* divided among 5 children is the same as____*rotis* divided among _____ children.

So, $\frac{7}{5} = \frac{\square}{\square}$.

In which group will each child get more *chikki*?

1 *chikki* divided between 2 children or 5 *chikkis* divided among 8 children.

Mukta: So, we must compare $\frac{1}{2}$ and $\frac{5}{8}$. Which is more?

Shabnam: Well, we have seen that $\frac{1}{2} = \frac{4}{8}$; and clearly $\frac{4}{8} < \frac{5}{8}$. So, the children for whom 5 *chikkis* is divided equally among 8 will get more than those children for whom 1 *chikki* is divided equally among 2. The children of the second group will get more *chikki* each.

What about the following groups? In which group will each child get more?

1 *chikki* divided between 2 children or 4 *chikkis* divided among 7 children.

Shabnam: The children of which group will get more chikki this time?

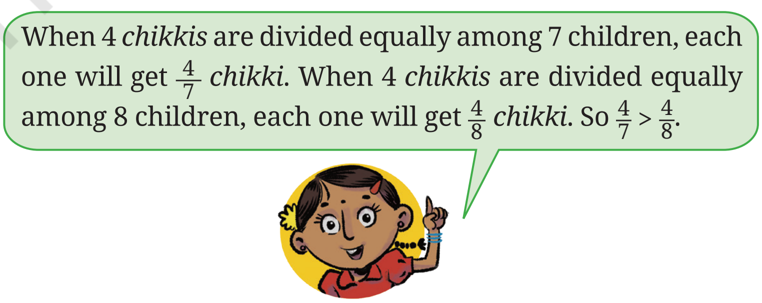
Mukta: We must compare $\frac{1}{7}$ and $\frac{4}{7}$.

Now

$\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$ so, $\frac{1}{2} = \frac{4}{8}$.

Shabnam: But why did you multiply the numerator and denominator by 4 again?

Mukta: You will see!



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If the number of units that are shared is the same, but the number of children among whom the units are shared is more, then the share is less.

Therefore, $\frac{4}{7} > \frac{4}{8}$ and $\frac{4}{8} = \frac{1}{2}$, so $\frac{4}{7} > \frac{1}{2}$.

Now I understood why you multiplied the numerator and denominator by 4.

Suppose the number of children is kept the same, but the number of units that are being shared is increased? What can you say about each child's share now? Why? Discuss how your reasoning explains $\frac{1}{5} < \frac{2}{5}$, $\frac{3}{7} < \frac{4}{7}$, and $\frac{1}{2} < \frac{5}{8}$.

Now, decide in which of the two groups will each child get a larger share:

Math Talk

1. **Group 1 :** 3 glasses of sugarcane juice divided equally among 4 children.
   Group 2: 7 glasses of sugarcane juice divided equally among 10 children.
2. **Group 1 :** 4 glasses of sugarcane juice divided equally among 7 children.
   Group 2: 5 glasses of sugarcane juice divided equally among 7 children.

Which groups were easier to compare? Why?

When the number of children is the same, it is easier to compare, isn't it?

Shabnam: To compare the first two groups, we have to find fractions equivalent to the fractions $\frac{3}{4}$ and $\frac{7}{10}$.

Mukta: How about $\frac{6}{8} = \frac{3}{4}$ and $\frac{21}{30} = \frac{7}{10}$?

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Shabnam: There is a condition. The fractional unit used for the two fractions have to be the same! Like $\frac{2}{6}$ and $\frac{3}{6}$ both use the same fractional unit $\frac{1}{6}$ (i.e., the denominators are the same). But $\frac{6}{8}$ and $\frac{21}{30}$ do not use the same fractional units (they have different denominators).

Mukta: Okay, so let us start making equivalent fractions then:

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20}$ … But when do I stop?

Shabnam: Got it! How about we go on till $4 \times 10 = 40$.

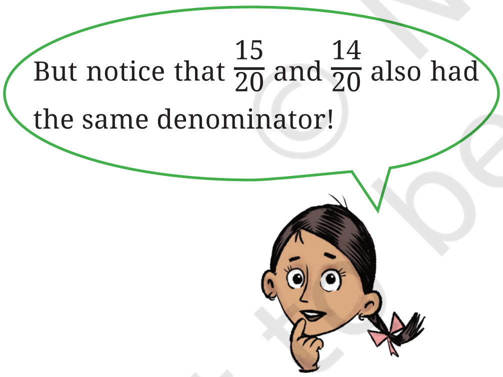
Mukta: You mean the product of the two denominators? Sounds good!

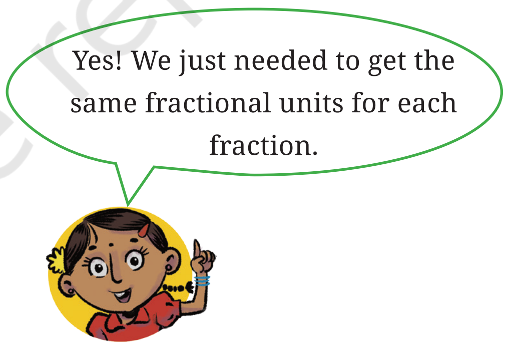
We have $\frac{3}{4}$ and $\frac{7}{10}$. The product of the two denominators (4 and 10) is 40.

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \ldots = \frac{27}{36} = \frac{30}{40}$.

$\frac{7}{10} = \frac{14}{20} = \frac{21}{30} = \frac{28}{40}$.

Go till we reach the denominator 40.





Shabnam: So, fractions equivalent to $\frac{3}{4}$ and $\frac{7}{10}$ with the same fractional unit (same denominators) are $\frac{30}{40}$ and $\frac{28}{40}$, or $\frac{15}{20}$ and $\frac{14}{20}$.

Since clearly $\frac{30}{40} > \frac{28}{40}$, we conclude that $\frac{3}{4} > \frac{7}{10}$.

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Find equivalent fractions for the given pairs of fractions such that the fractional units are the same.

a. $\frac{7}{2}$ and $\frac{3}{5}$ b. $\frac{8}{3}$ and $\frac{5}{6}$ c. $\frac{3}{4}$ and $\frac{3}{5}$ d. $\frac{6}{7}$ and $\frac{8}{5}$

e. $\frac{9}{4}$ and $\frac{5}{2}$ f. $\frac{1}{10}$ and $\frac{2}{9}$ g. $\frac{8}{3}$ and $\frac{11}{4}$ h. $\frac{13}{6}$ and $\frac{1}{9}$

Expressing a Fraction in Lowest Terms (or in its Simplest Form)

In any fraction, if its numerator and denominator have no common factor except 1, then the fraction is said to be in **lowest terms** or in its **simplest form**. In other words, a fraction is said to be in lowest terms if its numerator and denominator are as small as possible.

Any fraction can be expressed in lowest terms by finding an equivalent fraction whose numerator and denominator are as small as possible.

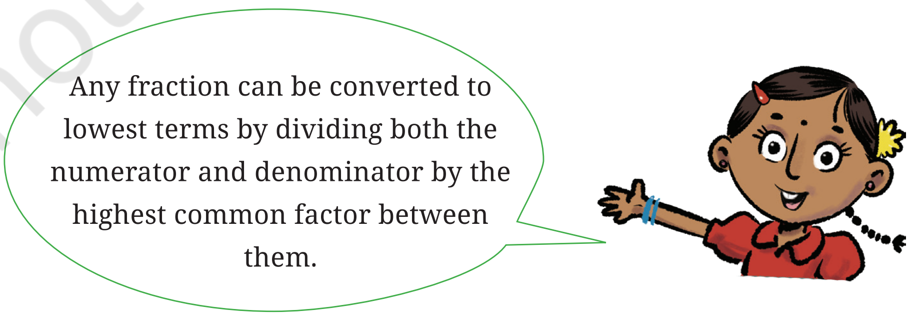
Let's see how to express fractions in lowest terms.

Example: Is the fraction $\frac{16}{20}$ in lowest terms? No, 4 is a common factor of 16 and 20. Let us reduce $\frac{16}{20}$ to lowest terms.

We know that both 16 (numerator) and 20 (denominator) are divisible by 4.

So, $\frac{16 \div 4}{20 \div 4} = \frac{4}{5}$.

Now, there is no common factor between 4 and 5. Hence, $\frac{16}{20}$ expressed in lowest terms is $\frac{4}{5}$. So, $\frac{4}{5}$ is called the simplest form of $\frac{16}{20}$, since 4 and 5 have no common factor other than 1.



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Expressing a fraction in lowest terms can also be done in steps.

Suppose we want to express $\frac{36}{60}$ in lowest terms. First, we notice that both the numerator and denominator are even. So, we divide both by 2, and see that $\frac{36}{60} = \frac{18}{30}$.

Both the numerator and denominator are even again, so we can divide them each by 2 again; we get $\frac{18}{30} = \frac{9}{15}$.

We now notice that 9 and 15 are both multiples of 3, so we divide both by 3 to get $\frac{9}{15} = \frac{3}{5}$.

Now, 3 and 5 have no common factor other than 1, so, $\frac{36}{60}$ in lowest terms is $\frac{3}{5}$.

Alternatively, we could have noticed that in $\frac{36}{60}$, both the numerator and denominator are multiples of 12 : we see that $36 = 3 \times 12$ and $60 = 5 \times 12$. Therefore, we could have concluded that $\frac{36}{60} = \frac{3}{5}$ straight away.

Either method works and will give the same answer! But sometimes it can be easier to go in steps.

Figure it Out

Express the following fractions in lowest terms:

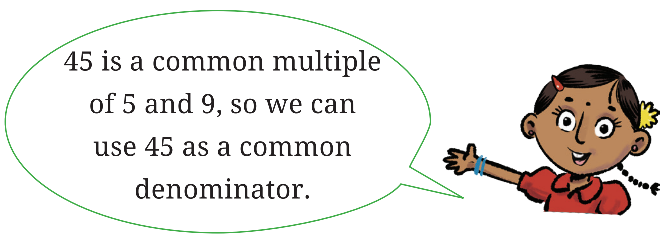
a. $\frac{17}{51}$ b. $\frac{64}{144}$ e. $\frac{126}{147}$ d. $\frac{525}{112}$

7.7 Comparing Fractions

Which is greater, $\frac{4}{5}$ or $\frac{7}{9}$? It can be difficult to compare two such fractions directly. However, we know how to find fractions equivalent to two fractions with the same denominator. Let us see how we can use it:

$$\frac{4}{5} = \frac{4 \times 9}{5 \times 9} = \frac{36}{45}$$

$$\frac{7}{9} = \frac{7 \times 5}{8 \times 5} = \frac{35}{45}.$$



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Clearly, $\frac{36}{45} > \frac{35}{45}$

So, $\frac{4}{5} > \frac{7}{9}$!

Let us try this for another pair: $\frac{7}{9}$ and $\frac{17}{21}$.

63 is a common multiple of 9 and 21. We can then write:

$\frac{7}{9} = \frac{7\times7}{9\times7} = \frac{49}{63}$, $\frac{17}{21} = \frac{17\times3}{21\times3} = \frac{51}{63}$.

Clearly, $\frac{49}{63} < \frac{51}{63}$. So, $\frac{7}{9} < \frac{17}{21}$!

Let's Summarise!

Steps to compare the sizes of two or more given fractions:

Step 1: Change the given fractions to equivalent fractions so that they all are expressed with the same denominator / same fractional unit.

Step 2: Now, compare the equivalent fractions by simply comparing the numerators, i.e., the number of fractional units each has.

Figure it Out

1. Compare the following fractions and justify your answers:

   a. $\frac{8}{3}$, $\frac{5}{2}$ b. $\frac{4}{9}$, $\frac{3}{7}$ c. $\frac{7}{10}$, $\frac{9}{14}$

   d. $\frac{12}{5}$, $\frac{8}{5}$ e. $\frac{9}{4}$, $\frac{5}{2}$

2. Write the following fractions in ascending order.

   a. $\frac{7}{10}$, $\frac{11}{15}$, $\frac{2}{5}$ b. $\frac{19}{24}$, $\frac{5}{6}$, $\frac{7}{12}$

3. Write the following fractions in descending order.

   a. $\frac{25}{16}$, $\frac{7}{8}$, $\frac{13}{4}$, $\frac{17}{32}$ b. $\frac{3}{4}$, $\frac{12}{5}$, $\frac{7}{12}$, $\frac{5}{4}$

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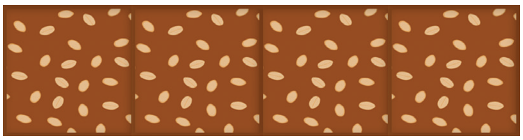
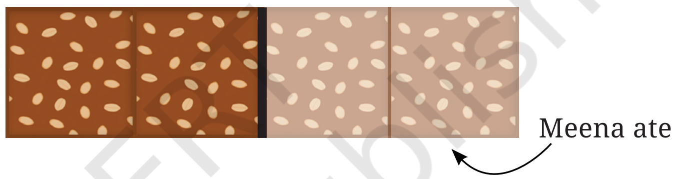
7.8 Addition and Subtraction of Fractions

Meena's father made some *chikki*. Meena ate $\frac{1}{2}$ of it and her younger brother ate $\frac{1}{4}$ of it. How much of the total *chikki* did Meena and her brother eat together?



We can arrive at the answer by visualising it. Let us take a piece of *chikki* and divide it into two halves first like this.

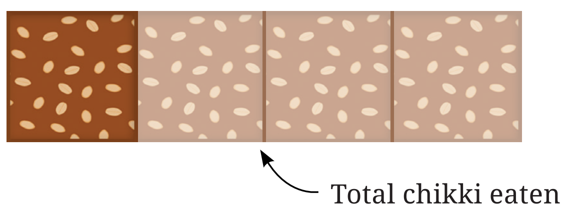
Meena ate $\frac{1}{2}$ of it as shown in the picture.



Let us now divide the remaining half into two further halves as shown. Each of these pieces is $\frac{1}{4}$ of the whole *chikki*.

Meena's brother ate $\frac{1}{4}$ of the whole *chikki*, as is shown in the picture.

The total *chikki* eaten is $\frac{1}{2}$ (by Meena) and $\frac{1}{4}$ (by her brother)



The total *chikki* eaten is $= \frac{1}{2} + \frac{1}{4}$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$= 3 \times \frac{1}{4} = \frac{3}{4}.$$

How much of the total *chikki* is remaining?

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Adding Fractions with the Same Fractional Unit or Denominator

Example: Find the sum of $\frac{2}{5}$ and $\frac{1}{5}$.

Let us represent both using the rectangular strips. In both fractions, the fractional unit is the same $\frac{1}{5}$, so, each strip will be divided into 5 equal parts.

So $\frac{2}{5}$ will be represented as—

And $\frac{1}{5}$ will be represented as—

Adding the two given fractions is the same as finding out the total number of shaded parts, each of which represent the same fractional unit $\frac{1}{5}$.

In this case, the total number of shaded parts is 3. Since, each shaded part represents the fractional unit $\frac{1}{5}$, we see that the 3 shaded parts together represent the fraction $\frac{3}{5}$.

Therefore, $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$!

Example: Find the sum of $\frac{4}{7}$ and $\frac{6}{7}$.

Let us represent both again using the rectangular strip model. Here in both fractions, the fractional unit is the same, i.e., $\frac{1}{7}$, so each strip will be divided into 7 equal parts.

Then $\frac{4}{7}$ will be represented as—

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and $\frac{6}{7}$ will be represented as—

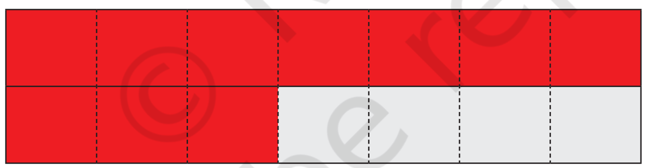
In this case, the total number of shaded parts is 10, and each shaded part represents the fractional unit $\frac{1}{7}$, so, the 10 shaded parts together represent the fraction $\frac{10}{7}$ as seen here.

While adding fractions with the same fractional unit, just add the number of fractional units from each fraction.

$$\text{Therefore, } \frac{4}{7} + \frac{6}{7} = \frac{10}{7}$$

$$= 1 + \frac{3}{7}$$

$$= 1\frac{3}{7}.$$



Try adding $\frac{4}{7} + \frac{6}{7}$ using a number line. Do you get the same answer?

Adding Fractions with Different Fractional Units or Denominators

Example: Find the sum of $\frac{1}{4}$ and $\frac{1}{3}$.

To add fractions with different fractional units, first convert the fractions into equivalent fractions with the same denominator/

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fractional unit. In this case, the common denominator can be made $3 \times 4 = 12$, i.e., we can find equivalent fractions with fractional unit $\frac{1}{12}$.

Let us write the equivalent fraction for each given fraction.

$$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}, \quad \frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}.$$

Now, $\frac{3}{12}$ and $\frac{4}{12}$ have the same fractional unit, i.e., $\frac{1}{12}$.

Therefore, $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$.

This method of addition, which works for adding any number of fractions, was first explicitly described in general by Brahmagupta in the year 628 CE! We will describe the history of the development of fractions in more detail later in the chapter . For now, we simply summarise the steps in Brahmagupta's method for addition of fractions.

Brahmagupta's method for adding fractions

1. Find equivalent fractions so that the fractional unit is common for all fractions. This can be done by finding a common multiple of the denominators (e.g., the product of the denominators, or the smallest common multiple of the denominators).
2. Add these equivalent fractions with the same fractional units. This can be done by adding the numerators and keeping the same denominator.
3. Express the result in lowest terms if needed.

Let us carry out another example of Brahmagupta's method.

Example: Find the sum of $\frac{2}{3}$ and $\frac{1}{5}$.

The denominators of the given fractions are 3 and 5. The lowest common multiple of 3 and 5 is 15. Then we see that

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}, \quad \frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \frac{3}{15}.$$

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Therefore, $\frac{2}{3} + \frac{1}{5} = \frac{10}{15} + \frac{3}{15} = \frac{13}{15}$.

Example: Find the sum of $\frac{1}{6}$ and $\frac{1}{3}$.

The smallest common multiple of 6 and 3 is 6.

$\frac{1}{6}$ will remain $\frac{1}{6}$.

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

Therefore, $\frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6}$.

The fraction $\frac{3}{6}$ can now be re-expressed in lowest terms, if desired. This can be done by dividing both the numerator and denominator by 3 (the biggest common factor of 3 and 6):

$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$.

Therefore, $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$.

Figure it Out

1. Add the following fractions using Brahmagupta's method:

   a. $\frac{2}{7} + \frac{5}{7} + \frac{6}{7}$ b. $\frac{3}{4} + \frac{1}{3}$ c. $\frac{2}{3} + \frac{5}{6}$ d. $\frac{2}{3} + \frac{2}{7}$ e. $\frac{3}{4} + \frac{1}{3} + \frac{1}{5}$

   f. $\frac{2}{3} + \frac{4}{5}$ g. $\frac{4}{5} + \frac{2}{3}$ h. $\frac{3}{5} + \frac{5}{8}$ i. $\frac{9}{2} + \frac{5}{4}$ j. $\frac{8}{3} + \frac{2}{7}$

   k. $\frac{3}{4} + \frac{1}{3} + \frac{1}{5}$ l. $\frac{2}{3} + \frac{4}{5} + \frac{3}{7}$ m. $\frac{9}{2} + \frac{5}{4} + \frac{7}{6}$

2. Rahim mixes $\frac{2}{3}$ litres of yellow paint with $\frac{3}{4}$ litres of blue paint to make green paint. What is the volume of green paint he has made?

3. Geeta bought $\frac{2}{5}$ meter of lace and Shamim bought $\frac{3}{4}$ meter of the same lace to put a complete border on a table cloth whose perimeter is 1 meter long. Find the total length of the lace they both have bought. Will the lace be sufficient to cover the whole border?

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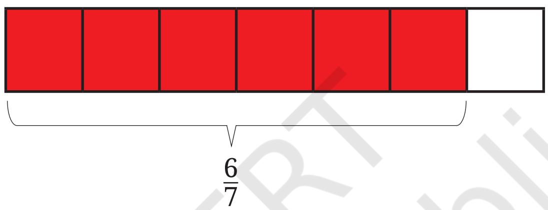
Subtraction of Fractions with the same Fractional Unit or Denominator

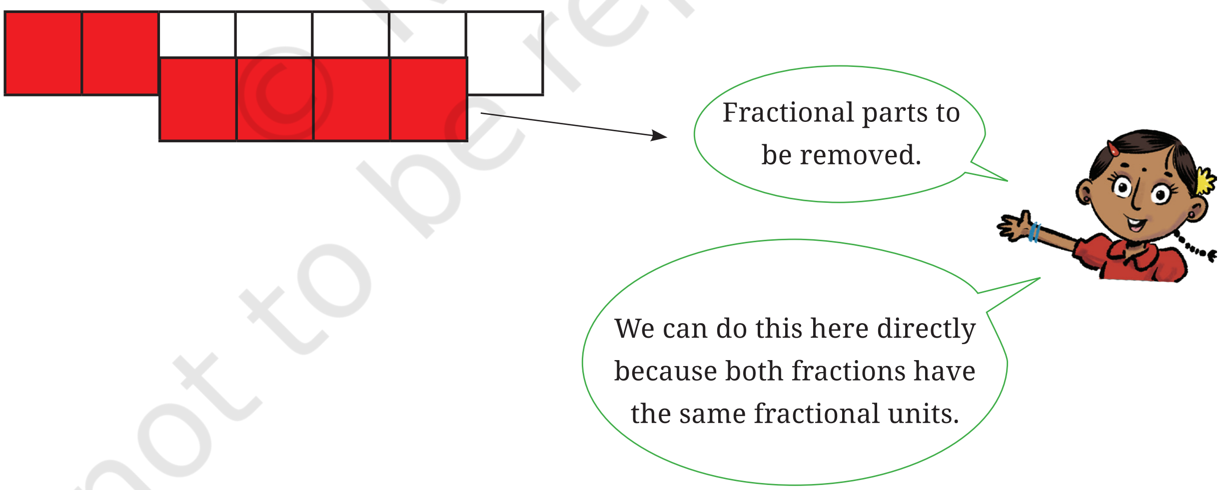
Brahmagupta's method also applies when subtracting fractions!

Let us start with the problem of subtracting $\frac{4}{7}$ from $\frac{6}{7}$, i.e., what is $\frac{6}{7} - \frac{4}{7}$?

To solve this problem, we can again use the rectangular strips. In both fractions, the fractional unit is the same i.e. $\frac{1}{7}$. Let us first represent the bigger fraction using a rectangular strip model as shown:



Each shaded part represents $\frac{1}{7}$. Now, we need to subtract $\frac{4}{7}$. To do this let us remove 4 of the shaded parts:



So, we are left with 2 shaded parts, i.e., $\frac{6}{7} - \frac{4}{7} = \frac{2}{7}$.

Try doing this same exercise using the number line.

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Figure it Out

1. $\frac{5}{8} - \frac{3}{8}$
2. $\frac{7}{9} - \frac{5}{9}$
3. $\frac{10}{27} - \frac{1}{27}$

Subtraction of Fractions with Different Fractional Units or Denominators

Example: What is $\frac{3}{4} - \frac{2}{3}$?

As we already know the procedure for subtraction of fractions with the same fractional units, let us convert each of the given fractions into equivalent fractions with the same fractional units.

$$\frac{3}{4} = \frac{(3\times3)}{(4\times3)} = \frac{9}{12}$$

Yes! By doing this we can easily subtract the two fractions.

Think! Why did we choose to multiply both the numerator and denominator by 3?

and similarly,

$$\frac{2}{3} = \frac{(2\times4)}{(3\times4)} = \frac{8}{12}.$$

Again! Why did we choose to multiply both the numerator and denominator here by 4?

Therefore, $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$.

Brahmagupta's method for subtracting two fractions—

1. Convert the given fractions into equivalent fractions with the same fractional unit, i.e., the same denominator.
2. Carry out the subtraction of fractions having the same fractional units. This can be done by subtracting the numerators and keeping the same denominator.
3. Simplify the result into lowest terms if needed.

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Figure it Out

1. Carry out the following subtractions using Brahmagupta's method:

   a. $\frac{8}{15} - \frac{3}{15}$ b. $\frac{2}{5} - \frac{4}{15}$ c. $\frac{5}{6} - \frac{4}{9}$ d. $\frac{2}{3} - \frac{1}{2}$

2. Subtract as indicated:

   a. $\frac{13}{4}$ from $\frac{10}{3}$ b. $\frac{18}{5}$ from $\frac{23}{3}$ c. $\frac{29}{7}$ from $\frac{45}{7}$

3. Solve the following problems:

   a. Jaya's school is $\frac{7}{10}$ km from her home. She takes an auto for $\frac{1}{2}$ km from her home daily, and then walks the remaining distance to reach her school. How much does she walk daily to reach the school?

   b. Jeevika takes $\frac{10}{3}$ minutes to take a complete round of the park and her friend Namit takes $\frac{13}{4}$ minutes to do the same. Who takes less time and by how much?

7.9 A Pinch of History

Do you know what a fraction was called in ancient India? It was called *bhinna* in Sanskrit, which means 'broken'. It was also called *bhaga* or *ansha* meaning 'part' or 'piece'.

The way we write fractions today, globally, originated in India. In ancient Indian mathematical texts, such as the *Bakshali manuscript* (from around the year 300 CE), when they wanted to write $\frac{1}{2}$, they wrote it as $\begin{matrix}1\\2\end{matrix}$ which is indeed very similar to the way we write it today! This method of writing and working with fractions continued to be used in India for the next several centuries, including by Aryabhata (499 CE), Brahmagupta (628 CE), Sridharacharya (c. 750 CE), and Mahaviracharya (c. 850 CE), among others. The line segment between the numerator and denominator in '$\frac{1}{2}$' and in other fractions

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was later introduced by the Moroccan mathematician Al-Hassar (in the 12th century). Over the next few centuries the notation then spread to Europe and around the world.

Fractions had also been used in other cultures such as the ancient Egyptian and Babylonian civilisations, but they primarily used only fractional units, that is, fractions with a 1 in the numerator. More general fractions were expressed as sums of fractional units, now called 'Egyptian fractions'. Writing numbers as the sum of fractional units, e.g., $\frac{19}{24} = \frac{1}{2} + \frac{1}{6} + \frac{1}{8}$, can be quite an art and leads to beautiful puzzles. We will consider one such puzzle below.

General fractions (where the numerator is not necessarily 1) were first introduced in India, along with their rules of arithmetic operations like addition, subtraction, multiplication, and even division of fractions. The ancient Indian treatises called the 'Sulba-sutras' shows that even during Vedic times, Indians had discovered the rules for operations with fractions. General rules and procedures for working with and computing with fractions were first codified formally and in a modern form by Brahmagupta.

Brahmagupta's methods for working with and computing with fractions are still what we use today. For example, Brahmagupta described how to add and subtract fractions as follows:

"By the multiplication of the numerator and the denominator of each of the fractions by the other denominators, the fractions are reduced to a common denominator. Then, in case of addition, the numerators (obtained after the above reduction) are added. In case of subtraction, their difference is taken." (Brahmagupta, Brahmasphuṭasiddhānta, Verse 12.2, 628 CE)

The Indian concepts and methods involving fractions were transmitted to Europe via the Arabs over the next few centuries and they came into general use in Europe in around the 17th century and then spread worldwide.

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Puzzle!

It is easy to add up fractional units to obtain the sum 1, if one uses the same fractional unit, e.g.,

$\frac{1}{2} + \frac{1}{2} = 1$, $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$, $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$, etc.

However, can you think of a way to add fractional units that are all different to get 1?

It is not possible to add two different fractional units to get 1. The reason is that ½ is the largest fractional unit, and $\frac{1}{2} + \frac{1}{2} = 1$.

To get different fractional units, we would have to replace at least one of the $\frac{1}{2}$'s with some smaller fractional unit - but then the sum would be less than 1! Therefore, it is not possible for two different fractional units to add up to 1.

We can try to look instead for a way to write 1 as the sum of three different fractional units.

Try This

1. Can you find three different fractional units that add up to 1?

   It turns out there is only one solution to this problem (up to changing the order of the 3 fractions)! Can you find it? Try to find it before reading further.

   Here is a systematic way to find the solution. We know that $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$. To get the fractional units to be different, we will have to increase at least one of the $\frac{1}{3}$'s, and decrease at least one of the other $\frac{1}{3}$'s to compensate for that increase. The only way to increase $\frac{1}{3}$ to another fractional unit is to replace it by $\frac{1}{2}$. So $\frac{1}{2}$ must be one of the fractional units.

   Now $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$. To get the fractional units to be different, we will have to increase one of the $\frac{1}{4}$'s and decrease the other $\frac{1}{4}$ to compensate for that increase. Now the only way to increase $\frac{1}{4}$

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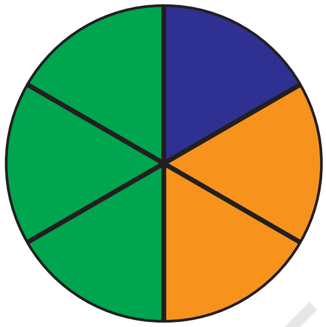
to another fractional unit, that is different from $\frac{1}{2}$, is to replace it by $\frac{1}{3}$. So two of the fractions must be $\frac{1}{2}$ and $\frac{1}{3}$! What must be third fraction then, so that the three fractions add up to 1?

This explains why there is only one solution to the above problem.



$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1.$

What if we look for four different fractional units that add up to 1?

2. Can you find four different fractional units that add up to 1?

Try This

It turns out that this problem has six solutions! Can you find at least one of them? Can you find them all? You can try using similar reasoning as in the cases of two and three fractional units – or find your own method!

Once you find one solution, try to divide a circle into parts like in the figure above to visualize it!

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SUMMARY

- **Fraction as equal share:** When a whole number of units is divided into equal parts and shared equally, a **fraction** results.
- **Fractional Units:** When one whole basic unit is divided into equal parts, then each part is called a **fractional unit.**
- **Reading Fractions:** In a fraction such as $\frac{5}{6}$, 5 is called the **numerator** and 6 is called the **denominator.**
- **Mixed fractions** contain a whole number part and a fractional part.
- **Number line:** Fractions can be shown on a number line. Every fraction has a point associated with it on the number line.
- **Equivalent Fractions:** When two or more fractions represent the same share/number, they are called **equivalent fractions**.
- **Lowest terms:** A fraction whose numerator and denominator have no common factor other than 1 is said to be in **lowest terms** or in its **simplest form.**
- **Brahmagupta's method for adding fractions:** When adding fractions, convert them into equivalent fractions with the same fractional unit (i.e., the same denominator), and then add the number of fractional units in each fraction to obtain the sum. This is accomplished by adding the numerators while keeping the same denominator.
- **Brahmagupta's method for subtracting fractions:** When subtracting fractions, convert them into equivalent fractions with the same fractional unit (i.e., the same denominator), and then subtract the number of fractional units. This is accomplished by subtracting the numerators while keeping the same denominator.

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8 PLAYING WITH CONSTRUCTIONS

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8.1 Artwork

Observe the following figures and try drawing them freehand.

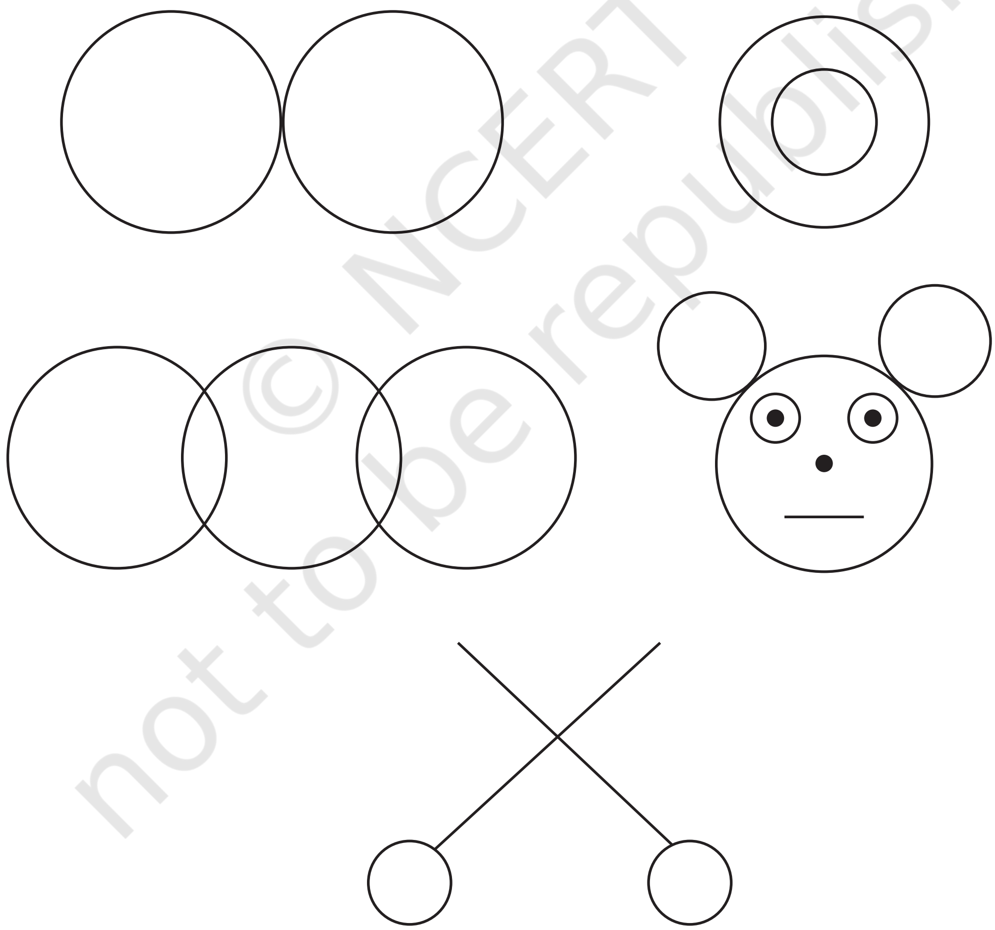

Fig. 8.1

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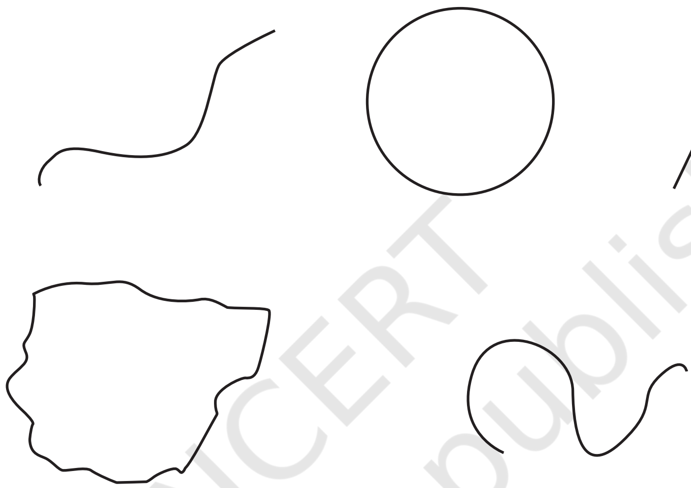
Now, arm yourself with a ruler and a compass. Let us explore if we can draw these figures with these tools and get familiar with a compass.

Observe the way a compass is made. What can one draw with the compass? Explore!

Do you know what curves are? They are any shapes that can be drawn on paper with a pencil, and include straight lines, circles and other figures as shown below.



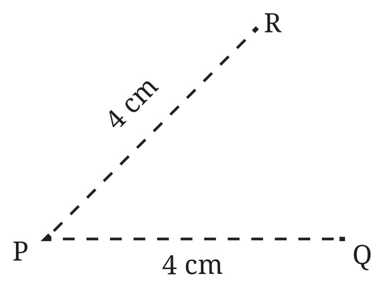
Mark a point 'P' in your notebook. Then, mark as many points as possible, in different directions, that are 4 cm away from P.

Think: Imagine marking all the points of 4 cm distance from the point P. How would they look?

Try to draw it and verify if it is correct by taking some points on the curve and checking if their distances from P are indeed 4 cm.

Explore, if you have not already done so, and see if a compass can be used for this purpose.

You can start by marking a few points of distance 4 cm from P using the compass. How can this be done?



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You will have to open up the compass against a ruler (see Fig. 8.2) such that the distance between the tip of the compass and the pencil is 4 cm.

Now, try to get the full curve.

Hint: Keep the point of the compass fixed moving only the pencil.

What is the shape of the curve? It is a circle!

Take a point on the circle. What will be its distance from P—equal to 4 cm, less than 4 cm or greater than 4 cm? Similarly, what will be the distance between P and another point on the circle?

As shown in the figure, the point P is called the **centre** of the circle and the distance between the centre and any point on the circle is called the **radius** of the circle.

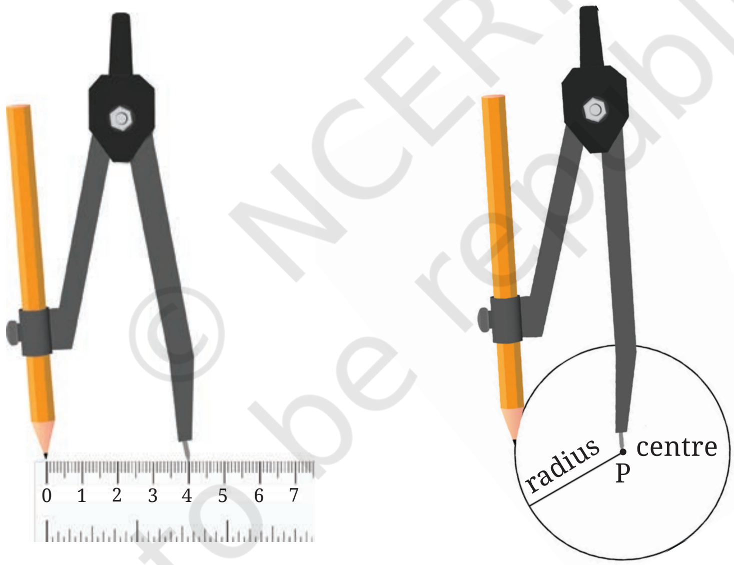

Fig. 8.2

Having explored the use of a compass, go ahead and recreate the images in Fig. 8.1.

Can you make the figures look as good as the figures shown there? Try again if you want to!

Also, has the use of instruments made the construction easier?

Now try constructing the following figures.

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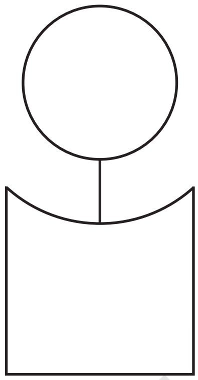
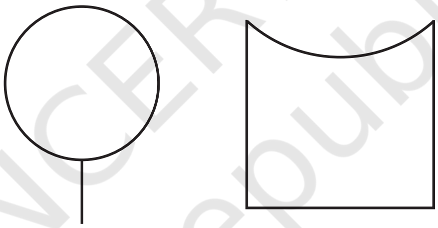
Construct

1. A Person

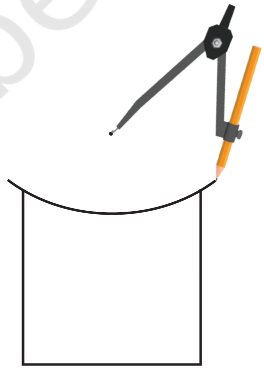
How will you draw this?



This figure has two components.



You might have figured out a way of drawing the first part. For drawing the second part, see this.



The challenge here is to find out where to place the tip of the compass and the radius to be taken for drawing this curve. You can fix a radius in the compass and try placing the tip of the

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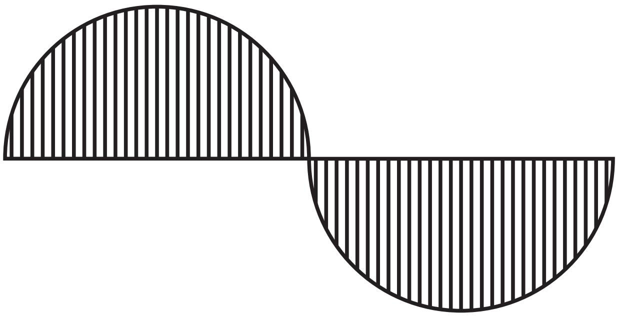
compass in different locations to see which point works for getting the curve. Use your Estimate where to keep the tip.

2. Wavy Wave

Construct this.



As the length of the central line is not specified, we can take it to be of any length.

Let us take AB to be the central line such that the length of AB is 8 cm. We write this as AB = 8 cm.

Here, the first wave is drawn as a half circle.

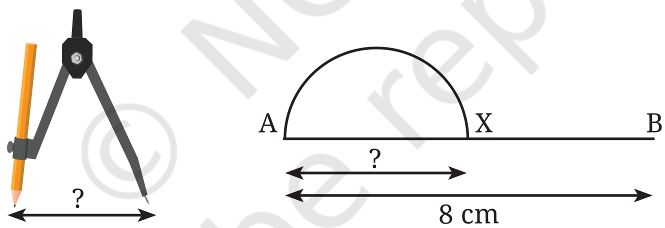


Figure it Out

1. What radius should be taken in the compass to get this half circle? What should be the length of AX?
2. Take a central line of a different length and try to draw the wave on it.
3. Try to recreate the figure where the waves are smaller than a half circle (as appearing in the neck of the figure 'A Person'). The challenge here is to get both the waves to be identical. This may be tricky!

Try This

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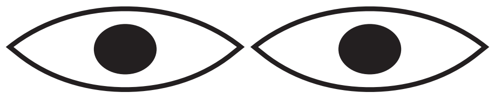
3. Eyes

How do you draw these eyes with a compass?



For a hint, go to the end of the chapter.

Make other artwork of your choice with a ruler and a compass.

8.2 Squares and Rectangles

Now, let us look at some basic figures having straight lines in their boundary.

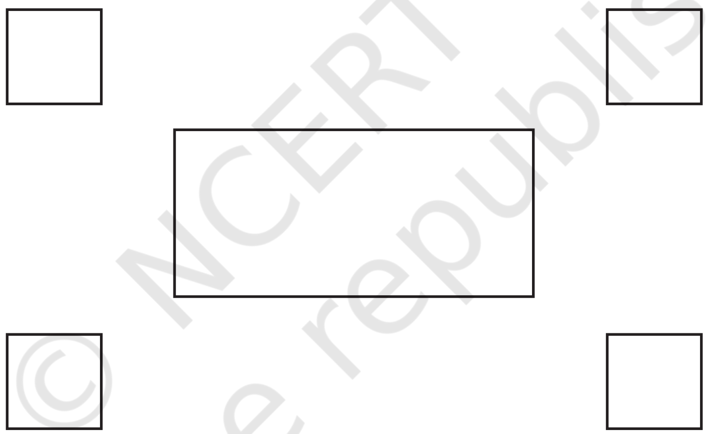


Fig. 8.3

What shapes are these? Yes, these are our familiar squares and rectangles. But what makes them squares and rectangles?

Consider this rectangle ABCD.

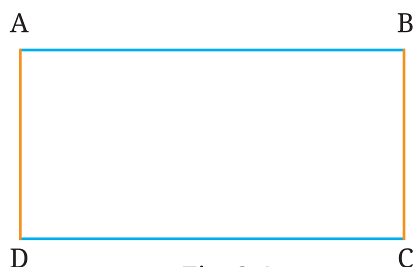


Fig. 8.4

The points A, B, C and D are the corners of the rectangle. Lines AB, BC, CD and DA are its sides. Its angles are ∠A, ∠B, ∠C and ∠D.

The blue sides AB and CD are called **opposite sides**, as they lie opposite to each other. Likewise, AD and BC is the other pair of opposite sides.

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Recall that, in a rectangle:

R1) The opposite sides are equal in length, and

R2) All the angles are 90°.

As in the case of rectangles, the corners and sides are defined for a square in the same manner.

A square satisfies the following two properties:

S1) All the sides are equal, and

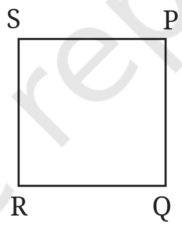
S2) All the angles are 90°.

See the rectangle in Fig. 8.4 and the name given to it: ABCD. This rectangle can also be named in other ways—BCDA, CDAB, DABC, ADCB, DCBA, CBAD and BADC. So, can a rectangle be named using any combination of the labels around its corners? No! For example, it cannot be named ABDC or ACBD. Can you see what names are allowed and what names are not?

In a valid name, the corners occur in an order of travel around the rectangle, starting from any corner.

Which of the following is not a name for this square?

1. PQSR
2. SPQR
3. RSPQ
4. QRSP



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Rotated Squares and Rectangles

Here is a square piece of paper having all its sides equal in length and all angles equal to 90°. It is rotated as shown in the figure. Is it still a square?

Let us check if the rotated paper still satisfies the properties of a square.

- Are all the sides still equal? Yes.
- Are all the angles still 90°? Yes.

Rotating a square does not change its lengths and angles.

Therefore, this rotated figure satisfies both the properties of a square and so, it is a square.

By the same reasoning, a rotated rectangle is still a rectangle.

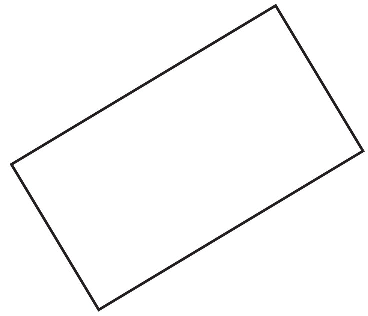
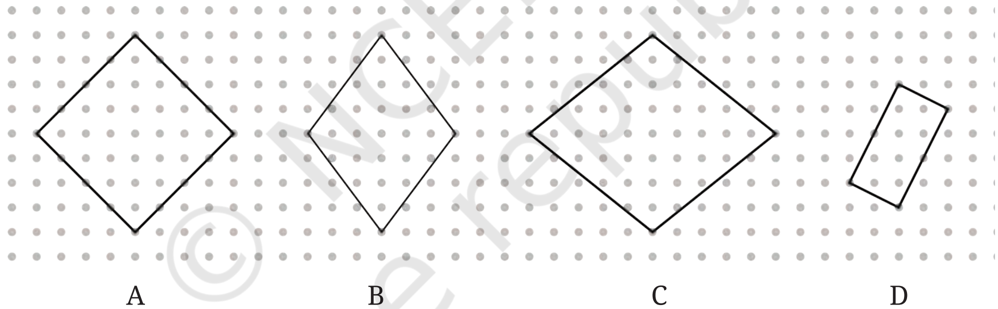


Figure it Out

1. Draw the rectangle and four squares configuration (shown in Fig. 8.3) on a dot paper.

   What did you do to recreate this figure so that the four squares are placed symmetrically around the rectangle? Discuss with your classmates.

2. Identify if there are any squares in this collection. Use measurements if needed.



Think: Is it possible to reason out if the sides are equal or not, and if the angles are right or not without using any measuring instruments in the above figure? Can we do this by only looking at the position of corners in the dot grid?

3. Draw at least 3 rotated squares and rectangles on a dot grid. Draw them such that their corners are on the dots. Verify if the squares and rectangles that you have drawn satisfy their respective properties.

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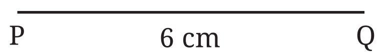
8.3 Constructing Squares and Rectangles

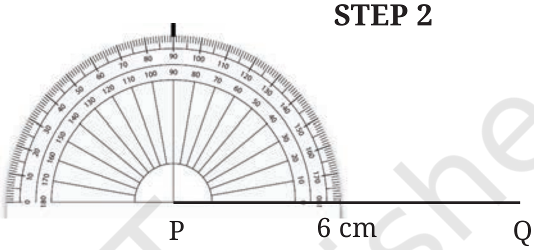
Now, let us start constructing squares and rectangles. How would you construct a square with a side of 6 cm?

For help, you can see the following figures. A square PQRS of sidelength 6 cm is constructed.

STEP 1

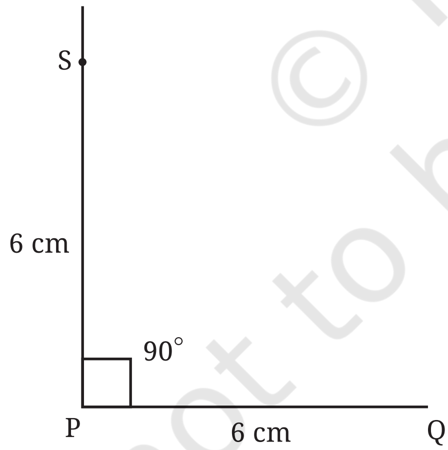


STEP 2



Mark a point to draw a perpendicular to PQ through P.

STEP 3
Method 1



Mark S on the perpendicular such that PS = 6 cm using a ruler.

Method 2

This can also be done using a compass.

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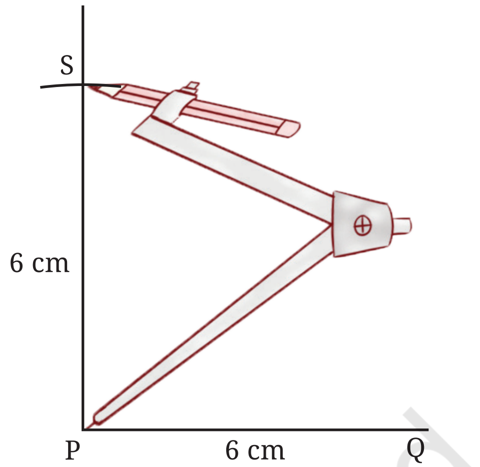
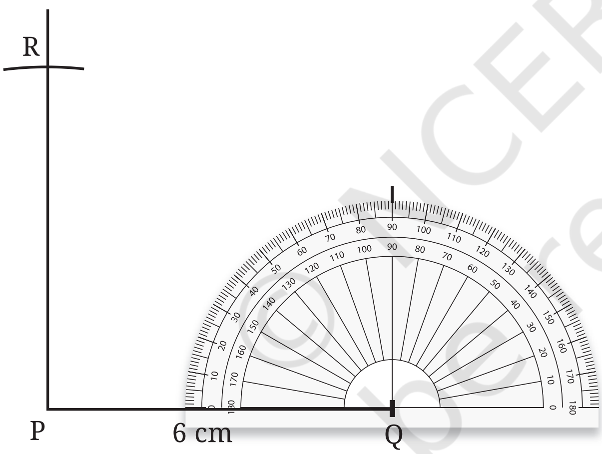


Can you see why PS should be 6 cm long?

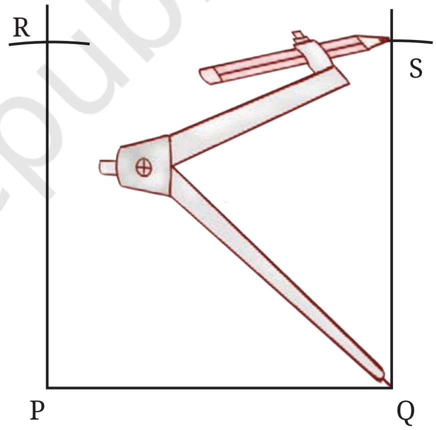
STEP 4

Draw a perpendicular to line segment PQ through Q.

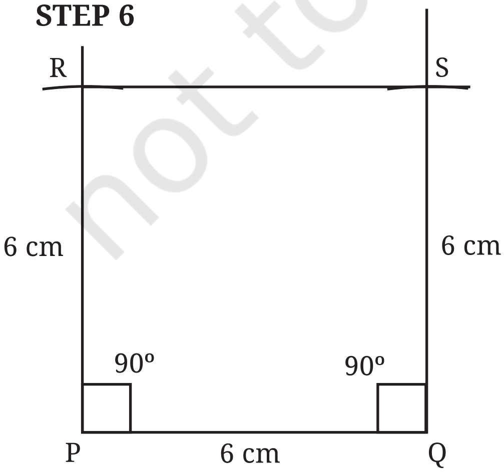


STEP 5

If we had used the compass, then the next point can easily be marked using it!



STEP 6



How long is the side RS and what are the measures of ∠R and ∠S?

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Construct

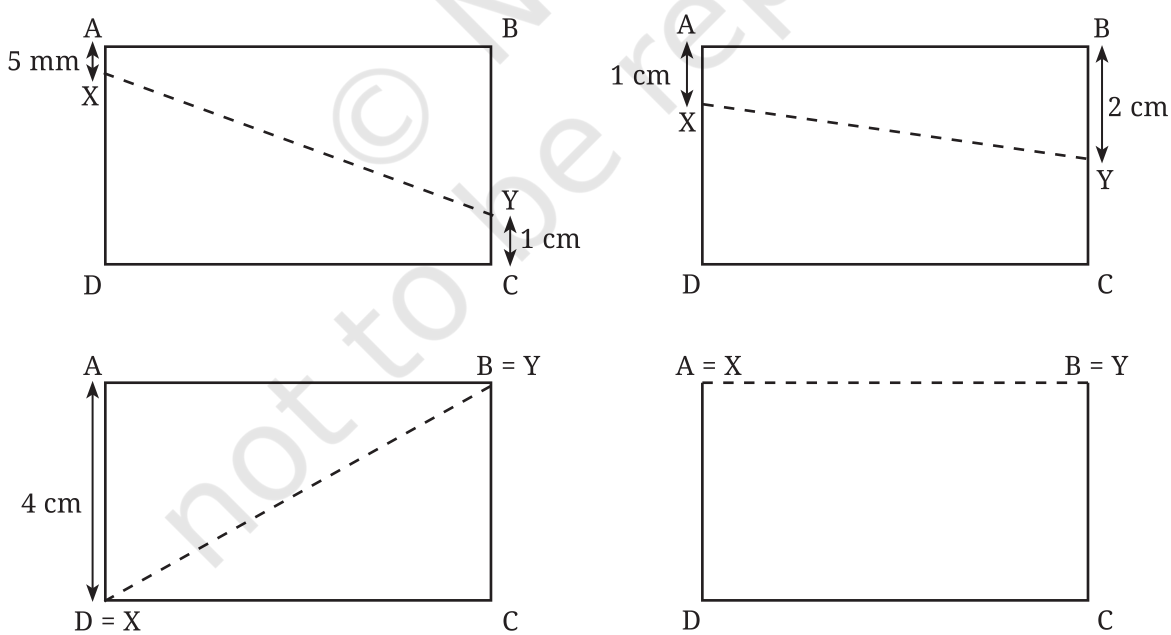
1. Draw a rectangle with sides of length 4 cm and 6 cm. After drawing, check if it satisfies both the rectangle properties.
2. Draw a rectangle of sides 2 cm and 10 cm. After drawing, check if it satisfies both the rectangle properties.
3. Is it possible to construct a 4-sided figure in which—
   - all the angles are equal to 90º but
   - opposite sides are not equal?

Try This

8.4 An Exploration in Rectangles

Construct a rectangle ABCD with AB = 7 cm and BC = 4 cm.

Imagine X to be a point that can be moved anywhere along the side AD. Similarly, imagine Y to be a point that can be moved anywhere along the side BC. Note that X can also be placed on the end point A or D. Similarly, Y can also be placed on the end point B or C.



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At which positions will the points X and Y be at their closest? When do you think they will be the farthest? What does your intuition say? Discuss with your classmates.

Math Talk

Now, verify your guesses by placing the points X and Y on the sides and measure how near or far they are.

The distance between X and Y can be obtained by measuring the length of the line XY.

How does the minimum distance between the points X and Y compare to the length of AB?

Change the positions of X and Y to check if there are other positions where they are at their nearest or farthest. You could construct multiple copies of the rectangle and try out various positions of X and Y.

How will you keep track of the lengths XY for different positions of X and Y?

Here is one way of doing it. Suppose here are some of the positions of X and Y that you have considered:

- When X is 5 mm away from A and Y is 3 cm away from B,
  XY = ___ cm __ mm
- When X is 1 cm away from A and Y is 1 cm away from B,
  XY = ___ cm ___ mm
- When X is 2 cm away from A and Y is 4 cm away from B,
  XY = ___ cm ___ mm and so on ...

Is there a shorthand way of writing it down? In all the sentences, only the position of X, Y and the length XY changes. So we could write this as:

Distance of X from A	Distance of Y from B	Length of XY

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Have you checked what happens to the length XY when X and Y are placed at the same distance away from A and B, respectively? For example, as in the cases like these:

Distance of X from A	Distance of Y from B	Length of XY
5 mm	5 mm	
1 cm	1 cm	
1 cm 5 mm	1 cm 5 mm	

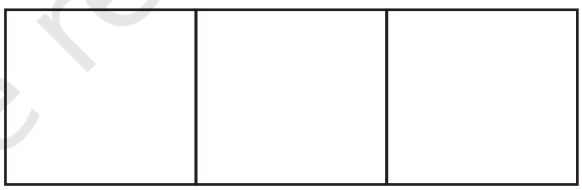
and so on ...

- In each of these cases, observe
  i. how the length XY compares to that of AB and
  ii. the shape of the 4-sided figure ABYX.
- How does the farthest distance between X and Y compare with the length of AC? BD?

Construct

Breaking Rectangles

Construct a rectangle that can be divided into 3 identical squares as shown in the figure.



Solution

If this seem difficult, let us simplify the problem.

Explore:

What about constructing a rectangle that can be divided into two identical squares? Can you try it?

It is wise to first plan and then construct. But how do we plan? Can you think of a way?

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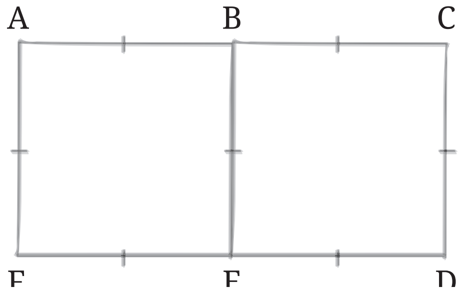
One way is to visualise the final figure by drawing a rough diagram of it.



What can we infer from this figure?

Can you identify the equal sides?

Since, the two squares are identical,

AB = BC and FE = ED

Since ABEF and BCDE are squares, all the sides in each of the squares are equal. This is written as—

AF = AB = BE = FE

BE = BC = CD = ED

So, all the shorter lines are equal!

A convention is followed to represent equal sides. It is done by putting a '|' on the line. Refer to the rough figure.

Using this analysis, can you try constructing it? Remember, all that was asked for is a rectangle that can be divided into two identical squares and with no measurements imposed.

To draw the rectangle *ACDF*, one could assign any length to *AF*. For example, if we assign *AF* = 4 cm, then what must the length of *AC* be?

Explore: Can the rectangle now be completed?

In fact, one could proceed by drawing AF without even measuring its length using a ruler. We could then construct a line perpendicular to AF that is long enough to contain the other side. As AB = AF, we need to somehow transfer the length of AF

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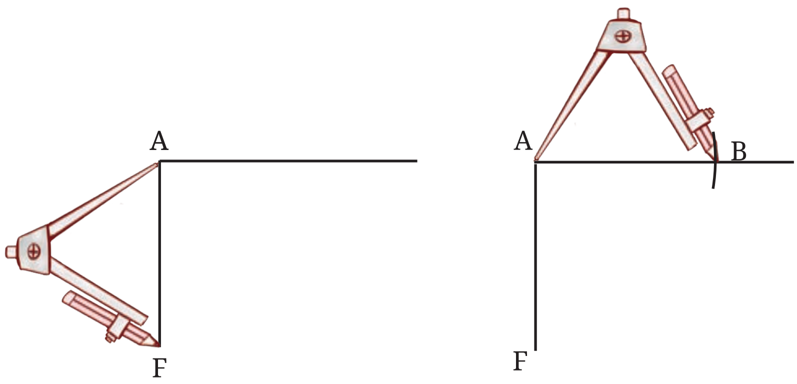
to get the point B. How do we do it without a ruler? Can it be done using a compass?
Observe, how the length of AF is measured using a compass.



Use it to mark out the points B and C, and complete the rectangle.

With this idea, try constructing a rectangle that can be divided into three identical squares.

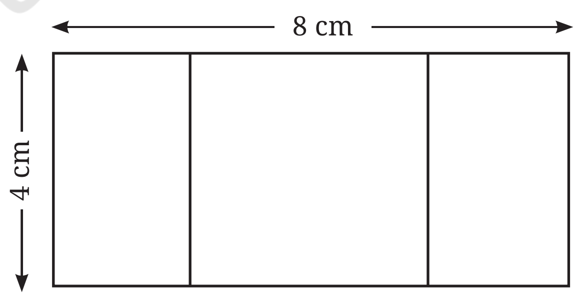
Give the lengths of the sides of a rectangle that cannot be divided into—

- two identical squares;
- three identical squares.

Construct

1. A Square within a Rectangle

Construct a rectangle of sides 8 cm and 4 cm. How will you construct a square inside, as shown in the figure, such that the centre of the square is the same as the centre of the rectangle?



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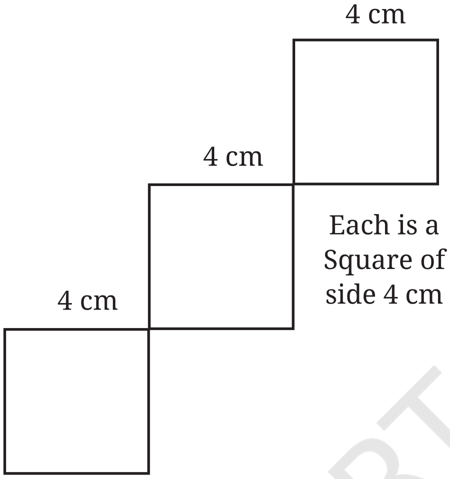
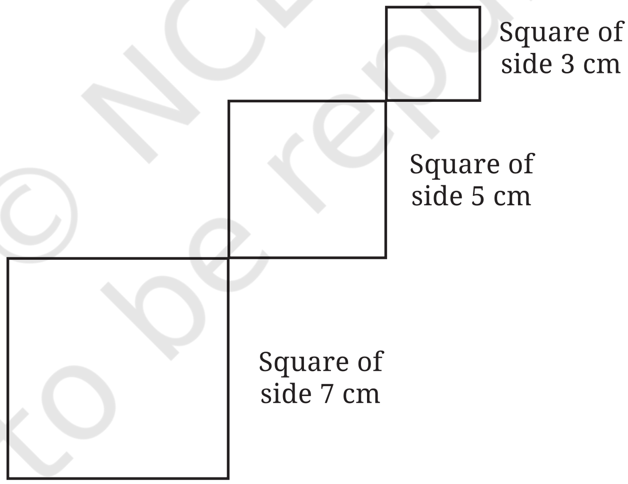
Hint: Draw a rough figure. What will be the sidelength of the square? What will be the distance between the corners of the square and the outer rectangle?

2. Falling Squares



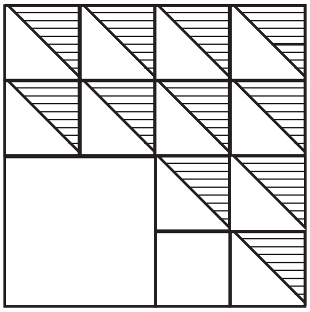
Make sure that the squares are aligned the way they are shown.

Now, try this.



3. Shadings

Construct this. Choose measurements of your choice. Note that the larger 4-sided figure is a square and so are the smaller ones.



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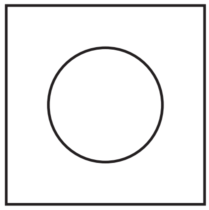
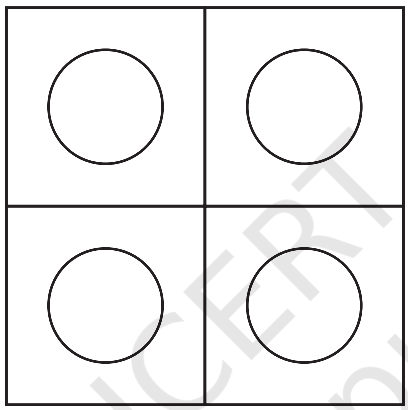
4. **Square with a Hole**



Observe that the circular hole is the same as the centre of the square.

Hint: Think where the centre of the circle should be.

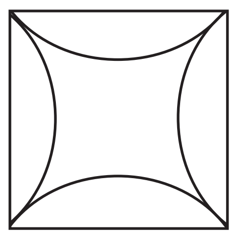
5. **Square with more Holes**



6. **Square with Curves**

This is a square with 8 cm sidelengths.

Hint: Think where the tip of the compass can be placed to get all the 4 arcs to bulge uniformly from each of the sides. Try it out!

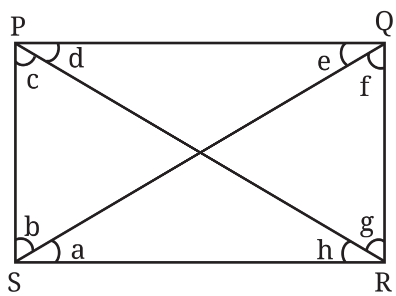


Try This

8.5 Exploring Diagonals of Rectangles and Squares

Consider a rectangle PQRS. Join PR and QS. These two lines are called the **diagonals** of the rectangle.

Compare the lengths of the diagonals. First predict the answer. Then construct a



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rectangle marking the points as shown and measure the diagonals. In rectangle PQRS, the right angles at P and R are referred to as opposite angles. The other pair of opposite angles are the right angles at Q and S.

Observe that a diagonal divides each of the pair of opposite angles into two smaller angles. In the figure, the diagonal PR divides angle R into two smaller angles which we simply call g and h. The diagonal also divides angle P into c and d. Are g and h equal? Are c and d equal?

First predict the answers, and then measure the angles. What do you observe? Identify pairs of angles that are equal.

Explore

How should the rectangle be constructed so that the diagonal divides the opposite angles into equal parts?

How will you record your observations? First, identify the parameters that need to be tracked. They are the sides of the rectangle and the 8 angles formed by the two diagonals. Are there any other measurements that you would want to keep track of?

Sides	A	B	C	D	E	F	G	H

In your experimentation, did you consider the case when all four sides of the rectangle are equal? That is, did you consider the case of a square? See what happens in this special case!

What general laws did you observe with respect to the angles and sides? Try to frame and discuss them with your classmates.

How can one be sure if the laws that you have observed will always be true?

Math Talk

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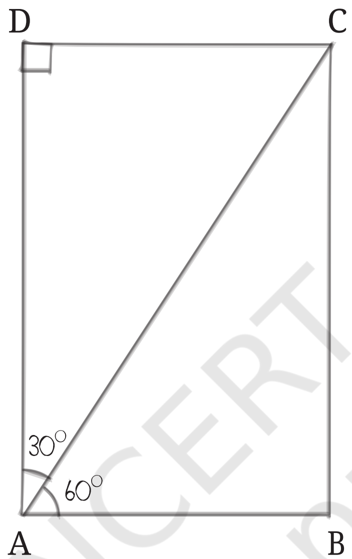
Construct

1. Construct a rectangle in which one of the diagonals divides the opposite angles into 60° and 30°.

Solution

Let us start with a rough diagram.



In what order should its parts be drawn?

We will briefly sketch a possible order of construction.

STEP 1

AB is drawn with an arbitrary length. What is the next point that can be located?

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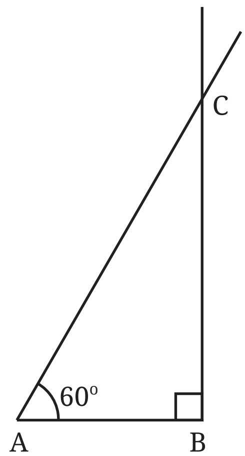
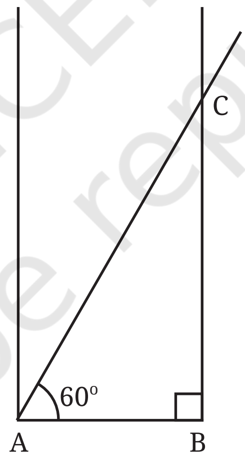
STEP 2



STEP 3

We know the line on which D lies. Draw a line through A perpendicular to AB.



Now $\angle A$ is divided into two angles. One measures 60°. Check what the other angle is.

There are at least two ways of finding the point D—

- One uses the fact that all the angles of a rectangle are right angles.
- The other uses the fact that opposite sides are equal.

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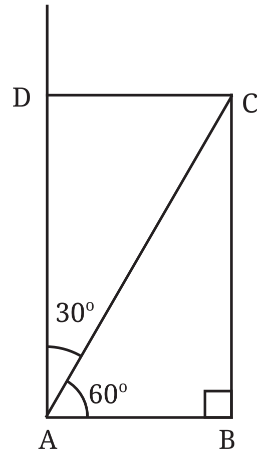
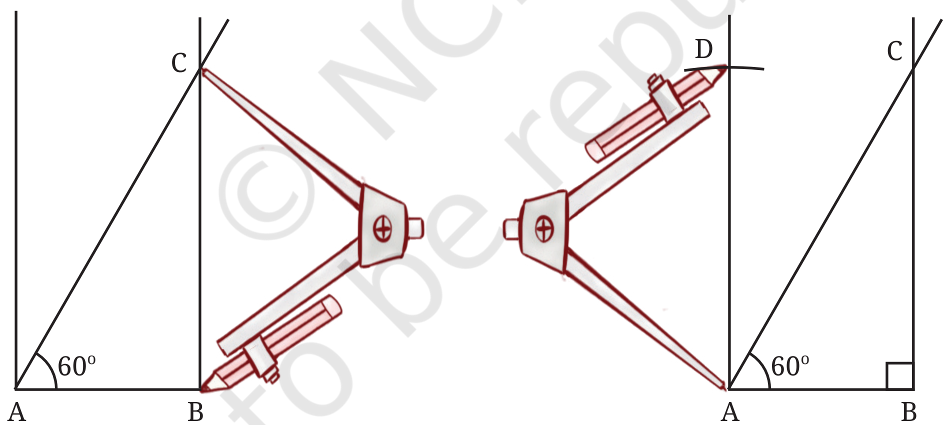
STEP 4

Method 1



Draw a line perpendicular to BC at C to get the point D.

Method 2



Using a compass, mark the point D such that AD = BC.
Join CD to get the required rectangle.

We have seen how to construct rectangles when their sides are given. But what do we do if a side and a diagonal is given?

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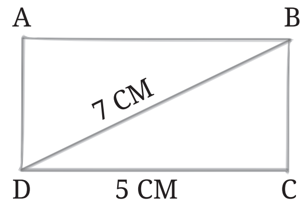
2. Construct a rectangle where one of its sides is 5 cm and the length of a diagonal is 7 cm.

Solution

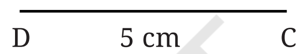
Let us draw a rough diagram.



Let us decide the steps of construction. Which line can be drawn first?

STEP 1

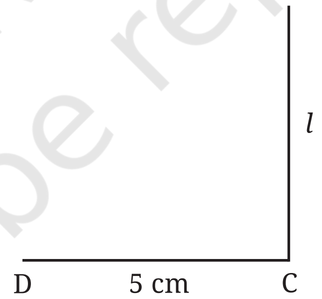
The base CD measuring length 5 cm can be easily constructed.



Next?

STEP 2

Draw a perpendicular to line DC at the point C. Let us call this line *l*.



This is easy as we know that this line is perpendicular to the base. The point B should be somewhere on this line *l*.

How do we spot it? What else do we know about the position of B?

We know that it is of distance 7 cm from the point D.

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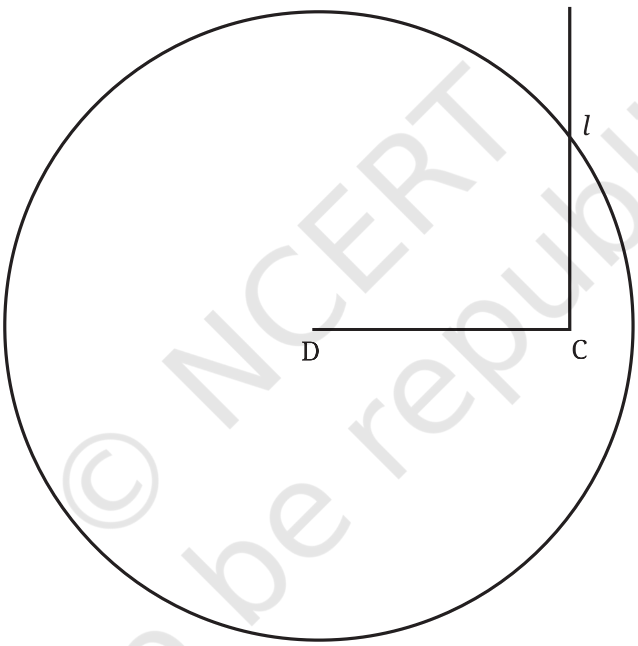
One of the ways of marking B is by taking a ruler and trying to move it around to get a point on line *l* that is 7 cm from point D. However, this requires trial and error. There is another efficient method which doesn't involve trial and error.

For this, instead of trying to get that one required point of distance 7 cm from D, let us explore a way of getting all the points of distance 7 cm from D.

We know what this shape is!

STEP 3
Method 1



Construct a circle of radius 7 cm with point D as the centre.

Can you spot the point B here? Remember that it is 7 cm away from point D and on the line *l*.

Consider the point at which the circle and the line intersect. What is its distance from point D? If needed, check your figure. What do you observe?

The point where the circle intersects the line *l* is the required point B.

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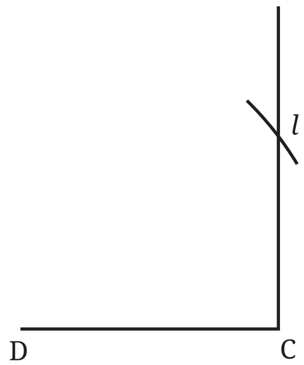
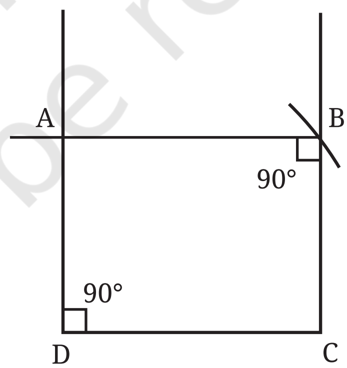
Method 2

To locate the point B, was it necessary to draw the entire circle? We can see that only the arc near the line *l* is needed. So, the third step can also be done as shown in the figure below.



Having marked the three points of the rectangle, we only need to complete it. Recall that we were in a similar situation in the previous problem also. We saw two methods of completing the rectangle from here. We could follow any one of those methods.

STEP 4



Construct perpendiculars to DC and BC passing through D and B, respectively. The point where these lines intersect is the fourth point A.

Check if ABCD is indeed a rectangle satisfying properties R1 and R2.

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Construct

1. Construct a rectangle in which one of the diagonals divides the opposite angles into 50° and 40°.
2. Construct a rectangle in which one of the diagonals divides the opposite angles into 45° and 45°. What do you observe about the sides?
3. Construct a rectangle one of whose sides is 4 cm and the diagonal is of length 8 cm.
4. Construct a rectangle one of whose sides is 3 cm and the diagonal is of length 7 cm.

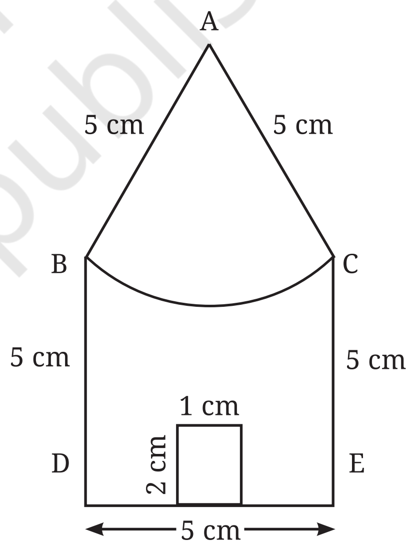
8.6 Points Equidistant from Two Given Points

Construct

House

Recreate this figure.

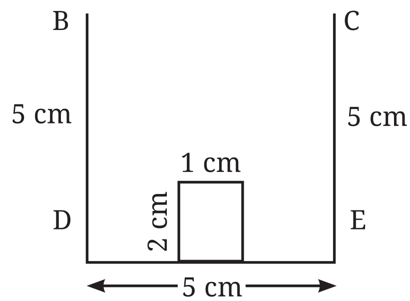
Note that all the lines forming the border of the house are of length 5 cm.



Solution

The first task is to identify in what sequence the lines and curve will have to be drawn

STEP 1



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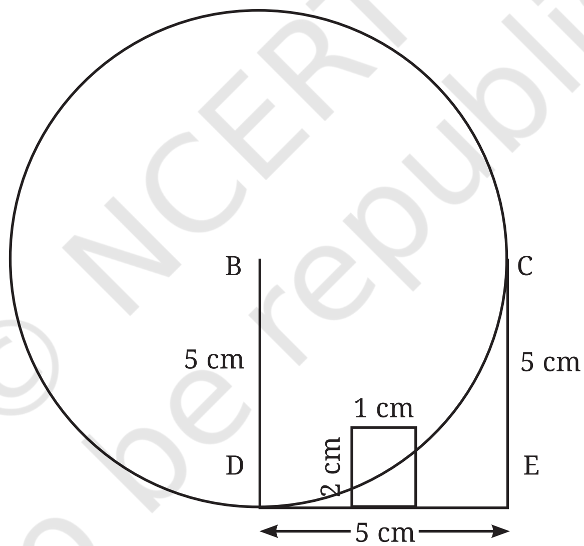
Can you complete the figure? Try!

We need to locate the point A that is of distance 5 cm from the points B and C. You might have realised that this can be done using a ruler. However, this leads to a lot of trial and error. This construction can be further simplified. How?

If you have guessed that this can be done by the use of compass, you are right! Go ahead and explore how the point A can be located without trial and error.

There is a similarity between the problem of finding point A in this problem and point B at step 3 of the second solved example of the previous section (see page 209).

STEP 2



Draw a curve that has all its points of 5 cm from the point B; the circle centred at B should be with 5 cm radius.

Does this help in spotting the point A? Construct and explore in the figure.

The point A can be located by finding the correct point on the circle that is of distance 5 cm from the point C. Again, this can be done using a ruler. But can we use a compass for this?

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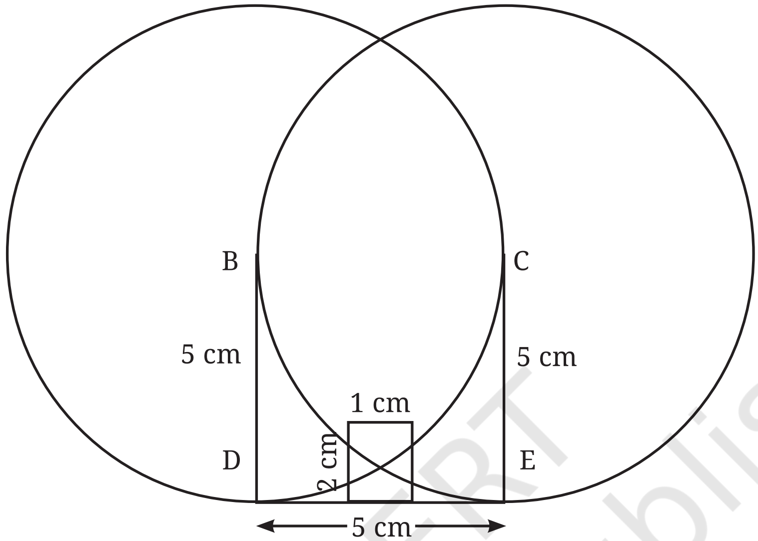
STEP 3
Method 1

Take a radius of 5 cm in the compass and with C as the centre, draw a circle.



Are you able to spot the point A? Check the figure on your notebook. What do you observe?

See the point at which both the circles intersect. How far is it from the point B?

How far is it from C?

Thus, this is the point A!

Think

Was it necessary to draw two full circles to get the point A? We only needed part of both the circles.

Method 2

So the point A could have been obtained just by drawing arcs of radius 5 cm from points B and C.

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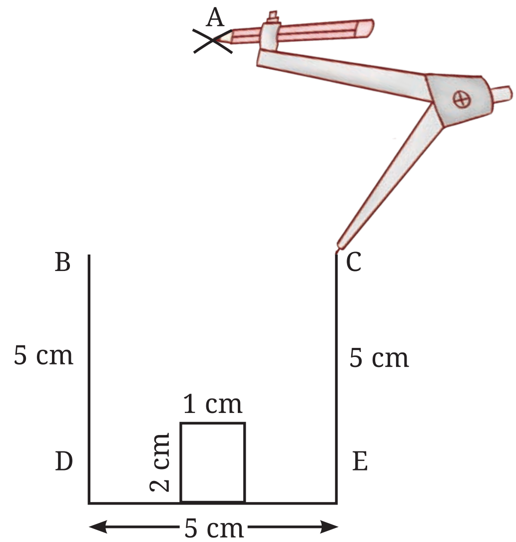


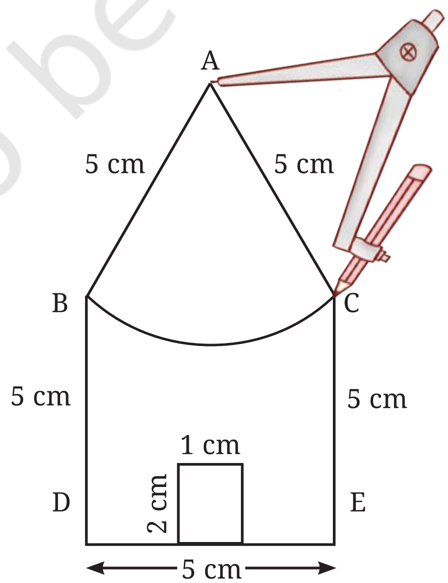
Join A to B and A to C by straight lines.

Having obtained point A, what remains is the construction of the remaining arc. How do we do it?

Can we use the fact that A is of distance 5 cm from both B and C?

STEP 4

Take 5 cm radius in the compass and from A, draw the arc touching B and C as shown in the figure.



The house is ready!

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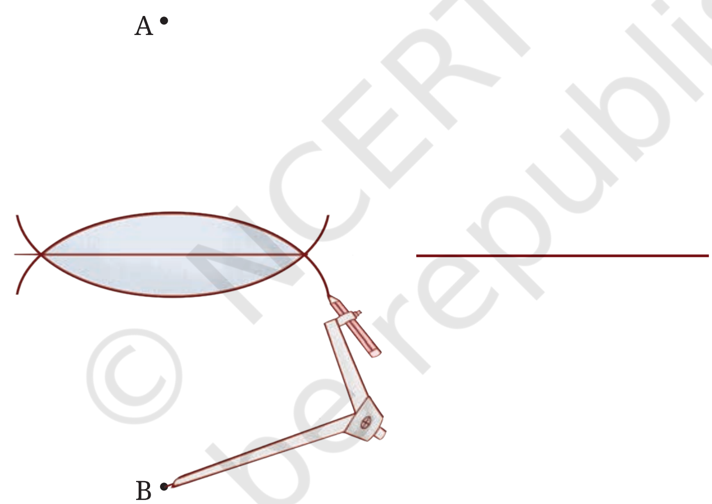
Construct

1. Construct a bigger house in which all the sides are of length 7 cm.
2. Try to recreate 'A Person', 'Wavy Wave' and 'Eyes' from the section Artwork, using ideas involved in the 'House' construction.
3. Is there a 4-sided figure in which all the sides are equal in length but is not a square? If such a figure exists, can you construct it?

Hints

A) Eyes (from 8.1 Artwork and construct 2 above)

Part of the construction is shown. Observe it carefully. You will see two horizontal lines drawn lightly. In geometric constructions, one often constructs supporting curves or figures that are not part of the given figure but help in constructing it.



The technique to draw the upper and the lower curves of the eye is the same as that used in the figure "A Person". Points A and B are the locations where the tip of the compass is placed when drawing the curves of the eye. Note that the upper curve and the lower curve should together form a symmetrical figure. For this to happen, where should these points A and B be placed? Make a good estimate.

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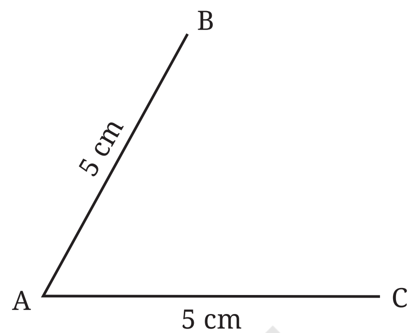
Try to get the eyes as symmetrical and identical as possible. This might need many trials.

B) (From Construct 3 above)

For the purpose of construction, let us take the side lengths to be of 5 cm. Consider this figure.



We need to identify only one more point to make this a 4-sided figure. That point, let us call it D, should be 5 cm from both B and C. How can such a point be found?

Can any of the ideas used in the 'House' problem be used here?

SUMMARY

- All the points of a circle are at the same distance from its **centre**. This distance is called the **radius** of the circle.
- A compass can be used to construct circles and their parts.
- A rough diagram can be useful in planning how to construct a given figure.
- A rectangle can be constructed given the lengths of its sides or that of one of its sides and a diagonal.

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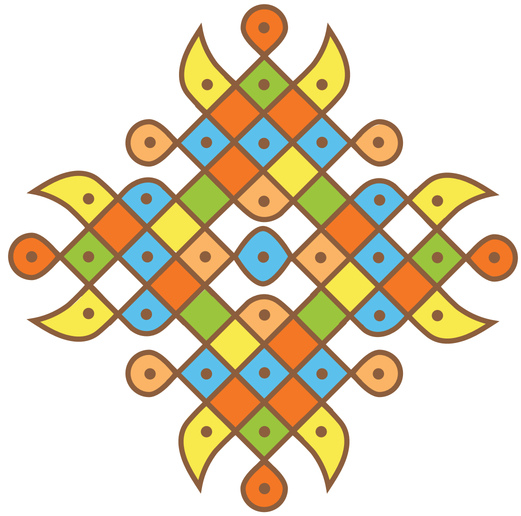
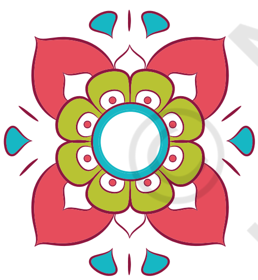


SYMMETRY

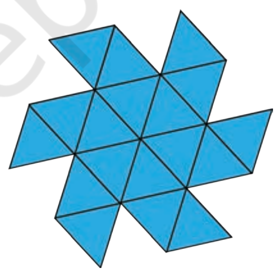
Look around you—you may find many objects that catch your attention. Some such things are shown below:

Flower

Butterfly



Rangoli



Pinwheel

There is something beautiful about the pictures above.

The flower looks the same from many different angles. What about the butterfly? No doubt, the colours are very attractive. But what else about the butterfly appeals to you?

In these pictures, it appears that some parts of the figure are repeated and these repetitions seem to occur in a definite pattern. Can you see what repeats in the beautiful *rangoli* figure? In the

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rangoli, the red petals come back onto themselves when the flower is rotated by 90° around the centre and so do the other parts of the *rangoli*.

What about the pinwheel? Can you spot which pattern is repeating? **Hint:** Look at the hexagon first.

Clouds

Now, can you say what figure repeats along each side of the hexagon? What is the shape of the figure that is stuck to each side? Do you recognise it? How do these shapes move as you move along the boundary of the hexagon? What about the other pictures—what is it about those structures that appeals to you and what are the patterns in those structures that repeat?

On the other hand, look at this picture of clouds. There is no such repetitive pattern.

We can say that the first four figures are symmetrical. and the last one is not symmetrical. A symmetry refers to a part or parts of a figure that are repeated in some definite pattern.

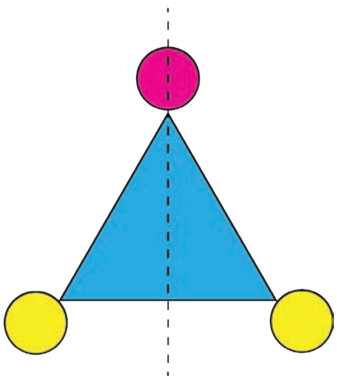
Taj Mahal

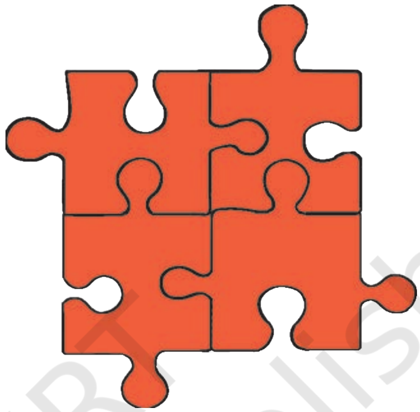
Gopuram

What are the symmetries that you see in these beautiful structures?

9.1 Line of Symmetry

Figure (a) shows the picture of a blue triangle with a dotted line. What if you fold the triangle along the dotted line? Yes, one half of the triangle covers the other half completely. These are called **mirror halves**!


(a)

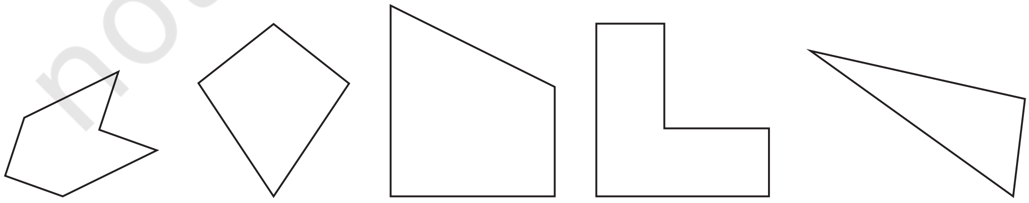

(b)

What about Figure (b) with the four puzzle pieces and a dotted line passing through the middle? Are they mirror halves? No, when we fold along the line, the left half does not exactly fit over the right half.

A line that cuts a figure into two parts that exactly overlap when folded along that line is called a **line of symmetry** of the figure.

Figure it Out

1. Do you see any line of symmetry in the figures at the start of the chapter? What about in the picture of the cloud?
2. For each of the following figures, identify the line(s) of symmetry if it exists.



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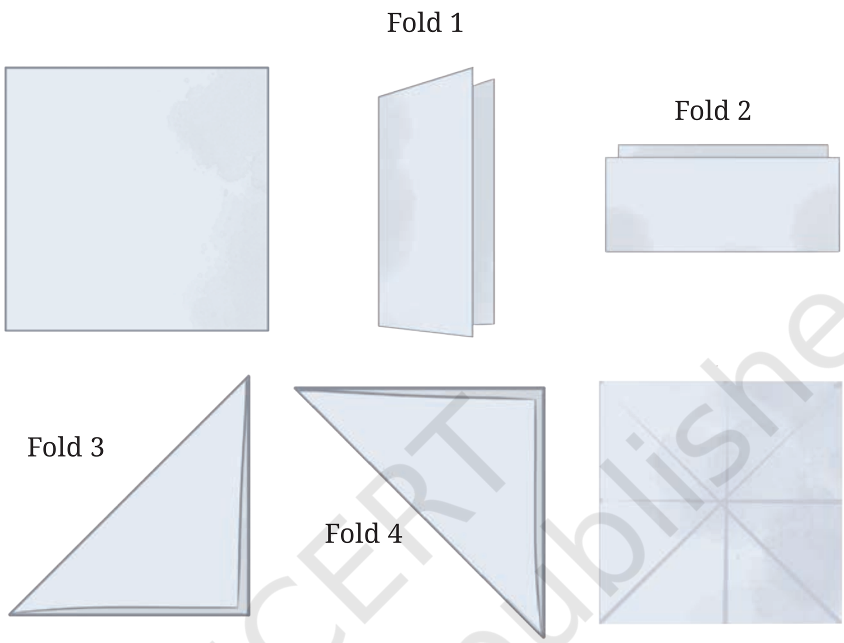
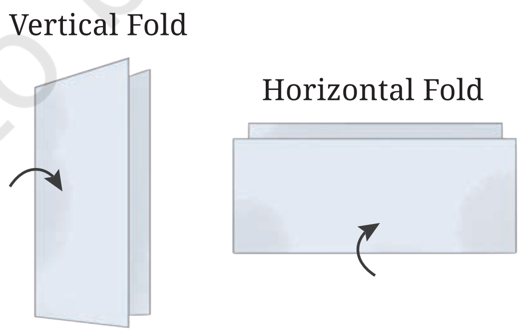
Figures with more than one line of symmetry

Does a square have only one line of symmetry?
Take a square piece of paper. By folding, find all its lines of symmetry.



Here are the different folds giving different lines of symmetry.

- Fold the paper into half vertically.
- Fold it again into half horizontally. (i.e. you have folded it twice). Now open out the folds.



Again fold the square into half (for a third time now), but this time along a diagonal, as shown in the figure. Again, open it.

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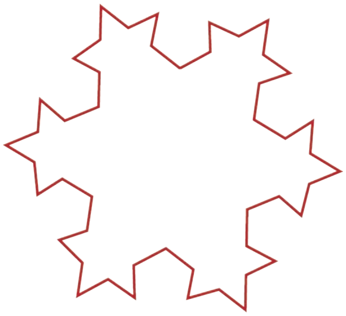
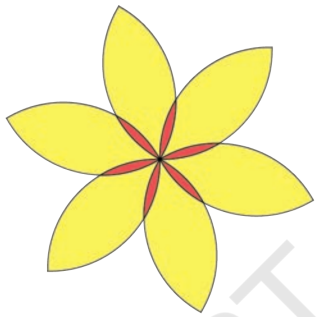
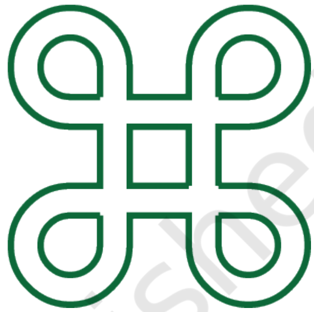
Fold it into half (for the fourth time), but this time along the other diagonal, as shown in the figure. Open out the fold.

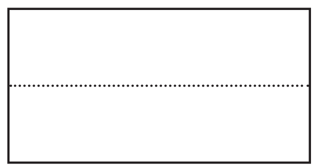
Is there any other way to fold the square so that the two halves overlap?How many lines of symmetry does the square shape have?

Thus, figures can have multiple lines of symmetry. The figures below also have multiple lines of symmetry. Can you find them all?.

We saw that the diagonal of a square is also a line of symmetry. Let us take a rectangle that is not a square. Is its diagonal a line of symmetry?

First, see the rectangle and answer this question. Then, take a rectangular piece of paper and check if the two parts overlap by folding it along its diagonal. What do you observe?



Reflection

So far we have been saying that when we fold a figure along a line of symmetry, the two parts overlap completely. We could also say that the part of the figure on one side of the line of symmetry is reflected by the line to the other side; similarly, the part of the figure on the other side of the line of symmetry is reflected to the first side! Let us understand this by labeling some points on the figure.

The figure shows a square with its corners labeled A, B, C and D. Let us first consider the vertical line of symmetry. When we reflect

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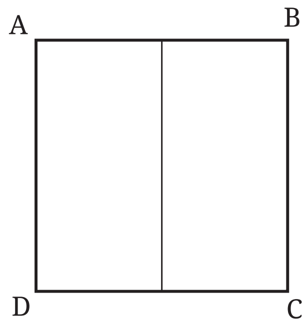
the square along this line, the points B, C on the right get reflected to the left side and occupy the positions occupied earlier by A, D. What happens to the points A, D? A occupies the position occupied by B and D that of C!



What if we reflect along the diagonal from A to C? Where do points A, B, C and D go? What if we reflect along the horizontal line of symmetry?

A figure that has a line or lines of symmetry is thus also said to have **reflection symmetry**.

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Generating Shapes having Lines of Symmetry

So far we have seen symmetrical figures and asymmetrical figures. How does one generate such symmetrical figures? Let us explore this.

Ink Blot Devils

You enjoyed doing this earlier in Class 5. Take a piece of paper. Fold it in half. Open the paper and spill a few drops of ink (or paint) on one half.

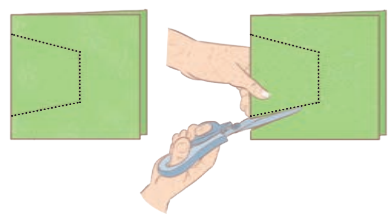
Now press the halves together and then open the paper again.

- What do you see?
- Is the resulting figure symmetric?
- If yes, where is the line of symmetry?
- Is there any other line along which it can be folded to produce two identical parts?
- Try making more such patterns.

Paper Folding and Cutting

Here is another way of making symmetric shapes!

In these two figures, a sheet of paper is folded and a cut is made along the dotted line shown. Draw a sketch of how the paper will look when unfolded.



Do you see a line of symmetry in this figure? What is it?

Make different symmetric shapes by folding and cutting.

There are more ways of folding and cutting pieces of paper to get symmetric shapes!

Use thin rectangular coloured paper. Fold it several times and create some intricate patterns by cutting the paper, like the one shown here. Identify the lines of symmetry in the repeating design. Use such decorative paper cut-outs for festive occasions.

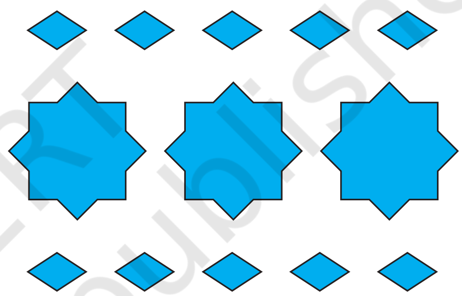
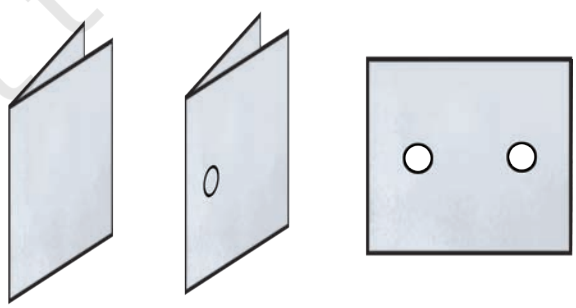


Figure it Out

Punching Game

The fold is a line of symmetry. Punch holes at different locations of a folded square sheet of paper using a punching machine and create different symmetric patterns.



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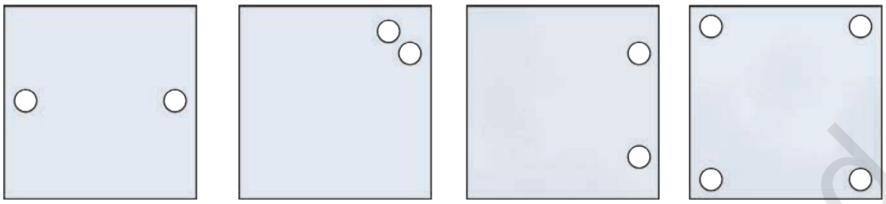
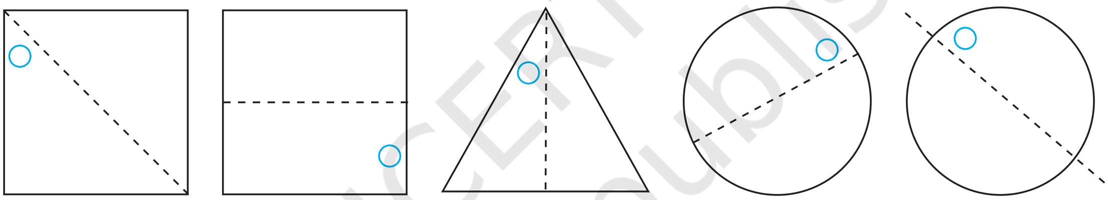
1. In each of the following figures, a hole was punched in a folded square sheet of paper and then the paper was unfolded. Identify the line along which the paper was folded.
   Figure (d) was created by punching a single hole. How was the paper folded?



a. b. c. d.

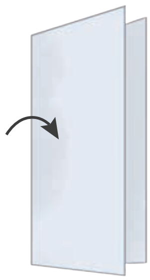
2. Given the line(s) of symmetry, find the other hole(s):



a. b. c. d. e.

3. Here are some questions on paper cutting.

Consider a vertical fold. We represent it this way:



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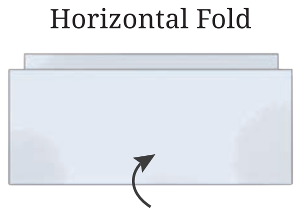
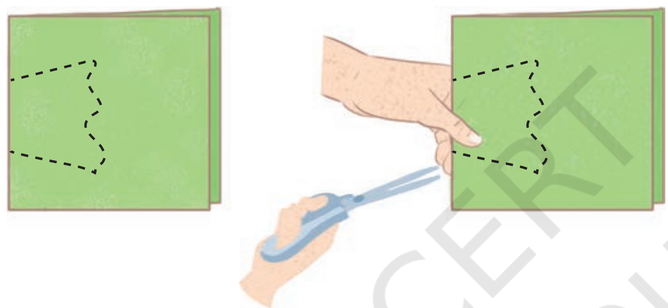
Similarly, a horizontal fold is represented as follows.

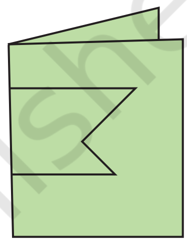


4. After each of the following cuts, predict the shape of the hole when the paper is opened. After you have made your prediction, make the cutouts and verify your answer.

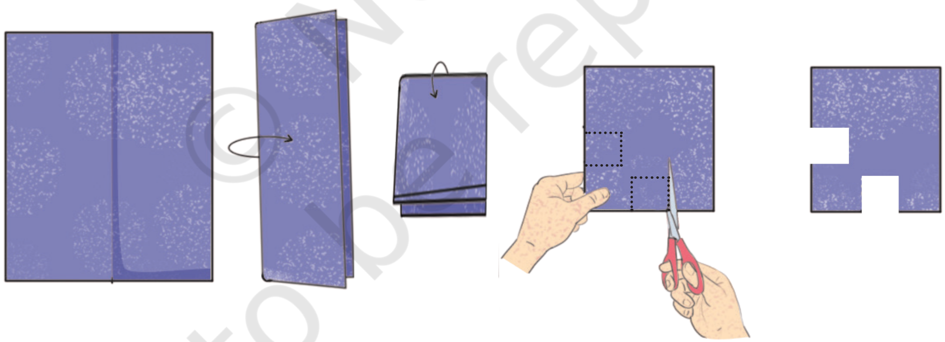
a.



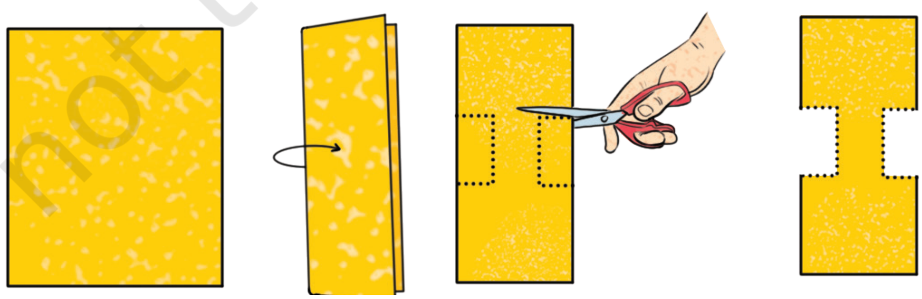
b.



c.



d.



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5 Suppose you have to get each of these shapes with some folds and a single straight cut. How will you do it?

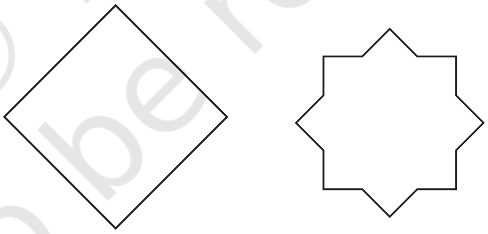
a. The hole in the centre is a square.

b. The hole in the centre is a square.

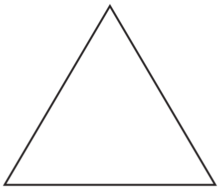
Note: For the above two questions, check if the 4-sided figures in the centre satisfy both the properties of a square.

6. How many lines of symmetry do these shapes have?

i.



ii. A triangle with equal sides and equal angles.



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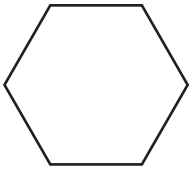
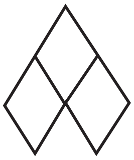
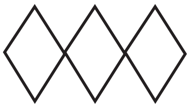
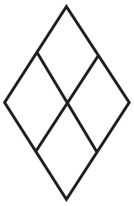
iii. A hexagon with equal sides and equal angles.

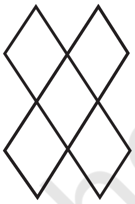


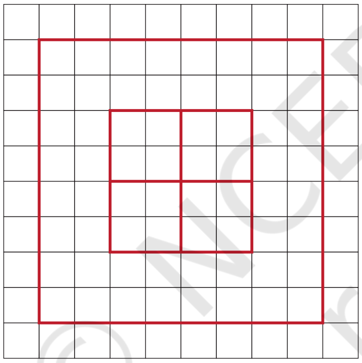
7. Trace each figure and draw the lines of symmetry, if any:

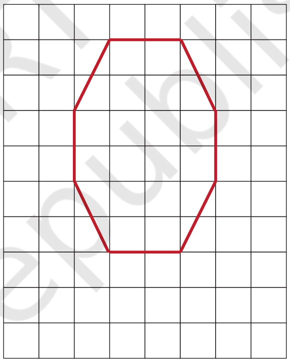


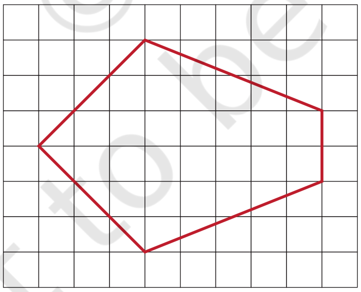


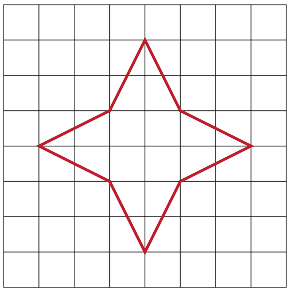












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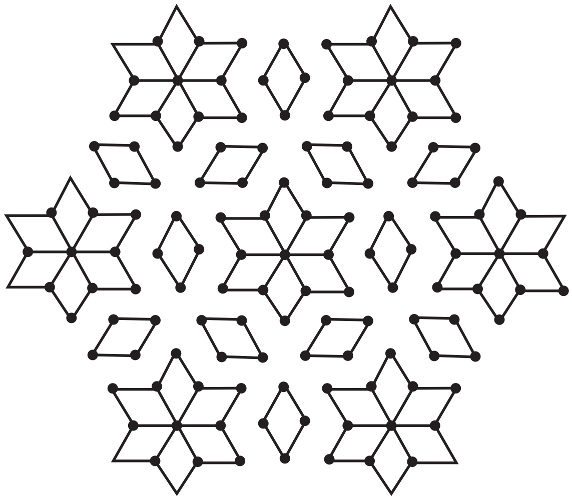
8. Find the lines of symmetry for the *kolam* below.

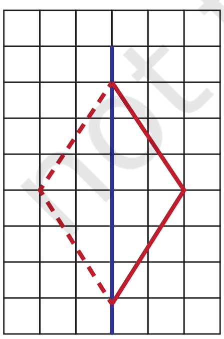


9. Draw the following.
   a. A triangle with exactly one line of symmetry
   b. A triangle with exactly three lines of symmetry
   c. A triangle with no line of symmetry

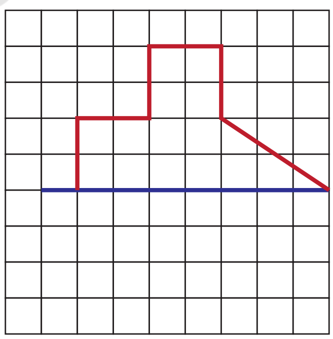
   Is it possible to draw a triangle with exactly two lines of symmetry?

10. Draw the following. In each case, the figure should contain at least one curved boundary.
    a. A figure with exactly one line of symmetry
    b. A figure with exactly two lines of symmetry
    c. A figure with exactly four lines of symmetry

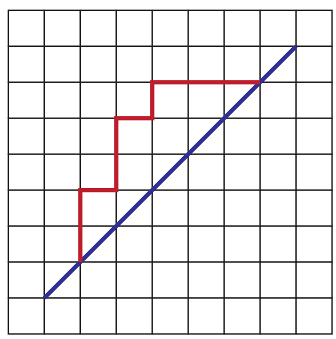
11. Copy the following on squared paper. Complete them so that the blue line is a line of symmetry. Problem (a) has been done for you.



(a)



(b)



(c)

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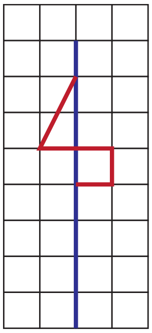
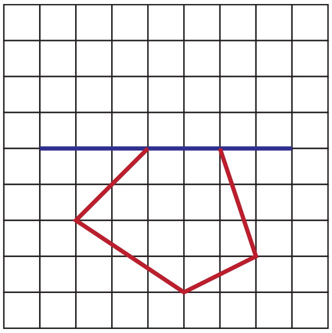
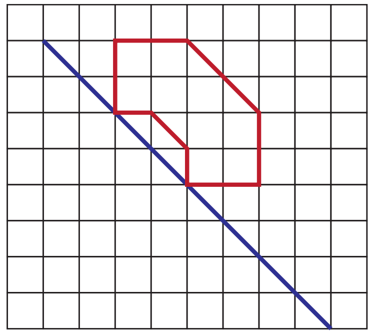


(d)



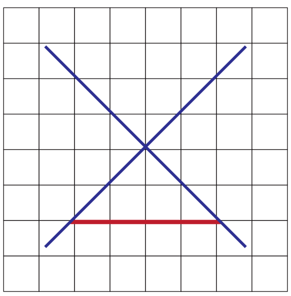
(e)



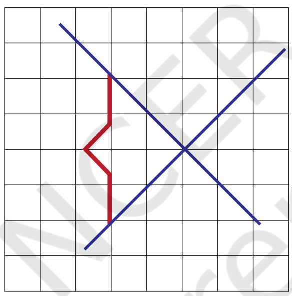
(f)

Hint: For (c) and (f), see if rotating the book helps!

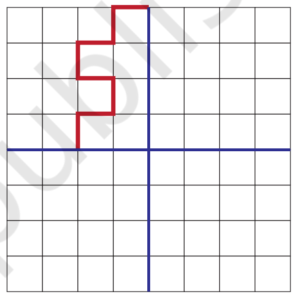
12. Copy the following drawing on squared paper. Complete each one of them so that the resulting figure has the two blue lines as lines of symmetry.



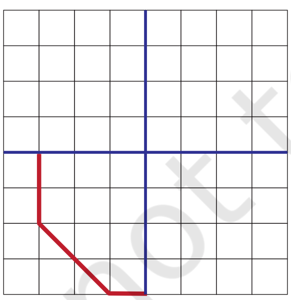
(a)



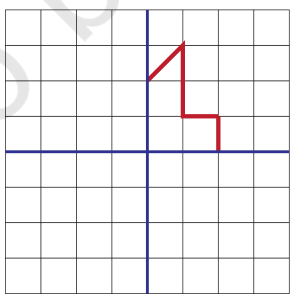
(b)



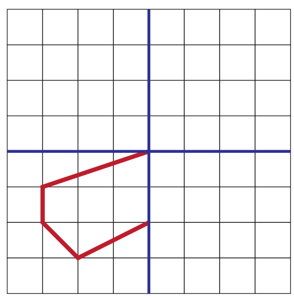
(c)



(d)



(e)



(f)

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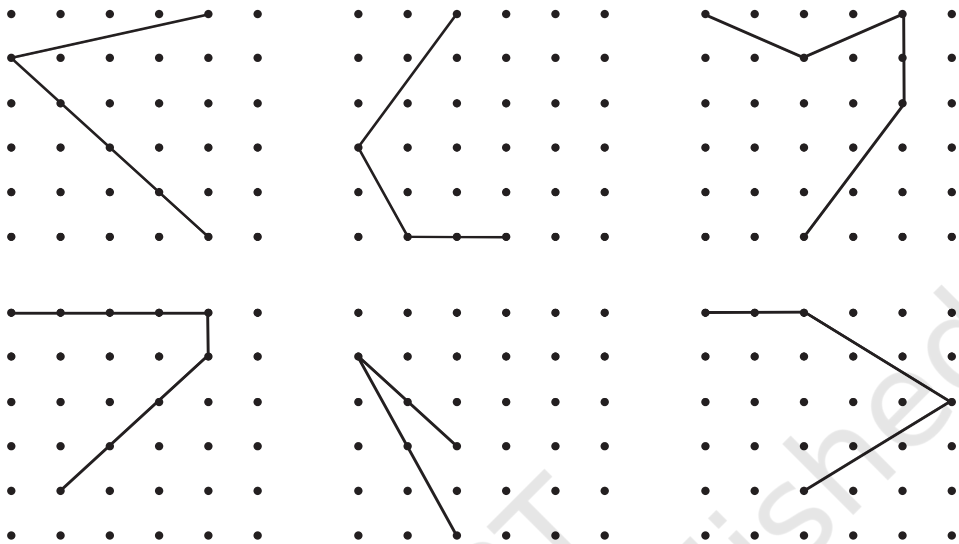
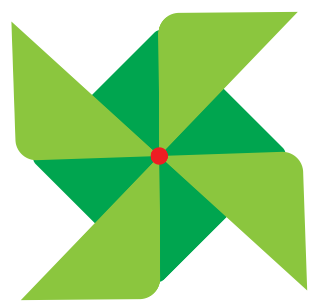
13. Copy the following on a dot grid. For each figure draw two more lines to make a shape that has a line of symmetry.



9.2 Rotational Symmetry



The paper windmill in the picture looks symmetrical but there is no line of symmetry! However you fold it, the two halves will not exactly overlap. On the other hand, if you rotate it by 90° about the red point at the centre, the windmill looks exactly the same.

We say that the windmill has **rotational symmetry**.

When talking of rotational symmetry, there is always a fixed point about which the object is rotated. This fixed point is called the **centre of rotation**.

Will the windmill above look exactly the same when rotated through an angle of less than 90°?

No!

An angle through which a figure can be rotated to look exactly the same is called an **angle of rotational symmetry**, or just an **angle of symmetry**, for short.

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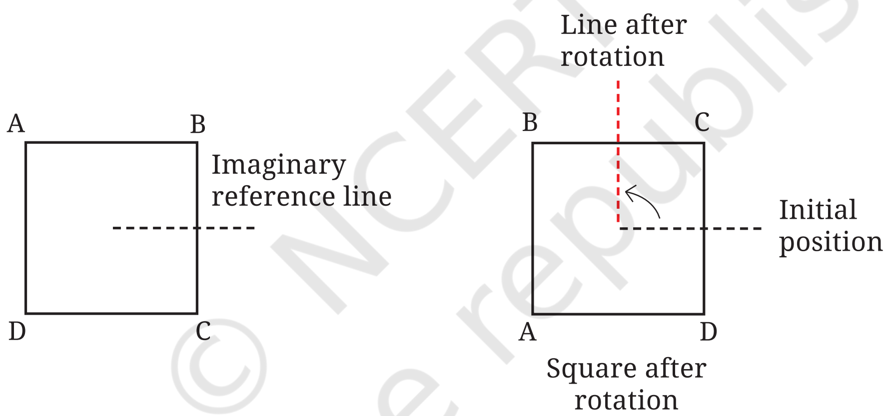
For the windmill, the angles of symmetry are 90° (quarter turn), 180° (half turn), 270° (three-quarter turn) and 360° (full turn). Observe that when any figure is rotated by 360°, it comes back to its original position, so 360° is always an angle of symmetry.

Thus, we see that the windmill has 4 angles of symmetry.

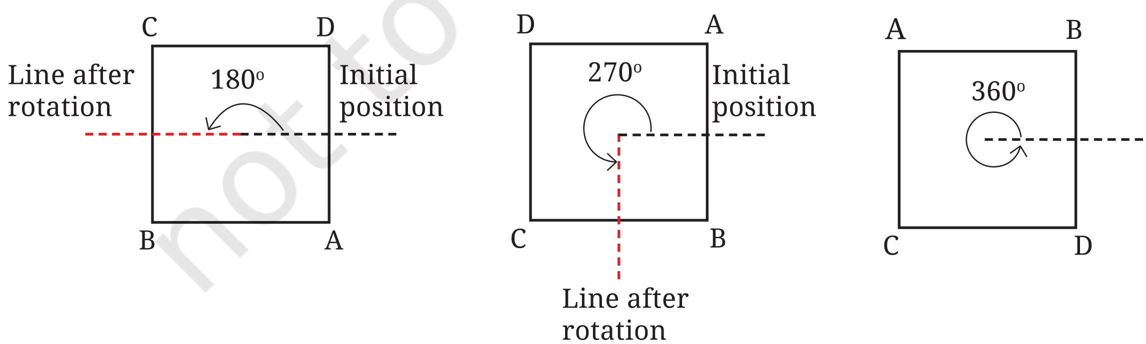
Do you know of any other shape that has exactly four angles of symmetry?

How many angles of symmetry does a square have? How much rotation does it require to get the initial square?

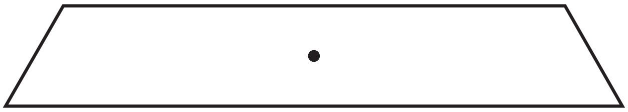
We get back a square overlapping with itself after 90° of rotation. This takes point A to the position of point B, point B to the position of point C, point C to the position of point D and point D back to the position of point A. Do you know where to mark the centre of rotation?



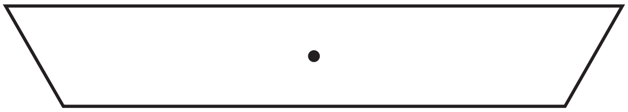
What are the other angles of symmetry?



Example: Find the angles of symmetry of the following strip.



Solution: Let us rotate the strip in a clockwise direction about its centre.



A rotation of 180° results in the figure above. Does this overlap with the original figure.

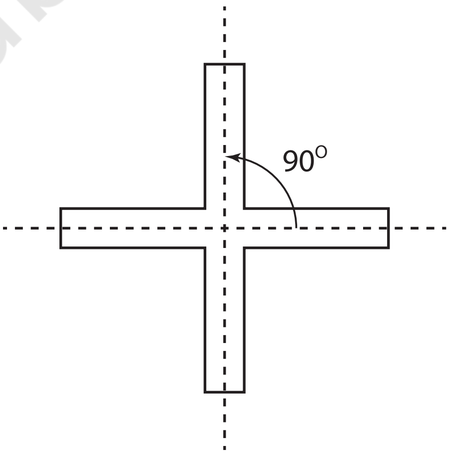
No. Why?

Another rotation through 180° from this position gives the original shape.

This figure comes back to its original shape only after **one** complete rotation through 360°. So we say that this figure **does not have rotational symmetry**

Rotational Symmetry of Figures with Radial Arms

Consider this figure, a picture with 4 radial arms. How many angles of symmetry does it have? What are they? Note that the angle between adjacent central dotted lines is 90°.



Can you change the angles between the radial arms so that the figure still has 4 angles of symmetry? Try drawing it.

To check if the figure drawn indeed has 4 angles of symmetry, you could draw the figure on two different pieces of paper. Cut out the radial arms from one of the papers. Keep the figure on the paper fixed and rotate the cutout to check for rotational symmetry.

How will you modify the figure above so that it has only two angles of symmetry?

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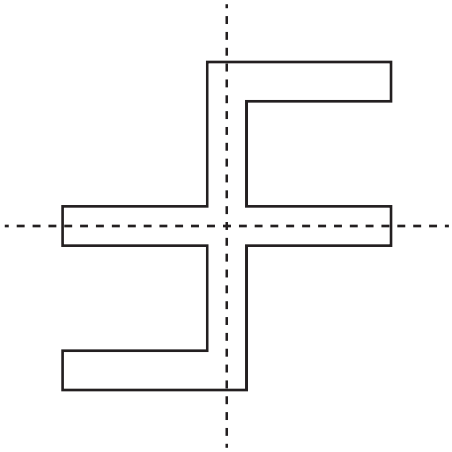
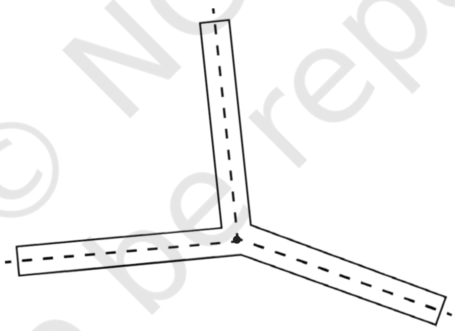
Here is one way:



We have seen figures having 4 and 2 angles of symmetry. Can we get a figure having exactly 3 angles of symmetry? Can you use radial arms for this?

Let us try with 3 radial arms as in the figure below. How many angles of symmetry does it have and what are they?

Here is a figure with three radial arms.



Trace and cut out a copy of this figure. By rotating the cutout over this figure determine its angles of rotation.

We see that only a full turn or a rotation of 360° will bring the figure back into itself. So this figure does not have rotational symmetry as 360 degrees is its only angle of symmetry.

However, can anything in the figure be changed to make it have 3 angles of symmetry?

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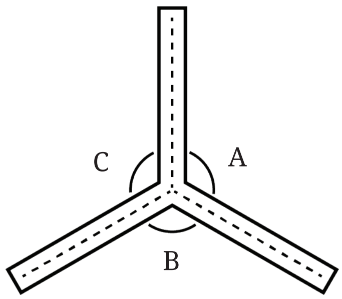
Can it be done by changing the angles between the dotted lines?

If a figure with three radial arms should have rotational symmetry, then a rotated version of it should overlap with the original. Here are rough diagrams of both of them.

If these two figures must overlap, what can you tell about the angles?



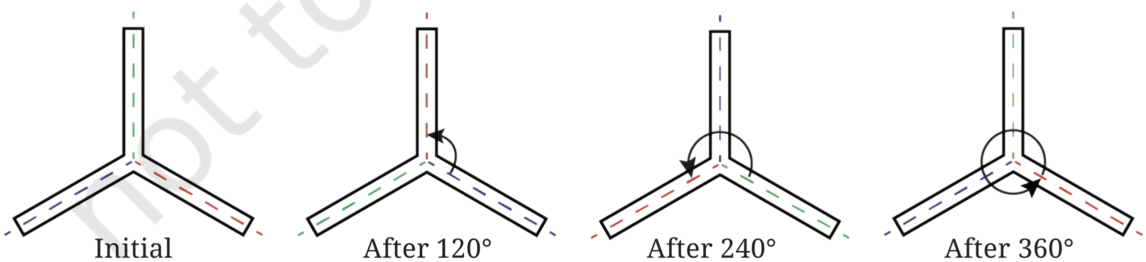
Observe that ∠A must overlap ∠B, ∠B must overlap ∠C and ∠C must overlap ∠A.

So, ∠A = ∠B = ∠C. What must this angle be?

We know that a full turn has 360 degrees. This is equally distributed amongst these three angles. So each angle must be 360°/3= 120°.

So, the radial arms figure with 3 arms shows rotational symmetry when the angle between the adjacent dotted lines is 120 deg. Use paper cutouts to verify this observation.

Now how many angles of rotation does the figure have and what are they?



Note: The colours have been added to show the rotations.

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Let us explore more figures.

Can you draw a figure with radial arms that has a) exactly 5 angles of symmetry, b) 6 angles of symmetry? Also find the angles of symmetry in each case.

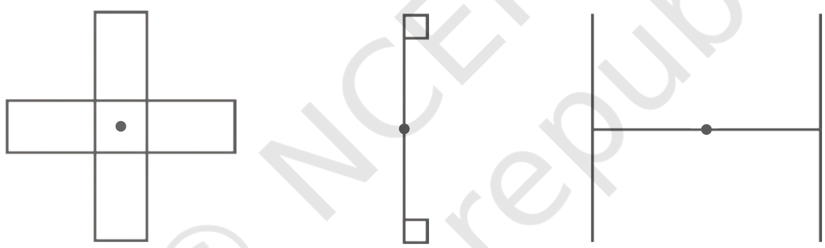
Hint: Use 5 radial arms for the first case. What should the angle between two adjacent radial arms be?

Consider a figure with radial arms having exactly 7 angles of symmetry. What will be its smallest angle of symmetry? Is the number of degrees a whole number in this case? If not, express it as a mixed faction.

Let us find the angles of symmetry for other kinds of figures.

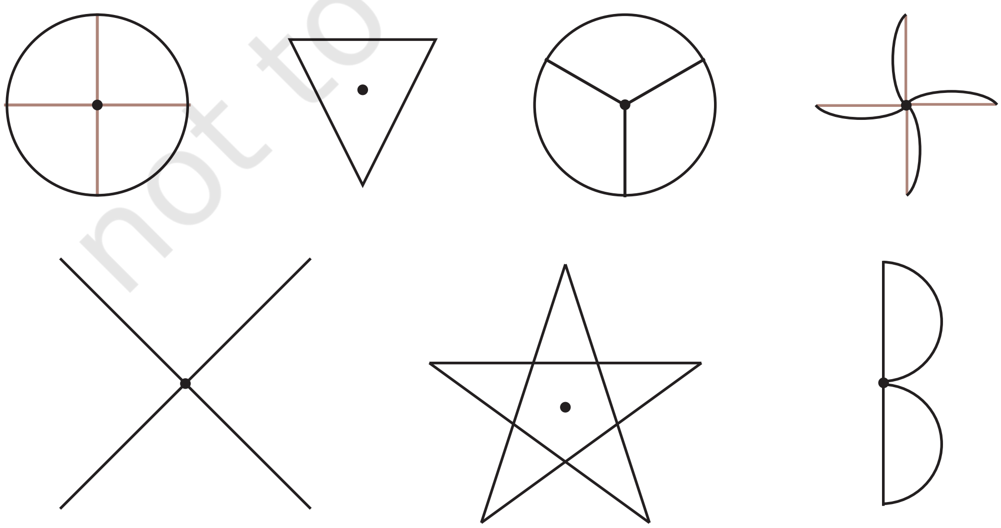
Figure it Out

1. Find the angles of symmetry for the given figures about the point marked •.



(a) (b) (c)

2. Which of the following figures have more than one angle of symmetry?



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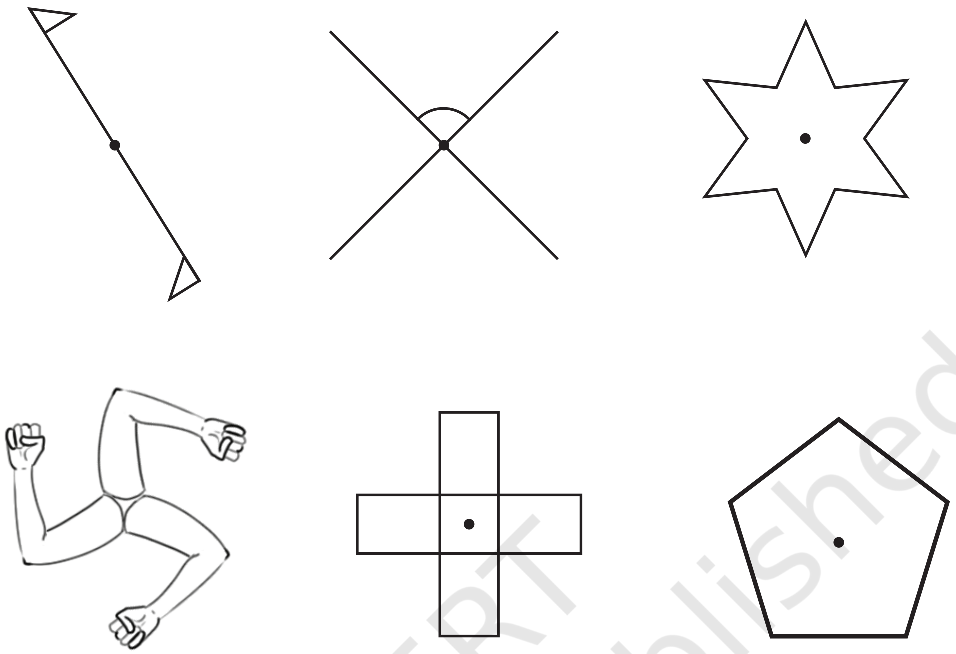
3. Give the order of rotational symmetry for each figure:



Let us list down the angles of symmetry for all the cases above.

- Angles of symmetry when there are exactly 2 of them: 180°, 360°.
- Angles of symmetry when there are exactly 3 of them: 120°, 240°, 360°.
- Angles of symmetry when there are exactly 4 of them: 90°, 180°, 270°, 360°.

Do you observe something common about the angles of symmetries in these cases? The first set of numbers are all multiples of 180. The second are all multiples of 120. The third are all multiples of 90.

In each case, the angles are the multiples of the smallest angle. You may wonder and ask if this will always happen. What do you think?

True or False

- Every figure will have 360 degrees as an angle of symmetry.

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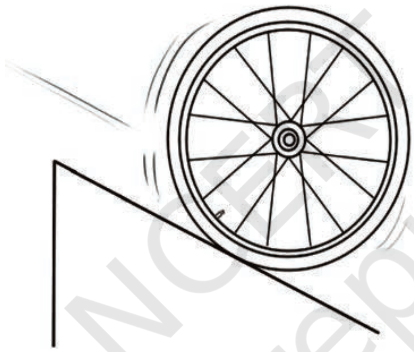
- If the smallest angle of symmetry of a figure is a natural number in degrees, then it is **a factor** of 360.

Is there a smallest angle of symmetry for all figures? It turns out that this is the case for most figures, except for the most symmetric shapes like the circle, whose symmetries we now discuss.

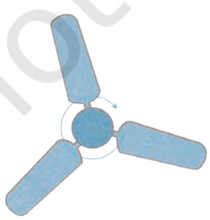
Symmetries of a circle

The circle is a fascinating figure. What happens when you rotate a circle clockwise about its centre? It coincides with itself. It does not matter what angle you rotate it by! So, for a circle, every angle is an angle of symmetry.

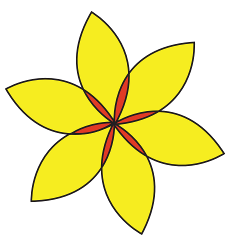


Now take a point on the rim of the circle and join it to the centre. Extend the segment to a diameter of the circle. Is that diameter a line of reflection symmetry? It is. Every diameter is a line of symmetry!

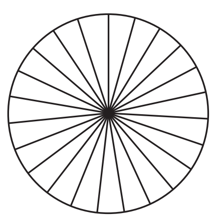
Like wheels, we can find other objects around us having rotational symmetry. Find them. Some of them are shown below:



Fan



Flower



Wheel

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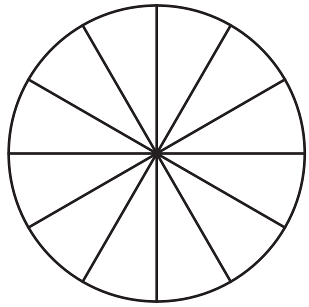
Figure it Out

1. Color the sectors of the circle below so that the figure has i) 3 angles of symmetry, ii) 4 angles of symmetry, iii) what are the possible numbers of angles of symmetry you can obtain by coloring the sectors in different ways?



2. Draw two figures other than a circle and a square that have both reflection symmetry and rotational symmetry.
3. Draw, wherever possible, a rough sketch of
   a. A triangle with at least two lines of symmetry and at least two angles of symmetry.
   b. A triangle with only one line of symmetry but not having rotational symmetry.
   c. A quadrilateral with rotational symmetry but no reflection symmetry.
   d. A quadrilateral with reflection symmetry but not having rotational symmetry.

Try This

4. In a figure, 60° is the smallest angle of symmetry. What are the other angles of symmetry of this figure?
5. In a figure, 60° is an angle of symmetry. The figure has two angles of symmetry less than 60°. What is its smallest angle of symmetry?
6. Can we have a figure with rotational symmetry whose smallest angle of symmetry is
   a. 45°?
   b. 17°?

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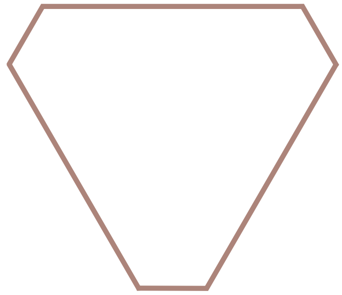
7. This is a picture of the new Parliament Building in Delhi.



   a. Does the outer boundary of the picture have reflection symmetry? If so, draw the lines of symmetries. How many are they?
   b. Does it have rotational symmetry around its centre? If so, find the angles of rotational symmetry.

8. How many lines of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?
9. How many angles of symmetry do the shapes in the first shape sequence in Chapter 1, Table 3, the Regular Polygons, have? What number sequence do you get?
10. How many lines of symmetry do the shapes in the last shape sequence in Chapter 1, Table 3, the Koch Snowflake sequence, have? How many angles of symmetry?
11. How many lines of symmetry and angles of symmetry does Ashoka Chakra have?

14. Playing with Tiles

   a. Use the color tiles  given at the end of the book to complete the following figure so that it has exactly 2 lines of symmetry.
   b. Use 16 such tiles to make figures that have exactly:
      1 line of symmetry,
      2 lines of symmetry
   c. Use these tiles in making creative symmetric designs.

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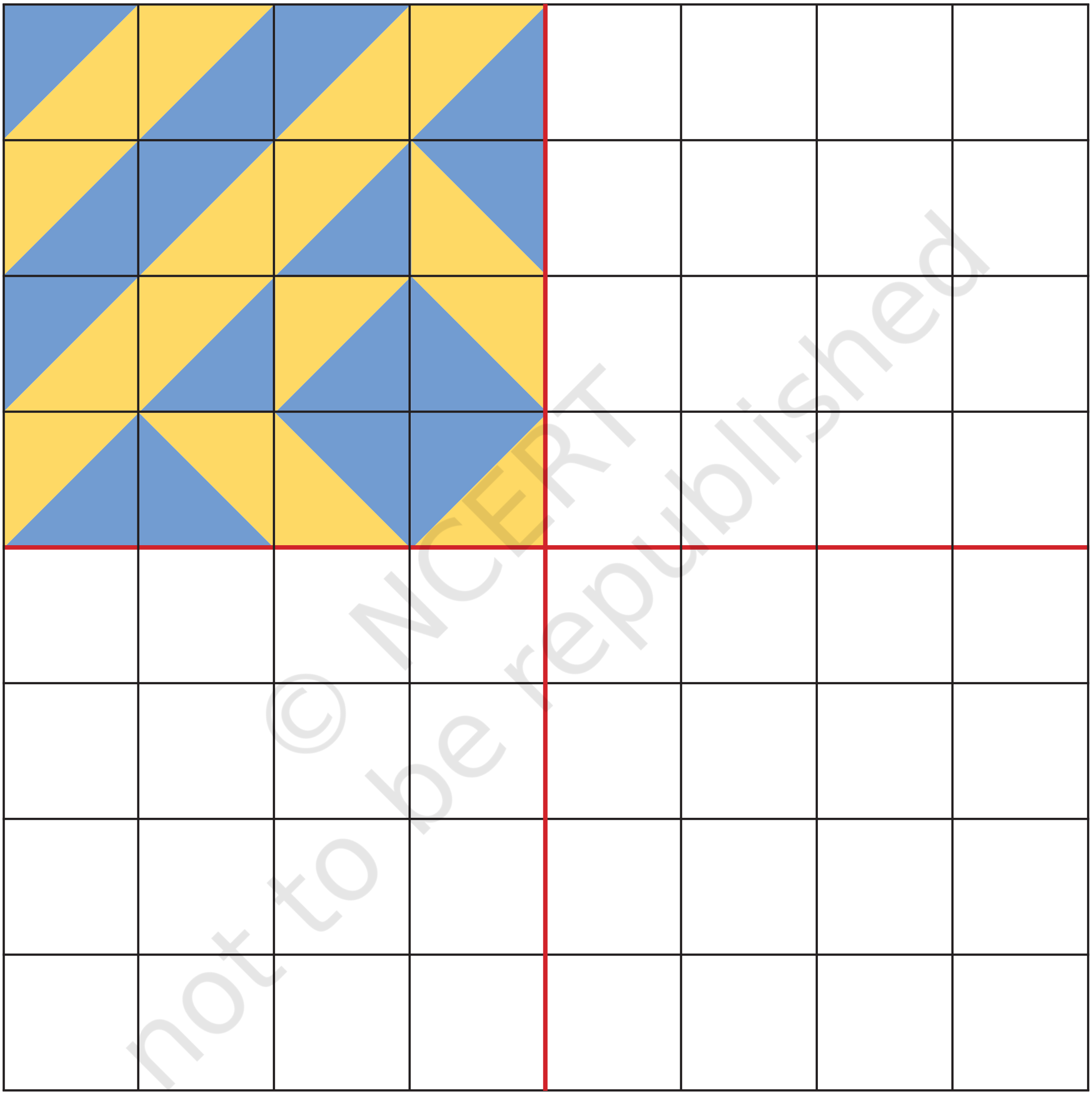

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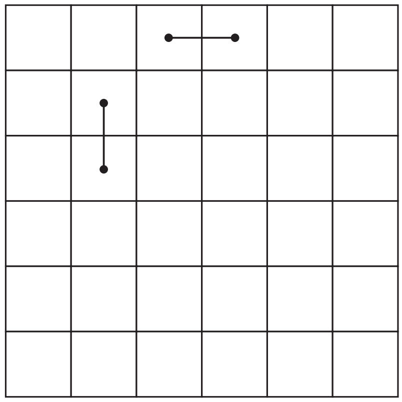
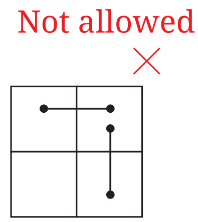
Game

Draw a 6 by 6 grid. Two players take turns covering two adjacent squares by drawing a line. The line can be placed either way: horizontally or vertically. The lines cannot overlap. The game goes on till a player is not able to place any more lines. The player who is not able to place a line loses.





With what strategy can one play to win this game?

SUMMARY

- When a figure is made up of parts that repeat in a definite pattern, we say that the figure has **symmetry**. We say that such a figures is **symmetrical**.
- A line that cuts a plane figure into two parts that exactly overlap when folded along that line is called a **line of symmetry** or **axis of symmetry** of the figure.
- A figure may have multiple lines of symmetry.
- Sometimes a figure looks exactly the same when it is rotated by an angle about a fixed point. Such an angle is called an **angle of symmetry** of the figure. A figure that has an angle of symmetry strictly between 0 and 360 degrees is said to have **rotational symmetry**. The point of the figure about which the rotation occurs is called the **centre of rotation**.
- A figure may have multiple angles of symmetry.
- Some figures may have a line of symmetry but no angle of symmetry, while others may have angles of symmetry but no lines of symmetry. Some figures may have both lines of symmetry as well as angles of symmetry.

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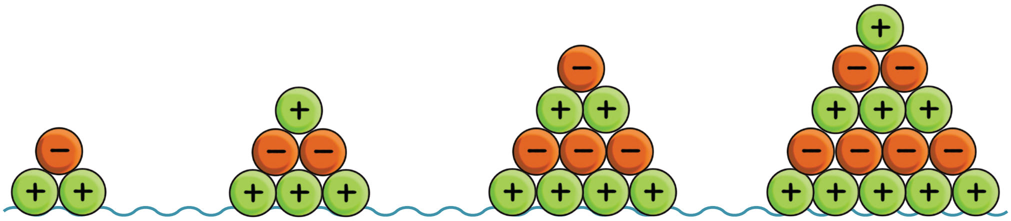

10 THE OTHER SIDE OF ZERO

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Integers

More and More Numbers!

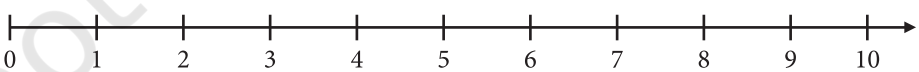
Recall that the very first numbers we learned about in the study of mathematics were the counting numbers 1, 2, 3, 4, ...

Then we learned that there are even more numbers! For example, there is the number 0 (zero), representing nothing, which comes before 1. The number 0 has a very important history in India and now in the world. For example, around the world we learn to write numbers in the Indian number system using the digits 0 to 9, allowing us to write numbers however large or however small using just these 10 digits.

We then learned about more numbers that exist between the numbers 0, 1, 2, 3, 4, ... , such as $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{13}{6}$. These are called *fractions*.

But are there still more numbers? Well, 0 is an additional number that we didn't know about earlier, and it comes before 1 and is less than 1. Are there perhaps more numbers that come before 0 and are less than 0?

Phrased another way, we have seen the number line:



However, this is actually only a number '*ray*', in the language we learned earlier in geometry; this ray starts at 0 and goes forever to the right. Do there exist numbers to the left of 0, so that this number ray can be completed to a true number line?

That is what we will investigate in this chapter!

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Can there be a number less than 0? Can you think of any ways to have less than 0 of something?

10.1 Bela's Building of Fun

Children flock to Bela's ice cream factory to see and taste her tasty ice cream. To make it even more fun for them, Bela purchased a multi-storied building and filled it with attractions. She named it *Bela's Building of Fun*.

But this was no ordinary building!

Observe that some of the floors in the 'Building of Fun' are below the ground. What are the shops that you find on these floors? What is there on the ground floor?

A lift is used to go up and down between the floors. It has two buttons: '+' to go up and '–' to go down. Can you spot the lift?

To go to the Art Centre from the Welcome Hall, you must press the '+' button twice.

We say that the button press is + + or +2.

To go down two floors, you must press the '–' button twice, which we write as – – or –2.

So if you press +1 (i.e., if you press the '+' button once), then you will go up one floor and if you press –1 (i.e., if you press the '–' button once), then you will go down 1 floor.

> Lift button presses and numbers:
> +++ is written as +3
> – – – – is written as – 4

What do you press to go four floors up? What do you press to go three floors down?

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Numbering the Floors in the Building of Fun

Entry to the 'Building of Fun' is at the ground floor level and is called the 'Welcome Hall'. Starting from the ground floor, you can reach the Food Court by pressing +1 and can reach the Art Centre by pressing +2. So, we can say that the Food Court is on Floor +1 and that the Art Centre is on Floor +2.

Starting from the ground floor, you must press –1 to reach the Toy Store. So, the Toy Store is on Floor –1 similarly starting from the ground floor, you must press –2 to reach the Video Games shop. So, the Video Games shop is on Floor –2.

The Ground floor is called Floor 0. Can you see why?

Number all the floors in the Building of Fun.

Did you notice that +3 is the floor number of the Book Store, but it is also the number of floors you move when you press +3? Similarly, –3 is the floor number but it is also the number of floors you go down when you press –3, i.e., when you press – – – .

A number with a '+' sign in front is called a **positive number**. A number with a '–' sign in front is called a **negative number**.

In the 'Building of Fun', the floors are numbered using the ground floor, Floor 0, as a reference or starting point. The floors above the ground floor are numbered with positive numbers. To get to them from the ground floor, one must press the + button some number of times. The floors below the ground are numbered with negative numbers. To get to them from the ground floor, one must press the – button some number of times.

Zero is neither a positive nor a negative number. We do not put a '+' or '–' sign in front of it.

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Addition to Keep Track of Movement

Start from the Food Court and press +2 in the lift. Where will you reach? ___________

We can describe this using an expression:

Starting floor + **Movement** = **Target floor.**

The starting floor is +1 (Food Court) and the number of button presses is +2. Therefore, you reach the target floor **(+1) + (+2) = +3** (Book Store).

Figure it Out

1. You start from Floor +2 and press –3 in the lift. Where will you reach? Write an expression for this movement.
2. Evaluate these expressions (you may think of them as Starting Floor + Movement by referring to the Building of Fun).
   a. (+1) + (+4) = ______ b. (+4) + (+1) = ______
   c. (+4) + (– 3) = ______ d. (–1) + (+2) = ______
   e. (–1) + (+1) = ______ f. 0 + (+2) = ________
   g. 0 + (–2) = ________
3. Starting from different floors, find the movements required to reach Floor –5. For example, if I start at Floor +2, I must press –7 to reach Floor –5. The expression is (+2) + (–7) = –5.
   Find more such starting positions and the movements needed to reach Floor –5 and write the expressions.

Combining Button Presses is also Addition

Gurmit was in the Toy Store and wanted to go down two floors. But by mistake he pressed the '+' button two times. He realised his mistake and quickly pressed the '–' button three times. How many floors below or above the Toy Store will Gurmit reach?

Gurmit will go one floor down. We can show the movement resulting from combining button presses as an expression: (+2) + (–3) = –1.

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Figure it out

Evaluate these expressions by thinking of them as the resulting movement of combining button presses:

a. (+1) + (+4) = ____________ b. (+4) + (+1) = ____________

c. (+4) + (– 3) + (–2) = ______ d. (–1) + (+2) + (–3) = ______

Back to Zero!

On the ground floor, Basant is in a great hurry and by mistake he presses +3. What can he do to cancel it and stay on the ground floor? He can cancel it by pressing –3. That is, (+3) + (–3) = 0.

We call –3 the inverse of +3. Similarly, the inverse of –3 is +3.

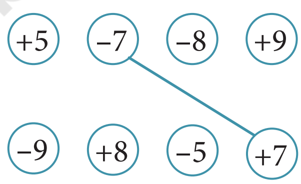
If Basant now presses +4 and then presses –4 in the lift, where will he reach?

Here is another way to think of the concept of inverse. If you are at Floor +4 and you press its inverse –4, then you are back to zero, the ground floor! If you are at Floor –2 and press its inverse +2, then you go to (–2) + (+2) = 0, again the ground floor!

- Write the inverses of these numbers:

  +4, –4, –3, 0, +2, –1.

- Connect the inverses by drawing lines.



Comparing Numbers using Floors

- Who is on the lowest floor?
  1. Jay is in the Art Centre. So, he is on Floor +2.
  2. Asin is in the Sports Centre. So, she is on Floor ___.
  3. Binnu is in the Cinema Centre. So, she is on Floor ____.
  4. Aman is in the Toys Shop. So, he is on Floor ____.

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Floor +3 is lower than Floor +4. So, we write +3 < +4. We also write +4 > +3.

Should we write –3 < –4 or –4 < –3?

Floor –4 is lower than Floor –3. So, –4 < –3. It is also correct to write –3 > –4

Figure it Out

1. Compare the following numbers using the Building of Fun and fill in the boxes with < or >.

   a. $-2 \ \boxed{}\ +5$ b. $-5 \ \boxed{}\ +4$ c. $-5 \ \boxed{}\ -3$

   d. $+6 \ \boxed{}\ -6$ e. $0 \ \boxed{}\ -4$ f. $0 \ \boxed{}\ +4$

> Notice that all negative number floors are below Floor 0. So, all negative numbers are less than 0. All the positive number floors are above Floor 0. So, all positive numbers are greater than 0.

2. Imagine the Building of Fun with more floors. Compare the numbers and fill in the boxes with < or >:

   a. $-10 \ \boxed{}\ -12$ b. $+17 \ \boxed{}\ -10$

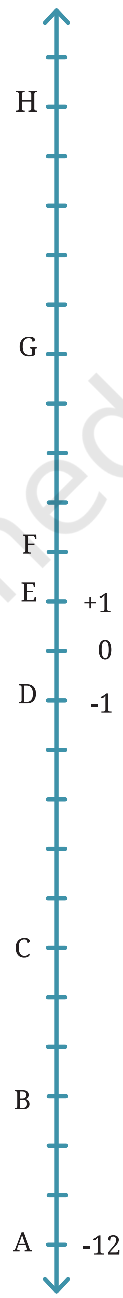
   c. $0 \ \boxed{}\ -20$ d. $+9 \ \boxed{}\ -9$

   e. $-25 \ \boxed{}\ -7$ f. $+15 \ \boxed{}\ -17$

3. If Floor A = –12, Floor D = –1 and Floor E = +1 in the building shown on the right as a line, find the numbers of Floors B, C, F, G and H.

4. Mark the following floors of the building shown on the right.

   a. –7   b. – 4   c. +3   d. – 10



Subtraction to Find which Button to Press

In earlier classes, we understood the meaning of subtraction as 'take away'. For example, "There are 10 books on the shelf. I take away 4 books. How many are left on the shelf?"

We can express the answer using subtraction: 10 – 4 = 6. Or 'Ten take away four is six.'

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You may also be familiar with another meaning of subtraction which is related to comparison or making quantities equal. For example, consider this situation: "I have ₹10 with me and my sister has ₹6."

Now, I can ask the question: 'How much more money should my sister get in order to have the same amount as me?'

We can write this in two ways: 6 + ? = 10   Or   10 – 6 = ?.

Here, we see the connection between 'finding the missing number to be added' and subtraction.

For subtraction of positive and negative numbers, we will use this meaning of subtraction as 'making equal' or 'finding the missing number to be added'.

Evaluate 15 – 5, 100 – 10 and 74 – 34 from this perspective.

Teachers' Note

In general, when there are two unequal quantities, subtraction can indicate the change needed to make the quantities equal. Subtraction shows how much the starting quantity should change in order to become the target quantity. In the context of different floor levels, what is the change required to reach the Target Floor from the Starting Floor? Notice that the change needed may be positive (for an increase) or negative (for a decrease).

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Your starting floor is the Art Centre and your target floor is the Sports Centre. What should be your button press?

You need to go three floors up, so you should press +3. We can write this as an expression using subtraction:

Target floor – Starting floor = Movement needed.

In the above example, the starting floor is +2 (Art Centre) and the target floor is +5. The button press to get to +5 from +2 is +3. Therefore,

$$(+5) - (+2) = +3.$$

Explanation:

Recall the connection between addition and subtraction. For $3 + ? = 5$, we can find the missing number using subtraction: $5 - 3 = 2$. That is, subtraction is the same as finding the missing number to be added.

We know that

Starting floor + Movement needed = Target Floor.

If the movement needed is to be found, then,

Starting floor + ? = Target Floor.

So

Target floor – Starting floor = ? = Movement needed.

More examples:

a. If the Target Floor is –1 and Starting Floor is –2, what button should you press?
You need to go one floor up, so, you should press +1.
Expression: **(–1) – (–2) = (+1).**

b. If the Target Floor is –1 and Starting floor is +3, what button should you press?
You need to go four floors down, so, you should press –4.
Expression: **(–1) – (+3) = (–4).**

c. If the Target Floor is +2 and Starting Floor is –2, what button should you press?
You need to go four floors up, so, you should press +4.
Expression: **(+2) – (–2) = (+4).**

Figure it Out

Complete these expressions. You may think of them as finding the movement needed to reach the Target Floor from the Starting Floor.

a. (+1) – (+4) = _______
b. (0) – (+2) = _______
c. (+4) – (+1) = _______
d. (0) – (–2) = _______
e. (+4) – (–3) = _______
f. (–4) – (–3) = _______
g. (–1) – (+2) = _______
h. (–2) – (–2) = _______
i. (–1) – (+1) = _______
j. (+3) – (–3) = _______

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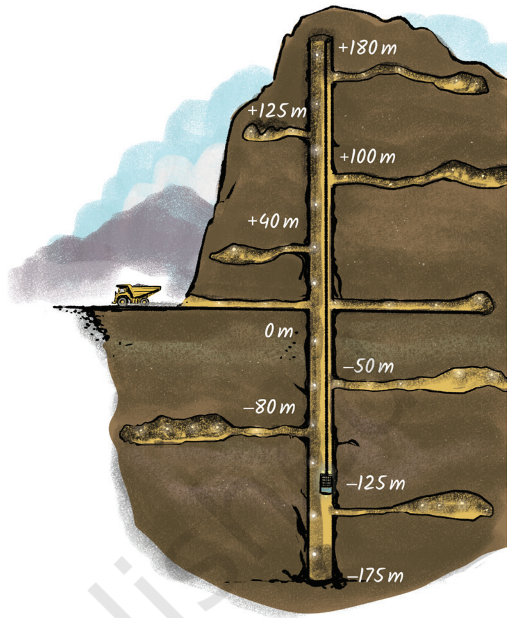
Adding and Subtracting Larger Numbers

The picture shows a mine, a place where minerals are extracted by digging into the rock. The truck is at the ground level, but the minerals are present both above and below the ground level. There is a fast moving lift which moves up and down in a mineshaft carrying people and ore.



Some of the levels are marked in the picture. The ground level is marked 0. Levels above the ground are marked by positive numbers and levels below the ground are marked by negative numbers. The number indicates how many meters above or below the ground level it is.

In the mine, just like in the Building of Fun:

Starting level + Movement = Target level.

For example:

$(+40) + (+60) = +100$ $\quad$ $(-90) + (-55) = -145$

Target level – Starting level = Movement needed.

For example:

$(+40) - (-50) = +90$ $\quad$ $(-90) - (+40) = -130$

How many negative numbers are there?

Bela's Building of Fun had only six floors above and five floors below. That is numbers –5 to +6. In the mine above, we have numbers from –200 to +180. But we can imagine larger buildings or mineshafts. Just as positive numbers +1, +2, +3, ... keep going up without an end, similarly, negative numbers –1, –2, –3, ... keep going down. Positive and negative numbers, with zero, are called **integers**. They go both ways from 0: ... –4, –3, –2, –1, 0, 1, 2, 3, 4, ...

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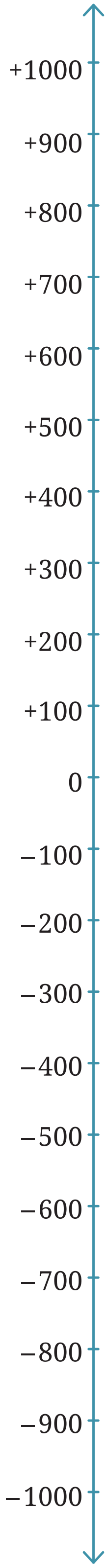

Figure it Out

Complete these expressions.

a. (+40) + ______ = +200
b. (+40) + ______ = –200
c. (–50) + ______ = +200
d. (–50) + ______ = –200
e. (–200) – (–40) = ______
f. (+200) – (+40) = ______
g. (–200) – (+40) = ______

Check your answers by thinking about the movement in the mineshaft.

Adding, Subtracting, and Comparing any Numbers

To add and subtract even larger integers, we can imagine even larger lifts! In fact, we can imagine a lift that can extend forever upwards and forever downwards, starting from Level 0. There does not even have to be any building or mine around – just an 'infinite lift'!

We can use this imagination to add and subtract any integers we like.

For example, suppose we want to carry out the subtraction +2000 – (–200). We can imagine a lift with 2000 levels above the ground and 200 below the ground. Recall that

Target level – Starting level = Movement needed.

To go from the Starting Floor –200 to the Target Floor +2000, we must press +2200 (+200 to get to zero, and then +2000 more after that to get to +2200). Therefore, (+2000) – (–200) = +2200.

Notice that (+2000) + (+200) is also +2200.

Try evaluating the following expressions by similarly drawing or imagining a suitable lift:

a. –125 + (–30)
b. +105 – (–55)
c. +105 + (+55)
d. +80 – (–150)
e. +80 + (+150)
f. –99 – (–200)
g. –99 + (+200)
h. +1500 – (–1500)

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In the above example, we saw that + 2000 – (– 200) = + 2000 + (+ 200) = + 2200. In other words, subtracting a negative number is the same as adding the corresponding positive number. That is, we can replace subtraction of a negative number by addition of a positive number!

In the other exercises that you did above, did you notice that subtracting a negative number was the same as adding the corresponding positive number?

Math Talk

Take a look at the 'infinite lift' above. Does it remind you of a number line? In what ways?

Back to the Number Line

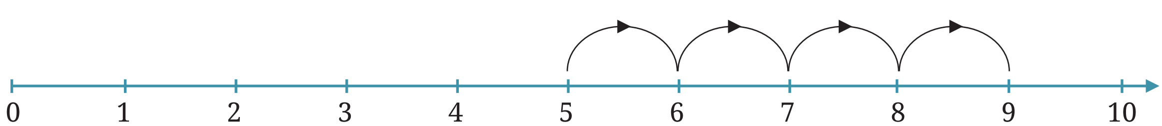
The 'infinite lift' we saw above looked very much like a number line, didn't it? In fact, if we rotate it by 90°, it basically becomes a number line. It also tells us how to complete the number *ray* to a number line, answering the question that we had asked at the beginning of the chapter. To the left of 0 are the negative numbers –1, –2, –3, ...

Usually we drop the + signs on positive numbers, and simply write them as 1, 2, 3, ...

Instead of traveling along the number line using a lift, we can simply imagine walking on it. To the right is the positive (forward) direction, and to the left is the negative (backward) direction.

Smaller numbers are now to the left of bigger numbers, and bigger numbers are to the right of smaller numbers. So 2 < 5; –3 < 2; and –5 < –3.

If, from 5 you wish to go over to 9, how far must you travel along the number line?



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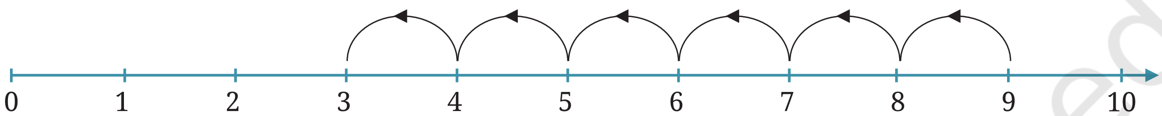
You must travel 4 steps. That is why $5 + 4 = 9$.

(Remember: **Starting Number** + **Movement** = **Target Number.**)

The corresponding subtraction statement is $9 - 5 = 4$.

(Remember: **Target Number – Starting Number = Movement Needed.**)

Now, from 9, if you wish to go to 3, how much must you travel along the number line?



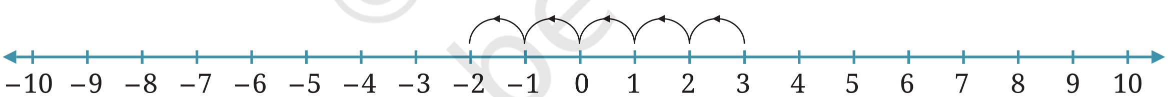
You must move 6 steps backward, i.e., you must move –6. Hence, we write $9 + (-6) = 3$.

(Remember again : **Starting number** + **Movement** = **Target number.**)

The corresponding subtraction statement is $3 - 9 = -6$.

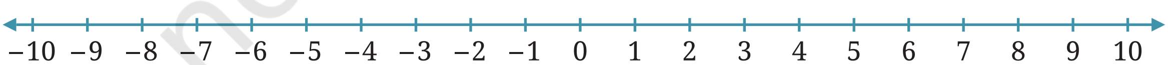
(Remember again: **Target number – Starting number = Movement needed.**)

Now, from 3, if you wish to go to –2, how far must you travel?



You must travel –5 steps, i.e., 5 steps backward. Thus $3 + (-5) = -2$. The corresponding subtraction statement is: $-2 - 3 = -5$.

Figure it Out



1. Mark 3 positive numbers and 3 negative numbers on the number line above.
2. Write down the above 3 marked negative numbers in the following boxes: $\boxed{}$ $\boxed{}$ $\boxed{}$

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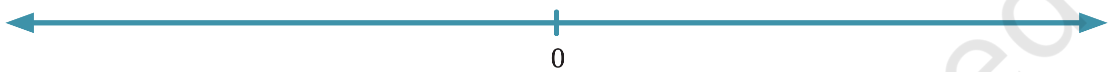
3. Is $2 > -3$? Why? Is $-2 < 3$? Why?
4. What are (i) $-5 + 0$ (ii) $7 + (-7)$ (iii) $-10 + 20$ (iv) $10 - 20$ (v) $7 - (-7)$ (vi) $-8 - (-10)$?

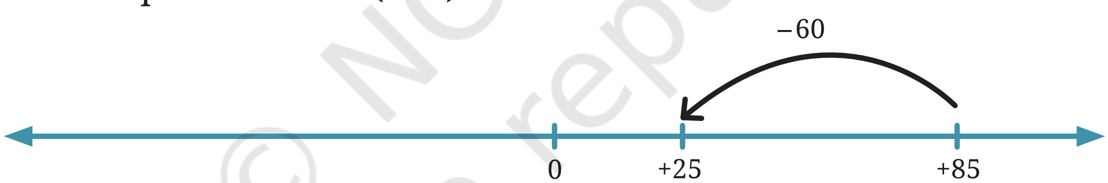
Using the Unmarked Number Line to Add and Subtract

Just as you can do additions, subtractions and comparisons with small numbers using the number line above, you can also do them with large numbers by imagining an 'infinite number line', or drawing an 'unmarked number line' as follows:



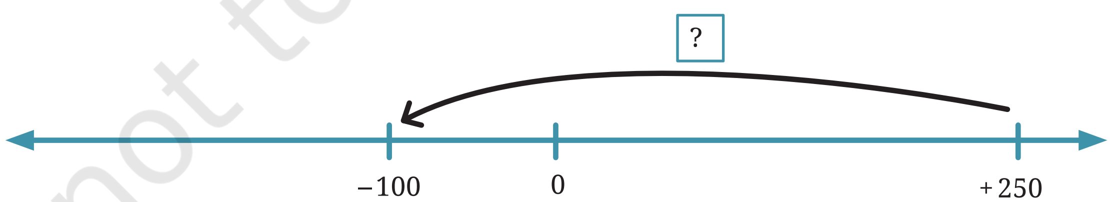
This line shows only the position of zero. Other numbers are not marked. It can be convenient to use this **u**nmarked **n**umber **l**ine to add and subtract integers. You can show, or simply imagine, the scale of the number line and the positions of numbers on it.

For example, this unmarked number line (UNL) shows the addition problem: $85 + (-60) = ?$:



We then can visualise that $85 + (-60) = 25$

The following UNL shows a subtraction problem which can also be written as a missing addend problem: $(-100) - (+250) = ?$ or $250 + ? = -100$.



We can then visualise that $? = -350$ in this problem.

In this way, you can carry out addition and subtraction problems, with positive and negative numbers, on paper or in your head using an unmarked number line.

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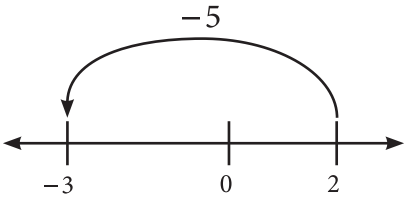
Use unmarked number lines to evaluate these expressions:

a. $-125 + (-30) =$ _______

b. $+105 - (-55) =$ _______

c. $+80 - (-150) =$ _______

d. $-99 - (-200) =$ _______



Converting subtraction to addition and addition to subtraction

Recall that **Target floor – Starting floor = Movement needed**

or

Target floor = Starting floor + Movement needed

If we start at 2 and wish to go to –3, what is the movement needed?

First method: Looking at the number line, we see we need to move –5 (i.e., 5 in the backward direction). Therefore, $-3 - 2 = -5$. The movement needed is –5.

Second method: Break the journey from 2 to –3 into two parts.

a. From 2 to 0, the movement is $0 - 2 = -2$.

b. From 0 to –3, the movement is $-3 - 0 = -3$.

The total movement is the sum of the two movements: $-3 + (-2) = -5$.

Look at the two coloured expressions. There is no subtraction in the second one!

In this way, we can always convert subtraction to addition. **The number that is being subtracted can be replaced by its inverse and then added instead.**

Similarly, a number that is being added can be replaced by its inverse and then subtracted. In this way, we can also always convert addition to subtraction.

Examples:

a. $(+7) - (+5) = (+7) + (-5)$

b. $(-3) - (+8) = (-3) + (-8)$

c. $(+8) - (-2) = (+8) + (+2)$

d. $(+6) - (-9) = (+6) + (+9)$

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10.2 The Token Model

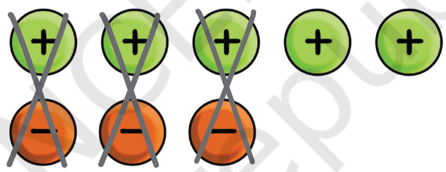
Using Tokens for Addition

In Bela's Building of Fun, the lift attendant is bored. To entertain himself, he keeps a box containing lots of positive (red) and negative (black) tokens. Each time he presses the '+' button, he takes a positive token from the box and puts it in his pocket. Similarly, each time he presses the '–' button, he takes a negative token and puts it in his pocket.

He starts on the ground floor (Floor 0) with an empty pocket. After one hour, he checks his pocket and finds 5 positive and 3 negative tokens. On which floor is he now?

He must have pressed '+' five times and '–' 3 times and $(+5)+(-3)=+2$. So he is at Floor $+2$ now.

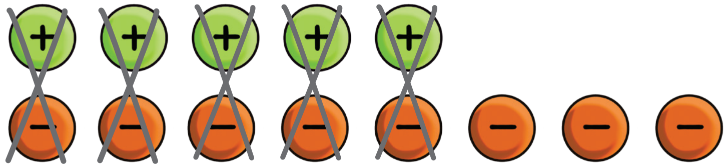
Here is another way to do the calculation.



A positive token and a negative token cancel each other, because the value of this pair of tokens together is zero. These two tokens in his pocket meant that he pressed '+' once and '–' once, respectively, and these cancel each other. We say that a positive and a negative token make a 'zero pair'. When you remove all the zero pairs, you are left with two positive tokens, so $(+5) + (-3) = +2$.

We can perform any such addition using tokens!

Example: Add $+5$ and -8.



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From the picture, we see that we can remove five zero pairs, and we are then left with -3. Therefore $(+5)+(-8)=-3$.

Figure it Out

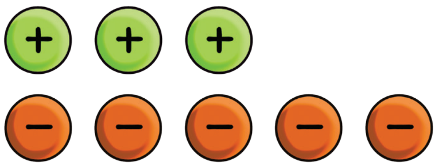
1. Complete the additions using tokens.

a. $(+6)+(+4)$ b. $(-3)+(-2)$

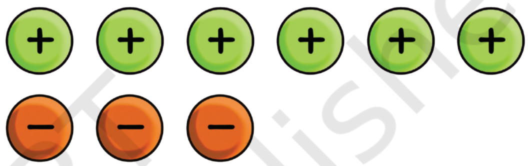
c. $(+5)+(-7)$ d. $(-2)+(+6)$

2. Cancel the zero pairs in the following two sets of tokens. On what floor is the lift attendant in each case? What is the corresponding addition statement in each case?

a.



b.



Using Tokens for Subtraction

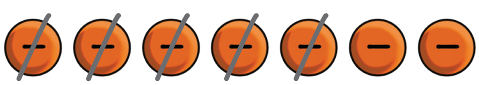
We have seen how to perform addition of integers with positive tokens and negative tokens. We can also perform subtraction using tokens!

Example: Let us subtract: $(+5)-(+4)$.

$(+5)-(+4)=+1$

This is easy to do. From 5 positives take away 4 positives to see the result.

Example: Let us subtract: $(-7)-(-5)$.

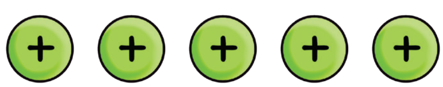


$(-7)-(-5)=-2$

Is $(-7)-(-5)$ the same as $(-7)+(+5)$?

Example: Let us subtract: $(+5)-(+6)$.

Put down 5 positives.



But there are not enough tokens to take out 6 positives!

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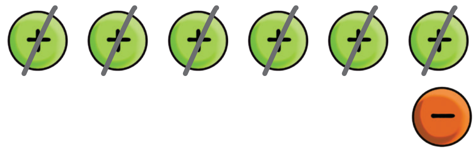
To get around this issue, we can put out an extra zero pair (a positive and a negative), knowing that this does not change the value of the set of tokens.

Now we can take out 6 positives! See what is left:



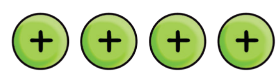
We conclude that (+5) – (+6) = –1.

Figure it Out

1. Evaluate the following differences using tokens. Check that you get the same result as with other methods you now know:
   a. (+10)–(+7) b. (–8)–(–4) c. (–9)–(–4)
   d. (+9)–(+12) e. (–5)–(–7) f. (–2)–(–6)
2. Complete the subtractions:
   a. (–5)–(–7) b. (+10)–(+13) c. (–7)–(–9)
   d. (+3)–(+8) e. (–2)–(–7) f. (+3)–(+15)

Example: +4 – (–6).

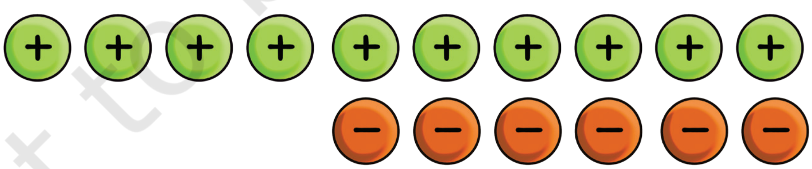
Start with 4 positives.



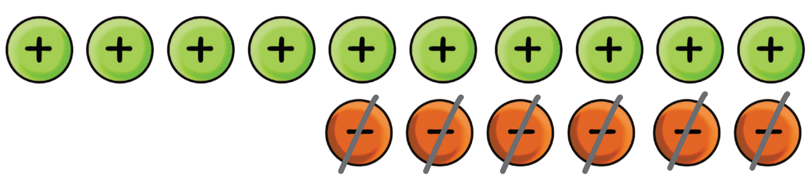
We have to take out 6 negatives from these. But there are not enough negatives.

This is not a problem. We add some zero pairs as this does not change the value of the set of tokens.

But how many zero pairs? We have to take away 6 negatives so we put down 6 zero pairs:



Now we can take away 6 negatives:



Therefore, +4–(–6) = +10.

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Figure it Out

1. Try to subtract: $-3-(+5)$.
   How many zero pairs will you have to put in? What is the result?
2. Evaluate the following using tokens.
   a. $(-3)-(+10)$ b. $(+8)-(-7)$ c. $(-5)-(+9)$
   d. $(-9)-(+10)$ e. $(+6)-(-4)$ f. $(-2)-(+7)$

10.3 Integers in Other Places

Credits and Debits

Suppose you open a bank account at your local bank with the ₹100 that you had been saving over the last month. Your bank balance therefore starts at ₹100.

Then you make ₹60 at your job the next day and you deposit it in your account. This is shown in your bank passbook as a 'credit'.

Your new bank balance is ______.

The next day you pay your electric bill of ₹30 using your bank account. This is shown in your bank passbook as a 'debit'.

Your bank balance is now ______.

The next day you make a major purchase for your business of ₹150. Again this is shown as a debit.

What is your bank balance now? ______

Is this possible?

(Yes, some banks do allow your account balance to become negative, temporarily! Some banks also charge you an additional amount if your balance becomes negative, in the form of 'interest' or a 'fee'.)

Your strategic large purchase the previous day allows you to make 200 rupees at your business the next day.

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- What is your balance now? ______

You can think of 'credits' as positive numbers and 'debits' as negative numbers. The total of all your credits (positive numbers) and debits (negative numbers) is your total bank account balance. This can be positive or negative!

In general, it is better to try to keep a positive balance in your bank account!

Figure it Out

1. Suppose you start with 0 rupees in your bank account, and then you have credits of ₹30, ₹40, and ₹50, and debits of ₹40, ₹50, and ₹60. What is your bank account balance now?
2. Suppose you start with 0 rupees in your bank account, and then you have debits of ₹1, 2, 4, 8, 16, 32, 64, and 128, and then a single credit of ₹256. What is your bank account balance now?
3. Why is it generally better to try and maintain a positive balance in your bank account? What are circumstances under which it may be worthwhile to temporarily have a negative balance?

As you can see, positive and negative numbers along with zero are extremely useful in the world of banking and accounting.

Geographical Cross-sections

We measure the height of geographical features like mountains, plateaus, and deserts from 'sea level'. The height at sea level is 0m. Heights above sea level are represented using positive numbers and heights below sea level are represented using negative numbers.

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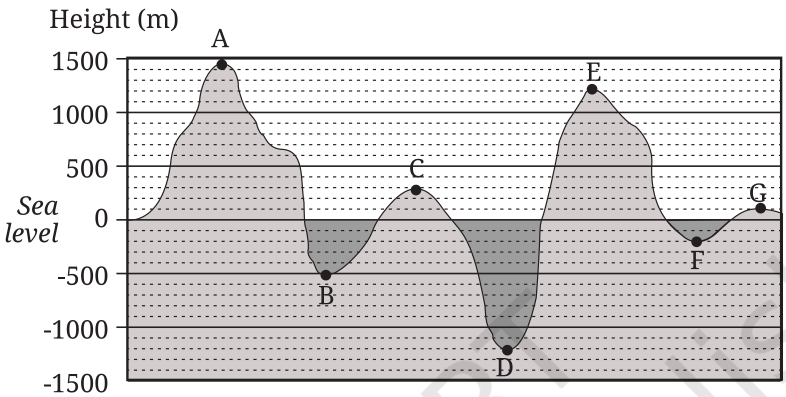
Figure it Out

1. Looking at the geographical cross section fill in the respective heights:

   A [ ] B [ ] C [ ] D [ ]

   E [ ] F: [ ] G [ ]



Teachers' Note

Ask what a geographical cross-section is by showing the figure in this page. It is like imagining a vertical slice taken out at some location on the earth. This is what would be seen from a side view. Discuss the notion of "sea level" for measuring heights and depths in geography.

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2. Which is the highest point in this geographical cross-section? Which is the lowest point?
3. Can you write the points A, B, ..., G in a sequence of decreasing order of heights? Can you write the points in a sequence of increasing order of heights?
4. What is the highest point above sea level on Earth? What is its height?
5. What is the lowest point with respect to sea level on land or on the ocean floor? What is its height? (This height should be negative).

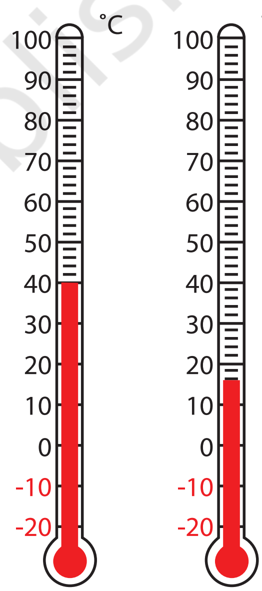
Temperature

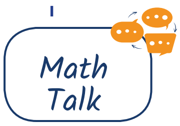
During summertime you would have heard in the news that there is a 'heat wave'. What do you think will be the temperature during the summer when you feel very hot? In winter we have cooler or colder temperatures.

What has been the maximum temperature during the summer and the minimum temperature during the winter last year in your area? Find out.

When we measure temperature, we use Celsius as the unit of measure (°C). The thermometers below are showing 40°C and 15°C temperatures.

Figure it Out





1. Do you know that there are some places in India where temperatures can go below 0°C? Find out the places in India where temperatures sometimes go below 0°C. What is common among these places? Why does it become colder there and not in other places?
2. Leh in Ladakh gets very cold during winter. The following is a table of temperature readings taken during different times of the day/night in Leh on a day in November. Match the temperature with the appropriate time of the day/night.

Temperature
14°C
8°C
-2°C
-4°C

Time
02:00 am
11:00 pm
02:00 pm
11:00 am

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Teachers' Note

Talk about thermometers and how they are used to measure temperature. Bring a laboratory thermometer to the class and measure the temperature of hot water and cold water. Point out to children that there are markings in the thermometer that are below 0°C. Have a discussion on what 0°C indicates, namely, the freezing point of water.

10.4 Explorations with Integers

A Hollow Integer Grid

4	–1	–3
–3		1
–1	–1	2

5	–3	–5
0		–5
–8	–2	7

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There is something special about the numbers in these two grids. Let us explore what that is.

Top row:	$4 + (-1) + (-3) = 0$	$5 + (-3) + (-5) =$ ____
Bottom row:	$(-1) + (-1) + 2 = 0$	$(-8) + (-2) + 7 =$ ____
Left column:	$4 + (-3) + (-1) = 0$	$5 + 0 + (-8) =$ ____
Right column:	$(-3) + 1 + 2 = 0$	$(-5) + (-5) + 7 =$ ____

In each grid, the numbers in each of the two rows (the top row and the bottom row) and the numbers in each of the two columns (the leftmost column and the rightmost column) add up to give the same number. We shall call this sum as the 'border sum'. The border sum of the first grid is '0'.

Figure it Out

1. Do the calculations for the second grid above and find the border sum.

2. Complete the grids to make the required border sum:

–10		
		–5
9		

Border sum is +4

6	8	
		–5
	–2	

Border sum is –2

7		
		–5

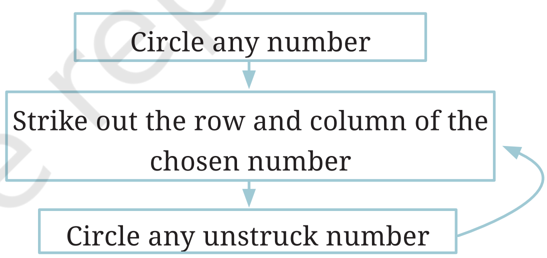
Border sum is –4

3. For the last grid above, find more than one way of filling the numbers to get border sum –4.
4. Which other grids can be filled in multiple ways? What could be the reason?
5. Make a border integer square puzzle and challenge your classmates.

An Amazing Grid of Numbers!

Below is a grid having some numbers. Follow the steps as shown until no number is left.

3	4	0	9
–2	–1	–5	4
1	2	–2	7
–7	–6	–10	–1



When there are no more unstruck numbers, STOP. Add the circled numbers.

In the example below, the circled numbers are –1, 9, –7, –2. If you add them, you get –1.

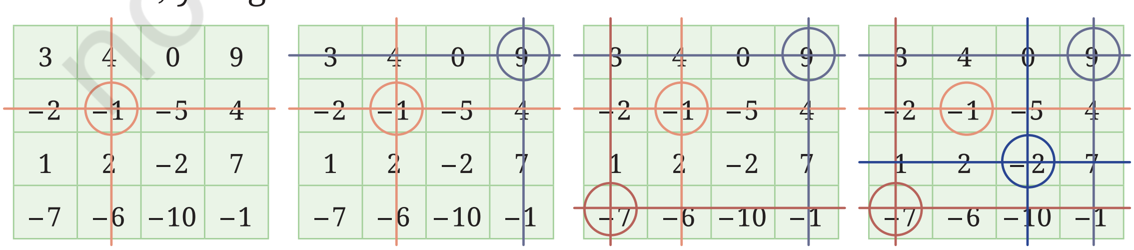


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Figure it Out

1. Try afresh, choose different numbers this time. What sum did you get? Was it different from the first time? Try a few more times!
2. Play the same game with the grids below. What answer did you get?

7	10	13	16
–2	1	4	7
–11	–8	–5	–2
–20	–7	–14	–11

–11	–10	–9	–8
–7	–6	–5	–4
–3	–2	–1	0
1	2	3	4

3. What could be so special about these grids? Is the magic in the numbers or the way they are arranged or both? Can you make more such grids?

Try This

Figure it Out

1. Write all the integers between the given pairs, in increasing order.
   a. 0 and –7
   b. –4 and 4
   c. –8 and –15
   d. –30 and –23
2. Give three numbers such that their sum is –8.
3. There are two dice whose faces have these numbers: –1, 2, –3, 4, –5, 6. The smallest possible sum upon rolling these dice is –10 = (–5) + (–5) and the largest possible sum is 12 = (6) + (6). Some numbers between (–10) and (+ 12) are not possible to get by adding numbers on these two dice. Find those numbers.
4. Solve these:

8 – 13	(–8) – (13)	(–13) – (–8)	(–13) + (–8)
8 + (–13)	(–8) – (–13)	(13) – 8	13 – (–8)

5. Find the years below.
   a. From the present year, which year was it 150 years ago? ________
   b. From the present year, which year was it 2200 years ago? _______
   (**Hint**: Recall that there was no year 0.)

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c. What will be the year 320 years after 680 BCE? ________

6. Complete the following sequences:

   a. (–40), (–34), (–28), (–22), _____, ______, ______

   b. 3, 4, 2, 5, 1, 6, 0, 7, _____, _____, _____

   c. _____, ______, 12, 6, 1, (–3), (–6), _____, ______, ______

7. Here are six integer cards: (+1), (+7), (+18), (–5), (–2), (–9).

   You can pick any of these and make an expression using addition(s) and subtraction(s).

   Here is an expression: (+18) + (+1) – (+7) – (–2) which gives a value (+14). Now, pick cards and make an expression such that its value is closer to (– 30).

8. The sum of two positive integers is always positive but a (positive integer) – (positive integer) can be positive or negative. What about

   a. (positive) – (negative)

   b. (positive) + (negative)

   c. (negative) + (negative)

   d. (negative) – (negative)

   e. (negative) – (positive)

   f. (negative) + (positive)

9. This string has a total of 100 tokens arranged in a particular pattern. What is the value of the string?

10.5 A Pinch of History

Like general fractions, general integers (including zero and the negative numbers) were first conceived of and used in Asia, thousands of years ago, before they eventually spread across the world in more modern times.

The first known instances of the use of negative numbers occurred in the context of accounting. In one of China's most important mathematical works, *The Nine Chapters on Mathematical Art (Jiuzhang Suanshu)*—which was completed by the first or second century CE—positive and negative numbers were represented using red and black rods, much like the way we represented them using red and black tokens!

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There was a strong culture of accountancy also in India in ancient times. The concept of credit and debit was written about extensively by *Kautilya* in his Arthaśhāstra (c. 300 BCE), including the recognition that an account balance could be negative. The explicit use of negative numbers in the context of accounting is seen in a number of ancient Indian works, including in the Bakśhālī Manuscript from around the year 300, where a negative number was written using a special symbol placed after the number (rather than before the number as we do today).

The first general treatment of positive numbers, negative numbers, and *zero* - all on an equal footing as equally-valid numbers on which one can perform the basic operations of addition, subtraction, multiplication and even division–was given by Brahmagupta in his *Brāhma-sphuṭa-siddhānta* in the year 628 CE. Brahmagupta gave clear and explicit rules for operations on all numbers–positive, negative, and zero–that essentially formed the modern way of understanding these numbers that we still use today!

Some of Brahmagupta's key rules for addition and subtraction of positive numbers, negative numbers, and zero are given below:

Brahmagupta's Rules for Addition (*Brāhma-sphuṭa-siddhānta* 18.30, 628 CE):

1. The sum of two positives is positive (e.g., $2 + 3 = 5$).
2. The sum of two negatives is negative. To add two negatives, add the numbers (without the signs), and then place a minus sign to obtain the result (e.g., $(-2) + (-3) = -5$).
3. To add a positive number and a negative number, subtract the smaller number (without the sign) from the greater number (without the sign), and place the sign of the greater number to obtain the result (e.g., $-5 + 3 = -2$, $2 + (-3) = -1$ and $-3 + 5 = 2$).
4. The sum of a number and its inverse is zero (e.g., $2 + (-2) = 0$).
5. The sum of any number and zero is the same number (e.g., $-2 + 0 = -2$ and $0 + 0 = 0$).

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Brahmagupta's Rules for Subtraction (Brāhma-sphuṭa-siddhānta 18.31-18.32):

1. If a smaller positive is subtracted from a larger positive, the result is positive (e.g., $3 - 2 = 1$).
2. If a larger positive is subtracted from a smaller positive, the result is negative (e.g., $2 - 3 = -1$).
3. Subtracting a negative number is the same as adding the corresponding positive number (e.g., $2 - (-3) = 2 + 3$).
4. Subtracting a number from itself gives zero (e.g., $2 - 2 = 0$ and $-2 - (-2) = 0$).
5. Subtracting zero from a number gives the same number (e.g., $-2 - 0 = -2$ and $0 - 0 = 0$). Subtracting a number from zero gives the number's inverse (e.g., $0 - (-2) = 2$).

Once you understand Brahmagupta's rules, you can do addition and subtraction with any numbers whatsoever - positive, negative, and zero!

Figure it Out

1. Can you explain each of Brahmagupta's rules in terms of Bela's Building of Fun, or in terms of a number line?
2. Give your own examples of each rule.

Brahmagupta was the first to describe zero as a number on an equal footing with positive numbers as well as with negative numbers, and the first to give explicit rules for performing arithmetic operations on all such numbers, positive, negative, and zero—forming what is now called a *ring*. It would change the way the world does mathematics.

However, it took many centuries for the rest world to adopt zero and negative numbers as numbers. These numbers were transmitted to, accepted, by and further studied by the Arab world by the 9th century, before making their way to Europe by the 13th century.

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Surprisingly, negative numbers were still not accepted by many European mathematicians even in the 18th century. Lazare Carnot, a French mathematician in the 18th century, called negative numbers 'absurd'. But over time, zero as well as negative numbers proved to be indispensable in global mathematics and science, and are now considered to be critical numbers on an equal footing with and as important as positive numbers—just as Brahmagupta had recommended and explicitly described way back in the year 628 CE! This abstraction of arithmetic rules on all numbers paved the way for the modern development of algebra, which we will learn about in future classes.

SUMMARY

- There are numbers that are less than zero. They are written with a '–' sign in front of them (e.g., –2), and are called **negative numbers**. They lie to the left of zero on the number line.
- The numbers ..., –4, –3, –2, –1, 0, 1, 2, 3, 4, ... are called **integers**. The numbers 1, 2, 3, 4, ... are called **positive integers** and the numbers ..., –4, –3, –2, –1 are called **negative integers**. Zero (0) is neither positive nor negative.
- Every given number has another number associated to it which when added to the given number gives zero. This is called the **additive inverse** of the number. For example, the additive inverse of 7 is –7 and the additive inverse of –543 is 543.
- Addition can be interpreted as **Starting Position** + **Movement** = **Target Position**.
- Addition can also be interpreted as the combination of movements or increases/decreases: **Movement 1** + **Movement 2** = **Total Movement**.
- Subtraction can be interpreted as **Target Position** – **Starting Position** = **Movement**.

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- In general, we can add two numbers by following Brahmagupta's Rules for Addition:
  a. If both numbers are positive, add the numbers and the result is a positive number (e.g., $2 + 3 = 5$).
  b. If both numbers are negative, add the numbers (without the signs), and then place a minus sign to obtain the result ($-2 + (-3) = -5$).
  c. If one number is positive and the other is negative, subtract the smaller number (without the sign) from the greater number (without the sign), and place the sign of the greater number to obtain the result (e.g., $-5 + 3 = -2$).
  d. A number plus its additive inverse is zero (e.g., $2 + (-2) = 0$).
  e. A number plus zero gives back the same number (e.g., $-2 + 0 = -2$).
- We can subtract two integers by converting the problem into an addition problem and then following the rules of addition. Subtraction of an integer is the same as the addition of its additive inverse.
- Integers can be compared: $\ldots -3 < -2 < -1 < 0 < +1 < +2 < +3 < \ldots$ Smaller numbers are to the left of larger numbers on the number line.
- We can give meaning to positive and negative numbers by interpreting them as credits and debits. We can also interpret positive numbers as distances above a reference point like the ground level. Similarly, negative numbers can be interpreted as distances below the ground level. When measuring temperatures in degrees Celsius, positive temperatures are those above the freezing point of water, and negative temperatures are those below the freezing point of water.

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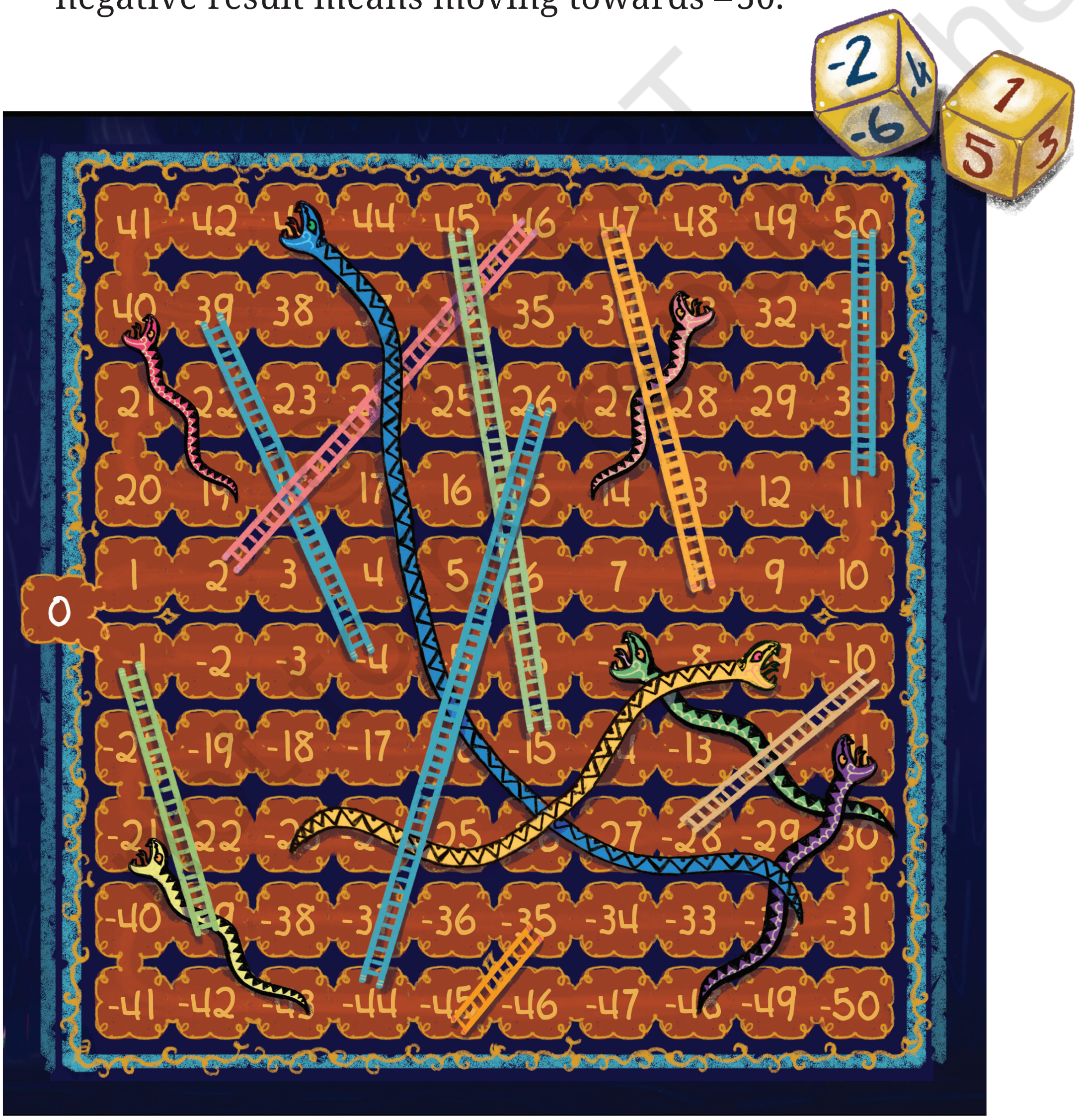
Integers: Snakes and Ladders

Rules

- This is a two player game. Each player has 1 pawn. Both players start at 0. Players can reach either –50 or +50 to win but need not decide or fix this before or during play.
- Each player rolls two dice at a time. One dice has numbers from +1 to +6 and the other dice has numbers from –1 to –6.
- After each roll of the two dice, the player can add or subtract them in any order and then move the steps that indicate the result. A positive result means moving towards +50 and a negative result means moving towards –50.



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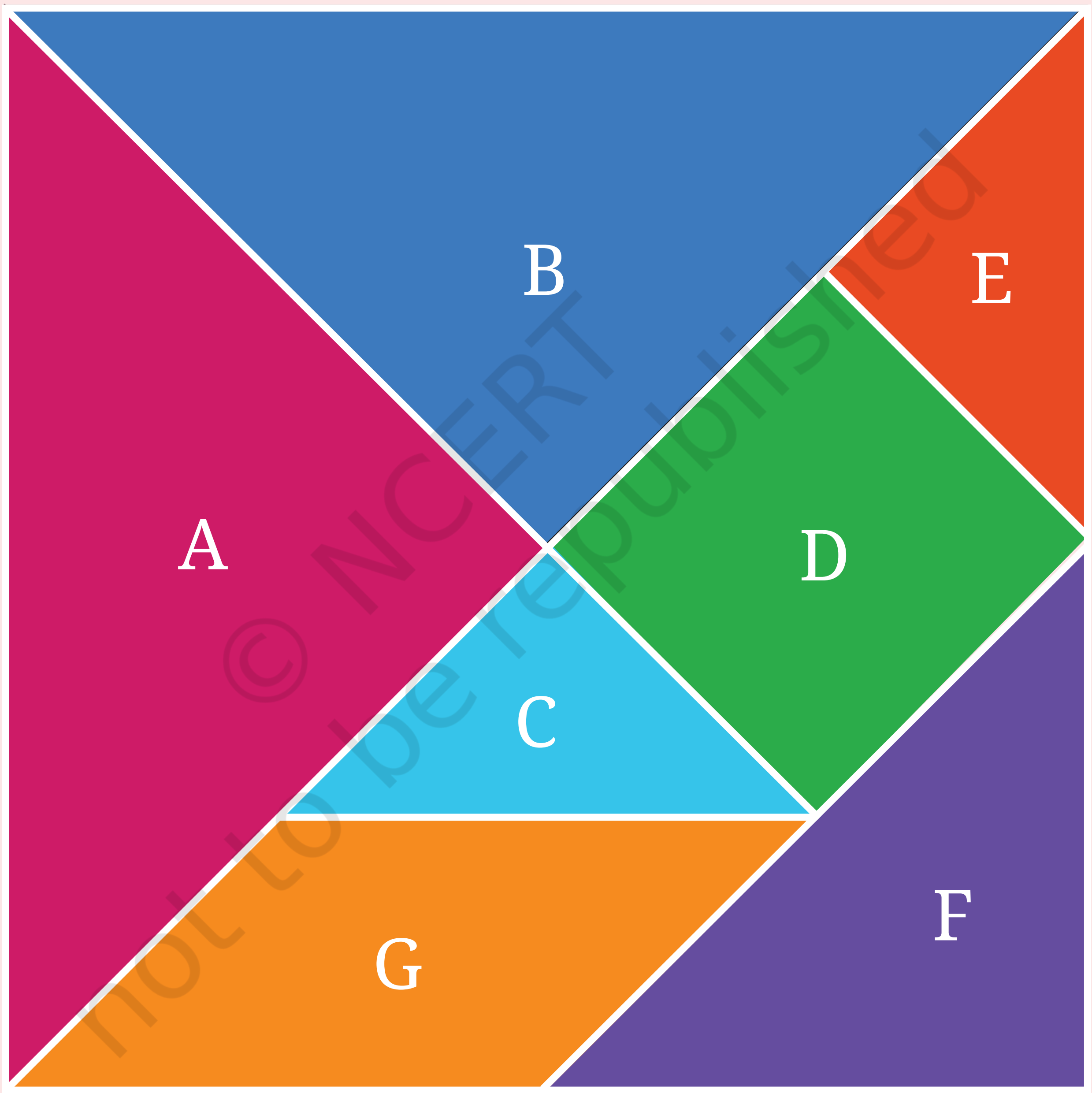
TANGRAM

Note: Cut each shape along the white border.



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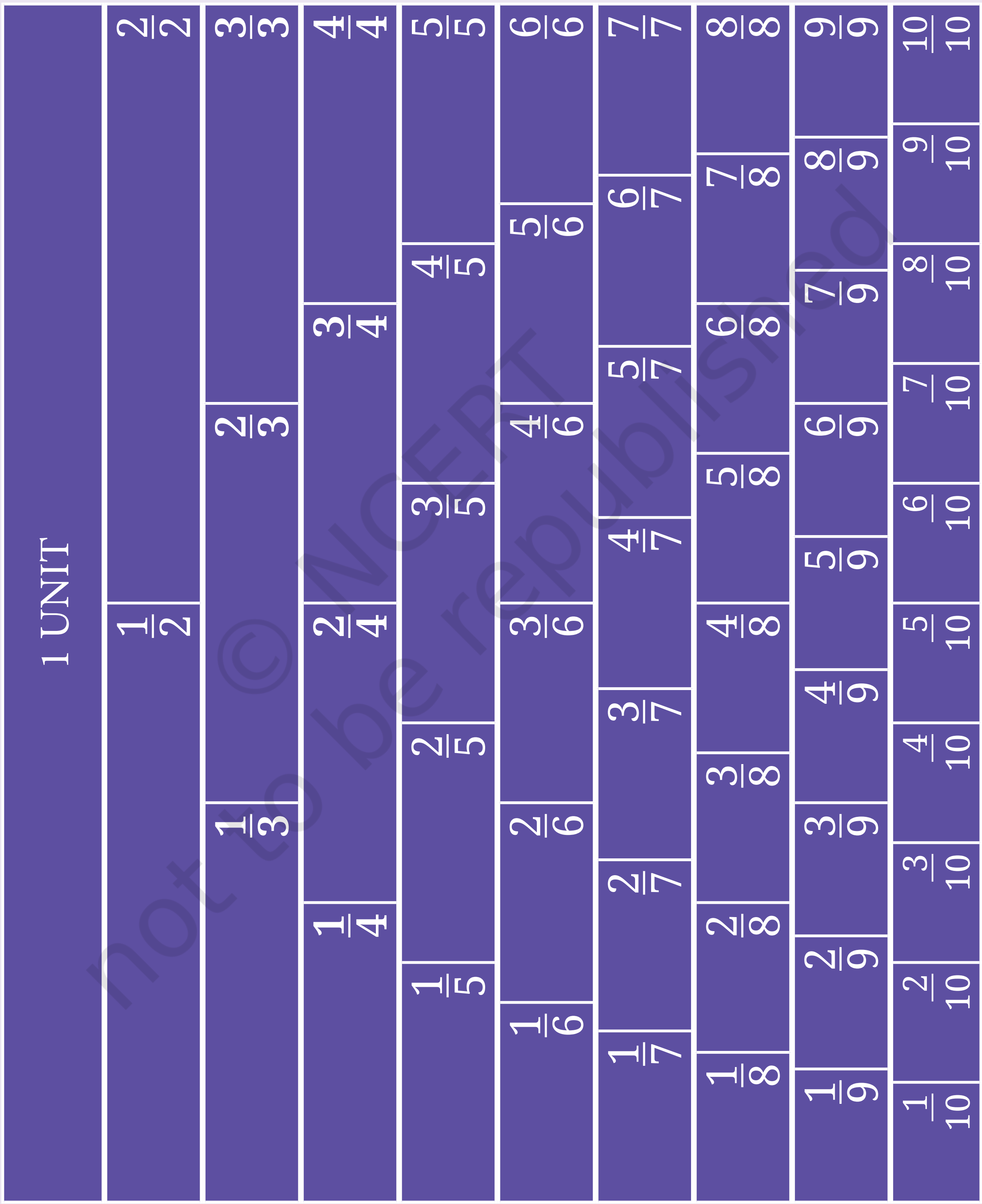
FRACTION WALL

Note: Cut each shape along the white border.



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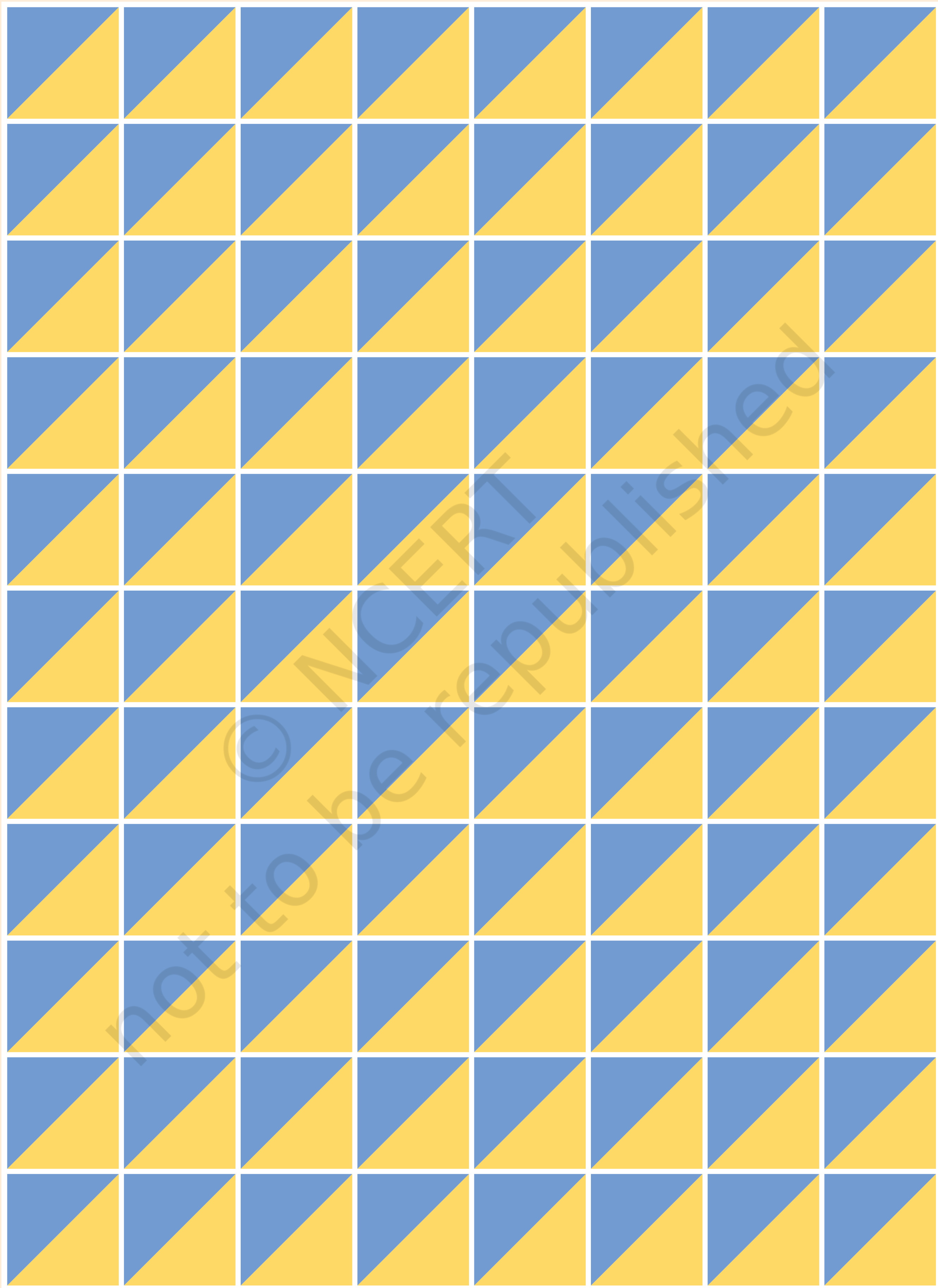
Note: Cut the tiles along the white border.



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