Combinatorics and Graph Theory

Marks: 100 Course Code: CSPE32 Time: 3 hrs

Instructions to the Students: Answer all questions.

1. Derive a formula for the number of diagonals in a polygon with n vertices. Then prove your formula using mathematical induction. [5]

2. During a month with 30 days, a baseball team plays at least one game a day, but no more than 40 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 19 games. [5]

3. Define the height h(T) of a complete binary tree T recursively. Let n(T) denote the number of vertices in T. Prove that, $n(T) \leq 2^{h(T)+1}-1$. [5]

4. Show that for every integer n there is a multiple of n that has only 0s and 1s in its decimal expansion. [5]

5. How many solutions are there to the equation x1+x2+x3+x4 =17, where x1, x2, x3 and x4 are nonnegative integers? [4]

6. Find all solutions of the recurrence relation:
$a_n = 7a_{n-1} - 16\,a_{n-2} + 12\,a_{n-3} + n4^n$ with $a_0 = -2$, $a_1 = 0$ and $a_2 = 5$. [6]

7. Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group. [Hint: Use generalized pigeonhole principle] [5]

8. In the Internet, each network connection of a computer is assigned an Internet address. In Internet Protocol (IPv4), an address is a string of 32 bits. It begins with a network number (netid) followed by a host number (hostid), which identifies a computer as a member of a particular network. Three forms of addresses are used, with different numbers of bits used for netids and hostids. Class A addresses, used for the largest networks, consist of 0, followed by a 7-bit netid and a 24-bit hostid. Class B addresses, used for medium-sized networks, consist of 10, followed by a 14-bit netid and a 16-bit hostid. Class C addresses, used for the smallest networks, consist of 110, followed by a 21-bit netid and an 8-bit hostid. There are several restrictions on addresses because of special uses: 1111111 is not

available as the netid of a Class A network, and the hostids consisting of all 0s and all 1s are not available for use in any network. A computer on the Internet has either a Class A, a Class B, or a Class C address. How many different IPv4 addresses are available for computers on the Internet? [5]

9. Find a recurrence relation for the number of bit strings of length n that contain a pair consecutive 0s. What are the initial conditions? How many bit strings of length seven contain two consecutive 0s? [3 +1+2]

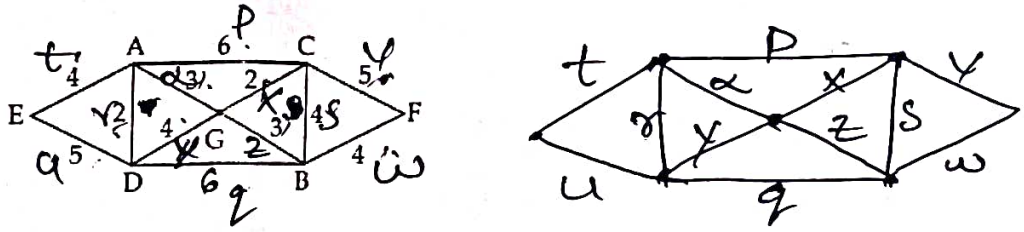

Fig. A

10. Find the dual of the graph of Fig. A. [5]

11. Find a Minimum Spanning Tree by Kruskal's algorithm (show steps). Find all fundamental cut-sets and fundamental circuits determined by this spanning tree. [5+3+3]

12. Define maximal independent set and minimal dominating set. Give examples with respect to the above graph. Find all minimal dominating set of the above graph using Boolean algebra. [2+2+2+5]

13. Define minimum vertex cover problem. Use a 2-approximation algorithm to find the minimum vertex cover for the above graph. Show the approximation. [2+4+2]

14. State and prove the five coloring theorem. [5]

15. What is a graphic sequence? Which of the following 6-tuples is NOT graphic? Show the derivation for each. [2+4*3]

a. (1, 1, 1, 1, 1, 1)
b. (3, 3, 3, 1, 0, 0)
c. (2, 2, 2, 2, 2, 2)
d. (3, 2, 1, 1, 1, 0)