

DAA Assignment 4

GROUP 17

PRAVALLIKA KODI (IIT2019234)

VISHWAM SHRIRAM MUNDADA (IIT2019235)

NOONSAVATH SRAVANA SAMYUKTA (IIT2019236)

B.tech Information Technology business Informatics
Indian Institute of Information Technology, Allahabad

4 April 2021

0.1 Question

Given a 2D array, find the maximum sum subarray in it.

0.2 Introduction

Here, different approaches are analysed and used to achieve results. These approaches are:

- 1) Brute force
- 2) Using Dynamic programming

In this report we will explain our solution approach. We explain our code in detail. We will discuss the time complexity analysis and the space complexity analysis. And last but not least, the conclusion.

0.3 Code Explanation

0.3.1 Brute force

In the brute force approach we try to check every possible rectangle in the given $n \times m$ 2D array (where n, m are number of rows and columns respectively). Set the position of the top-left and bottom-right corners of the sub-rectangle and adding the integers within it while iterating through all the rows sequentially. Parallely we try to find the maximum subarray sum value.

Algorithm 1 Brute force

```
int A[101][101]
```

```
function MAIN()
```

```
maxSum <- INT_MIN
tempSum <- 0
x <- 0, y <- 0, z <- 0, w <- 0
n <- 0, m <- 0
input n, m

for i <- 0 to n-1
  for j <- 0 to m-1
    input A[i][j]
  end
end

for i <- 0 to n-1
  for j <- 0 to m-1
    for k <- 0 to n-1
      for l <- 0 to m-1
        tempSum = FINDSUM(i,
                           j, k, l)

        if tempSum > maxSum
          x <- i
          y <- j
          z <- k
          w <- l
          maxSum = tempSum
        end if
      end
    end
  end
end

print x, y, z, w, maxSum
```

Algorithm 2 Maxsum Pseudo Code

```
function FINDSUM(x, y, z, w)
  sum <- 0

  for i <- x to z
    for j <- y to w
      sum <- sum + A[i][j]
    end
  end
```

```
end
```

```
return sum
```

0.3.2 Dynamic programming

The idea is to fix the left and right columns one by one and find the maximum sum contiguous rows for every left and right column pair.

We basically find top and bottom row numbers (which have maximum sum) for every fixed left and right column pair. To find the top and bottom row numbers, calculate the sum of elements in every row from left to right and store these sums in an array say temp[].

temp[i] indicates sum of elements from left to right in row i. If we apply Kadane's 1D algorithm on temp[], and get the maximum sum subarray of temp, this maximum sum would be the maximum possible sum with left and right as boundary columns. To get the overall maximum sum, we compare this sum with the maximum sum so far

We use Kadane's algorithm to reduce the time complexity to $O(n^2xm)$.

Algorithm 3 Dynamic programming

```
int A[101][101]
```

```
function MAIN()
```

```
  r <- 0, c <- 0, maxSum <-  
  INT_MIN
```

```
  x <- 0, y <- 0, z <- 0, w <- 0
```

```
input r, c
```

```
for i <- 0 to r-1
```

```
  for j <- 0 to c-1
```

```
    input A[i][j]
```

```
  end
```

```
end
```

```
for i <- 0 to r-1
```

```
  vector<int> sum(c)
```

```
  for j <- 0 to r-1
```

```
    for col <- 0 to c-1
```

```
      sum <- sum + A[j][col]
```

```
    end
```

```
  vector<int> res <-
```

```
    KADANE(sum)
```

```
  if maxSum < res[2]
```

```
    x <- i
```

```
    y <- res[0]
```

```
    z <- j
```

```
    w <- res[1]
```

```
    maxSum = res[2]
```

```
  end if
```

```
end
```

```
end
```

```
print x, y, z, w, maxSum
```

```
function KADANE(V)
```

```
  maxSum <- INT_MIN, tempSum <- 0
```

```
  st <- -1, end <- -1, localSt <-  
  0
```

```
for i <- 0 to length[V]
```

```
  tempSum <- tempSum + V[i]
```

```
if maxSum < tempSum
```

```
  st <- localSt
```

```
  end <- i
```

```
  maxSum <- tempSum
```

```
end if
```

```

    if tempSum < 0
      localSt <- i+1
      tempSum <- 0
    end if
  end
end

vector<int> res <- {st, end,
  maxSum}
return res

```

0.4 Algorithm Analysis

0.4.1 Brute force approach

In the brute force approach we try to check every possible rectangle in the given $n \times m$ 2D array (where n, m are number of rows and columns respectively).

This solution requires 6 nested loops :
2 for the summation of the sub-matrix $O(n \times m)$

4 for start and end coordinate of the 2 axis $O(n^2 \times m^2)$

The overall time complexity is $O(n^3 \times m^3)$

0.4.2 Dynamic programming

Basically, Dynamic algorithm (complexity: $O(n)$) is used inside a naive maximum sum subarray problem (complex-

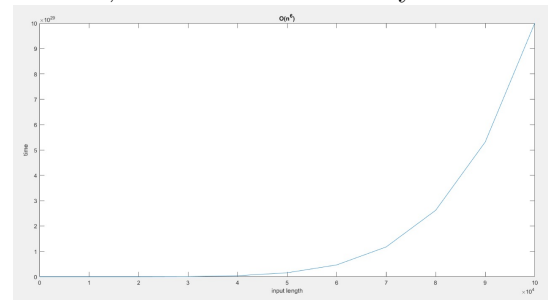
ity: $O(n \times m)$).

This gives a total complexity of $O(n^2 \times m)$

0.5 Graph analysis

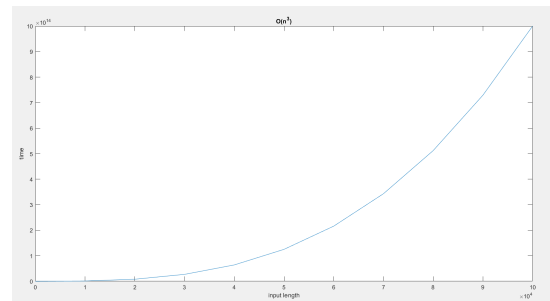
Brute force

In the brute force approach as n increases, the time increases by n^6 .



Dynamic programming

In the Dynamic programming approach as n increases, the time increases by n^3 .



0.6 Conclusion

From the experimental study we concluded that the average running time of dynamic algorithm is best, which can be observed from the mutual graph of dynamic algorithm and Brute Force algorithm as shown.

0.7 References

<https://www.geeksforgeeks.org/maximum-sum-rectangle-in-a-2d-matrix-dp-27/>
<https://www.geeksforgeeks.org/largest-sum-contiguous-subarray/>

0.8 code

<https://ideone.com/W0rVpg>
<https://ideone.com/Zw0Ape>