ARVR CT-1 Numericals: Prajwal Sundar

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(2) (a) Let the object be rotated at angle θ in clockwise direction about (x_f, y_f)

Steps to perform:

1. Perform translation with $t_x = -x_f \quad t_y = -y_f$
2. Perform rotation with angle $= -\theta$
3. Perform inverse translation with $t_x = x_f \quad t_y = y_f$

$$P' = T^{-1} \cdot R \cdot T \cdot P$$

$$P' = \begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} P$$

$$P' = \begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} P$$

$$P' = \begin{bmatrix} \cos\theta & \sin\theta & x_f \\ -\sin\theta & \cos\theta & y_f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} P$$

$$P' = \begin{bmatrix} \cos\theta & \sin\theta & (-x_f\cos\theta - y_f\sin\theta + x_f) \\ -\sin\theta & \cos\theta & (x_f\sin\theta - y_f\cos\theta + y_f) \\ 0 & 0 & 1 \end{bmatrix} P$$

$$P' = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta & x_f(1-\cos\theta) - y_f\sin\theta \\ -\sin\theta & \cos\theta & x_f\sin\theta + y_f(1-\cos\theta) \\ 0 & 0 & 1 \end{bmatrix}}_{\text{transformation matrix}} P$$

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② (b) case 1 :

1) Reflect about y-axis $(R_{x=0})$

2) Reflect about $y=-x$ $(R_{y=-x})$

Transformation matrix is given by

$$R_2 R_1 = R_{y=-x} \cdot R_{x=0}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

case 2 : Rotate by -270°

$\equiv$ Rotation by $-270^\circ + 360^\circ = 90^\circ$

Transformation matrix :

0] [0 -1 0

Transformation matrix.

$$\begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

since the transformation matrices obtained are the same, case 1 = case 2, the two transformations are equivalent

Performing transformations with $A(1,1)$ & $B(10,10)$

$$P' = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 10 \\ 1 & 10 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -10 \\ 1 & 10 \\ 1 & 1 \end{bmatrix}$$

$A' = (-1, 1)$ $\quad B' = (-10, 10)$

$\therefore$ verified that case 1 = case 2. Transformed points A' and B' are also obtained successfully.

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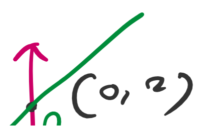
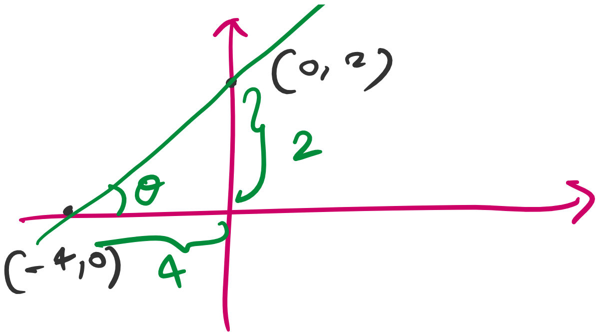
② (c) Reflection about $2y = x + 4$

$$x - 2y = -4 \qquad \frac{x}{(-4)} + \frac{y}{(2)} = 1$$





$\tan\theta = \frac{1}{2}$

$\sin\theta = \frac{1}{\sqrt{5}}$ $\cos\theta = \frac{2}{\sqrt{5}}$

Steps to perform reflection :

1. T : $t_x = 0$ $t_y = -2$
2. R : $-\theta$ rotation
3. RL : x axis reflection
4. R^{-1} : θ rotation
5. T^{-1} : $t_x = 0$, $t_y = 2$

Transformation matrix is given by

$$\boxed{T^{-1} \cdot R^{-1} \cdot RL \cdot R \cdot T}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & -1/\sqrt{5} & 0 \\ 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{5} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{5} \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 2\sqrt{5} \\ 0 & 0 & \sqrt{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 \\ -1 & 2 & -4 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$$

$$\frac{1}{5}\begin{bmatrix} \ldots \\ 0 & 0 & \sqrt{5} \end{bmatrix}\begin{bmatrix} \ldots \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \ldots \\ 0 & 0 & \sqrt{5} \end{bmatrix}$$

$$\frac{1}{5}\begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 2\sqrt{5} \\ 0 & 0 & \sqrt{5} \end{bmatrix}\begin{bmatrix} 2 & 1 & -2 \\ -1 & 2 & -4 \\ 0 & 0 & \sqrt{5} \end{bmatrix} = \underbrace{\frac{1}{5}\begin{bmatrix} 3 & 4 & -8 \\ 4 & -3 & 16 \\ 0 & 0 & 5 \end{bmatrix}}_{M}$$

Given points :

$A(6, 3, 3) \equiv A(2, 1, 1)$

$B(3, 2, 1) \equiv B(3, 2, 1)$

$C(2, 4, 2) \equiv C(1, 2, 1)$

performing the transformation

$$P' = M \cdot P$$

$$P' = \frac{1}{5}\begin{bmatrix} 3 & 4 & -8 \\ 4 & -3 & 16 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = \frac{1}{5}\begin{bmatrix} 2 & 9 & 3 \\ 21 & 22 & 14 \\ 5 & 5 & 5 \end{bmatrix}$$

$$P' = \begin{bmatrix} 2/5 & 9/5 & 3/5 \\ 21/5 & 22/5 & 14/5 \\ 1 & 1 & 1 \end{bmatrix}$$

... triangle has coordinates

reflected triangle has coordinates

$A'\left(\frac{2}{5}, \frac{21}{5}, 1\right)$ $B'\left(\frac{9}{5}, \frac{28}{5}, 1\right)$

$C'\left(\frac{3}{5}, \frac{14}{5}, 1\right)$

$\equiv$ $A'(2, 21, 5)$
$B'(9, 28, 5)$
$C'(3, 14, 5)$

(homogeneous coordinates of the reflected triangle)

8. Reflection completed