Q3 RMS path length difference
$= \lambda/2\pi$.

$\Rightarrow$ RMS phase difference =

$$\frac{\lambda}{2\pi} \times \frac{2\pi}{\lambda} = 1 \text{ radian.}$$

for freq 1 (1.4 GHz),
the phase = $\frac{2\pi}{\lambda_1}$ RMS (B. θ)

for freq 2, phase =

$\frac{2\pi}{\lambda_2}$ RMS (B. θ)

(considering θ to be the 1 FWHM at 1.4 GHz $\rightarrow$

then,

$$\frac{2\pi}{\lambda_1} RMS(B \cdot \theta) - \frac{2\pi}{\lambda_2} RMS(B \cdot \theta)$$

$$= 1 \text{ radian}$$

$$\Rightarrow 2\pi \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) RMS(B \cdot \theta)$$

$$= 1 \text{ radian}$$

$$\Rightarrow 2\pi \left(\frac{r_1}{c} - \frac{r_2}{c} \right) RMS(B \cdot \theta)$$

$$= 1 \text{ radian}$$

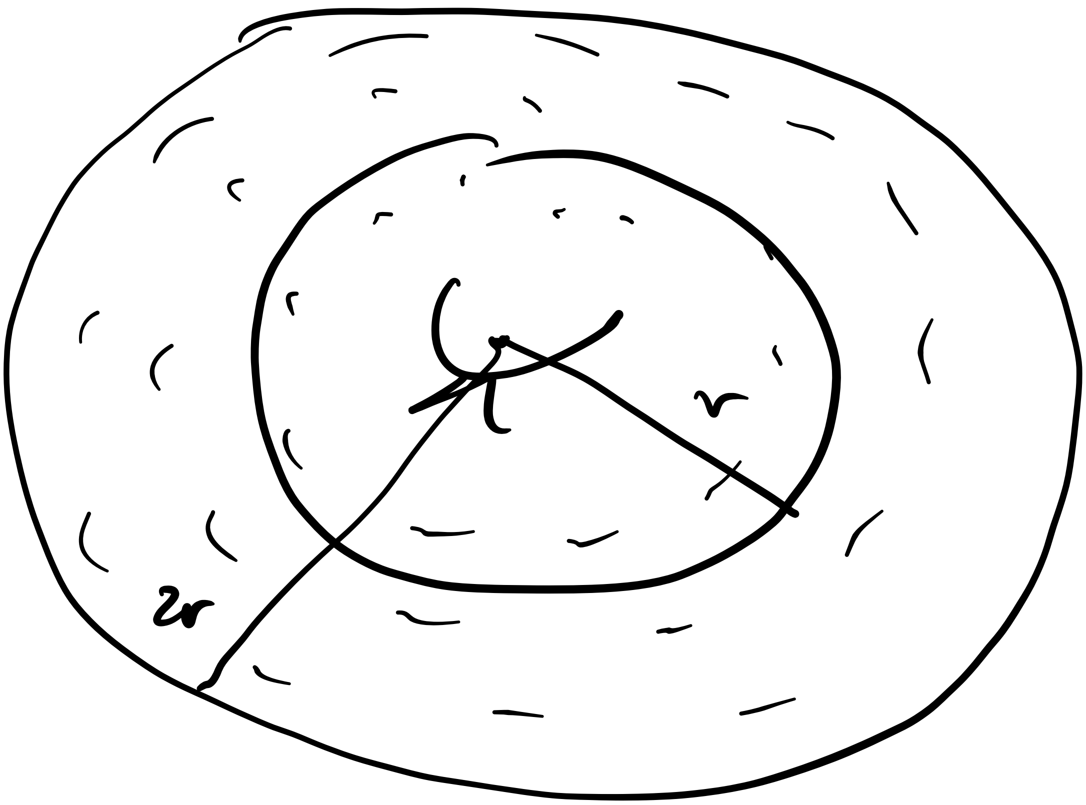
$$\Rightarrow \Delta r = \frac{c}{2\pi RMS(B \cdot \theta)}$$

$$\Rightarrow r_1 = r_2 + \frac{c}{2\pi RMS(B \cdot \theta)}$$

Q4

a) As explained by Jon in class:-

If we make a telescope 4 times more sensitive, that is increase it's area by 4 times, it detects sources with 4 times lesser flux, or which are twice the distance more farther.



Initial detection region of r, final detection region of 2r after sensitivity is increased by 4 times.

The number of sources now seen by the telescope would go as $\propto (2r)^3 \Rightarrow 8r^3 \Rightarrow$

8 times the original sources.

Using the same argument, we can say that the number of sources brighter than some flux limit detected by the telescope goes as $(\text{sensitivity})^{-3/2}$

$\Downarrow$

flux to which sources are detectable

b) 100 / square degree

1 GBT beam size at 1.4 GHz $= 1.22\lambda/d$
$\sim 9'$

100 sources / beam $\Rightarrow$

Sources in beam = $100 \times \left(\frac{9}{60}\right)^2$

$= 2.25$

Sources in 30 beams =

2.25×30

$= 67.5$

Confusion limit = $\left(\sqrt{67.5}\right)(1 mJ)$

$= 8.21$ mJ

2 Fact beam size at 1.4 GHz = $1.22 \lambda/d$

$= 1.22 \times \frac{0.214}{300}$

$\sim 3'$

Sources in beam = $100 \times \left(\frac{3}{60}\right)^2$

$= 0.25$

Sources in 30 beams = 7.5

Confusion limit = $(\sqrt{7.5})(\lambda(m))$

= 2.74 m3.

3 VLA A array has largest baseline of 36648 m

Beam size = $1.22\lambda/d$

$= \frac{1.22 \times 0.2f4}{36648}$

$= 0.02'$

Sources in beam = $100 \times \left(\frac{0.02}{60}\right)^2$

= 0.00001

Sources in 30 beams = 0.0003

Confusion limit :

$$\sqrt{0.0003}\,(>1\,mJ)$$

$$= 0.017\,mJ$$

(4) VLA D array has longest baseline of as 1031 m

Beam size = $1.22\lambda/d$

$$= \frac{1.22 \times 0.214}{1031}$$

$$= 0.87'$$

Sources in beam : $100 \times \left(\frac{0.87}{60}\right)^2$

$$= 0.02$$

Sources in 30 beams = 0.63

Confusion limit =

$\sqrt{0.63}$ (Jy/m J)

$= 0.79$ mJ.

c) $T_{sys} = 25$, $B = 500\,MHz$

aperture efficiency $= 70\%$

(1) Gain for GBT $\Rightarrow$

$(0.7)(\pi)(5000)^2/2k =$

$2e^{23} = 2$ K/Jy

$\Rightarrow$ for a confusion limit of 8.21 mJ, the equivalent $dT = 8.21 \times 2 = 16.42$ mK

The integration time required using the radiometer eq^n $\Rightarrow$

$$\frac{dT}{T} = \frac{1}{\sqrt{Bt}}$$

$$\frac{dT^2}{T^2} = \frac{1}{Bt}$$

$$t = \frac{T^2}{dT^2}\frac{1}{B}$$

$$t = \left(\frac{25}{0.016}\right)^2 \frac{1}{500e6}$$

$$t = 0.005\ s.$$

(2) Gain for FAST =

$$(0.7)(\pi)(15000)^2/2k =$$

$$18\ K/Jy$$

For confusion limit of 2.74 mJ, $dT = 49.32$ mJ

Integration time required =

$$t = \left(\frac{25}{0.049}\right)^2 \frac{1}{500e6}$$

$= 0.0005$ s

(3) Gain for VLA a array =

$$\frac{(0.7)(\pi)\left(\frac{36648 \times 100}{2}\right)^2}{2k}$$

$= 268615$ K/Jy

For confusion limit of

0.017 mJ, dT = 4566 mK

Integration time required as

$$t = \left(\frac{25}{4.566}\right)^2 \frac{1}{500e6}$$

$= 5.99e^{-8}$ s.

(4) Gain for VLA d-array =

$$\frac{(0.7)(\pi)\left(\frac{1031}{2}\times 100\right)^2}{2k}$$

$= 212$ kJ

For confusion limit θ_2

0.79 mJ, $dT = 168$ mK

Integration time required is

$$t = \left(\frac{25}{0.168}\right)^2 \frac{1}{500e6}$$

$= 0.00004$ s

d) 8 GHz $= 0.037$ m

Flux at 8 GHz $= 0.25 \times$ flux at 1.4 GHz

$B = 2$ GHz

1 GBT beam size at 8 GHz $= 1.22\lambda/d$

$\sim 1.56'$

100 sources / beam $\Rightarrow$

Sources in beam = $100 \times \left(\frac{1.56}{60}\right)^2$

$= 0.068$

Sources in 30 beams =

$= \frac{2.25 \times 30}{2}$

Confusion limit = $\sqrt{2} \times (> 0.25)$

$= 0.35$ mJ

for GBT gain of 2 K/J,

$dT = 0.35 \times 2 = 0.7$ mK

Integration time required =

$$\left(\frac{25}{0.7 \times 10^{-3}}\right)^2 \left(\frac{1}{2 \times 10^9}\right)$$

$= 0.64$ s

2 FAST beam size at 8 GHz

$= 0.525'$

Sources / beam $= 100\left(\frac{0.525}{60}\right)^2$

$= 0.008$

Sources in 30 beams $= 0.23$

Confusion limit $= \sqrt{0.23} \times 0.25$

$= 0.12$ mJ

Gain of FAST $=$ 18 K/J

Corresponding $dT = 0.12 \times 18$

$= 2.15$ mK

Integration time $=$

$$\left(\frac{25}{2.15\times10^{-3}}\right)^2 \frac{1}{2\times10^9}$$

$= 0.07\ s$

(3) VLA a array beam size at 8 GHz = 0.0035'

$$\text{Source/beam} = 100 \times \left(\frac{0.0035}{60}\right)^2$$

$$= 3.4 \times 10^{-7}$$

Sources in 30 beams = $1e^{-5}$

Confusion limit =

$$\sqrt{1e^{-5}} \times 0.25$$

$$= 8e^{-4}\ mJ$$

Gain of VLA a-array =

268615 K/Jy

$dT = 8e^{-4} \times 268615 =$

212 mK

Integration time =

$$\left(\frac{25}{0.212}\right)^2 \frac{1}{2e9}$$

$= 7e^{-7}$ s

(4) VLA d array beam size at 8 GHz = 0.15'

Sources / beam = $100 \times \left(\frac{0.15}{60}\right)^2$

= 0.0006

Source in 30 beams = 0.019

Confusion limit = $\sqrt{(0.019)}\,(0.25)$

= 0.034 mJ

Gain for ULA d-array = 212 K/J

$dT = 0.034 \times 212 = 7.25\ mK$

Integration time $= \left(\frac{25}{0.007}\right)^2 \frac{1}{2e9}$

$= 0.006\ s$