In [1]: import numpy as np import matplotlib.pyplot as plt a) In [2]: from astropy.io import fits hdul=fits.open('advact\_tt\_patch.fits') map=hdul[0].data hdul.close() map=np.asarray(map, dtype='float') width = 20patch = map[1999-width:1999+width,2999-width:2999+width]plt.imshow(map) plt.colorbar() plt.title("Original Map") Out[2]: Text(0.5, 1.0, 'Original Map') 2500 Original Map 0 2000 500 1500 1000 1000 1500 500 2000 2500 0 3000 -5003500 -10001000 2000 3000 4000 5000 0 -1500 In [3]: import numpy as np from astropy.io import fits from matplotlib import pyplot as plt plt.ion() def smooth\_map(map, npass): tmp=map.copy() for i in range(npass): tmp=tmp+np.roll(map,1,0)+np.roll(map,-1,0)+np.roll(map,1,1)+np.roll(map,-1,1)tmp=tmp/5 return tmp def pad\_map(map): map=np.hstack([map,np.fliplr(map)]) map=np.vstack([map,np.flipud(map)]) return map def gauss2d(pars,x): x0=pars[0]y0=pars[1]amp=pars[2] sig=pars[3] c=pars[4] dx=x-x0dy=x-y0dxmat=np.outer(dx,np.ones(len(dx))) dymat=np.outer(np.ones(len(dy)), dy) rsqr=dxmat\*\*2+dymat\*\*2 map=amp\*np.exp(-.5\*rsqr/sig\*\*2) + creturn map def get\_derivs\_ravel(fun, pars, dp, x): mymod=fun(pars,) npar=len(pars) dplus=[None]\*npar dminus=[None]\*npar for i in range(npar): pp=pars.copy() pp[i]=pp[i]+dp[i] dplus[i]=fun(pp,x) pp=pars.copy() pp[i]=pp[i]-dp[i] dminus[i]=fun(pp,x) n=dplus[0].size A=np.empty([n,npar]) #actually do the numerical derivatives for i in range(npar): dd=(dplus[i]-dminus[i])/(2\*dp[i]) A[:,i]=np.ravel(dd)return np.ravel(mymod), A def newton(pars,fun,data,x,dp,niter=10): for i in range(niter): mod, A=get\_derivs\_ravel(fun, pars, dp, x) r=data-mod  $n_{inv} = 1/np.std(data)$ lhs=n\_inv\*A.T@A rhs=n\_inv\*A.T@r dp=np.linalg.inv(lhs)@rhs pars=pars+dp print('on interation ',i,' parameters are ',pars) return pars hdul=fits.open('advact\_tt\_patch.fits') map=hdul[0].data hdul.close() map=np.asarray(map, dtype='float') #center of the original map x0=1999 y0=2999 #The patch I use for getting the best fit parameters width=10 patch=map[x0-width:x0+width,y0-width:y0+width] x=np.arange(0, patch.shape[0]) #The source in the center which corresponds to [width,width] pixels on the patch, givi #value on the map, sig = 1 and the constant c as 0 guess=np.asarray([width,width,patch.min(),1.0,0]) model=gauss2d(guess,x) dp=np.asarray([0.01,0.01,-1.0,0.01,0.01]) mod, derivs=get\_derivs\_ravel(gauss2d, guess, dp, x) fitp=newton(guess, gauss2d, np.ravel(patch), x, dp) modfit=gauss2d(fitp,x) print("Final best-fit parameters after 10 iterations of newton method ", fitp) plt.imshow(modfit-patch) plt.colorbar() plt.title("Residuals plot") on interation 0 parameters are [ 9.75881785 9.68323728 -39.98899538 1.6456 759 -134.22287183] on interation 1 parameters are [ 9.27102981 5.28343442 -130.56877873 8.7828 492 -123.20266329] on interation 2 parameters are [ 10.12087485 12.08461919 -228.41263435 5.6371 -5.72565158] 9.7402335 7.44525047 -148.62051402 on interation 3 parameters are [ 5.1831 6336 -80.59332514] on interation 4 parameters are [ 9.80153948 8.99525577 -172.63337126 3.5460 6699 -103.39284038] on interation 5 parameters are [ 9.75571846 8.44929035 -213.18579082 2.9764 4151 -111.47545657] on interation 6 parameters are [ 9.80740533 8.66008571 -210.98808778 3.1748 3822 -110.33333751] 8.63132911 -210.82843993 9.79223982 3.2187 on interation 7 parameters are [ 5394 -109.35931124] on interation 8 parameters are 9.79143876 8.63245772 -210.36883949 3.2333 4715 -109.12584924] 8.63153202 -210.23707758 9.79088592 on interation 9 parameters are [ 3.2372 828 -109.06575536] 9.79088592 Final best-fit parameters after 10 iterations of newton method [ 8.63 153202 -210.23707758 3.2372828 -109.06575536] Out[3]: Text(0.5, 1.0, 'Residuals plot') Residuals plot 0.0 150 2.5 100 5.0 50 7.5 0 10.0 -5012.5 -10015.0 17.5 -150 10 15 b) In [7]: def get\_gauss\_kernel(map, sig, amp, c, dx, dy, norm=False): nx=map.shape[0]x=np.fft.fftfreq(map.shape[0])\*map.shape[0] - dx y=np.fft.fftfreq(map.shape[1])\*map.shape[1] - dy rsqr=np.outer(x\*\*2,np.ones(map.shape[1]))+np.outer(np.ones(map.shape[0]),y\*\*2)kernel=np.exp((-0.5/sig\*\*2)\*rsqr)\*amp + ckernel=kernel/kernel.sum() return kernel def estimate\_ps2(pad\_fun, data): #pad the input map and take it's FT patch2 = pad\_fun(data) patchft=np.fft.fft2(patch2) #FT of patchft again as patchft has to be convolved with a gaussian patchft\_ft = np.fft.fft2(np.abs(patchft)\*\*2) #The gaussian template with the sigma as the sigma obtained from the best fit (3.2  $\#center = (0,0) \ and \ c=0$ gauss = get\_gauss\_kernel(patch2,3.23882668,1,0,0,0, norm=**True**) #the next lines of code to ensure that the gaussian and the FT(FT(padded map)) had x = patch2.shape[0]y = patch2.shape[1]width = gauss.shape[0]/2  $gauss_full = np.zeros([x,y])$ p = 0for i in np.arange(int(x/2-width),int(x/2+width)): for j in np.arange(int(y/2-width), int(y/2+width)): gauss\_full[i][j] = gauss[p][q] #FT of the gaussian and then IFT to get the convolved map gauss\_ft = np.fft.fft2(gauss\_full) smoothm = np.fft.irfft2(gauss\_ft\*patchft\_ft, patch2.shape) return smoothm In [30]: #For the analysis, I use a 2000x2000 patch from the original map, as my code takes to $\epsilon$ #original map (and this patch includes the galaxy that I need to identify for part 6) #center of the original map x0=1999 y0=2999 width = 1000 $map_new = map[x0-width:x0+width,y0-width:y0+width]$ smoothm = estimate\_ps2(pad\_map, map\_new) plt.imshow(np.log(smoothm)) plt.colorbar() plt.title("Log of the power spectrum") Out[30]: Text(0.5, 1.0, 'Log of the power spectrum') Log of the power spectrum 0 34 500 32 1000 1500 30 2000 - 28 2500 3000 26 3500 0 1000 2000 3000 c) In [31]: def filter\_map(map, ps, pad\_fun):  $pad_map = pad_fun(map)$  $n_{pix} = pad_{map.size}$ #Normalising the filtered map by dividing the power spectrum by the number of pixe Ninv=1/(ps/n\_pix) #This noise matrix is uncorrelated in the fourier space data\_ft=np.fft.fft2(pad\_map) filt\_map=np.fft.irfft2(data\_ft\*Ninv, pad\_map.shape) #Getting back into real space filt\_map= filt\_map[:map.shape[0],:map.shape[1]] #Getting the non-padded map from a return filt\_map In [32]: #The "ideal" best fit center of the cluster in part 1 should have been (10,10), but tl #(9.79073657, 8.6313414). So, the cluster is off from the center of the map by (dx, d) dx = 10 - 9.79073657dy = 10 - 8.6313414#Putting the sig, amp, c and center of the map obtained from the best-fit in part 1. gauss\_fit = get\_gauss\_kernel(map\_new,3.23882668,-210.18367169,-109.04243493,dx, dy) #Making sure that the gauss template and the map has the same dimensions  $x = map_new.shape[0]$  $y = map_new.shape[1]$ width = gauss\_fit.shape[0]/2  $gauss_full = np.zeros([x,y])$ p = 0for i in np.arange(int(x/2-width),int(x/2+width)): for j in np.arange(int(y/2-width),int(y/2+width)): gauss\_full[i][j] = gauss\_fit[p][q] q+=1 p+=1 #Getting the residuals from the fit resid = map\_new - gauss\_full #Getting the power spectrum from the residuals ps = estimate\_ps2(pad\_map, resid) plt.imshow(np.log(ps)) plt.colorbar() plt.title("Log of the power spectrum of the residuals") Out[32]: Text(0.5, 1.0, 'Log of the power spectrum of the residuals') Log of the power spectrum of the residuals 38 500 36 1000 34 1500 2000 30 2500 28 3000 3500 26 1000 2000 3000 0 In [33]: filt\_map = filter\_map(map\_new, ps, pad\_map) plt.imshow(filt\_map) plt.colorbar() plt.title("Filter map") Out[33]: Text(0.5, 1.0, 'Filter map') Filter map 0 0.15 250 500 0.10 750 0.05 1000 1250 0.00 1500 -0.051750 500 1000 1500 0 In [34]: #Patch of the filter map centered at the center of the original map #Map center x0=1000 y0=1000 width=40 filt\_patch=filt\_map[x0-width:x0+width,y0-width:y0+width] plt.imshow(filt\_patch) plt.colorbar() plt.title("Patch of the filtered map") plt.figure() plt.imshow(map\_new[x0-width:x0+width,y0-width:y0+width]) plt.colorbar() plt.title("Similar patch of the original map") Out[34]: Text(0.5, 1.0, 'Similar patch of the original map') Patch of the filtered map 0 0.04 0.03 10 0.02 20 0.01 30 0.00 40 -0.0150 -0.0260 -0.03 70 -0.0440 60 20 Similar patch of the original map 0 200 10 100 20 0 30 40 -10050 -200 60 -300 70 400 20 40 60 0 As can be seen, the filtered map has uncorrelated noise! d) In [36]: #Generating a map of the white noise, and getting it's filter map map\_test = np.zeros([map\_new.shape[0], map\_new.shape[1]]) map\_test = np.random.randn(map\_test.shape[0], map\_test.shape[1]).copy() ps\_test = estimate\_ps2(pad\_map, map\_test) filt\_map\_test = filter\_map(map\_test, ps\_test, pad\_map) plt.figure() plt.imshow(filt\_map\_test) plt.colorbar() plt.title("Filtered map for the input white noise map") Out[36]: Text(0.5, 1.0, 'Filtered map for the input white noise map') Filtered map for the input white noise map 250 500 750 1000 1250 1500 1750 0 500 1000 1500 In [37]:  $print("Mean and std of the original white noise map: m = {}, std = {}".format(np.mean)$ Mean and std of the original white noise map: m = -0.00012766840596836298, std = 1.000 3788986583046 In [38]: print("Mean and std of the filtered white noise map:  $m = \{\}$ , std =  $\{\}$ ".format(np.mean Mean and std of the filtered white noise map: m = -9.06758060581218e-05, std = 0.9925094004141256 As can be see, the std (and hence variance) of the filtered map is close to the expected value of 1, so the map is properly normalised! e) In [39]: #Getting the template gauss signal, simialar to what I did in part c dx = 10 - 9.79073657dy = 10 - 8.6313414padm = pad\_map(map\_new)  $n_{pix} = padm.size$ gauss\_fit = pad\_map(get\_gauss\_kernel(map\_new, 3.23882668, -210.18367169, -109.04243493, d) #FT of the gauss template tft=np.fft.fft2(gauss\_fit) #Normalising the power spectrum of the residuals Ninv=1/(ps/n\_pix) #FT of the input padded map datft=np.fft.fft2(padm) #Correlation of the input map with A^TN^-1 Ninvt=tft\*Ninv mf\_rhs=np.fft.irfft2(Ninvt\*np.conj(datft), datft.shape) #Getting lhs =  $A^TN^-1A$  (where A is the gaussian template) Ninv\_A = np.fft.irfft2(Ninvt, datft.shape) mf\_lhs = gauss\_fit.T@Ninv\_A #Getting the best-fit parameter which is the amplitude of the gaussian amp = np.linalg.pinv(mf\_lhs)@mf\_rhs #This will return the amp of the padded map. Next steps get the amp of the original  $m_{ ilde{c}}$ amp\_cut = amp[:map\_new.shape[0],:map\_new.shape[1]] #amp at the center of the map where our galaxy cluster is print("Best-fit amp at the center of the map is : {}".format(amp\_cut[1000,1000])) Best-fit amp at the center of the map is : -398396625.487954 In [40]: #Error in the amplitude error = np.sqrt(np.abs(np.linalg.pinv(mf\_lhs))) #Error in amp for the non-padded map error\_cut = error[:map\_new.shape[0],:map\_new.shape[1]] print("Error ar the center of the map is {}".format(error\_cut[1000,1000])) Error ar the center of the map is 330.72059101422695 In [60]: #The amplitude map around the center where there is a galaxy cluster plt.imshow(amp\_cut[1000-40:1000+40,1000-40:1000+40]) plt.colorbar() plt.title("Amplitude map around the center where there is a galaxy cluster") Out[60]: Text(0.5, 1.0, 'Amplitude map around the center where there is a galaxy cluster') Amplitude map around the center where there is a galaxy cluster 6 10 4 20 - 2 30 40 . 0 50 60 70 In [42]: #Std around the center of the map where there is a galaxy cluster print("Standard deviation of the matched filter output in a region around the cluster Standard deviation of the matched filter output in a region around the cluster = 19582 58094.789334 As returned by the matched filter fit, the amp at the center of the cluster is -398396625.487954 +-330.72059101422695 uK. The error is very small compared to the std I get in a region around the cluster, so I do not buy the error bars f) In [54]: plt.imshow(amp\_cut[1000-50:1000+50,1000-50:1000+250]) plt.colorbar() plt.title("Map with A2813 at (50,50)") Out[54]: Text(0.5, 1.0, 'Map with A2813 at (50,50)') le10 1.00 0.75 Map with A2813 at (50,50) 0 0.50 25 0.25 50 0.00 75 -0.25150 50 100 200 250 0 -0.50I can see the presence of a galactic cluster at about (200,50) but could not find a matching galaxy in the NED. In [59]: plt.imshow(amp\_cut[1000-100:1000+75,1000-100:1000+100]) plt.colorbar() plt.title("Map with A2813 at (100,100)") Out[59]: Text(0.5, 1.0, 'Map with A2813 at (100,100)') Map with A2813 at (100,100) 0 8 20 6 40 60 80 100 120 140 160 0 25 50 75 100 125 150 175 Similarly, I also see a galaxy cluster like feature at about (100,170), but do not find a matching galaxy in the NED. In [ ]: