In [135…
```python
import numpy as np
from matplotlib import pyplot as plt
```

In [136…
```python
lat=34.0784*np.pi/180

if False:
    antpos=np.loadtxt('vla_d_array.txt')
    du=2.0
    color='b.'
    array='d'
else:
    antpos=np.loadtxt('vla_a_array.txt')
    du=40.0
    color='k.'
    array='a'

antpos=antpos[:,:3]  #the last column is boring...
antpos=antpos*1e-9*3e8 #convert to meters, since the file is in ns
nant=antpos.shape[0]
nvis=nant*(nant-1)//2

#we can look at the antenna array in 2D coordinates by looking at the
#distance from the zenith.  The following math is useful for converting 3D
#positions to EW/NS coordinates on the Earth's surface
zenith=np.asarray([np.cos(lat),0,np.sin(lat)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

mat=np.vstack([east,north,zenith])
xyz=antpos[:,:3]@mat.T
```

All questions for a array

Q1

a

In [137…
```python
import numpy as np
from matplotlib import pyplot as plt


lat=34.0784*np.pi/180

if False:
    antpos=np.loadtxt('vla_d_array.txt')
    du=2.0
    color='b.'
    array='d'
else:
```

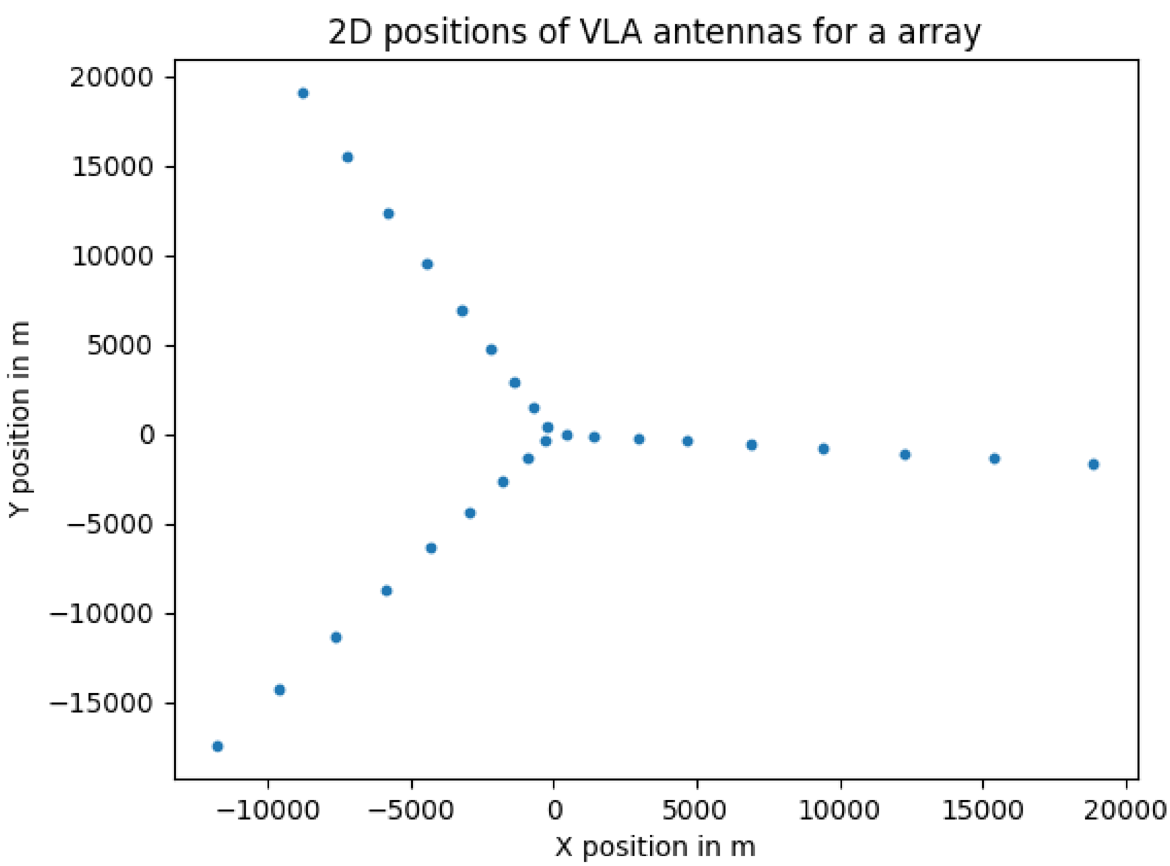
```
    antpos=np.loadtxt('vla_a_array.txt')
    du=40.0
    color='k.'
    array='a'

antpos=antpos[:,:3]  #the last column is boring...
antpos=antpos*1e-9*3e8 #convert to meters, since the file is in ns
nant=antpos.shape[0]
nvis=nant*(nant-1)//2

#we can look at the antenna array in 2D coordinates by looking at the
#distance from the zenith.  The following math is useful for converting 3D
#positions to EW/NS coordinates on the Earth's surface
zenith=np.asarray([np.cos(lat),0,np.sin(lat)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

mat=np.vstack([north,east,zenith])
xyz=antpos[:,:3]@mat.T
```

In [138…

```
x = xyz[:, 0]
y = xyz[:, 1]
z = xyz[:, 2]
plt.scatter(x, y, s =10)
plt.xlabel("X position in m")
plt.ylabel("Y position in m")
plt.title("2D positions of VLA antennas for a array")
```

Out[138]:

```
Text(0.5, 1.0, '2D positions of VLA antennas for a array')
```



In [139…
```
spread_x = max(x) - min(x)
spread_y = max(y) - min(y)
spread_z = max(z) - min(z)
print("Spread in x diection is {} m".format(spread_x))
print("Spread in y diection is {} m".format(spread_y))
print("Spread in z diection is {} m".format(spread_z))
```

```
Spread in x diection is 30627.32256038749 m
Spread in y diection is 36525.741 m
Spread in z diection is 81.35383558735688 m
```

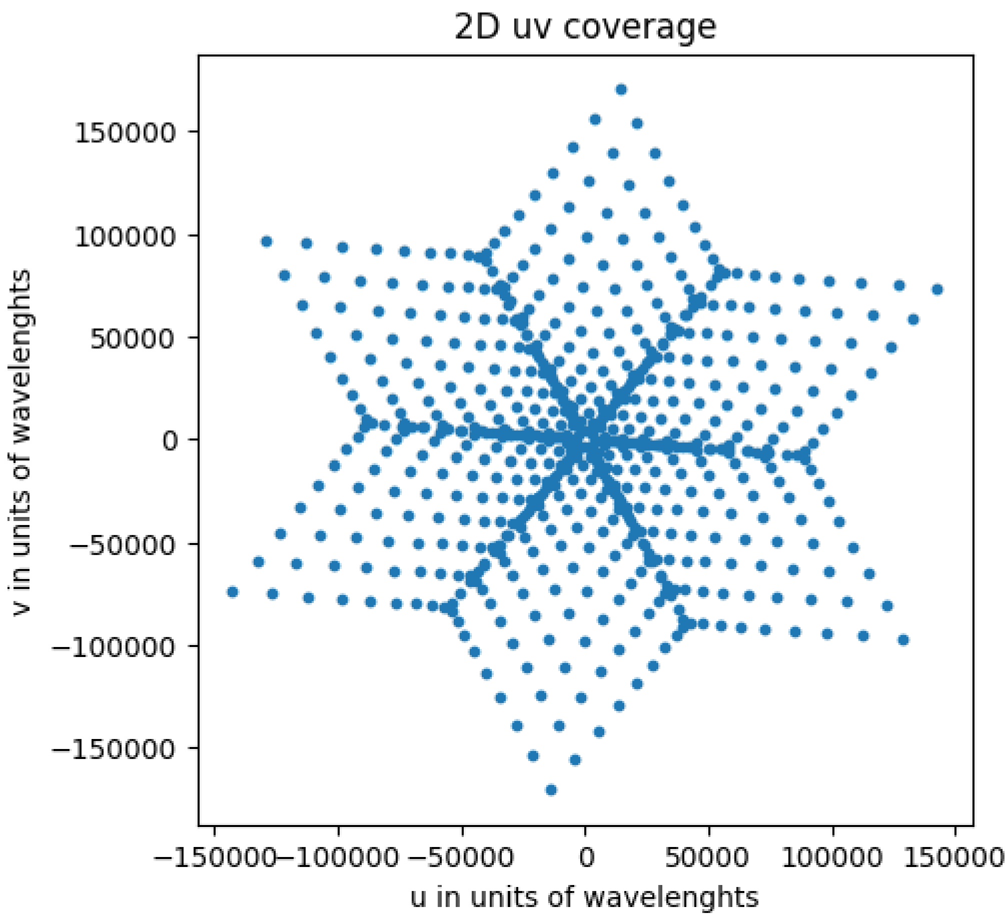
In [140…
```
rms_vert = np.sqrt(np.mean(z**2))
print("RMS vertical scatter is {} m".format(rms_vert))
```

```
RMS vertical scatter is 19.155695105333383 m
```

b

In [141…
```
uv=np.zeros([nvis,2])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv[icur,:]=xyz[i,:2]-xyz[j,:2]
        icur=icur+1
uv=np.vstack([uv,-1*uv])
nu = 1.4*10**9
wavelength = 3*10**8/nu
uv = uv/wavelength
plt.figure(figsize = (5,5))
plt.scatter(uv[:,0], uv[:,1], s=10)
plt.title("2D uv coverage")
plt.xlabel("u in units of wavelenghts")
plt.ylabel("v in units of wavelenghts")
```

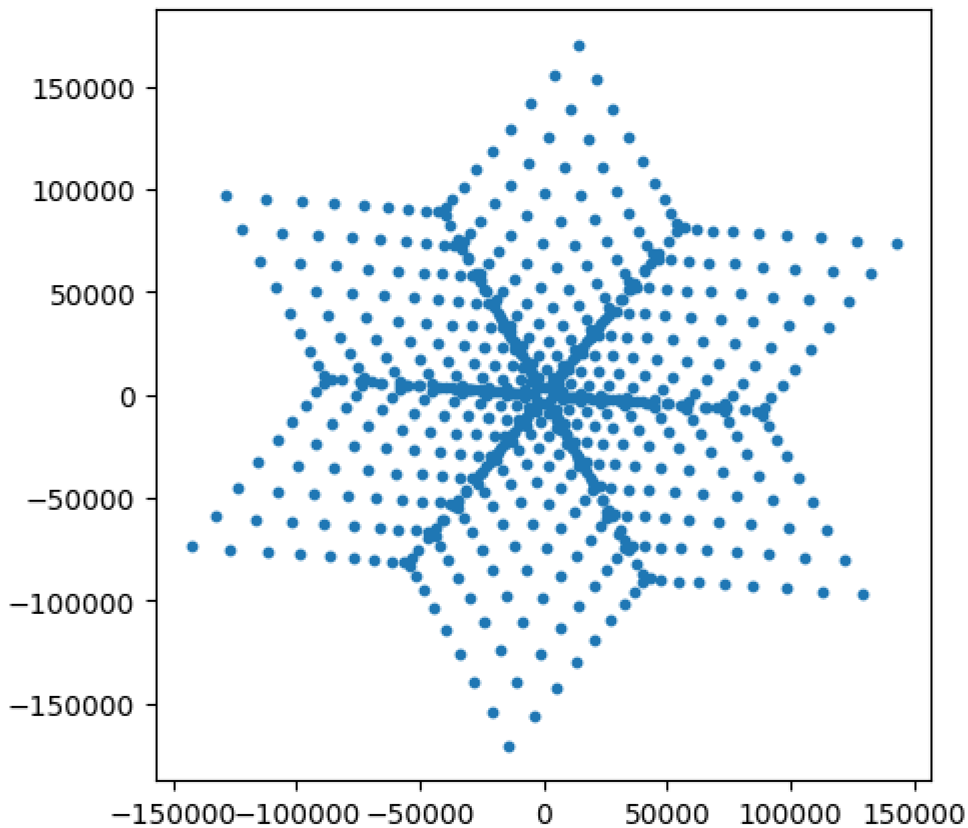
Out[141]:
```
Text(0, 0.5, 'v in units of wavelenghts')
```



In [142…

```
uv_3d=np.zeros([nvis,3])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv_3d[icur,:]=antpos[i,:]-antpos[j,:]
        icur=icur+1
uv_3d=np.vstack([uv_3d,-uv_3d])
uv_3d = uv_3d/wavelength
theta = 0.00*np.pi/180
dec=34.0784*np.pi/180
zenith=np.asarray([np.cos(dec),0,np.sin(dec)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

proj_mat=np.vstack([north, east])
rot_mat=np.zeros([3,3])
rot_mat[0,0]=np.cos(theta)
rot_mat[1,1]=np.cos(theta)
rot_mat[2,2]=1.0
rot_mat[0,1]=np.sin(theta)
rot_mat[1,0]=-np.sin(theta)
uv_rot=uv_3d@rot_mat
uv_snap=uv_rot@proj_mat.T
#if np.abs(theta)<0.001:
#    np.save('vla_uv_snap_'+array+'_array',uv_snap)
plt.figure(figsize=(5,5))
plt.scatter(uv_snap[:,0],uv_snap[:,1], s=10)
```

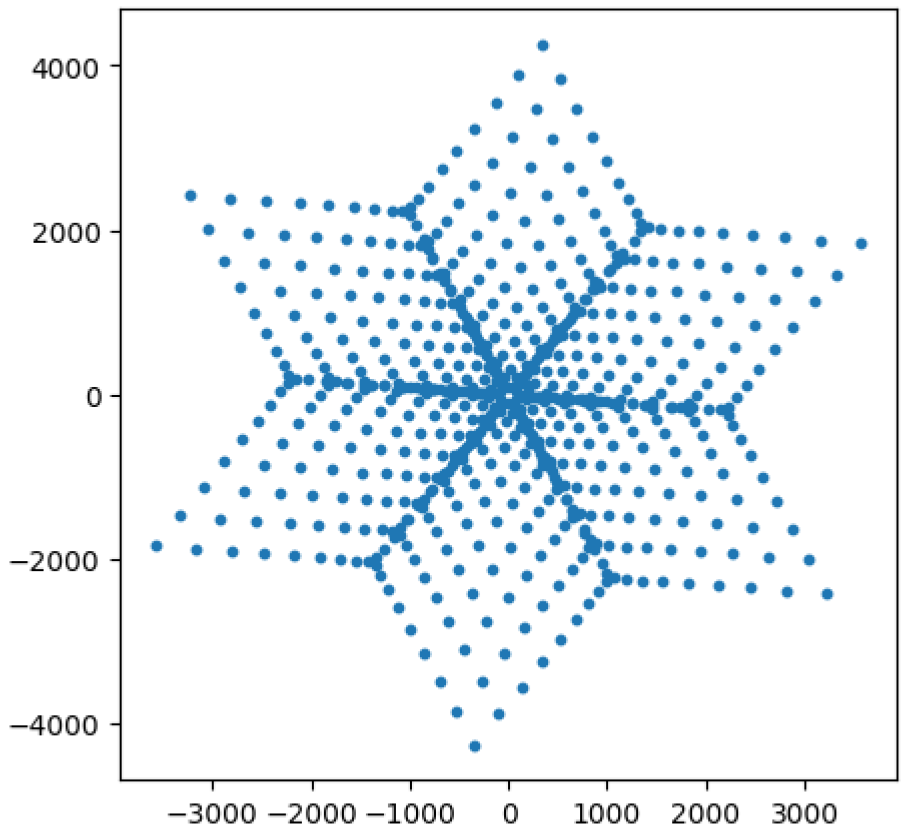
Out[142]: <matplotlib.collections.PathCollection at 0x12755ff40>



In [143…

```
du = 40
uv_int=np.asarray(uv_snap/du,dtype='int')
plt.figure(figsize=(5,5))
plt.scatter(uv_int[:,0], uv_int[:,1], s=10)
```

Out[143]: <matplotlib.collections.PathCollection at 0x127382ac0>



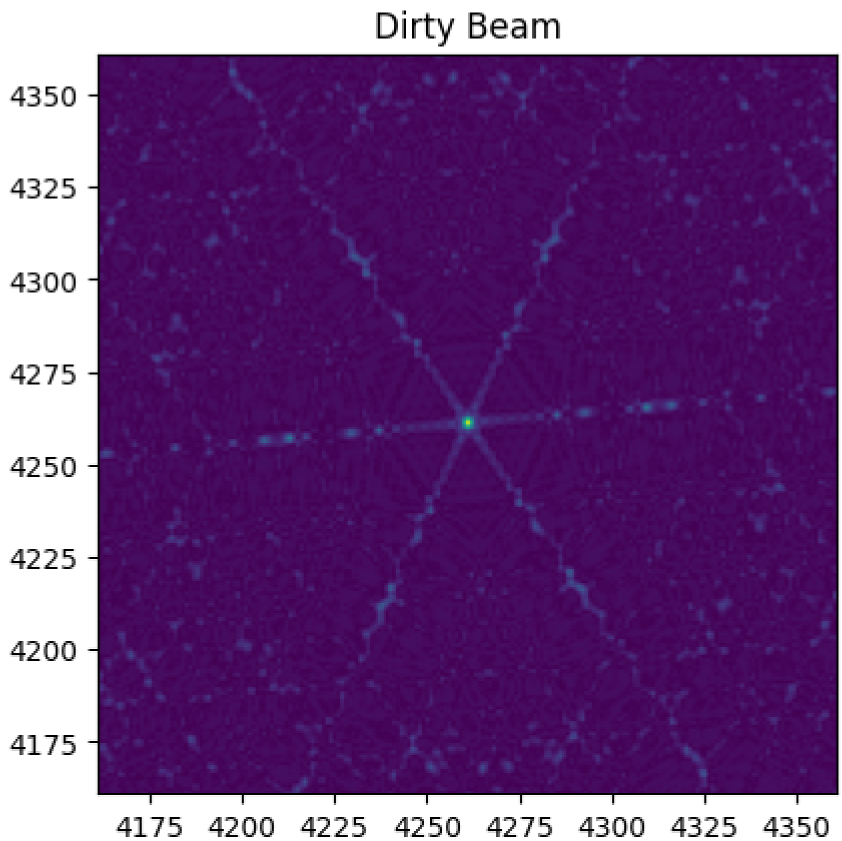
c)

In [144…

```
pad=1
sz=int(np.max(np.abs(uv_3d))/du)
uv_mat=np.zeros([pad*2*sz,2*pad*sz])
for i in range(uv_snap.shape[0]):
    uv_mat[uv_int[i,0],uv_int[i,1]]=uv_mat[uv_int[i,0],uv_int[i,1]]+1
beam=np.abs(np.fft.ifft2(uv_mat))
x0=beam.shape[0]//2
dx=100
plt.figure(2)
plt.clf()
plt.imshow(np.fft.fftshift(beam))
plt.xlim([x0-dx,x0+dx])
plt.ylim([x0-dx,x0+dx])
plt.title("Dirty Beam")
```

Out[144]:

```
Text(0.5, 1.0, 'Dirty Beam')
```

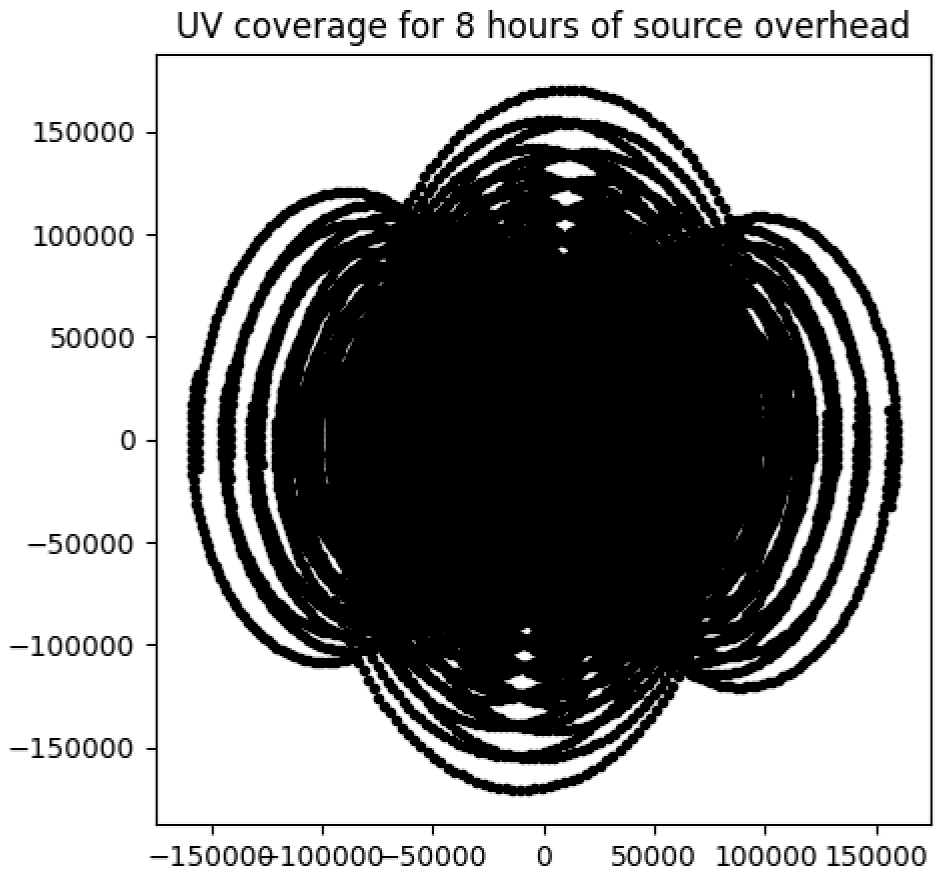


d

In [145…

```
dec=34.0784*np.pi/180
zenith=np.asarray([np.cos(dec),0,np.sin(dec)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

proj_mat=np.vstack([north, east])
t_range=np.linspace(-4,4,61)
theta_range=t_range*2*np.pi/24
pad=1
sz=int(np.max(np.abs(uv_3d))/du)
uv_mat_overhead=np.zeros([pad*2*sz,2*pad*sz])
plt.figure(figsize=(5,5))
plt.title("For source overhead")
for theta in theta_range:
    rot_mat=np.zeros([3,3])
    rot_mat[0,0]=np.cos(theta)
    rot_mat[1,1]=np.cos(theta)
    rot_mat[2,2]=1.0
    rot_mat[0,1]=np.sin(theta)
    rot_mat[1,0]=-np.sin(theta)
    uv_rot=uv_3d@rot_mat
    uv_snap=uv_rot@proj_mat.T
    if np.abs(theta)<0.001:
        np.save('vla_uv_snap_'+array+'_array',uv_snap)
    plt.plot(uv_snap[:,0],uv_snap[:,1],color)
    plt.title("UV coverage for 8 hours of source overhead")
```

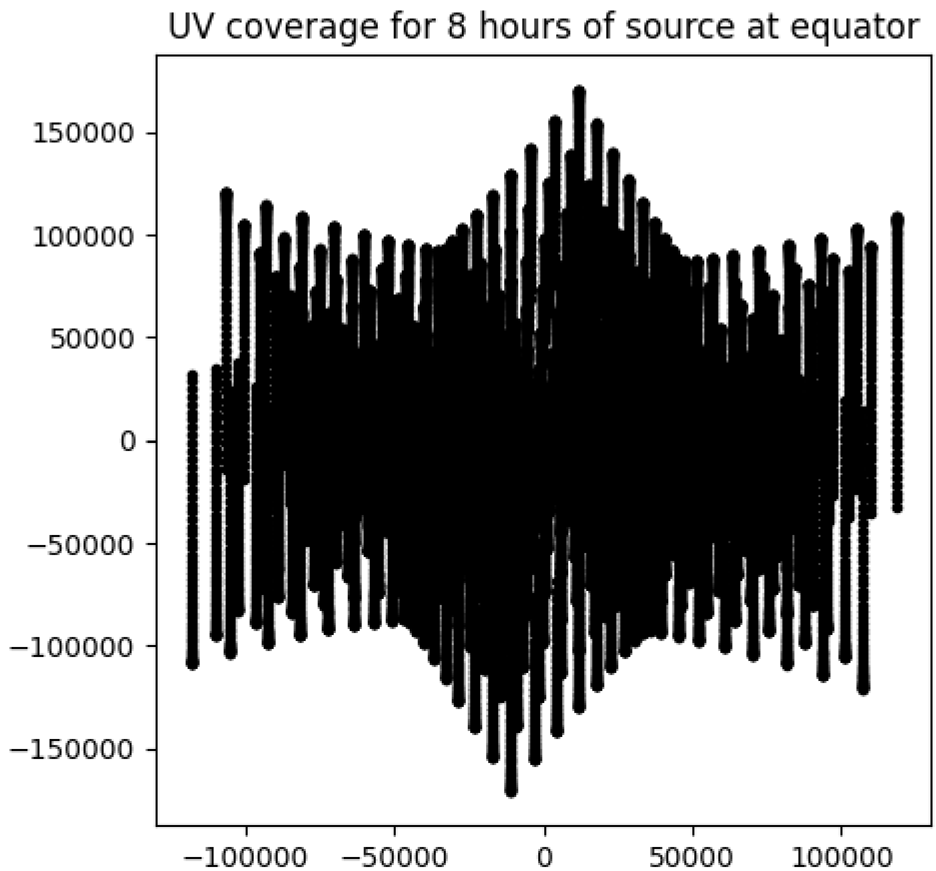
```
    uv_int=np.asarray(uv_snap/du,dtype='int')
    for i in range(uv_snap.shape[0]):
        uv_mat_overhead[uv_int[i,0],uv_int[i,1]]=uv_mat_overhead[uv_int[i,0]
```



In [146…

```
dec=0.00*np.pi/180
zenith=np.asarray([np.cos(dec),0,np.sin(dec)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

proj_mat=np.vstack([north, east])
t_range=np.linspace(-4,4,61)
theta_range=t_range*2*np.pi/24
pad=1
sz=int(np.max(np.abs(uv_3d))/du)
uv_mat_equator=np.zeros([pad*2*sz,2*pad*sz])
plt.figure(figsize=(5,5))
plt.title("For source at equator")
for theta in theta_range:
    rot_mat=np.zeros([3,3])
    rot_mat[0,0]=np.cos(theta)
    rot_mat[1,1]=np.cos(theta)
    rot_mat[2,2]=1.0
    rot_mat[0,1]=np.sin(theta)
    rot_mat[1,0]=-np.sin(theta)
    uv_rot=uv_3d@rot_mat
    uv_snap=uv_rot@proj_mat.T
    if np.abs(theta)<0.001:
        np.save('vla_uv_snap_'+array+'_array',uv_snap)
```

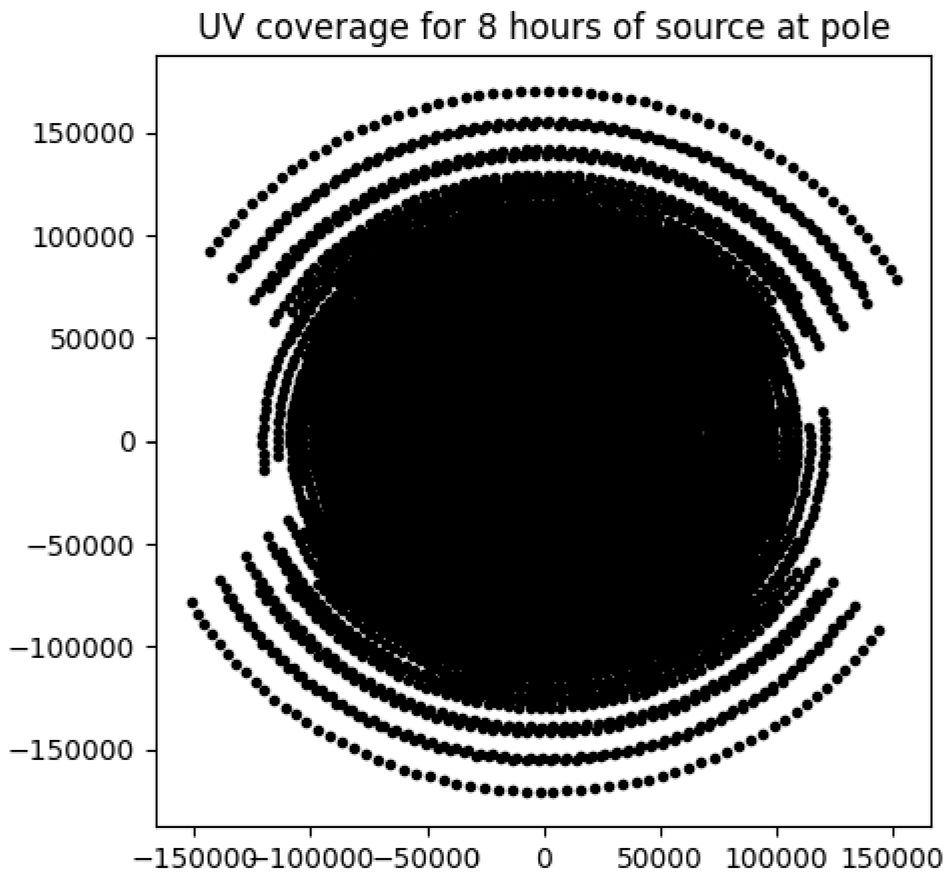
```
    plt.plot(uv_snap[:,0],uv_snap[:,1],color)
    plt.title("UV coverage for 8 hours of source at equator")
    uv_int=np.asarray(uv_snap/du,dtype='int')
    for i in range(uv_snap.shape[0]):
        uv_mat_equator[uv_int[i,0],uv_int[i,1]]=uv_mat_equator[uv_int[i,0],u
```



In [147…

```
dec=90.00*np.pi/180
zenith=np.asarray([np.cos(dec),0,np.sin(dec)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

proj_mat=np.vstack([north, east])
t_range=np.linspace(-4,4,61)
theta_range=t_range*2*np.pi/24
pad=1
sz=int(np.max(np.abs(uv_3d))/du)
uv_mat_pole=np.zeros([pad*2*sz,2*pad*sz])
plt.figure(figsize=(5,5))
plt.title("For source at north celestial pole")
for theta in theta_range:
    rot_mat=np.zeros([3,3])
    rot_mat[0,0]=np.cos(theta)
    rot_mat[1,1]=np.cos(theta)
    rot_mat[2,2]=1.0
    rot_mat[0,1]=np.sin(theta)
    rot_mat[1,0]=-np.sin(theta)
    uv_rot=uv_3d@rot_mat
    uv_snap=uv_rot@proj_mat.T
    if np.abs(theta)<0.001:
```

```
        np.save('vla_uv_snap_'+array+'_array',uv_snap)
    plt.plot(uv_snap[:,0],uv_snap[:,1],color)
    plt.title("UV coverage for 8 hours of source at pole")
    uv_int=np.asarray(uv_snap/du,dtype='int')
    for i in range(uv_snap.shape[0]):
        uv_mat_pole[uv_int[i,0],uv_int[i,1]]=uv_mat_pole[uv_int[i,0],uv_int[
```

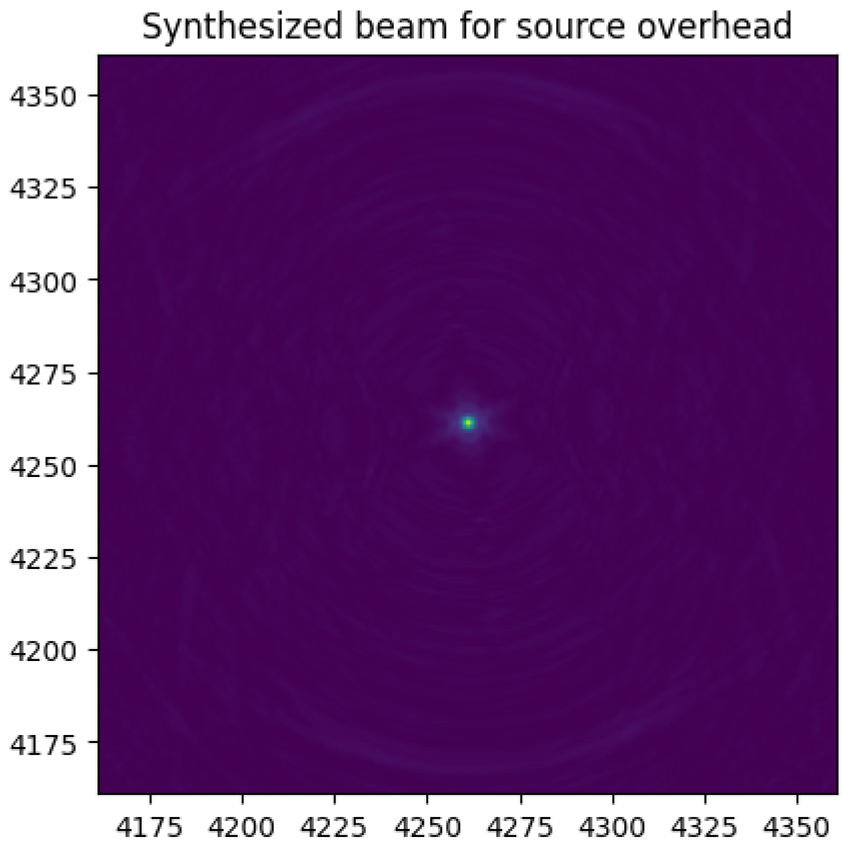


e

In [148…

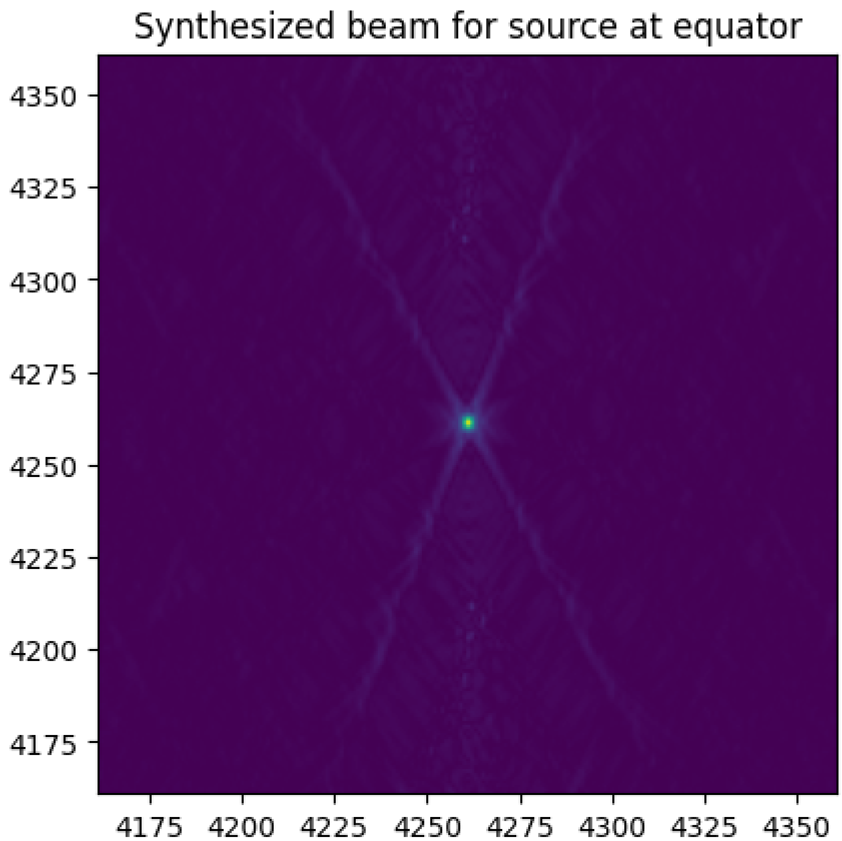
```
beam_overhead=np.abs(np.fft.ifft2(uv_mat_overhead))
x0=beam.shape[0]//2
dx=100
plt.figure(2)
plt.clf()
plt.imshow(np.fft.fftshift(beam_overhead))
plt.xlim([x0-dx,x0+dx])
plt.ylim([x0-dx,x0+dx])
plt.title("Synthesized beam for source overhead")
```

Out[148]: Text(0.5, 1.0, 'Synthesized beam for source overhead')



In [149…
```
beam_equator=np.abs(np.fft.ifft2(uv_mat_equator))
x0=beam.shape[0]//2
dx=100
plt.figure(2)
plt.clf()
plt.imshow(np.fft.fftshift(beam_equator))
plt.xlim([x0-dx,x0+dx])
plt.ylim([x0-dx,x0+dx])
plt.title("Synthesized beam for source at equator")
```

Out[149]:
```
Text(0.5, 1.0, 'Synthesized beam for source at equator')
```

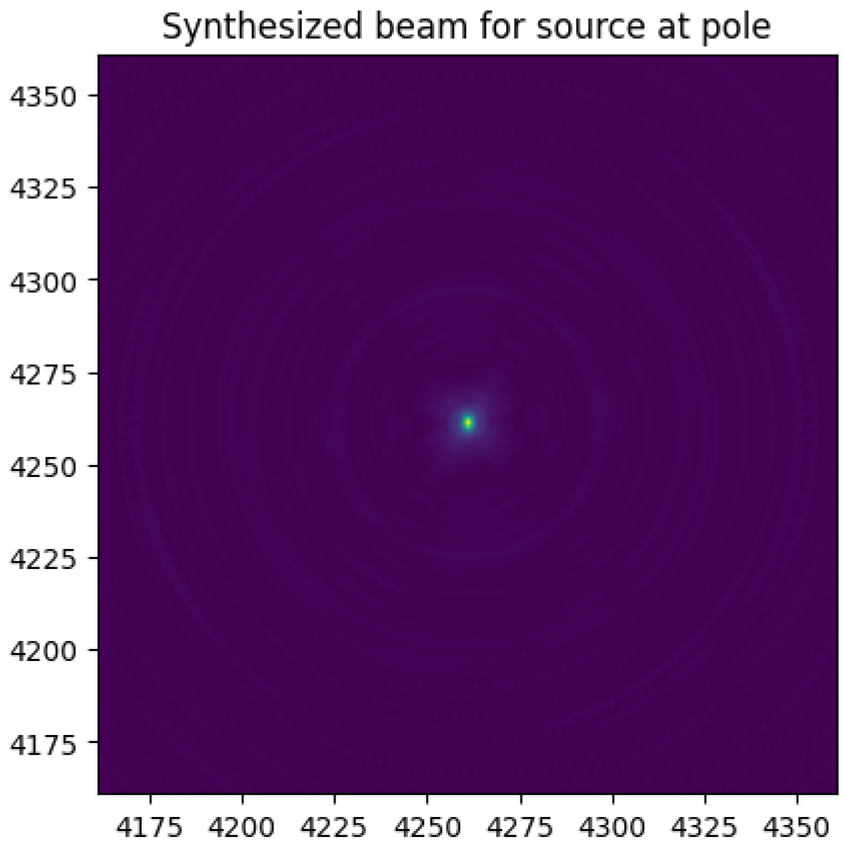


In [150…

```
beam_pole=np.abs(np.fft.ifft2(uv_mat_pole))
x0=beam.shape[0]//2
dx=100
plt.figure(2)
plt.clf()
plt.imshow(np.fft.fftshift(beam_pole))
plt.xlim([x0-dx,x0+dx])
plt.ylim([x0-dx,x0+dx])
plt.title("Synthesized beam for source at pole")
```

Out[150]:

```
Text(0.5, 1.0, 'Synthesized beam for source at pole')
```



Q2

a)

In [151…

```
D = 25 #diameter of each dish in VLA
nu1 = 1.4*10**9
wavelength1 = 3*10**8/nu1
nu2 = 8*10**9
wavelength2 = 3*10**8/nu2
FOV1 = 1.22*wavelength1/D
FOV1_arcmin = FOV1*180*60/np.pi
FOV2 = 1.22*wavelength2/D
FOV2_arcmin = FOV2*180*60/np.pi
print("Wavelength at 1.4 GHz is {} m".format(wavelength1))
print("Wavelength at 8 GHz is {} m".format(wavelength2))
print("FOV of VLA at 1.4GHz is {} arcmin".format(FOV1_arcmin))
print("FOV of VLA at 8GHz is {} arcmin".format(FOV2_arcmin))
```

```
Wavelength at 1.4 GHz is 0.21428571428571427 m
Wavelength at 8 GHz is 0.0375 m
FOV of VLA at 1.4GHz is 35.94900908877965 arcmin
FOV of VLA at 8GHz is 6.29107659053644 arcmin
```

b)

In [152…
```
uv=np.zeros([nvis,2])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv[icur,:]=(xyz[i,:2]-xyz[j,:2])
        icur=icur+1
#uv=np.vstack([uv,-1*uv])
```

In [153…
```
longest_baseline = 0
for i in range(nvis):
    len_baseline = np.sqrt(uv[i][0]**2 + uv[i][1]**2)
    if len_baseline > longest_baseline:
        longest_baseline = len_baseline
print("Longest baseline = {} m".format(longest_baseline))
```

```
Longest baseline = 36648.431756910744 m
```

In [154…
```
#For source overhead
path_diff = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-0.5)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
```

In [155…
```
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordi
nates: 52.256902030580804 m
```

c)

In [156…
```
uv_3d=np.zeros([nvis,3])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv_3d[icur,:]=(xyz[i,:]-xyz[j,:])
        icur=icur+1
#uv_3d=np.vstack([uv_3d,-uv_3d])
```

In [157…
```
path_diff_3d = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-0.5)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
```

In [158…
```
rms_3d = 0
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 3D uv
```

```
RMS path length difference between source 1 and source 2 using 3D uv coordi
nates: 52.22007059985015 m
```

In [159…
```
diff_of_diff = np.abs(rms-rms_3d)
print("The RMS difference between path length differences for 2D and 3D coor
```

```
The RMS difference between path length differences for 2D and 3D coordinate
s is 0.03683143073065054 m
```

d)

From the official VLA website, the synthesized FWHM for a array at 1.5 GHz is 1.3''. Since the synthesized beam resolution is proportiaonal to the wavelength, I use this information to get the FWHM at 1.4 GHz and 8 GHz.

In [160…
```
FWHM = 1.3
FWHM1 = 1.3*(1.5/1.4)*(1/3600) #FWHM at 1.4 GHz
FWHM2 = 1.3*(1.5/8)*(1/3600) #FWHM at 8 GHz
print("The FWHM at 1.4GHz is {} degree".format(FWHM1))
print("The FWHM at 8GHz is {} degree".format(FWHM2))
```

```
The FWHM at 1.4GHz is 0.00038690476190476186 degree
The FWHM at 8GHz is 6.770833333333334e-05 degree
```

For an overhead pointing center, I already calculated the uv coordiantes (where the projection matrix (mat) was calculated with the latitute of VLA.

For 1.4 GHz:

In [161…
```
path_diff = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-FWHM1)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
path_diff_3d = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-FWHM1)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
```

```
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
rms_3d = 0
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
diff_of_diff1 = np.abs(rms - rms_3d)
phase_diff1 = 360*diff_of_diff1/wavelength1
print("RMS path length difference between source 1 and source 2 using 3D uv
print("The RMS difference between path length differences for 2D and 3D coor
print("The corresponding RMS phase difference is {} degrees".format(phase_di
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordi
nates: 0.040178457370602945 m
RMS path length difference between source 1 and source 2 using 3D uv coordi
nates: 0.040149874824723486 m
The RMS difference between path length differences for 2D and 3D coordinate
s at 1.4GHz is 2.858254587945891e-05 m
The corresponding RMS phase difference is 0.04801867707749097 degrees
```

Since the RMS phase difference between the 2D and 3D coordinates is very small, we do not need to worry about the w term

For 8 GHz

In [162…

```
path_diff = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-FWHM2)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
path_diff_3d = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-FWHM2)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
rms_3d = 0
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
diff_of_diff1 = np.abs(rms - rms_3d)
phase_diff2 = 360*diff_of_diff1/wavelength2
print("RMS path length difference between source 1 and source 2 using 3D uv
print("The RMS difference between path length differences for 2D and 3D coor
print("The corresponding RMS phase difference is {} degrees".format(phase_di
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordi
nates: 0.007031201089687078 m
RMS path length difference between source 1 and source 2 using 3D uv coordi
nates: 0.007026199135027653 m
The RMS difference between path length differences for 2D and 3D coordinate
s at 8GHz is 5.001954659425434e-06 m
The corresponding RMS phase difference is 0.048018764730484165 degrees
```

Since the RMS phase difference between the 2D and 3D coordinates is very small, we do not need to worry about the w term

For source at equator:

For 1.4 GHz:

In [163…

```
path_diff = []
theta1 = np.radians(90)
theta2 = np.radians(90-FWHM1)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
path_diff_3d = []
theta1 = np.radians(90)
theta2 = np.radians(90-FWHM1)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
rms_3d = 0
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
diff_of_diff1 = np.abs(rms - rms_3d)
phase_diff1 = 360*diff_of_diff1/wavelength1
print("RMS path length difference between source 1 and source 2 using 3D uv
print("The RMS difference between path length differences for 2D and 3D coor
print("The corresponding RMS phase difference is {} degrees".format(phase_di
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordi
nates: 2.4210328714535286e-07 m
RMS path length difference between source 1 and source 2 using 3D uv coordi
nates: 0.0001851734404227878 m
The RMS difference between path length differences for 2D and 3D coordinate
s at 1.4GHz is 0.00018493133713564245 m
The corresponding RMS phase difference is 0.31068464638787935 degrees
```

Since the RMS phase difference between the 2D and 3D coordinates is very small, we do not need to worry about the w term

For 8 GHz:

In [164…

```
path_diff = []
theta1 = np.radians(90)
theta2 = np.radians(90-FWHM2)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
path_diff_3d = []
theta1 = np.radians(90)
theta2 = np.radians(90-FWHM2)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
rms_3d = 0
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
diff_of_diff1 = np.abs(rms - rms_3d)
phase_diff2 = 360*diff_of_diff1/wavelength2
print("RMS path length difference between source 1 and source 2 using 3D uv
print("The RMS difference between path length differences for 2D and 3D coor
print("The corresponding RMS phase difference is {} degrees".format(phase_di
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordi
nates: 7.414089405432098e-09 m
RMS path length difference between source 1 and source 2 using 3D uv coordi
nates: 3.241190299522957e-05 m
The RMS difference between path length differences for 2D and 3D coordinate
s at 8GHz is 3.2404488905824134e-05 m
The corresponding RMS phase difference is 0.3110830934959117 degrees
```

Since the RMS phase difference between the 2D and 3D coordinates is very small, we do not need to worry about the w term

Q3

As calculated in Q2 d), for an overhead source and a second source separated by 1 FWHM, the RMS path length difference is 0.04341473317231679 m (for 3D coordinates).

In [165…

```
delta_nu = 3*10**8/(2*np.pi*0.04341473317231679)
```

```
print("Using the calculation shown in the report, the amount of frequency sh
```

```
Using the calculation shown in the report, the amount of frequency shift re
quired is: 1.0997760308248061 GHz
```

Q4 is done in the report

All questions for d array

For d array:

In [166…

```
antpos=np.loadtxt('vla_d_array.txt')
du=2.0
color='b.'
array='d'
```

In [167…

```
antpos=antpos[:,:3]  #the last column is boring...
antpos=antpos*1e-9*3e8 #convert to meters, since the file is in ns
nant=antpos.shape[0]
nvis=nant*(nant-1)//2

#we can look at the antenna array in 2D coordinates by looking at the
#distance from the zenith.  The following math is useful for converting 3D
#positions to EW/NS coordinates on the Earth's surface
zenith=np.asarray([np.cos(lat),0,np.sin(lat)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

mat=np.vstack([east,north,zenith])
xyz=antpos[:,:3]@mat.T
```

In [168…

```
uv=np.zeros([nvis,2])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv[icur,:]=(xyz[i,:2]-xyz[j,:2])
        icur=icur+1
#uv=np.vstack([uv,-1*uv])
```

In [169…

```
longest_baseline = 0
for i in range(nvis):
    len_baseline = np.sqrt(uv[i][0]**2 + uv[i][1]**2)
    if len_baseline > longest_baseline:
        longest_baseline = len_baseline
print("Longest baseline = {} m".format(longest_baseline))
```

```
Longest baseline = 1031.9082462393058 m
```

All questions for d array

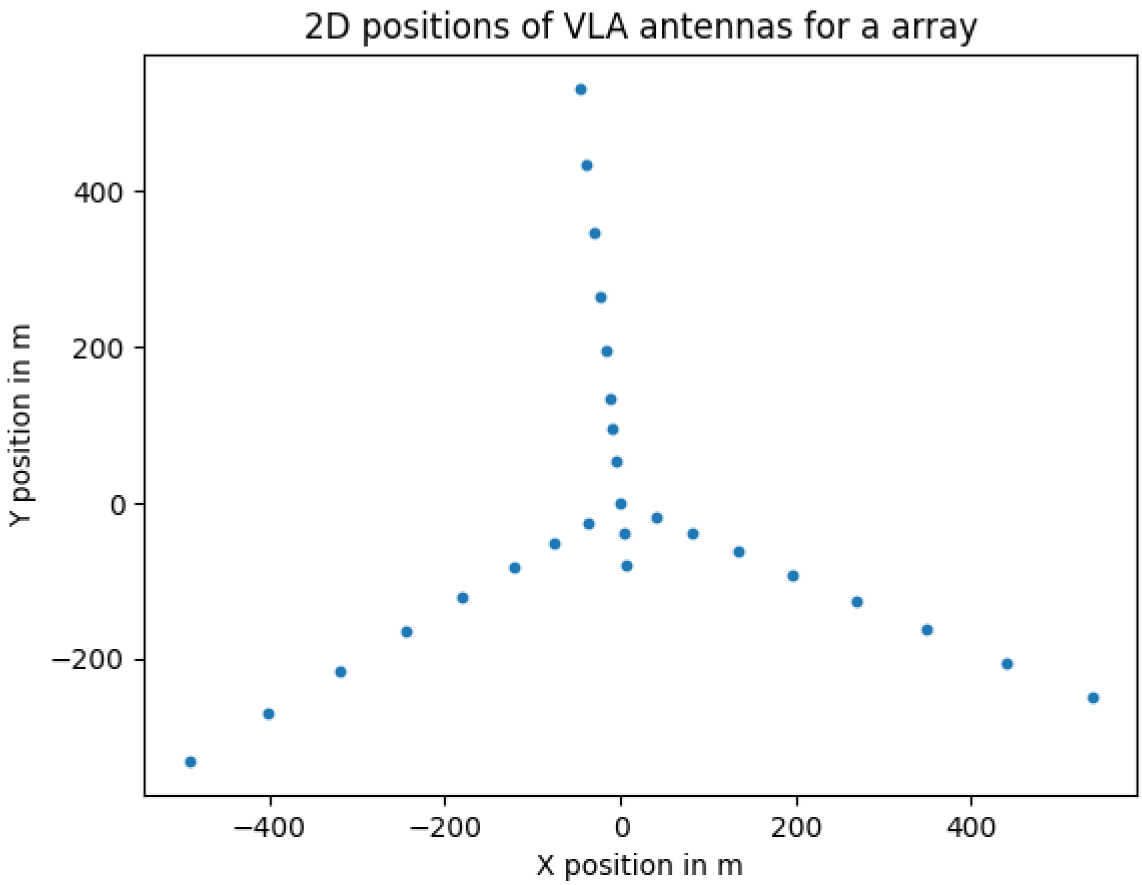
Q1

In [170…
```
lat=34.0784*np.pi/180

antpos=np.loadtxt('vla_d_array.txt')
du=2.0
color='b.'
array='d'

antpos=antpos[:,:3]  #the last column is boring...
antpos=antpos*1e-9*3e8 #convert to meters, since the file is in ns
nant=antpos.shape[0]
nvis=nant*(nant-1)//2

#we can look at the antenna array in 2D coordinates by looking at the
#distance from the zenith.  The following math is useful for converting 3D
#positions to EW/NS coordinates on the Earth's surface
zenith=np.asarray([np.cos(lat),0,np.sin(lat)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

mat=np.vstack([east,north,zenith])
xyz=antpos[:,:3]@mat.T
```

a)

In [171…
```
x = xyz[:, 0]
y = xyz[:, 1]
z = xyz[:, 2]
plt.scatter(x, y, s =10)
plt.xlabel("X position in m")
plt.ylabel("Y position in m")
plt.title("2D positions of VLA antennas for a array")

spread_x = max(x) - min(x)
spread_y = max(y) - min(y)
spread_z = max(z) - min(z)
print("Spread in x diection is {} m".format(spread_x))
print("Spread in y diection is {} m".format(spread_y))
print("Spread in z diection is {} m".format(spread_z))

rms_vert = np.sqrt(np.mean(z**2))
print("RMS vertical scatter is {} m".format(rms_vert))
```

```
Spread in x diection is 1028.7240000000002 m
Spread in y diection is 864.0059245685211 m
Spread in z diection is 2.1859881794338034 m
RMS vertical scatter is 0.8359471876940011 m
```

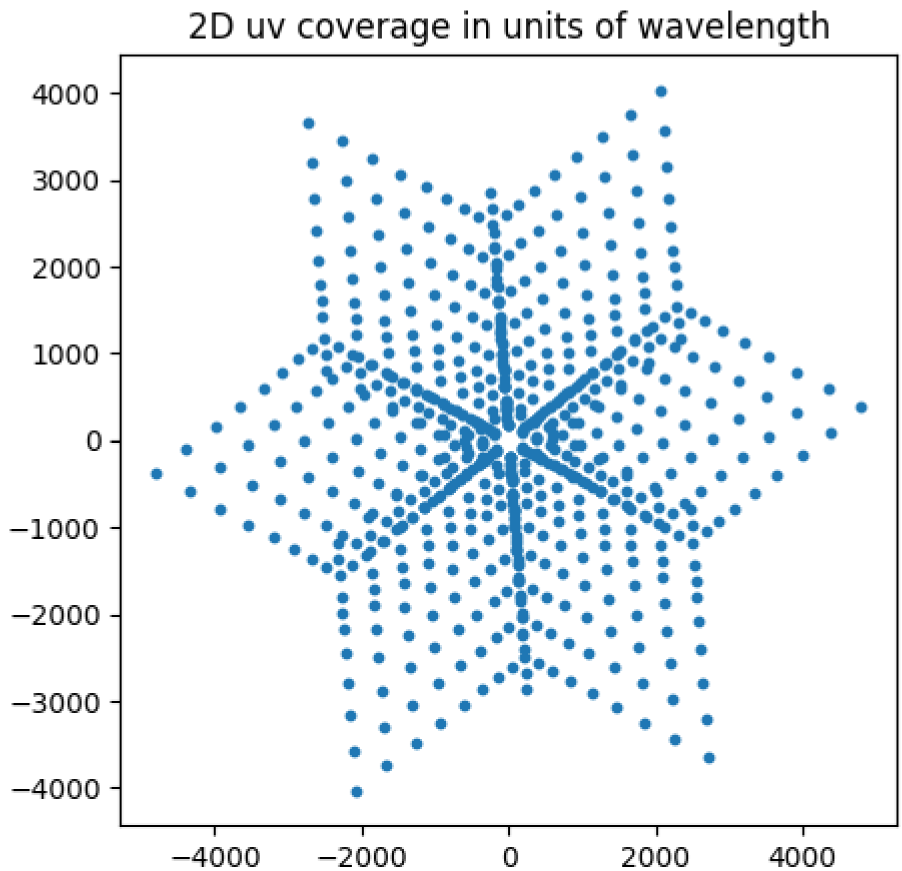


b)

In [172…

```
## b

uv=np.zeros([nvis,2])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv[icur,:]=xyz[i,:2]-xyz[j,:2]
        icur=icur+1
uv=np.vstack([uv,-1*uv])
nu = 1.4*10**9
wavelength = 3*10**8/nu
uv = uv/wavelength
plt.figure(figsize = (5,5))
plt.scatter(uv[:,0], uv[:,1], s=10)
plt.title("2D uv coverage in units of wavelength")
```

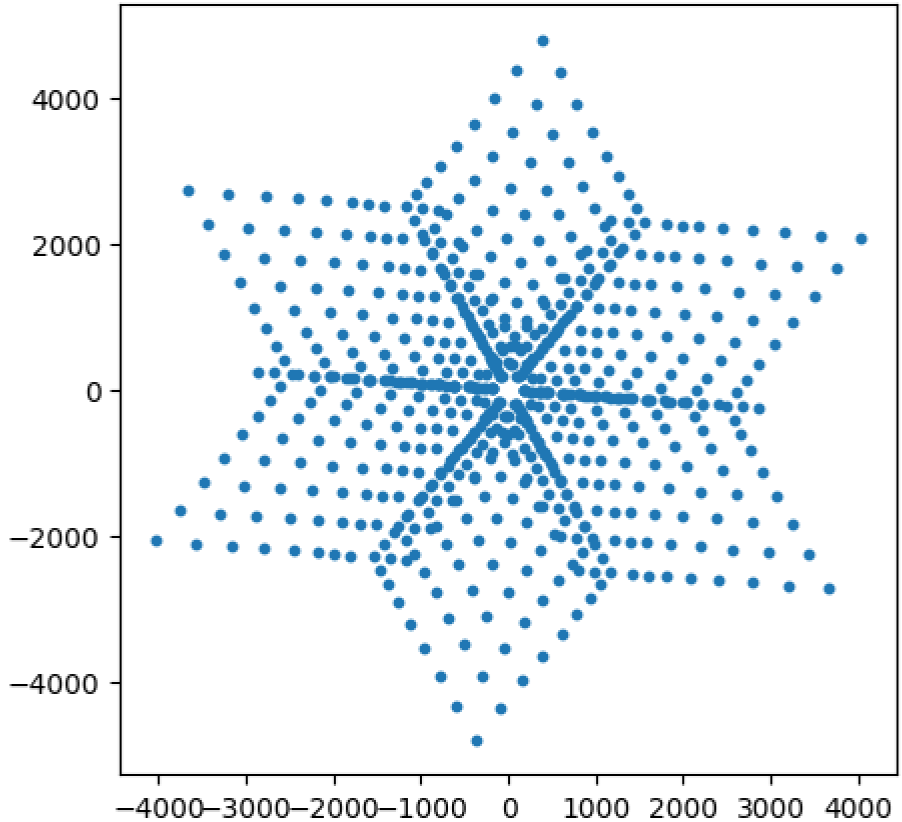
Out[172]: Text(0.5, 1.0, '2D uv coverage in units of wavelength')



In [173…

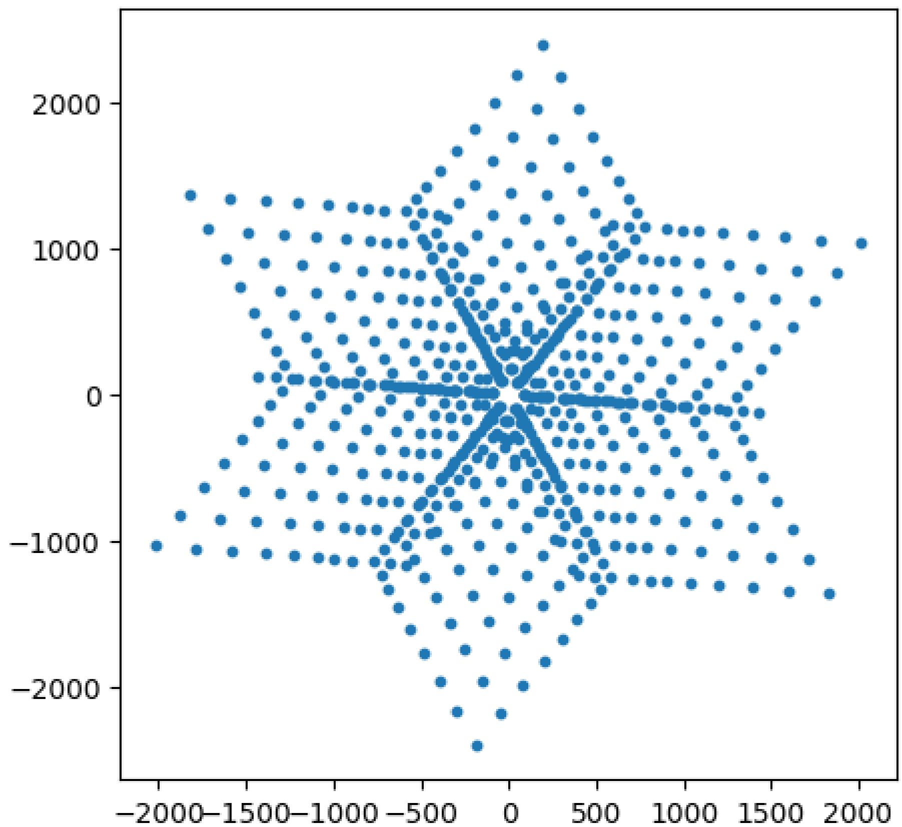
```
uv_3d=np.zeros([nvis,3])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv_3d[icur,:]=antpos[i,:]-antpos[j,:]
        icur=icur+1
uv_3d=np.vstack([uv_3d,-uv_3d])
uv_3d = uv_3d/wavelength
theta = 0.00*np.pi/180
dec=34.0784*np.pi/180
zenith=np.asarray([np.cos(dec),0,np.sin(dec)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

proj_mat=np.vstack([north, east])
rot_mat=np.zeros([3,3])
rot_mat[0,0]=np.cos(theta)
rot_mat[1,1]=np.cos(theta)
rot_mat[2,2]=1.0
rot_mat[0,1]=np.sin(theta)
rot_mat[1,0]=-np.sin(theta)
uv_rot=uv_3d@rot_mat
uv_snap=uv_rot@proj_mat.T
#if np.abs(theta)<0.001:
#    np.save('vla_uv_snap_'+array+'_array',uv_snap)
plt.figure(figsize=(5,5))
plt.scatter(uv_snap[:,0],uv_snap[:,1], s=10)
```

```
Out[173]: <matplotlib.collections.PathCollection at 0x1224d1220>
```




```
In [174…: du = 2
uv_int=np.asarray(uv_snap/du,dtype='int')
plt.figure(figsize=(5,5))
plt.scatter(uv_int[:,0], uv_int[:,1], s=10)
```

```
Out[174]: <matplotlib.collections.PathCollection at 0x122536c40>
```



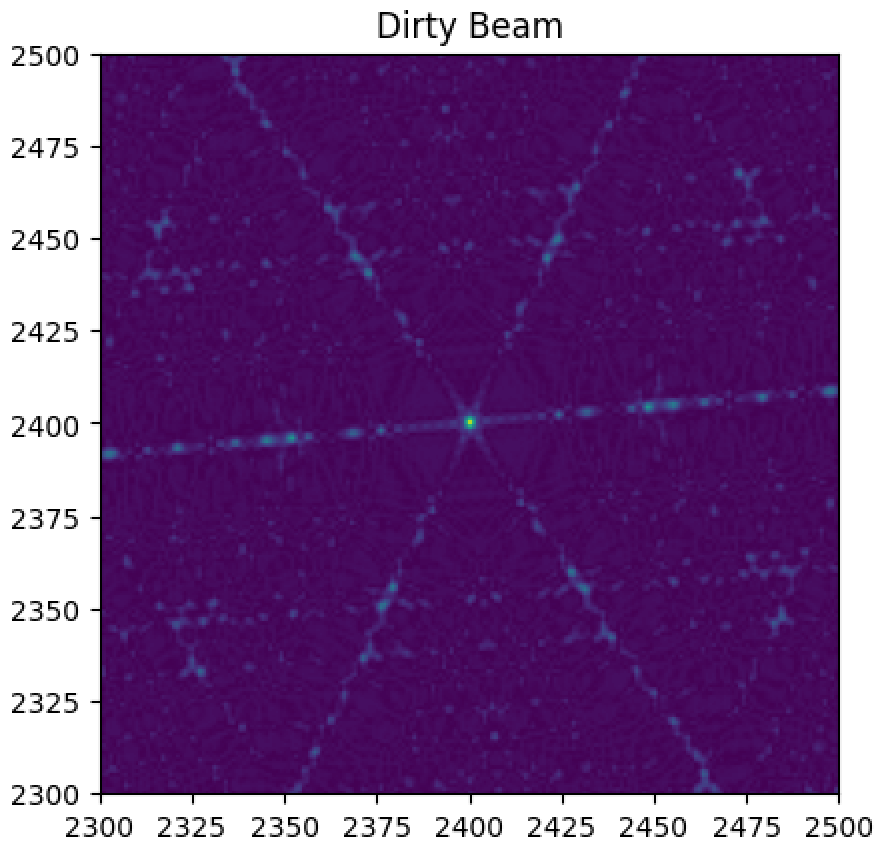
c)

In [175…

```
pad=1
sz=int(np.max(np.abs(uv_3d))/du)
uv_mat=np.zeros([pad*2*sz,2*pad*sz])
for i in range(uv_snap.shape[0]):
    uv_mat[uv_int[i,0],uv_int[i,1]]=uv_mat[uv_int[i,0],uv_int[i,1]]+1
beam=np.abs(np.fft.ifft2(uv_mat))
x0=beam.shape[0]//2
dx=100
plt.figure(2)
plt.clf()
plt.imshow(np.fft.fftshift(beam))
plt.xlim([x0-dx,x0+dx])
plt.ylim([x0-dx,x0+dx])
plt.title("Dirty Beam")
```

Out[175]:

```
Text(0.5, 1.0, 'Dirty Beam')
```



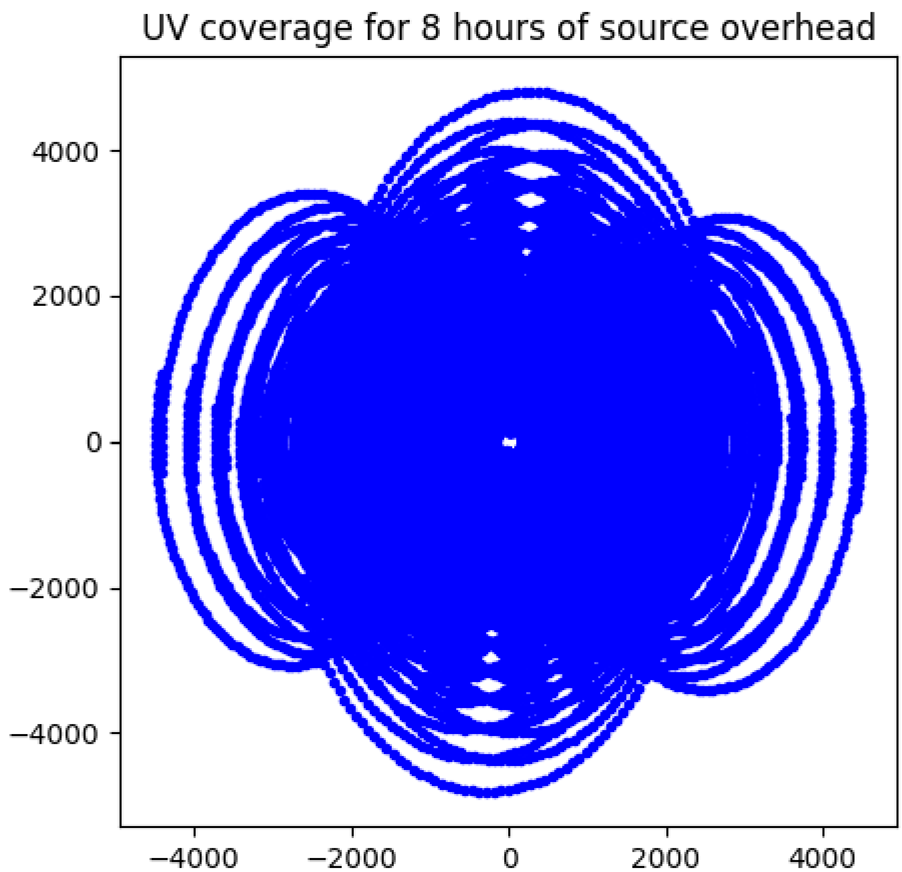
d)

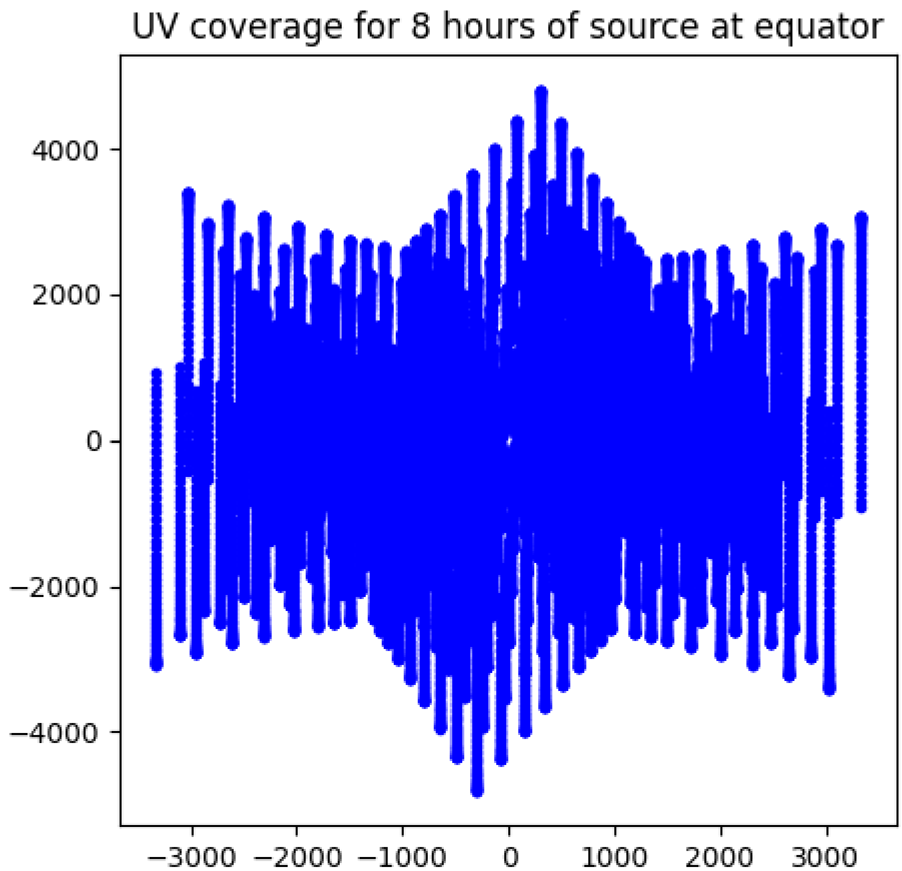
In [176…

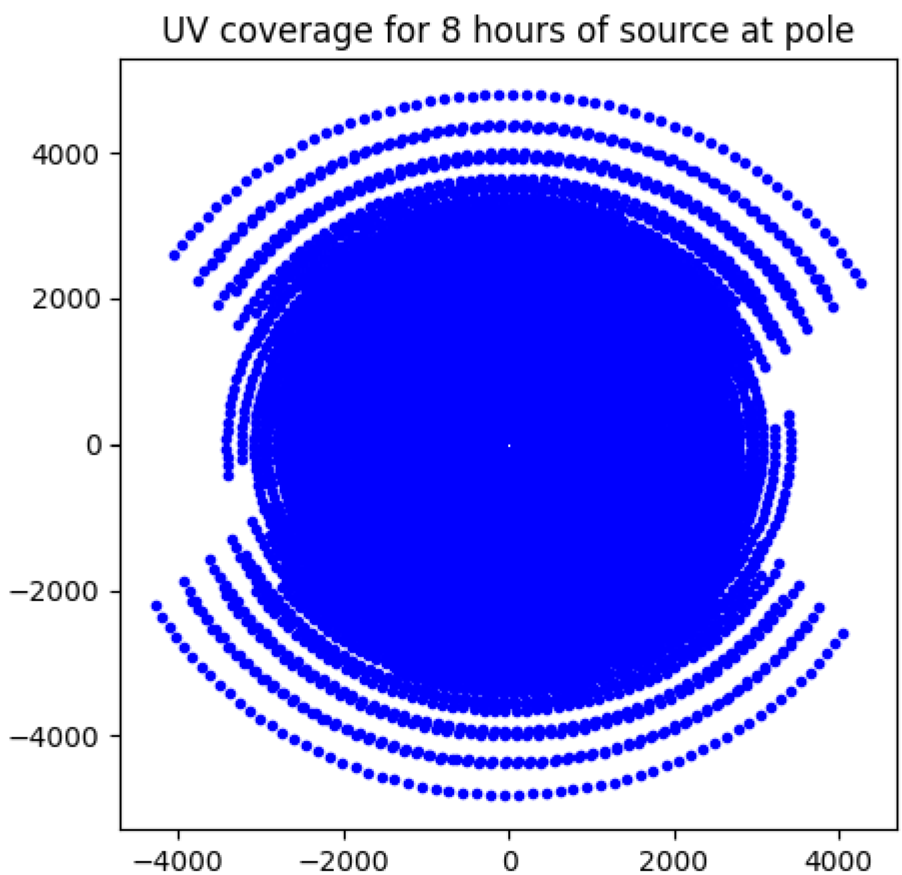
```
dec=34.0784*np.pi/180
zenith=np.asarray([np.cos(dec),0,np.sin(dec)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

proj_mat=np.vstack([north, east])
t_range=np.linspace(-4,4,61)
theta_range=t_range*2*np.pi/24
pad=1
sz=int(np.max(np.abs(uv_3d))/du)
uv_mat_overhead=np.zeros([pad*2*sz,2*pad*sz])
plt.figure(figsize=(5,5))
plt.title("For source overhead")
for theta in theta_range:
    rot_mat=np.zeros([3,3])
    rot_mat[0,0]=np.cos(theta)
    rot_mat[1,1]=np.cos(theta)
    rot_mat[2,2]=1.0
    rot_mat[0,1]=np.sin(theta)
    rot_mat[1,0]=-np.sin(theta)
    uv_rot=uv_3d@rot_mat
    uv_snap=uv_rot@proj_mat.T
    if np.abs(theta)<0.001:
        np.save('vla_uv_snap_'+array+'_array',uv_snap)
    plt.plot(uv_snap[:,0],uv_snap[:,1],color)
    plt.title("UV coverage for 8 hours of source overhead")
```

```
    uv_int=np.asarray(uv_snap/du,dtype='int')
    for i in range(uv_snap.shape[0]):
        uv_mat_overhead[uv_int[i,0],uv_int[i,1]]=uv_mat_overhead[uv_int[i,0]


dec=0.00*np.pi/180
zenith=np.asarray([np.cos(dec),0,np.sin(dec)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

proj_mat=np.vstack([north, east])
t_range=np.linspace(-4,4,61)
theta_range=t_range*2*np.pi/24
pad=1
sz=int(np.max(np.abs(uv_3d))/du)
uv_mat_equator=np.zeros([pad*2*sz,2*pad*sz])
plt.figure(figsize=(5,5))
plt.title("For source at equator")
for theta in theta_range:
    rot_mat=np.zeros([3,3])
    rot_mat[0,0]=np.cos(theta)
    rot_mat[1,1]=np.cos(theta)
    rot_mat[2,2]=1.0
    rot_mat[0,1]=np.sin(theta)
    rot_mat[1,0]=-np.sin(theta)
    uv_rot=uv_3d@rot_mat
    uv_snap=uv_rot@proj_mat.T
    if np.abs(theta)<0.001:
        np.save('vla_uv_snap_'+array+'_array',uv_snap)
    plt.plot(uv_snap[:,0],uv_snap[:,1],color)
    plt.title("UV coverage for 8 hours of source at equator")
    uv_int=np.asarray(uv_snap/du,dtype='int')
    for i in range(uv_snap.shape[0]):
        uv_mat_equator[uv_int[i,0],uv_int[i,1]]=uv_mat_equator[uv_int[i,0],u

dec=90.00*np.pi/180
zenith=np.asarray([np.cos(dec),0,np.sin(dec)])
east=np.asarray([0,1,0])
north=np.cross(zenith,east)

proj_mat=np.vstack([north, east])
t_range=np.linspace(-4,4,61)
theta_range=t_range*2*np.pi/24
pad=1
sz=int(np.max(np.abs(uv_3d))/du)
uv_mat_pole=np.zeros([pad*2*sz,2*pad*sz])
plt.figure(figsize=(5,5))
plt.title("For source at north celestial pole")
for theta in theta_range:
    rot_mat=np.zeros([3,3])
    rot_mat[0,0]=np.cos(theta)
    rot_mat[1,1]=np.cos(theta)
    rot_mat[2,2]=1.0
    rot_mat[0,1]=np.sin(theta)
    rot_mat[1,0]=-np.sin(theta)
    uv_rot=uv_3d@rot_mat
```

```
    uv_snap=uv_rot@proj_mat.T
    if np.abs(theta)<0.001:
        np.save('vla_uv_snap_'+array+'_array',uv_snap)
    plt.plot(uv_snap[:,0],uv_snap[:,1],color)
    plt.title("UV coverage for 8 hours of source at pole")
    uv_int=np.asarray(uv_snap/du,dtype='int')
    for i in range(uv_snap.shape[0]):
        uv_mat_pole[uv_int[i,0],uv_int[i,1]]=uv_mat_pole[uv_int[i,0],uv_int[
```




UV coverage for 8 hours of source at equator
4000
2000
0
−2000
−4000
−3000
−2000
−1000
0
1000
2000
3000


UV coverage for 8 hours of source at pole
4000
2000
0
−2000
−4000
−4000
−2000
0
2000
4000

e)

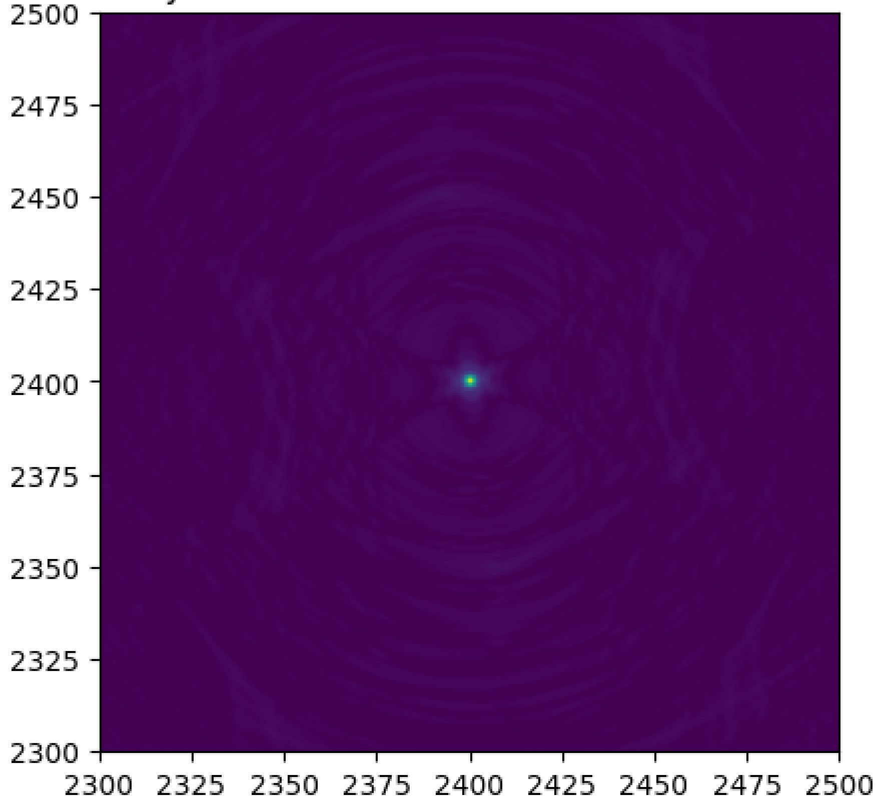
In [177…

```
beam_overhead=np.abs(np.fft.ifft2(uv_mat_overhead))
x0=beam.shape[0]//2
dx=100
plt.figure()
plt.clf()
plt.imshow(np.fft.fftshift(beam_overhead))
plt.xlim([x0-dx,x0+dx])
plt.ylim([x0-dx,x0+dx])
plt.title("Synthesized beam for source overhead")

beam_equator=np.abs(np.fft.ifft2(uv_mat_equator))
x0=beam.shape[0]//2
dx=100
plt.figure()
plt.clf()
plt.imshow(np.fft.fftshift(beam_equator))
plt.xlim([x0-dx,x0+dx])
plt.ylim([x0-dx,x0+dx])
plt.title("Synthesized beam for source at equator")

beam_pole=np.abs(np.fft.ifft2(uv_mat_pole))
x0=beam.shape[0]//2
dx=100
plt.figure()
plt.clf()
plt.imshow(np.fft.fftshift(beam_pole))
plt.xlim([x0-dx,x0+dx])
plt.ylim([x0-dx,x0+dx])
plt.title("Synthesized beam for source at pole")
```

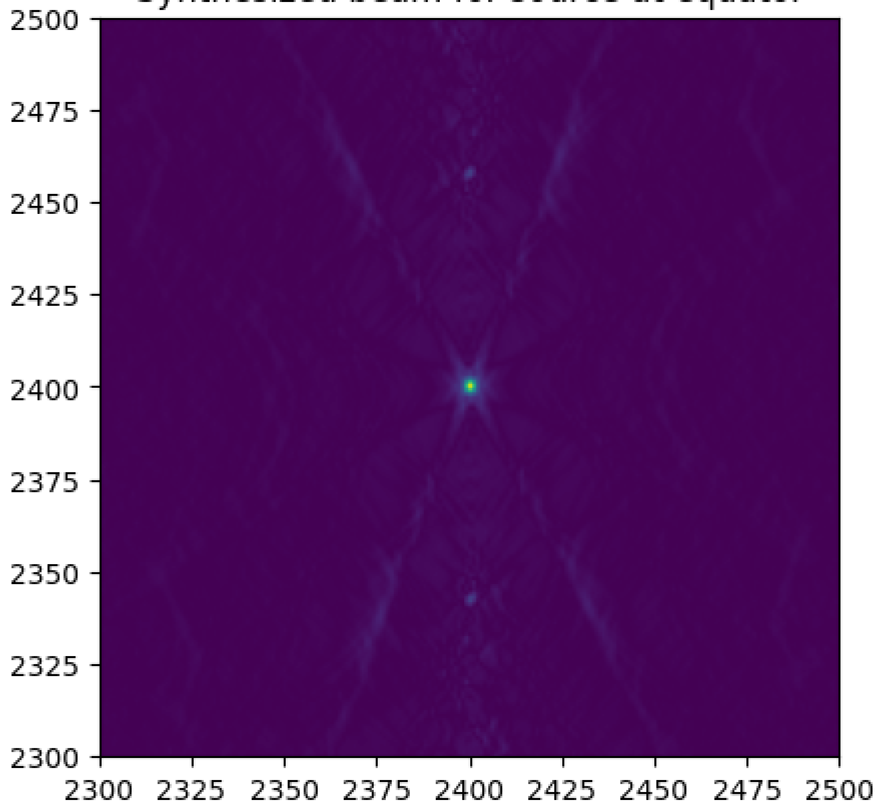
Out[177]:

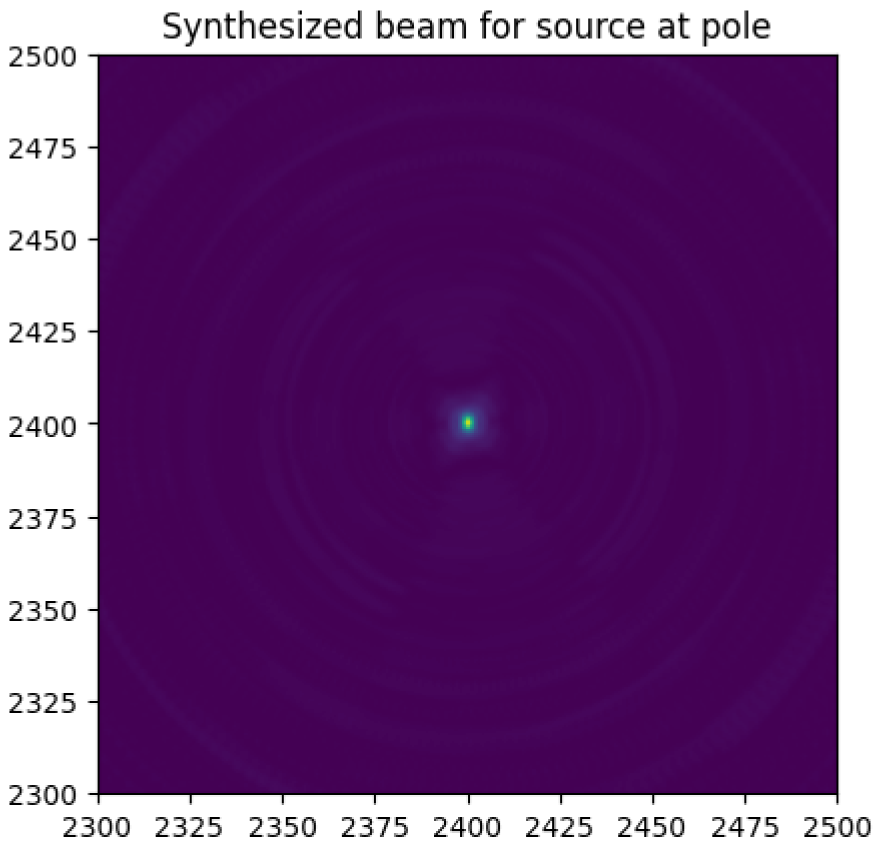
```
Text(0.5, 1.0, 'Synthesized beam for source at pole')
```

Synthesized beam for source overhead



Synthesized beam for source at equator





Q2

a)

In [178…

```
D = 25 #diameter of each dish in VLA
nu1 = 1.4*10**9
wavelength1 = 3*10**8/nu1
nu2 = 8*10**9
wavelength2 = 3*10**8/nu2
FOV1 = 1.22*wavelength1/D
FOV1_arcmin = FOV1*180*60/np.pi
FOV2 = 1.22*wavelength2/D
FOV2_arcmin = FOV2*180*60/np.pi
print("Wavelength at 1.4 GHz is {} m".format(wavelength1))
print("Wavelength at 8 GHz is {} m".format(wavelength2))
print("FOV of VLA at 1.4GHz is {} arcmin".format(FOV1_arcmin))
print("FOV of VLA at 8GHz is {} arcmin".format(FOV2_arcmin))
```

```
Wavelength at 1.4 GHz is 0.21428571428571427 m
Wavelength at 8 GHz is 0.0375 m
FOV of VLA at 1.4GHz is 35.94900908877965 arcmin
FOV of VLA at 8GHz is 6.29107659053644 arcmin
```

b)

In [179…
```
uv=np.zeros([nvis,2])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv[icur,:]=(xyz[i,:2]-xyz[j,:2])
        icur=icur+1
#uv=np.vstack([uv,-1*uv])

longest_baseline = 0
for i in range(nvis):
    len_baseline = np.sqrt(uv[i][0]**2 + uv[i][1]**2)
    if len_baseline > longest_baseline:
        longest_baseline = len_baseline
print("Longest baseline = {} m".format(longest_baseline))

#For source overhead
path_diff = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-0.5)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]

rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
```

```
Longest baseline = 1031.9082462393058 m
RMS path length difference between source 1 and source 2 using 2D uv coordi
nates: 1.594208525126613 m
```

c)

In [180…
```
uv_3d=np.zeros([nvis,3])
icur=0
for i in range(nant):
    for j in range(i+1,nant):
        uv_3d[icur,:]=(xyz[i,:]-xyz[j,:])
        icur=icur+1
#uv_3d=np.vstack([uv_3d,-uv_3d])

path_diff_3d = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-0.5)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.

rms_3d = 0
squared = 0
```

```
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 3D uv

diff_of_diff = np.abs(rms-rms_3d)
print("The RMS difference between path length differences for 2D and 3D coor
```

```
RMS path length difference between source 1 and source 2 using 3D uv coordi
nates: 1.5976998338162693 m
The RMS difference between path length differences for 2D and 3D coordinate
s is 0.003491308689656414 m
```

d)

From the official VLA website, the synthesized FWHM for a array at 1.5 GHz is 1.3''. Since the synthesized beam resolution is proportiaonal to the wavelength, I use this information to get the FWHM at 1.4 GHz and 8 GHz.

In [181…

```
FWHM = 1.3
FWHM1 = 1.3*(1.5/1.4)*(1/3600) #FWHM at 1.4 GHz
FWHM2 = 1.3*(1.5/8)*(1/3600) #FWHM at 8 GHz
print("The FWHM at 1.4GHz is {} degree".format(FWHM1))
print("The FWHM at 8GHz is {} degree".format(FWHM2))
```

```
The FWHM at 1.4GHz is 0.00038690476190476186 degree
The FWHM at 8GHz is 6.770833333333334e-05 degree
```

For an overhead pointing center, I already calculated the uv coordiantes (where the projection matrix (mat) was calculated with the latitute of VLA.

For 1.4 GHz:

In [182…

```
path_diff = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-FWHM1)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
path_diff_3d = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-FWHM1)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
rms_3d = 0
```

```
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
diff_of_diff1 = np.abs(rms - rms_3d)
phase_diff1 = 360*diff_of_diff1/wavelength1
print("RMS path length difference between source 1 and source 2 using 3D uv
print("The RMS difference between path length differences for 2D and 3D coor
print("The corresponding RMS phase difference is {} degrees".format(phase_di
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordi
nates: 0.0012257297462677597 m
RMS path length difference between source 1 and source 2 using 3D uv coordi
nates: 0.0012284394160251423 m
The RMS difference between path length differences for 2D and 3D coordinate
s at 1.4GHz is 2.7096697573826285e-06 m
The corresponding RMS phase difference is 0.004552245192402816 degrees
```

Since the RMS phase difference between the 2D and 3D coordinates is very small, we do not need to worry about the w term

For 8 GHz

In [183…

```
path_diff = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-FWHM2)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
path_diff_3d = []
theta1 = np.radians(90-34.0784)
theta2 = np.radians(90-34.0784-FWHM2)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
rms_3d = 0
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
diff_of_diff1 = np.abs(rms - rms_3d)
phase_diff2 = 360*diff_of_diff1/wavelength2
print("RMS path length difference between source 1 and source 2 using 3D uv
print("The RMS difference between path length differences for 2D and 3D coor
print("The corresponding RMS phase difference is {} degrees".format(phase_di
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordinates: 0.00021450182240909156 m
RMS path length difference between source 1 and source 2 using 3D uv coordinates: 0.00021497601551515643 m
The RMS difference between path length differences for 2D and 3D coordinates at 8GHz is 4.7419310606486645e-07 m
The corresponding RMS phase difference is 0.004552253818222718 degrees
```

Since the RMS phase difference between the 2D and 3D coordinates is very small, we do not need to worry about the w term

For source at equator:

For 1.4 GHz

In [184…

```python
path_diff = []
theta1 = np.radians(90)
theta2 = np.radians(90-FWHM1)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
path_diff_3d = []
theta1 = np.radians(90)
theta2 = np.radians(90-FWHM1)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
rms_3d = 0
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
diff_of_diff1 = np.abs(rms - rms_3d)
phase_diff1 = 360*diff_of_diff1/wavelength1
print("RMS path length difference between source 1 and source 2 using 3D uv
print("The RMS difference between path length differences for 2D and 3D coor
print("The corresponding RMS phase difference is {} degrees".format(phase_di
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordinates: 7.385879049512076e-09 m
RMS path length difference between source 1 and source 2 using 3D uv coordinates: 4.932509302280288e-06 m
The RMS difference between path length differences for 2D and 3D coordinates at 1.4GHz is 4.925123423230776e-06 m
The corresponding RMS phase difference is 0.008274207351027704 degrees
```

Since the RMS phase difference between the 2D and 3D coordinates is very small, we do not need to worry about the w term

For 8 GHz

```
In [185…
path_diff = []
theta1 = np.radians(90)
theta2 = np.radians(90-FWHM2)
theta_vec1 = np.array([np.sin(theta1),0])
theta_vec2 = np.array([np.sin(theta2),0])
for i in range(nvis):
    path_diff.append(np.abs(np.dot(uv[i], theta_vec1)) - np.abs(np.dot(uv[i]
rms = 0
squared = 0
for i in range(nvis):
    squared += path_diff[i]**2
rms = np.sqrt(squared/nvis)
print("RMS path length difference between source 1 and source 2 using 2D uv
path_diff_3d = []
theta1 = np.radians(90)
theta2 = np.radians(90-FWHM2)
theta_vec1_3d = np.array([np.sin(theta1),0,np.cos(theta1)])
theta_vec2_3d = np.array([np.sin(theta2),0,np.cos(theta2)])
for i in range(nvis):
    path_diff_3d.append(np.abs(np.dot(uv_3d[i], theta_vec1_3d)) - np.abs(np.
rms_3d = 0
squared = 0
for i in range(nvis):
    squared += path_diff_3d[i]**2
rms_3d = np.sqrt(squared/nvis)
diff_of_diff1 = np.abs(rms - rms_3d)
phase_diff2 = 360*diff_of_diff1/wavelength2
print("RMS path length difference between source 1 and source 2 using 3D uv
print("The RMS difference between path length differences for 2D and 3D coor
print("The corresponding RMS phase difference is {} degrees".format(phase_di
```

```
RMS path length difference between source 1 and source 2 using 2D uv coordi
nates: 2.2618278705436265e-10 m
RMS path length difference between source 1 and source 2 using 3D uv coordi
nates: 8.624816377892954e-07 m
The RMS difference between path length differences for 2D and 3D coordinate
s at 8GHz is 8.62255455002241e-07 m
The corresponding RMS phase difference is 0.008277652368021514 degrees
```

Since the RMS phase difference between the 2D and 3D coordinates is very small, we do not need to worry about the w term

Q3

As calculated in Q2 d), for an overhead source and a second source separated by 1 FWHM, the RMS path length difference is 0.0012284394160251423 m (for 3D coordinates).

```
In [186…
delta_nu = 3*10**8/(2*np.pi*0.0012284394160251423)
```

```
print("Using the calculation shown in the report, the amount of frequency sh
```

```
Using the calculation shown in the report, the amount of frequency shift re
quired is: 38.8675927397883 GHz
```