Q1

DFT of a function $f(x)$ =

$$F(k) = \sum_{x=0}^{N-1} f(x) e^{-2\pi i k x/N}$$

Inverse DFT of $F(k)$ =

$$f(x) = \sum_{k=0}^{N-1} F(k) e^{2\pi i k x/N}$$

If I multiply this by a phase →

$$f(x-x_0) = \sum_{k=0}^{N-1} F(k) e^{2\pi i k x/N} e^{-2\pi i k x_0/N}$$

$$= \sum_{k=0}^{N-1} F(k) e^{\frac{2\pi i k}{N}(x-x_0)}$$

Thus, multiplying by a phase in the fourier space causes a shift of the array in the real space.

Q4

a)

a)

$$\sum_{x=0}^{N-1} \exp(-2\pi i k x/N)$$

$$= \sum_{x=0}^{N-1} \exp\left(-\frac{2\pi i k}{N}\right)^{x}$$

$$= \frac{1 - \exp\left(-\frac{2\pi i k}{N}\right)^{N}}{1 - \exp\left(-\frac{2\pi i k}{N}\right)}$$

$$= \frac{1 - \exp\left(-2\pi i k\right)}{1 - \exp\left(-\frac{2\pi i k}{N}\right)}$$

b)

$$\sum_{x=0}^{N-1} \exp\left(-\frac{2\pi i k x}{N}\right) =$$

lim $k \to 0$

$\exp(0) + \exp(0) + \dots$ N times

$= N$

If k is an integer and not a multiple of $N \to$

$$\sum_{x=0}^{N-1} \exp\left(\frac{-2\pi i k x}{N}\right) =$$

$$\frac{1 - \exp(-2\pi i k)}{1 - \exp\left(\frac{-2\pi i k}{N}\right)}$$

(since k is an integer, $\exp(-2\pi i k) = 1$)

$$= \frac{1-1}{1-\exp\left(-\frac{2\pi i k}{N}\right)} = 0$$

c) Non-integer sine wave:

$$\sin(2\pi(10.33)x) =$$

$$\frac{e^{i2\pi(10.33x)}}{i}$$

DFT of this sine wave

$$= \sum_{x=0}^{N-1} \frac{e^{i2\pi(10.33x)}}{i} e^{-\frac{2\pi i k x}{N}}$$

$$= \sum_{x=0}^{N-1} e^{2\pi i x\left(10.33 - \frac{k}{N}\right)}$$

$$= \sum_{x=0}^{N-1} e^{2\pi i x\left(\frac{31N/3 - k}{N}\right)}$$

$$\left(10.33 = \frac{31N/3}{}\right)$$

$$(\qquad\qquad N)$$

$$= \sum_{x=0}^{N-1} e^{-2\pi i x \frac{\left(k - \frac{31N}{3}\right)}{N}}$$

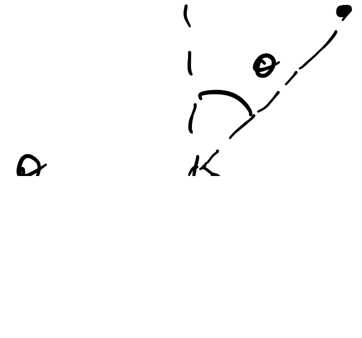
$$= \sum_{x=0}^{N-1} e^{-\frac{2\pi i x}{N}\left(k - \frac{31N}{3}\right)}$$

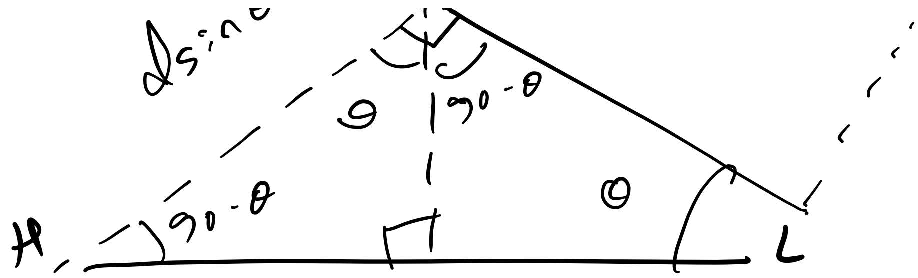
$$= \frac{1 - \exp\left(-2\pi i \left(k - \frac{31N}{3}\right)\right)}{1 - \exp\left(-2\pi i \frac{\left(k - \frac{31N}{3}\right)}{N}\right)}$$

Q5

f)

theta I measure in the code.





$d = 3030$ km

$$\frac{d \sin\theta}{\Delta t} = c$$

$$\theta = \sin^{-1}\left(\frac{c\Delta t}{d}\right)$$

Δt is the difference in the time of arrival of the gravitational wave at Hanford & Livingston detectors.

$$\theta = \sin^{-1}\left(\frac{c\Delta t}{d}\right)$$

$$\sin\theta = \frac{c\Delta t}{d}$$

I get the uncertainty for the TOA of the wave at each detector. Then the uncertainty in Δt is obtained as →

$$\delta \Delta t = \sqrt{\delta TOA_H^2 + \delta TOA_L^2}$$

(subtraction rule for uncertainty)

Then I get as

$$\sin(\theta \pm \delta\theta) = \frac{c(\Delta t \pm \delta\Delta t)}{d}$$

$$\theta \pm \delta\theta = \sin^{-1}\left(\frac{c(\Delta t \pm \delta\Delta t)}{d}\right)$$

$$\delta\theta = \sin^{-1}\left(\frac{c(\Delta t \pm \delta\Delta t)}{d}\right) - \theta$$