3. Methods

3.1. Shadow imaging sensor

For the experimental investigation of the 3D position measurement capability, a minimal setup with a light source and three shadow imaging sensors is used, see Fig. 3. Thereby, three sensors are applied to investigate the required measuring volume of one sensor. The light source, whose position is to be measured, is a surface-mounted device LED with a peak wavelength of 520 nm, a maximum luminous intensity of 1300 mcd and a beam angle of 140°. Each sensor consists of a 30 mm × 40 mm large mask with transparent and opaque parts, which is manufactured by laser exposure of a polyester film, and a DMM 37UX273-ML monochrome board camera from the company The Imaging Source. The camera has a resolution of 1440 px × 1080 px with a pixel size of 3.45 µm. The distance h between mask and camera is 20 mm.

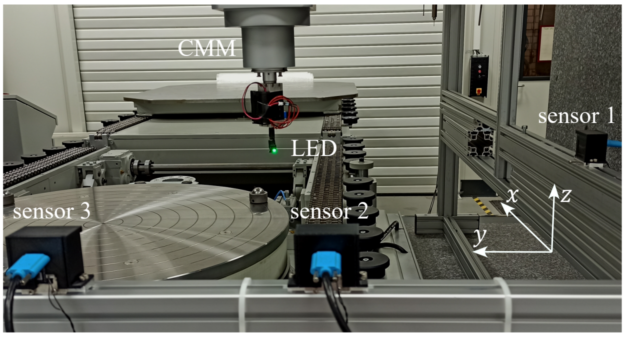

Figure 3: Experimental setup to investigate the 3D position measurement capability of a measurement system of several shadow imaging sensors in a measurement volume of 500 mm × 300 mm × 200 mm. The LED is positioned by a coordinate measuring machine (CMM), which also serves as reference system.

3.1.1. Mask

For measuring the absolute 3D position of the tool tip, i. e. the LED, a mask is required that contains features in horizontal and vertical direction and absolute features. A section of the used mask is shown in Fig. 4. The mask contains alternately arranged grids with vertical and horizontal stripes. Vertical stripes enable to determine the horizontal shadow position ξ_i and horizontal stripes allow the evaluation of the vertical shadow position ζ_i, respectively. In order to ensure that at least one full grid is always visible in the image while the LED is moved through the entire measurement volume, each grid has a size of 2.0 mm × 1.5 mm. Each stripe in a grid is 100 µm wide. The absolute feature is realized by 8-bit binary codes in each first transparent stripe of a grid. Eight adjacent squares are either transparent and provide a '0' or opaque and provide a '1' and so form an index of the grid. In the mask, each index is used twice, once for a vertical grid and once for a horizontal grid. The index defines where each grid is located with respect to the mask center. Therefore, the coded grid mask enables to determine the absolute shadow position of the mask center in horizontal and vertical direction so that the absolute 3D LED position can be measured by two or more sensors.

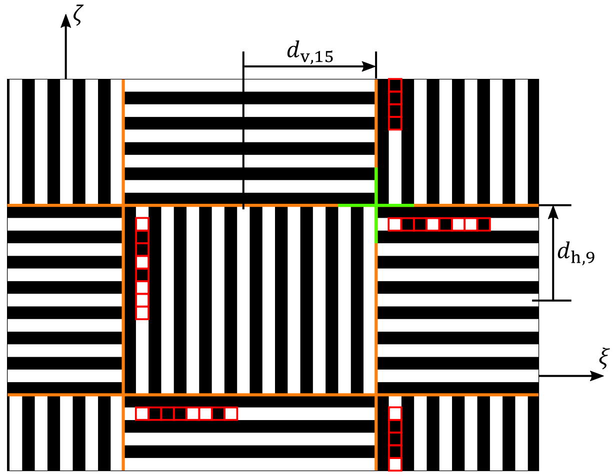

Figure 4: Section of the mask used in the shadow imaging sensors. The mask is the black and white structure wherein black areas represent opaque contents and white areas transparent contents. The orange lines highlight the borders between the horizontal and vertical grids. The red squares visualize the bits used to build the binary index of each grid. The center of the entire mask is marked by the green cross. The axes ξ and ζ are projected from the sensor coordinate system to the mask plane. The distances $d_{\mathrm{v},a}$ and $d_{\mathrm{h},b}$ in the mask plane between the mask center and the stripes with the the number a and b respectively are known. As an example, the distances $d_{\mathrm{v},a}$ and $d_{\mathrm{h},b}$ for the stripes a = 15 and b = 9 are visualized.

3.1.2. Image processing

To determine the position, where the shadow of the mask center occurs in the image plane, the grids must be segmented first. In a second step, the stripes in each grid are localized, and then, the index is read in the binary coded stripe. The position of the shadow of the mask center is then obtained by evaluating the location of the shadows of the stripes visible in the image, the location of these stripes in the mask with respect to the mask center and the magnification of the stripe spacing in the shadow image with respect to the stripe spacing in the mask.

To separate the grids, the detection of horizontal and vertical borders is necessary. For visualization, an example image with evaluated intensity profiles is shown in Fig. 5. Horizontal borders are located as the drop of the intensity after a bright vertical stripe. A vertical stripe is detected as a peak in the column-wise averaged intensity. A previously performed low-pass-filtering ensures robustness of the image processing against noise. Then, the horizontal borders are located in the row, where the filtered column intensity first passes through a threshold intensity after a plateau on a higher level. Thereby, the threshold intensity is the average intensity of the entire image and the intensity plateau indicates a stripe of a vertical grid. A respective intensity profile is given by the orange profile in Fig. 5. Similarly, vertical borders are detected. A right border of a horizontal grid is where the filtered intensity passes through a threshold on the right side of the high-level-plateau, i. e. a horizontal bright stripe in the image, see the blue intensity profile in Fig. 5. Accordingly, the left border of a horizontal grid is where the intensity passes through the threshold on the left side of a low-level-plateau, i. e. a dark horizontal stripe, as shown by the intensity profile in Fig. 5.

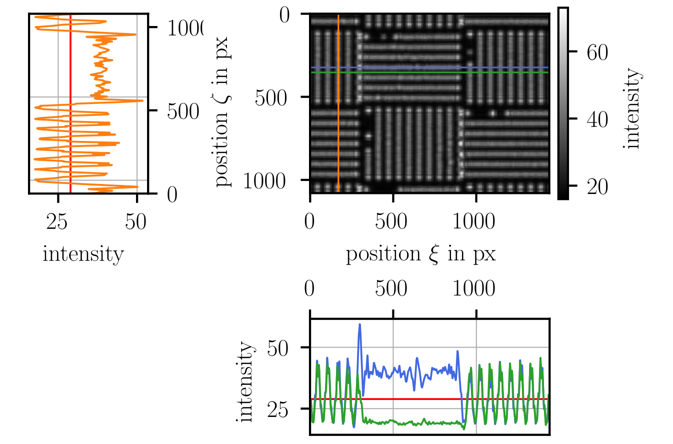

Figure 5: Camera image with intensity profiles used for grid segmentation. The lines in the image are the columns or rows where the filtered intensity profiles shown in the same color are taken. The red line in each intensity graph presents the threshold intensity. The intersections of the orange intensity profile and the threshold next to high-level-plateaus are horizontal borders. The left vertical border is located where the green intensity profile crosses the threshold on the left of the low-level-plateau and the right vertical border is located where the blue intensity profile crosses the threshold on the right of the high-level-plateau.

In each grid, the stripes are localized by approximating a model function because preliminary investigations have shown that this method provides more accurate results than a phase evaluation or a correlation [24]. Before the approximation, the image section is averaged in the direction of the stripes and a low-pass-filter is applied to smoothen interferences due to noise and diffraction. Then, the intensity profile of a vertical bright stripe a in the region between adjacent intensity minima is approximated by the model function

$$I_{\mathrm{M,v}}(\xi) = \begin{cases} I'_{\mathrm{M,v}}(\xi) & \text{for } I'_{\mathrm{M,v}}(\xi) < I_{\mathrm{max,v},a} \\ I_{\mathrm{max,v},a} & \text{for } I'_{\mathrm{M,v}}(\xi) \geq I_{\mathrm{max,v},a} \end{cases} \tag{8}$$

with

$$I'_{\mathrm{M,v}}(\xi) = I_{0,\mathrm{v},a} + A_{\mathrm{v},a} \cdot e^{-\left(\frac{|\xi - \mu_{\mathrm{v},a}|}{w_{\mathrm{v},a}}\right)^2}$$

and the intensity profile of each horizontal bright stripe b is approximated by the model function

$$I_{\mathrm{M,h}}(\zeta) = \begin{cases} I'_{\mathrm{M,h}}(\zeta) & \text{for } I'_{\mathrm{M,h}}(\zeta) < I_{\mathrm{max,h},b} \\ I_{\mathrm{max,h},b} & \text{for } I'_{\mathrm{M,h}}(\zeta) \geq I_{\mathrm{max,h},b} \end{cases} \tag{9}$$

with

$$I'_{\mathrm{M,h}}(\zeta) = I_{0,\mathrm{h},b} + A_{\mathrm{h},b} \cdot e^{-\left(\frac{|\zeta - \mu_{\mathrm{h},b}|}{w_{\mathrm{h},b}}\right)^2}, \tag{10}$$

respectively. The model function is a limited Gaussian function with an offset I_0, an amplitude A, a width w, a peak position μ and an intensity limit I_{max}. The index v refers to a vertical stripe and the index h to a horizontal stripe. By approximating the parameters, the determined peak position $\mu_{\mathrm{v},a}$ serves as ξ-stripe location for a vertical stripe a and the determined peak position $\mu_{\mathrm{h},b}$ is the ζ-stripe location of a horizontal stripe b.

To calculate the absolute shadow position, the index of one grid in the image is needed. The locations of stripes in adjacent grids are used to determine the borders of each code bit. The intensity averaged in the quadratic range of each bit of the coded line is compared to an empirical threshold, that adapts to the image intensity. Mean bit intensities higher than the threshold are associated with a '0' and lower intensities provide a '1', and thus, the index is composed of the code bits.

The determined index enables to calculate the shadow position of the mask center. Indeed, using the index, each stripe in the image can be associated with a stripe in the mask whose absolute position with respect to the mask center is known. To transfer the mask stripe position with respect to the mask center to the image plane, the magnification

$$k = \frac{l_{\mathrm{S}}}{l_{\mathrm{M}}} \tag{11}$$

of the stripe spacing l_{S} in the shadow on the camera chip with respect to the stripe spacing l_{M} in the mask is applied. Therefore, the horizontal mask center shadow position

$$\xi_{\mathrm{i}} = \frac{1}{s_{\mathrm{v},1} - s_{\mathrm{v},0} + 1} \cdot \sum_{a=s_{\mathrm{v},0}}^{s_{\mathrm{v},1}} (\mu_{\mathrm{v},a} + d_{\mathrm{v},a} \cdot k) \tag{12}$$

and the vertical mask center shadow position

$$\zeta_{\mathrm{i}} = \frac{1}{s_{\mathrm{h},1} - s_{\mathrm{h},0} + 1} \cdot \sum_{b=s_{\mathrm{h},0}}^{s_{\mathrm{h},1}} (\mu_{\mathrm{h},b} + d_{\mathrm{h},b} \cdot k) \tag{13}$$

are calculated from the stripe shadow positions $\mu_{\mathrm{v},a}$ of each vertical stripe a or $\mu_{\mathrm{h},b}$ of each horizontal stripe b visible in the image, the positions $d_{\mathrm{v},a}$ in horizontal direction or $d_{\mathrm{h},b}$ in vertical direction of each stripe in the mask and the magnification k. Here, $s_{\mathrm{v},0}$ is the first and $s_{\mathrm{v},1}$ the last index of the vertical stripes in the image and $s_{\mathrm{h},0}$ the first and $s_{\mathrm{h},1}$ the last index of the horizontal stripes. This way, the absolute shadow position $(\xi_{\mathrm{i}}, \zeta_{\mathrm{i}})$ is evaluated for each image. The shadow position is then inserted into Eq. (4) to (7) to obtain the direction vector $\boldsymbol{r}_{\mathrm{m},n}$ pointing from the sensor to the LED, and the measured directions to the LED from several sensors finally provide the sought LED position according to Eq. (2) and (3).

3.2. *Experimental setup with three sensors*

Sensor 2 and sensor 3 are arranged perpendicular to sensor 1 because it is expected that a lower sum of the squares of the position component uncertainties is achieved if the angle between between two measured direction vectors is close to 90° and if the angle of view from the sensor to the LED is close to 0° [22]. Sensor 1 is subsequently investigated in an axial measurement range of 500 mm beginning at a minimal measuring distance of 300 mm. The investigated lateral measurement range is 300 mm in horizontal direction and 200 mm in vertical direction, each centered in front of the sensor. Sensor 2 serves as second sensor for triangulation in the closer half of the axial measurement range of sensor 1 and sensor 3 covers the farther half. The investigated measurement range is located at a distance of 400 mm in front of sensor 2 and sensor 3. The LED is oriented in an angle of 45° towards the negative x- and y-axis so that the LED illuminates all sensors.