Solution: Q.NO 1

Let x_i be the number of nurses to report at ith period where $i = 1, 2, 3, 4, 5, 6$.

$$\min\ z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \quad — (½)$$

s.t.

$$x_6 + x_1 \geq 8$$
$$x_1 + x_2 \geq 16$$
$$x_2 + x_3 \geq 20 \quad — (1)$$
$$x_3 + x_4 \geq 14$$
$$x_4 + x_5 \geq 24$$
$$x_5 + x_6 \geq 8$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \quad — (½)$$

Solution: Q. No 2

Initial iteration:

C_{Bj}	Basic variables	Basic variable values x_{Bj}	x_1	x_2	x_3	s_1	s_2	s_3	Ratio $\frac{x_{Bi}}{a_{ir}} : a_{ir} > 0$
			-1	3	-3	0	0	0	
0	s_1	4	3	-1	1	1	0	0	—
0	s_2	6	-1	$\boxed{2}$	0	0	1	0	$6/2 = 3 \rightarrow$
0	s_3	10	-4	3	8	0	0	1	$10/3 = 3.3$
			1	-3 $\uparrow$	3	0	0	0	$\rightarrow (1\frac{1}{2})$

First Iteration

C_{Bj}	Basic variables	Basic variable values x_{Bj}	x_1	x_2	x_3	s_1	s_2	s_3	Ratio $\frac{x_{Bi}}{a_{ir}} : a_{ir} > 0$
			-1	3	-3	0	0	0	
0	s_1	10	$\boxed{5/2}$	0	1	1	$1/2$	0	$\frac{10}{5/2} = 4 \rightarrow$
3	x_2	3	$-1/2$	1	0	0	$1/2$	0	—
0	s_3	1	$-5/2$	0	8	0	$-\frac{3}{2}$	1	—
			$-1/2$ $\uparrow$	0	3	0	$3/2$	0	$\rightarrow (3\ M)$

Second iteration

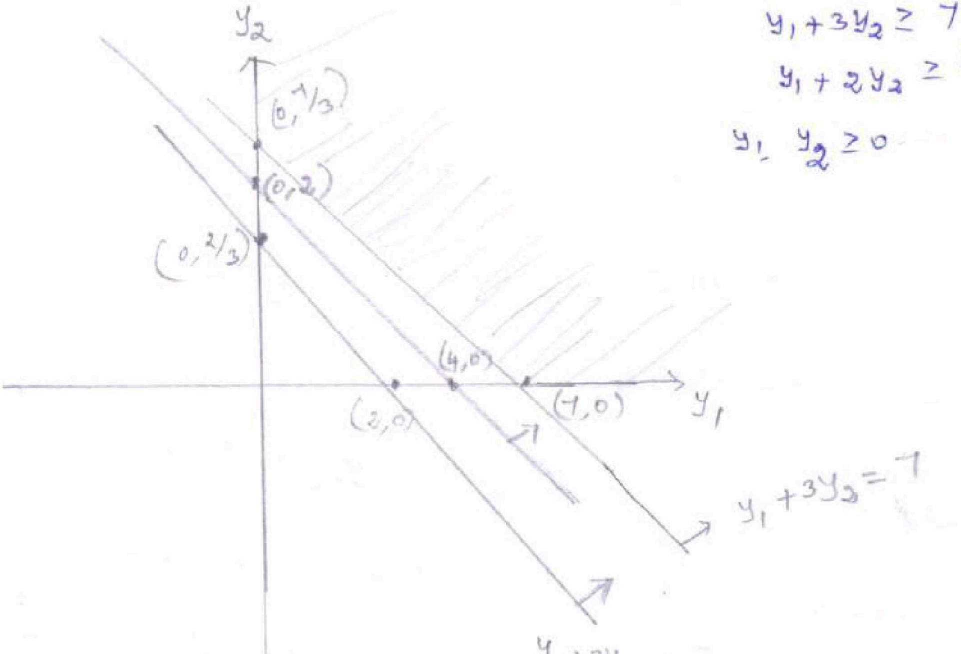
C_{Bj}	Basic variables	Basic variable values x_{Bj}	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
			-1	3	-3	0	0	0	
-1	x_1	4	1	0	$2/5$	$2/5$	$1/5$	0	
3	x_2	5	0	1	$1/5$	$\frac{1}{5}$	$3/5$	0	
0	s_3	11	0	0	9	1	-1	1	$\rightarrow (3\ \text{marks})$
		11	0	0	$\frac{16}{5}$	$\frac{1}{5}$	$8/5$	0	

An optimum solution is $x_1 = 4$, $x_2 = 5$, $x_3 = 0$. optimum value $= 11$ $\rightarrow (\frac{1}{2})$

Solution:

Q.NO 3 The dual problem is min $w = 10y_1 + 10y_2$

s. to
$$y_1 + 3y_2 \geq 2$$
$$y_1 + 3y_2 \geq 7$$
$$y_1 + 2y_2 \geq 4$$
$$y_1, y_2 \geq 0$$
— (1)



The dual problem has a feasible solution $(0, 7/3)$

The primal problem has a feasible solution $(0, 0, 0)$.

∴ Both the primal and dual have optimum solutions. (1)

The objective value corresponding to $(0, 7/3)$ is $10 \times 0 + 10 \times \frac{7}{3} = \frac{70}{3}$
$= 23.33$

By weak duality theorem
$$\max z \leq \min w$$
$$\leq 23.33$$

The optimum value of primal problem ≤ 23.33 — (1)

∴ The optimum value of the primal problem cannot exceed 25

Solution:

Q NO 4 From the optimal ~~value~~ table, the basic variables are x_2, x_4 and s_3. The Basis matrix $B = \begin{pmatrix} 3 & 4 & 0 \\ 8 & 5 & 0 \\ 0 & 1 & 1 \end{pmatrix}$.

$$B^{-1} = \begin{pmatrix} 0.47 & -0.18 & 0 \\ -0.29 & 0.24 & 0 \\ -0.47 & 0.18 & 1 \end{pmatrix}$$

(a) $\tilde{b} = (80, 120, 20)^T$

$$\begin{pmatrix} x_2 \\ x_4 \\ s_3 \end{pmatrix} = B^{-1}\tilde{b} = \begin{pmatrix} 0.47 & -0.18 & 0 \\ -0.29 & 0.24 & 0 \\ -0.47 & 0.18 & 1 \end{pmatrix}\begin{pmatrix} 80 \\ 120 \\ 20 \end{pmatrix} = \begin{pmatrix} 16 \\ 5.6 \\ 4 \end{pmatrix} \geq 0$$

optimum solution is $x_4 = 5.6$, $x_2 = 16$ → $(1\frac{1}{2})$

Q No 4 (b) $\hat{b} = (100, 60, 3)^T$ (5)

$$\begin{pmatrix} x_2 \\ x_1 \\ s_3 \end{pmatrix} = B^{-1}\hat{b} = \begin{pmatrix} 0.47 & -0.18 & 0 \\ -0.29 & 0.24 & 0 \\ -0.47 & 0.18 & 1 \end{pmatrix} \begin{pmatrix} 100 \\ 60 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 36.2 \\ -14.6 \\ -33.2 \end{pmatrix} \not\geq 0$$

From the optimal simplex table

			25	20	0	0	0	
c_{Bj}	Basic variable	x_{Bj}	x_1	x_2	s_1	s_2	s_3	Ratio
20	x_2	36.2	0	1	0.47	-0.18	0	
25	x_1	-14.6	1	0	-0.29	0.24	0	
0	s_3	-33.2	0	0	$\boxed{-0.47}$	0.18	1	→
			0	0	2.15	2.4	0	

(1)

$\min\{-14.6, -33.2\} = -33.2$. $\therefore$ s_3 is the leaving variable

$\max\left\{\frac{z_j - c_j}{a_{kj}} : a_{kj} < 0\right\} = \max\left\{\frac{2.15}{-0.47}\right\}$ $\therefore$ s_1 is the entering variable

c_{Bj}	Basic variable	x_{Bj}	x_1	x_2	s_1	s_2	s_3
20	x_2	3	0	1	0	0	1
25	x_1	5.885	1	0	0	0.129	-0.617
0	s_1	70.64	0	0	1	-0.383	-2.13

(2)

optimum solution $x_1 = 5.885$
$x_2 = 3$

(1/2)

optimum value = 207.125

Solution:

Q no 5 a) Consider the LPP max $z = c^T x$

s. to $Ax = b$

$x \geq 0$

where A is an $m \times n$ matrix with rank $= m < n$

By setting $n-m$ variables to zero we get a system of equations in m variables and m equations. If the resulting system gives unique solution then the associated m variables are called basic variables and the remaining $n-m$ variables are called non-basic variables. The resulting solution is a basic solution. (1)

(b) Put $x_2 = 0$

The resulting system is

$$x_1 + x_3 = 2$$

$$x_1 + x_3 = 2$$

$$2x_1 + 2x_3 - x_4 = 0$$

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 2 & -1 \end{vmatrix} = 0$$

$(1,0,1,4)$
$(2,0,0,4)$
are two solutions of the resulting system

The resulting system has more than one solution

$\therefore (1,0,1,4)$ is not a basic feasible solution (1)