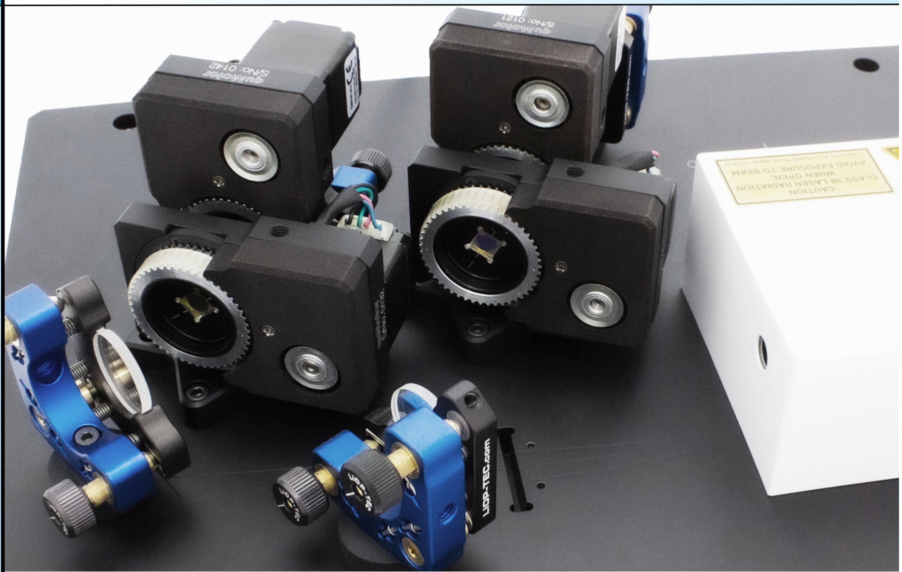
quED-TOM

Quantum State Tomography



quED-TOM Manual V1.0

quED-TOM Manual V1.1 (July 28, 2022)

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1 quED-TOM: Quantum State Tomography

1.1 Quickstart Manual

Figure 1.1: The quED-TOM motorized components.

The quED-Tom basically consists of two additional quarter wave plates and an easy-to-use software with which you can record the data for an overcomplete tomography scheme. Basically, this lets you determine the full quantum state density matrix that defines all properties of either a single photonic qubit or even the two-qubit entangled or non-entangled state that can be produced in the quED source. Like most of the other AddOns, the quED-TOM can be manual or motorized. You can use the software as data collection and analysis also with the manual version, whereas in the motorized version, the experiments can be run fully automated.

Please note that you need to have the quED-TOM in the same version as the basic quED-Setup in order to follow the experiments below.

After mounting the quarter wave plates into the beam paths of the quED optics as in Fig. 1.2, you're ready to go.

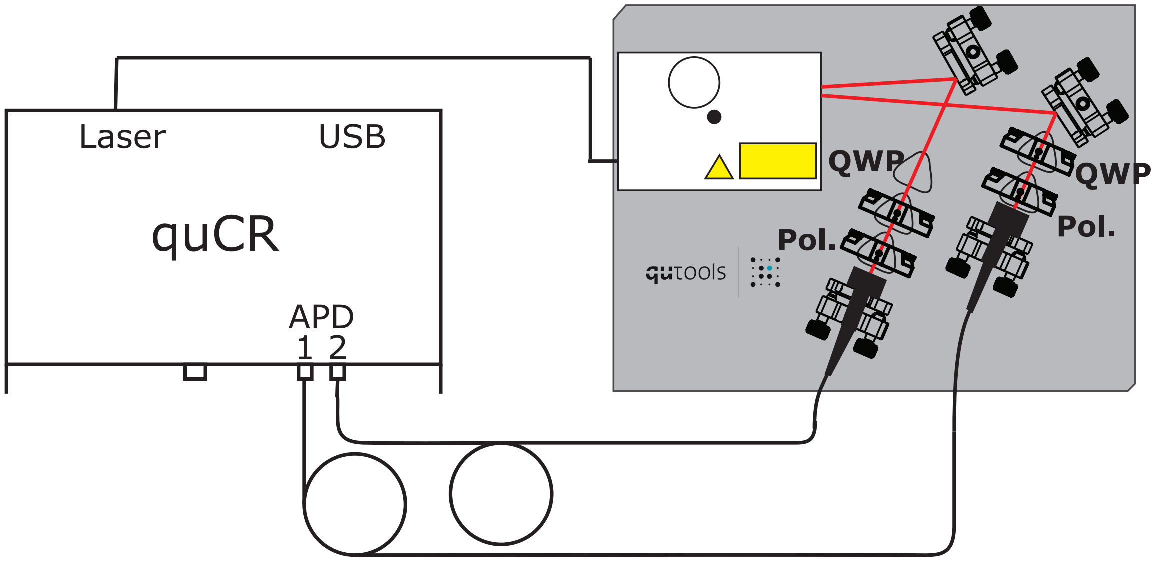


Figure 1.2: Schematic setup of the quED-TOM Add-On

2 Experiments with the quED-TOM

2.1 Single Photon Tomography

A quantum mechanical *state* can not be determined using only a single measurement. But, if you have an *ensemble* of equally prepared states, as, e.g., by our SPDC source, there is a procedure that makes the complete determination of the quantum state, i.e. its *density matrix*, possible.

2.1.1 Theoretical Background

Tomographic state reconstruction

To determine the full density matrix of an experimentally prepared state, it is necessary to perform measurements in different bases and combine the results accordingly. Commonly, a procedure like that is referred to as *quantum state tomography* (QST). This section focuses on a *linear inversion* scheme for a single qubit.

Measurements Measurements of the polarization of photons are performed by a *polarization analysis* (PA) setup consisting of a QWP and a polarizer, followed by an APD. For the measurement of circularly polarized photons, the quarter-wave plate at 45° rotates the polarization

$$U_{\mathrm{QWP}}(45°)|R\rangle = \mathrm{e}^{\mathrm{i}\phi_a}|V\rangle \tag{2.1}$$

(with arbitrary global phase ϕ_a) such that a passing of the polarizer at 90° corresponds to a projection onto $|R\rangle$. All angle settings for measuring the important single qubit states are listed in Table 2.1. In a typical experiment, the source produces a stream of identically prepared photons, so that one can count the detection events c_i^s in the APD i occurring during a given time for every setting $s \in \{1, 2, 3\} \equiv \{x, y, z\}$.

Table 2.1: **Settings of the QWP and polarizer for polarization analysis** A click in the APD while having the components set at certain angles corresponds to a projection onto a specific state. For every setting, the experiment runs for a fixed time and the number of counts in each APD can be measured.

Projection onto	Angle of QWP	Angle of polarizer	Measured quantity
$\|H\rangle$	0°	0°	c_h
$\|V\rangle$	0°	90°	c_v
$\|P\rangle$	45°	45°	c_p
$\|M\rangle$	45°	−45°	c_m
$\|R\rangle$	45°	90°	c_r
$\|L\rangle$	45°	0°	c_l

Performing all of these 6 measurements corresponds to an *overcomplete tomography* since it would be sufficient to determine 3 parameters to fully describe a single qubit state. A tomography scheme with only 4 (the fourth measurement is needed to exclude the influence of the total photon flux) projection measurements (projection onto $|H\rangle$, $|V\rangle$, $|P\rangle$ and $|R\rangle$) is presented in [1]. Nevertheless, the overcomplete scheme normally leads to better results[2] and the reconstruction process is easier to understand.

In the case of a single qubit, the density matrix is obtained by means of

$$\rho = \frac{1}{2}\left(\sigma_0 + T_x\sigma_x + T_y\sigma_y + T_z\sigma_z\right) = \frac{1}{2}\sum_{s=0}^{3} T_s\sigma_s \tag{2.2}$$

with

$$T_0 = 1\;; \qquad T_x = \frac{c_p - c_m}{c_p + c_m}\;; \qquad T_y = \frac{c_r - c_l}{c_r + c_l}\;; \qquad T_z = \frac{c_h - c_v}{c_h + c_v} \tag{2.3}$$

for the correlation tensor elements and the standard notation for the three *Pauli matrices*

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\;,\;\; \sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}\;\text{ and }\;\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\;. \tag{2.4}$$

[1] Measurement of Qubits, *James, Daniel F.V. et. al.*, PRA 64, pg. 052312, 2001

[2] Choice of measurement sets in qubit tomography, *de Burgh, Mark D. et. al.*, PRA 78, pg. 052122, 2008

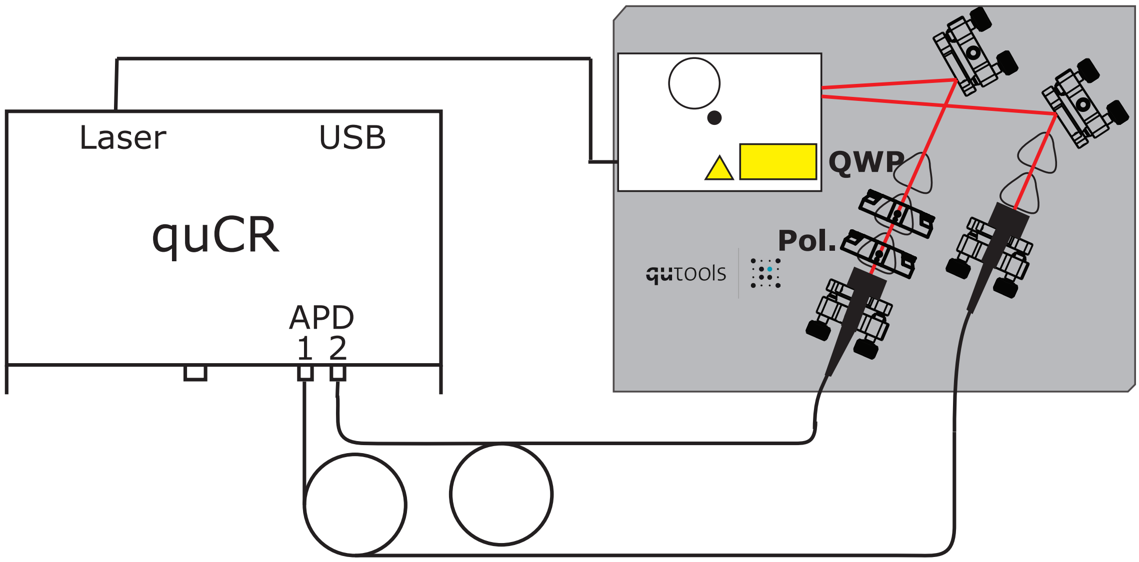


Figure 2.1: Setup of the tomography setup for single photons, manual version.

2.1.2 Realization with the quED

Necessary Components

- quED unit with quCR and polarizers
- manual: 1 QWP in manual rotation mount
- motorized: 1 QWP in quMotor + second motor driver qu3MD
- Optionally: unknown polarizing medium, e.g. sugar solution or 3D-cinema glasses.

Experimental description

As in Fig. 2.1 for the manual version or Fig. 2.2 for the motorized, put a polarizer and a quarterwave plate into one of the beam paths of the quED. With this setup, you can perform projection measurements for every necessary polarization. You can then perform the tomography scheme using different state preparation, e.g. with and without the waveplate in the source. If you put another polarizational-active component into the beam path, you can observe the changes.

After setting up, measure the count rate behind the polarization analysis setup for all 6 angle settings and calculate the resulting density matrix. You can use the *quAPP Single Qubit Tomography screen*, see Fig. 2.3 of the quCR for data acquisition and analysis even in the manual version by tapping on the box that corresponds to the set angles of the rotation mounts. In this screen, the second arm of the quED is used as a trigger in order to reduce influence of stray light and make sure you actually measure the single photons

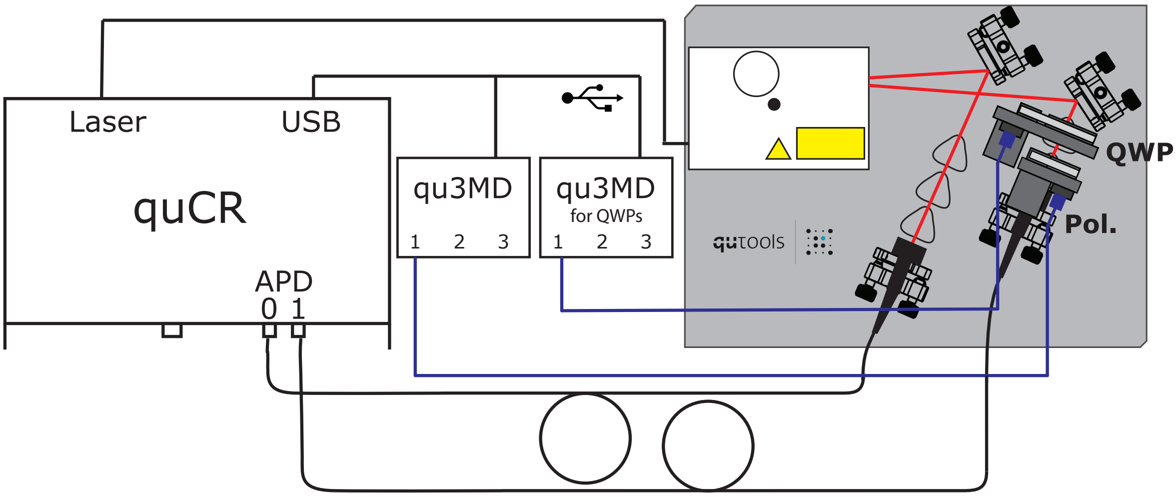


Figure 2.2: Setup of the tomography setup for single photons, motorized version.

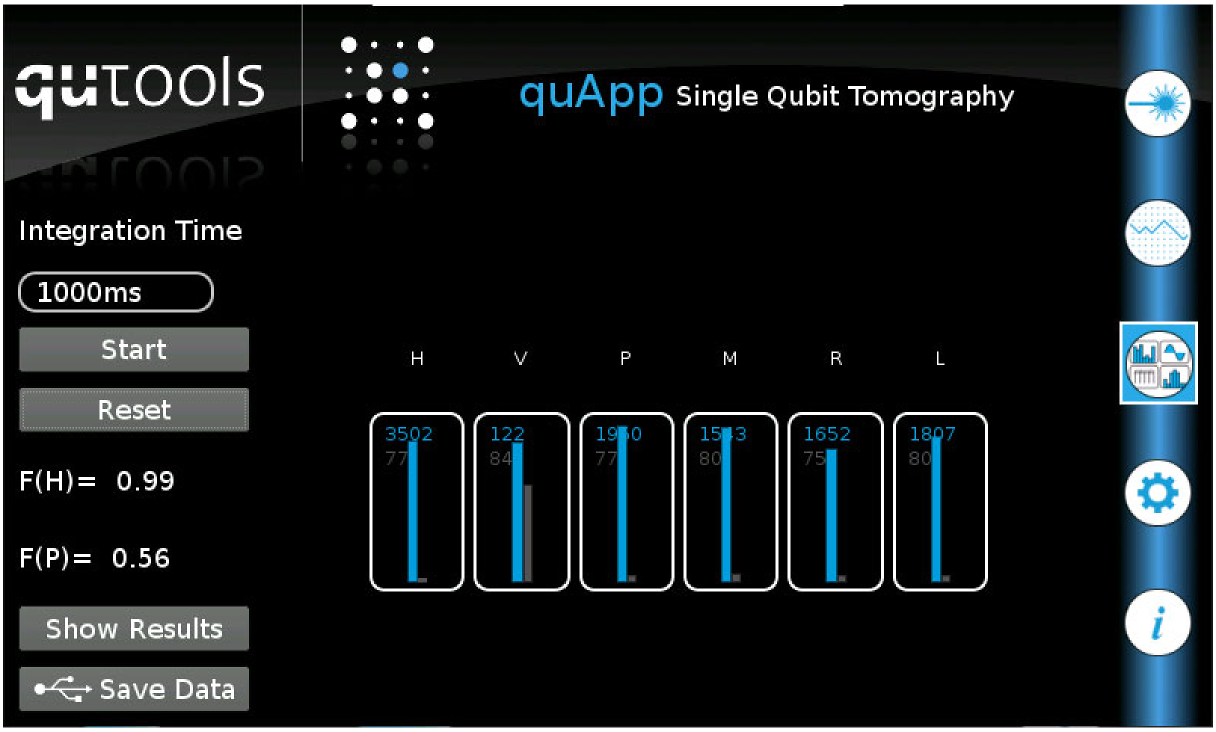


Figure 2.3: Screenshot of the software for data acquisition and automation.

from the source. This also means that you can use it together with an entangled state preparation to show the effect of measuring this second photon in comparison to tracing it out.

2.1.3 Didactic Material

1. Which are the angle settings for quarterwave plate and polarizer to project onto the 6 polarizations H, V, P, M, R and L?
2. Which results would you expect for an H-polarized prepared state?
3. Measure the single count rate for all 6 settings. Calculate the density matrix using your measurement results.
4. Calculate the Bloch vector and plot the state on the Bloch sphere.
5. Calculate purity, eigenstate of the largest eigenvalue and fidelity to the 6 pure states H, V, P, M, R and L.

Single Photon Tomography

Projection	QWP in °	Pol. In °	Count rate
H			
V			
P			
M			
R			
L			

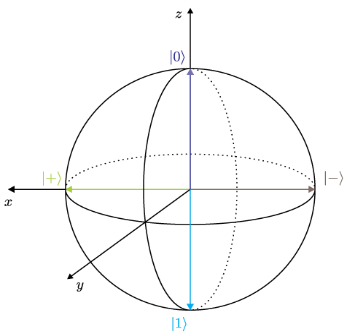
Density Matrix:

Purity:

State	Fidelity
H	
V	
P	
M	
R	
L	

Eigenstate of largest eigenvalue:

Bloch vector:



2.1.4 Sample Solution

For the sample solution please refer to the qutools quED-TOM page `http://qutools.com/quED-TOM`.

2.2 2-Qubit Tomography

A quantum physical state of a qubit can not be determined by a single measurement. Also, a series of measurements on the same quantum object is doomed as well, since the first measurement will influence the state. However, if you have access to an *ensemble* of equally prepared states, as is the case with our SPDC photon source, there is a procedure to measure the full quantum state of this ensemble, named *density matrix*.

2.2.1 Theoretical Background

Tomographic state reconstruction

To determine the full density matrix of an experimentally prepared state, it is necessary to perform measurements in different bases and combine the results accordingly. Commonly, a procedure like that is referred to as *quantum state tomography* (QST). This section focuses on a *linear inversion* scheme for 2 qubits.

Measurements Measurements of the polarization of photons are performed by a *polarization analysis* (PA) setup consisting of a quarterwave plate (QWP) and a polarizer, followed by an APD. For the measurement of circularly polarized photons, the quarterwave plate at 45° rotates the polarization

$$U_{\mathrm{QWP}}(45°)|R\rangle = \mathrm{e}^{\mathrm{i}\phi_a}|V\rangle \tag{2.5}$$

(with arbitrary global phase ϕ_a) such that a passing of the polarizer at 90° corresponds to a projection onto $|R\rangle$. All angle settings for measuring the important single qubit states are listed in Table 2.1. In a typical experiment, the source produces a stream of identically prepared photons, so that one can count the detection events c_i^s in the APD i occurring during a given time for every setting $s \in \{1,2,3\} \equiv \{x,y,z\}$.

Performing all of these 6 measurements on a single qubit corresponds to an *overcomplete tomography*, see above. It can be generalized to 2 spatially separated qubits encoded on the polarizational degree of freedom of a photon pair straightforwardly. One polarization analysis setup is needed for every photon, and the 6 settings given in Table 2.2 have to be combined in every possible way, resulting in $6^2 = 36$ measurement settings. Instead of only single counts c_i, coincidences $c_{i_1 i_2}$ are recorded.

Using the count rates, the respective frequencies

$$f_{i_1 i_2} = \frac{c_{i_1 i_2}}{N_{s_1 s_2}} \tag{2.6}$$

can be determined with total counts $N_{s_1 s_2} = \sum_{i_1, i_2} c_{i_1 i_2}$ of one basis. For example, in the zz basis there are the settings hh, hv, vh, and vv. Therefore, the frequency

$$f_{hh} = \frac{c_{hh}}{c_{hh} + c_{hv} + c_{vh} + c_{vv}} \tag{2.7}$$

is calculated with the use of all measurements in that basis.

Linear inversion For state reconstruction by *linear inversion* (LIN), the expression (**??**) for the density matrix

$$\rho = \frac{1}{2^n} \sum_{s_1, \cdots, s_n = 0}^{3} T_{s_1 \cdots s_n} \bigotimes_{j=1}^{n} \sigma_{s_j} \tag{2.8}$$

is used with the correlation tensor T. For the 2 qubit state that means

$$\rho = \frac{1}{4} \begin{pmatrix} T_{00}+T_{0z}+T_{z0}+T_{zz} & T_{0x}-\mathrm{i}T_{0y}+T_{zx}-\mathrm{i}T_{zy} & T_{x0}+T_{xz}-\mathrm{i}T_{y0}-\mathrm{i}T_{yz} & T_{xx}-\mathrm{i}T_{xy}-\mathrm{i}T_{yx}-T_{yy} \\ T_{0x}+\mathrm{i}T_{0y}+T_{zx}+\mathrm{i}T_{zy} & T_{00}-T_{0z}+T_{z0}-T_{zz} & T_{xx}+\mathrm{i}T_{xy}-\mathrm{i}T_{yx}+T_{yy} & T_{x0}-T_{xz}-\mathrm{i}T_{y0}+\mathrm{i}T_{yz} \\ T_{x0}+T_{xz}+\mathrm{i}T_{y0}+\mathrm{i}T_{yz} & T_{xx}-\mathrm{i}T_{xy}+\mathrm{i}T_{yx}+T_{yy} & T_{00}+T_{0z}-T_{z0}-T_{zz} & T_{0x}-\mathrm{i}T_{0y}-T_{zx}+\mathrm{i}T_{zy} \\ T_{xx}+\mathrm{i}T_{xy}+\mathrm{i}T_{yx}-T_{yy} & T_{x0}-T_{xz}+\mathrm{i}T_{y0}-\mathrm{i}T_{yz} & T_{0x}+\mathrm{i}T_{0y}-T_{zx}-\mathrm{i}T_{zy} & T_{00}-T_{0z}-T_{z0}+T_{zz} \end{pmatrix} \tag{2.9}$$

Full correlations can be calculated directly

$$T_{s_1 \cdots s_n} = \sum_{i_1, \cdots, i_n = 0}^{1} g(i_1, \cdots, i_n) f_{i_1 \cdots i_n} \tag{2.10}$$

Table 2.2: **Settings of the QWP and polarizer for polarization analysis** A click in the APD while having the components set at certain angles corresponds to a projection onto a specific state. For every setting, the experiment runs for a fixed time and the number of counts in each APD can be measured.

Projection onto	Angle of QWP	Angle of polarizer	Measured quantity
$\lvert H\rangle$	0°	0°	c_h
$\lvert V\rangle$	0°	90°	c_v
$\lvert P\rangle$	45°	45°	c_p
$\lvert M\rangle$	45°	−45°	c_m
$\lvert R\rangle$	45°	90°	c_r
$\lvert L\rangle$	45°	0°	c_l

from the observed frequencies, where $g = (-1)^{\sum_j i_j}$ ($h, p, r = 0$ and $v, m, l = 1$) is a parity function determining the sign of the contributions. For example,

$$T_{zx} = f_{hp} - f_{hm} - f_{vp} + f_{vm} \ . \tag{2.11}$$

Non-full correlations can be determined from multiple settings, e.g. T_{z0} could be calculated by using results from measuring either in the setting zx, zy or zz, because the measurement result on the second qubit just has to be neglected. Consequently, $T_{s_1 0}$ can be calculated using e.g. the setting $S = s_1 x$,

$$T_{s_1 0} = \sum_{i_1, i_2 = 0}^{1} g'(s_1, x, i_1, i_2) f_{i_1 i_2} \ , \tag{2.12}$$

with a different parity function $g' = (-1)^k$, where

$$k = \sum_{j=1}^{2} \left(\delta_{s_j, x} + \delta_{s_j, y} + \delta_{s_j, z}\right) i_j \tag{2.13}$$

counts only relevant results of nonzero settings s_j. To reduce the variance of these non-full correlations, one takes the mean over all possible ways to compute it. For example,

$$T_{z0} = \frac{1}{3}\left((f_{hp} + f_{hm} - f_{vp} - f_{vm}) + (f_{hr} + f_{hl} - f_{vr} - f_{vl}) + (f_{hh} + f_{hv} - f_{vh} - f_{vv})\right) \ . \tag{2.14}$$

The T_{00} component is always equal to 1.

2.2.2 Realization with the quED

Necessary Components

For the manual version, you need

- quED base unit with quCR and polarizers in manual rotation mounts
- 2 QWP in manual rotation mounts

For the motorized version, you need

- quED base unit with quCR and polarizers in motorized rotation mounts
- USB Hub
- 2 qu3MD motor drivers
- 2 QWP in motorized rotation mounts

Experimental description

Put a polarizer and a quarterwave plate in both beam paths of the quED as shown in Fig. 2.4 and Fig. 2.5 for the manual and motorized version, respectively. With this setup, the photon pairs can be projected onto every possible polarization state. Please note that with the motorized version, it is important how the motors are connected to the motor drivers. Both polarizers will be driven by one qu3MD, the QWPs by the other. Always connect the motor with lower serial number to socket 1 of the qu3MD and take care to connect the qu3MDs to the correct USB port of the USB-Hub. You can test the configuration by using the motor control tab in the quCR. Also, please make sure that the motors will initialize to the proper zero position when powering up the motor drivers.

Set the 4 optical components in the two arms to the 36 measurement settings and record a count rate for each setting. You can use the *quAPP 2-Qubit Tomography screen* (see Fig. 2.6 and Fig. 2.7) of the quCR for data acquisition and analysis even in the manual version by tapping on the box that corresponds to the set angles of the rotation mounts.

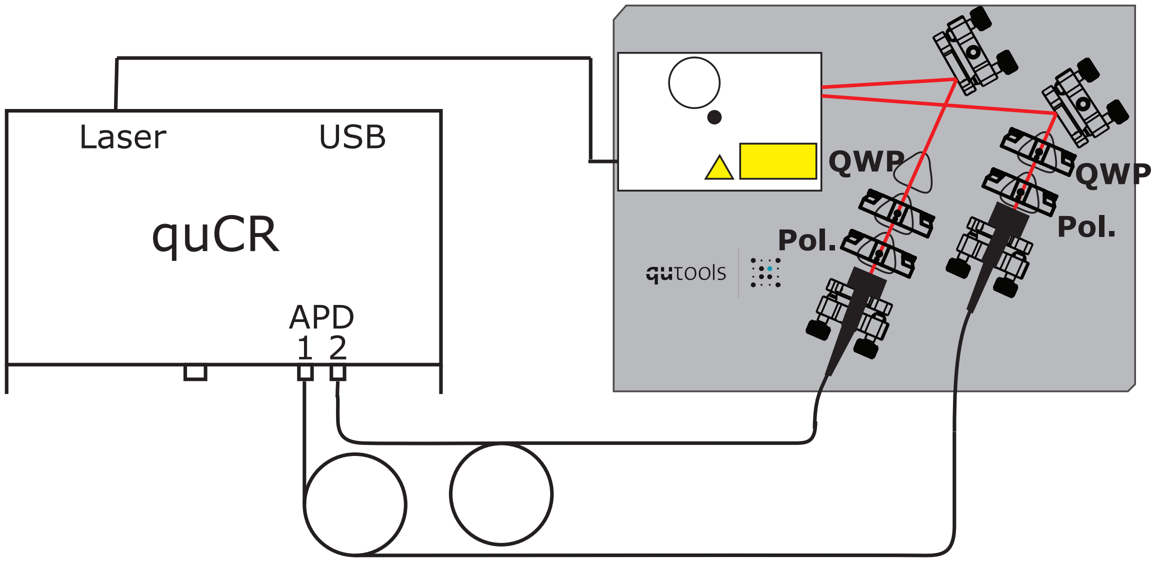


Figure 2.4: Setup of the manual 2-photon tomography with the quED.

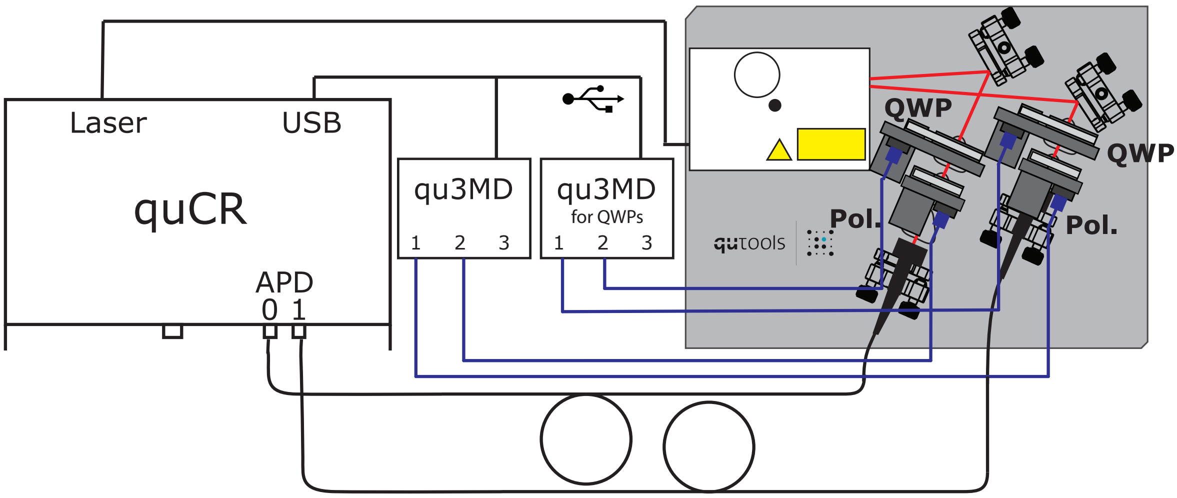


Figure 2.5: Setup of the motorized 2-photon tomography with the quED.

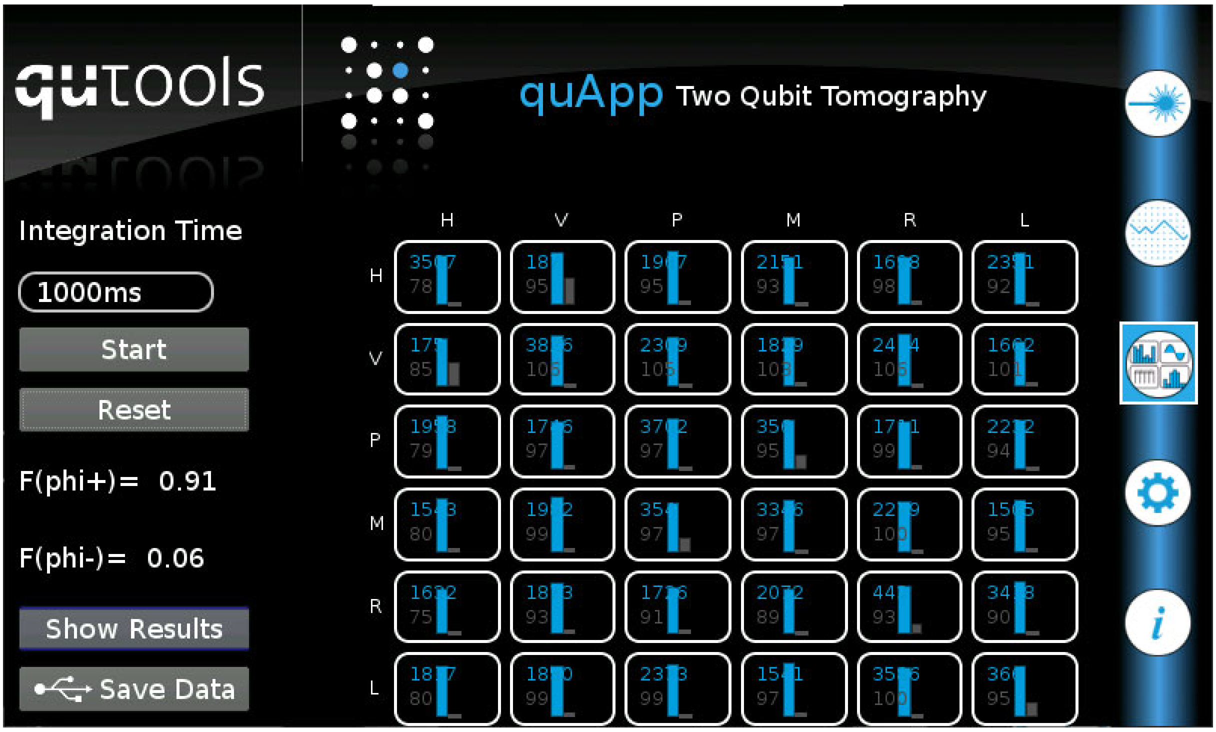


Figure 2.6: Data acquisition screen for the 2-qubit Tomography.

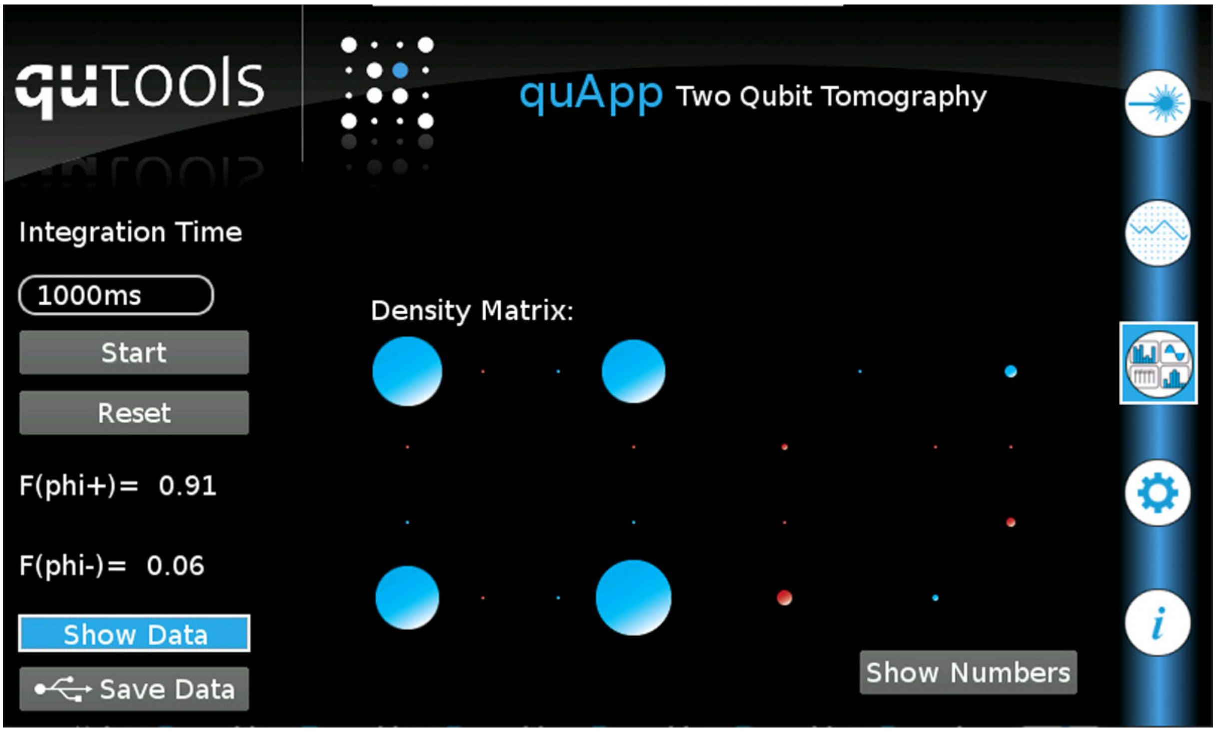


Figure 2.7: Graphical representation of the reconstructed density matrix of an entangled $|\phi_+\rangle$ state. On the left, the real part of the density is depicted while the imaginary part is on the right. The radius of a circle is determined by the absolute value of the density matrix element, while the color shows the sign: blue for positive values, red for negative values. One can also show and hide the actual numbers.

2.2.3 Didactic Material

1. What are the angle settings of the QWPs and the polarizers to determine c_{pr}?
2. What other measurements are necessary to determine the frequency f_{pr}?
3. How does one calculate the correlation tensor element T_{xy}?
4. Calculate the tensor product of the Pauli matrices σ_x and σ_y and show how this measurements acts on different elements of the density matrix.
5. Perform the full state tomography, reconstruct the density matrix and calculate the fidelity to the theoretical $|\phi_+\rangle$ state and the purity of the reconstructed state.

2.2.4 Sample Solution

For the sample solution please refer to the qutools quED-TOM page `http://qutools.com/quED-TOM`.