Automata Theory.

* Alphabet $\Sigma = \{0, 1\}$
* string. $x = 0101101$ $|x| = 7$

Empty string $\rightarrow$ ε

Powers of an alphabet.

$\Sigma^k \rightarrow$ set of all string of length k.

$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cdots$

$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \cdots$

$\Sigma^0 = \{\varepsilon\}$

$\Sigma^1 = \{0, 1\}$

$\Sigma^2 = \{00, 01, 10, 11\}$

$\Sigma^* \rightarrow$ Kneeli closure.

$\Sigma^+ \rightarrow$ Kneeli plus.

* Language

$L \subseteq \Sigma^*$

Finite Automata.

12-11-2022.

Application.

* In compilers.

Lexical analysis $\rightarrow$ separates statement into tokens.

Parse generators $\rightarrow$

Finite Automata.

- On/Off switch

Kinds of Automata.

Chomsky hierarchy.

Type 0 → Unrestricted grammar. Turing machine.
Type 1 → Context sensitive grammar. Linear bound.
Type 2 → Context free grammar. Push down automata
Type 3 → Regular grammar Finite automata.

Finite Automata

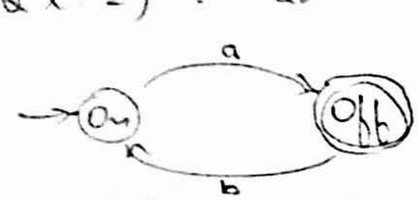
F : Quintuple.

M = { Q, Σ, δ, q0, F }

Q → set of states.
Σ → set of alphabets.
δ → set of transitions.
q_0 → start state.
F → final state/accept state

$\boxed{\delta(Q \times \Sigma) : Q}$

Eg



Q : {on, off}

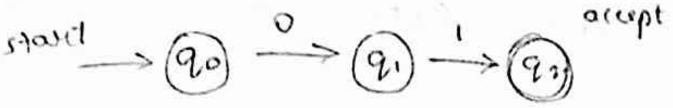
Σ : {a, b}

δ(On, a) : Off

δ(Off, b) : On

q_0 : On

F : Off



$M: \{Q, \Sigma, \delta, q_0, F\}$

$Q: \{q_0, q_1, q_2\}$

$\Sigma: \{0, 1\}$

$\delta: \quad (q_0, 0): q_1$

$\quad\quad (q_1, 1): q_2$

	0	1
$\rightarrow q_0$	q_1	-
q_1	-	q_2
$* \, q_2$	-	-

As there are blank spaces, this machine is non-deterministic.

* Language with strings of 'n' a's and 'n' b's

$L: \{\varepsilon, ab, aabb, abab, bbaa, baba, baab, abba, \ldots\}$

$L: \{w : \{a, b\}^*\}$ $\quad\quad * \rightarrow$ 0 or more.

* language of strings containing 'a' & 'b' and the string ends with 'a'

$L: \{w: ya, \; y: \{a, b\}^*\}$

* Language where all 'a's' preceed b's.

$L: \{w: a^i b^j, \; i \geq 1, j \geq 1\}$

14-11-2022

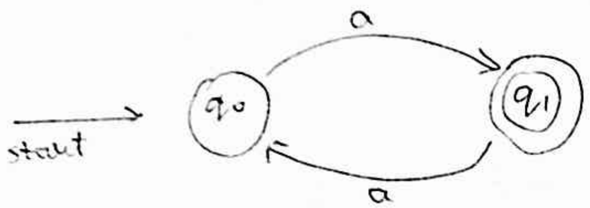
Configuration of Finite State Machine FSM.

* current state
* string to be processed.

(q, w)

Initial configuration $\rightarrow$ (q_0, w)

Final configuration $\rightarrow$ (q_f, ε)



"aaa"

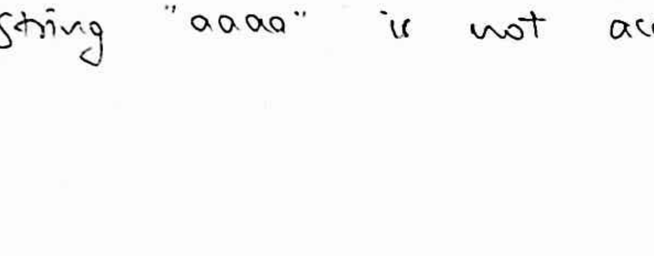
IC $(q_0, aaa) \vdash (q_1, aa)$

$(q_1, aa) \vdash (q_0, a)$

$(q_0, a) \vdash (q_1, \varepsilon)$ FC

Final configuration has the final state q_1 hence the string "aaa" is accepted by the machine.

Hence the machine is deterministic finite state machine



"aaaa"

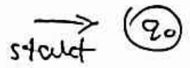
String "aaaa" is not accepted by the machine.

Pattern recognition.

DFSM

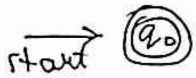
* Empty language

$L = \{\phi\}$



* Empty string

$L = \{\varepsilon\}$

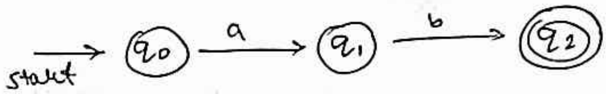


* Exactly 1 'a'.

$L = \{a\}$

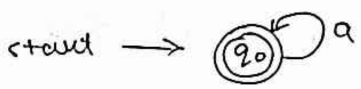
* Accepts "ab"

$L = \{ab\}$



Non-deterministic

* Any number of 'a's



Steps.

1. Minimum string.
2. I/p alphabets.
3. Skeleton of DFA.
4. Other transitions.
5. complete TD. Transition diagram.

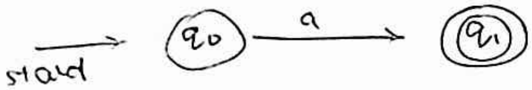
Draw a DFA to accept the strings of a's having atleast one 'a'

⇒ 1) Minimum string : a.

$L = \{ w : n_a \geq 1 \quad w \in (a)^+ \}$

2) Input alphabets $\Sigma : \{a\}$

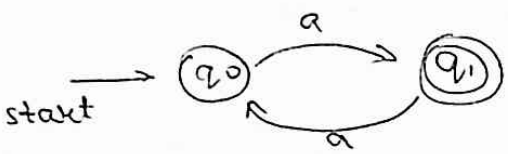
3) Skeleton



4) $\delta(q_0, a) = q_1$

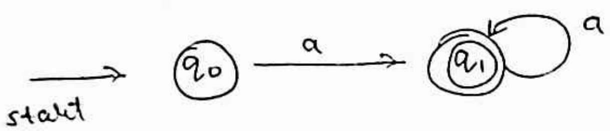
	a
→ q_0	q_1
* q_1	

if



given a string "aa" the machine does not accept the string as q_0 is not the final state.

hence



	a
→ q_0	q_1
* q_1	q_1

5) $M : (Q, \Sigma, \delta, q_0, F)$

$Q : \{q_0, q_1\}$

$\Sigma : \{a\}$

$\delta : \quad \delta(q_0, a) = q_1$

$\qquad \delta(q_1, a) = q_1$

$q_0 : q_0$

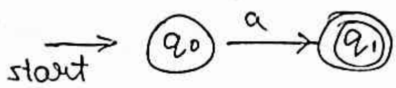
$F : q_1$

Draw a DFSM to accept strings of a's and b's having atleast 1 a.

⇒ 1) Minimum string : a

2) Input alphabet $\Sigma = \{a, b\}$

3) skeleton



Atleast one 'a'.

4) $\delta(q_0, a) = q_1$

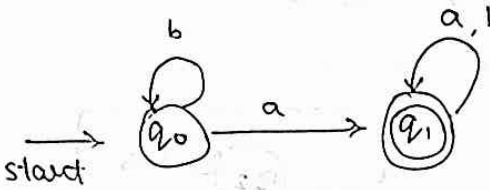
	a	b
q_0	q_1	
q_1		

The strings can be of order

$\boxed{b^* a (a+b)^*}$

Eg: baa
bbbbabab
bbab
aab

given b to q_0 the machine should not reach final state.



	a	b
q_0	q_1	q_0
q_1	q_1	q_1

16-11-2022

Language

$L = \{ \omega : n_a \geq 1 \ . \ \omega \in (a+b)^* \}$

5) $M = \{Q, \Sigma, \delta, q_0, F\}$

$Q = \{q_0, q_1\}$

$\Sigma = \{a, b\}$

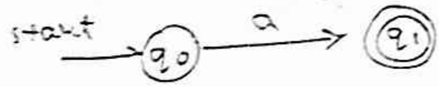
δ =

Design a DFA to accept the string having exactly one 'a' string with alphabets (a, b).

⇒ 1) Minimum string : a

2) $\Sigma = \{a, b\}$

3) Skeleton

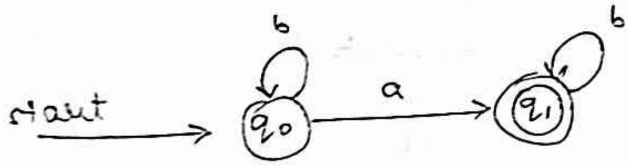


4) $\delta(q_0, a) = q_1$

	a	b
q_0	q_1	
q_1		

the string can be

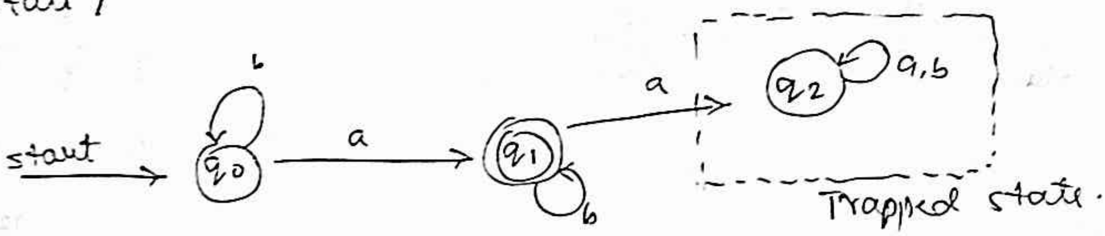
$\boxed{b^* a b^*}$



	a	b
q_0	q_1	q_0
q_1	—	q_1

q_1 cannot take 'a' so this state is called Trapped state / Dead state.

∴



∴ $L = \{w : n_a = 1, \ w \in (a, b)\}$

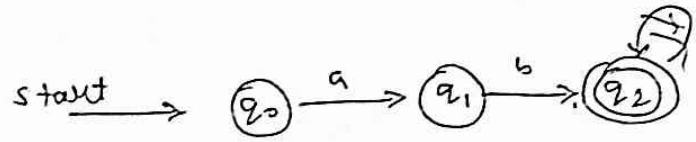
		a	b
→	q_0	q_1	q_0
*	q_1	q_2	q_1
TS	q_2	q_2	q_2

Design a DFA starting with string "ab."

→ 1) Minimum string = ab

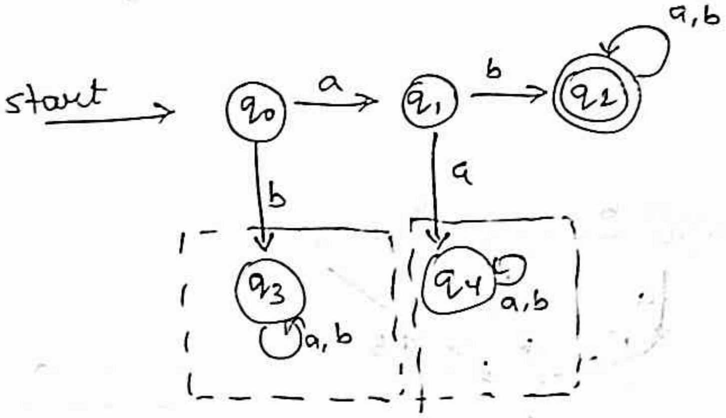
2) $\Sigma = \{a, b\}$

3) Skeleton



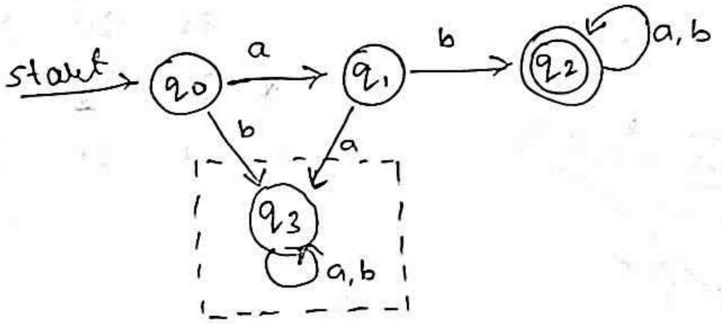
	a	b
q_0	q_1	
q_1		q_2
q_2		

$\boxed{ab(a+b)^*}$



Trapped states

		a	b
→	q_0	q_1	q_3
	q_1	q_4	q_2
*	q_2	q_2	q_2
TS	q_3	q_3	q_3
TS	q_4	q_4	q_4



Trapped state

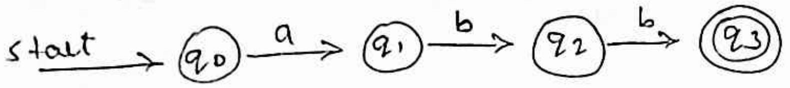
		a	b
→	q_0	q_1	q_3
	q_1	q_3	q_2
*	q_2	q_2	q_2
TS	q_3	q_3	q_3

Draw a DFA ending with string abb.

→ 1) Minimum string : abb

2) $\Sigma = \{a, b\}$

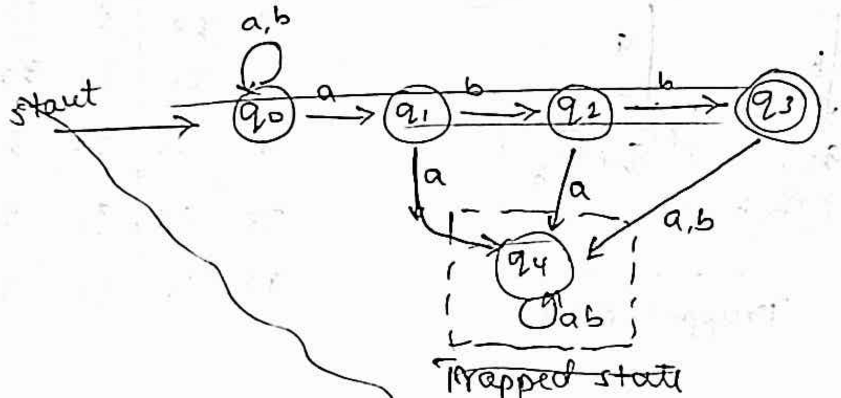
3) Skeleton



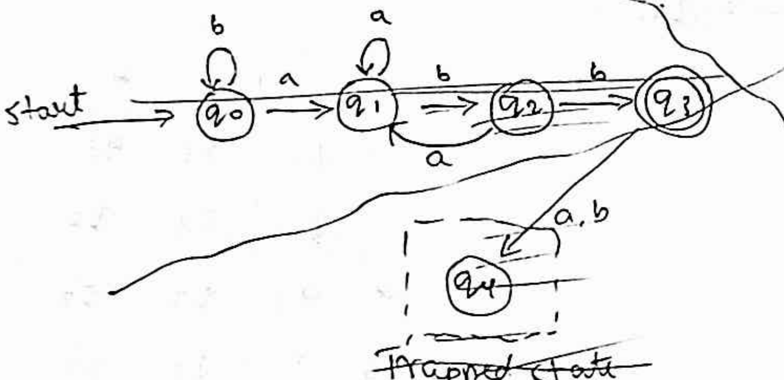
	a	b
q_0	q_1	q_0
q_1		q_2
q_2		q_3
q_3		

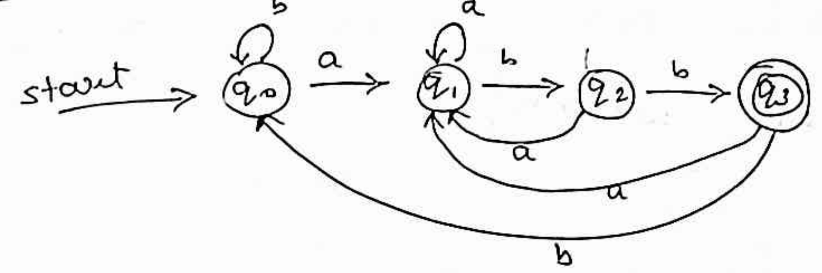
$(a+b)^* abb$

4)



		a	b
→	q_0	q_1	q_0
	q_1	q_1	q_2
	q_2	q_1	q_3
*	q_3	q_4	q_4
TS	q_4	q_4	q_4





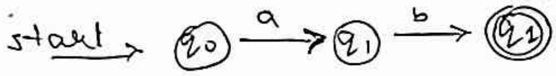
17-11-2022

Ending with string ab	Substring "ab"	substring "aab"
		no substring "aab"

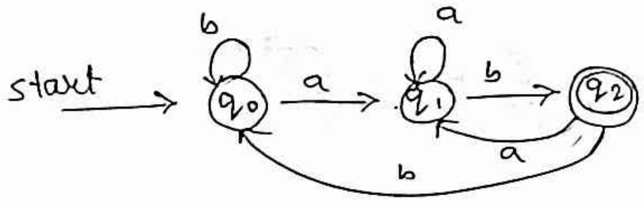
Ending with string 'ab'.

$\Rightarrow$ $L = \{ w = (a+b)^* ab : w \in (a,b) \}$

1) M = ab
2) Σ : {a, b}
3) skeleton



4)

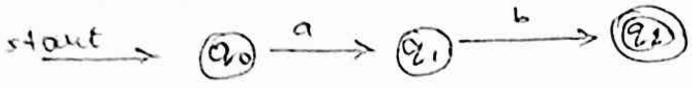


abab ✓
abaab ✓
abbab ✓
babbab

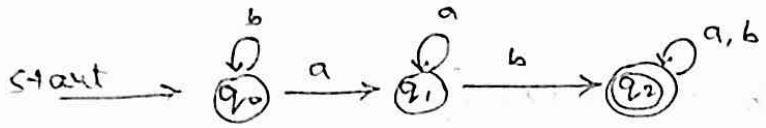
Having substring "ab"

$\Rightarrow$ $L = \{ w = (a+b)^* ab (a+b)^*$, $w \in (a,b) \}$

1) Min. = ab
2) $\Sigma = \{a, b\}$
3) Skeleton



4)

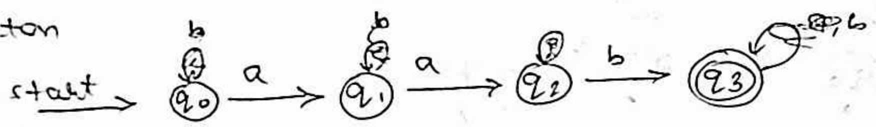


aaaa

Having substring "aab"

$\Rightarrow$ $L = \{ w = (a+b)^* aab (a+b)^*$, $w \in (a,b) \}$

1) Min : aab
2) $\Sigma = \{a, b\}$
3) skeleton



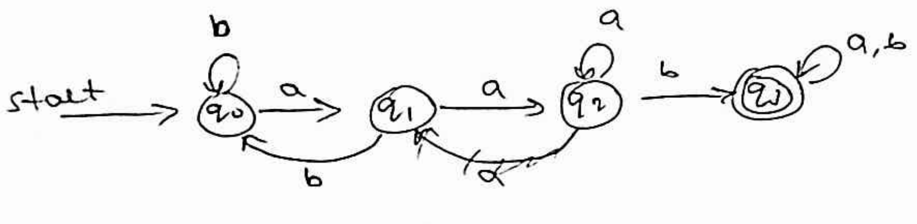
babab

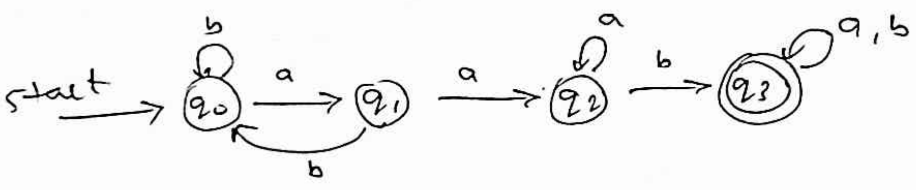
baabb

baabba

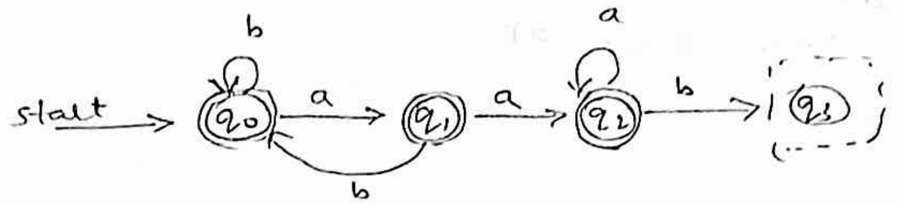
babaab

babaaab





Having no sub string 'aab'.



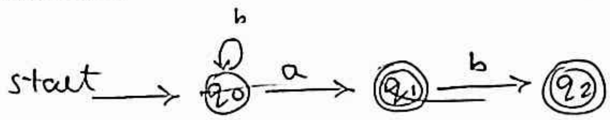
ababbaa

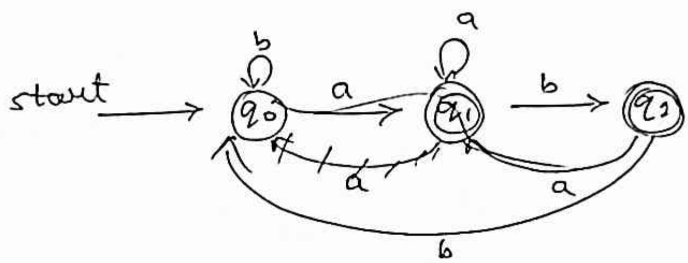
Obtain a DFA to accept strings of a's b's ending with 'ab' or 'ba'

⇒ 1) Min : ab / ba

2) Σ : {a, b}

3) skeleton





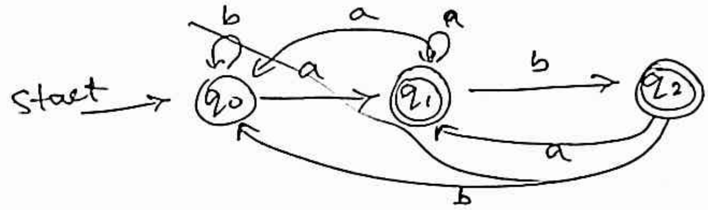
baab

babab

bababb

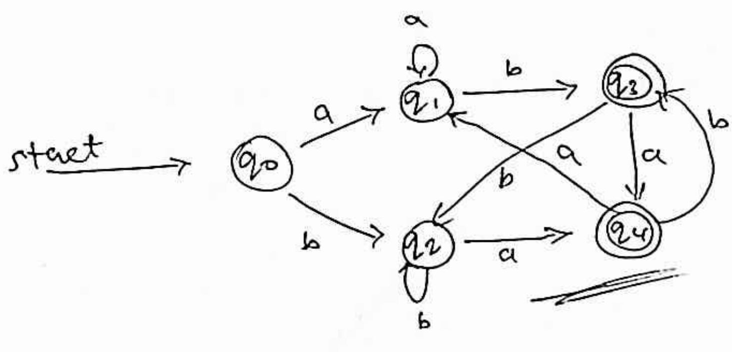
abaaab

baa



aabab

baabaa



Draw a DFA to accept strings of 0's and 1's having three consecutive 0's. and no three consecutive 0's

* having atleast 4 a's.
* having not more than three a's.
* having exactly 3 a's.

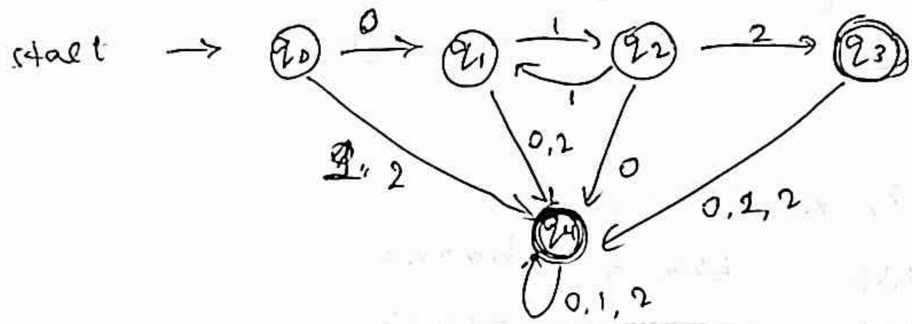
24-11-2022.

Draw a DFA with beginning string 0 followed by odd no. of 1's and ending with 2.

→ Minimum string : 012

$\Sigma = \{0, 1, 2\}$

Skeleton



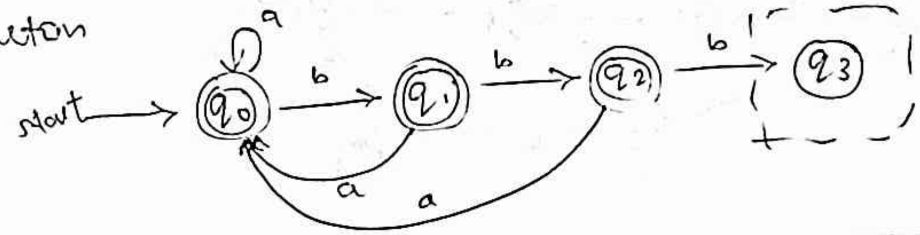
	0	1	2
→ q_0	q_1	q_4	q_4
q_1	q_4	q_2	q_4
q_2	q_4	q_1	q_3
* q_3	q_4	q_4	q_4
q_4	q_4	q_4	q_4

Draw DFA with a's & b's atmost two consecutive b's.

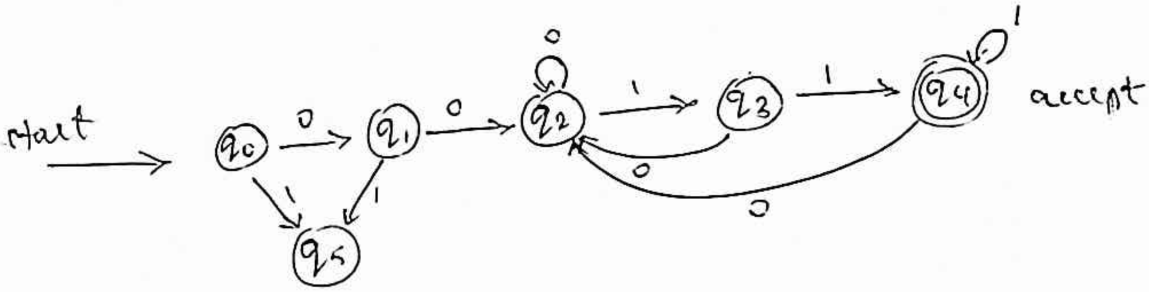
→ Minimum string : a

$\Sigma = \{a, b\}$

Skeleton



0's & 1's with atleast 2 0's & ending with atleast 2 1's.

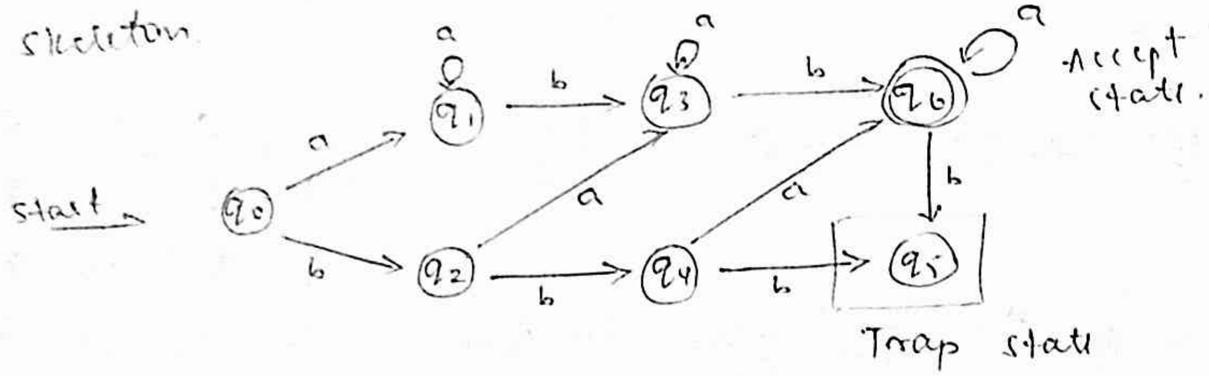


→ $L = \{w : n_a(w) \geq 1,\ n_b(w) = 2\}$

Minimum = abb, bab, bba

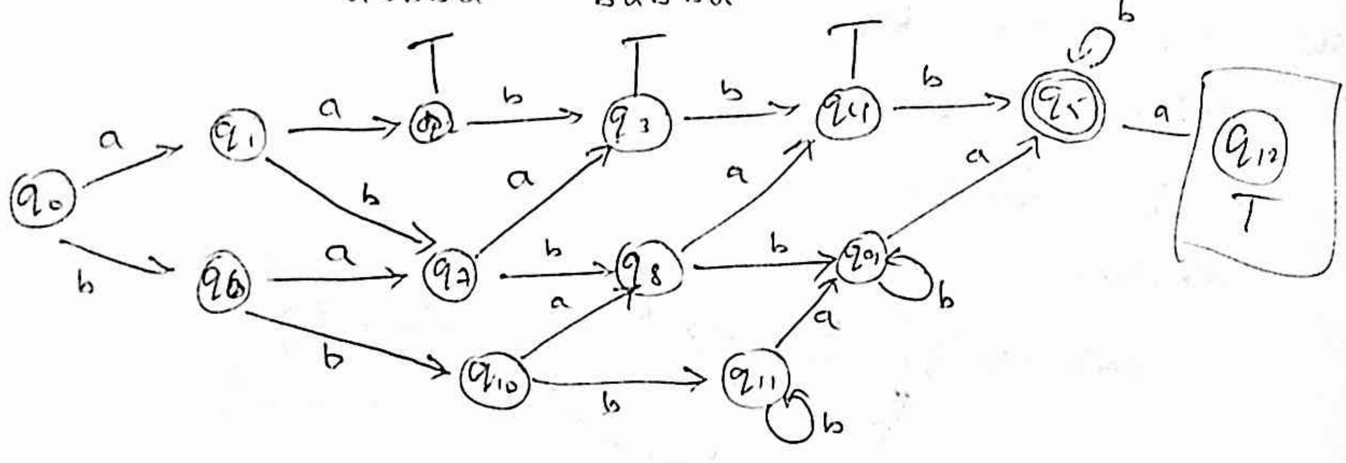
$\Sigma = \{a, b\}$

Skeleton



$L = \{w : n_a(w) = 2,\ n_b(w) \geq 3\}$

Minimum : aabbb bbbaa baabb
ababb bbaba babab
abbab bbaab
abbba babba



23-11-2022

ij

i → No. of scanned

j →

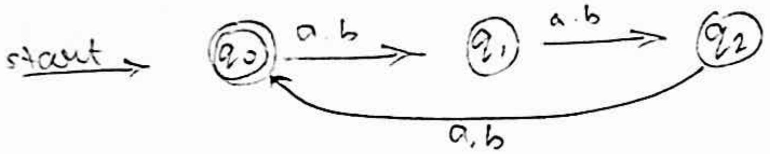
Draw DFA

- - 11- 2022

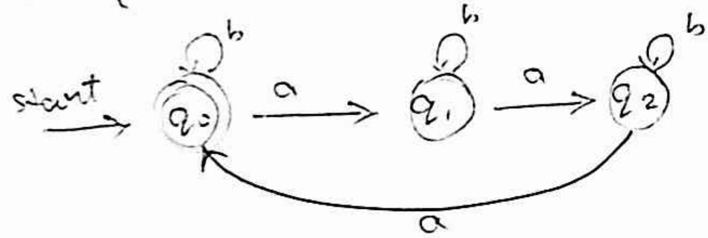
String length mod K

* $L = \{ w : |w| \bmod 3 = 0 \}$

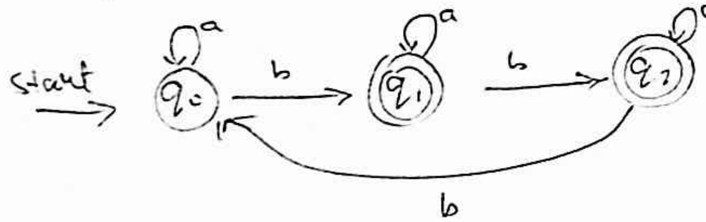
  $L = \{ \varepsilon, aaa, aab, aba, baa, bba, bab, abb, bbb, bbbbbb, aaaaaa - - - - \}$

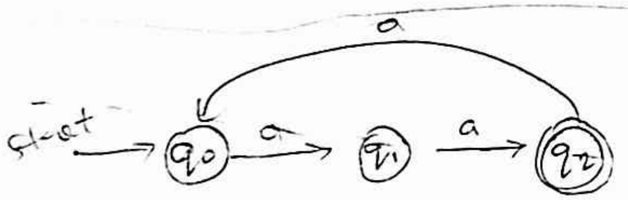


* $L = \{ w : n_a(w) \bmod 3 = 0, (a,b) \in w \}$



* $L = \{ w : n_b(w) \bmod 3 \neq 0, (a,b) \in w \}$





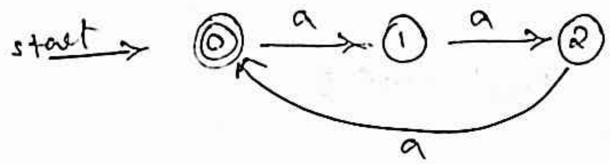
— — $n_a(w) \bmod 2 = 0$

$n_a(w)$ ~~mod 5~~ $= 0$

a) $|n_a| \bmod 3 = 0$

$|n_a| \bmod 2 = 0$

M_1



$Q_1 = \{0, 1, 2\}$

M_2

$Q_2 = \{0, 1\}$

$Q_1 \times Q_2 \; \{(0.0).(0,1)\ (1 0)\ \ (1,1)\ \ (2,0)\ \ (2,1)\}$

$\delta(\underset{M_1\ M_2}{(0.0)}, a) = \underset{1}{\delta(0,a)} \quad \underset{1}{\delta(0.a)} \qquad (1,1)$

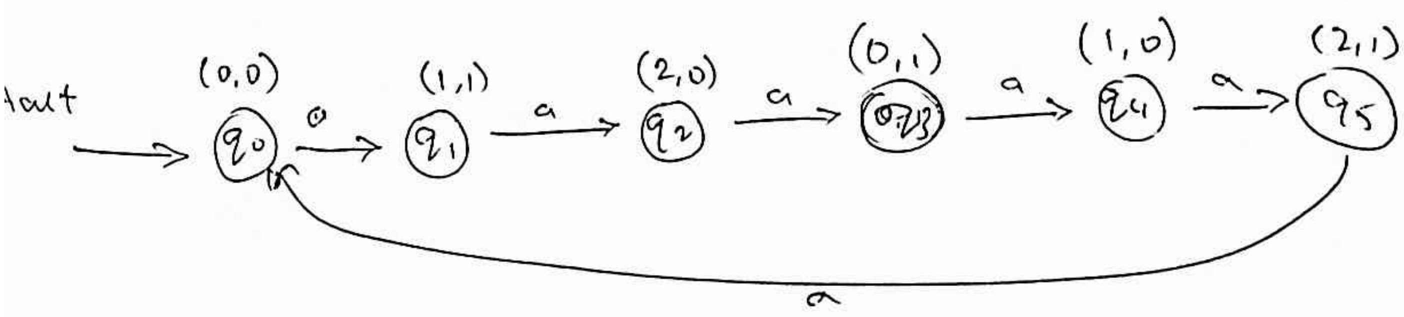
$\delta(\underset{M_1\ M_2}{(1.1)}, a) = \underset{2}{\delta(1,a)} \quad \underset{0}{\delta(1,a)} \qquad (2,0)$

$\delta((2,0).a) = \underset{0}{\delta(2,a)} \quad \underset{1}{\delta(0.a)} \qquad (0,1)$

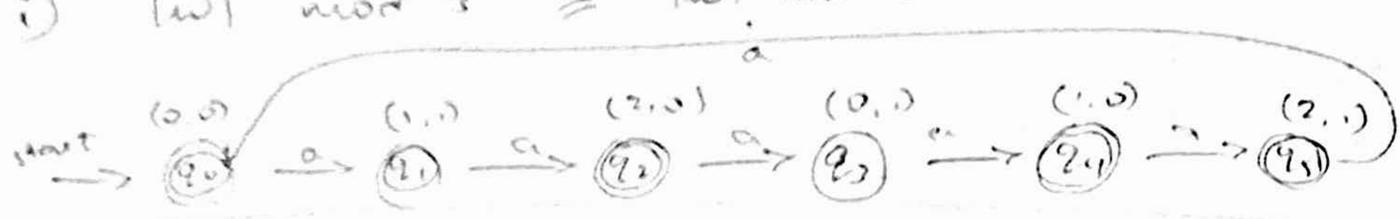
$\delta((0,1),a) : \underset{1}{\delta(0,a)} \quad \underset{0}{\delta(1,a)} \qquad (1,0)$

$\delta((1,0).a) = \underset{2}{\delta(1,a)} \quad \underset{1}{\delta(0.a)} \qquad (2,1)$

$\delta((2,1),a) = \underset{0}{\delta(2,a)} \quad \underset{0}{\delta(1,a)} \qquad (0,0)$

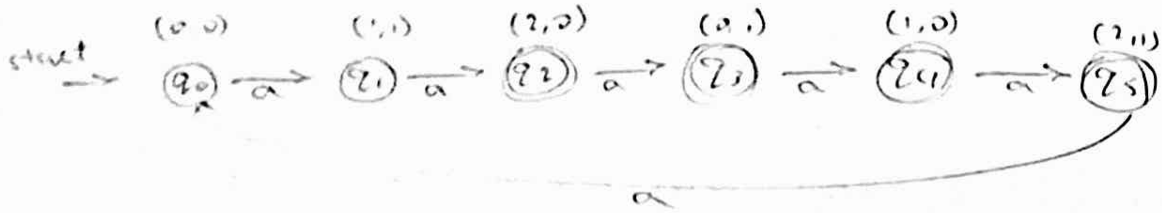


i) $|w| \bmod 3 \geq |w| \bmod 2$



ii) $|w| \bmod 3 \neq |w| \bmod 2$

0=0
1=1
2>0
0≠1
1>0



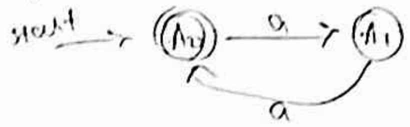
25-11-2022.

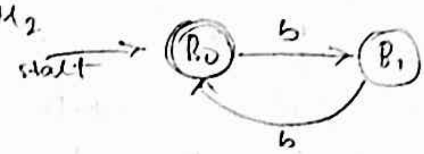
$N_a(w)$ mod k problems with logical operator.

1. $L = \{ n_a(w) \bmod 2 = 0$ && $n_b(w) \bmod 2 = 0 \}$

$\Sigma = \{a, b\}$

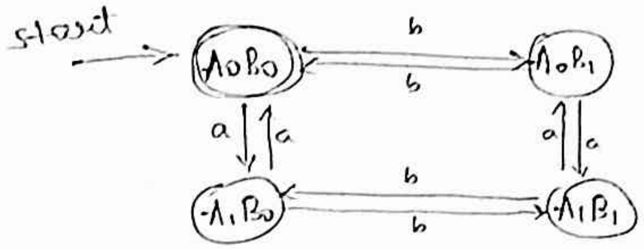
M_1 $\qquad\qquad\qquad\qquad\qquad$ M_2





$Q_1 = \{A_0, A_1\}$ $\qquad\qquad\qquad$ $Q_2 = \{B_0, B_1\}$

$Q = Q_1 \times Q_2 = \{(A_0 B_0), (A_0 B_1), (A_1 B_0), (A_1 B_1)\}$



even a's odd b's A_0B_1

odd a's even b's (A_1B_0)

odd a's odd b's (A_1B_1)

2. $L = \{ n_a(w) \bmod 3 = 0$ and $n_a(w) \bmod 2 = 0 \}$

$Q_1 = \{A_0 \; A_1 \; A_2\}$

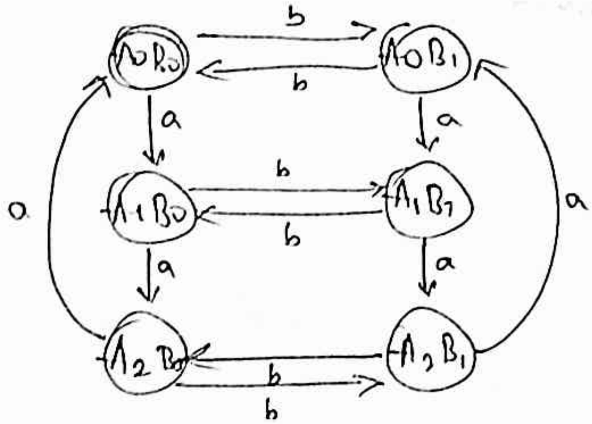
$Q_2 = \{B_0 \; B_1\}$

$Q = Q_1 \times Q_2$

$= \{(A_0 B_0) \; (A_0 B_1)$

$(A_1 B_0) \; (A_1 B_1)$

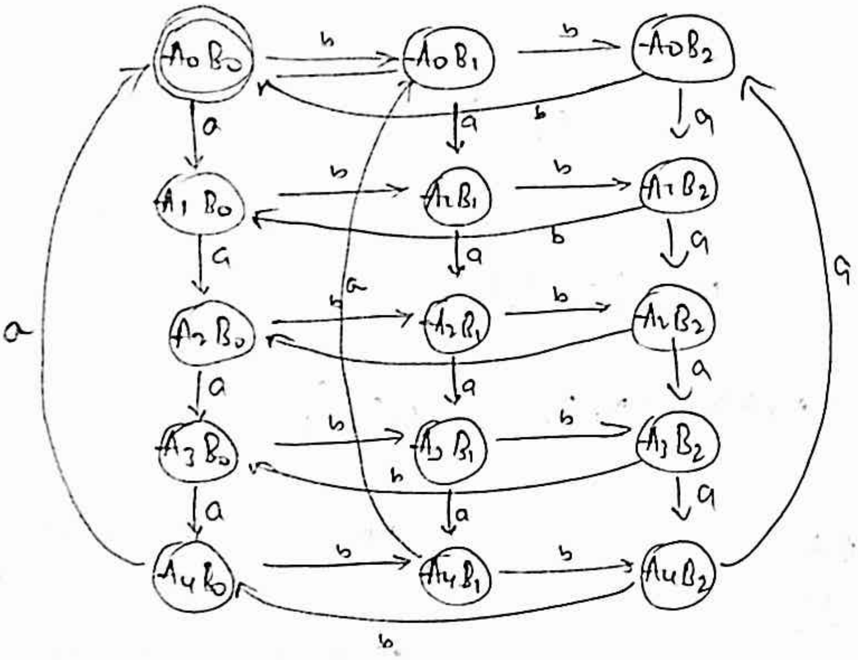
$(A_2 B_0) \; (A_2 B_1)\}$



$L = \{ n_a(w) \bmod 5 = 0$ and $n_b(w) \bmod 3 = 0 \}$

$Q_1 = \{A_0 \; A_1 \; A_2 \; A_3 \; A_4\}$

$Q_2 = \{B_0 \; B_1 \; B_2\}$



Getting DFA from NFA.

step-1 : Start state of NFA is also the start state of DFA.

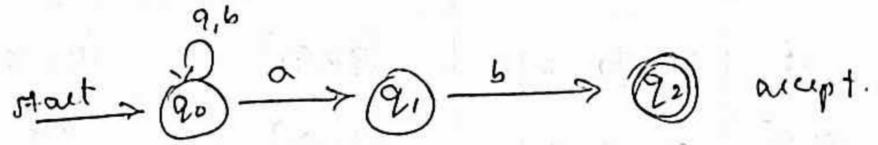
step-2 : $\Sigma =$

step-3 : Identifying all the states.

step-4 : Final state in DFA are the ones which contain the final state in NFA.

step-5 : Find all the transitions

29-11-2022.



T.T	a	b
q_0	$\{q_0, q_1\}$	q_0
q_1	ϕ	q_2
q_2	ϕ	ϕ

Step-1 start state $\Rightarrow q_0$

Step-2 $\Sigma = \{a, b\}$.

Step-3 $Q_D = \{ \{\phi\}, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_2\}, \{q_0, q_1, q_2\} \}$

Step 4 Final state.

$\{ \{q_2\}\ \{q_0 q_2\}\ \{q_1 q_2\}\ \{q_0 q_1 q_2\} \}$

Step-5 $\delta(\{\phi\}, a) = \phi \qquad \delta(\phi, b) = \phi$

$\delta(\{q_0\}, a) = \{q_0, q_1\} \qquad \delta(\{q_0\}, b) = q_0$

$\delta(\{q_1\}, a) = \phi \qquad \delta(\{q_1\}, b) = q_2$

$\delta(\{q_2\}, a) = \phi \qquad \delta(\{q_2\}, b) = \phi$

$\delta(\{q_0, q_1\}, a) = \delta(\{q_0\}, a) \cup \delta(\{q_1\}, a)$

$= \{q_0, q_1\} \cup \{\phi\}$

$= \{q_0, q_1\}$

$\delta(\{q_0, q_1\}, b) = \delta(\{q_0\}, b) \cup \delta(\{q_1\}, b)$

$= \{q_0\} \cup \{q_2\}$

$= \{q_0, q_2\}$

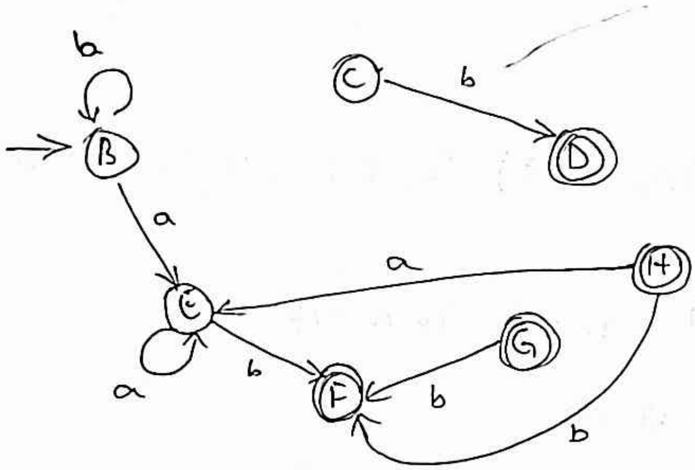
$\delta(\{q_1, q_2\}, a) = \phi \qquad \delta(\{q_1, q_2\}, b) = \{q_0, q_2\}$

$\delta(\{q_0, q_2\}, a) = \{q_0, q_1\} \qquad \delta(\{q_0, q_2\}, b) = \{q_0\}$

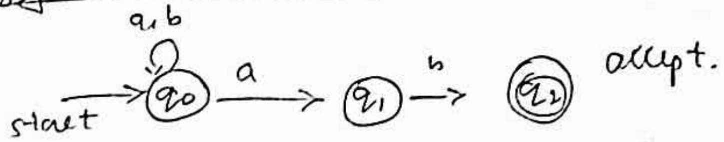
$\delta(\{q_0, q_1, q_2\}, a) = \{q_0, q_1\} \qquad \delta(\{q_0, q_1, q_2\}, b) = \{q_0, q_2\}$

	T.T	a	b
A	ϕ	ϕ	ϕ
B	$\rightarrow q_0$	$q_0 q_1$	$q_0 q_1$
C	q_1	ϕ	q_2
D	$* q_2$	ϕ	ϕ

		a	b
E	$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0\ q_2\}$
F	$* \{q_0, q_2\}$	$\{q_0\ q_1\}$	q_2
G	$* \{q_1, q_2\}$	ϕ	$\{q_0\ q_2\}$
H	$* \{q_0, q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$



Lazy evaluation.



	a	b
q_0	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
q_2	ϕ	ϕ

$\Sigma = \{a, b\}$

Start $\{q_0\}$ input a & b

$\delta(q_0, a) = \underline{\{q_0, q_1\}}$ $\quad \delta(q_0, b) = \{q_0\}$

$\{q_0, q_1\}$ input a & b

$\delta((q_0, a)) \cup \delta(q_1, a) = \{q_0, q_1\}$ $\quad \delta((q_0, b)) \cup \delta(q_1, b) = \underline{\{q_0, q_2\}}$

$\{q_0, q_2\}$ input a & b

$\delta(q_0, a) \cup \delta(q_2, a) = \{q_0, q_1\}$ $\quad \delta(q_0, b) \cup \delta(q_2, b) = \{q_0\}$

	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$

	a	b
A	B	A
B	B	C
C	B	A

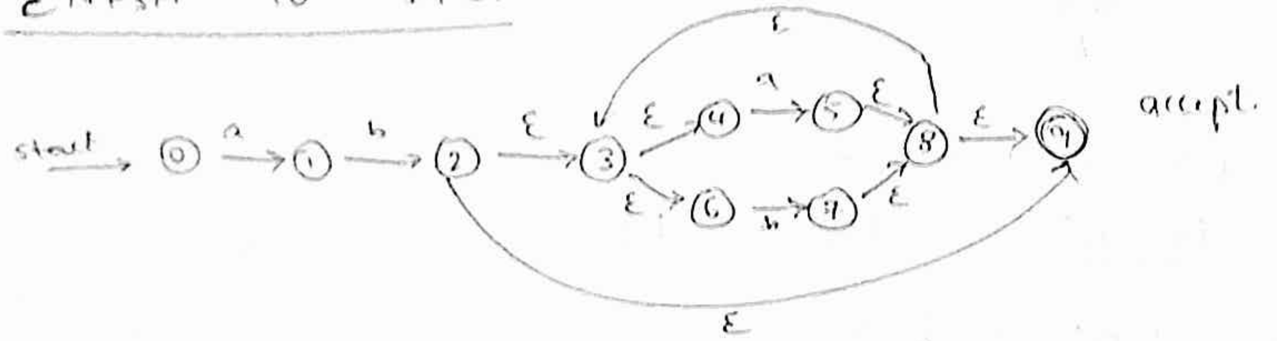
Definition

DFSM − $\delta : Q \times \Sigma$ to Q

NFSM − $\delta : Q \times \Sigma$ to 2^Q

ε-NFSM − $\delta : Q \times (\Sigma \cup \varepsilon)$ to 2^Q

ENFSM to DFSM



start state 0

$ECLOSE(0) : \{0\} \rightarrow A$

consider state A

$\delta(A, a) = ECLOSE(\delta_E(A, a))$
$= ECLOSE(\delta_E(0, a))$
$= \{1\} \rightarrow B$

$\delta(A, b) = ECLOSE(\delta_E(A, b))$
$= ECLOSE(\delta_E(0, b))$
$= \{\phi\}$

State B

$\delta(B, a) : ECLOSE(\delta_E(B, a))$
$: ECLOSE(\delta_E(1, a))$
$: \phi$

$\delta(B, b) : ECL(\delta_E(1, b))$
$: ECLOSE(\{2\})$
$: \{2, 3, 4, 6, 9\} \rightarrow C$

state C

$\delta(C, a) : ECLOSE(\delta_E(C, a))$
$: ECLOSE(\delta_E(\{2, 3, 4, 6, 9\}), a)$
$= ECLOSE(5)$
$: \{5, 8, 3, 6, 9, 4\} \rightarrow D$

$\delta(C, b) : ECLOSE(\delta_E(C, b))$
$: ECLOSE(\delta_E(\{2, 3, 4, 6, 9\}), b)$
$= ECLOSE(7)$
$= \{7, 8, 3, 9\} 4, 6\} \rightarrow E$

	a
A	$\{1\}$
B	ϕ
C	$\{5, 8, 3, 6, 9, 4\}$
D	$\{$
E	