In [246…

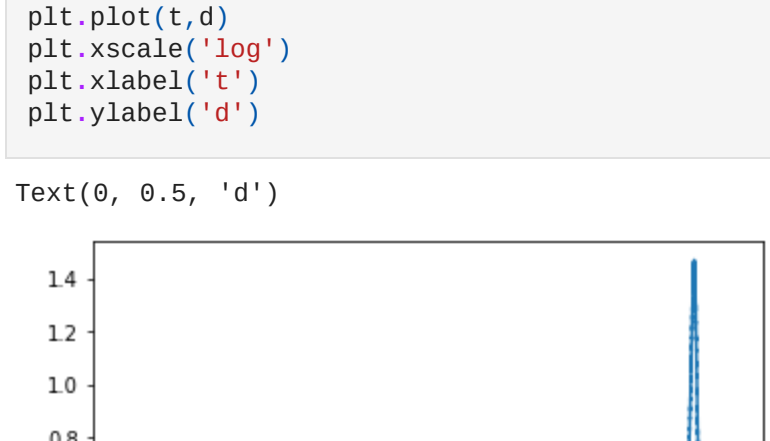
```python
import numpy as np
import matplotlib.pyplot as plt
```

Reading in and plotting the data

In [247…

```python
stuff=np.load('sidebands.npz')
t=stuff['time']
d=stuff['signal']
plt.plot(t,d)
plt.xscale('log')
plt.xlabel('t')
plt.ylabel('d')
```

Out[247… Text(0, 0.5, 'd')



a)

From the plot, guessing the initial parameters as: t0 = 2x10^{-4}, a = 1.4, w = 2x10^{-4}

Function to give the predicted data based on the guess of the parameters and also return A'. The array p conatins the parameters in the order [a, t0, w]

In [248…

```python
def calc_lorentz(p,t):
    y=p[0]/(1+(t-p[1])**2/p[2]**2)
    grad=np.zeros([t.size,p.size])
    #now partial differentiate w.r.t. all the parameters
    grad[:,0]=1.0/(1+(t-p[1])**2/p[2]**2)
    grad[:,1]=(2*p[0]*(t-p[1]))/((1+(t-p[1])**2/p[2]**2)**2*p[2]**2)
    grad[:,2]=(2*p[0]*(t-p[1])**2)/((1+(t-p[1])**2/p[2]**2)**2*p[2]**3)
    return y,grad
```

Starting with an initial guess of the parmeters. The function calc_lorentz returns the predicted data based on p, and the matrix A'. Then as derived in class $\delta\chi^2 = -2A'^T N^{-1} r$, but I am considering N^{-1} to be I. Since we want $\delta\chi^2$ to be 0, I set the convergence criteria so that the loop will run until $\delta\chi^2$ is greater than 10^{-3}. As we derived in class, the step dp will be $(A'^T A')^{-1}(A'^T r)$

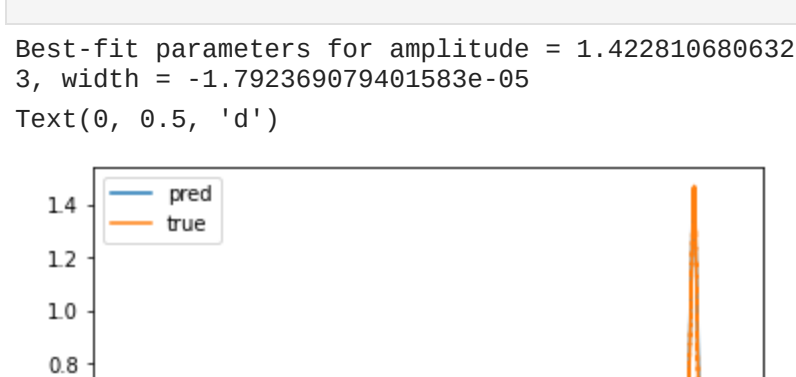
In [249…

```python
p0=np.array([1.4, 2*pow(10,-4), 2*pow(10,-4)])

p=p0.copy()
flag = True
while flag:
    pred,grad=calc_lorentz(p,t)
    r=d-pred
    r=np.matrix(r).transpose()
    grad=np.matrix(grad)
    delta_chi2 = np.array(-2*grad.transpose()*r)
    flag = False
    for i in range(len(delta_chi2)):
        if abs(delta_chi2[i][0]) > pow(10,-3):
            flag = True
            break
    if flag == True:
        lhs=grad.transpose()*grad
        rhs=grad.transpose()*r
        dp=np.linalg.inv(lhs)*(rhs)
        for jj in range(p.size):
            p[jj]=p[jj]+dp[jj]
print("Best-fit parameters for amplitude = {}, center = {}, width = {}".format(p[0], p
plt.plot(t,pred,label="pred")
plt.plot(t,d,label="true")
plt.legend()
plt.xscale('log')
plt.xlabel("t")
plt.ylabel("d")
plt.xlabel("t")
plt.ylabel("d")
```

```
Best-fit parameters for amplitude = 1.4228106806321144, center = 0.0001923586493756518
3, width = -1.792369079401583e-05
```

Out[249… Text(0, 0.5, 'd')



b)

Estimating the noise (or variance) to be the mean of the residual squared. Then, the error in the parameters would be the square root of the diagonal elemets of the covariance matrix $(A'^T N^{-1} A')^{-1}$.

In [250…

```python
A = np.zeros([len(t)])
A = p[0]/(1+(t-p[1])**2/p[2]**2)
N=np.mean((d-pred)**2)
par_errs=np.sqrt(N*np.diag(np.linalg.inv(lhs)))
print("Best-fit parameter error for amplitude = {}, center = {}, width = {}".format(pa
```

```
Best-fit parameter error for amplitude = 0.00042547904587075835, center = 5.3583455633
32885e-09, width = 7.588097244743478e-09
```

c)

Function to return partial derivative wrt the parameters. I am taking the dx in the partial derivative to be the value of the model parameter divided by 100.

In [251…

```python
def part_diff(fun, pars, t):
    pred = fun(pars, t)
    grad = np.zeros([t.size, pars.size])
    for i in range(len(pars)):
        pars_temp1 = pars.copy()
        pars_temp2 = pars.copy()
        h = pars[i]/pow(10, 2)
        #print(h)
        pars_temp1[i] = pars_temp1[i] + h
        pars_temp2[i] = pars_temp2[i] - h
        #print(pars_temp1, pars_temp2)
        grad[:,i] = (fun(pars_temp1, t) - fun(pars_temp2, t))/(2*h)
    return pred, grad
```

Function which returns the Lorentzian

In [252…

```python
def fun(p, t):
    y = p[0]/(1+(t-p[1])**2/p[2]**2)
    return np.array(y)
```

Plotting the predicted data from the initial guess of the parameters

In [253…

```python
p0 = np.array([1.4, 2*pow(10,-4), 2*pow(10,-4)])
pred, grad = part_diff(fun, p0, t)
plt.plot(t, pred, label = "pred")
plt.plot(t, d, label = "real")
plt.legend()
plt.xscale("log")
plt.xlabel("t")
plt.ylabel("d")
```

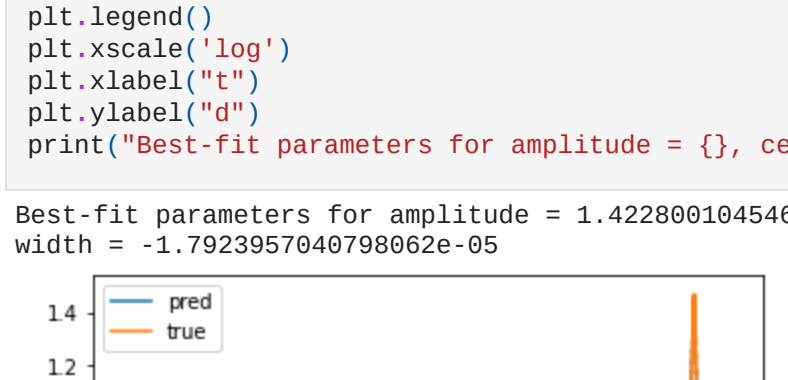
Out[253… Text(0, 0.5, 'd')

Doing the same thing as in a) but with the numerical partial derivative function.

In [254…

```python
p0 = np.array([1.4, 2*pow(10,-4), 2*pow(10,-4)])
p=p0.copy()
flag = True
while flag:
    #pred = fun(p, t)
    pred, grad = part_diff(fun, p, t)
    r=d-pred
    r=np.matrix(r).transpose()
    grad=np.matrix(grad)
    delta_chi2 = np.array(-2*grad.transpose()*r)
    flag = False
    for i in range(len(delta_chi2)):
        if abs(delta_chi2[i][0]) > pow(10,-3):
            flag = True
            break
    if flag == True:
        lhs=grad.transpose()*grad
        rhs=grad.transpose()*r
        dp=np.linalg.pinv(lhs)*(rhs)
        for jj in range(p.size):
            p[jj]=p[jj]+dp[jj]
plt.plot(t,pred,label="pred")
plt.plot(t,d,label="true")
plt.legend()
plt.xscale('log')
plt.xlabel("t")
plt.ylabel("d")
print("Best-fit parameters for amplitude = {}, center = {}, width = {}".format(p[0], p
```

```
Best-fit parameters for amplitude = 1.4228001045462348, center = 0.000192354414007016,
width = -1.7923957040798062e-05
```



In [255…

```python
N=np.mean((d-pred)**2)
par_errs=np.sqrt(N*np.diag(np.linalg.inv(lhs)))
print("Best-fit parameter error for amplitude = {}, center = {}, width = {}".format(pa
```

```
Best-fit parameter error for amplitude = 0.00042548791051056197, center = 5.3892107678
60868e-09, width = 7.588424876129845e-09
```

The best-fit parameters are not statistically significantly different from my answers in a)

d)

Defining the new model function with the array p having the parameters in the order [a, b, c, t0, dt, w] and doing the same thing as in c)

In [256…

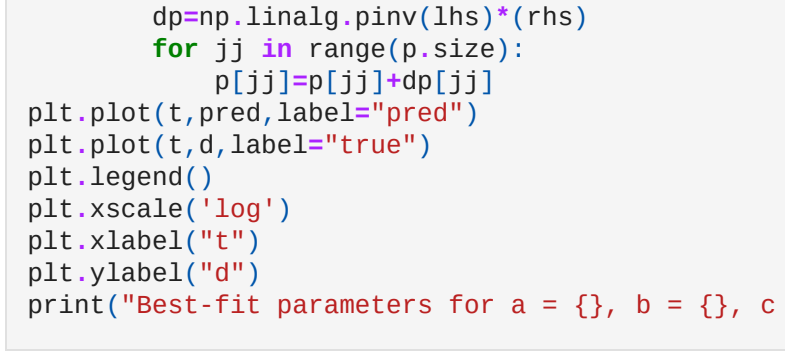
```python
# p = [a, b, c, t0, dt, w]
def fun2(p, t):
    y = p[0]/(1+(t-p[3])**2/p[5]**2) + p[1]/(1+(t-p[3]+p[4])**2/p[5]**2) + p[2]/(1+(t-
    return np.array(y)
```

For the initial parameters, using the best-fit values for a, t0 and w

In [257…

```python
p0 = np.array([1.42281068e+00, 0.3, 0.3, 1.92358649e-04, 0.5*pow(10, -4), -1.79236908e
p=p0.copy()
flag = True
while flag:
    #pred = fun(p, t)
    pred, grad = part_diff(fun2, p, t)
    r=d-pred
    r=np.matrix(r).transpose()
    grad=np.matrix(grad)
    delta_chi2 = np.array(-2*grad.transpose()*r)
    flag = False
    for i in range(len(delta_chi2)):
        if abs(delta_chi2[i][0]) > pow(10,-3):
            flag = True
            break
    if flag == True:
        lhs=grad.transpose()*grad
        rhs=grad.transpose()*r
        dp=np.linalg.pinv(lhs)*(rhs)
        for jj in range(p.size):
            p[jj]=p[jj]+dp[jj]
plt.plot(t,pred,label="pred")
plt.plot(t,d,label="true")
plt.legend()
plt.xscale('log')
plt.xlabel("t")
plt.ylabel("d")
print("Best-fit parameters for a = {}, b = {}, c = {}, t0 = {}, dt = {}, w = {}".forma
```

```
Best-fit parameters for a = 1.4429874309196413, b = 0.10382365393961594, c = 0.0647893
0144874528, t0 = 0.0001925742772637443, dt = 4.4570480060147 69e-05, w = -1.60655747189
23747e-05
```



In [258…

```python
N=np.mean((d-pred)**2)
par_errs=np.sqrt(N*np.diag(np.linalg.inv(lhs)))
print("Best-fit parameter errors for a = {}, b = {}, c = {}, t0 = {}, dt = {}, w = {}'
```

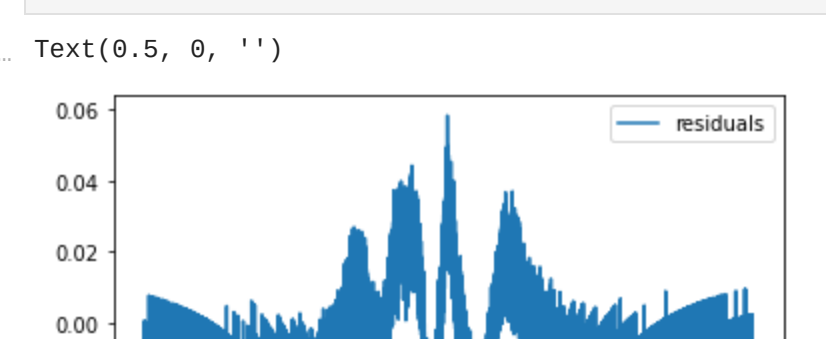
```
Best-fit parameter errors for a = 0.00026643414835356 88, b = 0.0002542262998720864, c
= 0.0002489206549916085, t0 = 3.1820022608659255e-09, dt = 3.8060597997087904e-08, w =
5.649755024223123e-09
```

e)

In [259…

```python
plt.plot(t, d-pred, label="residuals")
plt.legend()
plt.xlabel("")
```

Out[259… Text(0.5, 0, '')



As can be seen, there is structure in the residuals similar to the shape of the data. This means that the model doesn't fully describe the data. Also, since the residuals are not randomly distributed, we cannot say that the data are independent with uniform variance

f)

Getting the full covariance matrix for the error in parameters.

In [260…

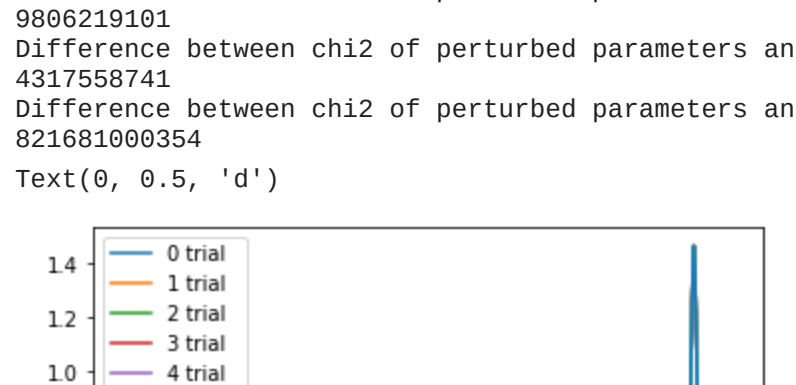
```python
N=np.mean((d-pred)**2)
cov=N*(np.linalg.inv(lhs))
```

In [261…

```python
bestfit_chi2 = np.sum((d - fun2(p,t))**2)
print("Best-fit chi2 is {}".format(bestfit_chi2))
for i in range(10):
    p_new = np.random.multivariate_normal(p, cov)
    pred_new = fun2(p_new, t)
    plt.plot(t, pred_new, label=str(i)+" trial")
    chi2 = np.sum((d-pred_new)**2)
    print("Difference between chi2 of perturbed parameters and best fit parameters is
plt.plot(t, fun2(p, t), label="Best-fit")
plt.legend()
plt.xscale("log")
plt.xlabel("t")
plt.ylabel("d")
```

```
Best-fit chi2 is 21.24764880700172
Difference between chi2 of perturbed parameters and best fit parameters is -0.00083902
71231029089
Difference between chi2 of perturbed parameters and best fit parameters is -0.00046142
66868046002
Difference between chi2 of perturbed parameters and best fit parameters is -0.00129308
06570068398
Difference between chi2 of perturbed parameters and best fit parameters is -0.00048080
83884810799
Difference between chi2 of perturbed parameters and best fit parameters is -0.00109117
68097656704
Difference between chi2 of perturbed parameters and best fit parameters is -0.00187972
08579890423
Difference between chi2 of perturbed parameters and best fit parameters is -0.00214075
38847029173
Difference between chi2 of perturbed parameters and best fit parameters is -0.00196048
9806219101
Difference between chi2 of perturbed parameters and best fit parameters is -0.00227223
4317558741
Difference between chi2 of perturbed parameters and best fit parameters is -0.00029088
821681000354
```

Out[261… Text(0, 0.5, 'd')



As can be seen, the $\chi^2_{best-fit}$ - $\chi^2_{perturbed}$ is -ve for all the iterations. This makes sense as we want our best-fit parameters to have as less of a χ^2 as possible.

g)

Function which returns the χ^2 (considering noise matrix N to to I)

In [264…

```python
def get_chi2(p, data):
    x=data['x']
    y=data['y']

    pred=fun2(p,x)
    chisq=np.sum((pred-y)**2)
    return chisq
```

Function which gives the offset in the parameters (dp) from a normal distribution having the varaiance as the covariance matrix of the error in model parameters. Multiplying this offset by 0.5 to have the convergence faster.

In [265…

```python
def get_step(p, cov):
    mean = np.zeros([len(p)])
    dp = np.random.multivariate_normal(mean, cov)*0.5
    return dp
```

The MCMC chain which startes with the best-fit parameters I got from d. Then, it adds an offset dp to p and calculates the χ^2 for the new p. The probability of acceptance of the new p is higher is the new χ^2 is lower. The p and χ^2 for each step is saved.

In [266…

```python
def run_chain(fun,pars,cov,data,nstep=40000,T=1):
    npar=len(pars)
    chain=np.zeros([nstep,npar])
    chisq=np.zeros(nstep)
    chain[0,:]=pars
    chi_cur=fun(pars,data)
    chisq[0]=chi_cur
    for i in range(1,nstep):
        pp=pars+get_step(pars, cov)
        new_chisq=fun(pp,data)
        accept_prob=np.exp(-0.5*(new_chisq-chi_cur)/T)
        if np.random.rand(1)<accept_prob:
            pars=pp
            chi_cur=new_chisq
        chain[i,:]=pars
        chisq[i]=chi_cur
    return chain,chisq
```

With the p and χ^2 saved for all the steps, the final p would be the weighted avg of the p for all steps. The weight is the difference between the χ^2 of a step from the minimum χ^2. The p with less difference (and hence less χ^2) has more weight. Similarly, the variance in p is calculated. (Here I am taking T=1, so the weight for all the steps is same)

In [267…

```python
def process_chain(chain,chisq,T=1.0):
    dchi=chisq-np.min(chisq)
    wt=np.exp(-0.5*dchi*(1-1/T))
    npar=chain.shape[1]
    tot=np.zeros(npar)
    totsqr=np.zeros(npar)
    for i in range(npar):
        tot[i]=np.sum(wt*chain[:,i])
        totsqr[i]=np.sum(wt*chain[:,i]**2)
    #divide by sum or weights
    mean=tot/np.sum(wt)
    meansqr=totsqr/np.sum(wt)

    #variance is <x^2>-<x>^2
    var=meansqr-mean**2
    return mean,np.sqrt(var),wt
```

In [268…

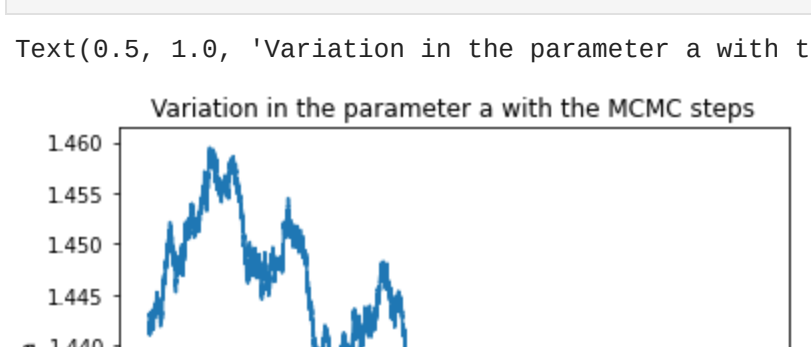
```python
data={}
data['x']=t
data['y']=d
chain,chivec=run_chain(get_chi2,p,cov,data)

mean,errs,wts=process_chain(chain,chivec)
```

In [269…

```python
steps = 1 + np.arange(40000)
plt.plot(steps, chain[:,0])
plt.xlabel("steps")
plt.ylabel("a")
plt.title("Variation in the parameter a with the MCMC steps")
```

Out[269… Text(0.5, 1.0, 'Variation in the parameter a with the MCMC steps')



In [270…

```python
print("Best-fit parameters for a = {}, b = {}, c = {}, t0 = {}, dt = {}, w = {}".forma
#print("Best-fit parameter errors for a = {}, b = {}, c = {}, t0 = {}, dt = {}, w = {]
```

```
Best-fit parameters for a = 1.4358313039601938, b = 0.10437690803596947, c = 0.0499967
4907790118, t0 = 0.00019273171650887786, dt = 4.721992715952323e-05, w = -1.6210301457
627292e-05
```

In [271…

```python
pred, grad = part_diff(fun2, mean, t)
N=np.mean((d-pred)**2)
grad = np.matrix(grad)
lhs = grad.transpose()*grad
par_errs=np.sqrt(N*np.diag(np.linalg.inv(lhs)))
print("Best-fit parameter errors for a = {}, b = {}, c = {}, t0 = {}, dt = {}, w = {}'
```

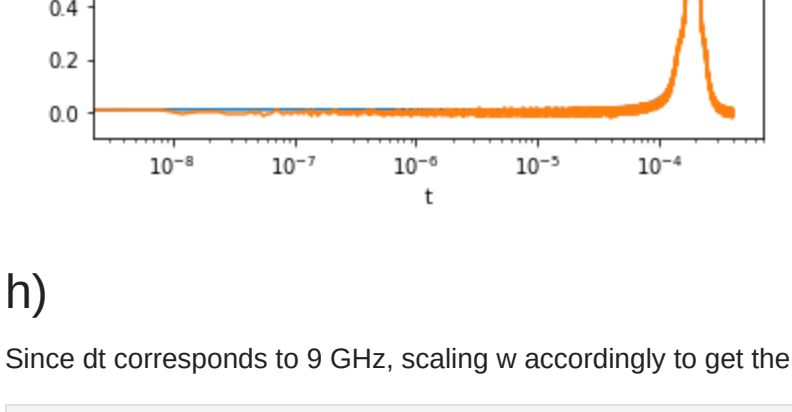
```
Best-fit parameter errors for a = 0.0002815938609723725, b = 0.0002586869177527817, c
= 0.0002519654484592520 4, t0 = 3.371114798420097e-09, dt = 4.2263460452428594e-08, w =
5.838623018827503e-09
```

The errors in the parameters are the same order as before

In [272…

```python
plt.plot(t, pred, label="pred")
plt.plot(t, d, label="real")
plt.legend()
plt.xscale("log")
plt.xlabel("t")
plt.ylabel("d")
```

Out[272… Text(0, 0.5, 'd')



h)

Since dt corresponds to 9 GHz, scaling w accordingly to get the width in GHz

In [273…

```python
dt = mean[4]
w = mean[5]
width_cavity = np.abs(w)*9/dt
print("Width of the cavity is {} GHz".format(width_cavity))
```

```
Width of the cavity is 3.0896429091424857 GHz
```

In [ ]: