

National Institute of Technology Tiruchirappalli
Department of Mathematics
July 2023 Session - B.Tech IInd Year (CSE) - A-section
Probability and Operations Research - MAIR31 - Final Assessment

Date: 11.12.2023

Duration: 3 Hour

Max. marks: 40

Attempt all the Eleven questions.

1. The contents of boxes A, B and C are as follows: 1 yellow, 2 black and 3 red balls: 2 yellow, 1 black and 1 red balls: 4 yellow, 5 black and 3 red balls. One box is chosen at random and two balls drawn. They happen to be yellow and red. What is the probability that they came from box A. [3]
2. Find the moment generating function of the Normal distribution and hence find its mean and variance. [3]
3. The joint probability mass function of a two-dimensional discrete random variable (X, Y) is given by $f(x, y) = \frac{3x + 2y}{45}$, where x and y assume integer values 0, 1 and 2. Find the conditional probability distribution of Y given $X = 2$. [3]
4. The continuous random variable X has pdf

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the pdf of $Y = 64X^3$.

[2]

5. The joint probability density function of the continuous random variables X and Y is given by:

[4]

$$f(x, y) = \begin{cases} e^{-y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Find (i) the conditional expectation of X for given $Y = y$.

(ii) the conditional expectation of Y for given $X = x$.

6. There are three firms A, B and C sharing a market of 40 %, 40 % and 20 % respectively on March. Over a month, the following developments take place:

A retains 80 % of its customers, loses 16 % to B and 4 % to C;

B retains 84 % of its customers, loses 12 % to A and 4 % to C;

C retains 76 % of its customers, loses 18 % to A and 6 % to B.

Assuming that the market does not change.

- (a) Obtain the transition probability matrix.
- (b) What share of the market shall be held by firm C on May?
- (c) What would the long-run shares of the firms A and B at equilibrium be, if the buying habits do not change?

[5]

[P.T.O]

7. Consider the following primal linear program

$$\text{Max } z = 2x_1 + 4x_2 + 7x_3 \text{ subject to}$$

$$x_1 + 4x_2 + 5x_3 \leq 2$$

$$-x_1 + x_2 + x_3 \leq -5$$

$$x_1 \leq 0, x_2 \text{ is unrestricted in sign, } x_3 \geq 0.$$

(i) Solve the dual problem using the graphical method.

(ii) Find a feasible solution and the optimum value of the primal problem.

8. Use Big-M method to solve the Linear Programming Problem (LPP)

$$\text{Minimize } z = 2x_1 - 5x_2 \text{ subject to}$$

$$x_1 - x_2 \leq 3$$

$$2x_1 + x_2 = 4$$

$$5x_1 + x_2 \geq 7$$

$$x_1 \geq 0, x_2 \geq 0.$$

9. Use the dual simplex method to solve the following LPP

$$\text{Maximize } z = -2x_1 - x_2 \text{ subject to}$$

$$2x_1 - x_2 - x_3 \geq 3$$

$$x_1 - x_2 + x_3 \geq 2$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

10. What is a transportation problem and give the mathematical formulation of it.

11. Consider a project consisting of ten activities (A, B, ..., J) with the following precedence relations and time estimates in weeks:

Activity	Immediate Predecessor(s)	a (optimistic time)	m (most likely time)	b (pessimistic time)
A	—	1	4	7
B	—	4	7	10
C	—	6	9	12
D	A, B, C	3	12	21
E	A, B, C	2	2	8
F	A	5	8	11
G	D	1	1	7
H	D, E, F	6	7	8
I	D, E, F	5	14	17
J	G, H	3	6	15

(i) Draw the project network with 8 nodes.

(ii) Identify the critical path and the expected project completion time.