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MAT2002 – Applications of Differential and Difference Equations

Experiment 1: Fourier Series and Harmonic Analysis

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Experiment 1–A: Fourier Series

MATLAB Commands Used:

`syms var1 var2`	Creates symbolic variables var1 and var2
`disp(x)`	Displays the contents of x without printing the variable name
`int(expr,var,a,b)`	Evaluates the definite integral of expr with respect to var from a to b.
`ezplot(fun,[xmin,xmax])`	Plot the function fun over the domain (xmin, xmax)

MATLAB Code:

```
clear all
clc
syms x
f=input('enter the function:');
i=input('enter interval in [a,b] form:')
m=input('enter number of harmonics:')
a=i(1);
b=i(2);
l=(b-a)/2;

a0=(1/l)*int(f,a,b);
fx=a0/2;
```

```
for n=1:m
    figure;
    an(n)=(1/l)*int(f*cos(n*pi*x/l),x,a,b);
    bn(n)=(1/l)*int(f*sin(n*pi*x/l),x,a,b);
    fx=fx+an(n)*cos(n*pi*x/l) + bn(n)*sin(n*pi*x/l);
    fx=vpa(fx,4);
    ezplot(fx,[a,b]);
    hold on
    ezplot(f,[a,b]);
    title(['Fourier Series with ',num2str(n), ' harmonic']);
    legend('Fourier Series', 'Function Plot');
    hold off
end
disp(strcat('Fourier Series with', num2str(n), 'harmonics is: ',
char(fx)))
```

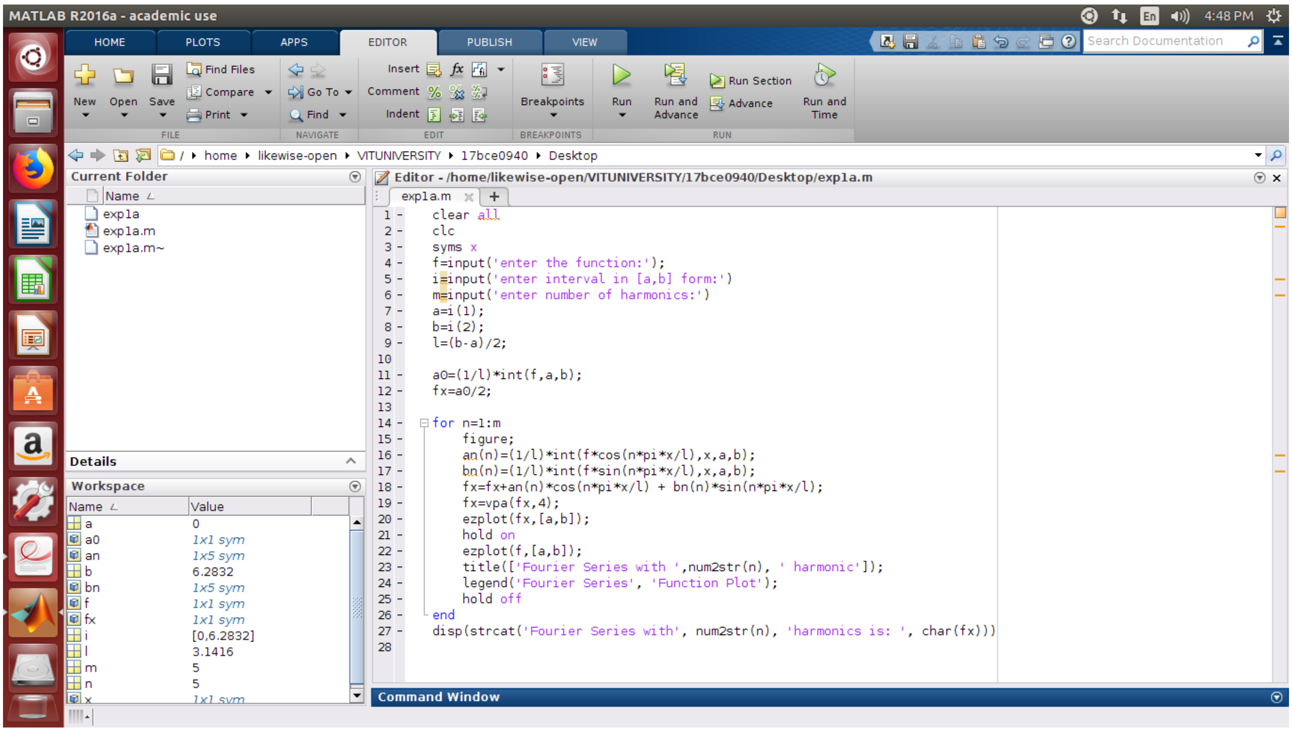
Exercise Problem 1:

Find the Fourier series expansion of the following functions:

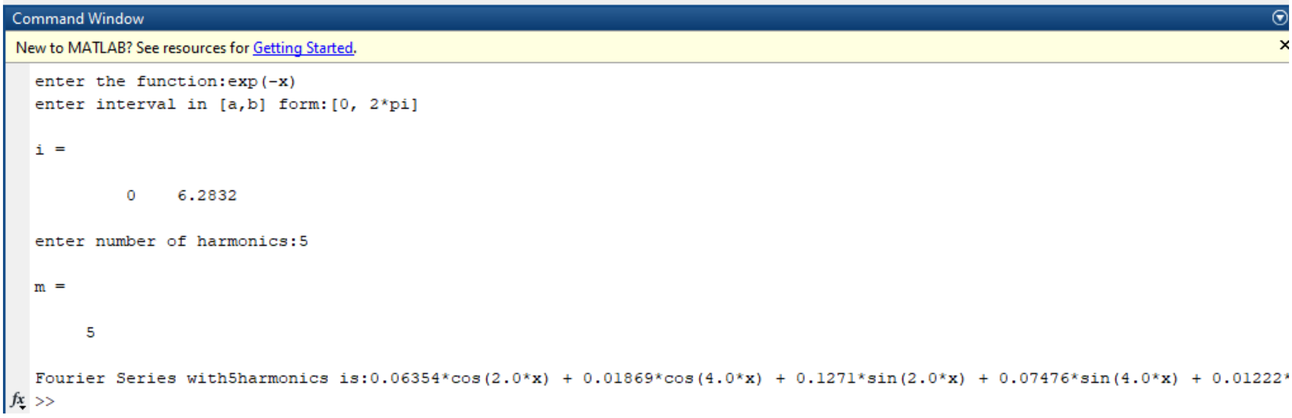
$f(x) = e^{-x}$ in the interval $0 < x < 2\pi$, given that $f(x + 2\pi) = f(x)$.

Solution:

Screenshots of MATLAB work area:



Input for given problem:

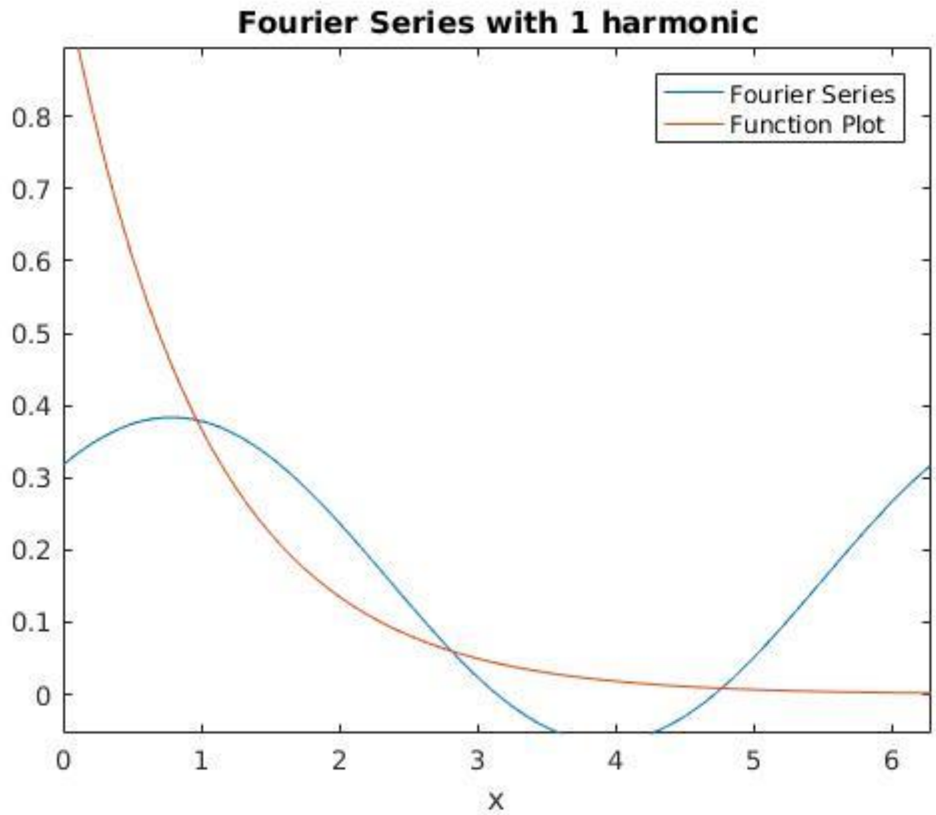


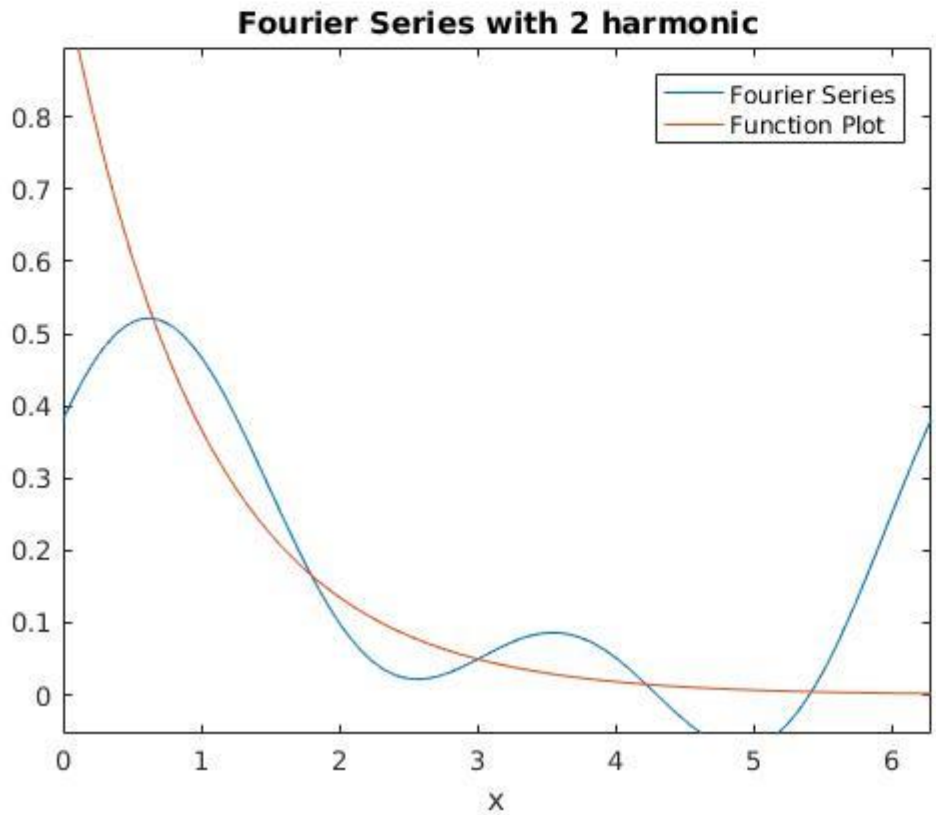
```
Command Window
New to MATLAB? See resources for Getting Started.
enter the function:exp(-x)
enter interval in [a,b] form:[0, 2*pi]

i =

         0    6.2832

enter number of harmonics:5

m =

     5

Fourier Series with5harmonics is:0.06354*cos(2.0*x) + 0.01869*cos(4.0*x) + 0.1271*sin(2.0*x) + 0.07476*sin(4.0*x) + 0.01222*
>>
```

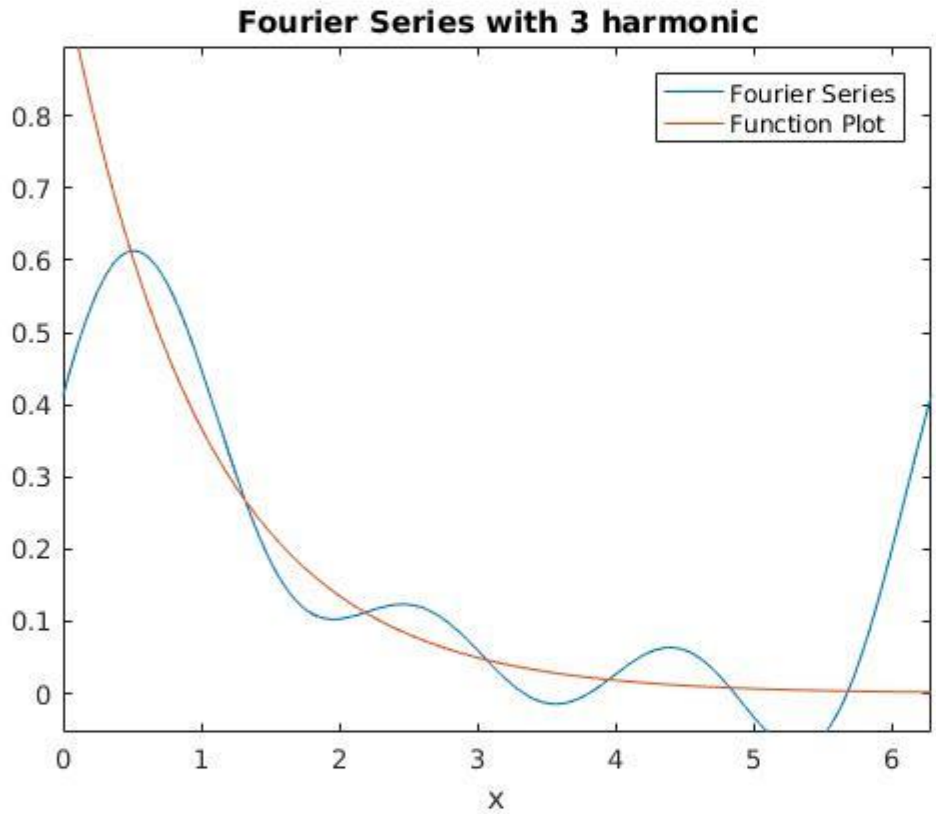
By giving above input, we got the following FS:
Fourier Series with5harmonics is:0.06354*cos(2.0*x) + 0.01869*cos(4.0*x) + 0.1271*sin(2.0*x) + 0.07476*sin(4.0*x) + 0.01222*cos(5.0*x) + 0.0611*sin(5.0*x) + 0.03177*cos(3.0*x) + 0.09531*sin(3.0*x) + 0.1589*cos(x) + 0.1589*sin(x) + 0.1589

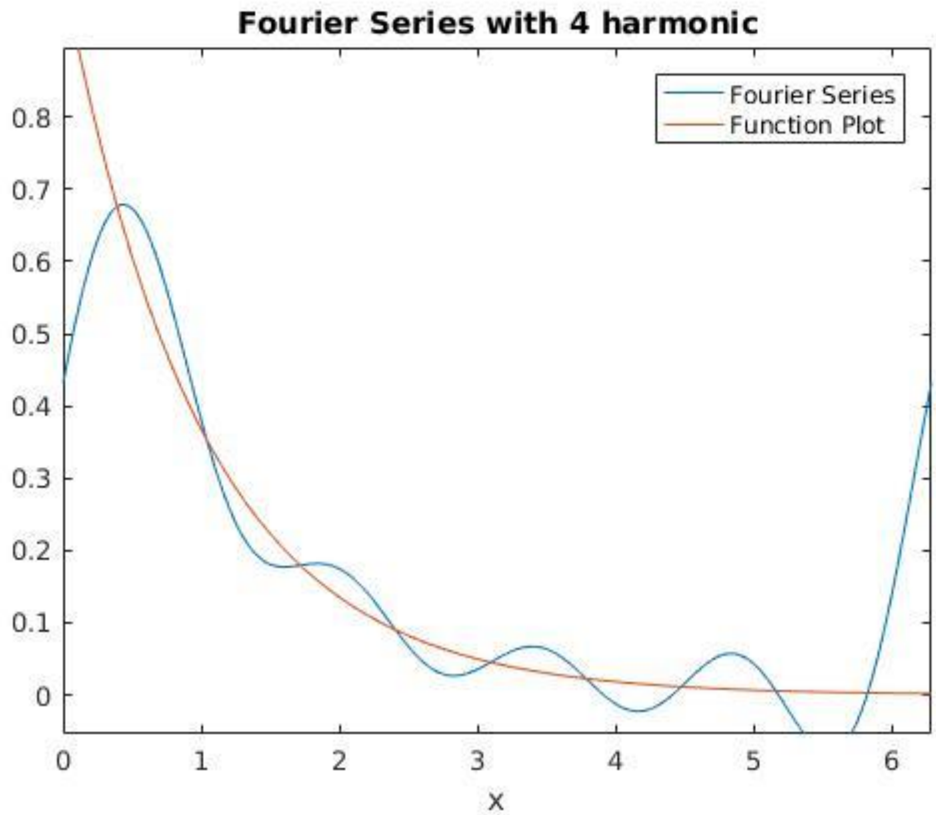
Output Plots:

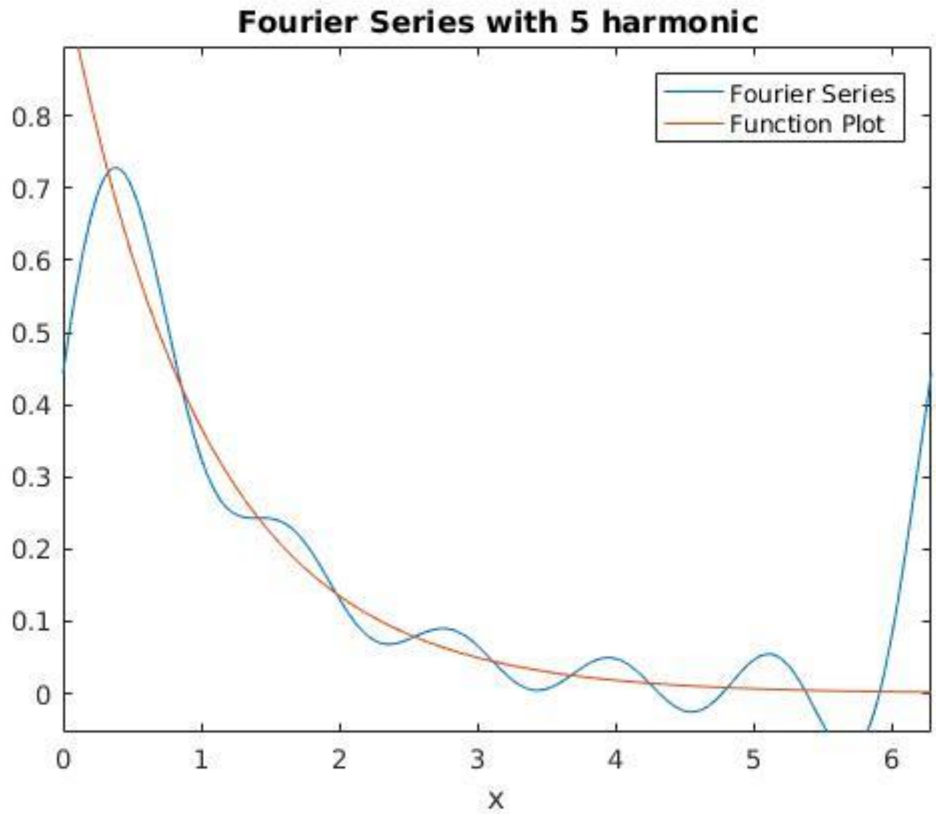
Since given input for ***5 harmonics***, the corresponding graphs of function and Fourier series are as following:




Fourier Series with 2 harmonic
Fourier Series
Function Plot
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
x


Fourier Series with 3 harmonic
Fourier Series
Function Plot
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
x


Fourier Series with 4 harmonic
Fourier Series
Function Plot
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
x


Fourier Series with 5 harmonic
Fourier Series
Function Plot
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
x

Analysis of the Result:

Fourier series is an infinite series. By adding a greater number of harmonics of given function, we can see in the plot that difference between both curves are decreasing. If we consider infinite harmonics, then function plot and Fourier series plot will be very closure.

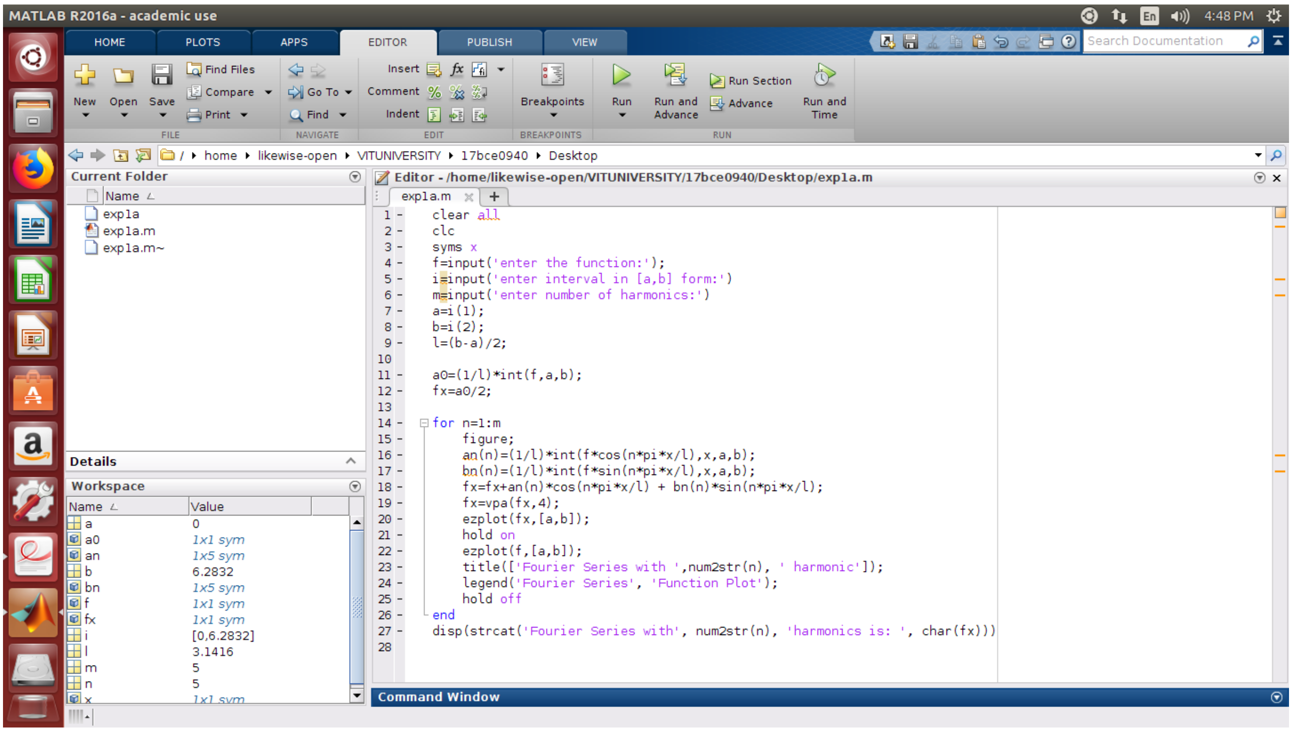
Exercise Problem 2:

Find the Fourier series expansion of the following functions:

$$f(x)=\begin{cases}-1; -2<x<0 \\ 1; 0<x<2\end{cases}, \quad f(x+4)=f(x)$$

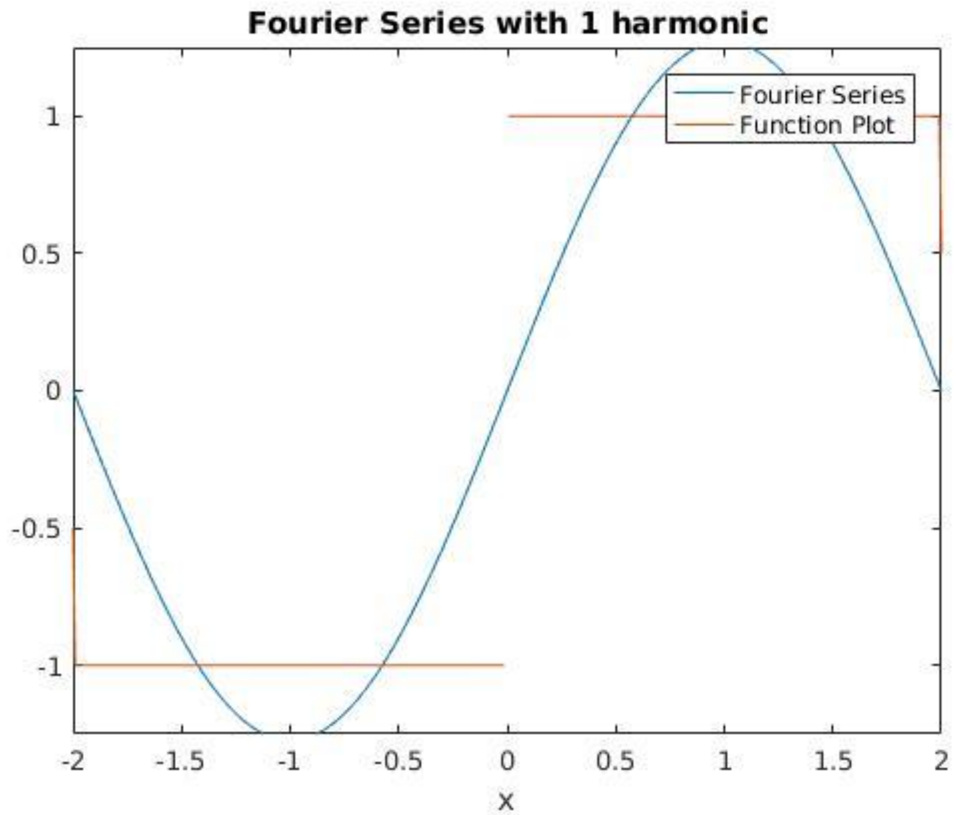
Solution:

Screenshots of MATLAB work area:

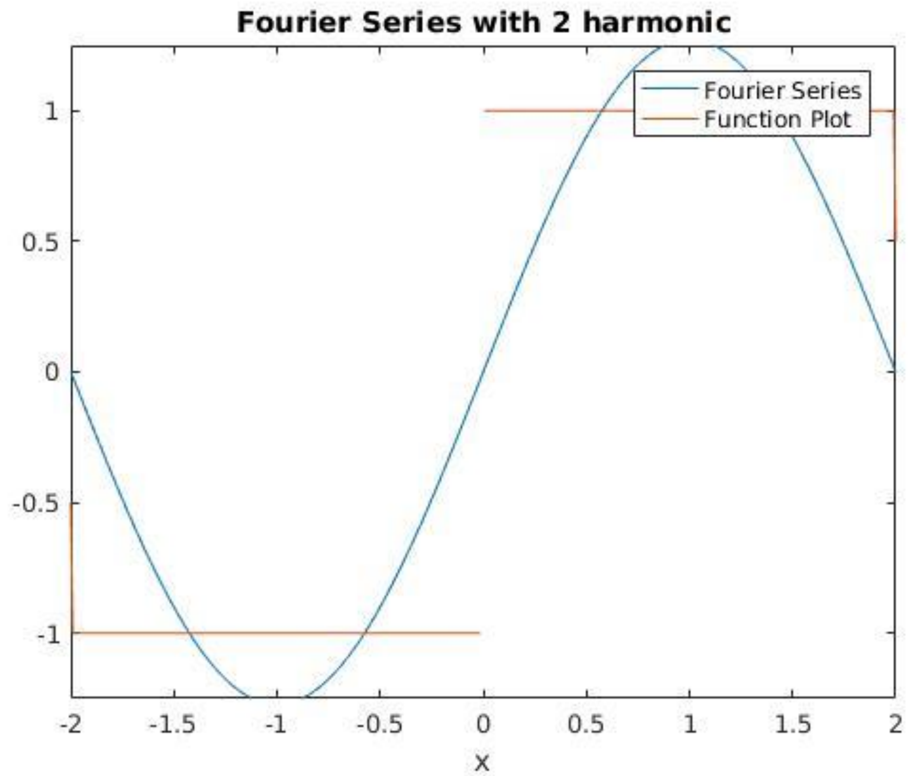


Output Plots:

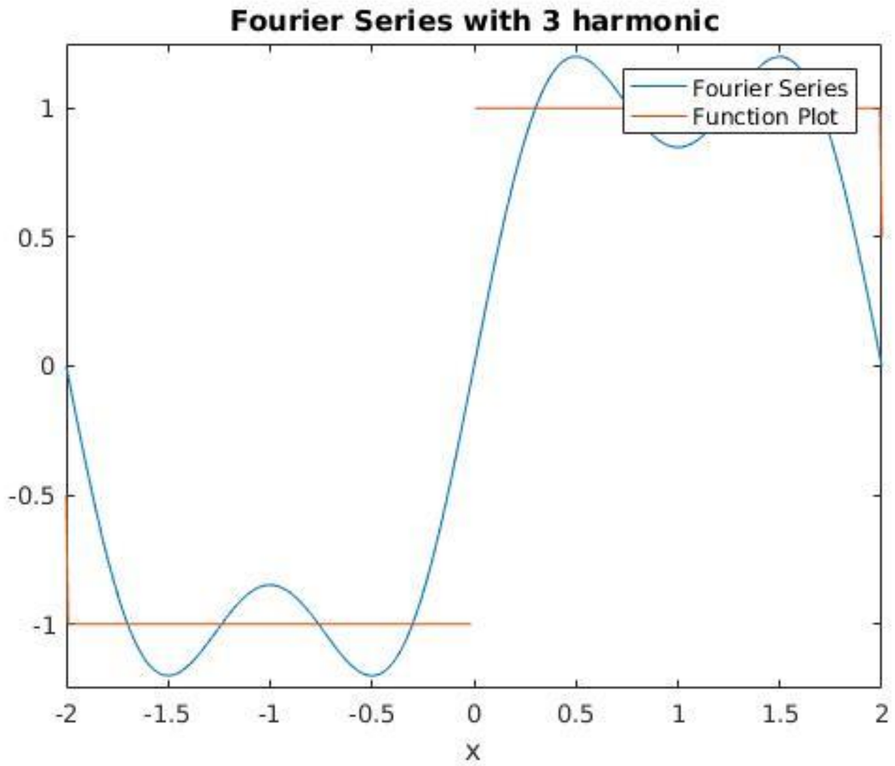
Since given input for ***3 harmonics***, the corresponding graphs of function and Fourier series are as following:



Fourier Series with 1 harmonic
Fourier Series
Function Plot
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x



Fourier Series with 2 harmonic
Fourier Series
Function Plot
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x



Analysis of the Result:

Fourier series is an infinite series. By adding a greater number of harmonics of given function, we can see in the plot that difference between both curves are decreasing. If we consider infinite harmonics, then function plot and Fourier series plot will be very closure.

Experiment 1–B: Harmonic Analysis

MATLAB Commands Used:

`syms var1 var2`	Creates symbolic variables var1 and var2
`disp(x)`	Displays the contents of x without printing the variable name
`length(X)`	returns the length of vector X
`plot(fun)`	Plots the discrete function fun whose domain and range are given.

MATLAB code:

```
clear all
clc
syms t
x=input('Enter the equally spaced values of x: ');
y=input('Enter the values of y=f(x): ');
m=input('Enter the number of harmonics required: ');
n=length(x);a=x(1);b=x(n);
h=x(2)-x(1);
L=(b-a+h)/2;
theta=pi*x/L;
a0=(2/n)*sum(y);
Fx=a0/2; x1=linspace(a,b,100);
for i=1:m
figure
an=(2/n)*sum(y.*cos(i*theta));
bn=(2/n)*sum(y.*sin(i*theta));
Fx=Fx+an*cos(i*pi*t/L)+bn*sin(i*pi*t/L) ;
Fx=vpa(Fx,4);
Fx1=subs(Fx,t,x1);
plot(x1,Fx1);
hold on
plot(x,y);
title(['Fourier Series with ',num2str( i ),'harmonics'])
legend('Fourier Series', 'Function Plot')
hold off;
end
disp(strcat('Fourier series with', num2str(i),'harmonics
is:',char(Fx)));
```

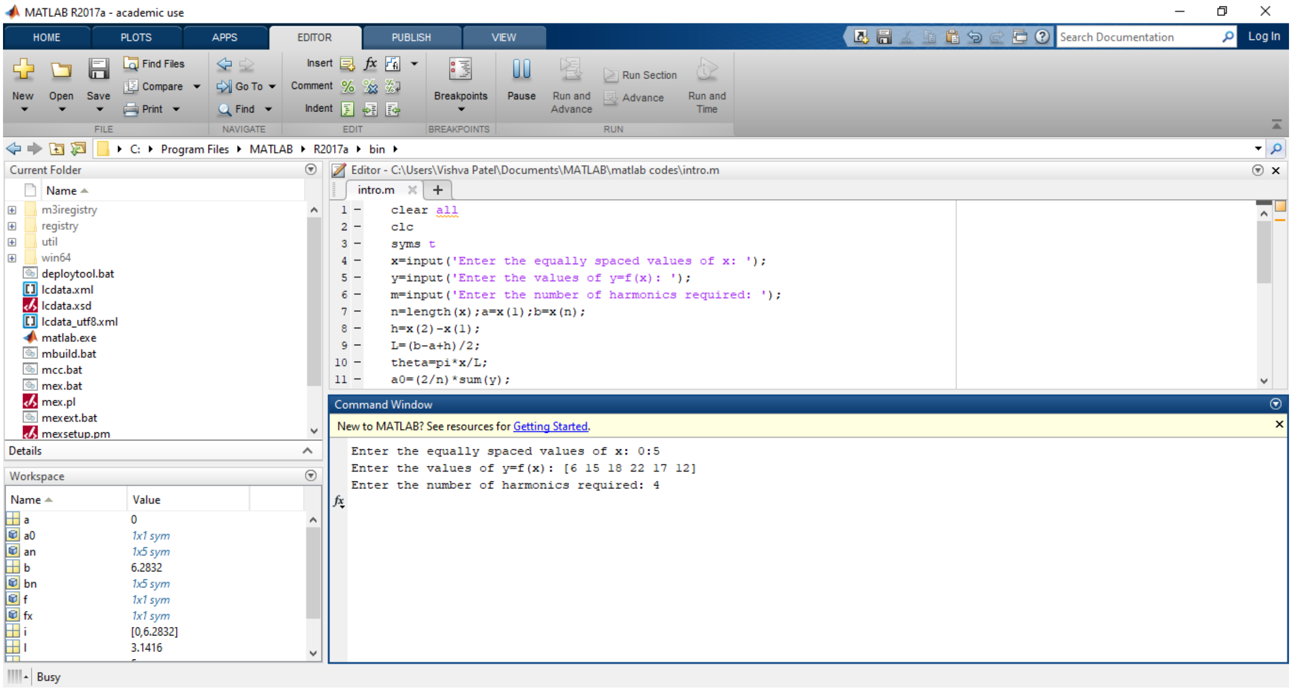
Exercise Problem 1:

Find the constant, the **four** sine and cosine terms in the Fourier series expansion of the function $y = f(x)$ tabulated below:

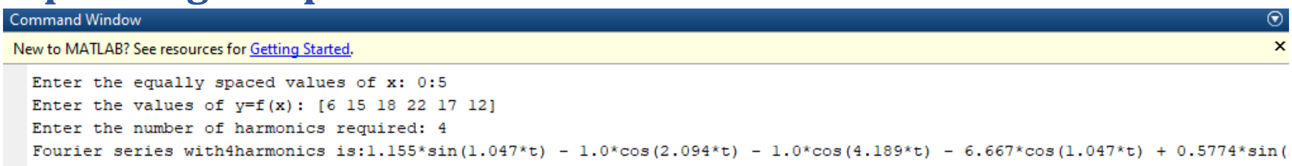
x	0	1	2	3	4	5
$y = f(x)$	6	15	18	22	17	12

Solution:

Screenshots of MATLAB work area:



Input for given problem:

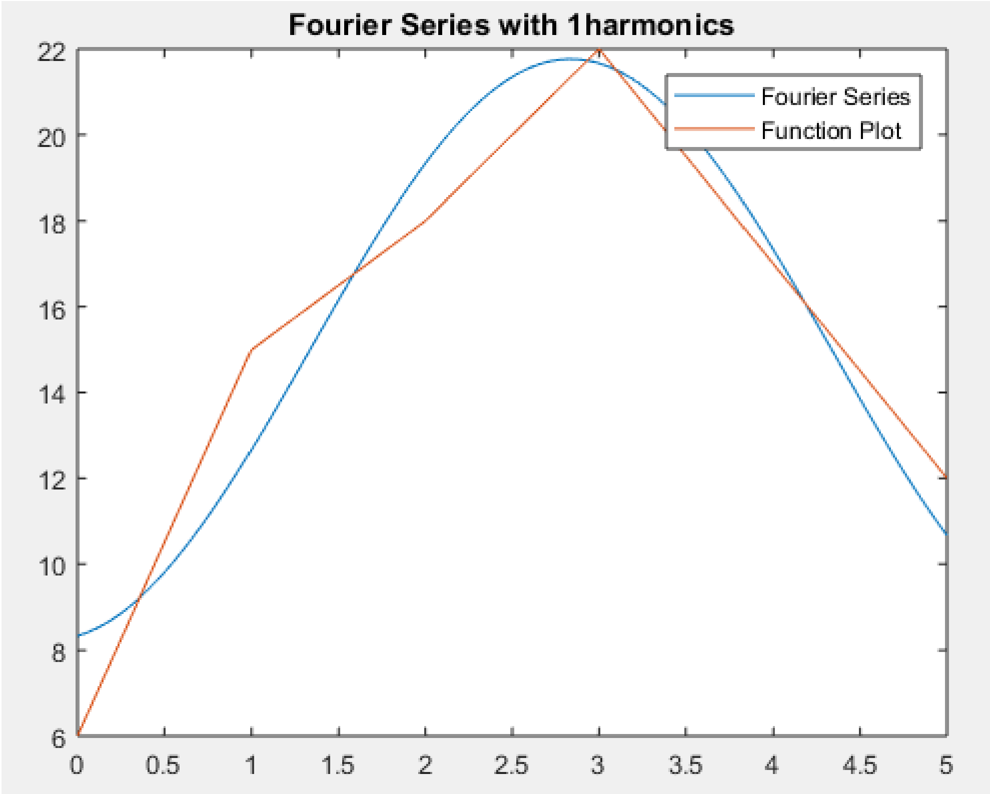


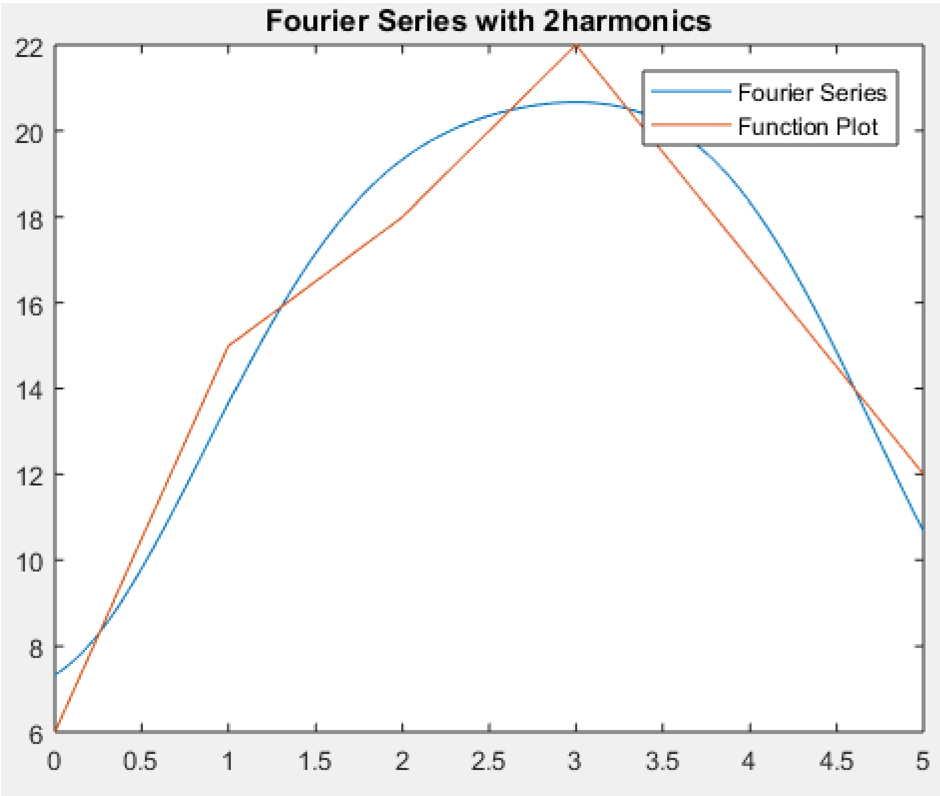
By giving above input, we got the following FS:

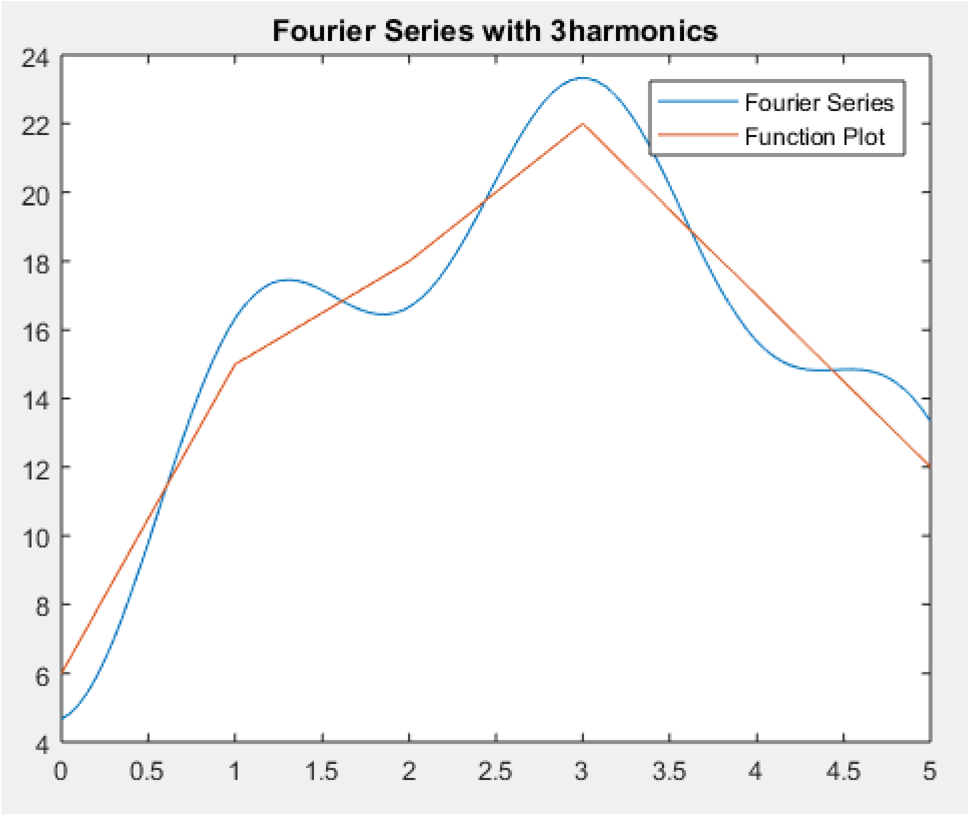
Fourier series with4harmonics is:1.155*sin(1.047*t) - 1.0*cos(2.094*t) - 1.0*cos(4.189*t) - 6.667*cos(1.047*t) + 0.5774*sin(2.094*t) - 0.5774*sin(4.189*t) - 2.667*cos(3.142*t) + 1.51e-15*sin(3.142*t) + 15.0

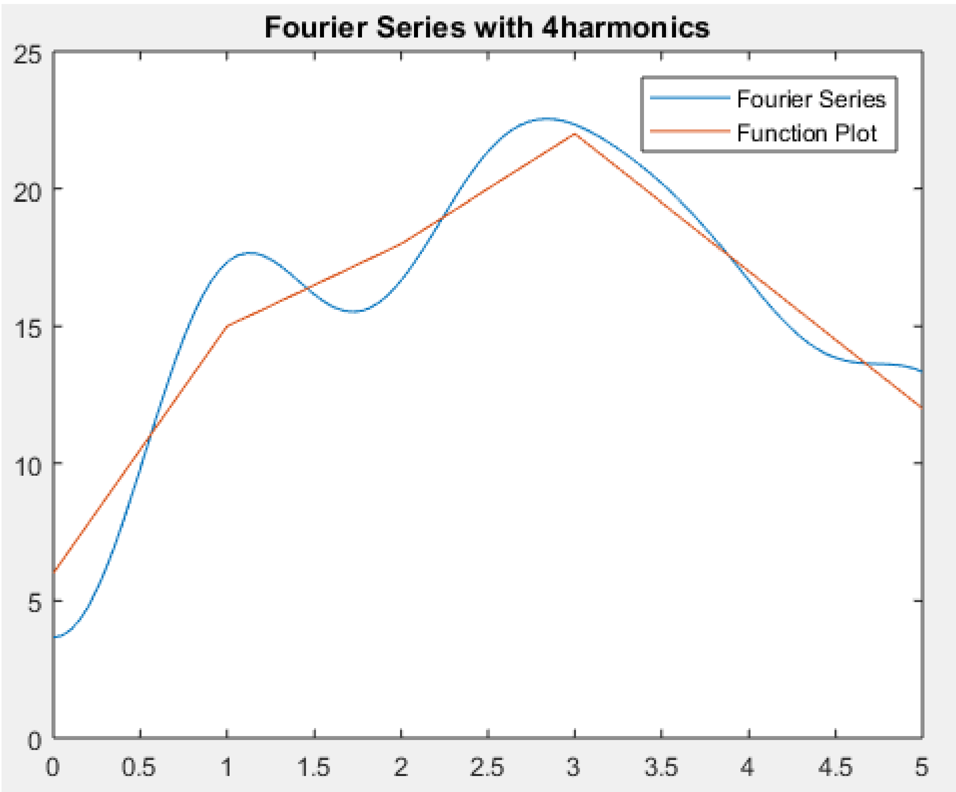
Output Plots:

Since given input for ***4 harmonics***, the corresponding graphs of function and Fourier series are as following:


Fourier Series with 1harmonics
Fourier Series
Function Plot
22
20
18
16
14
12
10
8
6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Fourier Series with 2harmonics
Fourier Series
Function Plot
22
20
18
16
14
12
10
8
6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Fourier Series with 3harmonics
Fourier Series
Function Plot
24
22
20
18
16
14
12
10
8
6
4
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Fourier Series with 4harmonics
Fourier Series
Function Plot
25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Analysis of the Result:

Fourier series is an infinite series. By adding a greater number of harmonics of given function, we can see in the plot that difference between both curves are decreasing. If we consider infinite harmonics, then function plot and Fourier series plot will be very closure.

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