ACTL3162 General Insurance Techniques
ACTL5106 Insurance Risk Models
Assignment, 2023 T3

1 Learning outcomes

The assignment aims at developing the course learning outcomes in relation to those stated in the course outline. It also assesses the program learning outcomes "Knowledge", "Problem solving and critical thinking", as well as "Communication". You are expected to demonstrate your ability to analyse an actuarial problem, apply appropriate theories and logic to interpret the problem, and develop solutions and conclusions. The communication of those will also be assessed.

2 Two tasks

Task 1. [40 marks]

You are an actuary at a general insurance company, has been tasked with analysing the loss severity data for a portfolio of property insurance claims. To improve the risk assessment and pricing strategies, data set containing 1,000 loss amounts are given for you to examine the data set and apply different probability distributions to capture the underlying patterns of the loss severity data.

Data: The claims amounts are stored in Loss.csv.

Your task is to use Maximum Likelihood Estimation (MLE) to fit an appropriate loss severity distribution. You are required to fit the **Log-normal, Gamma, and Pareto** distributions to the loss data and use appropriate goodness-of-fit tests to decide and subsequently justify which of the three distributions is the most appropriate to use for modelling the loss severity distribution. You may wish to further support your conclusions via graphical approaches.

You must briefly describe your methodology in reaching your MLE estimates of your parameters. Your answer should include the following steps:

- Estimate the model parameters for a given model and present the fitted model.
- Evaluate the quality of the given model by using graphical approaches and performing hypothesis tests (Note: when there is no grouping in the data, the K-S and A-D tests make more sense than the χ^2-test because no arbitrary decisions need to be made. So you do not have to perform the χ^2-test in this assignment.)
- Determine the model that fits best using the criteria introduced in the lectures.

However, providing detailed mathematical formulas and code snippets is not necessary (but the entire R code or the code of other software if you are not using R must be provided in a separate pdf or word file).

Task 2. [60 marks]

The ruin probability plays a crucial role in assessing the financial stability and solvency of insurance companies that exposed to risks, particularly in the context of extreme events. In the regulatory context, the appropriate level of capital that needs to be held to mitigate the risk of ruin, e.g. the probability of ruin within one year is no more than 0.005 (1 in 200 years event). Based on the recent experience, it is known that a Gamma distribution with shape $a = 5$ and rate $b = 1.5$ describes the individual claims sufficiently well. In addition, the claim arrival follows a Poisson Process with parameter $\lambda = 5$ *per month*. Therefore, the surplus of the direct insurer at time t (measured in months) can be described as

$$C(t) = c_0 + \pi t - \sum_{i=1}^{N(t)} Y_i, \quad t \geq 0$$

where c_0 is the initial surplus at time 0, $Y_i \sim \text{Gamma}(a = 5, b = 1.5)$ is the i-th claim amount and π is the constant rate of premium income paid continuously, and $N(t)$ is a number of claims up to time t. The insurer's premium is paid continuously at a constant rate π and is calculated so that the relative security loading is 7.5%.

Let $\psi_t(c_0)$ denote the probability that ruin occurs within t periods of time with initial surplus c_0, i.e. $\Pr(\min_{s \leq t} C(s) < 0 | C(0) = c_0)$.

For efficient use of capital, the minimum capital is determined to stay solvent at certain level. Specifically, it is often ensured that the 1-year survival probability is at least 99.5% and the 5-year survival probability is at least 99%, i.e. $\psi_{12}(c_0) \leq 0.005$ and $\psi_{60}(c_0) \leq 0.01$.

With the initial surplus $c_0 = 90$, the ruin probability $\psi_{60}(c_0)$ is estimated to be 2.21%, making the 5-year survival probability less than 99%. Therefore, the insurer considers two common types of reinsurance products in order to improve financial stability (i.e. reduce the ruin probability) in the following.

1. The insurer plans to purchase either

   - (A) a **proportional reinsurance** from another reinsurance company which charges a premium loading factor of 12.5% and the direct insurer retains $\alpha = 0.7$ of each claim or
   - (B) an **excess of loss (EoL) reinsurance** with a retention level $d = 2.57359$ and the reinsurance company charges a premium loading factor of 12.5% for this EoL reinsurance.

   For the above reinsurance products (A) and (B), perform the following analysis.

   (a) With the initial surplus $c_0 = 90$, find the approximated ruin probabilities within 5 years.

   (b) To avoid that ultimate ruin is certain, the insurer's net of reinsurance premium income per unit time must be larger than the expected aggregate claims per unit time. Find the range of α in (A) and d in (B) respectively.

   (c) Find (numerically) the direct insurer's retained proportion $\alpha \in [0, 1]$ in (A) and limit d in (B) that will maximize the adjustment coefficient for the direct insurer. Then calculate the upper bound for the probability of ruin with this choice of α in (A) and limit d in (B).

2. Explain how the above reinsurance products (A) and (B) can enhance an insurer's stability and discuss the trade-off between **stability** and **profitability** of the insurance company.

3. Discuss three various scenarios when reinsurance plays a crucial role.

3 Required document

You are asked to provide a **report and R code**. There will be **THREE** submission boxes (two business reports; one for Task 1 and one for Task 2, R code for Task 1 AND Task 2) in Moodle.

- The report should provide results for all of the above **two** tasks in word or pdf format. You do not need to provide a table of contents in your report. and should think of a clear and effective structure for your responses.
  - For Task 1, the main body of the report should be **no more than 3 pages** (i.e. maximum 3).
  - For Task 2, the main body of the report should be **no more than 3 pages** (i.e. maximum 3).

  You need to provide a reference list if any references are used in your report. Cover pages, appendices and reference lists are not counted towards the page limit. No page limit for the appendix. There is no specific formatting requirement; however, you should ensure that the report is professional in the business context.
- Intermediate steps for questions involving any form of derivation are required. Your comments and conclusions should be well justified and charts should be used to support your conclusions where applicable.
- You are **strongly recommended to use the software R for programming**, although the use of other software will also be accepted. Some sample R codes for fitting are available on the course website which may be of use. In addition, feel free find packages online to perform your computations (but always check that your answer is sensible!).
- When making a comment or conclusion based on R outputs (or other software outputs), you should include the relevant outputs in the main body of your report. You should make sure that the marker can read and understand your arguments and statements without referring to the separate R code file.
- Your R codes (or codes of other software) should be included in the separate file. The marker will choose a portion of the reports to check the code. He/she will need to copy the code, run it and check whether it is correct, implementable and consistent with the output presented in your answer. **Students will risk failing the assignment if the code cannot be run or the output provided in the answer is not consistent with the output generated by the code.**
- You should **not**
  - Include a chunk of programming codes in the main body of your report
  - Have figures or tables that are not referred to or analysed in the main body of your report
  - Include materials that are not highly relevant in the main body of your report

4 Assignment submission procedure

4.1 Report and R code: Turnitin submission through Moodle

Your assignment must be uploaded as a **unique document** (either pdf or word document) and all parts must be in **portrait format**. The R code must be provided as a separate file, in a format that we can copy and paste to check it. As long as the due date is still future, you can resubmit your work; the previous version of your assignment will be replaced by the new version.

Assignments must be submitted via the Turnitin submission box that is available on the course Moodle website. **There are THREE submission boxes for two business report and R code separately**. Turnitin reports on any similarities between their own cohort's assignments, and also with regard to other sources (such as the internet or all assignments submitted all around the world via Turnitin). More information is available at: [click]. Please read this page, as we will assume that you are familiar with its content.

Please refer to the course outline for special consideration and late submission. Students who are late must submit their assessment item to the Lecturer-in-Charge (LIC) via e-mail (j.k.woo@unsw.edu.au). The LIC will then upload documents to the relevant submission boxes. The date and time of reception of the e-mail determines the submission time for the purposes of calculating the penalty.

You need to check your document once it is submitted (check it on-screen). **We will not mark assignments that cannot be read on screen.**

Students are reminded of the risk that technical issues may delay or even prevent their submission (such as internet connection and/or computer breakdowns). Students should then consider either submitting their assignment from the university computer rooms or **allow enough time (at least 24 hours is recommended) between their submission and the due time.** The Turnitin module will not let you submit a late report. **No paper copy will be either accepted or graded.**

In case of a technical problem, the full document must be submitted to the LIC (j.k.woo@unsw.edu.au) before the due time by e-mail, with explanations about why the student was not able to submit on time. In principle, this assignment will not be marked. It is only in exceptional circumstances where the assignment was submitted before the due time by e-mail that it may be marked—and this only if a valid reason is established (and the LIC has the discretion in deciding whether a given reason is valid).

4.2 Plagiarism awareness

Students are reminded that the work they submit must be their own. While we have no problem with students discussing assignment problems if they wish, the material students submit for assessment must be their own. In particular, this means that any code you present are from your own computer, which you **yourself** developed, without any reference to any other student's work.

While some small elements of code are likely to be similar, big patches of identical code (even with different variable names, layout, or comments—Turnitin picks this up) *will* be considered as plagiarism. The best strategy to avoid any problem is *not* to share bits and pieces of code with other student outside your group.

Note however that you are allowed to use any R code that was made available during the course (either with the lectures or developed in the tutorial exercises). You don't need to reference them formally, and this will *not* be considered as plagiarism.

Students should make sure they understand what plagiarism is—cases of plagiarism have a very high probability of being discovered. For issues of collective work, having different persons marking the assignment does not decrease this probability. For more information on plagiarism, see [click].

Students should consult the "Write well; Learn deeply" website and consult the resources provided there. In particular, all students should do the quiz about plagiarism to make sure they know how to avoid any issue. For instance, did you know that sharing any part of your work with other students (outside your group) before the deadline is already considered as plagiarism? [1]

The course materials (e.g. R tutorial, R sample code) can be used, not regarded as plagiarism. However, any other references should be properly cited.

5 Assessment criteria

Please see the file, "Rubric".

[1] Yes, that's right, just sending it, even if the third party promises not to copy, is already plagiarism in the UNSW policy!