11) Given an array of [4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7
-4, 1, 9, -1, 0, -6, -8, 11, -9]

Sol/

Given array:-
[4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11,
-9]

Sorted array (ascending):-
[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,
10, 11]

Maximum Product:
Maximum Product be obtained by multiplying two ~~larges~~ largest Positive Number and two smallest negative numbers

Largest Positive number:- $10 \times 11 = 110$

Smallest negative number: $\cancel{9 \times 8}$ $-9 \times -8 = 72$

$(72 < 110)$

$\boxed{\text{Maximum Product:- } 110}$

Minimum Product

Minimum Product Multiply one largest Positive × Smallest largest negative = 11 × -9 = -99

$\boxed{\text{Minimum Product : -99}}$

12) Binary Search

arr [] = [2, 5, 8, 12, 16, 23, 38, 56, 72, 91]

0	1	2	3	4	5	6	7	8	9
2,	5,	8,	12,	16,	23,	38,	56,	72,	91

low = 0 Key = 23

high = 9

Mid = 0+9/2 = 4.5 = (4 or 5)

→ Consider it 4

0	1	2	3	4	5	6	7	8	9
2,	5,	8,	12,	$\boxed{16}$ Mid	23,	38,	56,	72,	91

Key > mid element

Move to right side of ~~mid~~ array element

5	6	7	8	9
23,	38,	$\boxed{56}$ mid	72,	91

high = 9

low = 5

mid = 5+9/2 = 7

Key < mid element (Move to left of the array)

14.

$\overset{5}{\boxed{23}}$ $\overset{6}{38}$
mid

low = 5

high = 6

mid = 5+6/2 = ~~5~~ 5.5 = 5 or 6

→ Take 5

Key = Mid

Element found at index 5

→ Take 6

23, $\boxed{38}$

~~Mid~~ Key < Mid , Move to ~~be~~ left

$\overset{5}{\boxed{23}}$

~~hi~~ hig = 5 , low = 5 , Mid = 5+5/2 = 5

→ Key = Mid .

→ Element found at index 5

→ Consider it (5)

0	1	2	3	4	5	6	7	8	9
2,	5,	8,	12,	16,	$\boxed{23}$	38,	56,	72,	91

~~lt low =~~ o Key = mid

→ Element is found at index 5

13) Merge sort

d = (45, 67, -12, 5, 22, 30, 50, 20)

$$\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}$$
$\rightarrow$ [45, 67, -12, 5, 22, 30, 50, 20]

$d = \frac{l+h}{2} = \frac{0+7}{2} = 3.5 = 3$

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \qquad \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array}$$
$\rightarrow$ [45, 67, -12, 5]   [22, 30, 50, 20]

$d = \frac{l+h}{2} = \frac{0+3}{2} = 1.5 = 1$

$d = l+h = \frac{4+7}{2} = 5$

$$\begin{array}{cc} 0 & 1 \end{array} \quad \begin{array}{cc} 2 & 3 \end{array} \qquad \begin{array}{cc} 4 & 5 \end{array} \quad \begin{array}{cc} 6 & 7 \end{array}$$
$\rightarrow$ [45, 67] [-12, 5]   [22, 30] [50, 20]

$d = \frac{l+h}{2} = \frac{0+3}{2} = 1$ $d = \frac{2+3}{2} = 2$

$d = \frac{4+5}{2} = 4$ $\frac{6+7}{2} = 6$

[45] [67] [-12] [5]   [22], [30] [50] [20]

unsorted array $\rightarrow$ [45, 67, -12, 5, 22, 30, 50, 20]

Divide

[45, 67, -12, 5]   [22, 30, 50, 20]

[45, 67]   [-12, 5]   [22, 30]   [50, 20]

[45]   [67]   [-12]   [5]   [22]   [30]   [50]   [20]

[45, 67]   [-12, 5]   [22, 30]   [20   50]

[-12, 5, 45, 67]   [20, 22, 30, 50]

Conquer.

Sorted array $\rightarrow$ [-12, 5, 20, 22, 30, 45, 50, 67] worst Case :- $(O(n^2))$

Best case :- $(O(n))$

Avg case :- $(O(n^2))$

6) Big omega notation : Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^2)$

Sol// we have to demonstrate that constant $C > 0$ and

$n_0 > 0$ Such that for all $n \geq n_0$

$g(n) \geq C \cdot n^3$

$n^3 + 2n^2 + 4n \geq C \cdot n^3$

Simplify by divide n^3

$1 + 2/n + 4/n^2 \geq C$.

As n grows larger, $2/n$ and $4/n^2$ become smaller

for all $n \geq 1$: $2/n > 0$, $4/n^2 > 0$

Hence for $n \geq 1$: $1 + 2/n + 4/n^2 \geq 1$

$\therefore C = 1$ for $n \geq 1$

$1 + 2/n + 4/n^2 \geq 1$

$C = 1$ and $n_0 = 1 \rightarrow n^3 + 2n^2 + 4n \geq n^3$ for all $n \geq 1$

$\therefore g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

$\therefore$ hence Proved.

15) Binary Search

arr[] = [2, 4, 6, 8, 10, 12, 14, 16, 18, 20]

target = 10

0	1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18	20

↓ Left Pointer (low)

↓ Rigth Point (high)

Middle Element $= \frac{l+h}{2} = \frac{0+9}{2} = 4.5 = 4$

Conditions:

target < mid = mid − 1

target > mid = mid + 1

target the Element = 10

Middle Element = 10

target = Mid

Element 10 is found at index 4

16) Merge Sort

	0	1	2	3	4	5	6	7	8	9	10	11
arr[] =	38	27	43	3	9	82	10	15	88	52	60	5

$d = \frac{l+h}{2} = \frac{0+11}{2} = 5.5$

[38, 27, 43, 3, 9, 82]   [10, 15, 88, 52, 60, 5]

$d = \frac{l+h}{2} = \frac{0+5}{2} = 2$ $d = \frac{6+11}{2} = 8$

[38, 27, 43] [3, 9, 82]   [10, 15, 88] [52, 60, 5]

$d = \frac{0+2}{2} = 1$ $d = \frac{3+5}{2} = 4$ $6+8/2 = 7$ $9+11/2 = 10$

[38, 27] [43] [3, 9] [82]   [10, 15] [88] [52, 60] [5]

$0+1/2 =$

[38], [27] [43] [3], [9]   [10] [15] [88] [52] [60] [5]

16

~~Bubble Sort~~ Merge Sort.

unsorted array

arr [] = [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5]

[38, 27, 43, 3, 9, 82] [10, 15, 88, 52, 60, 5]

[38, 27, 43] [3, 9, 82] [10, 15, 88] [52, 60, 5]

[38, 27] [43] [3, 9] [82] [10, 15] [88] [52, 60] [5]

[38] [27] [43] [3] [9] [82] [10] [15] [88] [52] [60] [5]

[27, 38] [43] [3, 9] [82] [10, 15] [88] [52, 60] [5]

[27, 38, 43] [3, 9, 82] [10, 15, 88] [52, 60, 5]

[3, 9, 27, 38, 43, 82] [5, 10, 15, 52, 60, 88]

sorted array :- [3, 5, 9, 10, 15, 27, 38, 43, 52, 60, 82, 88]

Time complexity :- (O(n/log n))

Space complexity :- (O(n))

5

2) Find the time complexity of the below recursive equation

i) $T(n) \begin{cases} 1 \\ 2T(n/2)+1 & \text{if } n>1 \end{cases}$

$2T(n/2)+1$ if $n>1$

using Master's theorem

$T(n) = aT(n/b) + f(n)$

$a=2, b=2, f(n)=1$

$\log_a^b = \log_2^2 = 1$

$f n = 1 = n^1 \cdot \log_a^b = n^1$

$f(n) = 1 = O(n^0)$ and $0<1 =$ case 1:

$f(n) = O(n^0)$ and $0 < \log_2^2$

$T(n) = \Theta(n^k \log_a^b) = \Theta(n) = \Theta(n)$

$T(n) = \Theta(n)$

(ii)

$$T(n) = \begin{cases} 2T(n-1) & \text{if } n > 0 \\ 1 & \end{cases}$$

$T(n) = 2T(n-1)$ if $n > 0$

$T(n) = 2T(n-1)$

$T(n-1) = 2T(n-2)$

$T(n-2) = 2T(n-3)$

$T(n) = 2T(n-1) = 2(2T(n-2))$

$= 2^2 . 2T(n-3) = 2^3 T(n-3)$

$T(n) = 2^k T(n-k)$

Base case, $T(n) = 2^n T(0)$
$(k=n)$

$T(0) = 1 \Rightarrow T(n) = 2^n (1) = 2^n$

$\therefore T(n) = O(2^n)$

5) Big Θ notation : Show that $f(n) = n^2 + 3n + 5$ is $O(n^2)$

Big $O \rightarrow f(n) \leq c \cdot g(n)$

$f(x) = n^2 + 3n + 5$

Let us assume $g(x) = 9n^2$

$= n^2 + 3n + 5 \leq c \cdot g(n^2)^2 \Rightarrow 1 + 3/n + 5/2^2 \leq c$

when $n = 1 \rightarrow 1^2 + 3\cancel{5} + 5 = g(1)^2 = 9$

$3/n \leq 3 \quad (n \geq 1)$

$5/n^2 \leq 5 \quad (n > 1)$

Hence for $n \geq 1$

$1 + 3/n + 5/n^2 \leq 1 + 3 + 5 = 9$

$n = 2$

$2^2 + 3(2) + 5 = 9(4)$

$= 4 + 6 + 5 = 36 \Rightarrow 15 < 36$

$n = 3 \Rightarrow 3^2 + 3(3) + 5 = 9(9) = 9 + 9 + 5 = 81$

$= 23 < 81$

$\therefore f(x) \geq c \cdot g(x)$

$\therefore$ Big (O) is Staisfied

Selection Sort

Set(S) = (12, 7, 5, -2, 18, 6, 13, 4)

$\boxed{12}$, 7, 5, $\boxed{-2}$, 18, 6, 13, 4

Swap - 1

-2, $\boxed{7}$, 5, 12, 18, 6, 13, $\boxed{4}$

Swap - 2

-2, 4, $\boxed{5}$ 12, 18, 6, 13, 7

No swap No smaller number than 5

-2, 4, 5, $\boxed{12}$, 18, $\boxed{6}$ 13, 7

Swap 3

-2, 4, 5, 6, $\boxed{18}$, 12, 13, $\boxed{7}$

Swap 4

[-2, 4, 5, 6, 7, 12, 13, 18] - sorted array

Number of Swap = 4

Time Complexity of Selection sort is

$O(n^2)$

5) 1) if $t_1(n) \in o(g_1(n))$ and $O(g_2(n))$, $t_1(n) +$ $t_2(n) \in O(\max\{g_1(n), g_2(n)\})$ Prove that assertion

(i) $t_1(n) \in o(g_1(n))$ means there exist constant $c_1 > 0$ and n_1 such that for all $n \geq n_1$:

$t_1(n) \leq c_1 g_1(n)$

(ii) $t_2(n) \in o(g_2(n))$ means there exist constant $c_2 > 0$ and n_2 such that for all $n \geq n_2$: $t_2(n) \leq c_2 g_2(n)$

wee need to Show that $t_1(n) + t_2(n) \in o(\max\{g(n), g_2(n)\})$

Proof :- Consider : $t_1(n) + t_2(n)$ and the defenition of

$\max\{g(n), g_2(n)\}$

let $g(n) = \max\{g_1(n), g_2(n)\}$.

By defenition of Maximum :- $g(n) = \max\{g(n), g_2(n)\} \geq g_1(n)$

$g(n) = \max\{g_1(n), g_2(n)\} \geq g_2(n)$

given the bound of $t_1(n)$ and $t_2(n)$:-

$t_1(n) \leq c_1 g_1(n) \leq c_1 g(n) \rightarrow n \geq n_1$

$t_2(n) \leq c_2 g_2(n) \leq c_2 g(n) \rightarrow n > n_2$

to bound $t_1(n) + t_2(n)$ consider $t_1(n) + t_2(n)$ $n \geq \max(n_1, n_2)$

$t_1(n) + t_2(n) \leq c_1 g(n) + c_2 c g(n) = (c_1 + c_2) g(n)$

By defenition of O notation $\rightarrow t_1(n) + t_2(n) \in O(g(n)$

Hence the asserstion is proved.

17) Bubble sort

arr[] = [64, 34, 25, 12, 22, 11, 90]

unsorted

$\boxed{64}, \boxed{34}$, 25, 12, 22, 11, 90

Pass 1

34, $\boxed{64}, \boxed{25}$, 12, 22, 11, 90

34, 25, $\boxed{64}, \boxed{12}$, 22, 11, 90

34, 25, 12, $\boxed{64}, \boxed{22}$, 11, 90

34, 25, 12, 22, $\boxed{64}, \boxed{11}$, 90

34, 25, 12, 22, 11, $\boxed{64}, \boxed{90}$ no swap.
64<90

Pass 2

$\boxed{34}, \boxed{25}$, 12, 22, 11, 64, 90

25, $\boxed{34}, \boxed{12}$, 22, 11, 64, 90

25, 12, $\boxed{34}, \boxed{22}$, 11, 64, 90

25, 12, 22, $\boxed{34}, \boxed{11}$, 64, 90

25, 12, 22, 11, $\boxed{34}, \boxed{64}$, 90 no swap 34<64

Pass 3:-

$\boxed{25}, \boxed{12}$, 22, 11, 34, 64, 90

12, $\boxed{25}, \boxed{22}$, 11, 34, 64, 90

12, 22, $\boxed{25}, \boxed{11}$, 34, 64, 90

12, 22, 11, $\boxed{25}, \boxed{34}$, 64, 90

no swap 25<34

Pass 4:-

$\boxed{12}, \boxed{22}$, 11, 25, 34, 64, 90

Noswap 12<22

Pass 5:-

12, $\boxed{22}, \boxed{11}$, 25, 34, 64, 90

12, 11, $\boxed{22}, \boxed{25}$, 34, 64, 90

no swap 22<25

Pass 6 :

$\boxed{12}, \boxed{11}$, 22, 25, 34, 64, 90

11, 12, 22, 25, 34, 64, 9 → Sorted array

Worst case time complexity:- $(O(n^2))$

Best case time complexity:- $(O(n))$

Avg case time complexity:- $(O(n^2))$

17) Selection Sort

arr [] = [64, 25, 12, 22, 11]

$\boxed{64}$, 25, 12, 22, $\boxed{11}$ → unsorted array

11, $\boxed{25}$, $\boxed{12}$, 22, 64

11, 12, $\boxed{25}$, $\boxed{22}$, 64

[11, 12, 22, 25, 64] → sorted array

worst case :- $(O(n^2))$

Best case :- $(O(n^2))$

Avg case :- $(O(n^2))$

Insertion Sort

arr[] = [38, 27, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5]

unsorted

$\boxed{38}$, $\boxed{27}$, 43, 3, 9, 82, 10, 15, 88, 52, 60, 5

27<38 38>27

27, $\boxed{38}$, $\boxed{43}$, 3, 9, 82, 10, 15, 88, 52, 60, 5

38<43 (no swap)

27, 38, $\boxed{43}$, $\boxed{3}$, 9, 82, 10, 15, 88, 52, 60, 5

3<43

27, 38, $\boxed{3}$, 43, 9, 82, 10, 15, 88, 52, 60, 5

3, 27, 38, $\boxed{43}$, $\boxed{9}$, 82, 10, 15, 88, 52, 60, 5

3, 27, 38, $\boxed{9}$, 43, 82, 10, 15, 88, 52, 60, 5

~~9, 3, 27, 38, 43, $\boxed{82}$, $\boxed{10}$, 15, 88, 52, 60, 5~~

3, 9, 27, 38, 43, $\boxed{82}$, $\boxed{10}$, 15, 88, 52, 60, 5

3, 9, 27, 38, 43, $\boxed{10}$, 82, 15, 88, 52, 60, 5

3, 9, 10, 27, 38, 43, $\boxed{82}$, $\boxed{15}$, 88, 52, 60, 5

3, 9, 10, 27, 38, 43, $\boxed{15}$, 82, 88, 52, 60, 5

3, 9, 10, 15, 27, 38, 43, $\boxed{82}$, $\boxed{88}$, 52, 60, 5 → No Shift

3, 9, 10, 15, 27, 38, 43, 82, $\boxed{88}$, $\boxed{52}$, 60, 5

3, 9, 10, 15, 27, 38, 43, 82, $\boxed{52}$, 88, 60, 5

8

3, 9, 10, 15, 27, 38, 43, 52, 82, $\boxed{88}$, $\boxed{60}$, 5

3, 9, 10, 15, 27, 38, 43, 52, 82, $\boxed{60}$, 88, 5

3, 9, 10, 15, 27, 38, 43, 52, 60, 82, $\boxed{88}$, $\boxed{5}$

3, 9, 10, 15, 27, 38, 43, 52, 60, 82, $\boxed{5}$, 88

[3, 5, 9, 10, 15, 27, 38, 43, 52, 60, 82, 88] Sorted Array

Worst Case :- $O(n^2)$

Best Case :- $O(n)$

Avg Case :- $O(n^2)$

2.) Insertion Sort

arr[] = 4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1
0, 1 -6, -8, 11, -9]

4, $\boxed{-2}$, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9

-2, $\boxed{4}$, $\boxed{5}$, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9

-2, 4, $\boxed{5}$, $\boxed{3}$, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9

-2, 3, 4, 5, $\boxed{10}$, $\boxed{-5}$, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9

-5, -2, 3, 4, 5, $\boxed{10}$, $\boxed{2}$, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9

-5, -2, 2, 3, 4, 5, $\boxed{10}$, $\boxed{8}$, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9

-5, -2, 2, 3, 4, 5, 8, $\boxed{10}$, $\boxed{-3}$, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, -9

-5, -3, -2, 2, 3, 4, 5, 8, $\boxed{10}$, $\boxed{6}$, 7, -4, 1, 9, -1, 0, -6, -8, 11, 9

-5, -3, -2, 2, 3, 4, 5, 6, 8, $\boxed{10}$, $\boxed{7}$ -4, 1, 9, -1, 0, -6, -8, 11, -9

-5, -3, -2, 2, 3, 4, 5, 6, 7, 8, $\boxed{10}$, $\boxed{-4}$, 1, 9, -1, 0, -6, -8, 11, 9

-5, -4, -3, -2, 2, 3, 4, 5, 6, 7, 8, $\boxed{10}$, $\boxed{1}$, 9, -1, 0, -6, -8, 11, 9

-5, -4, -3, -2, 1, 2, 3, 4, 5, 6, 7, 8, $\boxed{10}$, $\boxed{9}$, -1, 0, -6, -8, 11, -9

-5, -4, -3, -2, 1, 2, 3, 4, 5, 6, 7, 8, 9, $\boxed{10}$, $\boxed{-1}$, 0, -6, -8, 11, -9

-5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, $\boxed{10}$, $\boxed{0}$, -6, -8, 11, -9

-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, $\boxed{10}$, $\boxed{-6}$, -8, 11, -9

-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, $\boxed{10}$, $\boxed{-8}$, 11, -9

-8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, $\boxed{10}$, $\boxed{11}$, -9

No shift

-8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, $\boxed{11}$, $\boxed{-9}$

[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

Sorted array