National Institute of Technology Tiruchirappalli
Department of Mathematics
July 2023 Session - B.Tech IInd Year (CSE) - A-section
Probability and Operations Research - MAIR31 - Assessment-I

Date: 07.09.2023 Duration: 1 Hour Max. marks: 20

Attempt all the five questions.

1. A city hospital has the following minimal daily requirements for nurses:

period	Clock time (24 hour day)	Minimum no of nurses required
1	6 a.m. - 10 a.m.	8
2	10 a.m. - 2 p.m.	16
3	2 p.m. - 6 p.m.	20
4	6 p.m. - 10 p.m.	14
5	10 p.m. - 2 a.m.	24
6	2 a.m. - 6 a.m.	8

Period 1 follows immediately after period 6. Nurses report to the hospital at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the number of nurses to report at each period so that the total number of nurses who have to report for duty in a day is minimum. Formulate the problem as a linear programming problem (LPP). [2]

2. Solve the following LPP by the simplex method. [8]

Maximize $z = -x_1 + 3x_2 - 3x_3$ subject to

$$\begin{aligned} 3x_1 - x_2 + x_3 &\leq 7 \\ -x_1 + 2x_2 &\leq 6 \\ -4x_1 + 3x_2 + 8x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

3. Consider the primal problem [3]

Maximize $z = 2x_1 + 7x_2 + 4x_3$ subject to

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 10 \\ 3x_1 + 3x_2 + 2x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

Using duality, show that the optimal value of z for the primal problem cannot exceed 25.

[P.T.O]

4. Consider the following LPP: [5]

$$\text{Maximize } z = 25x_1 + 20x_2 \text{ subject to}$$

$$\begin{aligned} 3x_1 + 4x_2 &\le 70 \\ 8x_1 + 5x_2 &\le 150 \\ x_2 &\le 20 \\ x_1, x_2 &\ge 0. \end{aligned}$$

The optimal simplex tableau for the standard form of the above LPP (with slack variables s_1, s_2, s_3) is given below:

			25	20	0	0	0
c_{B_j}	Basic Variables	x_{B_j}	x_1	x_2	s_1	s_2	s_3
20	x_2	6.4706	0	1	0.47	−0.18	0
25	x_1	14.7059	1	0	−0.29	0.24	0
0	s_3	13.5294	0	0	−0.47	0.18	1
	$(z_j - c_j)$:		0	0	2.06	2.35	0

Using sensitivity analysis, find an optimum solution if

(a) $b = (70, 150, 20)^T$ is changed to $\widetilde{b} = (80, 120, 20)^T$.

(b) $b = (70, 150, 20)^T$ is changed to $\widehat{b} = (100, 60, 3)^T$.

5. (a) Define basic feasible solution of a LPP.

(b) Consider the problem of maximizing $z = 2x_1 + 3x_2 - 4x_3 + x_4$ subject to

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 - x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + 2x_3 - x_4 &= 0 \\ x_1, x_2, x_3, x_4 &\ge 0. \end{aligned}$$

Prove or disprove that $(1, 0, 1, 4)$ is a basic feasible solution. [2]

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