Q-1 Poisson distribution :

$$P(n, \lambda) = \frac{e^{-\lambda} \lambda^{n}}{n!}$$

let $x = \lambda - n \Rightarrow$ deviation from mean.

Then,

$$P(x, \lambda) = \frac{e^{-\lambda} \lambda^{\lambda - x}}{(\lambda - x)!}$$

Using sterling's approximation,

$$(\lambda - x)! = \left(\frac{\lambda - x}{e}\right)^{\lambda - x} \sqrt{2\pi(\lambda - x)}$$

$$\Rightarrow P(x, \lambda) = \frac{e^{-\lambda} \lambda^{\lambda - x}}{(\lambda - x)^{\lambda - x} e^{-(\lambda - x)} \sqrt{2\pi(\lambda - x)}}$$

$$= e^{-x} \left(\frac{\lambda}{\lambda - x}\right)^{\lambda - x} \frac{1}{\sqrt{2\pi(\lambda - x)}}$$

$$= e^{-x} \left(1 - \frac{x}{\lambda}\right)^{x - \lambda} \frac{1}{\sqrt{2\pi(\lambda - x)}}$$

$$= e^{-x} \left(1 - \frac{x}{\lambda}\right)^{x - \lambda} \frac{(\lambda - x)^{-1/2}}{\sqrt{2\pi}}$$

$$= \frac{e^{-x}}{\sqrt{2\pi}} \left(1 - \frac{x}{\lambda}\right)^{x-\lambda} \frac{1}{\sqrt{\lambda}} \left(1 - \frac{x}{\lambda}\right)^{-1/2}$$

$$= \frac{e^{-x}}{\sqrt{2\pi\lambda}} \left(1 - \frac{x}{\lambda}\right)^{x-\lambda-1/2}$$

$$\log\left(P(x,\lambda)\right) =$$

$$-x + \left(x - \lambda - \frac{1}{2}\right) \log\left(1 - \frac{x}{\lambda}\right)$$

$$- \log\left(\sqrt{2\pi\lambda}\right)$$

Taking the log expansion eq

$$\log\left(1 - \frac{x}{\lambda}\right) = -\frac{x}{\lambda} - \frac{x^2}{2\lambda^2}$$

$$\log\left(P(x,\lambda)\right) =$$

$$-x + \left(x - \lambda - \frac{1}{2}\right)\left(-\frac{x}{\lambda} - \frac{x^2}{2\lambda^2}\right)$$

$$- \log\left(\sqrt{2\pi\lambda}\right)$$

$$= -\cancel{x} + \left(-\frac{x^2}{\lambda} - \frac{x^3}{2\lambda^2} + \cancel{x} + \frac{x^2}{2\lambda} + \frac{x}{2\lambda} + \frac{x^2}{4\lambda^2} \right) - \log\left(\sqrt{2\pi\lambda}\right)$$

$$= -\frac{x^2}{2\lambda} \underbrace{- \frac{x^3}{2\lambda^2} + \frac{x}{2\lambda} + \frac{x^2}{4\lambda^2}}_{\text{approximately } 0 \text{ as } \lambda >> x} - \log\left(\sqrt{2\pi\lambda}\right)$$

$$\Rightarrow -\frac{x^2}{2\lambda} - \log\left(\sqrt{2\pi\lambda}\right)$$

$$\log\left(P(x,\lambda)\right) = -\frac{x^2}{2\lambda} - \log\left(\sqrt{2\pi\lambda}\right)$$

$$P(x,\lambda) = \frac{e^{-x^2/2\lambda}}{\sqrt{2\pi\lambda}}$$

Substituting $x = \lambda - n$

$$P(n,\lambda) = \frac{e^{-(n-\lambda)^2/2\lambda}}{\sqrt{2\pi\lambda}}$$

Thus, in the limit of large λ,
Poisson converges to Gaussian where
mean of the Gaussian $= \lambda$ and
σ^2 of the Gaussian $= \lambda$