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APS - PROBLEM SET-1

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Q1] a] 7 out of 10, no restrictions

Sol :- $n = 10$
$r = 7$

$$C(10,7) = {}^{10}C_7 = \frac{10!}{7!(10-7)!} = \frac{10!}{7!\,3!} = 120$$

So, 120 different choices

Q1] b] 7 out of 10, with exactly 2 of the last 4

Sol :- Two cases

Case 1 :- 2 out of last 4

$$C(4,2) = 4C_2 = \frac{4!}{2!\,2!} = 6$$

Case 2 :- 5 out of first 6

$$C(6,5) = 6C_5 = \frac{6!}{5! \times 1!} = \frac{6 \times 5!}{5!} = 6$$

∴ No of choices = Case 1 × Case 2
= 6 × 6
= 36 ways

KSSEM

Q1] c] 7 out of 10, with exactly 2 out of first 6

Sol:- Two cases :-

Case 1 :- exactly 2 out of first 6

$$c(6,2) = {}^6C_2 = \frac{6!}{2!\,(6-2)!}$$

$$= \frac{6!}{2!\,4!} = \frac{6\times5\times\cancel{4!}}{\cancel{4!}\times2}$$

$$= 15$$

Case 2 :- 5 out of next 4

$$c(4,5)$$

this is not possible as 5 questions cannot be selected within 4 remaining questions

$$= 0$$

$\therefore$ No. of choices = Case1 × Case 2

$$= 15 \times 0$$

$$= 0 \text{ ways}//$$

Q1] d] Atleast 3 out of first 5

~~[illegible]~~

Atleast 3 out of first 5 means 3, 4, or 5
& 4, 3, 2 qustion out of next 5

∴ Total possibilities:

$5C_3 * 5C_4 + 5C_4 * 5C_3 + 5C_5 * 5C_2$

$= \frac{5!}{3! \times 2!} * \frac{5!}{4! \times 1!} + \frac{5!}{4! \times 1!} * \frac{5!}{3! \times 2!} + \frac{5!}{5!\,0!} \times \frac{5!}{2!\,3!}$

$= 10 \times 5 + 5 \times 10 + 1 \times 10$

$= 50 + 50 + 10$

$= 110$ ways

Q2] a] order of antigens is not considered & replacement is allowed

the formula is

$$\binom{n+r-1}{r}$$

Possibilities for HLA - A antingens

$n = 18 \quad r = 2$

$\therefore \binom{n+r-1}{r} = \binom{18+2-1}{2} = \binom{19}{2}$

$C(19,2)$

II^{ly} for HLA - B antingens

$n = 40 \quad r = 2$

$\binom{40+2-1}{2} = \binom{41}{2} = C(41,2)$

II^{ly} for HLA - DR antingens

$n = 14 \quad r = 2$

$\binom{14+2-1}{2} = \binom{15}{2} = C(15,2)$

$\therefore$ Total antigen strings

$= C(19,2) \times C(41,2) \times C(15,2)$

$= 171 \times 820 \times 105$

$= \underline{1,47,23,100}$ possible antigen strings

Q3) Two cases, one is distribution without restriction

& another is each school must get atlest 1 so first case

$n = 8$

$r = 4$

$\therefore$ possible ways $= \binom{n+r-1}{r}$

since order

$= \binom{8+}{4} = \binom{8}{4}$

$= \dfrac{8!}{4!\,4!} = \dfrac{\cancel{11 \times 10 \times 9 \times 8 \times 7}}{7}$

$= 70$ ways

Second case:

with each recieves atleast 1 board

$\therefore$ we are left with 4 boards extra

as each of the 4 schools takes 1 board.

$\therefore$ possible ways $= \left[\begin{matrix} n+r-1 \\ r \end{matrix}\right]$

$= \left[\begin{matrix} 4+4-1 \\ 4 \end{matrix}\right] = C(7,4) = \dfrac{7!}{4! \times 3!} = \dfrac{7 \times \cancel{6}^{2} \times 5}{\cancel{3} \times 2 \times 1}$

$= 35$ ways

Q4] a] $n = 9$

3 pc's 4 Macs 2 Linux

Total ways $= \dfrac{n!}{q_1! \, q_2! \ldots q_n!}$

$= \dfrac{9!}{3! \, 4! \, 2!} = \dfrac{\cancel{362880}}{\cancel{6 \times 24 \times 2}}$

$= \dfrac{9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{3! \, \cancel{4!} \times 2!} = \dfrac{15120}{12}$

$= 1260$ ways

Q4] b] If first 5 machines serviced must ~~to~~ have 4 MAC's

$n = 5$
$r = 4$

No. of ways $= C(n+r-1, r)$

$= C(5+4-1, 4)$

$= C(8,4) = \dfrac{8!}{4! \, 4!} = \dfrac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}$

$= 70$ ways

Q4] c] Two cases :-

1st case :- 2 PC's in first 3

Number of ways $= \frac{3!}{2!\,1!} \times \frac{6!}{4!\,2!} = 90$ ways

2nd case :- 1 PC in last 3.

The remaining 5 can be filled with 3 MAC's and 2 Linux machines depending on ~~what~~ which was serviced earlier

$\therefore$ Number of ways $= \frac{6!}{4!\,2!} \times \frac{3!}{2!\,1!}$

$= 90$

$\therefore$ Total Number of ways $= 90 + 90$

$= 180$ ways

Q5] 100 on 5

vehicle 1 needs atleast 10

vehicle 2 & 3 needs atleast 12 each

vehicles 6 & 7 needs atleast 4 combined

$$= {}^{100}C_{10} \cdot {}^{90}C_{12} \cdot {}^{78}C_{12} \cdot {}^{66}C_{4} \cdot {}^{62}C_{5}$$

$$= \frac{100!}{10!\,\cancel{90!}} \cdot \frac{\cancel{90!}}{12!\,\cancel{78!}} \cdot \frac{\cancel{78!}}{12!\,\cancel{66!}} \cdot \frac{\cancel{66!}}{4!\,\cancel{62!}} \cdot \frac{\cancel{62!}}{5!\,57!}$$

$$= \frac{100!}{10!\,12!\,12!\,4!\,57!}$$

Q6 11 players, 4 groups

3 Forwards, 3 midfields, 4 Defences, 1 goalie

2 goalkeepers - 1 only selected

$= {}^{2}C_{1}$

Remaining goalie can play in any position

Total Number of ways =

$${}^{2}C_{1}\left({}^{6}C_{1} \cdot {}^{5}C_{3} \cdot {}^{6}C_{3} \cdot {}^{3}C_{3}\right) +$$
$$\left({}^{10}C_{1} \cdot {}^{9}C_{2} \cdot {}^{4}C_{4} \cdot {}^{3}C_{3}\right) +$$
$$\left({}^{3}C_{1} \cdot {}^{4}C_{2} \cdot {}^{7}C_{3} \cdot {}^{4}C_{4}\right)$$

$= 2(1200 + 360 + 1050)$

$= 5220$ ways

Q7] No. of ways to distribute r among n so that i recieves atleast m_i

$C(r - m_1 - m_2 - m_3 - m_n + n-1, n-1)$

Eg: $r = 10$

$n = 3$

$m_1 = 2$

$m_2 = 3$

$m_1 = 1$

$C(10 - 2 - 3 - 1 + 3-1, 3-1)$

$= 6C_2 = \frac{6!}{4!2!} = 15$ ways

Q1]

Judge	Correct	wrong
A	0.95	0.05
B	0.95	0.05
C	0.9	0.1
D	0.9	0.1
E	0.8	0.2

Q9] a] sample space $S = \{0, 1\}$

where $0 \rightarrow$ correct

$1 \rightarrow$ wrong

1st possibility

10111

$A \rightarrow 1$

$B \rightarrow 0$

$C \rightarrow 1$

$D \rightarrow 1$

$E \rightarrow 1$

here 4 of the 5 judges have made an incorrect or wrong decision.

$\therefore$ the court has made wrong decision.

2nd possibility

00011

$A \rightarrow 0$

$B \rightarrow 0$

$C \rightarrow 0$

$D \rightarrow 1$

$E \rightarrow 1$

Here 3/5 judges have made correct decision $\therefore$ the court has made correct decision b] Number of outcomes $= 2^n$

Here $n = 5$

$\therefore$ No. of outcomes $= 2^5$

$= 32$ //

c] $n(E) =$ atleast 3

i.e, $n(E) = 5C_3 + 5C_4 + 5C_5$

$= 10 + 5 + 1$

$= 16$

d] It cannot be calculated $n(E)/n$ as each judge has different probabilitys of giving a correct decision.

Q10] Type A → 1 } together

Type B → 1

10 Type A + 10 type B = 20

a] $P(\text{Pile A}) = \frac{10}{18} = 0.55$

$P(\text{Pile B}) = \frac{8}{18} = 0.44$

$P(\text{each tab from the pile in A\&B}) = \frac{10}{18} + \frac{8}{18} = 1$

Q10] b] P(Random)

$P(A)+P(B) = \frac{10}{20} \times \frac{9}{19} = \frac{90}{380} = P(B)+P(B)$

$P(A)+P(B) = P(B)+P(A)$

$= \frac{10}{20} \times \frac{10}{19} = \frac{100}{380}$

$P = \frac{90}{380} + \frac{100}{380} +$

$\frac{100}{380} + \frac{90}{380} = \frac{380}{380} = 1$

Q11] a) $P(A) \to 28\% = 0.28$ $P(B) \to 7\% = 0.07$ $P(A \cap B) \to 5\%$ 0.05

$P(A \cup B)^c = 1 - P(A \cup B)$

$= 1 - (P(A)+P(B) - P(A \cap B))$

$= 1 - (0.28 + 0.07 - 0.05)$

$(A \cup B)^c = 0.7$ 70%

$(A \cup B)^c$ represents that the random choosen person is not a cigarette or a cigar smoker.

6) $B \cap A^c$

$= P(B) - P(A \cap B)$

$= 0.07 - 0.05$

$P(B \cap A^c) = 0.02 = 2\%$

It represents that choosen person is a cigar smoker.

Q12) a) atleast one 6 in 4 rolls of single dice

$$P = 1 - \left(1 - \frac{1}{6}\right)^4$$

$$= 0.517 \quad -(1)$$

b) 1 12 in 24 rolls of pairs

$$P = 1 - \left(1 - \frac{1}{36}\right)^{24}$$

$$= 0.491 \quad -(2)$$

After comparing ① & ②
the chance that one 6 appears in 4 rolls of a single dice is highly likely to appear.

Q[ ] a] age ≤ 25 years

Total $= 952 + 1050 + 53 + 456 + 2055$
$+ 1570 + 54 + 952 + 1008$
$= 8150$

age $< 25 =$ ~~200~~ $952 + 1050 + 53$
$= 2055$

age $< 25 =$ ~~Total~~ $\dfrac{\text{age} < 25}{\text{total}} = \dfrac{2055}{8150}$

$= 25.21\%$

b] age > 25
Total $-$ age < 25
$= 8150 - 2055$
$= 6095$

$P(\text{age} > 25) = \dfrac{6095}{8150} = 0.7478$

c] Salary < 70000

$$= \frac{952+1050+456+2055+54+952}{8150}$$

$$= \frac{5519}{8150} = 0.6771 \text{ or } 67.71\%$$

d] age < 25 years & Salary > 70 k

$$= \frac{53}{8150} = 0.0065 \text{ or } 6.5 \times 10^{-3}$$

or 0.65%

e] ~~age < 25 years &~~

Salary < 25k & 25-45 years

$$\frac{456}{8150} = 0.0559$$

or 5.59%

f] ~~Salary > 45~~ age > 45 & Salary < 70k

$$= \frac{54+952}{8150} = \frac{1006}{8150} = 0.1234$$

or 12.34%