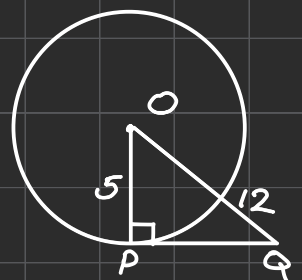
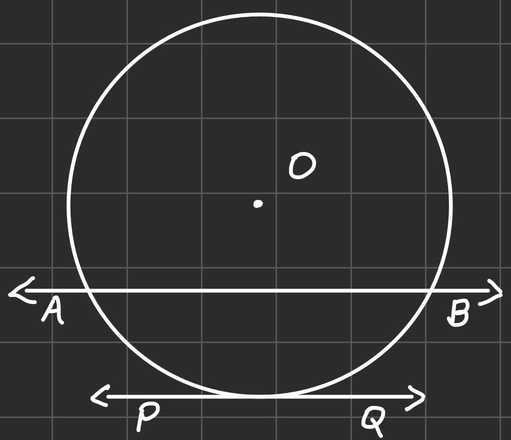
Chapter - 10 : Circles

1)



- (b) 13 cm

2)

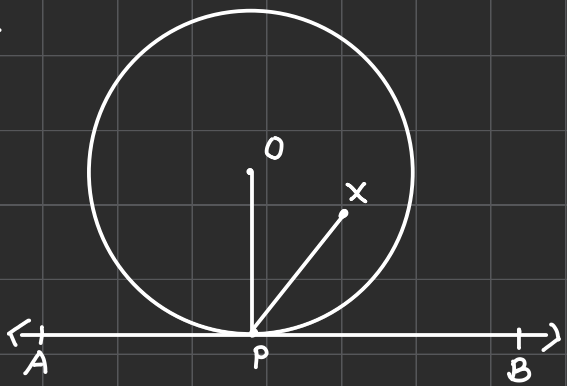


AB is secant
PQ is tangent

③ - ⓑ 70°

④ - ⓐ 50°

⑤ -



Given:

Let us assume with centre O.

AB is tangent at point P.

To prove:

PX passes through the centre.

Solution:

Tangent of circle is $\perp$ to radius at point of contact.

Hence, $OP \perp AB$

$\angle OPB = 90^\circ$ — ①

Now, lets assume some point X,
such that $XP \perp AB$

Hence, $\angle XPB = 90^\circ$ — ②

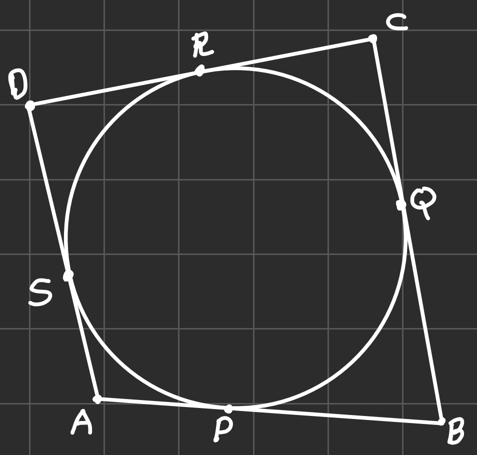
From ① & ②

$\angle OPB = \angle XPB = 90^\circ$

Which is only possible when XP passes through O.

Hence OP passes through the centre.

⑥



Given :

Let ABCD be a quadrilateral circumscribing the circle with centre O.

The quadrilateral touches the circle at points P, Q, R and S.

To prove :

$AB + CD = AD + BC$

Solution :

$AP = AS$ (Th - 10.2)

$BP = BQ$ (Th - 10.2)

$CQ = CR$ (Th - 10.2)

$DR = DS$ (Th - 10.2)

$AB + CD$

$AP + BP + CR + DR$

$AS + BQ + CQ + DS$

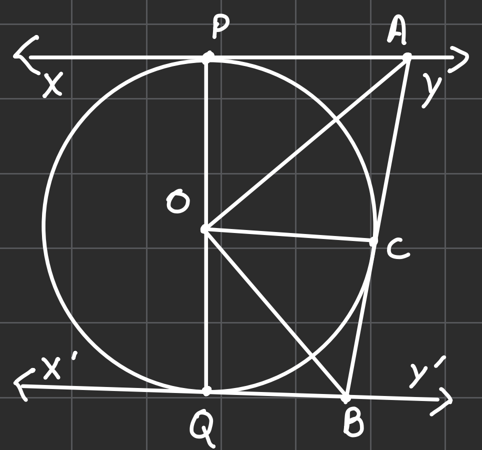
$(AS + DS) + (BQ + CQ)$

$AD + BC$

$\therefore AB + CD = AD + BC$

Hence Proved

7



Given :

$XY \parallel X'Y'$
XY is tangent at P
$X'Y'$ is tangent at Q
AB is tangent at C

To prove :
$\angle AOB = 90^\circ$

Solution:

$\triangle AOP$, $\triangle AOC$
$AO = AO$ (common)
$AP = AC$ (Th-10.2)
$OP = OC$ (Radii)

By SSS rule,
$\triangle AOP \cong \triangle AOC$

$\angle AOP = \angle AOC$ (By CPCT)

similarly, $\triangle BOQ \cong \triangle BOC$

$\angle BOQ = \angle BOC$ (By CPCT)

PQ is a line

$\angle AOP + \angle AOC + \angle BOC + \angle BOQ = 180^\circ$

$\angle AOC + \angle AOC + \angle BOC + \angle BOC = 180^\circ$

$2\angle AOC + 2\angle BOC = 180^\circ$

$2(\angle AOC + \angle BOC) = 180^\circ$

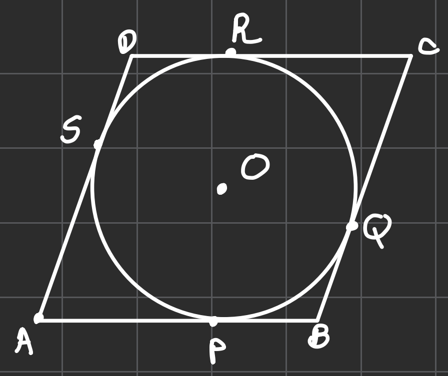
$\angle AOC + \angle BOC = \frac{\cancel{180^\circ}\ 90^\circ}{\cancel{2}\ 1}$

$\angle AOC + \angle BOC = 90^\circ$

$\boxed{\angle AOB = 90^\circ}$

Hence proved

8



Given :

A circle with centre O

A || gm ABCD touching the circle at points P, Q, R and S.

To prove :

ABCD is rhombus (AB = BC = CD = DA)

Solution :

AP = AS (Th - 10.2)

BP = BQ (Th - 10.2)

CQ = CR (Th - 10.2)

DR = DS (Th - 10.2)

$AB = CD$ (|| gm)

$AD = BC$ (|| gm)

$AB + CD$

$AP + BP + CR + DR$

$AS + BQ + CQ + DS$

$AD + BC$

$AB + CD = AD + BC$

$AB + AB = AD + AD$

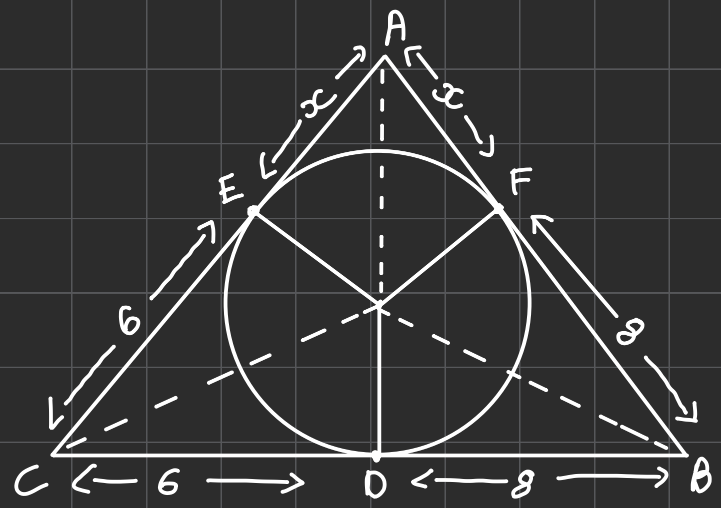
$2AB = 2AD$

$AB = AD$

$$\boxed{AB = BC = CD = AD}$$

Hence ABCD is a rhombus

⑨



Given :

Radius of circle with centre O.

OD = 4 cm

ΔABC circumscribe the centre

BD = 8cm

CD = 6cm

To find :

AC = ?

AB = ?

Contruction :

Jain OA, OC & OB

Let AC, AB intersect at E & F respectively

Solution:

$CE = CD = 6\,cm$

$BF = BD = 8\,cm$

$AE = AF = x\,cm$

$CB = CD + BD = 6 + 8 = 14\,cm$

$AC = AE + EC = (x + 6)\,cm$

$AB = AF + FC = (x + 8)\,cm$

$$s = \frac{AB + CB + AC}{2}$$

$$= \frac{14 + x + 6 + x + 8}{2}$$

$$= \frac{28 + 2x}{2}$$

$$= \frac{2(14 + x)}{}$$

2)

$S = 14 + x$

Heron's formula $= \sqrt{(14+x)(x)(6)(8)}$

$Ar(\triangle ABC) = \sqrt{48x(14+x)}$

$Ar(\triangle ABC) = Ar(\triangle AOB + \triangle BOC + \triangle AOC)$

$Ar\left(\frac{1}{\cancel{2}_1} \times (x+6) \times \cancel{4}^2 + \frac{1}{\cancel{2}_1} \times \cancel{4}^2 \times 14 + \frac{1}{\cancel{2}_1} \times \cancel{4}^2 \times (x+8)\right)$

$Ar(2x + 12 + 28 + 2x + 16)$

$Ar(4x + 56)$

$\sqrt{48x(14+x)} = 4x + 56$

Squaring both sides

$$48x(14+x) = 16x^2 + 3136 + 448x$$
$$672x + 48x^2 = 16x^2 + 3136 + 448x$$
$$32x^2 + 224x - 3136 = 0$$
$$32(x^2 + 7x - 98) = 0$$

$$x^2 + 7x - 98 = 0$$
$$x^2 + 14x - 7x - 98 = 0$$
$$x(x+14) - 7(x+14)$$

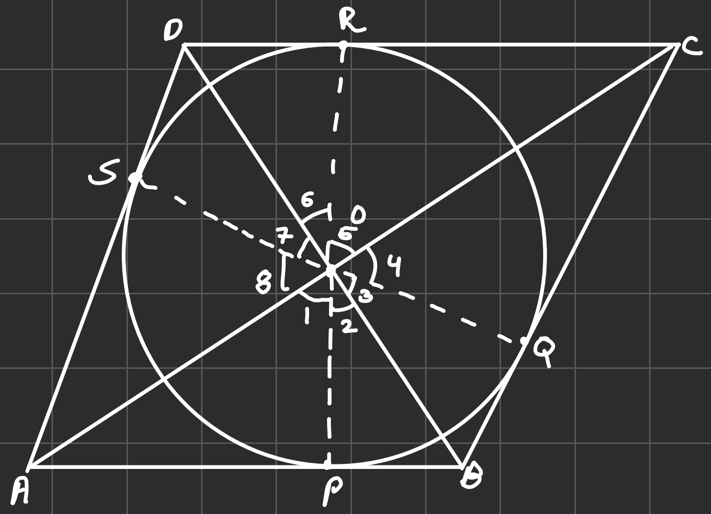
$$(x-7)(x+14)$$

$$x - 7 = 0$$
$$x = 7$$

$$AB = 6 + x$$
$$= 6 + 7$$
$$AB = 13cm$$

$$AC = 8 + 7$$
$$AC = 15\ cm$$

10



Given:

Let ABCD is quadrilateral circumscribing with centre O.
ABCD touches the circle at P, Q, R and S

To prove:

$\angle AOB + \angle DOC = 180°$

$\angle AOD + \angle BOC = 180°$

Construction:

Join OP, OQ & OS

Solution:

$\triangle AOS$, $\triangle AOP$

$AO = AO$ (common)
$AP = AS$ (Th - 10.2)
$PO = SO$ (Radii)

Similarly,
$\triangle BOP \cong \triangle BOQ$
$\triangle COQ \cong \triangle COR$
$\triangle DOR \cong \triangle DOS$

$\angle 2 = \angle 3$
$\angle 4 = \angle 5$
$\angle 6 = \angle 7$

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$
$$\angle 1 + \angle 2 + \angle 2 + \angle 5 + \angle 5 + \angle 6 + \angle 6 + \angle 1 = 360^\circ$$
$$2(\angle 1 + \angle 2 + \angle 5 + \angle 6) = 360^\circ$$
$$\angle 1 + \angle 2 + \angle 5 + \angle 6 = 180^\circ$$
$$\angle AOB + \angle COD = 180^\circ$$

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$\angle 8 + \angle 3 + \angle 3 + \angle 4 + \angle 4 + \angle 7 + \angle 7 + \angle 8 = 360^\circ$$

$$2(\angle 3 + \angle 4 + \angle 7 + \angle 8) = 360^\circ$$

$$\angle 3 + \angle 4 + \angle 7 + \angle 8 = 180^\circ$$

$$\angle AOC + \angle BOC = 180^\circ$$

Hence Proved