CSPE74 - Image Processing and Applications

14/7/25

PPT - unit I

15/7/25

PPT - digital image, color, model

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Basic Relationship b/w pixels

image element
picture element
pel

Adjacency of pixels

Two pixels
1) Neighbours
2) intensities value
should be same

1. 4 - Adjacency

$V = \{1\}$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}$$

2. 8 - Adjacency

$$\begin{matrix} 54 & 10 & 100 & 8 \\ 81 & 150 & 2 & 34 \\ 201 & 200 & 3 & 45 \\ 7 & 70 & 147 & 55 \end{matrix}$$

$V = \{1, 2, 3 \ldots\ldots 10\}$

3. M-adjacency (mixed)

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$V = \{1\}$

Connectivities

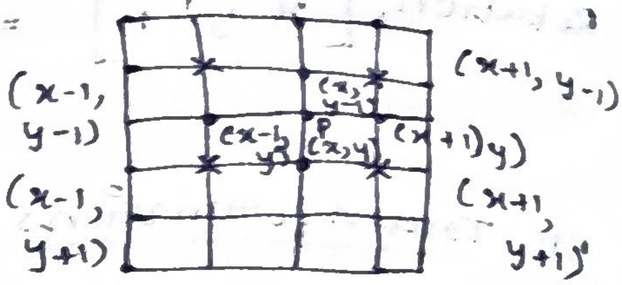
$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Region - subsets of pixels

Boundary

Neighbours of pixel

1) 4 - Neighbours ($N_4(P)$)
2) Diagonal neighbours ($N_D(P)$)
3) 8 - neighbours ($N_8(P)$)



$N_8(P) = N_4(P) + N_D(P)$

Region

$V = \{1\}$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{matrix}$$

Operations on pixels

1)

$$\underset{A}{\begin{bmatrix} 0 & 100 & 10 \\ 4 & 0 & 10 \\ 8 & 0 & 0 \end{bmatrix}} + \underset{B}{\begin{bmatrix} 10 & 100 & 5 \\ 2 & 0 & 0 \\ 0 & 10 & 10 \end{bmatrix}} = \begin{bmatrix} 10 & 200 & 15 \\ 6 & 0 & 10 \\ 8 & 10 & 10 \end{bmatrix}$$

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Homogeneous Coordinates

$P' = M_1 \cdot P + M_2$

Translation, $[x' \; y' \; 1] = [x \; y \; 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$

Scaling, $[x' \; y' \; 1] = [x \; y \; 1] \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotation, $[x' \; y' \; 1] = [x \; y \; 1] \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3D Transformations – check numerical problems

1) Translation

$x' = x + t_y$

$y' = y + t_y$

$z' = z + t_z$

2) Scaling

$x' = x \cdot s_x$

$y' = y \, s_y$

$z' = z s_y$

3) Rotation around z-axis

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

Fourier Transforms (continuous)

$f(x)$ – continous function of x.

$$\left.\begin{aligned} F(u) &= \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} \, dx \\ f(x) &= \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} \, du \end{aligned}\right\} 1D$$

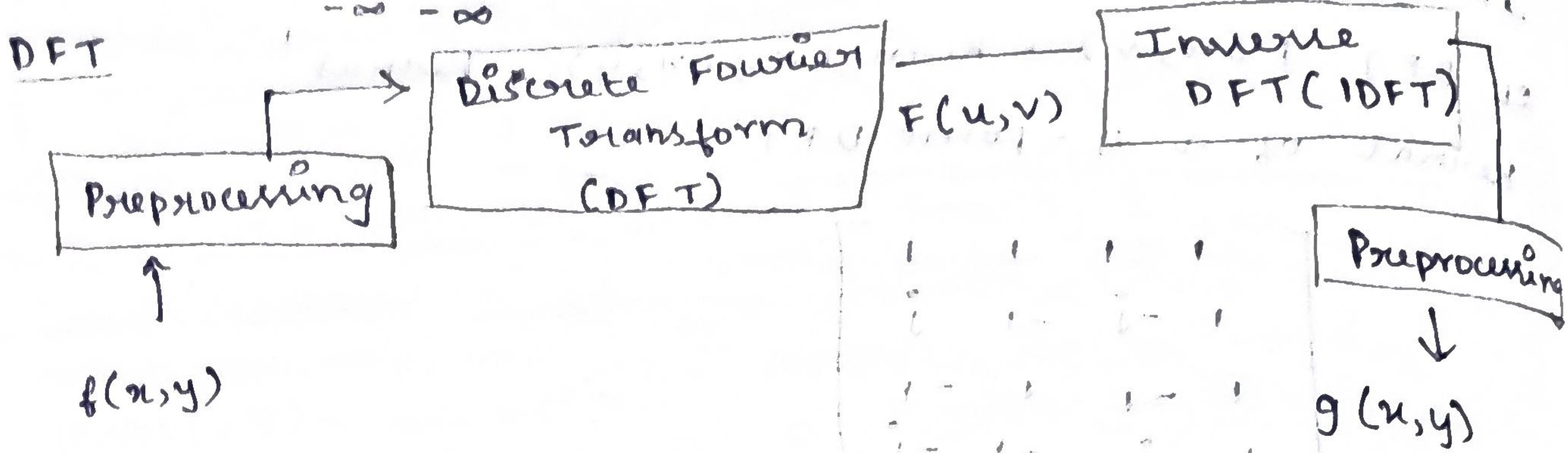
2D

$f(x, y)$

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux + vy)} \, dx dy$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\, dv$$

DFT



1D

$$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)\, e^{\frac{-j2\pi x}{N}}; \quad x = 0 \ldots N-1$$

$$f(x) = \sum_{u=0}^{N-1} F(u)\, e^{\frac{j2\pi x}{N}} \quad x = 0 \ldots N-1$$

2D

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$\left[\begin{array}{l} u = 0 \ldots M-1 \\ v = 0 \ldots N-1 \end{array}\right]$$

$$f(x,y) = \frac{1}{MN}\sum^{M-1}\sum^{N-1} F(u,v)\, e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$e^{j\theta} = \cos\theta + j\sin\theta$ | $e^{j\pi} = -1$ | $e^{-j\pi/2} = -j$

$e^{-j\theta} = \cos\theta - j\sin\theta$ | $e^{-j\pi} = -1$ | $e^{-j2\pi} = e^{j2\pi} = -1$

$e^{j\pi/2} = j$

1. compute a DFT of sequence, $f(x) = \{1,0,0,1\}$

$$F(u) \text{ [redacted] } = \sum^{N-1} f(x)\, e^{\frac{-j2\pi x}{N}} \quad x = 0 \ldots 3$$

$= \frac{1}{4}(1-j)$

$F(0) =$ [redacted] $(1+1) =$ [redacted] 2

$F(1) =$ [redacted] $(1+j)$

$F(2) = 0$

$F(3) = (1-j)$

1D DFT $F(u) = \text{Kernal} \times f(x)$

2D DFT $F(u, v) = \text{Kernal} \times f(x, y) \times \text{Kernal}^T$

Kernal of a 4-point DFT

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

1) Calculate the 4-point DFT sequence for $f(x) = \{0, 1, 2, 3\}$ using matrix method.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

2) Compute the 2D DFT of the grayscale image given,

$$f(x, y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Discrete Cosine transformer (DCT)

1D DCT - Signal $\times$ mask

$$C(u) = \sqrt{\frac{2}{N}}\, w(u) \sum_{x=0}^{N-1} f(x) \cos\left[\frac{2x+1}{2N} u\pi\right]$$

$$w(u) = \begin{cases} 1/\sqrt{2} & \text{if } u = 0 \\ 1 & \text{if } u \neq 0 \end{cases}$$

$$f(x) = \sqrt{\frac{2}{N}} \sum_{x=0}^{N-1} w(u) \cdot F(u) \cos\left[\frac{2x+1}{2N} u\pi\right]$$

2D DCT - mask × image × mask Transform

$$C(u,v) = \frac{1}{\sqrt{M}} \cdot \frac{1}{\sqrt{2}} \, w(u) \cdot w(v) \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \cdot \cos\left[\frac{2x+1}{2N} \cdot x\pi\right] \cdot \cos\left[\frac{2y+1}{2N} \cdot v\pi\right]$$

$$f(x,y) = \frac{1}{\sqrt{u}} \cdot \frac{1}{\sqrt{N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u,v)\, w(u)\, w(v) \cos\left[\frac{2x+1}{2N} \cdot x\pi\right] \cdot \cos\left[\frac{2y+1}{2N} \cdot v\pi\right]$$

4-point DCT kernel

$$\begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix}$$

3) Find 2D DFT of $f(x,y) = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix}$

$F(u,v)$ = Kernal × $f(x,y)$ × Kernal ^{T}

$$= \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0.65 & 0.5 & 0.27 \\ 0.5 & 0.27 & -0.5 & -0.65 \\ 0.5 & -0.27 & -0.5 & 0.65 \\ 0.5 & -0.65 & 0.5 & -0.27 \end{bmatrix}$$

$$= \Bigg[$$

2u
1D
2D
Ker

1) C
w

Discrete Sine Transform (DST)

1D

$$s(u) = \sqrt{\frac{2}{N+1}} \sum_{x=0}^{N-1} f(x) \sin\left[\frac{\pi (x+1)(u+1)}{N+1}\right]$$

$$f(x) = \sqrt{\frac{2}{N+1}} \sum_{x=0}^{N-1} s(u) \sin\left[\frac{\pi (x+1)(u+1)}{N+1}\right]$$

2D

$$s(u) = \sqrt{\frac{2}{M+1}} \sqrt{\frac{2}{N+1}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \sin\left[\frac{\pi (x+1)(u+1)}{M+1}\right] \cdot \sin\left[\frac{\pi (y+1)(v+1)}{N+1}\right]$$

$$f(x,y) = \sqrt{\frac{2}{M+1}} \sqrt{\frac{2}{N+1}} \sum_{x=0}^{M-1} \sum_{v=0}^{N-1} s(u,v) \sin\left[\frac{\pi (x+1)(u+1)}{M+1}\right] \sin\left[\frac{\pi (y+1)(v+1)}{N+1}\right]$$

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Hadamard Transform

$H(N)$ $1 \quad -1$

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad H_4 = \begin{bmatrix} 1\,[H_2] & 1\,[H_2] \\ 1\,[H_2] & -1\,[H_2] \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$f(x) = \{\ldots\}$

$H \cdot f(x)$

$F(u,v) = H \cdot f(x,y) \cdot H$

1) compute the hadamard transform for $f(x) = \{1, 2, 0, 1\}$

Ans $\quad 4 \quad -2 \quad 2 \quad 0$

2) compute the hadamard transform for image

$$f(x,x) = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 34 & 2 & -6 & -6 \\ 2 & 2 & 2 & 2 \\ -6 & 2 & 2 & 2 \\ -6 & 2 & 2 & 2 \end{bmatrix}$$

Haar Transform

$T = H \cdot F \cdot H \qquad H \cdot F \qquad H \cdot F \cdot H^T$

$$H_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad H_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

Discrete

1D $g(u)$

$f(x$

2D $s($

$f($

1) $f(m,n) = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}_{2\times 2}$

Ans:

$F(u,v) = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix}$

Slant Transform

$S_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$S_4 = \begin{bmatrix} S_2 & 0 \\ 0 & S_2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 & 0 \\ a & b & -a & b \\ 0 & 1 & 0 & -1 \\ -b & a & b & a \end{bmatrix}$

$a_n = \left[\frac{3N^2}{4(N^2-1)} \right]^{1/2}$ $\quad b_n = \left[\frac{N^2-4}{4(N^2-1)} \right]^{1/2}$

$n = \log N$

$$S_n = \frac{1}{\sqrt{2}} \left[\begin{array}{cc|c|cc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ a_n & b_n & & -a_n & b_n & \\ \hline 0 & & I_{N/2-2} & 0 & & I_{N/2-2} \\ \hline 0 & 1 & 0 & 0 & -1 & 0 \\ -b_n & a_n & & b_n & a_n & \\ \hline 0 & & I_{N/2-2} & 0 & & -I_{N/2-2} \end{array}\right] \times \begin{bmatrix} S_{n-1} & 0 \\ 0 & S_{n-1} \end{bmatrix}$$

1) Find the slant Transform of $f(m,n) = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

2) $f(m,n) = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

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Image enhancement

1) Spatial domain —— [① Intensity transformation. ② spatial filtering
2) Frequency domain
3) combination of both

$$g(x, y) = T\{f(x, y)\}$$

Gray level transformation

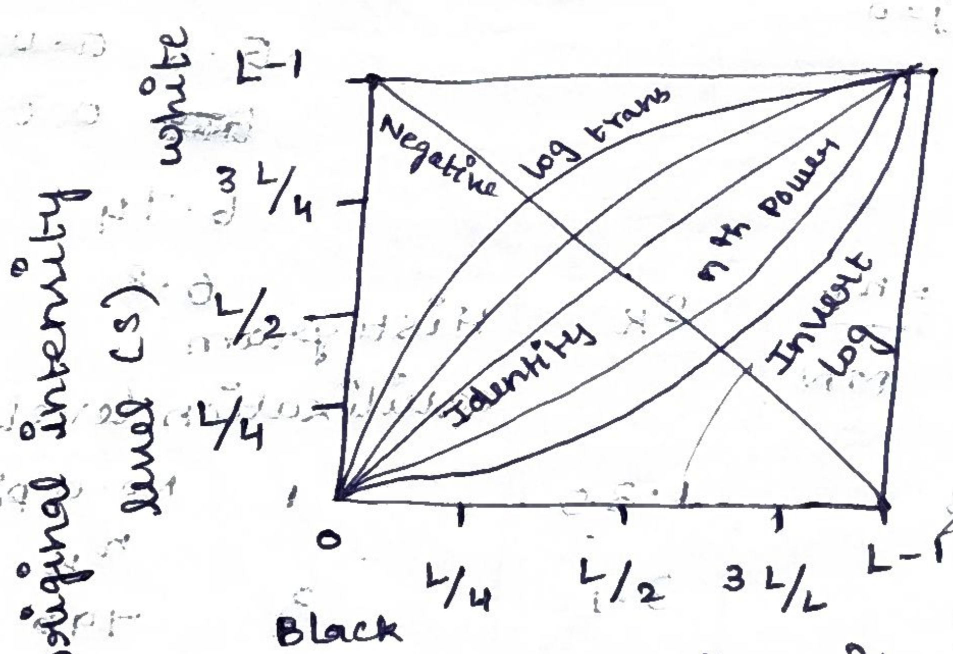
$$S = T(r)$$

Basic transformation function

1) Linear [Identity of negative].

2) Logarithm [log and inverse log],

3) Power law [n^{th} power & n^{th} root]



log

$$S = c \log(r + 1)$$

c is a contant

$r \geq 0$

↓

input image

negative

$$S = (L - 1) - r$$

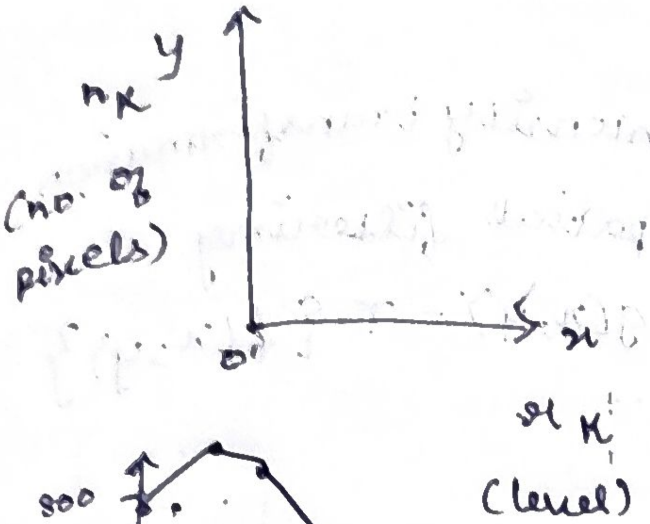
Power

$S = c \cdot r^{\gamma}$, $\gamma > 1$ n^{th} power

$\gamma < 1$ n^{th} root

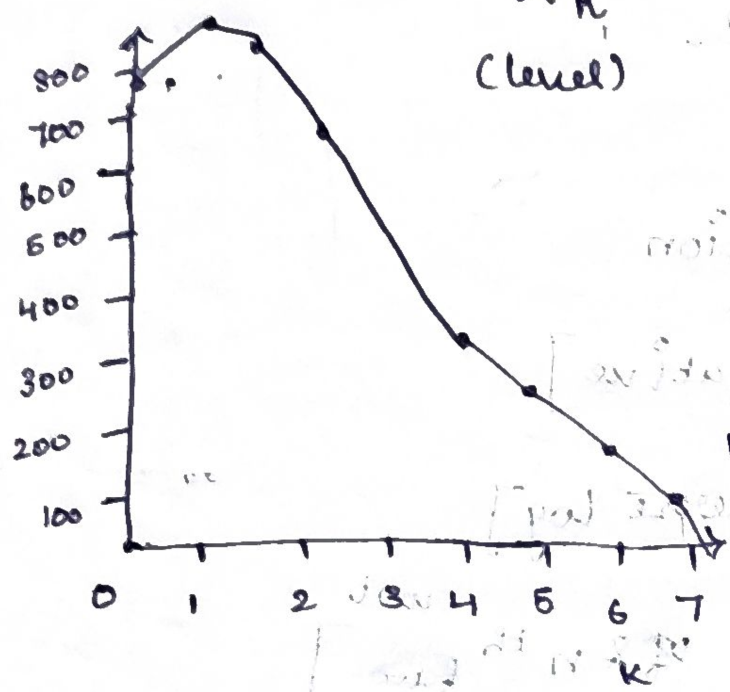
Graphical image enhancement

Histogram processing



- **Histogram equalization**

r_K	n_k	$Pr(r_k) = \frac{n_k}{MN}$
0	790	0.193
1	1023	0.25
2	850	0.2
3	656	0.16
4	329	0.08
5	245	0.06
6	122	0.02
7	81	0.01



MN = 4096

S_K [redacted]
1.35 [redacted]
3.1 [redacted]
[redacted]
4.5
5.67 [redacted]
6.2 [redacted]
6.6 [redacted]
[redacted]
6.74
6.8

$$S_K = (L-I) * \sum_{j=0}^{k} Pr(r_j)$$

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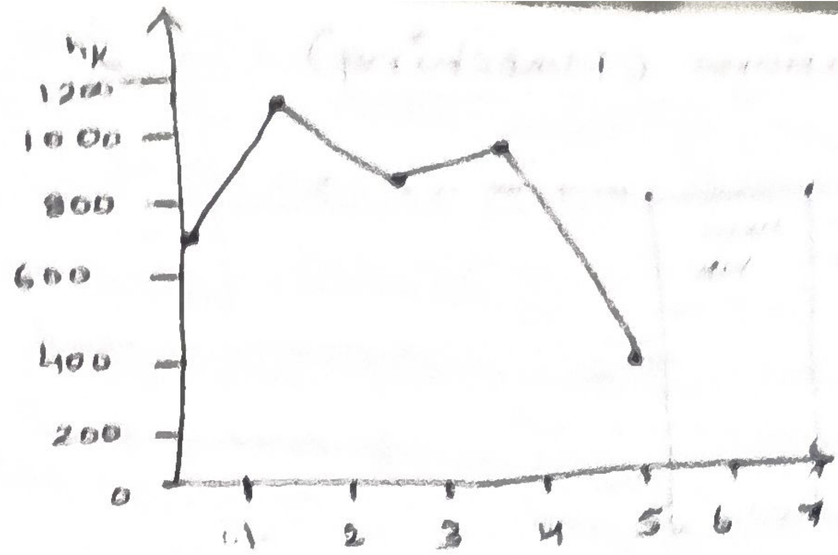
r_K	n_K	$Pr(r_K) = \frac{n_K}{MN}$	S_K	Histogram equilization level	No. of pixel n_K
0	790	0.193	1.35	1	790
1	1023	0.25	3.1	3	1023
2	850	0.2	4.5	5	850
3	656	0.16	5.67	6	985
4	329	0.08	6.2	6	
5	245	0.05	6.6	7	448
6	122	0.02	6.74	7	
7	81	0.01	6.8	7	



1) Perform the histrogram equalization for the image

$$f(x,y) = \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 2 & 5 & 3 & 5 & 2 \\ 2 & 5 & 5 & 5 & 2 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$$

r_K	n_K	P_r
0	0	
1	8	
2	6	
3	1	
4	0	
5	5	
6	0	
7	0	

Histogram specifications (Matching)

(ii)

rk	new rk

new rk	new nk

(1)

rk	0	1	2	3	4	5	6	7
nk	80	100	90	60	30	20	10	0

(2)

rk	0	1	2	3	4	5	6	7
nk	0	0	0	60	80	100	80	70

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ii

Gray level	Histogram equalication val
0	0
1	0
2	0
3	1
4	2
5	4
6	6
7	7

i

Histrogram equalication val	new gray level
1	80
3	100
5	90
6	90
6	
7	30
7	

Gray level	0	1	2	3	4	5	6	7
no. of pixels	0	0	0	80	80	100	90	30

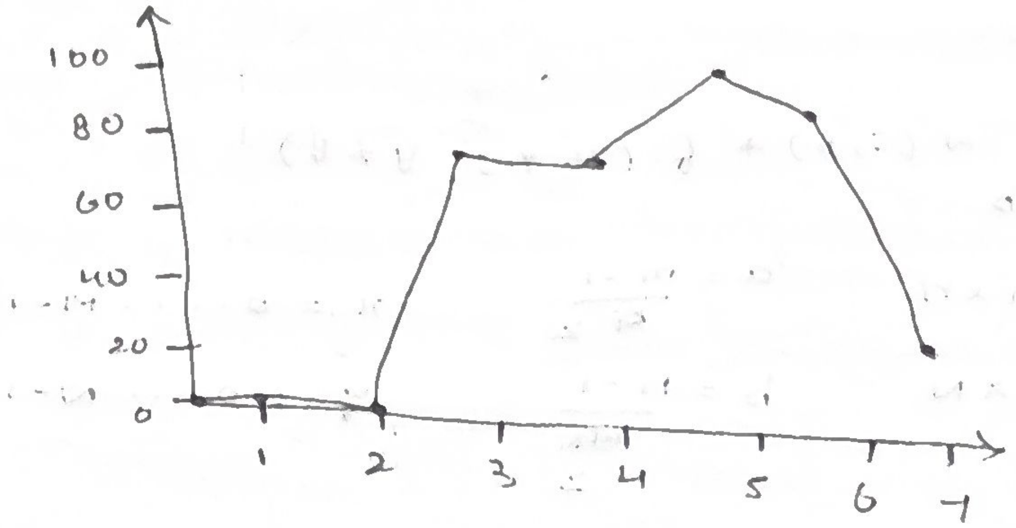


Image Subtraction & Image averaging

$$g(x,y) = h(x,y) - f_2(x,y)$$

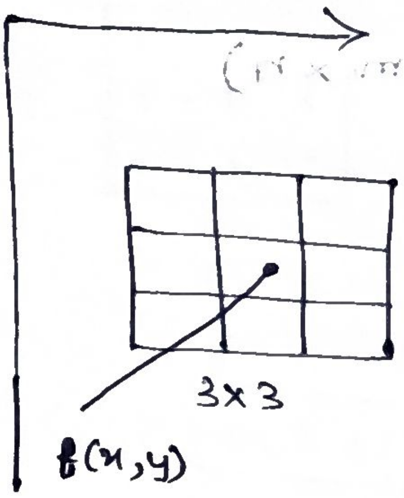
1) Levelling the unknown part of the image
2) Detecting changes b/w two images.
3) Separating the ROI from background.

Averging-
1) Remove the noise
2) Avoid Blurring

Spatial filtering

1) low pass filter
2) high pass filter

Image processing
- Spatial domain
  - Interests transformation
  - Spatial filtering
- Frequency domain
  - filter, mask, window kernel, template.



-1, +1	(-1,0)	(-1,1)
(0,-1)	w(0,0)	w(0,1)
(1,-1)	w(1,0)	w(1,1)

3 x 3 mask

x-1, y-1	x-1, y	x-1, y+1
x, y-1	x, y	x, y+1
x+1, y-1	x+1, y	x+1, y+1

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$$g(x,y) = \sum_{s=a}^{a} \sum_{t=b}^{b} w(s,t) * f(x+s, y+t)$$

size of mask : $m \times n$ $\quad a = \frac{m-1}{2}$ $\quad x = 0 \ldots M-1$

size of image = $M \times N$ $\quad b = \frac{n-1}{2}$ $\quad y = 0 \ldots N-1$

correlation & convolution

f(x) mask

1D

0 0 | 0 0 0 1 0 0 0 0 | 0 0 0 0 $\quad$ 1 2 3 2 8

1 2 3 2 8

Full correlation result

$\boxed{0\ 0\ 0\ 8\ 2\ 3\ 2\ 1\ 0\ 0\ 0\ 0}$

Cropped correlation result

$\boxed{0\ 8\ 2\ 3\ 2\ 1\ 0\ 0}$

mask

1 2 3 2 8

8 2 3 2 1

convolution result -

0 8 2 3 2 1 0 0

0 1 2 3 2 8 0 0

2D

Image $M \times N$ $\quad$ mask $m \times n$

m - 1 top & bottom

n - 1 left & right

f(x, y) $\quad$ w(x, y) (m × n)

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

1	2	3
4	5	6
7	8	9

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

convolution result -

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

mass for convolution,
$$\boxed{\begin{matrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{matrix}}$$

Types of spatial ~~domain~~ filter

- spatial filter
  - Smoothening spatial filter
    - Linear filter
      - mean/box filter
      - weighted ~~Sampled~~ average filter
    - non linear filter (order states filter)
      - median filter
      - max filter
      - min filter
  - Sharpening spatial filter.

1) Mean filter

$$\frac{1}{9}\boxed{\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}} \qquad \cancel{f(x,y)} \qquad \boxed{\begin{matrix} z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 \\ z_7 & z_8 & z_9 \end{matrix}}$$

1	2	3
4	5	6
7	8	9

$$R = \frac{1}{9}\sum_{i=1}^{9} w_i z_i$$

ii) weighted average filter

$$\frac{1}{16}\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix} \quad \begin{bmatrix}z_1 & z_2 & z_3\\ z_4 & z_5 & z_6\\ z_7 & z_8 & z_9\end{bmatrix} \quad R = \frac{1}{16}\sum_{i=1}^{9} w_i z_i$$

median filter

{10, 15, 20, 20, 15, 20, 20, 25, 100}

{10, 15, 15, 20, 20, 20, 20, 25, 100}

median = 20

max filter

$Res = \max \{Z_K,\ k = 0, 1, \ldots M\}$

min filter

$Res = \min \{Z_K,\ K = 0 \ldots M\}$

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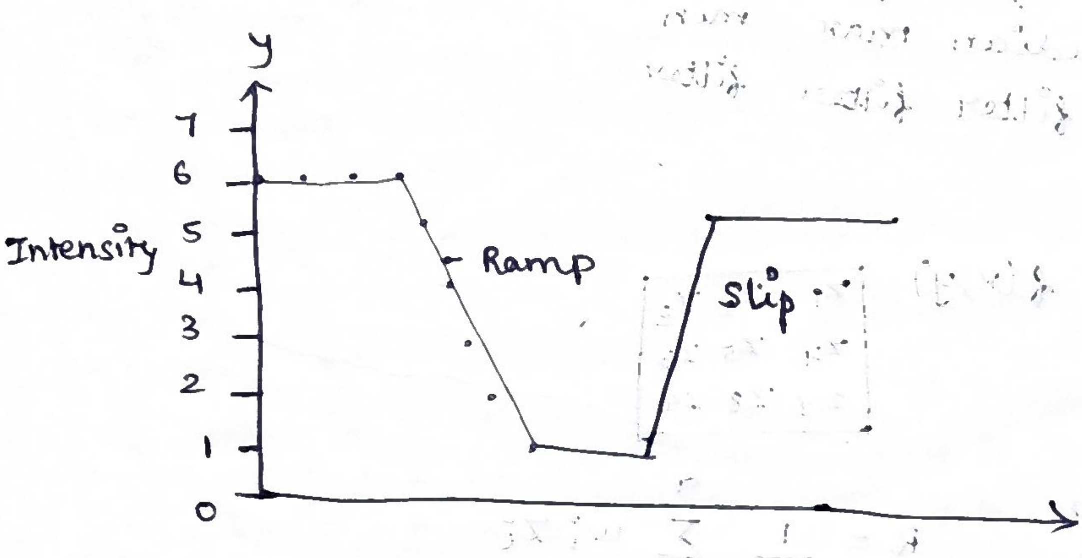
spatial sharpening filter

- highlights details
- enhance the blurring
- enhance the edge

Applications - Medical imaging, electronic printing, autonomous guidance in military.

spatial differentiation



Scan line

6	6	6	6	5	4	3	2	1	1	1	6	6	6	6	6	

First order derivative

0 0 0 -1 -1 -1 -1 -1 0 0 0 5 0 0 0 0 0

Second order derivative

- 0 0 -1 0 0 0 0 0 1 0 0 5 -5 0 0 0 0

1D ① First order derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x) \quad — ①$$

1) constant intensity - zero

2) onset of ramp step - non zero

3). ~~Alg~~ Along ramp - non zero

② Second order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x) \quad — ②$$

1) constant intensity - zero

2) ~~co~~ onset / end of ramp and stop - non-zero

3) Along the ramp - zero

2D

$f(x,y)$

Laplacian filter

Isotopic image - rotation invariant

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad — ①$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x,y) \quad — ②$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x,y) \quad — ③$$

$$\nabla f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$f(x-1, y-1)$	$f(x-1, y)$	$f(x-1, y+1)$
$f(x, y-1)$	$f(x, y)$	$f(x, y+1)$
$f(x+1, y-1)$	$f(x+1, y)$	$f(x+1, y+1)$

mask

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6
Z_7	Z_8	Z_9

General expression for constructing laplasion mask is

$$g(x,y) = f(x,y) + C \cdot \nabla^2 f(x,y)$$

$C \rightarrow -1 \,\&\, 1$

coefficients

	1	
1	-4	1
	1	

laplasion ~~latention~~ mask

$-4f(x,y)$

1) Apply the laplasion filter for the given image on central pixel of image.

8	5	4
0	6	2
1	3	4

mask

0	1	0
1	-4	1
0	1	0

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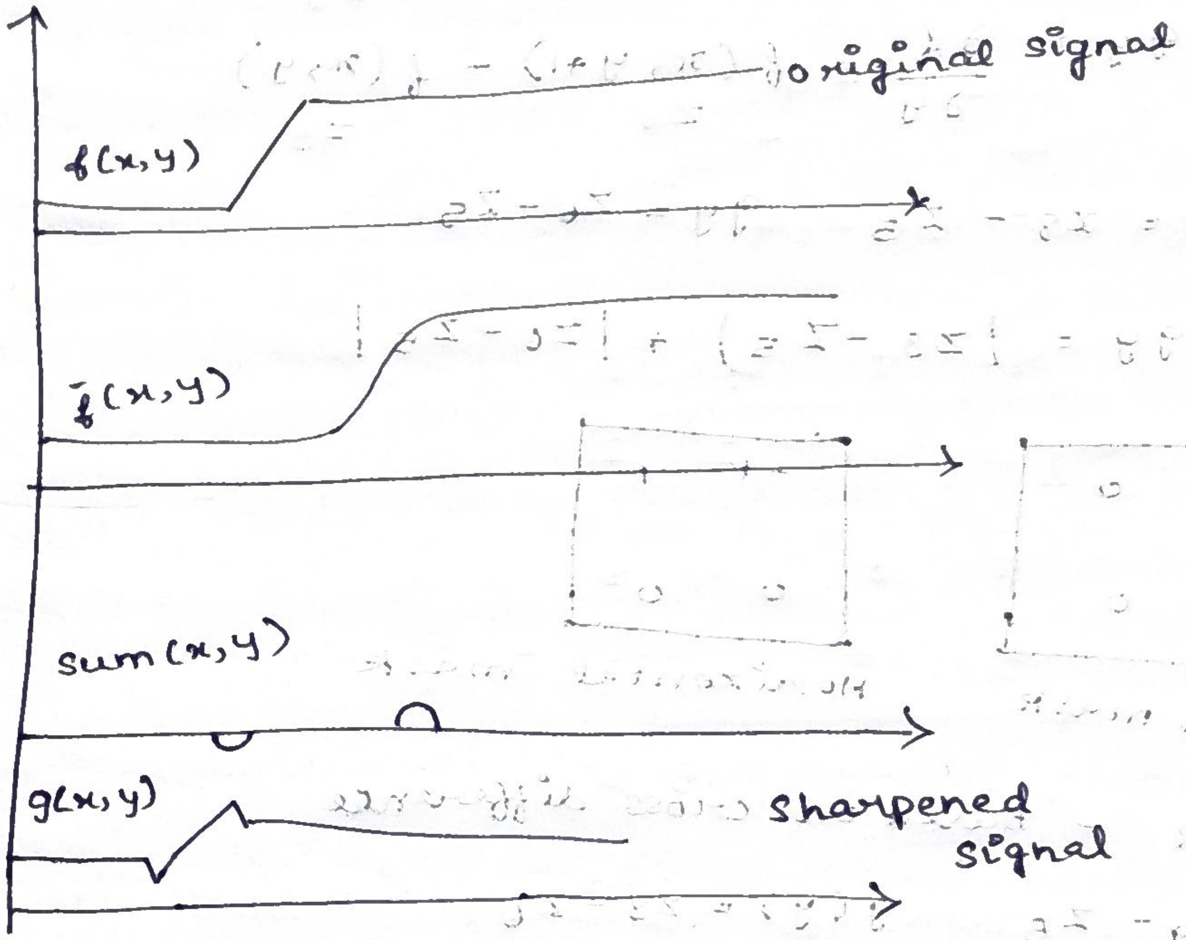
2) unsharp marking and high boost filtering



$f(x,y) = f(x,y) + \bar{f}(x,y)$

$g(x,y) = f(x,y) + K + g\,mask(x,y) \quad K \geq 0$

$K = 1$, unsharp masking $\quad K > 1$, High boost filtering

4) using first order derivative (max linear) the gradiant

$$\nabla f = grad[f(x,y)] = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \quad - (1)$$

$$\text{Magnitude}, M(x,y) = Mag[\Delta f] = \sqrt{g_x^2 + g_y^2}$$

$$M(x,y) \cong |g_x| + |g_y| \quad - (2)$$

$$f(x+1) - f(x)$$

$$g(x) = \frac{\partial f}{\partial x} = \underbrace{f(x+1, y)}_{z_8} - \underbrace{f(x, y)}_{z_5}$$

$$g_y = \frac{\partial f}{\partial y} = \underbrace{f(x, y+1)}_{z_6} - \underbrace{f(x, y)}_{z_5}$$

$$g_x = z_8 - z_5 \qquad g_y = z_6 - z_5$$

$$g_x + g_y = |z_8 - z_5| + |z_6 - z_5|$$

$$\boxed{\begin{matrix} -1 & 0 \\ 1 & 0 \end{matrix}} \qquad \boxed{\begin{matrix} -1 & 1 \\ 0 & 0 \end{matrix}}$$

vertical mask Horizontal mask

Robult's mask - Based on cross difference

$$g(x) = z_9 - z_5 \qquad g(y) = z_8 - z_6$$

$$M(x, y) = |z_9 - z_5| + |z_8 - z_6|$$

$$\boxed{\begin{matrix} -1 & 0 \\ 0 & 1 \end{matrix}} \qquad \boxed{\begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix}}$$

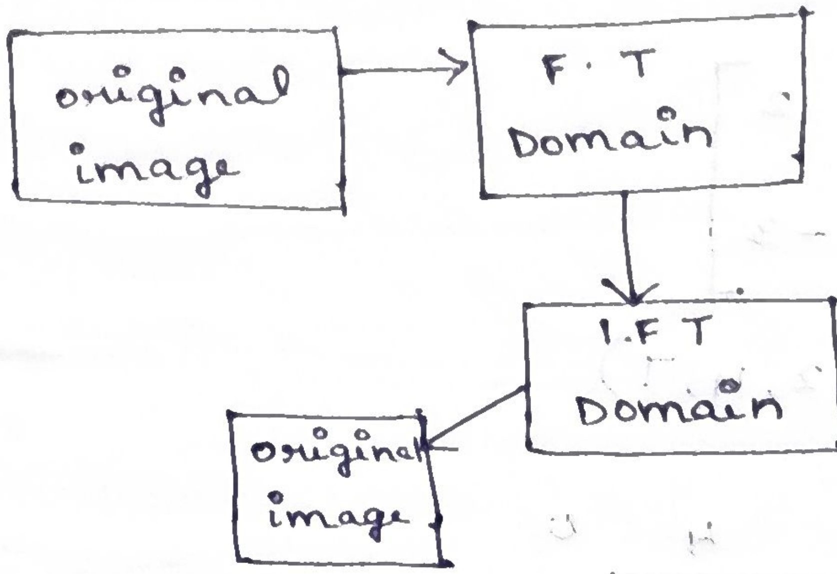
Sobal's operation

$$g(x) = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$g_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

$$\boxed{\begin{matrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{matrix}} \qquad \boxed{\begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}}$$

Filtering in frequency domain



Steps for filtering in frequency domain:

① Given an image $f(x,y)$ of size $M \times N$, obtain the padding parameters, P & Q ($P = 2M$ & $Q = 2N$)

② For getting padding $f_p(x,y)$ of size $P \times Q$ append Zero's

③ Multiply $f_p(x,y)$ by $(-1)^{x+y}$ to shift the transfor to its center i.e ($u = M/2$, $v = N/2$)

④ Compute DFT and obtain $F(u,v)$

⑤ Create a symmetric filter function $H(u,v)$ of size $P \times Q$ with center at $(P/2, Q/2)$ $|z_8 - z_5| + |z_6 - z_5|$

$$G(u,v) = H(u,v) \cdot F(u,v)$$

⑥ obtain the processed image for filtering in frequency domain - $g(x,y) = real\{\mathcal{F}^{-1}[G(u,v)]\}$

⑦ ~~obt~~ obtain the final processed $g(x,y)$ in $M \times N$ reg

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Q1) Haar ~~Haah~~ transform

$$f(x,y) = \begin{bmatrix} 4 & 2 \\ 2 & -3 \end{bmatrix}$$

Q2) DCT $f(x) = (1,2,4,7)$

Q3) (Sobel)

10	9	9	4	0
0	6	6	2	2
5	9	8	4	3
7	5	5	4	3
8	10	8	5	0

Apply formal mask for the image in both x - direction & y - direction.

1) i) Row wise transform

(4, 2)

Avg = $(4+2)/2 = 3$

dif = $(4-2)/2 = 1$

(2, -3)

Avg = $(2 + (-3))/2 = -0.5$

dif = $(2 - (-3))/2 = 2.5$

$$\begin{bmatrix} 3 & 1 \\ -0.5 & 2.5 \end{bmatrix}$$

ii) Column wise transform

(3, -0.5)

Avg = $(3 + (-0.5))/2$

$= 1.25$

diff = $(3 - (-0.5))/2$

$= 1.75$

(1, 2.5)

Avg = $(1+2.5)/2 = 1.75$

Diff = $(1-2.5)/2 = -0.75$

$$\begin{bmatrix} 1.25 & 1.75 \\ 1.75 & -0.75 \end{bmatrix}$$

2) $$f(u) = w(u) \sum_{x=0}^{N-1} f(x) \cos\left[\frac{\pi(2x+1)u}{2N}\right]$$

$$w(u) = \begin{cases} \sqrt{\frac{1}{N}}, & u = 0 \\ \sqrt{\frac{2}{N}} & u > 0 \end{cases}$$

$$= \begin{bmatrix} 7 \\ -4.44 \\ 1 \\ -0.32 \end{bmatrix}$$

3)

<u>x dir</u>

$$\begin{bmatrix} -6 & -2 & 2 \\ 4 & -1 & 4 \\ 5 & 2 & -1 \end{bmatrix}$$

<u>y dir</u>

$$\begin{bmatrix} 14 & -18 & -22 \\ 10 & -15 & -16 \\ -1 & -12 & -17 \end{bmatrix}$$