Chapter - 10 : Circles

* Ex-10.1

1. A circle can have infinite number of tangent.

2. (i) one

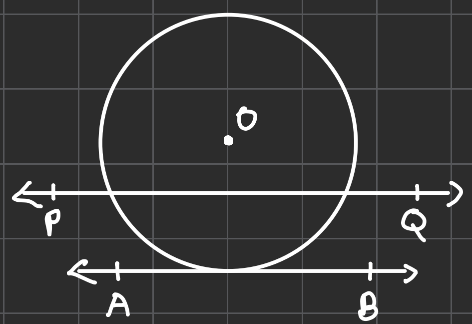
   (ii) secant

   (iii) 2

   (iv) Point of contact

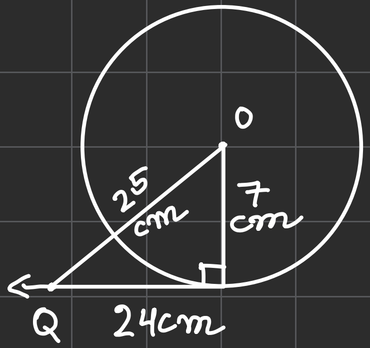
3. - (b) 13 cm

4. -



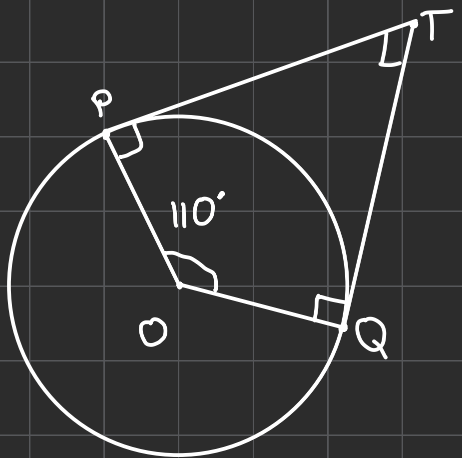
* Ex - 10.2

1.



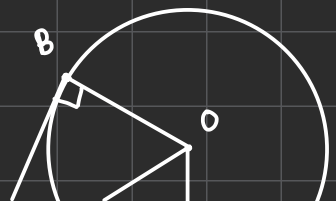
- ⓐ 7 cm

2.



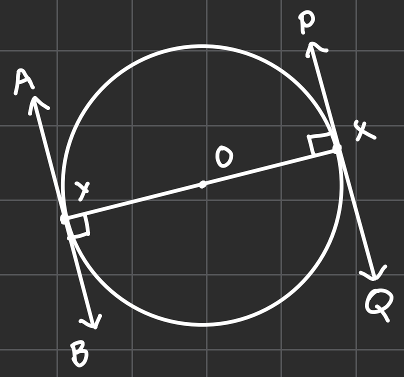
- ⓑ 70°

3.



- @ 50°

4.



Given:
AB is tangent at Y
PQ is tangent at X

To prove:
AB ∥ PQ

Solution:
$\angle AYO = 90^\circ$ (Th-10.1)
$\angle QXO = 90^\circ$ (Th-10.1)

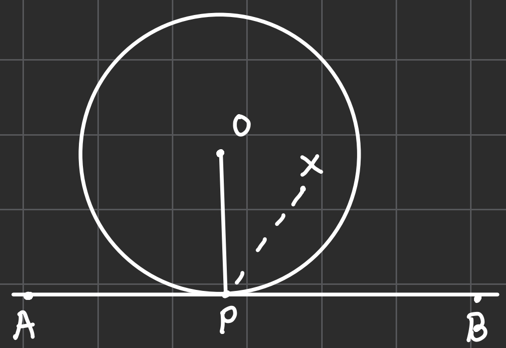
$\angle AYO = \angle QXO$

For lines AB & PQ,
and transversal XY,

$\angle AYO = \angle QXO$ that is aternate interior angle
so, the lines are $\parallel$.

$\therefore AB \parallel PQ$

5.



Given:
$OP \perp AB$

To prove:

OP passes through the centre.

Solution :

$OP \perp AB$

$\angle APO = 90^\circ$ — ①

Let us assume some point X,
such that $XP \perp AB$

Hence, $\angle XPA = 90^\circ$ — ②

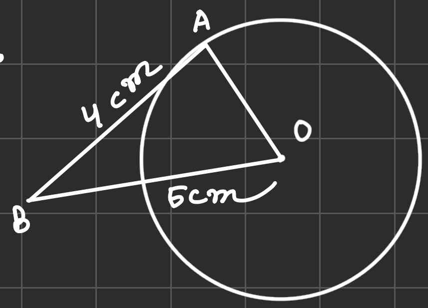
From ① & ②

$\angle APO = \angle XPA = 90^\circ$

Which is only possible when XP passes through O.

Hence, perpendicular to tangent to the circle passes through centre.

6.



Given :

AB is tangent at A

OB = 5cm

AB = 4cm

To find :

AO = ?

Solution

$\triangle AOB$

$\angle OAB = 90^\circ$ (Th-10.1)

By pythagoras theorem

$OB^2 = AB^2 + AO^2$

$5^2 = 4^2 + AO^2$

$25 = 16 + AO^2$

$25 - 16 = AO^2$

$9 = AO^2$

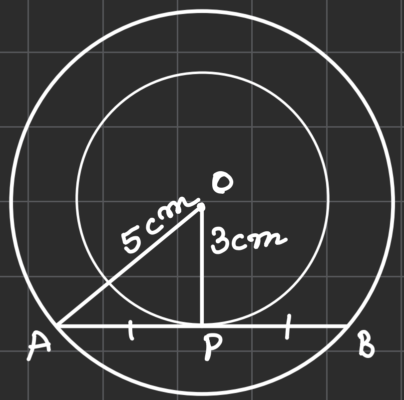
$\sqrt{9} = \sqrt{AO^{\cancel{2}}}$

$3 = AO$

$AO = 3cm$

Hence, the radius of the circle is 3 cm.

7.



Given :

$AO = 5cm$

$PO = 3cm$

To find:

$AB = ?$

Solution:

Since AB is tangent to smaller circle

$OP \perp AB$

Perpendicular from the centre of chord bisects the chord

$AP = BP$

$AB = AP + BP$

$AB = 2AP$

By pythagoras theorem

$AP^2 + OP^2 = AO^2$

$AP^2 + 3^2 = 5^2$

$AP^2 = 5^2 - 3^2$

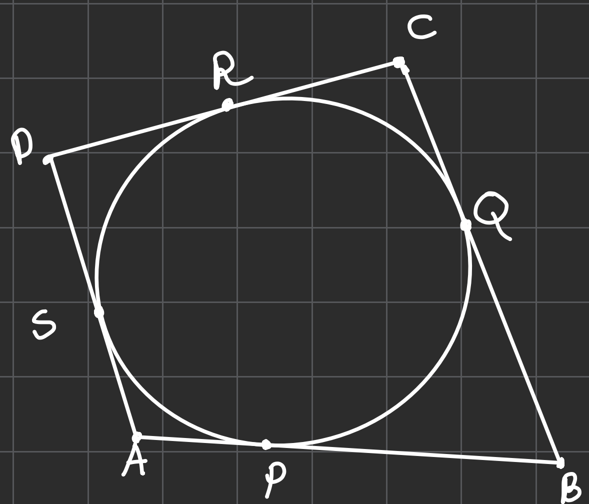
$AP^2 = 25 - 9$

$$AP^2 = 16$$
$$\sqrt{AP^{\cancel{2}}} = \sqrt{16}$$
$$AP = 4 \text{ cm}$$

$$AB = 2\,AP$$
$$= 2(4)$$

$$\boxed{AB = 8 \text{ cm}}$$

8.



Given :
Quadrilateral is circumscribing a circle.

To prove :

$AB + CD = AD + BC$

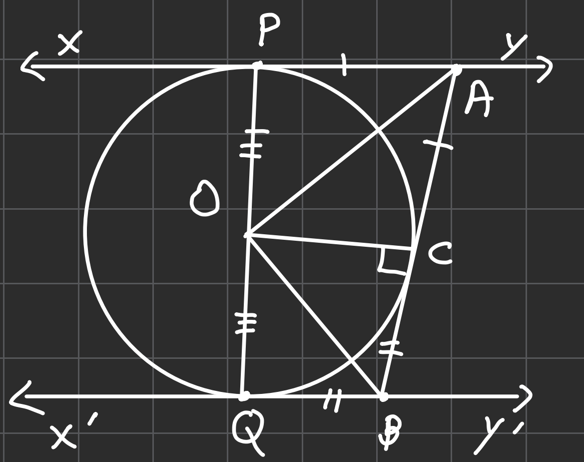
Salutian

AB is tangent at P
BC is tangent at Q
CD is tangent at R
DA is tangent at S

$AP = AS$ (Th-10.2)
$BP = BQ$ (Th-10.2)
$CQ = CR$ (Th-10.2)
$DR = DS$ (Th-10.2)

$AB + CD = AD + BC$ (To prave)
$AP + BP + CR + DR = AD + BC$
$AS + BQ + CQ + DS = AD + BC$
$(AS + DS) + (BQ + CQ) = AD + BC$
$AD + BC = AD + BC$
Hence Praved

9.



Given :

XY is tangent at P

X'Y' is tangent at Q

$XY \parallel X'Y'$

AB is tangent at C

To prove :

$\angle AOB = 90°$

Solution :

$\triangle AOP$, $\triangle AOC$

$AP = AC$ (Th-10.2)

$AO = AO$ (Camman)

$CO = PO$ (Radii)

$\triangle AOP \cong \triangle AOC$

$\angle AOP = \angle AOC$ (By CPCT)

Similarly, $\triangle BOQ \cong \triangle BOC$

$\angle BOQ = \angle BOC$

$\angle AOP + \angle AOC + \angle BOC + \angle BOQ = 180^\circ$

$2\angle AOC + 2\angle BOC = 180^\circ$

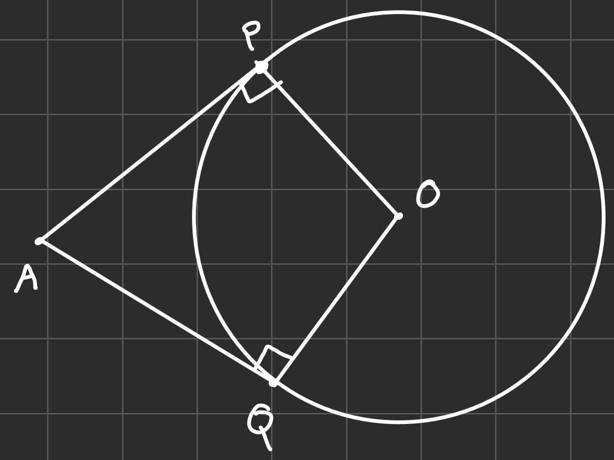
$2(\angle AOC + \angle BOC) = 180^\circ$

$\angle AOC + \angle BOC = 90^\circ$

$\boxed{\angle AOB = 90^\circ}$

Hence Proved

10.



Given :

AP is tangent at P

AQ is tangent at Q

To prove :

$\angle PAQ + \angle POQ = 180'$

Solution :

$\angle APO = 90^\circ$ (Th-10.1)

$\angle AQO = 90^\circ$ (Th-10.1)

$\angle APO + \angle AQO + \angle PAQ + \angle POQ = 360^\circ$

$$90^\circ + 90^\circ + \angle PAQ + \angle POQ = 360^\circ$$

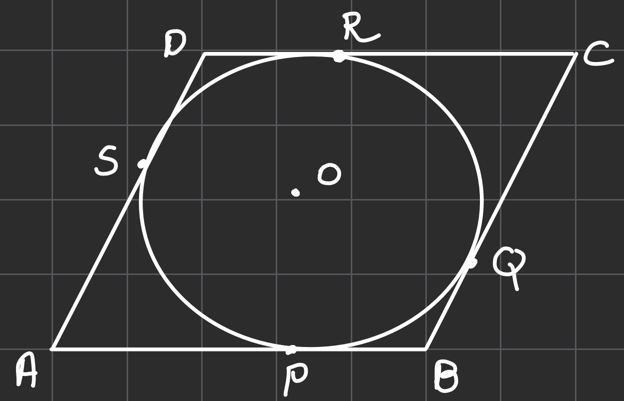
$$180^\circ + \angle PAQ + \angle POQ = 360^\circ$$

$$\angle PAQ + \angle POQ = 360^\circ - 180^\circ$$

$$\boxed{\angle PAQ + \angle POQ = 180^\circ}$$

Hence Proved

11.



Given:

ABCD is ||gm

$AB = CD$

$AD = BC$

To prove:

ABCD is rhombus

solution:

AB is tangent at P
BC is tangent at Q
CD is tangent at R
DA is tangent at S

$AP = AS$ (Th-10.2)
$BP = BQ$ (Th-10.2)
$CQ = CR$ (Th-10.2)
$DR = DS$ (Th-10.2)

$AB + CD = AD + BC$
$AP + BP + CR + DR = AD + BC$
$AS + BQ + CQ + DS = AD + BC$
$(AS + DS) + (BQ + CQ) = AD + BC$
$AD + BC = AD + BC$

$\therefore AD + BC = AB + CD$

$AD + AD = AB + AB$

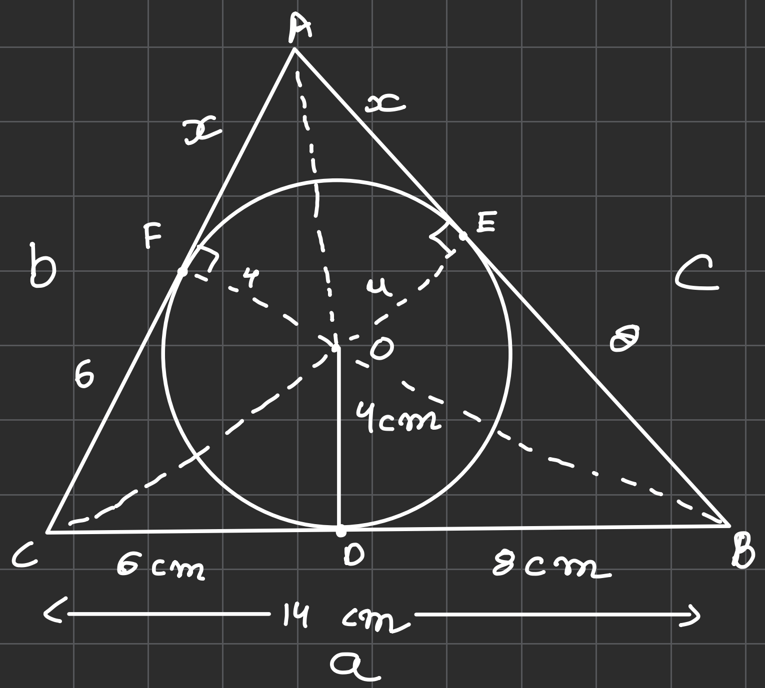
$2AD = 2AB$

$AD = AB$

$$\boxed{AB = CD = AD = BC}$$

Hence ABCD is rhombus

12.



Given:

$OB = 4cm$

$BD = 8cm$

$CD = 6cm$

To find :

$AC = ?$

$AB = ?$

Solution:

BC is tangent at D

AC is tangent at E

AB is tangent at F

$BF = BD = 8cm$ (Th-10.2)

$CD = CE = 6cm$ (Th-10.2)

$AF = AE = x cm$ (Th-10.2)

$BC = CD + BD$

$= 6 + 8$

$BC = 14cm$

$AC = AE + CE$

$AC = x + 6 \text{ cm}$

$AB = AF + BF$

$AB = x + 8 \text{ cm}$

Heron's formula:

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{14 + x + 6 + x + 8}{2}$$

$$= \frac{\cancel{28}^{14} + \cancel{2x}^{x}}{\cancel{2}_{1}}$$

$$s = 14 + x$$

$a = 14 \text{ cm}$

$b = x + 6 \text{ cm}$

$c = x + 8 \text{ cm}$

$$\begin{array}{r} \overset{3}{48} \\ \times\ 14 \\ \hline 192 \\ 480 \\ \hline 672 \end{array}$$

$$\Delta = \sqrt{(14+x)[14 + x - 14]\,[14 + \cancel{x} - \cancel{x} - 6]\,[14 + \cancel{x} - \cancel{x} + 8]}$$

$$= \sqrt{(14+x)(x)(8)(6)}$$

$$= \sqrt{14x + x^2 (8)(6)}$$

$$\Delta = \sqrt{672x + 48x^2} \quad - ①$$

$$\Delta = \text{Area of } \Delta ACO + Ar(\Delta ABO) + Ar(\Delta BOC)$$
$$= \frac{1}{2} \times OE \times AC + \frac{1}{2} \times OF \times AB + \frac{1}{2} \times OD \times BC$$
$$= \frac{1}{\cancel{2}} \times \cancel{4}^{2} \times (x+6) + \frac{1}{\cancel{2}} \times \cancel{4}^{2} \times (x+8) + \frac{1}{\cancel{2}} \times \cancel{4}^{2} \times 14$$
$$= 2x+12 + 2x+16 + 28$$
$$\Delta = 4x + 56 \qquad - (2)$$

$$(1) = (2)$$

$$\sqrt{672x + 48x^2} = (4x+56)^2$$

squaring both sides

$$672x + 48x^2 = 16x^2 + 3136 + 448x$$
$$224x + 32x^2 = 3136$$
$$32(7x + x^2) = 98$$
$$7x + x^2 = 98$$
$$x^2 + 7x - 98 = 0$$
$$x^2 + 14x - 7x - 98 = 0$$

$x(x+14) \quad -7(x+14)=0$

$(x+14)(x-7)=0$

$x=-14$ ✗

$x=7$ ✓

$\therefore$ the value of x is 7 cm

$AC = x+6$

$= 7+6$

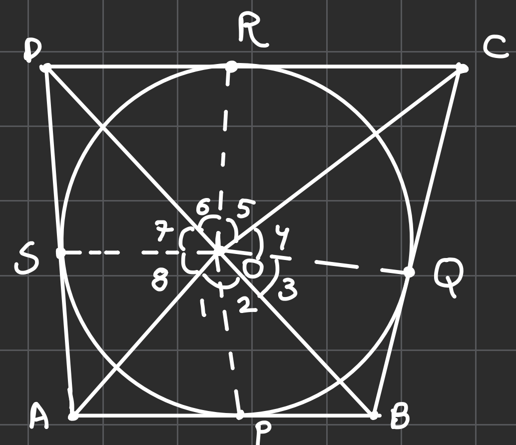
$AC = 13cm$

$AB = x+8$

$= 7+8$

$AB = 15cm$

13.



Given:

ABCD is quadrilateral

ABCD touches at P, Q, R and R

To prove:

$\angle AOB + \angle COD = 180^\circ$

$\angle AOD + \angle BOC = 180^\circ$

Solution:

$\triangle AOP, \triangle AOS$

$AP = AS$ (Th-10.2)

$AO = AO$ (Common)

$OP = OS$ (Radii)

$\triangle AOP \cong \triangle AOS$

$\angle 1 = \angle 8$ (By CPCT)

similarly,

$BOP \cong \triangle BOQ$

$COQ \cong \triangle COR$
$DOR \cong \triangle DOS$

$\angle 2 = \angle 3$ (By CPCT)
$\angle 4 = \angle 5$ (By CPCT)
$\angle 6 = \angle 7$ (By CPCT)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$
$\angle 1 + \angle 2 + \angle 2 + \angle 5 + \angle 5 + \angle 6 + \angle 6 + \angle 1 = 360^\circ$
$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^\circ$
$2(\angle 1 + \angle 2 + \angle 5 + \angle 6) = 360^\circ$
$\angle 1 + \angle 2 + \angle 5 + \angle 6 = 180^\circ$

$$\boxed{\angle AOB + \angle COD = 180^\circ}$$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$
$\angle 8 + \angle 3 + \angle 3 + \angle 4 + \angle 4 + \angle 7 + \angle 7 + \angle 8 = 360^\circ$
$2(\angle 8 + \angle 7 + \angle 4 + \angle 3) = 360^\circ$
$\angle 8 + \angle 7 + \angle 4 + \angle 3 = 180^\circ$
$\angle AOD + \angle BOC = 180^\circ$

The

End!!