

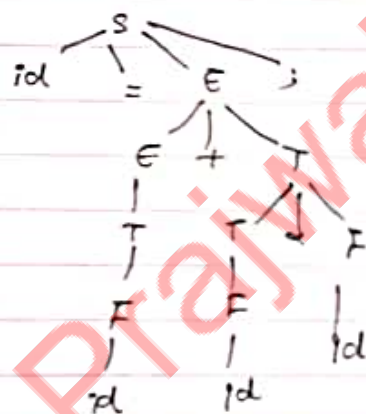
## Introduction to Parsers

outcome

- ✓ Definition of parser.
- ✓ ways of generating parse trees.
- ✓ classification of parsers.

Parser : A parser is a program that generates a parse tree for the given string, if the string is generated from the underlying grammar.

$x = a + b * c;$   $\xrightarrow{LA}$   $id = id + id * id;$



$\downarrow$  parser  
 $S \rightarrow id = E;$   
 $E \rightarrow E + T / T$   
 $T \rightarrow T * F / F$   
 $F \rightarrow id$

Generation of parse Tree :

- ✓ Top Down Approach,
- ✓ Bottom Up Approach

eg: generate string aabcde from the grammar

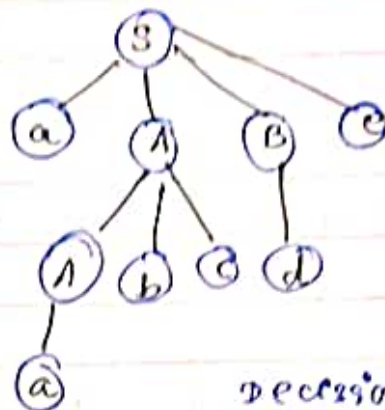
$S \rightarrow aABE$

$A \rightarrow Abc | a$

$B \rightarrow d$

using both ✓ top down  
✓ bottom up approaches

Top Down Approach :



$$S \rightarrow a \underline{A} B e$$

$$S \rightarrow a \underline{A} b c e$$

$$S \rightarrow a a b c \underline{B} e$$

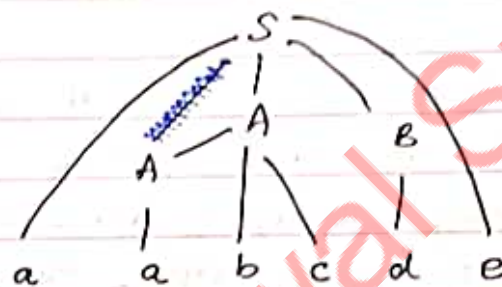
$$S \rightarrow a a b c d e$$

↓  
[Leftmost Derivation]

Decision :

which production to use ?

Bottom Up Approach :



$$S \rightarrow a \underline{A} B e$$

$$S \rightarrow a \underline{A} d e$$

$$S \rightarrow a \underline{A} b c d e$$

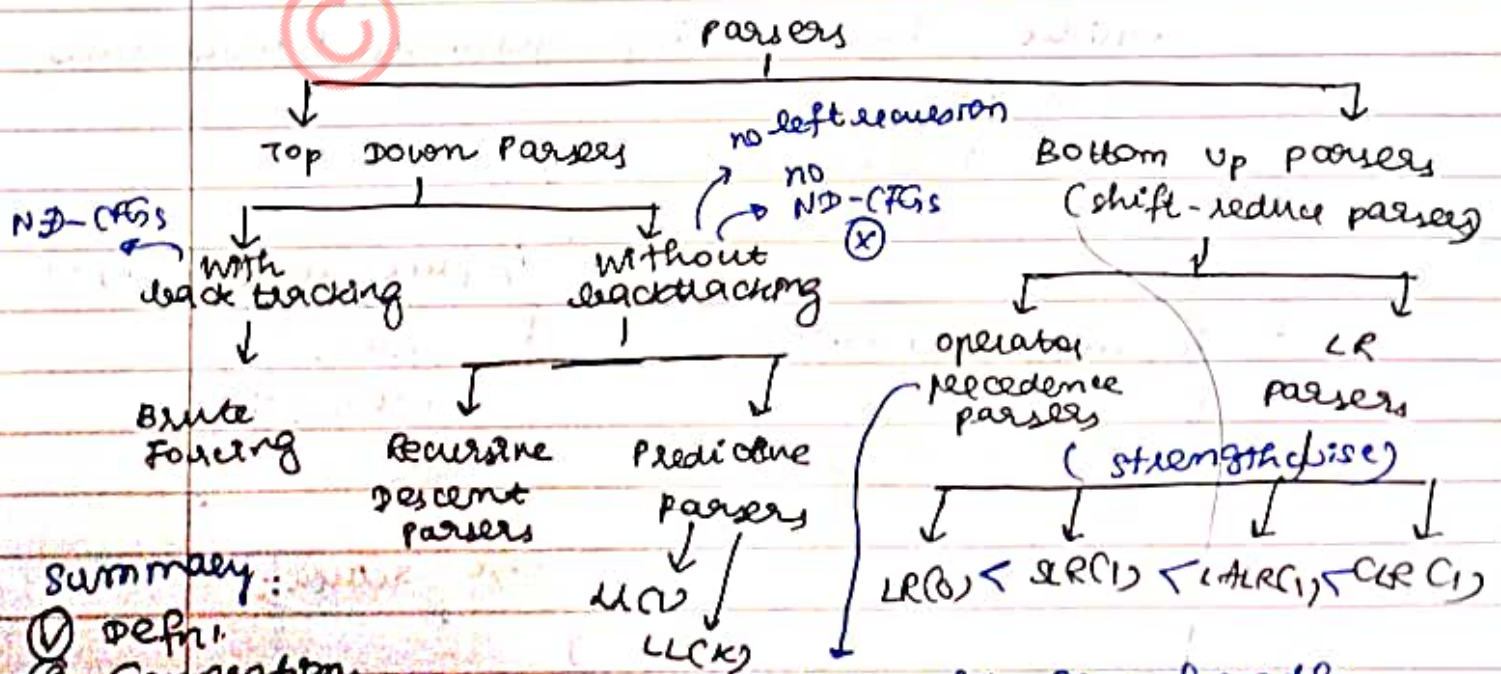
$$S \rightarrow a a b c d e$$

↓  
[Rightmost Derivation]  
(In reverse)

Decision :

When to reduce ?

Classification of parsers :



Summary :

- ① Defn.
- ② Generation
- ③ Classification

only this can handle ambiguous grammar



## Top-Down Parser: Recursive Descent Parser

outcome:

- ① Top Down Parsers.
- ② Examples of Recursive Descent parsers.

### Top-Down Parser:

In order to construct top down parsers, the context free grammars should not have: ① left recursion ② non-determinism

Recursive-Descent Parser: A recursive descent parser is a top-down parser built from a set of mutually recursive procedures (or a non-recursive equivalent) where each such procedure implements one of the non-terminals of the grammar. Thus the structure of the resulting program closely mirrors that of the grammar it recognizes.

↓  
consider the following grammar having rules,

$$\begin{cases} E \rightarrow iE' \\ E' \rightarrow +iE' \mid \epsilon \end{cases}$$

```
EC()
{
    if (look_ahead == 'i')
    {
        match('i');
        E'();
    }
}
```

```
E'()
{
    if (look_ahead == '+')
    {
        match('+');
        match('i');
        E'();
    }
    else return;
}
```

```

match(char c) → (match function)
{
    if (look_ahead == '$')
        look_ahead = getchar();
    else
        printf("ERROR!");
}

```

```

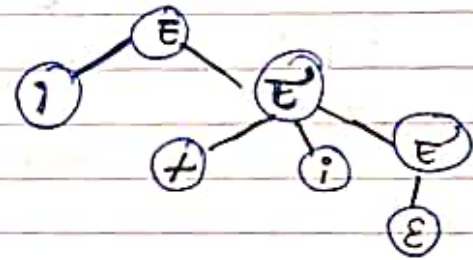
main() → (main function)
{
    E();
    if (look_ahead == '$')
        printf("parsing successful!");
}

```

(end of input symbol)

Eg: Input  $i + i \$$

output:  
parsing successful!



summary: ✓ Top down parser    ✓ Recursive descent parser

Top Down parser - LL(1) parser

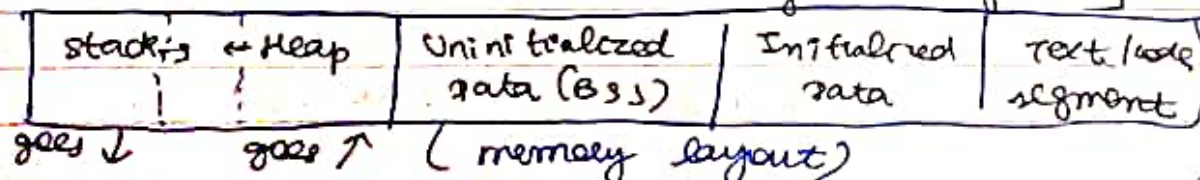
outcome:

- ✓ organization of LL(1) parser
- ✓ understanding of the concepts of FIRST & FOLLOW.

E()    E'()  
 match()    main()  
 (Recursive Descent parser)

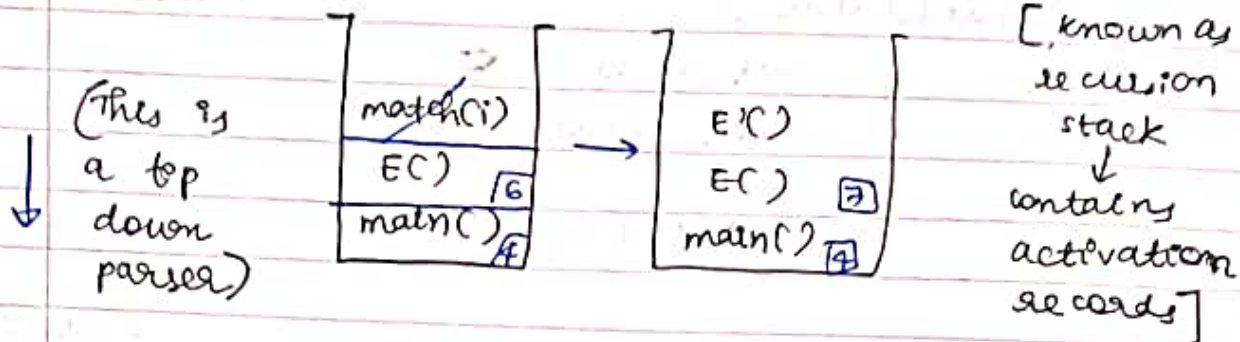
when this code was placed in the main memory for execution, it doesn't remain a code anymore. [i.e. a program anymore]

process

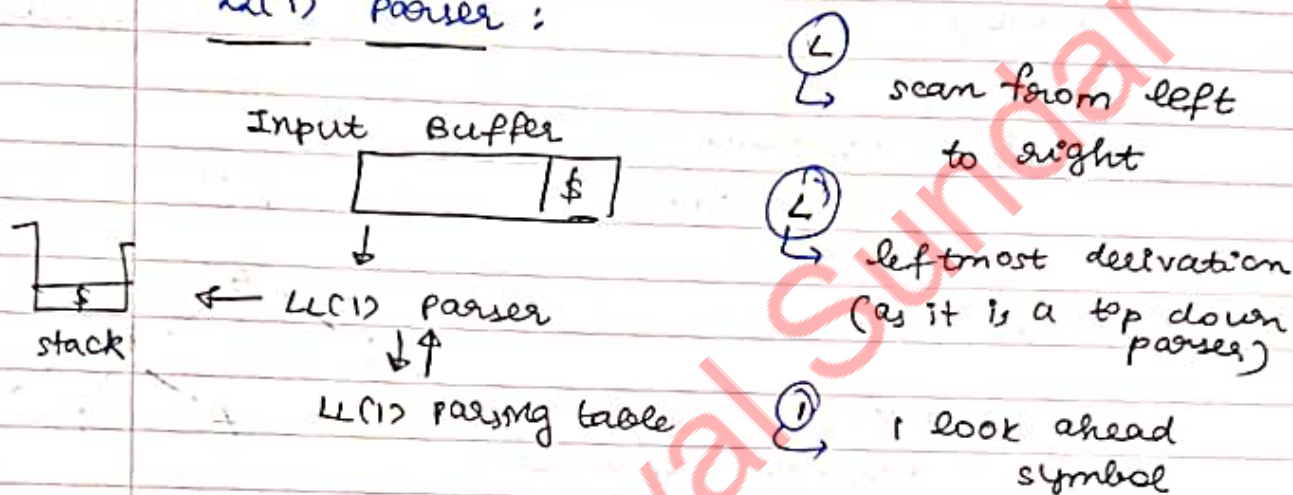




stack:



LL(1) parser:



① **FIRST()**: Given any non-terminal of a CFG, if we derive all the possible strings from it, the first terminal(s) is the FIRST() of the non-terminal.

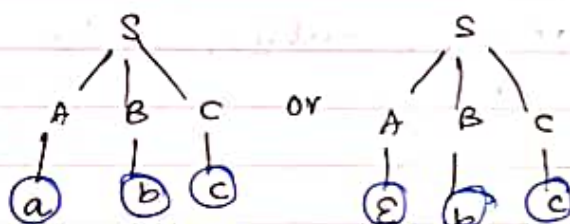
eg 1)  $S \rightarrow aABC$   
 $A \rightarrow b$   
 $B \rightarrow c$   
 $C \rightarrow d$

**FIRST(S)**

$\text{FIRST}(S) = a$

$\text{FIRST}(A) = b$   $\text{FIRST}(B) = c$   
 $\text{FIRST}(C) = d$

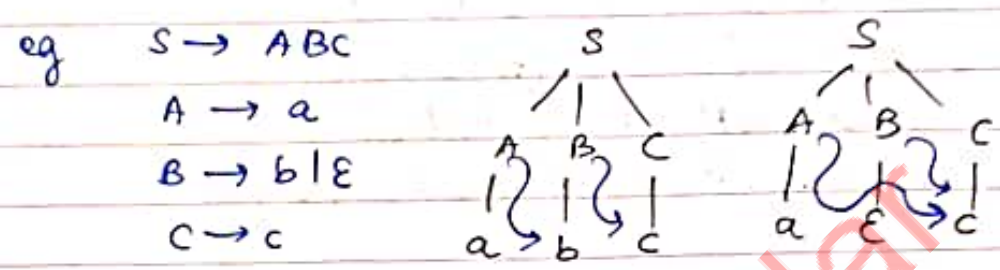
eg 2)  $S \rightarrow ABC$   
 $A \rightarrow a|e$   
 $B \rightarrow b$   
 $C \rightarrow c$



$\text{FIRST}(S) = \{a, b\}$   
 $\text{FIRST}(C) = \{c\}$

$\text{FIRST}(B) = \{b\}$

② FOLLOW(): During the process of derivation, the terminal(s) that could follow the non-terminal are to be considered as FOLLOW() of the non-terminal



$FOLLOW(S) : \{ \$ \}$        $FOLLOW(B) : \{ c \}$   
 $FOLLOW(A) : \{ b, c \}$        $FOLLOW(C) : \{ \$ \}$

Summary :

- ① organization of LL(1) parser
- ② FIRST() and FOLLOW()

FIRST() and FOLLOW() Functions

outcome : ① step by step derivation of FIRST and FOLLOW functions

Derivation of FIRST

eg  $E \rightarrow TE'$   
 $E' \rightarrow +TE' | \epsilon$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT' | \epsilon$   
 $F \rightarrow id | (E)$

Note : It is always a good idea to start with the bottom-most non-terminal.

same  
→  $FIRST(F) : \{ id, ( \}$   
→  $FIRST(T') : \{ *, \epsilon \}$   
→  $FIRST(T) : \{ id, ( \}$   
same  
→  $FIRST(E') : \{ +, \epsilon \}$   
→  $FIRST(E) : \{ id, ( \}$

$FIRST(F) = FIRST(T)$   
 $= FIRST(E)$



## Derivation of FOLLOW

(start with the start symbol)

same  $\rightarrow$

$$\begin{aligned}\text{FOLLOW}(E) &= \{ \$, ) \} \\ \text{FOLLOW}(E') &= \{ \$, ) \} \\ \text{FOLLOW}(T) &= \{ +, \$, ) \} \\ \text{FOLLOW}(T') &= \{ +, \$, ) \} \\ \text{FOLLOW}(F) &= \{ *, +, \$, ) \}\end{aligned}$$

$E' \rightarrow +TE' / E$   
same

### Rules :

- ① The following terminal symbol will be selected as follow.
- ② The FIRST of the following non-terminal will be selected as FOLLOW.
- ③ If it is the rightmost in the RHS, the FOLLOW of the LHS will be selected.

$$\text{FOLLOW}(E') = \text{FOLLOW}(E) = \{ \$, ) \}$$

$$\text{FOLLOW}(E') = \text{FOLLOW}(E') \quad \begin{matrix} \text{(RHS)} \quad \leftarrow \quad \text{(LHS)} \end{matrix}$$

$$\begin{aligned}\text{FOLLOW}(T) &= \text{FIRST}(E') = \{ +, \textcircled{\epsilon} \} \quad \begin{matrix} \times \text{ no } \epsilon \text{ in} \\ \text{FOLLOW} \\ \text{allowed} \end{matrix} \\ &\cup \text{FOLLOW}(E') = \{ +, \$, ) \}\end{aligned}$$

$$\text{FOLLOW}(T) = \text{FOLLOW}(T) = \{ +, \$, ) \}$$

$$\text{FOLLOW}(F) = \text{FIRST}(T') = \{ *, \textcircled{\epsilon} \}$$

$$\{ * \} \cup \text{FOLLOW}(T) \cup \text{FOLLOW}(T')$$

$$\text{FOLLOW}(F) = \{ *, +, \$, ) \}$$

Summary:  $\checkmark$  step by step derivation of FIRST and FOLLOW.

## First & Follow - solved problems (set 1)

outcome :

① 3 solved problems for determining FIRST and FOLLOW.

Q1  
GATE 2017

consider the following grammar :

First		Follow
$\{x\}$	$P \rightarrow xQRS$	$\{\$ \}$
$\{y, z\}$	$Q \rightarrow yz \mid z$	$\{w, y\}$
$\{w, \epsilon\}$	$R \rightarrow w \mid \epsilon$	$\{y\}$
$\{y\}$	$S \rightarrow y$	$\{\$ \}$

What is FOLLOW(Q) ?

(A)  $\{R\}$     (B)  $\{w\}$     (C)  $\{w, y\}$     (D)  $\{w, \epsilon\}$

Q2

Find the FIRST and FOLLOW of all the non-terminals :

FIRST		FOLLOW
$\{a, b, c\}$	$S \rightarrow ABCDE$	$\{\$ \}$
$\{a, \epsilon\}$	$A \rightarrow a \mid \epsilon$	$\{b, c\}$
$\{b, \epsilon\}$	$B \rightarrow b \mid \epsilon$	$\{c\}$
$\{c\}$	$C \rightarrow c$	$\{d, e, \$ \}$
$\{d, \epsilon\}$	$D \rightarrow d \mid \epsilon$	$\{e, \$ \}$
$\{e, \epsilon\}$	$E \rightarrow e \mid \epsilon$	$\{\$ \}$

Q3

Find the FIRST and FOLLOW :

FIRST		FOLLOW
$\{a, b, c, d\}$	$S \rightarrow Bb \mid Cd$	$\{\$ \}$
$\{a, \epsilon\}$	$B \rightarrow aB \mid \epsilon$	$\{b\}$
$\{c, \epsilon\}$	$C \rightarrow cC \mid \epsilon$	$\{d\}$

summary :

① 3 solved problems



outcome:  
 (✓) 2 solved problems

## First and Follow - solved Problems (set 2)

Q1: Find FIRST() and FOLLOW() of all non-terminals:

FIRST		FOLLOW
{a}	$S \rightarrow aBDh$	{ $\epsilon$ }
{c}	$B \rightarrow cC$	{g, f, h}
{b, $\epsilon$ }	$C \rightarrow bc   \epsilon$	{g, f, h}
{g, f, $\epsilon$ }	$D \rightarrow EF$	{h}
{g, $\epsilon$ }	$E \rightarrow g   \epsilon$	{f, h}
{f, $\epsilon$ }	$F \rightarrow f   \epsilon$	{h}

Q2: Find FIRST() and FOLLOW() of all non-terminals:

FIRST		FOLLOW
{d, g, h, $\epsilon$ }	$S \rightarrow ACB   CbB   Ba$	{ $\epsilon$ }
{d, g, h, $\epsilon$ }	$A \rightarrow da   Bc$	{h, g, $\epsilon$ }
{g, $\epsilon$ }	$B \rightarrow g   \epsilon$	{ $\epsilon$ , a, g, h}
{h, $\epsilon$ }	$C \rightarrow h   \epsilon$	{g, $\epsilon$ , b, h}

summary:

(✓) 2 solved problems

## LL(1) parsing Table

outcome: (✓) construction of LL(1) parsing table

construction of LL(1) parsing table:

FIRST		FOLLOW
{id, (}	$E \rightarrow TE'$	{ $\epsilon$ , )}
{+, $\epsilon$ }	$E' \rightarrow +TE'   \epsilon$	{ $\epsilon$ , )}
{id, (}	$T \rightarrow FT'$	{+, $\epsilon$ , )}
{*, $\epsilon$ }	$T' \rightarrow *FT'   \epsilon$	{+, $\epsilon$ , )}
{id, (}	$F \rightarrow id   (E)$	{ $\epsilon$ , +, $\epsilon$ , )}

↪ (construct using this data)

	id	(	)	*	+	\$
E	$E \rightarrow TE'$	$E \rightarrow TE'$			$E' \rightarrow +TE'$	$E' \rightarrow \epsilon$
E'			$E' \rightarrow \epsilon$			
T	$T \rightarrow FT'$	$T \rightarrow FT'$			$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
T'			$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$		
F	$F \rightarrow id$	$F \rightarrow (E)$				

Rules :

- ① All the  $\epsilon$ -productions are placed under FOLLOW sets.
- ② Remaining productions are placed under the FIRSTs.

Idea:

LL(1) parsers are predictive parsers. Basically, when any non-terminal is in the stack, we will take 1 look-ahead symbol [terminal symbols] in the input string, and then predict which of the production rules to follow in order to generate the parse tree. Basically, for a particular terminal, considering a non-terminal, the parser will predict which rule to use.

LL(1) parsing

Outcome :

- ① Illustration of LL(1) parsing procedure.

eg:

$S \rightarrow aABb$

FIRST

FOLLOW

$\{a\}$

$\{\epsilon, \$\}$

$A \rightarrow c | \epsilon$

$\{c, \epsilon\}$

$\{a, b\}$

$B \rightarrow d | \epsilon$

$\{d, \epsilon\}$

$\{b\}$

eg I/P:

$a | b | \$$

(match: pop & move ptr of I/P else, look at twice)

	a	b	c	d	\$
S	$S \rightarrow aABb$				
A		$A \rightarrow \epsilon$	$A \rightarrow c$	$A \rightarrow \epsilon$	
B		$B \rightarrow \epsilon$		$B \rightarrow d$	

(stack)

summary

① Illustration

\* only one rule in each cell → LL(1) parsing table



## LL(1) Parsing - solved Problems (set 1)

outcome :

① 5 solved problems on whether the grammar is LL(1)

Q1: Find out whether the grammar is LL(1) :  
 $S \rightarrow asbs \mid bsas \mid \epsilon$

$$\text{FIRST}(S) = \{a, b, \epsilon\}$$

$$\text{FOLLOW}(S) = \{\$, b, a\}$$

	a	b	\$
S	$S \rightarrow asbs$ $S \rightarrow \epsilon$	$S \rightarrow bsas$ $S \rightarrow \epsilon$	$S \rightarrow \epsilon$

not a unique production in each cell

② not an LL(1) grammar

Q2: Find out whether the grammar is LL(1) :  
 $S \rightarrow (S) \mid \epsilon$

$$\text{FIRST}(S) = \{ (, \epsilon \}$$

$$\text{FOLLOW}(S) = \{ \$, ) \}$$

	(	)	\$
S	$S \rightarrow (S)$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$

③ LL(1) grammar

Q3: Find out whether the grammar is LL(1) :

$$\{a, b\} \quad S \rightarrow AaAb \mid BbBa \quad \{\$ \}$$

$$\{\epsilon\} \quad A \rightarrow \epsilon \quad \{a, b\}$$

$$\{\epsilon\} \quad B \rightarrow \epsilon \quad \{b, a\}$$

	a	b	\$
S	$S \rightarrow AaAb$	$S \rightarrow BbBa$	
A	$A \rightarrow \epsilon$	$A \rightarrow \epsilon$	
B	$B \rightarrow \epsilon$	$B \rightarrow \epsilon$	

④ LL(1) grammar

Q4:

 $S \rightarrow A|a$ 

FIRST

 $\{a\}$ 

FOLLOW

 $\{\$ \}$ 

	a	\$
S	$S \rightarrow A$ $S \rightarrow a$	
A	$A \rightarrow a$	

(X)

 $A \rightarrow a$  $\{a\}$  $\{\$ \}$ 

Q5:

Find out whether the grammar is LL(1):

 $\{a, \epsilon\}$  $S \rightarrow AB| \epsilon$  $\{\$ \}$  $\{b, \epsilon\}$  $B \rightarrow bC| \epsilon$  $\{\$ \}$  $\{c, \epsilon\}$  $C \rightarrow cS| \epsilon$  $\{\$ \}$ 

	a	b	c	\$
S	$S \rightarrow AB$			$S \rightarrow \epsilon$
B		$B \rightarrow bC$		$B \rightarrow \epsilon$
C			$C \rightarrow cS$	$C \rightarrow \epsilon$

(V)  
LL(1)

summary: (V) 5 solved problems

LL(1) parsing - solved problems (set 2)

outcome: (V) 4 solved problems on whether the grammar is LL(1).

Q1:

Find out whether the following grammar is LL(1).

 $\{a, b, \epsilon\}$  $S \rightarrow AB$  $\{\$ \}$  $\{a, \epsilon\}$  $A \rightarrow a| \epsilon$  $\{b, \$ \}$  $\{b, \epsilon\}$  $B \rightarrow b| \epsilon$  $\{\$ \}$ 

	a	b	$\epsilon$
S	$S \rightarrow AB$	$S \rightarrow AB$	$S \rightarrow AB$
A	$A \rightarrow a$	$A \rightarrow \epsilon$	$A \rightarrow \epsilon$
B		$B \rightarrow b$	$B \rightarrow \epsilon$

(V) LL(1)  
grammar

→ This particular production will be used by the predictive LL(1) parser in case the grammar generates  $\epsilon$  or empty strings



Q2: Find out whether the grammar is LL(1):

$S \rightarrow aSA \mid \epsilon$      $\{a, \epsilon\}$      $\{\$, c\}$   
 $A \rightarrow c \mid \epsilon$      $\{c, \epsilon\}$      $\{\$, c\}$

	a	c	\$
S	$S \rightarrow aSA$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$
A		$A \rightarrow c$ $A \rightarrow \epsilon$	$A \rightarrow \epsilon$

⊗ not LL(1)

Q3: Find out whether the grammar is LL(1):

$S \rightarrow A$      $\{a, b, c, d\}$      $\{\$ \}$   
 $A \rightarrow Bb \mid Cd$      $\{a, b, c, d\}$      $\{\$ \}$   
 $B \rightarrow aB \mid \epsilon$      $\{a, \epsilon\}$      $\{b, d\}$   
 $C \rightarrow cC \mid \epsilon$      $\{c, \epsilon\}$      $\{d\}$

⊙  
LL(1)  
grammar

	a	b	c	d	\$
S	$S \rightarrow A$	$S \rightarrow A$	$S \rightarrow A$	$S \rightarrow A$	
A	$A \rightarrow Bb$	$A \rightarrow Bb$	$A \rightarrow Cd$	$A \rightarrow Cd$	
B	$B \rightarrow aB$	$B \rightarrow \epsilon$			
C			$C \rightarrow cC$	$C \rightarrow \epsilon$	

Q4: Find out whether the grammar is LL(1):

$S \rightarrow aAa \mid \epsilon$      $\{a, \epsilon\}$      $\{\$, a\}$   
 $A \rightarrow abs \mid \epsilon$      $\{a, \epsilon\}$      $\{a\}$

	a	b	\$
S	$S \rightarrow aAa$ $S \rightarrow \epsilon$		$S \rightarrow \epsilon$
A	$A \rightarrow abs$ $A \rightarrow \epsilon$		

⊗ not an  
LL(1) grammar

Hwa:

$S \rightarrow iEtSS' \mid a$      $\{i, a\}$      $\{\$ \}$   
 $S' \rightarrow es \mid \epsilon$      $\{e, \epsilon\}$      $\{\$ \}$   
 $E \rightarrow b$      $\{b\}$      $\{t\}$

Summary:  
⊙ 4Q +  
(1 Hwa)

⊙ LL(1)  
grammar

	i	a	b	e	\$
S	$S \rightarrow iEtSS'$	$S \rightarrow a$			
S'				$S' \rightarrow es$	$S' \rightarrow \epsilon$
E			$E \rightarrow b$		