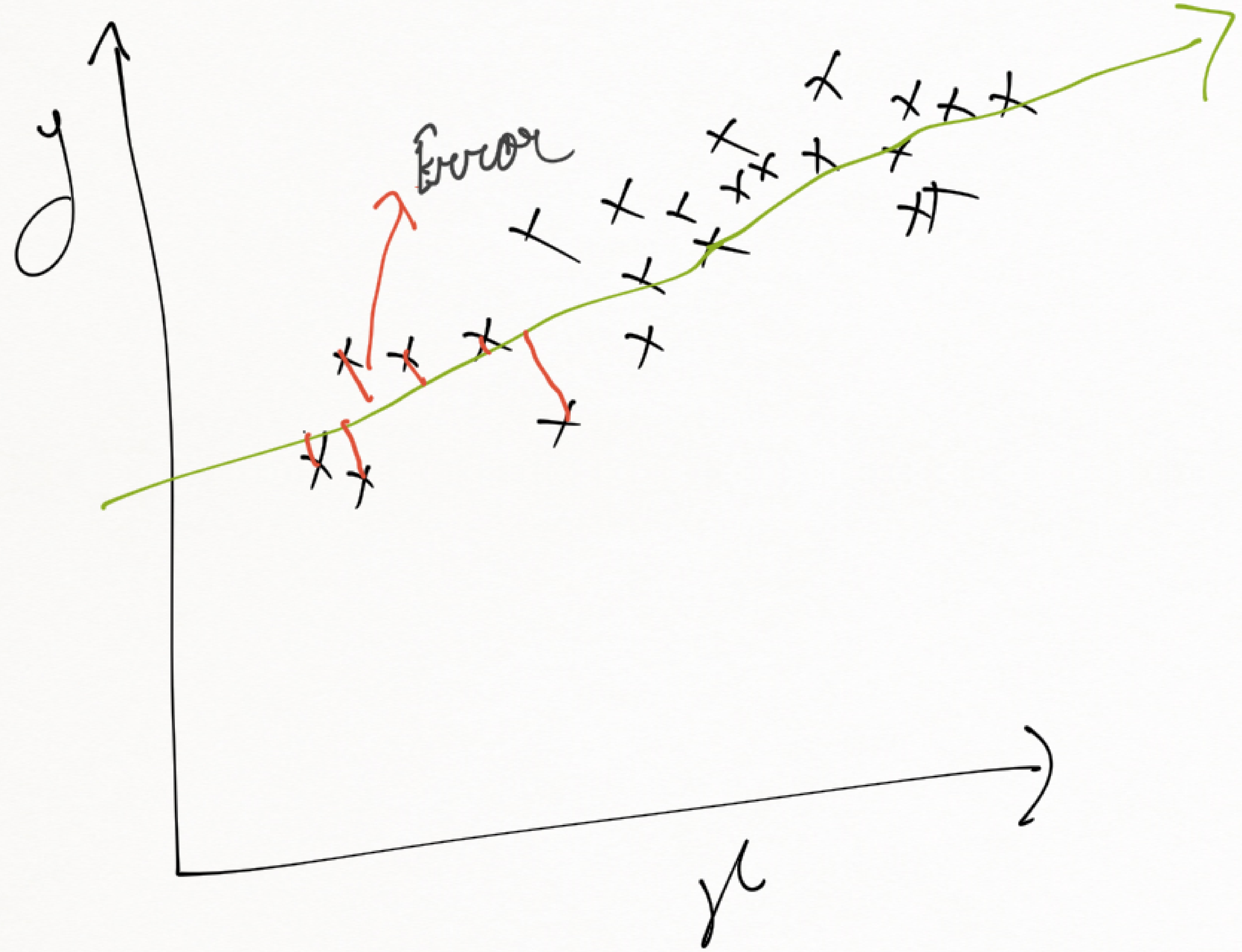
Linear Regression

Predict y, given x.



Best Fit Line

$x=1, \quad y=2.5$

$x=2, \quad y=4.5$

$x=3, \quad y=6.5$

$x=6, \quad y=\,?$

$y=2x+0.5$

12.5 ✓

$y = 2x + 0.5$

y is dependent on $\rightarrow$ x & 0.5 i.e. Constant (c)

$y = mx + c$ $\leftarrow$ Best Fit line

Our Task is to minimize Loss.

Best Fit Line

here c=0

y is dependent on "m".

Best Fit line

here m is same

y is dependent on "c"

Here the task is Loss Minimization:

So, we need to understand Loss first:

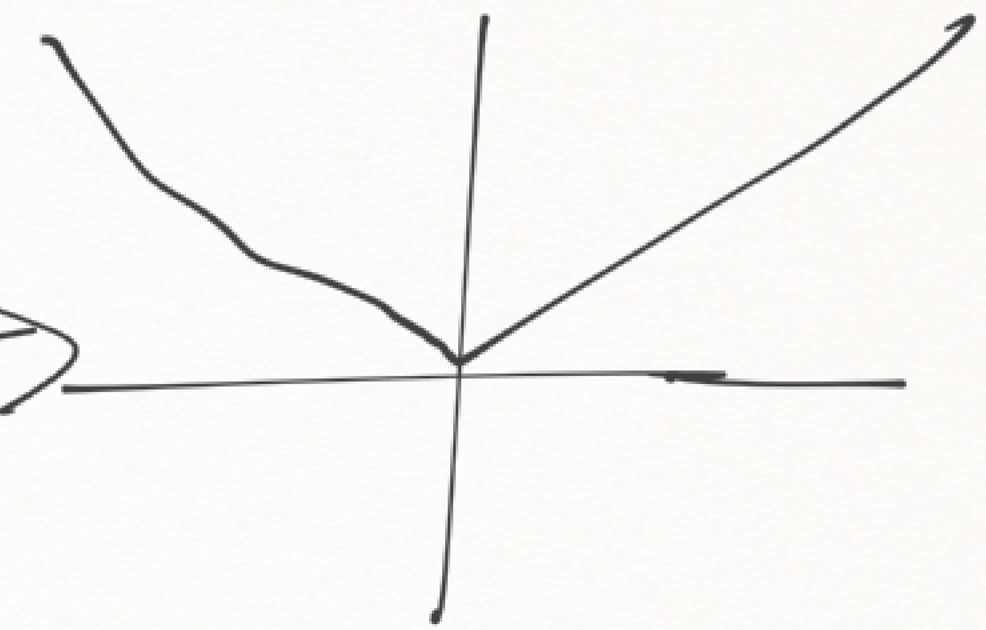
Loss $\Rightarrow$ MSE: $$\sum_{k=0}^{n-1} \frac{(y_k - \hat{y}_k)^2}{n}$$

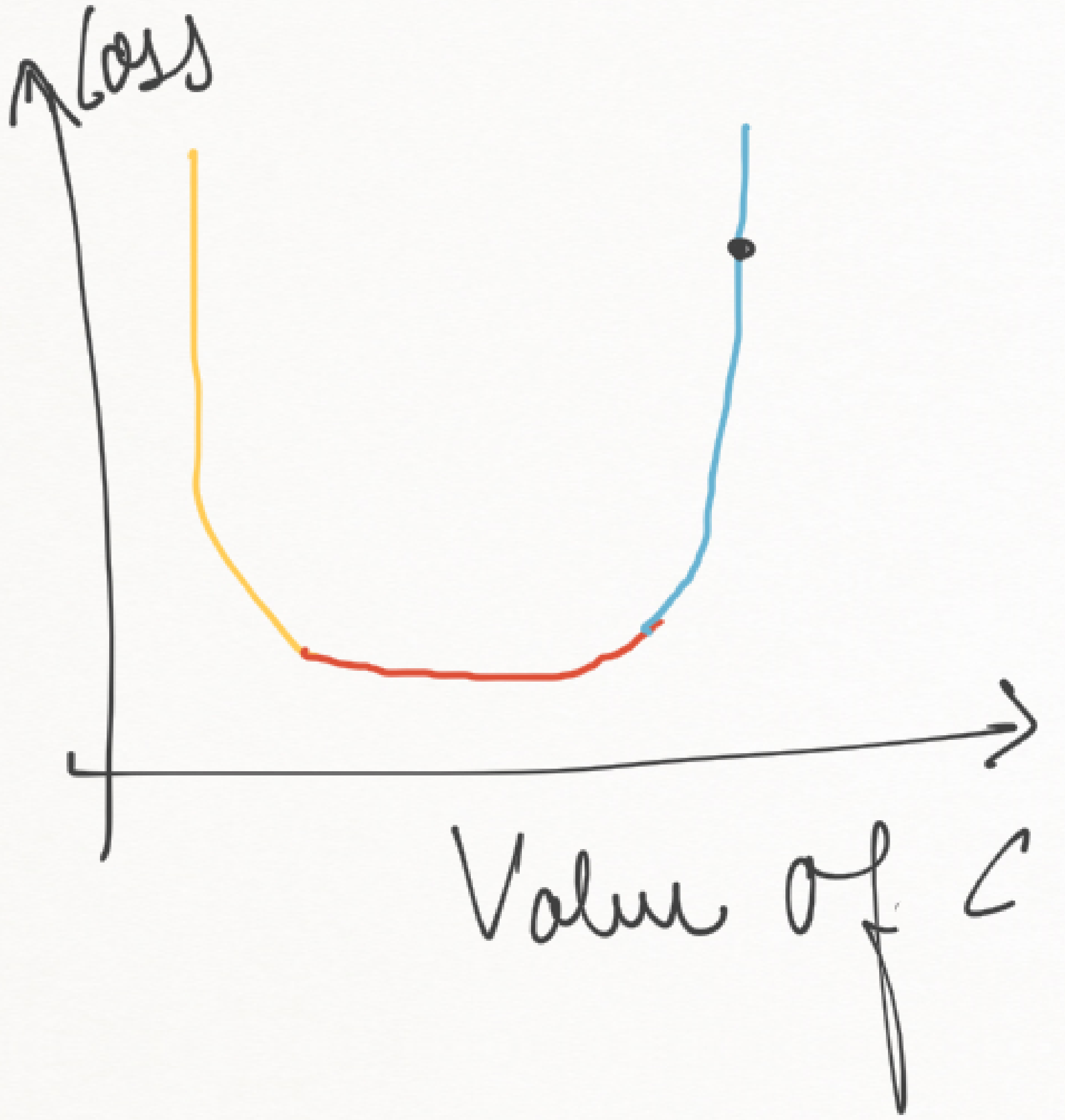
y_k = True label at k^{th} Index

$\hat{y}_k$ = Predicted label at k^{th} Index.

RMSE: $$\sqrt{\sum_{k=0}^{n-1} \frac{(y_k - \hat{y}_k)^2}{n}}$$

MAE: $$\sum_{k=0}^{n-1} \frac{\text{abs}(y_k - \hat{y}_k)}{n}$$





$$\text{minima} = \frac{dL}{dc} = 0$$

$$\frac{dL}{dw} = 0$$

x	y
0	1
1	3
2	5
3	7
4	9
...	...

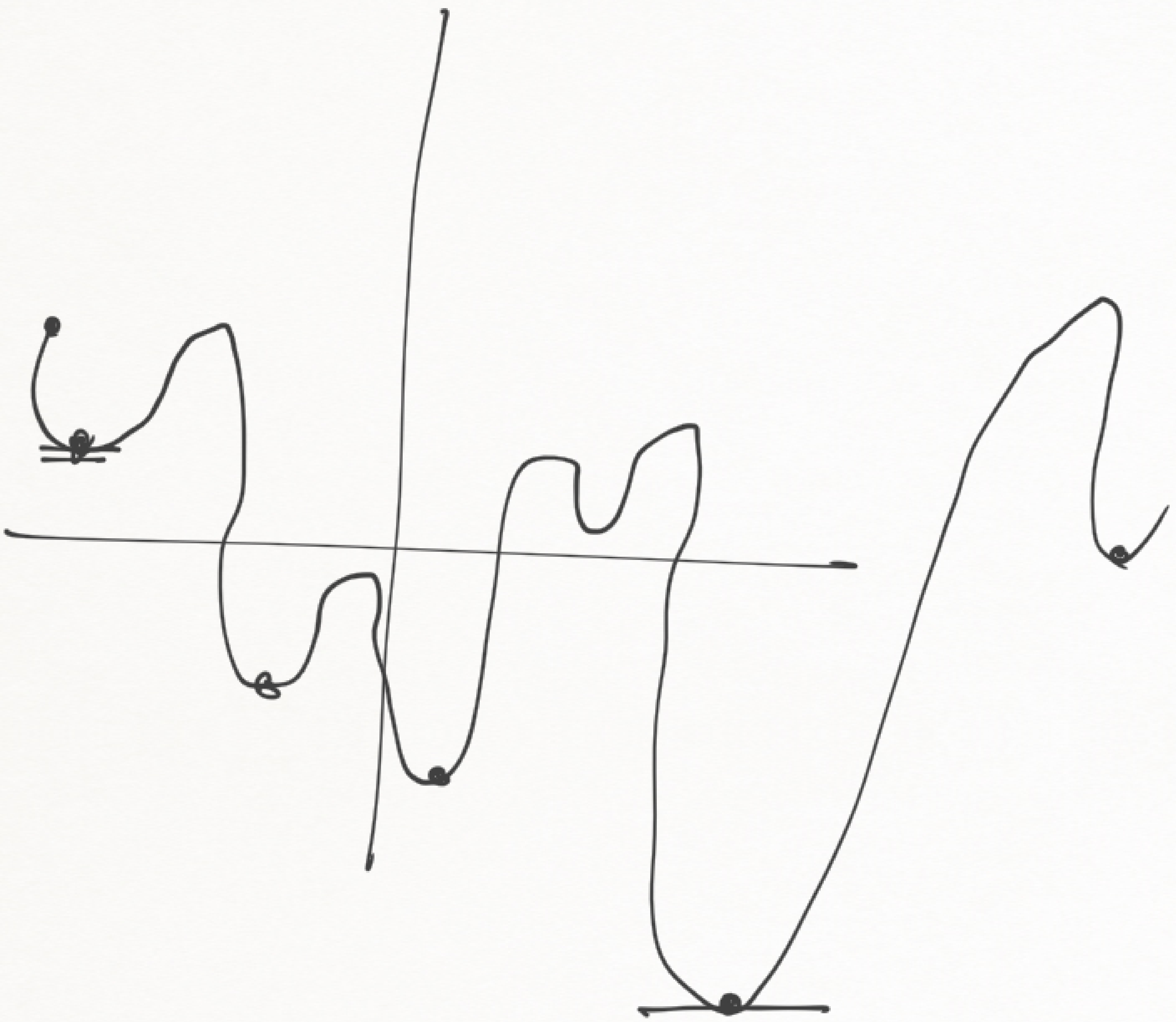
$y = 2x + 1$

$c = 11 \rightarrow 1$

$m = 5 \rightarrow 2$

Steps:

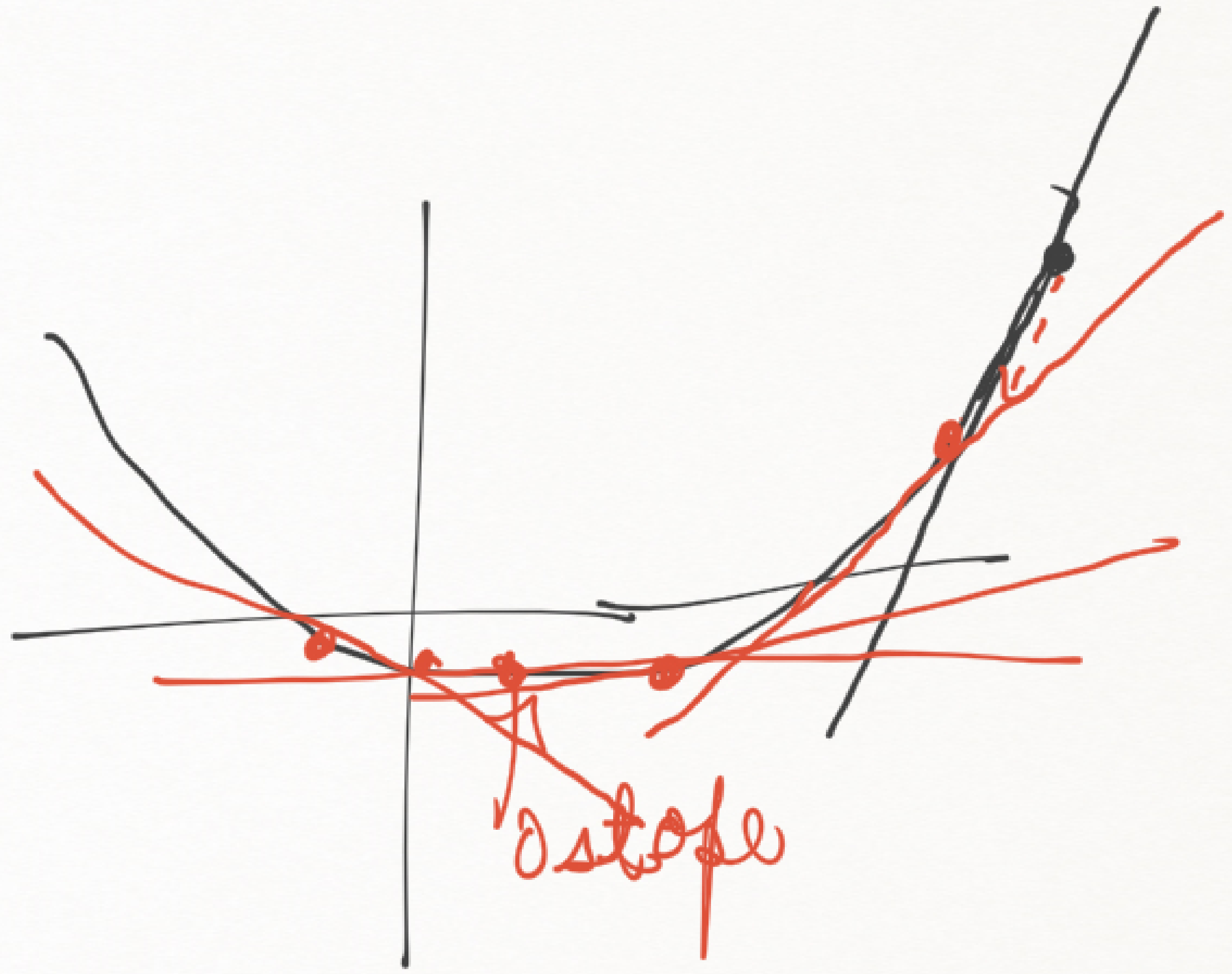
1. Randomly Initialize m & c
2. Calculate Loss
3. Calculate $\frac{dL}{dm}$, $\frac{dL}{dc}$
4. $m = m - \alpha \frac{dL}{dm}$ $\quad c = c - \alpha \frac{dL}{dc}$
5. Repeat step 2 - 4 until loss stops changing



Global Minima

$$m = m - \alpha \frac{\partial L}{\partial m}$$

$$\frac{\partial L}{\partial m} = 0$$



Step 3

$$L = \underline{mse} \rightarrow \frac{1}{N}\sum_{n=0}^{K-1} (y_K - \hat{y}_K)^2$$

1 $\frac{\partial L}{\partial m}$ & 2 $\frac{\partial L}{\partial c}$

$$\frac{\partial L}{\partial m} = \frac{1}{N} \times \frac{1}{\partial m} \times \partial(y_K - \hat{y}_K)^2$$

$$= \frac{1}{N} \times \frac{1}{\partial m} \times \partial(y_K - (\underline{m x_K + c}))^2$$

$$= \frac{1}{N} \times 2 \times (y_K - (m x_K + c)) \times (0 - x)$$

$$= \frac{2}{N} \times (y_K - \hat{y}_K) \times (-x) = \boxed{\frac{-2}{N} \times x_K (y_K - \hat{y}_K)}$$

$=$

2 $$\frac{\partial L}{\partial c} = \frac{-2}{N}(y_K - \hat{y}_K)$$

step 4

1 $m = m - \alpha \frac{\partial L}{\partial m}$

2 $c = c - \alpha \frac{\partial L}{\partial c}$

Standardization:

1. mean of x
2. std of x

$$x = \frac{x - mean}{std}$$