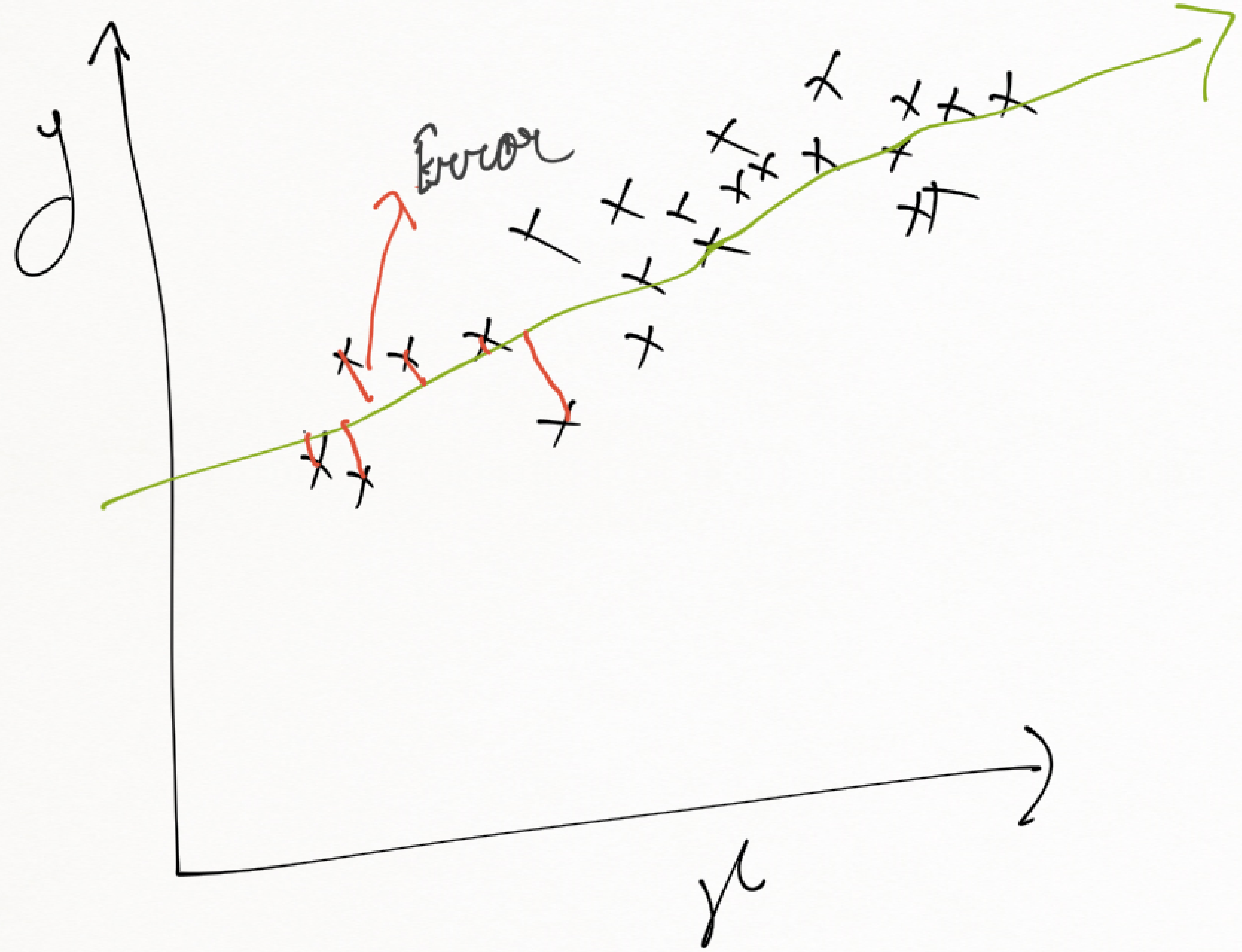
Linear Regression

Predict y, given x.



Best Fit Line

$x=1, \; y=2.5$

$x=2, \; y=4.5$

$x=3, \; y=6.5$

$x=6, \; y=?$

$y=2x+0.5$

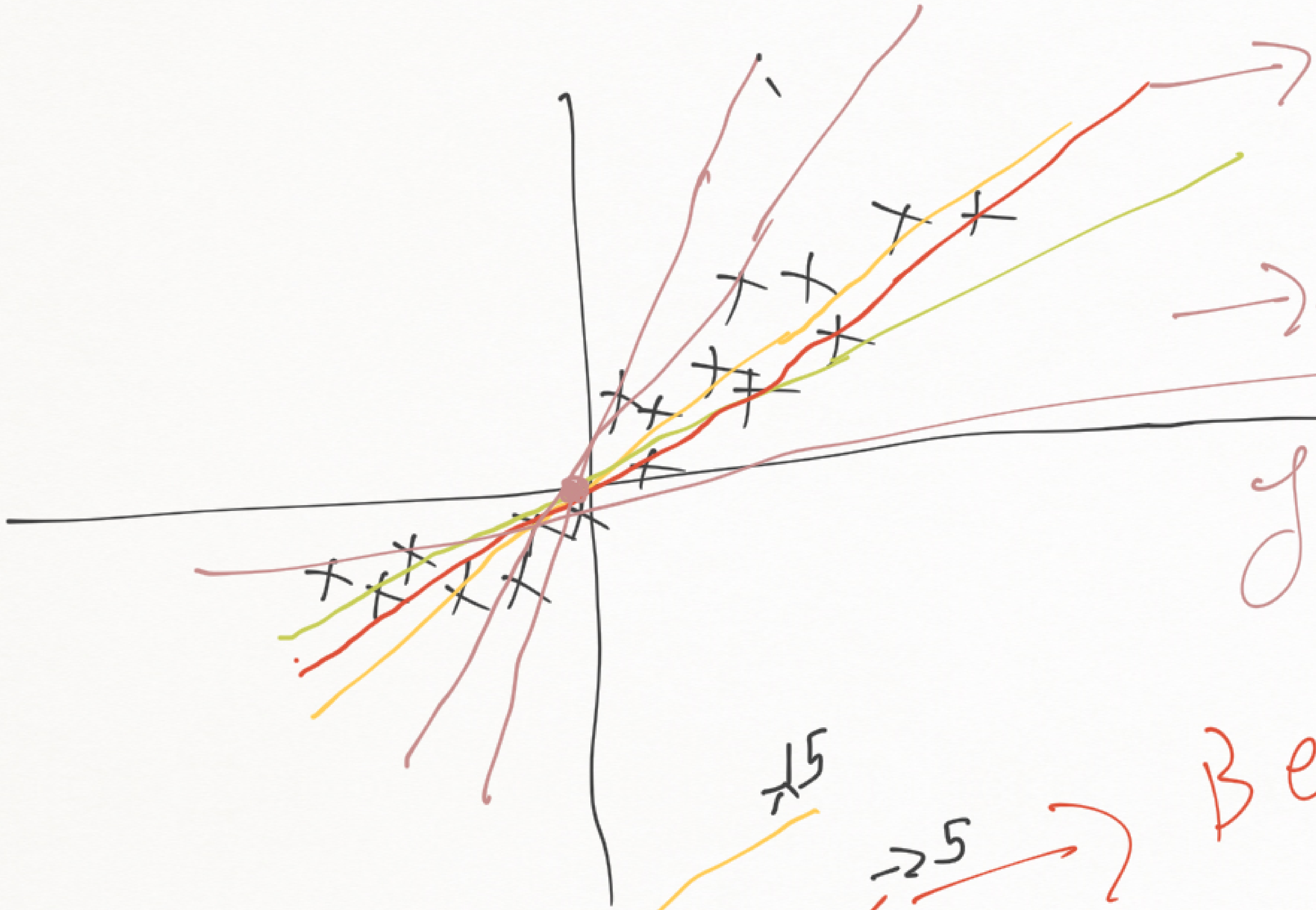
12.5 ✓

$y = 2x + 0.5$

y is dependent on $\rightarrow$ x & 0.5 i.e. Constant (c)

$y = mx + c$ $\leftarrow$ Best Fit line

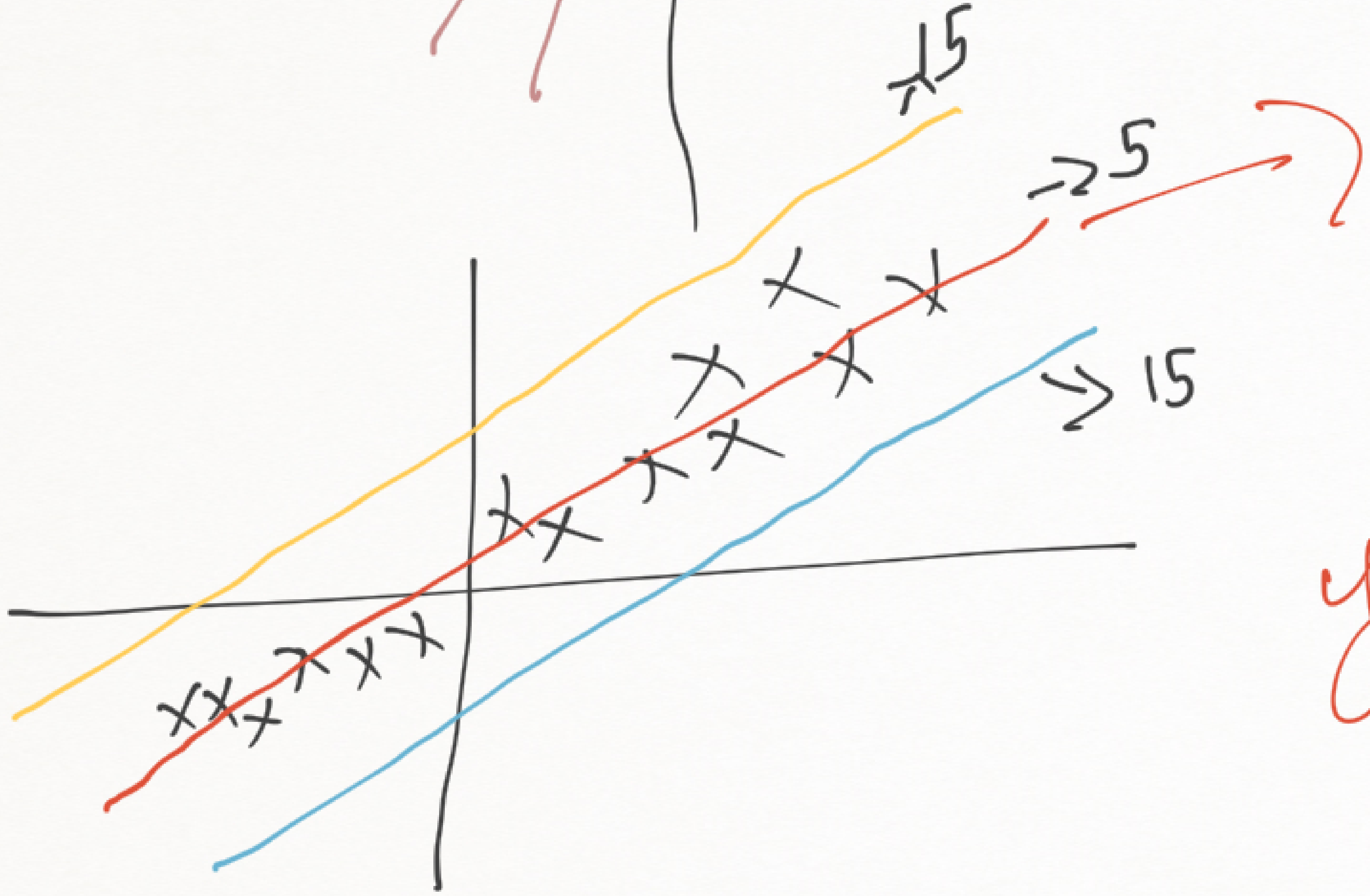
Our Task is to minimize Loss.



Best Fit Line

here c=0

y is dependent on "m".



Best Fit line

here m is same

y is dependent on "c"

Here the task is Loss Minimization:

So, we need to understand Loss first:

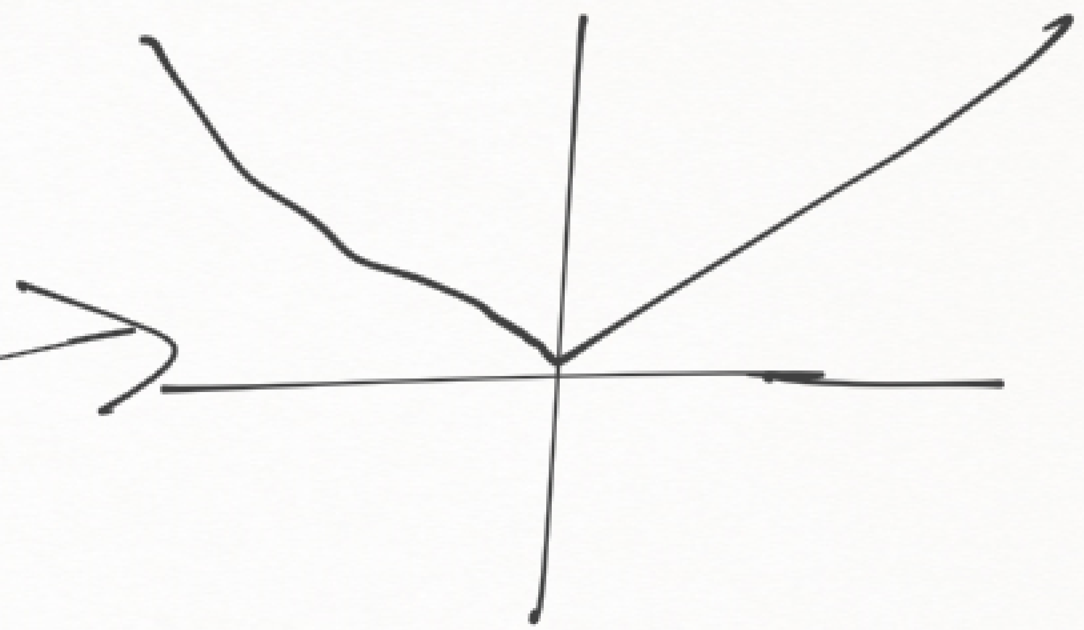
y_k = True label at k^{th} Index

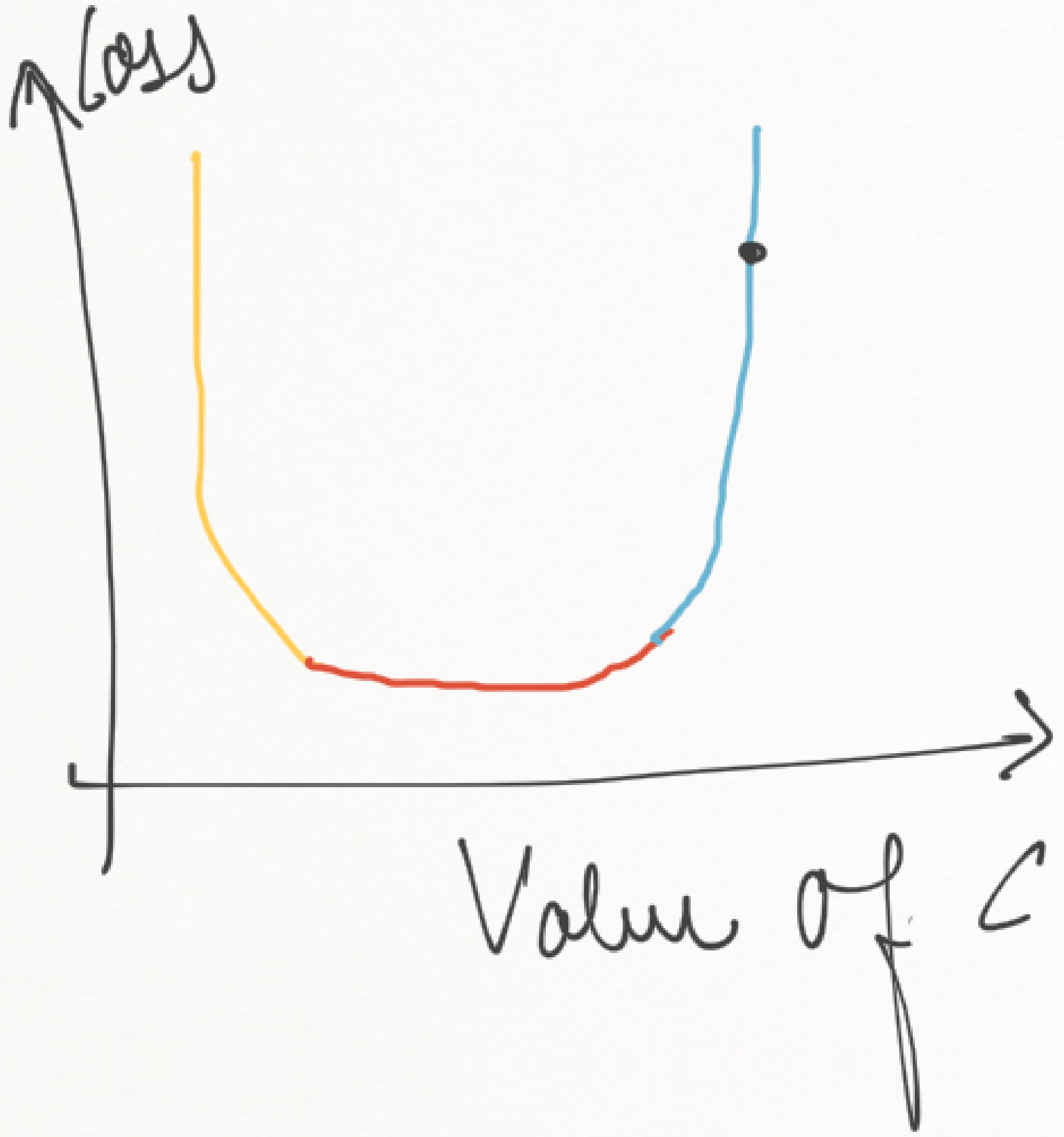
$\hat{y}_k$ = Predicted label at k^{th} Index.

Loss $\Rightarrow$ MSE: $$\sum_{k=0}^{n-1} \frac{(y_k - \hat{y}_k)^2}{n}$$

RMSE: $$\sqrt{\sum_{k=0}^{n-1} \frac{(y_k - \hat{y}_k)^2}{n}}$$

MAE: $$\sum_{k=0}^{n-1} \frac{abs(y_k - \hat{y}_k)}{n}$$





$$\text{minima} = \frac{dL}{dc} = 0$$

$$\frac{dL}{dw} = 0$$

x	y
0	1
1	3
2	5
3	7
4	9
⋮	⋮

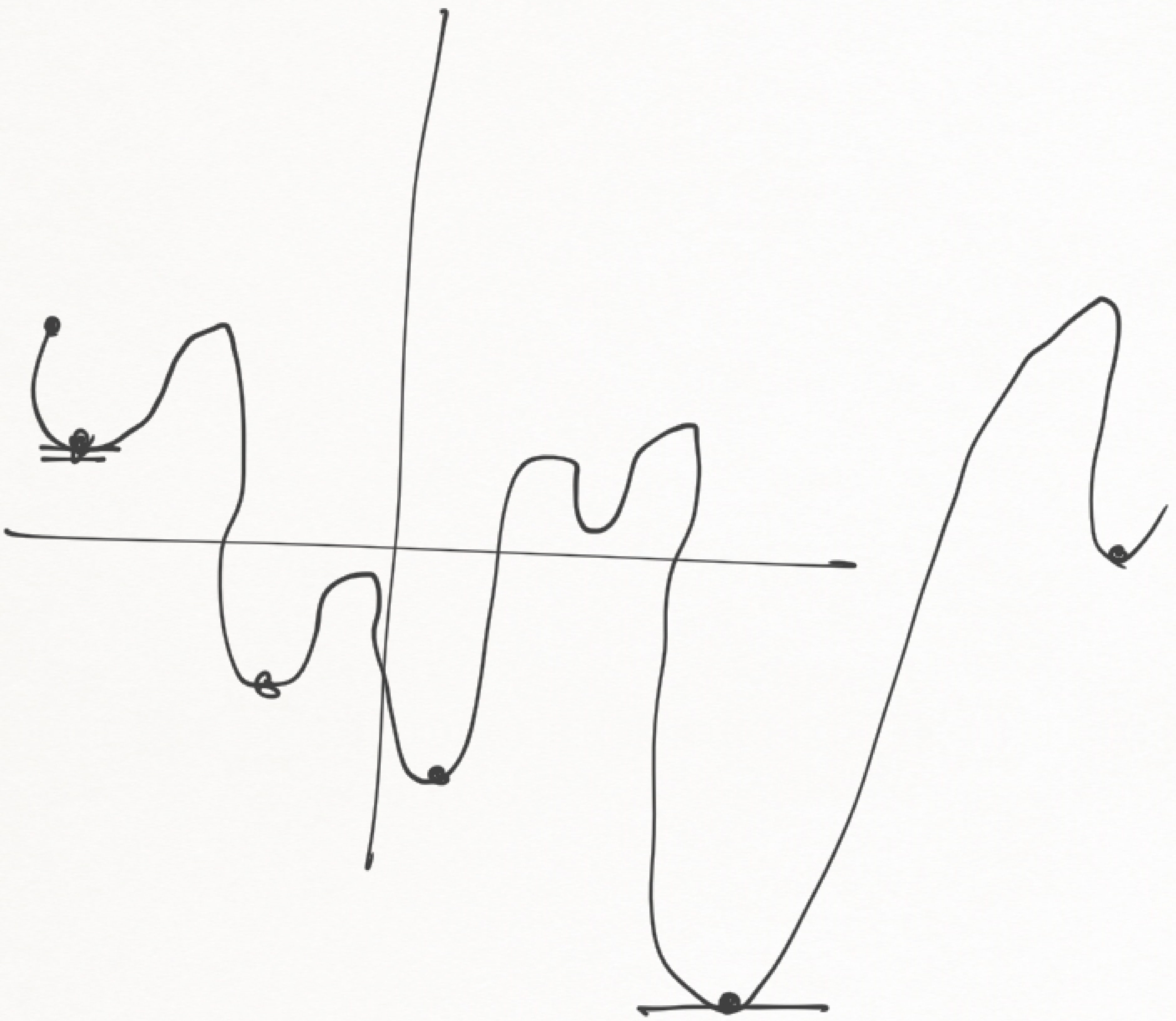
$$y = 2x + 1$$

$c = 11 \rightarrow 1$

$m = 5 \rightarrow 2$

Steps:

1. Randomly Initialize m & c
2. Calculate Loss
3. Calculate $\frac{dL}{dm}$, $\frac{dL}{dc}$
4. $m = m - \alpha \frac{dL}{dm}$ $\quad c = c - \alpha \frac{dL}{dc}$
5. Repeat step 2 - 4 until loss stops changing



Global Minima

$$m = m - \alpha \frac{\partial L}{\partial m}$$

$$\frac{\partial L}{\partial m} = 0$$