



NATIONAL INSTITUTE OF TECHNOLOGY, TIRUCHIRAPPALLI
Department Of Computer Science And Engineering

End Semester Examination

Combinatorics and Graph Theory

Marks: 50

Course Code: CSPE32

Time: 3hrs

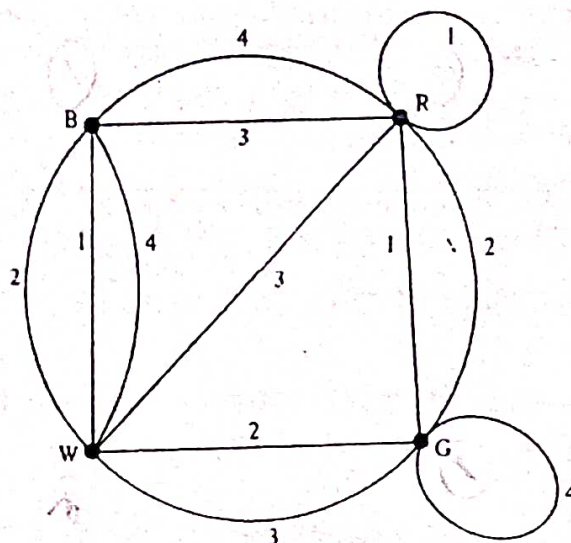
Section A: Answer any 20 marks

1. Let $P(n)$ be the statement that every set of n lines in the plane, no two of which are parallel, meet in a common point. Can you prove that $P(n)$ is true for all positive integers $n \geq 2$ by mathematical induction? If yes, show the proof. If no, show the incorrectness in the proof. [3]
2. Proof that if n is an integer greater than 1, then n can be written as the product of primes. [4]
3. In the Internet, each network connection of a computer is assigned an Internet address. In Internet Protocol (IPv4), an address is a string of 32 bits. It begins with a network number (netid) followed by a host number (hostid), which identifies a computer as a member of a particular network. Three forms of addresses are used, with different numbers of bits used for netids and hostids. Class A addresses, used for the largest networks, consist of 0, followed by a 7-bit netid and a 24-bit hostid. Class B addresses, used for medium-sized networks, consist of 10, followed by a 14-bit netid and a 16-bit hostid. Class C addresses, used for the smallest networks, consist of 110, followed by a 21-bit netid and an 8-bit hostid. There are several restrictions on addresses because of special uses: 1111111 is not available as the netid of a Class A network, and the hostids consisting of all 0s and all 1s are not available for use in any network. A computer on the Internet has either a Class A, a Class B, or a Class C address. How many different IPv4 addresses are available for computers on the Internet? [4]
4. What is the least number of area codes needed to guarantee that the 5 crores phones in a state can be assigned distinct 10-digit telephone numbers? (Assume that telephone numbers are of the form NXX-NXX-XXXX, where the first three digits form the area code, N represents a digit from 2 to 9 inclusive, and X represents any digit.) [4]
5. In a cashbox, there are notes of Rupees 5, 10, 20, 50, 100 and 500. How many ways are there to select five notes from this cashbox? Assume that the order in which the notes are chosen does not matter, the notes of each denomination are distinguishable, and that there are at least five notes of each type. [4]
6. Find an explicit formula for the Fibonacci numbers (Hint: solve recurrence). [5]



NATIONAL INSTITUTE OF TECHNOLOGY, TIRUCHIRAPPALLI
Department Of Computer Science And Engineering

Section B: Answer all questions



7. Find the shortest path from B to G by Dijkstra's algorithm. Show all steps. [3]
8. Find a Minimum Spanning Tree by Prim's algorithm (show steps). Find all fundamental cut-sets and fundamental circuits determined by this spanning tree. [3+3]
9. Define maximal independent set and ~~maximal~~ ^{minimal} dominating set. Give examples with respect to the above graph. Find all maximal independent sets of the above graph using Boolean algebra. What is the value of the coefficient of internal stability for this graph? [2+1+4+1]
10. Define minimum vertex cover problem. How is it related to matching? Use a 2-approximation algorithm to find the minimum vertex cover for the above graph. Show the approximation. [1+1+3+1]
11. State and prove Cayle's formula. [3]
12. State and prove the five coloring theorem. [4]