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National Institute of Technology Tiruchirappalli
Department of Computer Science and Engineering
Design and Analysis of Algorithms (CSPC42)

Cycle Test 2, Date: April 11, 2023

Duration: 1 Hour | II-B.Tech CSE-A, IV Semester | Max marks: 20

1. Consider a variant of the matrix-chain multiplication problem in which the goal is to parenthesize the sequence of matrices to maximize, rather than minimize, the number of scalar multiplications. Does this problem exhibit optimal substructure? If yes, provide the problem's recurrence equation and write a bottom-up dynamic programming algorithm to solve. Analyze the time complexity **(5M (CO1, CO2))**

2. Given a number of stairs and a frog, the frog wants to climb from the 0^{th} stair to the $(N-1)^{th}$ stair. At a time, the frog can climb either one or two steps. A $energy[N]$ array is also given. Whenever the frog jumps from a stair i to stair j, the energy consumed in the jump is $abs(energy[i] - energy[j])$, where $abs()$ means the absolute difference. You need to return the minimum energy the frog can use to jump from stair 0 to stair $N-1$. Provide a recurrence equation for the problem using dynamic programming approach and write a bottom-up dynamic programming algorithm to solve. Analyze the time complexity. **(5 M (CO1, CO2))**

3. Prove that the longest common subsequence problem can be solved using dynamic programming. Write a recurrence equation and bottom-up dynamic programming algorithm to solve the problem. Analyze the time complexity. Consider two strings $A =$ "$qpqrr$" and $B =$ "$pqprqrp$". Find the length of LCS for the strings A and B using dynamic programming approach and how many such LCS lengths are possible for given strings. **(5M (CO1, CO2))**

4. (a) What is Amortized analysis? How is it different from the Asymptotic analysis? Explain with a dynamic array.

   (b) Prove that the Randomized quick sort time complexity is $O(nlogn)$

   (5M (CO1, CO3))

Best Wishes