NATIONAL INSTITUTE OF TECHNOLOGY, TIRUCHIRAPPALLI
Department Of Computer Science And Engineering

First Class Test

Combinatorics and Graph Theory

Marks: 20   Course Code: CSPE32   Time: 1hr

Instructions to the Students: Answer all questions.

1. Derive a formula for the number of diagonals in a polygon with n vertices. Then prove your formula using mathematical induction.   [4]

2. During a month with 31 days, a baseball team plays at least one game a day, but no more than 50 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 11 games.   [3]

3. Find all solutions of the recurrence relation:

   $a_n = 7a_{n-1} - 16 a_{n-2} + 12 a_{n-3} + n4^n$ with $a_0 = -2$, $a_1 = 0$ and $a_2 = 5$.   [5]

4. Assume that in a group of six people, each pair of individuals consists of two friends or two enemies. Show that there are either three mutual friends or three mutual enemies in the group. [Hint: Use generalized pigeon hole principle]   [4]

5. Solve the following recurrence using Master's Theorem: [4]

   $T(n) = 2T(\sqrt{n}) + 1$ where $T(1) = 1$