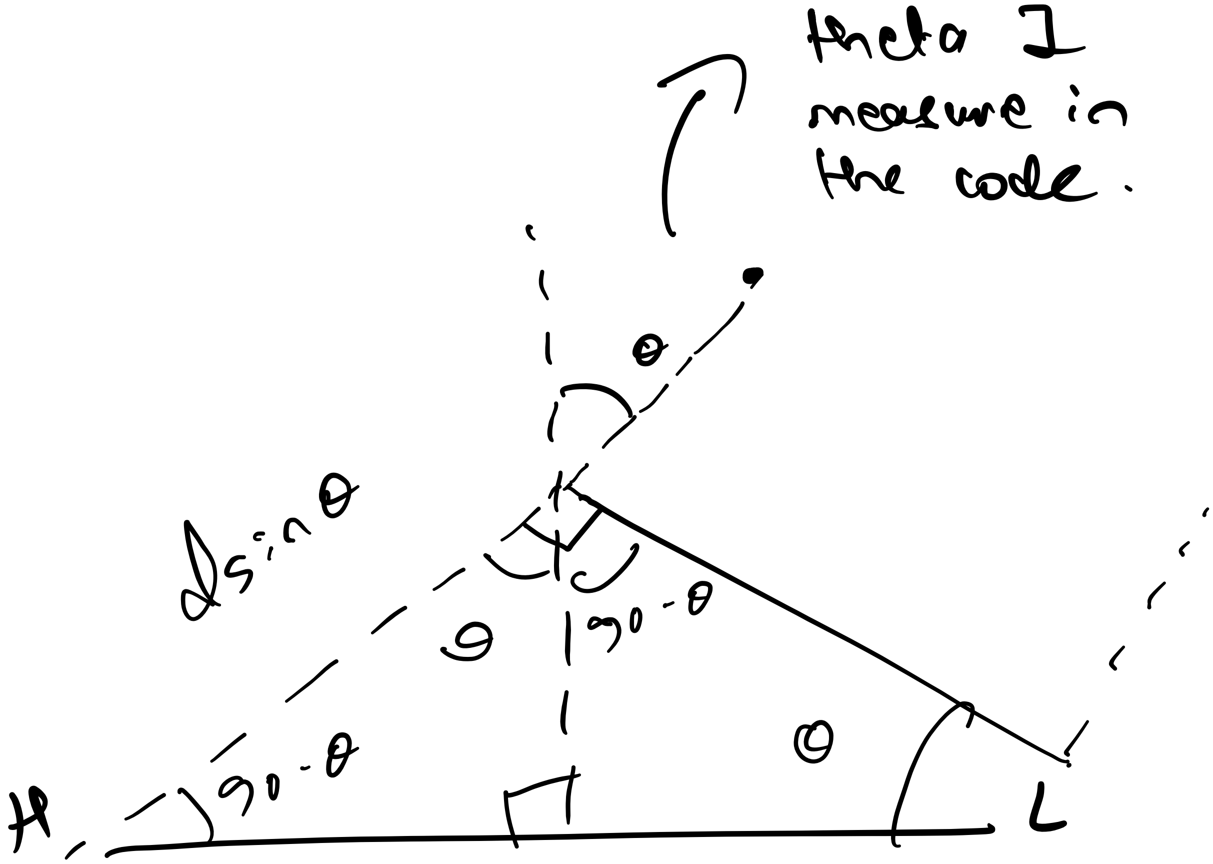
Q1 f)



$d = 3030$ km

$$\frac{d\sin\theta}{\Delta t} = c$$

$$\theta = \sin^{-1}\left(\frac{c\Delta t}{d}\right)$$

Δt is the difference in the

time of arrival of the gravitational wave at Hanford & Livingston detectors.

$$\theta = \sin^{-1}\left(\frac{c\Delta t}{d}\right)$$

$$\sin\theta = \frac{c\Delta t}{d}$$

I get the uncertainty for the TOA of the wave at each detector. Then the uncertainty in Δt is obtained as →

$$\delta\Delta t = \sqrt{\delta TOA_H^2 + \delta TOA_L^2}$$

(subtraction rule for uncertainty)

Then I get →

$$\sin(\theta + \delta\theta) = \frac{c(\Delta t + \delta\Delta t)}{d}$$

$$\theta + \delta\theta = \sin^{-1}\left(\frac{c(\Delta t + \delta\Delta t)}{d}\right)$$

$$\delta\theta = \sin^{-1}\left(\frac{c(\Delta t + \delta\Delta t)}{d}\right) - \theta$$

Q2 Since the beam pattern is gaussian,

$$A_{eff} = \int_0^R e^{-r^2/2\sigma^2}\, 2\pi r\, dr$$

Taking $r^2/2\sigma^2 = x$,

$$\frac{\cancel{2}r\, dr}{\cancel{2}\sigma^2} = dx$$

$$r\, dr = \sigma^2 dx$$

A_{eff} : $$\int_0^{R^2/2\sigma^2} e^{-x}\, 2\pi\sigma^2 dx$$

$$= -2\pi\sigma^2 \left[e^{-x} \right]_0^{R^2/2\sigma^2}$$

$$= 2\pi\sigma^2 \left[1 - e^{-R^2/2\sigma^2} \right]$$

$$\frac{A_{eff}}{A} = \frac{2\cancel{\pi}\sigma^2 \left[1 - e^{-R^2/2\sigma^2} \right]}{\cancel{\pi} R^2}$$

$$= \frac{2\sigma^2}{R^2} \left[1 - e^{-R^2/2\sigma^2} \right]$$

Fraction of the beam which stays on the dish (since the beam gaussian extends to ∞) =

$$dA_{dish} = \frac{\int_0^R e^{-r^2/2\sigma^2} 2\pi r dr}{\int_0^\infty e^{-r^2/2\sigma^2} 2\pi r dr}$$

$$-R^2/2\sigma^2 \,]$$

$$= \frac{2\pi\sigma^2 \{ 1 - e \quad \}}{2\pi\sigma^2}$$

$$= \left(1 - e^{-R^2/2\sigma^2}\right)$$

as Signal strength at the feed:

$$\frac{A_{eff}}{A} \times dA_{dish}$$

$$= \frac{2\sigma^2}{R^2}\left(1 - e^{-R^2/2\sigma^2}\right)^2$$

(For $R = 1$)

$$= 2\sigma^2 \left(1 - e^{-1/2\sigma^2}\right)^2$$