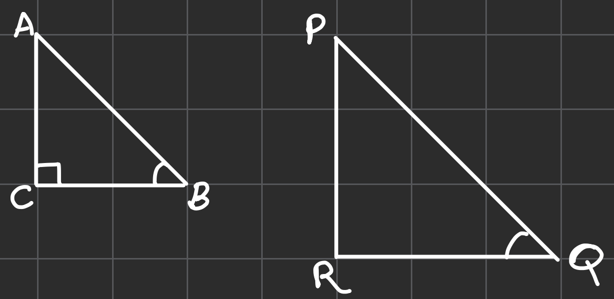
Chapter - 8

Introduction to Trigonometry

1. 

$\sin B = \sin Q$

$$\frac{AC}{AB} = \frac{PR}{PQ}$$

$$\frac{AC}{PR} = \frac{AB}{PQ} = K$$

$$BC^2 = AB^2 - AC^2$$

$$BC = \sqrt{AB^2 - AC^2} \quad - ①$$

$$QR^2 = PQ^2 - PR^2$$

$$QR = \sqrt{PQ^2 - PR^2}$$

$$\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}}$$

$$\frac{BC}{QR} = \frac{\sqrt{K^2PQ^2 - K^2PR^2}}{\sqrt{PQ^2 - PR^2}}$$

$$\frac{BC}{QR} = \frac{\sqrt{K^2(PQ^2 - PR^2)}}{\sqrt{PQ^2 - PR^2}}$$

$$\frac{BC}{QR} = \frac{K\cancel{\sqrt{PQ^2 - PR^2}}}{\cancel{\sqrt{PQ^2 - PR^2}}}$$

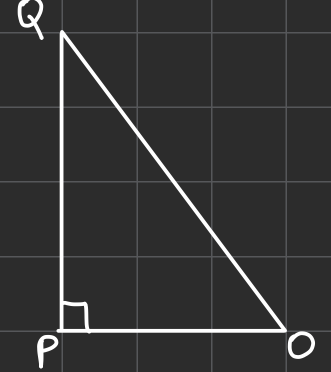
$$\frac{BC}{QR} = K$$

$\therefore \dfrac{AC}{PR} = \dfrac{AB}{PQ} = \dfrac{BC}{QR}$

By SSS rule

$\Delta ABC \sim \Delta PQR$

$\angle B = \angle Q$ ($\because$ By CPST)

2. 

$OP = 7\text{ cm}$

$OQ - PQ = 1\text{ cm}$ — 1

By pythagoras theorem

$OQ^2 = PQ^2 + OP^2$

$OQ^2 = PQ^2 + (7)^2$

$OQ^2 = PQ^2 + 49$

$49 = OQ^2 - PQ^2$ — 2

from 1

$OQ - PQ = 1$

$OQ = 1 + PQ$ — 3

Substituting OQ in 2

$49 = (1 + PQ)^2 - PQ^2$

$49 = 1 + \cancel{PQ^2} + 2PQ \cancel{- PQ^2}$

$49 - 1 = 2PQ$

$48 = 2PQ$

$\dfrac{48}{2} = PQ$

$\boxed{PQ = 24\text{ cm}}$

Putting value of PQ in ③

$OQ = 1 + 24$

$$\boxed{OQ = 25 \text{ cm}}$$

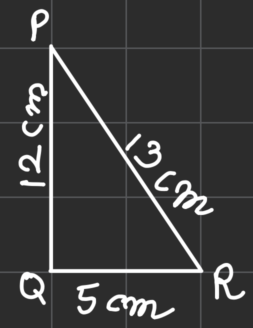
$\sin Q = \frac{OP}{OQ} = \frac{7}{25}$

$$\boxed{\sin Q = \frac{7}{25}}$$

$\cos Q = \frac{PQ}{OQ} = \frac{24}{25}$

$$\boxed{\cos Q = \frac{24}{25}}$$

3.



By pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$(13)^2 = (12)^2 + QR^2$

$169 = 144 + QR^2$

$169 - 144 = QR^2$

$25 = QR^2$

$QR = \sqrt{25}$

$QR = 5 \text{ cm}$

$\tan P - \cot R$

$\frac{QR}{PQ} - \frac{QR}{PQ}$

$\frac{7}{12} - \frac{7}{12} = 0$

$\therefore \tan P - \cot R = 0$

4. $15 \cot A = 8$

$\cot A = \frac{8}{15} = \frac{B}{P}$

Let $B = 8x$
Let $P = 15x$

By pythagoras theorem

$H^2 = P^2 + B^2$
$H^2 = (15x)^2 + (8x)^2$
$H^2 = 225x^2 + 64x^2$
$H^2 = 289x^2$

$H = \sqrt{289x^2}$

$\boxed{H = 17x}$

$\sin A = \frac{P}{H} = \frac{15\cancel{x}}{17\cancel{x}} = \frac{15}{17}$

$\boxed{\sin A = \frac{15}{17}}$

$\sec A = \frac{H}{B} = \frac{17\cancel{x}}{8\cancel{x}} = \frac{17}{8}$

$\boxed{\sec A = \frac{17}{8}}$

5. $\sec \theta = \frac{13}{12} = \frac{H}{B}$

Let $H = 13x$

Let $B = 12x$

$H^2 = B^2 + P^2$

$(13x)^2 = (12x)^2 + P^2$

$169x^2 = 144x^2 + P^2$

$169x^2 - 144x^2 = P^2$

$25x^2 = P$

$P = \sqrt{25x^2}$

$P = 5x$

$\sin \theta = \frac{P}{H} = \frac{5\cancel{x}}{13\cancel{x}} = \frac{5}{13}$

$\boxed{\sin \theta = \frac{5}{13}}$

$\cos \theta = \frac{B}{H} = \frac{12\cancel{x}}{13\cancel{x}} = \frac{12}{13}$

$\boxed{\cos \theta = \frac{12}{13}}$

$\tan \theta = \frac{P}{B} = \frac{5\cancel{x}}{12\cancel{x}} = \frac{5}{12}$

$\boxed{\tan \theta = \frac{5}{12}}$

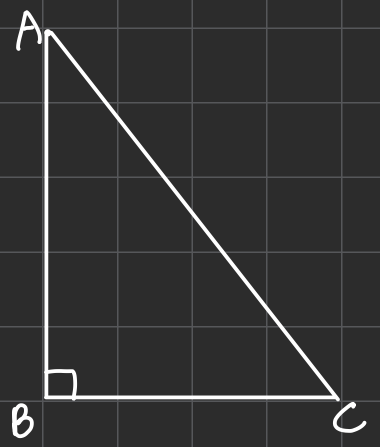
$\operatorname{cosec} \theta = \frac{H}{P} = \frac{13\cancel{x}}{5\cancel{x}} = \frac{13}{5}$

$\boxed{\operatorname{cosec} \theta = \frac{13}{5}}$

$$\cot\theta = \frac{B}{P} = \frac{12\cancel{x}}{5\cancel{x}} = \frac{12}{5}$$

$$\boxed{\cot\theta = \frac{12}{5}}$$

6.



$$\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$$

Let $BC = 1x$

Let $AB = \sqrt{3}\,x$

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = (\sqrt{3}\,x)^2 + (x)^2$$
$$AC^2 = 3x^2 + x^2$$
$$AC^2 = 4x^2$$
$$AC = \sqrt{4x^2}$$

$$AC = 2x$$

$$\sin A = \frac{BC}{AC} = \frac{1\cancel{x}}{2\cancel{x}} = \frac{1}{2}$$

$$\boxed{\sin A = \frac{1}{2}}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}\cancel{x}}{2\cancel{x}} = \frac{\sqrt{3}}{2}$$

$$\boxed{\cos A = \frac{\sqrt{3}}{2}}$$

$$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$

$$\boxed{\sin C = \frac{\sqrt{3}}{2}}$$

$$\cos C = \frac{BC}{AC} = \frac{1x}{2x} = \frac{1}{2}$$

$$\boxed{\cos C = \frac{1}{2}}$$

(i) $\sin A . \cos C + \cos A . \sin C$

$$\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$\frac{1}{4} + \frac{3}{4}$$

$$\frac{1 + 3}{4} = \frac{4}{4} = 1$$

$\therefore \sin A . \cos C + \cos A . \sin C = 1$

(ii) $\cos A . \cos C - \sin A . \sin C$

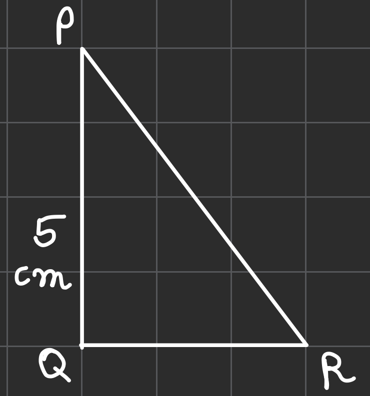
$$\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$\cancel{\frac{\sqrt{3}}{4}} - \cancel{\frac{\sqrt{3}}{4}}$$

$$= 0$$

$\therefore \cos A . \cos C - \sin A . \sin C = 0$

7.



$PR + QR = 25\,cm$ — ①

$PQ = 5\,cm$

By pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$PR^2 = (5)^2 + QR^2$

$PR^2 = 25 + QR^2$

$25 = PR^2 - QR^2$ — ②

from ①

$PR + QR = 25$

$PR = 25 - QR$ — ③

substituting PR in ②

$25 = (25 - QR)^2 - QR^2$

$25 = 625 \cancel{\mp QR^2} - 50QR \cancel{= QR^2}$

$25 - 625 = -50QR$

$-600 = -50QR$

$\frac{\cancel{-600}^{12}}{\cancel{-50}} = QR$

$\boxed{QR = 12\,cm}$

Putting value of QR in ③

$PR = 25 - 12$

$\boxed{PR = 13\,cm}$

$\sin P = \frac{QR}{PR} = \frac{12}{13}$

$$\boxed{\sin P = \frac{12}{13}}$$

$$\cos P = \frac{PQ}{PR} = \frac{5}{13}$$

$$\boxed{\cos P = \frac{5}{13}}$$

$$\tan P = \frac{QR}{PQ} = \frac{12}{5}$$

$$\boxed{\tan P = \frac{12}{5}}$$

8. (i) False

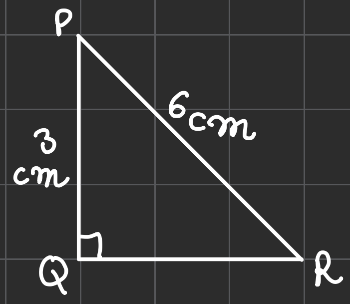
(ii) True

(iii) False

(iv) False

(v) False

9.



$$\sin R = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\sin R = \sin 30^\circ$$

On comparing sides

$\angle R = 30^\circ$

$$\boxed{\angle QRP = 30^\circ}$$

$\angle PQR + \angle QRP + \angle QPR = 180^\circ$

(∵ Angle sum Prop.)

$90^\circ + 30^\circ + \angle QPR = 180^\circ$

$120^\circ + \angle QPR = 180^\circ$

$\angle QPR = 180^\circ - 120^\circ$

$\boxed{\angle QPR = 60^\circ}$

10. $\sin(A - B) = \frac{1}{2}$

$\cos(A + B) = \frac{1}{2}$

$\sin(A - B) = \frac{1}{2} = \sin 30^\circ$

$\sin(A - B) = \sin 30^\circ$

On camparing sides

$A - B = 30^\circ \quad -①$

$\cos(A + B) = \frac{1}{2} = \cos 60^\circ$

$\cos(A + B) = \cos 60^\circ$

On camparing sides

$A + B = 60^\circ \quad -②$

① - ②

$$\begin{array}{r} A \cancel{- B} = 30^\circ \\ + \; A \cancel{+ B} = 60^\circ \\ \hline 2A = 90^\circ \end{array}$$

$A = \frac{90^\circ}{2}$

$\boxed{A = 45^\circ}$

Eram ①

$45^\circ - B = 30$

$B = 45^\circ - 30^\circ$

$\boxed{B = 15^\circ}$

11. $$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$\frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$\frac{\sqrt{3} + 2\sqrt{3} - 4}{4 + \sqrt{3} + 2\sqrt{3}}$$

$$\frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$$

$$\frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$\frac{(3\sqrt{3})^2 + (4)^2 - 2(3\sqrt{3})(4)}{27 - 16}$$

$$\frac{27 + 16 - 24\sqrt{3}}{11}$$

$$\boxed{\frac{43 - 24\sqrt{3}}{11}}$$

12. $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$\frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\frac{5\left(\frac{1}{4}\right) + 4\left(\frac{4}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$\frac{\frac{5}{4} + \frac{16}{3} - 1}{1}$$

$$\frac{5}{4} + \frac{16}{3} - 1$$

$$\frac{15 + 64 - 12}{12}$$

$$\boxed{\frac{67}{12}}$$

13. (i) - (a) $\sin 60^\circ$

(ii) - (d) 0

(iii) - (a) 0

(iv) - (c) $\tan 60^\circ$

14. $\tan(A+B) = \sqrt{3}$

$\tan(A-B) = \dfrac{1}{\sqrt{3}}$

$\tan(A+B) = \sqrt{3} = \tan 60^\circ$

$\tan(A+B) = \tan 60^\circ$

On comparing sides

$A + B = 60^\circ \quad - ①$

$\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$\tan(A-B) = \tan 30^\circ$

On comparing sides

$A - B = 30^\circ \quad - ②$

$$\begin{array}{rl} A \cancel{+B} & = 60^\circ \\ + \quad A \cancel{-B} & = 30^\circ \\ \hline 2A & = 90^\circ \end{array}$$

$2A = 90^\circ$

$A = \frac{90^\circ}{2}$

$\boxed{A = 45^\circ}$

Putting value of A in ①

$45^\circ + B = 60^\circ$

$B = 60^\circ - 45^\circ$

$\boxed{B = 15^\circ}$

15. (i) False

(ii) True

(iii) False

(iv) False

(v) True

15. $\sec A (1 - \sin A)(\sec A + \tan A) = 1$

LHS :

$\frac{1}{\cos A}(1 - \sin A)(\sec A + \tan A)$

$\left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)(\sec A + \tan A)$

$(\sec A - \tan A)(\sec A + \tan A)$

$(\sec A)^2 - (\tan A)^2$

$\sec^2 A - \tan^2 A = 1$

16. $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

$$\frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$

$$\frac{\cancel{\cos A}\left(\frac{1}{\sin A} - 1\right)}{\cancel{\cos A}\left(\frac{1}{\sin A} + 1\right)}$$

$$\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

17. $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \tan\theta + \sec\theta$

LHS:

Diuiding bath numeratar & Denaminatar by $\cos\theta$

$$\frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$\frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$\frac{\tan\theta + \sec\theta - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$\frac{\tan\theta + \sec\theta\,(1 - (\sec\theta - \tan\theta))}{\tan\theta - \sec\theta + 1}$$

$$\frac{\tan\theta + \sec\theta\,\cancel{(1 - \sec\theta + \tan\theta)}}{\cancel{\tan\theta - \sec\theta + 1}}$$

$$\tan\theta + \sec\theta$$

18. $\dfrac{\sec A}{1} = \dfrac{H}{B}$

$H = \sec A$

$B = 1$

$P = \sqrt{\sec^2 A - 1}$

$$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\cos A = \frac{1}{\sec A}$$

$\tan A = \sqrt{\sec^2 A - 1}$

$\operatorname{cosec} A = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

$\cot A = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

19. (i) - (b) 9

(ii) - (c) 2

(iii) - (d) cos A

(iv) - (d) $\tan^2 A$

20. $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

LHS:

$$(\operatorname{cosec}\theta - \cot\theta)^2$$

$$\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$\frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$\frac{(1-\cos\theta)^{\cancel{2}}}{(1+\cos\theta)\cancel{(1-\cos\theta)}}$$

$$\frac{1 - \cos\theta}{1 + \cos\theta} = RHS$$

21. $$\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

LHS:

$$\frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) . \cos A}$$

$$\frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A) . \cos A}$$

$$\frac{1 + 1 + 2 \sin A}{(1 + \sin A) . \cos A}$$

$$\frac{2 + 2 \sin A}{(1 + \sin A) . \cos A}$$

$$\frac{2\cancel{(1+\sin A)}}{\cancel{(1+\sin A)}\cdot\cos A}$$

$$\frac{2}{\cos A}$$

$$2 \times \frac{1}{\cos A}$$

$$2\sec A = RHS$$

22. $$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec.\operatorname{cosec}\theta$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$\left(\frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta-\cos\theta}\right) + \left(\frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta-\sin\theta}\right)$$

$$\frac{\sin^2\theta}{\cos\theta.(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta.(\cos\theta-\sin\theta)}$$

$$\frac{\sin^2\theta(\sin\theta) - \cos^2\theta(\cos\theta)}{\cos\theta.(\sin\theta-\cos\theta)\sin\theta}$$

$$\frac{\sin^3\theta - \cos^2\theta}{\cos\theta.(\sin\theta-\cos\theta)\sin\theta}$$

$$\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta.\cos\theta)}{\cos\theta(\sin\theta-\cos\theta)\sin\theta}$$

$$\frac{\cancel{(\sin\theta - \cos\theta)}\,(1 + \sin\theta.\cos\theta)}{\cos\theta\,\cancel{(\sin\theta - \cos\theta)}\,\sin\theta}$$

$$\frac{1 + \sin\theta.\cos\theta}{\cos\theta.\sin\theta}$$

$$\frac{1}{\cos\theta.\sin\theta} + \frac{\cancel{\sin\theta.\cos\theta}^{\,1}}{\cancel{\sin\theta.\cos\theta}}$$

$$\sec\theta.\operatorname{cosec}\theta + 1 = RHS$$

23. $$\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

LHS :

$$\frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$\frac{\frac{\cos A + 1}{\cancel{\cos A}}}{\frac{1}{\cancel{\cos A}}}$$

$$\cos A + 1$$

RHS :

$$\frac{\sin^2 A}{1 - \cos A}$$

$$\frac{1 - \cos^2 A}{1 - \cos A}$$

$$\frac{\cancel{(1 - \cos A)}(1 + \cos A)}{\cancel{1 - \cos A}}$$

$$1 + \cos A$$

24. $$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

$$\sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$\sqrt{\frac{(1+\sin A)^2}{(1)^2-(\sin A)^2}}$$

$$\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$\sqrt{\frac{(1+\sin A)^2}{(\cos A)^2}}$$

$$\cancel{\sqrt{\left(\frac{1+\sin A}{\cos A}\right)^{\cancel{2}}}}$$

$$\frac{1+\sin A}{\cos A}$$

$$\frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$\sec A + \tan A = RHS$$

25. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\sin^2 A + \operatorname{cosec}^2 A + 2 \sin A . \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cos A . \sec A$$

$$1 + \operatorname{cosec}^2 A + 2 \cancel{\sin A} \times \frac{1}{\cancel{\sin A}} + \sec^2 A + 2 \cancel{\cos A} \times \frac{1}{\cancel{\cos A}}$$

$$1 + \operatorname{cosec}^2 A + 2 + \sec^2 A + 2$$

$$5 + \operatorname{cosec}^2 A + \sec^2 A$$

$$5 + 1 + \cot^2 A + 1 + \tan^2 A$$

$$7 + \cot^2 A + \tan^2 A = RHS$$

26. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

LHS:

$$\left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$\left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$\left(\frac{\cos^{\cancel{2}} A}{\cancel{\sin A}}\right)\left(\frac{\sin^{\cancel{2}} A}{\cancel{\cos A}}\right)$$

$$\cos A \cdot \sin A$$

RHS

$$\frac{1}{\tan A + \cot A}$$

$$\frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$\frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A . \cos A}}$$

$$\frac{\sin A . \cos A}{1}$$

The End!