

Introduction to Bottom up parsers

outcome :

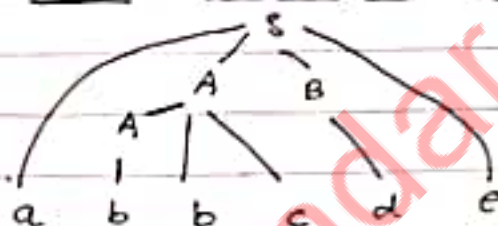
- ✓ understanding the Bottom-up approach of parsing
- ✓ Revisit the classification of parsers.

Generation of parse Tree - Bottom up Approach

$S \rightarrow aABe$

$A \rightarrow Abc | b$

$B \rightarrow d$



Decision :
when to
reduce

$S \rightarrow a \underline{A} B e$

$S \rightarrow a \underline{A} d e$

$S \rightarrow a \underline{A} b c d e$

$S \rightarrow a b b c d e$

(leftmost derivation in reverse)

classification of parsers :

points to note :

- ✓ Top down parsers with backtracking can handle non-determinism - but those without backtracking cannot.
- ✓ only operator precedence parsers can handle ambiguous grammar.
- ✓ power of $LR(0) < SLR(0) < LALR(1) < CLR(1)$

summary :

- ✓ understanding Bottom up parsing.
- ✓ classification of parsers

Bottom up : \rightarrow operator precedence
 $\rightarrow LR : LR(0), SLR(1),$
 $LALR(1), CLR(1)$

operator Precedence parser

outcome :

- ✓ understanding the operator Precedence Grammar.
- ✓ How to convert a CFG into operator Grammar.
- ✓ Generation of operator Relation Table
- ✓ operator Precedence Parsing.

operator Precedence Parser :

- ① It is mainly used for mathematical expressions.
- ② It processes operator grammar.
- ③ It can handle Ambiguous Grammars.
- ④ It uses operator Relation Table.

Why can it handle ambiguity ?

If a grammar is ambiguous, for the same string, we can generate many different parse trees → the reason behind that is the production rules of the ambiguous grammar are defined without considering the associativity and precedence of the operators involved in those production rules → that is why all other parsers need unambiguous grammar. On the other hand, due to the use of the operator relation table, operator precedence parser can smoothly handle ambiguous grammar - wherein the associativity and precedence of the operators are clearly defined.

↪ handles ✓

operator Grammar:

used to define operators

Restrictions:

- ① No adjacent non-terminals.
- ② No epsilon (ϵ) productions.

$$E \rightarrow EAE \mid id \Rightarrow E \rightarrow E + E \mid E * E \mid id$$
$$A \rightarrow + \mid *$$

conversion to operator Grammar:

$$S \rightarrow SAS \mid a \Rightarrow S \rightarrow sbbsbs \mid sbbs \mid a$$
$$A \rightarrow bsb \mid b \rightarrow \text{unreachable}$$

$$S \rightarrow sbbsbs \mid sbbs \mid a$$

construction of operator Relation Table

$$E \rightarrow E + E \mid E * E \mid id$$

Precedence	Associativity
$*, /, \%$	Left to Right
$+, -$	Left to Right

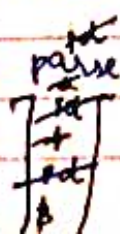
Note:

$id \rightarrow$ highest precedence
 $\$ \rightarrow$ lowest precedence

	id	$+$	$*$	$\$$
id	$-$	$>$	$>$	$>$
$+$	$<$	$>$	$<$	$>$
$*$	$<$	$>$	$>$	$>$
$\$$	$<$	$<$	$<$	$-$

summary
done

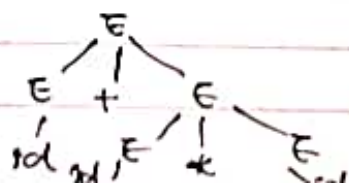
eg:



$$E + (E * E)$$

rules for parsing:

- ① If $TOS < \text{Look Ahead}$,
PUSH (shift) Look Ahead
into the stack
- ② If $TOS > \text{Look Ahead}$,
POP (Reduce)



Improved operator Precedence parser

Outcome :

- ✓ Disadvantage of using operator Relation Table.
- ✓ construction of operator Function Table.

For 'n' terminals, size of operator relation table = $\frac{O(n^2)}{\text{can be reduced}}$ $[(n+1)(n+1)]$

Construction of operator Function Table

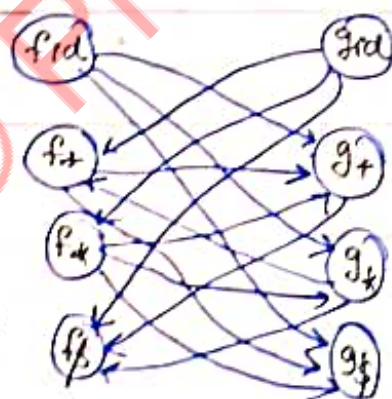
rows are defined by function (f)
columns are defined by function (g)

A directed graph is drawn, where all the terminals of f will be treated as one set of nodes, while the terminals of g will be treated as another set of nodes.

1822 operator relation table Rule :

Direction of edge :

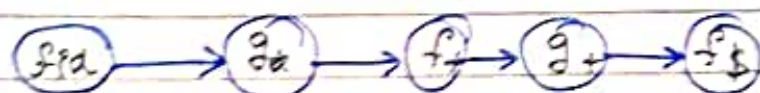
operator precedence \rightarrow level precedence



From this directed graph, operator function table will be constructed.

Restriction: we cannot draw it if the graph contains any cycle.

Procedure : find out the longest path starting at each individual node and note them down.



operator
function
table

	id	+	*	\$
f	4	2	4	0
g	5	2	3	0

rule for filling
No. of edges in longest
path starting from
every node.

Note: f_1 and d_1 will never contain any outgoing
edges, as id has the least precedence among
all other symbols. so $f \$ = 0$, $g \$ = 0$ in cell.

significance of cell data: More the value, more
the precedence.

Table size = $O(2n)$ only

now $f * = 4$, $g * = 3$

$E \rightarrow E + E \mid E * E \mid id$

$\begin{matrix} & & f & & g \\ & & \uparrow & & \uparrow \\ \text{left} & & & & \text{right} \end{matrix} \Rightarrow \text{left associativity}$
 (rows) (columns)

Summary:

- ① Disadvantages of operator Relation Table.
- ② construction of operator Function Table.

operator Precedence parsing - solved problem

outcome: construction of operator function table for a given CFG.

- Q. construct the operator function table for operator Precedence parsing from the following CFG,
- $P \rightarrow \overline{SR} \mid S$
 $R \rightarrow bSR \mid bS$
 $S \rightarrow WbS \mid W$
 $W \rightarrow L * W \mid L$
 $L \rightarrow id$
- sentence: word followed by a blank or a sentence or a word.
 word: a letter concatenated with a word or a letter.
 letter: identifier.
- Grammar description
- Paragraph: sentence followed by Recursive_# sentences or a sentence.
- Recursive_# sentences: a blank followed by a sentence & Recursive_# sentences (or) a blank followed by a sentence

converting into operator grammar:

$$P \rightarrow sb \overline{SR} \mid sbS \mid S$$

$$P \rightarrow sbP \mid sbS \mid S$$

$P \rightarrow$ useless

$$W \rightarrow L * W$$

$$P \rightarrow sbP$$

$$S \rightarrow WbS$$

$$S \rightarrow W$$

$$W \rightarrow L * W$$

$$L \rightarrow id$$

$$P \rightarrow sbP$$

$$S \rightarrow WbS$$

$$S \rightarrow W$$

$$W \rightarrow L * W$$

$$L \rightarrow id$$

precedence increase

$$P \rightarrow sbP \mid sbS \mid S$$

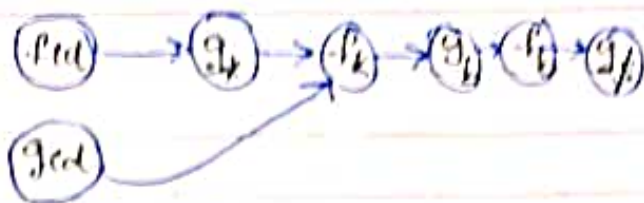
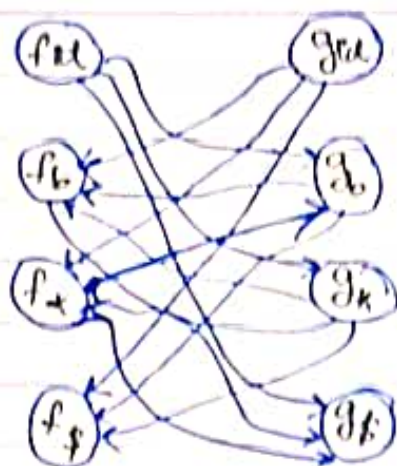
$$S \rightarrow WbS \mid W$$

$$W \rightarrow L * W \mid L$$

$$L \rightarrow id$$

	id	b	*	\$
id	-	>	>	>
b	<	<	<	>
*	<	>	<	>
\$	<	<	<	-

($a > b$ means a is right associative)



	id	r	x	f
f	5	1	3	0
g	4	2	4	0

Disadvantage of operator function table :

In the operator relation table, we never compared the ids, but here, we are comparing the ids. So although operator function table reduces the size of the table, which will eventually enable the parser to take less time for parsing, the error detection capacity will be lesser than that of the operator relation table.

summary : construction of operator function table.

Introduction to LR parsers

outcome :

- ✓ understanding the organization of LR parsers.
- ✓ what are LR(0) items?
- ✓ CLOSURE and GOTO properties.
- ✓ Derivation of canonical collection of LR(0) items.

LR Parsers :

① → scan from left to right

* if buffer

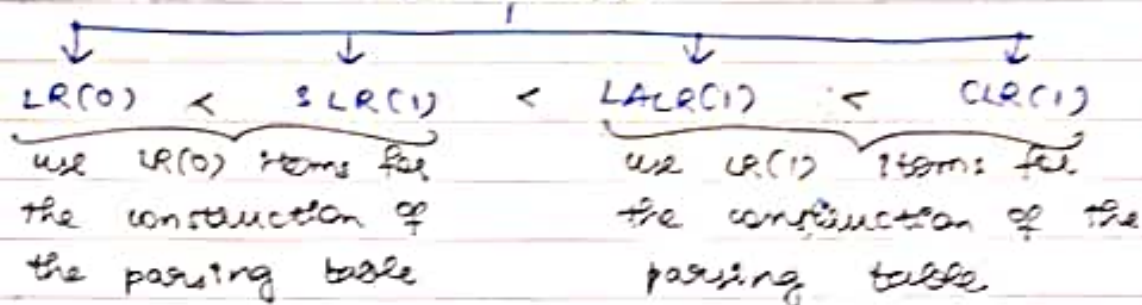
② → rightmost derivation in reverse

* Stack ← LR parser

↑

* LR parsing table

LR- parsers

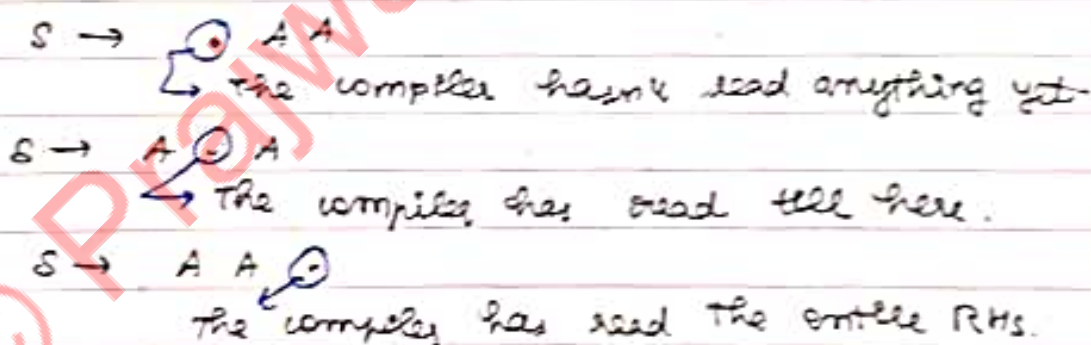


LR(0) parser :

- ① conversion to augmented Grammar.
- ② use CLOSURE and GOTO properties.

$$\left. \begin{array}{l} S \rightarrow AA \\ A \rightarrow aAb \end{array} \right\} \Rightarrow \left. \begin{array}{l} S' \rightarrow S \\ S \rightarrow AA \\ A \rightarrow aAb \end{array} \right\} \begin{array}{l} \text{new start symbol } S' \\ \text{introduced.} \\ \text{(augmented grammar)} \end{array}$$

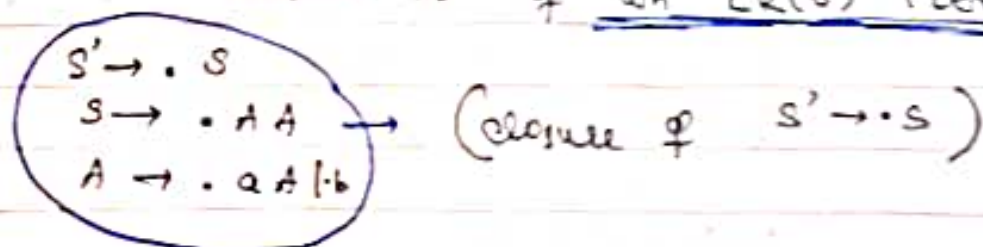
LR(0) items :



CLOSURE :

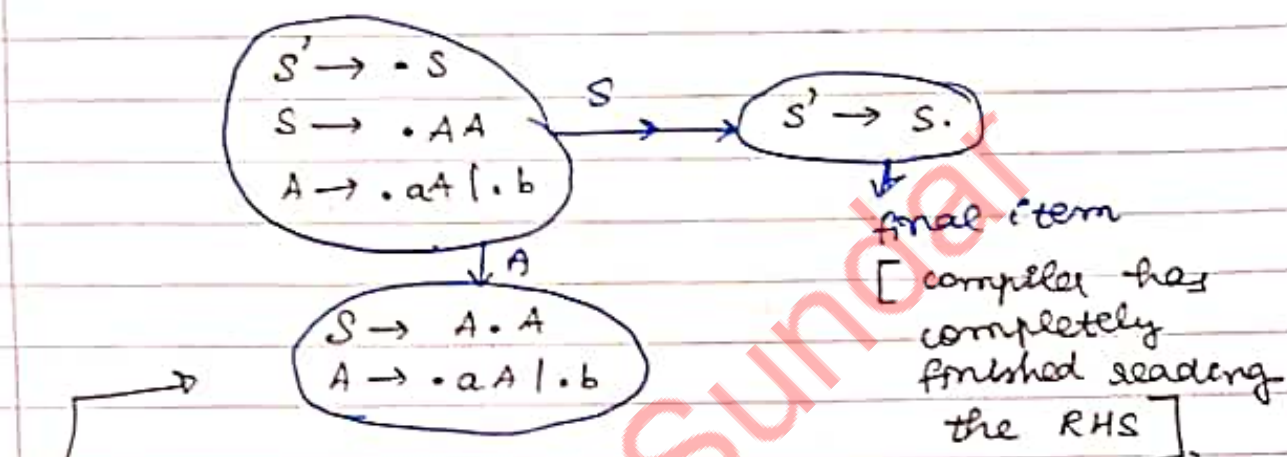
whenever there is a '.' to the left of a non-terminal, include all its productions in the set, adding a '.' preceding the rules.

↓
closure is of an LR(0) item



GOTO:

on a canonical collection of LR(0) items, we apply GOTO for every non-terminal by moving the \cdot to the right of the same, thus the new collection is obtained.

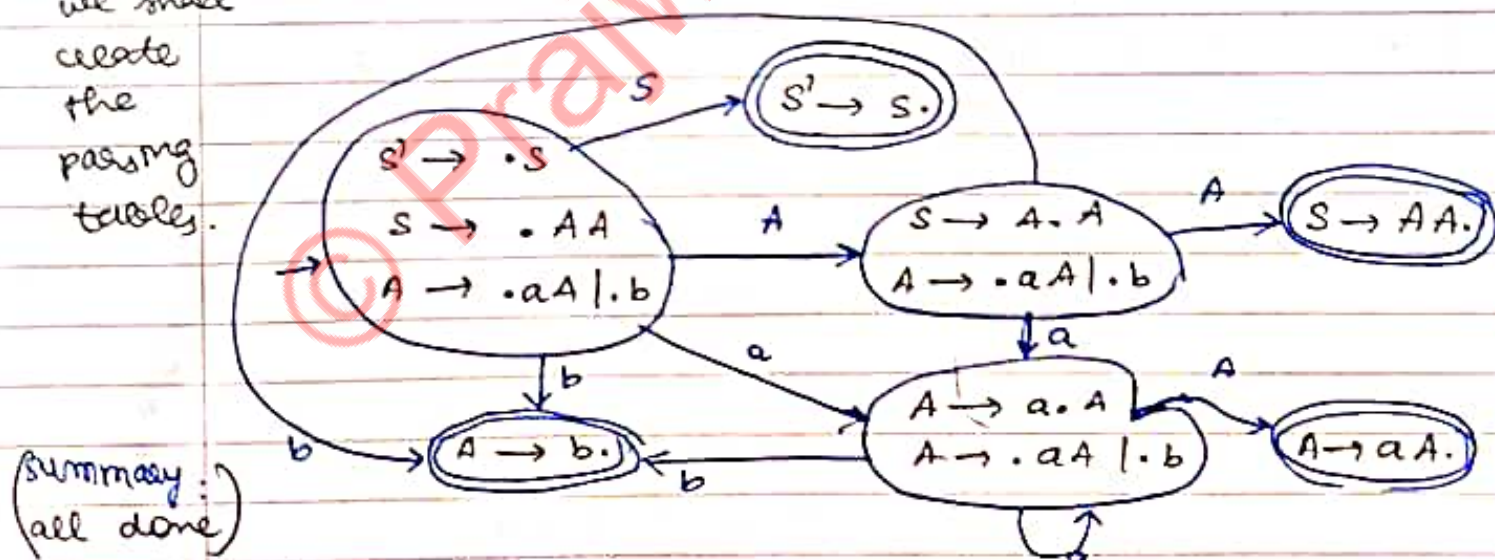


canonical collection of LR(0) items:

Take CLOSURE & GOTO.

They are nothing but finite state machines.

using this, we shall create the parsing tables.



(summary: all done)

- ① states where \cdot is placed at the end of every production rule are called final items.
- ② Moves involving non-terminals are called GOTO moves, while moving terminal symbols are known as the SHIFT moves.
- ③ Final states guarantee of reduction. When the compiler reaches any final state, now the compiler can reduce the RHS \rightarrow LHS.

— / — / —

```

graph LR
    I0((I0)) -- S --> I1((I1))
    I0 -- A --> I2((I2))
    I0 -- b --> I4((I4))
    I1 -- A --> I2
    I2 -- A --> I5((I5))
    I3((I3)) -- a --> I2
    I3 -- A --> I6((I6))
    I3 -- b --> I4
    I4 -- b --> I0
    I3 -- a --> I3
  
```

[sampling states from the previous canonical collection]

State	Action			GO TO	
	a	b	\$	A	S
0	s ₃	s ₄		2	1
1			Accept		
2	s ₃	s ₄		5	
3	s ₃	s ₄		6	
4	r _{iii}	r _{iii}	Accept r _{iii}		
5	r _i	r _i	Accept r _i		
6	r _{ii}	r _{ii}	Accept r _{ii}		

- (i) $S \rightarrow AA$
(ii) $A \rightarrow aA$
(iii) $A \rightarrow b$
rules numbered

Summary :
construction of $LC(0)$
passing table.

outcome :

LR(0) parsing procedure

$\$$

Sn : ① push Input
② push the state n

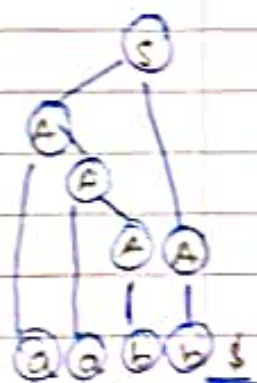
$|RHS| = 1, \quad 1 \times 2 = \underline{\underline{2}}$ pop twice

46
82 A
82 5
82 A
82 1
82 5
0

LR(0) parsing procedure.

Tabular Representation : PPL

Parse Tree :



Stack	Input	Action
	a a b b \$	shift 3
0 a 3	a b b \$	shift 3
0 a 3 a 3	b b \$	Shift 4
0 a 3 a 3 b 4	b \$	Reduce 3
0 a 3 a 3 A 6	b \$	Reduce 2
0 a 3 A 6	b \$	Reduce 2
0 A 2	b \$	Shift 4
0 A 2 b 4	\$	Reduce 3
0 A 2 A 5	\$	Reduce 1
0 S 1	\$	Accept

SLR(1) PARSERS

Outcome :

- ① Problem with LR(0) parsing procedure.
- ② Difference between LR(0) & SLR(1) parsers.
- ③ construction of SLR(1) parsing table.

LR(0) parsing :

a a b b \$
 ↑
 ↑
 ptr

We aren't reducing the b pointed by the pointer in the input buffer → rather we are reducing the previous one. In the parsing procedure, we got lucky because the other b also gets reduced to A. If the (b) ptr pointing variable didn't exist in the input buffer, the final reduction to S wouldn't have been possible.

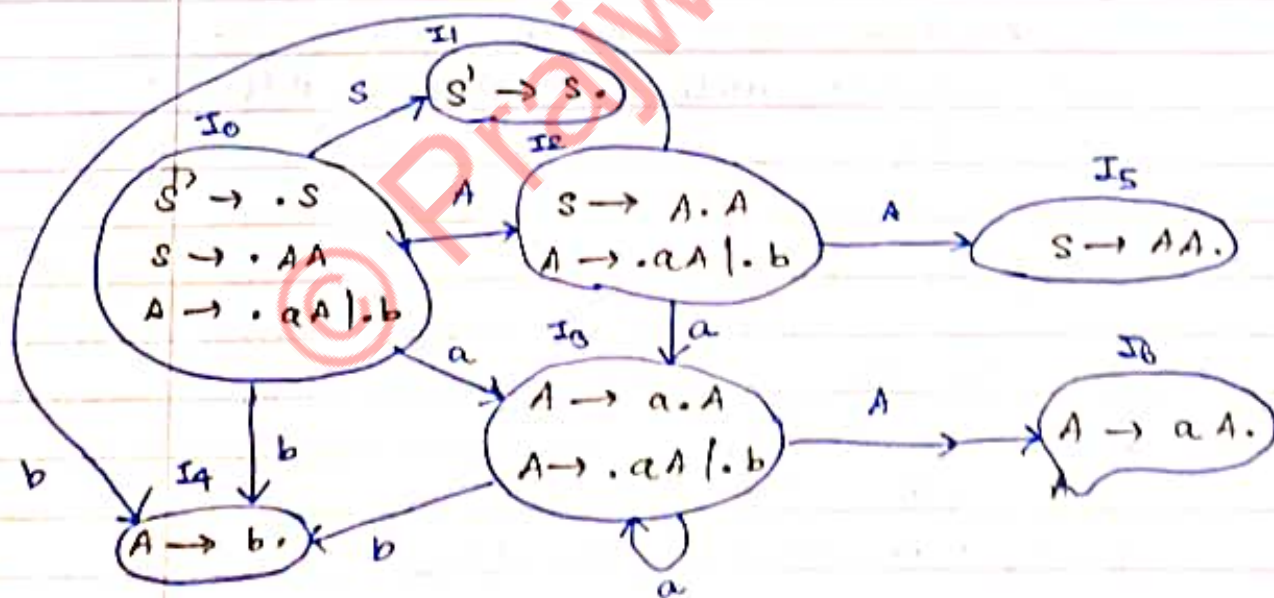
This is why it is called LR(0) parser → with 0 lookahead symbols. we were pointing to something in the input buffer, but reducing the previous symbol from the buffer → [which is also the reason why for the states containing the final items, we were using the same reduction rules for all the terminal symbols.]

This is the problem associated with the LR(0) parsing procedure.

SLR(1) parsing table

$S \rightarrow AA$
 $A \rightarrow aA$
 $A \rightarrow b$

$S' \rightarrow S$
 $S \rightarrow AA$
 $A \rightarrow aA | b$



$S \rightarrow AA$
 $A \rightarrow aA | b$
 $A \rightarrow b$

FIRST
 $\{a, b\}$
 $\{a, b\}$

FOLLOW
 $\{\$ \}$
 $\{\$, a, b\}$

$FOLLOW(A) = \{\$, a, b\}$

states	Action			GOTO	
	a	b	\$	A	S
0	S ₃	S ₄		2	1
1			Acc ept		
2	S ₃	S ₄		5	
3	S ₃	S ₄		6	
4	r _{iii}	r _{iii}	r _{iii}		
5			r _i		
6	r _{ii}	r _{ii}	r _{ii}		

$\text{FOLLOW}(A) = \{a, b, \$\}$ &
 → reduce iii in row 4, columns

similarly

$\text{FOLLOW}(S) = \{\$\}$ &
 → reduce (i) in row 5, column

row 6
r_{ii}

comparing SLR(1) and LR(0) parsing tables:
 content-wise, we have more space in
 the SLR(1) parsing table → Now we already
 know that during the parsing procedure,
 at any point, if we end up in any
 blank spaces on the table, that will
 signify error, and as spaces are more in
 case of SLR(1) parsing table, it means
 that SLR(1) parsing is more powerful than
 LR(0) parsing. The power has been achieved
 in SLR(1) parsing because in here for
 reduction, we are considering the look-
 ahead.

summary:

① LR(0)
problem

② LR(0) vs
SLR(1)
diff

③ SLR(1) parsing
table

conflicts in LR(0) and SLR(1) parsing

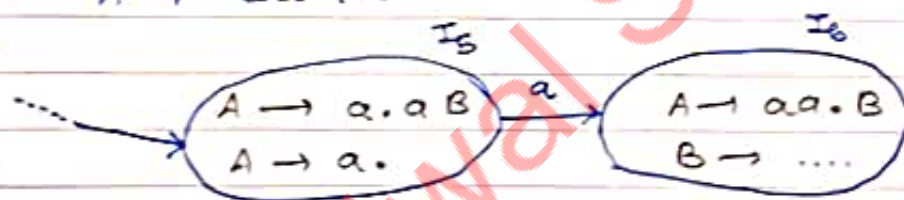
outcome :

- ① S-R and R-R conflicts in LR(0) parsing table.
- ② S-R and R-R conflicts in SLR(1) parsing table

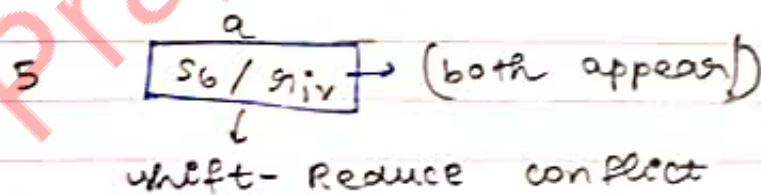
LR(0) parsing table :

In the previous example, we were lucky that we didn't encounter any conflicts, that is, no entry in the LR(0) parsing table has multiple entries. But in reality, this may happen.

- iii) $A \rightarrow aab$
- iv) $A \rightarrow a$

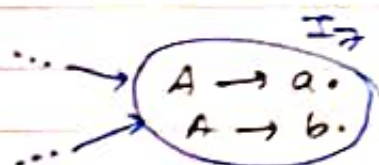


Here, in LR(0) parsing table.

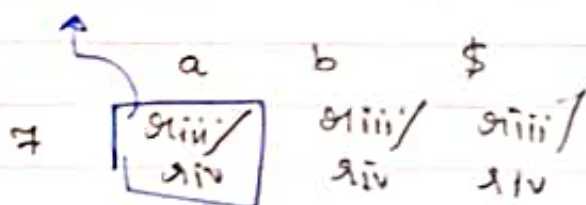


- iii) $A \rightarrow a$
- iv) $A \rightarrow b$

$A \rightarrow a | b$



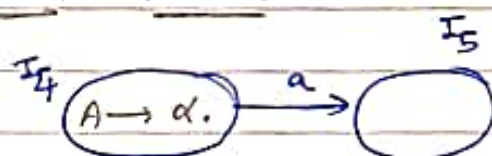
Reduce-Reduce conflict



If a grammar shows either S-R or R-R conflict, that grammar will not be an LR(0) grammar.

SLR(1) parser :

SR conflict

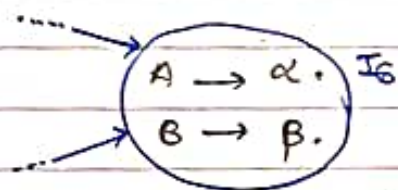


and if

$$\text{FOLLOW}(A) = \{ \dots, a, \dots \}$$

$$4 \quad \boxed{S_5 / x_a} \xrightarrow{a} \text{SR conflict}$$

RR conflict



$$\frac{\text{FOLLOW}(A) \cap \text{FOLLOW}(B) \neq \phi}{\downarrow} = \{ \dots, a, \dots \}$$

$$6 \quad \boxed{x_y / x_z} \xrightarrow{x} \text{RR conflict}$$

Also, a grammar that shows either S-R or R-R conflict will not be an SLR(1) grammar.

summary

- ① S-R and R-R conflicts in LR(0) PT.
- ② S-R and R-R conflicts in SLR(1) PT.

Determining the type of grammar - set 1

outcome : understanding how to determine the type of a given grammar.

Q1

Determine whether the following grammar is LL(1) or LR(0) or SLR(1) :

FIRST

{d, a}

{b, c}

{b, c}

$S \rightarrow dA \mid aB$

$A \rightarrow bA \mid c$

$B \rightarrow bB \mid c$

FOLLOW

{ \$ }

{ \$ }

{ \$ }

	a	b	c	d	\$
S	$S \rightarrow aB$			$S \rightarrow dA$	
A		$A \rightarrow bA$	$A \rightarrow c$		
B		$B \rightarrow bB$	$B \rightarrow c$		

✓ LL(1) grammar

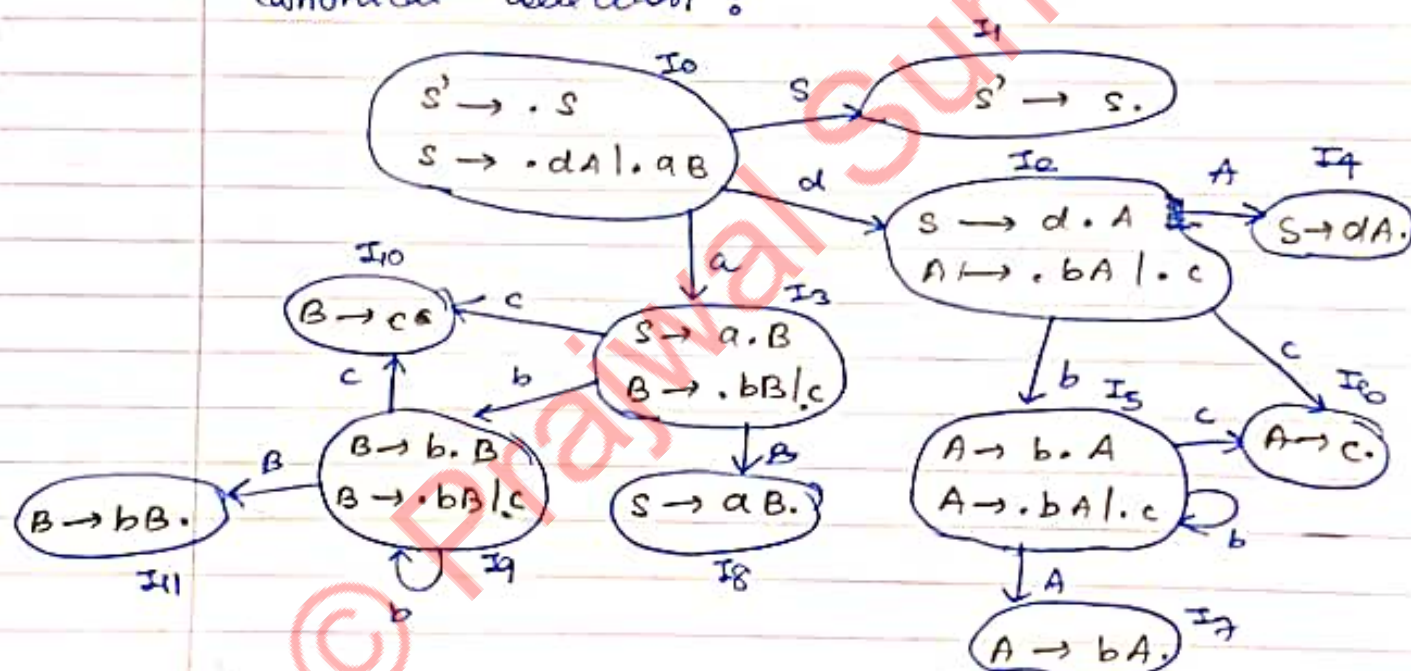
Augmented grammar:

$S' \rightarrow S$
 $S \rightarrow dA \mid aB$
 $A \rightarrow bA \mid c$
 $B \rightarrow bB \mid c$

After drawing CC:

- For all final items:
- ✓ 1 final item.
 - ✓ no shift move.

canonical collection:



As final items have

- ✓ 1 final item
 - ✓ no shift move
- RR (X)] safe! LR(0) (X) grammar

SLR(1) is a reduced version of LR(0)

LR(0) → definitely SLR(1) ✓

$\checkmark R(\checkmark)$ $\checkmark LR(1)$
 $\checkmark SLR(1)$

Summary: How to determine grammar type

Determining the type of grammar - set 2

outcome :

- ✓ 2 solved problems on how to determine the type of a given grammar.

Q1:

Determine whether the given grammar P_3 is LL(1) or LR(0) or SLR(1):

FIRST

{a}

{a}

$S \rightarrow A | a$

$A \rightarrow a$

FOLLOW

{ ϵ }

{ ϵ }

	a	ϵ
S	$S \rightarrow A$ $S \rightarrow a$	
A	$A \rightarrow a$	

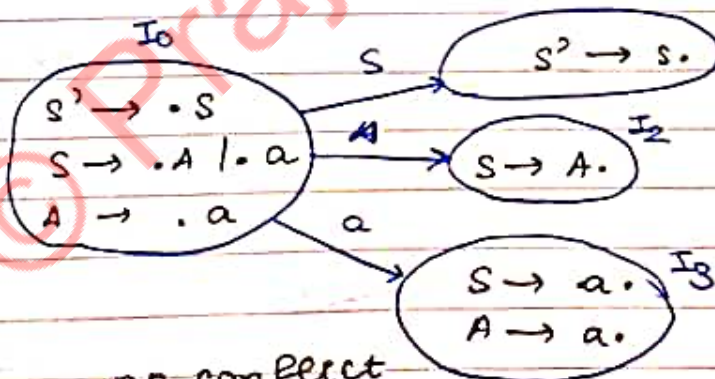
⊗ not an LR(1) grammar

Augmented grammar :

$S' \rightarrow S$
 $S \rightarrow A | a$
 $A \rightarrow a$

Ambiguous grammar

to derive @



RR conflict

I_3 has 2 final states \rightarrow LR(0) ⊗

now $\text{FOLLOW}(S) \cap \text{FOLLOW}(A) = \{\epsilon\}$

$\neq \emptyset$ NOT NULL

RR conflict exists in SLR(1) table also ⊗

LL(1)	LR(0)	SLR(1)
⊗	⊗	⊗

Q2: Determine the type of grammar:

FIRST

FOLLOW

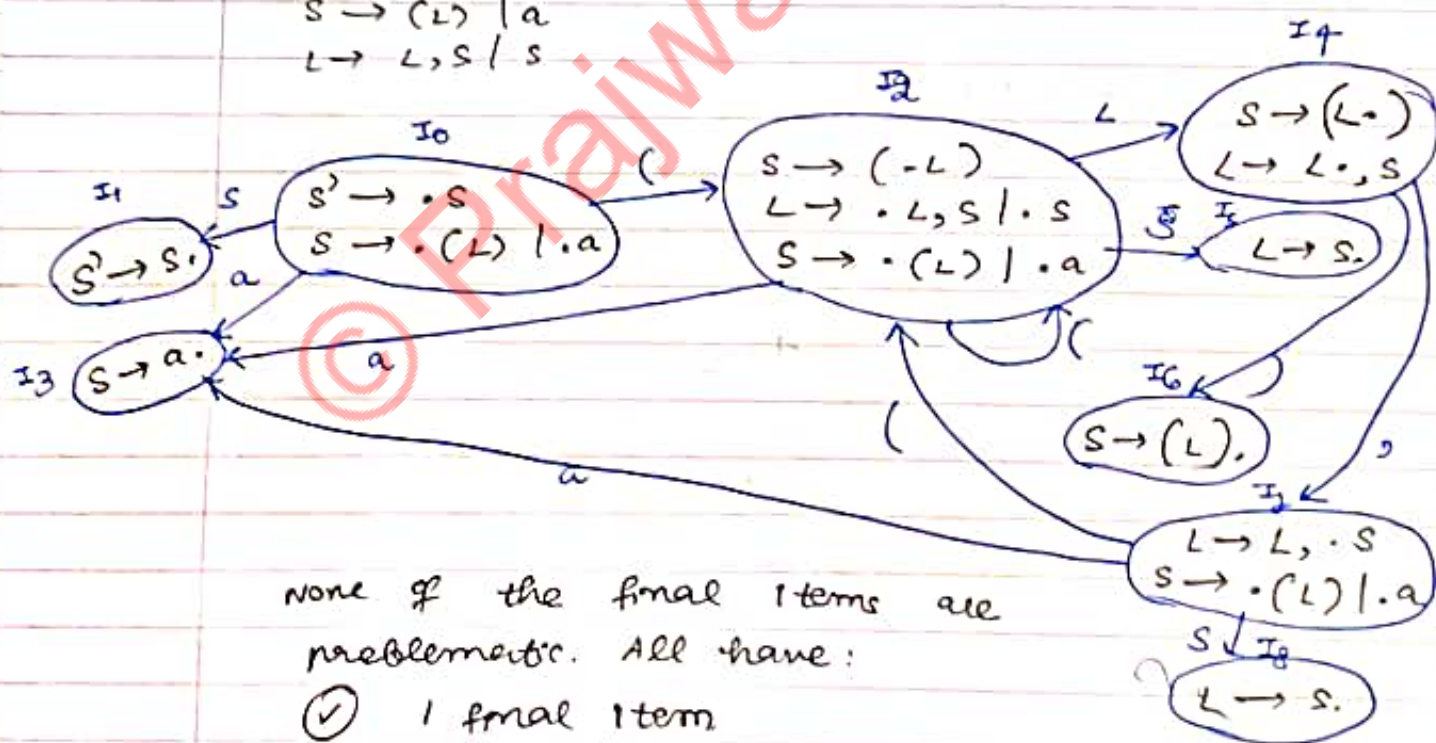
$\{ (, a \}$ $S \rightarrow (L) \mid a$
 $\{ (, a \}$ $L \rightarrow L, S \mid S$

$\{ \$,), > \}$
 $\{), > \}$

	()	,	a	\$
S	$S \rightarrow (L)$			$S \rightarrow a$	
L	$L \rightarrow S$			$L \rightarrow S$	

There are no multiple entries in any cell BUT STILL, it is not LL(1) \otimes grammar due to the presence of left recursion. $\underline{L} \rightarrow \underline{L}, S$

$S' \rightarrow S$
 $S \rightarrow (L) \mid a$
 $L \rightarrow L, S \mid S$



none of the final items are problematic. All have:

- ✓ 1 final item
- ✓ No shift move

LL(1)	LR(0)	SLR(1)
\otimes	\checkmark	\checkmark

Summary: 2 solved problems.

Determining the type of grammar - set 3

outcome: solved problem on how to determine the type of a given grammar.

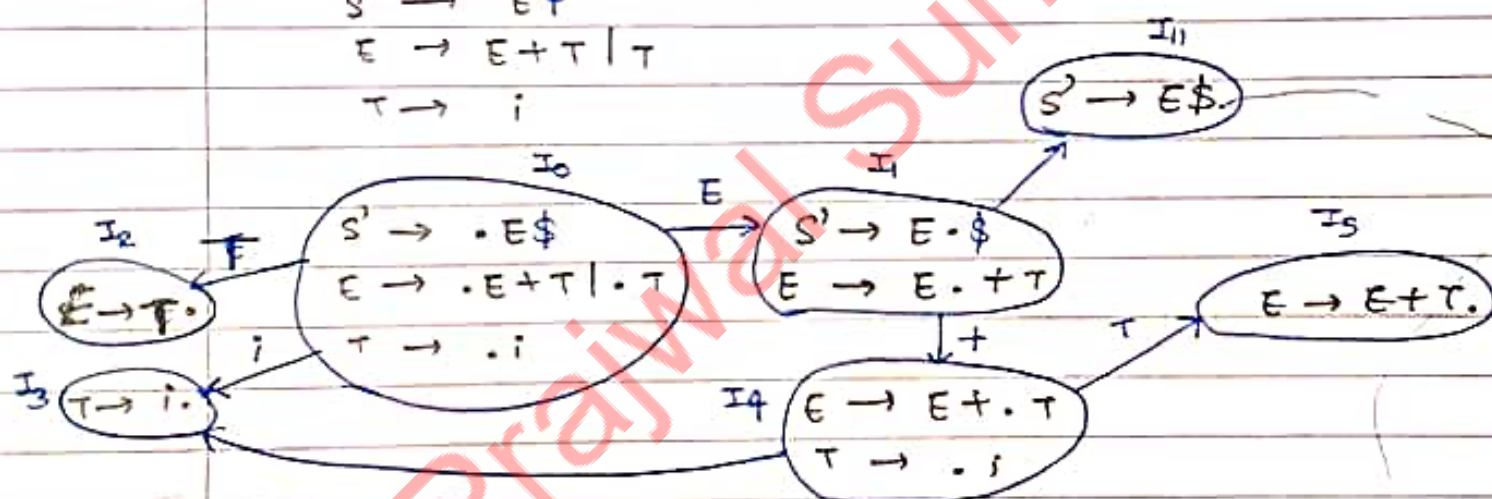
Q. Determine the grammar type:

$$\{i\} \quad E \rightarrow E + T \mid T \quad \{\$, +\}$$

$$\{i\} \quad T \rightarrow i \quad \{\$, +\}$$

$E \rightarrow E + T \mid T$ left recursion \rightarrow \otimes not LR(1)

$$\begin{aligned} S' &\rightarrow E\$ \\ E &\rightarrow E + T \mid T \\ T &\rightarrow i \end{aligned}$$



All final items: $\textcircled{1}$ 1 final item
 $\textcircled{2}$ no shift move

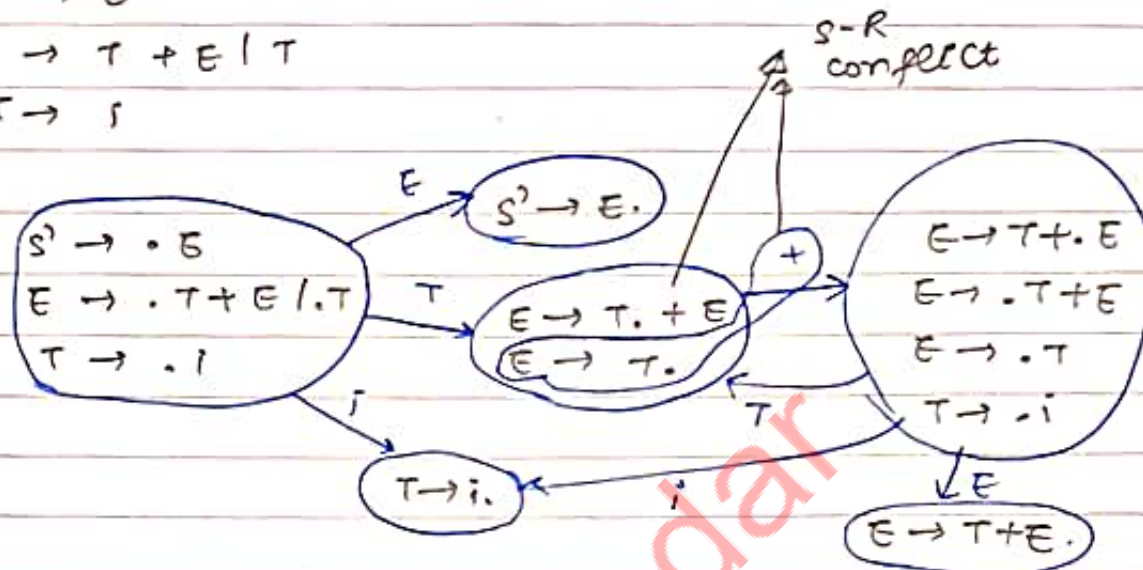
LL(1) \times LR(0) \checkmark SLR(1) \checkmark

Q. $\{i\} \quad E \rightarrow T + E \mid T \quad \{\$, \}$
 $\{i\} \quad T \rightarrow i \quad \{+, \$\}$

	+	i	\$
E		$E \rightarrow T + E$ $E \rightarrow T$	
T		$T \rightarrow i$	

\otimes not LR(1) grammar

$S' \rightarrow E$
 $E \rightarrow T + E \mid T$
 $T \rightarrow i$



$FOLLOW(E) = \{ \$ \}$
 \hookrightarrow not involved in shift(+)

no LR conflict in SLR(1) parser

$LL(1)$
 \otimes

$LR(0)$
 \otimes

$SLR(1)$
 \checkmark

Summary : solved problem on how to determine type of a given grammar.

Determining the type of grammar - set 4

outcome : \checkmark 2 solved problems

Q1: Determine if $LL(1)$, $LR(0)$, $SLR(1)$:

$E \rightarrow E + T \mid T$
 $T \rightarrow T F \mid F$
 $F \rightarrow F * \mid a \mid b$

$\left. \begin{array}{l} \underline{E} \rightarrow \underline{E} + T \\ \underline{T} \rightarrow \underline{T} F \\ \underline{F} \rightarrow \underline{F} * \end{array} \right\}$ left recursive
 $LL(1) \otimes$

FIRST

$\{a, b\}$

$\{a, b\}$

$\{a, b\}$

FOLLOW

$\{+, \$\}$

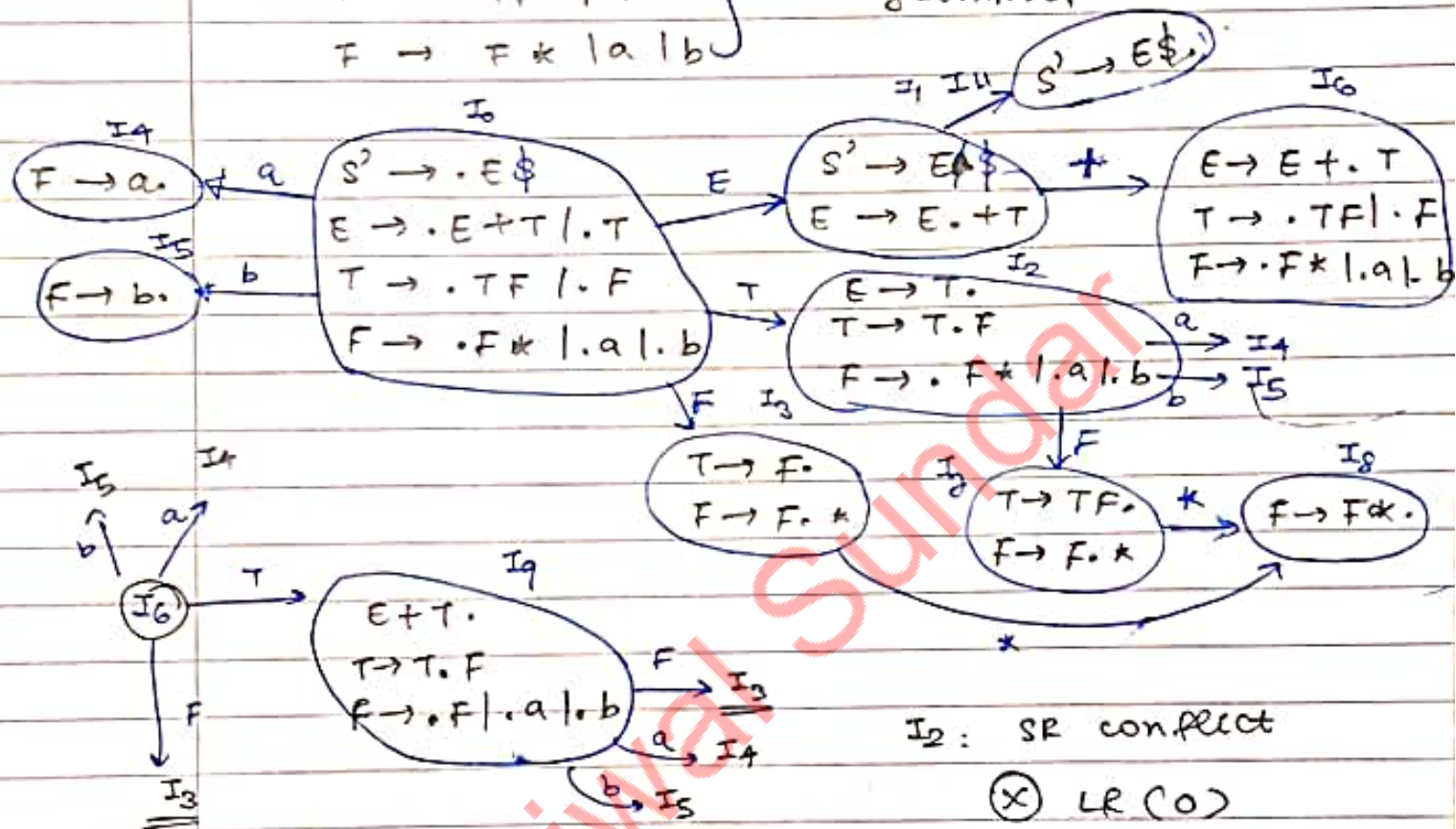
$\{t, \$, a, b\}$

$\{*, t, \$, a, b\}$

\downarrow
 [top down parser]

$S' \rightarrow E \$$
 $E \rightarrow E + T \mid T$
 $T \rightarrow TF \mid F$
 $F \rightarrow F * \mid a \mid b$

augmented grammar



I_2 : SR conflict

⊗ LR(0)

$\text{FOLLOW}(E) = \{+, \$\} \neq \{a, b\}$ ✓

I_3 : SR $\text{FOLLOW}(T) \neq *$ ✓

I_4, I_5, I_8 : single final items ✓

I_7 : $\text{FOLLOW}(T) \neq *$ ✓

I_9 : $\text{FOLLOW}(E) = \{ \cancel{a}, +, \$ \}$
 not there
 no problem

SR conflict
⊗ no conflict

LL(0)
⊗

LR(0)
⊗

SLR(1)
✓

∴ Grammar is only SLR(1).

Q2.

Determine type of grammar :

FIRST		FOLLOW
$\{a, b\}$	$S \rightarrow AaAb \mid BbBa$	$\{\$ \}$
$\{\epsilon\}$	$A \rightarrow \epsilon$	$\{a, b\}$
$\{\epsilon\}$	$B \rightarrow \epsilon$	$\{b, a\}$

	a	b	\$
S	$S \rightarrow AaAb$	$S \rightarrow BbBa$	
A	$A \rightarrow \epsilon$	$A \rightarrow \epsilon$	
B	$B \rightarrow \epsilon$	$B \rightarrow \epsilon$	

LL(1) \checkmark
grammar

$S' \rightarrow S$
 $S \rightarrow AaAb \mid BbBa$
 $A \rightarrow \epsilon$
 $B \rightarrow \epsilon$

note : For productions
 $A \rightarrow \epsilon$, the LR(0) items
 $A \rightarrow \cdot \epsilon \cong A \rightarrow \epsilon \cdot \cong A \rightarrow \cdot$

$S' \rightarrow \cdot S$
 $S \rightarrow \cdot AaAb \mid \cdot BbBa$
 $A \rightarrow \cdot$
 $B \rightarrow \cdot$

reduce-reduce
conflicts

LR(0) \times

$\text{FOLLOW}(A) \cap \text{FOLLOW}(B)$
 $\neq \emptyset$

\downarrow
 in SLR(1) parsing
 tables also, there
 will be R-R conflicts
 \times

LR(1) \checkmark

LR(0) \times

SLR(1) \times

LR(0) $\checkmark \rightarrow$ SLR(1) \checkmark

keypoint to note,

If LR(0) $\times \rightarrow$ again verify SLR(1)

summary..

& solved Q.

Determining the type of grammar - set 5

outcome : 2 related problems on how to determine the type of a given grammar.

Q1.

Determine the type of the following grammar :

FIRST

$\{a, b\}$

$\{a, b\}$

$S \rightarrow AS \mid b$

$A \rightarrow SA \mid a$

FOLLOW

$\{ \epsilon, a, b \}$

$\{a, b\}$

	a	b	ϵ
S	$S \rightarrow AS$	$S \rightarrow AS$ $S \rightarrow b$	
A	$A \rightarrow a$	$A \rightarrow SA$	

not LR(1) grammar

Ambiguous grammar :



Q2: Determine the type of grammar:

FIRST

FOLLOW

$\{a, b\}$

$S \rightarrow Aa/bAc/dc/bda$

$\{\$ \}$

$\{d\}$

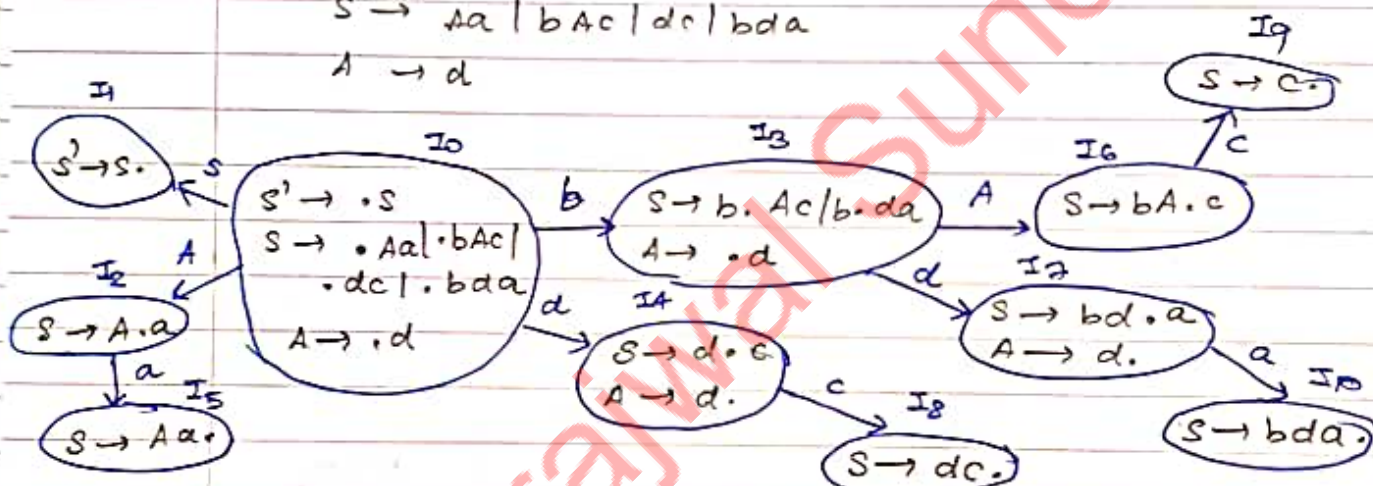
$A \rightarrow d$

$\{a, c\}$

	a	b	c	d	\$
S		$S \rightarrow bAc$ $S \rightarrow bda$		$S \rightarrow Aa$ $S \rightarrow dc$	
A				$A \rightarrow d$	

⊗ not an LL(1) grammar

$S' \rightarrow S$
 $S \rightarrow Aa/bAc/dc/bda$
 $A \rightarrow d$



$I_4: A \rightarrow d \cdot \xrightarrow{c}$

$I_7: A \rightarrow \cdot \xrightarrow{a}$

SR conflicts

⊗

not LR(0) grammar ⊗

$c \in \text{FOLLOW}(A)$

⊗

$a \in \text{FOLLOW}(A)$

⊗

⊗ not SLR(1) grammar also ⊗

LL(1)	LR(0)	SLR(1)
⊗	⊗	⊗

But grammar isn't ambiguous

key:

ambiguous →

LL(1)

LR(0)

SLR(1)

[not reverse]

Summary: 2 solved & done

canonical collection of LR(1) items

outcome :

- ✓ understanding LR(1) items.
- ✓ Derivation of the canonical collection of LR(1) items from a given grammar.

LR parsers :

organization wise, all LR parsers are the same. However, the only difference lies in how we are constructing the LR parsing table.

$$\text{LR(1) items} = \text{LR(0) items} + \text{1 look-ahead symbol}$$

(i) ,

(ii) $S \rightarrow aA$

(iii) :

LR(0) item

$S \rightarrow \cdot aA$

$S \rightarrow a \cdot A$

$S \rightarrow aA \cdot$

LR(1) item

$S \rightarrow aA \cdot, a \mid b$
 final item will be associated with the look-ahead as well

In

$S \rightarrow aA \cdot, a \mid b$
 \vdots

states	Action			
	a	b	c	\$
r	r1	r1		

In LR(1) items, the reduced moves, only will be placed under the look-ahead symbols which are already mentioned in the state itself.

↳ How to figure out the look-ahead symbols?

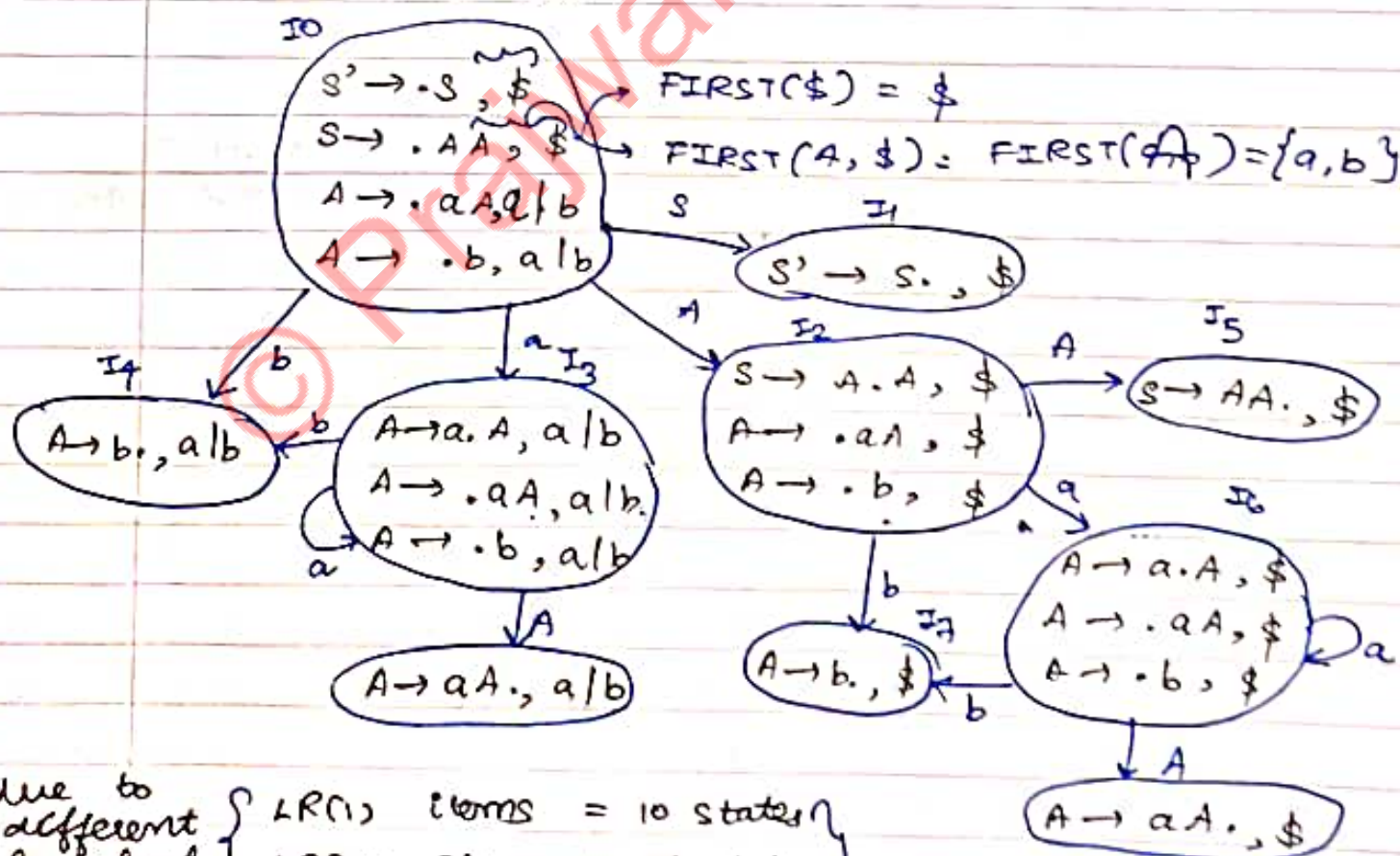
↓
 example :-

canonical collection of LR(1) items

$$\left. \begin{array}{l} S \rightarrow AA \\ A \rightarrow aA | b \end{array} \right\} \rightarrow \left. \begin{array}{l} S' \rightarrow S \\ S \rightarrow AA \\ A \rightarrow aA | b \end{array} \right\} \text{ augmented grammar}$$

Rules :

- ① For start symbol's production, the look.ahead is '\$'.
- ② In the closure set, the look.aheads of the LR(1) items are determined based on the rest of the items' FIRST set.
- ③ Look ahead will never change during transitions. However, they may change within state while determining the closure set.



due to different lookahead symbol

LR(1) items = 10 states
LR(0) items = 7 states

Summary : ☒ LR(1) items ☒ canonical collection of LR(1) items

CLRC(1) parser

outcome : construction of CLRC(1) parsing table.

$s' \rightarrow s$ (i) $s \rightarrow AA$ (ii) $A \rightarrow aA$ (iii) $A \rightarrow b$
 not numbered as it wasn't a part of the original grammar.

CLRC(1) parsing table :

(looking at the canonical collection of LR(1) items)

States	Action			GOTO	
	a	b	\$	A	S
0	s_3	s_4		2	1
1			Accept		
2	s_6	s_7		5	
3	s_3	s_4		8	
4	r_{iii}	r_{iii}			
5			r_i		
6	s_6	s_7		9	
7			r_{iii}		
8	r_{ii}	r_{ii}			
9			r_{ii}		

Filling in the same way as was filled for LR(0) and SLR(1) parsers.

↓
 shift & Goto → no difference.

Reduce → only fill in those columns as specified by look-ahead

LR(0) parsing table : 0 to 6 states : r in all

SLR(1) parsing table : 0 to 6 states : r in FOLLOW

CLRC(1) parsing table : 0 to 9 states : r in look-ahead

Summary : construction of CLRC(1) parsing table

LALR(1) parser

outcome: conversion from CLR(1) to LALR(1) parsing table.

canonical collection of LR(1) items:

If we merge the states which are similar w.r.t to the LR(0) items, but different because of the look aheads, the no. of states in the resulting collection will be equal to that of the canonical collection of LR(0) items.

The parsing table for that is known as LALR(1) or LOOK-Ahead LR(1) parsing table.

$$\begin{aligned} \text{No. of states in CLR(1)} &> \text{LR(0)} = \text{SLR(1)} \\ &= \text{LALR(1)} \end{aligned}$$

Taking canonical collection of LR(1) items

old states	New state
I ₃ , I ₆	I ₃₆
I ₄ , I ₇	I ₄₇
I ₈ , I ₉	I ₈₉

(merging of states)

LR(1) table →

states	Action			GOTO	
	a	b	\$	A	S
0	S ₃₆	S ₄₇	⊙	2	1
1	S ₃₆	S ₄₇		5	
36	S ₃₆	S ₄₇		89	
47	r _{iii}	r _{iii}			
5	S ₃₆	S ₄₇	r _i	89	
36	S ₃₆	S ₄₇	r _{iii}		
47	r _{ii}	r _{ii}	r _{iii}		
89	r _{ii}	r _{ii}	r _{ii}		
89					

remove
redundant
states

states	Action			GOTO	
	a	b	\$	A	S
0	S ₃₆	S ₄₇	⊙	2	1
1					
2	S ₃₆	S ₄₇		5	
36	S ₃₆	S ₄₇		89	
47	r _{iii}	r _{iii}	r _{iii}		
5			r _i		
89	r _{ii}	r _{ii}	r _{ii}		

To merge reduced states,
just perform union

summary: CLR(1) → LALR(1) PT ✓

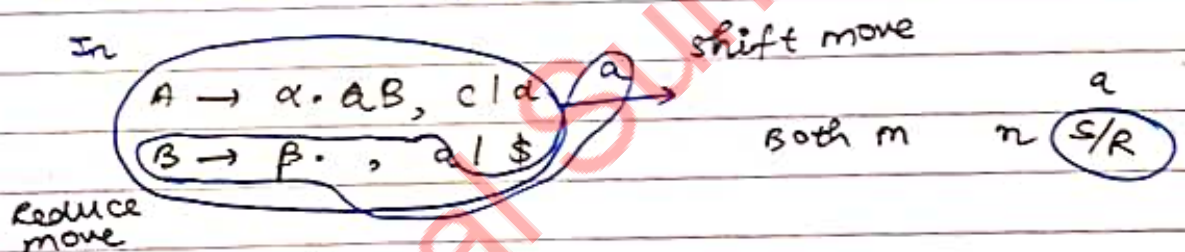
Conflicts in CLR(1) and LALR(1) parsers

outcome :

- ① S-R and R-R conflicts in CLR(1) and LALR(1) parsing tables.
- ② Difference between CLR(1) and LALR(1) parsers w.r.t R-R conflict.

Conflicts w.r.t LR(1) items

Shift-Reduce conflict :

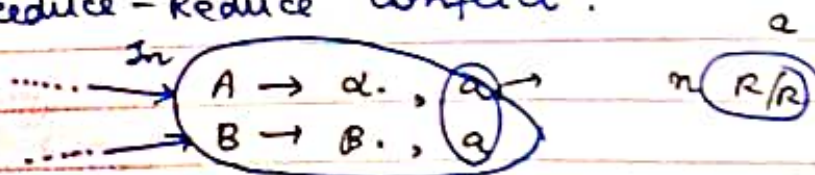


we generally use :

- ① uppercase letters for non-terminals
- ② lowercase letters for terminals.

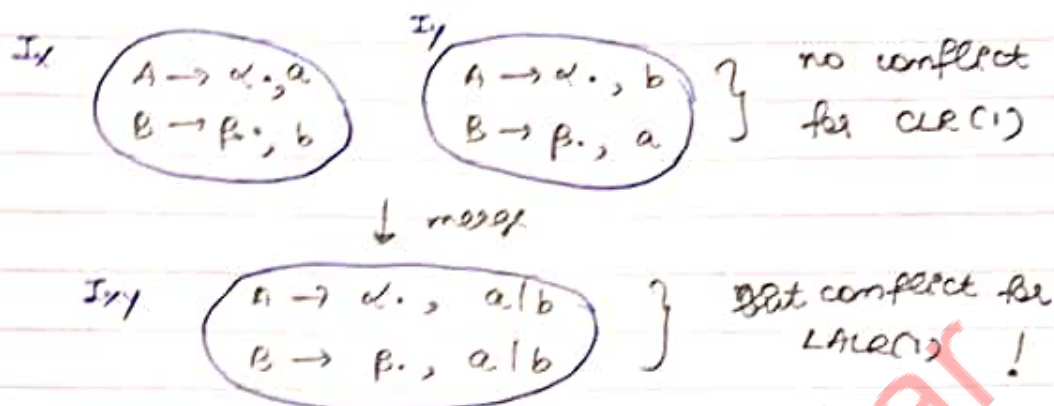
Then why notations like α, β, γ ?
→ when we are concerned about the presence of the element rather than the type of it, that is, we don't want to know whether the element is a terminal or non-terminal, rather we want the element to be in its place → in those cases, we use symbols like $\alpha, \beta, \gamma \dots$

Reduce-Reduce conflict :



Any grammar with such situation is neither CLR(1) nor LALR(1).

Reduce - Reduce conflict



Key points:

No S-R conflict in CLR(1) \Rightarrow No S-R conflict in LALR(1)

No R-R conflict in CLR(1) \nRightarrow No R-R conflict in LALR(1)

Summary:

✓ S-R, R-R conflicts

✓ CLR(1) & LALR(1) (with RR)

Determining the type of grammar - set G

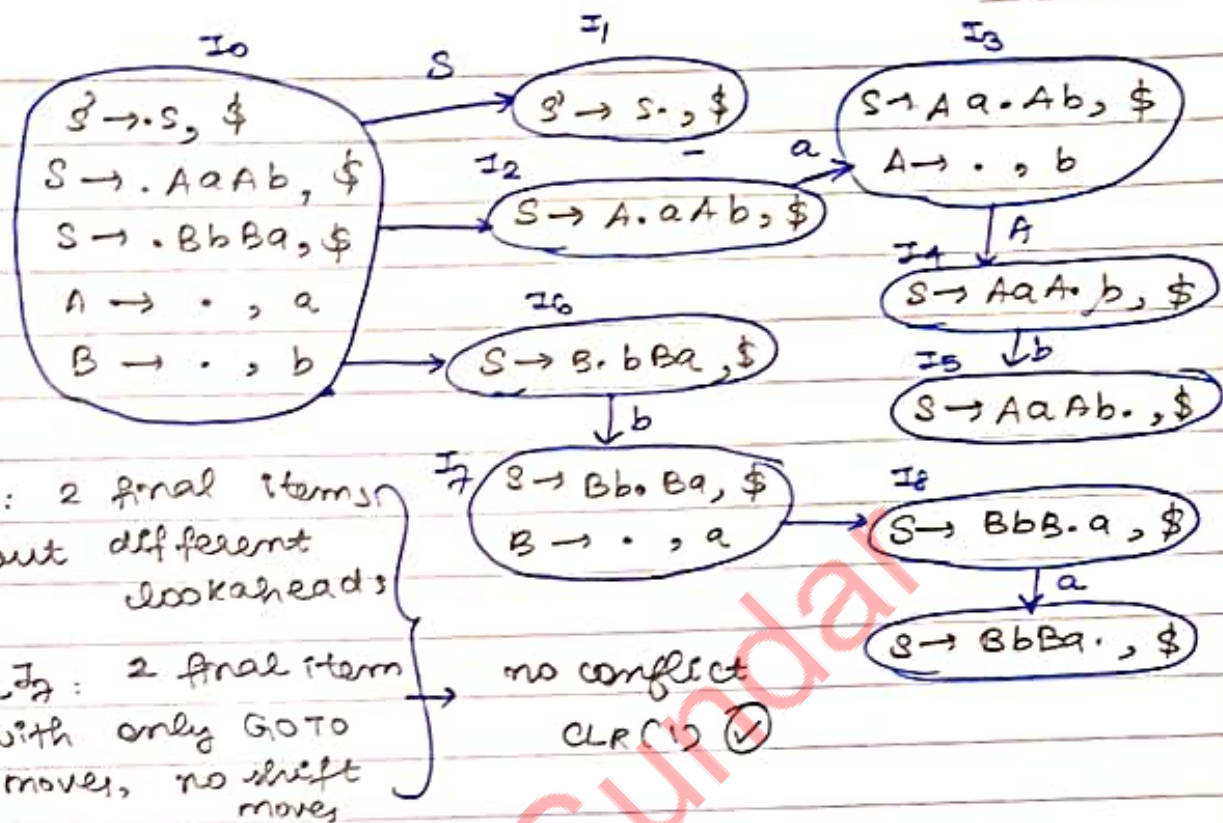
outcome: a solved problems on how to determine the type of a given grammar.

Q1: Determine whether the following grammar is CLR(1) or LALR(1):

$$\left. \begin{array}{l} S \rightarrow AaAb|BbBa \\ A \rightarrow \epsilon \\ B \rightarrow \epsilon \end{array} \right\}$$

Augmented grammar \rightarrow

$$\begin{array}{l} S' \rightarrow S \\ S \rightarrow AaAb|BbBa \\ A \rightarrow \epsilon \\ B \rightarrow \epsilon \end{array}$$



Also, there are no states which we can merge with LR(0) items \rightarrow so, CLR(1) and LALR(1) parsing tables will be the same for this grammar \rightarrow

CLR(1)	LALR(1)
✓	✓

Q2: Determine whether grammar is CLR(1) or LALR(1):

Grammar rules:

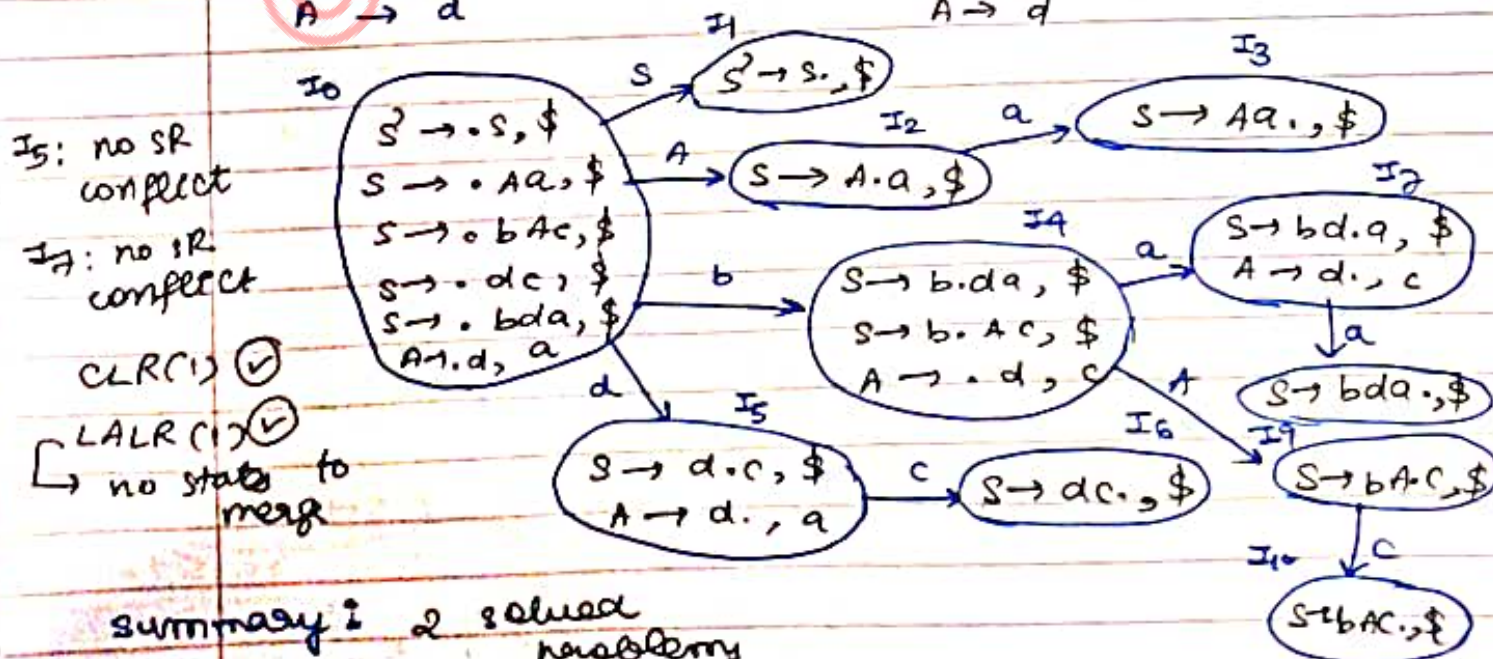
$$S \rightarrow Aa | bAc | dc | bda$$

$$A \rightarrow d$$

LR(0) items and transitions:

$$S' \rightarrow S$$

$$S \rightarrow .Aa | bAc | dc | bda$$

$$A \rightarrow ., d$$


Determining the type of grammar - set 7

outcome : solved problem on how to determine the type of a given grammar.

Q1. Determine whether the following grammar is LR(1) / LR(0) / SLR(1) / CLR(1) / LALR(1);

FIRST $\{d, b, a\}$ FOLLOW $\{\$, a, c\}$
 $S \rightarrow Aa | bAc | Bc | bBA$
 $A \rightarrow d$
 $B \rightarrow d$

	a	b	c	d	\$
S		$S \rightarrow bAc$ $S \rightarrow bBA$		$S \rightarrow Aa$ $S \rightarrow Bc$	
A				$A \rightarrow d$	
B				$B \rightarrow d$	

⊗ not an LR(1) grammar

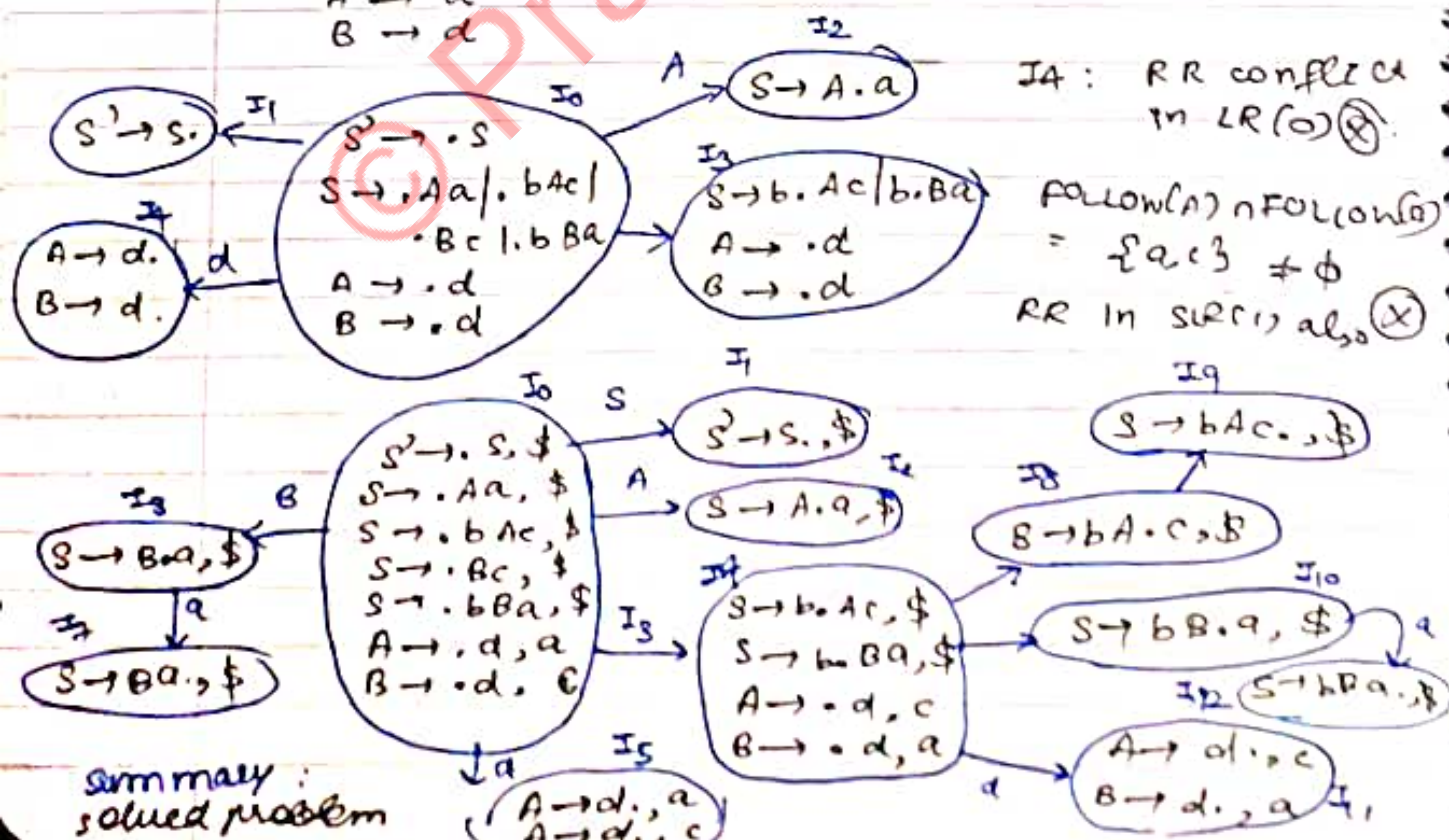
$S' \rightarrow S$
 $S \rightarrow Aa | bAc | Bc | bBA$
 $A \rightarrow d$
 $B \rightarrow d$

CLR(1) ✓

IS II, LR(1) RR conflict ⊗

IA: RR conflict in LR(0) ⊗

FOLLOW(A) ∩ FOLLOW(B) = $\{a, c\} \neq \emptyset$
 RR in SLR(1) also ⊗



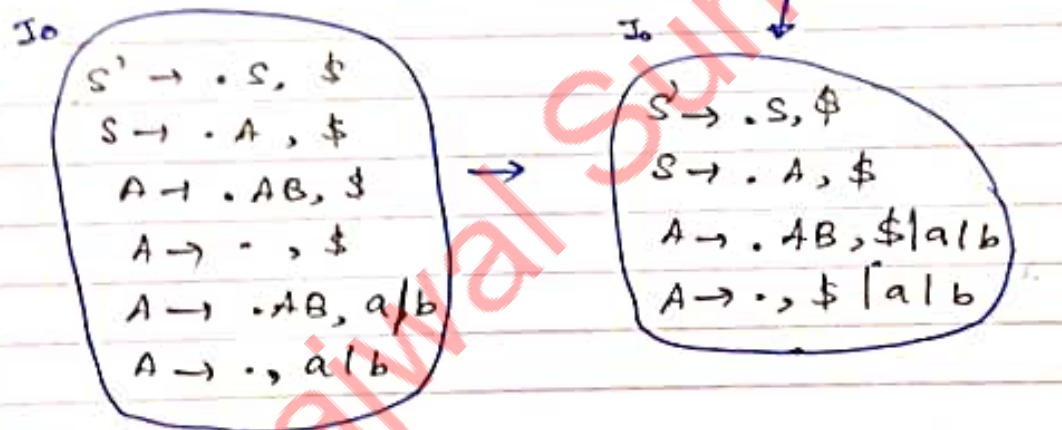
summary : solved problem

Interacting closure Representation of LR(0) Items

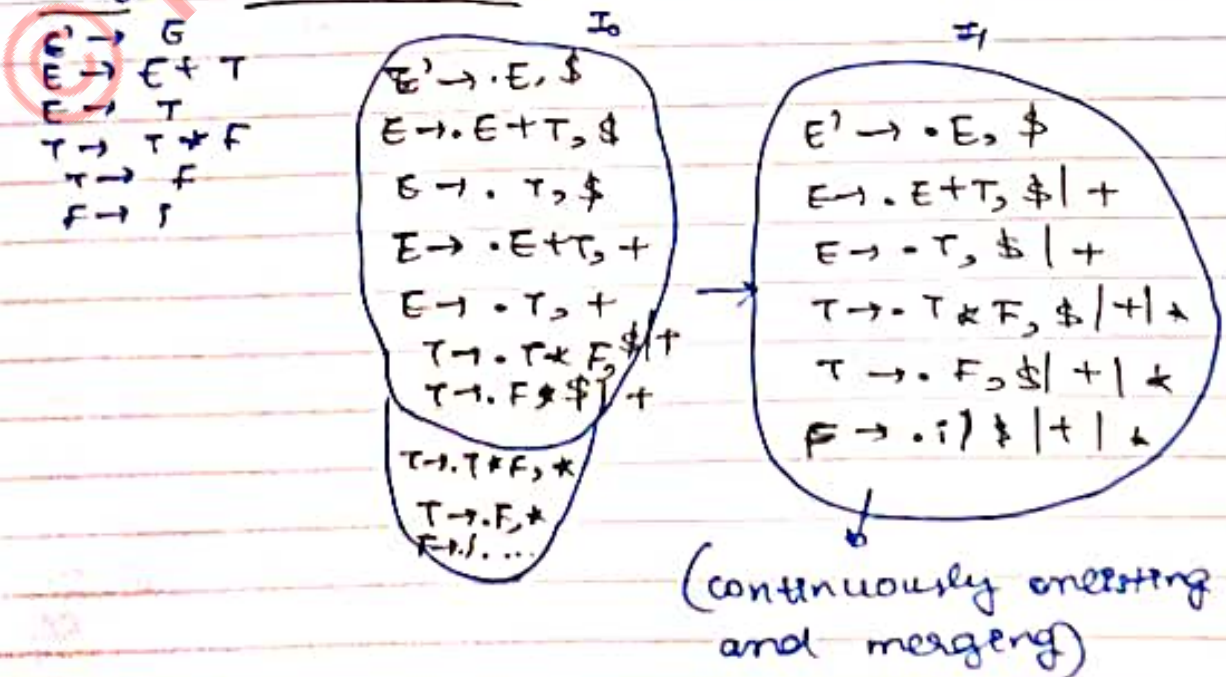
outcome: canonical collection of LR(0) items with merged look-aheads.

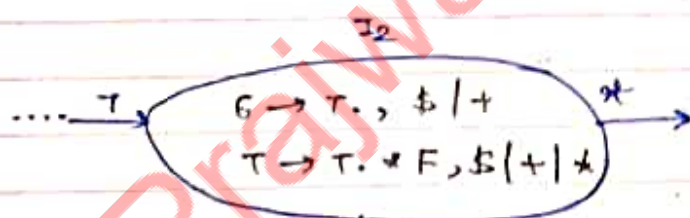
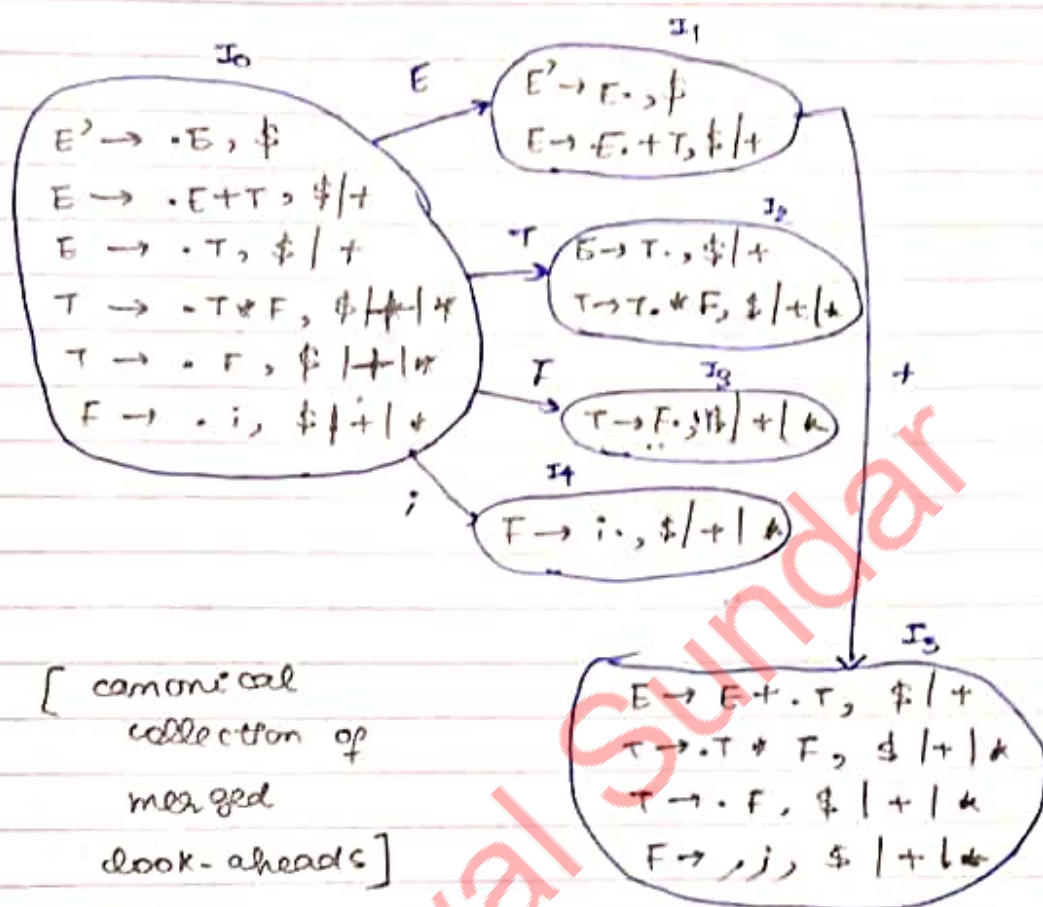
closure Representation of LR(0) items:

$S' \rightarrow S$
 $S \rightarrow A$
 $A \rightarrow AB|E$
 $B \rightarrow aB|b$



canonical collection of LR(0) items with merged look-aheads:





They might look like an S-R conflict, but it is NOT.

↓

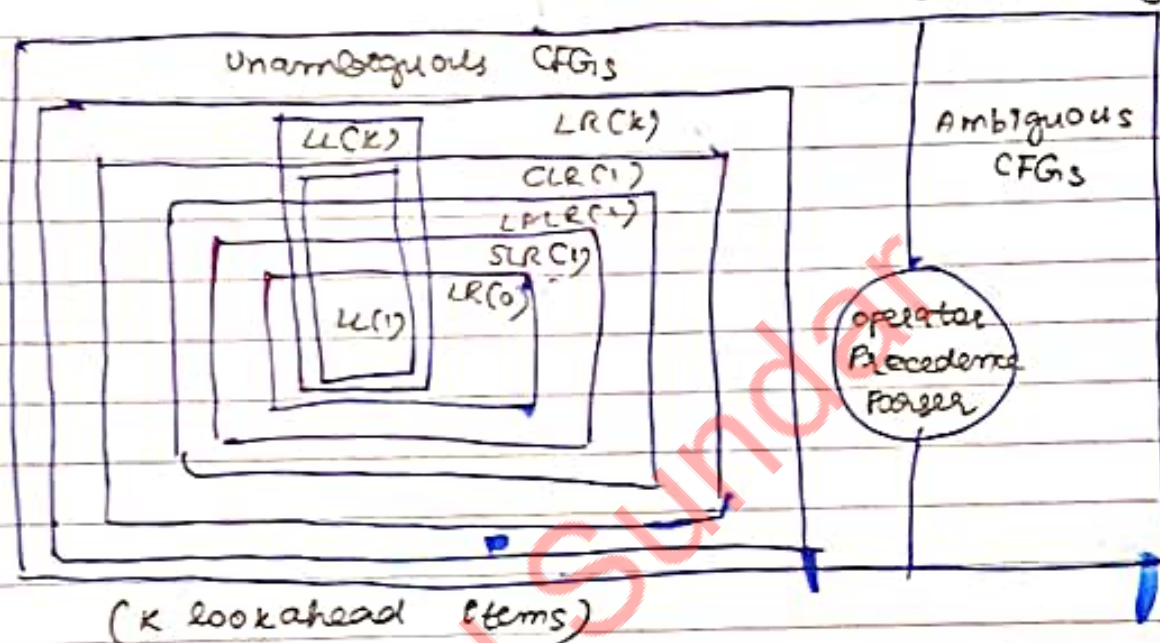
$E \rightarrow T$ will be a reduction rule underneath $\$$ and $+$. whereas the shift move will be entered underneath $*$.

not a conflict

Summary: canonical collection of LR(0) items with merged look-aheads.

CFEs - The Big Picture

outcome: Relationship of all context Free Grammar.



summary: Relationship of all CFGs.

CFGs - solved Problems (set - 1)

outcome: 4 solved problems on CFGs

$$[LL(0) < SLR(1) < LALR(1) < CLR(1)]$$

Q1. which of the following statements is true?
with power

(GATE 1998)

- X(A) $SLR > LALR$ X(B) $LALR > LR$ (canonical)
 ✓ (C) $CLR > LALR$ X(D) $SLR = CLR = LALR$

Q2. which of the following statements is false?

(GATE 2001)

- ✓ F(A) unambiguous grammar has same leftmost & rightmost derivation.
 T(B) An $LL(1)$ parser is a top-down parser.
 T(C) $LALR$ is more powerful than SLR .
 T(D) An ambiguous grammar can never be $LR(k)$ for any k .

Q3:
(GATE 2003)

considering the SLR(1) and LALR(1) parsing tables of a CFG, find out the false stmt.

(A) GOTO of both tables may be different.
(B) shift entries are identical in both tables.
(C) reduce entries in tables may be different.
(D) Error entries in tables may be different.

- * GOTO \rightarrow always same
- * SHIFT \rightarrow always same
- * REDUCE \rightarrow may / may not be same
- * ERROR \rightarrow may / may not be same

Q4: consider the grammar: $S \rightarrow (s) / a$.
Let the number of states in SLR(1), LR(1) and LALR(1) parsers for the grammar be

with LR(0): n_1, n_2 and n_3 respectively. The following relationship holds good:

$I_3 \equiv I_2$

$I_4 \equiv I_8$

$I_2 \equiv I_6$

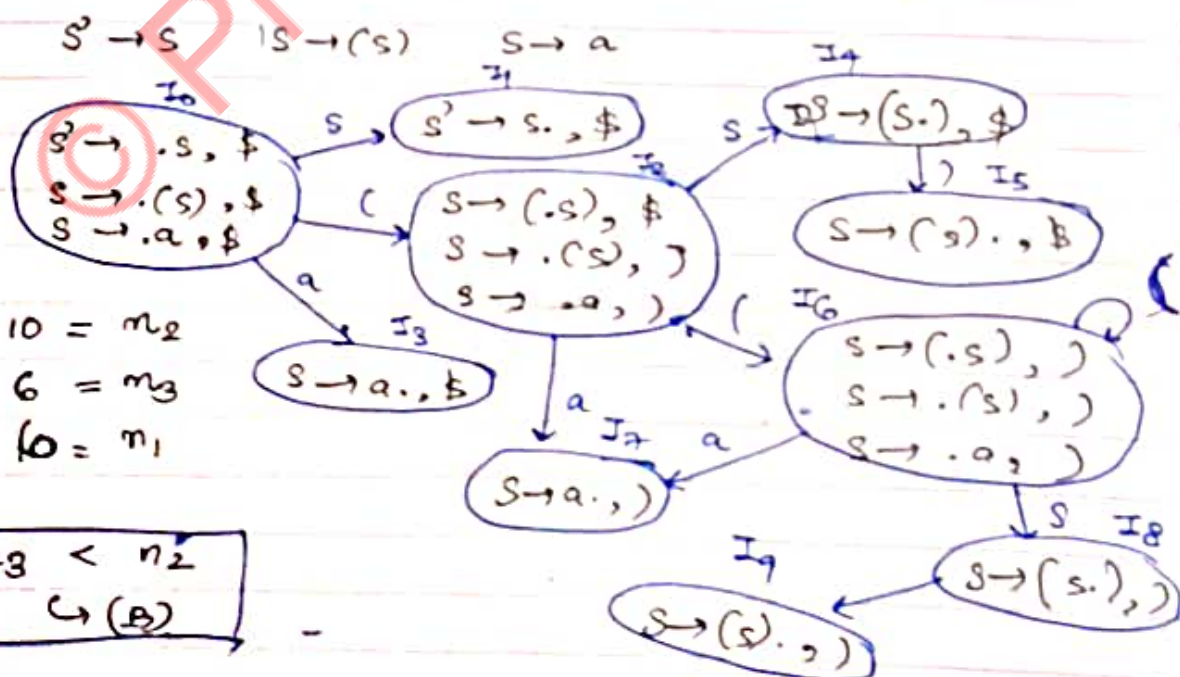
$I_5 \equiv I_9$

(A) $n_1 < n_2 < n_3$

(B) $n_1 = n_3 < n_2$

(C) $n_1 = n_2 = n_3$

(D) $n_1 > n_2 > n_3$



SLR(1) $\rightarrow 10 = n_2$

LALR(1) $\rightarrow 6 = n_3$

SLR(1) $\rightarrow 10 = n_1$

$$n_1 = n_3 < n_2$$

$\hookrightarrow (B)$

summary: 4 solved problems on CFGs

CFGs - solved problems - set 2

outcome: 4 solved problems on CFGs.

Q1:

consider the grammar shown below:

$$S \rightarrow cC$$

$$C \rightarrow cC \mid d$$

(A) $LL(1)$ (B) $SLR(1)$, not $LL(1)$
(C) $LALR(1)$, not $SLR(1)$ (D) $LR(1)$, not $LALR(1)$

FIRST

$\{c\}$

$$S \rightarrow cC$$

$\{c, d\}$

$$C \rightarrow cC \mid d$$

FOLLOW

$\{\$ \}$

$\{c, d, \$ \}$

	c	d	\$
S	$S \rightarrow cC$	$S \rightarrow cC$	
C	$C \rightarrow cC$	$C \rightarrow d$	

① $LL(1)$ grammar

Q2:

(GATE 2006)

consider the following grammar

$$S \rightarrow S * E \mid E$$

$$E \rightarrow F + E \mid F$$

$$F \rightarrow id$$

consider the following $LR(0)$ items corresponding to the grammar:

(i) $S \rightarrow S * \cdot E$

(ii) $E \rightarrow F \cdot + E$

(iii) $E \rightarrow F + \cdot E$

which two items will appear in the same state of the canonical sets of items for the grammar?

(A) (i) and (ii)

(B) (ii) and (iii)

(C) (i) and (iii)

(D) none of the above

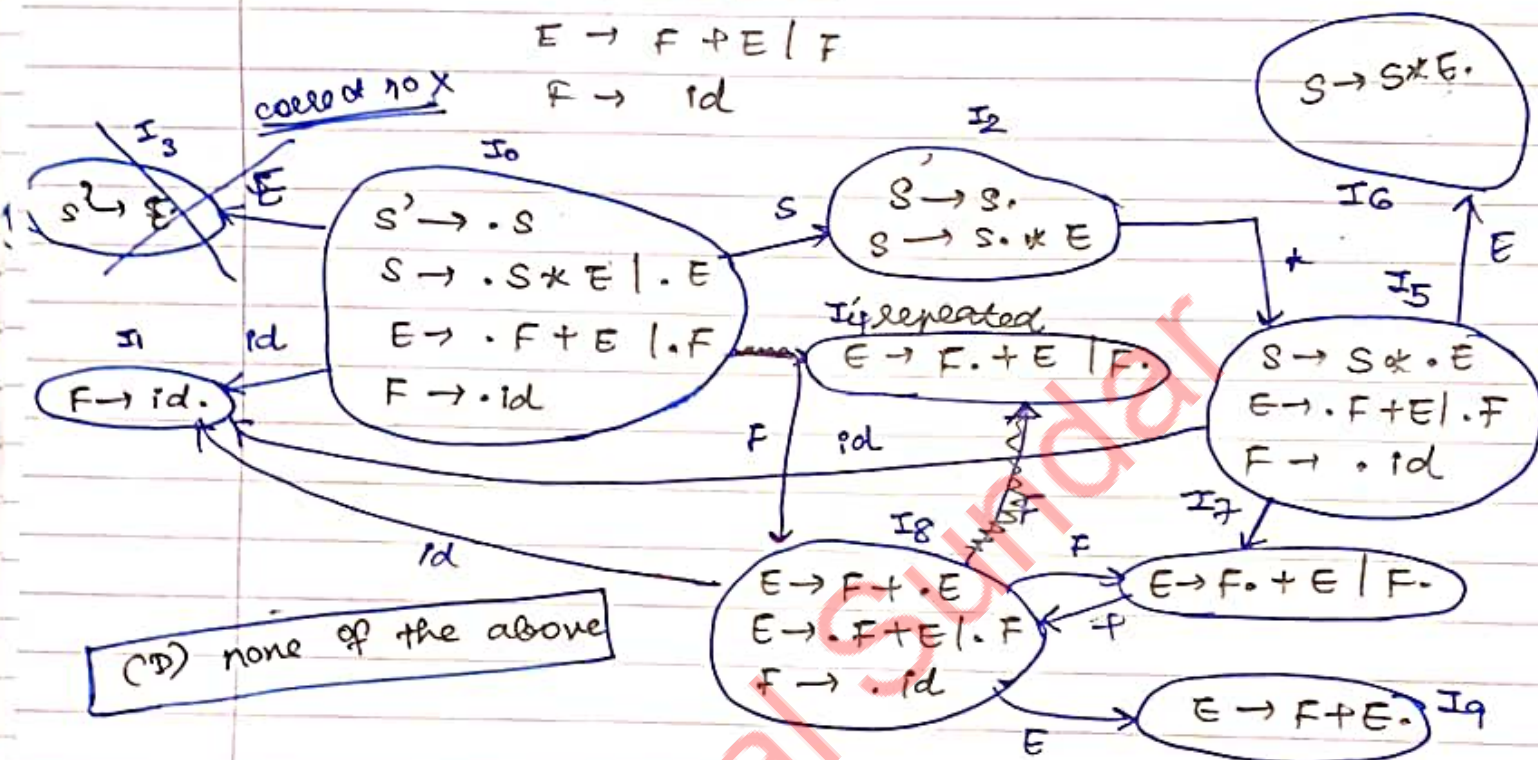
Augmented grammar :

$$S' \rightarrow S$$

$$S \rightarrow S * E \mid E$$

$$E \rightarrow F + E \mid F$$

$$F \rightarrow id$$



Q3: consider the grammar

$$E \rightarrow E + n \mid E * n \mid n$$

For a sentence $n + n * n$, the handles in the right-sentential form of reduction are :

- (A) $n, E + n, E + n * n$
- (B) $n, E + n, E + E * n$
- (C) $n, n + n, n + n * n$
- (D) $n, E + n, E * n$

right-sentential form

$$\begin{array}{c} E - E - E \\ | \quad | \quad | \\ n + n * n \end{array}$$

Q4: consider the grammar given below :

$$S \rightarrow SS \mid a \mid E$$

the no. of inadequate states in the DFA with LR(0) items are :

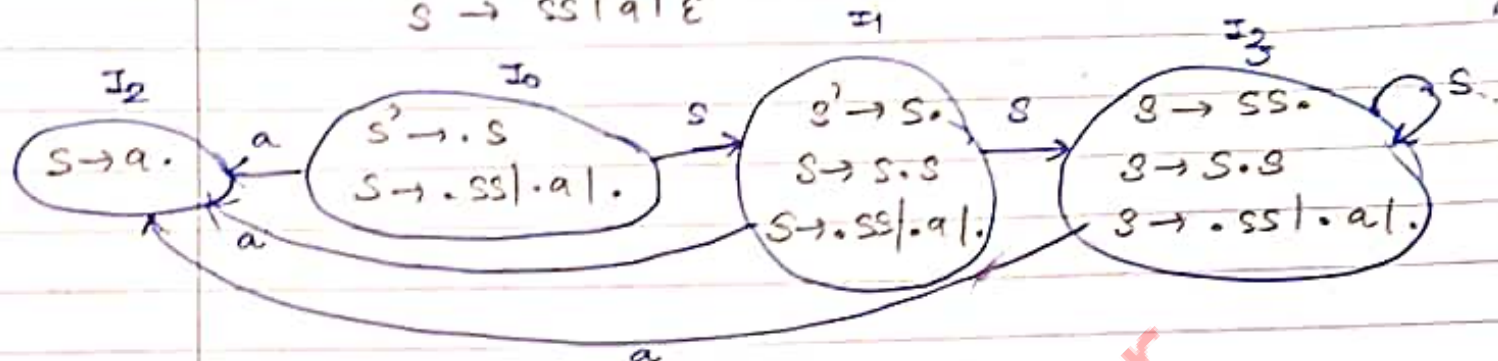
- (A) 1
- (B) 2
- (C) 3
- (D) 4

LR(0) items \rightarrow

Augmented grammar :

$$S' \rightarrow S$$

$$S \rightarrow SS | a | \epsilon$$



$S \rightarrow \cdot$ \xrightarrow{a} in I_0, I_1, I_3 (3) S-R conflict

also $S \rightarrow SS \cdot$ in I_3 (1) R-R conflict

No. of inadequate states = $(3) - 4$

Usefulness of Ambiguous Grammars

outcome : Advantages of using Ambiguous Grammars.

Ambiguous Grammar vs Unambiguous Grammar :

(X) $E \rightarrow E + E | E * E | id$

(associativity, precedence not specified)

(Y)
$$\left. \begin{aligned} G &\rightarrow E + T | T \\ T &\rightarrow T * F | F \\ F &\rightarrow id \end{aligned} \right\} \text{(unambiguous version of the above)}$$

For X: (✓) productions are less. } flexible
(✓) simple to understand }

For Y: (✓) associativity and precedence
is fixed \rightarrow not flexible

— / — / —

$$Y: E \rightarrow T \rightarrow F \rightarrow Yd$$

③

(steps of deduction)

however ↘

- x: causes numerous conflicts while passing

These can be resolved by taking meaningful decisions.

$Pd + Pd + Pd$

$$\begin{array}{c} E \\ \swarrow \quad \searrow \\ E \quad E \\ \uparrow \quad \uparrow \\ id + id + id \end{array}$$
$$E \rightarrow E + E$$

S/R conflict

choose \downarrow R due to left associativity

$$id + id \neq id$$
$$\tau \rightarrow \tau \cdot * \tau$$

S/R conflict

choose s due to higher multiplication precedence

A → Another

c → compiler

C → compiler

A blank coordinate plane with x and y axes. The x-axis is horizontal and the y-axis is vertical, intersecting at the origin. There are no tick marks or labels on the axes.

parser for

all grammar ambiguous & unambiguous

$$\rightarrow \textcircled{S} / R$$

known shift

favoured
first reduce

favoured
first reduce

- Conflicts can be resolved by taking meaningful decisions)

Summary :

Advantages of using Ambiguous Grammar