

$$2] \text{ Given: } \|v\| = 1$$

$$\frac{(x^{(1)})^T v}{\|v\|} = (x^{(1)})^T v$$

$$x^T v = \begin{bmatrix} (x^{(1)})^T v \\ (x^{(2)})^T v \\ (x^{(3)})^T v \\ (x^{(4)})^T v \end{bmatrix}$$

$$3] \mu = \frac{x^{(1)} + x^{(2)} + x^{(3)} + x^{(4)}}{4}$$

$$4] \frac{1}{4} x^T \mathbf{1}$$

$$5] \text{ Scalar projection} = \frac{x^{(1)} + x^{(2)} + x^{(3)}}{\sqrt{3}}$$

$$\mu = \frac{\frac{x^{(1)}}{\sqrt{3}} + \frac{x^{(2)}}{\sqrt{3}} + \frac{x^{(3)}}{\sqrt{3}}}{3} = \frac{x^{(1)} + x^{(2)} + x^{(3)}}{3\sqrt{3}}$$

$$6] \text{ Scalar projection of } \mu \text{ on } v$$

$$= \frac{\mu v}{\|v\|}$$

$$7] \frac{1}{4} [(x^{(1)})^T v + (x^{(2)})^T v + (x^{(3)})^T v + (x^{(4)})^T v]$$

$$= v^T \left[ \frac{x^{(1)} + x^{(2)} + x^{(3)} + x^{(4)}}{4} \right]$$

$$= v^T \mu$$



Mean of Scalar projection on to the direction of  $V$  is same as scalar projection of mean sample onto the same direction of vector

$$8) \frac{\bar{x} - \mu}{\sigma} = 0$$

$$9) \frac{1}{n} \sum_{i=1}^n (V^T x^{(i)} - V^T \mu)^2$$

Squared difference between the vector  $x^{(i)}$  along the direction of  $V$  and mean of projected samples along  $V$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (V^T)^2 [x^{(i)} - \mu]^2$$

$$\begin{aligned} 10) & \frac{1}{n} \sum_{i=1}^n (V^T x^{(i)} - V^T \mu)^2 \\ &= \frac{1}{n} \sum_{i=1}^n [V^T x^{(i)} - V^T \mu] \times [(x^{(i)})^T V - \mu^T V] \\ &= \frac{1}{n} \sum_{i=1}^n [V^T (x^{(i)} - \mu) \times ((x^{(i)})^T - \mu^T) V] \\ &= V^T \left[ \frac{1}{n} \sum_{i=1}^n (x^{(i)} - \mu)(x^{(i)} - \mu)^T V \right] \end{aligned}$$