

# From Stationarity to Deep Learning: A Comparative Framework for Stock Price and Volatility Forecasting

Vishvas Ranjan

## Table of contents

<b>Time-Series Model</b>	<b>3</b>
Introduction . . . . .	3
Zero-mean models . . . . .	4
IID noise: . . . . .	4
Simple Symmetric Random Walk: . . . . .	4
Models with trends and seasonality . . . . .	5
Population of the USA, 1790-1990 . . . . .	6
Level of Lake Huron 1875–1972 . . . . .	11
Stationary Models . . . . .	13
Examples: . . . . .	15
Sample Autocorrelation Function . . . . .	21
Illustration of Sample ACF through IID $N(0,1)$ noise . . . . .	22
Lake Huron Residuals Modeling Process . . . . .	29
<b>Estimation and Elimination of Trend and Seasonal Components</b>	<b>36</b>
Estimation and Elimination of Trend in the Absence of Seasonality . .	37
Trend Estimation . . . . .	37
Trend Elimination by Differencing . . . . .	39
Estimation and Elimination of Both Trend and Seasonality . . . . .	41
Estimation of Trend and Seasonal components . . . . .	41
Elimination of Trend and Seasonal Components by Differencing .	42

<b>Testing the Estimated Noise Sequence</b>	<b>44</b>
<b>Basic Properties of Stationary Processes</b>	<b>44</b>
Non-Negative Definite Functions . . . . .	44
Autocovariance Characterization . . . . .	44
Remarks: . . . . .	45
Properties of Strictly Stationary Series $\{X_t\}$ . . . . .	45
Constructing Strictly Stationary Time Series via Filtering: . . . . .	45
The MA(q) Process . . . . .	46
Proposition 1: . . . . .	46
<b>Linear Processes</b>	<b>46</b>
Properties of Linear Processes . . . . .	46
Proposition 2 . . . . .	47
Proof: . . . . .	47
Remark: . . . . .	48
AR(1) Process . . . . .	49
Remark . . . . .	51
<b>ARMA Processes</b>	<b>52</b>
Definition of ARMA(1,1) process . . . . .	52
Invertibility: . . . . .	53
<b>Properties of the sample mean and autocorrelation function</b>	<b>54</b>
Estimation of Sample mean $\mu$ . . . . .	54
Proposition 3 . . . . .	55
Estimation of Autocovariance $\gamma(\cdot)$ and Autocorrelation $\rho(\cdot)$ Functions	56
Example 1: IID Noise . . . . .	57
Example 2: An MA(1) Process . . . . .	60
<b>GARCH Process</b>	<b>61</b>
Definition: GARCH(p,q) . . . . .	61
<b>LSTM Neural Network</b>	<b>62</b>

<b>Comparing Forecasting through ARMA-GARCH with LSTM on Time-Series Data</b>	<b>62</b>
LSTM Model . . . . .	63
Data Preprocessing . . . . .	63
Model Training and Forecasting . . . . .	65
Visual Analysis of Actual vs Predicted (Forecasted) Closing Prices	66
ARMA Method . . . . .	72
Volatility Prediction . . . . .	76
Volatility Prediction through ARMA-GARCH: . . . . .	77
Volatility Prediction through LSTM-GARCH . . . . .	82
Conclusion: Methodological Synergy and Divergence . . . . .	87

## Time-Series Model

### Introduction

A time series model specifies how the random variables  $\{X_t\}$  behave jointly over time. This can be done in two main ways:

1. **Full Joint Distribution:** The model might completely specify the joint probability distribution of  $\{X_t\}$ .
2. **Moments (Means and Covariances):** In many practical situations, especially when dealing with linear processes or Gaussian assumption: it is sufficient to specify just the means and covariances (or autocovariances) of the process. This is because for Gaussian processes, the mean and covariance completely determine the joint distribution.

#### ! Definition

A **time series model** for the observed data  $\{x_t\}$  is a specification of the joint distributions (or possibly only the means and covariances) of a sequence of random variables  $\{X_t\}$  of which  $\{x_t\}$  is postulated to be a realization. Here,  $x_t$  is the single outcome of stochastic process  $X_t$ .