Formulae for Second Year Computer Engineering (SPPU 2019 Pattern)

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Contents

1 Unit III: Statistics (7 Hours)

1.1 Measures of Central Tendency

- **Mean (Arithmetic Mean):**

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \text{(ungrouped data)}$$

$$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{\sum_{i=1}^{k} f_i} \quad \text{(grouped data)}$$

- **Median:**

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f} \right) \times h \quad \text{(grouped data)}$$

- **Mode:**

$$\text{Mode} = L + \left(\frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \right) \times h \quad \text{(grouped data)}$$

1.2 Measures of Dispersion

- **Range:**

$$\text{Range} = \text{Maximum value} - \text{Minimum value}$$

- **Variance:**

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} \quad \text{(population)}$$

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \quad \text{(sample)}$$

- **Standard Deviation:**

$$\sigma = \sqrt{\text{Variance}}$$

- **Mean Deviation:**

$$\text{MD} = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} \quad \text{(about mean)}$$

1.3 Coefficient of Variation

$$\text{CV} = \left(\frac{\sigma}{\bar{x}} \right) \times 100 \quad \text{(as percentage)}$$

1.4 Moments

$$\mu_r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^r}{n} \quad (r\text{-th central moment})$$

1.5 Skewness and Kurtosis

- **Skewness (Pearsons Coefficient):**

$$\text{Skewness} = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

- **Kurtosis:**

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

1.6 Curve Fitting (Method of Least Squares)

- **Straight Line** ($y = a + bx$)**:**

$$\sum y = na + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

- **Parabola** ($y = a + bx + cx^2$)**:**

$$\sum y = na + b \sum x + c \sum x^2$$

$$\sum xy = a \sum x + b \sum x^2 + c \sum x^3$$

$$\sum x^2 y = a \sum x^2 + b \sum x^3 + c \sum x^4$$

1.7 Correlation and Regression

- **Pearsons Correlation Coefficient** (r)**:**

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$

- **Regression Line** (y **on** x)**:**

$$y - \bar{y} = b_{yx}(x - \bar{x}), \quad b_{yx} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

- **Regression Line** (x **on** y)**:**

$$x - \bar{x} = b_{xy}(y - \bar{y}), \quad b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

1.8 Reliability of Regression Estimates

$$S_y = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n - 2}}$$

2 Unit IV: Probability and Probability Distributions (7 Hours)

2.1 Probability

- **Basic Probability:**

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

- **Addition Theorem:**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- **Multiplication Theorem (Independent Events):**

$$P(A \cap B) = P(A) \cdot P(B)$$

- **Conditional Probability:**

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

2.2 Bayes Theorem

$$P(A_i|B) = \frac{P(A_i) \cdot P(B|A_i)}{\sum_{j=1}^{n} P(A_j) \cdot P(B|A_j)}$$

2.3 Random Variables and Mathematical Expectation

- **Expected Value:**

$$\mathbb{E}(X) = \sum x_i P(x_i) \quad \text{(discrete)}$$

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x)\, dx \quad \text{(continuous)}$$

- **Variance:**

$$\mathrm{Var}(X) = \mathbb{E}(X^2) - [\mathbb{E}(X)]^2$$

2.4 Probability Distributions

- **Binomial Distribution:**

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

  Mean: $\mathbb{E}(X) = np$, Variance: $\mathrm{Var}(X) = np(1-p)$

- **Poisson Distribution:**

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

  Mean: $\mathbb{E}(X) = \lambda$, Variance: $\mathrm{Var}(X) = \lambda$

- **Normal Distribution:**
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
Standard Normal: $Z = \frac{X-\mu}{\sigma}$

- **Hypergeometric Distribution:**
$$P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$
Mean: $\mathbb{E}(X) = n\frac{K}{N}$, Variance: $\mathrm{Var}(X) = n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}$

2.5 Sampling Distributions

- **Mean of Sampling Distribution:**
$$\mu_{\bar{x}} = \mu$$

- **Standard Error of the Mean:**
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

2.6 Test of Hypothesis

- **Chi-Square Test:**
$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

- **t-Test:**
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

3 Unit V: Numerical Methods (8 Hours)

3.1 Numerical Solution of Algebraic and Transcendental Equations

- **Bisection Method:**
$$c = \frac{a+b}{2}$$

- **Secant Method:**
$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

- **Regula-Falsi Method:**
$$x_{n+1} = \frac{x_{n-1}f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

- **Newton-Raphson Method:**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- **Successive Approximation:**

$$x_{n+1} = g(x_n)$$

3.2 Numerical Solutions of System of Linear Equations

- **Jacobi Method:**

$$x_i^{(k+1)} = \frac{b_i - \sum_{j \neq i} a_{ij} x_j^{(k)}}{a_{ii}}$$

- **Gauss-Seidel Method:**

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$$

4 Unit VI: Numerical Methods (8 Hours)

4.1 Interpolation

- **Newtons Forward Interpolation:**

$$f(x) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \cdots$$

- **Newtons Backward Interpolation:**

$$f(x) = f(x_n) + u\nabla f(x_n) + \frac{u(u+1)}{2!}\nabla^2 f(x_n) + \cdots$$

- **Lagranges Interpolation:**

$$f(x) = \sum_{i=0}^{n} f(x_i) \prod_{j \neq i} \frac{(x - x_j)}{(x_i - x_j)}$$

4.2 Numerical Differentiation

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} \quad \text{(central difference)}$$

4.3 Numerical Integration

- **Trapezoidal Rule:**

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right]$$

- **Simpsons 1/3 Rule:**

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,\text{odd}}^{n-1} f(x_i) + 2\sum_{i=2,\text{even}}^{n-2} f(x_i) + f(x_n)\right]$$

- **Truncation Error Bounds:**

$$\text{Trapezoidal Error} \le \frac{(b-a)^3}{12n^2}\max|f''(x)|$$

$$\text{Simpsons Error} \le \frac{(b-a)^5}{180n^4}\max|f^{(4)}(x)|$$

4.4 Solution of Ordinary Differential Equations

- **Eulers Method:**

$$y_{n+1} = y_n + hf(x_n, y_n)$$

- **Modified Eulers Method:**

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{\text{predictor}})\right]$$

- **Runge-Kutta 4th Order (RK4):**

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where:

$$k_1 = f(x_n, y_n), \quad k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right),$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right), \quad k_4 = f(x_n + h, y_n + hk_3)$$