Practical 01.

1. $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$= \lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$= \lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}$$

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3. $$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

By substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

where $h \to 0$.

$$\lim_{h \to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\cos h . \cos\frac{\pi}{6} - \sin h . \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\cos h . \frac{\sqrt{3}}{2} . \sin h \frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{\sqrt{2}}\right)}{\pi - 6h + \pi}$$

$$= \lim_{h \to 0} \frac{\cos \frac{\sqrt{3}h}{2} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}h}{2}}{-6h}$$

$$= \lim_{h \to 0} \frac{-\frac{\sin 4h}{2}}{-6h} = \lim_{h \to 0} \frac{\sin 4h}{3+2h}$$

1 lim sin ...

4. $$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$$

By rationalising numerator & denominator both

$$= \lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$= \lim_{x \to \infty} \left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$= \lim_{x \to \infty} \frac{8(\sqrt{x^2+3} + \sqrt{x^2+1})}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$= 4\lim_{x \to \infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit we get. $= 4$

5. $$f(x) = \begin{cases} \frac{\sin 2x}{\sqrt{1-\cos 2x}}, & \text{for } 0 < x < \pi/2 \\ = \frac{\cos x}{\pi - 2x}, & \text{for } \frac{\pi}{2} < x < \pi \end{cases}$$ at $x = \pi/2$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2\left(\frac{\pi}{2}\right)}} \qquad \therefore f\left(\frac{\pi}{2}\right) = 0.$$

f at $x = \frac{\pi}{2}$ define.

3. $\lim_{x \to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$

$x = h + \frac{\pi}{6}$

where $h \to 0$.

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{6}\right) - \sqrt{3}\sin\left(h + \frac{\pi}{6}\right)}{\pi - 6\left(h + \frac{\pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\cos h . \cos\frac{\pi}{6} - \sin h . \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\cos h . \frac{\sqrt{3}}{2} . \sin h \frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{\sqrt{2}}\right)}{\pi - 6h + \pi}$$

$$= \lim_{h \to 0} \frac{\cos \frac{\sqrt{3}}{2} h - \sin \frac{h}{2} - \sin \frac{3h}{2} - \cos \frac{\sqrt{3}h}{2}}{-6h}$$

$$= \lim_{h \to 0} \frac{-\sin \frac{4h}{2}}{-6h} = \lim_{h \to 0} \frac{\sin 4h}{3 + 2h}$$

$$\frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3} \times 1 = \frac{1}{3}.$$

ii) $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\cos x}{\pi - 2x}$

By substituting method.

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi-2\left(h+\frac{\pi}{2}\right)}$

$= \lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi-2\left(\frac{2h+\pi}{2}\right)}$

$= \lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{-2h}$ using $\cos(A+B) = \cos A.\cos B - \sin A.\sin B$

$= \lim_{h\to 0} \frac{\cos h.\cos\frac{\pi}{2} - \sin h.\sin\frac{\pi}{2}}{-2h}$

$= \lim_{h\to 0} \frac{\cos h.0 - \sin h}{-2h}$

$= \lim_{h\to 0} \frac{-\sin h}{-2h} = \frac{1}{2}\lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{2}$

6. $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$= \lim_{x\to\pi/2} \frac{2\sin x.\cos x}{\sqrt{2\sin^2 x}}$

$= \lim_{x\to\pi/2} \frac{2.\sin x.\cos x}{\sqrt{2}.\sin x}$

$= \lim_{x\to\pi/2} \frac{2\cos x}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} \lim_{x\to\pi/2} \cos x$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ f is not continuous at $x = \pi/2$

5.(ii) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x \le 6$

$= \frac{x^2+9}{x+3}$ $6 \le x < 9$

at $x = 3$ & $x = 6$.

at $x = 3$

I) $f(3) = \frac{x^2-9}{x-3} = 0.$

f at $x = 3$ define.

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ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$.

$f(3) = x+3 = 3+3 = 6$

f is defined at $x=3$.

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{(\cancel{x-3})(x+3)}{(\cancel{x-3})}$

$\therefore$ L.H.L = R.H.L.

f is continous at $x=3$.

For $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

2. $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)(\cancel{x+3})}{(\cancel{x+3})}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} x+3 = 3+6 = 9$.

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore f$ is not continous.

6.

i) $f(x) = \frac{1-\cos 4x}{x^2}$ $x<0$

$= k$ $x=0$ } at $x=0$.

$\rightarrow$ f is continous at $x=0$.

$\lim_{x\to 0} f(x) = f(0)$.

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$.

$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$

$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$.

$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$.

$2(2)^2 = k$

$\therefore k = 8$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x\neq 0$

$= k$ $x=0$ } at $x=0$

$\rightarrow$ $f(x) = (\sec^2 x)^{\cot^2 x}$

Using

$\tan^2 x - \sec^2 x = 1$.

$\therefore \sec^2 x = 1+\tan^2 x$ &

$\cot^2 x = \frac{1}{\tan^2 x}$

$$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$$

We know that

$$\lim_{x\to 0} (1+px)^{1/px} = e.$$

$$\therefore = e$$

$$K = e.$$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \pi/3$

$= K$ $x = \pi/3$ } at $x = \pi/3$

$$x - \frac{\pi}{3} = h$$

$$x = h + \frac{\pi}{3}$$

where $h \to 0$.

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

Using

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

7. $f(x) = \dfrac{1-\cos 3x}{x\tan x}$ $\quad x \neq 0.$

$\quad = 9 \quad\quad x = 0$ $\Big\}$ at $x=0$.

$f(x) = \dfrac{1-\cos^3 x}{x\tan x}$

$\lim\limits_{x\to 0} = \dfrac{2\sin^2 3/2\, x}{x\tan x}$

$\lim\limits_{x\to 0} = \dfrac{\dfrac{2\sin^2 3x/2}{x^2} \times x^2}{x.\dfrac{\tan x}{x^2} \times x^2}$

$= 2\lim\limits_{x\to 0} \dfrac{(3/2)^2}{1} = \dfrac{2\times 9}{4} = \dfrac{9}{2}$

$\lim\limits_{x\to 0} f(x) = \dfrac{9}{2}$ $\quad 9 = f(0).$

$\therefore$ f is not continoos $x \neq 0$.

$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = \dfrac{9}{2} & x = 0 \end{cases}$

Now $\lim\limits_{x\to 0} f(x) = f(0).$

f has removable discontinoity at $x=0$.

No

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 in Num & denominator.

$$= 1 + 2\times\frac{1}{4} = \frac{3}{2} = f(0).$$

$$f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \qquad x \neq \pi/2.$$

$f(0)$ is continous at $x = \pi/2$

Q. $$\lim_{x\to\pi/2} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$= \lim_{x\to\pi/2} \frac{2-1+\sin x}{\cos^2 x+(\sqrt{2}+\sqrt{1+\sin x})}.$$

$$= \lim_{x\to\pi/2} \frac{1+\sin x}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}.$$

$$= \frac{1}{2\cancel{(\sqrt{2}+\sqrt{2})}}.$$

$$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

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$$f(\pi/2) = \frac{1}{\ldots}.$$

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$$= \lim_{h \to 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \cdot \tan(a+h)}{\tan(a+h)\tan a}$$

$$= 1 - 1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= -\frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\operatorname{cosec}^2 a$.

$\therefore$ f is differentiable $\forall a \in R$.

ii). cosec x.

$$f(x) = \operatorname{cosec} x$$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\sin x - 1/\sin a}{x - a}$$

$$= \lim_{x \to 0} \frac{\sin a - \sin x}{(x-a)\sin x \sin a}$$

Put $x - a = h$,

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

Formula :

$-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$= \lim_{h\to 0} \frac{-2\sin(a+a+h/2)\sin((a-a-h)/2)}{h - \cos a \cos(a+h)}$

$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -h/2}{\cos a \cos(a+h) \times -h/2} \times -1/2$

$= \frac{-1}{2} \times \frac{2\sin(2a+0/2)}{\cos a \cos(a+0)}$

$= -1/2 \times \frac{-2\sin a}{\cos a \times \cos a}$

$= \tan a . \sec a.$

2) If $f(x) = 4x+1$; $x \le 2$.

$= x^2+5$, $x > 0$ at $x = 2$ then.

Find function is differentiable or not.

→ LHD.

$Df(2-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$

$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$

$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$

$= \lim_{x\to 2^-} \frac{4(x-2)}{(x-2)} = 4.$

$Df(2-) = 4$

RHD

$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$

$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$

$= \lim_{x\to 2^+} \frac{(x+2)(x-2)}{x-2}$

$= 2+2 = 4.$

$Df(2^+) = 4$

RHD = LHD.

f is differentiable at $x = 2$.

3) If $f(x) = 4x+7$, $x < 3$

$= x^2+3x+1$, $x \ge 3$ at $x = 3$, then

Find f is differentiable or not solution

RHD $Df(3^+) = \lim_{x\to 2^+} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$

$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$

$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$= \lim_{x\to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9.$

$Df(3^+) = 9.$

LHD = DF(3⁻).

$= \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$

$= \lim_{x\to 3^-} \frac{4(x-3)}{(x-3)}$.

DF(3⁺) = 4

RHD ≠ LHD.

f is not differentiable at 2.

4) If $f(x) = 8x-5$, $x \le 2$

$= 3x^2-4x+7$, $x>2$. at $x=2$, then.

find f is differentiable or not.

→ $f(2) = 8\times 2-5 = 16-5 = 11$.

RHD: $DF(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}$

$= \lim_{x\to 2^+} \frac{3x^2-\cancel{6x}\cancel{+}-4}{x-2}$

$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}$

$= \lim_{x\to 2^+} \frac{3x(x-2)-2(x-2)}{x-2}$

$= \lim_{x\to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$.

$= 3\times 2+2 = 8$ DF(2⁺) = 8.

LHD $DF(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$

$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$

$= \lim_{x\to 2^-} \frac{8(x-2)}{(x-2)}$

DF(2⁻) = 8 LHD = RHD.

f is differentiable at $x=3$.

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Practical No. 3

Topic :- Application of derivative.

1) Find the intervals in which function is increasing / decreasing.

→ (i) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\because f$ is increasing iff $f'(x) > 0$.

$3x^2 - 5 > 0$.

$3(x^2 - 5/3) > 0$.

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$.

+ $-\sqrt{5}/3$ − $\sqrt{5}/3$ +

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) < 0$.

$\therefore 3x^2 - 5 < 0$.

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$.

+ $-\sqrt{5}/3$ − $\sqrt{5}/3$ +

$x \in (-\sqrt{5}/3, \sqrt{5}/3)$

(ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$.

$\therefore 2x - 4 > 0$.

$\therefore 2(x - 2) > 0$.

$\therefore x - 2 > 0$.

$x \in (2, \infty)$.

and f is decreasing iff $f'(x) < 0$.

$\therefore 2x - 4 < 0$.

$\therefore 2(x - 2) < 0$.

$\therefore x - 2 < 0$.

$x \in (-\infty, 2)$.

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$.

$\therefore 6x^2 + 2x - 20 > 0$.

$\therefore 2(3x^2 + x - 10) > 0$.

$\therefore 3x^2 + x - 10 > 0$.

$\therefore 3x(x+2) - 5(x+2) > 0$.

$\therefore (x+2)(3x-5) > 0$.

+ -2 − $5/3$ +

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$.

$\therefore 6x^2 + 2x - 20 < 0$.

$\therefore 2(3x^2 + x - 10) < 0$.

$\therefore 3x^2 + x - 10 < 0$.

$\therefore 3x^2 + 6x - 5x - 10 < 0$.

$\therefore 3x(x+2) - 5(x+2) < 0$.

$\therefore (x+2)(3x-5) < 0$.

+ -2 − $5/3$ +

$x \in (-2, 5/3)$.

4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore$ f is increasing iff $f'(x) > 0$.

$\therefore 3(x^2-9) > 0$.

$\therefore (x-3)(x+3) > 0$.

+   −3   −   3   + $x \in (-\infty, -3) \cup (3, \infty)$.

and f is decreasing iff $f'(x) < 0$.

$\therefore 3x^2 - 27 < 0$.

$\therefore 3(x^2-9) < 0$.

$\therefore (x-3)(x+3) < 0$.

+   −3   −   3   + $\therefore x \in (-3, 3)$.

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$.

$f'(x) = 6x^2 - 18x - 24$

$\because$ f is increasing iff $f'(x) > 0$.

$\therefore 6x^2 - 18x - 24 > 0$.

$\therefore 6(x^2 - 3x - 4) > 0$.

$\therefore x^2 - 4x + x - 4 > 0$.

$\therefore (x-4)(x+1) > 0$.

+   −1   −   4   + $\therefore x \in (-\infty, -1) \cup (4, \infty)$.

and f is decreasing iff $f'(x) < 0$.

$\therefore 6x^2 - 18x - 24 < 0$.

$\therefore 6(x^2 - 3x - 4) < 0$.

$\therefore x^2 - 4x + x - 4 < 0$.

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$

+   −1   −   4   + $\therefore x \in (-1, 4)$.

Q.2 Find the intervals in which function is concave upwards.

(i) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$.

$\therefore (6 - 12x) > 0$.

$\therefore 12(6/12 - x) > 0$.

$x - 1/2 > 0$.

$x > 1/2$

$\because f''(x) > 0$

$\therefore x \in (1/2, \infty)$.

(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$.

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$.

f is concave upward if $f''(x) > 0$.

$\therefore 12x^2 - 36x + 24 > 0$.

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$.

$\therefore (x-2)(x-1) > 0$.

+   1   −   2   + $x \in (-\infty, 1) \cup (2, \infty)$.

(iii) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$.

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$.

$\therefore 6x > 0$.

$\therefore x > 0$.

$\therefore x \in (0, \infty)$.

(iv) $y = 69 - 24x - 9x^2 + 2x^3$.

$f(x) = 2x^3 - 9x^2 - 24x + 64$.

$f'(x) = 6x^2 - 18x - 24$.

$f''(x) = 12x - 18$.

f is concave upward iff $f''(x) > 0$.

$\therefore 12x - 18 > 0$.

$\therefore 12(x - 18/12) > 0$.

$\therefore x - 3/2 > 0$. $\therefore x > 3/2$.

$\therefore x \in (3/2, \infty)$.

(v) $y = 2x^3 + x^2 - 20x + 4$.

$f(x) = 2x^3 + x^2 - 20x + 4$.

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f''(x) = 12x + 2$.

f is concave upward iff $f''(x) > 0$.

$\therefore f''(x) > 0$.

$\therefore 12(x + 2/12) > 0$.

$\therefore x + 1/6 > 0$.

$\therefore x < -1/6$

$\because f''(x) \ngtr 0$.

$\therefore$ There exist no interval.

Q.1
(i) $F(x) = x^2 + \frac{16}{x^2}$

$F'(x) = 2x - \frac{32}{x^3}$

Now consider,

$F'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0.$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2.$

$F''(x) = 2 + \frac{96}{x^4}$

$= 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0.$

$\therefore$ F has maximum value at $x = 2$.

$\therefore F(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4$

$= 8.$

$F''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0.$

f has minimum value at $x = ...$

$\therefore$ function reaches minimum value at $x = 2$ and $x = -2$.

(ii) $f(x) = 3 - 5x^3 + 3x^5$

$F'(x) = -15x^2 + 15x^4$

consider

$F'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore F''(x) = -30x + 60x^3$

$F(1) = -30 + 60$

$= 30 > 0.$

$\therefore$ F has minimum value at $x = 1$.

$\therefore F(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1.$

now

$F''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0.$

$\therefore$ F has minimum value at $x = -1$

$\therefore F(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 6$

$\therefore$ F has the maximum value at $x = -1$. and has the minimum value at $x = 1$.

(iii) $f(x) = x^3 - 3x^2 + 1$.

$\therefore f'(x) = 3x^2 - 6x$

consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$.

$\therefore 3x(x-2) = 0$.

$\therefore 3x = 0$ & $x - 2 = 0$

$\therefore x = 0$ or $x = 2$.

$f''(x) = 6x - 6$.

$f''(0) = 6(0) - 6$

$= -6 < 0$.

$\therefore f$ has maximum value at $x = 0$.

$\therefore f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$.

$\therefore f$ has minimum value at $x = 2$.

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore f$ has maximum value 1 at $x = 0$ and

f has minimum value -3 at $x = 2$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

consider,

$f'(x) = 0$.

$\therefore 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$.

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x+1)(x-2) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore f$ has minimum value at $x = 2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$.

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$.

$\therefore f$ has maximum value at $x = -1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

$\therefore f$ has maximum value 8 at $x = -1$ &

f has minimum value -19 at $x = 2$

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Q. 2.

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\qquad x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\therefore x_1 = 0 + \frac{9.5}{55}$$

$x_1 = 0.1727.$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829.$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0829}{55.9467}$$

$= 0.1712.$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$= 3 - \frac{6}{23}$$

$$= 2.7392.$$

$$f(x_1) = (2.7392)^3 - 4(2.7392) - 9.$$

$$= 20.5528 - 10.9568 - 9$$

$$= 0.596.$$

$$f'(x_1) = 3(2.7392)^2 - 4.$$

$$= 22.5096 - 4.$$

$$= 18.5096.$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7392 - \frac{0.596}{18.5096}$$

$$= 2.7071$$

$$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$$

$$= 19.8386 - 10.8284 - 9$$

$$= 0.0102.$$

$$f'(x_2) = 3(2.7071)^2 - 4$$

$$= 21.9851 - 4.$$

$$= 17.9851.$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7071 - \frac{0.0102}{17.9851}.$$

$$= 2.7015.$$

$$f(x_3) = (2.7015)^3 - 4(2.7015) - 9.$$

$$= 19.7158 - 10.806 - 9$$

$$= -0.0901.$$

$$f'(x_3) = 3(2.7015)^2 - 4$$

$$= 21.8943 - 4$$

$$= 17.8943.$$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$

$$= 2.7015 + 0.0050$$

$$= 2.7065$$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17.$

$$f'(x) = 3x^2 - 3.6x - 10.$$

$$f(1) = (1)^3 - 1.8(1)^2 - 10 + 17$$

$$= 1 - 1.8 - 10 + 17$$

$$= 6.2$$

$$f(2) = (2)^3 \cancel{-1.8(2)^2} - 10(2) + 17$$

$$= 8 - 7.2 - 20 + 17$$

$$= -2.2$$

let $x_0 = 2$ be initial approximation

By Newton's method.

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \frac{2.2}{5.2}$

$= 2 - 0.4230$

$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6756$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.577 + \frac{0.6755}{8.2164}$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 3.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.6592 + \frac{0.0204}{7.7143}$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 1.6618 + \frac{0.0004}{7.6977}$

$= 1.6618$

$\therefore$ The root of equation is 1.6618.

Practical No. 6

Integration.

Q1) Solve the following integration.

i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x-3}} dx$.

$I = \int \frac{1}{\sqrt{x^2+2x-\cancel{3}-4}} dx$.

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx = \int \frac{1}{\sqrt{(x+1)^2-(2)^2}} dx$

$\because a^2+2ab+b^2 = (a+b)^2$

Substitute

$x+1 = t$

$dx = \frac{1}{t} \times dt$ where $t = 1$, $t = x+1$.

$\int \frac{1}{\sqrt{t^2-4}} dt$

$= \log(|t+\sqrt{t^2-4}|)$ $[\because \int \frac{1}{\sqrt{x^2-a^2}} dx = \log|x+\sqrt{x^2-a^2}|]$

$t = x+1$

$= \log(|x+1+\sqrt{(x+1)^2 \cancel{3}-4}|)$

$= \log(|x+1+\sqrt{x^2-2x-3}|)+c$

(ii) $\int (4e^x+1)dx$

$I = \int (4e^{3x}+1)dx$

$= \int 4e^{3x}dx + \int dx$

$= 4\int e^{3x}dx + \int 1 dx$

$= \frac{4e^{3x}}{3} + x$ $[\because \int e^{ax}dx = \frac{1}{a}e^{ax}]$

$= \frac{4e^{3x}}{3} + x + c$

(iii) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$I = \int 2x^2 - 3\sin x + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin x + 5x^{1/2} dx$.

$= \int 2x^2 dx - 3\int \sin x dx + 5\int x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\sqrt[3]{x}}{3} + c$ $[\because \int x^n dx = \frac{x^{n+1}}{n+1} + c]$

$= \frac{2x^3+10\sqrt[3]{x}}{3} + 3\cos x + c$.

(iv) $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

$I = \int \frac{x^3 + 3x + 4}{\sqrt{x}} dx.$

$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}} \right) dx$

$= \int \frac{x^3}{x^{1/2}} dx + \int \frac{3x}{x^{1/2}} dx + \int \frac{4x^0}{x^{1/2}} dx.$

$= \int x^{5/2} dx + 3\int x^{1/2} dx + 4\int x^{-1/2} dx$

$= \frac{x^{5/2+1}}{5/2+1} + 3 \frac{x^{1/2+1}}{1/2+1} + 4 \frac{x^{-1/2+1}}{-1/2+1}$

$= \frac{x^{7/2}}{7/2} + 3 \frac{x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2}$

$= \frac{2x^{7/2}}{7} + 2x^{3/2} + 8\sqrt{x} + C.$

(v) $\int t^7 \times \sin(2t^4)\, dt.$

PUT $U = 2t^4.$

$dU = 2 \times 4t^3$

$\cancel{= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3} dU}$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4} dU.$

$= \int t^4 \sin(2t^4) \times \frac{1}{8} dU.$

$= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

(vi) $\int \sqrt{x}(x^2-1)\,dx$

$I = \int \sqrt{x}(x^2-1)\,dx$

$= \int x^{1/2}(x^2-1)\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= \frac{x^{5/2+1}}{5/2+1} - \frac{x^{1/2+1}}{1/2+1}$

$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2}$

$= \frac{2x^{7/2}}{7} - \frac{2x^3}{3} + C$

(vii) $\int \frac{\cos x}{3\sqrt{\sin(x)^3}}\,dx$

$I = \int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \frac{\cos x}{\sin x^{3/2}}\,dx$

put $t = \sin x$

$dt = \cos x\,dx$

$= \frac{1}{(t)^{1/3}}\,dt$

$= (t)^{-2/3}\,dt$

$= \frac{t^{-2/3+1}}{-2/3+1}$

$= \frac{t^{1/3}}{1/3}$

$= 3\sqrt[3]{t} + C$

$= 3\sqrt[3]{\sin x} + C$

(ix) $\int e^{\cos^2 x} \sin 2x \, dx$.

$I = \int e^{\cos^2 x} \sin 2x \, dx$.

put $\cos^2 x = t$.

$2\cos x(-\sin x)dx = dt$.

$-\sin 2x \, dx = dt$

$\sin 2x \, dx = -dt$

$\therefore \int e^t (-dt)$.

$-\int e^t dt \quad [\because \int e^x dx = e^x + c]$.

Resubstituting $\cos^2 x = t$.

$= -e^{\cos^2 x} + c$.

(x) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$.

put $x^3 - 3x^2 + 1 = t$.

$(3x^2 - 6x)dx = dt$.

$3(x^2 - 2x)dx = dt$.

$(x^2 - 2x)dx = \frac{dt}{3}$.

$\therefore \int \left(\frac{1}{t}\right) \frac{dt}{3}$.

$\frac{1}{3}\int \left(\frac{1}{t}\right) dt$.

$= \frac{1}{3} \log|t| + c \quad [\because \int \left(\frac{1}{x}\right) dx = \log|x| + c]$.

Resubstituting $x^3 - 3x^2 + 1 = t$.

$\therefore \frac{1}{3} \log|x^3 - 3x^2 + 1| + c$.

AV 02/01/2020

PRACTICAL NO. 6.

Topic :- Application of integration & numeric integration.

Q.1) Find the length of the following.

1) $x = t\sin t$; $y = 1-\cos t$ $t \in [0, 2\pi]$

→ arclength $= \int_0^{4} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t + dt} = \sqrt{1-2\cos t + dt}$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$= \int_0^{2\pi} 2\sin t/2 \, dt$

$= \left[-4\cos t/2\right]_0^{2\pi}$

$= (-4\cos\pi) + 4\cos 0$

$= 8$ units.

2) $y = \sqrt{4-x^2}$ $x \in [-2, 2]$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$

$= \frac{-x}{\sqrt{4-x^2}}$

$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$

$= \int_{-2}^{2} \frac{1}{\sqrt{2^2 - x^2}}\, dx$

$= 2\left[\sin^{-1}(1) - \sin^{-1}(1)\right]$

$= 2\left[\pi/2 + \pi/2\right]$

$= 2\pi$

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3) $y = x^{3/2}$ in $[0, 4]$.

$\rightarrow \frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$$l = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx.$$

$$= \int_0^4 \sqrt{1+ 9x/4}\quad dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\quad dx.$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$$

$$= \frac{-1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$L = \frac{1}{27}\left(40^{3/2} - 8\right)$ units.

$$= \int_1^2 \sqrt{\frac{(y^4-1)+4\times y^4}{4y^4}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2 dy + \frac{1}{2}\int_1^2 y^{-2} dy.$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1/2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$= \frac{17}{12} \text{ units.}$$

Q II i) $\int_0^4 x^2 dx \quad x = 4.$

$$L = \frac{4-0}{4} = 1.$$

x	0	1	2	3	4
y	0	1	4	9	16

$$= \int_0^4 x^2 dx = \frac{1}{3}\left[(y_0+y_4) + 4(y_1+y_3)\,2(y_2)\right]$$

Practical 7.

TOPIC: Differential Equation.

Q.1 solve the following differential equation.

(i) $x\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

comparing with

$\frac{dy}{dx} + P(x)y = Q(x)$

$IF = e^{\int F dx}$

$= e^{\int 1/x \, dx}$

$= \int e^{\log x} dx = x.$

$y(IF) = \int Q(I.F) x \, dx + C$

$y(x) = \int \frac{e^x}{x} . x \, dx + C$

$y(x) = \int e^x + c$

$y(x) = e^x + c.$

(ii) $e^x dy + 2y = 1$

dividing by e^x

$\cancel{\frac{dy}{dx} + 2.y = \frac{1}{e^x}}$

comparing with

$\frac{dy}{dx} + P(x)y = Q(x)$

$= \int \cos x + c$

$x^3 y = \sin x + c.$

(iv) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\rightarrow \frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both sides).

$P(x) = 3/x$ $Q(x) = \sin x / x^3$

$IF = e^{\int P dx}$

$= e^{\int 3/x dx}$

$= e^{3 \ln x}$

$= e^{\ln x^3}$

$IF = x^3$

$y(I.F) = \int Q(x)\,(I.F)\,dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3 dx + c$

$= \int \sin x\, dx + c$

$x^3 y = -\cos x + c.$

(v) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\rightarrow \frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2$ $Q(x) = 2x / e^{2x} = 2x e^{-2x}$

$IF = e^{\int P(x) dx}$

$= e^{\int 2 dx}$

$= e^{2x}$

$y(IF) = \int Q(x)\,(I.F)\,dx + c$

$= \int 2x e^{-2x} e^{2x} dx + c$

$= \int 2x\,dx + c$

$\therefore y e^{2x} = x^2 + c$

(vi) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\rightarrow \sec^2 x \cdot \tan y\, dx = -\sec^2 y \cdot \tan x\, dy$

$\frac{\sec^2 x\, dx}{\tan x} = -\frac{\sec^2 y\, dy}{\tan y}$

$\int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x - \tan y| = c$

$\tan x \cdot \tan y = e^c.$

(vii) $\frac{dy}{dx} = \sin^2(x - y + 1)$

Put $x - y + 1 = v$

Differentiating on both sides

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$$\frac{dv}{dx} = \cos^2 v.$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v\,dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

(viii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+7y+6}$$

put $2x+3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{9}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3(v+1)}{v+2}$$

$$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$$

$$= \int \frac{v+1}{v}\,dx + \int \frac{v_1}{v+1}\,dv = 3x$$

$$v + \log(x) = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + 1$$

$$3y = x - \log|2x+3y+1| + c.$$

Practical No. 8.

Q.1 using Euler's method find the following.

1) $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$.

$f(x) = y + e^x - 2$, $x_0 = 0$

$y(0) = 2$, $h = 0.5$

$y(0.2) = ?$

n	x_n	$y(0)$	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.14787	3.5743
2	1	3.5443	4.2925	5.7205
3	1.5	5.7205	8.2021	9.3215

$y(2) = 9.8215$.

(2) $\frac{dy}{dx} = 1+y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$.

$x_0 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0.	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9934	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$.

(3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ find $y(1)$.

$y(0) = 1$, $x_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3313	0.7694	1.5051
5	1	1.5051		

$y(1) = 1.5051$.

(5) $\frac{dy}{dx} = \sqrt{xy}+2$, $y(1)=1$ find $y(1.2)$ with $h=0.2$

$y(0)=1$ $x(0)=1$ $h=0.2$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1) = 3.6$.

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$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x}$$

$$\frac{1+1\times1}{1^2} = 2.$$

Q.2 Find f_x, f_y for each of the following.

(i) $F(x,y) = xye^{x^2+y^2}$

$$f(x) = \frac{\partial f}{\partial x}$$

$$= \frac{\partial(xye^{x^2+y^2})}{\partial x}$$

$$= y\,\frac{\partial(xe^{x^2+y^2})}{\partial x}$$

$$= y\left[x.\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}\frac{d}{dx}(x)\right]$$

$$\left[\because \frac{d}{dx}(uv) = uv' + vu'\right]$$

$$= y[x.e^{x^2+y^2}.2x + e^{x^2+y^2}(1)]$$

$$= y.e^{x^2+y^2}[2x+1]$$

NOW, $f(y) = \frac{\partial f}{\partial y}$

$$= \frac{\partial(xye^{x^2+y^2})}{\partial y}$$

$$= x.\frac{\partial}{\partial y}(y.e^{x^2+y^2})$$

$$= x\left[y.\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}.\frac{d}{dy}(y)\right]$$

$$\left[\because \frac{d}{dx}(uv) = uv' + vu'\right]$$

$$= x.[2y^2.e^{x^2+y^2} + e^{x^2+y^2}]$$

$$= x.e^{x^2+y^2}[2y^2+1]$$

(ii) $F(x,y) = e^x \cos y$

$\therefore F(x) = e^x \cos y$

$$F(y) = e^x.\frac{\cos y}{\partial y}$$

$$= e^x(-\sin y)$$

$$= -e^x \sin y$$

(iii) $F(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$$F(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$$

$$= 3x^2y^2 - 3(2x)y$$

$$= 3x^2y^2 - 6xy$$

$$F(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$$

$$= x^3(2y) - 3(1)x^2 + 3y^2$$

$$= 2x^3y - 3x^2 + 3y^2.$$

Q.3 using defination find value of f_x, f_y at $(0,0)$.
for $f(x,y) = \dfrac{2x}{1+y^2}$

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

where $(a,b) = (0,0)$.

$$\therefore f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{2h - 0}{2} = 2$$

similarly

$$f_y(0,b) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0$$

$\therefore f_x = 2,\ f_y = 0$

~~[illegible]~~

Q.4 find all second order partial derivative of f also verify whether $f_{xy} = f_{yx}$

(i) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$$\therefore f(x) = \frac{\partial f}{\partial x} = \frac{\partial (y^2 - xy)}{x^2}$$

$$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

~~[illegible]~~

$$= \frac{6x^2y^2 - 2x^3y}{x^6} = \frac{x^2(6y^2 - 2xy)}{x^6}$$

$$= \frac{6y^2 - 2xy}{x^4}$$

$$F(y,y) = \partial\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{\frac{1}{x^2}\,\partial(2y - x)}{\partial y} = \frac{1}{x^2}(2) = \frac{2}{x^2}$$

$$F(xy) = \partial\left(\frac{xy - 2y^2}{x^3}\right) = \partial\left(\frac{xy}{x^3} - \frac{2y^2}{x^3}\right)$$

$$= \frac{\partial\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{1}{x^2} - \frac{1}{x^3}\,\partial(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3 - 4yx^2}{x^6}$$

$$= \frac{x^2(x - 4y)}{x^6}$$

$$= \frac{x - 4y}{x^4}$$

$$F(yx) = \frac{\partial\left(\frac{2y - x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(\frac{2y}{x^2} - \frac{1}{x}\right)}{\partial x}$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= \frac{-4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{-4yx^3 + x^3}{x^6}$$

$$= \frac{x^2(x - 4y)}{x^6}$$

$$= \frac{x - 4y}{x^4}$$

$$\therefore F(xy) = F(yx) = \frac{x.4y}{x^4}$$

Hence verified

(ii) $F(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$\therefore F(x) = \frac{\partial F}{\partial x} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial x}$$

$$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$$

$$F(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$F(y) = \frac{\partial F}{\partial y} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial y}$$

$$= 0 + 3(2y)(x^2) + 0$$

$$F(y) = 6x^2y.$$

: a

$$f(xx) = \frac{\partial fx}{\partial x} = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{(x^2+1)^2}\right]$$

$$\left[\because \frac{d}{dx}\frac{U}{V} = \frac{VU' - UV'}{V^2}\right]$$

$$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)}\right)$$

$$= 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$$

$$f(yy) = \frac{\partial fy}{\partial y} = \frac{\partial(6x^2y)}{\partial y}$$

$$= 6x^2(1) = 6x^2$$

$$f(xy) = \frac{\partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$$

$$= 0 + 6x(2y)$$

$$= 12xy$$

$$f(yx) = \frac{\partial fy}{\partial x}$$

$$= \frac{\partial(6x^2y)}{\partial x}$$

$$= 12xy$$

$$\therefore f(xy) = f(yx) = 12xy$$

Hence, verified.

$$f(y,x) = \frac{\partial f_y}{\partial x} = \frac{\partial (x\cos xy + e^{x+y})}{\partial x}$$

$= \cos xy(1) + x(-\sin(xy))(y) + e^{x+y}$

$= -xy\sin(xy) + \cos(xy) + e^{x+y}$

$\therefore f(xy) = f(yx) = -xy\sin(xy) + \cos(xy) + e^{x+y}$

Q.5) Find the linearization of $f(x,y)$ at given point.

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$.

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$f_y(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= 2 + x - 1 + y - 1$

$= \frac{2 + x + y - 2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

Practical No. 10.

Q.1 Find the directional derivative of the following functions at given point & in the direction of given vector.

i) $f(x,y) = x+2y-3 \quad a = (1,-1) \quad u = 3i - j$

Here, $u = 3i - j$ is not a unit vector.

$|\bar{u}| = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$.

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(\left(1+\frac{3}{\sqrt{10}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

a

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}.$$

ii) $f(x) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i + 5j$

Here, $u = i + 5j$ is not a unit vector.

$|u| = \sqrt{1^2 + 5^2} = \sqrt{26}$

unit along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}} (1,5)$.

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right).$$

$$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{\sqrt{26}} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$$

$$= 2+\frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8$$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$$

$$= \frac{18}{5}$$

Q.2 Find gradient vector for the following functions at given point

(i) $f(x,y) = x^y + y^x$, $a = (1,1)$

$f_x = y.x^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y,\ x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (-\tan^{-1} x)\, y^2$ $a = (1,-1)$

$f_x = \frac{1}{1+x^2} . y^2$ $f_y = 2y - \tan^{-1} x$

$\nabla f(x,y) = f(x), f(y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, -\tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi(-2)}{4}\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

(iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = \left((-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{(1+(-1)+0)}, (1)(1) - e^{1+(-1)+0}\right)$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q.3 Find the equation of tangent & normal to each of the following using curve at given points.

(i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y\, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} . x$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1, y_0 = 0$

eqⁿ of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$f_x(x_0,y_0) = \cos 0 \cdot 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0,y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ → It is the required eqⁿ of tangent

eqⁿ of normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$d = -1$

(ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2, y_0 = -2$

$f_x(x_0,y_0) = 2(2) - 2 = 2$

$f_y(x_0,y_0) = 2(-2) + 3 = -1$

eqⁿ of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ → It is required eqⁿ of tangent.

eqⁿ of normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$d = 6.$

Q4 Find the eqn of tangent & normal line to each of the following surface.

(i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$.

$f_x = 2x - 0 + 0 + z$.

$f_x = 2x + z$.

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$.

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$.

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

eqn of tangent

$f_x(x_0 - x_0) + f_y(y - y_0) + f_z(z_0 - z_0) = 0$.

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$\cancel{4x - 8 + 3y - 3 = 0}$

$= 4x + 3y - 11 = 0$ → This is required eqn of tangent.

eqn of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$ //

$3xyz - x - y + z = -4$ at $(1,-1,2)$.

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$.

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1,-1,2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2)(-1) = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$.

eqn of tangent.

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → This is required eqn of tangent.

eqn of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

05

Q.5 Find the local maxima & minima for the following

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \dots (1)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \dots (2)$

Multiply eq^(1) with 2

$\therefore\ 4x - 2y = -4$

$2y - 3x = 4$

substitute value of x in eq^(1).

$2(0) - y = -2$

$y = 2$

$\therefore$ critical points are $(0,2)$.

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here, $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$\therefore\ 3 > 0$

$\therefore$ f has maximum at $(0,2)$.

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$.

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$.

$0 + 4 - 0 + 0 - 8$

$= -4$

(ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore\ 8x^3 + 6xy = 0$

$\therefore\ 2x(4x^2 + 3y) = 0$.

$\therefore\ 4x^2 + 3y = 0 \dots (1)$.

$f_y = 0$

$3x^2 - 2y = 0 \dots (2)$.

Multiply eq^(1) with 3

(2) with 4.

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$y = 0$

Substitute value of y in eq^n (1)

$4x^2 + 8(0) = 0$

$4x^2 = 0$

$x = 0$

critical point is $(0,0)$.

$r = f_{xx} = 24x^2 + 6z$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - 5^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

$f(x,y)$ at $(0,0)$

$= 2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 + 0$

$= 0$

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$\therefore x = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = 4$

$\therefore$ critical point is $(-1,4)$.

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$.

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 47 - 70$

$= -23$.

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